

# OSCILLATIONS

## Important Points:

### 1. Simple Harmonic Motion:

a) The acceleration is directly proportional to the displacement of the body from the fixed point and it is always directed towards the fixed point in the path of the body.

$$a \propto -x \quad (\text{Or}) \quad a = -kx$$

$$\text{Acceleration (a)} = \frac{F}{m} = \frac{-k}{m}x.$$

$$a = -\omega^2 x \quad \text{Where } \omega = \sqrt{\frac{k}{m}}$$

$$(\text{Or}) \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

2. Every harmonic is periodic. But every periodic is not harmonic.

### 3. Amplitude:

The maximum displacement of a vibrating particle from its mean position is called amplitude (A). It is a vector quantity.

### 4. Time Period:

The time taken for one oscillation is called time period

$$T = \frac{2\pi}{\omega} \quad \text{Where } \omega \text{ is angular velocity}$$

### 5. Frequency:

The number of oscillations completed by a particle in one second is called frequency.

$$\text{Frequency } f = \frac{1}{T} \text{ cycles/sec.}$$

## 6. Phase:

Phase is that which denotes the state of vibration

## 7. Displacement:

Displacement of a particle executing simple harmonic motion is given by

$$y = A \sin(\omega t \pm \phi)$$

Where  $\phi$  = Initial phase

$\omega$  = Angular frequency

A = Amplitude

## 8. Velocity:

Velocity of the particle executing simple harmonic motion is given by

$$V = A\omega \cos \omega t \quad (\text{Or}) \quad V = \omega \sqrt{A^2 - y^2}$$

$$V_{\max} = A\omega$$

## 9. Acceleration:

Acceleration of a particle executing simple harmonic motion is given by

$$a = -A\omega^2 \sin(\omega t \pm \phi)$$

$$a = -\omega^2 y$$

$$a_{\max} = A\omega^2$$

For simple harmonic motion is given by

$$T = 2\pi \sqrt{\frac{y}{a}}$$

Where y = displacement and

a = acceleration

### 10. Energy of the particle in SHM:

$$a) \quad PE = \frac{1}{2} m \omega^2 y^2 = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t$$

$$b) \quad KE = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (A^2 - y^2) \\ = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$$

$$c) \quad \text{Total energy} = \frac{1}{2} m \omega^2 A^2 = 2\pi^2 n^2 m A^2$$

d) At the mean position KE is maximum and PE is minimum.

e) At the extreme position KE is minimum and PE is maximum.

### 11. Simple Pendulum:

Time period of simple pendulum executing S.H.M. is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Where  $l$  = length of the simple pendulum

$g$  = acceleration due to gravity

### 12. Seconds Pendulum:

A pendulum with time period 2 seconds is called second pendulum. Its length on earth is 100 cm.

### 13. Loaded Spring:

i) If a mass  $M$  is suspended vertically from a spring and if the spring elongates by 'x' then spring constant is

$$K = \frac{F}{x} \quad \text{Or} \quad K = \frac{Mg}{x}$$

ii) Time period of the loaded spring is

$$T = 2\pi\sqrt{\frac{M}{K}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{x}{g}}$$

iii) If two springs of force constants  $k_1$  and  $k_2$  are joined in series, the effective force

constant  $k = \frac{k_1 k_2}{k_1 + k_2}$

iv) If two springs of force constants  $k_1$  and  $k_2$  are joined in parallel, the effective force

constant  $k = k_1 + k_2$ .

v) When a spring of force constant  $k$  is cut into  $n$  equal parts, the spring constant of each part is

$nk$

vi) If a uniform spring of spring constant  $K$  is cut into two pieces of lengths in the ratio  $l_1 : l_2$ , then the force constants of the two springs will be

$$k_1 = \frac{k(l_1 + l_2)}{l_1} \quad \text{and} \quad k_2 = \frac{k(l_1 + l_2)}{l_2}$$

## Very Short Answer Questions

**1. Give two examples of periodic motion which are not oscillatory?**

- A.** 1) Motion of the seconds hand in a watch  
2) Revolution earth around the sun.

**2. The displacement in S.H.M. is given by  $y = a \sin (20t + 4)$ . What is the displacement when it is increased by  $2\pi / \omega$ ?**

- A.** In time equal to time period  $T$ , the particle comes to same position. So displacement of the particle remains same even after time is increased by  $\frac{2\pi}{\omega}$ .

