CS 577-2: Introduction to Algorithms Fall 2020

Practice Problems for Midterm Exam 1

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The following are some old exam problems that you can use as design practice. Don't forget to do the additional problems on each of the assignments!

1. Let P be a set of n points in the plane. A point (x, y) in P is called undominated if for every other point (x', y') in P, either $x' < x$ or $y' < y$ (or both).

   Design an $O(n \log n)$ algorithm for finding all of the undominated points.

2. To get in shape you have decided to start running to school. You want a route that goes entirely uphill and then entirely downhill. Your run starts at home and ends at school. You have a map detailing the roads with m road segments and n intersections. Each road segment has a positive length and each intersection has an elevation.

   Design an efficient algorithm to find a shortest route that meets your specifications, or report that none exists. You may assume that every road segment is either uphill or downhill, and that home and school lie at intersections. For full credit, your algorithm should run in linear time.

3. In the two-player game "Two Ends", n cards are laid out in a row. On each card, face up, is written a positive integer. Players take turns removing a card from either end of the row and placing the card in their pile, until all cards are removed. The score of a player is the sum of the integers of the cards in his/her pile.

   Design an algorithm that takes the sequence of n positive integers, and determines the score of each player when both play optimally. Your algorithm should run in time $O(n^2)$ and space $O(n)$.

4. You are given an arithmetic expression containing n integers and $n - 1$ operators, each either $+$, $-$, or $\times$. You are also given a positive integer m. Your goal is to find an order to perform the operations such that the result is a multiple of m, or report that no such order exists.

   For example, for the expression $6 \times 3 + 2 \times 5$, this is possible for $m = 7$, namely as follows: $(6 \times 3) + (2 \times 5) = 28$. For the same expression this is not possible for $m = 8$ as none of the five possible orderings yield a multiple of 8: $(6 \times 3) + (2 \times 5) = 28$, $((6 \times (3 + 2)) \times 5) = 150$, $((6 \times 3) + 2) \times 5 = 100$, $6 \times (3 + (2 \times 5)) = 78$, $6 \times ((3 + 2) \times 5) = 150$.

   Design an algorithm that runs in time polynomial in n and m.