विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

This is TYPE 1 Package please wait for Type 2

Subject : PHYSICS

Topic : PARTICLE DYNAMICS

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7. 34 Yrs. Que. from IIT-JEE
8. 10 Yrs. Que. from AIEEE

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KEY CONCEPT

FORCE

1. There are, basically, five forces, which are commonly encountered in mechanics.

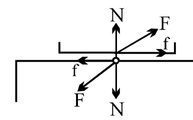
(i) **Weight :** Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

$$W = \frac{GMm}{R^2} = mg$$

(ii) **Contact Force :** When two bodies come in contact they exert forces on each other that is called contact forces.

(a) **Normal force (N) :** It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

(b) **Frictional force :** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.



(iii) **Tension :** The force exerted by the end of a taut string, rope or chain is called the tension. The direction of tension is to pull the body while that of normal reaction is to push the body.

(iv) **Spring force :** The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (units Nm^{-1}).

NEWTON'S LAWS

2. **Newton's First Law :** Every particle continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by the action of an applied force.

3. **Newton's Second Law :** $\vec{F}_{net} = m\vec{a}$

4. **Newton's Third Law :** Whenever two bodies interact they exert forces on each other which are equal in magnitude and opposite in direction. So whenever body A exerts a force F on body B, B exerts a force – F on A.

Inertial Reference Frame : A reference frame in which Newton's first law is valid is called an inertial reference frame. An inertial frame is either at rest or moving with uniform velocity.

Non-Inertial Frame : An accelerated frame of reference is called a non-inertial frame. Objects in non-inertial frames do not obey Newton's first law.

Pseudo Force : It is an imaginary force which is recognized only by a non-inertial observer to explain the physical situation according to Newton's law. The magnitude of this force F_P is equal to the product of the mass m of the object and acceleration a of the frame of reference. The direction of the force is opposite to the direction of acceleration.

$$F_P = -ma$$

The **force of friction** comes into action only when there is a relative motion between the two contact surfaces or when an attempt is made to have it.

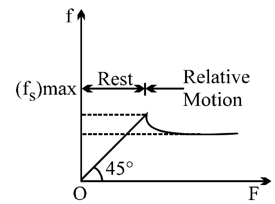
The force of friction on each body is in a direction opposite to its motion (existing or impending) relative to other body.

5. **Static friction :** The frictional force acting between any two surfaces at rest with respect to each other is called the force of static friction (f_s).

$$f_s \leq \mu_s N$$

where μ_s is the static coefficient of friction.

6. **Kinetic friction :** The frictional force acting between surfaces in relative motion with respect to each other is called the force of kinetic friction or sliding friction (f_k).

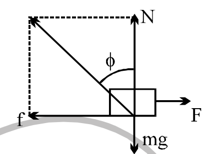


$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction.

$$\mu_s > \mu_k$$

Angle of friction (ϕ) : Mathematically, the angle of friction (ϕ) may be defined as the angle between the normal reaction N and the resultant of the maximum friction force f and the normal reaction.



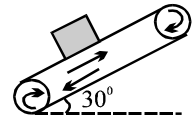
Thus $\tan\phi = \dfrac{f}{N}$

Since $f = \mu N$, therefore,

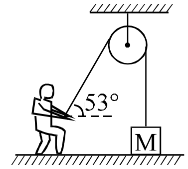
$$\tan \phi = \mu$$

(NEWTONS LAW FORCE & FRICTION)
EXERCISE–I

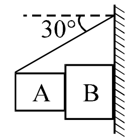
Q.1 A block of mass 1 kg is stationary with respect to a conveyor belt that is accelerating with 1 m/s^2 upwards at an angle of 30° as shown in figure. Determine force of friction on block and contact force between the block & bell.



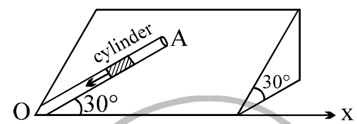
Q.2 A man of mass 63 kg is pulling a mass M by an inextensible light rope passing through a smooth and massless pulley as shown in figure. The coefficient of friction between the man and the ground is $\mu = 3/5$. Find the maximum value of M that can be pulled by the man without slipping on the ground.



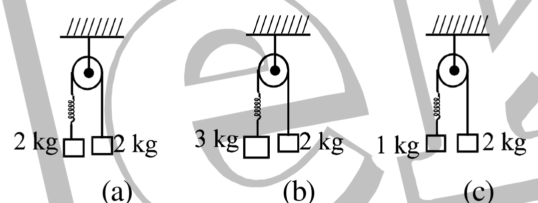
Q.3 Two blocks A and B of mass m 10 kg and 20 kg respectively are placed as shown in figure. Coefficient of friction between all the surfaces is 0.2. Then find tension in string and acceleration of block B. (g = 10 m/s^2)



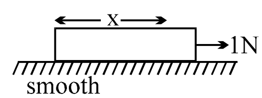
Q.4 An inclined plane makes an angle 30° with the horizontal. A groove OA = 5 m cut in the plane makes an angle 30° with OX. A short smooth cylinder is free to slide down the influence of gravity. Find the time taken by the cylinder to reach from A to O. (g = 10 m/s^2)



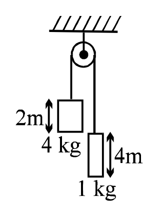
Q.5 Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown in figure. If x_1, x_2 and x_3 be the constan extensions in the spring in these three cases then find the ratio of their extensions.



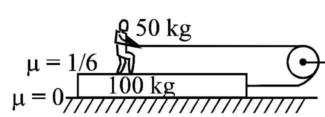
Q.6 A rope of length L has its mass per unit length λ varies according to the function $\lambda(x) = e^{x/L}$. The rope is pulled by a constant force of 1N on a smooth horizontal surface. Find the tension in the rope at x = L/2.



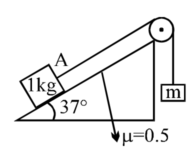
Q.7 In figure shown, both blocks are released from rest.
Find the time to cross each other?



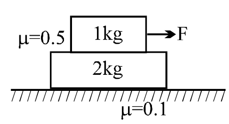
Q.8 A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.



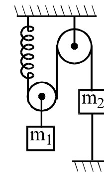
Q.9 In the figure, what should be mass m so that block A slide up with a constant velocity?



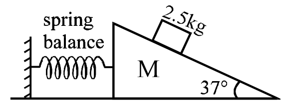
Q.10 What should be minimum value of F so that 2 kg slides on ground but 1 kg does not slide on it? [g = 10 m/sec^2]



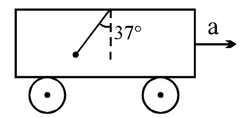
Q.11 In figure shown, pulleys are ideal $m_1 > 2\,m_2$. Initially the system is in equilibrium and string connecting m_2 to rigid support below is cut. Find the initial acceleration of m_2?



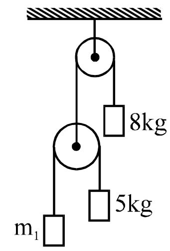
Q.12 Find the reading of spring balance as shown in figure. Assume that mass M is in equilibrium



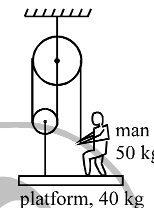
Q.13 At what acceleration of the trolley will the string makes an angle of 37° with vertical if a small mass is attached to bottom of string.



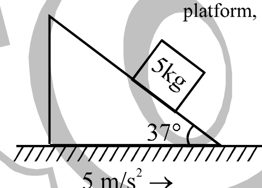
Q.14 At what value of m_1 will 8 kg mass be at rest.



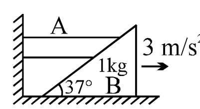
Q.15 What force must man exert on rope to keep platform in equilibrium?



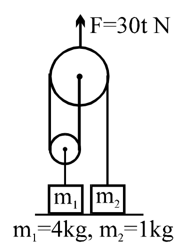
Q.16 Inclined plane is moved towards right with an acceleration of 5 ms^{-2} as shown in figure. Find force in newton which block of mass 5 kg exerts on the incline plane.



Q.17 Find force in newton which mass A exerts on mass B if B is moving towards right with 3 ms^{-2}. Also find mass of A.

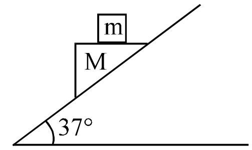


Q.18 Force F is applied on upper pulley. If F = 30t where t is time in second. Find the time when m_1 loses contact with floor.

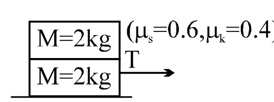


Q.19 A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has a $\mu = 0.5$. Plank rests on frictionless surface. Find the time when m_1 comes to rest w.r.t. plank.