**3. A girl is swinging seated in a swing. What is the effect on the frequency of oscillation if the stands?**

- A:** When the girl stands up, her centre of mass moves up and length of swing decreases. Hence frequency of oscillation increases, since  $\nu \propto \frac{1}{\sqrt{l}}$

**4. The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change, if the water begins to drain out of the hollow sphere?**

- A.** If water begins to drain out of the hollow sphere, its centre of mass begins to shift below the centre of the sphere. Hence the length of the pendulum increases and time period also increases. When the entire water is drained out of the sphere, the centre of mass shifts to centre of sphere and the time period  $T$  attains its initial value.

**5. The bob of a simple pendulum is made of wood. What will be the effect on the time period if the wooden bob is replaced by an identical bob of aluminum?**

- A:** Since the sizes of metal bob and wooden bob are equal, the position of centre of mass does not change. Therefore the length of the pendulum remains same so, time period remains constant.

**6. Will a pendulum clock gain or lose time when taken to the top of a mountain?**

**A:** On the top of mountain, value of 'g' is less. Period of oscillation  $T \propto \sqrt{\frac{l}{g}}$ , there period of oscillation increases and clock loses time.

**7. A pendulum clock gives correct time at the equator. Will it gain or lose time if it is taken to the poles? If so, why?**

**A .** Time period  $T = 2\pi\sqrt{\frac{l}{g}}$

The value of 'g' is maximum at the poles and hence time period of the pendulum clock decreases and hence gains time.

**8. What fraction of the total energy is K.E when the displacement is one half of a amplitude of particle executing S.H.M.**

$$\text{A: } \frac{\text{Kinetic energy}}{\text{Total energy}} = \frac{\frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)}{\frac{1}{2}m\omega^2 A^2} = \frac{3}{4} \left[ \because y = \frac{A}{2} \right]$$

$$\therefore \text{Fraction of total energy in the kinetic energy} = \frac{3}{4}$$

**9. What happens to the energy of a simple harmonic oscillator if its amplitude is doubled?**

**A.** Energy of simple harmonic oscillator  $E = \frac{1}{2}m\omega^2 A^2 \Rightarrow E \propto A^2$

If the amplitude is doubled, energy becomes four times.

**10. Can the pendulum clocks be used in an artificial satellite?**

**A.** No. In an artificial satellite, the bodies are weightless. Hence the pendulum clocks cannot be used in an artificial satellite.

## Short Answer Questions

1. Define simple harmonic motion. Give two examples?

A. Definition:

A body is said to be in simple harmonic motion, if the acceleration (a) is directly proportional to its displacement (y) from the fixed point and is always directed towards the fixed point in its path.  $a \propto -y$

Example:

1) Oscillations of a simple pendulum.

2) Vibrations of a tuning fork

2. Present graphically the variations of displacement, velocity and acceleration with time for a particle in S.H.M

A: Consider a particle is in SHM with amplitude A. The displacement of the particle is given by

$$y = A \sin(\omega t + \phi) \rightarrow (1)$$

Velocity of the particle at any instant can be obtained differentiating displacement equation with respect to time

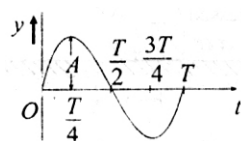
$$V = \frac{dy}{dt} = A\omega \cos(\omega t + \phi) \rightarrow (2)$$

To get acceleration of the particle again differentiate the velocity equation with respect to time

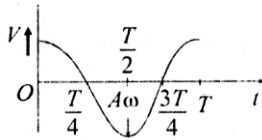
$$a = \frac{dV}{dt} = -A\omega^2 \sin(\omega t + \phi) \rightarrow (3)$$

GRAPHICAL REPRESENTATION:

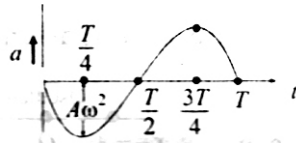
i) Displacement time graph



ii) Velocity –time graph



iii) Acceleration – time graph



3. What is phase? Discuss the phase relations between displacement, velocity and acceleration in simple harmonic motion?

A: Phase:

The physical quantity which gives the information about the position and direction of motion of particle in SHM at any instant from mean position is called phase.

In general, the displacement of a particle in SHM can be written as  $Y = A \sin(\omega t \pm \phi)$ , where  $(\omega t \pm \phi)$  is called phase, expressed in radian. When the particle starts from mean position,

$Y = A \sin \omega t$  velocity  $V = \frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$ . The phase difference between

displacement and velocity is  $\frac{\pi}{2}$ .

