BRILLIANT PUBLIC SCHOOL, SITAMARHI

(Affiliated up to +2 level to C.B.S.E., New Delhi)

Affiliation No. - 330419

XI-Maths Chapterwise Topicwise Worksheets with Solution

Session : 2014-15

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CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets

1. Describe the set in Roster form [1]

   $\{x : x$ is a two digit number such that the sum of its digit is 8 $\}$

2. Are the following pair of sets equal? Give reasons. [1]

   A = { x:x is a letter in the word FOLLOW}

   B = { y:y is a letter in the word WOLF}

3. Write down all the subsets of the set {1,2,3} [1]

4. Let A = { 1,2,{3,4,},5} is $\{\{3,4\}\} \Leftarrow$ A is incorrect. Give reason. [1]

5. Draw venn diagram for $(A \cap B)'$ [1]

6. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice. [4]

7. A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What % Indians like both apples and oranges? [4]

8. In a school there are 20 teachers who teach mathematics or physics. Of these 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics? [4]

9. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to [6]

   (1) chemical C_1 but not chemical C_2

   (2) chemical C_2 but not chemical C_1

   (4) chemical C_1 or chemical C_2

10. In a survey it was found that 21 peoples liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people like C and A, 14 people like B and C and 8 liked all the three products. Find now many liked product C only. [6]

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets [ANSWERS]

Ans 01. { 17, 26, 35, 44, 53, 62, 71, 80 }

Ans 02. A= {F, O, L, W}

B = {W, O, L, F }

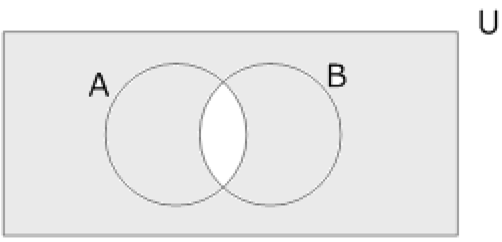
Hence A=B

Ans 03. ϕ, {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}

Ans 04. {3,4} is an elements of sets A, therefore $\{\{3,4\}\}$ is a set containing element {3,4} which is belongs to A

Hence $\{\{3,4\}\} \Leftarrow$ A is correct

Ans 05. $(A \cap B)' = U - (A \cap B)$



Ans 06. Let A denote the set of students taking apple juice and B denote the set of students taking orange juice

$n(\cup) = 400$, $n(A) = 100$, $n(B) = 150$ $n(A \cap B) = 75$

$n(A' \cap B') = n(A \cup B)'$

$= n(\cup) - n(A \cup B)$

$= n(\cup) - [n(A) + n(B) - n(A \cap B)]$

$= 400 - 100 - 150 + 75 = 225$

Ans 07. Let A=set of Indian who like apples

B= set of Indian who like oranges

n (A)= 73, n (B) = 65

n(AUB)=100

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

=73+65-100

=38

38% like both

Ans 08. $n(M \cup P) = 20$, $n(M)=12$

$n(M \cap P) = 4$

$n(M \cup P) = n(M)+n(P)- n(M \cap P)$

n(P)=12

Ans.09. A denote the set of individuals exposed to the chemical C_1 and B denote the set of individuals exposed to the chemical C_2

$n(U) = 200$, $n(A) = 120$, $n(B) = 50$, $n(A \cap B) = 30$

(i) $n(A-B) = n(A) - n(A \cap B)$

=120-30=90

(ii) $n(B-A) = n(B) - n(A \cap B)$

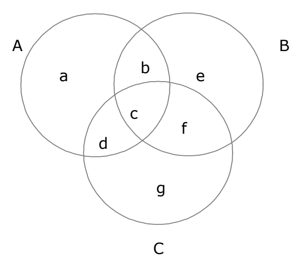
=50-30 = 20

(iii) $n(A \cup B) = n(A)+ n(B)-n(A \cap B)$

=120+50-30

=140

Ans.10. a + b + c + d = 21



b + c + e + f = 26

c + d + f + g = 29

b + c = 14, c + f =15, c + d = 12

c = 8

d = 4, c = 8, f = 7, b = 6 g = 10, e = 5, a = 3

like product c only = g = 10

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets

1. Write the set in roster form A = The set of all letters in the word T R I G N O M E T R Y [1]
2. Ate the following pair of sets equal? Give reasons [1]
   A, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".
3. Write down the power set of A [1]
   A = {1, 2, 3}
4. A = {1, 2, {3, 4}, 5} which is incorrect and why. (i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ [1]
5. Fill in the blanks. [1]
   (i) $A \cup A' =$--------
   (ii) $(A')'$ = ---------
   (iii) $A \cap A'=$ --------
6. Let U = {1, 2, 3, 4, 5, 6} A = {2, 3} and B = {3, 4, 5} [4]
   Find $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.
7. For any two sets A and B prove by using properties of sets that: [4]
   $(A \cap B) \cup (A - B) = A$
8. If A, B, and C, are three sets and U is the universe set such that n(U) = 1000, n(A) = 300, n (B) 300 and $n(A \cap B) = 200$ find $n(A' \cap B')$. [4]
9. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medal in all the three sports, how many received medals in exactly two of the three sports? [6]
10. In a survey of 60 people, it was found that 25 people read news paper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspaper. Find [6]
    (i) The no. of people who read at least one of the newspapers.
    (ii) The no. of people who read exactly one news paper.

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Sets)

[ANSWERS]

<u>Topic: - Sets</u>

Ans 01. A = {T, R, I, G, N, O, M, E, Y}

Ans 02. A = {A, L, O, Y}
B = {L, O, Y, A}
Hence A = B

Ans 03. P (A) = $\{\phi\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

Ans 04. {3, 4} is an element of set A.
Hence $\{3,4\}\in$ A is correct and
$\{3,4\}\subset$ A is incorrect.

Ans 05. (i) $\cup$
(ii) A
(iii) ϕ

Ans 06. A' = U – A
= { 1, 4, 5, 6}
B' = U – B
={1, 2, 6}
A U B = {2, 3, 4, 5}
$(A \cap B)'$ = U – (A $\cup$ B)
= {1, 6}
$A' \cap B'$ = {1, 6}
Hence proved.

Ans 07. L. H. S. = (A ∩B) $\cup$ (A – B)
= (A ∩ B) $\cup$ (A $\cap B'$)
= X $\cup$ (A ∩ B') [$\therefore$ X = A ∩ B
= (X $\cup$ A) $\cap$ (X ∪ B') = A ∩ (A $\cup B'$)
= A

Ans 08. n ($A' \cap B'$) = n ($A \cup B)'$

= n(U) – n (A $\cup$ B)

= n (U) – [n(A) + n (B) – n (A $\cap$B)]

= 1000 – [300+300-200]

= 1000 – 400

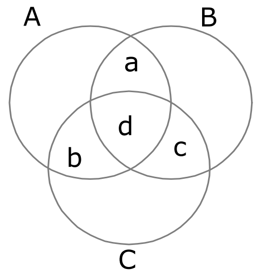
= 600

Ans 09. Let A, B and C denotes the set of men who received medals in football, basketball and cricket respectively.

n (A) = 38, n (B) = 15, n (C) = 20

n (A $\cup$ B $\cup$ C) = 58 and n (A $\cap$ B $\cap$ C) = 3

n (A $\cup$ B $\cup$ C) = n (A) + n (B) + n(C) – n (A $\cap$ B) – n (B $\cap$ C) – n (C $\cap$ A) + n (A $\cap$ B $\cap$ C)



58 = 38 + 15 + 20 – (a + d) – (d + c) – (b + d) + 3

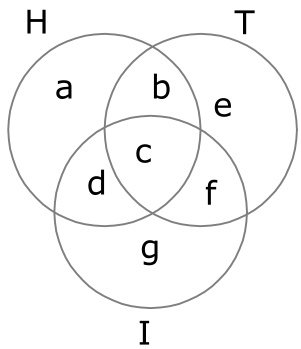
18 = a + d + c + b + d

18 = a + b + c + 3d

18 = a + b + c + 3 $\times$ 3

$\boxed{9 = a + b + c}$

Ans 10. a + b + c + d = 25



b + c + e + f = 26

c + d + f +g = 26

c + d = 9

b + c = 11

c + f = 8

c = 3

f = 5, b = 8, d = 6, c = 3, g = 12

e = 10, a = 8

(i) a + b + c + d + e + f + g = 52

(ii) a + e + g = 30

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets

1. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form. [1]
2. Is set C = { x : x – 5 = 0} and E = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$} [1]
3. Write down all possible proper subsets of the set {1, {2}}. [1]
4. State whether each of the following statement is true or false. [1]
   (i) {2, 3, 4, 5} and {3, 6} are disjoint
   (ii) {2, 6, 10} and {3, 7, 11} are disjoint sets
5. Fill in the blanks [1]
   (i) $(A \cup B)'$ = --------- (ii) $(A \cap B)'$ = ---------
6. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee. [4]
   Find how many drink coffee, How many drink coffee, but not tea.
7. If P (A) = P (B), Show that A = B [4]
8. In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the no. of students who have taken both mathematics and biology and the no. of those who have taken biology but not mathematics each student has taken either mathematics or biology or both. [4]
9. These are 20 students in a chemistry class and 30 students in a physics class. Find the number of students which are either in physics class or chemistry class in the following cases. [6]
   (i) Two classes meet at the same hour
   (ii) The two classes met at different hours and ten students are enrolled in both the courses.
10. In a survey of 25 students, it was found that 15 had taken mathematics, 12had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. [6]
    Find the no. of students that had taken
    (i) only chemistry (ii) only mathematics (iii) only physics
    (iv) physics and chemistry but mathematics (v) mathematics and physics but not chemistry (vi) only one of the subjects (vii) at least one of three subjects
    (viii) None of three subjects.

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Sets)

[ANSWERS]

Topic: - Sets

Ans 01. $\left\{x : x = \dfrac{n}{n+1},\ \text{where n is a nature no. and } 1 \le \text{n } \le 6\right\}$

Ans 02. C = {5}

$x^2 - 2x - 15 = 0$

$x^2 - 5x + 3x - 15 = 0$

$x(x-5) + 3(x-5) = 0$

$(x-5)(x+3) = 0$

$x = 5$

$x = -3 \quad [x = -3 \text{ reject}]$

$x = 5$

$E = \{5\}$

Henc C = E.

Ans 03. $\phi,\ \{1\},\ \{\{2\}\},\ \{1,\{2\}\}$

Ans 04. (i) $\{2,3,4,5\} \cap \{3,6\} = \{3\} \neq \phi$

Hence false

(ii) $\{2,6,10\} \cap \{3,7,11\} = \phi$

true

Ans 05. $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Ans 06. $n(T) = 100$

$n(T - C) = 65$

$n(T \cup C) = 210$

$n(T - C) = n(T) - n(T \cap C)$

$65 = 100 - n(T \cap C)$

$n(T \cap C) = 35$

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$210 = 100 + n(C) - 35$

$n(C) = 145.$

Ans 07. $\forall\, a \in A$

$\Rightarrow \{a\} \subset A$

$\Rightarrow \{a\} \in P(A)$

$\Rightarrow \{a\} \in P(B)$ $[\because P(A) = P(B)$

$\Rightarrow \{a\} \in B$

$\Rightarrow a \subset B$

$\Rightarrow A \subset B$

for all $b \in B$

$\Rightarrow \{b\} \subset B$

$\Rightarrow \{b\} \in P(B)$ $[\because P(A) = P(B)$

$\Rightarrow \{b\} \in P(A)$

$\Rightarrow \{b\} \subset A$

$\Rightarrow b \in A$

$\Rightarrow B \subset A$

Thus $A \subset B$

and $B \subset A$

$\Rightarrow A = B$

Ans 08. $n(M) = 12$, $n(M - B) = 8$

$n(M \cup B) = 25$

$n(M \cup B) = n(M) + n(B - M)$

$25 = 12 + n(B - M)$

$13 = n(B - M)$

$n(M \cup B) = n(M\text{-}B) + n(M \cap B) + n(B\text{-}M)$

$25 = 8 + n(M \cap B) + 13$

$n(M \cap B) = 4$

Ans 09. Let C be the set of students in chemistry class and P be the set of students in physics class.

$n(C) = 20, n(P) = 30$

(i) $C \cap P = \phi$

$n(C \cup P) = n(C) + n(P)$

$= 20 + 30$

$= 50$

(ii) $n(C \cap P) = 10$

$n(C \cup F) = n(C) + n(F) - n(C \cap P)$

$= 20 + 30 - 10$

$= 40$

Ans 10. $n(M) = a+b+d+e = 15$

$n(P) = b + c + e + f = 12$

$n(C) = d + e + f + g = 11$

$n(M \cap P) = b + e = 9$

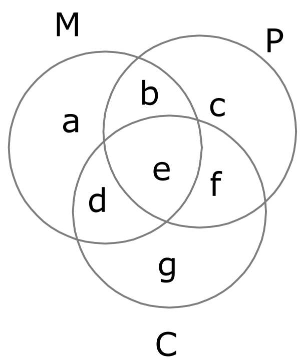
$n(M \cap C) = d + e = 5$

$n(P \cap C) = e + f = 4$

$e = 3$

so $b = 6, d = 2, f = 1$

$a = 4, g = 5, c = 2$



(i) $g = 5$,

(ii) $a = 4$,

(iii) $c = 2$

(iv) $f = 1$,

(v) $b = 6$,

(vi) $g + a + c = 11$

(vii) $a + b + c + d + e + f + g + = 23$

(viii) $25 - (a + b + c + d + e + f + g) = 25 - 23 = 2$

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets

1. Write the set of all vowels in the English alphabet which precede k. [1]
2. Is pair of sets equal? Give reasons. [1]
   A = {2, 3} B = x : x is solution of $x^2 + 5x + 6 = 0$}
3. Write the following intervals in set builder form: [1]
   (-3, 0) and [6, 12]
4. If X = {a, b, c, d} [1]
   Y = {f, b, d, g}
   Find X – Y and Y - X
5. If A and B are two given sets, Then represent the set $(A - B)'$, using Venn diagram. [1]
6. A and B are two sets such that n (A – B) = 20 + x, n (B – A) = 3x and n (A ∩ B) = x + 1. Draw a Venn diagram to illustrate this information. If n (A) = n (B), Find (i) the value of x (ii) n (A ∪ B) [4]
7. If A and B are two sets such that A ∪ B= A ∩ B, then prove that A = B [4]
8. Prove that if A ∪ B = C and A ∩ B = ϕ then A = C – B [4]
9. In a survey of 100 students, the no. of students studying the various languages were found to be English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find [6]
   (i) How many students were studying Hindi?
   (ii) How many students were studying English and Hindi?
10. In a class of 50 students, 30 students like Hindi, 25 like science and 16 like both. Find the no. of students who like [6]
    (i) Either Hindi or science
    (ii) Neither Hindi nor science.

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Sets)

[ANSWERS]

Ans 01. A = {b, c, d, f, g, h, j}

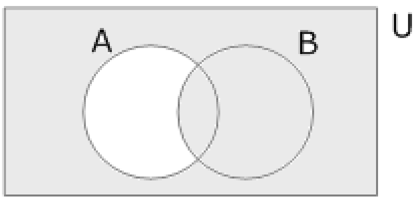
Ans 02. A = {2, 3}
B = {- 2, - 3}
A ≠ B

[∴ $x^2 + 5x + 6 = 0$
$x^2 + 3x + 2x + 6 = 0$
x = - 2 – 3

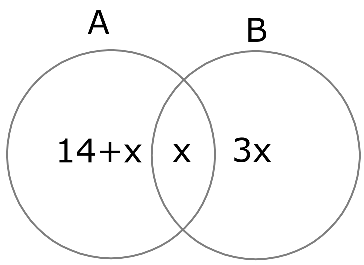
Ans 03. (- 3, 0) = {x : x ϵ R, - 3 < x < 0}
[6, 12] = {x : x ϵ R, 6 ≤ x ≤ 12}

Ans 04. X – Y = {a, b, c, d} – {f, b, d, g}
= {a, c}
Y – X = {f, b, d, g} – {a, b, c, d}
= {f, g}

Ans 05. $(A - B)' = U - (A - B)$



Ans 06. (i) n (A) = n (A – B) + n (A ∩ B)
= 14 + x + x
= 14 + 2x
N (B) = n (B – A) + n (A ∩ B)
= 3x + x
= 4x
but n (A) = n (B) (Given)
14 + 2x = 4x
x = 7
(ii) n (A ∪ B) = n (A – B) + n (B – A) + n (A ∩ B)
= 14 + x+ 3x + x
= 14 + 5x = 14 + 5 × 7 = 49



Ans 07. Let $a \in A$, then $a \in A \cup B$

Since $A \cup B = A \cap B$

$a \in A \cap B$. So $a \in B$

Therefore $A \subset B$

Similarly if $b \in B$,

Then $b \in A \cup B$. Since

$A \cup B = A \cap B$, $b \in A \cap B$

So $b \in A$

Therefore, $B \subset A$

Thus $\boxed{A = B}$

Ans 08. $C - B = A$

$= (A \cup B) - B$

$= (A \cup B) \cap B'$

$= B' \cap (A \cup B)$

$= (B' \cap A) \cup (B' \cap B)$

$= (B' \cap A) \cup \phi$

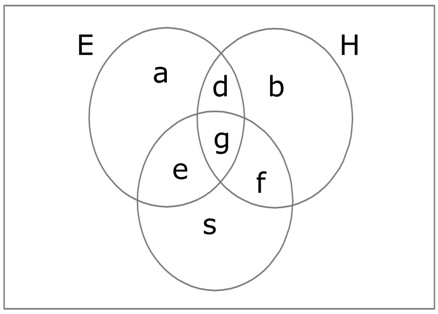
$= B' \cap A$

$= A \cap B'$

$= A - B$

$= A$ (Proved)

Ans 09. $\cup = 100$, $a = 18$



$a + e = 23$, $e + g = 8$

$a + e + g + d = 26$

$e + g + f + c = 48$

$g + f = 8$

so, $e = 5$, $g = 3$, $d = 0$, $f = 5$, $c = 35$

(i) $d + g + f + b = 0 + 3 + 5 + 10 = 18$

(ii) $d + g = 0 + 3 = 3$

Ans 10. Let $\cup$ = all the students of the class , H = students who like Hindi

S = Students who like Science

(i) $n(H \cup S) = n(H) + n(S) - n(H \cap S)$

$= 30 + 25 - 16$

$= 39$

(ii) $n(H' \cap S') = n(H \cup S)'$

$= \cup - n(H \cup S)$

$= 50 - 39$

$= 11$

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Sets)

Topic: -Sets

1. List all the element of the set A = { x : x is an integer $x^2 \le 4$} [1]
2. From the sets given below pair the equivalent sets. [1]

   A = { 1, 2, 3}, B = {x, y, z, t}, C = {a, b, c} D = {0, a}
3. Write the following as interval [1]

   (i) $\{x : x \in R, -4 < x \le 6\}$

   (ii) $\{x : x \in R, 3 \le x \le 4\}$
4. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} [1]

   Find $(A \cap B) \cap (B \cup C)$
5. Write the set $\left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \frac{11}{13}\right\}$ in set builder form. [1]
6. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis? [4]
7. Let A, B and C be three sets $A \cup B = A \cup C$ and $A \cap B = A \cap C$ show that B = C [4]
8. If ∪ = {a, e, i. o. u} [4]

   A = {a, e, i}

   And B = {e, o, u}

   C = {a, i, u}

   Then verity that $A \cap (B - C) = (A \cap B) - (A \cap C)$
9. In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three papers. Find the no. of families which buy [6]

   (i) A only (ii) B only (iii) none of A, B, and C.
10. Two finite sets have m and n elements. The total no. of subsets of the first set is 56 more than the total no. of subsets of second set. Find the value of m and n. [6]

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Sets)

[ANSWERS]

Ans 01. {-2, -1, 0, 1, 2}

Ans 02. A = {1, 2, 3} B = {a, b, c} are equivalent sets $[\because n(A) = n(B)]$

Ans 03. (i) (-4, 6]
(ii) [3, 4]

Ans 04. $A \cap B = \{7, 9, 11\}$
$B \cup C = \{7, 9, 11, 13, 15\}$
$(A \cap B) \cap (B \cup C) = \{7, 9, 11\}$

Ans 05. $\left\{\frac{2n-1}{2n+1}: \text{ n is a natural no. less than 7}\right\}$

Ans 06. Let C = the set of people who like cricket and
T = the set of people who like tennis.
$n(C \cup T) = 56$, $n(C) = 40$
$n(C \cap T) = 10$
$n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$65 = 40 + n(T) - 10$
$n(T) = 35$

Ans 07. Let $b \in B \Rightarrow b \in A \cup B$
$\Rightarrow b \in A \cup C$ $[\because A \cup B = A \cup C$
$\Rightarrow b \in A$ or $b \in c$
if $b \in c$ then $B \subset C$
if $b \in A$, then $b \in A \cap B$ $[\because A \cap B = A \cap C$
$\Rightarrow b \in A \cap C$
$\Rightarrow b \in C \Rightarrow B \subset C$
thus in both cases $B \subset C$
Simiularly $C \subset B$
Hence B = C

Ans 08. $B - C = \{e, o\}$

$A \cap (B - C) = e$

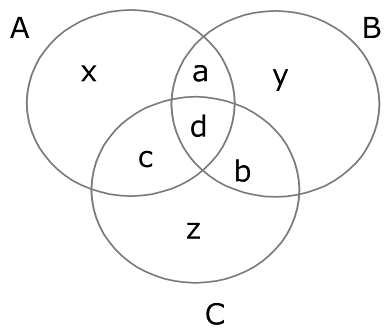
$A \cap B = \{e\}$

$A \cap C = \{a\}$

$(A \cap B) - (A \cap C) = e$

Hence proved.

Ans 09. $x + a + c + d = 4000$



$y + a + d + b = 2000$

$z + b + c + d = 1000$

$a + d = 500$, $b + d = 300$, $C + d = 400$ $d = 200$

On Solving $a = 300$, $b = 100$, $c = 200$

(i) $x = 4000 - 300 - 200 - 200 = 3300$

(ii) $y = 2000 - 300 - 200 - 100 = 1400$

(iii) $z = 1000 - 100 - 200 - 200 = 500$

None of these $= 10{,}000 - (3300 + 1400 + 500 + 300 + 100 + 200 + 200)$

$= 10{,}000 - 6000$

$= 4000$

Ans 10. Let A and B be two sets having m and n elements respectively

no of subsets of A $= 2^m$

no of subsets of B $= 2^n$

According to question

$2^m = 56 + 2^n$

$2^m - 2^n = 56$

$2^n (2^{m-n} - 1) = 56$

$2^n (2^{m-n} - 1) = 2^3 (2^3 - 1)$

$2^n = 2^3$

$n = 3$

$m - n = 3$

$m - 3 = 3$

$m = 6$

TEST PAPER-01
CLASS - XI MATHEMATICS (Relations and functions)

1. If the ordered Pairs $(x-1, y+3)$ and $(2, x+4)$ are equal, find x and y [1]

(i) $(3,3)$ (ii) $(3,4)$ (iii) $(1,4)$ (iv) $(1,0)$

2. If, $n(A)=3, n(B)=2, A$ And B are two sets Then no. of relations of $A\times B$ have. [1]

(i) (6) (ii) (12) (iii) (32) (iv) (64)

3. Let $f(x)=-|x|$ then Range of function [1]

(i) $(0,\infty)$ (ii) $(-\infty,\infty)$ (iii) $(-\infty,0)$ (iv) none of there

4. A real function f is defined by $f(x)=2x-5$. Then the Value of $f(-3)$ [1]

(i) -11 (ii) 1 (iii) 0 (iv) none of there

5. Let $R=\{(x,-y): x, y =\in W, 2x+y=8\}$ then [4]

(i) Find the domain and the range of R (ii) Write R as a set of ordered pairs.

6. Let R be a relation from Q to Q defined by $R=\{(a,b): a,b\in Q$ and $a-b\in z,\}$ [4]

show that $(i)(a,a)\in R$ for all $a\in Q$ $(ii)(a,b)\in R$ implies that $(b,a)\in R$

$(iii)(a,b)\in R$ and $(b,c)\in R$ implies that $(a,c)\in R$

7. If $f(x)=\dfrac{x^2-3x+1}{x-1}$, find $f(-2)+f\left(\dfrac{1}{3}\right)$ [4]

8. Find the domain and the range of the function $f(x)=3x^2-5$. Also find $f(-3)$ and the numbers which are associated with the number 43 m its range. [4]

9. If $f(x)=x2-3x+$, find x such that $f(2x)=f(x)$ [4]

10. Find the domain and the range of the function $f(x)=\sqrt{x-1}$ [4]

11. Draw the graphs of the following real functions and hence find their range [6]

$f(x)=\dfrac{1}{x}, x\in R, x\neq 0$

12. If $f(x)=x-\dfrac{1}{x}$, Prove that $\left[f(x)\right]^3=f\left(x^3\right)+3f\left(\dfrac{1}{x}\right)$ [6]

TEST PAPER-01
CLASS - XI MATHEMATICS (Relations and functions)

[ANSWERS]

Ans1 $(3,4)$

Ans2 64

Ans3 $(-\infty,0)$

Ans4 -11

Ans5 (i) Given $2x+y=8$ and $x.y\in w$

$x=0,2\times0+y=8\Rightarrow y=8,$

$x=1,2\times1+y=8\Rightarrow y=6,$

Put $x=2,2\times2+y=8\Rightarrow y=4,$

$x=3,2\times3+y=8\Rightarrow y=2,$

$x=4,2\times4+y=8\Rightarrow y=0$

for all other values of $x\in w,$ we do not get $y\in w$

$\therefore$ Domain of $R=\{0,1,2,3,4,\}$ and range of $R=\{8,6,4,2,0\}$

(ii) R as a set of ordered pairs can be written as

$R=\{(0,8),(1,6),(2,4),(3,2),(4,0)\}$

Ans6 $R=[(a,b):a,b\in Q \text{ and } a-b\in z]$

(i) For all $a\in Q, a-a=0$ and $0\in z,$ it implies that $(a,a)\in R.$

(ii) Given $(a,b)\in R\Rightarrow a-b\in z\Rightarrow-(a-b)\in z$

$\Rightarrow b-a\in z\Rightarrow(b,a)\in R.$

(iii) Given $(a,b)\in R$ and $(b,c)\in R\Rightarrow a-b\in z$ and $b-c\in z\Rightarrow(a-b)+(b-c)\in z$

$\Rightarrow a-c\in z\Rightarrow(a-c)\in R.$

Ans7 **Given** $f(x)=\dfrac{x^2-3x+1}{x-1}, Df=R-\{1\}$

$\therefore f(-2)=\dfrac{(-2)^2-3(-2)+1}{-2-1}-\dfrac{4+6+1}{-3}=1\dfrac{1}{3}$ and

$$f\left(\frac{1}{3}\right)=\frac{\left(\frac{1}{3}\right)^2-3\times\frac{1}{3}+1}{\frac{1}{3}-1}=\frac{\frac{1}{9-1+1}}{-\frac{2}{3}}=\frac{\frac{1}{9}}{-\frac{2}{3}}=\frac{1}{9}\times\left(-\frac{3}{2}\right)=-\frac{1}{6}$$

$$\therefore f(-2)+f\left(\frac{1}{3}\right)=-\frac{11}{3}-\frac{1}{6}=\frac{-22-1}{6}=\frac{-23}{6}=3\frac{5}{6}.$$

Ans8 Given $f(x)=3x^2-5$

For Df, $f(x)$ must be real number

$\Rightarrow 3x^2-5$ must be a real number

Which is a real number for every $x\in R$

$\Rightarrow Df=R..........(i)$

$for\ Rf$, let $y=f(x)=3x^2-5$

We know that for all $x\in R, x^2\geq 0\Rightarrow 3x^2\geq 0$

$\Rightarrow 3x^2-5\geq -5\Rightarrow y\geq -5\Rightarrow Rf=[-5,\infty]$

Funthes, as $-3\in Df$, $f(-3)$ exists is and $f(-3)$

$=3(-3)^2-5=22.$

As $43\in Rf$ on putting $y=43$ is (i) weget

$3x^2\text{-}5\text{=}43\Rightarrow 3x^2\text{=}48\Rightarrow \mathrm{x}^2\text{=}16\Rightarrow x=-4,4.$

There fore -4 and 4 are number

(is Df) which are associated with the number 43 in Rf

Ans9 Given $f(x)=x^2-3x+1, Df=R$

$\therefore f(2x)=(2x)^2-3(2x)+1=4x^2-6x+1$

As $f(2x)=f(x)$(Given)

$\Rightarrow 4x^2-6x+1=x^2-3x+1$

$\Rightarrow 3x^2-3x=0\Rightarrow x^2-x=0\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1.$

Ans10 Given $f(x)=\sqrt{x-1},$

for Df, $f(x)$ must be a real number

$\Rightarrow \sqrt{x-1}$must be a real number

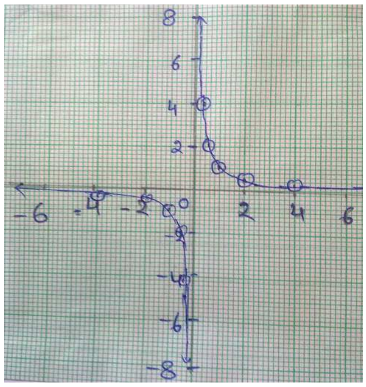
$\Rightarrow x-1\geq 0\Rightarrow x\geq 1$

$\Rightarrow Df = [1, \infty]$

for Rf, let $y = f(x) = \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

$\Rightarrow Rf = [0, \infty]$



(Fig for Answer 11)

Ans11 Given $f(x) = \frac{1}{x}, x \in R, x \neq 0$

Let $y = f(x) = i\ell y = \frac{1}{x}, x \in R, x \neq 0$

x	-4	-2	-1	-0.5	-0.25	0.5	1	2	4
$y = \frac{1}{x}$	-0.25	-0.5	-1	-2	-4	2	1	0.5	0.25

Plot the points shown is the above table and join there points by a free hand drawing. Portion of the graph are shown the right margin

From the graph, it is clear that $Rf = R - [0]$

This function is called reciprocal function.

Ans12 If $f(x) = x - \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + f\left(\frac{1}{x}\right)$

Given $f(x) = x - \frac{1}{x}, Df = R - [0]$

$\Rightarrow f(x^3) = x^3 - \frac{1}{x^3}$ and $f\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x \ldots\ldots\ldots(i)$

$\therefore [f(x)]^3 = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x - \frac{1}{x}\right)$

$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$

$= f(x^3) + 3f\left(\frac{1}{x}\right)$ [using (i)]

TEST PAPER-02

CLASS - XI MATHEMATICS (Relations and functions)

1. If $P=\{a,b,c\}$ and $Q=\{d\}$, form the sets $P\times Q$ and $Q\times P$ are these two Cartesian products equal? [1]

2. If A and B are finite sets such that $n(A)=m$ and $n(B)=k$ find the number of relations from A to B [1]

3. Let $f=\{(1,1),(2,3),(0,-1),(-1,3),......\}$ be a function from z to z defined by $f(x)=ax+b$, for same integers a and b determine a and b. [1]

4. Express $\{(x,y): y+2x=5, xy\in w\}$ as the set of ordered pairs [1]

5. Let a relation $R=\{(0,0),(2,4),(-1,2),(3,6),(1,2)\}$ then [4]

   (i) write domain of R (ii) write range of R

   (iii) write R the set builder form (iv) represent R by an arrow diagram

6. Let $A=\{1,2,3\}$, $B=\{1,2,3,4\}$ and $R=\{(x,y):(x,y)\in A\times B, y=x+1\}$ [4]

   (i) find $A\times B$ (ii) write R in roster form

   (iii) write domain & range of R (iv) represent R by an arrow diagram

7. The cartesian product $A\times A$ has a elements among which are found $(-1,0)$ and $(0,1)$. find the set and the remaining elements of $A\times A$ [4]

8. Find the domain and the range of the following functions $f(x)=\dfrac{1}{\sqrt{5-x}}$ [4]

9. Let $f(x)=x+1$ and $g(x)=2x-3$ be two real functions. Find the following functions $(i)\ f+g$ $(ii)\ f-g$ $(iii)\ fg$ $(iv)\ \dfrac{f}{g}$ $(v)\ f^2-3g$ [4]

10. Find the domain and the range of the following functions [4]

    $(i)\ f(x)=\dfrac{x-3}{2x+1}$ $(ii)\ f(x)=\dfrac{x^2}{1+x^2}$ $(iii)\ f(x)=\dfrac{1}{1-x^2}$

11. Draw the graphs of the following real functions and hence find their range [6]

    $(i)\ f(x)=2x-1$ $(ii)\ f(x)=\dfrac{x^2-1}{x-1}$

12. Let f be a function defined by $F: x\to 5x^2+2, x\in R$ [6]

    (i) find the image of 3 under f (ii) find $f(3)+f(2)$

    (iii) find x such that $f(x)=22$

TEST PAPER-02

CLASS - XI MATHEMATICS (Relations and functions)

[ANSWERS]

Ans1. Given $P=\{a,b,c\}$ and $Q=\{d\}$, by definition of cartesion product, we set

$P\times Q=\left[(a,d),(b,d),(c,d)\right]$ and $Q\times P=\left[(d,a),(d,b),(d,c)\right]$

By definition of equality of ordered pains the pair (a,d) is not equal to the pair (d,a) therefore $p\times Q\neq Q\times P$.

Ans2. Linen $n(A)=n$ and $n(B)=k$

$\therefore n(A\times B)=\cap(A)\times\cap(B)=MK$

$\therefore$ the number of subsets of $A\times B=2^{MK}$

$\because n(A)=m$, then the number of subsets of $A=2^{m}$

Since every subset of $A\times B$ is a relation from A to B therefore the number of relations from A to B = 2^{mk}

Ans3. Given $fx=ax+b$

Since $(1,1)\in f_1 f(1)=1\Rightarrow a+b=1......(i)$

$(2,3)\in f.f(2)=3\Rightarrow 2a+b=3........(ii)$

Subtracting (i) from(ii) we set a=2

Substituting a=2 is (ii) we get 2+b=1

$\Rightarrow$ b = -1

Hence a = 2, b = -1

Ans4. Since $y+2x=5$ and $x,y\in w,$

Put $x=0, y+0=5\Rightarrow y=5$

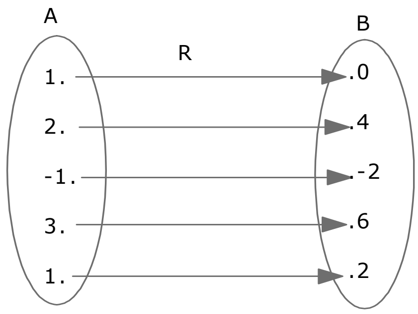
$x=1, y+2\times1=5\Rightarrow y=3$

$x=2, y+2\times2=5\Rightarrow y=1$

For anther values of $x\in w,$ we do not get $y\in w.$

Hence the required set of ordered peutes is $\{(0'5),(1,3),(2,1)\}$

Ans5. Given $R=[(0,0),(2,4),(-1,-2,),(3,6),(1,2)]$



(i) Domain of $R=[0,2,-1,3,1]$

(ii) Rang of $R=[0,4,-2,6,2]$

(iii) R in the builder from can be written as

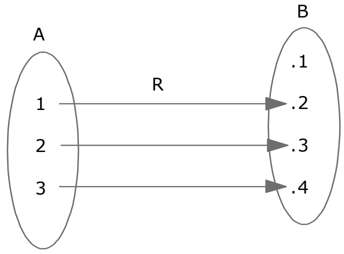
$R=[(x,y):x\in I,-1\leq x\leq 3,y=2x]$

(iv) The reaction R can be represented by the arrow diagram are shown.

Ans6 (i) $\{(1,1),(1,2),(1,3),(1,4)$

$(2,1),(2,2),(2,3),(2,4)$

$(3,1),(3,2),(3,3),(3,4)\}$



(ii) $R=[(1,2),(2,3),(3,4)]$

(iii) Domain of $R=\{1,2,3\}$ and range of $R=\{2,3,4\}$

(iv) The relation R can be represented by the are arrow diagram are shown.

Ans7. Let $n(A)=m$

Given $n(A\times A)=9\Rightarrow n(A).n(A)=9$

$\Rightarrow m.m=9\Rightarrow m^2=9\Rightarrow m=3 \quad (\because m>0)$

Given $(-1,0)\in A\times A\Rightarrow -1\in A$ and $0\in A$

Also $(0,1)\in A\times A\Rightarrow 0\in A$ and $1\in A$

This $-1,0,1\in A$ but $n(A)=3$

Therefore $A=[-1,0,1]$

The remaining elements of $A\times A$ are

$(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)$

Ans8. Given $f(x)=\dfrac{1}{\sqrt{5-x}}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \dfrac{1}{\sqrt{5-x}}$ Must be a real number

$\Rightarrow 5-x>0\Rightarrow 5>x\Rightarrow x<5$

$\Rightarrow D_F=(-\infty,5)$

For R_F let $y=\dfrac{1}{\sqrt{5-x}}$

As $x<5, 0<5-x$

$\Rightarrow 5-x>0 \Rightarrow \sqrt{5-x}>0$

$\Rightarrow \frac{1}{\sqrt{5x}}>0 \qquad \left(\because \frac{1}{a}>0 \text{ if and only if } a>0\right)$

$\Rightarrow y>0$

$\Rightarrow R_F=(0,\infty)$

Ans9 Given $f(x)=x+1$ and $g(x)=2x-3$ we note that $D_F=R$ and $D_g=R$ so there functions have the same Domain R

(i) $(f+g)(x)=f(x)+g(x)=(x+1)+(2x-3)=3x-2,$ for $x\in R$

(ii) $(f-g)(x)=f(x)-g(x)=(x+1)-(2x-3)-x+4,$ for all $x\in R$

(iii) $(fg)(x)=f(x)=(x+1)(2x-3)=2x^2-x-3,$ for all $x\in R$

(iv) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{x+1}{2s-3}, x\neq\frac{3}{2}, x\in R$

(v) $(f^2-3g)(x)=(f^2)(x)-(3f)(x)=(f(x))^2-3g(x)$

$=(x+1)^2-3(2x-3)=x^2+2x+1-6x+9$

$=x^2-4x+10,$ for all $x\in R$

Ans10 (i) Given $f(x)=\frac{x-3}{2x+1}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \frac{x-3}{2x+1}$ must be a real number

$\Rightarrow 2x+1\neq 0 \Rightarrow x\neq -\frac{1}{2}$

$\Rightarrow D_F=$ set of all real number except

$-\frac{1}{2} i.e. R-\left[-\frac{1}{2}\right]$

For R_F, let $y=\frac{x-3}{2x+1} \Rightarrow 2xy+y=x-3$

$\Rightarrow (2y-1)x=-y-3 \Rightarrow x=\frac{y+3}{1-2y}$ but $x\in R$

$\Rightarrow \frac{y+3}{1-2y}$ Must be a real number $\Rightarrow 1-2y\neq 0 \Rightarrow y\neq\frac{1}{2}$

$\Rightarrow R_F=$ Set of all real number except $\frac{1}{2}$ $R-\left[\frac{1}{2}\right]$

(ii) Given $f(x)=\frac{x^2}{1+x^2}$

For D_F, $f(x)$ must be a real number $\Rightarrow \frac{x^2}{1+x^2}$

Must be a real number

$\Rightarrow D_F = R \qquad (\because x^2 + 1 \neq 0 \text{ for all } x \in R)$

For R_F let $y = \frac{x^2}{1+x^2} \Rightarrow x^2 y + y = x^2$

$\Rightarrow (y-1)x^2 = -y \Rightarrow x^2 = \frac{-y}{y-1}, y \neq 1$

But $x2 \geq 0$ for all $x \in R \Rightarrow \frac{-y}{y-1} \geq 0, y \neq 1$

Multiply both sides by $(y-1)^2$, a positive real number

$\Rightarrow -y(y-1) \geq 0$

$\Rightarrow y(y-1) \leq 0 \Rightarrow (y-0)(y-1) \leq 0$

$\Rightarrow 0 \leq y \leq 1$ but $y \neq 1$

$\Rightarrow 0 \leq y < 1$

$\Rightarrow R_F = (0,1)$

(iii) Given $f(x) = \frac{1}{1-x^2}$

For $D_{F,} f(x)$ must be a real number

$\Rightarrow \frac{1}{1-x^2}$ Must be a real number

$\Rightarrow 1 - x^2 \neq 0 \Rightarrow x \neq -1, 1$

$\Rightarrow D_F =$ Set of all real number except $-1, 1 i.e D_F = R - [-1,1]$

For R_F let $y = \frac{1}{1-x^2}, y \neq 0$

$\Rightarrow 1 - x^2 = \frac{1}{y} \Rightarrow x^2 = 1 - \frac{1}{y} \neq 0$

But $x^2 \geq 0$ for all $\in D_F \Rightarrow 1 - \frac{1}{y} \geq 0$

But $y^2 > 0, y \neq 0$

Multicity bath sides by y^2 a positive real number

$\Rightarrow y2\left(1 - \frac{1}{y}\right) \geq 0 \Rightarrow y.(y-1) \geq 0 \Rightarrow (y-0)(y-1) \geq 0$

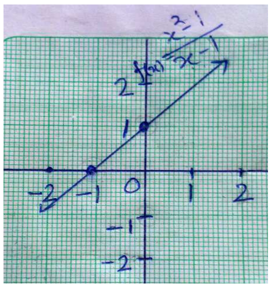
Either $y \leq 0$ or $y \geq 1$ but $y \neq 0$

$\Rightarrow R_F = (-\infty, 0) \cup (1, \infty)$.

