विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

This is TYPE 1 Package please wait for Type 2

Subject : PHYSICS

Topic : FLUID MECHENICS

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Index

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KEY CONCEPTS

PART (A) - HYDROSTATICS

1. DENSITY AND RELATIVE DENSITY :

Density is mass per unit volume. Densities of solids and liquids are frequently compared with density of water and the relative density of a substance is defined as follows :

Relative Density with respect to water $= \dfrac{\text{density of the substance}}{\text{density of water}}$

$$= \frac{\text{mass of any volume of substance}}{\text{mass of an equal volume of water}}$$

(Also known as specific gravity of the substance) .

Note that relative density being a ratio of same type of physical quantities, it is a number only , without any physical dimension .

2. PRINCIPLE OF ARCHIMEDES :

The principle of Archimedes states that any body , totally or partially immersed in a fluid , experiences an upward force or thrust which is equal to the weight of fluid it displaces and acts vertically up through the C. G. of the displaced fluid . The term fluid covers liquids and gases .

3. LAW OF FLOTATION :

If a body floats in equilibrium in a fluid , its entire weight is supported by the upward thrust of the fluid . Hence , the weight of a floating body is equal to the weight of the fluid displaced by the body at the equilibrium state .

4. FLUID PRESSURE :

Pressure at any point in a fluid is defined as the normal force (or thrust) exerted by the liquid on the surface per unit area .

Pressure is measured in dyne cm^{-2} in C G S units and in N m^{-2} in SI units , (also known as pascal) .

[Note that pressure is a scalar quantity] .

When a plane surface is placed inside a liquid , the liquid exerts hydrostatic pressure on the surface , because of the weight of the liquid column above the surface . The total force exerted normally on the plane surface is called the thrust. The thrust over the surface is the vector sum of the thrusts over small area of the surface , over which the pressure can be considered to be uniform. Then :

(i) If the plane surface is horizontal, the pressure over the surface is uniform and the thrust = (area) × (the uniform pressure)

(ii) If the plane surface is rectangular with its plane vertical and a pair of sides horizontal , the thrust = (area) × (pressure at the centre of the area).

5. PRESSURE IN LIQUIDS :

The hydrostatics pressure 'p' at any point in a liquid varies directly.

(i) As the vertical height (h) of the point below the surface &

(ii) As the density (d) of the liquid.

It can be shown that $p = hdg$.

When a liquid is at rest, the pressure is same at all points at the same horizontal level . The pressure at a point in a liquid does not depend on either the shape of the vessel or the area of cross - section of the vessel.

6. PASCAL'S LAW :

Pressure applied to a liquid (at rest) at one point is transmitted equally in all directions throughout the liquid. This is known as Pascal's Law.

The compressibility of all liquids is exceedingly small & for all practical purposes, liquids may be considered incompressible. Hence, the density is constant throughout the liquid.

Pressure at a point in a liquid is the same in all directions and is perpendicular to the surface upon which its acts.

7. GUAGE PRESSURE AND TOTAL PRESSURE (ABSOLUTE PRESSURE) :

The total pressure at any point in a liquid is the addition of :

(i) The pressure due to the liquid above the point. It is called guage pressure &

(ii) The atmospheric pressure acting on the surface of the liquid.

Thus total pressure = pressure due to liquid + atmospheric pressure

The total pressure is called absolute pressure.

8. THRUST DUE TO PRESSURE :

Total thrust on a horizontal surface immersed in a liquid = **(Pressure On Surface)** × **(Area Of Surface)**

Total thrust on a vertical surface immersed in a liquid = **(Pressure At C.G. Of Area)** × **(Area Of Surface)**

PART (B) - HYDRODYNAMICS

1. The study of fluids in flow is called Hydrodynamics.

2. VELOCITY - FIELD :

It is that space, where at every point in that space, there is a definite velocity. Then the space, where a fluid is in flow is a **velocity - field**.

3. FLOW - LINE :

In a velocity field is an imaginary line in that space, where the tangent to the line at any point on the line gives the direction of the velocity at that point. A flow line is also called a **velocity - line** or a **stream - line**.

4. TYPES OF FLOW OF A FLUID :

Then the flow of a fluid can be classified as :

(i) A **Stream-Line Flow**, the stream lines in the flow space remains steady as time progresses.

(ii) A **Trubulent-Flow**, the stream lines in the flow space shift their positions as time progresses.

