KEY CONCEPTS

THINGS TO REMEMBER :

RESULT – 1

(i) ***SAMPLE–SPACE*** **:** The set of all possible outcomes of an experiment is called the **SAMPLE–SPACE(S)**.

(ii) ***EVENT*** **:** A sub set of sample–space is called an **EVENT**.

(iii) ***COMPLEMENT OF AN EVENT A*** **:** The set of all out comes which are in S but not in A is called the **COMPLEMENT OF THE EVENT A DENOTED BY** $\overline{A}$ **OR** A^c.

(iv) ***COMPOUND EVENT*** **:** If A & B are two given events then $A \cap B$ is called **COMPOUND EVENT** and is denoted by $A \cap B$ or AB or A & B .

(v) ***MUTUALLY EXCLUSIVE EVENTS*** **:** Two events are said to be **MUTUALLY EXCLUSIVE** (or disjoint or incompatible) if the occurence of one precludes (rules out) the simultaneous occurence of the other . If A & B are two mutually exclusive events then P (A & B) = 0.

(vi) ***EQUALLY LIKELY EVENTS*** **:** Events are said to be **EQUALLY LIKELY** when each event is as likely to occur as any other event.

(vii) ***EXHAUSTIVE EVENTS*** **:** Events A,B,C .... L are said to be **EXHAUSTIVE EVENTS** if no event outside this set can result as an outcome of an experiment . For example, if A & B are two events defined on a sample space S, then A & B are exhaustive $\Rightarrow A \cup B = S \Rightarrow P(A \cup B) = 1$.

(viii) ***CLASSICAL DEF. OF PROBABILITY*** **:** If n represents the total number of equally likely, mutually exclusive and exhaustive outcomes of an experiment and m of them are favourable to the happening of the event A, then the probability of happening of the event A is given by P(A) = m/n .

Note : **(1)** $0 \le P(A) \le 1$

(2) $P(A) + P(\overline{A}) = 1$, Where $\overline{A}$ = Not A .

(3) If x cases are favourable to A & y cases are favourable to $\overline{A}$ then $P(A) = \frac{x}{(x+y)}$ and $P(\overline{A}) = \frac{y}{(x+y)}$ We say that **ODDS IN FAVOUR OF A** are x: y & odds against A are y : x

Comparative study of Equally likely , Mutually Exclusive and Exhaustive events.

	Experiment	Events	E/L	M/E	Exhaustive
1.	Throwing of a die	A : throwing an odd face {1, 3, 5} B : throwing a composite face {4,. 6}	No	Yes	No
2.	A ball is drawn from an urn containing 2W, 3R and 4G balls	E_1 : getting a W ball E_2 : getting a R ball E_3 : getting a G ball	No	Yes	Yes
3.	Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more {46, 64, 55, 56, 65, 66}	Yes	No	No
4.	From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	Yes	Yes	Yes
5.	From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	No	No	No

RESULT – 2

AUB = A+ B = A or B denotes occurence of at least A or B. For 2 events A & B : **(See fig.1)**

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$P(A.\overline{B}) + P(\overline{A}.B) + P(A.B) = 1 - P(\overline{A}.\overline{B})$

(ii) Opposite of′ "atleast A or B" is **NIETHER A NOR B**

i.e. $\overline{A+B} = 1\text{-}(A \text{ or } B) = \overline{A} \cap \overline{B}$

Note that $P(A+B) + P(\overline{A} \cap \overline{B}) = 1$.

(iii) If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

(iv) For any two events A & B, P(exactly one of A , B occurs)

$$= P(A \cap \overline{B}) + P(B \cap \overline{A}) = P(A) + P(B) - 2P(A \cap B)$$
$$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$$

(v) If A & B are any two events $P(A \cap B) = P(A).P(B/A) = P(B).P(A/B)$, Where $P(B/A)$ means conditional probability of B given A & $P(A/B)$ means conditional probability of A given B. (This can be easily seen from the figure)

(vi) **De Morgan's Law** : – If A & B are two subsets of a universal set U , then
(a) $(A \cup B)^c = A^c \cap B^c$ & **(b)** $(A \cap B)^c = A^c \cup B^c$

(vii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

RESULT – 3

For any three events A,B and C we have **(See Fig. 2)**

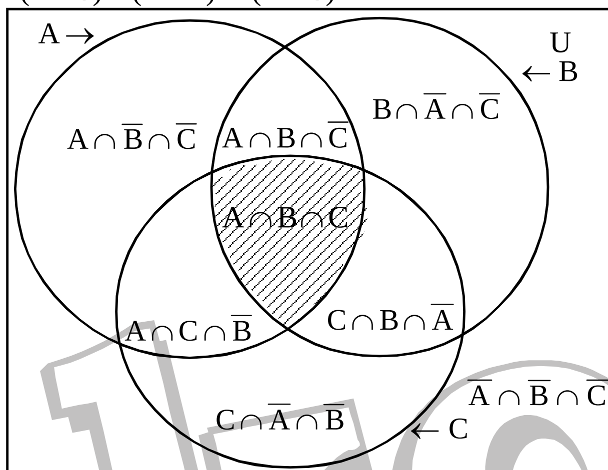

Fig. 2

(i) P(A or B or C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A,B,C occurs) = $P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

Note :
If three events A, B and C are pair wise mutually exclusive then they must be mutually exclusive. i.e $P(A \cap B) = P(B \cap C) = P(C \cap A) = 0 \Rightarrow P(A \cap B \cap C) = 0$. However the converse of this is not true.

RESULT – 4

Independent Events **:** Two events A & B are said to be independent if occurence or non occurence of one does not effect the probability of the occurence or non occurence of other.

(i) If the occurence of one event affects the probability of the occurence of the other event then the events are said to be **Dependent** or **Contingent** . For two independent events A and B : $P(A \cap B) = P(A). P(B)$. Often this is taken as the definition of independent events.