Ans11 (i) Given $f(x) i.e. y = x - 1$, which is first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine straight lint uniquely

Table of values

x	0	1
y	-1	1



A portion of the graph is shown in the figure from the graph, it is clear that y takes all real values. It therefore that $R_F = R$

(ii) Given $f(x) = \dfrac{x^2-1}{x-1} \Rightarrow D_F = R-(1)$

Let $y = f(x) = \dfrac{x^2-1}{x-1} = x+1 (\because x \neq 1)$

i.e $y = x+1$, which is a first degree equation is x, y and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

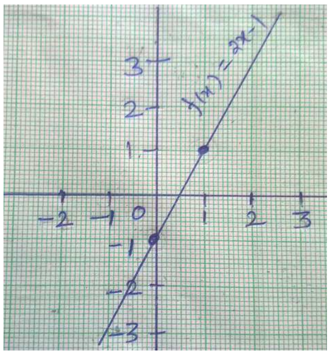


Table of values

x	-1	0
y	0	1

A portion of the graph is shown is the figure from the graph it is clear that y takes all real values except 2. It fallows that $R_F = R-[2]$.

Ans12. Given $f(x) = 5x^2 + 2, x \in R$

(i) $f(3) = 5\times 3^2 + 2 = 5\times 9 + 2 = 47$

(ii) $f(2) = 5\times 2^2 + 2 = 5\times 4 + 2 = 22$

$\therefore f(3)\times f(2) = 47\times 22 = 1034$

(iii) $f(x) = 22$

$\Rightarrow 5x^2 + 2 = 22$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2, \ -2$

TEST PAPER-03

CLASS - XI MATHEMATICS (Relations and functions)

1. If $A=\{1,2\}$, find $(A\times A\times A)$ [1]

2. If A and B are two sets containing m and n elements respectively how many different relations can be defined from A to B? [1]

3. A Function f is defined by $f(x)=2x-3$ find $f(5)$ [1]

4. Let $f=\{(0,-5),(1,-2),(2,1),(3,4),(4,7)\}$ be a linear function from z into z find f [1]

5. If $A=\{1,2,3\}$ $B=\{3,4\}$ and $c=\{4,5,6\}$ [4]

   find (i) $A\times(B\cup C)$ (ii) $A\times(B\cap C)$ (iii) $(A\times B)\cap(B\times C)$

6. For non empty sets A and B prove that $(A\times B)=(B\times A)\Leftrightarrow A=B$ [4]

7. Let m be a given fixed positive integer. [4]

   let $R=[(a,b):a,b\in z$ and $(a-b)$ is divisible by $m]$

   show that R is an equivalence relation on Z.

8. Let $A=\{1,2,3,4,5\}$ and $B=\{1,3,4\}$ [4]

   let R be the relation, is greater than from A to B. Write R as a a set of ordered pairs. find domain (R) and range (R)

9. Define modulus function Draw graph. [4]

10. Let $f(x)=\begin{Bmatrix} x^2, \text{when } 0\le x\le 3 \\ 3x, \text{when } 3\le x\le 10 \end{Bmatrix}$ $g(x)\begin{Bmatrix} x^2, 0\le x\le 3 \\ 2x, 3\le x\le 10 \end{Bmatrix}$ [4]

    Show that f is a function, while g is not a function.

11. The function $f(x)=\dfrac{9x}{5}+32$ is the formula to connect x^0C to Fahrenheit [6]

    units find (i) $f(0)$ (ii) $f(-10)$ (iii) the value of x

    $f(x)=212$ interpret the result is each case

12. Draw the graph of the greatest integer function, $f(x)=[x]$. [6]

TEST PAPER-03

CLASS - XI MATHEMATICS (Relations and functions)

[ANSWERS]

Ans01. We have

$A\times A\times A=\{(1,1,1),(1,1,2),(1,2,1),(2,1,1),(2,2,1),(2,2,2)\}$

Ans02. 2^{m+n}

Ans03. $Here\ f(x)=2x-3$

$f(x)=(2\times 5-3)=7$

Ans04. $f(x)=3x-5$

Ans05. We have

$(i)\ (B\cup C)=\{3,4\}\cup\{4,5,6\}=\{3,4,5,6\}$

$\therefore A\times(B\cup C)$

$=\{1,2,3\}\times\{3,4,5,6\}$

$=\{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),$

$(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

$(ii)\ (B\cap C)=\{3,4\}\cap\{4,5,6\}=\{4\}$

$\therefore A\times(B\cap C)=\{1,2,3\}\times\{4\}=\{(1,4),(2,4),(3,4)\}$

$(iii)\ (A\times B)=\{1,2,3\}\times\{3,4\}$

$=\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

$(B\times C)=\{3,4\}\times\{4,5,6\}$

$=\{(3,4),(3,5),(3,6),(4,4),(4,5),(4,6)\}$

$\therefore (A\times B)\cap(B\times C)=\{(3,4)\}$

Ans06. First we assume that $A=B$

Then $(A\times B)=(A\times A)$ and $(B\times A)=(A\times A)$

$\therefore (A\times B)=(B\times A)$

This, when $A = B$, then $(A \times B) = (B \times A)$

Conversely, Let $(A \times B) = (B \times A)$, and let be $x \in A$.

Then, $x \in A \Rightarrow (x, b) \in A \times B$ for same $b \in B$

$\Rightarrow (x, b) \in B \times A \qquad [\therefore A \times B = B \times A]$

$\Rightarrow x \in B.$

$\therefore A \subseteq B$

similarly, $B \subseteq A$

Hence, $A = B$

Ans07. $R = \{(a,b) : a, b \in Z$ and $(a-b)$ is divisible by $m\}$

(i) Let $a \in Z$.Then,

$a - a$ =0, which is divisible by m

$\therefore (a,a) \in R$ for all $a \in Z$

so R is refleseive

(ii) Let $(a,b) \in R$ Then

$(a,b) \in R \Rightarrow (a-b)$ is divisible by m

$\Rightarrow -(a-b)$ is divisible by m

$\Rightarrow (b-a)$ is divisible m

$\Rightarrow (b,a) \in R$

Then $(a,b) \in R \Rightarrow (b,a) \in R.$

So R is symmetric.

(iii) Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a-b)$ is divisible by m and $(b-c)$ is divisible by m

$\Rightarrow [(a-b)+(b-c)]$ is divisible by m

$\Rightarrow (a-c)$ is divisible by m

$\Rightarrow (a,c) \in R$

$\therefore (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R.$

So, R is transitive this R is reflexive symmetric and transitive Hence, R is an equivalence relation and Z .

Ans08. $R=\{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,1),(5,2),(5,3),(5,4)\}$

Domain of R $=\{2,3,4,5\}$ Range of R $=\{1,2,3,4\}$

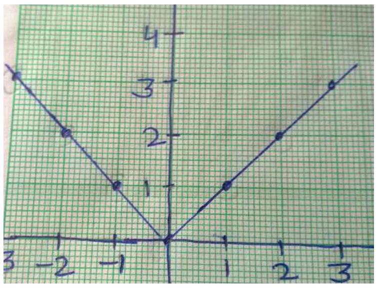
Ans09. let $f:R\to R: f(x)=|x|$ for each $x\in R$. then $f(x)=|x|=\begin{Bmatrix} x, \text{ when } x\geq 0 \\ -x, \text{when } x<0\end{Bmatrix}$

we know that $|x|>0$ for all x

$\therefore$ dom$(f)=R$ and range$(f)=$ set of non negative real number

Drawing the graph of modulus function defined by

$f:R\to R: f(x)=|x|=\begin{Bmatrix} x \text{ when } x\geq 0 \\ -x \text{ when } x<0\end{Bmatrix}$



We have

x	3	-2	-1	0	1	2	3	4
$f(x)$	3	2	1	0	1	2	3	4

Scale: 5 small divisions = 1 unit

On a graph paper, we plot the points

$A(-3,3), B(-2,2), C(-1,1), o(0,0), D(1,1), \in(2,2), F(3,3)$ and $G(4,4)$

Join them successively to obtain the graph lines AO and OG, as show in the figure above.

Ans10. Each element in $\{0,10\}$ has a unique image under f.

But, $g(3)=3^2=9$ and

$g(3)=(2\times 3)=6$

So g is not a function

Ans11. $f(x)=\dfrac{9x}{5}+32(given)$

(i) $f(0)=\left(\dfrac{9\times 0}{5}+32\right)=32 \Rightarrow f(0)=32 \Rightarrow 0^\circ c=32^\circ F$

(ii) $f(-10)=\left(\dfrac{9\times(-10)}{5}+32\right)=14 \Rightarrow f(-10)=14^\circ \Rightarrow (-10)^\circ c=14^\circ F$

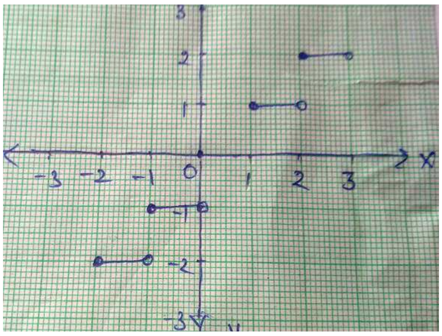
(iii) $f(x)=212 \Leftrightarrow \dfrac{9x}{5}+32=212 \Leftrightarrow 9x=5\times(180)$

$\Leftrightarrow x=100$

$\therefore 212^\circ f=100^\circ c$

Ans12. Clearly, we have

$$f(x)=\begin{cases}-2, \text{ when } x\in[-2,-1)\\ -1, \text{ when } x\in[-1,0)\\ 0, \text{ when } x\in[0,1)\\ 1, \text{ when } x\in[1,2)\end{cases}$$



x	……	$-2\le x<1$	$-1\le x<0$	$0\le x<1$	$1\le x<2$	$2\le x<3$	……
y	……	-2	-1	0	1	2	……

TEST PAPER-04

CLASS - XI MATHEMATICS (Relations and functions)

1. If the ordered pairs $(x-2, 2y+1)$ and $(y-1, x+2)$ are equal, find x & y [1]

2. Let $A=\{-1,2,5,8\}$, $B=\{0,1,3,6,7\}$ and R be the relation, is one less than from A to B then find domain and Range of R [1]

3. Let R be a relation from N to N define by $R=\left[(a,b): a,b\in N \text{ and } a=b^2\right]$. [1]

   Is the following true $a,b\in R$ implies $(b,a)\in R$

4. Let N be the set of natural numbers and the relation R be define in N by $R=\left[(x,y): y=2x, x, y\in N\right]$. what is the domain, co domain and range of R ? Is this relation a function? [1]

5. Let $A=\{1,2\}$ and $b=\{3,4\}$ write $A\times B$ how many subsets will $A\times B$ have? List them. [4]

6. Let $A=\{1,2\}$, $B=\{1,2,3,4\}$, $C=\{5,6\}$ and $D=\{5,6,7,8\}$ verify that [4]

   $(i)\ A\times(B\cap C)=(A\times B)\cap(A\times C)$ $(ii)\ A\times C$ is subset of $B\times D$

7. Find the domain and the range of the relation R defined by $R=\left[(x+1, x+3): x\in(0,1,2,3,4,5)\right]$ [4]

8. Find the linear relation between the components of the ordered pairs of the relation R where $R=\{(2,1),(4,7),(1,-2),.......\}$ [4]

9. Let $A=\{1,2,3,4,5,6\}$ define a relation R from A to A by [4]

   $R=\{(x,y): y=x+1, x, y\in A\}$

   (i) write R in the roaster form

   (ii) write down the domain co domain and range of R

   (iii) Represent R by an arrow diagram

10. A relation 'f' is defined by $f: x\to x^2-2$ where $x\in\{-1,-2,0,2\}$ [4]

    (i) list the elements of f

    (ii) is f a function?

11. Find the domain and the range of the following functions: [6]

    $(i)\ f(x)=\sqrt{x^2-4}$ $(ii)\ f(x)=\sqrt{16-x^2}$ (iii) $f(x)=\dfrac{1}{\sqrt{9-x^2}}$

12. Draw the graphs of the following real functions and hence find range: $f(x)=x^2$ [6]

TEST PAPER-04
CLASS - XI MATHEMATICS (Relations and functions)

[ANSWERS]

Ans.1 $x=3$, $y=2$

Ans.2 Given $A == \{-1,2,5,8\}$, $B=\{0,1,3,6,7\}$, and R is the relation 'is one less than' from A to B therefore $R=[(-1,0),(2,3),(5,6)]$

Domain of $R=\{-1,2,5\}$ and range of $R=\{0,3.6\}$

Ans.3 No; let $a=4, b=2$. As $4=2^2$, so $(4,2)\in R$ but $2\neq 4^2$. so $(2,4)\in R$

Ans.4 Given $R=[(x,y): y=2x, x, y\in N]$

$\therefore$ Domain of $R=N$. co domain of $R=N$. and Range of R is the set of even natural numbers.

Since every natural number x has a unique image $2x$ therefore, the relation R is a function.

Ans.5 $A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$; 16 Subsets of $A\times B$ have

Subsets $=\phi, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}$,

$\{(1,4)\}, \{(1,3),(2,3)\}, \{(1,3),(2,4)\}$,

$\{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3)\}$,

$\{(2,4)\}, \{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),\}$

$\{(2,4),\}, \{(1,3),(2,3),(2,4)\}, \{(1,4),(2,3)\}$,

$\{(2,4)\}$;

Ans6. $L.H.S.\ \ B\cap C=\phi$

Part-I

L.H.S $A\times(B\cap C)=\phi$

$$R.H.S.\ \ A\times B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4)\\(2,1),(2,2),(2,3),(2,4)\end{Bmatrix}$$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$(A\times B)\cap(A\times C)=\phi \qquad L.H.S=R.H.S$

Part-II

$$B\times D=\begin{Bmatrix}(1,5),(1,6),(1,7),(1,8) \\ (2,5),(2,6),(2,7),(2,8)\end{Bmatrix}$$

$$(A\times C)\subset(B\times D)$$

Ans7. Given $x\in\{0,1,2,3,4,5\}$

put $x=0, x+1=0+1=1$ and $x+3=0+3=3$

$x=1, x+1=1+1=2$ and $x+3=1+3=4,$

$x=2, x+1=2+1=3$ and $x+3=2+3=5,$

$x=3, x+1=3+1=4$ and $x+3=3+3=6,$

$x=4, x+1=4+1=5$ and $x+3=4+3=7$

$x=5, x+1=6$ and $x+3=5+3=8$

Hence $R=[(1,3),(2,4),(3,5),(4,6),(5,7),(6,8)]$

$\therefore$ Domain of $R=[1,2,3,4,5,6]$ and range of $R=[3,4,5,6,7,8]$

Ans8. Given $R=\{(2,1),(4,7),(1,-2),....\}$

Let $y=ax+b$ be the linear relation between the components of R

Since $(2,1)\in R, \therefore y=ax+b\Rightarrow 1=2a+b........(i)$

Also $(4,7)\in R, \therefore y=ax+b\Rightarrow 7=4a+b........(ii)$

Subtracting (i) from (ii), we get $2a=6\Rightarrow a=3$

Subtracting $a=3$ is (i), we get $1=6+b\Rightarrow b=-5$

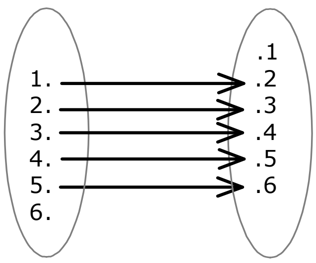
Subtracting there values of a and b in $y=ax+b$, we get

$y=3x-5$, which is the required linear relation between the components of the given relation.

Ans9 (i) $\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

(ii) Domain $=\{1,2,3,4,5\}$ co domain $=A$, range $=\{2,3,4,5,6\}$

(iii)



Ans10. Reletion f is defined by $f: x \to x^2 - 2$

(i) is $f(x) = x^2 - 2$ whene $x \in \{-1, -2, 0, 2\}$

$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

$f(0) = 0^2 - 2 = 0 - 2 = -2$

$f(2) = 2^2 - 2 = 4 - 2 = 2$

$\therefore f = \{(-1,-1), (-2,2), (0,-2), (2'2)\}$

(ii) We note that each element of the domain of f has a unique image; therefore, the relation f is a function.

Ans11. (i) Given $f(x) = \sqrt{x^2 - 4}$

For $Df_1 f(x)$ must be a real number

$\Rightarrow \sqrt{x^2 - 4}$ Must be a real number

$\Rightarrow x^2 - 4 \geq 0 \Rightarrow (x+2)(x-2) \geq 0$

$\Rightarrow$ either $x \leq -2$ or $x \geq 2$

$\Rightarrow D_F = (-\infty, -2] \cup [2, \infty)$.

For R_F, let $y = \sqrt{x^2 - 4}$.........(i)

As square root of a real number is always non-negative, $y \geq 0$

On squaring (i), we get $y^2 = x^2 - 4$

$\Rightarrow x^2 = y^2 + 4$ but $x^2 \geq 0$ for all $x \in D_F$

$\Rightarrow y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4$, which is true for all $y \in R$. also $y \geq 0$

$\Rightarrow R_F = [0, \infty)$

(ii) Given $f(x) = \sqrt{16 - x^2}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \sqrt{16 - x^2}$ must be a real number

$\Rightarrow \sqrt{16 - x^2} \geq 0 \Rightarrow -(x^2 - 16) \geq 0$

$\Rightarrow x^2 - 16 \leq 0$

$\Rightarrow (x+4)(x-4) \leq 0 \Rightarrow -4 \leq x \leq 4$

$\Rightarrow D_F = [-4, 4]$.

For R_F, let $y = \sqrt{16 - x^2}$........(i)

As square root of real number is always non-negative, $y \geq 0$

Squaring (i) we get

$y^2 = 16 - x^2$

$\Rightarrow x^2 = 16 - y^2$ but $x^2 \geq 0$ for all $x \in D_f$

$\Rightarrow 16 - y^2 \geq 0 \Rightarrow -\left(y^2 - 16\right) \geq 0 \Rightarrow y^2 - 16 \leq 0$

$\Rightarrow (y+4)(y-4) \leq 0 \Rightarrow -4 \leq y \leq 4$ but $y \geq 0$

$\Rightarrow R_F = [0,4]$

(iii) Given $f(x) = \dfrac{1}{\sqrt{9-x^2}}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \dfrac{1}{\sqrt{9-x^2}}$ must be a real number

$\Rightarrow 9 - x2 > 0 \Rightarrow -\left(x^2 - 9\right) > 0 \Rightarrow x^2 - 9 < 0$

$\Rightarrow (x+3)(x-3) < 0 \Rightarrow -3 < x < 3 \Rightarrow D_F = (-3,3)$

For $R_{f,}$ let $y = \dfrac{1}{\sqrt{9-x^2}}, y \neq 0.........(i)$

Also as the square root of a real number is always non-negative, $y > 0$.

on squaring (i) we get

$y2 = \dfrac{1}{9-x^2} \Rightarrow 9 - x^2 = \dfrac{1}{y^2} \Rightarrow x^2 = 9 - \dfrac{1}{y^2}$

But $x^2 \geq 0$ for all $x \in D_F \Rightarrow 9 - \dfrac{1}{y^2} \geq 0$

$y^2 > 0$

(Multiply bath sides by y^2, a positive real number)

$\Rightarrow 9y^2 - 1 \geq 0 \Rightarrow y^2 - \dfrac{1}{9} \geq 0$

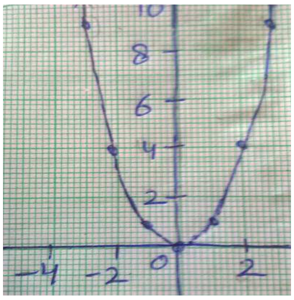
$\Rightarrow \left(y + \dfrac{1}{3}\right)\left(y - \dfrac{1}{3}\right) \geq 0$

$\Rightarrow$ either $y \leq -\dfrac{1}{3}$ or $y \geq \dfrac{1}{3}$

$y > 0 \Rightarrow y \geq \dfrac{1}{3}$

$\Rightarrow R_F = [\dfrac{1}{3}, \infty)$.

Ans12. Given $f(x)=x^2 \Rightarrow D_F = R$



Let $y=f(x)=x^2, x\in R$

x	-4	-3	-2	-1	0	1	2	3	4
$y=x^2$	16	9	4	1	0	1	4	9	16

Plot the points

$(-4,16),(-3,9),(-2,4),(-1,1),(0,0),(1,1)(2,4),(3,9),(4,16)$.....,

And join these points by a free hand drawing. A portion of the graph is shown in sigma (next)

From the graph, it is clear that y takes all non-negative real values, if follows that $R_F = [0,\infty)$

TEST PAPER-05

CLASS - XI MATHEMATICS (Relations and functions)

1. Let $R=\{(x,y): y=x+1\}$ and $y\in\{0,1,2,3,4,5\}$ list the element of R [1]

2. Let f be the subset of $Q\times Z$ defined by [1]

   $f=\left\{\left(\frac{m}{n},m\right): m.n\in Z, n\neq 0\right\}$. Is f a function from Q to Z? Justify your answer

3. The function `f' which maps temperature in Celsius into temperature in [1]

   Fahrenheit is defined by $f(c)=\frac{9}{5}c+32$ find $f(0)$

4. If $f\left(x=x^3-\frac{1}{x^3}\right)$ Prove that $f(x)+f\left(\frac{1}{x}\right)=0$ [1]

5. If $y=\frac{6x-5}{5x-6}$. Prove that $f(y)=x,\ x\neq\frac{6}{5}$ [4]

6. Let $f: X\rightarrow Y$ be defined by $f(x)=x^2$ for all $x\in X$ where $X=\{-2,-1,0,1,2,3\}$ [4]

   and $y=\{0,1,4,7,9,10\}$ write the relation f in the roster farm. It f a function?

7. Determine a quadratic function `f' defined by [4]

   $f(x)=ax^2+bx+c$ if $f(0)=6,\ f(2)=11$ and $f(-3)=6$

8. Find the domain and the range of the function f defied by $f(x)=\frac{x+2}{|x+2|}$ [4]

9. Find the domain and the range of $f(x)=\frac{x^2}{1+x^2}$ [4]

10. $A=\{1,2,3\}, B=\{1,2,3,4\}$. and [4]

    If $R=\{(x,y):(x,y)\in A\times B, y=x+1\}$ then

    (i) find $A\times B$ (ii) write domain and Range

11. Define polynomial function. Draw the graph of $f(x)=x^3$ find domain and range [6]

12. (a) If A, B are two sets such that $n(A\times B)=6$ and some elements of $A\times B$ are [6]

    $(-1,2),(2,3),(4,3)$, than find $A\times B$ and $B\times A$

    (b) Find domain of the function $f(x)=\frac{1}{\sqrt{x+[x]}}$

TEST PAPER-05

CLASS - XI MATHEMATICS (Relations and functions)

[ANSWERS]

Ans1 $R=\{(-1,0),(0,1),(1,2),(2,3),(3,4),(4,5)\}$

Ans2 f Is not a function from Q to Z

$f\left(\frac{1}{2}\right)=1 \; and \; f\left(\frac{2}{4}\right)=2$

$But \; \frac{1}{2}=\frac{2}{4}$

$\therefore$ One element $\frac{1}{2}$ have two images

$\therefore$ f is not function

Ans3 $f(0)=\frac{9}{5}\times 0+32$

$f(0)=32$

Ans4 $f(x)=x^3-\frac{1}{x^3}$

$f\left(\frac{1}{x}\right)=\frac{1}{x^3}-x^3$

$f(x)+f\left(\frac{1}{x}\right)=\cancel{x^3}-\cancel{\frac{1}{x^3}}+\cancel{\frac{1}{x^3}}-\cancel{x^3}$

$=0$

Ans5 $y=\frac{6x-5}{5x-6}$

$y=f(x)=\frac{6x-5}{5x-6}$

$$f(y)=\frac{6\left[\frac{6x-5}{5x-6}\right]-5}{5\left[\frac{6x-5}{5x-6}\right]-6}$$

$$f(y)=\frac{\frac{36x-\cancel{30}-25x+\cancel{30}}{\cancel{5x-6}}}{\frac{\cancel{30}x-25-\cancel{30}x+36}{\cancel{5x-6}}}$$

$$f(y)=\frac{11x}{11}=x, x\neq\frac{6}{5}$$

Ans6 $f: X\rightarrow Y$ defined by

$f(x)=x^2, x\in X$

$and\ X=\{-2,-1,0,1,2,3\}$

$y=\{0,1,4,7,9,10\}$

$f(-2)=(-2)^2=4$

$f(-1)=(-1)^2=1$

$f(0)=0^2=0$

$f(1)=1^2=1$

$f(2)=2^2=4$

$f(3)=3^2=9$

$\therefore f=\{(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9)\}$

f is a function because different elements of X have different imager in y

Ans7 $f(x)=ax^2+bx+c$

$f(0)=6$

$a\times0^2+b\times0+c=6$

$c=6$

$f(2)=11$

$a\times2^2+b\times2+c=11$

$4a+2b+c=11$

$4a+2b+6=11$

$4a+2b=11-6$

$[4a+2b=5]----(i)$

$(-3)=6$

$a\times(-3)^2+b\times(-3)+c=0$

$9a-3b+6=0$

$[9a-3b=-6]----(ii)$

Multiplying eq. (i) by 3 and eq. (ii) by 2

$$12a+\cancel{6b}=15$$
$$\underline{18a-\cancel{6b}=-12}$$
$$30a=3$$
$$a=\frac{3}{30}=\frac{1}{10}$$
$$^{2}\cancel{4}\times\frac{1}{_{5}\cancel{10}}+2b=5$$
$$2b=5\frac{-2}{5}$$
$$2b=\frac{25-2}{5}=\frac{23}{5}$$
$$b=\frac{23}{10}$$
$$\therefore\ f(x)=\frac{1}{10}x^2+\frac{23}{10}x+6$$

Ans8 $f(x)=\dfrac{x+2}{|x+2|}$

For Df, $f(x)$ must be a real no.

$\Rightarrow |x+2|\neq 0 \Rightarrow x+2\neq 0 \Rightarrow x\neq -2$

$\therefore$ Domain of f = set of all real numbers

$except -2 i.e. Df = R-\{-2\}$

$for\ Rf$

$caseI\ \ if\ \ x+2>0\ then\ |x+2|=x+2$

$$\therefore\ f(x)=\frac{x+2}{|x+2|}=1$$

$caseII\ \ if\ \ x+2<0, |x+2|=-(x+2)$

$$\therefore\ f(x)=\left(\cancel{\frac{x+2}{-x+2}}\right)=-1$$

$\therefore$ $Range\ of\ f=\{-1,1\}$

Ans9 $f(x)=\dfrac{x^2}{1+x^2}$

$Domain\ of\ f=all\ real\ no.=R$

$for\ Range\ let\ f(x)=y$

$$y=\frac{x^2}{1+x^2}$$

$$y\left(1+x^2\right)=x^2$$
$$y+yx^2=x^2$$
$$y=x^2-yx^2$$
$$y=(1-y)x^2$$
$$x^2=\frac{y}{1-y}$$
$$x=\sqrt{\frac{y}{1-y}}$$
$$\frac{y}{1-y}\geq 0 \qquad 1-y\neq 0$$
$$y\neq 1$$
$$also\ y\geq 0\ and\ 1-y>0$$
$$\mathrm{y}<1$$
$$\therefore Range\ of\ f=[0,1).$$

Ans10 (i) $A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4)\}$

(ii) $R=\{(1,2),(2,3),(3,4)\}$

$Domain\ of\ R=\{1.2.3\}$

$Range\ of\ R=\{2,3,4\}$

Ans11 A function $f:R\to R$ define by

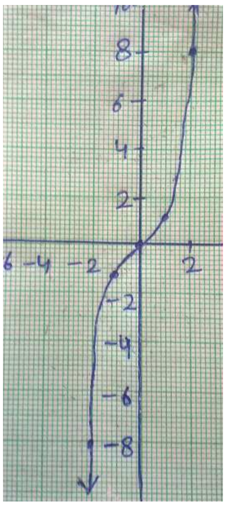
$f(x)=a_0+a_1x+a_2x^2+---+a_nx^n$

$where\ a_0,a_1,a_2,---a_n \in R$

And n is non negative integer is called polynomial function

Graph of $f(x)=x^3$

x	0	1	2	-1	-2
$f(x)$	0	1	8	-1	-8



Domain of $f=R$

Range of $f=R$

Ans12 (a) Given A and B are two sets such that

$n(A\times B)=6$

Some elements of $A\times B$ are

$(-1,2),(2,3)$ *and* $(4,3)$

then $A=\{-1,2,4\}$ *and* $B=\{2,3\}$

$A\times B=\{(-1,2),(-1,3),(2,2),(2,3),(4,2),(4,3)\}$

$B\times A=\{(2,-1),(3,-1),(2,2),(3,2),(2,4),(3,4)\}$

(b) $f(x)=\dfrac{1}{\sqrt{x+[x]}}$

we knowe that

$x+[x]>0$ for all $x>0$

$x+[x]=0$ for all $x=0$

$x+[x]<0$ for all $x<0$

also $f(x)=\dfrac{1}{\sqrt{x+[x]}}$ is defined for all

x satisfying $x+[x]>0$

Hence, Domain $(f)=(0,\infty)$

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Trigonometric Functions)

1. Convert into radian measures. $-47^0\ 30'$ [1]

2. Evaluate $\tan 75^0$. [1]

3. Prove that $\text{Sin}(40+\theta).\text{Cos}(10+\theta) - \text{Cos}(40+\theta).\text{Sin}(10+\theta) = \frac{1}{2}$ [1]

4. Find the principal solution of the eq. $\text{Sin } x = \frac{\sqrt{3}}{2}$ [1]

5. Prove that $\text{Cos}\left(\frac{\pi}{4}+x\right) + \text{Cos}\left(\frac{\pi}{4}-x\right) = \sqrt{2}\ \text{Cos } x$ [1]

6. The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute? [4]

7. Show that $\tan 3x.\tan 2x.\tan x = \tan 3x - \tan 2x - \tan x$ [4]

8. Find the value of $\tan \frac{\pi}{8}$. [4]

9. If $\text{Sin }\alpha + \text{Sin }\beta = a$ and $\text{Cos }\alpha + \text{Cos }\beta = b$ show that $\text{Cos}(\alpha+\beta) = \frac{b^2-a^2}{b^2+a^2}$ [6]

10. Prove that $\text{Cos }\alpha + \text{Cos }\beta + \text{Cos }\gamma + \text{Cos}(\alpha+\beta+\gamma)$ [6]

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Trigonometric Functions)

[ANSWERS]

Ans 01. $-47^0\,30' = -\left(47+\frac{30}{60}\right)^0$

$$=-\left(47\frac{1}{2}\right)^0$$

$$=-\left(\frac{95}{2}\times\frac{\pi}{180}\right) \text{ radians}$$

$$= -\frac{19\pi}{72} \text{ radians.}$$

Ans 02. tan 75 = tan (45 + 30)

$$=\frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$$

$$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Ans 03. L. H. S = Sin (40 + θ). Cos (10 + θ) – Cos (40 + θ). Sin (10 +θ)

$$= \text{Sin}\left[40+\cancel{\theta}-10-\cancel{\theta}\right] = \text{Sin } 30 = \frac{1}{2}$$

Ans 04. $\text{Sin } x = \frac{\sqrt{3}}{2}$

$$\text{Sin } x = \text{Sin } \frac{\pi}{3}$$

$$x = \frac{\pi}{3}$$

$$Sin\ x = \text{Sin}\left(\pi-\frac{\pi}{3}\right)$$

$$x = \frac{2\pi}{3}$$

Ans 05. L. H. S = $\text{Cos}\left(\frac{\pi}{4}+x\right) + \text{Cos}\left(\frac{\pi}{4}-x\right)$

$2 \text{ Cos } \frac{\pi}{4}. \text{Cos } x$ [$\because$ Cos (A + B) + Cos (A - B) = 2 Cos A. Cos B

$= 2 \frac{1}{\sqrt{2}}. \text{Cos x} \qquad = \sqrt{2} \text{ Cos x}$

Ans 06. r = 1.5 cm

Angle made in 60 mint = 360^0

Angle made in 1 min = $\frac{360}{60} = 6^0$

Angle made in 40 mint = 6×40

$= 240^0$

$\Theta = \frac{l}{r}$

$\cancel{240}^{\,4} \times \frac{\pi}{\cancel{180}\ 3} = \frac{l}{1.5}$

$\frac{4 \times 3.14}{\cancel{3}} = \frac{l}{\frac{\cancel{1.5}\ \cancel{5}}{\cancel{10}\ 2}}$

$2 \times 3.14 = l$

$6.28 = l$

$l = 6.28$ cm

Ans 07. Let 3x = 2x + x

tan 3x = tan (2x + x)

$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x. \tan x}$

tan 3x (1 - tan 2x. tan x) = tan 2x + tan x

tan 3x - tan 3x. tan 2x. tan x = tan 2x + tan x

tan 3x. tan 2x. tan x = tan 3x - tan 2x - tan x

Ans 08. Let $x = \frac{\pi}{8}$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$\tan\left(\cancel{2} \frac{\pi}{\cancel{8}\ 4}\right) = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$

$1 = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$

put $\tan \pi/8 = \iota$

$\frac{1}{1} = \frac{2\iota}{1 - \iota^2}$

$2\iota = 1 - \iota^2$

$\iota^2 + 2\iota - 1 = 0$

$\iota = \frac{-2 \pm 2\sqrt{2}}{2 \times 1}$

$= -1 \pm \sqrt{2}$

$= \pm \sqrt{2} - 1$

$= \sqrt{2} - 1$ or $-\sqrt{2} - 1$

$\tan \pi/8 = \sqrt{2} - 1$

Ans 09. $b^2 + a^2 = (\text{Cos}\ \alpha + \text{Cos}\ \beta)^2 + (\text{Sin}\ \alpha + \text{Sin}\ \beta)^2$

$= \text{Cos}^2\ \alpha + \text{Cos}^2\ \beta + 2\ \text{Cos}\ \alpha.\ \text{Cos}\ \beta + \text{Sin}^2\ \alpha + \text{Sin}^2\ \beta + 2 Sin\ \alpha.\ \text{Sin}\ \beta$

$= 1 + 1 + 2\ (\text{Cos}\ \alpha.\ \text{Cos}\ \beta + \text{Sin}\ \alpha\ .\ \text{Sin}\ \beta)$

$= 2 + 2\ \text{Cos}\ (\alpha - \beta) \quad (1)$

$b^2 - a^2 = (\text{Cos}\ \alpha + \text{Cos}\ \beta)^2 - (\text{Sin}\ \alpha + \text{Sin}\ \beta)^2$

$= (\text{Cos}^2\ \alpha - \text{Sin}^2\ \beta) + (\text{Cos}^2\ \beta - \text{Sin}^2\ \alpha) + 2\ \text{Cos}\ (\alpha + \beta)$

$= \text{Cos}\ (\alpha + \beta)\ \text{Cos}\ (\alpha - \beta) + \text{Cos}\ (\beta + \alpha) \text{Cos}\ (\alpha - \beta) + 2\ \text{Cos}\ (\alpha + \beta)$

$= 2\ \text{Cos}\ (\alpha + \beta).\ \text{Cos}\ (\alpha - \beta) + 2\ \text{Cos}\ (\alpha + \beta)$

$= Cos\ (\alpha + \beta)\ [2\ \text{Cos}\ (\alpha - \beta) + 2]$

$= \text{Cos}\ (\alpha + \beta).\ (b^2 + a^2)$ [from (1)

$$\frac{b^2 - a^2}{b^2 + a^2} = \text{Cos}\ (\alpha + \beta)$$

Ans 10. L. H. S.

$= \text{Cos}\ \alpha + \text{Cos}\ \beta + \text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)$

$= \text{Cos}\ \alpha + \text{Cos}\ \beta) + [\text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)]$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\ \text{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\alpha+\gamma}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$$

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Trigonometric Functions)

1. Convert into radian measures. $-37^0 30'$ (1)

2. Prove Sin (n+1) x Sin (n+2) x + Cos (n+1) x. Cos (n+2) x = Sin (x + y) (1)

3. Find the value of Sin $\frac{31\pi}{3}$ (1)

4. Find the principal solution of the eq. tan x = $\frac{-1}{\sqrt{3}}$ (1)

5. Prove that $\frac{Sin(x+y)}{Sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$ (4)

6. If in two circles, arcs of the same length subtend angles 60^0 and 75^0 at the centre find the ratio of their radii. (4)

7. Prove that Cos 6x= 32 $Cos^6 x$ – 48 Cos^4 x + 18 Cos^2 x-1 (4)

8. Solve Sin2x-Sin4x+Sin6x=o (4)

9. Prove that Sin3x +Sin2x-Sinx=4Sinx.Cos $\frac{x}{2}$. Cos $\frac{3x}{2}$ (6)

10. Prove that 2Cos $\frac{\pi}{13}$.Cos $\frac{9\pi}{13}$ + Cos $\frac{3\pi}{13}$ + Cos $\frac{5\pi}{13}$ =0 (6)

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Trigonometric Functions)

[ANSWERS]

Ans.01. $-37^0\,30' = -\left(37+\frac{\cancel{30}^{\,1}}{\cancel{60}^{\,2}}\right)^0$

$= -\left(\frac{75}{2}\right)^0$

$= -\frac{75}{2}\times\frac{\pi}{180}\,radien$

$= -\frac{5\pi}{24}$

Ans.02. L.H.S = Cos (n+1) x Cos (n+2) x + Sin (n+1) x Sin (n+2) x

$=\text{Cos}\left[(n+1)x-(n+2)x\right]$

=Cos x

Ans.03. $\text{Sin}\ \frac{31\pi}{3} = \text{Sin}\left(10\pi+\frac{\pi}{3}\right)$

$= \sin\left(2\pi\times5+\frac{\pi}{3}\right)$

$= \text{Sin}\ \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

Ans.04. $\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(\pi-\frac{\pi}{6}\right)$

$x = \frac{5\pi}{6}$

$\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(2\pi-\frac{\pi}{6}\right)$

$x = \frac{11\pi}{6}$

Ans.05. L.H.S $= \dfrac{Sin(x+y)}{Sin(x-y)}$

$$= \frac{Sinx.Cosy + Cosx.Siny}{Sinx.Cosy - Cosx.Siny}$$

$$DividingNandDbyCosx.Cosy$$

$$= \frac{\tan x + \tan y}{\tan x - \tan y}$$

.Ans.06. $\theta = \dfrac{l}{r_1}$

$$60 \times \frac{\pi}{18} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

(1) ÷ (2)

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}} = \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5:4$$

Ans.7. L.H.S. = Cos 6x

$$\cos 2(3x) = 2\cos^2 3x - 1$$

$$= 2(4\cos^3 x - 3\cos x)^2 - 1$$

= cos 2(3x) = $= 2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$

$$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$$

$$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

Ans.08

$$Sin\ 6x+\sin 2x-\sin 4x=0$$
$$2\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right)-\sin 4x=0$$
$$2\sin 4x.\cos 2x-\sin 4x=0$$
$$\sin 4x\left[2\cos 2x-1\right]=0$$
$$\sin 4x=0$$
$$4x=n\pi$$
$$x=\frac{n\pi}{4}$$
$$2\cos 2x-1=0$$
$$\cos 2x=\cos\frac{\pi}{3}$$
$$2x=2n\pi\pm\frac{\pi}{3}$$
$$x=n\pi\pm\frac{\pi}{6}$$

Ans.09

$$\left(\sin 3x-\sin x\right)+\sin 2x$$
$$=2\cos\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right)+\sin 2x$$
$$=2\cos 2x.\sin x+\sin 2x$$
$$=2\cos 2x.\sin x+2\sin x.\cos x$$
$$=2\sin x\left[2\cos 2x+\cos x\right]$$
$$=2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$$

Ans10. L.H.S.

$$=2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$
$$=\cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$
$$=\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$
$$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$
$$=\cancel{-\cos\frac{3\pi}{13}}-\cancel{\cos\frac{5\pi}{13}}+\cancel{\cos\frac{3\pi}{13}}+\cancel{\cos\frac{5\pi}{13}}$$
$$=0$$

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Trigonometric Functions)

1. Convert into radian measures. $5^0 37' 30''$ [1]

2. Prove Cos 70^0. Cos 10^0 + Sin 70^0. Sin 10^0 = $\frac{1}{2}$ [1]

3. Evaluate 2 Sin $\frac{\pi}{12}$. [1]

4. Find the solution of Sin x = $-\frac{\sqrt{3}}{2}$ [1]

5. Prove that $\frac{\text{Cos } 9^0 \text{ - Sin } 9^0}{\text{Cos } 9^0 \text{ + Sin } 9^0} = \tan 36^0$ [1]

6. In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor are of the chord. [4]

7. Prove that $\tan 4x = \frac{4 \tan x\,(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$ [4]

8. Prove that (Cos x + Cos y)^2 + (Sin x – Sin y)^2 = 4 Cos^2 $\left(\frac{x+y}{2}\right)$ [4]

9. Find the value of tan (α + β) Given that [6]

   Cot $\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and Sec $\beta = -\frac{5}{3},\ \beta \in \left(\frac{\pi}{2}, \pi\right)$

10. Prove that $\frac{\text{Sec } 8A - 1}{\text{Sec } 4A - 1} = \frac{\tan 8A}{\tan 2A}$ [6]

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Trigonometric Functions)

[ANSWERS]

Ans 01. $5^0\ 37^1\ 30^{11} = 5^0 + \left(37+\frac{30}{60}\right)^1$

$$= 5^0 + \left(\frac{75}{2}\right)^1$$

$$= 5^0 + \left(\frac{75}{2}\times\frac{1}{60}\right)^0$$

$$= 5^0 + \left(\frac{5}{8}\right)^0$$

$$= \left(5\frac{5}{8}\right)^0$$

$$= \left(\frac{45}{8}\right)^0$$

$$= \frac{45}{8}\times\frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{32} \text{ radian.}$$

Ans 02. L. H. S. = Cos (70 – 10) = Cos 60 = $\frac{1}{2}$

Ans 03. $2 \text{ Sin } \frac{\pi}{12} = 2 \text{ Sin } \left[\frac{\pi}{4} - \frac{\pi}{6}\right]$

$$= 2\left[\text{Sin } \frac{\pi}{4}.\text{ Cos } \frac{\pi}{6} - \text{Cos } \frac{\pi}{6}.\text{ Sin } \frac{\pi}{4}\right]$$

$$=2\left[\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$=\cancel{2}\left(\frac{\sqrt{3}-1}{\cancel{2}\sqrt{2}}\right) = \frac{\sqrt{3}-1}{\sqrt{2}}$$

Ans 04. Sin x = $-\frac{\sqrt{3}}{2}$

Sin x = Sin $\left(\pi+\frac{\pi}{3}\right)$

Sin x = Sin $\frac{4\pi}{3}$

if Sin θ = Sin α

θ = n π + $(-1)^n$. α

$$x = \text{n}\pi + (-1)^n . \frac{4\pi}{3}$$

Ans 05. L. H. S = tan 36^0

$= \tan(45^0 - 9^0)$

$= \frac{1 - \tan 9^0}{1 + \tan 9^0}$

$= \frac{\text{Cos } 9^0 - \text{Sin } 9^0}{\text{Cos } 9^0 + \text{Sin } 9^0}$

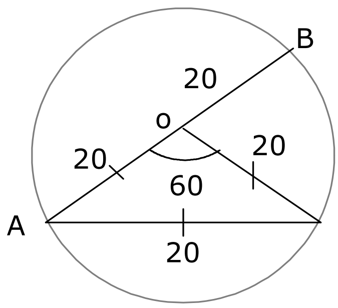
Ans 06.

