Sample Paper-03
Mathematics
Class – XI

Answer

Section A

1. **Solution:**
   Domain of is in the open interval $(-2, 2)$
2. **Solution:**
   $(A \cap B) = \{\phi\}$
3. **Solution**
   Max value is 2
4. **Solution:** Ellipse

Section B

5. **Solution**
   Condition for co-linearity is not satisfied here since
   $\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$
6. **Solution:**
   $b^2 + c^2 - 4ad > 0$
7. **Solution:**

$$\cos 3x = \cos \frac{2\pi}{3}$$

$$3x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in Z$$

8. **Solution:**

Let $P(n)$ be the statement given by $1 + 2 + 3 + \ldots\ldots\ldots + n = \frac{n(n+1)}{2}$

$$P(1) = \frac{1(1+1)}{2}$$

=1, True

Let it be true for n=m

$$1 + 2 + 3 + \ldots\ldots + m = \frac{m(m+1)}{2}$$

$$1 + 2 + 3 + \ldots\ldots + m + (m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m^2 + 3m + 2}{2}$$

$$P(m+1)=\frac{(m+1)(m+2)}{2}$$

$Thus\ P(m)\ is\ true \Rightarrow P(m+1)\ is\ True$

9. Solution:

Let $\sqrt{z} = \sqrt{-8i}$

$$\sqrt{z}=\pm\{\frac{\sqrt{|z|-Re(z)}}{\sqrt{2}}\}-i\{\frac{\sqrt{|z|-Re(z)}}{\sqrt{2}}\},\ Im(z)<0$$

$$\sqrt{-8i}=\pm\{\frac{\sqrt{8+0}}{\sqrt{2}}-i\frac{\sqrt{8-0}}{\sqrt{2}}\},\ Im(z)<0$$

$$=\pm(2-2i)$$

10. Solution

$$\frac{2x+5}{x-2}-3\geq 0$$

$$=\frac{2x+5-3x+6}{x-2}\geq 0$$

$$=\frac{-x+11}{x-2}\geq 0$$

$$=\frac{x-11}{x-2}\leq 0$$

$$=(x-11)(x-2)\leq 0$$

$$x\in(2,11]$$

11. Solution

$$x+x+4=12$$

$$2x=8$$

$$x=4$$

12. Solution

Let p be the probability of winning Car C, $P(C)$

P(C) = p

P(B) = 2p

P(A) = 6p

P(A) + P(B) + P(C) = 1

p + 2p + 6p = 1

9p = 1

$$p=\frac{1}{9}$$

$$P(C)=\frac{1}{9}$$

$$P(B)=\frac{2}{9}$$

$$P(A)=\frac{6}{9}$$

Section C

13. Solution :

Let a satisfy the relation $f(a)=3$

$$f(f(a)).(1+f(a))=-f(a)$$
$$f(3).(4)=-3$$
$$f(3)=-\frac{3}{4}$$

14. Solution:

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}}$$
$$=1$$
$$A+B=45$$
$$2(A+B)=90$$
$$\sin 90=1$$

15. Solution:

Form a quadratic equation sum of whose roots are 30 and product of the roots is 81

$$x^2-x(30)+81=0$$
$$x^2-3x-27x+81=0$$
$$x(x-3)-27(x-3)$$
$$(x-3)(x-27)=0$$

Hence the numbers are 3 and 27

16. Solution:

Let $f:R\to R$ be a function given by $f(x)=x^2+2$ find $f^{-1}(27)$

$$f(x)=x^2+2$$
$$x^2+2=27$$
$$x^2=25$$
$$x=\pm 5$$
$$f^{-1}(27)=\{-5,5\}$$

17. Solution:

The function is defined for all values of x where the denominator is not equal to zero

$$a+1-x\neq 0$$

Hence domain =

$$R-\{(a+1)\}$$

Range of f

Let $y=f(x)$

$$y=\frac{x-a}{a+1-x}$$
$$(a+1)y-xy=x-a$$
$$x(y+1)=(a+1)y+a$$
$$x=\frac{(a+1)y+2}{y+1}$$

