Sample Paper-04

Mathematics
Class – XI

Answers

Section A

1. Solution

Number of subsets

$^{10}C_0 + ^{10}C_1 + ^{10}C_2 + ^{10}C_3 + ^{10}C_4 + ^{10}C_5 + ^{10}C_6 + ^{10}C_7 + ^{10}C_8 + ^{10}C_9 + ^{10}C_{10} = 2^{10}$

2. Solution

$^{3}C_1 + ^{3}C_2 + ^{3}C_3 + = 2^3 - 1 \ = 7$

3. Solution

1. Each card can be drawn in 52 ways and so the total number of ways =

   $52 \times 52 \times 52 = 52^3$

2. If there is no replacement the first card can be drawn in 52 ways, the second by 51 ways and the third by 50 ways. Hence the total number of ways is

   $52 \times 51 \times 50 = 132600$

4. Solution:

1. None of the factors are zero
2. Factors must be of the form $(a+ib); k(b+ia)$ where k is a real number

Section B

5. Solution

Length of arc = $r\theta$

Hence length of arc==2units

6. Solution

1 Full rotation is $2\pi radians$

500 radians = $\frac{500}{2\pi}$ rotations

$$\frac{500}{2\pi} = 79.57 rotations$$

79 full rotations and 0.57 of a rotation

$$0.5 < 0.57 < 0.75$$

The incomplete rotation is between $\frac{1}{2}$ *and* $\frac{3}{4}$ of a rotation. Hence 500 radians is in third quadrant. So $\cos\theta$ is negative

7. Solution:

$$\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1$$

cos2x + cos4x = 0

2 cos3x cos x = 0

Cos3x = 0

$$x=\frac{\pi}{6}+\frac{\pi}{3}n$$
$$Cosx=0$$
$$x=\frac{\pi}{2}+\pi k=\frac{\pi}{6}+\frac{\pi}{3}n \quad n \text{ is integer}$$

8. Solution:

$$i^{30}+i^{40}+i^{60}=(i^4)^7.i^2+(i^4)^{10}+(i^4)^{15}$$
$$i^4=1=-1+1+1=1$$

9. Solution:

Substituting the points (0, 0) and (5, 5) on the given line
x + y – 8 = 0
0 + 0 – 8 = -8
5 + 5 – 8 = 2
Since the signs of the resulting numbers are different the given points lie on opposite sides of the given line.

10. Solution :

$$\tan^{-1}x=A$$
$$\tan A=x$$
$$\cot^{-1}x=B$$
$$\cot B=x$$
$$\tan(\frac{\pi}{2}-B)=x$$
$$\tan^{-1}x=\frac{\pi}{2}-B$$

$$\tan^{-1}x=A$$

$A=\frac{\pi}{2}-B$

$A+B=\frac{\pi}{2}$

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

11. Solution:

$11^{n+2}+12^{2n+1}$ is divisible by 133

n = 1

$11^3+12^3=(11+12)(11^2-11.12+12^2)$

=23.133

Let it be true for k

$11^{k+2}+12^{2k+1}$ is divisible by 133

For k = k + 1

$11^{k+3}+12^{2k+3}=11.11^{k+2}+12^2.12^{2k+1}$
$=11.11^{k+2}+144.12^{2k+1}$

$=11.11^{k+2}+133.12^{2k+1}+11.12^{2k+1}$
$=11.11^{k+2}+11.12^{2k+1}+133.12^{2k+1}$

Is divisible by 133 since $11^{k+2}+12^{2k+1}$ is divisible by 133

12. Solution:

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$
$=n(U)-[n(A)+n(B)-n(A\cap B)]$
$800-[200+300-100]$
$=400$

Section C

13. Solution: $\alpha+\beta=b$

$\alpha\beta=c$
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=b^2-2c$

14. Solution:

$(x + a)^n = P + Q$

$(x-a)^n = P-Q$
$(P+Q)(P-Q) = (x+a)(x-a)$
$P^2 - Q^2 = (x^2 - a^2)^n$

15. Solution:

Discriminant of numerator = 9 – 24 < and

Coefficient of x^2 is positive. Hence Numerator is always positive

Hence dividing by the numerator on both sides of

The equality does not change the sign of the inequality

Hence we need only consider $\frac{1}{3x+4} < 0$

$x < \frac{-4}{3}$

$x \in (-\infty, -\frac{4}{3})$

16. Solution:

$$\cot(A+15) - \tan(A-15) = \frac{\cos(A+15)}{\sin(A+15)} - \frac{\sin(A-15)}{\cos(A-15)}$$
$$= \frac{\cos(A+15)\cos(A-15) - \sin(A+15)\sin(A-15)}{\sin(A+15)\cos(A-15)}$$
$$= \frac{\cos 2A}{\frac{1}{2}(\sin 2A + \frac{1}{2})}$$
$$= \frac{2\cos 2A}{\sin 2A + \frac{1}{2}}$$
$$= \frac{4\cos 2A}{1 + 2\sin 2A}$$

