CS 577- Intro to Algorithms

Computational Intractability (Part 3)

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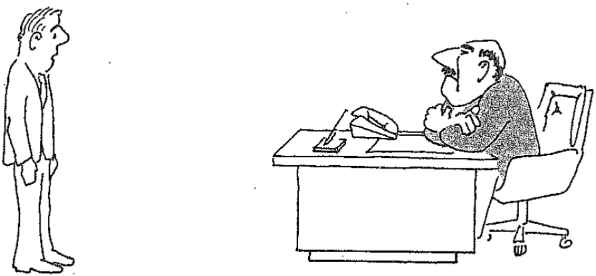
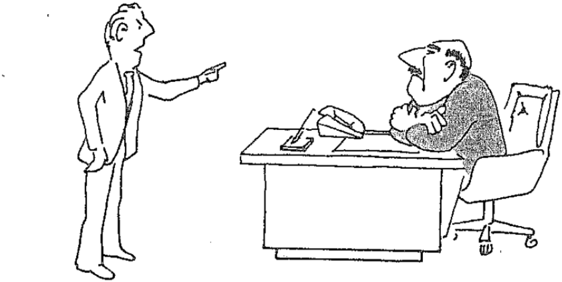
December 1, 2020

Motivation



"I can't find an efficient algorithm, I guess I'm just too dumb."

Motivation



"I can't find an efficient algorithm, because no such algorithm is possible!"

Motivation

"I can't find an efficient algorithm, but neither can all these famous people."

Recap

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- P: decision problems that have polynomial-time algorithms

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- NP: decision problems with yes-instances that have polynomial-time verifiable certificates

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- Fact: $P \subseteq NP$
- Conjecture: $P \neq NP$

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- Fact: $\text{P} \subseteq \text{NP}$
- Conjecture: $\text{P} \neq \text{NP}$
- Definition: B is NP-hard if $(\forall A \in \text{NP})\, A \leq^p B$.

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- P: decision problems that have polynomial-time algorithms
- NP: decision problems with yes-instances that have polynomial-time verifiable certificates
- Fact: P $\subseteq$ NP
- Conjecture: P $\neq$ NP
- Definition: B is NP-hard if $(\forall A \in \text{ NP})\, A \leq^p B$.
- Assume P $\neq$ NP. If B is NP-hard then $B \notin$ P.

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- P: decision problems that have polynomial-time algorithms
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- Fact: P $\subseteq$ NP
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- Definition: B is NP-hard if $(\forall\, A \in \text{ NP})\, A \leq^p B$.
- Assume P $\neq$ NP. If B is NP-hard then $B \notin$ P.
- Theorem: Circuit-SAT is NP-hard.

Establishing NP-Hardness

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- 3-SAT $\leq^p$ 3-Coloring

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Satisfiability and Coloring

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Input: 3-CNF formula φ

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Input: graph $G = (V, E)$

Output: whether G has a 3-coloring, i.e., a mapping $c : V \to [3]$ such that $(\forall\,(u, v) \in E)\, c(u) \neq c(v))$.

3-SAT $\leq^p$ 3-Coloring

3-SAT $\leq^p$ 3-Coloring – variable gadgets

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- Bijection between assignments to variables $x_1, \ldots, x_n$ and valid colorings with $\{\text{red}, \text{green}, \text{blue}\}$.

3-SAT $\leq^p$ 3-Coloring – clause gadgets & connections

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- For each 3-clause C_j, include a complete graph on 3 new vertices, each labeled with a unique literal of C_j.

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Conclusion

φ is satisfiable $\Leftrightarrow$ G is 3-colorable

Satisfiability and Subset Sum

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E.g.: $\varphi = (x_1 \vee \overline{x_2} \vee x_4) \wedge (\overline{x_1} \vee x_2)$

Output: whether φ has a satisfying assignment.

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Subset Sum

Input: $a_1, a_2, \ldots, a_k \in \mathbb{N}$; $t \in \mathbb{N}$

Satisfiability and Subset Sum

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E.g.: $\varphi = (x_1 \lor \overline{x_2} \lor x_4) \land (\overline{x_1} \lor x_2)$

Output: whether φ has a satisfying assignment.

Subset Sum

Input: $a_1, a_2, \ldots, a_k \in \mathbb{N}$; $t \in \mathbb{N}$

Output: whether there exists $I \subseteq [k]$ such that $\sum_{i \in I} a_i = t$.

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- Subset Sum $\leq^p$ Knapsack

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- For each variable x_i, include two numbers $a_{2i-1} = a_{2i} = 2^{i-1}$.

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- For each variable x_i, include two numbers $a_{2i-1} = a_{2i} = 2^{i-1}$.
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- Pick bit two new consecutive bit positions B_j.

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For each clause C_j with k_j literals:

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- Consider subset $I \subseteq [2n]$ such that $\sum_{i \in I}^{n} a_i = t \bmod 2^n$.

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- Consider subset $I \subseteq [2n]$ such that $\sum_{i \in I}^{n} a_i = t \bmod 2^n$. I can be extended with subset of new numbers to I' such that $\sum_{i \in I'} a_i$ equals t in positions B_j iff underlying assignment satisfies C_j.

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Conclusion

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  3-D Matching

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Input: disjoint sets X, Y, Z; $T \subseteq X \times Y \times Z$

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Input: disjoint sets X, Y, Z; $T \subseteq X \times Y \times Z$

Output: whether there exists $S \subseteq T$ such that each element of $X \cup Y \cup Z$ appears exactly once

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- Numerical: Subset Sum, Knapsack