In a **Stream-Line Flow**, a group of stream lines form a tubular volume of the flow space, the surface of which is tangential to the stream lines, forming the lateral boundary of that tubular volume. Such a tubular volume in the flow space is a **Tube Of Flow**.

(iii) A steady state flow is the flow in which the fluid properties at any point in the velocity field do not change with time.

5. Equation Of Continuity :

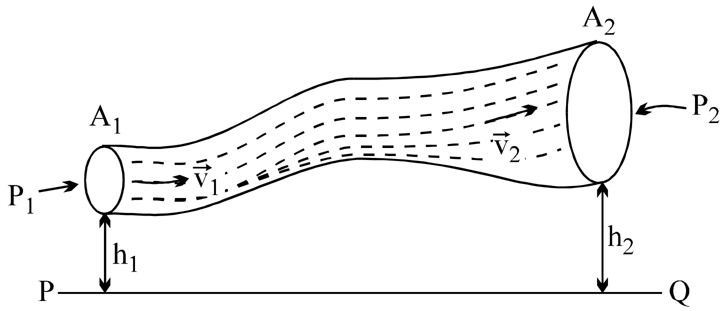
Equation of continuity states that for a steady state flow of a fluid in a pipe, the rate of mass flow across any cross section is constant.

$$\frac{dM}{dt} = \rho AV = \text{constant} .$$

If the fluid is incompressible density is constant at all points, hence, equation of continuity is AV = constant.

6. BERNOULLI 'S EQUATION :

Consider a tube of flow in the space of the stream line flow of a fluid , in a uniform gravitational field . The flow is steady state .



Then :

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

Generalising and removing suffixes $\frac{P}{\rho}+\frac{V_1^2}{2}+gz$ = constant .

This equation is called **BERNOULLI 'S EQUATION** for steady, non-viscous and incompressible fluid flow.

SURFACE TENSION

1. Surface Tension : Surface tension of a liquid is the normal force acting per unit length on either side of an imaginary line drawn the free surface of a liquid. The direction of this force is perpendicular to the line and tangential to the free surface of liquid.

$$T = \frac{F}{L}$$

Note: The surface tension of a liquid varies with temperature as well as dissolved impurities, etc. When soap mixed with water, the surface tension of water decrease.

2. Surface Energy : If the area of the liquid surface has to be increased work has to be done against the force of surface tension. The work done to form a film is stored as potential energy in the surface.

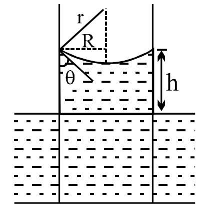
$$W = T\,\Delta A$$

3. Excess Pressure : Excess pressure inside a liquid drop

$$\Delta p = \frac{2T}{r}$$

For a soap bubble in air, there are two surfaces, and so,

$$\Delta p = 2 \times \frac{2T}{r} = \frac{4T}{r}$$



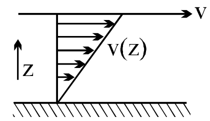
4. Capillarity : Water in the capillary rises to a height

$$h = \frac{2T}{rg\rho}$$

where r is the radius of meniscus, and $r = \frac{R}{\cos\theta}$ where θ is the angle of contact and thus $h = \frac{2T\cos\theta}{R\rho g}$

VISCOSITY

If a glass plate in contact with a water column of height h is moved with constant velocity v. Forces of viscosity appear between the solid surface and the layer in contact.



$$F = -\eta\, A \frac{dv}{dz}$$

where h is a consant called coefficient of viscosity, its cgs unit is poise.
Dimension is $ML^{1}T^{-1}$. The SI units of viscosity equal to 10 poise.

Stoke's Law and Terminal Velocity

When a sphere of radius r moves with a velocity v through a fluid of viscosity h, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta r v$$

If for a sphere viscous force become equal to the net weight acing downward, the velocity of the body become constant and is known as terminal velocity.

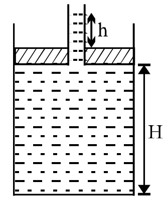
$$6\pi\eta r v_T = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$\Rightarrow \qquad v_T = \frac{2}{9} r^2 \left\{\frac{\rho - \sigma}{\eta}\right\} g$$

EXERCISE # I

Q.1 A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

Q.2 A piston of mass M = 3kg and radius R = 4cm has a hole into which a thin pipe of radius r = 1cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750gm of water is now poured into the pipe so that the piston & pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in the pipe.



