

SA

KVPY QUESTION PAPER –STREAM SA

November 2, 2014

PART I

One-Mark Questions

MATHEMATICS

- 1 Let r be a root of the equation $x^2 + 2x + 6 = 0$. The value of $(r+2)(r+3)(r+4)(r+5)$ is equal to
A. 51 B. -51
C. -126 D. 126
- 2 Let \mathbb{R} be the set of all real numbers and let f be a function from \mathbb{R} to \mathbb{R} such that
- $$f(x) + \left(x + \frac{1}{2}\right)f(1-x) = 1,$$
- for all $x \in \mathbb{R}$. Then $2f(0) + 3f(1)$ is equal to
A. 2 B. 0
C. -2 D. -4
- 3 The sum of all positive integers n for which
- $$\frac{1^3 + 2^3 + \dots + (2n)^3}{1^2 + 2^2 + \dots + n^2}$$
- is also an integer is
A. 8 B. 9
C. 15 D. Infinite
- 4 Let x and y be two 2-digit numbers such that y is obtained by reversing the digits of x . Suppose they also satisfy $x^2 - y^2 = m^2$ for some positive integer m . The value of $x+y+m$ is
A. 88 B. 112
C. 144 D. 154

- 5 Let $p(x) = x^2 - 5x + a$ and $q(x) = x^2 - 3x + b$, where a and b are positive integers. Suppose $\text{hcf}(p(x), q(x)) = x - 1$ and $k(x) = \text{lcm}(p(x), q(x))$. If the coefficient of the highest degree term of $k(x)$ is 1, the sum of the roots of $(x - 1) + k(x)$ is

- A. 4 B. 5
C. 6 D. 7

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- A. 4

- B. 5

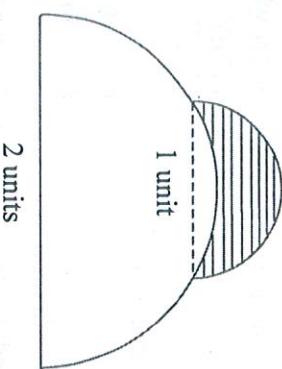
- C. 6

- D. 7

- 6 In a quadrilateral $ABCD$, which is not a trapezium, it is known that $\angle DAB = \angle ABC = 60^\circ$. Moreover,

- $\angle CAB = \angle CBD$. Then
- A. $AB = BC + CD$
B. $AB = AD + CD$
C. $AB = BC + AD$
D. $AB = AC + AD$

- 7 A semi-circle of diameter 1 unit sits at the top of a semi-circle of diameter 2 units. The shaded region inside the smaller semi-circle but outside the larger semi-circle is called a *lune*. The area of the lune is



2 units

1 unit

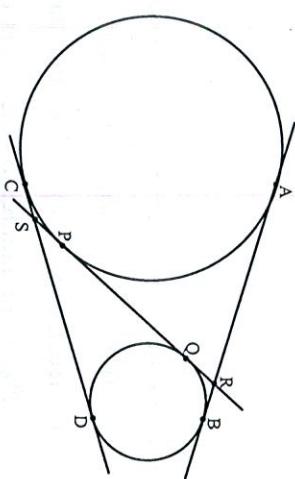
- A. $\frac{\pi - \sqrt{3}}{6}$
B. $\frac{\sqrt{3} - \pi}{4}$
C. $\frac{\sqrt{3} - \pi}{4}$
D. $\frac{\sqrt{3} - \pi}{8}$

- 8 The angle bisectors BD and CE of a triangle ABC are divided by the incentre I in the ratios $3:2$ and $2:1$ respectively. Then the ratio in which I divides the angle bisector through A is

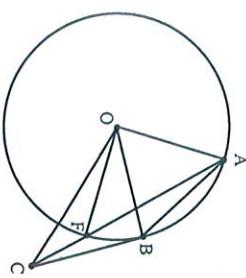
- A. 3:1 B. 11:4
C. 6:5 D. 7:4

9 Suppose S_1 and S_2 are two unequal circles; AB and CD are the direct common tangents to these circles. A transverse common tangent PQ cuts AB in R and CD in S . If $AB = 10$, then RS is

