lnx# Jh j.kNkMnkl th egkjkt FUNCTIONS (ASSERTION AND REASON)

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.

(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.

(C) **Statement – 1** is True, **Statement – 2** is False.

(D) **Statement – 1** is False, **Statement – 2** is True.

1. Let $f(x) = \cos 3\pi x + \sin \sqrt{3}\pi x$.

Statement – 1 : f(x) is not a periodic function.

Statement – 2 : L.C.M. of rational and irrational does not exist

2. **Statement – 1:** If f(x) = ax + b and the equation $f(x) = f^{-1}(x)$ is satisfied by every real value of x, then $a \in R$ and $b = -1$.

Statement – 2: If f(x) = ax + b and the equation $f(x) = f^{-1}(x)$ is satisfied by every real value of x, then $a = -1$ and $b \in R$.

3. **Statements-1:** If f(x) = x and $F(x) = \frac{x^2}{x}$, then F(x) = f(x) always

Statements-2: At x = 0, F(x) is not defined.

4. **Statement–1 :** If $f(x) = \frac{1}{1-x}$, $x \neq 0, 1$, then the graph of the function y = f (f(f(x)), x > 1 is a straight line **Statement–2 :** f(f(x)))) = x

5. Let f(1 + x) = f(1 – x) and f(4 + x) = f(4– x)

Statement–1 : f(x) is periodic with period 6

Statement–2 : 6 is not necessarily fundamental period of f(x)

6. **Statement–1 :** Period of the function $f(x) = \sqrt{1+\sin 2x} + e^{\{x\}}$ does not exist

Statement–2 : LCM of rational and irrational does not exist

7. **Statement–1 :** Domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$ is $(-\infty, 0)$ **Statement–2:** $|x| - x > 0$ for $x \in R^-$

8. **Statement–1 :** Range of $f(x) = \sqrt{4-x^2}$ is [0, 2]

Statement–2 : f(x) is increasing for $0 \le x \le 2$ and decreasing for $-2 \le x \le 0$.

9. Let a, b $\in$ R, a $\ne$ b and let $f(x) = \dfrac{a+x}{b+x}$.

Statement–1 : f is a one–one function. **Statement–2** : Range of f is R – {1}

10. **Statement–1** : $\sin x + \cos(\pi x)$ is a non–periodic function.

Statement–2 : Least common multiple of the periods of sin x and cos (πx) is an irrational number.

11. **Statement–1:** The graph of f(x) is symmetrical about the line x = 1, then, f(1 + x) = f(1 – x).

Statement–2 : even functions are symmetric about the y-axis.

12. **Statement–1 :** Period of $f(x) = \sin\dfrac{\pi x}{(n-1)!} + \cos\dfrac{\pi x}{n!}$ is 2(n)!

Statement–2 : Period of $|\cos x| + |\sin x| + 3$ is π.

13. **Statement–1 :** Number of solutions of $\tan(|\tan^{-1}x|) = \cos|x|$ equals 2 **Statement–2** : ?

14. **Statement–1 :** Graph of an even function is symmetrical about y–axis

Statement–2 : If f(x) = cosx has x (+)ve solution then total number of solution of the above equation is 2n. (when f(x) is continuous even function).

15. If f is a polynomial function satisfying $2 + f(x).f(y) = f(x) + f(y) + f(xy)\ \forall\ x, y \in R$

Statement-1: f(2) = 5 which implies f(5) = 26

Statement-2: If f(x) is a polynomial of degree 'n' satisfying f(x) + f(1/x) = f(x). f(1/x), then $f(x) = 1\, x^n + 1$

16. **Statement-1:** The range of the function $\sin^{-1} + \cos^{-1}x + \tan^{-1}x$ is [π/4, 3π/4]

Statement-2: $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \le 1$ and $\tan^{-1}x$ is defined for all 'x'.

17. A function f(x) is defined as $f(x) = \begin{cases} 0 & \text{where } x \text{ is rational} \\ 1 & \text{where } x \text{ is irrational} \end{cases}$

Statement-1 : f(x) is discontinuous at xll x$\in$R

Statement-2 : In the neighbourhood of any rational number there are irrational numbers and in the vincity of any irrational number there are rational numbers.

18. Let $f(x) = \sin\left(2\sqrt{3}\,\pi x\right) + \cos\left(3\sqrt{3}\,\pi x\right)$

Statement-1 : f(x) is a periodic function

Statement-2: LCM of two irrational numbers of two similar kind exists.

