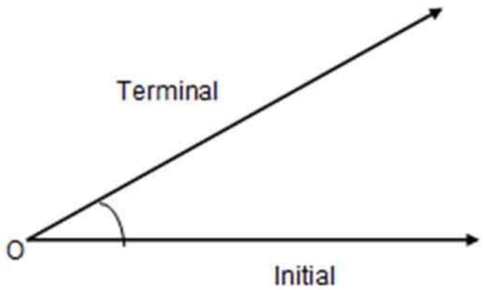
Class XI: Maths
Ch 3: Trigonometric Function

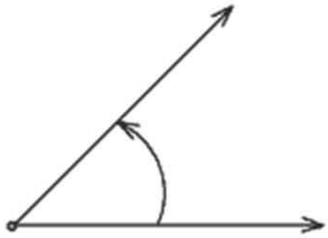
Chapter Notes

Top Concepts

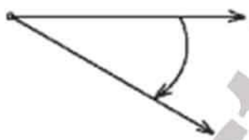
1. An angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.



2. If the direction of the rotation is anticlockwise, the angle is said to be positive and if the direction of the rotation is clockwise, then the angle is negative.



Positive Angle- Anticlockwise



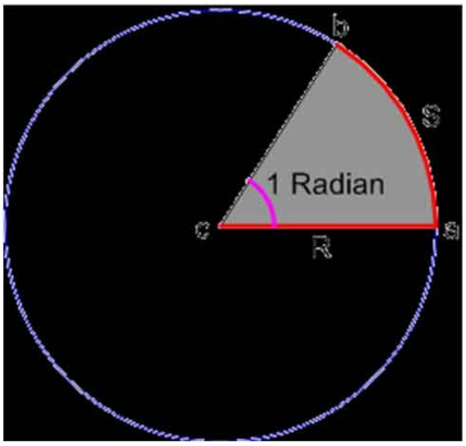
Negative Angle- Clockwise

3. If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, I t is denoted by 1°.

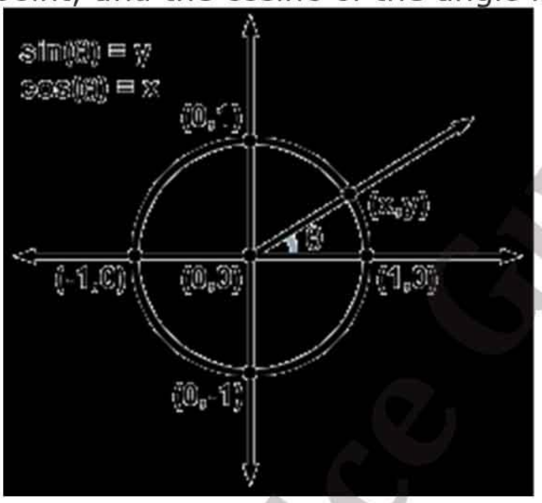
4. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. One sixtieth of a degree is called a minute, written as 1′, and one sixtieth of a minute is called a second, written as 1″

Thus, $1^{\circ} = 60'$, $1' = 60''$

5. Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian



6. If a point on the unit circle is on the terminal side of an angle in standard position, then the sine of such an angle is simply the y-coordinate of the point, and the cosine of the angle is the x-coordinate of that point.



7. All the angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles .Values of quadrantal angles are as follows:

$\cos 0 = 1, \sin 0 = 0$

$\cos \frac{\pi}{2} = 0, \sin \frac{\pi}{2} = 1$

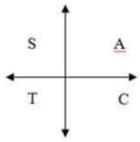
$\cos \pi = -1, \sin \pi =$

$\cos \frac{3\pi}{2} = 0, \sin \frac{\pi}{2} = -1$

$\cos 2\pi = 1, \sin 2\pi = 0$

8 .Cosine is even and sine is odd function
$\cos(-x) = \cos x$
$\sin(-x) = -\sin x$

9. Signs of Trigonometric functions in various quadrants
In quadrant I, all the trigonometric functions are positive.
In quadrant II, only sine is positive. In quadrant III, only tan is positive, quadrant IV, only cosine function is positive. This is depicted as follows



10. In quadrants where Y-axis is positive (i.e. I and II), sine is positive and in quadrants where X-axis is positive (i.e. I and IV), cosine is positive

11. A function f is said to be a periodic function if there exists a real number $T>0$, such that $f(x + T) = f(x)$ for all x. This T is the period of function.

12. $\sin(2\pi + x) = \sin x$ so the period of sine is 2π. Period of its reciprocal is also 2π

13. $\cos(2\pi + x) = \cos x$ so the period of cos is 2π. Period of its reciprocal is also 2π

14. $\tan(\pi + x) = \tan x$ Period of tangent and cotangent function is π

15.The graph of cos x can be obtained by shifting the sin function by the factor $\frac{\pi}{2}$

16. The tan function differs from the previous two functions in two ways
(i)tan is not defined at the odd multiples of $\pi/2$ (ii) tan function is not bounded.