- A. 8 B. 9
C. 10 D. 11



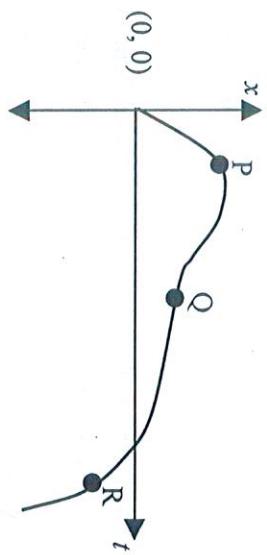
- 10 On the circle with center O , points A, B are such that $OA = AB$. A point C is located on the tangent at B to the circle such that A and C are on the opposite sides of the line OB and $AB = BC$. The line segment AC intersects the circle again at F . Then the ratio $\angle BOF : \angle BOC$ is equal to



- 11 In a cinema hall, the charge per person is Rs. 200. On the first day, only 60% of the seats were filled. The owner decided to reduce the price by 20% and there was an increase of 50% in the number of spectators on the next day. The percentage increase in the revenue on the second day was
- A. 50 B. 40 C. 30 D. 20
- 12 The population of cattle in a farm increases so that the difference between the population in year $n+2$ and that in year n is proportional to the population in year $n+1$. If the populations in years 2010, 2011 and 2013 were 39, 60 and 123, respectively, then the population in 2012 was
- A. 81 B. 84 C. 87 D. 90
- 13 The number of 6-digit numbers of the form $ababab$ (in base 10) each of which is a product of exactly 6 distinct primes is
- A. 8 B. 10 C. 13 D. 15
- 14 The houses on one side of a road are numbered using consecutive even numbers. The sum of the numbers of all the houses in that row is 170. If there are at least 6 houses in that row and a is the number of the sixth house, then
- A. $2 \leq a \leq 6$ B. $8 \leq a \leq 12$ C. $14 \leq a \leq 20$ D. $22 \leq a \leq 30$
- 15 Suppose $a_2, a_3, a_4, a_5, a_6, a_7$ are integers such that $\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!}$, where $0 \leq a_j < j$ for $j = 2, 3, 4, 5, 6, 7$. The sum $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is
- A. 8 B. 9 C. 10 D. 11

PHYSICS

- 16 In the following displacement (x) vs time (t) graph, at which among the points P, Q, and R is the object's speed increasing?



- A. R only
- B. P only
- C. Q and R only
- D. P, Q, R

- 17 A box, when hung from a spring balance shows a reading of 50 kg. If the same box is hung from the same spring balance inside an evacuated chamber, the reading on the scale will be

- A. 50 kg because the mass of the box remains unchanged.
- B. 50 kg because the effect of the absence of the atmosphere will be identical on the box and the spring balance.
- C. less than 50 kg because the weight of the column of air on the box will be absent.
- D. more than 50 kg because the atmospheric buoyancy force will be absent.

- 18 Two positively charged spheres of masses m_1 , and m_2 are suspended from a common point at the ceiling by identical insulating massless strings of length l . Charges on the two spheres are q_1 and q_2 , respectively. At equilibrium both strings make the same angle θ with the vertical. Then
- A. $q_1m_1 = q_2m_2$
 - B. $m_1 = m_2$
 - C. $m_1 = m_2 \sin\theta$
 - D. $q_2m_1 = q_1m_2$
- 19 A box when dropped from a certain height reaches the ground with a speed v . When it slides from rest from the same height down a rough inclined plane inclined at an angle 45° to the horizontal, it reaches the ground with a speed $\sqrt{3}v$. The coefficient of sliding friction between the box and the plane is (acceleration due to gravity is 10 ms^{-2})
- A. $\frac{8}{9}$
 - B. $\frac{1}{9}$
 - C. $\frac{2}{3}$
 - D. $\frac{1}{3}$
- 20 A thin paper cup filled with water does not catch fire when placed over a flame. This is because
- A. the water cuts off oxygen supply to the paper cup.
 - B. water is an excellent conductor of heat.
 - C. the paper cup does not become appreciably hotter than the water it contains.
 - D. paper is a poor conductor of heat.