19. **Statements-1:** The domain of the function $f(x) = \cos^{-1}x + \tan^{-1}x + \sin^{-1}x$ is [-1, 1]

Statements-2: $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \le 1$and $\tan^{-1}x$ is defined for all x.

20. **Statement-1 :** The period of f(x) = = sin2x cos [2x] – cos2x sin [2x] is 1/2

Statements-2: The period of x – [x] is 1, where [·] denotes greatest integer function.

21. **Statements-1:** If the function f : R $\to$ R be such that f(x) = x – [x], where [·] denotes the greatest integer less than or equal to x, then $f^{-1}(x)$ is equals to [x] + x

Statements-2: Function ‘f ’ is invertible iff is one-one and onto.

22. **Statements-1 :** Period of f(x) = sin 4π {x} + tan π [x] were, [·] & {·} denote we G.I.F. & fractional part respectively is 1.

Statements-2: A function f(x) is said to be periodic if there exist a positive number T independent of x such that f(T + x) = f(x). The smallest such positive value of T is called the period or fundamental period.

23. **Statements-1:** $f(x) = \dfrac{x+1}{x-1}$ is one-one function

Statements-2: $\dfrac{x+1}{x-1}$ is monotonically decreasing function and every decreasing function is one-one.

24. **Statements-1:** f(x) = sin2x (|sinx| - |cosx|) is periodic with fundamental period π/2

Statements-2: When two or more than two functions are given in subtraction or multiplication form we take the L.C.M. of fundamental periods of all the functions to find the period.

25. **Statements-1:** e^x = lnx has one solution.

Statements-2: If f(x) = x ⇒ f(x) = $f^{-1}(x)$ have a solution on y = x.

26. **Statements-1:** F(x) = x + sinx. G(x) = -x

H(x) = F(X) + G(x), is a periodic function.

Statements-2: If F(x) is a non-periodic function & g(x) is a non-periodic function then h(x) = f(x) ± g(x) will be a periodic function.

27. **Statements-1:** $f(x) = \begin{cases} x+1, & x \ge 0 \\ x-1, & x < 0 \end{cases}$ is an odd function.

Statements-2: If y = f(x) is an odd function and x = 0 lies in the domain of f(x) then f(0) = 0

28. **Statements-1:** $f(x) = \begin{cases} x; & x \in Q \\ -x; & x \in Q^C \end{cases}$ is one to one and non-monotonic function.

Statements-2: Every one to one function is monotonic.

29. **Statement–1 :** Let f : [1, 2] ∪ [5, 6] → [1, 2] ∪ [5, 6] defined as $f(x) = \begin{cases} x+4, & x \in [1, 2] \\ -x+7, & x \in [5, 6] \end{cases}$ then the equation f(x) = $f^{-1}(x)$ has two solutions.

Statements-2: f(x) = $f^{-1}(x)$ has solutions only on y = x line.

30. **Statements-1:** The function $\dfrac{px+q}{rx+s}$ (ps – qr ≠ 0) cannot attain the value p/r.

Statements-2: The domain of the function g(y) = $\dfrac{q-sy}{ry-p}$ is all real except a/c.

31. **Statements-1:** The period of f(x) = sin [2] xcos [2x] – cos2x sin [2x] is 1/2

Statements-2: The period of x – [x] is 1.

32. **Statements-1:** If f is even function, g is odd function then $\dfrac{b}{g}$ (g ≠ 0) is an odd function.

Statements-2: If f(–x) = –f(x) for every x of its domain, then f(x) is called an odd function and if f(–x) = f(x) for every x of its domain, then f(x) is called an even function.

33. **Statements-1:** $f : A \to B$ and $g : B \to C$ are two function then $(gof)^{-1} = f^{-1} og^{-1}$.
Statements-2: $f : A \to B$ and $g : B \to C$ are bijections then f^{-1} & g^{-1} are also bijections.

34. **Statements-1:** The domain of the function $f(x) = \sqrt{\log_2 \sin x}$ is $(4n + 1)\dfrac{\pi}{2}$, $n \in N$.
Statements-2: Expression under even root should be ≥ 0

35. **Statements-1:** The function $f : R \to R$ given $f(x) = \log_a (x + \sqrt{x^2 + 1})$ $a > 0$, $a \neq 1$ is invertible.
Statements-2: f is many one into.