Q.3 A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enter water. Upto what depth will the ball go? How much time will it take to come again to the water surface? Neglect air resistance & velocity effects in water.

Q.4 Place a glass beaker, partially filled with water, in a sink. The beaker has a mass 390 gm and an interior volume of 500cm^3. You now start to fill the sink with water and you find, by experiment, that if the beaker is less than half full, it will float; but if it is more than half full, it remains on the bottom of the sink as the water rises to its rim. What is the density of the material of which the beaker is made?

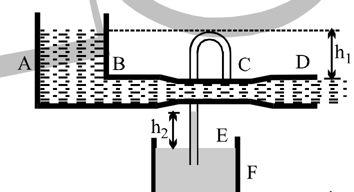
Q.5 Two spherical balls A and B made up of same material having masses 2m and m are released from rest. Ball B lies at a distance h below the water surface while A is at a height of 2h above water surface in the same vertical line, at the instant they are released.

(a) Obtain the position where they collide.

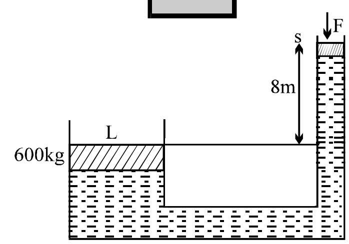
(b) If the bodies stick together due to collision, to what maximum height above water surface does the combined mass rise?

Specific gravity of the material of the balls is 2/3. Neglect viscosity and loss due to splash.

Q.6 Two very large open tanks A and F both contain the same liquid. A horizontal pipe BCD, having a constriction at C leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section at C is one half that at D and if D is at a distance h_1 below the level of liquid in A, to what height h_2 (in terms of h_1) will liquid rise in pipe E ?



Q.7 For the system shown in the figure, the cylinder on the left at L has a mass of 600kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25cm^2 and negligible weight. If the apparatus is filled with oil.(ρ = 0.75 gm/cm^3) Find the force F required to hold the system in equilibrium.

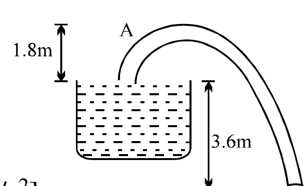


Q.8 A siphon has a uniform circular base of diameter $\frac{8}{\sqrt{\pi}}$ cm with its crest A 1.8 m above water level as in figure. Find

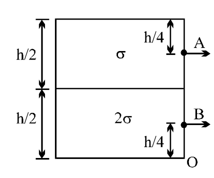
(a) velocity of flow

(b) discharge rate of the flow in m^3/sec.

(c) absolute pressure at the crest level A. [Use $P_0 = 10^5$ N/m^2 & g = 10m/s^2]



Q.9 A large tank is filled with two liquids of specific gravities 2σ and σ. Two holes are made on the wall of the tank as shown. Find the ratio of the distances from O of the points on the ground where the jets from holes A & B strike.

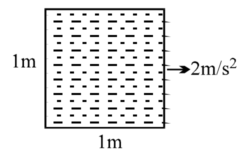


Q.10(a) A spherical tank of 1.2m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of $10 m/s^2$. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

(b) The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water. Find the depth of the lake.

Q.11 A test tube of thin walls has some lead shots in it at its bottom and the system floats vertically in water, sinking by a length $l_o = 10$ cm. A liquid of density less than that of water, is poured into the tube till the levels inside and outside the tube are even. If the tube now sinks to a length $l_o = 40$ cm, the specific gravity of the liquid is ________.

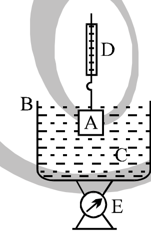
Q.12 An open cubical tank completely filled with water is kept on a horizontal surface. Its acceleration is then slowly increased to $2 m/s^2$ as shown in the Fig. The side of the tank is 1m. Find the mass of water that would spill out of the tank.



Q.13 In air an object weighs 15N, when immersed completely in water the same object weighs 12N. When immersed in another liquid completely, it weighs 13N. Find
(a) the specific gravity of the object and
(b) the specific gravity of the other liquid.

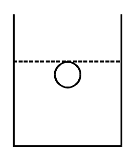
Q.14 Compute the work which must be performed to slowly pump the water out of a hemispherical reservoir of radius R = 0.6 m.

Q.15 Block A in figure hangs by a cord from spring balance D and is submerged in a liquid C contained in a beaker B. The mass of the beaker is 1kg & the mass of the liquid is 1.5 kg. The balance D reads 2.5 kg & balance E reads 7.5 kg.. The volume of block A is $0.003 m^3$.
(i) What is the density of block & the liquid.
(ii) What will each balance read if block is pulled out of the liquid.