$\Theta = 60^0$

$\theta = \frac{l}{r}$

$\cancel{60} \times \frac{\pi}{\cancel{180}\ 3} = \frac{l}{20}$

$l = \frac{20\pi}{3}$ cm.



Ans 07. L. H. S = tan 4x

$$= \frac{2\tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2.\frac{2\tan x}{1-\tan^2 x}}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\frac{4\tan x}{1-\tan^2 x}}{\frac{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}{\left(1-\tan^2 x\right)^2}}$$

$$= \frac{4\tan x}{\left(1-\tan^2 x\right)} \times \frac{\left(1-\tan^2 x\right)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

Ans 08. L. H. S = $(\text{Cos } x + \text{Cos } y)^2 + (\text{Sin } x - \text{Sin } y)^2$

$$=\left(2\text{ Cos } \frac{x+y}{2}.\text{ Cos } \frac{x-y}{2}\right)^2 + \left(2\text{ Cos } \left(\frac{x+y}{2}\right).\text{ Sin } \left(\frac{x-y}{2}\right)\right)^2$$

$$= 4\text{ Cos}^2 \frac{x+y}{2}.\text{ Cos}^2\left(\frac{x-y}{2}\right) + 4\text{ Cos}^2 \frac{x+y}{2}.\text{ Sin}^2 \frac{x-y}{2}$$

$$= 4\text{ Cos}^2 \left(\frac{x+y}{2}\right)\left[\text{Cos}^2 \frac{x-y}{2} + Sin^2 \frac{x-y}{2}\right]$$

$$= 4\text{ Cos}^2 \left(\frac{x+y}{2}\right)$$

Ans 09. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$ (1)

$\text{Cot } \alpha = \dfrac{1}{2},$

$\Rightarrow \tan\alpha = 2$

$1 + \tan^2 \beta = \text{Sec}^2 \beta$

$1 + \tan^2 \beta = \left(\dfrac{-5}{2}\right)2 \quad \left[\because \text{Sec } \beta = \dfrac{-5}{3}\right.$

$\tan\beta = \pm \dfrac{4}{3}$

$\tan\beta = -\dfrac{4}{3} \quad \left[\because \beta \in \left(\dfrac{\pi}{2}\Big/x\right)\right.$

put $\tan\alpha$, and $\tan\beta$ in eq. (1)

$$\tan(\alpha+\beta) = \frac{2-\frac{4}{3}}{1-2\left(-\frac{4}{3}\right)}$$

$$= \frac{2}{11}$$

Ans 10. L. H. S

$$= \frac{\frac{1}{Cos\ 8A} - 1}{\frac{1}{\text{Cos } 4A} - 1}$$

$$= \frac{\text{1 - Cos 8A}}{\text{1 - Cos 4A}} \times \frac{\text{Cos 4A}}{\text{Cos 8A}}$$

$$= \frac{2\ \text{Sin}^2\ 4A}{2\ \text{Sin}^2\ 2A} . \frac{\text{Cos 4A}}{\text{Cos 8A}}$$

$$= \frac{(2\ \text{Sin 4A. Cos 4A}).\ \text{Sin 4A}}{2\ \text{Sin}^2\ 2A.\ \text{Cos 8A}}$$

$$= \frac{\text{Sin 8A. } (\cancel{2}\ \cancel{\text{Sin}}\ \cancel{2}A\ .\ \text{Cos 2A})}{\cancel{2}\ \cancel{\text{Sin}}^{\cancel{2}}\ 2A.\ \text{Cos 8A}}$$

$$= \frac{\text{Sin 8A Cos 2A}}{\text{Sin 2A. Cos 8A}}$$

$$= \frac{\tan\ 8A}{\tan\ 2A}$$

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Trigonometric Functions)

1. Find the value of tan $\frac{19\pi}{3}$. [1]

2. Prove Cos 4x = 1 – 3 Sin^2 x. Cos^{2}x [1]

3. Prove $\frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cdot \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$ [1]

4. Prove that tan 56^0 = $\frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0}$ [1]

5. Prove that Cos 105^0 + Cos 15^0 = Sin 75^0 – Sin 15^0 [1]

6. If Cot x = - $\frac{5}{12}$, x lies in second quadrant find the values of other five trigonometric functions. [4]

7. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$ [4]

8. Prove that Sin x + Sin 3x + Sin 5x + Sin 7x = 4 Cos x. Cos 2x. Sin 4x [4]

9. Prove that $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$ [6]

10. Prove that Cos 2x. Cos $\frac{x}{2}$ - Cos 3x. Cos $\frac{9x}{2}$ = Sin 5x Sin $\frac{5x}{2}$ [6]

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Trigonometric Functions)

[ANSWERS]

Ans 01. $\tan \frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\left(3\times 2\pi + \frac{\pi}{3}\right)$

$= \tan \frac{\pi}{3} = \sqrt{3}$

Ans 02. L. H. S = Cos 4x

$= 1 - 2\,\text{Sin}^2\,2x \quad \left[\because \text{Cos}\,2\theta = 1 - 2\text{Sin}^2\,\theta\right.$

$= 1 - 2\,(\,\text{Sin}2x)^2$

$= 1 - 2\,(2\,\text{Sin}\,x.\,\text{Cos}\,x)^2$

$= 1 - 2\,(4\,\text{Sin}^2\,x\,.\,\text{Cos}^2\,x)$

$= 1 - 8\,\text{Sin}^2\,x.\,\text{Cos}^2\,x.$

Ans 03. L. H. S $= \frac{-\text{Cos}\,x.\,\text{Cos}\,x}{-\text{Sin}\,x.\,\text{Sin}\,x} = \text{Cot}^2\,x.$

Ans 04. L. H. S = tan 56^0

= tan (45 + 11)

$= \frac{\tan 45 + \tan 11}{1 - \tan 45.\,\tan 11}$

$= \frac{1 + \tan 11}{1 - \tan 11}$

$= \frac{\text{Cos}\,11 + \text{Sin}\,11}{\text{Cos}\,11 - \text{Sin}\,11}$ = R H S

Ans 05. L. H. S = Cos 105^0 + Cos 15^0

$= \text{Cos}\,(90^0 + 15^0) + \text{Cos}\,(90^0 - 75^0)$

$= -\,\text{Sin}\,15^0 + \text{Sin}\,75^0$

$= \text{Sin}\,75 - \text{Sin}\,15^0$

Ans 06. Cot x = $-\frac{5}{12}$

tan x = - $\frac{12}{5}$

$\text{Sec}^2 x = 1 + \tan^2 x$

Sec x = $\pm\frac{13}{5}$

Sec x = - $\frac{13}{5}$ [$\because$ x lies in IInd quad.

Cos x = - $\frac{5}{13}$

Sin x = tan x. Cos x

$$= \frac{-12}{5} \times \left(\frac{-5}{13}\right) \qquad = \frac{12}{13}$$

Cosec x = $\frac{13}{12}$

Ans 07. L. H. S = $\dfrac{\text{Sin } 5x + \text{Sinx} - 2 \text{ Sin } 3x}{\text{Cos } 5x - \text{Cos } x}$

$$= \frac{2\text{Sin } 3x. \text{Cos } 2x - 2 \text{ Sin } 3x}{- 2 \text{ Sin } 3x. \text{Sin } 2x}$$

$$= \frac{\cancel{2 \text{ Sin } 3x} (\text{Cos } 2x - 1)}{-\cancel{2 \text{ Sin } 3x}. \text{Sin } 2x}$$

$$= \frac{\cancel{-}(1 - \text{Cos } 2x)}{\cancel{-}\text{Sin } 2x}$$

$$= \frac{\cancel{2}\ \cancel{\text{Sin}}^{\cancel{2}}\ x}{\cancel{2}\ \cancel{\text{Sin}}\ x. \text{Cos } x}$$

$$= \frac{\text{Sin } x}{\text{Cos } x} = \tan x$$

Ans 08. L. H. S. = Sin x + Sin 3x + Sin 5x + Sin 7x

= Sin x + Sin 7x + Sin 3x + Sin 5x

$$= 2 \text{ Sin } \left(\frac{x+7x}{2}\right). \text{Cos } \left(\frac{x-7x}{2}\right) + 2 \text{ Sin } \left(\frac{3x+5x}{2}\right) \text{Cos } \left(\frac{3x-5x}{2}\right)$$

= 2 Sin 4x. Cos 3x + 2 Sin 4x. Cos x

= 2 Sin 4x [Cos 3x + Cos x]

$$= 2 \text{ Sin } 4x \left[2 \text{ Cos } \left(\frac{3x+x}{2}\right). \text{Cos } \left(\frac{3x-x}{2}\right)\right]$$

= 2 Sin 4x [2 Cos 2x. Cos x]

= 4 Cos x. Cos 2x. Sin 4x

Ans 09. L. H. S = $\frac{1 + \text{Cos } 2x}{2} + \frac{1 + \text{Cos}\left(2x+\frac{2\pi}{3}\right)}{2} + \frac{1 + \text{Cos}\left(2x-\frac{2\pi}{3}\right)}{2}$

$$= \frac{1}{2}\left[1+1+1+\text{Cos } 2x + \text{Cos}\left(2x+\frac{2\pi}{3}\right) + \text{Cos}\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3+Cos2x+Cos\left(2x+\frac{2\pi}{3}\right)+Cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3+Cos2x+2Cos\left(\frac{2x+\frac{\cancel{2}\cancel{\pi}}{\cancel{3}}+2x-\frac{\cancel{2}\cancel{\pi}}{\cancel{3}}}{2}\right). \text{Cos}\left(\frac{\cancel{2}\cancel{x}+\frac{2\pi}{3}-\cancel{2}\cancel{x}+\frac{2\pi}{3}}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos } \frac{{}^{2}\cancel{4}\cancel{\pi}}{\cancel{6}_{3}}\right]$$

$$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos } \frac{2\pi}{3}\right]$$

$$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos}\left(\pi-\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \text{Cos } 2x + \cancel{2} \text{ Cos } 2x. \left(\frac{-1}{\cancel{2}}\right)\right]$$

$$= \frac{3}{2}.$$

Ans 10. L. H. S = $\frac{1}{2}\left[2 \text{ Cos } 2x \text{ Cos } \frac{x}{2} - 2\text{Cos } 3x. \text{ Cos } \frac{9x}{2}\right]$

$$= \frac{1}{2}\left[\text{Cos}\left(2x+\frac{x}{2}\right)+Cos\left(2x-\frac{x}{2}\right)-Cos\left(\frac{9x}{2}+3x\right)-Cos\left(\frac{9x}{2}-3x\right)\right]$$

$$= \frac{1}{2}\left[Cos\frac{5x}{2}+\cancel{Cos}\frac{\cancel{3}\cancel{x}}{\cancel{2}}-Cos\frac{15x}{2}-\cancel{Cos}\frac{\cancel{3}\cancel{x}}{\cancel{2}}\right]$$

$$= \frac{1}{2}\left[Cos\frac{5x}{2}-Cos\frac{15x}{2}\right]$$

$$= \frac{1}{2}\left[-2Sin\left(\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right).Sin\left(\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right)\right]$$

$$= -\text{ Sin } 5x. \text{ Sin}\left(\frac{-5x}{2}\right)$$

$$= \text{ Sin } 5x. \text{ Sin } \frac{5x}{2}.$$

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Trigonometric Functions)

1. Find the value of Cos (- 1710^0). [1]

2. A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second. [1]

3. Prove $\text{Sin}^2 6x - \text{Sin}^2 4x = \text{Sin}2x. \text{Sin} 10x$. [1]

4. Prove that $\frac{\tan 69 + \tan 66}{1 - \tan 69. \tan 66} = -1$ [1]

5. Prove that $\frac{\text{Sin x}}{1 + \text{Cos x}} = \tan \frac{x}{2}$ [1]

6. Find the angle between the minute hand and hour hand of a clock when the time is 7. 20. [4]

7. Prove that Cot 4x (Sin 5x + Sin 3x) = Cot x (Sin 5x – Sin 3x) [4]

8. Show that $\sqrt{2+\sqrt{2 + 2 \text{ Cos } 4\theta}} = 2 \text{ Cos } \theta$ [4]

9. Prove that $\text{Cos } 20^0. \text{Cos } 40^0. \text{Cos } 60^0 \text{ Cos } 80^0 = \frac{1}{16}$ [6]

10. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, Find the value of Sin $\frac{x}{2}$, Cos $\frac{x}{2}$ and tan $\frac{x}{2}$. [6]

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Trigonometric Functions)

[ANSWERS]

Ans 01. Cos (-1710^0) = Cos (1800-90) [Cos ($-\theta$) = Cos θ

= Cos [$5 \times 360 + 90$]

= Cos $\frac{\pi}{2}$ = 0

Ans 02. N. of revolutions made in 60 sec. = 360

N. of revolutions made in 1 sec = $\frac{360}{60} = 6$

Angle moved in 6 revolutions = $2\pi \times 6 = 12\pi$

Ans 03. L. H. S = $Sin^2\, 6x - Sin^2\, 4x$

= Sin (6x + 4x). Sin (6x – 4x)

= Sin 10x . Sin 2x

Ans 04. L. H. S = tan (69 + 66)

= tan (135)

= tan (90 + 45)

= - tan 45

= -1

Ans 05. L. H. S = $\frac{2Sin\frac{x}{2}.Cos\frac{x}{2}}{2Cos^x\frac{x}{2}}$ = tan $\frac{x}{2}$

Ans 06. Angle made by mint hand in 15 mint= $15 \times 6 = 90^0$

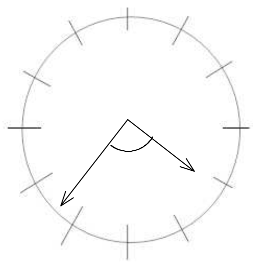
Angle made by hour hand in 1 hr = 30^0

in 60 minute = $\frac{30}{60} = \frac{1}{2}$

[$\because$ Angle Traled by hr hand in 12 hr = 360^0

in 20 minute = $\frac{1}{2} \times 20 = 10^0$

Angle made = 90 + 10 = 100^0



Ans 07. L. H. S = Cot 4x (Sin 5x + Sin 3x)

$$= \frac{\text{Cos } 4x}{\text{Sin } 4x}\left[2 \text{ Sin } \frac{5x+3x}{2}. \text{ Cos } \frac{5x-3x}{2}\right]$$

$$= \frac{\text{Cos } 4x}{\cancel{\text{Sin } 4x}}\ 2\ \cancel{\text{Sin } 4x}. \text{ Cos } x$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

R. H. S = Cot x (Sin 5x – Sin 3x)

$$= \frac{\text{Cos } x}{\text{Sin } x}\left[2 \text{ Cos } \frac{5x+3x}{2}. \text{ Sin } \frac{5x-3x}{2}\right]$$

$$= \frac{\text{Cos } x}{\cancel{\text{Sin } x}}\left[2 \text{ Cos } 4x. \cancel{\text{Sin } x}\right]$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

L. H. S = R. H. S

Ans 08. L H. S = $\sqrt{2+\sqrt{2+2\text{Cos } 4\theta}}$

$$= \sqrt{2+\sqrt{2(1+\text{Cos } 4\theta)}}$$

$$= \sqrt{2+\sqrt{2.2 \text{ Cos}^2\ 2\theta}}$$

$$= \sqrt{2+2 \text{ Cos } 2\theta}$$

$$= \sqrt{2(1+\text{Cos } 2\theta)}$$

$$= \sqrt{2.2 \text{ Cos}^2\theta}$$

$$= 2 \text{ Cos } \theta$$

Ans 09. L. H. S = Cos 20^0. Cos 40^0. Cos 60^0. Cos 80^0.

= Cos 60. Cos 20^0. Cos 40^0. Cos 80.

$$= \frac{1}{2}.\frac{1}{2} \text{ Cos}40^0\ (2 \text{ Cos}20. \text{ Cos}80)$$

$$= \frac{1}{4}\text{Cos}40^0\ [\text{Cos } (80 + 20) + \text{Cos } (80 - 20)]$$

$$= \frac{1}{4}\text{Cos}40^0\left[\text{Cos } 100^0 + \text{Cos } 60\right]$$

$$= \frac{1}{4}\text{Cos}40^0\left[\text{Cos } 100+\frac{1}{2}\right]$$

$$= \frac{1}{8}\left(2 \text{ Cos } 100. \text{ Cos } 40^0\right)+\frac{1}{8} \text{ Cos } 40^0$$

$$= \frac{1}{8}\left[\text{Cos }(100 + 40^0) + \text{Cos }(100 - 40^0)\right] + \frac{1}{8}Cos 40^0$$

$$= \frac{1}{8}\left[\text{Cos } 140^0 + \text{Cos } 60\right] + \frac{1}{8}\text{ Cos } 40^0$$

$$= \frac{1}{8}\left[\text{Cos } 140 + \frac{1}{2}\right] + \frac{1}{8}\text{ Cos } 40^0$$

$$= \frac{1}{8}\text{ Cos } 140^0 + \frac{1}{16} + \frac{1}{8}\text{ Cos } 40^0$$

$$= \frac{1}{8}\text{ Cos }(180 - 40) + \frac{1}{16} + \frac{1}{8}\text{ Cos } 40^0$$

$$= -\frac{1}{8}\cancel{\text{Cos } 40^0} + \frac{1}{16} + \frac{1}{8}\cancel{\text{Cos } 40^0}$$

$$= \frac{1}{16}$$

Ans 10.

$\pi < x < \frac{3\pi}{2}$ [Given

Cos x is – tive

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Sin $\frac{x}{2}$ is + tive and Cos $\frac{x}{2}$ is - tive.

$1 + \tan^2 x = \text{Sec}^2 x$

$1 + \left(\frac{3}{4}\right)^2 = \text{Sec}2x$

Sec x = $\pm \frac{5}{4}$

Cos x = $\pm \frac{5}{4}$

Cos x = $-\frac{5}{4}$ $\left[\because \pi < x < \frac{3\pi}{2}\right.$

$$\text{Sin } \frac{x}{2} = \sqrt{\frac{1 - Cosx}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\text{Cos } \frac{x}{2} = -\sqrt{\frac{1 + Cosx}{2}}$$

$$= -\sqrt{\frac{1 - \frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}}$$

$$\tan \frac{x}{2} = \frac{\frac{3}{\sqrt{10}}}{\frac{-1}{\sqrt{10}}} = -3$$

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

1. For every integer n, prove that $7^n - 3^n \mathit{is}$ divisible by 4. [4]

2. Prove that $n(n+1)(n+5)$ is multiple of 3. [4]

3. Prove that $10^{2n-1}+1$ is divisible by 11 [4]

4. Prove $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{n}\right)=(n+1)$ [4]

5. Prove 1.2+2.3+3.4+--+n$(n+1)=\frac{n(n+1)(n+2)}{3}$ [4]

6. Prove $(2n+7)<(n+3)^2$ [4]

7. Prove $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$ [4]

8. Prove $1.2+2.2^2+3.2^3+...+n.2^n = (n-1)2^{n+1}+2$ [4]

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Principal of mathematical Induction)
[ANSWERS]

Ans.01. P(n) : $7^n - 3^n$ is divisible by 4

For n=1

P(1) : $7^1 - 3^1 = 4$ which is divisible by 4. Thus, P(1) is true

Let P(k) be true

$7^k - 3^k$ *is* divisible by 4

$7^k - 3^k = 4\lambda$, where $\lambda \in N$ (i)

we want to prove that P (k+1) is true whenever P(k) is true

$7^{k+1} - 3^{k+1} = 7^k.7 - 3^k.3$

$= (4\lambda + 3^k).7 - 3^k.3$(from i)

$= 28\lambda + 7.3^k - 3^k.3$

$= 28\lambda + 3^k(7-3)$

$= 4(7\lambda + 3^k)$

Hence

$7^{k+1} - 3^{k+1}$ is divisible by 4

thus P (k+1) is true when P(k) is true.

Therefore by P.M.I. the statement is true for every positive integer n.

Ans.02. $P(n): n(n+1)(n+5)$ is multiple of 3

for n=1

P(1) : 1(1+1) (1+5) = 12 is multiple of 3

let P(k) be true

P(k) : K (k+1) (k+5) is muetiple of 3

$\Rightarrow$ k(k+1)(k+5)=3λ where $\lambda \in$ N (i)

we want to prove that result is true for n=k+1

P(k+1) : (k+1)(k+2)(k+6)

$\Rightarrow$ (K+1)(k+2)(k+6)= $\left[(k+1)(k+2)\right](k+6)$

= k(k+1)(k+2)+6(k+1)(k+2)

=k(k+1)(k+5-3)+6(k+1)(k+2)

=k(k+1)(k+5)-3k(k+1)+6(k+1)(K+2)

=k(k+1)(k+5)+(k+1)$\left[6\left(k+2\right)-3k\right]$

=k(k+1)(k+5)+(k+1)(3k+12)

=k(k+1)(k+5)+3(k+1)(k+4)

=3λ+3 (k+1)(k+4) (from i)

=3$\left[\lambda+(K+1)(K+4)\right]$ which is multiple of three

Hence P(k+1) is multiple of 3 .

Ans .03. $P(n): 10^{2n-1}$ +1 is divisible by 11

for n=1

P(1) = $10^{2\times1-1}+1=11$ is divisible by 11 Hence result is true for n=1

let P(k) be true

P(k) : $10^{2k-1}+1$ is divisible by 11

$\Rightarrow$ $10^{2k-1}+1 = 11\lambda$ where $\lambda \in$ N(i)

we want to prove that result is true for n= k+1

$=10^{2(k+1)-1}+1=10^{2k+2-1}+1$

$=10^{2k+1}+1$

$=10^{2k}.10^{1}+1$

$=(110\lambda-10).10+1(\mathit{from}\text{ i})$

$=1100\lambda-100+1$

$=1100\lambda-99$

$=11(100\lambda-9)$ *is* divisible by 11

Hence by P.M.I. P (k+1) is true whenever P(k) is true.

Ans 04. $\mathit{let}P(n):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{n}\right)=\left(n+1\right)$

for n=1

P(1): $\left(1+\frac{1}{1}\right)=\left(1+1\right)=2$

which is true

let P(k) be true

P(k) : $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{k}\right)=\left(k+1\right)$

we want to prove that P(k+1) is true

P(k+1):$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)..\left(1+\frac{1}{k+1}\right)=\left(k+2\right)$

$L.H.S. = \left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)....\left(1+\frac{1}{k}\right)\left(1+\frac{1}{k+1}\right)$

$=(k+1)\left(1+\frac{1}{k+1}\right) \quad [from(1)]$

$= (k+1)\left(\frac{k+1+1}{K+1}\right)$

$=(K+2)$

thus P(k+1) is true whenever

P (K) is true.

Ans.05 $p(n):1.2+2.3+--n(n+1)=\frac{n(n+1)(n+2)}{3}$

for $n=1$

$p(1):1(1+1)=\frac{1(1+1)(1+2)}{3}$

$p(1)=2=2$

hence $p(1)$ be true

$p(k):1.2+2.3+---+k(k+1)=\frac{k(k+1)(k+2)}{3}.........(i)$

we want to prove that

$p(k+1):$

$1.2+2.3+---+(k+1)(k+2)=\frac{(k+1)(k+2)(k+3)}{3}$

$L.H.S.$

$=1.2+2.3+---+k(k+1)+(k+1)(k+2)$

$=\frac{k(k+1)(k+2)}{3}+\frac{(k+1)(k+2)}{1} \quad [from(i)]$

$\frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$

$\frac{(k+1)(k+2)[k+3]}{3}$

hence p$(k+1)$ is true whenenes $p(k)$ is true

Ans.06 $p(n):(2n+7)<(n+3)^2$

for $n=1$

$9<(4)^2$

$9 < 16$

which is true

let p(k) be true

$(2k+7) < (k+3)^2$

now

$2(k+1)+7 = (2k+7)+2$

$< (k+3)^2 + 2 = k^2 + 6k + 11$

$< k^2 + 8k + 16 = (k+4)^2$

$= (k+3+1)^2$

$\therefore\ p(k+1) : 2(k+1)+7 < (k+1+3)^2$

$\Rightarrow\ p(k+1)$ is true, when ever $p(k)$ is true

hence by *PMI* $p(k)$ is true for all $n \in N$

Ans.07 $p(n) : \frac{1}{1.4} + \frac{1}{4.7} + ---+ \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$

for $n = 1$

$p(1) : \frac{1}{(3-2)(3+1)} = \frac{1}{(3+1)} = \frac{1}{4}$

which is true

let $p(k)$ be true

$p(k) : \frac{1}{1.4} + \frac{1}{4.7} + ---+ \frac{1}{(3k-2)(3k+1)} = \frac{k}{(3k+1)}$.........$(i)$

we want to prove that p$(k+1)$ is true

$p(k+1) : \frac{1}{1.4} + \frac{1}{4.7} + ---+ \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{(3k+4)}$

L.H.S.

$= \frac{1}{1.4} + \frac{1}{4.7} + ---+ \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$

$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$ $\quad\quad [\textit{from}.......(i)]$

$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)} = \frac{\cancel{(3k+1)}(k+1)}{\cancel{(3k+1)}(3k+4)}$$

$p(k+1)$ is true whenever $p(k)$ is true.

Ans. 08 $p(n): 1.2+2.2^2+3.2^3+---+n.2^n=(n-1)2^{n+1}+2$

$p(n): 1.2+2.2^2+3.2^3+---+n.2^n=(n-1)2^{n+1}+2$

for $n=1$

$p(1): 1.2^1=(1-1)2^2+2$

$2=2$ which is true

let $p(k)$ be true

$p(k): 1.2+2.2^2+---+k.2^k=(k-1)2.^{k+1}+2..........(i)$

we want to prove that $p(k+1)$ is true

$p(k+1): 1.2+2.2^2+---+(k+1)2^{k+1}=k.2^{k+2}+2$

$L.H.S.$

$1.2+2.2^2+---+k.2^k+(k+1)2^{k+1}$ $\quad\left[\text{from}...........(i)\right]$

$=(k-1)2^{k+1}+2+(k+1)2^{k+1}$

$=2^{k+1}(k-\cancel{1}+k+\cancel{1})+2$

$=2^{k+2}.k+2$

This $p(k+1)$ is true whenever $p(k)$ is true

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

Topic: - Principle of mathematical Induction

1. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 $\forall$ $n \in N$ [4]

2. Prove that $41^n - 14^n$ is a multiple of 27 [4]

3. Using induction, prove that $10^n + 3.4^{n+2} + 5$ is divisible by 9 $\forall$ $n \in N$. [4]

4. Prove that $1^2 + 2^2 + 3^2 + \text{---} + n^2 = \frac{n(n+1)(2n+1)}{6}$ [4]

5. Prove that $1+3+3^2 + \text{--} + 3^{n-1} = \frac{3^n - 1}{2}$ [4]

6. By induction, prove that $1^2 + 2^2 + 3^2 + \text{--} + n^2 > \frac{n^3}{3} \forall$ $n \in N$ [4]

7. Prove by PMI $(ab)^n = a^n b^n$ [4]

8. Prove by PMI $a + ar + ar^2 + \text{----} + ar^{n-1} = \frac{a\left(r^n - 1\right)}{r - 1}$ [4]

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

[ANSWERS]

Topic: - Principle of mathematical Induction

Ans 01. $P(n) : 2.7^n + 3.5^n - 5$ is divisible by 24

for n = 1

$P(1) : 2.7^1 + 3.5^1 - 5 = 24$ is divisible by 24

Hence result is true for n = 1

Let P (K) be true

$P(K) : 2.7^K + 3.5^K - 5$

$\Rightarrow 2.7^K + 3.5^K - 5 = 24\lambda$ when $\lambda \in N$ (i)

we want to prove that P (K+!) is True whenever P (K) is true

$2.7^{K+1} + 3.5^{K+1} - 5 = 2.7^K .7^1 + 3.5^K . 5^1 - 5$

$$= 7\left[2.7^K + 3.5^K - 5 - 3.5^K + 5\right] + 3.5^K.5^1 - 5$$

$$= 7\left[24\lambda - 3.5^K + 5\right] + 15.5^K - 5 \text{ (from i)}$$

$$= 7\times 24\lambda - 21.5^K + 35 + 15.5^K - 5$$

$$= 7\times 24\lambda - 6.5^K + 30$$

$$= 7\times 24\lambda - 6(5^K - 5)$$

$$= 7\times 24\lambda - 6.4p$$

$$= 24(7\lambda - p), \quad 24 \text{ is divisible by } 24$$

Hence by P M I p (n) is true for all $n \in N$.

Ans 02. $P(n) : 41^n - 14^n$ is a multiple of 27

for n = 1

$P(1) : 41^1 - 14 = 27$, which is a multiple of 27

Let P (K) be True

$P(K) : 41^K - 14^K$

$\Rightarrow 41^K - 14^K = 27\lambda$, where $\lambda \in N$ (i)

we want to prove that result is true for n = K + 1

$41^{K+1} - 14^{k+1} = 41^K.41 - 14^K. 14$

$= (27\lambda + 14^K).41 - 14^K.14(\textit{ from } \text{i})$

$= 27\lambda.41 + 14^K.41 - 14^K.14$

$= 27\lambda.41 + 14^K(41 - 14)$

$= 27\lambda.41 + 14^K(27)$

$= 27(41\lambda + 14^K)$

is a multiple of 27

Hence by PMI p (n) is true for ace n ∈ N.

Ans 03. P (n) : $10^n + 3.4^{n+2} + 5$ is divisible by 9

For n = 1

p (1) : $10^1 + 3.4^{1+2} + 5 = 207$, divisible by 9

Hence result is true for n = 1

Let p (K) be true

p (K) : $10^K + 3.4^{K+2} + 5$ is divisible by 9

$\Rightarrow 10^K + 3.4^{K+2} + 5 = \lambda$ where $\lambda \in$ N (i)

we want to prove that result is true for n = K + 1

$10^{(K+1)} + 3.4^{K+1+2} + 5 = 10^{K+1} + 3.4^{K+3} + 5$

$= 10^K.10 + 3.4^K.4^3 + 5$

$= (9\lambda - 3.4^{K+2} - 5).10 + 3.4^K.4^3 + 5$ (from i)

$= 90\lambda - 30.4^{K+2} - 50 + 3.4^{K+3} + 5$

$= 90\lambda - 30.4^{K+2} - 45 + 3.4.\ 4^{K+2}$

$= 90\lambda - 18.4^{K+2} - 45$

$= 9(10\lambda - 2.4^{K+2} - 5)$

which is divisible by 9.

Ans 04. Let P(n) : $1^2 + 2^2 + 3^2 + \text{--} + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

for n = 1

$P(1): 1^2 = \dfrac{1(2)(3)}{6} = 1$

which is true

Let P (K) be true

$$P(K): 1^2 + 2^2 + -- + K^2 = \frac{K(K+1)(2K+1)}{6} \quad (1)$$

we want to prove that P(K+1) is true

$$P(K+1): 1^2 + 2^2 + -- + (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$$

$$L.H.S = \underbrace{1^2 \_+ 2^2 + --+ K^2} + (K+1)^2$$

$$= \frac{K(K+1)(2K+1)}{6} + \frac{(K+1)^2}{1} \qquad [\text{from } (1)$$

$$= \frac{K(K+1)(2K+1) + 6(K+1)^2}{6}$$

$$= \frac{(K+1)[K(2K+1) + 6(K+1)]}{6}$$

$$= \frac{(K+1)(2K^2 + K + 6K + 6)}{6}$$

$$= \frac{(K+1)(2K^2 + 7K + 6)}{6}$$

$$= \frac{(K+1)(K+2)(2K+3)}{6}$$

Thus P (K+1) is true, whenever P (K) is true.

Hence, from PMI, the statement P (n) is true for all natural no. n.

Ans 05. Let

$$P(n): 1 + 3 + 3^2 + -- + 3^{n-1} = \frac{3^n - 1}{2} \text{ for } n = 1$$

$$P(1)\ 3^{1-1} = \frac{3^1 - 1}{2} = 1$$

which is true

Let P (K) be true

$$P(K): 1 + 3 + 3^2 + ---- + 3^{K-1} = \frac{3^K - 1}{2} \quad (1)$$

we want to prove that P (K+1) is true

$$P(K+1): 1 + 3 + 3^2 + --- + 3^K = \frac{3^{K+1} - 1}{2}$$

L.H.S = $1+3+3^2+----+3^{K-1}+3K$

$$= \frac{3^K - 1}{2} + 3^K \qquad [\text{From } (1)$$

$$= \frac{3^K - 1 + 2.3^K}{2}$$

$$= \frac{3^K(1+2) - 1}{2}$$

$$= \frac{3^K.3 - 1}{2}$$

$$= \frac{3^{K+1} - 1}{2}$$

Hence p (K+1) is true whenever p (K) is True

Ans 06. Let P (n) : $1^2 + 2^2 + 3^2 + --- + n^2 > \frac{n^3}{3}$

for n = 1

$12 > \frac{1}{3}$ which is true

Let P (K) be true

P (K) : $1^2 + 2^2 + 3^2 + -- + K^2 > \frac{K^3}{3}$ (1)

we want to prove that P (K + 1) is true

P (K+1) : $1^2 + 2^2 + --- + (K+1)^2$

$$= 1^2 + 2^2 + -- + K^2 + (K+1)^2$$

$$> \frac{K^3}{3} + (K+1)^2$$

$$= \frac{1}{3}\left[K^3 + 3(K+1)^2\right]$$

$$= \frac{1}{3}\left[K^3 + 3K^2 + 3 + 6K\right]$$

$$= \frac{1}{3}\left[(K+1)^3 + (3K+2)\right]$$

$$> \frac{1}{3}(K+1)^3$$

$$\Rightarrow P(K+1) \text{is true}$$

Hence by PMI P (n) is true $\forall$ n $\in$ N

Ans 07. Let P (n) : $(ab)^n = a^n b^n$

for n = 1

ab = ab which is true

Let P (K) be true

$(ab)^K = a^K b^K$ (1)

we want to prove that P (K+1) is true

$(ab)^{K+1} = a^{K+1}. b^{K+1}$

L.H.S = $(ab)^{K+1}$

$$= (ab)^K.\ (ab)^1$$
$$= a^K\ b^K.\ (ab)^1 \text{ [from (1)}$$
$$= a^{K+1}.\ b^{K+1}$$
$$\Rightarrow P(K+1) \text{is true.}$$

Ans 08. Let P (n) : $a + ar + ar^2 + -- + ar^{n-1} = \dfrac{a(r^n - 1}{r-1}$

for n = 1

P (1) = a = a which is true

Let P (K) be true

P (K) : $a + ar + ar^2 + -- + ar^{K-1} = \dfrac{a(r^K - 1)}{r-1}$ (1)

we want to prove that

P (K+1) : $a + ar + ar^2 + -- + ar^K = \dfrac{a(r^{K+1} - 1)}{r-1}$

L.H.S $= \underbrace{a + ar + ar^2 + - - + ar^{K-1}} + ar^K$

$$= \frac{a\left(r^K - 1\right)}{r-1} + ar^K \quad \left[\text{ from (1)}\right.$$

$$= \frac{a\left(r^K\text{-}1\right)\text{+}ar^{k+1}\text{-}ar^K}{r\text{-}1}$$

$$= \frac{\cancel{ar^K}\text{-}a\text{+}ar^{k+1}\text{-}\cancel{ar^K}}{r\text{-}1} = \frac{a\left(r^{K+1}\text{-}1\right)}{r\text{-}1}$$

Thus P (K+1) is true whenever P(K) is true

Hence by PMI P(n) is true for all n ∈ N

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

Topic: - Principle of mathematical Induction

1. Prove that $x^{2n} - y^{2n}$ is divisible by x + y. [4]

2. Prove that n (n + 1) (2n + 1) is divisible by 6. [4]

3. Show that $2^{3n} - 1$ is divisible by 7 . [4]

4. Prove by P M I. [4]

   $1.\,2.\,3 + 2.\,3.\,4 + \text{---} + n\,(n+1)\,(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

5. Prove that $\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+.+\dfrac{1}{n(n+1)}=\dfrac{n}{n+1}$ [4]

6. Show that the sum of the first n odd natural no is n^2. [4]

7. Prove by P M I [4]

   $1^3+2^3+3^3+---+n^3=\left(\dfrac{n(n+1)}{2}\right)^2$

8. Prove. $\left(1+\dfrac{3}{1}\right)\left(1+\dfrac{5}{4}\right)\left(1+\dfrac{7}{9}\right)\text{----}+\left(1+\dfrac{(2n+1)}{n^2}\right)=(n+1)^2$ [4]

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

[ANSWERS]

Ans 01. P (n) : $x^{2n} - y^{2n}$ is divisible by x + y

for n = 1

p (1) : $x^2 - y^2 = (x - y)(x + y)$, which is divisible by x + y

Hence result is true for n = 1

Let P (K) be true

p (K) : $x^{2K} - y^{2K}$ is divisible by x + y

$\Rightarrow x^{2K} - y^{2K} = (x+y)\lambda$, where $\lambda \in N$ (i)

we want to prove the result is true for n = K + 1

$$x^{2(K+1)} - y^{2(K+1)} = x^{2K+2} - y^{2K+2}$$

$$= x^{2K}.x^2 - y^{2K}.y^2$$

$$= \left((x+y)\lambda + y^{2K}\right).x^2 - y^{2K}.y^2 (\textit{ from } i)$$

$$= (x+y)\lambda x^2 + x^2 y^{2K} - y^{2K}.y^2$$

$$= (x+y)\lambda x^2 + y^{2K}(x^2 - y^2)$$

$$= (x+y)\lambda x^2 + y^{2K}(x+y)(x-y)$$

$$= (x+y)\left[x^2\lambda + y^{2K}(x-y)\right] \text{ is divisible by } (x+y)$$

$\Rightarrow$ p (K+1) is true whenever p (K) is true

Hence by P.M.I, p (n) is true $\forall\, n \in N$

Ans 02. P (n) : n (n+1) (2n+1) is divisible by 6 for n = 1

P (1) : (1) (2) (3) = 6 is divisible by 6

Hence result is true for n = 1

Let P (K) be true

P (K) : K (K+1) (2K+1) is divisible by 6

$\Rightarrow K(K+1)(2K+1) = 6\lambda$ where $\lambda \in N$ (i)

we want to prove that result is true for n = K+1

$$(K+1)(K+2)(2k+3) = (K+1)(K+2)\left[(2K+1)+2\right]$$

$$= (K+1)(K+2)(2K+1) + 2(K+1)(K+2)$$

$= (K+2)\left[(K+1)(2K+1)\right]$ + 2 (K+1) (K+2)

= K (K+1) (2K+1) + 2 (K+1) (2K+1) + 2 (K+1) (K+2)

$= 6\lambda + 2(K+1)(2K+1) + 2(K+1)(K+2)$ (by i)

$= 6\lambda$ + 2(K+1) $\left[2K+1+K+2\right]$

$= 6\lambda + 2(K+1)(3K+3)$

$= 6\lambda + 6(K+1)(K+1)$

$= 6\left[\lambda + (K+1)(K+1)\right]$

is divisible by 6.