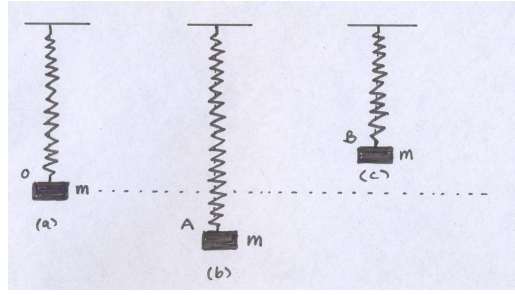
Acceleration  $a = \frac{dV}{dt} = -A\omega^2 \sin \omega t = A\omega^2 \sin(\omega t + \pi)$ . The phase difference between displacement

and acceleration is  $\pi$  and phase difference between velocity and acceleration is  $\frac{\pi}{2}$ .

4. Obtain an equation for frequency of oscillation of a spring of force constant K to which a mass m is attached.

A. Consider a mass less spring of force constant K is loaded with mass 'm'. Let it be pulled a little down and left to make vertical oscillations. Let 'y' be the displacement of the mass.





Restoring force  $F = -K y$

From Newton's second law,  $ma = -ky$  Or  $a = \frac{-k}{m} \cdot y$

Or  $a \propto -y$

Hence the oscillations of a loaded spring are simple harmonic.

Acceleration of the particle in S.H.M at any time is given by  $a = -\omega^2 y$ ,

$$\omega = \sqrt{\frac{k}{m}}.$$

But,  $n = \frac{\omega}{2\pi}$

$$\therefore n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

**5. Derive the expression for the kinetic energy and potential energy of simple harmonic oscillator?**

**A. Kinetic Energy:**

The velocity of the particle executing simple harmonic motion at any position  $V = \omega \sqrt{A^2 - y^2}$

$$\therefore \text{The kinetic energy of the particle } K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 (A^2 - y^2)$$

Also, the velocity of the particle executing simple harmonic motion at any position is given by,  $v = A\omega \cos \omega t$

$$\therefore K.E = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$$

### Potential Energy:

Consider a particle of mass 'm' executing SHM. If 'F' is the force at the displacement 'y'

$$\text{Restoring force } F = ma = -m\omega^2 y$$

Work is done in displacing the body through a small displacement dy is given by

$$W = F dy = -m\omega^2 y dy$$

Now the total work done in displacing the body from 0 to y is given by

$$W = \int dW = \int_0^y -m\omega^2 y dy = -\frac{1}{2}m\omega^2 y^2$$

This work done is stored as potential energy

$$\therefore P.E = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

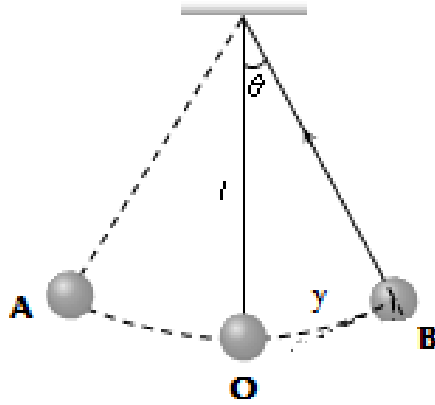
### 6. How does the energy of a simple pendulum vary as it moves from one extreme position to the other during its oscillation?

A. Consider a simple pendulum of length  $l$  and let  $m$  be the mass of the bob. Let A and B are extreme positions and 'O' be the mean position. Let A be its amplitude and  $y$  be the displacement of the bob.

**At the Extreme Positions A or B ( $y = A$ ):**

$$PE = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \text{ And } KE = \frac{1}{2}m\omega^2 (A^2 - y^2) = 0$$

$$\text{Total energy} = PE + KE = \frac{1}{2}m\omega^2 A^2 + 0 = \frac{1}{2}m\omega^2 A^2 \text{ --- (1)}$$



**At the mean position 'O' (y = 0):**

$$P.E = \frac{1}{2} m \omega^2 y^2 = 0$$

$$KE = \frac{1}{2} m \omega^2 (A^2 - y^2) = \frac{1}{2} m \omega^2 A^2$$

$$\text{Total energy} = PE + KE = 0 + \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} m \omega^2 A^2 \text{ --- (2)}$$

