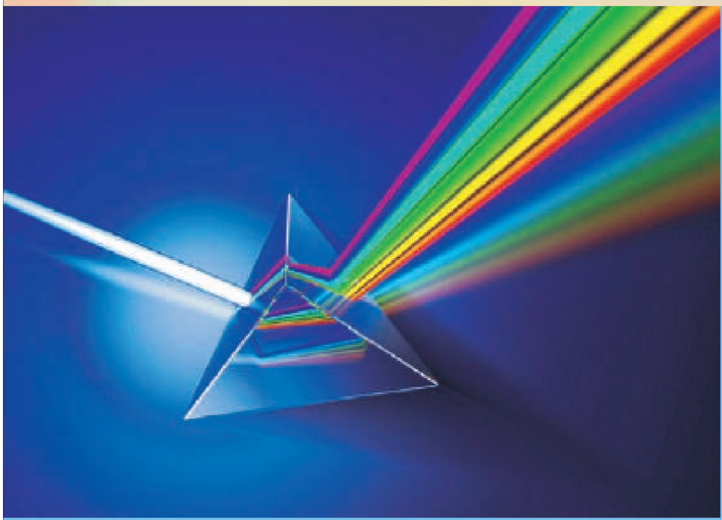
PHYSICS



UNIT & DIMENSION
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UNITS AND DIMENSIONS

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity.

UNITS

All physical quantities are measured w.r.t. standard magnitude of the same physical quantity and these standards are called UNITS. eg. second, meter, kilogram, etc.

So the four basic properties of units are:—

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

SET OF FUNDAMENTAL QUANTITIES

A set of physical quanties which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

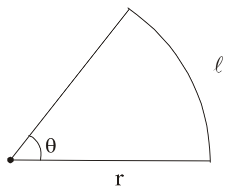
Physical Quantity	Units(SI)	Units(CGS)	Notations
Mass	kg (kilogram)	g	M
Length	m (meter)	cm	L
Time	s (second)	s	T
Temperature	K (kelvin)	°C	θ
Current	A (ampere)	A	I or A
Luminous intensity	cd (candela)	—	cd
Amount of substance	mol	—	mol

Physical Quantity (SI Unit)	Definition
Length (m)	The distance travelled by light in vacuum in $\frac{1}{299,792,458}$ second is called 1 metre.
Mass (kg)	The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is defined as 1 kilogram.
Time (s)	The second is the duration of 9,192,631,770 periods of

	the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Electric Current (A)	If equal currents are maintained in the two parallel infinitely long wires of negligible cross-section, so that the force between them is 2×10^{-7} newton per metre of the wires, the current in any of the wires is called 1 Ampere.
Thermodynamic Temperature (K)	The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of triple point of water is called 1 Kelvin
Luminous Intensity (cd)	1 candela is the luminous intensity of a blackbody of surface area $\frac{1}{600{,}000} m^2$ placed at the temperature of freezing platinum and at a pressure of 101,325 N/m^2, in the direction perpendicular to its surface.
Amount of substance (mole)	The mole is the amount of a substance that contains as many elementary entities as there are number of atoms in 0.012 kg of carbon-12.

There are two supplementary units too:

1. Plane angle (radian) — angle = arc / radius
   $\theta = \ell / r$



2. Solid Angle (steradian)

DERIVED PHYSICAL QUANTITIES

The physical quantities those can be expressed in terms of fundamental physical quantities are called derived physical quantities.eg. speed = distance/time.

DIMENSIONS AND DIMENSIONAL FORMULA

All the physical quantities of interest can be derived from the base quantities.

DIMENSION

The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base.

To make it clear, consider the physical quantity "force".

Force = mass × acceleration

$$= \text{mass} \times \frac{\text{length / time}}{\text{time}}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and –2 in time. Thus

$$[\text{Force}] = MLT^{-2}$$

Similarly energy has dimensional formula given by

$$[\text{Energy}] = ML^2T^{-2}$$

i.e. energy has dimensions, 1 in mass, 2 in length and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

DIMENSIONAL EQUATION

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

PRINCIPLE OF HOMOGENEITY

According to this principle, we can multiply physical quantities with same or different dimensional formulae at our convenience, however no such rule applies to addition and substraction, where only like physical quantites can only be added or substracted. e.g. If P + Q $\Rightarrow$ P & Q both represent same physical quantity.