(ii) Three events A , B & C are independent if & only if all the following conditions hold ;
$P(A \cap B) = P(A) . P(B)$; $P(B \cap C) = P(B) . P(C)$
$P(C \cap A) = P(C) . P(A)$ & $P(A \cap B \cap C) = P(A) . P(B) . P(C)$
i.e. they must be pairwise as well as mutually independent .
Similarly for n events $A_1, A_2, A_3, ... A_n$ to be independent , the number of these conditions is equal to ${}^nc_2 + {}^nc_3 + ... + {}^nc_n = 2^n - n - 1$.

(iii) The probability of getting exactly r success in n independent trials is given by
$P(r) = {}^nC_r\, p^r q^{n-r}$ where : p = probability of success in a single trial .
q = probability of failure in a single trial. note : p + q = 1 .
Note : Independent events are not in general mutually exclusive & vice versa.
Mutually exclusiveness can be used when the events are taken from the same experiment & independence can be used when the events are taken from different experiments .

RESULT – 5 : Baye's Theorem Or Total Probability Theorem :

If an event A can occur only with one of the n mutually exclusive and exhaustive events $B_1, B_2, ... B_n$ & the probabilities $P(A/B_1), P(A/B_2) .... P(A/B_n)$ are known then,

$$P(B_1/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

PROOF :

The events A occurs with one of the n mutually exclusive & exhaustive events $B_1, B_2, B_3, ... B_n$

$A = AB_1 + AB_2 + AB_3 + ... + AB_n$

$$P(A) = P(AB_1) + P(AB_2) + ... + P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

NOTE : A ≡ event what we have ; B_1 ≡ event what we want ;
$B_2, B_3, ...B_n$ are alternative event .

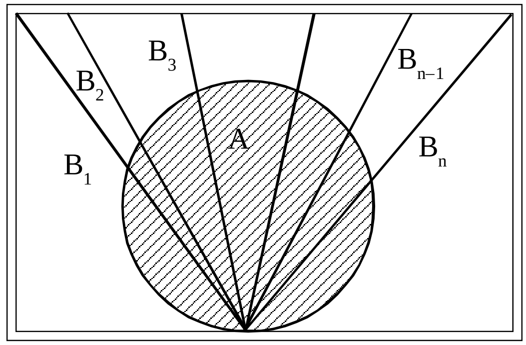

Fig. 3

Now,

$P(AB_i) = P(A) . P(B_i/A) = P(B_i) . P(A/B_i)$

$$P(B_i / A) = \frac{P(B_i) . P(A/B_i)}{P(A)} = \frac{P(B_i) . P(A/B_i)}{\sum_{i=1}^{n} P(AB_i)}$$

$$\boxed{P(B_i / A) = \frac{P(B_i) . P(A/B_i)}{\sum P(B_i) . P(A/B_i)}}$$

RESULT – 6

If p_1 and p_2 are the probabilities of speaking the truth of two indenpendent witnesses A and B then

$$P \text{ (their combined statement is true)} = \frac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)} .$$

In this case it has been assumed that we have no knowledge of the event except the statement made by A and B.

However if p is the probability of the happening of the event before their statement then

$$P \text{ (their combined statement is true)} = \frac{p\, p_1 p_2}{p\, p_1 p_2 + (1-p)(1-p_1)(1-p_2)} .$$

Here it has been assumed that the statement given by all the independent witnesses can be given in two ways only, so that if all the witnesses tell falsehoods they agree in telling the same falsehood.

If this is not the case and c is the chance of their coincidence testimony then the

Pr. that the statement is true = $P\, p_1 p_2$

Pr. that the statement is false = $(1-p).c\,(1-p_1)(1-p_2)$

However chance of coincidence testimony is taken only if the joint statement is not contradicted by any witness.

RESULT – 7

(i) A **PROBABILITY DISTRIBUTION** spells out how a total probability of 1 is distributed over several values of a random variable .

(ii) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\sum p_i x_i}{\sum p_i} = \sum p_i x_i \quad (\text{Since } \Sigma p_i = 1)$$

(iii) Variance of a random variable is given by, $\sigma^2 = \sum (x_i - \mu)^2 . p_i$

$$\sigma^2 = \sum p_i x_i^2 - \mu^2 \qquad (\text{Note that SD} = +\sqrt{\sigma^2})$$

(iv) The probability distribution for a binomial variate 'X' is given by ;

$P(X = r) = {}^nC_r p^r q^{n-r}$ where all symbols have the same meaning as given in result 4.

The recurrence formula $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} . \frac{p}{q}$, is very helpful for quickly computing

P(1) , P(2). P(3) etc. if P(0) is known .

(v) Mean of BPD = np ; variance of BPD = npq .

(vi) If p represents a persons chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

expectations = pM

RESULT – 8 : GEOMETRICAL APPLICATIONS :

The following statements are axiomatic :

(i) If a point is taken at random on a given staright line AB, the chance that it falls on a particular segment PQ of the line is PQ/AB .

(ii) If a point is taken at random on the area S which includes an area σ, the chance that the point falls on σ is σ/S .

EXERCISE-1

Q.1 Let a die be weighted so that the probability of a number appearing when the die is tossed is proportional to that number. Find the probability that,
(i) An even or a prime number appears (ii) An odd prime number appears
(iii) An even composite number appears (iv) An odd composite number appears.

Q.2 Numbers are selected at random , one at a time, from the two digit numbers 00, 01, 02, ... , 99 with replacement. An event E occurs if & only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

Q.3 In a box , there are 8 alphabets cards with the letters : S, S, A, A,A, H, H, H . Find the probability that the word 'ASH' will form if :
(i) the three cards are drawn one by one & placed on the table in the same order that they are drawn.
(ii) the three cards are drawn simultaneously .

Q.4 There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects . An unbiased die is cast . If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd} group . Find the probability that an engg. subject is selected.