17.

Function	**Period**
$y=\sin x$	2π
$y=\sin(ax)$	$\frac{2\pi}{a}$
$y=\cos x$	2π
$y=\cos(ax)$	$\frac{2\pi}{a}$
$y=\cos 3x$	$\frac{2\pi}{3}$
$y=\sin 5x$	$\frac{2\pi}{5}$

18. For a function of the form
 y= kf(ax+b) range will be k times the range of function x, where k is any real number if f(x)= sine or cosine function
range will be equal to R-[-k, k] if function is of the form sec x or cosec x,
Period is equal to the period of function f by a.
The position of the graph is b units to the right/left of y=f(x) depending on whether b>0 or b<0

19. The solutions of a trigonometric equation for which $0 \le x \le 2\pi$ are called principal solutions.

20.The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution.

21. The numerical smallest value of the angle (in degree or radian) satisfying a given trigonometric equation is called the Principal Value. If there are two values, one positive and the other negative, which are numerically equal, then the positive value is taken as the Principal value.

Top Formulae

1. 1 radian = $\frac{180^\circ}{\pi} = 57^\circ 16'$ approximately

2. 1° = $\frac{\pi}{180^\circ}$ radians $= 0.01746$ radians approximately

3.

s= r θ
Length of arc= radius × angle in radian
This relation can only be used when θ is in radians

4. Radian measure= $\frac{\pi}{180} \times$ Degree measure

5. Degree measure = $\frac{180}{\pi} \times$ Radian measure

6. Trigonometric functions in terms of sine and cosine

$\operatorname{cosec} x = \frac{1}{\sin x}, x \neq n\pi$, where n is any integer

$\sec x = \frac{1}{\cos x}, x \neq (2n+1)\frac{\pi}{2}$, where n is any integer

$\tan x = \frac{\sin x}{\cos x}, x \neq (2n+1)\frac{\pi}{2}$, where n is any integer

$\cot x = \frac{1}{\tan x}, x \neq n\pi$, where n is any integer

7. Fundamental Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

8 Values of Trigonometric ratios:

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	10	0	−1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0

9. Domain and range of various trigonometric functions:

Function	**Domain**	**Range**
y = sin x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	[−1, 1]
y = cos x	$[0, \pi]$	[−1, 1]
y = cosec x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	R – (−1,1)
y = sec x	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	R – (−1, 1)

y = tan x	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	R
y = cot x	$(0, \pi)$	R

10. Sign Convention

	I	II	III	IV
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosec x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

11. Behavior of Trigonometric Functions in various Quadrants

	I quadrant	II quadrant	III quadrant	IV quadrant
sin	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0
cos	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0	increases from 0 to 1
tan	increases from 0 to ∞	increases from $-\infty$ to 0	increase from 0 to $-\infty$	increases from $-\infty$ to 0
cot	decrease from ∞ to 0	decreases from 0 to $-\infty$	decreases from ∞ to 0	decreases from 0 to $-\infty$
sec	increases from 1 to ∞	increase from $-\infty$ to -1	decreases from -1 to $-\infty$	decreases from ∞ to 1
cosec	decreases from ∞ to 1	increases from 1 to ∞	increases from $-\infty$ to -1	decreases from -1 to $-\infty$

12. Basic Formulae

(i) $\cos(x + y) = \cos x \cos y - \sin x \sin y$

(ii) cos (x - y) = cos x cos y + sin x sin y

(iii) sin (x + y) = sin x cos y + cos x sin y

(iv) sin (x − y) = sin x cos y − cos x sin y

If none of the angles x, y and (x + y) is an odd multiple of $\frac{\pi}{2}$, then

(v) tan (x + y) = $\frac{\tan x + \tan y}{1 - \tan x \tan y}$

(vi) tan (x − y) = $\frac{\tan x - \tan y}{1 + \tan x \tan y}$

If none of the angles x, y and (x + y) is a multiple of π, then

(vii) cot (x + y) = $\frac{\cot x \cot y - 1}{\cot x \cot y}$

(viii) cot (x − y) = $\frac{\cot x \cot y - 1}{\cot y - \cot x}$

13. Allied Angle Relations

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\text{con}\left(\frac{\pi}{2} + x\right) = -\sin x$

