Sample Paper–4

Class 11th, Mathematics

Time: 3 hours **Max. Marks 100**

General Instructions

1. All questions are compulsory.
2. Use of calculator is not permitted. However you may use log table, if required.
3. Q.No. 1 to 12 are of very short answer type questions, carrying 1 mark each.
4. Q.No.13 to 28 carries 4 marks each.
5. Q.No. 29 to 32 carries 6 marks each.

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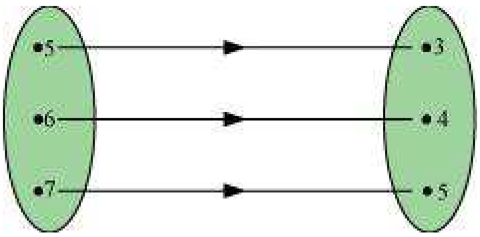
1. Find the union of each of the following pairs of sets:
   (i) X = {1, 3, 5} Y = {1, 2, 3}
   (ii) A = {a, e, i, o, u} B = {a, b, c}

2. The given figure shows a relationship between the sets P and Q. write this relation
   (i) in set-builder form (ii) in roster form.
   What is its domain and range?



3. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by
   {(a, b): a, b ∈ A, b is exactly divisible by a}.
   (i) Write R in roster form
   (ii) Find the domain of R
   (iii) Find the range of R.

4. Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, x lies in second quadrant.

5. Find the value of the trigonometric function $\tan\frac{19\pi}{3}$

6. Express the given complex number in the form $a + ib$: $\left(-2-\frac{1}{3}i\right)^3$

7. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

8. Find the distance between the following pairs of points:
   (i) (2, 3, 5) and (4, 3, 1) (ii) (–3, 7, 2) and (2, 4, –1)

9. Evaluate the given limit: $\lim_{x\to 0}\frac{\cos x}{\pi - x}$

10. Evaluate the Given limit: $\lim\limits_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

11. An experiment consists of recording boy-girl composition of families with 2 children.
    (i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?
    (ii) What is the sample space if we are interested in the number of girls in the family?

12. Two dice are thrown. The events A, B and C are as follows:
    A: getting an even number on the first die.
    B: getting an odd number on the first die.
    C: getting the sum of the numbers on the dice ≤ 5
    Describe the events
    (i) B and C (ii) $A \cap B' \cap C'$

13. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \Phi$.

14. Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$ be a function from **R** into **R**. Determine the range of *f*.

15. Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

16. Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

17. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find
    (i) $\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right)$, (ii) $\text{Im}\left(\frac{1}{z_1 \overline{z}_1}\right)$

18. Solve the inequalities and represent the solution graphically on number line: $5x+1>-24$, $5x-1<24$

19. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

20. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

21. If *a* and *b* are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever *n* is a positive integer.

22. If the first and the n^{th} term of a G.P. are *a* ad *b*, respectively, and if *P* is the product of *n* terms, prove that $P^2 = (ab)^n$.

23. A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

24. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

25. Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

26. Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $(ax^2 + \sin x)(p + q\cos x)$

27. Which of the following statements are true and which are false? In each case give a valid reason for saying so.
    (i) *p*: Each radius of a circle is a chord of the circle.
    (ii) *q*: The centre of a circle bisects each chord of the circle.
    (iii) *r*: Circle is a particular case of an ellipse.
    (iv) *s*: If *x* and *y* are integers such that $x > y$, then $-x < -y$.
    (v) *t*: $\sqrt{11}$ is a rational number.

28. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine
    (i) P(2) (ii) P(1 or 3) (iii) P(not 3)

29. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

30. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, *x* in quadrant II

31. Find the sum of the following series up to *n* terms: $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \ldots$

32. The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

Solutions

1. **(i)** X = {1, 3, 5} Y = {1, 2, 3}

   X∪ Y= {1, 2, 3, 5}

   (ii) A = {*a*, *e*, *i*, *o*, *u*} B = {*a*, *b*, *c*}

   A∪ B = {*a*, *b*, *c*, *e*, *i*, *o*, *u*}

2. According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

   (i) R = {(*x*, *y*): *y* = *x* – 2; *x* ∈ P} or R = {(*x*, *y*): *y* = *x* – 2 for *x* = 5, 6, 7}

   (ii) R = {(5, 3), (6, 4), (7, 5)}

   Domain of R = {5, 6, 7}

   Range of R = {3, 4, 5}

3. A = {1, 2, 3, 4, 6}, R = {(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}

   (i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

   (ii) Domain of R = {1, 2, 3, 4, 6}

   (iii) Range of R = {1, 2, 3, 4, 6}

4. $\sin x = \frac{3}{5}$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since *x* lies in the 2^{nd} quadrant, the value of cos *x* will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

5. It is known that the values of tan x repeat after an interval of π or 180°.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$$

6.

$$\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$= -\left[8 + \frac{i^3}{27} + 2i\left(2+\frac{i}{3}\right)\right]$$
$$= -\left[8 - \frac{i}{27} + 4i + \frac{2i^2}{3}\right] \qquad \left[i^3 = -i\right]$$
$$= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right] \qquad \left[i^2 = -1\right]$$
$$= -\left[\frac{22}{3} + \frac{107i}{27}\right]$$
$$= -\frac{22}{3} - \frac{107}{27}i$$

7. 3-digit numbers have to be formed using the digits 1 to 9.
Here, the order of the digits matters.
Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digit numbers $= {}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!}$

$$= \frac{9\times8\times7\times6!}{6!} = 9\times8\times7 = 504$$

8. The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$= \sqrt{(2)^2+(0)^2+(-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

$$= \sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$$
$$= \sqrt{(5)^2+(-3)^2+(-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

9. $\lim_{x\to 0}\frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

10. $\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1}$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1} = \lim_{x\to 0}\frac{1-2\sin^2 x - 1}{1 - 2\sin^2\frac{x}{2} - 1} \qquad \left[\cos x = 1 - 2\sin^2\frac{x}{2}\right]$$

$$= \lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}} = \lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\times x^2}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$= 4\frac{\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$= 4\frac{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2}{\left(\lim_{\frac{x}{2}\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2} \qquad \left[x\to 0 \Rightarrow \frac{x}{2}\to 0\right]$$

$$= 4\frac{1^2}{1^2} \qquad \left[\lim_{y\to 0}\frac{\sin y}{y} = 1\right]$$

$$= 4$$

11. (i) When the order of the birth of a girl or a boy is considered, the sample space is given by S = {GG, GB, BG, BB}

(ii) Since the maximum number of children in each family is 2, a family can either have 2 girls or 1 girl or no girl. Hence, the required sample space is S = {0, 1, 2}

12. (i) B and C = B ∩ C $\{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(ii)

$$C' = \begin{Bmatrix}(1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2),\\(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

$$\therefore A\cap B'\cap C' = A\cap A\cap C' = A\cap C'$$

$$= \begin{Bmatrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),\\(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

13. Let A = {0, 1}, B = {1, 2}, and C = {2, 0}.
Accordingly, $A \cap B = \{1\}$, $B \cap C = \{2\}$, and $A \cap C = \{0\}$.
$\therefore$ $A \cap B$, $B \cap C$, and $A \cap C$ are non-empty.
However, $A \cap B \cap C = \Phi$

14. $f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$

$$=\left\{(0,\ 0),\ \left(\pm 0.5,\ \frac{1}{5}\right),\ \left(\pm 1,\ \frac{1}{2}\right),\ \left(\pm 1.5,\ \frac{9}{13}\right),\ \left(\pm 2,\ \frac{4}{5}\right),\ \left(3,\ \frac{9}{10}\right),\ \left(4,\ \frac{16}{17}\right),\ \ldots\right\}$$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[Denominator is greater numerator]
Thus, range of *f* = [0, 1)

15. $\cos 3x + \cos x - \cos 2x = 0$

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x(2\cos x - 1) = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

16. Let the given statement be P(*n*), i.e.,

$$P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

For *n* = 1, we have

$$P(1): \frac{1}{3.5} = \frac{1}{3(2.1+3)} = \frac{1}{3.5}$$
, which is true.