- 21 Ice is used in a cooler in order to cool its contents. Which of the following will speed up the cooling process?

A. Wrap the ice in a metal foil.

B. Drain the water from the cooler periodically.

C. Put the ice as a single block.

D. Crush the ice.

- 22 The angle of a prism is 60° . When light is incident at an angle of 60° on the prism, the angle of emergence is 40° .

The angle of incidence i for which the light ray will deviate the least is such that

A. $i < 40^\circ$

B. $40^\circ < i < 50^\circ$

C. $50^\circ < i < 60^\circ$

D. $i > 60^\circ$

- 23 A concave lens made of material of refractive index 1.6 is immersed in a medium of refractive index 2.0. The two surfaces of the concave lens have the same radius of curvature 0.2 m. The lens will behave as a

A. divergent lens of focal length 0.4 m.

B. divergent lens of focal length 0.5 m.

C. convergent lens of focal length 0.4 m.

D. convergent lens of focal length 0.5 m.

- 24 A charged particle, initially at rest at O, when released follows a trajectory as shown. Such a trajectory is possible in the presence of



- A. electric field of constant magnitude and varying direction.
- B. magnetic field of constant magnitude and varying direction.
- C. electric field of constant magnitude and constant direction.
- D. electric and magnetic fields of constant magnitudes and constant directions which are parallel to each other.

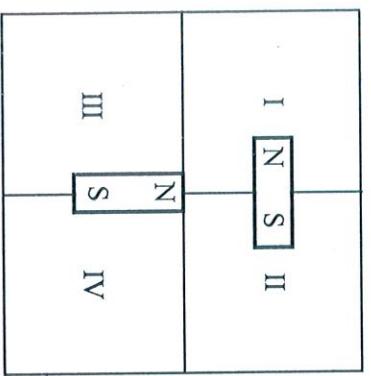
- 25 Two equal charges of magnitude Q each are placed at a distance d apart. Their electrostatic energy is E . A third charge $-Q/2$ is brought midway between these two charges. The electrostatic energy of the system is now

- A. $-2E$
- B. $-E$
- C. 0
- D. E

- 26 A bar magnet falls with its north pole pointing down through the axis of a copper ring. When viewed from above, the current in the ring will be

- A. clockwise while the magnet is above the plane of the ring, and counter clockwise while below the plane of the ring.
- B. counter clockwise throughout.
- C. counter clockwise while the magnet is above the plane of the ring, and clockwise while below the plane of the ring.
- D. clockwise throughout.

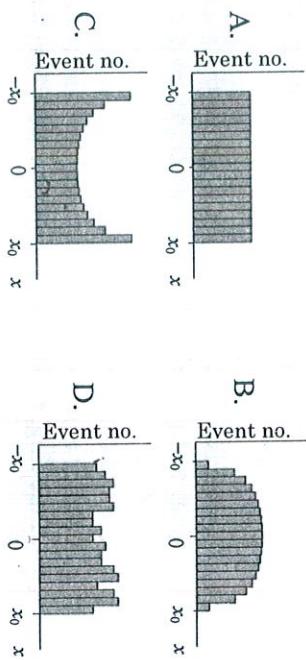
- 27 Two identical bar magnets are held perpendicular to each other with a certain separation, as shown below. The area around the magnets is divided into four zones.