36. **Statements-1:** $\phi(x) = \sin(\cos x)$ $x \in \left[0, \dfrac{\pi}{2}\right]$ is a one-one function.
Statements-2: $\phi'(x) \leq \forall x \in \left[0, \dfrac{\pi}{2}\right]$

37. **Statements-1:** For the equation $kx^2 + (2 - k)x + 1 = 0$ $k \in R - \{0\}$ exactly one root lie in (0, 1).
Statements-2: If $f(k_1)\, f(k_2) < 0$ (f(x) is a polynomial) then exactly one root of $f(x) = 0$ lie in (k_1, k_2).

38. **Statements-1:** Domain of $f(x) = \sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$ is $\{-1, 1\}$
Statements-2: $x + \dfrac{1}{x} \geq 2$ when $x > 0$ and $x + \dfrac{1}{x} \leq -2$ when $x < 0$.

39. **Statements-1:** Range of $f(x) = |x|(|x| + 2) + 3$ is $[3, \infty)$
Statements-2: If a function f(x) is defined $\forall\, x \in R$ and for $x \geq 0$ if $a \leq f(x) \leq b$ and f(x) is even function than range of f(x) f(x) is [a, b].

40. **Statements-1:** Period of $\{x\} = 1$. **Statements-2:** Period of $[x] = 1$

41. **Statements-1:** Domain of $f = \phi$. If $f(x) = \dfrac{1}{\sqrt{[x] - x}}$
Statements-2: $[x] \leq x \;\forall\; x \in R$

42. **Statements-1:** The domain of the function $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is $[-1, 1]$
Statements-2: $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \leq 1$ and $\tan^{-1}x$ is defined for all 'x'

ANSWER KEY

1. A	**2. D**	**3. A**	**4. C**	**5. A**	**6. A**	**7. A**
8. C	**9. B**	**10. C**	**11. A**	**12. C**	**13. B**	**14. A**
15. A	**16. A**	**17. A**	**18. A**	**19. A**	**20. A**	**21. D**
22. A	**23. A**	**24. A**	**25. D**	**26. C**	**27. D**	**28. C**
29. C	**30. A**	**31. A**	**32. A**	**33. D**	**34. A**	**35. C**
36. A	**37. C**	**38. A**	**39. A**	**40. A**	**41. A**	**42. A**

SOLUTIONS

4. $f(f(x)) = \dfrac{1}{1-f(x)} = \dfrac{1}{1-\dfrac{1}{1-x}} = \dfrac{x-1}{x}$

$\therefore f(f(f(x))) = \dfrac{1}{1-f(f(x))} = \dfrac{1}{1-\dfrac{x-1}{x}} = x$ Ans. C

5. $f(1 + x) = f(1 - x)$ ... (1)

$f(4 + x) = f(4 - x)$ ... (2)

$x \to 1 - x$ in (1) $\Rightarrow f(1 - x) = f(x)$ ... (3)

$x \to 4 - x$ in (2) $\Rightarrow f(2 - x)\ f(8 - x) = f(x)$ ... (4)

(1) and (4) $\Rightarrow f(2 - x) = f(8 - x)$ .... (5)

Use $x \to x - x$ in (5), we get

$f(x) = f(6 + x)$

$\Rightarrow$ f(x) is periodic with period 6

Obviously 6 is not necessary the fundamental period. Ans. A

6. L.C.M. of $\{\pi, 1\}$ does not exist $\therefore$ (A) is the correct option.

7. (a) Clearly both are true and statement – II is correct explantion of Statement – I .

8. (c) $f'(x) = \dfrac{-x}{\sqrt{4-x^2}}$ $\therefore$ f(x) is increasing for $-2 \le x \le 0$ and decreasing for $0 \le x \le 2$.

9. Suppose a > b. Statement – II is true as $f'(x) = \dfrac{b-a}{(b+x)^2}$, which is always negative and hence monotonic in its continuous part. Also $\lim\limits_{x\to -b^+} f(x) = \infty$ and $\lim\limits_{x\to -b^-} f(x) = -\infty$. Moreover $\lim\limits_{x\to\infty} f(x) = 1+$ and $\lim\limits_{x\to-\infty} f(x) = -1-$. Hence range of f is R – {1}.