Q.20 Block M slides down on frictionless incline as shown. Find the minimum friction coefficient so that m does not slide with respect to M.



Q.21 The coefficient of static and kinetic friction between the two blocks and also between the lower block and the ground are $\mu_s = 0.6$ and $\mu_k = 0.4$. Find the value of tension T applied on the lower block at which the upper block begins to slip relative to lower block.



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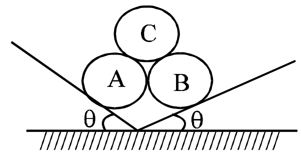
Q.22 Three identical rigid circular cylinders A, B and C are arranged on smooth inclined surfaces as shown in figure. Find the least value of θ that prevent the arrangement from collapse.



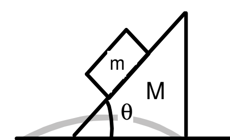
Q.23 Two men A and B of equal mass held on to the free ends of a massless rope which passes over a frictionless light pulley. Man A climbs up the rope with acceleration a relative to the rope while man B hangs on without climbing. Find the acceleration of the man B with respect to ground.

Q.24 A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.

Q.25 A body of mass M = 5kg rests on a horizontal plane having coefficient of fiction $\mu = 0.5$. At t = 0 a horizontal force F is applied that varies with time as F = 5t. Find the time instant t_0 at which motion starts and also find the distance of particle from starting point at t = 6 second.

EXERCISE–II

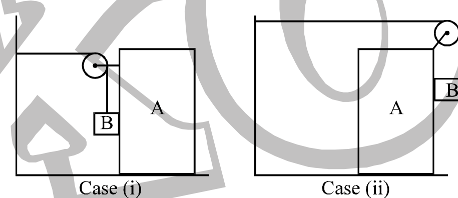
Q.1 A block of mass m lies on wedge of mass M as shown in figure. Answer following parts separately.



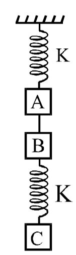
(a) With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

(b) Find the minimum friction coefficient required between wedge M and ground so that it does not move while block m slips down on it.

Q.2 A 20 kg block B is suspended from a cord attached to a 40 kg cart A. Find the ratio of the acceleration of the block in cases (i) & (ii) shown in figure immediately after the system is released from rest. (neglect friction)

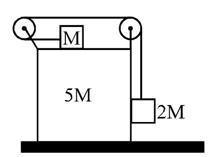


Q.3 The system shown adjacent is in equilibrium. Find the acceleration of the blocks A, B & C all of equal masses m at the instant when (Assume springs to be ideal)

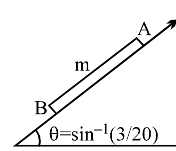


(a) The spring between ceiling & A is cut.
(b) The string (inextensible) between A & B is cut.
(c) The spring between B & C is cut.
Also find the tension in the string when the system is at rest and in the above 3 cases.

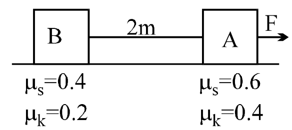
Q.4 In the system shown. Find the initial acceleration of the wedge of mass 5M. The pulleys are ideal and the cords are inextensible. (there is no friction anywhere).



Q.5 A plank of mass m is kept on a smooth inclined plane. A man of mass η times the mass of plank moves on the plank, starts from A, such that the plank is at rest, w.r.t. the inclined plane. If he reaches the other end B of the plank in t = 5sec. Then find the acceleration & the value of η, if the length of the plank is 50m.

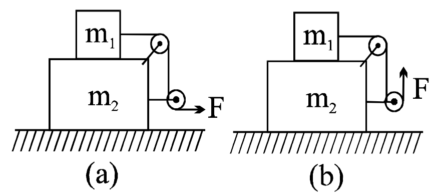


Q.6 Two horizontal blocks each of mass 1/2 kg are connected by a massless, inextensible string of length 2m and placed on a long horizontal table. The coefficient of static & kinetic friction are shown in the figure. Initially the blocks are at rest. If the leading block is pulled with a time dependent horizontal force $F = kt\,\hat{i}$ where k=1N/sec., determine

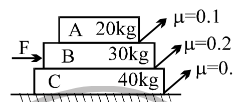


(a) The plots of acceleration of each block with time from t = 0 to t = 10sec.
(b) Velocity of blocks at t = 10sec.
(c) Distance transversed by the blocks in the time interval t = 0 to t = 10sec.
(d) If F stops acting at t = 10sec. find after how much further time would B collide with A.

Q.7 $m_1 = 20$kg, $m_2 = 30$kg. m_2 is on smooth surface. Surface between m_1 and m_2 has $\mu_s = 0.5$ and $\mu_k = 0.3$. Find the acceleration of m_1 and m_2 for the following cases
(a) (i) F = 160 N, (ii) F = 175 N ; (b) F = 160 N



Q.8 A system of masses is shown in the figure with masses & co-efficients of friction indicated. Calculate :
(i) the maximum value of F for which there is no slipping anywhere .
(ii) the minimum value of F for which B slides on C.
(iii) the minimum value of F for which A slips on B.



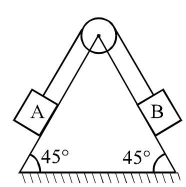
Q.9 A car begins to move at time t = 0 and then accelerates along a straight track with a speed given by $V(t) = 2t^2\ ms^{-1}$ for $0 \le t \le 2$
After the end of acceleration, the car continues to move at a constant speed. A small block initially at rest on the floor of the car begins to slip at t = 1sec. and stops slipping at t = 3 sec. Find the coefficient of static and kinetic friction between the block and the floor.

Q.10 A smooth right circular cone of semi vertical angle $\alpha = \tan^{-1}(5/12)$ is at rest on a horizontal plane. A rubber ring of mass 2.5kg which requires a force of 15N for an extension of 10cm is placed on the cone. Find the increase in the radius of the ring in equilibrium.

EXERCISE–III

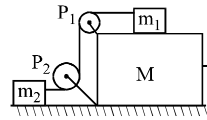
Q.1 A block of mass 0.1kg is held against a wall by applying a horizontal force of 5N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is
(A) 2.5N (B) 0.98N (C) 4.9N (D) 0.49N [JEE 1997]

Q.2 Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a massless inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the system of A and B is released from rest , find (i) the acceleration of A, (ii) tension in the string, (iii) the magnitude and the direction of friction acting on A. [JEE 1997]



Q.3 A spring of force constant k is cut into two pieces such that one piece such that one piece is double the length of the other. Then the long piece will have a force constant of
(A) (2/3) k (B) (3/2) k (C) 3k (D) 6k [JEE 1999]

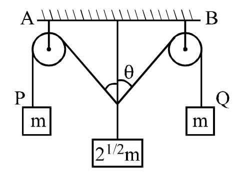
Q.4 In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The co-efficient of friction between M and ground is zero. The co-efficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless . The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2 . The string is perfectly vertical between P_1 and P_2.An external horizontal force F is applied to the mass M. Take g = 10 m/s^2.



(a) Draw a free-body diagram for mass M, clearly showing all the forces.

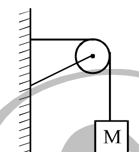
(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2 f_2$. Find f_1 and f_2 . Write down equations of motion of all the masses . Find F, tension in the string and accelerations of the masses. [JEE 2000]

Q.5 The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be [JEE (Scr) 2001]



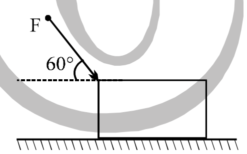
(A) 0° (B) 30°
(C) 45° (D) 60°

Q.6 A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given [JEE (Scr) 2001]



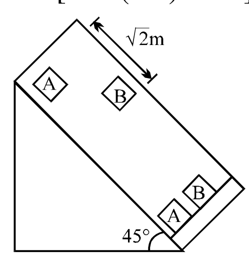
(A) $\sqrt{2}$ Mg (B) $\sqrt{2}$ mg
(C) $\sqrt{(M+m)^2+m^2}\ g$ (D) $\sqrt{(M+m)^2+M^2}\ g$

Q.7 A block of mass $\sqrt{3}$ kg is placed on a rough horizontal surface whose coefficient of friction is $1/2\sqrt{3}$ minimum value of force F (shown in figure) for which the block starts to slide on the surface. (g=10m/s^2)



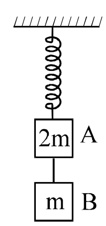
(A) 20 N (B) $20\sqrt{3}$ N
(C) $10\sqrt{3}$ N (D) None of these [JEE (Scr) 2003]

Q.8 Two blocks A and B of equal masses are released from an inclined plane of inclination 45° at t = 0. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially, the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line. [Take g = 10 m/s^2].



