Class XI: Mathematics

Chapter 5

Complex Numbers & Quadratic Equations

Chapter Notes

Top Definitions

1. A number of the form a + ib, where a and b are real numbers, is said to be a complex number.

2. In complex number z = a + ib, a is the real part, denoted by Re z and b is the imaginary part denoted by Im z of the complex number z.

3 $\sqrt{-1}$ =i is called the iota the complex number.

4. For any non – zero complex number z = a + ib (a ≠ 0, b ≠ 0), there exists a complex number $\frac{a}{a^2+b^2}+i\frac{-b}{a^2+b^2}$, denoted by $\frac{1}{z}$ or z^{-1}, called the multiplicative inverse of z such that (a + ib) $\left(\frac{a^2}{a^2+b^2}+i\frac{-b}{a^2+b^2}\right)=1+i0=1$.

5. Modulus of a complex number z = a+ib , denoted by |z|, is defined to be the non – negative real number $\sqrt{a^2+b^2}$, i.e $|z|=\sqrt{a^2+b^2}$

6. Conjugate of a complex number z =a+ib, denoted as $\bar{z}$, is the complex number a – ib.

7. z=r(cos θ +isin θ) is the polar form of the complex number z=a+ib.

here r = $\sqrt{a^2+b^2}$ is called the modulus of z and θ = $\tan^{-1}\left(\frac{b}{a}\right)$ is called the argument or amplitude of z, denoted by arg z.

8. The value of θ such that −π < θ ≤ π, called principal argument of z.

9 The plane having a complex number assigned to each of its points is called the complex plane or the Argand plane.

10.Fundamental Theorem of Algebra states that "A polynomial equation of degree n has n roots."

Top Concepts

1. **Addition of two complex numbers**:If $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers then, the sum
$z_1 + z_2 = (a + c) + i(b + d)$.
2. Sum of two complex numbers is also a complex number. this is known as the closure property.

3. The addition of complex numbers satisfy the following properties:
i. Addition of complex numbers satisfies the commutative law. For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$.
ii. Addition of complex numbers satisfies associative law for any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.
iii. There exists a complex number 0 + i0 or 0, called the additive identity or the zero complex number, such that, for every complex number z, $z + 0 = 0+z = z$.
iv. To every complex number $z = a + ib$, there exists another complex number $-z = -a + i(-b)$ called the additive inverse of z. $z+(-z)=(-z)+z=0$

4 **Difference of two complex numbers:** Given any two complex numbers If $z_1 = a + ib$ and $z_2 = c + id$ the difference $z_1 - z_2$ is given by

$$z_1 - z_2 = z_1 + (-z_2) = (a - c) + i(b - d).$$

5 **Multiplication of two complex numbers** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows:

$$z_1 z_2 = (ac - bd) + i(ad + bc)$$

6. **Properties of multiplication of complex numbers**: Product of two complex numbers is a complex number, the product z_1 z_2 is a complex number for all complex numbers z_1 and z_2.

i. Product of complex numbers is commutative i.e for any two complex numbers z_1 and z_2,

$$z_1 z_2 = z_2 z_1$$

ii. Product of complex numbers is associative law For any three complex numbers z_1, z_2, z_3,

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$

iii. There exists the complex number 1 + i0 (denoted as 1), called the multiplicative identity such that z.1 = z for every complex number z.

iv. For every non- zero complex number z = a + ib or a + bi (a ≠ 0, b ≠ 0), there is a complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$, called the multiplicative inverse of z such that

$$z \times \frac{1}{z} = 1$$

v. The distributive law: For any three complex numbers z_1, z_2, z_3,

a. $z_1 (z_2 + z_3) = z_1.z_2 + z_1.z_3$

b. $(z_1 + z_2) z_3 = z_1.z_3 + z_2.z_3$

7.Division of two complex numbers Given any two complex numbers z_1 = a + ib and z_2 = c + id z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by

$$\frac{z_1}{z_2} = z_1.\frac{1}{z_2} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

8. Identities for the complex numbers

i. $(z_1 + z_2)^2 = z_1^2 + z_2^2 = 2z_1.z_2$, for all complex numbers z_1 and z_2.

ii $(z_1 - z_2)^2 = z_1^2 - 2z_1z_2 + z_2^2$

iii.$(z_1 + z_2)^3 = z_1^3 + 3z_1^2z_2 + 3z1z_2^2 + z_2^3$

iv $(z_1 - z_2)^3 = z_1^3 = 3z_1^2z_2 + 3z_1z_2^3 - z_2^3$

v $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$

9. **Properties of modulus and conjugate of complex numbers**

For any two complex numbers z_1 and z_2,

i. $|z_1 z_2| = |z_1||z_2|$

ii. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ provided $|z_2| \neq 0$

iii. $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

iv. $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

v. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ provided $z_2 \neq 0$

10. For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -$

i. $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ when a<0and b<0.

11.The polar form of the complex number $z = x + iy$ is $r(\cos\theta + i\sin\theta)$, where r is the modulus of z and θ is known as the argument of z.

12.For a quadratic equation $ax^2 + bx + c = 0$ with real coefficient a, b, c and $a \neq 0$.