Q.5 A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

Q.6 In a building programme the event that all the materials will be delivered at the correct time is M, and the event that the building programme will be completed on time is F . Given that P (M) = 0.8 and P (M $\cap$ F) = 0.65, find P (F/M) . If P (F) = 0.7, find the probability that the building programme will be completed on time if all the materials are not delivered at the correct time .

Q.7 In a given race, the odds in favour of four horses A, B, C & D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

Q.8 A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

Q.9 Least number of times must a fair die be tossed in order to have a probability of at least 91/216, of getting atleast one six.

Q.10 Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a **"BEST OF THREE GAMES"** or a **"BEST OF 5 GAMES"** match against B , which option should A choose so that the probability of his winning the match is higher? (No game ends in a draw).

Q.11 A room has three electric lamps . From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.

Q.12 A bomber wants to destroy a bridge . Two bombs are sufficient to destroy it.
If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.

Q.13 The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd} . Find the chances of each. (assume that both events are neither sure nor impossible).

Q.14 A box contains 5 radio tubes of which 2 are defective . The tubes are tested one after the other until the 2 defective tubes are discovered . Find the probability that the process stopped on the
(i) Second test; **(ii)** Third test. If the process stopped on the third test , find the probability that the first tube is non defective.

Q.15 Anand plays with Karpov 3 games of chess. The probability that he wins a game is 0.5, looses with probability 0.3 and ties with probability 0.2. If he plays 3 games then find the probability that he wins atleast two games.

Q.16 An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane .

Q.17 In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective , the whole batch is rejected Find the probability that the batch will be rejected.

Q.18 Given P(A$\cup$B) = 5/6 ; P(AB) = 1/3 ; P($\overline{B}$) = 1/2. Determine P(A) & P(B). Hence show that the events A & B are independent.

Q.19 One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world and 30 read Business Today. Five students read all the three magazines. A student was selected randomly. Find the probability that he reads exactly two magazines.

Q.20 An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

Q.21 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, Which one is more likely to solve the puzzle correctly.

Q.22 A uniform unbised die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(i) exactly 6 on each of 3 successive throws.

(ii) more than 4 on at least one of the three successive throws.

Q.23 Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that one of them is a red card & the other is a queen.

Q.24 A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

Q.25 Consider the following events for a family with children

A = {of both sexes} ; B = {at most one boy}

In which of the following (are/is) the events A and B are independent.

(a) if a family has 3 children (b) if a family has 2 children

Assume that the birth of a boy or a girl is equally likely mutually exclusive and exhaustive.

Q.26 A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}) \qquad n \geq 3$$

Also compute P_1 and P_2 and hence deduce the pr that he scores exactly 4.

Q.27 Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being p_o. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

Q.28 The difference between the mean & variance of a Binomial Variate 'X' is unity & the difference of their square is 11. Find the probability distribution of 'X'.

Q.29 An examination consists of 8 questions in each of which the candidate must say which one of the 5 alternatives is correct one. Assuming that the student has not prepared earlier chooses for each of the question any one of 5 answers with equal probability.

(i) prove that the probability that he gets more than one correct answer is $(5^8 - 3 \times 4^8)/5^8$.

(ii) find the probability that he gets correct answers to six or more questions.

(iii) find the standard deviation of this distribution.

Q.30 Two bad eggs are accidently mixed with ten good ones. Three eggs are drawn at random without replacement, from this lot. Compute mean & S.D. for the number of bad eggs drawn.

EXERCISE-2

Q.1 The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)

(a) Find the probability that they all : **(i)** hit the target ; **(ii)** miss the target

(b) Find the probability that the target is hit : **(i)** at least once, **(ii)** exactly once.

(c) If only one hits the target, what is the probability that it was the first man?

Q.2 Let A & B be two events defined on a sample space . Given P(A) = 0.4 ; P(B) = 0.80 and $P\left(\overline{A}/\overline{B}\right) = 0.10$. Then find ; (i) $P\left(\overline{A} \cup B\right)$ & $P\left[\left(\overline{A} \cap B\right) \cup \left(A \cap \overline{B}\right)\right]$.

Q.3 Three shots are fired independently at a target in succession. The probabilities that the target is hit in the first shot is 1/2, in the second 2/3 and in the third shot is 3/4. In case of exactly one hit, the probability of destroying the target is 1/3 and in the case of exactly two hits, 7/11 and in the case of three hits is

1.0. Find the probability of destroying the target in three shots.

Q.4 In a game of chance each player throws two unbiased dice and scores the difference between the larger and smaller number which arise. Two players compete and one or the other wins if and only if he scores atleast 4 more than his opponent. Find the probability that neither player wins.

Q.5 A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "THE DRUG IS TOXIC" be denoted by H & the event "THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC" be denoted by S. Let $P(H) = a$, $P(S/H) = P(\overline{S}/\overline{H}) = 1 - a$. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic, is free from 'a'.

Q.6 A plane is landing. If the weather is favourable, the pilot landing the plane can see the runway. In this case the probability of a safe landing is p_1. If there is a low cloud ceiling, the pilot has to make a blind landing by instruments. The reliability (the probability of failure free functioning) of the instruments needed for a blind landing is P. If the blind landing instruments function normally, the plane makes a safe landing with the same probability p_1 as in the case of a visual landing. If the blind landing instruments fail, then the pilot may make a safe landing with probability $p_2 < p_1$. Compute the probability of a safe landing if it is known that in K percent of the cases there is a low cloud ceiling. Also find the probability that the pilot used the blind landing instrument, if the plane landed safely.

Q.7 A train consists of n carriages, each of which may have a defect with probability p. All the carriages are inspected, independently of one another, by two inspectors; the first detects defects (if any) with probability p_1, & the second with probability p_2. If none of the carriages is found to have a defect, the train departs. Find the probability of the event; "THE TRAIN DEPARTS WITH ATLEAST ONE DEFECTIVE CARRIAGE".

Q.8 A is a set containing n distinct elements. A non-zero subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A non-zero subset Q of A is again chosen at random. Find the probability that P & Q have no common elements.