F is obviously one–one as $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

However statement – II is not a correct reasoning for statement – I

Hence (b) is the correct answer.

10. Statement – I is true, as period of sin x and cos πx are 2π and 2 respectively whose L.C.M does not exist.

Obviously statement – II is false Hence (c) is the correct answer.

11. Graph of f(x) is symmetric about the line x = 0 if f(- x) = f(x) i.e. if f(0 – x) = f(0 + x)

$\therefore$ Graph of y = f(x) is symmetric about x = 1, if f(1 + x) = f(1 – x).

Hence (a) is the correct answer.

12. Period of $\sin \dfrac{\pi x}{(n-1)!} = 2(n-1)!$ Period of $\cos \dfrac{\pi x}{n!} = 2(n)!$

$\Rightarrow$ Period of f(x) = L.C.M of 2(n – 1)! And 2(n)! = 2(n!)

Now, $f(x) = |\text{copsx}| + |\sin x| + 3 = \sqrt{1 + |\sin 2x|} + 3$

$\therefore$ f(x) is periodic function with period = $\frac{\pi}{2}$. Hence (c) is the correct answer.

13. $\tan(|\tan^{-1}x|) = |x|$, since $|\tan^{-1}x| = \tan^{-1}|x|$

Obviously cos|x| and |x| meets at exactly two points

$\therefore$ (B) is the correct option.

14. (A)Since cos n is also even function. Therefore solution of cosx = f(x) is always sym. also out y–axis.

19. (a) Both A and R are obviously correct. **20.** (a) f(x) = x [x]

f(x + 1) = x + 1 – ([x] + 1) = x – [x]

So, period of x – [x] is 1.

Let f(x) = sin (2x – [2x])

$$f\left(x+\frac{1}{2}\right)=\sin\left(2\left(x+\frac{1}{2}\right)-\left[2\left(x+\frac{1}{2}\right)\right]\right)$$

= sin (2x + 1 – [2x] – 1) = sin (2x – [2x]) So, period is 1/2

21. f(1) = 1 – 1 = 0 f(0) = 0

$\therefore$ f is not one-one $\therefore$ $f^{-1}(x)$ is not defined Ans. (D)

22. Clearly $\tan \pi[x] = 0 \ \forall \ x \in R$ and period of $\sin 4\pi\{x\} = 1$. Ans. (A)

23. $f(x) = \frac{x+1}{x-1}$ $f'(x) = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2} < 0$

So f(x) is monotonically decreasing & every monotonic function is one-one.

So ‘a’ is correct.

24. f(x) = sin2x (|sinx| -|cosx|) is periodic with period π/2 because f(π/2 + x) = sin 2 (π/2 + x) (|sin (π/2 + x)| -|cos (π/2 + x)|)

= sin (π + 2x) (|cosx| - |sinx|) = -sin2x (|cosx| - |sinx|)

= sin2x (|sinx| - |cosx|)

Sometimes f(x + r) = f(x) where r is less than the L.C.M. of periods of all the function, but according to definition of periodicity, period must be least and positive, so ‘r’ is the fundamental period.

So ‘f’ is correct.

27. (D) If f(x) is an odd function, then $f(x) + f(-x) = 0 \ \forall \ x \in D_f$

28. (C) For one to one function if $x_1 \neq x_2$

$\Rightarrow f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in D_f$ $\sqrt{3} > 1$

but $f(\sqrt{3}) < f(1)$ and 3 > 1

f(5) > f(1) f(x) is one-to-one but non-monotonic

29. (C) $\left(\frac{3}{2}, \frac{11}{2}\right)$ and $\left(\frac{11}{2}, \frac{3}{2}\right)$ both lie on y = f(x) then they will also lie on $y = f^{-1}(x) \Rightarrow$ there are two solutions and they do not lie on y = x.

30. If we take $y = \frac{px+q}{rx+s}$ then $x = \frac{q-sx}{rx-p} \Rightarrow$ x does not exist if y = p/r

Thus statement-1 is correct and follows from statement-2 (A)

31. f(x) = sin(2x – [2x] $\qquad$ f(x + 1/2) = $\sin\left(2x+1-\left[2\left(x+\frac{1}{2}\right)\right]\right)$

= sin (2x + 1 – [2x] – 1] $\qquad$ = sin (2x – [2x].) i.e., period is 1/2.

f(x) = x – [x]

f(x + 1) = x + 1 – ([x] + 1) = x – [x] $\qquad$ i.e., period is 1. $\qquad$ (A)

32. (A) Let h(x) = $\frac{f(x)}{g(x)}$

$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$

$\therefore$ h(x) = $\frac{f}{g}$ is an odd function.

