Class XI

Mathematics

Chapter:14 Mathematical Reasoning

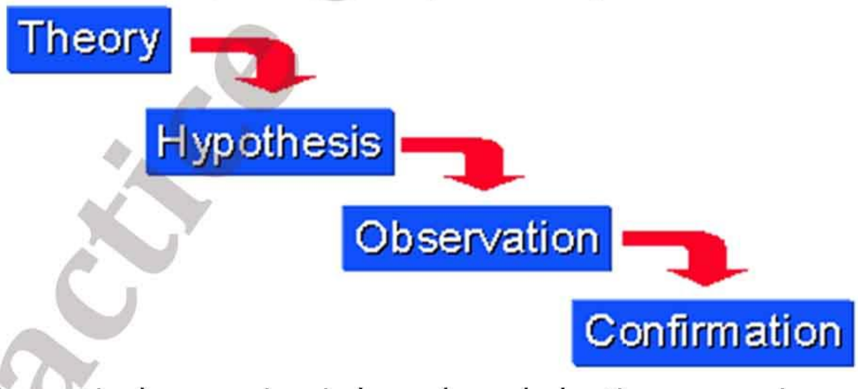
Chapter Notes

Key Concepts

1. There are two types of reasoning the **deductive** and **inductive**. Deductive reasoning was developed by **Aristotle, Thales, Pythagoras in the** classical Period (600 to 300 B.C.).

2. In deduction, given a statement to be proven, often called a conjecture or a theorem, valid deductive steps are derived and a proof may or may not be established. Deduction is the application of a general case to a particular case.

3. Inductive reasoning depends on working with each case, and developing a conjecture by observing incidence till each and every case is observed.
4. Deductive approach is known as the top-down" approach". Given the theorem which is narrowed down to specific *hypotheses* then to *observation*. Finally the hypotheses is tested with specific data to get the *confirmation* (or not) of original theory.



5. Mathematical reasoning is based on deductive reasoning.
   The classic example of deductive reasoning, given by Aristotle, is

- All men are mortal.
- Socrates is a man.
- Socrates is mortal.

6. The basic unit involved in reasoning is mathematical statement.

7. A sentence is called a mathematically acceptable statement if it is either true or false but not both. A sentence which is both true and false simultaneously is called a paradox.

8. Sentences which involve tomorrow, yesterday, here, there etc i.e variables etc are not statements.

9. The sentence expresses a request, a command or is simply a question are not statements.

10. The denial of a statement is called the negation of the statement.

11. Two or more statements joined by words like "and" "or" are called Compound statements. Each statement is called a **component statement.** **"**and" "or" are connecting words.

12. An " And" statement is true if each of the component statement is true and it is false even if one component statement is false.

13. An "OR" statement is will be true when even one of its components is true and is false only when all its components are false

14. The word "OR" can be used in two ways (i) Inclusive OR (ii) Exclusive OR. If only one of the two options is possible then the OR used is Exclusive OR.
 If any one of the two options or both the options are possible then the OR used is Inclusive OR.

15. There exists "$\exists$" and "For all" $\forall$ are called quantifiers.

16. A statement with quantifier "There exists" is true, if it is true for at least one case.

17. If p and q are two statements then a statement of the form '**If p then q'** is known as a conditional statement. In symbolic form p **implies** q is denoted by $p \Rightarrow q$.

18. The conditional statement $p \Rightarrow q$ can be expressed in the various other forms:
(i) q if p (ii) p only if q (iii) p is sufficient for q (iv) q is necessary for p.

19. A statement formed by the combination of two statements of the form if p then q and if q then p is p if and only if q. It is called biconditional statement.

20. Contrapositive and converse can be obtained by a if then statement
    The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
    The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$

21. Truth values of various statement

p	q	p and q	p or q	$p \Rightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

22. Two prove the truth of an if p- then q statement . there are two ways : the first is assume p is true and prove q is true. This is called the direct method.
Or assume that q is false and prove p is false. This is called the Contrapositive method.

23. To prove the truth of " p if and only if q" statement , we must prove two things , one that the truth of p implies the truth of q and the second that the truth of q implies the truth of p.

24. The following methods are used to check the validity of statements:
    (i) Direct method
    (ii) Contra positive method
    (iii) Method of contradiction
    (iv) Using a counter example

25. To check whether a statement p is true , we assume that it is not true, i.e. $\sim p$ is true . Then we arrive at some result which contradicts our assumption.