EXERCISE–5

Part : (A) Only one correct option

1. The locus of a point P which moves such that $PA^2 - PB^2 = 2k^2$ where A and B are (3, 4, 5) and (– 1, 3 – 7) respectively is
(A) $8x + 2y + 24z - 9 + 2k^2 = 0$ (B) $8x + 2y + 24z - 2k^2 = 0$
(C) $8x + 2y + 24z + 9 + 2k^2 = 0$ (D) none of these

2. The position vectors of three points A, B, C are $\hat{i} + 2\hat{j} + 3\hat{k} \cdot 2\hat{i} + 3\hat{j} + \hat{k}$ & $3\hat{i} + \hat{j} + 2\hat{k}$. A unit vector perpendicular to the plane of the triangle ABC is:
(A) $\left(-\frac{1}{\sqrt{3}}\right)\left(\hat{i} + \hat{j} + \hat{k}\right)$ (B) $\left(\frac{1}{\sqrt{3}}\right)\left(\hat{i} - \hat{j} + \hat{k}\right)$ (C) $\left(\frac{1}{\sqrt{3}}\right)\left(\hat{i} + \hat{j} - \hat{k}\right)$ (D) none

3. The square of the perpendicular distance of a point P (p, q, r) from a line through A(a, b, c) and whose direction cosine are ℓ, m, n is
(A) $\Sigma \{(q - b)\, n - (r - c)\, m\}^2$ (B) $\Sigma \{(q + b)\, n - (r + c)\, m\}^2$
(C) $\Sigma \{(q - b)\, n + (r - c)\, m\}^2$ (D) none of these

4. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is:
(A) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$ (B) $x^2 + 2y^2 + 3z^2 - x - 2y - 3z = 0$
(C) $x^2 + 4y^2 + 9z^2 + x + 2y + 3 = 0$ (D) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

5. The equation of the plane which bisects the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$ which contains the origin is
(A) $33x - 13y + 32z + 45 = 0$ (B) $x - 3y + z - 5 = 0$ (C) $33x + 13y + 32z + 45 = 0$ (D) None

6. The distance of the point of intersection of the line $x - 3 = (1/2)(y-4) = (1/2)(z-5)$ and the plane $x + y + z = 17$ from the point (3, 4, 5)
(A) 2 (B) 3 (C) 1/3 (D) 1/2

7. The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be mutually perpendicular provided
(A) $(a + a')(b + b')(c + c')$ (B) $aa' + cc' + 1 = 0$
(C) $aa' + bb' + cc' + 1 = 0$ (D) $(a + a')(b + b')(c + c') + 1 = 0$

8. A straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r}\,.\,\hat{n} = p$ in the point P whose position vector is
(A) $\vec{a} + \left(\frac{\vec{a}.\hat{n}}{\vec{b}.\hat{n}}\right)\vec{b}$ (B) $\vec{a} + \left(\frac{p - \vec{a}.\hat{n}}{\vec{b}.\hat{n}}\right)\vec{b}$ (C) $\vec{a} - \left(\frac{\vec{a}.\hat{n}}{\vec{b}.\hat{n}}\right)\vec{b}$ (D) $\vec{a} - \left(\frac{p - \vec{a}\,.\,\hat{n}}{\vec{b}\,.\,\hat{n}}\right)\vec{b}$

9. Equation of the angle bisector of the angle between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ & $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{-1}$ is
(A) $\frac{x-1}{2} = \frac{y-2}{2}$; $z - 3 = 0$ (B) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$
(C) $x - 1 = 0$; $\frac{y-2}{1} = \frac{z-3}{1}$ (D) None of these

10. The distance of the point, (–1, –5, –10) from the point of intersection of the line, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane, $x - y + z = 5$, is:
(A) 10 (B) 11 (C) 12 (D) 13

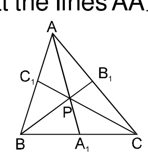
11. If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes, then the area of the triangle ABC =
(A) $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$ (B) $\frac{1}{2}$ (bc + ca + ab) (C) $\frac{1}{2}$ abc (D) $\frac{1}{2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

12. The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

13. If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines and θ is the angle between the lines then tan θ is equal to
(A) $\frac{\sqrt{\Sigma(b_1c_2 - b_2c_1)^2}}{a_1b_1 + a_2b_2 + c_1c_2}$ (B) $\frac{\sqrt{\Sigma(b_1c_2 - b_2c_1)^2}}{a_1a_2 + b_1b_2 + c_1c_2}$ (C) $\frac{\sqrt{\Sigma(b_1c_2 + b_2c_1)^2}}{a_1a_2 + b_1b_2 + c_1c_2}$ (D) none of these

14. A point moves so that the sum of the squares of its distances from the six faces of a cube given by $x = \pm 1$, $y = \pm 1$, $z = \pm 1$ is 10 units. The locus of the point is
(A) $x^2 + y^2 + z^2 = 1$ (B) $x^2 + y^2 + z^2 = 2$ (C) $x + y + z = 1$ (D) $x + y + z = 2$

15. In the adjacent figure 'P' is any arbitrary interior point of the triangle ABC such that the lines AA_1, BB_1 and CC_1 are concurrent at P. Value of $\frac{PA_1}{AA_1} + \frac{PB_1}{BB_1} + \frac{PC_1}{CC_1}$ is always equal to .



