
Sample Paper-05 (solved)
Mathematics
Class – XI

ANSWERS

Section A

1. Solution

$$\text{Length of latus rectum of the ellipse} = \frac{2a^2}{b}$$

2. Solution

$$f(x+5) = 5$$

3. Solution

The number of weights that can be measured = number of subsets can be formed excluding the null set

$$2^4 - 1 = 15$$

4. Solution

$$x = a + ib$$

$$|x| + x = \sqrt{a^2 + b^2} + a + ib$$

$$\sqrt{a^2 + b^2} + a = 2$$

$$a^2 + b^2 = (2 - a)^2$$

$$b = 1$$

$$a^2 + 1 = 4 + a^2 - 4a$$

$$a = \frac{3}{4}$$

$$x = \frac{3}{4} + i$$

5. Solution

$$S = 1 + 3 + 5 + \dots$$

$$S = \frac{n}{2}[2 + (n-1)(2)]$$

$$S = n^2$$

6. Solution

$$\text{First term} = 5$$

$$\text{Sum of first and second term} = 14$$

$$\text{Second term} = 9$$

$$\text{Common Difference} = 9 - 5 = 4$$

$$n^{\text{th}} \text{ term} = 5 + (n-1)4$$

$$= 4n + 1$$

Section B

7. Solution.

$$\frac{X^2}{k^2} + \frac{y^2}{\frac{k^2}{3}} = 1$$

$$\text{Latus rectum is} = \frac{\left(\frac{2k^2}{3}\right)}{k}$$

$$= \frac{2k}{3}$$

$$e = \sqrt{\frac{k^2 - \frac{k^2}{3}}{k^2}}$$

$$= \sqrt{\frac{2}{3}}$$

$$= \frac{\sqrt{6}}{3}$$

Coordinates of foci are $(ae, 0)$ and $(-ae, 0)$

Coordinates are $\left(\frac{\sqrt{6}}{3}k, 0\right)$ and $\left(-\frac{\sqrt{6}}{3}k, 0\right)$

8. Solution

Slope of line AB joining the points $(-8, 0)$ and $(12, 0) = 0$

Its midpoint = $(2, 0)$

Equation to the line perpendicular to AB and passing through $(2, 0)$ is $x = 2$

Slope of line AC joining the points $(-8, 0)$ and $(0, 8) = 1$

Its midpoint = $(-4, 4)$

Equation to the line perpendicular to AC and passing through $(-4, 4)$ is $y = -x$

So the center of the circle will be the point of intersection of line AB and line AC . Center of circle at point $(2, -2)$

$$\text{Radius} = \sqrt{(2-0)^2 + (-2-8)^2} = \sqrt{104}$$

$$\text{Area} = 104\pi$$

9. Solution

$$2S_1 = n[2a + (n-1)d]$$

$$2S_2 = 2n[2a + (2n-1)d]$$

$$2S_3 = 3n[2a + (3n-1)d]$$

$$\frac{2S_1}{n} = 2a + (n-1)d$$

$$\frac{2S_3}{3n} = 2a + (n-1)d$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + d(n-1+3n-1)$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + 2(2n-1)d$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 2 \cdot \frac{2S_2}{2n}$$

$$\frac{2S_3}{3n} = \frac{2S_2}{2n} - \frac{2S_1}{n}$$

$$\frac{2S_3}{3n} = \frac{4S_2}{2n} - \frac{4S_1}{2n}$$

$$S_3 = 3(S_2 - S_1)$$

10. Solution

$$f(x) = 3x^2 - 6x - 11$$

$$f(x) = 3\left(x^2 - 2x - \frac{11}{3}\right)$$

$$f(x) = 3\left(x^2 - 2x + 1 - 1 - \frac{11}{3}\right)$$

$$f(x) = 3\left[(x-1)^2 - \frac{11}{3} - 1\right]$$

$$f(x) = 3\left[(x-1)^2 - \frac{14}{3}\right]$$

$$f(x) = 3(x-1)^2 - 14$$

$f(x)$ will attain minimum when $x = 1$

Minimum value of $f(x) = -14$

11. Solution

$$f(x) = \frac{a^x}{a^x + \sqrt{a}}$$

$$f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}}$$

$$f(x) + f(1-x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{a^{1-x}}{a^{1-x} + \sqrt{a}}$$

$$= 1$$

12. Solution

$$\frac{\tan 2x \tan x}{\tan 2x - \tan x} = \frac{\frac{2 \tan x}{1 - \tan^2 x} \tan x}{\frac{2 \tan x}{1 - \tan^2 x} - \tan x}$$

$$= \frac{2 \tan^2 x}{\tan x + \tan^2 x}$$

$$= \frac{2 \tan x}{1 + \tan^2 x}$$

$$= \sin 2x$$

13. Solution

$$\lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!} = \lim_{n \rightarrow \infty} \frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)!}$$