$\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos(\pi - x) = -\cos x$

$\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$

$\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$

$\sin(2\pi - x) = -\sin x$

$\cos(2n\pi + x) = \cos x$

$\sin(2n\pi + x) = \sin x$

14. Sum and Difference Formulae

(i) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

(ii) $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

(iii) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

(iv) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

(v) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

(vi) $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$

(vii) $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

(viii) $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

15. Multiple Angle Formulae

(i) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

(ii) $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$

(iii) $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

(iv) $\sin 3x = 3\sin x - 4\sin^3 x$

(v) $\cos 3x = 4\cos^3 x - 3\cos x$

(vi) $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

16.Trigonometric Equations

No.	Equations	General Solution	Principal value

1	$\sin\theta = 0$	$\theta = n\pi, n \in Z$	$\theta = 0$
2	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}$, $n \in Z$	$\theta = \frac{\pi}{2}$
3	$\tan\theta = 0$	$\theta = n\pi$	$\theta = 0$
4	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n \alpha$ $n \in Z$	$\theta = \alpha$
5	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$ $n \in Z$	$\theta = 2\alpha, \alpha > 0$
6	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha$ $n \in Z$	$\theta = \alpha$

14. (i) $\sin\theta = k = \sin(n\pi + (-1)^n \alpha), n \in Z$

$\theta = n\pi + (-1)^n \alpha, n \in Z$

$\operatorname{cosec}\theta = \operatorname{cosec}\alpha \Rightarrow \sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n \alpha, n \in Z$

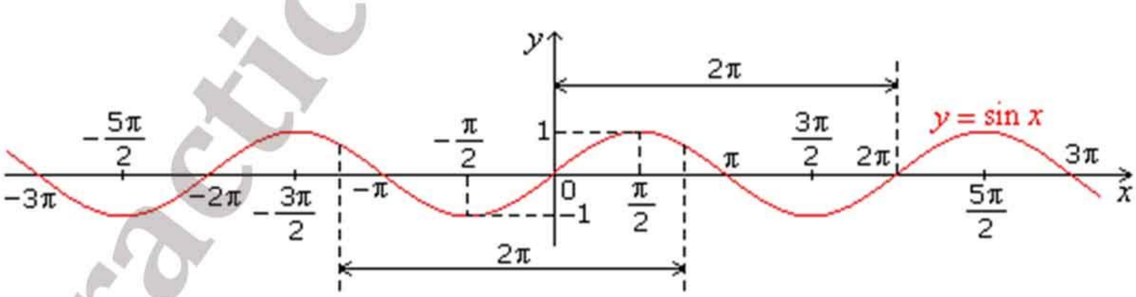
(ii) $\cos\theta = k = \cos(2n\pi \pm \alpha), n \in Z$

$\theta = 2n\pi \pm \alpha, n \in Z$

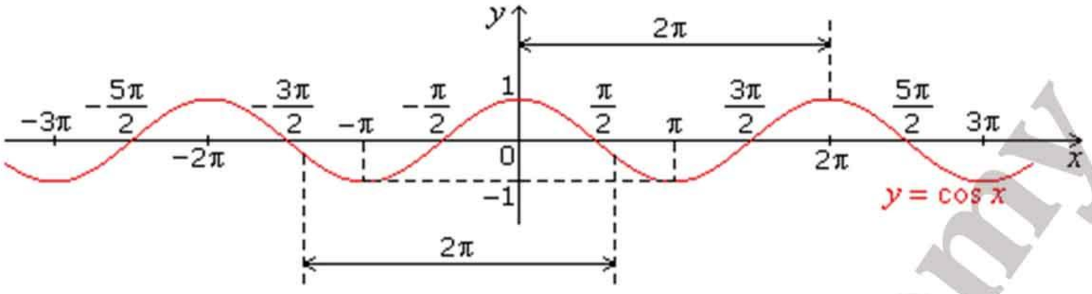
Top Diagrams

1. Graphs helps in visualization of properties of trigonometric functions. The graph of $y = \sin\theta$ can be drawn by plotting a number of points $(\theta, \sin\theta)$ as θ takes a series of different values. Since the sine function is continuous, these points can be joined with a smooth curve. Following similar procedures graph of other functions can be obtained.