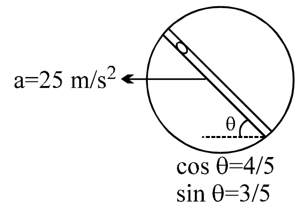
[JEE 2004]

Q.9 Two blocks A and B of masses 2m and m, respectively, are connected by a massless and inextensible string.The whole system is suspended by a massless spring as shown in the figure.The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively [JEE 2006]



(A) g, g (B) g, g/2
(C) g/2, g (D) g/2, g/2

Q.10 A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove in contact is μ = 2/5. The disc has an acceleration of 25 m/s^2. Find the acceleration of the block with respect to disc. [JEE 2006]

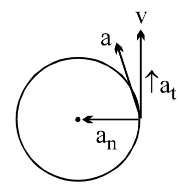


CIRCULAR MOTION & WORK POWER ENERGY

1. A body moving with constant speed in a circular path is continuously accelerated towards the centre of rotation. The magnitude of this normal acceleration is given by

$$a_n = \frac{v^2}{r} = \omega^2 r$$

where v is the constant speed ($v = \omega r$) and r is the radius of the circular path



Tangential area : $a_t = \frac{dv}{dt}$, $a = \sqrt{at^2 + a_n^2}$

2. **Radius of curvature :** $r = \frac{v^2}{a_n}$

3. According to Newton's second law, a body moving in a circular path with constant speed must be acted upon by an unbalanced force which is always directed towards the centre. This necessary unbalanced force is called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

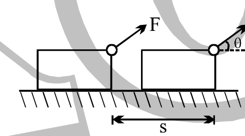
4. Centrifugal force is a pseudo force which is observed an observer in rotating frame.

$$\vec{F}_{cf} = m\omega_{frame}^2 \vec{r}$$

Work (W) :
The **work W** done by a constant force F when its point of application undergoes a displacement s is defined as

$$W = F.s = Fs \cos\theta$$

where θ is the angle between F and s. Work is a scalar quantity and its SI units is N-m or joule (J).



Note: Only the component ($F \cos\theta$) of the force F which is along the displacement contributes to the work done.

If $F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $s = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

then $W = \vec{F}\cdot\vec{s} = F_x\Delta x + F_y\Delta y + F_z\Delta z$

5. **Work done by a Variable Force :** When the magnitude and direction of a force varies with position, The work done by such a force for an infinitesimal displacement ds is given by

$$dW = \vec{F}\cdot d\vec{s}$$

In terms of rectangular components,

$$W_{AB} = \int_{X_A}^{X_B} F_x dx + \int_{Y_A}^{Y_B} F_y dy + \int_{Z_A}^{Z_B} F_z dz$$

6. **Work Done by a Spring Force :** The work done by the spring force for a displacement from x_i to x_f is given by

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

7. **Work Energy theorem :**
Work done on a body can produce a change in its kinetic energy. Work is required to produce motion and it is also required to destroy motion.

$$W = \Delta K = K_f - K_i$$

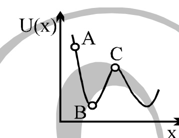
8. **Conservative Force :** The force which does work in complete independence of the path followed the body is called a conservative force. The gravitational force, spring force and electrostatic force are the examples of conservative forces.

9. **Non-Conservative Force :** The work done by a non-conservative force not only depends on the initial and final positions but also on the path followed. The common examples of such forces are : frictional force and drag force of fluids.

10. **Potential Energy :** The potential energy is defined only for conservative forces.

$$U_B - U_A = -\int_A^B F_c . ds$$

11. Conservative force : $F_c = -\dfrac{dU}{dx}$



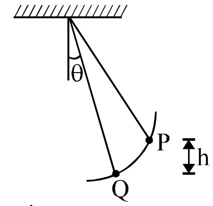
At equilibrium, $\dfrac{dU}{dx} = 0$

The point B is the position of stable equilibrium, because $\dfrac{d^2U}{dx^2} > 0$

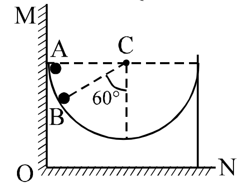
The point C is the position odf unstable equilibrium, because $\dfrac{d^2U}{dx^2} < 0$

(CIRCULAR MOTION & WORK POWER ENERGY)
EXERCISE–I

Q.1 The bob of a simple pendulum of length l is released from point P. What is the angle made by the net acceleration of the bob with the string at point Q.

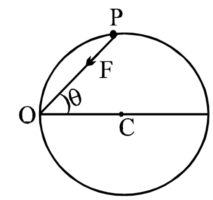


Q.2 A ball of mass 1 kg is released from position A inside a wedge with a hemispherical cut of radius 0.5 m as shown in the figure. Find the force exerted by the vertical wall OM on wedge, when the ball is in position B. (neglect friction everywhere). Take ($g = 10\ m/s^2$)



Q.3 A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of

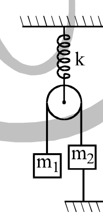
$$\frac{d^2\theta}{dt^2} \quad \& \quad \left(\frac{d\theta}{dt}\right)^2.$$



Q.4 A particle is moving in x direction, under the influence of force $F = \pi \sin \pi x$. Find the work done by another external agent in slowly moving a particle from $x = 0$ to $x = 0.5$ m.

Q.5 A particle moves in a circle of radius R with a constant speed v. Then, find the magnitude of average acceleration during a time interval $\frac{\pi R}{2v}$.

Q.6 In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring ($m_1 > m_2$).

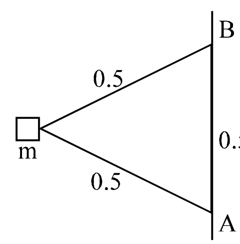


Q.7 A spring of mass m is pulled such that a given instant, velocity of both of its end is v in the opposite direction. Find the kinetic energy of the spring.

Q.8 A particle of mass 3 kg is rotating in a circle of radius 1 m such that the angle rotated by its radius is given by $\theta = 3(t + \sin t)$. Find the net force acting on the particle when $t = \pi/2$.

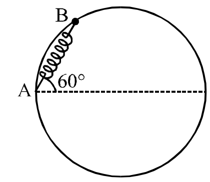
Q.9 For a particle rotating in a vertical circle with uniform speed, the maximum and minimum tension in the string are in the ratio 5 : 3. If the radius of vertical circle is 2m, then find the speed of revolving body.

Q.10 Two strings of length $l = 0.5$ m each are connected to a block of mass $m = 2$ kg at one end and their ends are attached to the point A and B 0.5 m apart on a vertical pole which rotates with a constant angular velocity $\omega = 7$ rad/sec. Find the ratio $\frac{T_1}{T_2}$ of tension in the upper string (T_1) and the lower string (T_2). [Use $g = 9.8\ m/s^2$]



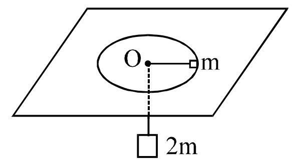
Q.11 A force $\vec{F} = -k(x\,\hat{i} + y\,\hat{j})$ [where k is a positive constant] acts on a particle moving in the x-y plane. Starting from origin, the particle is taken to (a, a) and then to $\left(a/\sqrt{2}, 0\right)$. Find the total work done by the force F on the particle.

Q.12 A bead of mass m is attached to one end of a spring of natural length $\sqrt{3}$ R and spring constant $k = \frac{(\sqrt{3}+1)\,mg}{R}$. The other end of the spring is fixed at point A on a smooth fixed vertical ring of radius R as shown in the figure. What is the normal reaction at B just after the bead is released?

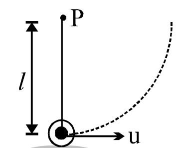


Q.13 Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2} m^2 upto a height of 10 m. If it is needed to deliver a volume 0.2 m^3 per second, find the power required. [Use g = 10 m/s^2]

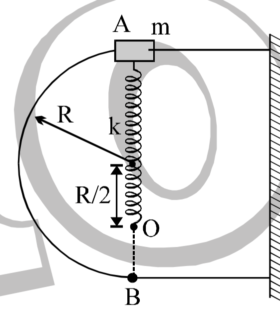
Q.14 A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



Q.15 Consider the shown arrangement when a is bob of mass 'm' is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity $\vec{u}$ having magnitude $\sqrt{3gl}$, find the minimum speed of the bob in subsequent motion.