Ans 03. P (n) : $2^{3n} - 1$ is divisible by 7

for n = 1

P (1) : $2^3 - 1 = 7$ which is divisible by 7

Let P (K) be true

P (K) : $2^{3K} - 1$ is divisible by 7

$\Rightarrow$ $2^{3K}-1 = 7\lambda$ where $\lambda \in$ N (i)

we want to prove that P(K+1) is true whenever P(K) is true

$2^{3(K+1)} - 1 = 2^{3K+3} - 1$

$= 2^{3K}.\ 2^3 - 1$

$= (7\lambda+1).\ 8 - 1$(from i)

$= 56\lambda + 8 - 1$

$= 56\lambda + 7$

$= 7(8\lambda+1)$ which is divisible by 7

Thus P (K+1) is true

Hence by P.M.I P (n) is true $\forall$ n $\in$ N

Ans 04. Let P (n) : 1. 2. 3 + 2. 3. 4 + --- + n (n+1) (n+2) = $\dfrac{n(n+1)(n+2)(n+3)}{4}$

For n = 1

P (1) = 1 (2) (3) = $\dfrac{(1)(2)(3)(4)}{4}$

P (1) = 6 = 6 which is true

Let P (K) be true

P (K) : 1. 2. 3 + 2. 3. 4 + -- + K (K+1) (K+2) = $\frac{K(K+1)(K+2)(K+3)}{4}$ (1)

we want to prove that

P (K+1) n: 1. 2. 3 + 2. 3. 4 + -- + (K+1) (K+2) (K+3) = $\frac{(K+1)(K+2)(K+3)(K+4)}{4}$

L.H.S = 1. 2. 3 + 2. 3. 4 + -- + K (K+1) (K+2) + (K+1) (K+2) (K+3)

$$= \frac{K(K+1)(K+2)(K+3)}{4} + \frac{(K+1)(K+2)(K+3)}{1} \qquad [\text{from } (1)$$

$$= \frac{(K+1)(K+2)(K+3)[K+4]}{4},$$

Thus P (K+1) is true whenever P(K) is true.

Ans 05. P(n) : $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + . + \frac{1}{n(n+1)} = \frac{n}{n+1}$

For n = 1

P (1) = $\frac{1}{2} = \frac{1}{2}$ which is true

Let P (K) be true

$P(K): \frac{1}{1.2} + \frac{1}{2.3} + \text{ --- } + \frac{1}{K(K+1)} = \frac{K}{K+1}$ (1)

we want to prove that P (K+1) is true

P (K+1) : $\frac{1}{1.2} + \frac{1}{2.3} + \text{ --- } + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$

L.H.S = $\frac{1}{1.2} + \frac{1}{2.3} + \text{ --- } + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)}$

$$= \frac{K}{K+1} + \frac{1}{(K+1)(K+2)} \qquad [\text{from } (1)$$

$$= \frac{K(K+2)+1}{(K+1)(K+2)}$$

$$= \frac{K^2+2K+1}{(K+1)(K+2)} = \frac{(K+1)^{\cancel{2}}}{\cancel{(K+1)}(K+2)}$$

$$= \frac{K+1}{K+2},$$

Thus P (K+1) is for whenever P (K) is true.

Ans 06. Let P (n) : 1 + 3 + 5 + --- + (2n-1) = n^2

For n = 1

P (1) = 1 = 1 which is true

Let P (K) be true

P (K) : 1 + 3 + 5 + --- + (2K-1) = K^2 (1)

we want to prove that P (K+1) is true

P (K+1) : 1 + 3 + 5 + --- + (2K+1) = $(K+1)^2$

L.H.S = $\underbrace{1+3+5+--+(2K-1)}+(2K+1)$

$$= K^2 + 2K + 1 \qquad [\text{From (1)}$$

$$= (K+1)^2$$

Thus P (K+_1) is true whenever P(K) is true.

Hence by PMI, P(n) is true for all n $\in$ N.

Ans 07. P (n) : $1^3 + 2^3 + 3^3 + ---- + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

For n = 1

P (1) : $1^3 = 1^3$ which is true

Let P (K) be true

$$P(K): 1^3 + 2^3 + --+K^3 = \left(\frac{K(K+1)}{2}\right)^2 \qquad (1)$$

we want to prove that P (K+1) is true

P (K+1) : 1^3+2^3 + -- + $(K+1)^3$ = $\left(\frac{(K+1)(K+2)}{2}\right)^2$

$$L.H.S = \underbrace{1^3 + 2^3 + --+K^3} + (K+1)^3$$

$$= \left(\frac{K(K+1)}{2}\right)^2 + (K+1)^3 \qquad [\text{from (1)}$$

$$= \frac{K^2(K+1)^2}{4} + \frac{(K+1)^3}{1}$$

$$= \frac{K^2(K+1)^2 + 4(K+1)^3}{4}$$

$$= \frac{(K+1)^2\left[K^2 + 4(K+1)\right]}{4}$$

$$=\frac{(K+1)^2(K^2+4K+4)}{4}$$

$$=\frac{(K+1)^2(K+2)^2}{4}$$

$$=\frac{(K+1)(K+2)}{2}$$

Thus P (K+1) is true whenever P (K) is true.

Ans 08. P (n) : $\left(1+\frac{3}{1}\right)\ \left(1+\frac{5}{4}\right)$ +----+ $\left(1+\frac{(2n+1)}{n^2}\right)$ = $(n+1)^2$

For n = 1

P (1) : 4 = 4 which is true

Let P (K) be true

P (K) : $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)+---+\left(1+\frac{(2K+1)}{K^2}\right)=(K+1)^2 \quad (1)$

We want to power that P (K+1) is true

$$P(K+1):\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)+--+\left(1+\frac{(2K+3)}{(K+1)^2}\right)=(K+2)^2$$

$$L.H.S=\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)+---+\left(1+\frac{(2K+1)}{K^2}\right)+\left(1+\frac{(2K+3}{(K+1)^2}\right)$$

$$=(K+1)^2+\left(1+\frac{(2K+3}{(K+1)^2}\right) \qquad [\because \text{ from } (1)$$

$$=(K+1)^2\left[\frac{(K+1)^2+2K+3}{(K+1)^2}\right]$$

$$=\frac{(K+1)^2(K^2+4K+4)}{(K+1)^2}$$

$$=\frac{\cancel{(K+1)^2}(K+2)^2}{\cancel{(K+1)^2}}$$

$$=(K+2)^2$$

Thus P (K+1) is true whenever P (K) is true.

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

1. Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 [4]

2. Prove by PMI. [4]

$x^n - y^n$ is divisible by $(x-y)$ whenever $x-y \neq 0$

3. Prove $(x^{2n}-1)$ is divisible by $(x-1)$. [4]

4. Prove $1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+--+\frac{1}{(1+2+--+n)}=\frac{2n}{(n+1)}$ [4]

5. Prove $1.3 + 3.5 + 5.7 + -- + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$ [4]

6. Prove by PMI [4]

$3.2^2 + 3^2.2^3 + 3^3.2^4 + -- + 3^n.2^{n+1} = \frac{12}{5}(6^n-1)\, n \in N$.

7. Prove $1.3 + 2.3^2 + 3.3^3 + --- + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$ [4]

8. Prove $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+--+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$ [4]

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

[ANSWERS]

Ans 01. P(n) : $3^{2n+2} - 8n - 9$ is divisible by 8

For n = 1

P (1) : $3^{2+2} - 8 - 9 = 64$

which is divisible by 8

Hence result is true for n = 1

Let P (K) be true

P (K) : $3^{2K+2} - 8K - 9$ is divisible by 8

$\Rightarrow 3^{2K+2} - 8K - 9 = 8\lambda,$ where $\lambda \in$ N (i)

we want to prove that result is true for n = K+1

$3^{2(K+1)+2} - 8(K+1) - 9 = 3^{2K+2+2} - 8K - 8 - 9$

$= 3^{2K+4} - 8K - 17$

$= 3^{2K}.3^{4} - 8K - 17$

$= 3^{2k+2}.3^{2} - 8K - 17$

$= (8\lambda + 8K + 9).9 - 8K - 17$ (from i)

$= 72\lambda + 72K + 81 - 8K - 17$

$= 64\lambda + 64K + 64$

$= 8(8\lambda + 8K + 8)$

which is divisible by 8

Hence P(K+1) is true whwnever P (K) is true.

Hence by P.M.I P (n) is true $\forall$ n $\in$ N

Ans 02. P (n) : $x^{n}-y^{n}$ is divisible by (x-y)

For n = 1

P (1) : x – y is divisible by (x – y)

Let P (K) be true

P (K) : $x^{K} - y^{K}$ is divisible by (x – y)

$\Rightarrow x^{K} - y^{K} = \lambda(x-y)(i)$

we want to prove that P (K+1) is true whenever P (K) is true

$x^{K+1} - y^{K+1} = x^{K}.x - y^{K}.y$

$= (\lambda(x-y) + y^{K}).x - y^{K}.y$ (from i)

$$= \lambda(x-y).x + y^K.x - y^K y$$

$$= \lambda(x-y).x + y^K(x-y)$$

$$= (x-y)\left[\lambda x + y^K\right]$$

which is divisible by x-y

Hence P (K+1) is true

Ans 03. P (n) : $(x^{2n}-1)$ is divisible by (x-1).

For n = 1

P (1) : $(x^2 - 1) = (x - 1)(x + 1)$

which is divisible by (x – 1)

Let P (K) be true

$P(K):\left(x^{2K}-1\right)$ is divisible by x-1 (i)

$\Rightarrow$ $x^{2K}-1=\lambda(x-1)$

we want to prove that P (K+1) is true

P(K+1) : $x^{2(K+1)}-1$

L.H.S

$$=x^{2K+2}-1$$

$$=x^{2K}.x^2-1$$

$$=\left(\lambda(x-1)+1\right).x^2-1(\text{from i })$$

$$=\lambda(x-1).x^2+x^2-1$$

$$=\lambda\left(x-1\right).x^2+\left(x-1\right)\left(x+1\right)$$

$$=(x-1)\left[\lambda x^2+\left(x+1\right)\right]$$

which is divisible by (x-1)

Hence p(K+1) is true whenever p(k) is true

Ans 04. P (n) : $1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+--+\frac{1}{(1+2+--+n)}=\frac{2n}{(n+1)}$

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+---+\frac{1}{\frac{n(n+1)}{2}}=\frac{2n}{n+1}$$

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+---+\frac{2}{n(n+1)}=\frac{2n}{n+1}$$

for n = 1

P (1) : $\frac{2}{2}=\frac{2}{2}=1$

which is true

Let p(k) be true

$$p(k) : 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+---+\frac{2}{k(k+1)}=\frac{2k}{k+1} \quad (1)$$

we want to prove that p (k + 1) is true

$$p(k+1) : 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+---+\frac{2}{(k+1)(k+2)}=\frac{2(k+1)}{k+2}$$

$$L.H.S = 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+---+\frac{2}{k(k+1)}=\frac{2}{(k+1)(k+2)}$$

$$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)} \qquad [\text{from (1)}$$

$$=\frac{2k(k+2)+2}{(k+1)(k+2)}$$

$$=\frac{2k^2+4k+2}{(k+1)(k+2)}$$

$$=\frac{2(k^2+2k+1)}{(k+1)(k+2)}$$

$$=\frac{2(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)}$$

$$=\frac{2(k+1)}{(k+2)}$$

thus p (k+1) is true whenever p(k) is true

Hence by PMI p(n) is true $\forall$ n $\in$ N.

Ans 05. Let p (n) : 1.3 + 3.5 + -- + (2n-1) (2n+1) = $\frac{n(4n^2+6n-1)}{3}$

For n = 1

P (1) = (1) (3) = $\frac{1(4+6-1)}{3}$

P(1) = 3 = 3 Hence p (1) is true

Let (k) be true

P(k) : 1.3 + 3.5 + -- + (2k - 1) (2K + 1) = $\frac{k(4k^2+6k-1)}{3}$ (1)

we want to prove that p (k+1) is true

p (k+1) : 1.3 + 3.5 + -- + (2k+1) (2k+3) = $\frac{(k+1)\left[4(k+1)^2+6(k+1)-1\right]}{3}$

L. H. S

$$1.3+3.5+--+(2k-1)(2k+1)+(2k+1)(2k+3)$$

$$=\frac{k(4k^2+6k-1)}{3}+\frac{(2k+1)(2k+3)}{1} \qquad [\text{from (1)}$$

$$=\frac{k\left(4k^2+6k-1\right)+3(2k+1)(2k+3)}{3}$$

$$=\frac{4k^3+18k^2+23k+9}{3}[\text{put k = -1 (k+1) is one fator}]$$

$$=\frac{(k+1)\left(4k^2+14k+9\right)}{3}$$

Thus p (k+1) is true whenever p (k) is true.

Ans 06. Let p (n) : $3.2^2 + 3^2.2^3 + 3^3.2^4 + --- + 3^n.\ 2^{n+1} = \frac{12}{5}\left(6^n-1\right)$

For n = 1

$$p(1): 3^1.2^2=\frac{12}{5}\left(6^1-1\right)$$

$$p(1)=12=12$$

p(1) is true

Let p(k) be true

$$\text{p(k)}:3.2^2+3^2.2^3+\text{--}+3^k.\ 2^{k+1}=\frac{12}{5}\left(6^k-1\right) \quad (1)$$

we want to prove that p (k+1) is true

$$\text{p(k+1)}:\ 3.2^2+3^2.2^3\ +\text{---}+\ 3^{k+1}.\ 2^{k+2}\ =\ \frac{12}{5}\left(6^{k+1}-1\right)$$

$$L.H.S=3.2^2+3^2.2^3+--+3^k.2^{k+1}+3^{k+1}.2^{k+2}$$

$$=\frac{12}{5}\left(6^k-1\right)+3^{k+1}.2^{k+2} \qquad [\text{from (1)}$$

$$=\frac{2}{5}.6.6^k-\frac{12}{5}+3^{k+1}.2^{k+1}.2^1$$

$$=\frac{2}{5}6^{k+1}-\frac{12}{5}+6^{k+1}.2$$

$$=6^{k+1}\left(\frac{2}{5}+2\right)-\frac{12}{5}$$

$$=6^{k+1}\left(\frac{12}{5}\right)-\frac{12}{5}=\frac{12}{5}\left[6^{k+1}-1\right]$$

Thus p(k+1) is truewhenever p(k) is true.

Ans 07. P (n) : $1.3 + 2.3^2 + 3.3^3 + --- + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$

For n = 1

P (1) : $1.\ 31 = \frac{(2-1).3^2+3}{4}$

$$p(1): 3 = \frac{\cancel{12}\ 3}{\cancel{4}}$$

hence p(1) is true

Let p(k) be true

p(k) : $1.3+2.3^2+3.3^3+--+k.3^k = \frac{(2k-1)3^{k+1}+3}{4}$ (1)

we want to prove that p (k+1) is true

p(k+1) : $1.3+2.3^2+3.3^3+--+(k+1).3^{k+1} = \frac{(2k+1)3^{k+2}+3}{4}$

$$L.H.S = 1.3+2.3^2+3.3^3+--+k.3^k+(k+1).3^{k+1}$$

$$= \frac{(2k-1).3^{k+1}+3}{4} + \frac{(k+1)3^{k+1}}{1} \quad \text{[from (1)}$$

$$= \frac{(2k-1).3^{k+1}+3+4(k+1)3^{k+1}}{4}$$

$$= \frac{(2k-1+4k+4).3^{k+1}+3}{4}$$

$$= \frac{(6k+3).3^{k+1}+3}{4}$$

$$= \frac{3(2k+1).3^{k+1}+3}{4}$$

$$= \frac{(2k+1)3^{k+2}+3}{4}$$

Thus p (k+1) is true whenever p(k) is true.

Ans 8. P(n) : $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+--+\frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

For n = 1

$$p(1): \frac{1}{(2+1)(2+3)} = \frac{1}{3(2+3)}$$

$p(1) = \frac{1}{.15} = \frac{1}{15}$ Hence p (1) is true

Let p (k) be true

p(k) : $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+--+\frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$ (1)

we want to prove that p (k+1) is true

p(k+1): $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+--+\frac{1}{(2k+3)(2k+5)}=\frac{(k+1)}{3(2k+5)}$

$$L.H.S=\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+--+\frac{1}{(2k+1)(2k+3)}+\frac{1}{(2k+3)(2k+5)}$$

$$=\frac{k}{3(2k+3)}+\left(\frac{1}{(2k+3)}\right)\left(\frac{1}{(2k+5)}\right) \quad \text{[from (1)}$$

$$=\frac{k(2k+5)+3}{3(2k+3)(2k+5)}$$

$$=\frac{k+1}{3(2k+5)}$$

Thus p (k+1) is true whenever p (k) is true

Hence p (n) is true for all n $\in$ N.

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

1. The sum of the cubes of three consecutive natural no. is divisible by 9. [4]

2. Prove that $12^n + 25^{n-1}$ is divisible by 13 [4]

3. Prove $11^{n+2} + 12^{2n+1}$ is divisible by 133. [4]

4. Prove $1^3 + 2^3 + 3^3 + --- + n^3 = \frac{n^2(n+1)^2}{4}$ [4]

5. Prove $(a) + (a + d) + (a + 2d) + -- + [a + (n - 1)d] = \frac{n}{2}[2a+(n-1)d]$ [4]

6. Prove that $2^n > n \ \forall$ positive integers n. [4]

7. Prove $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+---+\frac{1}{n(n+1)}=\frac{n}{n+1}$ [4]

8. Prove $\frac{1}{3.6}+\frac{1}{6.9}+\frac{1}{9.12}+---+\frac{1}{3n(3n+3)}=\frac{n}{9(n+1)}$. [4]

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Principle of mathematical Induction)

[ANSWERS]

Ans 01. P(n) $\left[k^3+(k+1)^3+(k+2)^3\right]$ is divisible by 9

For n = 1

P (1) : 1 + 8 + 9 = 18

which is divisible by 9

Let p (k) be true

$p(k):\left[k^3+(k+1)^3+(k+2)^3\right]$ is divisible by 9

$\Rightarrow k^3+(k+1)^3+(k+2)^3=9\lambda(i)$

we want to prove that p (k+1) is true

p(k+1) : $(k+1)^3+(k+2)^3+(k+3)^3$

L.H.S = $(k+1)^3+(k+2)^3+(k+3)^3$

$$=(k+1)^3+(k+2)^3+k^3+9k^2+27k+27$$

$$=\underbrace{k^3+(k+1)^3+(k+2)^3}+9(k^2+3k+3)$$

$$=9\lambda+9(k^2+3k+3) \text{ (from i)}$$

$$=9\left[\lambda+(k^2+3k+3)\right] \text{ which is } \div \text{ by 9.}$$

Ans 02. P(n) : 12^n+25^{n-1} is divisible by 13

For n = 1

P(1) : $12+(25)^0=13$

which is divisible by 13

Let p (k) be true

P(k) : 12^k+25^{k-1} is divisible by 13

$\Rightarrow 12^k+25^{k-1}=13\lambda(i)$

we want to prove that result is true for n = k+1

$12^{(k+1)}+25^{k+\cancel{1}\cancel{-1}}=12^k.12^1+25^k$

$$=\left(13\lambda-25^{k-1}\right).12+25^k(\textit{from } i)$$

$$=13\times12\lambda-25^{k-1}.12+25^k$$

$$=13\times12\lambda+25^{k-1}\left(-12+25\right)$$

$$=13\left(12\lambda+25^{k-1}\right)$$

which is divisible by 13.

Ans 03. P(n) : $11^{n+2} + 12^{2n+1}$ is divisible by 133.

For n = 1

P(1) : $11^3 + 12^3 = 3059$

which is divisible by 133

Let p (k) be true

$p(k): 11^{k+2} + 12^{2k+1}$ is divisible by 133

$\Rightarrow 11^{k+2} + 12^{2k+1} = 133\lambda$ (i)

we want to prove that

result is true for n = k+1

L.H.S = $11^{k+1+2} + 12^{2(k+1)+1}$

$=11^{k+3}+12^{2k+2+1}$

$=11^{k+3}+12^{2k+3}$

$=11^{k}.11^{3}+12^{2k}.12^{3}$

$=11^{k+2}.11+12^{2k}.12^{3}$

$=\left(133\lambda - 12^{2k+1}\right).11+12^{2k}.12^{3}$ (from i)

$=133\times11\lambda\text{-}12^{2k+1}.11+12^{2k}.12^{3}$

$=133\times11\times\lambda+12^{k}(-12\times11+12^{3})$

$=133\left[(11\lambda+12^{k}(1596)\right]$

which is $\div$ 133.

Ans 04. P(n) : $1^3 + 2^3 + 3^3 + \text{---} + n^3 = \dfrac{n^2(n+1)^2}{4}$

For n = 1

$p(1): 1^3 = \dfrac{1^2(2)^2}{4} = 1$

which is true

Let p(k) be true

p(k) : $1^3 + 2^3 + \text{---} + k^3 = \dfrac{k^2(k+1)^2}{4}$ (1)

we want to prove that p (k+1) is true

p(k+1) : $1^3 + 2^3 + \text{---} + (k+1)^3 = \dfrac{(k+1)^2(k+2)^2}{4}$

L.H.S = $1^3+2^3+\text{---}+k^3+(k+1)^3$ [from (1)

$$=\frac{k^2(k+1)^2}{4}+\frac{(k+1)^3}{1}$$

$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2[k^2 + 4(k+1)}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}$$

Thus p(k+1) is true whenever p(k) is true.

Ans 05. P(n) : (a)+ (a + d) + (a + 2d) + -- + [a + (n – 1)d] = $\frac{n}{2}\left[2a+(n-1)d\right]$

For n = 1

p(1) : a + (1-1) d = $\frac{1}{2}$ 2a + (1-1) d = a

which is true

Let p (k) be true

$$p(k):(a)+(a+d)+(a+2d)+--+\left(a+\left(k-1\ \right)d\right)=\frac{k}{2}\left[2a+(k-1)d\right] \quad (1)$$

we want to prove that p (k+1) is true

$$p(k+1) : (a) + (a+d) + -- + (a+kd) = \frac{k+1}{2}\left[2a+kd\right]$$

L.H.S = a + (a+d)+--+ a+kd

$= a + (a + d) + -- + a + (k-1)d + a + kd$

$=\frac{k}{2}\left[2a+(k-1)d\right]+a+kd$ [from (1)

$= ka + \frac{k}{2}(k-1)d+a+kd$

$$= \frac{2ak+k^2d-kd+2a+2kd}{2}$$

$$= \frac{2a(k+1)+kd(k+1)}{2} = \frac{(k+1)(2a+kd)}{2}$$

proved.

Ans 06. Let p (n) : $2^n > n$

For n = 1

P (1) : $2^1 > 1$

Which is true

Let p (k) be true

P (k) : $2^k > k$ (1)

we want to prove that p (k+1) is true

$2^k > k$ by (1)

$\Rightarrow\ 2^k.2 > 2k$

$2^{k+1} > 2k$

$2^{k+1} > 2k = k+k > k+1$

Hence provtd.

Ans 07. P(n) : $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+---+\frac{1}{n(n+1)}=\frac{n}{n+1}$

For n = 1

$p(1)=\frac{1}{2}=\frac{1}{2}$ which is true

Let p (k) be true

p(k) : $\frac{1}{1.2}+\frac{1}{2.3}+--+\frac{1}{k(k+1)}=\frac{k}{k+1}$ (1)

we want to prove that p (k+1) is true

p(k+1) : $\frac{1}{1.2}+\frac{1}{2.3}+--+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

L.H.S = $\frac{1}{1.2}+\frac{1}{2.3}+--+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$ [from (1)

$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$

$=\frac{k(k+2)+1}{(k+1)(k+2)}$

$=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^{\cancel{2}}}{(\cancel{k+1})(k+2)}$

$=\frac{k+1}{k+2}$

proved.

Ans 08. P(n) : $\frac{1}{3.6}+\frac{1}{6.9}+\frac{1}{9.12}+---+\frac{1}{3n(3n+3)}=\frac{n}{9(n+1)}$.

For n = 1

$p(1):\frac{1}{3(6)}=\frac{1}{9(2)}=\frac{1}{18}$ which is true

Let p(k) be true

p(k) : $\frac{1}{3.6}+\frac{1}{6.9}+---+\frac{1}{3k(3k+3}=\frac{k}{9(k+1)}$ (1)

we want to prove that p (k+1) is true

$$p(k+1): \frac{1}{3.6}+\frac{1}{6.9}+---+\frac{1}{3(k+1)(3k+6)}=\frac{k+1}{9(k+2)}$$

$$L.H.S=\frac{1}{3.6}+\frac{1}{6.9}+---+\frac{1}{3k(3k+3)}+\frac{1}{3(k+1)(3k+6)}$$

$$=\frac{\mathrm{k}}{9(\mathrm{k}+1)}+\frac{1}{3(\mathrm{k}+1)3(\mathrm{k}+2)} \qquad [\text{from } (1)$$

$$=\frac{\mathrm{k}(\mathrm{k}+2)+1}{9(\mathrm{k}+1)(\mathrm{k}+2)}$$

$$=\frac{\mathrm{k}^2+2\mathrm{k}+1}{9(\mathrm{k}+1)(\mathrm{k}+2)}$$

$$=\frac{(\mathrm{k}+1)^{\cancel{2}}}{9\cancel{(\mathrm{k}+1)}(\mathrm{k}+2)}$$

$$=\frac{\mathrm{k}+1}{9(\mathrm{k}+2)}$$

proved.

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

Topic: - Quadratic Equations

1. Evaluate i^{-39} [1]

2. Solved the quadratic equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$ [1]

3. If $\left(\frac{1+i}{1-i}\right)^m$ = 1, then find the least positive integral value of m. [1]

4. Evaluate $(1+ i)^4$ [1]

5. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ [1]

6. If x + ί y = $\frac{a+ib}{a-ib}$ Prove that $x^2 + y^2 = 1$ [4]

7. Find real θ such that $\frac{3+2i\ \text{Sin}\theta}{1-2i\ \text{Sin}\theta}$ is purely real. [4]

8. Find the modulus of $\frac{(1+i)(2+i)}{3+i}$ [4]

9. If $|a+ib| = 1$, then Show that $\frac{1+b+ai}{1+b-ai} = b+ai$ [4]

10. If z = x + i y and w = $\frac{1-i^2}{z-i}$ Show that |w| = 1 $\Rightarrow$ z is purely real. [6]

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

[ANSWERS]

Topic: - Quadratic Equation

Ans 01. $i^{-39}=\frac{1}{i^{39}}=\frac{1}{(i^4)^9.i^3}$

$=\frac{1}{1\times(-i)}\qquad \left[\begin{array}{l}\because\ i^4=1\\ \ \ i^3=-i\end{array}\right.$

$=\frac{1}{-i}\times\frac{i}{i}$

$=\frac{i}{-i^2}=\frac{i}{-(-1)}=i\qquad [\because i^2=-1$

Ans 02. $\frac{x^2}{1}+\frac{x}{1}+\frac{1}{\sqrt{2}}=0$

$\frac{\sqrt{2}x^2+\sqrt{2}x+1}{\sqrt{2}}=\frac{0}{1}$

$\sqrt{2}x^2+\sqrt{2}x+1=0$

$x=\frac{-b\pm\sqrt{D}}{2a}$

$=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}$

$=\frac{-\sqrt{2}\pm\sqrt{2}\sqrt{1-2\sqrt{2}}}{2\sqrt{2}}$

$=\frac{-1\pm\sqrt{2\sqrt{2}-1}\ i}{2}$

Ans 03. $\left(\frac{1+i}{1-i}\right)^m=1$

$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m=1$

$$\left(\frac{1+i^2+2i}{1-i^2}\right)^m = 1$$

$$\left(\frac{\cancel{1}-\cancel{1}+2i}{2}\right)^m = 1 \qquad \left[\because i^2 = -1\right.$$

$$i^m = 1$$

m=4

Ans 04. $(1+i)^4 = \left[(1+i)^2\right]^2$

$$= \left(1+i^2+2i\right)^2$$

$$= (1-1+2i)^2$$

$$= (2i)^2 = 4i^2$$

$$= 4(-1) = -4$$

Ans 05. Let z = $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

$$= \frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$= \frac{4i}{2}$$

$$= 2i$$

$$z = 0+2i$$

$$|z| = \sqrt{(0)^2+(2)^2}$$

$$= 2$$

Ans 06. $x+iy = \frac{a+ib}{a-ib}$ (i) (Given)

taking conjugate both side

$x-iy = \frac{a-ib}{a+ib}$ (ii)

(i) × (ii)

$$(x+iy)(x-iy) = \left(\frac{a+ib}{a-ib}\right) \times \left(\frac{a-ib}{a+ib}\right)$$

$$(x)^2 - (iy)^2 = 1$$

$$\boxed{x^2 + y^2 = 1} \qquad [i^2 = -1$$

Ans 07. $\frac{3+2i\ Sin\theta}{1-2i\ Sin\theta}=\frac{3+2i\ Sin\theta}{1-2i\ Sin\theta}\times\frac{1+2i\ Sin\theta}{1+2i\ Sin\theta}$

$$=\frac{3+6i\ Sin\theta+2i\ Sin\theta-4Sin^2\theta}{1+4Sin^2\theta}$$

$$=\frac{3-4\ Sin^2\theta}{1+4\ Sin^2\theta}+\frac{8i\ Sin\theta}{1+4\ Sin^2\theta}$$

For purely real

Im (z) = 0

$$\frac{8Sin\theta}{1+4Sin^2\theta}=0$$

$$Sin\theta=0$$

$$\theta=n\pi$$

Ans 08. $\left|\frac{(1+i)(2+i)}{3+i}\right|=\frac{|(1+i)||2+i|}{|3+i|}$

$$=\frac{\left(\sqrt{1^2+1^2}\right)\left(\sqrt{4+1}\right)}{\sqrt{(3)^2+(1)^2}}$$

$$=\frac{\left(\sqrt{2}\right)\left(\sqrt{5}\right)}{\sqrt{10}}$$

$$=\frac{\sqrt{2}\times\sqrt{5}}{\sqrt{2}\times\sqrt{5}}$$

$$=1$$

Ans 09. $|a+ib|=1$

$$\sqrt{a^2+b^2}=1$$

$$a^2+b^2=1$$

$$\frac{1+b+ai}{1+b-ai}=\frac{(1+b)+ai}{(1+b)-ai}\times\frac{(1+b)+ai}{(1+b)+ai}$$

$$=\frac{(1+b)^2+(ai)^2+2(1+b)(ai)}{(1+b)^2-(ai)^2}$$

$$=\frac{1+b^2+2b-a^2+2ai+2abc}{1+b^2+2a-a^2}$$

$$=\frac{\left(a^2+b^2\right)+b^2+2b-a^2+2ai+2abi}{\left(a^2+b^2\right)+b^2+2b-a^2}$$

$$=\frac{2b^2+2b+2ai+2abi}{2b^2+2b}$$

$$= \frac{b^2 + b + ai + abi}{b^2 + b}$$

$$= \frac{b(b+1) + ai(b+1)}{b(b+1)}$$

$$= b + ai$$

Ans 10. w $= \frac{1 - iz}{z - i}$

$$= \frac{1 - i(x + iy)}{x + iy - i}$$

$$= \frac{1 - ix - i^2 y}{x + i(y-1)}$$

$$= \frac{(1+y) - ix}{x + i(y-1)}$$

$\therefore \quad |w| = 1$

$$\Rightarrow \left| \frac{(1+y) - ix}{x + i(y-1)} \right| = 1$$

$$\frac{|(1+y) - ix|}{|x + i(y-1)|} = 1$$

$$\frac{\sqrt{(1+y)^2 + (-x)^2}}{\sqrt{x^2 + (y-1)^2}} = 1$$

$$1 + y^2 + 2y + x^2 = x^2 + y^2 + 1 - 2y$$

$$4y = 0$$

$$y = 0$$

$\therefore$ z = x + a i

is purely real

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

Topic: - Quadratic Equation

1. Express in the form of a + ib. $(1+3i)^{-1}$ [1]

2. Explain the fallacy in $-1 = i.\ i. = \sqrt{-1}.\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$ [1]

3. Find the conjugate of $\frac{1}{2-3i}$ [1]

4. Find the conjugate of – 3i – 5. [1]

5. Let $z_1 = 2 - i$, $z_2 = -2+i$ Find Re $\left(\frac{z_1 z_2}{\overline{z_1}}\right)$ [1]

6. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ Prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$ [4]

7. If $a+ib = \frac{c+i}{c-i}$, where a, b, c are real prove that $a^2+b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$ [4]

8. If $z_1 = 2-i$ and $Z_2 = 1+i$ Find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ [4]

9. If $(p + iq)^2 = x + iy$ Prove that $(p^2 + q^2)^2 = x^2 + y^2$ [4]

10. Convert into polar form $\frac{-16}{1+i\sqrt{3}}$ [6]

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

[ANSWERS]

Topic: - Quadratic Equation

Ans 01. $(1+3i)^{-1} = \frac{1}{1+3i} \times \frac{1-3i}{1-3i}$

$= \frac{1-3i}{(1)^2-(3i)^2}$

$= \frac{1-3i}{1-9i^2}$

$= \frac{1-3i}{1-9} \quad \left[i^2 = -1 \right.$

$= \frac{1-3i}{-8}$

$= \frac{1}{-8} + \frac{3i}{8}$

Ans 02 $1 = \sqrt{1} = \sqrt{(-1)(-1)}$ is okay but

$\sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1}$ is wrong.

Ans 03. Let z = $\frac{1}{2-3i}$

$z = \frac{1}{2-3i} \times \frac{2+3i}{2+3i}$

$= \frac{2+3i}{(2)^2-(3i)^2}$

$= \frac{2+3i}{4+9}$

$= \frac{2+3i}{13}$

$z = \frac{2}{13} + \frac{3}{13}i$

$\bar{z} = \frac{2}{13} - \frac{3}{13}i$

Ans 04. Let z = 3i – 5

$$\overline{z} = 3i - 5$$

Ans 05. $z_1 z_2 = (2 - i)(-2 + i)$

$$= -4 + 2i + 2i - i^2$$

$$= -4 + 4i + 1$$

$$= 4i - 3$$

$$\overline{z_1} = 2 + i$$

$$\frac{z_1 z_2}{\overline{z}_1} = \frac{4i-3}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{8i - 6 - 4i^2 + 3i}{4 - i^2}$$

$$= \frac{11i - 2}{5}$$

$$\frac{z_1 z_2}{z_1} = \frac{11}{5} i - \frac{2}{5}$$

$$\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = -\frac{2}{5}$$

Ans 06. $x - iy = \sqrt{\frac{a-ib}{c-id}}$ (1) (Given)

Taking conjugate both side

$x + iy = \sqrt{\frac{a+ib}{c+id}}$ (ii)

(i) × (ii)

$$(x-iy) \times (x+iy) = \sqrt{\frac{a-ib}{c-id}} \times \sqrt{\frac{a+ib}{c+id}}$$

$$(x)^2 - (iy)^2 = \sqrt{\frac{(a)^2 - (ib)^2}{(c)^2 - (id)^2}}$$

$$x^2 + y^2 = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$$

squaring both side

$$\boxed{\left(x^2 + y^2\right)^2 = \frac{a^2 + b^2}{c^2 + d^2}}$$

Ans 07. $a+ib=\frac{c+i}{c-i}$ (Given) (i)

$$a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}$$

$$a+ib=\frac{c^2+2ci+i^2}{c^2-i^2}$$

$$a+ib=\frac{c^2-1}{c^2+1}+\frac{2c}{c^2+1}i$$

$$a=\frac{c^2-1}{c^2+1},\ \text{b}=\frac{2c}{c^2+1}$$

$$a^2+b^2=\left(\frac{c^2-1}{c^2+1}\right)^2+\frac{4c^2}{\left(c^2+1\right)^2}$$

$$=\frac{\left(c^2+1\right)^2}{\left(c^2+1\right)^2}$$

$$\boxed{a^2+b^2=1}$$

$$\frac{a}{b}=\frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}} \qquad \boxed{\frac{a}{b}=\frac{2c}{c^2-1}}$$

Ans 08. $z_1 + z_2 + 1 = 2 - i + 1 + i + 1 = 4$

$$z_1-z_2+i=2-i-1-i+i=1-i$$

$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|=\left|\frac{4}{1-i}\right|$$

$$=\frac{|4|}{|1-i|}$$

$$=\frac{4}{\sqrt{1^2+(-1)^2}}$$

$$=\frac{4}{\sqrt{2}}$$

$$=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{4\sqrt{2}}{2}$$

$$=2\sqrt{2}$$

Ans 09. $(p + iq)^2 = x + iy$ (i)

Taking conjugate both side

$(p - iq)^2 = x - iy$ (ii)

(i) × (ii)

$$\left(p+iq\right)^2\left(p-iq\right)^2 = \left(x+iy\right)\left(x-iy\right)$$

$$\left[\left(p+iq\right)\left(p-iq\right)\right]^2 = (x)^2-(iy)^2$$

$$\left[(p)^2-(iq)^2\right]^2 = x^2-i^2y^2 \qquad \boxed{\left(p^2+q^2\right)^2 = x^2+y^2}$$

Ans 10. $$\frac{-16}{1+i\sqrt{3}} = \frac{-16}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$=\frac{-16\left(1-i\sqrt{3}\right)}{(1)^2-\left(i\sqrt{3}\right)^2}$$

$$=\frac{-16\left(1-i\sqrt{3}\right)}{1+3}$$

$$=-4\left(1-i\sqrt{3}\right)$$

$$z=-4+i4\sqrt{3}$$

$$r = \ |z| = \sqrt{(-4)^2+\left(4\sqrt{3}\right)^2}$$

$$=\sqrt{16+48}$$

$$=\sqrt{64}$$

$$=8$$

Let α be the acute ∠S

$$\tan\ \alpha = \left|\frac{\cancel{4}\sqrt{3}}{-\cancel{4}}\right|$$

$$\tan\alpha = \tan\frac{\pi}{3}$$

$$\alpha = \pi/3$$

Since Re (z) < o, and Im (z) > o

$$\theta = \pi-\alpha$$

$$=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

$$z = 8\left(Cos\ \frac{2\pi}{3}+i\ \text{Sin}\ \frac{2\pi}{3}\right)$$

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

Topic: - Quadratic Equation

1. Express in the form of a + ib (3i-7) + (7-4i) – (6+3i) + i^{23} [1]

2. Find the conjugate of $\sqrt{-3}+4i^2$ [1]

3. Solve for x and y 3x + (2x-y) i= 6 – 3i [1]

4. Find the value of $1+i^2 + i^4 + i^6 + i^8 + ---- + i^{20}$ [1]

5. Multiply 3-2i by its conjugate. [1]

6. If $a+ib=\dfrac{(x+i)^2}{2x^2+1}$ Prove that $a^2+b^2 = \dfrac{(x^2+1)^2}{(2x^2+1)^2}$ [4]

7. If $(x+iy)^3=u+iv$ then show that $\dfrac{u}{x}+\dfrac{v}{y}=4(x^2-y^2)$ [4]

8. Solve $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$ [4]

9. Find the modulus $i^{25}+(1+3i)^3$ [4]

10. Find two numbers such that their sum is 6 and the product is 14. [6]

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

[ANSWERS]

Topic: - Quadratic Equation

Ans 01. Let

$Z = \cancel{3}\cancel{i} - \cancel{7} + \cancel{7} - 4i - 6 - \cancel{3}\cancel{i} + \left(i^4\right)^5 .i^3$

$= -4i - 6 - i \qquad \left[\because i^4 = 1, \; i^3 = -i\right.$

$= -5i - 6$

$= -6 + \left(-5i\right)$

Ans 02. Let z = $\sqrt{-3} + 4i^2$

$= \sqrt{3}\ i - 4$

$\bar{z} = -\sqrt{3}\ i - 4$

Ans 03. 3x = 6

$\boxed{x = 2}$

2x – y = - 3

2 × 2 – y = - 3

- y = - 3 – 4

$\boxed{y = 7}$

Ans 04. $1 + i^2 + (i^2)^2 + (i^2)^3 + (i^2)^4 + ----+(i^2)^{10} = 1 \qquad \left[\because i^2 = -1\right.$

Ans 05. Let z = 3 – 2i

$\bar{z} = 3 + 2i$

$z\ \bar{z} = \left(3 - 2i\right)\left(3 + 2i\right)$

$= 9 + \cancel{6i} - \cancel{6i} - 4i^2$

= 9 - 4 (-1)

= 13

Ans 06. $a+ib=\frac{(x+i)^2}{2x^2+1}$ (i) (Given)

Taking conjugate both side

$a-ib=\frac{(x-i)^2}{2x^2+1}$ (ii)

(i) $\times$ (ii)

$$(a+ib)(a-ib)=\left(\frac{(x+i)^2}{2x^2+1}\right)\times\left(\frac{(x-i)^2}{2x^2+1}\right)$$

$$(a)^2-(ib)^2=\frac{\left(x^2-i^2\right)^2}{\left(2x^2+1\right)^2}$$

$$a^2+b^2=\frac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2} \quad \text{proved.}$$

Ans 07. $(x+iy)^3=4+iv$

$$x^3+(iy)^3+3x^2(iy)+3.x(iy)^2=u+iv$$

$$x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$x^3-3xy^2+\left(3x^2y-y^3\right)i=u+iv$$

$$x\left(x^2-3y^2\right)+y\left(3x^2-y^2\right)=u+iv$$

$$x\left(x^2-3y^2\right)=u,\ \mathrm{y}\left(3x^2-y^2\right)=v$$

$$x^2-3y^2=\frac{u}{x}\ \text{(i)}\ \Big|\ 3x^2-y^2=\frac{v}{y}\ \text{(ii)}$$

(i) + (ii)

$$4\mathrm{x}^2\text{-}4\mathrm{y}^2=\frac{u}{x}+\frac{v}{y}$$

$$\boxed{4\left(x^2-y^2\right)=\frac{u}{x}+\frac{v}{y}}$$

Ans 08. $\sqrt{3}\,x^2-\sqrt{2}\,\mathrm{x}+3\sqrt{3}=0$

a = $\sqrt{3},b=-\sqrt{2},c=3\sqrt{3}$

$$D=b^2-4ac$$

$$=\left(-\sqrt{2}\right)^2-4\times\sqrt{3}\left(3\sqrt{3}\right)$$

$$=2-36$$

$$= -34$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{34}\ \mathrm{i}}{2\sqrt{3}}$$

Ans 09. $i^{25} + (1+3i)^3$

$$= \left(i^4\right)^6 .i + 1 + 27i^3 + 3(1)(3i)(1+3i)$$

$$= i + \left(1 - 27i + 9i + 27i^2\right)$$

$$= i + 1 - 18i - 27$$

$$= -26 - 17i$$

$$\left|i^{25} + \left(1+3i\right)^3\right| = \left|-26 - 17i\right|$$

$$= \sqrt{\left(-26\right)^2 + \left(-17\right)^2}$$

$$= \sqrt{676 + 289}$$

$$= \sqrt{965}$$

Ans 10. Let x and y be the no.

x + y = 6

xy = 14

$$x^2 - 6x + 14 = 0$$

$$D = -20$$

$$x = -\frac{(-6) \pm \sqrt{-20}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{5}\ \mathrm{i}}{2}$$

$$= 3 \pm \sqrt{5}\ \mathrm{i}$$

x = 3+$\sqrt{5}$ i

$$y = 6 - \left(3 + \sqrt{5}\ \mathrm{i}\right)$$

$$= 3 - \sqrt{5}\ \mathrm{i}$$

when x = 3-$\sqrt{5}$ i

$$y = 6 - \left(3 - \sqrt{5}\ \mathrm{i}\right)$$

$$= 3 + \sqrt{5}\ \mathrm{i}$$

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

Topic: - Quadratic Equation

1. Find the multiplicative inverse 4 – 3i. [1]

2. Express in term of a + ib $\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$ [1]

3. Evaluate $i^{n}+i^{n+1}+i^{n+2}+i^{n+3}$ [1]

4. If 1, w, w^2 are three cube root of unity, show that $(1 - w + w^2)(1 + w - w^2) = 4$ [1]

5. Find that sum product of the complex number $-\sqrt{3}+\sqrt{-2}$ and $2\sqrt{3}-i$ [1]

6. If $a+ib=\frac{(x+i)^2}{2x-i}$ prove that $a^2+b^2=\frac{\left(x^2+1\right)^2}{4x^2+1}$ [4]

7. Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$ [4]

8. Find that modulus and argument $\frac{1+i}{1-i}$ [4]

9. Foe what real value of x and y are numbers equal $(1+i)\,y^2 + (6+i)$ and $(2+i)\,x$ [4]

10. Convert into polar form $z=\frac{i-1}{\cos\frac{\pi}{3}+i\ \text{Sin}\frac{\pi}{3}}$ [6]

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

[ANSWERS]

Topic: - Quadratic Equation

Ans 01. Let z = 4 – 3i

$\bar{z} = 4 + 3i$

$|z| = \sqrt{16+9} = 5$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$= \frac{4+3i}{25}$

$= \frac{4}{25} + \frac{3}{25}i$

Ans 02. $= \frac{(3)^2 - \left(i\sqrt{5}\right)^2}{\sqrt{\cancel{3}} + \sqrt{2}i - \sqrt{\cancel{3}} + i\sqrt{2}}$

$= \frac{9+5}{2\sqrt{2}i} = \frac{\cancel{14}\ 7}{\cancel{2}\sqrt{2}i}$

$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{7\sqrt{2}i}{-2}$

Ans 03. $= i^n + i^n.i^1 + i^n.i^2 + i^n.i^3$

$= i^n + i^n.i - i^n + i^n.(-i) \quad \left[\begin{array}{l} i^3 = -i \\ i^2 = -1 \end{array}\right.$

$= 0$

Ans 04. (1 – w + w²) (1 + w – w²)

(1 + w² - w) (1 + w – w²)

$(-w-w)\left(-w^2-w^2\right) \quad \left[\begin{array}{l} \because 1+w = -w^2 \\ 1+w^2 = -w \end{array}\right.$

$(-2w)\left(-2w^2\right)$

$4w^3$ $\qquad [w^3 = 1$

4×1

$= 4$

Ans 05. $z_1 + z_2 = -\sqrt{3} + \sqrt{2}i + 2\sqrt{3} - i$

$= \sqrt{3} + \left(\sqrt{2} - 1\right)i$

$z_1 z_2 = \left(-\sqrt{3} + \sqrt{2}i\right)\left(2\sqrt{3} - i\right)$

$= -6 + \sqrt{3}i + 2\sqrt{6}i - \sqrt{2}\ i^2$

$= -6 + \sqrt{3}\ i + 2\sqrt{6}\ i + \sqrt{2}$

$= \left(-6 + \sqrt{2}\right) + \left(\sqrt{3} + 2\sqrt{6}\right)i$

Ans 06. $a + ib = \dfrac{\left(x+i\right)^2}{2x-i}$ (i) (Given)

a - b = $\dfrac{\left(x \ -i\right)^2}{2x+i}$ (ii) [taking conjugate both side

(i) × (ii)

$\left(a+ib\right)\left(a-ib\right) = \dfrac{(x+i)^2}{(2x-i)} \times \dfrac{(x-i)^2}{(2x+i)}$

$a^2 + b^2 = \dfrac{(x^2+1)^2}{4x^2+1}$ proved.