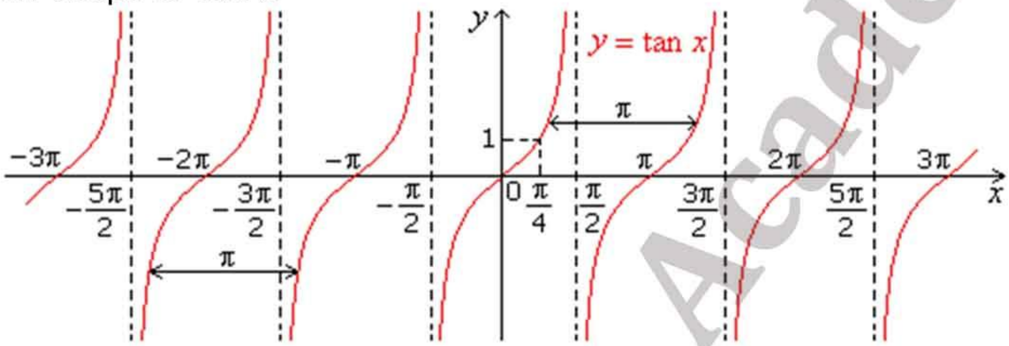
i. Graph of sin x



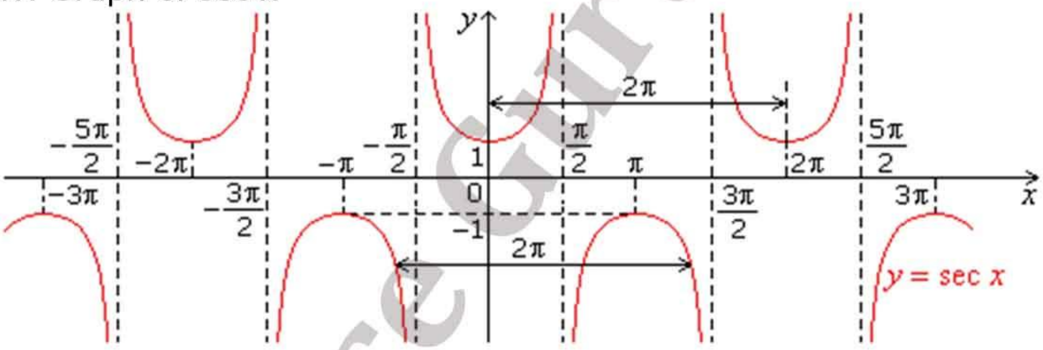
ii. Graph of cos x



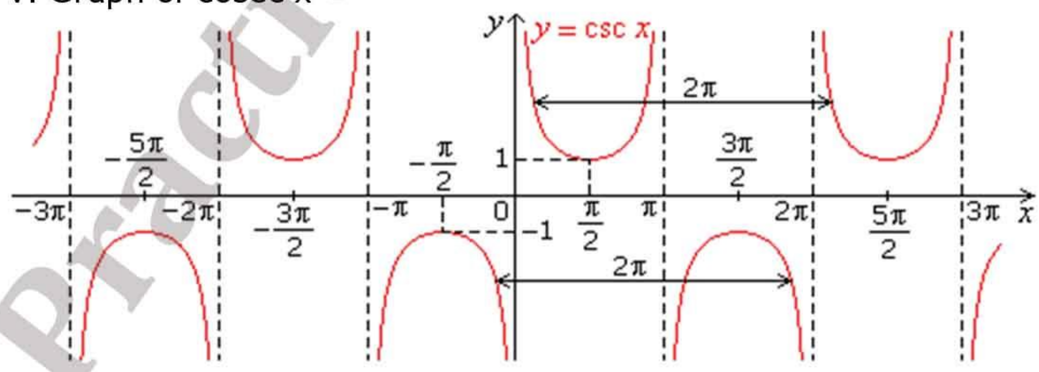
iii. Graph of tan x



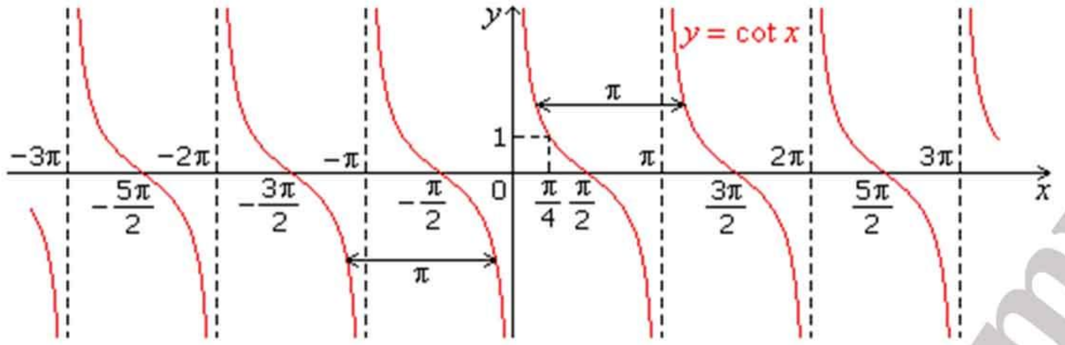
iv. Graph of sec x



v. Graph of cosec x



vi. Graph of cot x



y = cot x
y
x
π
1
−3π
−2π
−π
0
π
2π
3π
−5π/2
−3π/2
−π/2
π/4
π/2
3π/2
5π/2