Q.9 In a multiple choice question there are five alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks the correct answers. The candidate ticks the answers at random. If the probability of the candidate getting marks on the question is to be greater than or equal to 1/3 find the least number of chances he should be allowed.

Q.10 n people are asked a question successively in a random order & exactly 2 of the n people know the answer :

(a) If $n > 5$, find the probability that the first four of those asked do not know the answer.

(b) Show that the probability that the r^{th} person asked is the first person to know the answer is : $\left[\frac{2(n-r)}{n(n-1)}\right]$, if $1 < r < n$.

Q.11 A box contains three coins two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it is two headed coin.

(iii) Find the probability that tail appears twice.

Q.12 The ratio of the number of trucks along a highway, on which a petrol pump is located, to the number of cars running along the same highway is 3 : 2. It is known that an average of one truck in thirty trucks and two cars in fifty cars stop at the petrol pump to be filled up with the fuel. If a vehicle stops at the petrol pump to be filled up with the fuel, find the probability that it is a car.

Q.13 A batch of fifty radio sets was purchased from three different companies A, B and C. Eighteen of them were manufactured by A, twenty of them by B and the rest were manufactured by C.

The companies A and C produce excellent quality radio sets with probability equal to 0.9 ; B produces the same with the probability equal to 0.6.

What is the probability of the event that the excellent quality radio set chosen at random is manufactured by the company B?

Q.14 The contents of three urns 1, 2 & 3 are as follows :

1 W,	2 R,	3B balls
2 W,	3 R,	1B balls
3 W,	1 R,	2B balls

An urn is chosen at random & from it two balls are drawn at random & are found to be "**1 RED & 1 WHITE**". Find the probability that they came from the 2^{nd} urn.

Q.15 Suppose that there are 5 red points and 4 blue points on a circle. Let $\frac{m}{n}$ be the probability that a convex polygon whose vertices are among the 9 points has at least one blue vertex where m and n are relatively prime. Find $(m + n)$.

Q.16 There are 6 red balls & 8 green balls in a bag . 5 balls are drawn out at random & placed in a red box ; the remaining 9 balls are put in a green box . What is the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number?

Q.17 Two cards are randomly drawn from a well shuffled pack of 52 playing cards, without replacement. Let x be the first number and y be the second number.
Suppose that Ace is denoted by the number 1; Jack is denoted by the number 11 ; Queen is denoted by the number 12 ; King is denoted by the number 13.
Find the probability that x and y satisfy $\log_3(x + y) - \log_3 x - \log_3 y + 1 = 0$.

Q.18(a) Two numbers x & y are chosen at random from the set {1,2,3,4,....3n}. Find the probability that $x^2 - y^2$ is divisible by 3 .

(b) If two whole numbers x and y are randomly selected, then find the probability that $x^3 + y^3$ is divisible by 8.

Q.19 A hunter's chance of shooting an animal at a distance r is $\frac{a^2}{r^2}$ (r > a) . He fires when r = 2a & if he misses he reloads & fires when r = 3a, 4a, ... If he misses at a distance 'na', the animal escapes. Find the odds against the hunter.

Q.20 An unbiased normal coin is tossed 'n' times. Let :
E_1 : event that both **Heads and Tails** are present in 'n' tosses.
E_2 : event that the coin shows up **Heads** atmost once.
Find the value of 'n' for which E_1 & E_2 are independent.

Q.21 A coin is tossed (m + n) times (m>n). Show that the probability of at least m consecutive heads is $\frac{n+2}{2^{m+1}}$

Q.22 There are two lots of identical articles with different amount of standard and defective articles. There are N articles in the first lot, n of which are defective and M articles in the second lot, m of which are defective. K articles are selected from the first lot and L articles from the second and a new lot results. Find the probability that an article selected at random from the new lot is defective.

Q.23 *m* red socks and *n* blue socks (m > n) in a cupboard are well mixed up, where $m + n \le 101$. If two socks are taken out at random, the chance that they have the same colour is 1/2. Find the largest value of *m*.

Q.24 With respect to a particular question on a multiple choice test (having 4 alternatives with only 1 correct) a student knows the answer and therefore can eliminate 3 of the 4 choices from consideration with probability 2/3, can eliminate 2 of the 4 choices from consideration with probability 1/6, can eliminate 1 choice from consideration with probability 1/9, and can eliminate none with probability 1/18. If the student knows the answer, he answers correctly, otherwise he guesses from among the choices not eliminated.
If the answer given by the student was found correct, then the probability that he knew the answer is $\frac{a}{b}$ where a and b are relatively prime. Find the value of (a + b).

Q.25 In a knockout tournament 2^n equally skilled players; $S_1, S_2, ... S_{2^n}$ are participating. In each round players are divided in pairs at random & winner from each pair moves in the next round. If S_2 reaches the semifinal then find the probability that S_1 wins the tournament.

EXERCISE-3

Q.1 If p & q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. [JEE '97, 5]

Q.2 There is 30% chance that it rains on any particular day . What is the probability that there is at least one rainy day within a period of 7 – days ? Given that there is at least one rainy day, what is the probability that there are at least two rainy days ? [REE '97, 6]

Q.3 Select the correct alternative(s) . [JEE '98, 6 × 2 = 12]

(i) 7 white balls & 3 black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals :
(A) 1/2 (B) 7/15 (C) 2/15 (D) 1/3

(ii) If from each of the 3 boxes containing 3 white & 1 black, 2 white & 2 black, 1 white & 3 black balls, one ball is drawn at random, then the probability that 2 white & 1 black ball will be drawn is :
(A) 13/32 (B) 1/4 (C) 1/32 (D) 3/16

(iii) If $\overline{E}$ & $\overline{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then :

(A) $P(E|F) + P(\overline{E}|F) = 1$ (B) $P(E|F) + P(E|\overline{F}) = 1$
(C) $P(\overline{E}|F) + P(E|\overline{F}) = 1$ (D) $P(E|\overline{F}) + P(\overline{E}|\overline{F}) = 1$

(iv) There are 4 machines & it is known that exactly 2 of them are faulty . They are tested, one by one, in a random order till both the faulty machines are identified . Then the probability that only 2 tests are needed is :
(A) 1/3 (B) 1/6 (C) 1/2 (D) 1/4

(v) If E & F are events with $P(E) \le P(F)$ & $P(E \cap F) > 0$, then :
(A) occurrence of E $\Rightarrow$ occurrence of F
(B) occurrence of F $\Rightarrow$ occurrence of E
(C) non – occurrence of E $\Rightarrow$ non – occurrence of F
(D) none of the above implications holds.