Q.16 A solid cube, with faces either vertical or horizontal, is floating in a liquid of density 6 g/cc. It has two third of its volume submerged. If enough water is added from the top so as to completely cover the cube, what fraction of its volume will remain immersed in the liquid?

Q.17 A ball is given velocity v_0 (greater than the terminal velocity v_T) in downward direction inside a highly viscous liquid placed inside a large container. The height of liquid in the container is H. The ball attains the terminal velocity just before striking at the bottom of the container. Draw graph between velocity of the ball and distance moved by the ball before getting terminal velocity.

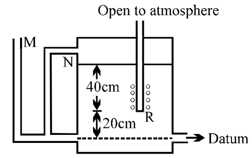


Q.18 Two arms of a U-tube have unequal diameters $d_1 = 1.0$ mm and $d_2 = 1.0$ cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.

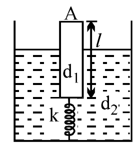
Q.19 A spherical ball of radius 1×10^{-4} m and density $10^4 kg/m^3$ falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2.

Q.20 Calculate the rate of flow of glycerine of density 1.25×10^3 kg/m^3 through the conical section of a pipe if the radii of its ends are 0.1m & 0.04m and the pressure drop across its length is 10N/m^2.

Q.21 The tank in fig discharges water at constant rate for all water levels above the air inlet R. The height above datum to which water would rise in the manometer tubes M and N respectively are __________ & _________.



Q.22 A uniform cylindrical block of length l density d_1 and area of cross section A floats in a liquid of density d_2 contained in a vessel ($d_2 > d_1$). The bottom of the cylinder just rests on a spring of constant k. The other end of the spring is fixed to the bottom of the vessel. The weight that may be placed on top of the cylinder such that the cylinder is just submerged in the liquid is ____



Q.23 Find the speed of rotation of 1 m diameter tank, initially full of water such that water surface makes an angle of 45^0 with the horizontal at a radius of 30cm. What is the slope of the surface at the wall of the tank.

Q.24 A vertical uniform U tube open at both ends contains mercury. Water is poured in one limb until the level of mercury is depressed 2cm in that limb. What is the length of water column when this happens.

Q.25 An expansible balloon filled with air floats on the surface of a lake with 2/3 of its volume submerged. How deep must it be sunk in the water so that it is just in equilibrium neither sinking further nor rising ? It is assumed that the temperature of the water is constant & that the height of the water barometer is 9 meters.

List of recommended questions from I.E. Irodov.

Fluid Mechanics

1.315, 1.319 to 1.322, 1.324, 1.326, 1.327, 1.329

Surface Tension

2.161, 2.162, 2.167, 2.168, 2.169, 2.174, 2.176, 2.178, 2.181, 2.182

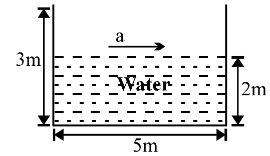
Viscosity

1.331, 1.332, 1.334, 1.336, 1.337, 1.339

EXERCISE # II

Q.1 A solid block of volume $V=10^{-3}m^3$ and density $d=800kg/m^3$ is tied to one end of a string, the other end of which is tied to the bottom of the vessel. The vessel contains 2 immiscible liquids of densities $\rho_1=1000kg/m^3$ and $\rho_2=1500kg/m^3$. The solid block is immersed with 2/5th of its volume in the liquid of higher density & 3/5th in the liquid of lower density. The vessel is placed in an elevator which is moving up with an acceleration of $a=g/2$. Find the tension in the string. [$g=10m/s^2$]

Q.2 An open rectangular tank $5m \times 4m \times 3m$ high containing water upto a height of 2m is accelerated horizontally along the longer side.

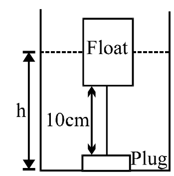


(a) Determine the maximum acceleration that can be given without spilling the water.

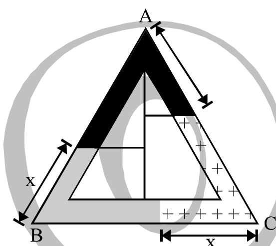
(b) Calculate the percentage of water split over, if this acceleration is increased by 20%.