(A) 1 (B) 2 (C) 3 (D) None of these

16. The plane $ax + by + cz = d$, meets the coordinate axes at the points A, B and C respectively. Area of triangle ABC is equal to
(A) $\frac{d^2\sqrt{a^2 + b^2 + c^2}}{|abc|}$ (B) $\frac{d^2\sqrt{a^2 + b^2 + c^2}}{2|abc|}$ (C) $\frac{d^2\sqrt{a^2 + b^2 + c^2}}{4|abc|}$ (D) None of these

17. The length of projection, of the line segment joining the points (1, –1, 0) and (–1, 0, 1), to the plane $2x + y + 6z = 1$, is equal to

(A) $\sqrt{\frac{255}{61}}$ (B) $\sqrt{\frac{237}{61}}$ (C) $\sqrt{\frac{137}{61}}$ (D) $\sqrt{\frac{155}{61}}$

18. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a_1, b_1, c_1 from the origin, then

(A) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{a_1^2}+\frac{1}{b_1^2}+\frac{1}{c_1^2}$ (B) $\frac{1}{a^2}-\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{a_1^2}-\frac{1}{b_1^2}+\frac{1}{c_1^2}$

(C) $a^2 + b^2 + c^2 = a_1^2+b_1^2+c_1^2$ (D) $a^2 - b^2 + c^2 = a_1^2-b_1^2+c_1^2$

19. The angle between the plane $2x - y + z = 6$ and a plane perpendicular to the planes $x + y + 2z = 7$ and $x - y = 3$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

20. The non zero value of 'a' for which the lines $2x - y + 3z + 4 = 0 = ax + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are co-planar is :

(A) – 2 (B) 4 (C) 6 (D) 0

21. The equation of the plane through the point (–1, 2 , 0) and parallel to the lines

$$\frac{x}{3} = \frac{y+1}{0} = \frac{z-2}{-1} \text{ and } \frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1} \text{ is -}$$

(A) $x + 2y + 3z - 1 = 0$ (B) $x - 2y + 3z + 5 = 0$

(C) $x + y - 3z + 1 = 0$ (D) $x + y + 3z - 1 = 0$

22. The equation of the plane bisecting the acute angle between the planes $2x + y + 2z = 9$ and $3x - 4y + 12z + 13 = 0$ is :

(A) $11x + 33y - 34z - 172 = 0$ (B) $11x + 33y - 34z - 182 = 0$

(C) $41x - 7y + 86z - 52 = 0$ (D) $41x - 7y + 86z - 62 = 0$

23. The base of the pyramid AOBC is an equilateral triangle OBA with each side equal to $4\sqrt{2}$, 'O' is the origin of reference, AO is perpendicular to the plane of Δ OBC and $|\vec{AO}| = 2$. Then the cosine of the angle between the skew straight lines one passing through A and the mid point of OB and the other passing through O and the mid point of BC is :

(A) $-\frac{1}{\sqrt{2}}$ (B) 0 (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{\sqrt{2}}$

24. The coplanar points A , B , C , D are (2 – x , 2 , 2) , (2 , 2 – y , 2) , (2 , 2 , 2 – z) and (1 , 1 , 1) respectively . Then :

(A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$ (B) $x + y + z = 1$ (C) $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$ (D) none of these

25. Let the centre of the parallelopiped formed by $\vec{PA} = \hat{i} + 2\hat{j} + 2\hat{k}$; $\vec{PB} = 4\hat{i} - 3\hat{j} + \hat{k}$; $\vec{PC} = 3\hat{i} + 5\hat{j} - \hat{k}$ is given by the position vector (7, 6, 2). Then the position vector of the point P is:

(A) (3, 4, 1) (B) (6, 8, 2) (C) (1, 3, 4) (D) (2, 6, 8)

26. Taken on side $\vec{AC}$ of a triangle ABC, a point M such that $\vec{AM} = \frac{1}{3}\vec{AC}$. A point N is taken on the side $\vec{CB}$ such that $\vec{BN} = \vec{CB}$ then, for the point of intersection X of $\vec{AB}$ & $\vec{MN}$ which of the following holds good?

(A) $\vec{XB} = \frac{1}{3}\vec{AB}$ (B) $\vec{AX} = \frac{1}{3}\vec{AB}$ (C) $\vec{XN} = \frac{3}{4}\vec{MN}$ (D) $\vec{XM} = 3\,\vec{XN}$

27. If the acute angle that the vector, $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ makes with the plane of the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ & $\hat{i} - \hat{j} + 2\hat{k}$ is $\cot^{-1}\sqrt{2}$ then:

(A) $\alpha(\beta + \gamma) = \beta\gamma$ (B) $\beta(\gamma + \alpha) = \gamma\alpha$ (C) $\gamma(\alpha + \beta) = \alpha\beta$ (D) $\alpha\beta + \beta\gamma + \gamma\alpha = 0$

28. Locus of the point P for which $\vec{OP}$ represents a vector with direction cosine $\cos\alpha = \frac{1}{2}$ ('O' is the origin) is:

(A) A circle parallel to y z plane with centre on the x – axis

(B) a cone concentric with positive x – axis having vertex at the origin and the slant height equal to the magnitude of the vector

(C) a ray emanating from the origin and making an angle of 60º with x – axis

(D) a disc parallel to y z plane with centre on x – axis & radius equal to $|\vec{OP}|$ sin 60º

29. Equation of the plane passing through $A(x_1, y_1, z_1)$ and containing the line $\frac{x-x_2}{d_1} = \frac{y-y_2}{d_2} = \frac{z-z_2}{d_3}$ is

(A) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$ (B) $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x-d_1 & y-d_2 & z-d_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$ (D) $\begin{vmatrix} x & y & z \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

30. The equations of the line of shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-2}{2}$ are

(A) $3(x-21) = 3y + 92 = 3z - 32$ (B) $\frac{x-(62/3)}{1/3} = \frac{y+31}{1/3} = \frac{z-(31/3)}{1/3}$

(C) $\frac{x-21}{1/3} = \frac{y+(92/3)}{1/3} = \frac{z-(32/3)}{1/3}$ (D) $\frac{x-2}{1/3} = \frac{y+3}{1/3} = \frac{z-1}{1/3}$