Illustration :

Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$.

Sol. *Dimensionally,* $E = \text{mass} \times (\text{velocity})^2$.

Since $\frac{1}{2}$ *is a number and has no dimension.*

or, $[E] = M \times \left(\frac{L}{T}\right)^2 = ML^2T^{-2}$.

Illustration :

Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ *where s is the distance travelled by the particle. Determine dimensions of* α.

Sol. $K = \alpha s^2$

$$[\alpha] = \frac{(ML^2T^{-2})}{(L^2)}$$

$$[\alpha] = M^1 L^0 T^{-2}$$

$$[\alpha] = (M T^{-2})$$

Illustration :

The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, *where* v_0 *is a constant and* $\alpha > 0$. *The dimensions of* v_0 *&* α *are respectively.*

(A) $M^0 L^1 T^0$ & T^{-1} (B) $M^0 L^1 T^{-1}$ & T

(C*) $M^0 L^1 T^{-1}$ & T^{-1} (D) $M^1 L^1 T^{-1}$ & LT^{-2}

Sol. $[V_0] = [x]\ [\alpha]$ & $[\alpha]\ [t] = M^0L^0T^0$

$= M^0L^1T^{-1}$ $[\alpha] = M^0L^0T^{-1}$

Illustration :

The distance covered by a particle in time t is going by $x = a + bt + ct^2 + dt^3$ *; find the dimensions of a, b, c and d.*

Sol. *The equation contains five terms. All of them should have the same dimensions. Since [x] = length, each of the remaining four must have the dimension of length.*

Thus, $[a] = \text{length} = L$

$[bt] = L$, or $[b] = LT^{-1}$

$[ct^2] = L$, or $[c] = LT^{-2}$

and $[dt^3] = L$ or $[d] = LT^{-3}$

USES OF DIMENSIONAL ANALYSIS

(I) TO CONVERT UNITS OF A PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER :

It is based on the fact that,

Numerical value × unit = constant

So on changing unit, numerical value will also gets changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Illustration

Young's modulus of steel is 19×10^{10} *N/m*^{2}*. Express it in dyne/cm*^{2}*. Here dyne isthe CGS unit of force.*

Sol. *The unit of Young's modulus is N/m*^{2}*.*

This suggest that it has dimensions of $\frac{\text{Force}}{(\text{dis tan ce})^2}$.

Thus, $[Y] = \frac{[F]}{L^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

N/m^2 *is in SI units,*

So, $1\ N/m^2 = (1\ kg)(1m)^{-1}\ (1s)^{-2}$

and $1\ dyne/cm^2 = (1g)(1cm)^{-1}\ (1s)^{-2}$

so, $$\frac{1N/m^2}{1dyne/cm^2} = \left(\frac{1kg}{1g}\right)\left(\frac{1m}{1cm}\right)^{-1}\left(\frac{1s}{1s}\right)^{-2} = 1000 \times \frac{1}{100} \times 1 = 10$$

or, $1\ N/m^2 = 10\ dyne/cm^2$

or, $19 \times 10^{10}\ N/m^2 = 19 \times 10^{11}\ dyne/m^2$.

Illustration :

The dimensional formula for viscosity of fluids is,

$$\eta = M^1L^{-1}T^{-1}$$

Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $\eta = M^1 L^{-1} T^{-1}$

1 CGS units = $g\ cm^{-1}\ s^{-1}$

1 SI units = $kg\ m^{-1}\ s^{-1}$

$= 1000\ g\ (100\ cm)^{-1}\ s^{-1}$

$= 10\ g\ cm^{-1}\ s^{-1}$

Thus, 1 Poiseuilli = 10 poise

Illustration :

. *A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kg* m^2/s^2*. Suppose we employ a system of units in which the unit of mass equals* α *kg, the unit of length equals* β *metre, the unit of time is* γ *second. Show that a calorie has a magnitude* $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ *in terms of the new units.*

