CS 577-2: Introduction to Algorithms 10/13/2020

Alternate Midterm Exam 1

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Guidelines:

- The exam consists of a single problem setting with three parts. When solving a later part, you can assume a solution for the earlier parts if that is helpful.
- You can use all the results we showed in class. Clearly state the results you use.
- Unlike for the homework, you are not expected to argue correctness and analyze the complexity of every algorithm you design. You only need to provide the elements that are explicitly asked for.
- Pay attention to the Piazza thread "Clarifications on Alternate Midterm Exam 1" for potential clarifications. Due to the time difference, Piazza will unfortunately not be monitored during the exam.
- The exam is open-book and open-notes. You are free to use scratch paper.
- You are not allowed to communicate with anyone but the instructors throughout the duration of the exam.
- The exam is proctored by Honorlock. During the exam you are not allowed to use any electronic devices other than the computer on which you are taking the exam.
- The only URLs you are allowed to access are `http://www.piazza.com` for Piazza, `https://wisconsin-madison.instructure.com/courses/205209` for Canvas, and `http://jeffe.cs.illinois.edu/teaching/algorithms` for the recommended online textbook. Only use Google Chrome, and follow the above links directly (right click to open in a new tab) so you don't access other websites (such as Google) in the process.
- You may take one 5-minute break maximum. Otherwise, you need to have your face within view of the webcam for the duration of the exam. You cannot wear headphones, earbuds or hats but are welcome to wear earplugs.
- The exam ends at 4:15. You need to stop working on the exam no later than that time.
- Your solutions can be typed or handwritten. You have till 4:20 to upload them in PDF format to Canvas. You are welcome to use your phone or other electronic devices for PDF conversion once you stopped working on the exam.
- Canvas will continue accepting uploads until 4. Anyone who submits between 4:20 and 4:30 may only do so if they stopped working by 4:15 and had submission issues uploading. We will review the Honorlock videos from 4:15 onward for any student who submits after 4:20.
- Good luck!

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You are given an $n \times m$ grid G in which each cell is either clear or occupied. The goal is to clear the entire grid using as few as possible of the following moves:

- [row move:] If one or more consecutive cells in the same row are occupied, clear all of them.
- [column move:] If *all* n cells in a given column are occupied, clear all of them.

You are asked to determine the minimum number of moves needed.

For example, consider the following grid with $(n, m) = (4, 9)$; the occupied cells are marked by "x".

	1	2	3	4	5	6	7	8	$9 = m$
1	x	x				x	x	x	x
2		x	x			x			
3		x	x	x		x			
$n = 4$	x	x	x	x	x	x			

There is a sequence of 6 moves that clears the grid: column 6, cells 1-2 of row 1, cells 7-9 of row 1, cells 2-3 of row 2, cells 2-4 of row 3, cells 1-5 of row 4. There is no sequence with fewer moves. Thus, the answer is 6.

Note that at the start the cells in row 4 can be cleared in a single move. However, if we start by clearing column 2, then clearing the rest of row 4 cannot be done in a single move.

For all of the problems below, the *only* way you can access the input grid G is through *column queries* C. For every $j \in [m]$, $C(j)$ returns in time $O(n)$ and space $O(n)$ an array with the contents of column j of G.

(a) [4 points] Let $R[0..m]$ be an array where $R[k]$ denotes the minimum number of moves to clear the subgrid $[n] \times [k]$ using *row moves only*.

In the above example $R[0..9] = (0, 2, 4, 4, 4, 4, 7, 7, 7, 7)$.

Describe an algorithm that computes $R[0..m]$ in time $O(nm)$ and space $O(n + m)$.

Analyze the space complexity.

(b) [3 points] Given integers k and ℓ with $1 \le k \le \ell \le m$, and given access to the array $R[0..m]$, show how to compute the following quantity $S(k, \ell)$ in time $O(n)$ using $O(1)$ queries to R and $O(1)$ column queries: the minimum number of moves to clear the subgrid $[n] \times \{k, k+1, \ldots, \ell\}$ using *row moves only*.

Argue correctness.

(c) [7 points] Design an algorithm that solves the problem in time $O(nm^2)$ and space $O(n + m)$.

Argue correctness and analyze the time complexity.

Extra credit: Design an algorithm that is more time and space efficient for (c).