31. A line passes through a point A with p.v. $3\hat{i} + \hat{j} - \hat{k}$ & is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the p.v. of the point P is:

(A) $13\hat{i} + 4\hat{j} - 9\hat{k}$ (B) $13\hat{i} - 4\hat{j} + 9\hat{k}$ (C) $7\hat{i} - 6\hat{j} + 11\hat{k}$ (D) $-7\hat{i} + 6\hat{j} - 11\hat{k}$

32. The equations of the planes through the origin which are parallel to the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$ and distant $\frac{5}{3}$ from it are

(A) $2x + 2y + z = 0$ (B) $x + 2y + 2z = 0$ (C) $2x - 2y + z = 0$ (D) $x - 2y + 2z = 0$

33. The value(s) of k for which the equation $x^2 + 2y^2 - 5z^2 + 2kyz + 2zx + 4xy = 0$ represents a pair of planes passing through origin is/are

(A) 2 (B) – 2 (C) 6 (D) – 6

34. The equation of lines AB is $\frac{x}{2} = \frac{y}{-3} = \frac{2}{6}$. Through a point P(1, 2, 5), line PN is drawn perpendicular to AB and line PQ is drawn parallel to the plane $3x + 4y + 5z = 0$ to meet AB is Q. Then

(A) coordinate of N is $\left(\frac{52}{49}, -\frac{78}{49}, \frac{156}{49}\right)$ (B) the coordinates of Q is $\left(3, -\frac{9}{2}, 9\right)$

(C) the equation of PN is $\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$ (D) the equation of PQ is $\frac{x-1}{4} = \frac{y-2}{-13} = \frac{z-5}{8}$

35. Let a perpendicular PQ be drawn from P (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ when Q is the foot. Then

(A) Q is (9, 13, – 15) (B) PQ = 14

(C) the equation of plane containing PQ and the given line is $9x - 4y - z - 14 = 0$ (D) none

EXERCISE–6

1. Find the equation of the plane which contains the origin and the line of intersection of the planes $\vec{r}\cdot\vec{a} = p$ and $\vec{r}\cdot\vec{b} = q$

2. If the lines $\frac{x-a}{a'} = \frac{y-b}{b'} = \frac{z-c}{c'}$ and $\frac{x-a'}{a} = \frac{y-b'}{b} = \frac{z-c'}{c}$ intersect at a point then the coordinate of the point of intersection.

3. The locus of a point which is a equidistant from the two given points with position vectors $\vec{a}$ and $\vec{b}$ is the plane $\left[\vec{r} - \frac{1}{2}(\vec{a}+\vec{b})\right] \cdot (\vec{a} - \vec{b}) = 0$ bisecting the line joining the points normally.

4. The foot of the perpendicular from (a, b, c) on the line x = y = z is the point (r, r, r) where 3r = a + b + c.

5. **Match the following :**

	Column A	Column B	
(a)	Sum of the square of the direction cosines of line is	(P)	0
(b)	All the points on the z-axis have their x and y coordinate equal to	(Q)	1
(c)	Distance between the points (1, 3, 2) and (2, 3, 1) is	(R)	9
(d)	Shortest distance between the lines $\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2}$ is	(S)	$\sqrt{2}$

6. Show that the angle between the straight lines whose direction cosines are given by the equations $\ell + m + n = 0$ and $amn + bn\ell + c\ell m = 0$ is $\frac{\pi}{3}$ if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

7. Prove that the two lines whose direction cosines are given by the relations. $p\ell + qm + rn = 0$ & $a\ell^2 + bm^2 + cn^2 = 0$ are perpendicular if, $p^2(b + c) + q^2 (c + a) + r^2 (a + b) = 0$ and parallel if $\frac{p^2}{a}+\frac{q^2}{b}+\frac{r^2}{c}=0$.

8. Find the plane π passing through the points of intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and is perpendicular to the plane $3x - y - 2z = 4$. Find the image of point (1, 1, 1) in plane π.

9. Given parallel planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ for what values of α, planes $\vec{r} \cdot (\mu\hat{i} - \alpha\hat{j} + 3\hat{k}) = 0$ & $\vec{r} \cdot (\alpha\hat{i} - 3\hat{j} + 2\lambda\hat{k}) = 0$ would be perpendicular.

10. The edges of a rectangular parallelepiped are a, b, c; show that the angles between the four diagonals are given by $\cos^{-1}\frac{a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}$.

11. Prove that the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$ and $\vec{r} \cdot (3\hat{i} + 2\hat{j} + \hat{k}) = 0$ is $\vec{r} = t(\hat{i} - 2\hat{j} + \hat{k})$. Show that the line is equally inclined to $\hat{i}$ and $\hat{k}$ and makes an angle (1/2) $\sec^{-1} 3$ with $\hat{j}$.

12. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ & $\frac{x+1}{3} = (y-2)$; $z = 2$

13. Show that the line L whose equation is, $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane π whose vector $\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$. Find the distance between them.

14. A sphere has an equation $\left|\vec{r}-\vec{a}\right|^2 + \left|\vec{r}-\vec{b}\right|^2 = 72$ where $\vec{a} = \hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 2\hat{k}$. Find:
(i) the centre of the sphere **(ii)** the radius of the sphere
(iii) perpendicular distance from the centre of the sphere to the plane $\vec{r} \cdot (2\hat{i} + 2\hat{j} - \hat{k}) = -3$.

15. Find the equation of the sphere which is tangential to the plane $x - 2y - 2z = 7$ at $(3, -1, -1)$ and passes through the point $(1, 1, -3)$.

16. P_1 and P_2 are planes passing through origin. L_1 and L_2 also passes through origin. L_1 lies on P_1 not on P_2 and L_2 lies on P_2 but not on P_1. Show that there exists points A, B, C and whose permutation A′.B′.C′ can be chosen such that **[IIT - 2004]**
(i) A is on L_1, B on P_1 but not on L_1 and C not on P_1. (ii) A′ in on L_2, B′ on P_2 but not on L_2 and C′ not on P_2.