Sol. $1\ cal = 4.2\ kg\ m^2 s^{-2}$

SI	*New system*
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1\ kg$	$M_2 = \alpha\ kg$
$L_1 = 1\ m$	$L_2 = \beta$ *metre*
$T_1 = 1\ s$	$T_2 = \gamma$ *second*

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$*, we find that* $a = 1,\ b = 2,\ c = -2$

Now, $$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\ kg}{\alpha\ kg}\right]^1\left[\frac{1\ m}{\beta\ m}\right]^2\left[\frac{1\ s}{\gamma\ s}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

(II) TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION:

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

(i) Powers are dimensionless

(ii) $\sin\theta$, e^{θ},, $\cos\theta$, $\log\theta$ gives dimensionless value and in above expression θ is dimensionless

(iii) We can add or subtract quantity having same dimensions.

Illustration :

Let us check the dimensional correctness of the relation $v = u + at$.

Here 'u' represents the initial velocity, 'v' represents the final velocity, 'a' the uniform acceleration and 't' the time.

Dimensional formula of 'u' is $[M^0LT^{-1}]$

Dimensional formula of 'v' is $[M^0LT^{-1}]$

Dimensional formula of 'at' is $[M^0LT^{-2}][T] = [M^0LT^{-1}]$

Here dimensions of every term in the given physical relation are the same, hence the given physical relation is dimensionally correct.

Illustration :

Let us check the dimensional correctness of the relation

$$x = ut + \frac{1}{2}at^2$$

Here 'u' represents the initial velocity, 'a' the uniform acceleration, 'x' the displacement and 't' the time.

Sol. $[x] = L$

$$[ut] = velocity \times time = \frac{length}{time} \times time = L$$

$$\left[\frac{1}{2}at^2\right] = [at^2] = accelecration \times (time)^2$$

$(\therefore \frac{1}{2}$ *is a number hence dimentionless)*

$$= \frac{velocity}{time} \times (time)^2 = \frac{length/tim\ e}{time} \times (time)^2 = L$$

Thus, the equation is correct as far as the dimensions are concerned.

(III) TO ESTABLISH A RELATION BETWEEN DIFFERENT PHYSICAL QUANTITIES :

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

Illustration :

Let us find an expression for the time period t of a simple pendulum. The time period t may depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. *Let (i)* $t \propto m^a$ *(ii)* $t \propto \ell^b$ *(iii)* $t \propto g^c$

Combining all the three factors, we get

$$t \propto m^a \ell^b g^c \quad or \quad t = Km^a \ell^b g^c$$

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$$[T] = [M^a][L^b][LT^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

Comparing dimensions, $a = 0,\ b + c = 0,\ -2c = 1$

$\therefore \quad a = 0,\ c = -1/2,\ b = 1/2$

From equation (i) $t = Km^0 \ell^{1/2} g^{-1/2}$ *or* $t = K\left(\frac{\ell}{g}\right)^{1/2} = K\sqrt{\frac{\ell}{g}}$

Illustration :

When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. *Suppose the formula is* $F = k\,\eta^a r^b v^c$

Then, $MLT^{-2} = [ML^{-1}T^{-1}]^a L^b \left(\frac{L}{T}\right)^c$

$= M^a L^{-a+b+c} T^{-a-c}$

Equating the exponents of M, L and T from both sides,

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

Solving these, $a = 1$, $b = 1$ *and* $c = 1$

Thus, the formula for F is $F = k\eta r v$.

Illustration :

If P is the pressure of a gas and ρ *is its density, then find the dimension of velocity in terms of P and* ρ .