Range of $f \; = R-\{-1\}$

18. Solution

Rationalize the numerator

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x}$$

$$=\lim_{x\to 0}\frac{(\sqrt{a+x}-\sqrt{a})(\sqrt{a+x}+\sqrt{a})}{x(\sqrt{a+x}+\sqrt{a})}$$

$$=\lim_{x\to 0}\frac{x}{x(\sqrt{a+x}+\sqrt{a})}$$

$$=\frac{1}{2\sqrt{a}}$$

19. Solution:

$$\sin 75^0+\cos 75^0$$

$$=\sqrt{2}(\frac{1}{\sqrt{2}}\sin 75^0+\frac{1}{\sqrt{2}}\cos 75^0$$

$$=\sqrt{2}(\cos 45^0\sin 75^0+\sin 45^0\cos 75^0)$$

$$=\sqrt{2}\sin(75^0+45^0)$$

$$=\sqrt{2}\sin 120^0$$

Hence sign is positive and value is $\frac{\sqrt{2}.\sqrt{3}}{2}$ = $\frac{\sqrt{6}}{2}$

20.Solution:

There are 4 groups and four groups can be arranged in $4!$ ways. Class 12 can be arranged in $3!$ ways , Class 11 can be arranged in $4!$ Class 10 can be arranged in $4!$. Class 9 can be arranged in $2!$ ways

Hence Total number of ways that they can be arranged in a row $4!\times 3!\times 4!\times 4!\times 2!=165888$

In a circular seating arrangement the four groups can be arranged only in $3!$ ways only.

Hence the total number of ways that they can be seated at a round table = $3!\times 3!\times 4!\times 4!\times 2!=41472$

21. Solution

The new coordinates of the centre in the new position are

$(a+4\pi r\,,b)$

$\{x-(a+4\pi r)\}^2+(y-b)^2=r^2$

22. Solution

$$x^2+4y^2+4x+16y+16=0$$

$$x^2+4x+4+4y^2+16y+16=4$$

$$(x+2)^2+4(y+2)^2=4$$

$$\frac{(x+2)^2}{2^2}+\frac{(y+2)^2}{1^2}=1$$

This equation represents an ellipse.

23. Solution

x_i	f_i	$f_i x_i$	$\lvert x_i-15\rvert$	$f_i\lvert x_i-15\rvert$
2	12	24	13	156
15	6	90	0	0
17	12	204	2	24

23	9	207	8	72
27	5	135	12	60
	$N=\Sigma f_i=44$	$\Sigma f_i\ x_i=660$		$f_i\Sigma\|x_i-15\|=312$

$$Mean=\overline{X}=\frac{1}{N}(\Sigma f_i\ x_i)=\frac{660}{44}=15$$

$$MeanDeviation=M.D=\frac{1}{N}(\Sigma f_i\ |x_i-15|)=\frac{312}{44}=7.0909$$

Section D

24. Solution

Let the ratios be

$a\ :\ b$

$x^2+px+q=0$

$a\alpha+b\alpha=-p$

$a\beta+b\beta=-p_1$

$a\alpha\times b\alpha=q$

$a\beta\times b\beta=q_1$

$(a+b)\alpha=-p$

$(a+b)\beta=-p_1$

$ab\alpha^2=q$

$ab\beta^2=q_1$

$$\frac{(a+b)^2\alpha^2}{(a+b)^2\beta^2}=\frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2}=\frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2}=\frac{q}{q_1}$$

$$\frac{p^2}{p_1^2}=\frac{q}{q_1}$$

$p^2q_1=p_1^2q$

25. Solution :

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\infty=\frac{1}{1-\frac{1}{2}}=\frac{1}{\frac{1}{2}}=2$$

$$a.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{8}}\ldots\ldots\infty=a^2$$

26. Solution

It is given that

$n(U)=700\ ,n(A)=200\ ,n(B)=295\ ,n(A\cap B)=115$

We need to find out

$n(A'\cap B')$

$n(A' \cap B') = n(A \cup B)'$

$= n(U) - n(A \cup B)$

$= n(U) - \{n(A) + n(B) - n(A \cap B)\}$

= 700 – {200 + 295 - 115} = 320