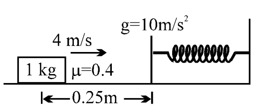


Q.16 A bead of mass m is tied at one end of a spring of spring constant $\frac{mg}{R}$ and unstretched length $\frac{R}{2}$ and other end to fixed point O. The smooth semicircular wire frame is fixed in vertical plane. Find the normal reaction between bead and wire just before it reaches the lowest point.



Q.17 A particle of mass m is hanging with the help of an elastic string of unstretched length a and force constant $\frac{mg}{a}$. The other end is fixed to a peg on vertical wall. String is given an additional extension of $2a$ in vertical downward direction by pulling the mass and released from rest. Find the maximum height reached by it during its subsequent motion above point of release. (Neglect interaction with peg if any)

Q.18 A particle of mass 1 kg is given a horizontal velocity of 4 m/s along a horizontal surface, with which it has a coefficient of friction (both static and kinetic) of 0.4. The particle strikes a fixed ideal spring of force constant 6 N/m after travelling a distance of 0.25 m. Assume acceleration due to gravity is 10 m/s^2. Find the final displacement of the particle from its starting point.



Q.19 A point moves along a circle having a radius 20 cm with a constant tangential acceleration 5 cm/s^2. How much time is needed after motion begins for the normal acceleration of the point to be equal to tangential acceleration ?

Q.20 A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by U (r) = 2r^3 Joule. If the body is moving in a circular orbit of 5m,then find its energy.

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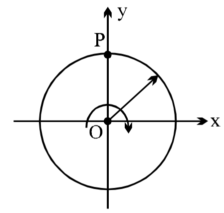
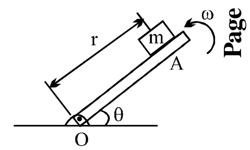
Q.21 A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $(6\hat{i}-8\hat{j})$ m/s^2. find the angular acceleration of the ring & the angular velocity at that instant. Radius of the ring is 2m.

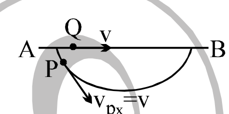


Q.22 A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At t = 0, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rpm in 2 sec. Particle then continues to move with attained speed. Calculate
(a) angular acceleration of the particle,
(b) tangential velocity of the particle as a function of time.
(c) acceleration of the particle at t = 0.5 second and at t = 3 second
(d) angular displacement at t = 3 second.

Q.23 The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity $\omega = 3$ rad/sec. As it passes the position $\theta = 0$, a small mass m is placed upon it at a radial distance r = 0.5 m. If the mass is observed to slip at $\theta = 37°$, find the coefficient of friction between the mass & the member.



Q.24 A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at t = 0. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at t=0 along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Which bead reaches point B earlier?



Q.25 The blocks are of mass 2 kg shown is in equilibrium. At t = 0 right spring in fig (i) and right string in fig (ii) breaks. Find the ratio of instantaneous acceleration of blocks?

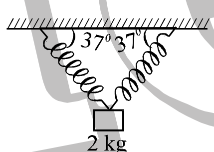


figure (i)

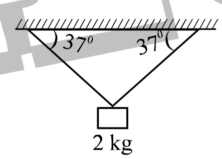
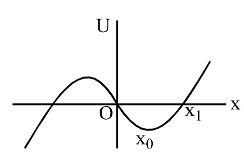


figure (ii)

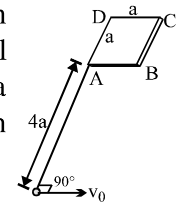
EXERCISE–II

Q.1 A particle is confined to move along the +x axis under the action of a force F(x) that is derivable from the potential $U(x) = ax^3 - bx$.
(a) Find the expression for F(x)
(b) When the total energy of the particle is zero, the particle can be trapped with in the interval x=0 to $x=x_1$. For this case find the values of x_1.
(c) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b.

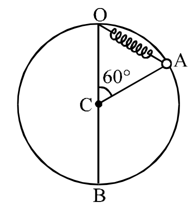


Q.2 A particle of mass 2kg is subjected to a two dimensional conservative force given by $F_x = -2x+2y$, $F_y = 2x-y^2$. (x,y in m and F in N) If the particle has kinetic energy of (8/3) J at point (2,3), find the speed of the particle when it reaches (1,2).

Q.3 A square plate is firmly atached to a frictionless horizontal plane. One end of a taut cord is attached to point A of the plate and the other end is attached to a sphere of mass m. In the process, the cord gets wrapped around the plate. The sphere is given an initial velocity v_0 on the horizontal plane perpendicular to the cord which causes it to make a complete circuit of the plate and return to point A. Find the velocity of the sphere when it hits point A again after moving in a circuit on the horizontal plane. Also find the time taken by the sphere to complete the circuit.

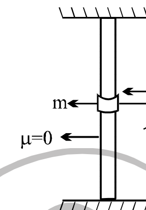


Q.4 A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle$ OCA = 60°, C being the centre of the ring. The natural length of the spring is also equal to r = 20cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.

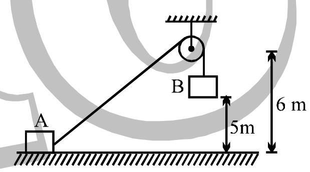


Q.5 A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M (> m). The ring is held on a level with the peg and released : Show that it first comes to rest after falling a distance:

$$\frac{2mMa}{M^2-m^2}$$

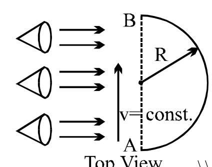


Q.6 A block of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6 m from the floor. A light inextensible string of length 16 m, connected with A passes over the pulley and another identical block B is hung from the string. Initial height of B is 5m from the floor as shown in Fig. When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.

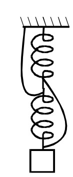


(i) If at an instant string makes an angle θ with horizontal, calculate relation between velocity u of A and v of B.
(ii) Calculate v when B strikes the floor.

Q.7 A small block can move in a straight horizontal linea along AB. Flash lights from one side projects its shadow on a vertical wall which has horizontal cross section as a circle. Find tangential & normal acceleration of shadow of the block on the wall as a function of time if the velocity of the block is constant (v).

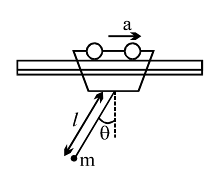


Q.8 In fig two identical springs, each with a relaxed length of 50cm and a spring constant of 500N/m, are connected by a short cord of length 10cm. The upper string is attached to the ceiling, a box that weighs 100N hangs from the lower spring. Two additional cords, each 85cm long , are also tied to the assembly; they are limp (i.e. slack).

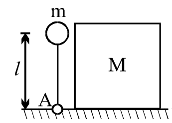


(a) If the short cord is cut, so that the box then hangs from the springs and the two longer cords, does the box move up or down?
(b) How far does the box move before coming to rest again?

Q.9 The small pendulum of mass m is suspended from a trolley that runs on a horizontal rail. The trolley and pendulum are initially at rest with $\theta = 0$. If the trolley is given a constant acceleration a = g determine the maximum angle θ_{max} through which the pendulum swings. Also find the tension T in the cord in terms of θ.

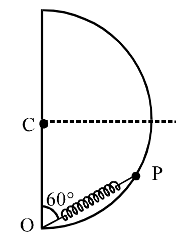


Q.10 A weightless rod of length l with a small load of mass m at the end is hinged at point A as shown in the figure and occupies a strictly vertical position, touching a body of mass M. A light jerk sets the system in motion. For what mass ratio M/m will the rod form an angle $\alpha = \pi/6$ with the horizontal at the moment of the separation from the body? What will be the velocity u of the body at this moment? Friction should be neglected.



EXERCISE–III

Q.1 A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length (3R/4) is attached to the lowest point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant K = mg/R. Consider the instant when the ring is released and



(i) draw the free body diagram of the ring.