(A) $P^{1/2}\rho^{-1/2}$ (B) $P^{1/2}\rho^{1/2}$ (C) $P^{-1/2}\rho^{1/2}$ (D) $P^{-1/2}\rho^{-1/2}$

[Sol. $v \propto P^a \rho^b$

$v = kP^a \rho^b$

$[LT^{-1}] = [ML^{-1}T^{-2}]^a\,[ML^{-3}]^b$ *(Comparing dimensions)*

$a = \frac{1}{2}$, $b = -\frac{1}{2}$ $\Rightarrow$ $[V] = [P^{1/2}\rho^{-1/2}]$

UNITS AND DIMENSIONS OF SOME PHYSICAL QUANTITIES

Quantity	SI Unit	Dimensional Formula
Density	kg/m^3	M/L^3
Force	Newton (N)	ML/T^2
Work	Joule (J)(=N-m)	ML^2/T^2
Energy	Joule(J)	ML^2/T^2
Power	Watt (W) (=J/s)	ML^2/T^3
Momentum	kg-m/s	ML/T
Gravitational constant	$N\text{-}m^2/kg^2$	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	$radian/s^2$	T^{-2}
Angular momentum	$kg\text{-}m^2/s$	ML^2/T
Moment of inertia	$kg\text{-}m^2$	ML^2
Torque	N-m	ML^2/T^2
Angular frequency	radian/s	T^{-1}
Frequency	Hertz (Hz)	T^{-1}
Period	s	T
Surface Tension	N/m	M/T^2
Coefficient of viscosity	$N\text{-}s/m^2$	M/LT
Wavelength	m	L
Intensity of wave	W/m^2	M/T^3

Temperature	kelvin (K)	K
Specific heat capacity	J/(kg-K)	L^2/T^2K
Stefan's constant	$W/(m^2–K^4)$	M/T^3K^4
Heat	J	ML^2/T^2
Thermal conductivity	W/(m-K)	ML/T^3K
Current density	A/m^2	I/L^2
Electrical conductivity	1/Ω-m(= mho/m)	I^2T^3/ML^3
Electric dipole moment	C-m	LIT
Electric field	V/m (=N/C)	ML/IT^3
Potential (voltage)	volt (V) (=J/C)	ML^2/IT^3
Electric flux	V-m	ML^3/IT^3
Capacitance	farad (F)	I^2T^4/ML^2
Electromotive force	volt (V)	ML^2/IT^3
Resistance	ohm (Ω)	ML^2/I^2T^3
Permittivity of space	$C^2/N\text{-}m^2$ (=F/m)	I^2T^4/ML^3
Permeability of space	N/A^2	ML/I^2T^2
Magnetic field	Tesla (T) (= Wb/m^2)	M/IT^2
Magnetic flux	Weber (Wb)	ML^2/IT^2
Magnetic dipole moment	N-m/T	IL^2
Inductance	Henry (H)	ML^2/I^2T^2

LIMITATIONS OF DIMENSIONAL ANALYSIS

(i) Dimension does not depend on the magnitude. Due to this reason the equation $x = ut + at^2$ is also dimensionally correct. Thus, a dimensionally correct equation need not be actually correct.
(ii) The numerical constants having no dimensions connot be deduced by the method of dimensions.
(iii) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.

SI Prefixes : The magnitudes of physical quantites vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain power of 10. Table shows these prefixes :

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da
–1	deci	d

–2	centi	c
–3	milli	m
–6	micro	μ
–9	nano	n
–12	pico	p
–15	femto	f

ORDER-OF MAGNITUDE CALCULATIONS

If value of phycal quantity P satisfy

$0.5\times10^{x} < P \le 5 \times 10^{x}$

x is an integer

x is called order of magnitude

Illustration :

The diameter of the sun is expressed as 13.9×10^{9} m. Find the order of magnitude of the diameter ?

Sol. *Diameter = 13.9×10^{9} m*

Diameter = 1.39×10^{10} m

order of magnitude is 10.

SYMBOLS AND THERE USUAL MEANINGS

The scientific group in Greece used following symbols.

θ	Theta
α	Alpha
β	Beta
γ	Gamma
δ	Delta
Δ	Delta
μ	Mu
λ	Lambda
ω, Ω	Omega
π	Pi
ϕ, φ	Phi
ε	epsilon

ψ	Psi
ρ	Roh
ν	Nu
η	Eta
σ	Sigma
τ	Tau
κ	Kappa
χ	chi
≅	Approximately equal to

Solved Examples

Q.1 Find the dimensional formulae of follwoing quantities :

(a) The surface tension S,

(b) The thermal conductivity k and

(c) The coefficient of vescosity η.