Let P(*k*) be true for some positive integer *k*, i.e.,

$$P(k): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \qquad \ldots (1)$$

We shall now prove that P(*k* + 1) is true.
Consider

$$\left[\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}\right]+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$$

$$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)} \qquad [\text{Using }(1)]$$

$$=\frac{1}{(2k+3)}\left[\frac{k}{3}+\frac{1}{(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{3(2k+5)}\right]$$

$$=\frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$$

$$=\frac{(k+1)}{3\{2(k+1)+3\}}$$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

17. $z_1=2-i,\ z_2=-2+i$

(i) $z_1z_2=(2-i)(-2+i)=-4+2i+2i-i^2=-4+4i-(-1)=-3+4i$

$\bar{z}_1=2+i$

$\therefore \frac{z_1z_2}{\bar{z}_1}=\frac{-3+4i}{2+i}$

On multiplying numerator and denominator by (2 – i), we obtain

$$\frac{z_1z_2}{\bar{z}_1}=\frac{(-3+4i)(2-i)}{(2+i)(2-i)}=\frac{-6+3i+8i-4i^2}{2^2+1^2}=\frac{-6+11i-4(-1)}{2^2+1^2}$$

$$=\frac{-2+11i}{5}=\frac{-2}{5}+\frac{11}{5}i$$

On comparing real parts, we obtain

$$\text{Re}\left(\frac{z_1z_2}{\bar{z}_1}\right)=\frac{-2}{5}$$

(ii) $$\frac{1}{z_1\bar{z}_1}=\frac{1}{(2-i)(2+i)}=\frac{1}{(2)^2+(1)^2}=\frac{1}{5}$$

On comparing imaginary parts, we obtain

$$\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)=0$$

18. $5x + 1 > -24$

$\Rightarrow 5x > -25$

$\Rightarrow x > -5 \ldots (1)$

$5x - 1 < 24$

$\Rightarrow 5x < 25$

$\Rightarrow x < 5 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is (–5, 5). The solution of the given system of inequalities can be represented on number line as

19. In the word DAUGHTER, there are 3 vowels namely, A, U, and E, and 5 consonants namely, D, G, H, T, and R.

Number of ways of selecting 2 vowels out of 3 vowels = $\underline{{}^3C_2 = 3}$

Number of ways of selecting 3 consonants out of 5 consonants = ${}^5C_3 = 10$

Therefore, number of combinations of 2 vowels and 3 consonants = 3 × 10 = 30

Each of these 30 combinations of 2 vowels and 3 consonants can be arranged among themselves in 5! ways.

Hence, required number of different words = 30 × 5! = 3600

20. There are 9 courses available out of which, 2 specific courses are compulsory for every student. Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme

$$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35$$

21. In order to prove that $(a - b)$ is a factor of $(a^n - b^n)$, it has to be proved that

$a^n - b^n = k(a - b)$, where k is some natural number

It can be written that, $a = a - b + b$

$$\therefore a^n = (a-b+b)^n = \left[(a-b)+b\right]^n$$

$$= {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + {}^nC_nb^n$$

$$= (a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + b^n$$

$$\Rightarrow a^n - b^n = (a-b)\left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$$

$$\Rightarrow a^n - b^n = k(a-b)$$

where, $k = \left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$ is a natural number

This shows that $(a - b)$ is a factor of $(a^n - b^n)$, where n is a positive integer.

22. The first term of the G.P is a and the last term is b.
Therefore, the G.P. is a, ar, ar^2, ar^3, … ar^{n-1}, where r is the common ratio.
$b = ar^{n-1}$ … (1)
P = Product of n terms
$= (a)(ar)(ar^2) \ldots (ar^{n-1})$
$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$
$= a^n r^{1+2+\ldots(n-1)}$ … (2)
Here, 1, 2, …(n – 1) is an A.P.