Given that there is a neutral point it is located in

- A. Zone I
- B. Zone II
- C. Zone III
- D. Zone IV

- 28 A large number of random snap shots using a camera are taken of a particle in simple harmonic motion between $x = -x_0$ and $x = +x_0$ with origin $x = 0$ as the mean position. A histogram of the total number of times the particle is recorded about a given position (Event no.) would most closely resemble



- 29 In 1911, the physicist Ernest Rutherford discovered that atoms have a tiny, dense nucleus by shooting positively charged particles at a very thin gold foil. A key physical property which led Rutherford to use gold was that it was

- A. electrically conducting
- B. highly malleable
- C. shiny
- D. non-reactive

- 30 Consider the following statements:

- (I) All isotopes of an element have the same number of neutrons.
 - (II) Only one isotope of an element can be stable and non-radioactive.
 - (III) All elements have isotopes.
 - (IV) All isotopes of Carbon can form chemical compounds with Oxygen-16.
- The correct option regarding an isotope is

- A. (III) and (IV) only.
- B. (II), (III) and (IV) only.
- C. (I), (II) and (III) only.
- D. (I), (III) and (IV) only.

CHEMISTRY

31 The isoelectronic pair is

- A. CO, N₂
- B. O₂, NO
- C. C₂, HF
- D. F₂, HCl

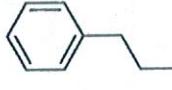
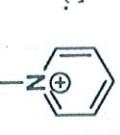
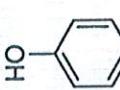
32 The numbers of lone pairs and bond pairs in hydrazine are, respectively

- A. 2 and 4
- B. 2 and 6
- C. 2 and 5
- D. 1 and 5

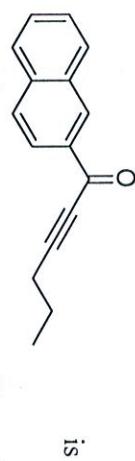
33 The volume of oxygen at STP required to burn 2.4 g of carbon completely is

- A. 1.12 L
- B. 8.96 L
- C. 2.24 L
- D. 4.48 L

34 The species that exhibits the highest R_f value in a thin layer chromatogram using a nonpolar solvent on a silica gel plate is

- A. 
- B. 
- C. 
- D. 

35 The number of C-C sigma bonds in the compound



is

- A. 16
- B. 17
- C. 18
- D. 11

36 If the radius of the hydrogen atom is 53 pm, the radius of the He⁺ ion is closest to

- A. 108 pm
- B. 81 pm
- C. 27 pm
- D. 13 pm

37 The diamagnetic species is

- A. NO
- B. NO₂
- C. O₂
- D. CO₂

38 The pH of 0.1 M aqueous solutions of NaCl, CH₃COONa and NH₄Cl will follow the order

- A. NaCl < CH₃COONa < NH₄Cl
- B. NH₄Cl < NaCl < CH₃COONa
- C. NH₄Cl < CH₃COONa < NaCl
- D. NaCl < NH₄Cl < CH₃COONa

39 At room temperature, the average speed of Helium is higher than that of Oxygen by a factor of

- A. $2\sqrt{2}$
- B. $6/\sqrt{2}$
- C. 8
- D. 6

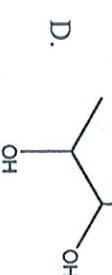
40 Ammonia is NOT produced in the reaction of

- A. NH₄Cl with KOH
- B. AlN with water
- C. NH₄Cl with NaNO₂
- D. NH₄Cl with Ca(OH)₂

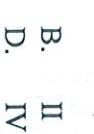
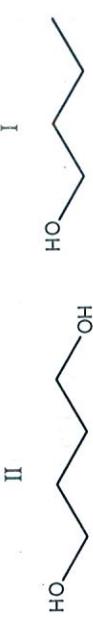
41 The number of isomers which are ethers and having the molecular formula $C_4H_{10}O$, is

- A. 2
- B. 3
- C. 4
- D. 5

42 The major product of the reaction of 2-butene with alkaline $KMnO_4$ solution is



43 Among the compounds I-IV, the compound having the lowest boiling point is



44 Of the following reactions

- (i) $A \rightleftharpoons B, \Delta G^\circ = 250 \text{ kJ mol}^{-1}$
- (ii) $D \rightleftharpoons E, \Delta G^\circ = -100 \text{ kJ mol}^{-1}$

- (iii) $F \rightleftharpoons G, \Delta G^\circ = -150 \text{ kJ mol}^{-1}$

the reaction with the largest equilibrium constant is

- A. (i)
- B. (ii)
- C. (iii)
- D. (iv)