Some equation involving these quntities are

$$S = \frac{\rho\, g\, r\, h}{2} \qquad Q = k\frac{A(\theta_2 - \theta_1)t}{d} \quad \text{and} \quad F = -\eta A \frac{v_2 - v_1}{x_2 - x_1}\ ;$$

where the symbols have their usual meanings. (ρ - density, g - acceleration due to gravity, r - radius, h - height, A - area, θ_1 & θ_2 - temperatures, t - time, d - thickness, v_1 & v_2 - velocities, x_1 & x_2 - positions.)

Sol. (a) $S = \frac{\rho\, g\, r\, h}{2}$

or $[S] = [\rho]\,[g]L^2 = \frac{M}{L^2} \cdot \frac{L}{T^2} \cdot L^2 = MT^{-2}$.

(b) $Q = k\ \frac{A(\theta_2 - \theta_1)t}{d}$

or $k = \frac{Qd}{A(\theta_2 - \theta_1)t}$.

Here, Q is the heat energy having dimension ML^2T^{-2}, $\theta_2 - \theta_1$ is temperature, A is area, d is thickness and t is time. Thus,

$$[k] = \frac{ML^2T^{-2}}{L^2KT} = MLT^{-3}\,K^{-1}\,.$$

(d) $F = -h\,A\frac{v_2 - v_1}{x_2 - x_1}$

or $MLT^{-2} = [\eta]L^2\ \frac{L/T}{L} = [\eta]\ \frac{L^2}{T}$

or, $[\eta] = ML^{-1}T^{-1}$.

Q.2 Suppose $A = B^nC^m$, where A has dimensions LT, B has dimensions L^2T^{-1}, and C has dimensions LT^2. Then the exponents n and m have the values:

(A) 2/3; 1/3 (B) 2; 3 (C) 4/5; -1/5 (D*) 1/5; 3/5

(E) 1/2; 1/2

Sol. $LT = [L^2T^{-1}]^n\,[LT^2]^m$

$LT = L^{2n+m}T^{2m-n}$

$2n + m = 1$ ....(i)

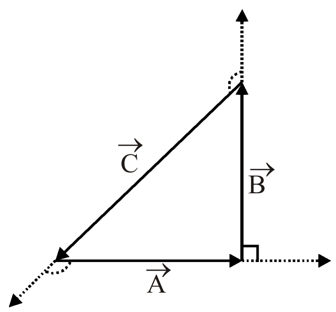
$-n + 2m = 1$ ....(ii)

Solved Example

Q.1 Given that $\vec{A}+\vec{B}+\vec{C}=\vec{0}$, but of three two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the vectors two having equal magnitude. Then the angles between vectors are given by -

(A) 30°, 60°, 90° (B) 45°, 45°, 90° (C) 45°, 60°, 90° (D) 90°, 135°, 135°

Sol. (D) From polygon law, there vectors having summation zero, should from a closed polygon (triangle). Since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle.



Angle between A and B is 90°

Angle between B and C is 135°

Angle between A and C is 135°

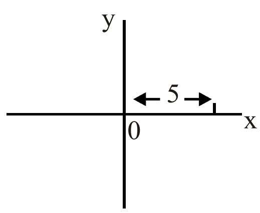
Q.2 If a particle moves 5m in + x-direction. Show the displacement of the particle-

(A) $5\hat{j}$ (B) $5\hat{i}$ (C) $-5\hat{j}$ (D) $5\hat{k}$

Sol. Magnitude of vector = 5

Unit vector in + x direction is $\hat{i}$

displacement = $5\hat{i}$



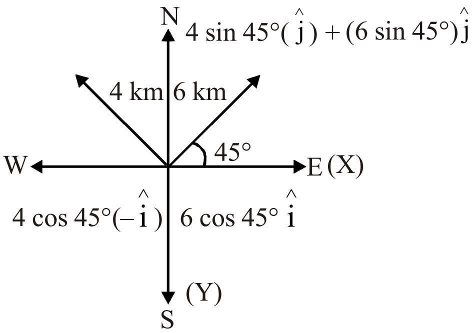
Hence correct answer is (B).