(ii) determine the tangential acceleration of the ring and the normal reaction. [JEE 96]

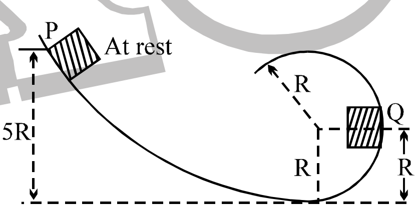
Q.2 Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1. [JEE 97]

Q.3 A small block of mass m slides along a smooth frictional track as shown in the fig. (i) If it starts from rest at P, what is is the resultant force acting on it at Q? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight. [REE 97]



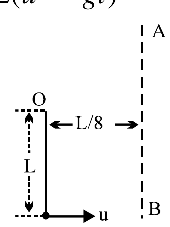
Q.4 A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a,0) and then parallel to the y-axis to the pint (a,a). The total work done by the force $\vec{F}$ on the particle is [JEE 98]

(A) – 2Ka^2 (B) 2Ka^2 (C) – Ka^2 (D) Ka^2

Q.5 A stone is tied to a string of length l is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity at it reaches a position where the string is horizontal is [JEE98]

(A) $\sqrt{(u^2 - 2gl)}$ (B) $\sqrt{2gl}$ (C) $\sqrt{(u^2 - gl)}$ (D) $\sqrt{2(u^2 - gl)}$

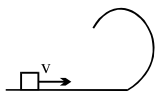
Q.6 A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance L/8 from O as shown. The object given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. [JEE'99, 10]

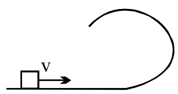


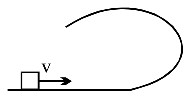
Q.7 A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end of A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and gravity is neglected, then the time after which the bead starts slipping is [JEE'2000]

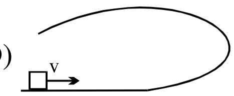
(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

Q.8 A small block is shot into each of the four tracks as shown below. Each of the tracks risks to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in [JEE(Scr)'2001]

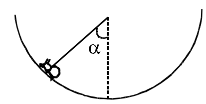
(A) 

(B) 

(C) 

(D) 

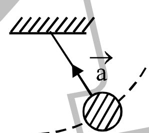
Q.9 An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by [JEE(Scr.)'2001]

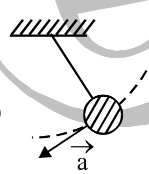


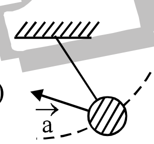
(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$ (C) $\sec \alpha = 3$ (D) $\operatorname{cosec} \alpha = 3$

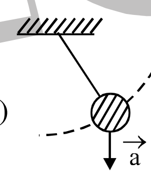
Q.10 A small ball of mass 2×10^{-3} Kg having a charge of 1 μc is suspended by a string of length 0.8m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. [JEE'2001]

Q.11 A simple pendulum is oscillating without damping. When the displacement of the bob is less that maximum, its acceleration vector $\vec{a}$ is correctly shown in [JEE (Scr.)'2002]

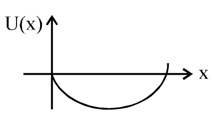
(A) 

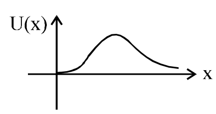
(B) 

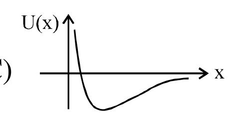
(C) 

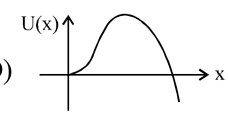
(D) 

Q.12 A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle x of the particle from the origin as $F(x) = -kx + ax^2$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy U (x) of the particle is [JEE (Scr.)'2002]

(A) 

(B) 

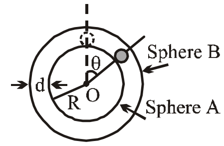
(C) 

(D) 

Q.13 An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is [JEE (Scr.)'2002]

(A) 4 Mg/k (B) 2 Mg/k (C) Mg/k (D) Mg/2k

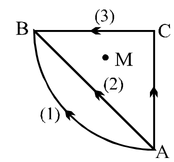
Q.14 A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). [JEE' 2002]



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

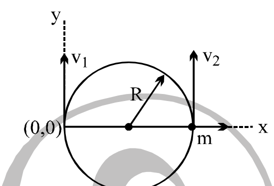
(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

Q.15 In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then [JEE (Scr.)'2003]

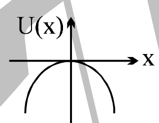


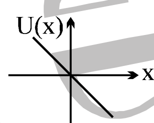
(A) $W_1 = W_2 = W_3$ (B) $W_1 = W_2 > W_3$
(C) $W_1 > W_2 > W_3$ (D) $W_1 < W_2 < W_3$

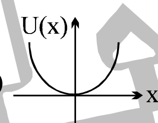
Q.16 A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time. [JEE' 2003]

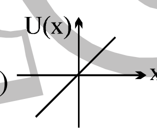


Q.17 A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function)

(A) 

(B) 

(C) 

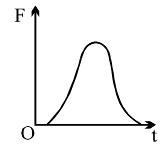
(D) 

[JEE' 2004(Scr)]

CENTRE OF MASS MOMENTUM & COLLISION

The action of force with respect to time is defined in terms of Impulse, that is,

$$\int F dt = mv_f - mv_i = \Delta p$$



Graphically, impulse is the area under the F-t graph

In the absence of a net external force, the momentum of a system is conserved.

i.e. $$\frac{dP}{dt} = F_{ext} = 0$$

$$p = p_1 + p_2 + \ldots\ldots\ldots\ldots + p_N = \text{constant}$$

1. **Collision** is a kind of interaction between two or more bodies which come in contact with each other for a very short time interval.

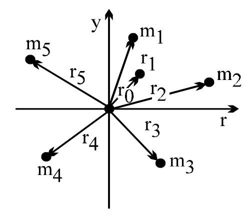
2. **Types of collision: Elastic and Inelastic**
Collisions may be either elastic or inelastic. Linear momentum is conserved in both cases.
(i) A perfectly elastic collision is defined as one in which the total kinetic energy of the system is conserved.
(ii) In an inelastic collision, the total kinetic energy of the system changes.
(iii) In a completely inelastic collision, the two bodies couple or stick togehter.

3. **Coefficient of Restitution :** It is defined as the ratio of the velocity of separation to the velocity of approach of the two colliding bodies.

$$e = \frac{\text{rel. velocity of separation}}{\text{rel. velocity of approach}}$$

For a perfectly elastic collision, $e = 1$
For an inelastic collision, $0 < e < 1$
For completely inelastic collision, $e = 0$
Note that the velocity of approach and the velocity of separation are always taken along the normal to the striking surface.

CENTRE OF MASS



1. **Discrete System :** The position vector of the centre of mass is

$$r_c = \frac{m_1 r_1 + m_2 r_2 + \ldots\ldots\ldots + m_n r_n}{m_1 + m_2 + \ldots\ldots\ldots m_n}$$

where $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ are the position vectors of masses m_1, m_2, ..., m_n respectively.
The components of the position vector of centre of mass are defined as

$$x_c = \frac{\sum m_i x_i}{M}; \qquad y_c = \frac{\sum m_i y_i}{M}; \qquad z_c = \frac{\sum m_i z_i}{M}$$

2. **Continuous system :** The centre of mass of a continuous body is defined as

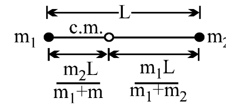
$$\vec{r}_c = \frac{1}{M}\int r\, dm$$

In the component form

$$x_c = \frac{1}{M}\int x\, dm; \qquad y_c = \frac{1}{M}\int y\, dm; \qquad z_c = \frac{1}{M}\int z\, dm$$

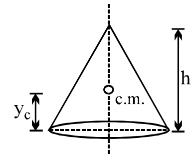
3. **Centre of Mass of Some Common Systems :**

(i) A system of two point masses.
The centre of mass lie closer to the heavier mass.



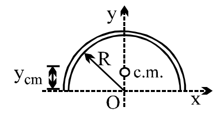
(ii) A circular cone

$y_c = \dfrac{h}{4}$



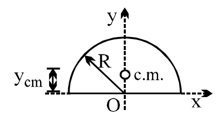
(iii) A semi-circular ring

$y_c = \dfrac{2R}{\pi}$; $x_c = 0$



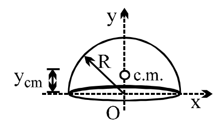
(iv) A semi-circular disc

$y_c = \dfrac{4R}{3\pi}$; $x_c = 0$



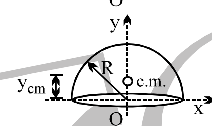
(v) A hemispherical shell

$y_c = \dfrac{R}{2}$; $x_c = 0$



(vi) A solid hemisphere

$y_c = \dfrac{3R}{8}$; $x_c = 0$



4. **Motion of the centre of mass :**

(i) **Velocity :** The instantaneous velocity of the centre of mass is defined as

$$v_c = \frac{\sum m_i v_i}{M}$$

(ii) **Acceleration :** The acceleration of the centre of mass is defined as

$$a_c = \frac{\sum m_i a_i}{M}$$

(iii) **Momentum :** The total momentum of a system of particles is

$$p = Mv_c$$

(iv) **Kinetic Energy :** The kinetic energy of a system of particles consisits of two parts.

$$K = K_c + K'$$

where $K_c = \frac{1}{2}Mv_c^2$, kinetic energy due to motion of c.m. relative to the fixed origin O,

and $K' = \sum \frac{1}{2} m_i v_i^2$, kinetic energy of the particles relative to the c.m.