45 The first ionization enthalpies for three elements are 1314, 1680, and 2080 kJ mol^{-1} , respectively. The correct sequence of the elements is

- A. O, F, and Ne
- B. F, O, and Ne
- C. Ne, F, and O
- D. F, Ne, and O

BIOLOGY

- 46 Individuals of one kind occupying a particular geographic area at a given time are called
A. community B. population
C. species D. biome
- 47 What fraction of the assimilated energy is used in respiration by the herbivores?
A. ~10 percent B. ~60 percent
C. ~30 percent D. ~80 percent
- 48 Athletes are often trained at high altitude because
A. training at high altitude increases muscle mass
B. training at high altitude increases the number of red blood cells
C. there is less chance of an injury at high altitude
D. athletes sweat less at high altitude
- 49 In human brain, two cerebral hemispheres are connected by a bundle of fibers which is known as
A. medulla oblongata B. cerebrum
C. cerebellum D. corpus callosum
- 50 Which one of the following hormones is produced by the pancreas?
A. Prolactin
B. Glucagon
C. Leutinizing hormone
D. Epinephrine
- 51 The stalk of a plant leaf is derived from which one of the following types of plant tissue?
A. Sclerenchyma B. Parenchyma
C. Chlorenchyma D. Collenchyma
- 52 Which of the following muscle types CANNOT be used voluntarily?
A. Both striated and smooth
B. Both cardiac and striated
C. Both smooth and cardiac
D. Cardiac, striated and smooth
- 53 The pulmonary artery carries
A. deoxygenated blood to the lungs
B. oxygenated blood to the brain
C. oxygenated blood to the lungs
D. deoxygenated blood to the kidney

54 Both gout and kidney stone formation is caused by

- A. calcium oxalate
- B. uric acid
- C. creatinine
- D. potassium chloride

55 The auditory nerve gets its input from which of the following?

- A. The sense cells of the cochlea
- B. Vibration of the last ossicle
- C. Eustachian tube
- D. Vibration of the tympanic membrane

56 Which of the following *organelles* contain circular DNA?

- A. Peroxisomes and Mitochondria
- B. Mitochondria and Golgi complex
- C. Chloroplasts and Lysosomes
- D. Mitochondria and Chloroplast

57 A reflex action does NOT involve

- A. neurons
- B. brain
- C. spinal cord
- D. muscle fiber

58 Which one of the following options is true in photosynthesis?

- A. CO_2 is oxidized and H_2O is reduced
- B. H_2O is oxidized and CO_2 is reduced
- C. Both CO_2 and H_2O are reduced
- D. Both CO_2 and H_2O are oxidized

59 Human mature red blood cells (RBCs) do NOT contain

- A. Iron
- B. Cytoplasm
- C. Mitochondria
- D. Haemoglobin

60 A person was saved from poisonous snake bite by anti-venom injection. Which of the following immunity explains this form of protection?

- A. Naturally acquired active immunity
- B. Artificially acquired active immunity
- C. Naturally acquired passive immunity
- D. Artificially acquired passive immunity

PART II

Two-Mark Questions

MATHEMATICS

- 61 Let a, b, c be non-zero real numbers such that $a+b+c=0$; let $q = a^2 + b^2 + c^2$ and $r = a^4 + b^4 + c^4$.

Then

- A. $q^2 < 2r$ always
- B. $q^2 = 2r$ always
- C. $q^2 > 2r$ always
- D. $q^2 - 2r$ can take both positive and negative values

- 62 The value of

$$\sum_{n=0}^{1947} \frac{1}{2^n + \sqrt{2^{1947}}}$$

is equal to

- A. $\frac{487}{\sqrt{2^{1945}}}$
- B. $\frac{1946}{\sqrt{2^{1947}}}$
- C. $\frac{1947}{\sqrt{2^{1947}}}$
- D. $\frac{1948}{\sqrt{2^{1947}}}$