17. Solution:

$4 - x^2 \geq 0$
$x^2 - 4 \leq 0$
Domain of $x \in [-2, 2]$

$y^2 = 4 - x^2$
$x^2 = 4 - y^2$
$x = \sqrt{4 - y^2}$
$4 - y^2 \geq 0$
$y^2 - 4 \leq 0$
$y \in [-2, 2]$

Also for all values of $x \in [-2, 2]$

$$y=\sqrt{4-x^2}\ge 0$$
$$Range\ y\in[0,2]$$

18. Solution:

$$\cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$
$$\cos\theta=\frac{1-4}{1+4}=\frac{-3}{5}$$
$$\sin\theta=\frac{2\tan\theta 2}{1+\tan^2\frac{\theta}{2}}=\frac{2.2}{1+4}=\frac{4}{5}$$
$$\frac{1}{2+\cos\theta+\sin\theta}=\frac{1}{2-\frac{3}{5}+\frac{4}{5}}=\frac{11}{5}$$

19. Solution:

$$\lim_{x\to 0}\frac{\sin 5x}{x+x^3}=\lim_{x\to 0}\frac{5\sin 5x}{5x(1+x^2)}$$
$$=\lim_{x\to 0}\frac{5\sin 5x}{5x}\lim_{x\to 0}\frac{1}{(1+x^2)}$$
$$=5.1.1$$
$$=5$$

20. Solution :

$$y=\log_{10}x$$
$$x=10^y$$
$$\log_e x=y\log_e 10$$
$$y=\frac{\log_e x}{\log_e 10}$$
$$\frac{dy}{dx}=\left(\frac{1}{\log_e 10}\right)\frac{1}{x}$$

21. Solution:

There are 3 even numbers 2, 4, 6

So the units place, 10th places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2\times 5p_4=720$

22. Solution:

Let the origin be shifted to (h,k)

$$x = x' + h$$
$$y = y' + k$$

Then

$$(x'+h)^2 + (y'+k)^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + y'^2 + x'(2h-4) + y'(2k+6) + h^2 + k^2 - 4h + 6k - 36 = 0$$

2h – 4 = 0

h = 2

2k + 6 = 0

k = -3

$$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$$
$$x'^2 + y'^2 + 13 - 62 = 0$$
$$x'^2 + y'^2 = 49$$

23. Solution:

$$\frac{2+4+12+14+11+x+y}{7} = 8$$
$$43 + x + y = 56$$
$$x + y = 13$$

$$\frac{2^2+4^2+12^2+14^2+11^2+x^2+y^2}{7} - (mean)^2 = 19$$
$$\frac{4+16+144+196+121+x^2+y^2}{7} - 64 = 19$$
$$\frac{481+x^2+y^2}{7} = 83$$

$$481 + x^2 + y^2 = 581$$
$$x^2 + y^2 = 100$$
$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$169 + (x-y)^2 = 200$$
$$(x-y)^2 = 31$$
$$x - y = 5.57$$
$$x + y = 13$$
$$x = 9.285$$
$$y = 3.715$$

Section D

24. Solution:

$$\frac{1}{\log_a b} = \log_b a$$
$$\frac{1}{\log_{2a} b} = \log_b 2a$$
$$\frac{1}{\log_{4a} b} = \log_b 4a$$
$$\frac{\log_b a + \log_b 4a}{2} = \frac{\log_b (2a)^2}{2}$$
$$= 2\frac{\log_b 2a}{2}$$
$$= \log_b 2a$$

Thus, $\frac{1}{\log_{2a} b}$ is, the, AM, between $\frac{1}{\log_a b}, \frac{1}{\log_{4a} b}$

25. Solution:

Probability of surviving = $\frac{9}{10}$

Required to find out the probability of 4 are safe or 5 are safe

Probability of 5 is safe = $\left(\frac{9}{10}\right)^5$

Probability of 4 is safe = ${}^5C_4\left(\frac{9}{10}\right)^4 \frac{1}{10}$

Required Probability = $\left(\frac{9}{10}\right)^5 + 5\left(\frac{9}{10}\right)^4 \frac{1}{10} = \frac{45927}{5000}$

26. Solution:

$$T_{2r+1} = {}^{40}C_{2r}$$
$$T_{r+2} = {}^{40}C_{r+1}$$
$${}^{40}C_{2r} = {}^{40}C_{r+1}$$
$$2r + r + 1 = 40$$
$$3r = 39$$
$$r = 13$$