**At any position 'P' (OP = y):**

$$P.E = \frac{1}{2} m \omega^2 y^2$$

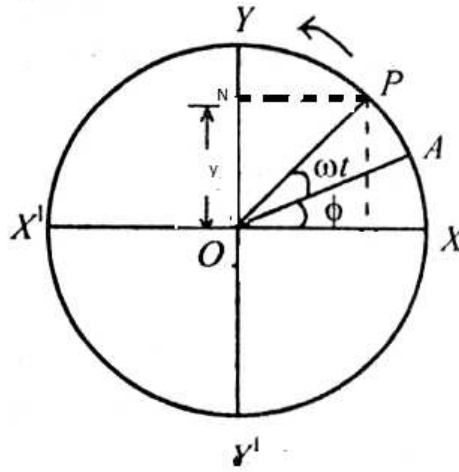
$$KE = \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$\text{Total energy} = PE + KE = \frac{1}{2} m \omega^2 y^2 + \frac{1}{2} m \omega^2 (A^2 - y^2) = \frac{1}{2} m \omega^2 A^2 \text{ --- (3)}$$

Hence from equations (1), (2) & (3) it is clear that the total energy of a simple pendulum is constant at any point on its path.

## 7. Derive the expression for displacement, velocity and acceleration of a particle executes SHM

**A:** Consider a particle moving along the circumference of a circle of radius A and centre 'O' with a uniform speed 'V' or with an angular velocity ' $\omega$ '. Suppose at time  $t = 0$ , the particle is at point A such that  $\angle XOA = \phi$ . At any time t, suppose the particle reaches the point P such that  $\angle AOP = \omega t$ . Draw  $PN \perp YY^1$



Clearly, displacement of projection N from centre O at any instant t is  $y = ON$ . In right angle triangle  $\triangle ONP$ ,  $\angle PON = \omega t + \phi$

$$\therefore \frac{ON}{OP} = \cos(\omega t + \phi) \text{ or } \frac{y}{A} = \cos(\omega t + \phi)$$

This equation gives displacement of a particle in S.H.M at any instant. To find velocity, differentiate the displacement with time.

$$V = \frac{dy}{dt} = -A\omega \sin(\omega t + \phi) = -\omega \sqrt{A^2 - x^2} \quad [\because y = A \cos(\omega t + \phi)]$$

The negative sign shows that the velocity of N is directed towards left. To find acceleration, differentiate the velocity with time

$$a = \frac{dV}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$$

## Long Answer Questions

### 1. Define Simple Harmonic Motion?

Show that the projection of uniform circular motion on any diameter is simple harmonic.

#### A. Definition:

A body is said to be in S.H.M, if the acceleration is directly proportional to its displacement from the fixed point and is always directed towards a fixed point in its path.

#### Expression:

Consider a particle 'P' moving on the circumference of a circle of radius 'A' with uniform angular velocity ' $\omega$ '. Let 'O' be the centre of the circle. Let PN be the perpendicular drawn to the diameter YY' from P. As 'P' moves on the circumference of the circle, 'N' moves on the diameter about the 'O'. Let the position of 'N' at any time 't' so that  $\angle XOP = \theta$

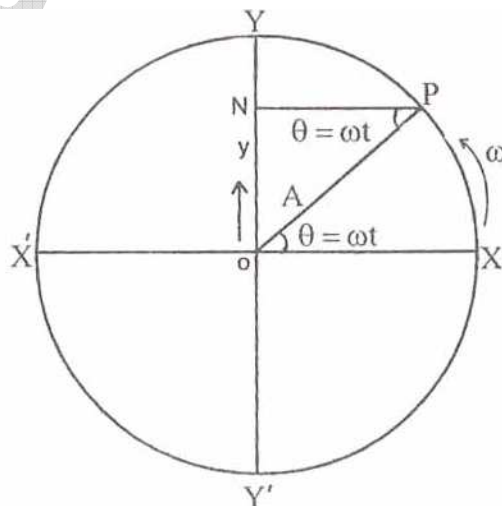
The displacement of 'N' with respect to the fixed point 'O' in the path is given by

$$ON = Y = A \sin \theta$$

(Or)

$$Y = A \sin \omega t \quad (\because \theta = \omega t) \dots\dots\dots (1)$$

The rate of change of displacement is called velocity.



$$\therefore v = \frac{dy}{dt} = A\omega \cos \omega t$$

The rate of change of velocity is called Acceleration.

$$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t$$

Or  $a = -\omega^2 y$  ( $\because y = A \sin \omega t$ )

$$\therefore \text{Acceleration } a = -\omega^2 y$$

Hence the acceleration of 'N' is directly proportional to the displacement in magnitude but in opposite direction, always towards the fixed point 'O' in the path.