$\therefore 1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}\left[2+(n-1-1)\times 1\right] = \frac{n-1}{2}[2+n-2] = \frac{n(n-1)}{2}$

$$P = a^n r^{\frac{n(n-1)}{2}}$$
$$\therefore P^2 = a^{2n} r^{n(n-1)}$$
$$= \left[a^2 r^{(n-1)}\right]^n$$
$$= \left[a \times ar^{n-1}\right]^n$$
$$= (ab)^n \qquad [\text{Using } (1)]$$

Thus, the given result is proved.

23. The equations of the given lines are
$2x - 3y + 4 = 0$ … (1)
$3x + 4y - 5 = 0$ … (2)
$6x - 7y + 8 = 0$ … (3)
The person is standing at the junction of the paths represented by lines (1) and (2).

On solving equations (1) and (2), we obtain $x = -\frac{1}{17}$ and $y = \frac{22}{17}$.

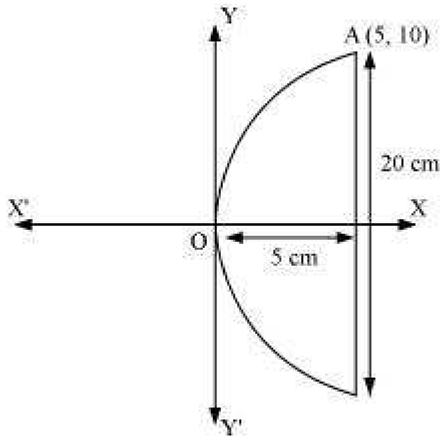
Thus, the person is standing at point $\left(-\frac{1}{17}, \frac{22}{17}\right)$.
The person can reach path (3) in the least time if he walks along the perpendicular line to (3) from point $\left(-\frac{1}{17}, \frac{22}{17}\right)$.

Slope of the line (3) $= \frac{6}{7}$

$\therefore$Slope of the line perpendicular to line (3) $= -\frac{1}{\left(\frac{6}{7}\right)} = -\frac{7}{6}$

The equation of the line passing through $\left(-\frac{1}{17}, \frac{22}{17}\right)$ and having a slope of $-\frac{7}{6}$ is given by
$$\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$
$$6(17y - 22) = -7(17x + 1)$$
$$102y - 132 = -119x - 7$$
$$119x + 102y = 125$$
Hence, the path that the person should follow is $119x + 102y = 125$.

24. The origin of the coordinate plane is taken at the vertex of the parabolic reflector in such a way that the axis of the reflector is along the positive x-axis.
This can be diagrammatically represented as



The equation of the parabola is of the form $y^2 = 4ax$ (as it is opening to the right).
Since the parabola passes through point A (10, 5), $10^2 = 4a(5)$
$\Rightarrow 100 = 20a$
$\Rightarrow a = \frac{100}{20} = 5$
Therefore, the focus of the parabola is $(a, 0) = (5, 0)$, which is the mid-point of the diameter.
Hence, the focus of the reflector is at the mid-point of the diameter.

25. The given points are A (2, –3, 4), B (–1, 2, 1), and $C\left(0, \frac{1}{3}, 2\right)$.
Let P be a point that divides AB in the ratio k:1.
Hence, by section formula, the coordinates of P are given by
$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$
Now, we find the value of k at which point P coincides with point C.
By taking $\frac{-k+2}{k+1} = 0$, we obtain $k = 2$.
For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.
i.e., $C\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.
Hence, points A, B, and C are collinear.

26. Let $f(x) = (ax^2 + \sin x)(p + q\cos x)$
By product rule,

$$f'(x) = (ax^2 + \sin x)\frac{d}{dx}(p + q\cos x) + (p + q\cos x)\frac{d}{dx}(ax^2 + \sin x)$$

$$= (ax^2 + \sin x)(-q\sin x) + (p + q\cos x)(2ax + \cos x)$$

$$= -q\sin x(ax^2 + \sin x) + (p + q\cos x)(2ax + \cos x)$$

27. (i) The given statement p is false.