Q.3 A car travels 6 km towards north at an angle of 45° to the east then travels distnace of 4 km towards north at an angle of 135° to the east. How far is its final position due east and due north? How far is the point from the strating point ? What angle does the straight line joining its initial and final position makes with the east ? What is the total distnace travelled by the car ?

Sol. Net movement along X - direction

$$= (6-4)\cos 45°\ \hat{i}$$

$$= 2\times\frac{1}{\sqrt{2}} = \sqrt{2}\ \text{km}$$



Net movement along Y – direction

$$= (6+4)\sin 45°\ \hat{j}$$

$$= 10\times\frac{1}{\sqrt{2}} = 5\sqrt{2}\ \text{km}$$

Net movement form starting point (Total distance travelled)

$$= 6 + 4 = 10\ \text{km}$$

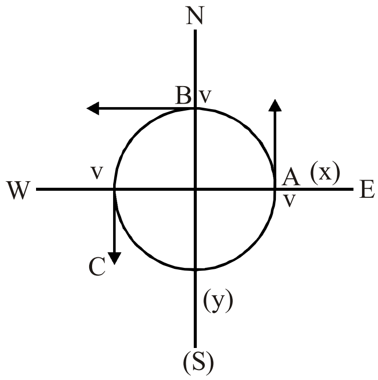
Angle which makes with the east direction

$$\tan\theta = \frac{Y-\text{component}}{X-\text{component}}$$

$$= \frac{5\sqrt{2}}{\sqrt{2}}$$

$$\theta = \tan^{-1}(5)$$

Q.4 A body is moving with uniform speed v on a horizontal circle in anticlockwise direction from A as shown in figure. What is the change in velocity in (a) half revolution (b) first quarter revolution.



Sol. Change in velocity in half revolution

$$\Delta\vec{v} = \vec{v}_C - \vec{v}_A$$

$$= v(-\hat{j}) - v(\hat{j})$$

EXERCISE-1 (Exercise for JEE Main)

[SINGLE CORRECT CHOICE TYPE]

Q.1 In the S.I. system, the unit of temperature is-
(A) degree centigrade (B) Kelvin
(C) degree Celsius (D) degree Fahrenheit
[1010110687]

Q.2 In the S.I. system the unit of energy is-
(A) erg (B) calorie (C) joule (D) electron volt
[1010111385]

Q.3 The dimensions of the ratio of angular momentum to linear momentum is
(A) $[M^0LT^0]$ (B) $[MLT^{-1}]$ (C) $[ML^2T^{-1}]$ (D) $[M^{-1}L^{-1}T^{-1}]$
[1010111480]

Q.4 If Force = (x/density) + C is dimensionally correct, the dimension of x are -
(A) MLT^{-2} (B) MLT^{-3} (C) ML^2T^{-3} (D) $M^2L^{-2}T^{-2}$
[1010113500]

Q.5 The dimensional formula for angular momentum is -
(A) ML^2T^{-2} (B) ML^2T^{-1} (C) MLT^{-1} (D) $M^0L^2T^{-2}$
[1010113444]

Q.6 For $10^{(at+3)}$, the dimension of a is-
(A) $M^0 L^0 T^0$ (B) $M^0 L^0 T^1$ (C) $M^0 L^0 T^{-1}$ (D) None of these
[1010113287]

Q.7 The velocity of a moving particle depends upon time t as $v = at + \frac{b}{t+c}$. Then dimensional formula for b is -
[1010112939]
(A) $[M^0 L^0 T^0]$ (B) $[M^0 L^1 T^0]$ (C) $[M^0 L^1 T^{-1}]$ (D) $[M^0 L^1 T^{-2}]$

Q.8 The pairs having same dimensional formula -
(A) Angular momentum, torque
(B) Torque, work
(C) Plank's constant, boltzman's constant
(D) Gas constant, pressure
[1010112800]

Q.9 If $F = ax + bt^2 + c$ where F is force, x is distance and t is time. Then what is dimension of $\frac{axc}{bt^2}$?
(A) $[M L^2 T^{-2}]$ (B) $[M L T^{-2}]$ (C) $[M^0 L^0 T^0]$ (D) $[M L T^{-1}]$
[1010112784]