17. A parallelopiped 'S' has base points A, B, C and D and upper face points A′, B′, C′ and D′. This parallelopiped is compressed by upper face A′B′C′D′ to form a new parallelopiped 'T' having upper face points A″, B″, C″ and D″. Volume of parallelpiped T is 90 percent of the volume of parallelopiped S. Prove that the locus of 'A″' is a plane. **[IIT - 2004]**

EXERCISE–5

1.	C	**2.**	A	**3.**	A
4.	A	**5.**	D	**6.**	B
7.	B	**8.**	B	**9.**	A
10.	D	**11.**	A	**12.**	C
13.	B	**14.**	B	**15.**	A
16.	B	**17.**	B	**18.**	A
19.	D	**20.**	A	**21.**	A
22.	C	**23.**	D	**24.**	A
25.	A	**26.**	C	**27.**	A
28.	B	**29.**	AB	**30.**	ABC
31.	AB	**32.**	AD	**33.**	BC
34.	ABCD	**35.**	BC		

EXERCISE–6

1. $\vec{r} \cdot (\vec{a}q - p\vec{b}) = 0$

2. $(a+a', b+b', c+c')$

3. True **4.** True

5. (a) $\rightarrow$ (Q), (b) $\rightarrow$ (P), (c) $\rightarrow$ (S), (d) $\rightarrow$ (R)

8. $7x + 13y + 4z - 9 = 0$; $\left(\frac{12}{117}, \frac{-78}{117}, \frac{57}{117}\right)$

9. $\alpha = +3$ **12.** $\frac{3}{\sqrt{59}}$ **13.** $\frac{10}{3\sqrt{3}}$

14. (i) (0, 5, 5) (ii) 9 (iii) $\frac{8}{3}$

15. $(x - 2)^2 + (y - 1)^2 + (z - 1)^2 = 5$

EXERCISE–7

Part : (A) Only one correct option

1. The lengths of the diagonals of a parallelogram constructed on the vectors $\vec{p} = 2\vec{a} + \vec{b}$ & $\vec{q} = \vec{a} - 2\vec{b}$, where $\vec{a}$ & $\vec{b}$ are unit vectors forming an angle of 60º are:
(A) 3 & 4 (B) $\sqrt{7}$ & $\sqrt{13}$ (C) $\sqrt{5}$ & $\sqrt{11}$ (D) none

2. $\left[\frac{\vec{a}}{|\vec{a}|^2} - \frac{\vec{b}}{|\vec{b}|^2}\right]^2 =$
(A) $|\vec{a}|^2 - |\vec{b}|^2$ (B) $\left[\frac{\vec{a} - \vec{b}}{|\vec{a}|\,|\vec{b}|}\right]^2$ (C) $\left[\frac{\vec{a}\,|\vec{a}| - \vec{b}\,|\vec{b}|}{|\vec{a}|\,|\vec{b}|}\right]^2$ (D) none

3. A, B, C & D are four points in a plane with pv's $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ respectively such that $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$. Then for the triangle ABC, D is its:
(A) incentre (B) circumcentre (C) orthocentre (D) centroid

4. Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1, |\vec{b}| = 2$ then $\left\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right\}^2 =$
(A) 225 (B) 250 (C) 275 (D) 300

5. Consider a tetrahedron with faces f_1, f_2, f_3, f_4. Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 & whose directions are perpendicular to these faces in the outward direction. Then,
(A) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = 0$ (B) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$ (C) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (D) none

6. For non–zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|\vec{a} \times \vec{b}.\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds if and only if;
(A) $\vec{a}.\vec{b} = 0, \vec{b}.\vec{c} = 0$ (B) $\vec{c}.\vec{a} = 0, \vec{a}.\vec{b} = 0$ (C) $\vec{a}.\vec{c} = 0, \vec{b}.\vec{c} = 0$ (D) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} =$
(A) 2 (B) 4 (C) 16 (D) 64

8. If $\vec{a}, \vec{b}$ & $\vec{c}$ are any three vectors, then $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ is true if:
(A) $\vec{b}$ & $\vec{c}$ are collinear (B) $\vec{a}$ & $\vec{c}$ are collinear (C) $\vec{a}$ & $\vec{b}$ are collinear (D) none

9. $(\vec{r}\,.\,i)(i \times r) + (\vec{r}\,.\,j)(j \times r) + (\vec{r}\,.\,k)(k \times r) =$
(A) 0 (B) $\vec{r}$ (C) $2\vec{r}$ (D) $3\vec{r}$

10. A point taken on each median of a triangle divides the median in the ratio 1 : 3, reckoning from the vertex. Then the ratio of the area of the triangle with vertices at these points to that of the original triangle is:
(A) 5 : 13 (B) 25 : 64 (C) 13 : 32 (D) none

11. Given a parallelogram ABCD. If $|\vec{AB}| = a, |\vec{AD}| = b$ & $|\vec{AC}| = c$, then $\vec{DB}\,.\,\vec{AB}$ has the value:
(A) $\frac{3a^2 + b^2 - c^2}{2}$ (B) $\frac{a^2 + 3b^2 - c^2}{2}$ (C) $\frac{a^2 - b^2 + 3c^2}{2}$ (D) none

12. The points whose position vectors are $p\hat{i} + q\hat{j} + r\hat{k}$; $q\hat{i} + r\hat{j} + p\hat{k}$ & $r\hat{i} + p\hat{j} + q\hat{k}$ are collinear if:
(A) p + q + r = 0 (B) $p^2 + q^2 + r^2 - pq - qr - rp = 0$
(C) $p^3 + q^3 + r^3 - 3pqr = 0$ (D) none of these