(c) If initially, the tank is closed at the top and is accelerated horizontally by $9m/s^2$, find the gauge pressure at the bottom of the front and rear walls of the tank.

Q.3 A level controller is shown in the figure. It consists of a thin circular plug of diameter 10cm and a cylindrical float of diameter 20cm tied together with a light rigid rod of length 10cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3kg and the plug may be assumed as massless.

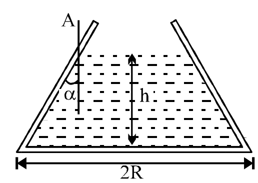


Q.4 A closed tube in the form of an equilateral triangle of side l contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find x in the figure if the densities of the liquids are in A.P.

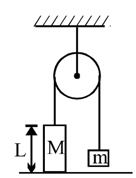


Q.5 A ship sailing from sea into a river sinks X mm and on discharging the cargo rises Y mm. On proceeding again into sea the ship rises by Z mm. Assuming ship sides to be vertical at water line, find the specific gravity of sea water.

Q.6 A conical vessel without a bottom stands on a table. A liquid is poured with the vessel & as soon as the level reaches h, the pressure of the liquid raises the vessel. The radius of the base of the vessel is R and half angle of the cone is α and the weight of the vessel is W. What is the density of the liquid ?



Q.7 As the arrangement shown in the fig is released the rod of mass M moves down into the water. Friction is negligible and the string is inextensible.



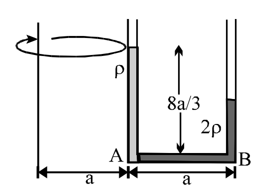
(a) Find the acceleration of the system w.r.t. the distance moved by each mass.

(b) Find the time required to completely immerse the rod into water

if $\frac{m}{M} = \frac{\rho - \rho_{water}}{\rho}$.

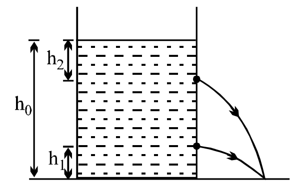
ρ = density of rod ; ρ_{water} = density of water

Q.8 The interface of two liquids of densities ρ and 2ρ respectively lies at the point A in a U tube at rest. The height of liquid column above A is 8a/3 where AB=a. The cross sectional area of the tube is S. With what angular velocity the tube must be whirled about a vertical axis at a distance 'a' such that the interface of the liquids shifts towards B by 2a/3.



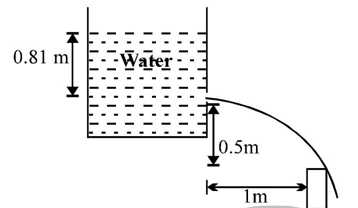
Q.9 A closed cylindrical tank 2m high & 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20.0 rad/s, how much of the bottom of the tank is uncovered? (The cylinder is rotated about vertical axis of symmetry passing through its length.)

Q.10 A cylinder of height H is filled with water to a height h_o ($h_o < H$), & is placed on a horizontal floor. Two small holes are punched at time t = 0 on the vertical line along the length of the cylinder, one at a height h_1 from the bottom & the other a depth h_2 below the level of water in the cylinder. Find the relation between h_1 & h_2 such that the instantaneous water jets emerging from the cylinder from the two holes will hit the ground at the same point.

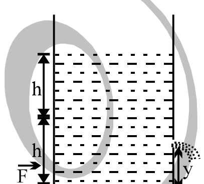


Q.11 A cylindrical tank with a height of h = 1m is filled with water up to its rim. What time is required to empty the tank through an orifice in its bottom? The cross sectional area of the orifice is (1/400)th of the tank. Find the time required for the same amount of water to flow out of the tank if the water level in the tank is maintained constant at a height of h = 1m from the orifice.

Q.12 For the arrangement shown in the figure. Find the time interval after which the water jet ceases to cross the wall.
Area of the tank = 0.5 m^2.
Area of the orifice = 1 cm^2.



Q.13 A cylindrical tank having cross-sectional area A = 0.5 m^2 is filled with two liquids of densities $\rho_1 = 900\ kgm^{-3}$ & $\rho_2 = 600\ kgm^{-3}$, to a height h = 60cm as shown in the figure. A small hole having area a = 5 cm^2 is made in right vertical wall at a height y=20cm from the bottom. Calculate



(i) velocity of efflux.
(ii) horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane.
(iii) minimum and maximum value of F to keep the cylinder at rest. The coefficient of friction between cylinder and the plane is $\mu = 0.01$.
(iv) velocity of the top most layer of the liquid column and also the velocity of the boundary separating the two liquids.