Note that the term K' may involve translational, rotational or vibrational energies relative to the centre of mass.

5. **Newon's Laws of a system of particles :** The first and second laws of motion for a system of particles are modified as :

First law : The centre of mass of an isolated system is at rest or moves with constant velocity.

Second law : The net external force acting on a system of total of mass M is related to the acceleration of centre of mass of the system.

$$\sum \vec{F}_{ext} = M\,\vec{a}_{cm}$$

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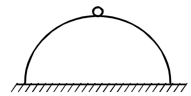
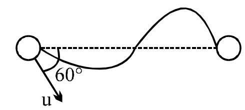
(CENTRE OF MASS MOMENTUM & COLLISION)
EXERCISE–I

Q.1 A hemisphere of radius R and of mass 4m is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the top of the hemisphere. Find the angular velocity of the particle relative to hemisphere at an angular displacement θ when velocity of hemisphere has become v.



Q.2 A man whose mass is m kg jumps vertically into air from a sitting position in which his centre of mass is at a height h_1 from the ground. When his feet are just about to leave the ground his centre of mass is h_2 from the ground and finally rises to h_3 when he is at the top of the jump. **(a)** What is the upward force exerted by the ground on him treating it as a constant? **(b)** Find work done by normal reaction from ground.

Q.3 In the figure shown, each tiny ball has mass m, and the string has length L. One of the ball is imparted a velocity u, in the position shown, in which the initial distance between the balls is $L/\sqrt{3}$. The motion of ball occurs on smooth horizontal plane. Find the impulse of the tension in the string when it becomes taut .

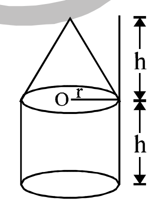


Q.4 Two trolleys A and B are free to move on a level frictionless track, and are initially stationary. A man on trolley A throws a bag of mass 10 kg with a horizontal velocity of 4 m/s with respect to himself on to trolley B of mass 100 kg. The combined mass of trolley A (excluding bag) and the man is 140 kg. Find the ratio of velocities of trolleys A and B, just after the bag lands on trolley B.

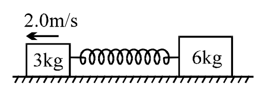
Q.5 A bob of mass m attached with a string of length l tied to a point on ceiling is released from a position when its string is horizontal. At the bottom most point of its motion, an identical mass m gently stuck to it. Find the angle from the vertical to which it rises.

Q.6 Two balls of equal masses are projected upward simultaneously, one from the ground with speed 50 m/s and other from a 40 m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.

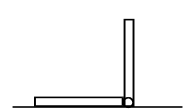
Q.7 Find the distance of centre of mass from O of a composite solid cone and solid cylinder made of same material.



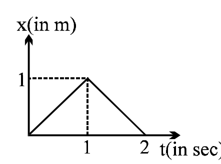
Q.8 Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring. Initially the spring is unstretched and the velocity of 2 m/s is imparted to 3 kg block as shown. Find the maximum velocity of 6 kg block during subsequent motion.



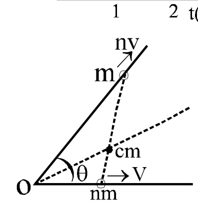
Q.9 Two planks each of mass m and length L are connected by a frictionless, massless hinge as shown in the figure. Initially the system is at rest on a level frictionless surface. The vertical plank falls anticlockwise and finaly comes to rest on the top of the horizontal plank. Find the displacement of the hinge till the two planks come in contact.



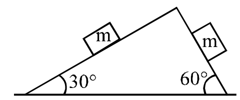
Q.10 2 bodies m_1 & m_2 of mass 1 and 2 kg respectively are moving along x-axis under the influence of mutual force only. The velocity of their centre of mass at a given instant is 2 m/s. The x coordinate of m_1 is plotted against time. Then plot the x coordinate of m_2 against time. (Both are initially located at origin)



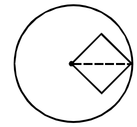
Q.11 Two masses, nm and m, start simultaneously from the intersection of two straight lines with velocities v and nv respectively. It is observed that the path of their centre of mass is a straight line bisecting the angle between the given straight lines. Find the magnitude of the velocity of centre of inertia.
(here θ = angle between the lines)



Q.12 Two blocks of equal masses m are released from the top of a smooth fixed wedge as shown in the figure. Find the magnitude of the acceleration of the centre of mass of the two blocks.



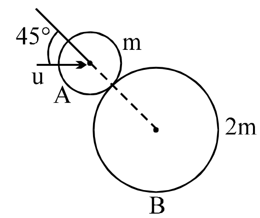
Q.13 From a uniform circular disc of radius R, a square is cut out with radius R as its diagonal. Find the centre of mass of remainder is at a distance.(from the centre)



Q.14 A sphere of mass m_1 in motion hits directly another sphere of mass m_2 at rest and sticks to it, the total kinetic energy after collision is 2/3 of their total K.E. before collision. Find the ratio of $m_1 : m_2$.

Q.15 Two bodies of same mass tied with an inelastic string of length l lie together. One of them is projected vertically upwards with velocity $\sqrt{6gl}$. Find the maximum height up to which the centre of mass of system of the two masses rises.

Q.16 Disc A of mass m collides with stationary disk B of mass 2m as shown in figure. Find the value of coefficient of restitution for which the two disks move in perpendicular direction after collision.

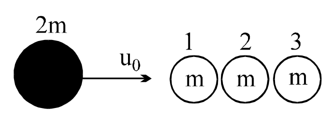


Q.17 A platform of mass m and a counter weight of mass (m + M) are connected by a light cord which passes over a smooth pulley. A man of mass M is standing on the platform which is at rest. If the man leaps vertically upwards with velocity u, find the distance through which the platform will descend. Show that when the man meets the platform again both are in their original positions.

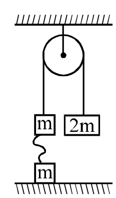
Q.18 The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

Q.19 After scaling a wall of 3 m height a man of weight W drops himself to the ground. If his body comes to a complete stop 0.15 sec. After his feet touch the ground, calculate the average impulsive force in the vertical direction exerted by ground on his feet. ($g = 9.8\ m/s^2$)

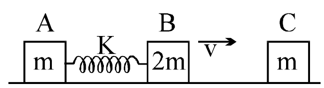
Q.20 A heavy ball of mass 2m moving with a velocity u_0 collides elastically head-on with a cradle of three identical balls each of mass m as shown in figure. Determine the velocity of each ball after collision.



Q.21 The Atwood machine in fig has a third mass attached to it by a limp string. After being released, the 2m mass falls a distance x before the limp string becomes taut. Thereafter both the mass on the left rise at the same speed. What is the final speed ? Assume that pulley is ideal.



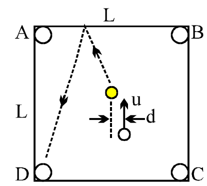
Q.22 Two blocks A and B of masses m and 2m respectively are connected by a spring of force constant k. The masses are moving to the right with uniform velocity v each, the heavier mass leading the lighter one. The spring in between them is of natural length during the motion. Block B collides with a third block C of mass m, at rest. The collision being completely inelastic. Calculate the maximum compression of the spring.



EXERCISE–II

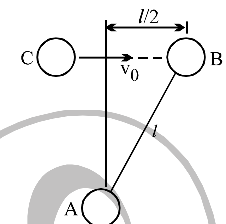
Q.1 A billiard table is 15 cm by 20 cm. A smooth ball of coefficient of restitution $e = 4/9$ is projected from a point on the shorter side so as to describe a rectangle and return to the point of projection after rebounding at each of the other three cushions. Find the position of the point and the direction of projection.

Q.2 In a game of Carom Board, the Queen (a wooden disc of radius 2 cm and mass 50 gm) is placed at the exact center of the horizontal board. The striker is a smooth plastic disc of radius 3 cm and mass 100 gm. The board is frictionless. The striker is given an initial velocity 'u' parallel to the sides BC or AD so that it hits the Queen inelastically with coefficient of restitution = 2/3. The impact parameter for the collision is 'd' (shown in the figure). The Queen rebounds from the edge AB of the board inelastically with same coefficient of restitution = 2/3 and enters the hole D following the dotted path shown. The side of the board is L.

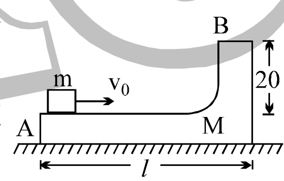


Find the value of impact parameter 'd' and the time which the Queen takes to enter hole D after collision with the striker.