(vi) A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :
(A) 1/2 (B) 1/32 (C) 31/32 (D) 1/5

Q.4 3 players A, B & C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows . Let p be the probability that the coin shows a head. Let α, β & γ be respectively the probabilities that A, B and C gets the first head . Prove that
$\beta = (1 - p)\alpha$. Determine α, β & γ (in terms of p). [JEE '98, 8]

Q.5 Each co–efficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die . Find the probability that the equation will have equal roots. [REE '98, 6]

Q.6(a) If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals
(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

(b) The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two, which of the following relations are true?
(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$ (C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

(c) Eight players P_1, P_2, P_3, ...P_8 play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. [JEE ' 99, 2 + 3 + 10 (out of 200)]

Q.7 Four cards are drawn from a pack of 52 playing cards. Find the probability (correct upto two places of decimals) of drawing exactly one pair. [REE'99, 6]

Q.8 A coin has probability ' p ' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that,
$$p_1 = 1,\ p_2 = 1 - p^2\ \&\ p_n = (1 - p)\, p_{n-1} + p\,(1 - p)\, p_{n-2},\ \text{ for all } n \ge 3.$$

Q.9 A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately.

Q.10 Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. [REE ' 2001 (Mains), 3]

Q.11(a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. [JEE ' 2001 (Mains), 5 + 5]

Q.12 A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? [JEE ' 2002 (mains)]

Q.13(a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3 , 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. Find the probability that B hits his target whereas C does not. [JEE' 2003, Mains-2 + 2 out of 60]

Q.14(a) Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(A) $\frac{4}{25}$ (B) $\frac{4}{35}$ (C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

(b) If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A or B occurs.

(c) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^{n}C_{r}$). [JEE 2004, 3 + 2 + 4]

Q.15(a) A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even number of trials is

(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

(b) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. [JEE 2005 (Mains), 2]

Comprehension (3 questions)

There are *n* urns each containing *n* + 1 balls such that the i^{th} urn contains *i* white balls and (n + 1 – i) red balls. Let u_i be the event of selecting i^{th} urn, *i* = 1, 2, 3, ..., *n* and *w* denotes the event of getting a white ball.

Q.16(a) If $P(u_i) \propto i$ where i = 1, 2, 3,....., n then $\lim_{n\to\infty} P(w)$ is equal to

(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

(b) If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If *n* is even and E denotes the event of choosing even numbered urn ($P(u_i) = \frac{1}{n}$), then the value of $P(w/E)$, is

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (B) $\frac{1}{n+1}$

[JEE 2006, 5 marks each]

ANSWER KEY

EXERCISE-1

Q 1. (i) $\frac{20}{21}$ (ii) $\frac{8}{21}$ (iii) $\frac{10}{21}$ (iv) 0 **Q 2.** $97/(25)^4$ **Q 3.** **(i)** 3/56 **(ii)** 9/28

Q 4. 13/24 **Q 5.** 5/9 **Q 6.** $P(F/M) = \frac{13}{16}$; $P(F/\overline{M}) = \frac{1}{4}$

Q 7. 319/420 **Q 8.** 120 **Q 9.** 3

Q 10. best of 3 games **Q 11.** $\frac{29}{30}$ **Q 12.** $\frac{328}{625}$ **Q 13.** $\frac{1}{9}, \frac{1}{3}$

Q 14. (i) 1/10, **(ii)** 3/10, **(iii)** 2/3 **Q 15.** 1/2 **Q 16.** 0.6976

Q 17. 19/42 **Q 18.** P(A) = 2/3, P(B) = 1/2 **Q 19.** 1/2

Q 20. 407/576 **Q 21.** Both are equally likely **Q 22.** (i) $\frac{125}{16^3}$; (ii) $\frac{63}{64}$

Q 23. 101/1326 **Q 24.** $\frac{{}^{24}C_2}{{}^{64}C_2}$ or $\frac{23}{168}$

Q25. Independent in (a) and not independent in (b) **Q 26.** $P_1 = 1/2$, $P_2 = 3/4$

Q 27. $(1-p)^{n-1} \cdot [\, p_o(1-p) + np(1-p_0)]$ **Q 28.** $\left(\frac{5}{6}+\frac{1}{6}\right)^{36}$

Q 29. $\frac{481}{5^8}, \frac{4\sqrt{2}}{5}$ **Q 30.** mean = 0.5

EXERCISE-2

Q 1. (a) 6%, 21% ; (b) 79%, 44%, (c) 9/44 ≈ 20.45% **Q 2. (i)** 0.82, **(ii)** 0.76 **Q 3.** $\frac{5}{8}$

Q 4. 74/81 **Q 5.** $P\left(\overline{H}/S\right) = 1/2$

Q6. $P(E) = (1-\frac{K}{100})p_1 + \frac{K}{100}[P\,p_1 + (1-P)\,p_2]$; $P(H_2/A) = \dfrac{\frac{K}{100}[P\,p_1+(1-P)\,p_2]}{\left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}[P\,p_1+(1-P)\,p_2]}$

Q 7. $1 - [\,1-p\,(1-p_1)\,(1-p_2)]^n$ **Q 8.** $(3^n-2^{n+1}+1)/(4^n-2^{n+1}+1)$ **Q 9.** 11