Q.10 If force, time and velocity are treated as fundamental quantities then dimensional formula of energy will be
(A) $[FTV]$ (B) $[FT^2V]$ (C) $[FTV^2]$ (D) $[FT^2V^2]$
[1010112441]

Q.11 Which of the following physical quantities do not have the same dimensions
(A) Pressure, Yongs modulus, stress (B) Electromotive force, voltage, potential
(C) Heat, Work, Energy (D) Electric dipole, electric field, flux
[1010112085]

EXERCISE-2 (Miscellaneous Exercise)

Q.1 Taking force, length and time to be the fundamental quantities find the dimensions of
(I) Density (II) Pressure (III) Momentum and (IV) Energy

[1010112351]

Q.2 The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m. Guess the expression for its frequency from dimensional analysis.

[1010111605]

Q.3 The intensity of X-rays decreases exponentially according to the law $I = I_0 e^{-\mu x}$, where I_0 is the initial intensity of X-rays and I is the intensity after it penetrates a distance x through lead. If μ be the absorption coefficient, then find the dimensional formula for μ.

[1010110571]

Q.4 Find the dimensions of Planck's constant h from the equation E = hν where E is the energy and ν is the frequency.

[1010110859]

Q.5 If the velocity of light (c), gravitational constant (G) and the Planck's constant (h) are selected as the fundamental units, find the dimensional formulae for mass, length and time in this new system of units.

[1010110105]

Q.6 The distance moved by a particle in time from centre of ring under the influence of its gravity is given by $x = a\sin\omega t$ where a and ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitation constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

[1010111222]

Q.7 Find the dimensions of
(a) the specific heat capacity c,
(b) the coefficient of linear expansion α and
(c) the gas constant R.
Some of the equations involving these quantities are $Q = mc\,(T_2 - T_1)$ $l_t = l_0[1 + \alpha\,(T_2 - T_1)]$ and PV = nRT.(Where Q = heat enegry, m= mass , T_1 & T_2 = temperatures, l_t = length at temperature t ºC, l_0 = length at temperature 0 ºC, P = pressure, v = volume, n = mole)

[1010111672]

Q.9 A particle is in a uni-directional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k(1 - \cos ax)$ and k and a are constants. Find the physical dimensions of a and k.

[1010113021]

Q.10 Consider a planet of mass (m), revolving round the sun. The time period (T) of revolution of the planet depends upon the radius of the orbit (r), mass of the sun (M) and the gravitational constant (G). Using dimensional analysis, verify Kepler's third law of planetary motion.

[1010112624]

EXERCISE-3

SECTION-A
(IIT JEE Previous Year's Questions)

PASSAGE (1 to 5)

The van-der Waals equation is

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT,$$

where P is pressure, V is molar volume and T is the temperature of the given sample of gas. R is called molar gas constant, a and b are called van-der Wall constants.

[1010110877]

Q.1 The dimensional formula for b is same as that for
(A) P (B) V (C) PV^2 (D) RT

Q.2 The dimensional formula for a is same as that for
(A) V^2 (B) P (C) PV^2 (D) RT

Q.3 Which of the following does not possess the same dimensional formula as that for RT
(A) PV (B) Pb (C) a/V^2 (D) ab/V^2

Q.4 The dimensional formula for ab/RT is
(A) ML^5T^{-2} (B) $M^0L^3T^0$ (C) $ML^{-1}T^{-2}$ (D) $M^0L^6T^0$

Q5 The dimensional formula of RT is same as that of
(A) energy (B) force (C) specific heat (D) laten heat

Q.6 Match the physical quantities in column A with their dimensional formulae expressed in column B.