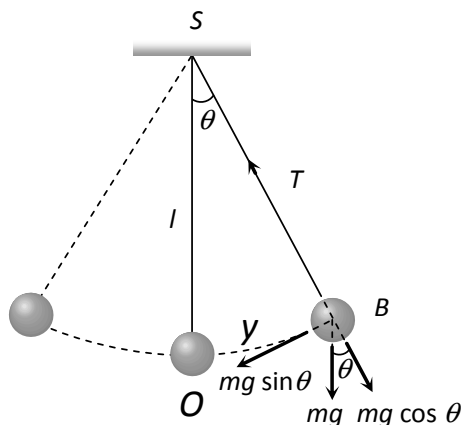
Hence the motion of 'N' is simple harmonic.

## 2. Show that the motion of a simple pendulum is simple harmonic and hence derive an equation for its time period. What is second's pendulum?

### A. Simple Pendulum:

Let  $l$  be the length and  $m$  be the mass of the bob of a simple pendulum. Let the bob be given a small angular displacement ' $\theta$ ' and released..Let  $OB = y =$  Displacement from the mean position.

$$\therefore \theta = \left( \frac{y}{l} \right) \dots\dots\dots(1)$$



At the extreme positions 'B' the tangential acceleration 'a' =  $g \sin \theta$

The torque about the point of suspension  $\tau = -l(mg \sin \theta)$

The restoring torque  $\tau = I\alpha$  Where I is the moment of inertia and  $\tau$  is the angular acceleration

$$I\alpha = -l(mg \sin \theta) \text{ or } \alpha = -\left(\frac{mgl}{I}\right)\theta$$

(When  $\theta$  is very small,  $\sin \theta \approx \theta$ )

Comparing this with the equation for SHM,  $\omega = \sqrt{\frac{mgl}{I}}$

Hence the bob of the pendulum executes SHM.

But,  $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgl}}$  and  $I = ml^2$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

### Second's Pendulum:

A simple pendulum whose time period is two seconds is called a second's pendulum.

3. **Derive the equation for the kinetic energy and potential energy of a simple harmonic oscillator and show that the total energy of a particle in simple harmonic motion is constant at any point on its path**

**A:** i) Expression of K.E (K): The displacement of a simple harmonic oscillator is given by

$$x = A \sin \omega t$$

Velocity of SHO is,  $V = \frac{dx}{dt} = A\omega \cos \omega t = \omega\sqrt{A^2 - x^2}$

The K.E of SHO is given by,  $K = \frac{1}{2}mV^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$

$$(Or) K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

ii) Expression of P.E (U) : When the displacement of SHO (particle) from its equilibrium position is x, the restoring force acting on it is  $F = -Kx$

If we displace the particle further through a small distance dx, then work done against the restoring force is given by  $dw = -Edx = +Kx dx$

The total work done in moving the SHO from mean position ( $x = 0$ ) to displacement 'x' is given by

$$W = \int dW = \int_a^x Kx dx = K \left[ \frac{x^2}{2} \right]_0^x = \frac{1}{2} Kx^2$$

This work done against the restoring force is stored as P.E of SHO

$$\text{Hence, P.E of SHO} = \frac{1}{2} Kx^2 = \frac{1}{2} m\omega^2 x^2$$

$$(Or) \text{ P.E of SHO } U = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)$$

$$\text{The total energy of oscillator, } E = K + U = \frac{1}{2} m\omega^2 (A^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

If the particle is at, (left extreme position)  $x = -A$ ,  $K = 0$ ,  $U = \frac{1}{2} m\omega^2 A^2$ , then

$$E = K + U = \frac{1}{2} m\omega^2 A^2 \rightarrow (1)$$

If the particle is at (mean position)  $x = 0$ ,  $K = \frac{1}{2} m\omega^2 A^2$ ,  $U = 0$

$$\text{Then } E = K + U = \frac{1}{2} m\omega^2 A^2 \rightarrow (2)$$

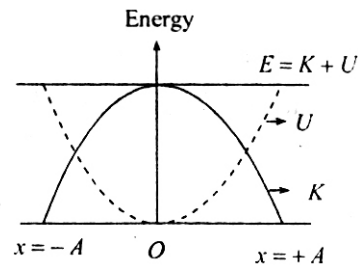
If the particle is at (right extreme position)

$$x = A, K = 0, U = \frac{1}{2} m\omega^2 A^2 \text{ then } E = K + U = \frac{1}{2} m\omega^2 A^2 \rightarrow (3)$$



Hence the total mechanical energy ( $E$ ) remains constant at any point

The variation of K.E ( $K$ ), P.E ( $U$ ) and T.E ( $E$ ) with displacement  $x$  is shown in figure.