33. (D) Assertion : f : A → B, g : B → C are two functions then $(gof)^{-1} \neq f^{-1}\,og^{-1}$ (since functions need not posses inverses. Reason : Bijective functions are invertibles.

34. (A) for f(x) to be real $\log_2(\sin x) \geq 0$

$\Rightarrow \sin x \geq 2^o$ $\qquad$ $\Rightarrow \sin x = 1 \quad \Rightarrow \quad x = (4n+1)\frac{\pi}{2}, n \in N.$

35. (C) f is injective since x ≠ y (x, y ∈ R)

$\Rightarrow \log_a\left\{x+\sqrt{x^2+1}\right\} \neq \log_a\left\{y+\sqrt{y^2+1}\right\}$

$\Rightarrow f(x) \neq f(y)$

f is onto because $\log_a\left(x+\sqrt{x^2+1}\right) = y$ $\qquad$ $\Rightarrow x = \frac{a^y - a^{-y}}{2}$.

40. Since {x} = x – [x]

$\therefore$ {x + 1} = x + 1 – [x + 1]

= x + 1 – [x] – 1 $\qquad$ = x – [x] = [x]

Period of [x] = 1 $\qquad$ Ans (A)

41. f(x) = $\frac{1}{\sqrt{[x]-x}}$ $[x] - x \neq 0$

$[x] \neq x \to [x] > x$ It is imposible or $[x] \leq x$

So the domain of f is ϕ $\qquad$ because reason $[x] \leq x$ $\qquad$ Ans. (A)

Imp. Que. From Competitive exams

1. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$ for $x \in R$, then $f(2002) =$ **[EAMCET 2002]**

(a) 1 $\qquad$ (b) 2 $\qquad$ (c) 3 $\qquad$ (d) 4

2. If $f: R \to R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is **[AIEEE 2003]**

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $7n(n+1)$ (d) $\frac{7n(n+1)}{2}$

3. Suppose $f:[2, 2] \to R$ is defined by $f(x) = \begin{cases} -1 & \text{for } -2 \le x \le 0 \\ x-1 & \text{for } 0 \le x \le 2 \end{cases}$, then $\{x \in (-2, 2) : x \le 0$ and $f(|x|) = x\} =$

(a) $\{-1\}$ (b) $\{0\}$ (c) $\{-1/2\}$ (d) ϕ **[EAMCET 2003]**

4. If $f(x) = \text{sgn}(x^3)$, then **[DCE 2001]**

(a) f is continuous but not derivable at $x = 0$ (b) $f'(0^+) = 2$

(c) $f'(0^-) = 1$ (d) f is not derivable at $x = 0$

5. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = |x|$ and $g(x) = |x|$ for each $x \in R$, then $\{x \in R : g(f(x)) \le f(g(x))\} =$

(a) $Z \cup (-\infty, 0)$ (b) $(-\infty, 0)$ (c) Z (d) R **[EAMCET 2003]**

6. For a real number x, $[x]$ denotes the integral part of x. The value of

$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + ... + \left[\frac{1}{2} + \frac{99}{100}\right]$ is **[IIT Screening 1994]**

(a) 49 (b) 50 (c) 48 (d) 51

7. If function $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$; $(-1 < x < 1)$ and $g(x) = \sqrt{3 + 4x - 4x^2}$, then the domain of gof is **[IIT 1990]**

(a) $(-1, 1)$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left[-1, \frac{1}{2}\right]$ (d) $\left[-\frac{1}{2}, -1\right]$

8. The domain of the function $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is **[DCE 2000]**

(a) $]-3, -2.5[\cup]-2.5, -2[$ (b) $[-2, 0[\cup]0, 1[$ (c) $]0, 1[$ (d) None of these

9. The domain of definition of the function $y(x)$ given by $2^x + 2^y = 2$ is **[IIT Screening 2000; DCE 2001]**

(a) (0, 1] (b) [0, 1] (c) $(-\infty, 0]$ (d) $(-\infty, 1)$

10. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and $m(b)$ the minimum value of $f(x)$ for a given b. As b varies, the range of $m(b)$ is **[IIT Screening 2001]**