$$= \lim_{n \rightarrow \infty} \frac{(n+3)}{(n+1)}$$

$$\lim_{n \rightarrow \infty} \frac{1 + \frac{3}{n}}{1 + \frac{1}{n}}$$

$$= 1$$

14. Solution

When $f(x) = x^2$

$$f(x) = x^2$$

$$f'(x) = 2x$$

$$f'(a+b) = 2(a+b)$$

$$f'(a) = 2a$$

$$f'(b) = 2b$$

$$f'(a) + f'(b) = 2(a+b)$$

$$= f'(a+b)$$

When $f(x) = x^3$

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f'(a+b) = 3(a+b)^2$$

$$f'(a) = 3a^2$$

$$f'(b) = 3b^2$$

$$f'(a) + f'(b) = 3(a^2 + b^2)$$

$$\neq f'(a+b)$$

15. Solution

$$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] = -p[p^2 - 3q]$$

16. Solution

Total number of 3 digit numbers with 0 in units place = 90

The digits that can go into tens place for the number to be divisible by 4 = 0, 2, 4, 6, 8

100th place can be formed with any of the 9 digits excepting 0

Hence total number of 3 digits number divisible by 4 is $9 \times 5 = 45$

$$\text{Probability} = \frac{45}{90} = \frac{1}{2}$$

17. Solution

$$\tan(45^\circ + x) = \frac{1 + \tan x}{1 - \tan x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$= \frac{\cos^2 x + \sin^2 x + 2 \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{1 + \sin 2x}{\cos 2x} = \sec 2x + \tan 2x$$

18. Solution

$$P(n) = n(n+1)$$

$$P(1) = 2, \text{ even}$$

$$P(k) = k(k+1) \text{ let this be true}$$

$$P(k+1) = (k+1)(k+2)$$

$$= k^2 + 3k + 2$$

$$= k^2 + k + 2k + 2$$

$$= k(k+1) + 2(k+1) \text{ True}$$

19. Solution

$$n[(A \cup B \cup C)] = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n[(A \cup B \cup C)] = 4000 + 2000 + 1000 - 400 - 400 - 400 + 200$$

$$n[(A \cup B \cup C)] = 6000$$

Section C

20. Solution

$$\text{Let } \frac{a}{y} = x$$

$$by = \frac{ab}{x}$$

$$f\left(1 + \frac{a}{y}\right)^{by} = f\left[\left(1 + x\right)^{\frac{1}{x}}\right]^{ab}$$

21. Solution:

$$\text{Probability of all the three hitting the target} = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{5}$$

$$\text{Probability of A alone missing the target} = \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{10}$$

$$\text{Probability of B alone missing the target} = \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} = \frac{2}{15}$$

$$\text{Probability of C alone missing the target} = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{5}$$

$$\text{The probability that the target being hit at least two} = \frac{2}{5} + \frac{1}{10} + \frac{2}{15} + \frac{1}{5} = \frac{5}{6}$$

22. Solution

Let T_{r+1} be the term that is independent of x

Then

$$T_{r+1} = {}^9C_r (ax^2)^r \left(-\frac{b}{x}\right)^{9-r}$$

$$2r + (r - 9) = 0$$

$$r = 3$$

4th term is independent of x

$$T_4 = {}^9C_3 (a)^3 (-b)^6$$

$$= {}^9C_3 (a)^3 (b)^6$$

23. Solution

$$\frac{dy}{dx} = \sin^n x \cdot \{-\sin nx\} \cdot (n) + \cos nx \cdot \{n \cdot \sin^{n-1} x \cdot \cos x\}$$

$$\frac{dy}{dx} = n \sin^{n-1} x (\cos nx \cdot \cos x - \sin x \cdot \sin nx)$$

$$\frac{dy}{dx} = n \sin^{n-1} [\cos(n+1)x]$$

24. Solution

$$(a-b)^2 = 5ab$$

$$a^2 + b^2 - 2ab = 5ab$$

$$a^2 + b^2 = 7ab$$

$$(a+b)^2 = 9ab$$

$$a+b = 3\sqrt{ab}$$

$$\frac{1}{3}(a+b) = \sqrt{ab}$$

$$\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}(\log a + \log b)$$

25. Solution

$$\text{First term} = \frac{1}{10}$$

$$\text{Second term} = \frac{7}{10^2}$$

$$\text{Third term} = \frac{7^2}{10^3}$$

$$r = \frac{7}{10}$$

This is a GP

$$\text{Sum to infinity} = \frac{\frac{1}{10}}{1 - \frac{7}{10}} = \frac{1}{3}$$

26. Solution

$$\text{Mean} = 14 = \frac{8+12+13+15+22+14}{6}$$

x_i	$x_i - \text{Mean}$	$(x_i - \text{Mean})^2$
8	-6	36
12	-2	4
13	-1	1
15	1	1
22	8	64
14	0	0
		$\Sigma(x_i - \text{Mean})^2 = 106$

$$\text{Variance} = \frac{1}{n} \Sigma(x_i - \text{Mean})^2 = \frac{106}{6} = 17.66$$

$$\text{SD} = \sqrt{\text{Variance}} = \sqrt{17.66} = 4.2$$