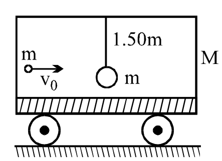
Q.3 Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible inelastic cord of length l and are at rest in the position shown when sphere B is struck directly by sphere C which is moving to the right with a velocity v_0. Knowing that the cord is taut when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, determine the velocity of each sphere immediately after impact.



Q.4 A wedge of mass M=2m rests on a smooth horizontal plane. A small block of mass m rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity $v_0 = 6\ ms^{-1}$. At highest point of its trajectory, the block collides with a particle of same mass m moving vertically downwards with velocity $v=2\ ms^{-1}$ and gets stuck with it. If the combined body lands at the end point A of body of mass M, calculate length l. Neglect friction $(g=10\ ms^{-2})$



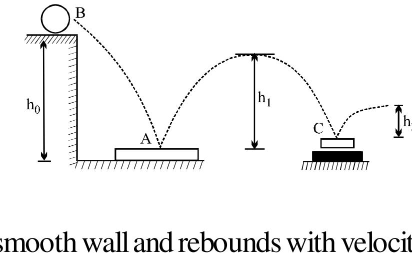
Q.5 A ball of mass = 1Kg is hung vertically by a thread of length $l = 1.50$ m. Upper end of the thread is attached to the ceiling of a trolley of mass M = 4 kg. Initially, trolley is stationary and it is free to move along horizontal rails without friction. A shell of mass m = 1 kg moving horizontally with velocity $v_0 = 6ms^{-1}$ collides with the ball and gets stuck with it. As a result, thread starts to deflect towards right. Calculate its maximum deflection with the vertical. $(g = 10m\ s^{-2})$



Q.6 A 70g ball B droped from a height $h_0 = 9$ m reaches a height $h_2 = 0.25$m after bouncing twice from identical 210g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine

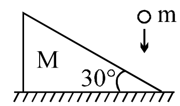
(a) the coefficient of resitution between the ball and the plates,

(b) the height h_1 of the ball's first bounce.



Q.7 A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

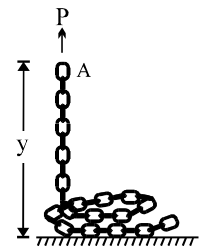
Q.8 A ball of mass m = 1 kg falling vertically with a velocity $v_0 = 2$ m/s strikes a wedge of mass M = 2kg kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is e = 1/2. Find the velocity of the wedge and the ball immediately after collision.



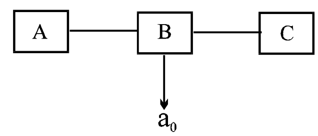
Q.9 A chain of length l and m lies in a pile on the floor. It its end A is raised vertically at a constant speed v_0, express in terms of the length y of chain which is off the floor at any given instant.

(a) the magnitude of the force P applied to end A.

(b) the reaction of the floor. (c) energy lost during the lifting of the chain.



Q.10 3 blocks of mass 1kg each kept on horizontal smooth ground are connected by 2 taut strings of length l as shown. B is pulled with constant acceleration a_0 in direction shown. Find the relative velocity of A & C just before striking.

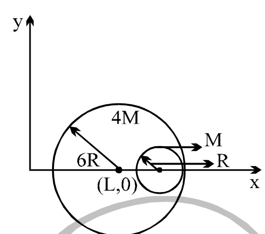


EXERCISE–III

Q.1 A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time t = 0. All colisions are completely inelastic, then
(i) the last block starts moving at t = n(n –1)L/(2v)
(ii) the last block starts moving at t= (n –1)L/v
(iii) the centre of mass of the system will have a final speed v/n
(iv) the centre of mass of the system will have a final speed v. [IIT 95]

Q.2 A small bucket of mass M (= 10^{-2}kg) is attached to a long cord of length L (= 5×10^{-2} m). The bucket is released from rest when the cord is in a horizontal position. In its lowest position the bucket scoops up m(=10^{-3}kg) of water, what is the height of the swing above the lowest position [REE 95]

Q.3 A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R. The masses of large and small spheres are 4M and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the large sphere when the smaller sphere reaches the other extreme position. [IIT 96]



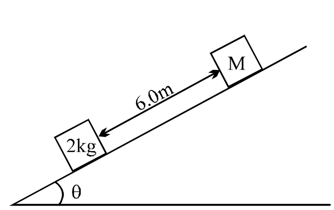
Q.4 A body of mass 5kg moves along the x axis with a velocity 2m/s. A second body of mass 10kg moves along the y axis with a velocity $\sqrt{3}$ m/s. They collide at the origin and stick together. Calculate
(i) the final velocity of the combined mass after collision
(ii) the amount of heat liberated in the collision. [REE 96]

Q.5 An isolated particle of mass m is moving in a horizontal plane (x-y) along the x-axis at a certain height above the ground. It suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later the smaller fragment is at y = + 15 cm. The larger fragment at this instant is at [IIT 97]
(A) y = – 5cm (B) y = + 20cm (C) y = + 5cm (D) y = – 20cm

Q.6 A cart is moving along +x direction with a velocity of 4m/s. A person in the cart throws a stone with a velocity of 6m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with the vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine
(a) the speed of the combined mass immediately after the collision with respect to an observer on the ground.
(b) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass. [IIT 97]

Q.7 A particle of mass m and velocity v collides elastically and obliquely with a stationary particle of mass m. Calculate the angle between the velocity vectors of the two particles after the collision. [REE 97]

Q.8 Two blocks of mass 2kg and M are at rest on an indiclined plane and are separated by a distance of 6.0m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10.0m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0m/s when it reaches its initial position. The other block M after the collision moves 0.5m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.
[Take sinθ ≈ tanθ = 0.05 and g = 10m/s^2] [IIT 99]



Q.9 Two trolleys A and B of equal masses M are moving in oppsite directions with velocities $\vec{v}$ and $-\vec{v}$ respectively on separate horizontal frictionless parallel tracks. When they start crossing each other, a ball of mass m is thrown from B to A and another of same mass is thrown from A to B with velocities normal to $\vec{v}$. The balls may be thrown in following two ways:

(i) balls from A to B and B to A are thrown simultaneously.

(ii) ball is thrown from A to B after the ball thrown from B reaches A.

Which procedure would lead to a larger change in the velocities of the trolleys? [REE 2000]

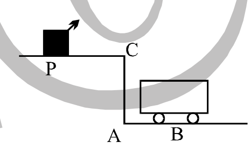
Q.10 A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to: [IIT (Scr) 2000]

(A) v (B) v^2 (C) v^3 (D) v^4

Q.11 Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time t = 0. They collide at time t_0. Their velocities become $\vec{v}_1^{\,'}$ and $\vec{v}_2^{\,'}$ at time $2t_0$ while still moving in air. The value of $\left|\left(m_1\vec{v}_1^{\,'} + m_2\vec{v}_2^{\,'}\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\right|$ is [IIT (Scr) 2001]

(A) zero (B) $(m_1 + m_2)gt_0$ (C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$

Q.12 A car P is moving with a uniform speed of $5(3^{1/2})$ m/s towards a carriage of mass 9 Kg at rest kept on the rails at a point B as shown in fig. The height AC is 120 m. Cannon balls of 1 Kg are fired from the car with an initial velocity 100 m/s at an angle 30^0 with the horizontal. The first canon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage. What will be the horizontal velocity of the carriage just after the second impact? [IIT 2001]

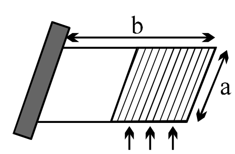


Q.13 Two block of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is : [IIT (Scr) 2002]

(A) 30 m/s (B) 20 m/s (C) 10 m/s (D) 5 m/s

Q.14 There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?

It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1m; g = 10 m/s^2.