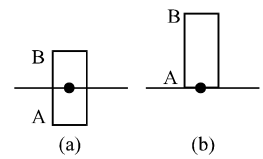
Q.14 A cylindrical wooden float whose base area S = 4000 cm² & the altitude H = 50 cm drifts on the water surface. Specific weight of wood d = 0.8 gf/cm^3.
(a) What work must be performed to take the float out of the water ?
(b) Compute the work to be performed to submerge completely the float into the water.

Q.15 A 10cm side cube weighing 5N is immersed in a liquid of relative density 0.8 contained in a rectangular tank of cross sectional area 15cm x 15cm. If the tank contained liquid to a height of 8cm before the immersion, determine the levels of the bottom of the cube and the liquid surface.

Q.16 A jug contains 15 glasses of orange juice. When you open the tap at the bottom it takes 12 sec to fill a glass with juice. If you leave the tap open. how long will it take to fill the remaining 14 glasses and thus empty the jug?

Q.17 An interstellar explorer discovers a remarkable planet made entirely of a uniform incompressible fluid on density ρ. The radius of the planet is R and the acceleration of gravity at its surface is g. What is the pressure at the center of the planet.

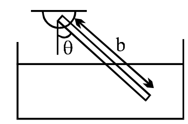
Q.18 A cylindrical rod of length $l = 2$m & density $\frac{\rho}{2}$ floats vertically in a liquid of density ρ as shown in Fig (a).



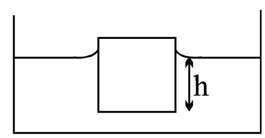
(a) Show that it performs SHM when pulled slightly up & released & find its time period. Neglect change in liquid level.

(b) Find the time taken by the rod to completely immerse when released from position shown in (b). Assume that it remains vertical throughout its motion. (take $g = \pi^2$ m/s^2)

Q.19 A uniform rod of length b capable of tuning about its end which is out of water, rests inclined to the vertical. If its specific gravity is 5/9, find the length immersed in water.



Q.20 A cube with a mass 'm' completely wettable by water floats on the surface of water. Each side of the cube is 'a'. What is the distance h between the lower face of cube and the surface of the water if surface tension is S. Take density of water as ρ_w. Take angle of contact m zero.

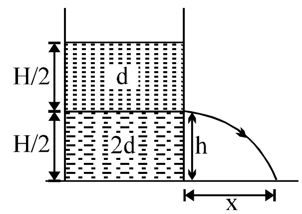


EXERCISE # III

Q.1 A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm², the water velocity is 1 m s^{-1} & the pressure is 2000 Pa. The pressure of water at another point where the cross sectional area is 5 cm², is ___________ pa.
[Density of water = 10^3 kg. m^{-3}] **[JEE '94 , 2]**

Q.2 A container of large uniform cross–sectional area A resting on a horizontal surface, holds two immiscible, non–viscous & incompressible liquids of densities d & 2d , each of height $\frac{H}{2}$ as shown in figure . The lower density liquid is open to the atmosphere having pressure P_0.

(a) A homogeneous solid cylinder of length L $\left(L < \frac{H}{2}\right)$ cross–sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid–liquid interface with the length $\frac{L}{4}$ in the denser liquid. Determine :



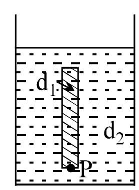
(i) The density D of the solid &
(ii) The total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored . A tiny hole of area s (s << A) is punched on the vertical side of the container at a height h $\left(h < \frac{H}{2}\right)$. Determine :

(i) The initial speed of efflux of the liquid at the hole ;
(ii) The horizontal distance x travelled by the liquid initially &
(iii) The height h_m at which the hold should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m . [Neglect the air resistance in these calculations]. **[JEE '95 , 10]**

Q.3 A cylindrical tank 1 m in radius rests on a platform 5 m high . Initially the tank is filled with water to a height of 5 m . A plug whose area is 10^{-4} m² is removed from an orifice on the side of the tank at the bottom . Calculate the following :
(i) initial speed with which the water flows from the orifice ;
(ii) initial speed with which the water strikes the ground &
(iii) time taken to empty the tank to half its original value . **[REE '95 , 5]**

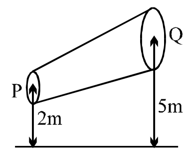
Q.4 A thin rod of length L & area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



[JEE '96 , 5]

Q.5 A large open top container of negligible mass & uniform cross–sectional area A has a small hole of cross–sectional area A/100 in its side wall near the bottom . The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0 . Assuming that the liquid starts flowing out horizontally through the hole at t = 0, calculate
(i) the acceleration of the container and
(ii) its velocity when 75 % of the liquid has drained out . **[JEE '97 , 5]**

Q.6 A nonviscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3}m^2 and 8×10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q.