(a) [0, 1] (b) $\left(0, \frac{1}{2}\right]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) $(0, 1]$

11. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is **[AIEEE 2004]**

(a) {1, 2, 3, 4, 5} (b) (1, 2, 3, 4, 5, 6) (c) {1, 2, 3, 4} (d) {1, 2, 3}

12. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval **[IIT 1994]**

(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$ (c) $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2\right)$

13. Let $f(x) = (x+1)^2 - 1$, $(x \ge -1)$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ is **[IIT 1995]**

(a) Empty (b) {0, –1} (c) {0, 1, –1} (d) $\left\{0, -1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$

14. If f is an even function defined on the interval (–5, 5), then four real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are

(a) $\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}$ (b) $\frac{-5+\sqrt{3}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

(c) $\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}, \frac{5+\sqrt{3}}{2}$ (d) $-3-\sqrt{5}, -3+\sqrt{5}, 3-\sqrt{5}, 3+\sqrt{5}$ **[IIT 1996]**

15. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right)=1$, then $(gof)(x)=$ **[IIT 1996]**

(a) –2 (b) –1 (c) 2 (d) 1

16. If $g(f(x))=|\sin x|$ and $f(g(x))=(\sin\sqrt{x})^2$, then **[IIT 1998]**

(a) $f(x)=\sin^2 x, g(x)=\sqrt{x}$ (b) $f(x)=\sin x, g(x)=|x|$ (c) $f(x)=x^2, g(x)=\sin\sqrt{x}$ (d) *f* and *g* cannot be determined

17. If $f(x)=3x+10$, $g(x)=x^2-1$, then $(fog)^{-1}$ is equal to **[UPSEAT 2001]**

(a) $\left(\frac{x-7}{3}\right)^{1/2}$ (b) $\left(\frac{x+7}{3}\right)^{1/2}$ (c) $\left(\frac{x-3}{7}\right)^{1/2}$ (d) $\left(\frac{x+3}{7}\right)^{1/2}$

18. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=2x+3$ and $g(x)=x^2+7$, then the values of *x* such that $g(f(x))=8$ are

(a) 1, 2 (b) –1, 2 (c) –1, –2 (d) 1, –2 **[EAMCET 2000, 03]**

19. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)=$ **[IIT 1978, 84; RPET 1997, 2001; UPSEAT 2003; Pb. CET 2003]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{2}{\pi}$ (d) 0

20. True statement for $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{2+3x}-\sqrt{2-3x}}$ is **[BIT Ranchi 1982]**

(a) Does not exist (b) Lies between 0 and $\frac{1}{2}$ (c) Lies between $\frac{1}{2}$ and 1 (d) Greater then 1

21. $\lim_{x\to\infty}\frac{x^n}{e^x}=0$ for **[IIT 1992]**

(a) No value of *n* (b) *n* is any whole number (c) $n=0$ only (d) $n=2$ only

22. $\lim_{n\to\infty}\sin[\pi\sqrt{n^2+1}]=$

(a) ∞ (b) 0 (c) Does not exist (d) None of these

23. If [.] denotes the greatest integer less than or equal to *x*, then the value of $\lim_{x\to 1}(1-x+[x-1]+[1-x])$ is

(a) 0 (b) 1 (c) –1 (d) None of these

24. The values of *a* and *b* such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, are **[Roorkee 1996]**

(a) $\frac{5}{2},\frac{3}{2}$ (b) $\frac{5}{2},-\frac{3}{2}$ (c) $-\frac{5}{2},-\frac{3}{2}$ (d) None of these

25. If $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=-1$, then **[EAMCET 2003]**

(a) $a=1$ (b) $a=0$ (c) $a=e$ (d) None of these

26. If $x_1=3$ and $x_{n+1}=\sqrt{2+x_n}, n\ge 1$, then $\lim_{n\to\infty}x_n$ is equal to

(a) –1 (b) 2 (c) $\sqrt{5}$ (d) 3

27. The value of $\lim_{x\to\frac{\pi}{2}}\frac{\int_{\pi/2}^{x}t\,dt}{\sin(2x-\pi)}$ is **[MP PET 1998]**

(a) ∞ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

The $\lim_{x\to 0}(\cos x)^{\cot x}$ is **[RPET 1999]**(a)–1(b)0c)1(d)None of these