Q 10. (a) $\frac{(n-4)\,(n-5)}{n((n-1)}$ **Q 11.** 1/2, 1/2, 1/12 **Q 12.** $\frac{4}{9}$

Q.13 $\frac{4}{13}$ **Q 14.** 6/11 **Q 15.** 458

Q 16. 213/1001 **Q.17** $\frac{11}{663}$ **Q 18. (a)** $\frac{(5n-3)}{(9n-3)}$ **(b)** $\frac{5}{16}$

Q 19. n+1 : n–1 **Q 20.** n = 3 **Q 22.** $\frac{K\,n\,M + L\,m\,N}{M\,N\,(K+L)}$

Q.23 55 **Q.24** 317 **Q.25** $\frac{3}{4\left(2^n-1\right)}$

EXERCISE-3

Q.1 31/50 **Q.2** $[1-(7/10)^7 - {}^7C_1\,(3/10)\,(7/10)^6]\,/\,1-(7/10)^7$

Q.3 (i) B **(ii)** A **(iii)** A, D **(iv)** A **(v)** D **(vi)** A

Q.4 $\alpha = \frac{p}{1-(1-p)^3}$, $\beta = \frac{(1-p)p}{1-(1-p)^3}$, $\gamma = \frac{(1-p)^2 p}{1-(1-p)^3}$ **Q.5** 5/216

Q.6 (a) A **(b)** B, C **(c)** 4/35 **Q.7** 0.31 **Q.9** $\frac{1}{2}$ & $\frac{1}{3}$ or $\frac{1}{3}$ & $\frac{1}{2}$

Q.10 $\frac{1}{26}$ **Q.11 (a)** $\frac{m}{m+n}$; **(b)** $\frac{{}^6C_3\left(3^n - 3.2^n + 3\right)}{6^n}$ **Q.12** $\frac{9m}{m+8N}$

Q.13 (a) $p^2\,(2-p)$; (b) 1/2 **Q.14** **(a)** D , **(c)** $\frac{{}^{12}C_2\,{}^6C_4\,{}^{10}C_1\,{}^2C_1 + {}^{12}C_1\,{}^6C_5\,{}^{11}C_1\,{}^1C_1}{{}^{12}C_2\left({}^{12}C_2\,{}^6C_4 + {}^{12}C_1\,{}^6C_5 + {}^{12}C_0\,{}^6C_6\right)}$

Q.15 **(a)** A, **(b)** $\frac{1}{7}$ **Q.16** (a) B, (b) A, (c) B

EXERCISE-4

Part : (A) Only one correct option

1. If A, B, C are 3 events, then the probability that exactly 2 of them occur is given by:
(A) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
(B) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
(C) $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
(D) none of these

2. In a series of 3 independent trials the probability of exactly 2 success is 12 times as large as the probability of 3 successes. The probability of a success in each trial is:
(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

3. There are two urns. There are m white & n black balls in the first urn and p white & q black balls in the second urn. One ball is taken from the first urn & placed into the second. Now, the probability of drawing a white ball from the second urn is:
(A) $\frac{pm + (p+1)n}{(m+n)(p+q+1)}$ (B) $\frac{(p+1)m + pn}{(m+n)(p+q+1)}$ (C) $\frac{qm + (q+1)n}{(m+n)(p+q+1)}$ (D) $\frac{(q+1)m + qn}{(m+n)(p+q+1)}$

4. Box–I contains 3 red and 2 blue balls whilst box-II contains 2 red and 6 blue balls. A fair coin is tossed. If it turns up head, a ball is drawn from Box-I, else a ball is drawn from Box–II. Find the probabiliy of event 'ball drawn is from Box-I, if it is red'.
(A) $\frac{12}{17}$ (B) $\frac{10}{17}$ (C) $\frac{17}{20}$ (D) $\frac{3}{5}$

5. A local post office is to send M telegrams which are distributed at random over N communication channels, (N > M). Each telegram is sent over any channel with equal probability. Chance that not more than one telegram will be sent over each channel is:
(A) $\frac{{}^{N}C_{M} \,.\, M!}{N^{M}}$ (B) $\frac{{}^{N}C_{M} \,.\, N!}{M^{N}}$ (C) $1 - \frac{{}^{N}C_{M} \,.\, M!}{M^{N}}$ (D) $1 - \frac{{}^{N}C_{M} \,.\, N!}{N^{M}}$

6. A mapping is selected at random from all the mappings defined on the set A consisting of three distinct elements. The probability that the mapping selected is one to one is:
(A) 1/9 (B) 1/3 (C) 1/4 (D) 2/9

7. A bag contains 7 tickets marked with the numbers 0, 1, 2, 3, 4, 5, 6 respectively. A ticket is drawn & replaced. Then the chance that after 4 drawings the sum of the numbers drawn is 8 is:
(A) 165/2401 (B) 149/2401 (C) 3/49 (D) none

8. A biased coin with probability p, 0 < p < 1 of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals
(A) 1/3 (B) 2/3 (C) 2/5 (D) 3/5

9. If 4 whole numbers taken at random are multiplied together, then the chance that the last digit in the product is 1, 3, 7 or 9 is:
(A) 16/625 (B) 4/125 (C) 8/81 (D) none

10. A letter is known to have come either from "KRISHNAGIRI" or "DHARMAPURI". On the post mark only the two consecutive letters "RI" are visible. Then the chance that it came from Krishnagiri is:
(A) 3/5 (B) 2/3 (C) 9/14 (D) none

11. If $\frac{(1+3p)}{3}, \frac{(1-p)}{4} \,\&\, \frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events then the set of all values of p is.
(A) $\left[\frac{1}{2}, \frac{2}{3}\right]$ (B) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (C) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (D) $\left[\frac{1}{3}, \frac{2}{3}\right]$

12. Let p be the probability that a man aged x years will die in a year time. The probability that out of 'n' men $A_1, A_2, A_3, \ldots\ldots, A_n$ each aged 'x' years. A_1 will die & will be the first to die is:
(A) $\frac{1-p^n}{n}$ (B) $\frac{p}{n}$ (C) $\frac{p\,(1-p)^{n-1}}{n}$ (D) $\frac{1-(1-p)^n}{n}$