[JEE '97]

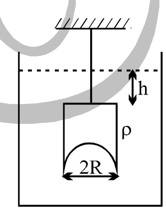
Q.7 Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross–sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross–sectional area of the stream 0.15 m below the tap is **[JEE '98, 2]**
(A) 5.0×10^{-4} m^2 (B) 1.0×10^{-5} m^2 (C) 5.0×10^{-5} m^2 (D) 2.0×10^{-5} m^2

Q.8 A wooden stick of length l, and radius R and density ρ has a small metal piece of mass *m* (of negligible volume) attached to its one end. Find the minimum value for the mass *m* (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ (>ρ). **[JEE '99 , 10]**

Q.9 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to:
(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$ (C) L (D) $\frac{L}{2\pi}$

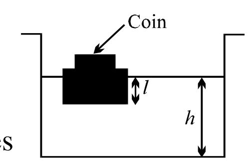
[JEE 2000 (Scr.)]

Q.10 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **[JEE 2001 (Scr.)]**



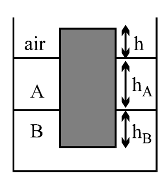
(A) Mg (B) Mg – v ρ g
(C) Mg + π R^{2}h ρ g (D) ρg (V + πR^{2}h)

Q.11 A wooden block, with a coin placed on its top, floats in water as shown in figure. The distances *l* and *h* are shown there. After some time the coin falls into the water. Then **[JEE 2002 (Scr.)]**



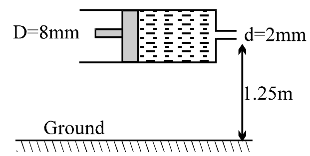
(A) *l* decreases and *h* increases (B) *l* increases and *h* decreases
(C) both *l* and *h* increase (D) both *l* and *h* decrease

Q.12 A uniform solid cylinder of density 0.8 gm/cm^3 floats in equilibrium in a combination of two non mixing liquids A and b with its axis vertical. The densities of the liquids A and B are 0.7 gm/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is h_A = 1.2 cm. The length of the part of the cylinder immersed in liquid B is h_B = 0.8 cm.



(a) Find the toal force exerted by liquid A on the cylinder.
(b) Find h, the length of the part of the cylinder in air.
(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. **[JEE 2002]**

Q.13 Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take g = 10 m/s2. **[JEE 2004]**



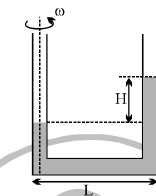
Q.14 A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[JEE 2004]**

Q.15 Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A}=0.1$ then v^2 is (where v is the velocity of water coming out of the hole)

(A) 48 (B) 51 (C) 50 (D) 51.5

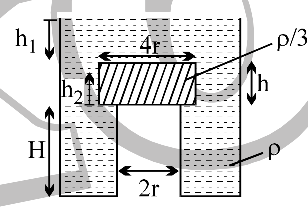
[JEE' 2005 (Scr)]

Q.16 A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L. **[JEE 2005]**



Comprehension –I

A wooden cylinder of diameter 4r, height h and density $\rho/3$ is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



Q.17 If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$ (C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$ **[JEE 2006]**

Q.18 Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$ (C) $\frac{2h}{3}$ (D) h **[JEE 2006]**

Q.19 If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position.

(B) for $h_2 = h/3$, cylinder again starts moving up

(C) for $h_2 = h/4$, cylinder again starts moving up

(D) for $h_2 = h/5$ cylinder again starts moving up **[JEE 2006]**

ANSWER KEY

EXERCISE # I

Q.1 $45^0, 9600\sqrt{2}$ (gauge)N/m^2

Q.2 $h = \dfrac{2m}{\pi}$, $H = \dfrac{11}{32\pi}$ m

Q.3 19.6 m, 4 sec

Q.4 2.79 gm/cc

Q.5 at the water surface, h/2

Q.6 $h_2 = 3\,h_1$

Q.7 37.5 N

Q.8 (a) $6\sqrt{2}$ m/s, (b) $9.6\sqrt{2} \times 10^{-3}$ M^3/sec, (c) 4.6×10^4 N/m^2