13. If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$ & $\vec{r}\,.\,\vec{s} = 0$, then $\vec{r} =$
(A) $\vec{p}\,.\,\vec{s}$ (B) $\vec{q} - \left(\frac{\vec{q}\,.\,\vec{s}}{\vec{p}\,.\,\vec{s}}\right)\vec{p}$ (C) $\vec{q} + \left(\frac{\vec{q}\,.\,\vec{p}}{\vec{p}\,.\,\vec{s}}\right)\vec{p}$ (D) $\vec{q} + \mu\vec{p}$ for all scalars μ

14. If a, b, c are pth, qth, rth terms of an H.P. and

$$\vec{u} = (q - r)\vec{i} + (r - p)\vec{j} + (p - q)\vec{k}, \vec{\upsilon} = \frac{\vec{i}}{a} + \frac{\vec{j}}{b} + \frac{\vec{k}}{c}, \text{ then:}$$

(A) $\vec{u}, \vec{\upsilon}$ are parallel vectors (B) $\vec{u}, \vec{\upsilon}$ are orthogonal vectors
(C) $\vec{u}.\vec{\upsilon} = 1$ (D) $\vec{u} \times \vec{\upsilon} = \vec{i} + \vec{j} + \vec{k}$

15. If $\vec{p}, \vec{q}$ are two noncollinear and nonzero vectors such that $(b-c)\vec{p}\times\vec{q}+(c-a)\vec{p}+(a-b)\vec{q}=0$, where a, b, c are the length of the sides of a triangle, then the triangle is
(A) right angled (B) obtuse angled (C) equilateral (D) isoceles

16. If $\cos\alpha\,\hat{i}+\hat{j}+\hat{k}, \hat{i}+\cos\beta\,\hat{j}+\hat{k}$ & $\hat{i}+\hat{j}+\cos\gamma\,\hat{k}$ $(\alpha\neq\beta\neq\gamma\neq 2n\pi)$ are coplanar then the value of $\left[\operatorname{cosec}^2\frac{\alpha}{2}+\operatorname{cosec}^2\frac{\beta}{2}+\operatorname{cosec}^2\frac{\gamma}{2}\right]=$
(A) 1 (B) 2 (C) 3 (D) none of these

17. If $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ & $\vec{r}\cdot\vec{a}=0$ where $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+\hat{k}$ & $\vec{c}=\hat{i}+\hat{j}+3\hat{k}$, then $\vec{r}$ is equal to:
(A) $2(\hat{i}-\hat{j}+\hat{k})$ (B) $2(\hat{i}+\hat{j}-\hat{k})$ (C) $2(-\hat{i}+\hat{j}+\hat{k})$ (D) $2(\hat{i}+\hat{j}+\hat{k})$

18. The value of $[\vec{d}\ \vec{b}\ \vec{c}]\vec{a}+[\vec{d}\ \vec{c}\ \vec{a}]\vec{b}+[\vec{d}\ \vec{a}\ \vec{b}]\vec{c}-\vec{d}[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to:
(A) 0 (B) $2[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$ (C) $-2[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$ (D) none of these

19. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to these three planes form a parallelopiped of volume:
(A) $\frac{1}{3}$ (B) 4 (C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{4}{3\sqrt{3}}$

20. For any four points P, Q, R, S, $\left|\vec{PQ}\times\vec{RS}-\vec{QR}\times\vec{PS}+\vec{RP}\times\vec{QS}\right|$ is equal to 4 times the area of the triangle:
(A) PQR (B) QRS (C) PRS (D) PQS

21. If $\vec{a}, \vec{b}, \vec{c}$ are three non−coplanar & $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors, then:
$(\ell\vec{a}+m\vec{b}+n\vec{c})\cdot(\ell\vec{p}+m\vec{q}+n\vec{r})$ is equal to:
(A) $\ell^2+m^2+n^2$ (B) $\ell m+mn+n\ell$ (C) 0 (D) none of these

22. In a quadrilateral ABCD, $\vec{AC}$ is the bisector of the $(\vec{AB}\wedge\vec{AD})$ which is $\frac{2\pi}{3}$, $15|\vec{AC}|=3|\vec{AB}|=5|\vec{AD}|$ then $\cos(\vec{BA}\wedge\vec{CD})$ is:
(A) $-\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $-\frac{\sqrt{21}}{7\sqrt{3}}$ (C) $\frac{2}{\sqrt{7}}$ (D) $\frac{2\sqrt{7}}{14}$

23. In the isosceles triangle ABC $|\vec{AB}|=|\vec{BC}|=8$, a point E divides AB internally in the ratio 1: 3, then the cosine of the angle between $\vec{CE}$ & $\vec{CA}$ is (where $|\vec{CA}|=12$)
(A) $-\frac{3\sqrt{7}}{8}$ (B) $\frac{3\sqrt{8}}{17}$ (C) $\frac{3\sqrt{7}}{8}$ (D) $\frac{-3\sqrt{8}}{17}$

24. If $\vec{p}=3\vec{a}-5\vec{b}$; $\vec{q}=3\vec{a}+\vec{b}$; $\vec{r}=\vec{a}+4\vec{b}$; $\vec{s}=-\vec{a}+\vec{b}$ are four vectors such that $\sin(\vec{p}\wedge\vec{q})=1$ and $(\vec{r}\wedge\vec{s})=1$ then $\cos(\vec{a}\wedge\vec{b})$ is:
(A) $-\frac{19}{5\sqrt{43}}$ (B) 0 (C) $\frac{19}{5\sqrt{43}}$ (D) 1