Ans 07. $\left[i^{18} + \left(\dfrac{1}{i}\right)^{25}\right]^3$

$\left[\left(i^4\right)^4 . i^2 + \dfrac{1}{i^{25}}\right]^3$

$\left[i^2 + \dfrac{1}{\left(i^4\right)^6 . i}\right]^3$

$\left[-1 + \dfrac{1}{i}\right]^3$

$\left[-1 + \dfrac{i^3}{i^4}\right]^3$

$[-1-i]^3 = -(1+i)^3$

$= -\left[1^3 + i^3 + 3.1.i(1+i)\right]$

$= -\left[1 - i + 3i + 3i^2\right]$

$= -[1 - i + 3i - 3]$

$= -[-2 + 2i] = 2 - 2i$

Ans 08. $\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{(1+i)^2}{1^2 - i^2}$

$= \dfrac{1 + i^2 + 2i}{1+1}$

$= \dfrac{2i}{2}$

$= i$

$z = 0 + i$

$r = |z| = \sqrt{(0)^2 + (1)^2} = 1$

Let α be the acute $\angle$s

$\tan\alpha = \left|\dfrac{1}{0}\right|$

$\alpha = \pi/2$

arg (z) = $\pi/2$

$r = 1$

Ans 09. $(1+i)\, y^2 + (6 + i) = (2 + i)\, x$

$y^2 + iy^2 + 6 + i = 2x + xi$

$(y^2 + 6) + (y^2 + 1)\, i = 2x + xi$

$y^2 + 6 = 2x$

$y^2 + 1 = x$

$y^2 = x - 1$

$x - 1 + 6 = 2x$

$\boxed{5 = x}$

$\boxed{y = \pm 2}$

Ans 10. $z=\dfrac{i-1}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}$

$$=\frac{2(i-1)}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$$

$$r=|z|=\left(\frac{\sqrt{3}-1}{2}\right)^2+\left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$r=2$$

Let α be the acule $\angle$s

$$\tan\ \alpha=\left|\frac{\frac{\sqrt{3}+1}{\cancel{2}}}{\frac{\sqrt{3}-1}{\cancel{2}}}\right|$$

$$=\left|\frac{\sqrt{\cancel{3}}\left(1+\frac{1}{3}\right)}{\sqrt{\cancel{3}}\left(1-\frac{1}{3}\right)}\right|$$

$$=\left|\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right|$$

$$\tan\alpha=\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right|$$

$$\alpha=\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{12}$$

$$z=2\left(Cos\frac{5\pi}{12}+i\ \text{Sin}\ \frac{5\pi}{12}\right)$$

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

Topic: - Quadratic Equation

1. Write the real and imaginary part $1 - 2i^2$ [1]

2. If two complex number z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$ [1]

3. Find the conjugate and modulus of $\overline{9-i}+\overline{6+i^3}-\overline{9+i^2}$ [1]

4. Find the number of non zero integral solution of the equation $|1-i|^x = 2^x$ [1]

5. If (a + ib) (c + ib) (e + if) (g + ih) = A + iB then show that [1]

6. If $x + iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$ [4]

7. Convert in the polar form $\frac{1+7i}{(2-i)^2}$ [4]

8. Find the real values of x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$ [4]

9. If $|z_1|=|z_2|=1$, prove that $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|=|z_1+z_2|$ [4]

10. If α and β are different complex number with $|\beta| = 1$ Then find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$ [6]

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Complex Numbers and Quadratic Equation)

[ANSWERS]

Topic: - Quadratic Equation

Ans 01. Let z = 1 – 2i^2

=1 – 2 (-1)

= 1 + 2

= 3

= 3 + 0.i

Re (z) = 3, Im (z) = 1

Ans 02. Let z_1 = a + ib

$|z_1| = \sqrt{a^2+b^2}$

$z_2 = b+ia$

$|z_2| = \sqrt{b^2+a^2}$

Hence $|z_1|=|z_2|$ but $z_1 \neq z_2$

Ans 03. Let $z = \overline{9-i} + \overline{6-i} - \overline{9-1}$

$= 9+i+6+i-8$

$= 7+2i$

$\bar{z} = 7-2i$

$|z| = \sqrt{(7)^2+(-2)^2}$

$= \sqrt{49+4}$

$= \sqrt{53}$

Ans 04. $|1-i|^x = 2^x$

$\left(\sqrt{(1)^2+(-1)^2}\right)^x = 2^x$

$\left(\sqrt{2}\right)^x = 2^x$

$(2)^{\frac{1}{2}x} = 2^x$

$\frac{1}{2}x = x$

$$\frac{1}{2}=1$$

$$1=2$$

Which is false no value of x satisfies.

Ans 05. $(a+ib)(c+id)(e+if)(g+ih)=A+iB$

$$\Rightarrow \left|(a+ib)(c+id)(e+if)(g+ih)\right|=\left|A+iB\right|$$

$$|a+ib||c+id||e+if||g+ih|=|A+iB|$$

$$\left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(\sqrt{g^2+h^2}\right)=\sqrt{A^2+B^2}$$

sq. both side

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$$

Ans 06. $x+iy=\sqrt{\dfrac{1+i}{1-i}}$ (i) (Given)

taking conjugate both side

$$x-iy=\sqrt{\frac{1-i}{1+i}} \quad \text{(ii)}$$

(i) × (ii)

$$(x+iy)(x-iy)=\sqrt{\frac{1+i}{1-i}}\times\sqrt{\frac{1-i}{1+i}}$$

$$(x)^2-(iy)^2=1$$

$$\boxed{x^2+y^2=1}$$ Proved.

Ans 07. $\dfrac{1+7i}{(2-i)^2}=\dfrac{1+7i}{4+i^2-4i}=\dfrac{1+7i}{3-4i}$

$$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}$$

$$=\frac{3+4i+21i+28i^2}{9+16}$$

$$=\frac{25i-25}{25}=i-1$$

$$=-1+i$$

$$r=|z|=\sqrt{(-1)^2+1^2}=\sqrt{2}$$

Let α be the acute ∠s

$$\text{ten } \alpha = \left|\frac{1}{-1}\right|$$

$\alpha = \pi/4$

since Re (z) < 0, Im (z) > 0

$\theta = \pi - \alpha$

$= \pi - \frac{\pi}{4} = 3\pi/4$

$z = r\left(\text{Cos}\,\theta + \text{i Sin}\,\theta\right)$

$= \sqrt{2}\left(\text{Cos }\frac{3\pi}{4} + iSin\frac{3\pi}{4}\right)$

Ans 08.

(x – iy (3 + 5i) = - 6 + 24i

3x + 5xi – 3yi – 5yi^2 = - 6 + 24i

$\left(3x+5y\right)+\left(5x-3y\right)i = -6+24i$

$3x+5y = -6$

$5x-3y = 24$

$$\boxed{\begin{aligned} x &= 3 \\ y &= -3 \end{aligned}}$$

Ans 09. If $|z_1| = |z_2| = 1$ (Given)

$\Rightarrow |z_1|^2 = |z_2|^2 = 1$

$\Rightarrow z_1\ \overline{z_1} = 1$

$\overline{z_1} = \frac{1}{z_1}$ (1)

$z_2\ \overline{z_2} = 1$

$\overline{z_2} = \frac{1}{z_2}$ (2)

$\left[\ \because z\ \overline{z} = |z|^2\right.$

$\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = \left|\overline{z_1} + \overline{z_2}\right|$

$= \left|\overline{z_1 + z_2}\right|$

$= \left|z_1 + z_2\right|$

$\left[\because |\overline{z}| = |z|\right.$ proved.

Ans 10. $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2=\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right)$ $\left[\because |z|^2=z\bar{z}\right.$

$$=\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right)$$

$$=\left(\frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}\right)$$

$$=\left(\frac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}\right)$$

$$=\left(\frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\because\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2}\right)$$ $\left[\because |\beta|=1\right.$

$$=1$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\sqrt{1}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

TEST PAPER-01

CLASS - XI MATHEMATICS (Linear inequalities)

1. Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ [1]

2. Solve $3x+8>2$ when x is a real no. [1]

3. Solve the inequality $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ [4]

4. Solve $3x-6 \geq 0$ graphically [4]

5. Ravi obtained 70 and 75 mark in first unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks. [4]

6. A plumber can be paid under two schemes as given below. [4]

   I: Rs 600 and Rs 50 per hr.

   II: Rs 170 per hr.

   If the job takes n hr. for what values of n does the scheme I gives the plumber the better wages.

7. IQ of a person is given by the formula $IQ = \frac{MA}{CA} \times 100$ [6]

   Where MA is mental age and CA is chronological age. If 80 $\leq$IQ$\leq$140 for a group of 12yr old children, fond the range of their mental age.

8. Solve graphically $4x+3y \leq 60 \quad y \geq 2x \quad x \geq 3 \quad x, y \geq 0$ [6]

TEST PAPER-1
CLASS - XI MATHEMATICS (Linear inequalities)

[ANSWERS]

Ans1. $\frac{3x-4}{2} \geq \frac{x+1}{4} - \frac{1}{1}$

$\frac{3x-4}{2} \geq \frac{x+1-4}{4}$

$\frac{3x-4}{2} \geq \frac{x-3}{4}$

$2(3x-4) \geq (x-3)$

$6x-8 \geq x-3$

$x \geq 1$

Ans2. $3x+8 > 2$

$3x > 2-8$

$3x > -6$

$x > -2$

$(-2, \infty)$

Ans3. $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

$\frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\frac{x}{4} < \frac{4x-1}{15}$

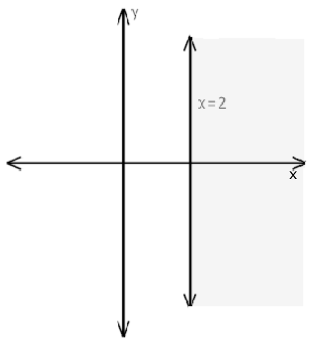
$15x < 16x-4$

$-x < -4$

$x > 4$

$(4, \infty)$

Ans4. $3x-6 \geq 0 ......(i)$



$3x-6 = 0$

$x = 2$

Put (0,0)in eq. (i)

$0-6 \geq 0$

$0 > 6$ false.

Ans5. Let Ravi secure x marks in third test

ATQ $\dfrac{70+75+x}{3} \geq 60$

$x \geq 135$

Ans6. For better wages earning should be more then

$600+50n > 170n$

$n < 5$

Thus for better wages scheme working hr. should be less than 5 hr.

Ans7. $80 \leq \text{IQ} \leq 140$ (Given)

$80 \leq \dfrac{MA}{CA} \times 100 \leq 140$

$80 \leq \dfrac{MA}{12} \times 100 \leq 140$

$80 \times \dfrac{12}{100} \leq MA \times \dfrac{\cancel{100}}{\cancel{12}} \times \dfrac{\cancel{12}}{\cancel{100}} \leq 140 \times \dfrac{12}{100}$

$\dfrac{96}{10} \leq \text{MA} \leq \dfrac{168}{10}$

$9.6 \leq \text{MA} \leq 16.8$

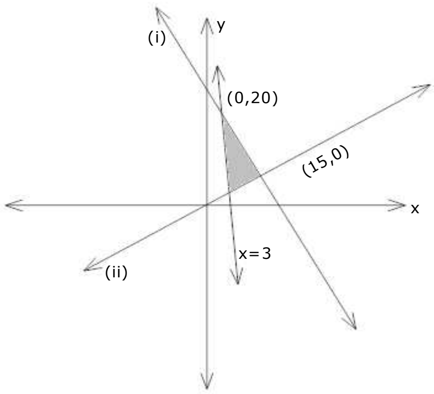
Ans8. $4x+3y=60$

x	0	15
y	20	0

$y=2x$

x	0	20
y	0	40

$x=3$



TEST PAPER-02

CLASS - XI MATHEMATICS (Linear inequalities)

1. If $4x > -16$ then $x \square - 4$. [1]

2. Solve $5x - 3 < 3x + 1$ when x is an integer. [1]

3. Solve the inequality $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$ [4]

4. Solve $3x + 2y > 6$ graphically [4]

5. Find all pairs of consecutive odd natural no. both of which are larger than 10 such that their sum is less than 40. [4]

6. A company manufactures cassettes and its cost equation for a week is C=300+1.5 x and its revenue equation is R=2 x, where x is the no. of cassettes sold in a week. How many cassettes must be sold by the company to get some profit? [4]

7. A manufacturer has 600 litre of a 12% sol. Of acid. How many litres of a 30% acid sol. Must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%. [6]

8. Solve graphically $x - 2y \leq 3 \quad 3x + 4y \geq 12 \quad x \geq 0 \quad y \geq 1$ [6]

TEST PAPER-2

CLASS - XI MATHEMATICS (Linear inequalities)

[ANSWERS]

Ans.1 $x > -4.$

Ans.2 $5x - 3 < 3x + 1$

$2x < 4$

$x < 2$

When x is an integer the solutions of the given inequality are……-4,-3,-2,-1, 0, 1

Ans3. $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$

$\frac{3x}{10} + 2 \geq \frac{x}{3} - 2$

$\frac{3x}{10} - \frac{x}{3} \geq -4$

$\frac{9x - 10x}{30} \geq -4$

$\frac{-x}{30} \geq -4$

$-x \geq -120$

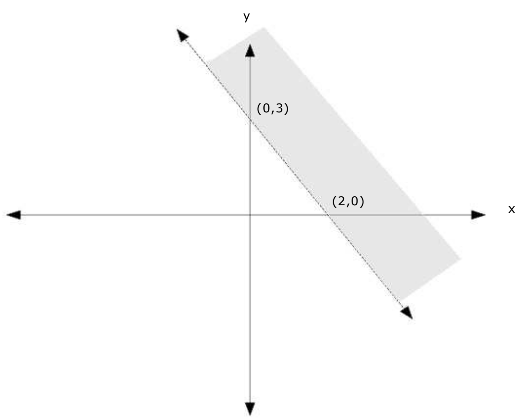
$x \leq 120$

$(-\infty, 120]$

Ans.4 $3x + 2y > 6 \ldots\ldots(i)$

$3x + 2y = 6$

x	0	2
y	3	0



Put $(0,0)$ in eq. ……….(i)

$0 + 0 > 6$

$0 > 6$ which is false

Ans5. Let x and $x+2$ be consecutive odd natural no.

ATQ $x>10.......(i)$

$(x)+(x+2)<40$

$x<19......(ii)$

From (i) and (ii)

$(11,13)\quad(13,15),\quad(15,17)\quad(17,19)$

Ans6. Profit = revenue-cost

$R>C \qquad$ [for to get some profit]

$2x>300+1.5x$

$\frac{1}{2}x>300$

$x>600$

Ans7. Let x litres of 30% acid sol. Is required to be added.

$30\%x+12\%$ of $600>15\%$ of $(x+600)$ and

$30\%x+12\%$ of $600<18\%$ of $(x+600)$

$\frac{30x}{100}+\frac{12}{100}(600)>\frac{15}{100}(x+600)$

$\frac{30x}{100}+\frac{12}{100}(600)<\frac{18}{100}(x+600)$

$x>120$ and $x<300$

i.e. $120<x<300$.

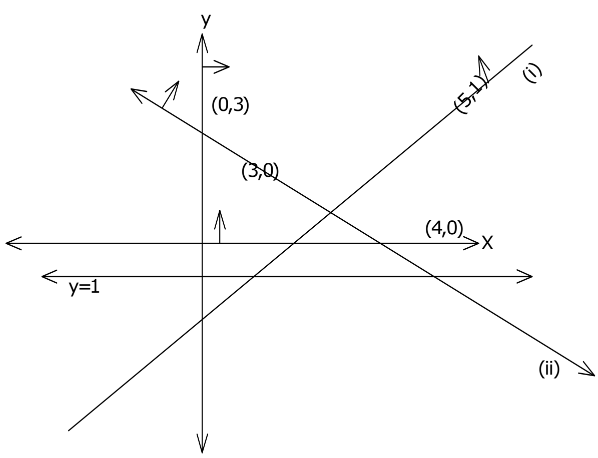
Ans8. $x-2y=3$

x	3	5
y	0	1

$3x+4y=12$

x	0	4
y	3	0

$y=1$



TEST PAPER-03

CLASS - XI MATHEMATICS (Linear inequalities)

1. Solution set of the in inequations $2x-1\leq 3$ and $3x+1\geq -5$ is. [1]

2. Solve $7x+3<5x+9$. Show the graph of the solution on number line. [1]

3. Solve the inequality. $\frac{2x-1}{3}\geq\frac{3x-2}{4}-\frac{2-x}{5}$ [4]

4. Solve the inequalities $3x+4y\leq 12$ graphically [4]

5. The longest side of a Δ is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the Δ is at least 61 cm find the minimum length of the shortest side. [4]

6. In drilling world's deepest hole it was found that the temperature T in degree Celsius, x km below the surface of earth was given by $T=30+25(x-3), 3<x<15$ At what depth will the tempt. Be between 200^0 c and $300^0 c$ [4]

7. A sol. Of 8% boric acid is to be diluted by adding a 2% boric acid sol. to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% sol. how many litre of the 2% sol. will have to be added. [6]

8. Solve graphically $x+2y\leq 10$ $x+y\geq 1$ $x-y\leq 0$ [6]

$x\geq 0,\qquad y\geq 0$

TEST PAPER-3

CLASS - XI MATHEMATICS (Linear inequalities)

[ANSWERS]

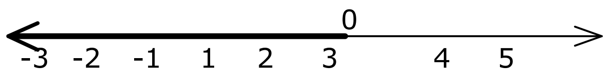
Ans1. $2x-1\le 3\ ,\ \ 3x+1\ge -5$

$\Rightarrow 2x\le 4, \qquad 3x\ge -6$

$\Rightarrow x\le 2, \qquad x\ge -2$

$\Rightarrow -2\le x\le 2$

Ans2. $7x+3<5x+9$



$2x<6$

$x<3$

Ans3 $\dfrac{2x-1}{3}\ge \dfrac{5(3x-2)-4(2-x)}{20}$

$20(2x-1)\ge 3(19x-18)$

$40x-20\ge 57x-54$

$-17x\ge -34$

$x\le 2$

$(-\infty,2]$

Ans4. $3x+4y\ge 12.......(i)$

$3x+4y=12$

x	0	4
y	3	0

Put $(0,0)$ in eq. ....(i)

$0+0\ge 12$ false

Ans5. Let shortest side be x cm then the longest side is $3x$ cm and the third side $(3x-2)$ cm.

ATQ $(x)+(3x)+(3x-2)\ge 61$

$x\ge 9$

Length of shortest side is 9 cm.

Ans6. Let x km is the depth where the tempt lies between $200^0 C$ and $300^0 C$

$$200^0 C < T < 300^0 C$$

$$200 < 30 + 25(x-3) < 300$$

$$\frac{49}{5} < x < \frac{69}{5} \qquad \Rightarrow 9.8 < x < 13.8$$

Ans7. Let x be added

ATQ 2% of $x + 8\%$ of $640 > 4\%$ of $(640 + x)$

$$\frac{2x}{100} + \frac{8 \times 640}{100} > \frac{4}{100}(640 + x)$$

$$x < 1280 \ldots\ldots.(i)$$

And 12% of $x + 8\%$ of $640 < 6\%$ of $(640 + x)$

$$\frac{2x}{100} + \frac{8 \times 640}{100} < \frac{6}{100}(640 + x)$$

$$x > 320 \ldots\ldots.(ii)$$

From (i) and (ii)

$$320 < x < 1280$$

Ans8. $x + 2y = 10$

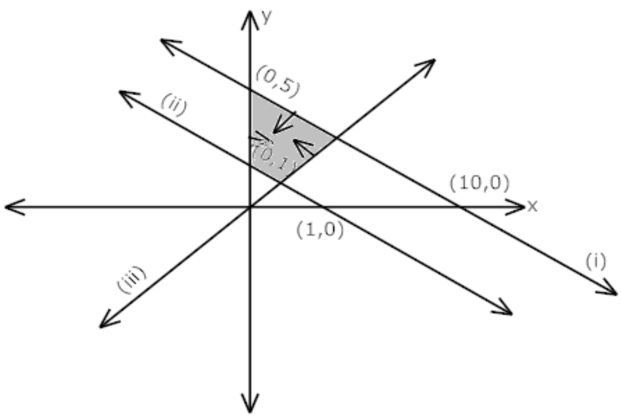
x	0	10
y	5	0

$x + y = 1$

x	0	1
y	1	0

$x - y = 0$

x	0	2
y	0	2



TEST PAPER-04

CLASS - XI MATHEMATICS (Linear inequalities)

1. Solve $5x-3\le 3x+1$ when x is an integer. [1]

2. Solve $30x<200$ **when** x is a natural no. [1]

3. Solve the inequality $\frac{x}{2}\ge \frac{5x-2}{3}-\frac{7x-3}{5}$ [4]

4. Solve graphically $x-y\le 0$ [4]

5. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second. [4]

6. The water acidity in a pool is considered normal when the average Ph reading of three daily measurements is between 7.2 and 7.8 If the first Ph reading are 7.48 and7.85, find the range of Ph value for the third reading that will result in the acidity level being normal. [4]

7. How many litres of water will have to be added to 1125 litres of the 45% sol. Of acid so that the resulting mixture will contain more than 25%but less than 30% acid content. [6]

8. Solve graphically $3x+2y\le 150$ $x+4y\le 80$ $x\le 15$ $y\ge 0$ $x\ge 0$ [6]

TEST PAPER-4

CLASS - XI MATHEMATICS (Linear inequalities)

[ANSWERS]

Ans1. $5x-3\le 3x+1$

$5x-3x\le 4$

$2x\le 4$

$x\le 2$

$\{\ldots,-3,-2,-1,0,1,2\}$

Ans2. $30x<200$

$x<\frac{200}{30}$

$x<\frac{20}{3}$

Solution set of the inequality $\{1,2,3,4,5,6\}$

Ans3. $\frac{x}{2}\ge\frac{5(5x-2)-3(7x-3)}{15}$

$\frac{x}{2}\ge\frac{25x-10-21x+9}{15}$

$\frac{x}{2}\ge\frac{4x-1}{15}$

$15x\ge 8x-2$

$7x\ge -2\Rightarrow x\ge -\frac{2}{7}$

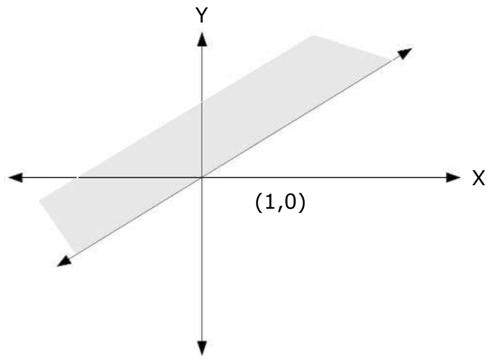
Ans4. $x-y\le 0.....(i)$

$x=y$

Put (1,0) in eq. (i)

$1-0\le 0$

$1\le 0$ false



Ans5. Let the shortest length be x cm, then second length is (x+3) cm and the third length is 2x cm.

ATQ $4x+3\le 91$

$$x\le \frac{88}{4}$$

$$x\le 22$$

Again ATQ

$$2x\ge 5+(x+3)$$

$$x\ge 8$$

$$x\in[8,22]$$

Ans6. Let third reading be x then

$$7.2<\frac{7.48+7.85+x}{3}<7.8$$

$$21.6<15.33+x<23.4$$

$$6.27<x<7.07$$

Ans7. Let x litre of water be added to 1125 litre of 45 acid sol.

$$45\% \text{ of } 1125>25\% \text{ of } (x+1125)$$

$$30\% \text{ of } 1125<30\% \text{ of } (x+1125)$$

$$900>x>562.5$$

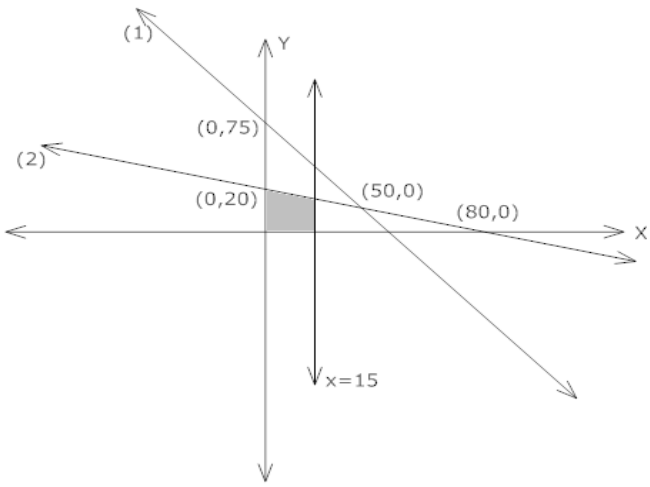
Ans8. $3x+2y=150$

$x+4y=80$

$x=15$

x	0	50
y	75	0

x	0	80
y	20	0



TEST PAPER-01

CLASS - XI MATHEMATICS (Permutation and Combinations)

1. Evaluate $4! - 3!$ [1]
2. If ${}^{n}c_{a} = {}^{n}c_{b}$ find n [1]
3. The value of 0! [1]
4. Given 5 flags of different colours here many different signals can be generated if each signal requires the use of 2 flags. One below the other [1]
5. How many 4 letter code can be formed using the first 10 letter of the English alphabet, if no letter can be repeated? [4]
6. How many words, with or without meaning can be made from the letters of the word MONDAY. Assuming that no. letter is repeated, it [4]
   (i) 4 letters are used at a time
   (ii) All letters are used but first letter is a vowel?
7. Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$ [4]
8. A bag contains 5 black and 6 red balls determine the number of ways in which 2 black and 3 red balls can be selected. [4]
9. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together? [4]
10. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE [4]
11. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has : [6]
    (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?
12. Find the number of words with or without meaning which can be made using all the letters of the word. AGAIN. If these words are writer as in a dictionary, what will be the 50^{th} word? [6]

TEST PAPER-01

CLASS - XI MATHEMATICS (Permutation and Combinations)

[ANSWERS]

Ans1. $4!-3!=4.3!-3!$

$=(4-1).3!$

$=3.3!=3\times3\times2\times1$

$=18$

Ans2. ${}^nC_a={}^nC_b \Rightarrow {}^nC_a={}^nC_{n-b}$

$a=n-b$

$n=a+b$

Ans3. $0!=1$

Ans4. First flag can be chosen is 5 ways

Second flag can be chosen is 4 ways

By $F.P.C.$ total number of ways $=5\times4=20$

Ans5. First letter can be used in 10 ways

Second letter can be used in 9 ways

Third letter can be used in 8 ways

Forth letter can be used in 7 ways

By $F.P.C.$ total no. of ways $=10.9.8.7$

$=5040$

Ans6. Part-I In the word MONDAY there are 6 letters

$\therefore n=6$

4 letters are used at a time

$\therefore r=4$

Total number of words $={}^n\mathrm{P}_\mathrm{r}$

$$={}^6P_4=\frac{\lfloor 6}{\lfloor 6-4}$$

$$=\frac{\lfloor 6}{\lfloor 2}=\frac{6.5.4.3.\cancel{\lfloor 2}}{\cancel{\lfloor 2}}=360$$

Part-II All letters are used at a time but first letter is a vowel then OAMNDY

2 vowels can be arranged in 2! Ways

4 consonants can be arranged in 4! Ways

$\therefore$ Total number of words $= 2! \times 4!$

$= 2 \times 4.3.2.1 = 48$

Ans7. Proof L.H.S.

$${}^{n}C_r + {}^{n}C_{r-1} = \frac{\lfloor n}{\lfloor n-r \lfloor r} + \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor (n-r)\, r \lfloor r-1} + \frac{\lfloor n}{(n-r+1)\lfloor n-r \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \qquad \left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \qquad \left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$= \frac{\lfloor n (n+1)}{\lfloor n-r (n-r+1) \lfloor r-1\, r}$$

$$= \frac{\lfloor n+1}{\lfloor n+1-r \lfloor n-r} = {}^{n+1}C_r$$

Ans8. No. of black balls =5

No. of red balls =6

No. of selecting black balls = 2

No. of selecting red balls = 3

Total no. of selection $= {}^{5}C_2 \times {}^{6}C_3$

$$= \frac{\lfloor 5}{\lfloor 5-2 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 6-2\ \lfloor 3}$$

$$\frac{5.\cancel{4}^{2}.\cancel{\lfloor 3}}{\cancel{\lfloor 3}\ \cancel{2}.1} \times \frac{\cancel{6}.5.\cancel{\lfloor 4}}{\cancel{\lfloor 4}\ \cancel{\lfloor 3.2}.1} = 50$$

Ans9. Let us first seat 0 the 5 girls. This can be done in 5! Ways

X G X G X G X G X G X

There are 6 cross marked placer and the three boys can be seated in ${}^{6}P_3$ ways

Hence by multiplication principle

The total number of ways

$$= 5! \times {}^{6}\mathrm{P}_3 = 5! \times \frac{6!}{3!}$$

$= 4 \times 5 \times 2 \times 3 \times 4 \times 5 \times 6$

$= 14400$

Ans10 In the INVOLUTE there are 4 vowels, namely I.O.E.U and 4 consonants namely M.V.L and T

The number of ways of selecting 3 vowels

Out of $4 = {}^4C_3 = 4$

The number of ways of selecting 2 consonants

Out of $4 = {}^4C_2 = 6$

$\therefore$ No of combinations of 3 vowels and 2 consonants $= 4 \times 6 = 24$

5 letters 2 vowel and 3 consonants can be arranged in 5! Ways

Therefore required no. of different words $= 24 \times 5! = 2880$

Ans11. Number of girls = 4

Number of boys = 7

Number of selection of members = 5

(i) If team has no girl

We select 5 boys

$\therefore$ Number of selection of 5 members

$= {}^7C_5 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} = 21$

(ii) At least one boy and one girl the team consist of

Boy	Girls
1	4
2	3
3	2
4	1

The required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140$

$= 441$

(iii) At least 3 girls

Girls	Boys
3	2
4	1

The required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7$

$= 91$

Ans12. In the word ‘AGAIN’ there are 5 letters in which 2 letters (A) are repeated

Therefore total no. of words $\frac{5!}{2!} = 60$

If these words are written as in a dictionary the number of words starting with Letter A. [A A G I N] $= 4! = 24$

The no. of wards starting with G [G A A I N] $= \frac{4!}{2!} = 12$

The no. of words starting with I [I A A G N] $= \frac{4!}{2!} = 12$

Now

Total words $= 24 + 12 + 12 = 48$

49^{th} Words = N A A G I

50^{th} Words = N A A I G

TEST PAPER-02
CLASS - XI MATHEMATICS (Permutation and Combinations)

1. A coin is tossed 3 times and the outcomes are recorded. How many possible out comes are there? [1]

2. Compute $\frac{8!}{6! \times 2!}$ [1]

3. If ${}^{n}C_{8} = {}^{n}C_{2}$. find ${}^{n}C_{2}$. [1]

4. In how many ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colours. [1]

5. Find r, if $5{}^{4}p_{r} = 6{}^{5}p_{r-1}$ [4]

6. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements [4]
   (i) do the words start with P
   (ii) do all the vowels always occur together

7. Find n if ${}^{n-1}P_{3} : {}^{n}P_{4} = 1:9$ [4]

8. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers? [4]

9. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4? [4]

10. In how many ways can the letters of the word ASSA SS IN ATION be arranged so that all the S's are together? [4]

11. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of there [6]
    (i) Four cards one of the same suit
    (ii) Four cards belong to four different suits
    (iii) Are face cards.
    (iv) Two are red cards & two are black cards.
    (v) Cards are of the same colour?

12. If ${}^{n}\mathrm{P}_{\mathrm{r}} = {}^{n}\mathrm{P}_{\mathrm{r+1}}$ and ${}^{n}c_{r} = {}^{n}c_{r-1}$ find the value of n and r [6]

TEST PAPER-02

CLASS - XI MATHEMATICS (Permutation and Combinations)

[ANSWERS]

Ans1. Total no. of possible out comes $= 2 \times 2 \times 2 = 8$

Ans2.
$$\frac{8!}{6!2!} = \frac{8.7.\cancel{6!}}{\cancel{6!}.2.1}$$
$$= 4 \times 7 = 28$$

Ans3. Given
$$^nC_8 = {}^nC_2 \Rightarrow {}^nC_{n-8} = {}^nC_2$$
$$n - 8 = 2$$
$$n = 10$$
$$\therefore n_{c2} = 10_{c2} = \frac{\lfloor 10}{\lfloor 10-2 \lfloor 2}$$
$$= \frac{10.9.\cancel{\lfloor 8}}{\cancel{\lfloor 8} \times 2.1} = 5 \times 9 = 45$$

Ans4. No. of ways of selecting 9 balls
$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3$$
$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$
$$= \frac{6.5.4\lfloor 3}{6.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3}$$
$$= 20 \times 10 \times 10 = 2000$$

Ans5.
$$5.\ {}^4p_r = 6.\ {}^5p_{r-1}$$
$$\Rightarrow 5.\frac{\lfloor 4}{\lfloor 4-r} = 6.\frac{\lfloor 5}{\lfloor 5-r+1}$$
$$\Rightarrow \frac{5.\lfloor 4}{\lfloor (4-r)} = \frac{6.5.\lfloor 4}{\lfloor 6-r}$$
$$\Rightarrow \frac{1}{\cancel{\lfloor 4-r}} = \frac{6}{(6-r)(5-r)\cancel{\lfloor 4-r}}$$
$$\Rightarrow (6-r)(5-r) = 6$$

$\Rightarrow 30-6r-5r+r^2=6$

$\Rightarrow r^2-11r-5r+r^2=6$

$\Rightarrow r^2-8r-3r+24=0$

$\Rightarrow r(r-8)-3(r-8)=0$

$\Rightarrow (r-3)(r-8)=0$

$r=3$ or $r=8$

$\therefore \boxed{r=3}$

$r=8$ Rejected. Because if we put $r=8$ the no. in the factorial is –ve.

Ans6. The number of letters in the word INDE PENDENCE are 12 In which E repeated in 4 times. N repeated 3 times. D repeated 2 times

(i) If the word starts with P The position of P is fined

Then the no. of arrangements $=\dfrac{11!}{4!3!2!}$

$=138600$

(ii) All the vowels always occur together There are 5 vowels in which 4 E's and 1 I

$\boxed{\text{EEEEI}}$ NDPNDNC

$\therefore$ Total letters are 8.

letters Can be arranged in $=\dfrac{8!}{3!2!}$ ways

Also 5 vowels can be arranged in $=\dfrac{5!}{4!}$ ways.

$\therefore$ required number of arrangements

$=\dfrac{8!}{3!2!}\times\dfrac{5!}{4!}=16800$

Ans7. Given

${}^{n-1}P_3 : {}^{n}P_4 = 1:9$

$\dfrac{\lfloor n-1}{\lfloor n-1-3} : \dfrac{\lfloor n}{\lfloor n-4} = 1:9$

$\dfrac{\dfrac{\lfloor n-1}{\cancel{\lfloor n-4}}}{\dfrac{\lfloor n}{\cancel{\lfloor n-4}}} = \dfrac{1}{9}$

$\dfrac{\cancel{\lfloor n-1}}{n\cancel{\lfloor n-1}} = \dfrac{1}{9}$

$n=9$

Ans8. Given total no. of players =17

5 players can bowl.