The graphs for  $K$  and  $U$  are parabolic while that for  $E$  is a straight line parallel to displacement axis. At  $x = 0$ , the energy is all kinetic and for  $x = \pm A$ , the energy is all potential.

## Problems

1. The bob of a pendulum is made of a hollow brass sphere. What happens to the time period of the pendulum, if the bob is filled with water completely? Why?

**Sol:**  $T = 2\pi\sqrt{\frac{L}{g}}$

Where  $L$  = length of the pendulum

When a hollow brass sphere is completely filled with water, centre of mass does not change.

Hence length of the pendulum remains same

Therefore, time period of hollow brass sphere filled with water completely has 'no change'.

2. Two identical springs of force constant 'K' are joined one at the end of the other (in series). Find the effective force constant of the combination.

A. Total elongation  $x = x_1 + x_2 \Rightarrow \frac{F}{K_{eff}} = \frac{F}{K} + \frac{F}{K} \Rightarrow K_{eff} = \frac{K}{2}$

Effective force constant of combination  $K_{eff} = \frac{K}{2}$

3. What are physical quantities having maximum value at the mean position in SHM?

A. Velocity and kinetic energy.

4. A particle executes SHM such that, the maximum velocity during the oscillation is numerically equal to half the maximum acceleration. What is the time period?

**A:**  $V_{max} = \frac{1}{2} a_{max}$

But  $V_{max} = A\omega$ ;  $a_{max} = A\omega^2 \Rightarrow A\omega = \frac{1}{2} A\omega^2 \Rightarrow \omega = 2$

$\therefore$  Time period  $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ sec}$

5. A mass of 2kg attached to a spring of force constant  $260Nm^{-1}$  makes 100 oscillations. What is the time taken?

A.  $m = 2kg, k = 260Nm^{-1}$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2 \times \frac{22}{7} \times \sqrt{\frac{2}{260}} = 0.5513$$

$$\text{Time for 100 oscillations} = 100 \times 0.5513 = 55.13s$$

6. A simple pendulum in a stationary lift has time period T. What would be the effect on the time period when the lift i) Move Up with uniform velocity ii) Moves Down with uniform velocity iii) Moves Up with uniform acceleration a iv) Moves Down with uniform acceleration 'a' v) Begins To fall freely under gravity?

A: Time period of a simple pendulum in a stationary lift;  $T = 2\pi\sqrt{\frac{L}{g}} \rightarrow (1)$

Time period of a simple pendulum in a lift moving up with uniform acceleration is

$$T_{up} = 2\pi\sqrt{\frac{L}{g+a}} \rightarrow (2)$$

Time period of a simple pendulum in a lift moving down with uniform acceleration is

$$T_{down} = 2\pi\sqrt{\frac{L}{g-a}} \rightarrow (3)$$

i) Moves up with uniform velocity, acceleration of lift at = 0

$$\text{Then } T_{op} = 2\pi\sqrt{\frac{L}{g}} = T \quad [\because (1)]$$

$\therefore$  No change in time period

ii) Moves down with uniform velocity a = 0

$$\text{Then } T_{down} = 2\pi\sqrt{\frac{L}{g}} = T \quad [\because (1)]$$

$\therefore$  No change in time period

$$\text{iii) } \frac{(2)}{(1)} \Rightarrow \frac{T_{up}}{T} = \sqrt{\frac{g}{g+a}} \Rightarrow T_{up} < T$$

∴ The lift moves up with uniform acceleration a, time period decreases

$$\text{iv) } \frac{(3)}{(1)} \Rightarrow \frac{T_{down}}{T} = \sqrt{\frac{g}{g-a}} \Rightarrow T_{down} > T$$

∴ The lift moves down with uniform acceleration a, time period increases

v) When the lift falls freely under gravity  $a = g$

$$\Rightarrow T_{down} = \sqrt{\frac{L}{g-g}} = \sqrt{\frac{L}{0}} = \text{infinity}$$