- 63 The number of integers a in the interval $[1, 2014]$ for which the system of equations

$$x+y=a, \quad \frac{x^2}{x-1} + \frac{y^2}{y-1} = 4$$

has finitely many solutions is

- A. 0
- B. 1007
- C. 2013
- D. 2014

- 64 In a triangle ABC with $\angle A = 90^\circ$, P is a point on BC such that $PA:PB = 3:4$. If $AB = \sqrt{7}$ and $AC = \sqrt{5}$, then $BP:PC$ is

- A. 2 : 1
- B. 4 : 3
- C. 4 : 5
- D. 8 : 7

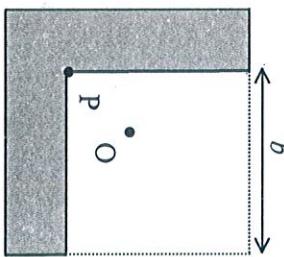
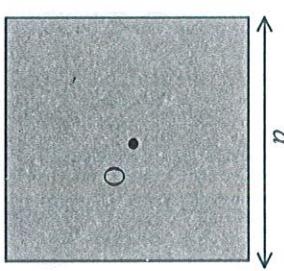
- 65 The number of all 3-digit numbers abc (in base 10) for which $(a \times b \times c) + (a \times b) + (b \times c) + (c \times a) + a + b + c = 29$ is

- A. 6
- B. 10
- C. 14
- D. 18

PHYSICS

66

A uniform square wooden sheet of side a has its center of mass located at point O as shown in the figure on the left. A square portion of side b of this sheet is cut out to produce an L-shaped sheet as shown in the figure on the right.



The center of mass of the L-shaped sheet lies at the point P (in the diagram) when

- A. $a/b = (\sqrt{5} - 1)/2$
- B. $a/b = (\sqrt{5} + 1)/2$
- C. $a/b = (\sqrt{3} - 1)/2$
- D. $a/b = (\sqrt{3} + 1)/2$

68

An aluminum piece of mass 50 g initially at 300 °C is dipped quickly and taken out of 1 kg of water, initially at 30 °C. If the temperature of the aluminum piece immediately after being taken out of the water is found to be 160 °C, what is the temperature of the water then? (Specific heat capacities of aluminum and water are 900 J kg⁻¹ K⁻¹ and 4200 J kg⁻¹ K⁻¹, respectively.)

- A. 165 °C
- B. 45 °C
- C. 31.5 °C
- D. 28.5 °C

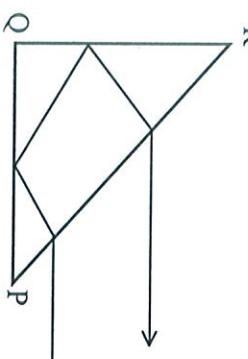
67

A machine is blowing spherical soap bubbles of different radii filled with helium gas. It is found that if the bubbles have a radius smaller than 1 cm, then they sink to the floor in still air. Larger bubbles float in the air. Assume that the thickness of the soap film in all bubbles is uniform and equal. Assume that the density of soap solution is same as that of water ($= 1000 \text{ kg m}^{-3}$). The density of helium inside the bubbles and air are 0.18 kg m^{-3} and 1.23 kg m^{-3} , respectively. Then the thickness of the soap film of the bubbles is (note $1 \mu\text{m} = 10^{-6} \text{ m}$)

- A. 0.50 μm
- B. 1.50 μm
- C. 7.00 μm
- D. 3.50 μm

69

A ray of light incident parallel to the base PQ of an isosceles right-angled triangular prism PQR suffers two successive total internal reflections at the faces PQ and QR before emerging reversed in direction as shown.



If the refractive index of the material of the prism is μ , then

- A. $\mu > \sqrt{5}$
- B. $\sqrt{3} < \mu < \sqrt{5}$
- C. $\sqrt{2} < \mu < \sqrt{3}$
- D. $\mu < \sqrt{2}$

70 Consider the circuit shown below where all resistors are of $1\text{ k}\Omega$.



If a current of magnitude 1 mA flows through the resistor marked X, what is the potential difference measured between points P and Q?