13. 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15, then the probability that end seats are occupied by the girls and between any two girls an odd number of boys sit is:
(A) $\frac{20 \times 10! \times 5!}{15!}$ (B) $\frac{10 \times 10! \times 5!}{15!}$ (C) $\frac{20 \times 10! \times 30}{15!}$ (D) $\frac{10 \times 10! \times 5!}{25!}$

14. Two dice are rolled simultaneously. The probability that the sum of the two numbers on the top faces will be atleast 10 is:
(A) 1/6 (B) 1/12 (C) 1/18 (D) none

15. There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i = 1, 2, 3, 4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is :

(A) $\frac{569}{1496}$ (B) $\frac{27}{56}$ (C) $\frac{8}{73}$ (D) none of these

16. 2/3[rd] of the students in a class are boys & the rest girls. It is known that probability of a girl getting a first class is 0.25 & that of a boy is 0.28. The probability that a student chosen at random will get a first class is:

(A) 0.26 (B) 0.265 (C) 0.27 (D) 0.275

17. The contents of urn I and II are as follows,

Urn I: 4 white and 5 black balls

Urn II: 3 white and 6 black balls

One urn is chosen at random and a ball is drawn and its colour is noted and replaced back to the urn. Again a ball is drawn from the same urn, colour is noted and replaced. The process is repeated 4 times and as a result one ball of white colour and 3 of black colour are noted. Find the probability the chosen urn was I.

(A) $\frac{125}{287}$ (B) $\frac{64}{127}$ (C) $\frac{25}{287}$ (D) $\frac{79}{192}$

18. The sides of a rectangle are chosen at random, each less than 10 cm, all such lengths being equally likely. The chance that the diagonal of the rectangle is less than 10 cm is

(A) 1/10 (B) 1/20 (C) $\pi/4$ (D) $\pi/8$

19. The sum of two positive quantities is equal to 2n. The probability that their product is not less than 3/4 times their greatest product is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) none of these

20. The probability that 4[th] power of a positive integer ends in the digit 6 is:

(A) 10 % (B) 20 % (C) 25 % (D) 40 %

21. Posteriory probability of the occurrance of the event 'The coin turns head and the die turns up an odd number' is

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

22. Expectation of D is

(A) $\frac{225}{7}$ (B) $\frac{150}{7}$ (C) $\frac{200}{7}$ (D) $\frac{300}{7}$

23. For the three events A, B & C, P(exactly one of the events A or B occurs) = P(exactly one of the events B or C occurs) = P(exactly one of the events C or A occurs) = p & P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events A, B & C occurring is: **[IIT -1996]**

(A) $\frac{3p+2p^2}{2}$ (B) $\frac{p+3p^2}{4}$ (C) $\frac{p+3p^2}{2}$ (D) $\frac{3p+2p^2}{4}$

Part : (B) May have more than one options correct

24. In throwing a die let A be the event 'coming up of an odd number', B be the event 'coming up of an even number', C be the event 'coming up of a number ≥ 4' and D be the event 'coming up of a number < 3', then

(A) A and B are mutually exclusive and exhautive (B) A and C are mutually exclusive and exhautive

(C) A, C and D form an exhautive system (D) B, C and D form an exhautive system

25. Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A). P(B)$, then:

(A) $P(B/A) = P(B) - P(A)$ (B) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(C) $P((A \cup B)^c) = P(A^c). P(B^c)$ (D) $P(A/B) = P(A)$

26. For any two events A & B defined on a sample space,

(A) $P(A/B) \geq \frac{P(A)+P(B)-1}{P(B)}$, $P(B) \neq 0$ is always true

(B) $P(A \cup \overline{B}) = P(A) - P(A \cap B)$

(C) $P(A \cup B) = 1 - P(A^c). P(B^c)$, if A & B are independent

(D) $P(A \cup B) = 1 - P(A^c). P(B^c)$, if A & B are disjoint

27. If A, B & C are three events, then the probability that none of them occurs is given by:

(A) $P(\overline{A}) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

(B) $P(\overline{A}) + P(\overline{B}) + P(\overline{C})$

(C) $P(\overline{A}) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$

(D) $P(\overline{A} \cup \overline{B} \cup \overline{C}) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

28. A student appears for tests I, II & III. The student is successful if he passes either in tests I & II or tests I & III. The probabilities of the student passing in the tests I, II & III are p, q &

1/2 respectively. If the probability that the student is successful is 1/2, then:
(A) p = 1, q = 0 (B) p = 2/3, q = 1/2
(C) p = 3/5, q = 2/3 (D) there are infinitely many values of p & q.

29. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals **[IIT - 1999]**
(A) 1/4 (B) 1/7 (C) 1/8 (D) 1/49

EXERCISE-5

1. A letter is known to have come either from London or Clifton; on the postmark only the two consecutive letters ON are legible; what is the chance that it came from London?

2. A speaks the truth 3 out of 4 times, and B 5 out of 6 times; what is the probability that they will contradict each other in starting the same fact?

3. A pair of dice is thrown 5 times. Find the mean and variance of the probability distribution of appearance of doublets on the throws.

4. If on a straight line 10 cm. two length of 6 cm and 4 cm are measured at random, find the probability that their common part does not exceed 3 cms.

5. Let p be the probability that a man aged x years will die in a year time. Find the probability that out of 'n' men $A_1, A_2, A_3, \ldots\ldots, A_n$ each aged 'x' years. A_1 will die & will be the second to die.

6. A car is parked by an owner amongst 25 cars in a row, not at either end. On his return he finds that exactly 15 placed are still occupied. Find the probability that both the neighbouring places are empty.

7. A gambler has one rupee in his pocket. He tosses an unbiased normal coin unless either he is ruined or unless the coin has been tossed for a maximum of five times. If for each head he wins a rupee and for each tail he looses a rupee, then find the probability that the gambler is ruined.

8. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine and solemnly says. “French”. Find the probability that the wine he tasted was Californian.

9. In ten trials of an experiment, if the probability of getting '4 successes' is maximum, then show that probability of failure in each trial can be equal to $\frac{3}{5}$.

10. Mean and variance of a Binomial variate are in the ratio of 3 : 2. Find the most probable number of happening of the variable in 10 trials of the experiment.

11. In a Nigerian hotel, among the english speaking people 40% are English & 60% Americans. The English & American spellings are **"Rigour" & "Rigor"** respectively. An English speaking person in the hotel writes this word. A letter from this word is chosen at random & found to be a vowel. Find the probability that the writer is an Englishman.

12. There is a group of k targets, each of which independently of the other targets, can be detected by a radar unit with probability p. Each of 'm' radar units detects the targets independently of other units. Find the probability that not all the targets in the group will be detected.

13. 2 positive real numbers x and y satisfy $x \le 1$ and $y \le 1$ are chosen at random. Find the probability that $x + y \le 1$, given that $x^2 + y^2 \ge \frac{1}{4}$.

14. There are two lots of identical articles with different amounts of standard & defective articles. There are N articles in the first lot, n of which are defective & M articles in the second lot, m of which are defective. K articles are selected from the first lot & L articles from the second & a new lot results. Find the probability that an article selected at random from the new lot is defective.

15. The odds that a book will be favorably reviewed by three independent crities are 5 to 2, 4 to 3, and 3 to 4 respectively : what is the probability that of the three reviews a majority will be favourable?

16. Find the chance of throwing 10 exactly in one throw with 3 dice.

17. If 12 tickets numbered 0, 1, 2, ...11 are placed in a bag, and three are drawn out, show that the chance that the sum of the numbers on them is equal to 12 is $\frac{3n}{(6n-1)(6n-2)} = \frac{3}{55}$

18. A man has 10 coins and one of them is known to have two heads. He takes one at random and tosses it 5 times and it always falls head : what is the chance that it is the coins with two heads?

19. A purse contains five coins, each of which may be a rupees coin or a 50 ps coin ; two are drawn and found to be shillings : find the probable value of the remaining coins.

20. One of a pack of 52 cards has been lost; from the remainder of the pack two cards are drawn and are found to be spades; find the chance that the missing card is a spade.

21. A, B are two inaccurate arithmeticians whose chance of solving a given question correctly are $\frac{1}{8}$ and $\frac{1}{12}$ respectively; if they obtain the same result, and if it is 1000 to 1 against their making the same mistake, find the chance that the result is correct.

22. If n integers taken at random are multiplied together, show that the chance that the last digit of the product is 1, 3, 7, or 9 is $\frac{2^n}{5^n}$; the chance of its being 2, 4, 6 or 8 is $\frac{4^n-2^n}{5^n}$; of its being 5 is $\frac{5^n-4^n}{10^n}$; and of its being 0 is $\frac{10^n-8^n-5^n+4^n}{10^n}$.

23. A player tosses a coin and is to score one point for every head and 2 points if every tail turned up. He is to play until he reaches 'n'. If p_n is the chance of obtaining exactly 'n' crores, find p_n for n = 1, 2, 3, 4. Also show that $p_n = \frac{1}{2}(p_{n-1} + p_{n-2})$.

24. A lot contains 50 defective & 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as: **[IIT - 1992]**
A = { the first bulb is defective} ; B = { the second bulb is non defective}
C = { the two bulbs are both defective or both non defective}
Determine whether (i) A,B,C are pair wise independent (ii) A,B,C are independent

25. Eight players $P_1, P_2, P_3, \ldots\ldots\ldots P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if i < j. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final. **[IIT - 1999]**

26. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **[IIT - 2002]**

27. A person has to go through three successive tests. Probability of his passing first exam is P. Probability of passing successive test is P or P/2 according as he passed the last test or not. He is selected if he passes atleast two tests. Find the probability of his selection. **[IIT - 2003]**

28. Prove that $P(A \cup B)\, P\left(\overline{A} \cap \overline{B}\right) \le P(C)$ where A and B are independent events and P(C) is the probability of exactly one of A or B occurs. **[IIT - 2004]**

29. A person goes to office either by car, scooter, bus or train, the probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. **[IIT - 2005]**

EXERCISE-4

1. B **2.** A **3.** B **4.** A **5.** A **6.** D **7.** B
8. A **9.** A **10.** C **11.** B **12.** D **13.** A **14.** A
15. A **16.** C **17.** A **18.** C **19.** B **20.** D **21.** B
22. A **23.** A **24.** AC **25.** CD **26.** AC **27.** CD
28. ABCD **29.** AD

EXERCISE-5

1. $\frac{12}{77}$ **2.** $\frac{1}{3}$ **3.** mean = $\frac{5}{6}$, variance = $\frac{25}{36}$

4. $\frac{9}{24}$ **5.** $\frac{1}{n}[1-(1-p)^n - np(1-p)^{n-1}]$

6. $\frac{15}{92}$ **7.** $\frac{11}{16}$ **8.** $\frac{14}{41}$ **10.** 3 **11.** $\frac{5}{11}$

12. $1-\{1-(1-p)^m\}^k$ **13.** $\frac{8-\pi}{16-\pi}$ **14.** $\frac{KnM+LmN}{MN(K+L)}$

15. 209/343 **16.** 1/5 **18.** (32/41) **19.** 2.25 Rs

20. 11/50 **21.** (13/14)

23. $p_1 = \frac{1}{2}$, $p_2 = \frac{3}{4}$, $p_3 = \frac{5}{8}$, $p_4 = \frac{11}{16}$.

24. (i) A, B, C are pairwise independent
(ii) A, B, C are not independent

25. 4/35 **26.** $\frac{9m}{8N+m}$ **27.** $2P^2 - P^3$ **29.** 1/7