**7. A particle executing SHM has amplitude of 4cm and its acceleration at a distance of 1cm from the mean position is  $2\text{cms}^{-2}$ . What will be its velocity when it is at a distance of 2cm from its mean position?**

A.  $A = 4\text{cm}$ ,  $a_1 = 3\text{cms}^{-2}$ ,  $x_1 = 1\text{cm}$ ,  $x_2 = 2\text{cm}$ ;  $V = ?$

$$\text{But, } a_1 = \omega^2 x_1 \Rightarrow 3 = \omega^2 \times 1 \Rightarrow \omega = \sqrt{3}$$

$$V = \omega \sqrt{A^2 - x_2^2} = \sqrt{3} \sqrt{4^2 - 2^2} = \sqrt{3} \sqrt{12}$$

**8. A simple harmonic oscillator has a time period of 2s. What will be the change in the phase 0.25 s after leaving the mean position?**

A.  $T = 2\text{s}$ ,  $t = 0.25\text{ s}$

$$\text{Phase change } \phi = \omega t = \frac{2\pi}{T} t$$

$$\therefore \phi = \frac{2\pi}{2} \times 0.25 = \frac{\pi}{4} \text{ rad}$$

9. A body describes simple harmonic motion with an amplitude of 5cm and a period of 0.2s. Find the acceleration and velocity of the body when the displacement is

a) 5cm

b) 3cm

c) 0 cm

A:  $A = 5\text{cm}$ ,  $T = 0.2\text{s}$

a) If  $y = 5$ ,  $a = -\omega^2 y = -\frac{4\pi^2}{0.04} \times 5 = -5\pi^2 \text{ms}^{-2} \left[ \because \omega = \frac{2\pi}{T} \right]$

$$V = \omega \sqrt{A^2 - y^2} = \omega \sqrt{5^2 - 5^2} = 0$$

b) If  $y = 3$ ,  $a = -\omega^2 y = -\frac{4\pi^2}{0.04} \times 3 = -3\pi^2 \text{ms}^{-2}$

$$V = \omega \sqrt{A^2 - y^2} = \frac{2\pi}{0.2} \sqrt{5^2 - 3^2}$$

$$V = \frac{\pi}{0.1} \times 4 = 40\pi \text{cms}^{-1} = 0.4\pi \text{ms}^{-1}$$

c) If  $y = 0\text{cm}$ ,  $a = -\omega^2 y = -\omega^2 (0) = 0$

$$V = \omega \sqrt{A^2 - y^2} = \frac{2\pi}{0.2} \sqrt{5^2 - 0^2} = \frac{\pi}{0.1} \times 5$$

$$\therefore V = 0.5\pi \text{ms}^{-2}$$

10. The mass and radius of a planet are double that of the earth. If the time period of a simple pendulum on the earth is T, Find the time period on the planet?

A:  $T_E = 2\pi \sqrt{\frac{L}{g_E}} = T$  where  $g_E = \frac{GM_E}{R_E^2}$

$$T_p = 2\pi \sqrt{\frac{L}{g_p}} \text{ But } g_p = \frac{GM_p}{R_p^2} = \frac{G2M_E}{4R_E^2} = \frac{g_E}{2}$$

$$\Rightarrow T_p = 2\pi \sqrt{\frac{L}{\left(\frac{g_E}{2}\right)}} = \sqrt{2} \times 2\pi \sqrt{\frac{L}{g_E}} \therefore T_p = \sqrt{2}T$$

11. Calculate the change in the length of a simple pendulum of length 1m, when its period of oscillation changes from 2s to 1.5s?

**Sol:**  $L_1 = 1\text{m}$  ;  $T_1 = 2\text{s}$  ,  $T_2 = 1.5\text{s} = \frac{3}{2}\text{s}$

$$L_2 = ?$$

$$L_2 - L_1 = ?$$

$$\frac{L_2}{L_1} = \frac{T_2^2}{T_1^2} \Rightarrow \frac{L_2}{L_1} = \frac{9}{4} = \frac{9}{16}$$

$$\text{Change in length, } L_2 - L_1 = \frac{9}{16} - 1 = \frac{-7}{16}$$

$$\therefore \text{Change in length is decreased by } \frac{7}{16}\text{m}$$

12. A freely falling body takes 2 seconds to reach the ground on a planet, when it is dropped from a height of 8m. If the period of a simple pendulum is  $\pi$  seconds on the planet, calculate the length of the pendulum?