- A. 21 V
- B. 68 V
- C. 55 V
- D. 34 V

CHEMISTRY

71

10 moles of a mixture of hydrogen and oxygen gases at a pressure of 1 atm at constant volume and temperature, react to form 3.6 g of liquid water. The pressure of the resulting mixture will be closest to

- A. 1.07 atm
- B. 0.97 atm
- C. 1.02 atm
- D. 0.92 atm

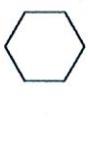
72 The ammonia evolved from 2 g of a compound in Kjeldahl's estimation of nitrogen neutralizes 10 mL of $2\text{ M H}_2\text{SO}_4$ solution. The weight percentage of nitrogen in the compound is

- A. 28
- B. 14
- C. 56
- D. 7

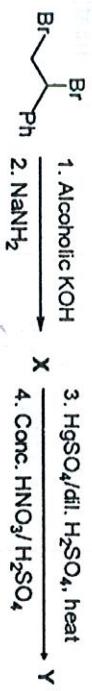
73 Complete reaction of 2.0 g of calcium (at. wt. = 40) with excess HCl produces 1.125 L of H₂ gas. Complete reaction of the same quantity of another metal "M" with excess HCl produces 1.85 L of H₂ gas under identical conditions. The equivalent weight of "M" is closest to

- A. 23
- B. 9
- C. 7
- D. 12

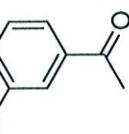
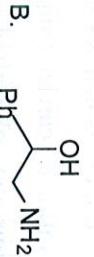
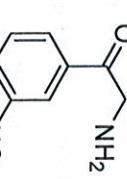
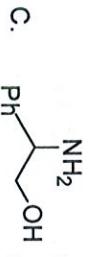
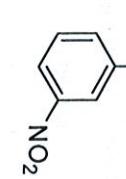
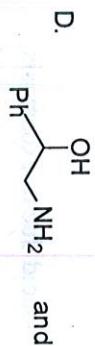
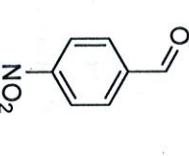
74 A compound X formed after heating coke with lime reacts with water to give Y which on passing over red-hot iron at 873 K produces Z. The compound Z is

- A. 
- B. 
- C. 
- D. 

75 In the following reaction sequence



X and Y are, respectively

- A.  and 
- B.  and 
- C.  and 
- D.  and 

BIOLOGY

- 79 Children suffering from phenylketonuria are given food low in phenylalanine and supplemented with tyrosine. This is because they
- A. are unable to utilize phenylalanine
 - B. do not require phenylalanine
 - C. have increased tyrosine anabolism
 - D. have increased tyrosine catabolism
- 76 In which of the following cellular compartment(s) do respiratory reactions occur?
- A. Cytoplasm and endoplasmic reticulum
 - B. Mitochondria and Golgi complex
 - C. Mitochondria and cytoplasm
 - D. Mitochondria only
- 77 A woman heterozygous for color blindness marries a color blind man. What would be the ratios of carrier daughters, color blind daughters, normal sons and color blind sons in the F₁ generation?
- A. 1:2:2:1
 - B. 2:1:1:2
 - C. 1:1:1:1
 - D. 1:1:2:2
- 78 Two semi-permeable bags containing 2% sucrose are placed in two beakers, 'P' containing water and 'Q' containing 10% sucrose. Which one of the following outcomes is true?
- A. Bag in 'P' becomes flaccid due to exosmosis
 - B. Bag in 'P' becomes turgid due to endosmosis
 - C. Bag in 'Q' becomes turgid due to endosmosis
 - D. Concentration of sucrose remains unchanged in both
- 80 Two bottles were half filled with water from Ganga ('P') and Kaveri ('Q') and kept under identical airtight conditions for 5 days. The oxygen was determined to be 2 % in bottle ('P') and 10 % in bottle ('Q'). What could be the cause of this difference?
- A. Ganga is more polluted than Kaveri
 - B. Both the rivers are equally polluted
 - C. Kaveri is more polluted than Ganga
 - D. Kaveri has more minerals than Ganga

ROUGH WORK