Q.9 $\sqrt{3} : \sqrt{2}$

Q.10 (a) $9600\sqrt{2}$, (b) nH

Q.11 0.75

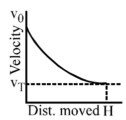
Q.12 100kg

Q.13 (a) 5, (b) 2/3

Q.14 101.8 Kgf–m

Q.15 (i) 2500 kg/m^3, $\dfrac{5000}{3}$ kg/m^3, (ii) $R_D = 7.5$ kg, $R_E = 2.5$ kg

Q.16 3/5

Q.17 

Q.18 2.5 cm

Q.19 20.4 m

Q.20 6.43×10^{-4} m^3/s

Q.21 20cm, 60cm

Q.22 $\ell(d_2 - d_1)\left(\dfrac{k}{d_2} + Ag\right)$

Q.23 $\omega = \dfrac{10}{\sqrt{3}}$ rad/s, $\tan\alpha = \dfrac{5}{3}$

Q.24 54.4 cm

Q.25 4.5m

EXERCISE # II

Q.1 6N

Q.2 4m/s^2, 10%, 0, 45kPa

Q.3 $h_1 = \dfrac{2(3+\pi)}{15\pi} = 0.26$; $h_2 = \dfrac{3+\pi}{10\pi} = 0.195$

Q.4 x = 1/3

Q.5 $\dfrac{Y}{y - x + z}$

Q.6 $\rho = \dfrac{W}{\pi h^2 g\tan\alpha\left(R - \frac{1}{3}h\tan\alpha\right)}$

Q.7 (a) $\left(\dfrac{M-m}{M+m}\right)g - \dfrac{(M-m)gx}{(M+m)L}$ (b) $t = \dfrac{\pi}{2}\sqrt{\dfrac{L}{g}\left(\dfrac{M+m}{M-m}\right)}$

Q.8 $\sqrt{\dfrac{18g}{19a}}$

Q.9 $\dfrac{\pi}{80}$ m^2

Q.10 $h_1 = h_2$

Q.11 $80\sqrt{5}$ sec, $40\sqrt{5}$ sec

Q.12 431 sec

Q.13 (i) 4m/s, (ii) F = 7.2N, (iii) $F_{min} = 0$, $F_{max} = 52.2$ N, (iv) both 4×10^{-3} m/s

Q.14 (a) $\frac{d^2H^2S}{2\rho g}$ = 32 Kgf – m , (b) $\frac{1}{2}$ $SH^2(1-d)^2$ = 2 Kgf –m

Q.15 $\frac{163}{36}$ cm, $\frac{388}{36}$ cm   Q.16 $t = \frac{12\sqrt{14}}{\sqrt{15}-\sqrt{14}}$ Q.17 $\frac{\rho g R}{2}$

Q.18 2 sec., 1 sec   Q.19 b/3   Q.20 $h = \frac{mg + 4sa}{\rho_w a^2 g}$

EXERCISE # III

Q.1 500 Pa

Q.2 (a)(i) $D = \frac{5}{4}d$, (ii) $p = P_0 + \frac{1}{4}(6H+L)dg$; (b)(i) $v = \sqrt{\frac{g}{2}(3H-4h)}$, (ii) $x = \sqrt{h(3H-4h)}$ (iii) $x_{max} = \frac{3}{4}H$

Q.3 (i) 10 m/s, (ii) 14.1 m/s, (iii) 2.5 hr   Q.4 $w = \sqrt{\frac{3g}{2L}\left(\frac{d_2-d_1}{d_1}\right)}$

Q.5 (i) 0.2 m/s^2, (ii) $\sqrt{2g\frac{m_0}{A\rho}}$ Q.6 + 29625 J/m^3, – 30000 J/m^3

Q.7 C

Q.8 $m_{min} = \pi r^2 l\,(\sqrt{\rho\sigma} - \rho)$; if tilted then it's axis should become vertical. C.M. should be lower than centre of bouyancy.   Q.9 A   Q.10 D

Q.11 D   Q.12 (a) 0, (b) h = 0.25 cm, (c) a= g/6 (upward)

Q.13 x = 2 m   Q.14 $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$ Q.15 C   Q.16 $H = \frac{L^2\omega^2}{2g}$

Q.17 C   Q.18 B   Q.19 A