According to the definition of chord, it should intersect the circle at two distinct points.

(ii) The given statement q is false.

If the chord is not the diameter of the circle, then the centre will not bisect that chord.

In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

If we put $a = b = 1$, then we obtain

$x^2 + y^2 = 1$, which is an equation of a circle

Therefore, circle is a particular case of an ellipse.

Thus, statement r is true.

(iv) $x > y$

$\Rightarrow -x < -y$ (By a rule of inequality)

Thus, the given statement s is true.

(v) 11 is a prime number and we know that the square root of any prime number is an irrational number. Therefore, $\sqrt{11}$ is an irrational number.

Thus, the given statement t is false.

28. Total number of faces = 6

(i) Number faces with number '2' = 3

$$\therefore \text{P}(2)=\frac{3}{6}=\frac{1}{2}$$

(ii) P (1 or 3) = P (not 2) = 1 − P (2) $=1-\frac{1}{2}=\frac{1}{2}$

(iii) Number of faces with number '3' = 1

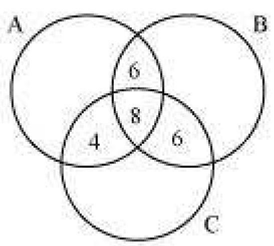
$$\therefore \text{P}(3)=\frac{1}{6}$$

$$\text{Thus, P(not 3)}=1-\text{P}(3)=1-\frac{1}{6}=\frac{5}{6}$$

29. Let A, B, and C be the set of people who like product A, product B, and product C respectively.

Accordingly, $n(\text{A}) = 21$, $n(\text{B}) = 26$, $n(\text{C}) = 29$, $n(\text{A} \cap \text{B}) = 14$, $n(\text{C} \cap \text{A}) = 12$, $n(\text{B} \cap \text{C}) = 14$, $n(\text{A} \cap \text{B} \cap \text{C}) = 8$

The Venn diagram for the given problem can be drawn as



It can be seen that number of people who like product C only is
{29 – (4 + 8 + 6)} = 11

30. Here, x is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$, and $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ [cos$x$ is negative in quadrant II]

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} \qquad \left[\because \cos\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}=\frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$.

31. The n^{th} term of the given series is $\frac{1^3+2^3+3^3+...+n^3}{1+3+5+...+(2n-1)}=\frac{\left[\frac{n(n+1)}{2}\right]^2}{1+3+5+...+(2n-1)}$

Here, $1,3,5,...(2n-1)$ is an A.P. with first term a, last term $(2n-1)$ and number of terms as n

$$\therefore 1+3+5+.....+(2n-1)=\frac{n}{2}\left[2\times1+(n-1)2\right]=n^2$$

$$\therefore a_n=\frac{n^2(n+1)^2}{4n^2}=\frac{(n+1)^2}{4}=\frac{1}{4}n^2+\frac{1}{2}n+\frac{1}{4}$$

$$\therefore S_n=\sum_{K=1}^{n}a_K=\sum_{K=1}^{n}\left(\frac{1}{4}K^2+\frac{1}{2}K+\frac{1}{4}\right)$$

$$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$$

$$=\frac{n\left[(n+1)(2n+1)+6(n+1)+6\right]}{24}$$

$$=\frac{n\left[2n^2+3n+1+6n+6+6\right]}{24}$$

$$=\frac{n\left(2n^2+9n+13\right)}{24}$$

32. The mean and the standard deviation of goals scored by team A are calculated as follows.

No. of goals scored	**No. of matches**	f_ix_i	x_i^2	$f_ix_i^2$
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28

3	5	15	9	45
4	3	12	16	48
	25	50		130

$$\text{Mean} = \frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{5} f_i} = \frac{50}{25} = 2$$

Thus, the mean of both the teams is same.

$$\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

$$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$$

$$= \frac{1}{25}\sqrt{750}$$

$$= \frac{1}{25} \times 27.38$$

$$= 1.09$$

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2. Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.