Motion in a plane

- **Scalar quantities:** These are the physical quantities that are not affected by the change in the coordinate systems used to define them. They do not have any direction. Example: Speed, charge, temperature, etc.

- **Vector quantities:** These are physical quantities that have both direction and magnitude. They change with change in the coordinate systems used to define them. Example: Displacement, velocity, etc.

- **Position vector:** Position vector of a point in a coordinate system is the straight line that joins the origin and the point.

- **Displacement Vector:** It is the straight line that joins the initial and the final position.

- **Equality of Vectors:** Two vectors are said to be equal only if they have the same magnitude and the same direction.

- **Negative vector:** Negative vector is a vector whose magnitude is the same as that of a given vector, but whose direction is opposite to that of the given vector.

- **Zero vector:** Zero vector is a vector whose magnitude is zero and have an arbitrary direction.

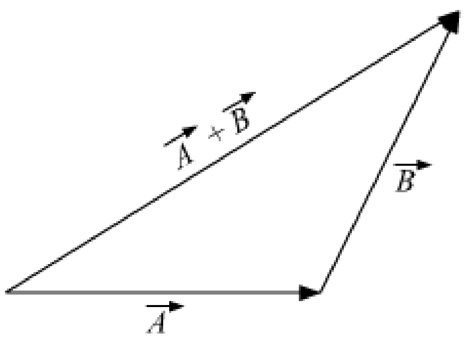
- **Resultant vector:** The resultant vector of two or more vectors is the vector which produces the same effect as produced by the individual vectors together.

- **Multiplication of Vectors by Real Numbers**

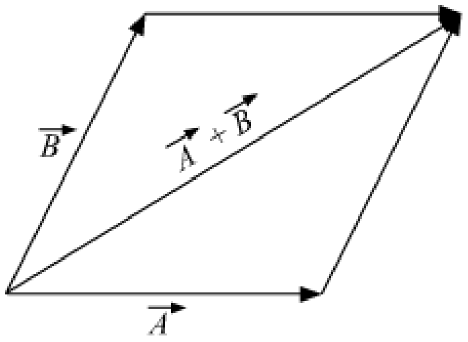
    - Multiplication of a vector with a positive number *k* only changes the magnitude of the vector keeping its direction unchanged.

    - Multiplication of a vector with a negative number –*k* changes the magnitude and direction of the vector.

- **Addition of vectors:**
    - Head-to-tail/ triangle method



- Parallelogram method



- **Vector addition follows commutative and associative laws**:
  - $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

$$Commutative$$

  - $\left(\vec{A}+\vec{B}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$

$$Associative$$

- **Subtraction of vector:**

$$\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)$$

- **Polygon law of vector addition:**
  - According to this law, if a number of vectors acting in a plane are represented in magnitudes and directions by the sides of an open polygon taken in order, then resultant vector is represented in magnitude and direction by the closing side of the polygon taken in the opposite order. The direction of the resultant vector is from the starting point of the first vector to the end point of the last vector.

- **Unit vector:** Unit vector is a vector of unit magnitude along the direction of the vector.

$$\hat{a}=\frac{\vec{a}}{|a|}, \quad \vec{a}=\hat{a}|a|$$

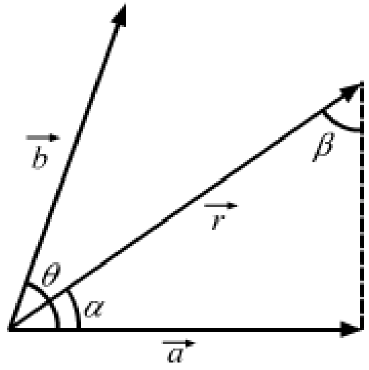
- In 2-D vector, $\vec{a}$ can be expressed as $\vec{a}=a_x\hat{i}+a_y\hat{j}$
- If $\vec{a}$ makes θ angle with X axis, then

$$a_x = a\cos\theta \quad \text{and } a_y = a\sin\theta$$

$$a=|a|=\sqrt{a_x^2+a_y^2}, \tan\theta=\frac{a_y}{a_x}$$

- The same process is used to resolve a vector into three components along X-axis, Y-axis, and Z-axis.