25. If $\vec{p},\vec{q},\vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p}\times((\vec{x}-\vec{q})\times\vec{p})+\vec{q}\times((\vec{x}-\vec{r})\times\vec{q})+\vec{r}\times((\vec{x}-\vec{p})\times\vec{r})=\vec{0}$, then $\vec{x}$ is given by **[IIT - 1997]**
(A) $\frac{1}{2}(\vec{p}+\vec{q}-2\vec{r})$ (B) $\frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$ (C) $\frac{1}{3}(\vec{p}+\vec{q}+\vec{r})$ (D) $\frac{1}{3}(2\vec{p}+\vec{q}-\vec{r})$

26. Let $\vec{a}$ & $\vec{b}$ be two non–collinear unit vectors. If $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$ & $\vec{v}=\vec{a}\times\vec{b}$, then $|\vec{v}|$ is **[IIT - 1999]**
(A) $|\vec{u}|$ (B) $|\vec{u}|+|\vec{u}\cdot\vec{a}|$ (C) $|\vec{u}|+|\vec{u}\cdot\vec{b}|$ (D) $\vec{u}+\vec{u}\cdot(\vec{a}+\vec{b})$

27. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non coplanar vectors and $\vec{b}_1=\vec{b}-\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2=\vec{b}+\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1 = \vec{c} - \frac{\vec{c}\,.\,\vec{a}}{|\vec{a}|^2}\,\vec{a} + \frac{\vec{b}\,.\,\vec{c}}{|\vec{c}|^2}\,\vec{b}_1$, $\vec{c}_2 = \vec{c} - \frac{\vec{c}\,.\,\vec{a}}{|\vec{a}|^2}\,\vec{a} - \frac{\vec{b}_1.\vec{c}}{|\vec{b}_1|^2}\,\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}\,.\,\vec{a}}{|\vec{c}|^2}\,\vec{a} + \frac{\vec{b}\,.\,\vec{c}}{|\vec{c}|^2}\,\vec{b}_1$,

$\vec{c}_4 = \vec{c} - \frac{\vec{c}\,.\,\vec{a}}{|\vec{c}|^2}\,\vec{a} - \frac{\vec{b}\,.\,\vec{c}}{|\vec{b}|^2}\,\vec{b}_1$, then the set of orthogonal vectors is **[IIT - 2005]**

(A) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (B) $(\vec{a}, \vec{b}_1, \vec{c}_2)$ (C) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (D) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

28. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin, P_1 is parallel to the vectors $2\hat{i} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is **[IIT - 2006]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

Part : (B) May have more than one options correct

29. If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are linearly independent set of vectors & $K_1\vec{a} + K_2\vec{b} + K_3\vec{c} + K_4\vec{d} = 0$ then:
(A) $K_1 + K_2 + K_3 + K_4 = 0$ (B) $K_1 + K_3 = K_2 + K_4 = 0$ (C) $K_1 + K_4 = K_2 + K_3 = 0$ (D) none of these

30. Given three vectors $\vec{a}, \vec{b}, \vec{c}$ such that they are non – zero, non – coplanar vectors, then which of the following are coplanar.
(A) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ (B) $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ (C) $\vec{a}+\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}+\vec{a}$ (D) $\vec{a}+\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}-\vec{a}$

31. Let $\vec{p} = 2\hat{i} + 3\hat{j} + a\hat{k}$, $\vec{q} = b\hat{i} + 5\hat{j} - \hat{k}$ & $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$. If $\vec{p}, \vec{q}, \vec{r}$ are coplanar and $\vec{p}\,.\,\vec{q} = 20$, a & b have the values:
(A) 1, 3 (B) 9, 7 (C) 5, 5 (D) 13, 9

32. If $\vec{z}_1 = a\hat{i} + b\hat{j}$ & $\vec{z}_2 = c\hat{i} + d\hat{j}$ are two vectors in $\hat{i}$ & $\hat{j}$ system where $|\vec{z}_1| = |\vec{z}_2| = r$ & $\vec{z}_1 . \vec{z}_2 = 0$ then $\vec{w}_1 = a\hat{i} + c\hat{j}$ & $\vec{w}_2 = b\hat{i} + d\hat{j}$ satisfy:
(A) $|\vec{w}_1| = r$ (B) $|\vec{w}_2| = r$ (C) $\vec{w}_1 . \vec{w}_2 = 0$ (D) none of these

33. If $\vec{a}$ & $\vec{b}$ are two non colinear unit vectors & $\vec{a}, \vec{b}, x\vec{a} - y\vec{b}$ form a triangle, then:

(A) $x = -1; y = 1$ & $|\vec{a} + \vec{b}| = 2\cos\left(\frac{\widehat{\vec{a}\,\vec{b}}}{2}\right)$

(B) $x = -1; y = 1$ & $\cos\left(\widehat{\vec{a}\,\vec{b}}\right) + |\vec{a} + \vec{b}|\cos\left[\widehat{\vec{a}, -(\vec{a}+\vec{b})}\right] = -1$

(C) $|\vec{a} + \vec{b}| = -2\cot\left(\frac{\widehat{\vec{a}\,\vec{b}}}{2}\right)\cos\left(\frac{\widehat{\vec{a}\,\vec{b}}}{2}\right)$ & $x = -1, y = 1$ (D) none

34. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with the Z-axis and the vectors $\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\,\hat{k}$ and $\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\,\hat{k}$ are orthogonal, is/are:
(A) $\tan^{-1} 3$ (B) $\pi - \tan^{-1} 2$ (C) $\pi + \tan^{-1} 3$ (D) $2\pi - \tan^{-1} 2$

35. A parallelogram is constructed on the vectors $\vec{p}$ & $\vec{q}$. A vector which coincides with the altitude of the parallelogram & perpendicular to the side $\vec{p}$ expressed in terms of the vectors $\vec{p}$ & $\vec{q}$ is:

(A) $\vec{q} - \frac{\vec{q}\,.\,\vec{p}}{(\vec{p})^2}\,\vec{p}$ (B) $\frac{(\vec{p} \times \vec{q}) \times \vec{p}}{\vec{p}^2}$ (C) $\frac{\vec{q}\,.\,\vec{p}}{\vec{p}^2}\,\vec{p} - \vec{q}$ (D) $\frac{\vec{p} \times (\vec{p} \times \vec{q})}{\vec{p}^2}$

36. Identify the statement(s) which is/are incorrect?

(A) $\vec{a} \times \left[\vec{a} \times (\vec{a} \times \vec{b})\right] = (\vec{a} \times \vec{b})\,(\vec{a}^2)$

(B) If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and $\vec{v}\,.\,\vec{a} = \vec{v}\,.\,\vec{b} = \vec{v}\,.\,\vec{c} = 0$ then $\vec{v}$ must be a null vector

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{c}$ and $\vec{d}$ then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$

(D) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then $\vec{a}\,.\,\vec{b}' + \vec{b}\,.\,\vec{c}' + \vec{c}\,.\,\vec{a}' = 3$

37. If $a = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 4\hat{j} - 4\hat{k}$, then the vector $\vec{a} \times (\vec{b} \times \vec{c})$ is orthogonal to:

(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{c}$ (D) $\vec{a}+\vec{b}+\vec{c}$

38. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero, non-collinear vectors such that a vector $\vec{p}$ = a b $\cos\left(2\pi - \left(\vec{a} \wedge \vec{b}\right)\right)\ \vec{c}$ and a vector $\vec{q}$ = a c $\cos\left(\pi - \left(\vec{a} \wedge \vec{c}\right)\right)\ \vec{b}$ then p + $\vec{q}$ is

(A) parallel to $\vec{a}$ (B) perpendicular to $\vec{a}$ (C) coplanar with $\vec{b}$ & $\vec{c}$ (D) none of these

39. Which of the following statement(s) is/are true?

(A) If $\vec{n}\cdot\vec{a} = 0, \vec{n}\cdot\vec{b} = 0$ & $\vec{n}\cdot\vec{c} = 0$ for some non zero vector $\vec{n}$, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

(B) there exist a vector having direction angles $\alpha = 30º$ & $\beta = 45º$

(C) locus of point for which x = 3 & y = 4 is a line parallel to the z - axis whose distance from the z axis is 5

(D) the vertices of a regular tetrahedron are OABC where ' O ' is the origin. The vector $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ is perpendicular to the plane ABC.

40. In a Δ ABC, let M be the mid point of segment AB and let D be the foot of the bisector of $\angle$C. Then the ratio $\dfrac{\text{Area } \Delta \text{ CDM}}{\text{Area } \Delta \text{ ABC}}$ is:

(A) $\dfrac{1}{4}\ \dfrac{a-b}{a+b}$ (B) $\dfrac{1}{2}\ \dfrac{a-b}{a+b}$ (C) $\dfrac{1}{2}\tan\dfrac{A-B}{2}\cot\dfrac{A+B}{2}$ (D) $\dfrac{1}{4}\cot\dfrac{A-B}{2}\tan\dfrac{A+B}{2}$

41. The vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are of the same length & pairwise form equal angles. If $\vec{a} = \hat{i}+\hat{j}$ & $\vec{b} = \hat{j}+\hat{k}$, the pv's of $\vec{c}$ can be:

(A) (1, 0, 1) (B) $\left(-\dfrac{4}{3}, \dfrac{1}{3}, -\dfrac{4}{3}\right)$ (C) $\left(\dfrac{1}{3}, -\dfrac{4}{3}, \dfrac{1}{3}\right)$ (D) $\left(-\dfrac{1}{3}, \dfrac{4}{3}, -\dfrac{1}{3}\right)$

EXERCISE–8

1. Through the middle point M of the side AD of a parallelogram ABCD the straight line BM is drawn cutting AC at R and CD produced at Q prove that QR = 2RB

2. Show that the perpendicular distance of the point $\vec{c}$ from the line joining $\vec{a}$ & $\vec{b}$ is,

$$\frac{\left|\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}.$$

3. If $\vec{\alpha} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{\beta} = 2\hat{i} - \hat{j} + \hat{k}$; $\vec{\gamma} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{\alpha}\times\left(\vec{\beta}\times\vec{\gamma}\right) = p\vec{\alpha} + q\vec{\beta} + r\vec{\gamma}$ then find the values of p, q, r

4. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{c} = 2\hat{i} + \hat{j} - \hat{k}$ & $\vec{d} = 3\hat{i} - \hat{j} - 2\hat{k}$, then find the value of $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}).\vec{d}$

5. Show that $\vec{a}\times\left(\left(\vec{q}\times\vec{c}\right)\times\left(\vec{p}\times\vec{b}\right)\right) = \vec{b}\times\left(\left(\vec{p}\times\vec{c}\right)\times\left(\vec{q}\times\vec{a}\right)\right) + \vec{c}\times\left(\left(\vec{p}\times\vec{a}\right)\times\left(\vec{q}\times\vec{b}\right)\right)$

6. It is given that $\vec{x} = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{y} = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{z} = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ where $\vec{a},\vec{b},\vec{c}$ are non – coplanar vectors. Show that $\vec{x},\vec{y},\vec{z}$ also forms a non – coplanar system. Find the value of $\vec{x}.(\vec{a}+\vec{b}) + \vec{y}.(\vec{b}+\vec{c}) + \vec{z}.(\vec{c}+\vec{a})$.

7. The median AD of a triangle ABC is bisected at E and BE is produced to meet the side AC in F. Prove that AF = (1/3) AC and EF = (1/4) BF.

8. Points X and Y are taken on the sides QR and RS, respectively of a parallelogram PQRS, so that QX = 4XR and RY = 4YS. The line XY cuts the line PR at Z. Find the ratio PZ: ZR.