28. The integer *n* for which $\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number is **[IIT Screening 2002]**

(a) 1 (b) 2 (c) 3 (d) 4

29. If *f* is strictly increasing function, then $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is equal to **[IIT Screening 2004]**

(a) 0 (b) 1 (c) –1 (d) 2

30. If $f(x)=\begin{cases}x^2-3, 2<x<3\\ 2x+5, 3<x<4\end{cases}$, the equation whose roots are $\lim_{x\to 3^-}f(x)$ and $\lim_{x\to 3^+}f(x)$ is **[Orissa JEE 2004]**

(a) $x^2-7x+3=0$ (b) $x^2-20x+66=0$ (c) $x^2-17x+66=0$ (d) $x^2-18x+60=0$

31. The function $f(x)=[x]\cos\left[\frac{2x-1}{2}\right]\pi$, where $[.]$ denotes the greatest integer function, is discontinuous at **[IIT 1995]**

(a) All *x* (b) No *x* (c) All integer points (d) *x* which is not an integer

32. Let $f(x)$ be defined for all $x>0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right)=f(x)-f(y)$ for all *x*, *y* and $f(e)=1$, then **[IIT 1995]**

(a) $f(x)=\ln x$ (b) $f(x)$ is bounded (c) $f\left(\frac{1}{x}\right)\to 0$ as $x\to 0$ (d) $x\,f(x)\to 1$ as $x\to 0$

33. The value of p for which the function $f(x)=\begin{cases}\frac{(4^x-1)^3}{\sin\frac{x}{p}\log\left[1+\frac{x^2}{3}\right]}, x\neq 0\\ 12(\log 4)^3,\ x=0\end{cases}$ may be continuous at $x=0$, is **[Orissa JEE 2004]**

(a) 1 (b) 2 (c) 3 (d) None of these

34. The function $f(x)=[x]^2-[x^2]$, (where [*y*] is the greatest integer less than or equal to *y*),is discontinuous at **[IIT 1999]**

(a) All integers (b) All integers except 0 and 1 (c) All integers except 0 (d) All integers except 1

35. If $f(x)=\begin{cases}xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0\\ 0, & x=0\end{cases}$, then $f(x)$ is **[AIEEE 2003]**

(a) Continuous as well as differentiable for all *x* (b) Continuous for all *x* but not differentiable at $x=0$

(c) Neither differentiable nor continuous at $x=0$ (d) Discontinuous every where

36. Let $f(x)=\frac{1-\tan x}{4x-\pi}$, $x\neq\frac{\pi}{4}$, $x\in\left[0,\frac{\pi}{2}\right]$, If $f(x)$ is continuous in $\left[0,\frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is **[AIEEE 2004]**

(a) –1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

37. Let $g(x)=x.f(x)$, where $f(x)=\begin{cases}x\sin\frac{1}{x}, x\neq 0\\ 0, x=0\end{cases}$ at $x=0$ **[IIT Screening 1994; UPSEAT 2004]**

(a) *g* is differentiable but *g*' is not continuous (b) *g* is differentiable while *f* is not

(c) Both *f* and *g* are differentiable (d) *g* is differentiable and *g*' is continuous

38. The function $f(x)=\max[(1-x),(1+x),2]$, $x\in(-\infty,\infty)$, is **[IIT 1995]**

(a) Continuous at all points (b) Differentiable at all points (c) Differentiable at all points except at $x=1$ and $x=-1$

(d) Continuous at all points except at $x=1$ and $x=-1$ where it is discontinuous

39. The function $f(x)=|x|+|x-1|$ is **[RPET 1996; Kurukshetra CEE 2002]**

(a) Continuous at $x=1$, but not differentiable at $x=1$ (b) Both continuous and differentiable at $x=1$

(c) Not continuous at $x=1$ (d) Not differentiable at $x=1$

ANSWER: Imp. Que. From Competitive exams

1	a	2	d	3	c	4	d	5	d
6	b	7	a	8	b	9	d	10	d
11	d	12	d	13	d	14	a	15	d
16	a	17	a	18	c	19	c	20	b
21	b	22	b	23	c	24	c	25	a
26	b	27	c	28	c	29	c	30	c
31	c	32	c	33	a	34	d	35	d
36	b	37	c	38	a,b	39	a,c	40	a