[JEE 2006]

ANSWER KEY
(NEWTONS LAW FORCE & FRICTION)
EXERCISE – I

Q.1 contact force between the block and the belt is 10.5 N Q.2 35 kg

Q.3 306 N , 4.7 m/s^2

Q.4 2 sec Q.5 $x_2 > x_1 > x_3$ $x_1 : x_2 : x_3 : 15 : 18 : 10$ Q.6 $\frac{1}{\sqrt{e}+1}$

Q.7 1 sec

Q.8 $\frac{100}{3}$ N towards left Q.9 1 kg Q.10 3 N Q.11 $\left(\frac{m_1 - 2m_2}{2m_2}\right)g$

Q.12 12 N

Q.13 7.5 ms^{-2} Q.14 10/3 kg Q.15 300 N Q.16 55

Q.17 5N, 16/31 kg Q.18 2 sec Q.19 $\frac{4}{3}$ sec Q.20 $\frac{3}{4}$

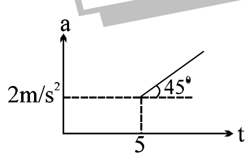
Q.21 40 N Q.22 $\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)$ Q.23 $\frac{a}{2}$ Q.24 1/2 sec Q.25 5 sec and $\frac{1}{6}$ m

EXERCISE – II

Q.1 (a) $a = g\cot\theta$, (b) $\mu_{min} = \frac{m\sin\theta\cos\theta}{m\cos^2\theta + M}$ Q.2 $\frac{3}{2\sqrt{2}}$

Q.3 (a) $a_A = \frac{3g\downarrow}{2} = a_B$; $a_C = 0$; T=mg/2; (b) $a_A = 2g\uparrow$, $a_B = 2g\downarrow$, $a_c = 0$, T=0; (c) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, T=$\frac{3mg}{2}$; T=2mg

Q.4 2g/23 Q.5 (a) $\eta = \frac{3}{5}$; (b) acceleration = 4 m/s^2

Q.6 (a) 

(b) 22.5 m/s ; (c) $\frac{275}{6}$ m; (d) $\sqrt{2}$ sec

Q.7 (a) (i) $a_1 = a_2 = 3.2$ m/s^2 , (ii) $a_1 = 5.75$ m/s^2 , $a_2 = 2$m/s^2 ; (b) $a_1 = 5$ m/s^2 , $a_2 = -10/3$ m/s^2

Q.8 (i)90N,(ii)112.5N(iii)150N Q.9 $\mu_s = 0.4$, $\mu_k = 0.3$

Q.10 $\Delta r = \frac{mg\cot\alpha}{4\pi^2 k}$, 1cm

EXERCISE – III

Q.1 B Q.2 (i) zero , (ii) can't be determined, (iii) can't be determined Q.3 B

Q.4 (b) a = 3/5 m/s^2 , T = 18 N, F = 60N Q.5 C Q.6 D Q.7 A

Q.8 11.313 m Q.9 C Q.10 10 m/s^2

(CIRCULAR MOTION & WORK POWER ENERGY)

EXERCISE – I

Q.1 $\tan^{-1}\left\{\frac{l\sin\theta}{2h}\right\}$ Q.2 $\frac{15\sqrt{3}}{2}$ N Q.3 2 tanθ Q.4 – 1 J

Q.5 $\frac{2\sqrt{2}\,v^2}{\pi R}$ Q.6 $\frac{2m_1^2g^2}{k}$ Q.7 $\frac{1}{6}mv^2$ Q.8 $9\sqrt{10}$ m/s^2

Q.9 $4\sqrt{5}$ m/s Q.10 9 Q.11 $-ka^2/4$ Q.12 $\left(1-\sqrt{3}/2\right)$mg

Q.13 80 kW Q.14 $\sqrt{2g}$ rad/s Q.15 $\frac{1}{3}\sqrt{\frac{gl}{3}}$ Q.16 6mg

Q.17 9*a*/2 Q.18 $\frac{7}{12}$m Q.19 2 sec Q.20 625 J

Q.21 $-3\hat{k}$ rad/s^2, $-2\hat{k}$ rad/s

Q.22 (a) 2 rad/s^2, (b) 12+2t for $t \le 2$s, 16 for $t \ge 2$s, (c) $\sqrt{28565} \simeq 169$, 256 m/s^2 (d) 44 rad

Q.23 0.1875 Q.24 P Q.25 $\frac{25}{24}$

EXERCISE – II

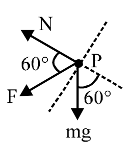
Q.1 $F = -3ax^2 + b$, $x = \sqrt{\frac{b}{a}}$, $KE_{max} = \frac{2b}{3\sqrt{3}}\sqrt{\frac{b}{a}}$ Q.2 2 m/s Q.3 $v = v_0$, $5\pi a/v_0$

Q.4 500N/m Q.6 $u = v\sec\theta$, $v = \frac{40}{\sqrt{41}}$ m/s

Q.7 $a_N = \frac{vR}{(2Rt - vt^2)}$, $a_t = \frac{R(vt-R)v^{1/2}}{(2Rt-vt^2)^{3/2}}$ Q.8 up, 10cm

Q.9 $\theta_{max} = \pi/2$, T=mg(3sinθ+3cosθ–2) Q.10 4, $\sqrt{gl/8}$

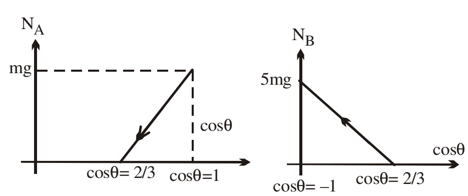
EXERCISE – III

Q.1 

, $a_t = 5\sqrt{3}\,g/8$, N=3mg/8 Q.2 (i) 36N, (ii) 11.66rad/sec ,(iii) 0.1m, 0.2m

Q.3 F=-8mgi-mgj, h=3R Q.4 C Q.5 D Q.6 $u = \sqrt{gL\left(\frac{3\sqrt{3}}{2}+2\right)}$ Q.7 A

Q.8 A Q.9 A Q.10 5.79 m/s Q.11 C Q.12 D Q.13 B

Q.14 (a) N=3mg cosθ – 2mg, (b) 

Q.15 A

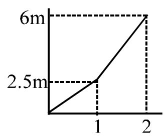
Q.16 $\vec{P}_{PM} = m\,\bar{v}_{PM} = -mv_2 \sin \omega t\,\hat{i} + m(v_2 \cos \omega t - v_1)\,\hat{j}$ Q.17 A

(CENTRE OF MASS & MOMENTUM)

EXERCISE – I

Q.1 $\dfrac{5v}{R\cos\theta}$ Q.2 (a) $\dfrac{mg(h_3 - h_2)}{(h_2 - h_1)}$; (b) 0 Q.3 $\dfrac{[mu\sqrt{3}]}{4}$ Q.4 11/14

Q.5 $\cos^{-1}(3/4)$ Q.6 100 m Q.7 $\dfrac{5h}{16}$ Q.8 4/3 m/s

Q.9 L/4 Q.10 

Q.11 $\dfrac{2nv\cos(\theta/2)}{n+1}$ Q.12 $g/2$ Q.13 $\dfrac{R}{4\pi - 2}$ Q.14 2 : 1

Q.15 l Q.16 $\dfrac{1}{2}$ Q.17 $\dfrac{Mu^2}{2g(M+2m)}$ Q.18 x = 6m Q.19 6.21 W

Q.20 $v_{\text{heavy ball}} = \dfrac{u_0}{27}$, $v_{\text{first ball}} = \dfrac{4u_0}{27}$, $v_{\text{second ball}} = \dfrac{4u_0}{9}$, $v_{\text{third ball}} = \dfrac{4u_0}{3}$ Q.21 $\sqrt{\dfrac{3gx}{8}}$

Q.22 $\sqrt{\dfrac{mv^2}{12k}}$

EXERCISE – II

Q.1 x = 3 units, $\tan\theta = 2/3$ Q.2 $5/\sqrt{17}$ cm, 153L/80u

Q.3 $v_c = -\dfrac{v_0}{15}$, $v_B = \dfrac{\sqrt{208}\,v_0}{15}$, $v_A = \dfrac{4\,v_0}{15}$ Q.4 40 cm

Q.5 37^0 Q.6 (a) 0.66, (b) 4 m

Q.7 impulse = $m(-3\hat{i} + 4\hat{j})$, $e = \dfrac{9}{16}$ Q.8 $v_1 = \dfrac{1}{\sqrt{3}}$ m/s, $v_2 = \dfrac{2}{\sqrt{3}}$ m/s

Q.9 (a) $\dfrac{m}{l}(gy + v_0^2)$, (b) $mg\left(1 - \dfrac{y}{l}\right)$, (c) $\dfrac{m{v_0}^2 y}{2l}$ Q.10 $2\sqrt{2a_0 l}$

EXERCISE – III

Q.1 (i), (iii) Q.2 4.13×10^{-2}m Q.3 (L + 2R, 0) Q.4 4/3 m/s, 35/3 J

Q.5 A Q.6 2.5 m, 0, 319 m Q.7 90^o

Q.8 $e = \dfrac{5+\sqrt{3}}{8}$, $M = \dfrac{26}{\sqrt{3}}$ kg, Q.9 2 in case I Q.10 C Q.11 C

Q.12 $t_0 = 12$ sec, $v = \dfrac{100\sqrt{3}}{11}$ Q.13 C Q.14 10 m/s