9. Forces $\vec{P},\vec{Q}$ act at O & have a resultant $\vec{R}$. If any transversal cuts their line of action at A,B,C respectively, then show that $\dfrac{P}{OA} + \dfrac{Q}{OB} = \dfrac{R}{OC}$.

10. In a tetrahedron, if two pairs of opposite edges are perpendicular, then show that the third pair of opposite edges is also perpendicular & in this case the sum of the squares of two opposite edges is the same for each pair. Also show that the segment joining the mid points of opposite edges bisect one another.

11. Use vectors to prove that the diagonals of a trapezium having equal non parallel sides are equal & conversely.

12. Given four non zero vectors $\vec{a},\vec{b}$, $\vec{c}$ and $\vec{d}$. The vectors $\vec{a},\vec{b}$ & $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ & $\vec{c}$ and $(\vec{a} \wedge \vec{b}) = (\vec{b} \wedge \vec{c}) = \frac{\pi}{3}$, $(\vec{d} \wedge \vec{a}) = \alpha$, $(\vec{d} \wedge \vec{b}) = \beta$, prove that $(\vec{d} \wedge \vec{c}) = \cos^{-1}(\cos\beta - \cos\alpha)$.

13. If $\vec{p}, \vec{q}$ & $\vec{r}$ are three non-coplanar vectors, prove that,

$$\vec{a} \times \vec{b} = \frac{1}{\sqrt{[\vec{q} \times \vec{r}\ ,\ \vec{r} \times \vec{p}\ ,\ \vec{p} \times \vec{q}]}} \begin{vmatrix} \vec{p} & \vec{q} & \vec{r} \\ \vec{p}.\vec{a} & \vec{q}.\vec{a} & \vec{r}.\vec{a} \\ \vec{p}.\vec{b} & \vec{q}.\vec{b} & \vec{r}.\vec{b} \end{vmatrix}$$

14. Consider the non zero vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ such that no three of which are coplanar then prove that $\vec{a}\left[\vec{b}\,\vec{c}\,\vec{d}\right] + \vec{c}\left[\vec{a}\,\vec{b}\,\vec{d}\right] = \vec{b}\left[\vec{a}\,\vec{c}\,\vec{d}\right] + \vec{d}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$. Hence prove that $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ represent the position vectors of the vertices of a plane quadrilateral if and only if $\frac{\left[\vec{b}\,\vec{c}\,\vec{d}\right] + \left[\vec{a}\,\vec{b}\,\vec{d}\right]}{\left[\vec{a}\,\vec{c}\,\vec{d}\right] + \left[\vec{a}\,\vec{b}\,\vec{c}\right]} = 1$.

15. Solve the following equation for the vector $\vec{p}$; $\vec{p} \times \vec{a} + (\vec{p}.\vec{b})\vec{c} = \vec{b} \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left[\vec{p} \times \vec{a} + \frac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\vec{a}.\vec{c}}\vec{c}\right]$ is perpendicular to $\vec{b} - \vec{c}$.

16. If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then prove that:

(i) $[\vec{a}\ \vec{b}\ \vec{c}]\ [\vec{a}'\ \vec{b}'\ \vec{c}'] = 1$ **(ii)** $(\vec{a}' \times \vec{b}') + (\vec{b}' \times \vec{c}') + (\vec{c}' \times \vec{a}') = \frac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a}\vec{b}\vec{c}]}$.

17. Let $\vec{A} = 2i + k$; $\vec{B} = i + j + k$ & $\vec{C} = 4i - 3j + 7k$. Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ & $\vec{R}.\vec{A} = 0$

18. For any two vectors $\vec{u}$ & $\vec{v}$, prove that **[IIT - 1998]**

(a) $(\vec{u}.\vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2$ & **(b)** $(1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = (1 - \vec{u}.\vec{v})^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$

19. Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent. **[IIT - 2000]**

20. Find 3 – dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying

$\vec{v}_1.\vec{v}_1 = 4$, $\vec{v}_1.\vec{v}_2 = -2$, $\vec{v}_1.\vec{v}_3 = 6$, $\vec{v}_2.\vec{v}_2 = 2$, $\vec{v}_2.\vec{v}_3 = -5$, $\vec{v}_3.\vec{v}_3 = 29$. **[IIT - 2001]**

21. If $\hat{u}, \hat{v}, \hat{w}$ be three non-coplanar unit vectors with angles between $\hat{u}$ & $\hat{v}$ is α, between $\hat{v}$ & $\hat{w}$ is β and between $\hat{w}$ & $\hat{u}$ is γ. If $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors along angle bisectors of α, β, γ respectively, then prove that, $\left[\vec{a} \times \vec{b}\ ,\ \vec{b} \times \vec{c}\ ,\ \vec{c} \times \vec{a}\right] = \frac{1}{16}\left[\hat{u}\ \hat{v}\ \hat{w}\right]^2 \sec^2\left(\frac{\alpha}{2}\right) \sec^2\left(\frac{\beta}{2}\right) \sec^2\left(\frac{\gamma}{2}\right)$. **[IIT - 2003]**

EXERCISE–7

1. B **2.** B **3.** C **4.** D **5.** A **6.** D
7. C **8.** B **9.** A **10.** B **11.** A **12.** B
13. B **14.** B **15.** C **16.** B **17.** C **18.** A
19. D **20.** B **21.** A **22.** C **23.** C **24.** C
25. B **26.** A **27.** B **28.** D **29.** ABC
30. BCD **31.** AD **32.** ABC **33.** AB **34.** BD
35. BD **36.** ACD **37.** AD **38.** BC **39.** ACD
40. BC **41.** AD

EXERCISE–8

3. p = 0; q = 10; r = – 3

4. – 98 **6.** 3

20. $\vec{v}_1 = 2\hat{i}$, $\vec{v}_2 = -\hat{i} \pm \hat{j}$, $\vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$ are some possible values