$\therefore$ no. of bat's man =17-5=12

Out of 5 bowers 4 can be choose in ${}^{5}C_{4}$ ways

Out of 12 bat's man (11-4)=7 bat's man can be choose in ${}^{12}C_{7}$ ways

Total no. of selection of 11 players

$= {}^{5}C_{4} \times {}^{12}C_{7}$

$= \frac{\lfloor 5}{\lfloor 5-4 \lfloor 4} \times \frac{\lfloor 12}{\lfloor 12-7 \lfloor 7}$

$5 \times \frac{12.11.10.9.8.\lfloor 7}{5.4.3.2.1\lfloor 7} = 3960$

Ans9. Given digits $1, 2, 0, 2, 4, 2, 4$ Are 7

The no. of arrangements of 7 digits $= \frac{7!}{3!21.} = 420$

If 0 is in extreme left position.

The no. of arrangements of 6 digits $= \frac{6!}{3!2!} = 60$

$\therefore$ Required numbers $= 420 - 60 = 360$

Ans10. In the word ASSA SSI N ATION there are 13 letters

But all S are together. Then no. of letters 10 [4 S take 1]then required no. of arrangements

$= \frac{10!}{3!2!2!} \quad = 151200$

Ans11 The no. of ways of choosing 4 cards form 52 playing cards.

${}^{52}C_{4} = \frac{52!}{4!48!} = 270725$

(i) If 4 cards are of the same suit there are 4type of suits. [diamond club, spade and heart] 4 cards of each suit can be selected in ${}^{13}C_{4}$ ways

$\therefore$ Required no. of selection $= {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4}$

$= 4 \times {}^{13}C_{4} = 2860$

(ii) If 4 cards belong to four different suits then each suit can be selected in 13_{c1} ways required no. of selection $= {}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1} = 13^{4}$

(iii) If all 4 cards are face cards. Out of 12 face cards 4 cards can be selected in ${}^{12}C_{4}$ ways.

$\therefore$ required no. of selection ${}^{12}C_4 = \frac{\underline{|12}}{\underline{|8}\underline{|4}} = 495$

(iv) If 2 cards are red and 2 are black then. Out of 26 red card 2 cards can be selected in ${}^{26}C_2$ ways similarly 2 black card can be selected in ${}^{26}C_2$ ways

$$= {}^{26}C_2 \times {}^{26}C_2$$

$$\therefore \text{ required no. of selection } = \frac{26!}{2!4!} \times \frac{26!}{2!4!} = (325)^2$$

$$= 105625$$

(v) If 4 cards are of the same colour each colour can be selected in ${}^{26}C_4$ ways

Then required no. of selection

$$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!22!}$$

$$= 29900$$

Ans12 Given that

$${}^{n}P_r = {}^{n}P_{r+1}$$

$$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}} = \frac{\underline{|n}}{\underline{|n-r}-1}$$

$$\Rightarrow \frac{1}{(n-r)\underline{|n-r-1}} = \frac{1}{\underline{|n-r-1}}$$

$$\Rightarrow n-r = 1..........(i)$$

also $n_{cr} = n_{cr-1}$

$$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}\underline{|r}} = \frac{\underline{|n}}{\underline{|n-r+1}\underline{|r-1}}$$

$$\Rightarrow \frac{1}{\underline{|n-r}\ r\underline{|r-1}} = \frac{1}{(n-r+1)\underline{|n-r}\underline{|r-1}}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$$

$$\Rightarrow n-2r = -1.........(ii)$$

Solving eq (i) and eq (ii) we get $n = 3$ and $r = 2$

TEST PAPER-03

CLASS - XI MATHEMATICS (Permutation and Combinations)

1. If $\frac{1}{6!}+\frac{1}{7!}+=\frac{x}{8!}$ find x [1]

2. Write relation between ${}^{n}p_{r}$. and ${}^{n}C_{r}$ [1]

3. What is $\underline{|n}$ [1]

4. If ${}^{n}C_{o}=1$ what is the value of ${}^{99}C_{o}$ [1]

5. How many words, with or with not meaning each of 2 vowels and 3 consonants can be flamed from the letter of the word DAUGHTER? [4]

6. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be flamed from the alphabet? [4]

7. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together? [4]

8. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colors are in distinguishable? [4]

9. Find the number of permutations of the letters of the word ALLAHABAD. [4]

10. How many 4 letter code can be formed using the first 10 letters of the English alphabet if no letter can be repeated? [4]

11. Find the value of n such that [6]

    $(i)\ {}^{n}P_{5}=42\,{}^{n}P_{3},\ n>4$ $\qquad$ $(ii)\ \frac{{}^{n}P_{4}}{{}^{n-1}P_{4}}=\frac{5}{3},\ n>4$

12. A committee of 7 has to be farmed from 9 boys and 4 girls in how many ways can this be done when the committee consists of [6]

    (i) Exactly 3 girls? (ii) Attest 3 girls? (iii) Almost 3 girls?

TEST PAPER-03

CLASS - XI MATHEMATICS (Permutation and Combinations)

[ANSWERS]

Ans1. $\frac{1}{6!}+\frac{1}{7.6!}=\frac{x}{8.7.6!}$

$\frac{1}{6!}\left[1+\frac{1}{7}\right]=\frac{x}{56.6!}$

$\frac{8}{7}=\frac{x}{56}$

$x=8\times 8=64$

Ans2. ${}^{n}\mathrm{P}_{\mathrm{r}}={}^{n}C_{r}\times \underline{|r}$

Ans3 $\underline{|n}$ Is multiplication of n consecutive natural number

$\underline{|n}=n(n-1)(n-2)(n-3)......5.4.3.2.1$

Ans4. ${}^{n}C_{o}=1$ then ${}^{99}C_{o}=1$

Ans5. In the word DAUGHTER There are 3 vowels and 5 consonants out of 3 vowels, 2 vowels can be selected in ${}^{3}C_{2}$ ways and 3 consonants can be selected in ${}^{5}C_{3}$ ways

5 letters 2 vowel and 3 consonant can be arranged in 5! Ways

$\therefore$ Total no. of words $={}^{3}C_{2}\times {}^{5}C_{3}\times 5!$

$=\frac{\underline{|3}}{\underline{|1}\underline{|2}}\times\frac{\underline{|5}}{\underline{|2}\times\underline{|3}}\times\underline{|5}$

$=\frac{3.\cancel{\underline{|2}}}{\cancel{\underline{|2}}}\times\frac{5.4.\cancel{\underline{|3}}}{\cancel{\underline{|2}}.\cancel{\underline{|3}}}\times 5.4.3.\cancel{2}.1$

$=3600$

Ans6. No. of vowels = 5

No. of consonants = 21

2 vowels can be selected in ${}^{5}C_{2}$ ways

2consonents can be selected in ${}^{21}C_{2}$ ways

No. of arrangements of 4 letters [2 vowel and 2 consonants] =4!

$\therefore$ Required no. of wards $={}^{5}C_{2}\times {}^{21}C_{2}\times 4!$

$= \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \frac{\lfloor 21}{\lfloor 19 \lfloor 2} \times 4.\lfloor 3$

$= \frac{5.4.3.\cancel{\lfloor 2} \times 21.20.\cancel{\lfloor 19} \times 4.\cancel{\lfloor 3}}{\cancel{\lfloor 3}.\cancel{\lfloor 2} \quad \cancel{\lfloor 19} \times \cancel{2}}$

$= 50400$

Ans7. In the word MISSISSIPPI no. of letters = 11

In which 4 I's, 4 S's and 2P

$\therefore$ Total no. of words $= \frac{11!}{4!4!2!} = 34650$

When four I's come together

Then four I's as one letter

And other letters are 7

Then no. of words when four I's come together

$= \frac{8!}{4!2!} = 840$

Then the no. of permutations when four I's do not come together

$= 34650 - 840 = 33810$

Ans8. Total no. of discs $= 4 + 3 + 2 = 9$

Out of 9 discs 4 are of some kind, 3are of same kind, 2 are of same kind

There for the number of arrangements $= \frac{9!}{4!3!2!} = 1260$

Ans9 In the word ALLAHABAD no. of letters = 9

In which four A's and two L's

$\therefore$ The no. of permutations $= \frac{9!}{4!2!} = 7560$

Ans10 There are 10 letters of English alphabet

For making 4 letter code

First letter can be choose in 10 ways

Second letter can be choose in 9 ways

Third letter can be choose in 8 ways

Forth letter can be choose in 7 ways

By fundamental principle of multiplication

Total no. of code $= 10 \times 9 \times 8 \times 7 \quad = 5040$

Ans11 (i) $^{n}P_5 = 42\,^{n}P_3$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-5} = 42\frac{\lfloor n}{\lfloor n-3}$$

$$\Rightarrow \frac{1}{\lfloor n-5} = \frac{42}{(n-3)(n-4)\lfloor n-5}$$

$$\Rightarrow (n-3)(n-4) = 42$$

$$\Rightarrow n^2 - 4n - 3n + 12 = 42$$

$$\Rightarrow n^2 - 7n - 30 = 0$$

$$\Rightarrow n^2 - 10n + 3n - 30 = 0$$

$$\Rightarrow n(n-10) + 3(n-10) = 0$$

$$\Rightarrow (n+3)(n-10) = 0$$

$n = -3$ or $n = 10$

$n = -3$ Is rejected

Because negative factorial is not defined $\therefore n = 10$

(ii) $\frac{^{n}P_4}{^{n-1}P_4} = \frac{5}{3} \qquad n > 4$

$$\Rightarrow \frac{\frac{\lfloor n}{\lfloor n-4}}{\frac{\lfloor n-1}{\lfloor n-5}} = \frac{5}{3}$$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-4} \times \frac{\lfloor n-5}{\lfloor n-1} = \frac{5}{3}$$

$$\Rightarrow \frac{n\cancel{\lfloor n-1}}{(n-4)\cancel{\lfloor n-5}} \times \frac{\cancel{\lfloor n-5}}{\cancel{\lfloor n-1}} = \frac{5}{3}$$

$$\Rightarrow 3n = 5n - 20$$

$$\Rightarrow -2n = -20 \quad \Rightarrow n = 10$$

Ans12. No. of boys = 9

No. of girls = 4

But committee has 7 members

(i) When committee consists of exactly 3 girls

	Boys	Girls	
	9	4	
Selecting member	4	3	=7

$\therefore$ Required no. of selection $= {}^{9}C_4 \times {}^{4}C_3 = 504$

(ii) Attest 3 girls

	Boys	Girls	
	9	4	
Selecting member	4	3	=7
	3	4	=7

The required no. of selections $= {}^9C_4 \times {}^4C_3 + {}^9C_3 \times {}^4C_4$

$= 504 + 84$

$= 588$

(iii) Almost 3 girls

	Boys	Girls	
	9	4	
	7	0	=7
Selecting member	6	1	=7
	5	2	=7
	4	3	=7

Then required no. of selection

$= {}^9C_7 \times {}^4C_0 + {}^9C_6 \times {}^4C_1 + {}^9C_5 \times {}^4C_2 + {}^9C_4 \times {}^4C_3$

$= 36 + 336 + 756 + 504$

$= 1632$

TEST PAPER-04

CLASS - XI MATHEMATICS (Permutation and Combinations)

1. Convert the following products into factorials $5\times6\times7\times8\times9$ [1]

2. Evaluate $\frac{n!}{(n-r)!}$, when $n=5,\ r=2$ [1]

3. Evaluate ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 + {}^{15}C_7$ [1]

4. What is the value of ${}^nC_o + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n$ [1]

5. Find n if ${}^{2n}C_3 : {}^nC_3 = 11:1$ [4]

6. Determine the number of ways of choosing 5 cards out of Adele of 52 cards which include exactly one ace. [4]

7. How many numbers greater than 56000 and farmed by using the digits 4,5,6,7,8, no digit being repeated in any number? [4]

8. Find n, if $\frac{\lfloor n}{\lfloor 2\ \lfloor n-2}$ and $\frac{\lfloor n}{\lfloor n-4}$ are in the ratio 2:1 [4]

9. Prove that $\lfloor 2n = 1.3.5\ldots\ldots(2n-1).2^n\ .\lfloor n$ [4]

10. How many 4 letter words with or without meaning, can be formed out of the letters of the word 'LOGARITHMS', if repetition of letters is not allowed? [4]

11. In how many ways can final eleven be selected from 15 cricket players' if [6]
(i) there is no restriction
(ii) one of then must be included
(iii) cone of them, who is in bad form, must always be excluded
(iv) Two of then being leg spinners, one and only one leg spinner must be included?

12. How many four letter words can be formed using the letters of the letters of the word 'FAILURE' so that [6]
(i) F is included in each word
(ii) F is excluded in each word.

TEST PAPER-04

CLASS - XI MATHEMATICS (Permutation and Combinations)

[ANSWERS]

Ans1. $\dfrac{1\times2\times3\times4\times5\times6\times7\times8\times9}{1\times2\times3\times4}$

$=\dfrac{\underline{|9}}{\underline{|4}}$

Ans2. $\dfrac{\underline{|n}}{\underline{|n-r}}=\dfrac{\underline{|5}}{\underline{|5-2}}=\dfrac{\underline{|5}}{\underline{|3}}$

$=\dfrac{5.4.\cancel{\underline{|3}}}{\cancel{\underline{|3}}}=20$

Ans3. $\left({}^{15}C_8+{}^{15}C_9\right)-\left({}^{15}C_6+{}^{15}C_7\right)\quad\left[{}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r\right]$

$={}^{16}C_9-{}^{16}C_7=0\qquad\left[\because\ {}^{16}C_9={}^{16}C_7\right]$

Ans4. 2^n

Ans5. Given ${}^{2n}C_3:{}^{n}C_3=11:1\Rightarrow\dfrac{\underline{|2n}}{\underline{|3}\ \underline{|2n-3}}\times\dfrac{\underline{|3}\ \underline{|n-3}}{\underline{|n}}=\dfrac{11}{1}$

$\Rightarrow\dfrac{2n(2n-1)(2n-2)\underline{|2n-3}}{\underline{|2n-3}}\times\dfrac{\underline{|n-3}}{n(n-1)(n-2)\underline{|n-3}}=\dfrac{11}{1}$

$\Rightarrow\dfrac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}=\dfrac{11}{1}$

$\Rightarrow\dfrac{4(2n-1)(n-1)}{(n-1)(n-2)}=\dfrac{11}{1}\Rightarrow\dfrac{4(n-1)}{n-2}=\dfrac{11}{1}$

$\Rightarrow 11n-22=8n-4\Rightarrow 3n=18$

$\Rightarrow n=b$

Ans6. In a deck of 52 cards, there are 4 aces and 48 other cards. Here, we have to choose exactly one ace 4 other cards.

The number of ways of choosing one ace out of 4 aces = ${}^{4}C_{1}$.

The number of ways of choosing 4 cards out of the other 48 cards = ${}^{48}C_{4}$.

Corresponding to one way of choosing an ace, there are ${}^{48}C_{4}$ ways of choosing 4 other cards. But there are ${}^{4}C_{1}$ ways of choosing aces, there fore, the required number of ways

$$= {}^{4}C_{1} \times {}^{48}C_{4} = \frac{4}{1} \times \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} = 778320$$

Ans7 Number greater than 56000 and farmed by using the digits 4,5,6,7,8 are of types

$5(6/7/8)\times\times\times$ or $(6/7/8)\times\times\times$

Now numbers of type $5(6/7/8)\times\times\times$ are $1\times3\times3\times2\times1=18$ in number

Number of type $(6/7/8)\times\times\times\times$ are $3\times4\times3\times2\times1=72$ in number

Hence required number of numbers is $18+72=90$

Ans8. Given $\frac{\lfloor n}{\lfloor 2 \lfloor n-2} : \frac{\lfloor n}{\lfloor 4 \lfloor n-4} = 2:1$

$$\Rightarrow \frac{\lfloor n}{\lfloor 2 \lfloor n-2} \times \frac{\lfloor 4 \lfloor n-4}{\lfloor n} = \frac{2}{1}$$

$$\Rightarrow \frac{4\times3\times\lfloor 2 \times \lfloor n-4}{\lfloor 2 \times (n-2)\times(n-3)\times\lfloor n-4} = \frac{2}{1}$$

$$\Rightarrow \frac{4\times3}{(n-2)(n-3)} = \frac{2}{1} \Rightarrow (n-2)(n-3) = 6$$

$$\Rightarrow n^2 - 5n = 0 \Rightarrow n(n-5) = 0$$

$$\Rightarrow n = 0 \text{ or } n = 5$$

But, for $n=0$, $\lfloor n-2$ and $\lfloor n-4$ are not meaningful, therefore, $n=5$.

Ans9. $\lfloor 2n = 1.2.3.4.5.6......(2n-1)(2n)$

$$= \left[1.3.5......(2n-1)\right]\left[2.4.6......2n\right]$$

$=\left[1.3.5.......(2n-1)\right]\left[(2.1)(2.2)(2.3)......(2.n)\right]$

$=1.3.5.......(2n-1).2^n.(1.2.3......n)$

$=1.3.5.......(2n-1).2^n.\underline{|n}$, as desired,

Ans10. There are 10 letters in the word 'LOGARITHMS'

For making 4 letter word we take 4 at a time

$\therefore$ No. of arrangements 10 letters taken 4 at a time

$= {}^{10}P_4 = 5040$

Ans11. (i) 11 players can be selected out of 15 in ${}^{15}C_{11}$ ways

$= {}^{15}C_4 \text{ ways} = \dfrac{15\times14\times13\times12}{1\times2\times3\times4} \text{ ways}$ =1365 ways

(ii) Since a particular player must be included, we have to select 10 more out of remaining 14 players.

This can be done in ${}^{14}C_{10}$ ways ${}^{14}C_4$ ways

$= \dfrac{14\times13\times12\times11}{1\times2\times3\times4}$ ways = 1001 ways

(iii) Since a particular player must be always excluded, we have to choose 11 ways out of remaining 14

This can be done in ${}^{14}C_{11} \text{ ways} = {}^{14}C_3 \text{ ways}$

$= \dfrac{14\times13\times12\times11}{1\times2\times3} \text{ ways} = 364 \text{ ways.}$

(iv) One leg spinner can be chosen out of 2 in ${}^{2}C_1$ ways. Then we have to select 10 more players get of 13 (because second leg spinner can't be included). This can be done in ${}^{13}C_{10}$ ways of choosing 10 players. But these are ${}^{2}C_1$ ways of choosing a leg spinner, there fore, by multiplication principle of counting the required number of ways

$= {}^{2}C_1 \times {}^{13}C_{10}$

$= {}^{2}C_1 \times {}^{13}C_3 = \dfrac{2}{1} \times \dfrac{13\times12\times11}{1\times2\times3} = 572$

Ans12. There are 7 letters in the word 'FAILVRE'

(i) F is included for making each word using 4 letters

$\therefore$ F is already selected

Then other 3 letters can be selected out of 6 are ${}^{6}C_{3}$ ways

Also arrangements of 4 letters are 4:

$\therefore$ Total no. of words $= {}^{6}C_{3} \times 4: \; = 480$

(ii) F is excluded in each word

$\therefore$ Out of 6 letters are choose 4 letters in ${}^{6}C_{4}$ ways

Also arrangement of 4 letters are 4:

$\therefore$ Total no. of words $= {}^{6}C_{4} \times 4: \; = 360$

TEST PAPER-05

CLASS - XI MATHEMATICS (Permutation and Combinations)

1. Evaluate ${}^{10}C_7 + {}^{10}C_6$ [1]

2. If $1 \le r \le n$ then what is the value of $\frac{n}{r}\ {}^{n-1}C_{r-1}$ [1]

3. How many 3 digit numbers can be formed by using the digits 1 to 9 if no. digit is repeated [1]

4. Convert into factorial 2.4.6.8.10.12 [1]

5. How many words with or without meaning can be formed using all the letters of the word 'EQUATION' at a time so that vowels and consonants occur together [4]

6. From a class of 25 students 10 are to be chosen for an excursion Party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can excursion party be chosen? [4]

7. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour. [4]

8. Find the number of 3 digit even number that can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no digit is repeated? [4]

9. If ${}^{n}\mathrm{P}_{\mathrm{r}} = {}^{n}\mathrm{P}_{\mathrm{r+1}}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$ find the values of n and r [4]

10. Prove that the product r of consecutive positive integer is divisible by $\lfloor r$ [4]

11. A committee of 5 is to be formed out of 6 gents and 4 Ladies. In how many ways this can be done, when [6]

    (i) at least two ladies are included?

    (ii) at most two ladies are included?

12. In how many ways can the letters of the word PERMUTATIONS be arranged if the [6]

    (i) words start with P and with S

    (ii) vowels are all together

    (iii) There are always 4 letters between P and S?

TEST PAPER-05

CLASS - XI MATHEMATICS (Permutation and Combinations)

[ANSWERS]

Ans1. $^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}$

$^{10}C_{7}+{}^{10}C_{6}={}^{10+1}C_{7}$

$={}^{11}C_{7}=\dfrac{\underline{|11}}{\underline{|11-7}\,\underline{|7}}$

$=\dfrac{\underline{|11}}{\underline{|4}\;\underline{|7}}=\dfrac{11.10.\cancel{9}_{3}.\cancel{8}^{2}.\cancel{\underline{|7}}}{\cancel{4}.\cancel{3}.\cancel{2}.1.\cancel{\underline{|7}}}=330$

Ans2. $^{n}C_{r}$

Ans3.

H	T	U
7	8	9

$9\times8\times7=504$

Ans4. $2\times1.\ 2\times2.\ 2\times3.\ 2\times4.\ 2\times5.\ 2\times6$

$=2^{6}\left[1.2.3.4.5.6\right]\ =2^{6}\ \underline{|6}$

Ans5. In the word 'EQUATION' there are 5 vowels [A.E.I.O.U.] and 3 consonants [Q.T.N]

Total no. of letters = 8

Arrangement of 5 vowels = $\underline{|5}$

Arrangements of 3 consonants = $\underline{|3}$

Arrangements of vowels and consonants = $\underline{|2}$

$\therefore$ Total number of words $=\underline{|5}\times\underline{|3}\times\underline{|2}$

$=5.4.3.2.1\times3.2.1\times2.1=1440$

Ans6. Total no. of students = 25

No. of students to be selected = 10

I case :

3 students all of them will join the excursion party.

Then remaining 7 students will be selected out of (25-3 = 22) in $^{22}C_{7}$ ways

II case :

All 3 students will not join the party then 10 students will be selected in ${}^{22}C_{10}$ ways

Total no. of selection $= {}^{22}C_7 + {}^{22}C_{10}$

$$= \frac{\underline{|22}}{\underline{|15}\ \underline{|7}} + \frac{\underline{|22}}{\underline{|12}\ \underline{|10}} \quad = 817190$$

Ans7. No. of red balls = 6

No. of white balls = 5

No. of blue balls = 5

No. of selecting each colour balls = 3

$\therefore$ required no. of selection = ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$$= \frac{\underline{|6}}{\underline{|3}\ \underline{|3}} \times \frac{\underline{|5}}{\underline{|2}\ \underline{|3}} \times \frac{\underline{|5}}{\underline{|2}\ \underline{|3}}$$

$$= \frac{\cancel{6}.5.4.\cancel{\underline{|3}}}{\cancel{\underline{|3}}.\cancel{3}.\cancel{2}.1} \times \frac{5.\cancel{4}^{2}.\underline{|3}}{\cancel{2}.\cancel{1}.\underline{|3}} \times \frac{5.{}^{2}\cancel{4}.\cancel{\underline{|3}}}{\cancel{2}.\cancel{1}.\cancel{\underline{|3}}}$$

$$= 5 \times 4 \times 5 \times 2 \times 5 \times 2$$

$$= 20 \times 10 \times 10 = 2000$$

Ans8. For making 3 digit even numbers unit place of digit can be filled in 3 ways. Ten's place of digit can be filled in 5 ways. Hundred place of digit can be filled in 4 ways. $\therefore$ Required number of 3 digit even number $= 3 \times 5 \times 4 \ = 60$

Ans9. Given that ${}^n\mathrm{P}_{\mathrm{r}} = {}^n\mathrm{P}_{\mathrm{r+1}}$

$$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}} = \frac{\underline{|n}}{\underline{|n-r-1}}$$

$$\Rightarrow \frac{1}{(n-r)\ \underline{|n-r-1}} = \frac{1}{\underline{|n-r-1}}$$

$$= n - r = 1 \ldots\ldots (i)$$

And ${}^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}\ \underline{|r}} = \frac{\underline{|n}}{\underline{|n-r+1}\ \underline{|r-1}}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$$

$$\Rightarrow n - r + 1 = r \Rightarrow n - 2r = -1 \ldots\ldots (ii)$$

Solving eq. (i) and (ii)

$n = 3, \quad r = 2$

Ans10. Suppose & consecutive positive integers are $(n+1),\ (n+2),\ ......(n+r)$

Then product $=(n+1).(n+2).(n+3)......(n+r)$

$$=\frac{\lfloor n\ \ (n+1).(n+2).(n+3)......(n+r)}{\lfloor n}$$

$$=\frac{1.2.3......n.(n+1)(n+2)(n+3)......(n+r)}{\lfloor n}$$

$$=\frac{\lfloor n+r}{\lfloor n}=\frac{\lfloor n+r}{\lfloor r\ \lfloor n+r-r}\quad \lfloor r$$

$=\left({}^{n+r}C_r\right)\ \lfloor r$ which is divisible by $\lfloor r$

Ans11. No of person to form committee = 5

No. of gents = 6 and No. of ladies = 4

(i) At least two ladies are included

Ladies [4] Gents [6]

Either we Select $\rightarrow 2$ and 3

or

$\rightarrow 3$ and 2

Or

$\rightarrow$ 4 and 1

$\therefore$ required number of selection

$= {}^4C_2\times{}^6C_3+{}^4C_3\times{}^6C_2+{}^4C_4\times{}^6C_1$

$120+60+6=186$

(ii) At most two ladies are included?

		Ladies [4]		Gents [6]
Either we select		0	and	5
	Or	1	and	4
	Or	2	and	3

$\therefore$ required no. of selection.

$= {}^4C_0\times{}^6C_5\times{}^4C_1\times{}^6C_4+{}^4C_2\times{}^6C_3$

$6+60+120=186$

In the word PERMUTATIONS there are 12 letters

It the word start with P and end with S then position of P and S will be fixed. Then other 10 letters can be arranged in $\lfloor 10$ ways. But T occurs twice.

$\therefore$ no. of arrangements $=\dfrac{\lfloor 10}{\lfloor 2}$

=1814400 ways

(iii) Vowels are together?

No. of vowels in the word PERMUTATIONS are 5 which are [A, E, I, O, U]

$\therefore$ Vowels can be arranged is $|\underline{5}$ ways other letters are consonants out of 8 consonants 2 are repeated

$\therefore$ No. of arrangements of consonants $= \frac{|\underline{8}}{|\underline{2}}$

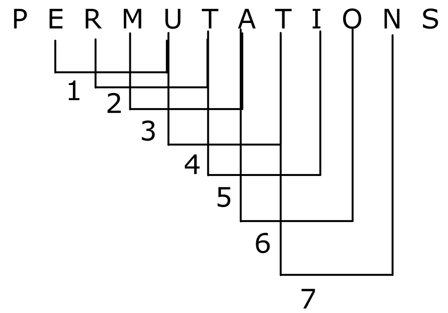
$\therefore$ requires no. arrangements $= \frac{|\underline{8}}{|\underline{2}} \times |\underline{5} = 2419200$

Ans12. There are always 4 letters between P and S. in the word 'PERMUTATIONS'

If 4 letters between P and S

Then P and S can be arranged in 2 ways other 10 letters can be arranged in $\frac{|\underline{10}}{|\underline{2}}$ ways

There are 7 pair 4 letters in the words PERMUTATIONS between P and S



$\therefore$ Required no. of arrangements = $\frac{|\underline{10}}{\cancel{2}} \times \cancel{2} \times 7$

= 2 540100

TEST PAPER-01

CLASS - XI MATHEMATICS (Binomial theorem)

1. Which is larger $(1.01)^{10,00000}$ or $10,000$ [4]

2. Prove that $\sum_{r=0}^{n} 3^r \ {}^nC_r = 4^n$ [4]

3. Using binomial theorem, prove that $6^n - 5^n$ always leaves remainder 1 when divided by 25. [4]

4. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$ [4]

5. Find the term independent of x in the expansion of $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}$, $x > 0$ [4]

6. Find the coefficient of x^5 in the expansion of the product $(1+2x)^5(1-x)^7$ [4]

7. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$ [6]

8. The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n [6]

TEST PAPER-01

CLASS - XI MATHEMATICS (Binomial theorem)

[ANSWERS]

Ans1. $(1.01)^{10,00000} = (1+0.01)^{10,00000}$

$= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other positive term}$

$=1+10,00000\times 0.01+\text{other positive term}$

$=1+10,000$

$=10,001$

Hence $(1.01)^{10,00000} > 10,000$

Ans2. $\sum_{r=0}^{n} 3^r \ {}^nC_r = \sum_{r=0}^{n} {}^nC_r .3^r$

$= {}^nC_0 + {}^nC_1 .3 + {}^nC_2 .3^2 + ...+ {}^nC_n 3^n$

$\left[\because (1+a)^n = 1+ {}^nC_1 .a + {}^nC_2\, a^2 + {}^nC_3\, a^3 + ...+ a^n\right]$

$=(1+3)^n$

$=(4)^n$

$H.P$

Ans3. Let $6^n = (1+5)^n$

$=1+ {}^nC_1 5^1 + {}^nC_2 5^2 + {}^nC_3 5^3 + ...+ 5^n$

$=1+5n+5^2\left({}^nC_2 + {}^nC_3 .5 + ...+ 5^{n-2}\right)$

$6^n - 5n = 1+25\left({}^nc_2 + {}^nc_3 .5 + ...+ 5^{n-2}\right)$

$=1+25k\left[\text{where } k = {}^nc_2 + {}^nc_3 .5 + ...5^{n-2}\right]$

$=25k+1$

$H.P$

Ans4. The general term in the expansion of

$$\left(9x-\frac{1}{3\sqrt{x}}\right)^{18} \text{ is}$$

$$T_{r+1}={}^{18}c_{r}(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^{r}$$

For 13th term, $r+1=13$

$r=12$

$$={}^{18}c_{12}(9x)^{6}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$={}^{18}c_{12}(3)^{12}.x^{6}\left(-\frac{1}{3}\right)^{12}.(x)^{-6}$$

$$={}^{18}c_{12}(3)^{12}.(-1)^{12}.(3)^{-12}$$

$$={}^{18}c_{12}$$

$$=18564$$

Ans5.

$$T_{r+1}={}^{18}c_{r}\left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^{r}$$

$$={}^{18}c_{r}(x)^{\frac{18-r}{3}}.\left(\frac{1}{2}\right)^{r}.x^{\frac{-r}{3}}$$

$$={}^{18}c_{r}(x)^{\frac{18-r-r}{3}}.\left(\frac{1}{2}\right)^{r}$$

For independent term $\frac{18-2r}{3}=0$

$r=9$

The req. term is ${}^{18}c_{9}\left(\frac{1}{2}\right)^{9}$

Ans6.

$$(1+2x)^{6}(1-x)^{7}=\left(1+{}^{6}c_{1}(2x)+{}^{6}c_{2}.(2x)^{2}+{}^{6}c_{3}(2x)^{3}+{}^{6}c_{4}(2x)^{4}+{}^{6}c_{5}(2x)^{5}+{}^{6}c_{6}(2x)^{6}\right)$$

$$\left(1-{}^{7}c_{1}x+{}^{7}c_{2}(x)^{2}-{}^{7}c_{3}(x)^{3}+{}^{7}c_{4}(x)^{4}-{}^{7}c_{5}(x)^{5}+{}^{7}c_{6}(x)^{6}-{}^{7}c_{7}(x)^{7}\right)$$

$$=\left(1+12x+60x^2+160x^3+240x^4+192x^5+64x^6\right).\left(1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7\right)$$

Coeff of x^5 is

$$=1\times(-21)+12\times35+60(-35)+160\times21+240\times(-7)+192\times1$$

$$=171$$

Ans7. Fifth term from the beginning in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is

$$T_{4+1}={}^nc_4\left(\sqrt[4]{2}\right)^{n-4}.\left(\frac{1}{\sqrt[4]{3}}\right)^4$$

$$T_5={}^nc_4(2)^{\frac{n-4}{4}}.(3)^{-1}\,......(i)$$

How fifth term from the end world be equal to $(n-5+2)$ in term from the beginning

$$T_{(n-4)+1}={}^nc_{n-4}\left(\sqrt[4]{2}\right)^{n-(n-4)}.\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$={}^nc_{n-4}(2)^1(3)^{\frac{n-4}{4}}\,.......(ii)$$

ATQ $\quad \dfrac{{}^nc_4.(2)^{\frac{n-4}{4}}(3)^{-1}}{{}^nc_{n-4}(2)^1(3)^{\frac{n-4}{-4}}}=\dfrac{\sqrt{6}}{1}$

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}}=(6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}}=(6)^{\frac{1}{2}}$$

$$\frac{n-8}{4}=\frac{1}{2}$$

$$\Rightarrow 2n-16=4$$

$$n=10$$

Ans8. Let three consecutive terms in the expansion of $(1+a)^n$ are $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ term

$$T_{r+1}={}^nc_r(1)^{n-r}.(a)^r$$

$$T_{r+1}={}^nc_r\ (a)^r$$

Coefficients are

${}^{n}c_{r-2},\ {}^{n}c_{r-1}$ and ${}^{n}c_{r}$ respectively

ATQ $\dfrac{{}^{n}c_{r-2}}{{}^{n}c_{r-1}} = \dfrac{1}{7}$

$\Rightarrow n - 8r + 9 = 0 \ldots\ldots.(i)$

$\dfrac{{}^{n}c_{r-1}}{{}^{n}c_{r}} = \dfrac{7}{42}$

$\Rightarrow n - 7r + 1 = 0 \ldots\ldots(ii)$

On solving eq. (i) and (ii) we get $n = 55$

TEST PAPER-02

CLASS - XI MATHEMATICS (Binomial theorem)

1. Compute $(98)^5$ [4]

2. Expand $\left(x+\frac{1}{x}\right)^6$ [4]

3. Find the fourth term from the end in the expansion of $\left(\frac{x}{x^2}-\frac{x^3}{3}\right)^9$. [4]

4. Find the middle term of $\left(2x-\frac{x^2}{4}\right)^9$. [4]

5. Find the coefficient of a^5b^7 in $(a-2b)^{12}$. [4]

6. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6. [4]

7. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$. [4]

8. Find a if the coeff. of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal [4]

9. The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 240, 720 and 1080 respectively. Find x, a and n. [6]

10. If a and b are distinct integers, prove that a-b is a factor of a^n-b^n, whenever n is positive. [6]

TEST PAPER-02

CLASS - XI MATHEMATICS (Binomial theorem)

[ANSWERS]

Ans1. $(98)^5 = (100-2)^5$

$$= {}^5C_0(100)^5 - {}^5C_1(100)^4.2 + {}^5C_2(100)^3.2^2$$
$$- {}^5C_3(100)^2.(2)^3 + {}^5C_4(100)(2)^4 - {}^5C_5(2)^5$$

$$= 1000000000 - 5\times100000000\times2 + 10\times1000000\times4$$
$$-10\times10000\times8 + 5\times100\times16 - 32$$

$$= 100\ 4\ 000\ 8000 - 1000\ 8000\ 32 = 9039207968$$

Ans2. $\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x^5)\left(\frac{1}{x}\right) + {}^6C_2(x^4)\left(\frac{1}{x}\right)^2 +$

$${}^6C_3(x^3)\left(\frac{1}{x}\right)^3 + {}^6C_4(x^2)\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\left(\frac{1}{x^2}\right) + 20x^3.\left(\frac{1}{x^3}\right) + 15x^2.\frac{1}{x^4} + 6x.\frac{1}{x^5} + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

Ans3. Fourth term from the end would be equal to $(9-4+2)^{th}$ term from the beginning

$$T_7 = T_{6+1} = {}^9C_6\left(\frac{3}{x^2}\right)^{9-6}.\left(\frac{-x^3}{3}\right)^6$$

$$= {}^9C_6(3)^3.(x)^{-6}.(x)^{18}.(3)^{-6}$$

$$= \frac{9!}{6!3!}.(3)^{-3}.(x)^{12}$$
$$= \frac{28}{9}x^{12}$$

Ans4. $n=9$ so there are two middle term

i.e $\left(\frac{9+1}{2}\right)^{th}$ term and $\left(\frac{9+1}{2}+1\right)^{th}$ term

$T_5 = T_{4+1} = {}^9C_4 (2x)^{9-4} . \left(\frac{-x^2}{4}\right)^4$

$= \frac{63}{4} x^{13}$

$T_6 = T_{5+1} = {}^9C_5 (2x)^{9-5} \left(\frac{-x^2}{4}\right)^5$

$= -{}^9C_4 (2)^4 . x^4 \ \frac{(x)^{10}}{(4)^5}$

$= \frac{-63}{32} \ x^{14}$

Ans5. $T_{r+1} = {}^{12}C_r (a)^{12-r} . (-2b)^r$

Put $12-r=5$

$r=7$

$T_8 = {}^{12}C_7 (a)^5 . (-2b)^7$

$= {}^{12}C_7 (a)^5 . (-2)^7 . b^7$

coeff. Of a^5 b^7 is ${}^{12}C_7 \ (-2)^7$

Ans6. $T_{r+1} = {}^mC_r (1)^{m-r} . (x)^r$

$= {}^mC_r (x)^r$

Put $r=2$

ATQ ${}^mC_2 = 6$

$\frac{m!}{2!(m-2)!} = \frac{6}{1}$

$$\frac{m(m-1)\cancel{(m-2)!}}{2\times1\times\cancel{(m-2)!}}=\frac{6}{1}$$

$m^2-m=12$

$m^2-m-12=0$

$m(m-4)=3(m-4)=0$

$(m-4)(m-3)=0$

$m=4$

$m=-3(\text{neglect})$

Ans7. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2}+1\right)^{th}$ term

i.e $(n+1)^{th}$ term

$T_{r+1}={}^{2n}C_r\ (1)^{2n-r}.(x)^r$

Coeff. of x^n is ${}^{2n}C_r$

And $(2n-1)$ is odd so two middle term

$\left(\frac{2n-1+1}{2}\right)^{th}$ and $\left(\frac{2n-1+1}{2}+1\right)^{th}$

i.e n^{th} and $(n+1)^{th}$ term

The coefficients of these terms are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

Now ATQ

${}^{2n-1}C_{n-1}+{}^{2n-1}C_n={}^{2n}C_n$ $\qquad\left[\because {}^{n}C_{r-1}+{}^{n}C_r={}^{n+1}C_r\right]$

$H.P$

Ans8. $T_{r+1}={}^{9}C_r(3)^{9-r}.(ax)^r$

ATQ

${}^{9}C_2(3)^7.a^2={}^{9}C_3(3)^6.a^3$

${}^{9}C_2(3)^1={}^{9}C_3.a$

$$\frac{9!}{2!7!}\times 3=\frac{9!}{3!6!}a$$

$$\frac{3!6!\times 3}{2!7!}=a$$

$$\frac{3\times 2\times 1\times 6!\times 3}{2\times 1\times 7\times 6!}=a$$

$$\frac{9}{7}=a$$

Ans9. $T_2=240$

$${}^nC_1\, x^{n-1}.a=240........(i)$$

$${}^nC_2\, x^{n-2}.a^2=720.......(ii)$$

$${}^nC_3\, x^{n-3}.a^3=1080........(iii)$$

Divide (ii) by (i) and (iii) by (ii)

We get

$$\frac{a}{x}=\frac{6}{n-1} \text{ and } \frac{a}{x}=\frac{9}{2(n-2)}$$

$\Rightarrow n=5$

On solving we get

$x=2$

$a=3$

Ans10. Let $a^n=(a-b+b)^n$

$$a^n=(b+a-b)^n$$

$$={}^nC_0\, b^n+{}^nC_1\, b^{n-1}(a-b)+{}^nC_2\, b^{n-2}.(a-b)^2+{}^nC_3\, b^{n-3}.(a-b)^3+....+{}^nC_n(a-b)^n$$

$$a^n=b^n+(a-b)\left[{}^nC_0\, b^n+{}^nC_1\, b^{n-1}(a-b)+{}^nC_2\, b^{n-2}.(a-b)^2+{}^nC_3\, b^{n-3}.(a-b)^3+....+{}^nC_n(a-b)^n\right]$$

$a^n-b^n=(a-b)k$ Where

$${}^nC_1\, b^{n-1}+{}^nC_2\, b^{n-2}(a-b)+......+(a-b)^{n-1}=k$$

$H.P$

TEST PAPER-03

CLASS - XI MATHEMATICS (Binomial theorem)

1. Find $(a+b)^4-(a-b)^4$. Hence evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4+\left(\sqrt{3}-\sqrt{2}\right)^4$. [4]

2. Show that $9^{n+1}-8n-9$ is divisible by 64, whenever n is positive integer. [4]

3. Find the general term in the expansion of $\left(x^2-yx\right)^{12}$ [4]

4. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal. [4]

5. Expand $\left(1-x+x^2\right)^4$ [4]

6. Find the sixth term of the expansion $\left(y^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third term from the end is 45. [4]

7. The sum of the coeff. 0f the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m$ m being natural no. is 559. Find the term of expansion. [6]

8. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.....(2n-1)}{n!}2^n.x^n$. [6]

TEST PAPER-03

CLASS - XI MATHEMATICS (Binomial theorem)

[ANSWERS]

Ans1. $(a+b)^4-(a-b)^4=\left({}^4C_0\,a^4+{}^4C_1\,a^3b+{}^4C_2\,a^2b^2+{}^4C_3\,ab^3+{}^4C_4\,b^4\right)$

$$-\left(a^4-{}^4C_1\,a^3b+{}^4C_2\,a^2b^2-{}^4C_3\,ab^2+{}^4C_4\,b^4\right)$$

$$=2\left({}^4C_1\,a^3b+{}^4C_3\,ab^3\right)$$
$$=2\left(4a^3b+4ab^3\right)$$
$$=8ab\left(a^2+b^2\right)$$

Put $a=\sqrt{3}, \quad b=\sqrt{2}$

$$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=8\sqrt{3}.\sqrt{2}\,(3+2)$$
$$=40\sqrt{6}$$

Ans.2 $(9)^{n+1}=(1+8)^{n+1}$

$$=1+{}^{n+1}C_1\,8^1+{}^{n+1}C_2\,.8^2+{}^{n+1}C_3\,8^3+........+{}^{n+1}C_{n+1}\,8^{n+1}$$

$$=1+(n+1).8+8^2\left[{}^{n+1}C_2+{}^{n+1}C_3\,.8+.....+8^{n-1}\right]$$

$$9^{n+1}-8n-9=64\left[{}^{n+1}C_2+{}^{n+1}C_3\,.+.......+8^{n-1}\right]$$

$$9^{n+1}-8n-9=64k,\ \text{where } k=\left[{}^{n+1}C_2+{}^{n+1}C_3\,.8+....+8^{n+1}\right]$$

Ans3. $T_{r+1}={}^{12}C_r\left(x^2\right)^{12-r}.(-yx)^r$

$$={}^{12}C_r\,(x)^{24-r}.(-1)^r.y^r.x^r$$
$$={}^{12}C_r\,(-1)^r.y^r.(x)^{24-2r}$$

Ans4. $T_{r+1}={}^{m+n}C_r\,(1)^{m+n-r}.(a)^r$

$T_{r+1} = {}^{m+n}C_r (a)^r \text{.........}(i)$

Put $r = m$ and $r = n$ respectively

$T_{m+1} = {}^{m+n}C_m a^m$

Coeff of a^m is ${}^{m+n}C_m \quad \Rightarrow \dfrac{(m+n)!}{m!n!}$

Coeff of a^n is ${}^{m+n}C_m \quad \Rightarrow \dfrac{(m+n)!}{n!m!}$ H.P

Ans5. $(1-x+x^2)^4 = \left[(1-x)+x^2\right]^4$

$= {}^4C_0(1-x)^4 + {}^4C_1(1-x)^3.(x^2) + {}^4C_2(1-x)^2.(x^2)^2 + {}^4C_3(1-x)^1(x^2)^3 + {}^4C_4(x^2)^4$

$= (1-x)^4 + 4(1-x)^3.x^2 + 6(1-x)^2.x^4 + 4(1-x).x^6 + 1.x^8$

$= (1-4x+6x^2-4x^3+x^4) + 4(1-3x+3x^2-x^3)x^2 + 6(1-2x+x^2)(x^4) + 4(1-x).x^6 + x^8$

$= 1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$

Ans6. The binomial coeff of the third term from and = binomial coeff of the third term from beginning = ${}^nC_2$

${}^nC_2 = 45$

$\dfrac{n(n-1)}{1.2} = 45$

$n^2 - n - 90 = 0$

$n = 10$

$T_{r+1} = {}^{10}C_r \left(y^{\frac{1}{2}}\right)^{10-r} . \left(x^{\frac{1}{3}}\right)^r$

$r = 5$

$T_6 = {}^{10}C_5 \left(y^{\frac{1}{2}}\right)^5 . \left(x^{\frac{1}{3}}\right)^5$

$= 252 y^{\frac{5}{2}} . x^{\frac{5}{3}}$

Ans7. The coeff. Of the first three terms of $\left(x-\frac{3}{x^2}\right)^m$ are ${}^mC_0, (-3)\,{}^mC_1$ and $9\ {}^mC_2$.