- **Resultant of two vectors**



$$\vec{r} = \vec{a} + \vec{b}$$

- **Law of cosines**

$$|\vec{r}| = \sqrt{|\vec{a}| + |\vec{b}| + 2|\vec{a}||\vec{b}| \cos \theta}$$

- **Law of sines**

$$\frac{|r|}{\sin \theta} = \frac{|a|}{\sin \beta} = \frac{|b|}{\sin \alpha}$$

- $\vec{a} = \frac{\overrightarrow{v'} - \vec{v}}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$ Displacement vector can be written as

$$\Delta \vec{r} = \overrightarrow{r'} - \vec{r} = (x' - x)\hat{i} + (y' - y)\hat{j} + (z' - z)\hat{k} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

- **Velocity:**
  - Average velocity, $\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$
  - Instantaneous velocity, $\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

  Where, $v_x = \frac{dx}{dt}$, $v_y = \frac{dx}{dt}$, $v_z = \frac{dx}{dt}$

- **Acceleration:**
  - Average acceleration, $\vec{a} = \frac{\overrightarrow{v'} - \vec{v}}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$
  - Instantaneous acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{\mathrm{i}} + a_y\hat{j} + a_z\hat{k}$

  Where, $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$

- Motion in a plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.
- Relative Velocity in Two Dimensions**:** If two objects A and B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$.
  - velocity of object A relative to that of B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

- velocity of object B relative to that of A is

$$\vec{v}_{\mathrm{BA}} = \vec{v}_{\mathrm{B}} - \vec{v}_{\mathrm{A}}$$

- If the man wants to protect himself from rain, then he should hold an umbrella at an angle $\theta = \tan^{-1}\left(\frac{v_{\mathrm{m}}}{v_{\mathrm{r}}}\right)$ towards his motion with the vertical.

- The motion of a projectile may be thought of as the result of horizontal and vertical components.
- Vertical component comes under accelerated motion because of acceleration due to gravity acting downwards but horizontal component is under uniform motion.
-
- Both the components act independently.
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- **Equation of the path of a projectile:**

$$y = x \tan\theta - \left(\frac{1}{2}\frac{g}{u^2 \cos^2\theta}\right)x^2$$

This is the equation of a parabola. Hence, the path of a projectile is a parabola.

- **Time of flight T:** It is total time for which an object is in flight.

Total time of flight = Time of ascent + Time of descent

$$T = \frac{2u\sin\theta}{g}$$

- **Maximum height:** Maximum height h reached by the projectile,

$$h = \frac{u^2\sin^2\theta}{2g}$$

- **Horizontal Range:** It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground. It is denoted by R.

$$R = \frac{u^2\sin 2\theta}{g}$$

Horizontal range is maximum if the angle of projection is 45°.

- **Angle of projection:** It is the angle made by velocity of projection with the horizontal.

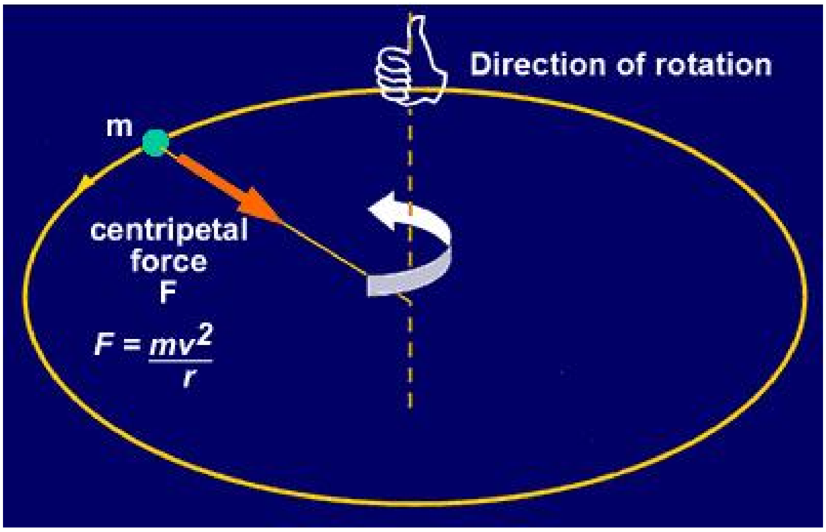
Angle of projection,
$\theta_0 = \tan^{-1}\left(\frac{4H}{R}\right)$
Cases :
When $\theta_o = 45^\circ$,
$R = \frac{u^2}{g}$
$R_{max} = 4H_{max}$

When $\theta_o = 90^o$,

$H = \frac{u^2}{2g}$

$R = 0$

Uniform Circular Motion:



- Speed = |Velocity| = v
- Time period, T = $\frac{2\pi r}{v}$
- Frequency, $f = \frac{v}{2\pi r}$
- Angular velocity, ω = $\frac{v}{r}$
- Centripetal force, F = $\frac{mv^2}{r}$
- Centripetal acceleration = $\frac{v^2}{r} = \omega^2 r$
- Angular velocity, ω = $\frac{v}{r}$
- Centripetal force is a real force that acts on a particle performing circular motion along the radius of a circle. The force is directed towards the centre of the circle.
- The magnitude of centripetal force is given by $F = \frac{mv^2}{r}$.
- Centrifugal force is a pseudo force in uniform circular motion. It acts along the radius and is directed away from the centre of the circle.
- Magnitude of centrifugal force = Mass × Acceleration of the reference frame
  $F_{CF} = mv\omega = \frac{mv^2}{r} = mr\omega^2$