A. For a freely falling body,  $h = \frac{1}{2}gt^2 \Rightarrow 8 = \frac{1}{2}g(2)^2$

$$\therefore g = 4\text{ms}^{-2}$$

$$l = \frac{gT^2}{4\pi^2} = \frac{4 \times \pi^2}{4\pi^2} = 1\text{m}$$

13. The period of a simple pendulum is found to increase by 50% when the length of the pendulum is increased by 0.4m. Calculate the initial length and the initial period of oscillation at a place where  $g = 9.8\text{ms}^{-2}$

**A:**  $\frac{T_2 - T_1}{T_1} = 50\% = \frac{1}{2} \Rightarrow T_2 = \frac{3T_1}{2}$

$$L_2 - L_1 = 0.6\text{m} \text{ and } g = 9.8\text{ms}^{-2}$$

$$\text{Initial time period } T_1 = 2\pi\sqrt{\frac{L_1}{g}} \Rightarrow T_1^2 = 4\pi^2 \frac{L_1}{g} \rightarrow (1)$$

$$\text{Final time period } T_2 = 2\pi\sqrt{\frac{L_2}{g}} \Rightarrow T_2^2 = 4\pi^2 \frac{L_2}{g} \rightarrow (2)$$

$$(2) - (1) \Rightarrow T_2^2 - T_1^2 = \frac{4\pi^2}{g}(l_2 - l_1) \Rightarrow \frac{9T_1^2}{4} - T_1^2 = \frac{4\pi^2 \times 0.6}{9.8}$$

$$\Rightarrow \frac{5T_1^2}{4} = \frac{4\pi^2 \times 0.6}{9.8} \Rightarrow T_1^2 = \frac{4 \times 4\pi^2 \times 0.6}{5 \times 9.8} = 1.931 \quad \therefore T_1 = \sqrt{1.931} = 1.391s$$

$$\text{Initial length } L_1 = \frac{gT_1^2}{4\pi^2} = \frac{9.8 \times 4 \times 4\pi^2 \times 0.6}{4\pi^2 \times 5 \times 9.8} \quad \therefore L_1 = \frac{2.4}{5} = 0.48m$$

- 14. A clock regulated by a second's pendulum keeps correct time. During summer the length of the pendulum increases to 1.02m. How much will the clock gain or lose in one day?**

**A:** The correct time of a clock,  $T = 2s$

$$\text{Time period of a clock during summer, } T_1 = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.02}{9.8}} = 2.0273s$$

$$\text{Increase in time period for each oscillation } T_1 - T = 0.0273s$$

$$\therefore \text{The clock loses time for each oscillation} = 0.0273s$$

No of oscillations made by pendulum keeping correct time is a day

$$= \frac{24 \times 60 \times 60s}{T} = \frac{24 \times 60 \times 60s}{2} = 43,200s$$

$$\therefore \text{Loss of time by clock in a day} = 0.0273 \times 43,200 = 1180s$$

- 15. The time period of a body suspended from a spring is T. What will be the new time period if the spring is cut into two equal parts and the mass is suspended**

**i) From one part                      ii) Simultaneously from both the parts?**

**A:** The time period of a body of mass m suspended from a spring of spring constant K is

$$T = 2\pi\sqrt{\frac{m}{K}} \rightarrow (1)$$

If the spring of length l is cut into two equal parts, length of each part  $l_1 = \frac{l}{2}$

Let the spring constant of each part is  $K_1$

The spring constant  $\propto \frac{1}{\text{elongation}} \propto \frac{1}{\text{length of the spring}}$ ,  $\frac{K_1}{K} = \frac{l}{l_1} = \frac{l}{\frac{l}{2}}$ ,  $\therefore K_1 = 2K$

i) With one part of the spring, time period  $T_1 = 2\pi\sqrt{\frac{m}{K_1}} \rightarrow (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{T_1}{T} = \sqrt{\frac{K}{K_1}} = \sqrt{\frac{K}{2K}} \quad \therefore T_1 = \frac{T}{\sqrt{2}}$$

ii) The same mass is suspended simultaneously from both the parts, time period

$$T_2 = 2\pi\sqrt{\frac{m}{K_2}} \rightarrow (2)$$

$$\frac{(3)}{(1)} \Rightarrow \frac{T_2}{T} = \sqrt{\frac{K}{K_2}} \quad \text{But } K_2 = K_1 + K_1 = 2K_1 = 2 \times 2K = 4K$$

$$\therefore T_2 = T\sqrt{\frac{K}{4K}} = \frac{T}{2}$$