Therefore, by the given condition

$${}^mC_0 - 3\,{}^mC_1 + 9\,{}^mC_2 = 559$$

$$1-3m+\frac{9m(m-1)}{2}=559$$

On solving we get $m=12$

$$T_{r+1} = {}^{12}C_r (x)^{12-r}\left(\frac{-3}{x^2}\right)^r$$

$$= {}^{12}C_r (x)^{12-r}.(-3)^r.(x)^{-2r}$$

$$= {}^{12}C_r (x)^{12-3r}.(-3)^r$$

$12-3r=0 \quad \Rightarrow \quad r=3, \quad$ *req.* term is -5940 x^3

Ans8. As $2n$ is even, the middle term of the expansion $(1+x)^{2n}$ is $(n+1)^{th}$ term

$$T_{n+1} = {}^{2n}C_n (1)^{2n-n}.x^n$$

$$= {}^{2n}C_n x^n$$

$$=\frac{(2n)!}{n!n!}.x^n$$

$$=\frac{(2n)(2n-1)(2n-2)......4.3.2.1}{n!n!}x^n$$

$$=\frac{1.2.3.4......(2n-2)(2n-1)(2n)}{n!n!}x^n$$

$$=\frac{[1.3.5.....(2n-1)][2.4.6.....(2n)]}{n!n!}x^n$$

$$=\frac{[1.3.5....(2n-1).2^n.(1.2.3.....n)]}{n!n!}x^n$$

$$=\frac{[1.3.5....(2n-1)].2^n.\cancel{n!}}{\cancel{n!}n!}.x^n$$

$$=\frac{[1.3.5...(2n-1)].2^n.x^n}{n!}$$

TEST PAPER-04

CLASS - XI MATHEMATICS (Binomial theorem)

1. What is The middle term in the expansion of $(1+x)^{2n+1}$ [1]

2. When n is a positive integer, the no. of terms in the expansion of $(x+a)^n$ is [1]

3. Write the general term $\left(x^2-y\right)^6$ [1]

4. In the expansion of $\left(x+\frac{1}{x}\right)^6$, find the 3rd term from the end [1]

5. Expand $(1+x)^n$ [1]

6. Find a if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal. [4]

7. Find the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$ [4]

8. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^6$ [4]

9. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7th term from the beginning to the 7th term the end is 1:6 find n [6]

10. If the coeff. Of 5th 6th and 7th terms in the expansion of $(1+x)^n$ are in A.P, then find the value of n. [6]

TEST PAPER-04

CLASS - XI MATHEMATICS (Binomial theorem)

[ANSWERS]

Ans1. Since $(2n+1)$ is odd there is two middle term

$i.e\ {}^{2n+1}C_n\, x^{n+1}$ and ${}^{2n+1}C_{n+1}.x^n$

Ans2. The no. of terms in the expansion of $(x+a)^n$ is one more than the index

$n.\quad i.e\quad (n+1).$

Ans3. $T^{r+1} = {}^6C_r\left(x^2\right)^{6-r}.(-y)^r$

$= {}^6C_r(x)^{12-2r}.(-1)^r.(y)^r$

Ans4. 3rd term form end $=(6-3+2)^{th}$ term from beginning

i.e

$T_5 = {}^6C_4(x)^{6-4}.\left(\frac{1}{x}\right)^4$

$= {}^6C_4\, x^2.x^{-4}$

$=15^{x-2}$

$=\frac{15}{x^2}$

Ans5. $(1+x)^n = 1+{}^nC_1(x)^1+{}^nC_2(x)^2+{}^nC_3(x)^3+......x^n$

Ans6. $T_{r+1} = {}^{50}C_r(2)^{50-r}.(a)^r$

ATQ put $r=16$ and 17

$\Rightarrow {}^{50}C_{16}(2)^{34}.a^{16} = {}^{50}C_{17}(2)^{33}.a^{17}$

$a = \frac{{}^{50}C_{16}\times 2}{{}^{50}C_{17}}$

$a=1$

Ans7. $T_{r+1} = {}^{18}C_r (9x)^{18-r} . \left(\frac{-1}{3\sqrt{x}}\right)^r$

Put $r = 12$

$T_{13} = {}^{18}C_{12} (9x)^{18-12} . \left(\frac{-1}{3}\right)^{12} . (x)^{\frac{-12}{2}}$

$= {}^{18}C_{12} (3)^{12} . x^6 . (3)^{-12} . (-1)^{12} . x^{-6}$

$= {}^{18}C_{12}$

Ans8. $T_{r+1} = {}^{6}C_r \left(\frac{3}{2}x^2\right)^{6-r} . \left(\frac{-1}{3x}\right)^r$

$= {}^{6}C_r \left(\frac{3}{2}\right)^{6-r} . (x)^{12-2r} . \left(\frac{-1}{3}\right)^r x^{-r}$

$= {}^{6}C_r \left(\frac{3}{2}\right)^{6-r} . \left(\frac{-1}{3}\right)^r . (x)^{12-3r}$

Put $12 - 3r = 0$

$r = 4$

$= {}^{6}C_4 \left(\frac{3}{2}\right)^2 . \left(\frac{-1}{3}\right)^4 .$

$= \frac{5}{12}$

Ans9. $T_7 = {}^{n}C_6 \left(\sqrt[3]{2}\right)^{n-6} . \left(\frac{1}{\sqrt[3]{3}}\right)^6$

$= {}^{n}C_6 (2)^{\frac{n-6}{3}} . (3)^{-2}$ ....(i)

7[th] term from end $= (n-7+2)$ term from beginning

$T_{n-6+1} = {}^{n}C_{n-6} \left(\sqrt[3]{2}\right)^{n-n+6} . \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$

$= {}^{n}C_{n-6} (2)^2 . (3)^{\frac{n-6}{-3}}$ ......(ii)

ATQ $\frac{{}^{n}C_6 (2)^{\frac{n-6}{2}} . (3)^{-2}}{{}^{n}C_{n-6} (2)^2 (3)^{\frac{6-n}{3}}} = \frac{1}{6}$

$$\frac{(2)^{\frac{n-12}{3}}}{(3)^{\frac{12-n}{3}}}=\frac{1}{6}$$

$$(6)^{\frac{n-12}{3}}=(6)^{-1}$$

$$\frac{n-12}{3}=\frac{-1}{1}$$

$$n-12=-3$$

$$n=9$$

Ans10. $T_{r+1}={}^{n}C_{r}(1)^{n-r}.(x)^{r}$

$T_{r+1}={}^{n}C_{r}x^{r}........(i)$

Coeff of 5^{th}, 6^{th}, 7^{th} terms in the expansion of $(1+x)^n$ are ${}^{n}C_{4}$, ${}^{n}C_{5}$, and ${}^{n}C_{6}$

ATQ $2.{}^{n}C_{5}={}^{n}C_{4}+{}^{n}C_{6}$

$$2.\frac{n!}{5!(n-5)!}=\frac{n!}{4!(n-4)!}+\frac{n!}{6!(n-6)!}$$

$$n=7,14$$

TEST PAPER-05

CLASS - XI MATHEMATICS (Binomial theorem)

1.	The middle term in the expansion of $(1+x)^{2n}$ is	[1]
2.	Find the no. of terms in the expansions of $\left(1-2x+x^2\right)^7$	[1]
3.	Find the coeff of x^5 in $(x+3)^9$	[1]
4.	Find the term independent of x $\left(x+\dfrac{1}{x}\right)^{10}$	[1]
5.	Expand $(a+b)^n$	[1]
6.	If the coeff of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal find r	[4]
7.	Show that the coeff of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coeff of two middle terms in the expansion of $(1+x)^{2n-1}$	[4]
8.	Find the value of r, if the coeff of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal.	[4]
9.	If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$ prove that (i) $P^2-Q^2=\left(x^2-a^2\right)^n$ (ii) $4PQ=(x+a)^{2n}-(x-a)^{2n}$ (iii) $2\left(P^2+Q^2\right)=\left[(x+a)^{2n}+(x-a)^{2n}\right]$	[6]
10.	If three successive coeff. In the expansion of $(1+x)^n$ are 220,495 and 792 then find n	[6]

TEST PAPER-05

CLASS - XI MATHEMATICS (Binomial theorem)

[ANSWERS]

Ans1. $^{2n}C_n . x^n$

Ans2. $\left(1-2x+x^2\right)^7$

$=\left(x^2-2x+1\right)^7$

$=\left[\left(x-1\right)^2\right]^7$

$=\left(x-1\right)^{14}$

No. of term is 15

Ans3. $T_{r+1} = {}^9C_r\left(x\right)^{9-r}.\left(3\right)^r$

Put $9-r=5$

$r=4$

$T_5 = {}^9C_4\left(x\right)^5.\left(3\right)^4$

Coeff of x^5 is ${}^9C_4\left(3\right)^4$

Ans4. $T_{r+1} = {}^{10}C_r\left(x\right)^{10-r}.\left(\frac{1}{x}\right)^r$

$= {}^{10}C_r\left(x\right)^{10-r}.\left(x\right)^{-r}$

$= {}^{10}C_r\left(x\right)^{10-2r}$

Put $10-2r=0$

$r=5$

Independent term is ${}^{10}C_5$

Ans5. $\left(a+b\right)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + ...+ {}^nC_n b^n$

Ans.6 $T_{r+1} = {}^{34}C_r (1)^{34-r}.(x)^r$

$T_{r+1} = {}^{34}C_r (x)^r ....(i)$

Coeff are

${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$

ATQ ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

$r-6 = 2r-2$

$r = -4$ (neglect)

$r-6 = 34-(2r-2)$ $\left[\begin{array}{l}\because {}^{n}C_r = {}^{n}C_p \\ r = p \text{ or } n = r+p\end{array}\right]$

$r = 14$

Ans7. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is $\left(\frac{2n}{2}+1\right)^{th}$ $i.e(n+1)^{th}$ term

Coeff of x^n is ${}^{2n}C_n$

Similarly $(2n-1)$ being odd the other expansion has two middle term i.e

$\left(\frac{2n-1+1}{2}\right)^{th}$. and $\left(\frac{2n-1+1}{2}+1\right)^{th}$ term

i.e n^{th} and $(n+1)^{th}$

The coeff are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

${}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$ $\left[\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right]$

Ans8. $T_{r+1} = {}^{18}C_r (1)^{18-r}.(x)^r$

$T_{r+1} = {}^{18}C_r x^r$

Put $r = r-3$

And $2r+3$

ATQ ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$18 = 2r + \cancel{3} + r - \cancel{3}$

$r = 6$

Ans9. $(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}a + {}^nC_2 x^{n-2}a^2 + ...+ {}^nC_n a^n$

$= t_1 + t_2 + t_3 + ...+ t_n + t_{n+1}$

$= (t_1 + t_3 + t_5 + ...) + (t_2 + t_4 + t_6 + ...)$

$= P + Q.......(i)$

$(x-a)^n = (t_1 - t_2 + t_3 - t_4 + ...)$

$= (t_1 + t_3 + t_5) - (t_2 + t_4 + t_6.....)$

$= P - Q.......(ii)$

$(i)\times(ii)$

$P^2 - Q^2 = (x^2 - a^2)^n$

Sq. (i) and (ii) and subt.

$\left[(x+a)^{2n} - (x-a)^{2n}\right] = 4PQ$

Sq. and adding we get

$\left[(x+a)^{2n} + (x-a)^{2n}\right] = 2(P^2 + Q^2)$

Ans10. Let coeff are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

ATQ ${}^nC_{r-1} = 220......(i)$

${}^nC_r = 495......(ii)$

${}^nC_{r+1} = 792........(iii)$

Dividing (ii) by (i)

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{495}{220}$$

$$\frac{n-r+1}{r} = \frac{9}{4}$$

$$4n - 13r + 4 = 0 \ldots\ldots (iv)$$

Dividing (iii) by (ii)

$$\frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{792}{495}$$

$$\frac{n-r}{r+1} = \frac{8}{5}$$

$$5n - 13r - 8 = 0 \ldots\ldots (v)$$

On solving (iv) and (v) we get $n = 12$

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Sequences and Series)

1. The sum of n terms of two A. P are in the ratio (3n+8) : (7n+15). Find the ratio of their 12^{th} terms. [4]

2. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+----$ [4]

3. Find the sum to n terms of the series $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+----$ [4]

4. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A. P. is equal to twice the m^{th} term. [4]

5. Fins the of all two digit no. which when divided by 4 yields 1 as remainder [4]

6. If a $\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A. P. prove that a, b, c are in A. P. [4]

7. Fins the sum of the products of the corresponding terms of the sequence 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$ [4]

8. Show that the ratio of the sum of first n terms of a G. P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$ [4]

9. If S_1, S_2, S_3 are the sum of first n natural no. their squares and their cubes respectively, show that $9S_2^{\ 2}=S_3\left(1+8S_1\right)$ [4]

10. 150 workers were engaged to finish a job in a certain no. of days 4 workers dropped out on the second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work find the no. of days in which the work was completed [6]

CBSE TEST PAPER-01

CLASS - XI MATHEMATICS (Sequences and Series)

[ANSWERS]

Ans 01. Let a_1, a_2 and d_1, d_2 are the first term and common difference of two A. P. S respectively.

$$\text{A T Q } \frac{\frac{\cancel{n}}{\cancel{2}}[2a_1+(n-1)d_1]}{\frac{\cancel{n}}{\cancel{2}}[2a_2+(n-1)d_2]}=\frac{3n+8}{7n+15}$$

$$\frac{\text{12th term of Ist A. P}}{\text{12th term of 2nd A. P}}=\frac{a_1+11\,d_1}{a_2+11\,d_2}$$

put n = 23 in eq (1)

$$\frac{2a_1+22\,d_1}{2a_2+11\,d_2}=\frac{3\times 23+8}{7\times 23+15}$$

$$\frac{a_1+11\,d_1}{a_2+11\,d_2}=\frac{7}{16}$$

Ans 02. $a=1,\ r=\frac{2}{3}$

$$S_n=\frac{a\left(1-r^n\right)}{1-r}$$

$$=\frac{1\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}}$$

$$=3\left[1-\left(\frac{2}{3}\right)^n\right]$$

$$S_5=3\left[1-\left(\frac{2}{3}\right)^5\right]=\frac{211}{81}$$

Ans 03. $a_n = 1^2 + 2^2 + 3^2 + \text{---} + n^2$

$$a_n=\frac{n\left(n+1\right)\left(2n+1\right)}{6}$$

$$S_n = \frac{1}{6}\left[2\sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + \sum k\right]$$

$$= \frac{1}{6}\left[2.\frac{n^2(n+1)^2}{4} + \frac{3.n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}[n(n+1)+(2n+1)]$$

$$= \frac{n(n+1)}{12}\left(n^2+n+2n+1+1\right)$$

$$= \frac{n(n+1)(n^2+3n+2)}{12}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

Ans 04. $a_{m+n} = a + (m+n-1)\ d$

$a_{m-n} = a + (m-n-1)\ d$

$$a_{m+n} + a_{m-n} = 2a + \left(m + \cancel{n} - 1 + m - \cancel{n} - 1\right)d$$

$$= 2a + (2m-2)d$$

$$= 2\left[a + (m-1)d\right]$$

$= 2\ a_m$ proved.

Ans 05. 13, 17, 21 --- 97

a = 13, d = 17 – 13 = 4

$a_n = 97$

$a_n = a + (n-1)\ d$

97 = 13 + (n-1) (4)

n = 22

$$S_{22} = \frac{22}{2}[13+97]$$

= 1210

Ans 06. $a\left(\frac{b+c}{bc}\right), b\left(\frac{c+a}{ca}\right), c\left(\frac{a+b}{ab}\right)$ are in A. P

$\Rightarrow \frac{ab+ac}{bc}, \frac{bc+ab}{ca}, \frac{ca+cb}{ab}$ are in A. P

$\Rightarrow 1+\frac{ab+ac}{bc}, 1+\frac{bc+ab}{ca}, 1+\frac{ca+cb}{ab}$ are in A. P

$\Rightarrow \frac{ab+ac+bc}{bc}, \frac{ab+bc+ca}{ca}, \frac{ab+bc+ca}{ab},$ are in A. P

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A. P

$\Rightarrow \frac{abc}{bc}, \frac{abc}{ca}, \frac{abc}{ab}$ are in A. P

a, b, c are in A. P prove

Ans 07. 256, 128, 64, 32, 16

a = 256

$r = \frac{128}{256} = \frac{1}{2}$

$n = 5$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$S_5 = \frac{256\left[1 - \left(\frac{1}{2}\right)^5\right]}{1 - \frac{1}{2}}$$

= 496

Ans 08. $Sn = \frac{a\left(r^n - 1\right)}{r - 1}$ (1)

Sum of terms from (n+1)th to (2n)th terms

$= S_{2n} - S_n$

$$= \frac{a\left(r^{2n} - 1\right)}{r - 1} - \frac{a\left(r^n - 1\right)}{r - 1}$$

A T Q $\frac{S_n}{S_{2n} - S_n} = \frac{\frac{a\left(r^n - 1\right)}{r - 1}}{\frac{a\left(r^{2n} - 1\right)}{r - 1} - \frac{a\left(r^n - 1\right)}{r - 1}}$

$= \frac{1}{r^n}$

Ans 09. $S_1 = \frac{n(n+1)}{2}$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$R.H.S = S_3(1+8S_1)$$

$$= \frac{\left(n(n+1)\right)^2}{2}\left[1+8\frac{n(n+1)}{2}\right]$$

$$= 9\left(\frac{n(n+1)(2n+1)}{6}\right)^2$$

$$= 9S_2^{\ 2}$$

Ans 10. a = 150, d = -4

$$S_n = \frac{n}{2}\left[2\times150+(n-1)(-4)\right]$$

If total works who would have worked all n days 150(n-8)

$$\text{A T Q } \frac{n}{2}\left[300+(n-1)(-4)\right]=150(n-8)$$

$$n = 25$$

CBSE TEST PAPER-02

CLASS - XI MATHEMATICS (Sequences and Series)

1. Insert 6 numbers between 3 and 24 such that the resulting sequence is an A. P. [4]

2. The sum of first three terms of a G. P is 13/12 and their product is -1. Find the common ratio and the terms. [4]

3. Find the Sum to n terms of $n^2 + 2^n$ [4]

4. If f is a function satisfying f (x+y) = f (x) f (y) for all x, y $\in$ N such that f (1) = 3 and $\sum_{x=1}^{n} f(x) = 120$ find the value of n [4]

5. The Sum of three no. in G.P is 56. If we subtract 1, 7, 21 from these no. in that order we obtain an A. P. find the no. [4]

6. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ Than show that a, b, c, and d are in G. P. [4]

7. In an A. P. is m^{th} term is n and n^{th} term is m, find the p^{th} term. [4]

8. If a, b, c are in G. P and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ prove that x, y, z are in A. P. [4]

9. If p, q, r are in G. P and the equation $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, that show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A. P. [4]

10. Prove that the sum to n terms of the series $11 + 103 + 1005 + ---- is\ \frac{10}{9}\left(10^n - 1\right) + n^2$ [6]

CBSE TEST PAPER-02
CLASS - XI MATHEMATICS (Sequences and Series)
[ANSWERS]

Ans 01. Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six number b/w 3 and 24

$3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in A.P

$a = 3, a_n = 24, n = 8$

$a_n = a + (n-1) d$

$24 = 3 + (8-1) d$

$d = 3$

$A_1 = a + d = 6$

$A_2 = a + 2d = 9$

$A_3 = a + 3d = 12$

$A_4 = a + 4d = 15$

$A_5 = a + 5d = 18$

$A_6 = a + 6d = 21$

Ans 02. Let $\frac{a}{r}$, r. ar be the first three term

$$\frac{a}{r}+a+ar=\frac{13}{12}$$

$$and\ \frac{a}{r}\times a\times ar=-1$$

$$a=-1$$

$$\frac{-1}{r}-1-r=\frac{13}{12}$$

$$12r^2+25r+12=0$$

$$r=\frac{-3}{4},\frac{-4}{3}$$

Hence G.P is $\frac{4}{3},-1,\frac{3}{4}$ and $\frac{3}{4},-1,\frac{4}{3}$

Ans 03. $a_n = n^2+2^n$

$S_n = (1^2 + 2^2 + 3^2 + \text{-----} + n^2) + (2^1 + 2^2 + \text{----} + 2^n)$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{2(2^n-1)}{2-1}$$

$$=\frac{n(n+1)(2n+1)}{6}+2.\left(2^n-1\right)$$

Ans 04. f (1) = 3

$f(1+2) = f(1)\,f(2) = 3\times 9 = 27$

$f(1+3) = f(1)\,f(3) = 3\times 27 = 81$

$L.H.S$

$= f(1)+f(2)+f(3)+---+f(n)$

$=3+9+27+81+---+n$ terms

$=\dfrac{3(3^n-1)}{3-1}=\dfrac{3}{2}\left(3^n-1\right)$

A T Q

$\dfrac{3}{2}\left(3^n-1\right)=120$

$3^n-1=80$

$3^n=81$

$n=4$

Ans 05. Let G. P be a, ar, ar^2

a + ar + ar^2 = 56 (I)

A T Q a – 1, ar – 7, ar^2 – 21 is an AP

2 (ar – 7) = (a – 1) + ar^2 – 21

ar^2 – 2ar + a = 8 (II)

from (I) and (II)

$3ar = 48$

$ar = 16$

$a=\dfrac{16}{r}$

put in eq (1)

$\dfrac{16}{r}+16+16r=56$

$2r^2-5r+2=0$

$r=2$ and r = $\dfrac{1}{2}$

when r = 2, a = 8

no. 8, 16, 32

when r = $\dfrac{1}{2}$

$no.32,16,8$

Ans 06. by c and d

$$\frac{a+\cancel{bx}+a-\cancel{bx}}{\cancel{a}+bx-\cancel{a}+bx}=\frac{b+cx+b-cx}{b+cx-b+cx}=\frac{c+dx+c-dx}{c+dx-c+dx}$$

$$\frac{2a}{2bx}=\frac{2b}{2cx}=\frac{2c}{2dx}$$

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$$

a, b, c, d are in G. P

Ans 07. $a_m = a + (m - 1)\, d$

$$n=\cancel{a}+(m-1)d \qquad (i)$$

$$\underline{{}_-m={}_-\cancel{a}+{}_-(n-1)d \qquad (ii)}$$

$$n-m=(m-\cancel{1}-n+\cancel{1})d$$

$$\frac{-(\cancel{m-n})}{(\cancel{m-n})}=d, \quad d=-1$$

a = n + m - 1

a_p = a + (p - 1)d

= n + m - 1 + (p - 1) (- 1)

= n + m - $\cancel{1}$ - p + $\cancel{1}$

= n + m - p

Ans 08. Let $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{2}}=k$

$a=k^x, b=k^y, c=k^z$

since a, b, c are in G. P

b^2 = ac

k^{2y} = k^{x+z}

$2y=x+z$

x, y, z are in A.P

Ans 09. $px^2 + 2qx + r = 0$ has root given by

$$x=\frac{-2q\pm\sqrt{4q^2-4rp}}{2p}$$

since p, q, r in G. P.

$q^2 = pr$

$$x=\frac{-q}{p}$$

but $\frac{-q}{p}$ is also root of

$dx^2 + 2ex + f = 0$

$$d\left(\frac{-q}{p}\right)^2 + 2e\left(\frac{-q}{p}\right) + f = 0$$

$$dq^2 - 2eqp + fp^2 = 0$$

÷ by pq^2

and using $q^2 = pr$

$$\frac{d}{p} - \frac{2e}{q} + \frac{fp}{pr} = 0$$

$$\frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$$

Henc $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A. P

Ans 10. $S_n = 11 + 103 + 1005 + \text{----} + n$ terms

$S_n = (10+1) + (10^2 + 3) + (10^3 + 5) + \text{-----} + [10^n + (2n-1)]$

$$S_n = \frac{10(10^n - 1)}{10 - 1} + \frac{n}{2}(\cancel{1} + 2n - \cancel{1})$$

$$= \frac{10}{9}(10^n - 1) + n^2$$

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Sequences and Series)

1. In an A. P. the first term is 2 and the sum of the first five terms is one fourth of next five terms. Show that 20^{th} term is -112. [4]

2. Find the sum of the sequence 7, 77, 777, ------ to n terms [4]

3. Find the sum to n terms of the series $3\times8+6\times11+9\times14+---$ [4]

4. Let the sum of n, 2n, 3n terms of an A. P. be S_1, S_2 and S_3 respectively show that $S_3 = 3\,(S_2 - S_1)$ [4]

5. Find the sum 0.6 + 0.66 + 0.666 + --- [4]

6. If a, b, c are in A.P, b, c, d are in G. P and $\frac{1}{c},\frac{1}{d},\frac{1}{e}$ are in A. P prove that a, c, e are in G. P. [4]

7. If a, b, c and d are in G.P show that $(a^2+b^2+c^2)\,(b^2+c^2+d^2) = (ab + bc + cd)^2$ [4]

8. The number of bacteria in a certain culture double every hour. If there were 30 bacteria in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour. [4]

9. If a and b are the roots of $x^2-3x+p = 0$ and c, d are roots of $x^2 - 12x + 9 = 0$ where a, b, c , d from G.P, prove that (q+p): (q-p) = 17:15 [4]

10. The ratio of A M and G. M of two positive no. a and b is m : n show that [6]

$$a:b=\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$$

CBSE TEST PAPER-03

CLASS - XI MATHEMATICS (Sequences and Series)

[ANSWERS]

Ans 01. ATQ $a_1 + a_2 + a_3 + a_4 + a_5 = \frac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$

$4(a_1 + a_2 + a_3 + a_4 + a_5) = (a_6 + a_7 + a_8 + a_9 + a_{10})$

adding $(a_1 + a_2 + a_3 + a_4 + a_5)$ both side

$5(a_1 + a_2 + a_3 + a_4 + a_5) = a_1 + a_2 + ----+ a_{10}$

$$5.\frac{5}{2}\left[2\times2+(5-1)d\right]=\frac{10}{2}\left[2\times2+(10-1)d\right]$$

$$\frac{5}{2}[4+4d]=[4+9d]$$

$$20+20d=8+18d$$

$$2d=-12$$

$$d=-6$$

$$a_{20}=a+19d$$

$$=2+19(-6)=2-114$$

$$=-112$$

Ans 02. $S_n = 7 + 77 + 777 + ---- + n \text{ terms}$

$$=\frac{7}{9}[9+99+999+-----+n\text{ terms}]$$

$$=\frac{7}{9}\left[(10-1)+(10^2-1)+(10^3-1)+----+n\text{ terms}\right]$$

$$=\frac{7}{9}\left[\left(10^1+10^2+----+n\text{ terms}\right)-\left(1+1+1--+n\text{ terms}\right)\right]$$

$$=\frac{7}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$=\frac{7}{9}\left[\frac{10\left(10^n-1\right)}{9}-n\right]$$

Ans 03. a_n = (n^{th} term of 3, 6, 9, ---) (nth term of 8, 11, 14, ---)

= (3 + (n – 1) 3) (8 + (n-1) 3)

$= (3n)(3n+5)$

$= 9n^2 + 15n$

$$S_n = 9\sum_{k=1}^{n} K^2 + 15\sum_{k=1}^{n} K$$

$$= \frac{9n(n+1)(2n+1)}{6} + \frac{15n(n+1)}{2}$$

$$= \frac{3}{2}.n(n+1)(2n+6)$$

$$= 3n(n+1)(n+3)$$

Ans 04. $S_1 = \frac{n}{2}[2a+(n-1)d]$ (1)

$S_2 = \frac{2n}{2}[2a+(2n-1)d]$ (2)

$S_3 = \frac{3n}{2}[2a+(3n-1)d]$ (3)

$R.H.S. = 3(S_2 - S_1)$

$$= 3\left[\frac{2n}{2}(2a+(2n-1)d)\right] - \frac{n}{2}[2a+(n-1)d]$$

$$= 3\left[\frac{n}{2}[2a+(3n-1)d]\right]$$

$$= \frac{3n}{2}(2a+(3n-1)d)$$

$$= L.H.S$$

Ans 05. Sn = 6 [0.1 + 0.11 + 0.111 + ---- + to n terms]

$$= \frac{6}{9}[0.9+0.99+0.999+---+n \text{ terms}]$$

$$= \frac{6}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+---n \text{ terms}\right]$$

$$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+---+ \text{ n terms}\right]$$

$$= \frac{6}{9}\left[n\times 1-\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+---+to \text{ n terms}\right]$$

$$=\frac{6}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$$

$$=\frac{6}{9}\left[n-\frac{1}{10}\times\frac{10}{9}\left(1-\frac{1}{10^n}\right)\right]$$

$$=\frac{6}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$$

Ans 06. a, b, c are in A. P.

$b=\frac{a+c}{2}$ (1)

b, c, d are in G. P.

$c^2 = bd$ (2)

$\frac{1}{c},\frac{1}{d},\frac{1}{e}$ are in A. P

$\frac{2}{d}=\frac{1}{c}+\frac{1}{e}$

$d=\frac{2ce}{c+e}$

put the value of b, and d

th eq (2)

$c^2 = \frac{a+c}{2}.\frac{2ce}{c+e}$

$c^2 = ae$

Ans 07. a, b, c, d are in G. P

b = ar

c = ar^2

d = ar^3

L. H. S = $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= $(a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$

= $a^4r^2(1 + r^2 + r^4)^2$

R. H. S. = $(ab + bc + cd)^2$

= $(a^2r + a^2r^3 + a^2r^5)^2$

= $a^4r^2(1 + r^2 + r^4)^2$

H. p

Ans 08. a = 30

r = 2

$a_3 = ar^2 = 30\ (2)^2$

$a_5 = ar^4 = 30\ (2)^4$

$a_{n+1} = ar^n = 30\ (2^n)$

Ans 09. a + b = 3, ab = p

c + d = 12, cd = q

a, b, c, d, are in GS. P

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=r$$

$$b=ar$$

$$c=ar^2$$

$$d=ar^3$$

$$a+ar=3 \quad \left|a.ar=p\right.$$

$$ar^2+ar^3=12\left|ar^2.ar^3=9\right.$$

on solving

$$r=2$$

$$\frac{p}{q}=\frac{16}{1}$$

by C and D

$$\frac{p+q}{p-q}=\frac{17}{15}$$

Ans 10.

$$\frac{\frac{a+b}{2}}{\sqrt{ab}}=\frac{m}{n}$$

$$\frac{a+b}{2\sqrt{ab}}=\frac{m}{n}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{m+n}{m-n}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{m+n}{m-n}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{m+n}}{\sqrt{m-n}}$$

by C and D

$$\frac{\sqrt{a}}{\sqrt{b}}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

Sq both side

$$\frac{a}{b}=\frac{m+\cancel{n}+m-\cancel{n}+2\sqrt{m^2-n^2}}{m+\cancel{n}+m-\cancel{n}-2\sqrt{m^2-n^2}}$$

$$\frac{a}{b}=\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Sequences and Series)

1. If the Sum of n terms of an A. P is $(pn + qn^2)$, where p and q are constants find the common difference. [4]

2. Insert three no. between 1 and 256 so that the resulting sequence is a G.P. [4]

3. Find the sum to n terms of $3\times1^2+5\times2^2+7\times3^2+\ldots$ 4]

4. Evaluate $\sum_{K=1}^{11}\left(2+3^K\right)$. [4]

5. The Sum of first three terms of a G.P. is 16 and the Sum of the next three term is 128. determine the first term, the common ratio and the Sum to n terms of the G.P. [4]

6. If the first and the n^{th} terms of a G.P. are a and b respectively and P is the product of n terms prove that $P^2 = (ab)^n$. [4]

7. The ratio of the sums of m and n terms of an AP is $m^2:n^2$. Show that the rates of m^{th} and n^{th} term is (2m-1) : (2n-1). [4]

8. If the Sum of n terms of an A.P. is $3n^2+n$ and its m^{th} term is 164, fins the value of m. [4]

9. The difference between any two consecutive interior angles of a polygon is 5^0. If the smallest angle is 120^0, find the no. of the sides of the polygon. [4]

10. Between 1 and 31, m number have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7^{th} and $(m-1)^{th}$ no. is 5:9 find the value of m. [6]

CBSE TEST PAPER-04

CLASS - XI MATHEMATICS (Sequences and Series)

[ANSWERS]

Ans 01. $s_n = p_n + qn^2$

put n = 1, 2, 3, ----

$s_1 = p + q$

$s_2 = 2p + 4q$

$s_3 = 3p + 9q$

$a_1 = s_1 = p + q$

$a_2 = s_2 - s_1 = 2p + 4q - p - q$

$= p + 3q$

$d = a_2 - a_1$

$= p + 3q - p - q$

$= 2q$

Ans 02. Let G_1 G_2 G_3 be three no between 1 and 256 such that

1, G_1, G_2, G_3, 256 is a G. P.

$a_n = 256$

$ar^{n-1} = 256$

$n = 5$

$r^4 = 256$

$r = \pm 4$

Number be 4, 16, 64

Ans 03. a_n = (n^{th} term of 3, 5, 7, ---) (n^{th} term of 1, 2, 3, ---)^2

$= (3 + (n - 1) 2) (1 + (n - 1). 1)^2$

$= (2n + 1) (n)^2$

$= 2n^3 + n^2$

$$s_n = 2\sum_{K=1}^{n} K^3 + \sum_{K=1}^{n} K^2$$

$$= 2\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \cancel{2}\frac{n^2(n+1)^2}{\cancel{4}\ 2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(3n^2+5n+1)}{6}$$

Ans 04. $\sum_{K=1}^{11}\left(2+3^{K}\right)=\left(2+3^{1}\right)+\left(2+3^{2}\right)+\left(2+3^{3}\right)+--+\left(2+3^{11}\right)$

$$=2\times 11+3^{1}+3^{2}+---+3^{11}$$

$$=22+\frac{3\left(3^{11}-1\right)}{3-1}$$

$$=22+\frac{3}{2}\left(3^{11}-1\right)$$

Ans 05. $s_3=16$

$$\frac{a\left(1-r^{3}\right)}{1-r}=16 \quad (1)$$

s_6-s_3=128

$$\frac{a\left(1-r^{6}\right)}{1-r}-16=128$$

$$\frac{a\left(1-r^{6}\right)}{1-r}=144 \quad (2)$$

(2) $\div$ (1)

$$\frac{1-r^{6}}{1-r^{3}}=\frac{144}{16}$$

$$1+r^{3}=9$$

$$r^{3}=8$$

$$r=2$$

$$s_3=\frac{a\left(r^{3}-1\right)}{r-1}=16$$

$$a=16/7$$

$$s_n=\frac{a\left(r^{n}-1\right)}{r-1}=\frac{16}{7}\frac{\left(2^{n}-1\right)}{2-1}=\frac{16}{7}\left(2^{n}-1\right)$$

Ans 06. $a_1=a$

$$a_n=b$$

$$p=a_1.a_2.a_3....a_n$$

$$=a.ar.ar^{2}----ar^{n-1}$$

$$=a^{n}.r^{1+2+3+---+(n-1)}$$

$$=a^{n}.r\frac{(n-1)(n)}{2}$$

$$\Rightarrow p^2 = \left(a^n . r \frac{(n-1).n}{2} \right)^2$$

$$= a^{2n} . r^{n(n-1)}$$

$$= \left(a.ar^{n-1} \right)^n$$

$$p^2 = (a.b)^n$$

Ans 07. $\frac{s_m}{s_n} = \frac{m^2}{n^2}$

$$\frac{\frac{\cancel{m}}{\cancel{2}}\left[2a+(m-1)d\right]}{\frac{\cancel{n}}{\cancel{2}}\left[2a+(n-1)d\right]} = \frac{m^{\cancel{2}}}{n^{\cancel{2}}}$$

$$\frac{2a+md-d}{2a+nd-d} = \frac{m}{n}$$

$$2an + \cancel{mnd} - dn = 2am + \cancel{mnd} - dm$$

$$2an - 2am - dn + dm = 0$$

$$2a(n-m) - d(n-m) = 0$$

$$2a = d$$

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1).2a}{a+(n-1).2a} \quad (as\ 2a = d\ )$$

$$= \frac{1+2m-2}{1+2n-2} = \frac{2m-1}{2n-1} \qquad \text{H.P}$$

Ans 08. $s_n = 3n^2 + 5n$

$n = 1, 2, 3 \ldots$

$s_1 = 8$

$s_2 = 22$

$a_1 = 8$

$a_2 = s_2 - s_1 = 22 - 8 = 16$

$d = 16 - 8 = 8$

$a_m = 164$

$a + (m-1)d = 164$

$8 + (m-1)8 = 164$

$m = 27$

Ans 09. a = 120^0, d = 5^0

$$s_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$s_n = \frac{n}{2}\left[2\times 120+(n-1)5\right]$$

$$= \frac{n}{2}\left[240+5n-5\right]$$

$$= \frac{n}{2}\left[235+5n\right]$$

Also

sum of interior angles of a polygon with n sides = (2n-4)×90

ATQ $\frac{n}{2}\left[235+5n\right] = (2n-4)\times 90$

$n = 9,16$

but n = 16 not possible

n = 9

Ans 10. $1, A_1, A_2, A_3, ---A_m, 31$ are in AP

a = 1

a_n = 31

$a_{m+2} = 31$

$a_n = a+(n-1)d$

$31 = a+(m+2-1)d$

$$d = \frac{30}{m+1}$$

$\frac{A7}{A_{m-1}} = \frac{5}{9}$ (Given)

$$\frac{1+7\left(\frac{30}{m+1}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9}$$

$m = 14$

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Sequences and Series)

1. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A. M. between a and b. Then find the value of n. [4]

2. If A. M. and G. M of two positive no. a and b are 10 and 8 respectively find the no. [4]

3. Find the sum to n terms of the series 5+11+19+29+41 + ---- [4]

4. Find the sum of integers from 1 to 100 that are divisible by 2or5 [4]

5. The Sum of the first four terms of an A.P. is 56. The Sum of the last four term is 112. If its first term is 11, then find the no. of terms. [4]

6. Let S be the Sum, P the product and R the sum of reciprocals of n terms in a G.P Prove that $P^2 R^n = S^n$ [4]

7. Find a G.P. for which sum of the first two term is -4 and the fifth term is 4 times the third term. [4]

8. If the p^{th}, q^{th} and r^{th} term of a G.P are a, b, c respectively prove that $a^{q-r} b^{r-p} c^{p-q} = 1$ [4]

9. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b. [4]

10. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio $(3 + 3\sqrt{2}) : (3 - 2\sqrt{2}$. [6]

CBSE TEST PAPER-05

CLASS - XI MATHEMATICS (Sequences and Series)

[ANSWERS]

Ans 01. $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}=\dfrac{a+b}{2}$

$2a^n+2b^n=(a+b)(a^{n-1}+b^{n-1})$

$2a^n+2b^n=a^n+ab^{n-1}+b.a^{n-1}+b^n$

$a^n+b^{\ n}=a.b^{n-1}+b.a^{n-1}$

$a^n-b.a^{n-1}=a.b^{n-1}-b^n$

$a^{n-1}(a-b)=b^{n-1}(a-b)$

$\left(\dfrac{a}{b}\right)^{n-1}=\dfrac{a-b}{a-b}$

$\left(\dfrac{a}{b}\right)^{n-1}=\left(\dfrac{a}{b}\right)^{0}\ \left(\because\left(\dfrac{a}{b}\right)^{0}=1\right)$

$n-1=0$

$n=1$

Ans 02. $A.M=\dfrac{a+b}{2}=10$

$G.M=\sqrt{ab}=8$

$a+b=20$

$ab=64$

$(a-b)^2=(a+b)^2-4ab$

$(a-b)^2=400-256$

$a-b=\sqrt{144}$

$a-b=\pm 12$

$a=4, b=16$

$or\ \ a=16, b=4$

Ans 03. S_n = 5 + 11 + 19 + 29 + --- + a_{n-1}+ a_n

S_n = 5 + 11 + 19 + ----- + a_{n-1} + a_{n-2} + a_n

On subtracting

0 = 5 + [6+8+10+12+---+(n-1) terms] – a_n

$$a_n = 5 + \frac{(n-1)[12+(n-2)\times 2]}{2}$$
$$= 5 + (n-1)(n+4)$$
$$= n^2 + 3n + 1$$
$$S_n = \sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + n$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$$
$$= \frac{n(n+2)(n+4)}{3}$$

Ans 04. Divisible by 2

2, 4, 6, ------ 100

a = 2, d = 2, a_n = 100

100 = 2 + (n-1) × 2

n = 50

$$S_{50} = \frac{50}{2}[2+100] = 2550$$

divisible by 5

$a = 5, d = 5, a_n = 100$

$5 + (n-1)\times 5 = 100$

$n = 20$

$S_{20} = 1050$

divisible by both 2 or 5

10, 20, 30, ----- 100

a = 10, d = 10, a_n = 100

100 = 10 + (n-1) × 10

n = 10

$$S_{10} = \frac{10}{2}[10+100]$$
$$= 550$$

A T Q Sum = 2550 + 1050 - 550

= 3050

Ans 05. A T Q a = 11

(a) + (a + d) + (a + 2d) + (a + 3d) = 56

2a + 3d = 28 (1)

$a_n + a_{n-1} + a_{n-2} + a_{n-3} = 112$

2nd – 5d = 14

d = 2

n = 11

Ans 06. Let G. P. be a, ar, ar^2, ...