[1010110434]

Column A	**Column B**
(1) Angular Momentum	(a) ML^2T^{-2}
(2) Latent Heat	(b) $ML^2T^{-2}A^{-2}$
(3) Torque	(c) ML^2T^{-1}
(4) Capacitance	(d) $ML^3T^{-3}A^{-2}$
(5) Inductance	(e) $M^{-1}L^{-2}T^4A^2$
(6) Resistivity	(f) $ML^2T^{-2}A^{-1}$
(7) Magnetic Flux	(g) $ML^{-1}T^{-2}$
(8) Magnetic Energy Density	(h) L^2T^{-2}

ANSWER KEY

EXERCISE-1

Q.1	B	Q.2	C	Q.3	A	Q.4	D	Q.5	B
Q.6	C	Q.7	B	Q.8	B	Q.9	B	Q.10	A
Q.11	D	Q.12	C	Q.13	B	Q.14	D	Q.15	B
Q.16	C	Q.17	B	Q.18	B	Q.19	C	Q.20	B

EXERCISE-2

Q.1 (I) $FL^{-4}T^2$ (II) FL^{-2} (III) FT (IV) FL

Q.2 $\frac{k}{L}\sqrt{\frac{F}{m}}$

Q.3 L^{-1}

Q.4 $M\,L^2\,T^{-1}$

Q.5 $M = c^{\frac{1}{2}}\,G^{-\frac{1}{2}}\,h^{\frac{1}{2}},\ L = c^{\frac{-3}{2}}\,G^{\frac{1}{2}}\,h^{\frac{1}{2}},\ T = c^{\frac{-5}{2}}\,G^{\frac{1}{2}}\,h^{\frac{1}{2}}$

Q.6 $\sqrt{\frac{GM}{r^3}}$

Q.7 (a) $L^2\,T^{-2}K^{-1}$ (b) K^{-1} (c) $ML^2\,T^{-2}\,K^{-1}\,(mol)^{-1}$

Q.9 L^{-1}, $M\,L^2\,T^{-2}$

Q.10 $T^2 = \frac{kr^3}{GM}$

Q.11 Equation is dimensionally correct

Q.13 $F\,\mu\,v^2$

Q.14 $T = k\sqrt{\frac{\rho r^3}{S}}$

EXERCISE-3

SECTION-A

Q.1	B	Q.2	C	Q.3	C	Q.4	D	Q5	A

Q.6 [1 – (c), 2 – (h), 3 – (a), 4 – (e), 5 – (b), 6 – (d), 7 – (f), 8 – (g)]

Q.7 (A) P, Q ; (B) R, S ; (C) R, S ; (D) R, S]

SECTION-B

Q.1 B

HINTS AND SOLUTION

EXERCISE-1 (Exercise for JEE Main)

[SINGLE CORRECT CHOICE TYPE]

3 $\left[\frac{l}{p}\right] = \left[\frac{mvr}{mv}\right] = [r] = L$

4 $[x] = [\text{force} \times \text{density}] = MLT^{-2}\,\frac{M}{L^3}$

$[x] = M^2L^{-2}T^{-2}$

5 $[l] = [mvr] = (MLT^{-1}.L) = ML^2T^{-1}$

6 $[a] = T^{-1}$

7 $\left[\frac{b}{t}\right] = [v]$

$[b] = LT^{-1}\,.\,T = L$

9 $\left[\frac{ax\,c}{bt^2}\right] = \frac{MLT^{-2} \times MLT^{-2}}{MLT^{-2}} = MLT^{-2}$

10 $[E] = ML^2T^{-2} = (MLT^{-2})\,.\,(L\,.\,T^{-1})\,(T) = [FTV]$

11 Flux $\phi = \vec{E} \times \vec{A}$

so $[\phi] \neq [E]$

& $\vec{p} = q\vec{d}$

so $[p] \neq [E]$

12 (13) $I = Mr^2$

$[I] = ML^2 \rightarrow$ moment of inertia

$[\tau] = [r\,.\,F] = [L\,.\,MLT^{-2}] = ML^2T^{-2}$

$[\tau] \rightarrow$ moment of force

13 $f = Cm^x\,.\,k^y$

$[\,f\,] = [m^x]\,[k^y]$

$\Rightarrow T^{-1} = M^x\,M^y\,T^{-2y}$

$x + y = 0$

$y = \frac{1}{2} \Rightarrow x = \frac{-1}{2}$