If discriminant $D = b^2 - 4ac \geq 0$ then the equation has two real roots given by

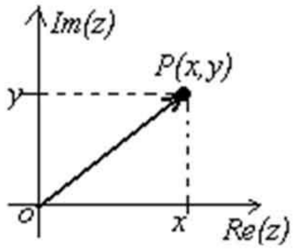
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b}{2a}$$

13. Roots of the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$,when discriminant $b^2-4ac < 0$, are imaginary given by

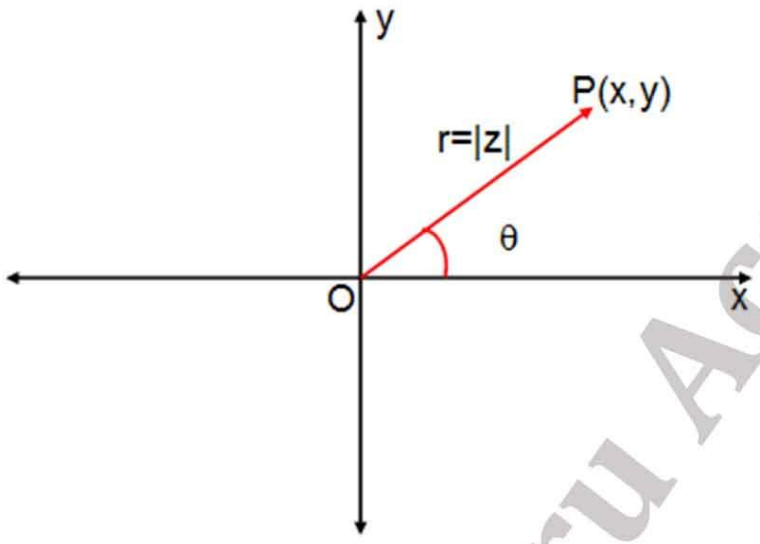
$$x = \frac{-b \pm \sqrt{4ac - b^2}\,i}{2a}$$.Complex roots occurs in pairs.

14. A polynomial equation of n degree has n roots. These n roots could be real or complex.

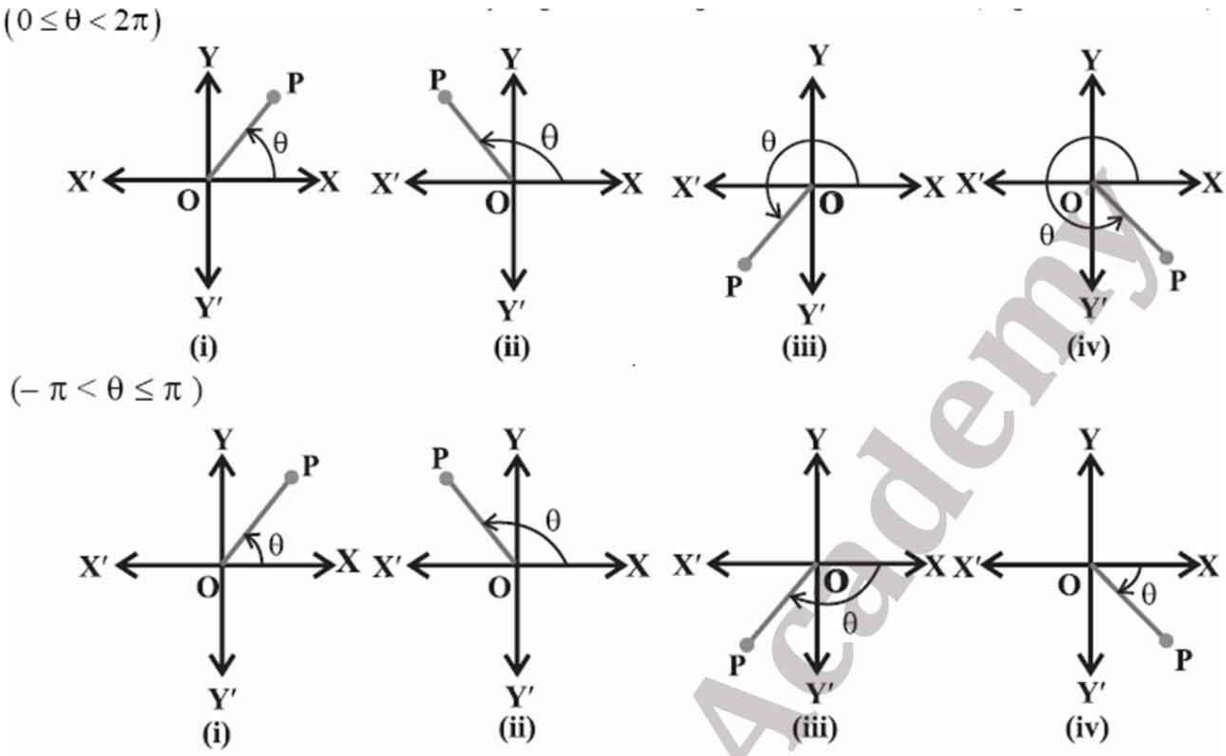
15. Complex numbers are represented in Argand plane with x axis being real and y axis imaginary



16. Representation of complex number z=x+iy in Argand Plane



17. Argument θ of the complex number z can take any value in the interval $[0, 2\pi)$. Different orientations of z are as follows


$(0 \le \theta < 2\pi)$
Y
P
θ
X′
X
O
Y′
(i)
(ii)
(iii)
(iv)
$(-\pi < \theta \le \pi)$
(i)
(ii)
(iii)
(iv)