Where r < 1

$$S=\frac{a\left(1-r^{n}\right)}{1-r}$$

$$R=\frac{1}{a}+\frac{1}{ar}+--+n \text{ terms}$$

$$=\frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^{n}-1\right]}{\frac{1}{r}-1} \qquad \left[\because r<1 \text{ then } \frac{1}{r}>1\right.$$

$$=\frac{1}{a}.\frac{1-r^{n}}{r^{n}}.\frac{r}{1-r}$$

$$=\frac{1-r^{n}}{ar^{n-1}\left(1-r\right)}$$

$$P=a.ar.ar^{2}.....ar^{n-1}$$

$$=a^{n}.r^{1+2+---+(n-1)}$$

$$=a^{n}.r\frac{n(n-1)}{2}$$

$$=a^{n}.r\frac{n(n-1)}{2}$$

$$L.H.S=P^{2}R^{n}$$

$$=a^{2n}r^{n(n-1)}.\ \frac{\left(1-r^{n}\right)^{n}}{ar^{n-1}(1-r)^{n}}$$

$$=S^{n}$$

Ans 07. S_2 = -4, a_5 = 4 a_3

$$\frac{a\left(1-r^{2}\right)}{1-r}=-4$$

$$a(1+r)=-4$$

$$ar^{4}=4ar^{2}$$

$$r=\pm 2$$

$$when\ r=2$$

$$a=-4/3$$

sequence is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, ...$

when r = -2

a = 4

sequence is

4, 8, 16, 32, 64, ...

Ans 08. a = AR^{p-1}

b = AR^{q-1}

c = AR^{r-1}

L. H. S = a^{a-r}. b^{r-p}. c^{p-q}

$$= \left(AR^{p-1}\right)^{a-r} . \left(AR^{q-1}\right)^{r-p} . \left(AR^{r-1}\right)^{p-q}$$

$$= A^0 R^0$$

$$= 1$$

Ans 09. $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a^{\frac{1}{2}} b^{\frac{1}{2}}}{1}$$

$$a^{n+1} + b^{n+1} = a^{\frac{1}{2}} b^{\frac{1}{2}} \left(a^n + b^n\right)$$

$$a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}\, \mathrm{b}^{\frac{1}{2}} + a^{\frac{1}{2}}\, \mathrm{b}^{\mathrm{n}+\frac{1}{2}}$$

$$a^{n+1} - a^{n+\frac{1}{2}}\, \mathrm{b}^{\frac{1}{2}} = a^{\frac{1}{2}}\, \mathrm{b}^{\mathrm{n}+\frac{1}{2}} - b^{n+1}$$

$$a^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{1/2} \right) = \mathrm{b}^{\mathrm{n}+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{1/2} \right)$$

$$\left(\frac{a}{b}\right)^{\mathrm{n}+\frac{1}{2}} = 1$$

$$\left(\frac{a}{b}\right)^{\mathrm{n}+\frac{1}{2}} = \left(\frac{a}{b}\right)^0$$

$$n + \frac{1}{2} = 0$$

$$n = \frac{-1}{2}$$

Ans 10. $a+b=6\sqrt{ab}$

$$\frac{a+b}{2\sqrt{ab}}=\frac{3}{1}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{3+1}{3-1}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{2}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{2}}{1}$$

again by C and D

$$\frac{\sqrt{a}+\cancel{\sqrt{b}}+\sqrt{a}-\cancel{\sqrt{b}}}{\cancel{\sqrt{a}}+\sqrt{b}-\cancel{\sqrt{a}}-\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{a}{b}=\frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2}$$ (on squaring both side)

$$\frac{a}{b}=\frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\frac{a}{b}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a:b=\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$$

TEST PAPER-01

CLASS - XI MATHEMATICS (Straight Line)

1. Find the slope of the lines passing through the point (3,-2) and (-1,4) [1]

2. Three points $P(h,k)$, $Q(x_1,y_1)$ and $R(x_2,y_2)$ lie on a line. Show that $(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$ [1]

3. Write the equation of the line through the points $(1,-1)$ and $(3,5)$ [1]

4. Find the measure of the angle between the lines $x+y+7=0$ and $x-y+1=0$ [1]

5. Find the equation of the line that has y-intercept 4 and is $\perp$ to the line $y=3x-2$ [1]

6. If p is the length of the $\perp$ from the origin on the line whose intercepts on the axes are a and b. show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ [4]

7. Find the value of p so that the three lines $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$ may intersect at one point. [4]

8. Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign. [4]

9. By using area of Δ. Show that the points $(a,b+c),(b,c+a)$ and $(c,a+b)$ are collinear. [4]

10. Find the values of k for the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ [6]

(a). Parallel to the x-axis (b). Parallel to y-axis (c).Passing through the origin

TEST PAPER-01

CLASS - XI MATHEMATICS (Straight Line)

[ANSWERS]

Ans1. Slope of line through (3,-2) and (-1, 4)

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{4-(-2)}{-1-3}$$

$$=\frac{6}{-4}=\frac{-3}{2}$$

Ans2. Since P,Q,R are collinear

Slope of PQ= slope of QR

$$\frac{y_1-k}{x_1-h}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{\cancel{-}(k-y_1)}{\cancel{-}(h-x_1)}=\frac{y_2-y_1}{x_2-x_1}$$

$$(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$$

Ans3. Req. eq. $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y+1=\frac{5+1}{2}(x-1)$$

$$-3x+y+4=0$$

Ans4. $x+y+7=0$

$$m_1=\frac{-1}{1}$$

$$x-y+1=0$$

$$m_2=\frac{-1}{-1}=1$$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

Ans5. $y = 3x - 2$

Slope $(m) = \frac{-3}{-1} = 3$, slope of any line $\perp$ it is $-\frac{1}{3}$

$C = 4$

Req. eq. is $y = mx + c$

$$y = \frac{-1}{3}x + 4$$

Ans6. Equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

The distance of this line from the origin is P

$$\therefore P = \frac{\left|\frac{0}{a} + \frac{0}{b} - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} \qquad \left[d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}\right]$$

$$\frac{P}{1} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$\frac{1}{P} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

Sq. both side

$$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Ans7. $3x + y - 2 = 0 \ldots\ldots..(i)$

$px + 2y - 3 = 0 \ldots\ldots.(ii)$

$2x - y + 3 = 0 \ldots\ldots.(iii)$

On solving eq. (i) and (iii)

$x = 1$, And $y = -1$

Put x, y in eq. (ii)

$$P(1)+2(-1)-3=0$$
$$p-2-3=0$$
$$p=5$$

Ans8. Let intercept be a and –a the equation of the line is

$$\frac{x}{a}+\frac{y}{-a}=1$$

$$\Rightarrow x-y=a.......(i)$$

Since it passes through the point (3, 4)

$$3-4=a$$
$$a=-1$$

Put the value of a in eq. (i)

$$x-y=-1$$
$$x-y+1=0$$

Ans9. Area of $\Delta=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$$=\frac{1}{2}|a(c+a)-b(b+c)+b(a+b)-c(c+a)+c(b+c)-a(a+b)|$$

$$=\frac{1}{2}.0=0$$

Ans10. (a) The line parallel to x-axis if coeff. Of x =0

$$k-3=0$$
$$k=3$$

(b) The line parallel to y-axis if coeff. Of y =0

$$4-k^2=0$$
$$k=\pm 2$$

(c) Given line passes through the origin if (0, 0) lies on given eq.

$$(k-3).(0)-(4-k^2)(0)+k^2-7k+6=0$$
$$(k-6)(k-1)=0$$
$$k=6,1$$

TEST PAPER-02

CLASS - XI MATHEMATICS (Straight Line)

1.	Find the equation of the line, which makes intercepts -3 and 2 on the x and y -axis respectively.	[1]
2.	Equation of a line is $3x-4y+10=0$ find its slope.	[1]
3.	Find the distance between the parallel lines $3x-4y+7=0$ and $3x-4y+5=0$	[1]
4.	Find the equation of a straight line parallel to y -axis and passing through the point (4,-2)	[1]
5.	If $3x-by+2=0$ and $9x+3y+a=0$ represent the same straight line, find the values of a and b.	[1]
6.	Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point $p(0,-4)$ and $Q(8,0)$	[4]
7.	Find equation of the line passing through the point $(2,2)$ and cutting off intercepts on the axes whose sum is 9	[4]
8.	Reduce the equation $\sqrt{3}x+y-8=0$ into normal form. Find the values p and ω.	[4]
9.	If p and q are the lengths of $\perp$ from the origin to the lines. $x\cos\theta-y\sin\theta=k\cos 2\theta$, and $x\sec\theta+y\operatorname{cosec}\theta=k$ respectively, prove that $p^2+4q^2=k^2$	[6]
10.	Prove that the product of the $\perp$ drawn from the points $\left(\sqrt{a^2-b^2},0\right)$ and $\left(-\sqrt{a^2-b^2},0\right)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2.	[6]

TEST PAPER-2

CLASS - XI MATHEMATICS (Straight Line)

[ANSWERS]

Ans1. Req. eq. $\frac{x}{a}+\frac{y}{b}=1$

$a=-3, b=2$

$\therefore \frac{x}{-3}+\frac{y}{2}=1$

$2x-3y+6=0$

Ans2. $m=\frac{-\text{coff. of } x}{\text{coff. of } y}$

$=\frac{-3}{-4}=\frac{3}{4}$

Ans3. $A=3, B=-4, C_1=7$ and $c_2=5$

$d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

$=\frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}$

$=\frac{2}{5}$

Ans4. Equation of line parallel to y-axis is $x=a.....(i)$

Eq. (i) passing through (-4,2)

$a=-4$

So $x=-4$

$x+4=0$

Ans5. ATQ

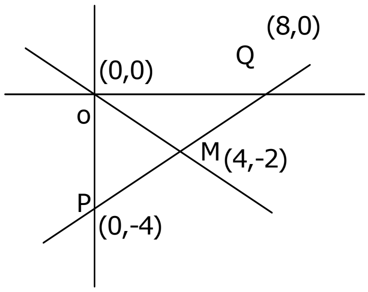
$$\frac{3}{9}=\frac{-b}{3}=\frac{2}{a}$$

$b=-1$

$\Rightarrow a=6$

Ans6. Let m be the midpoint of segment PQ then $M=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$

$=(4,-2)$



Slope of $OM=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-2-0}{4-0}=\frac{-1}{2}$

Ans7. Req. eq. be $\frac{x}{a}+\frac{y}{b}=1......(i)$

$a+b=9$

$b=9-a$

$\Rightarrow\frac{x}{a}+\frac{y}{9-a}=1$

This line passes through $(2,2)$

$\therefore\frac{2}{a}+\frac{2}{9-a}=1$

$a^2-9a+18=0$

$a^2-6a-3a+18=0$

$a(a-6)-3(a-6)=0$

$(a-6)(a-3)=0$

$a=6,3$

$a=6$	$a=3$
$b=3$	$b=6$
$\frac{x}{6}+\frac{y}{3}=1$	$\frac{x}{3}+\frac{y}{6}=1$

Ans8. $\sqrt{3}x+y-8=0$

$\sqrt{3}x+y=8......(i)$

$\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=2$

Dividing (i) by 2

$\frac{\sqrt{3}}{2}x+\frac{y}{2}=4$

$x\cos 30^0+y.\sin 30=4.......(ii)$

Comparing (ii) with

$x\cos\omega+y\sin\omega=p$

$p=4$

$\omega=30^0$

Ans9. $P=\frac{|0.\cos\theta-0\sin\theta-k\cos 2\theta|}{\sqrt{(\cos\theta)^2+(-\sin\theta)^2}}$ $\left[\begin{array}{l}\perp \textit{ from} \text{ origin} \\ \because (0,0)\end{array}\right.$

$P=K\cos 2\theta.....(i)$

$q=\frac{|0.\sec\theta+0\cos ec\theta-k|}{\sqrt{\sec^2\theta+\cos ec^2\theta}}$

$=\frac{K}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}$

$=\frac{k\cos\theta.\sin\theta}{\sqrt{\sin^2\theta+\cos^2\theta}}=\frac{1}{2}k.\sin\theta.\cos\theta$

$2q=k.\sin 2\theta......(ii)$

Squaring (i) and (ii) and adding

$P^2+(2q)^2=K^2\cos^2 2\theta+K^2\sin^2 2\theta$

$P^2+4q^2=K^2\left(\cos^2 2\theta+\sin^2 2\theta\right)$

$p^2+4q^2=k^2$

Ans10. Let

$$p_1 = \frac{\left|\frac{\sqrt{a^2-b^2}}{a}.\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}} \qquad \left[\because \perp \text{ from the points} \sqrt{a^2-b^2}, 0\right]$$

Similarly p_2 be the distance from $\left(-\sqrt{a^2-b^2}, 0\right)$ to given line

$$p_2 = \frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}}$$

$$p_1 p_2 = \frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\left(\frac{a^2-b^2}{a^2}\right).\cos^2\theta - 1\right|}{\frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2b^2}}$$

$$= \frac{\left|a^2\cos^2\theta - b^2\cos^2\theta - a^2\right| a^2b^2}{a^2(a^2\sin^2\theta + b^2\cos^2\theta)}$$

$$= \frac{\left|-(a^2\sin^2\theta + b^2\cos^2\theta)\right| b^2}{a^2\sin^2\theta + b^2\cos^2\theta} \qquad \left[\because a^2\cos^2\theta - a^2 = a^2(\cos^2\theta - 1)\right.$$

$$= \frac{(a^2\sin^2\theta + b^2\cos^2\theta)\, b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$= b^2$$

TEST PAPER-03

CLASS - XI MATHEMATICS (Straight Line)

1. Find the distance between $P(x_1 y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to the y-axis. [1]

2. Find the slope of the line, which makes an angle of 30^0 with the positive direction of y-axis measured anticlockwise. [1]

3. Determine x so that the inclination of the line containing the points $(x, -3)$ and $(2, 5)$ is 135. [1]

4. Find the distance of the point $(4,1)$ from the line $3x-4y-9=0$ [1]

5. Find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear. [1]

6. Without using the Pythagoras theorem show that the points $(4,4),(3,5)$ and $(-1,-1)$ are the vertices of a right angled Δ. [4]

7. The owner of a milk store finds that, he can sell 980 liters of milk each week at 14 liter and 1220 liter of milk each week at Rs 16 liter. Assuming a linear relationship between selling price and demand how many liters could he sell weekly at Rs 17 liter? [4]

8. The line through the points (h,3) and (4,1) intersects the line $7x-9y-19=0$ at right angle. Find the value of h. [4]

9. Find the equations of the lines, which cut off intercepts on the axes whose sum and product are 1 and -6 respectively. [4]

10. Find equation of the line mid way between the parallel lines $9x+6y-7=0$ and $3x+2y+6=0$ [6]

TEST PAPER-3

CLASS - XI MATHEMATICS (Straight Line)

[ANSWERS]

Ans1. When PQ is parallel to the y-axis,

Then $x_1 = x_2$

$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$

$= |y_2 - y_1|$

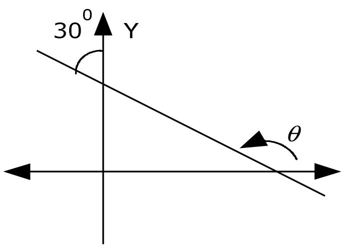
Ans2. Let θ be the inclination of the line

$\theta = 120^0$

$m = \tan 120^0$

$= \tan(90 + 30)$

$= -\sqrt{3}$



Ans3. $\frac{5-(-3)}{2-x} = \tan 135$

$\left[\begin{array}{l} \because m = \tan\theta \\ m = \frac{y_2 - y_1}{x_2 - x_1} \end{array}\right]$

$\frac{5+3}{2-x} = -1$

$x = 10$

Ans4. Let d be the req. distance

$d = \frac{|3.(4) - 4(1) - 9|}{\sqrt{(3)^2 + (-4)^2}}$

$= \frac{|-1|}{5} = \frac{1}{5}$

Ans5. Let $A(x,-1), B(2,1), C(4,5)$

Slope of AB= Slope of BC

$$\frac{1+1}{2-x} = \frac{5-1}{4-2}$$

$$\frac{2}{2-x} = \frac{\cancel{4}^{2}}{\cancel{2}}$$

$$2-x=1$$

$$-x=-1$$

$$x=1$$

Ans6. The given points are $A(4,4), B(3,5)$ and $C(-1,-1)$

Slope of $AB = \frac{5-4}{3-4} = -1$

Slope of $BC = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of $AC = \frac{-1-4}{-1-4} = +1$

Slope of $AB \times$ slope of $AC = -1$

$\Rightarrow AB \perp AC$

Hence ΔABC is right angled at A.

Ans7. Assuming sell along x-axis and cost per litre along y-axis, we have two points $(980,14)$ and $(1220,16)$ in $x\ y$ plane

$$y-14 = \frac{16-14}{1220-980}(x-980)$$

$$y-14 = \frac{\cancel{2}}{{}_{120}\cancel{240}}(x-980)$$

$$120y - 14\times120 = x-980$$

$$120y - 1680 = x - 980$$

$$x - 120y = -700$$

When $y = 17$

$x - 120\times17 = -700 \qquad \Rightarrow x = 1340$ litres.

Ans8. Slope of line joining (h,3) and (4,1)

$=\frac{1-3}{4-h} \qquad =\frac{-2}{4-h}$

Given line is $7x-9y-19=0$

Slope of this line = $\frac{-7}{-9}$

ATQ $\left(\frac{-2}{4-h}\right)\times\left(\frac{7}{9}\right)=-1 \qquad \Rightarrow h=\frac{22}{9}$

Ans9. ATQ $a+b=1.......(i)$

$ab=-6........(ii)$

$b=1-a \quad \left[from(i)\right]$

Put b in eq. (ii)

$a(1-a)=-6$

$a-a^2=-6$

$a^2-a-6=0$

$(a-3)(a+2)=0$

$a=3,-2$

When $a=3$

$b=-2$

Eq. of the line is

$\frac{x}{3}+\frac{y}{-2}=1$

$2x-3y-6=0$

When $a=-2$

$b=3$

Eq. of the line is

$\frac{x}{-2}+\frac{y}{3}=1$

$3x-2y+6=0$

Ans10. The equations are

$$9x+6y-7=0$$

$$3\left(3x+2y-\frac{7}{3}\right)=0$$

$$3x+2y-\frac{7}{3}=0.......(i)$$

$$3x+2y+6=0.....(ii)$$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$$3x+2y+k=0.......(iii)$$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left|\frac{K+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{K-6}{\sqrt{9+4}}\right| \qquad \left[\because d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right]$$

$$K+\frac{7}{3}=K-6$$

$$K=\frac{11}{6}$$

Req. eq. is

$$s\,3x+2y+\frac{11}{6}=0$$

TEST PAPER-04

CLASS - XI MATHEMATICS (Straight Line)

1. Find the angle between the x-axis and the line joining the points $(3,-1)$ and $(4,-2)$ [1]

2. Using slopes, find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear. [1]

3. Find the value of K so that the line $2x+ky-9=0$ may be parallel to $3x-4y+7=0$ [1]

4. Find the value of K, given that the distance of the point $(4,1)$ from the line $3x-4y+K=0$ is 4 units. [1]

5. Find the equation of the line through the intersection of $3x-4y+1=0$ $5x+y-1=0$ which cuts off equal intercepts on the axes. [1]

6. The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines. [4]

7. Point $R(h,k)$ divides a line segment between the axes in the ratio 1:2. Find equation of the line. [4]

8. The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K=273 when F=32 and that K= 373 when F=212 Express K in terms of F and find the value of F when K=0 [4]

9. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line $x-3y+4=0$ [6]

10. A person standing at the junction (crossing) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$ wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Find equation of the path that he should follow. [6]

TEST PAPER-4

CLASS - XI MATHEMATICS (Straight Line)

[ANSWERS]

Ans1. $m_1 = 0$ [Slope of x-axis]

$m_2 =$ slope of line joining points $(3,-1)$ and $(4,-2)$

$$= \frac{-2-(-1)}{4-3} = -1$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{0+1}{1+0\times(-1)}\right|$$

$$\tan\theta = 1$$

$$\theta = 45^0$$

Ans2. Since the given points are collinear slope of the line joining points $(x,-1)$ and $(2,1)$ =slope of the line joining points $(2,1)$ and $(4,5)$

$$\Rightarrow \frac{2}{2-x} = \frac{2}{1}$$

$$x = 1$$

Ans3. ATQ

Slope of 1st line = slope of 2nd line

$$\frac{-2}{k} = \frac{-3}{-4}$$

$$\Rightarrow k = \frac{-8}{3}$$

Ans4. We are given that distance of (4,0) from the line $3x-4y+k=0$ is 4

$$\frac{|3(4)-4(1)+k|}{\sqrt{(3)^2+(-4)^2}} = 4$$

$$|k+8| = 4\times 5$$

$$k = 12, -28$$

Ans5. Slope of a line which makes equal intercept on the axes is -1any line through the intersection of given lines is

$$(3x-4y+1)+K(5x-y-1)=0$$

$$(3+5K)x+y(K-4)+1-K=0$$

$$m=-\frac{(3+5K)}{K-4}=-1$$

$$K=\frac{-7}{4}$$

Ans6. Let the slope of one line is m and other line is $2m$

$$\frac{1}{3}=\left|\frac{2m-m}{1+(2m)(m)}\right|$$

$$\frac{1}{3}=\left|\frac{m}{1+2m^2}\right|$$

$$\pm\frac{1}{3}=\frac{m}{1+2m^2}$$

$$\frac{1}{3}=\frac{m}{1+2m^2}$$

$$2m^2-3m+1=0$$

$$2m^2-2m-m+1=0$$

$$2m(m-1)-1(m-1)=0$$

$$(m-1)(2m-1)=0$$

$$m=1, m=\frac{1}{2}$$

$$\frac{-1}{3}=\frac{m}{1+2m^2}$$

$$-1-2m^2=3m$$

$$2m^2+3m+1=0$$

$$2m^2+2m+m+1=0$$

$$2m(m+1)+1(m+1)=0$$

$$(m+1)(2m+1)=0$$

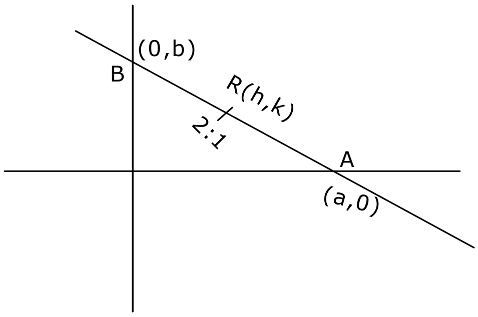
$$m=-1$$

$$m=\frac{-1}{2}$$

Ans7. Let eq. be $\frac{x}{a}+\frac{y}{b}=1.......(i)$

It is given that $R(h,k)$ divides AB in the ratio 1:2

$\therefore (h,k)=\left(\frac{2a}{3},\frac{b}{3}\right)$



$\frac{2a}{3}=h$

$a=\frac{3h}{2}$

$k=\frac{b}{3}$

$b=3k$

Put a and b in eq..........(i)

$$\frac{x}{\frac{3h}{2}}+\frac{y}{3k}=1$$

$$\frac{2x}{h}+\frac{y}{k}=3$$

Ans8. Let F along x-axis and K along y-axis

$$K-273=\frac{373-273}{212-32}(F-32) \qquad \left[\because y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$

$$K-273=\frac{10\cancel{0}}{18\cancel{0}}(F-32)$$

$$K=\frac{5}{9}(F-32)+273$$

Ans9. Let $Q(h,k)$ is the image of the point $p(1,2)$ in the line.

$x-3y+4=0......(i)$

Coordinate of midpoint of $PQ=\left(\frac{h+1}{2},\frac{k+2}{2}\right)$

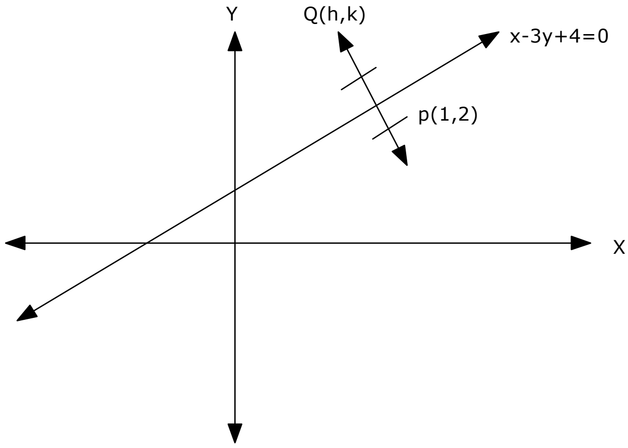
This point will satisfy the eq. ......(i)

$$\left(\frac{h+1}{2}\right)-3\left(\frac{k+2}{2}\right)+4=0$$

$h-3k=-3.......(i)$

(Slope of line PQ) $\times$ (slope of line $x-3y+4=0$) $=-1$

$$\left(\frac{k-2}{h-1}\right)\left(\frac{\cancel{-}1}{\cancel{-}3}\right)=-1$$



$3h+k=5......(ii)$

On solving (i) and (ii)

$h=\frac{6}{5}$ and $k=\frac{7}{5}$

Ans10. $2x-3y-4=0......(i)$

$3x+4y-5=0......(ii)$

$6x-7y+8=0.......(iii)$

On solving eq. (i) and (ii)

We get $\left(\frac{31}{17},\frac{-2}{17}\right)$

To reach the line (iii) in least time the man must move along the $\perp$ from crossing point $\left(\frac{31}{17},\frac{-2}{17}\right)$ to (iii) line

Slope of (iii) line is $\frac{6}{7}$

Slope of required path $=\frac{-7}{6}$ $\qquad [\because m_1\times m_2=-1]$

$$y-\left(-\frac{2}{17}\right)=\frac{-7}{6}\left(x-\frac{31}{17}\right)$$

$119x+102y=205$

TEST PAPER-05

CLASS - XI MATHEMATICS (Straight Line)

1. Find the distance of the point $(2,3)$ from the line $12x-5y=2$ [1]

2. Find the equation of a line whose perpendicular distance from the origin is 5 units and angle between the positive direction of the x-axis and the perpendicular is 30^0. [1]

3. Write the equation of the lines for which $\tan\theta=\frac{1}{2}$, where Q is the inclination of the line and x intercept is 4. [1]

4. Find the Angle between the x-axis and the line joining the points $(3,-1)$ and $(4,-2)$ [1]

5. Find the equation of the line intersecting the x-axis at a distance of 3 unit to the left of origin with slope -2. [1]

6. If three points $(h,0)(a,b)$ and $(0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$ [4]

7. $p(a,b)$ is the mid point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$ [4]

8. The line $\perp$ to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$ find the equation of the line. [4]

9. A line is such that its segment between the lines $5x-y+4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$ obtain its equation. [6]

10. Find the equations of the lines which pass through the point $(4,5)$ and make equal angles with the lines $5x-12y+6=0$ and $3x-4y-7=0$ [6]

TEST PAPER-5

CLASS - XI MATHEMATICS (Straight Line)

[ANSWERS]

Ans1.

$$d=\frac{|12x-5y-2|}{\sqrt{(12)^2+(-5)^2}}$$

$$d=\frac{|12\times 2-5\times 3-2|}{\sqrt{169}}$$

$$=\frac{|-41|}{13}=\frac{41}{13}$$

Ans2. $p=5, \alpha=30^0$

Req. eq. $x\cos\alpha+y\sin\alpha=p$

$$x\cos 30^0+y\sin 30^0=5$$

$$\sqrt{3}x+y-10=0$$

Ans3. $m=\tan\theta=\frac{1}{2}$ and $d=4$

$$y=\frac{1}{2}(x-4) \qquad [\because y=m(x-d)]$$

$$2y-x+4=0$$

Ans4. Let $A(3,-1)$ $B(4,-2)$

Slope of $AB=\frac{-2-(-1)}{4-2}$

$$=-1$$

$$\tan\theta=-1$$

$$\theta=135 \qquad \left[\begin{array}{l}\text{where } \theta \text{ is the angle which AB makes} \\ \text{with positive direction of } x-axis\end{array}\right]$$

Ans5. The line passing through (-3,0) and has slope = -2

Req. eq. is

$$y-0=-2(x+3)$$

$$2x+y+6=0$$

Ans6. Let $A(h,0)\,B(a,b)$ and $C(0,k)$

Slope of AB = slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$
$$\frac{b}{a-h}=\frac{h-b}{-a}$$

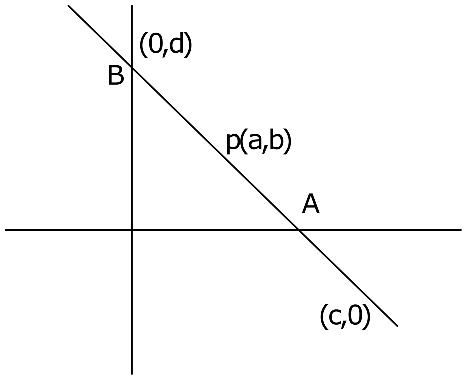
$$(a-h)(k-b)=-ab$$

$$ak-\cancel{ab}-hk+hb=-\cancel{ab}$$

$$ak+hb=hk$$
$$\frac{ak}{hk}+\frac{hb}{hk}=1$$

$$\frac{a}{h}+\frac{b}{k}=1$$

Ans7. Req. eq. be



$$\frac{x}{c}+\frac{y}{d}=1\ldots\ldots(i)$$

P is the mid point

Coordinate of $p=\left(\frac{c}{2},\frac{d}{2}\right)$

$$(a,b)=\left(\frac{c}{2},\frac{d}{2}\right)$$

$$\frac{a}{1}=\frac{c}{2}$$
$$c=2a$$

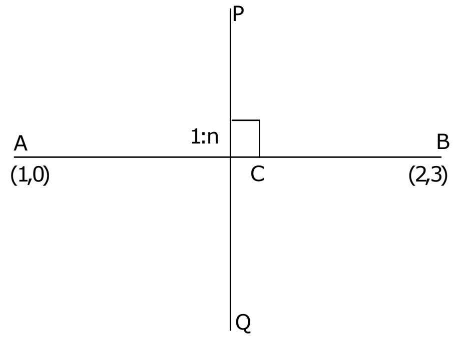
$$\frac{b}{1}=\frac{d}{2}$$
$$d=2b$$

Put the value of C and D in eq. (i)

$$\frac{x}{2a}+\frac{y}{2b}=1$$

$$\frac{x}{a}+\frac{y}{b}=2$$

Ans8. Coordinate of $c\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$



$m_{AB} = 3$

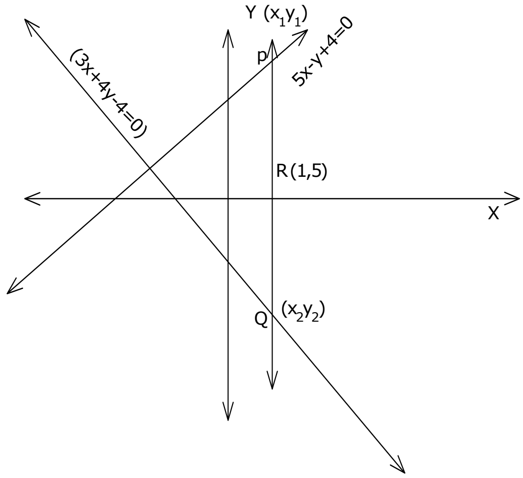
$m_{PQ} = -\frac{1}{3}$

Eq. of PQ is

$$\frac{y}{1} - \frac{3}{1+n} = -\frac{1}{3}\left(\frac{x}{1} - \frac{2+n}{1+n}\right)$$

$$(n+1)x + 3(n+1)y - (n+11) = 0$$

Ans9. $p(x_1 y_1)$ lies on $5x - y + 4 = 0$



$\Rightarrow 5x_1 - y_1 + 4 = 0$

And $Q(x_2 y_2)$ lies on $3x + 4y - 4 = 0$

$3x_2 + 4y_2 - 4 = 0$

On solving

$y_1 = 5x_1 + 4$

$$y_2 = \frac{4 - 3x_2}{4}$$

Since R is the mid point of PQ

$$\frac{x_1 + x_2}{2} = 1, \qquad \frac{y_1 + y_2}{2} = 5$$

$x_1 + x_2 = 2, \qquad y_1 + y_2 = 10$

On solving

$$x_1 = \frac{26}{23}, \qquad x_2 = \frac{20}{23}$$

And $y_1 = \frac{222}{23}, \qquad y_2 = \frac{8}{23}$

Eq. of PQ

$$y-\frac{222}{23}=\frac{\frac{8}{23}-\frac{222}{23}}{\frac{20}{23}-\frac{26}{23}}\left(x-\frac{26}{23}\right)$$

$$107x-3y-92=0$$

Ans10. The slopes of the given lines are $\frac{5}{12}$ and $\frac{3}{4}$

Let m be the slope of a required line

ATQ $\left|\frac{m-\frac{5}{12}}{1+m.\frac{5}{12}}\right|=\left|\frac{m-\frac{3}{4}}{1+m.\frac{3}{4}}\right|$

$$\Rightarrow\left|\frac{12m-5}{12+5m}\right|=\left|\frac{4m-3}{4+3m}\right|$$

$$\frac{12m-5}{12+5m}=\frac{4m-3}{4+3m}$$

$$16m^2=-16$$

$$m^2=-1$$

Neglect

$$\frac{12m-5}{12+5m}=-\frac{4m-3}{4+3m}$$

$$m=\frac{4}{7},\frac{-7}{4}$$

Req. eq. are

$$y-5=\frac{4}{7}(x-4)$$

$$4x-7y+19=0$$

$$y-5=\frac{-7}{4}(x-4)$$

$$7x+4y-48=0$$

TEST PAPER-01

CLASS - XI MATHEMATICS (Conic Section)

1. Find the equation of a circle with centre (P,Q) & touching the y axis [1]
$(A) x^2+y^2+2Qy+Q^2=0$ $(B) x^2+y^2-2px+2Qy+Q^2=0$
$(C) x^2+y^2-2px+2Qy+Q^2=0$ (D) none of these

2. Find the equations of the directrix & the axis of the parabola $\Rightarrow 3x^2=8y$ [1]
$(A) 3y-4=0, x=0$ $(B) 3x-4=0, y=0$
$(C) 3y-4x=0$ (D) none of these

3. Find the coordinates of the foci of the ellipse $\Rightarrow x^2+4y^2=100$ [1]
$(A) F\left(\pm 5\sqrt{3}, 0\right)$ $(B) F\left(\pm 3\sqrt{5}, 0\right)$
$(C) F\left(\pm 4\sqrt{5}, 0\right)$ (D) none of these

4. Find the eccentricity of the hyperbola: $3x^2-2y^2=6$ [1]
$(A) e=\sqrt{\frac{5}{2}}$ $(B) e=\frac{\sqrt{5}}{2}$ $(C) e=\frac{\sqrt{2}}{5}$ (D) none of these

5. Show that the equation $x^2+y^2-6x+4y-36=0$ represent a circle, also find its centre & radius? [4]

6. Find the equation of an ellipse whose foci are $(\pm 8, 0)$ & the eccentricity is $\frac{1}{4}$? [4]

7. Find the equation whose vertices are $(0, \pm 10)$ & $e=\frac{4}{5}$ [4]

8. Find the equation of hyperbola whose length of latus rectum is 36 & foci are $(0, \pm 12)$ [4]

9. Find the equation of a circle drawn on the diagonal of the rectangle as its diameter, whose sides are $x=6,\ x=-3,\ y=3$ & $y=-1$ [4]

10. Find the coordinates of the focus & vertex, the equations of the diretrix & the axis & length of latus rectum of the parabola $x=-8y$ [4]

11. Find the length of major & minor axis- coordinate's of vertices & the foci, the eccentricity & length of latus rectum of the ellipse $16x^2+y^2=16$ [6]

12. Find the lengths of the axis , the coordinates of the vertices & the foci the eccentricity & length of the lat us rectum of the hyperbola $25x^2-9y^2=225$ [6]

TEST PAPER-01

CLASS - XI MATHEMATICS (Conic Section)

[ANSWERS]

Ans1. $x^2+y^2-2px+2Qy+Q^2=0$

Ans2. $3y-4=0, x=0$

Ans3. $F\left(\pm 5\sqrt{3}, 0\right)$

Ans4. $e=\sqrt{\dfrac{5}{2}}$

Ans5. This is of the form $x^2+y^2+2gx+2Fy+c=0,$

where $2g=-6, 2f=4 \,\&\, c=-36$

$\therefore q=-3, f=2 \,\&\, c=-36$

So, centre of the circle $=(-g,-f)=(3,-2)$

&

Radius of the circle $=\sqrt{q^2+f^2-c}=\sqrt{9+4+36}$

$=7$ units

Ans6. Let the required equation of the ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1,$ where $a^2>b^2$

let the foci be $(\pm c, 0), c=8$

& $e=\dfrac{c}{a} \Leftrightarrow a=\dfrac{c}{e}=\dfrac{8}{\frac{1}{4}}=32$

Now $c^2=a^2-b^2 \Leftrightarrow b^2=a^2-c^2=1024-64=960$

$\therefore a^2=1024 \quad \& \quad b^2=960$

Hence equation is $\dfrac{x^2}{1024}+\dfrac{y^2}{960}=1$

Ans7. Let equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

& its vertices are $(0,\pm a)$ & $a=10$

Let $c^2=a^2-b^2$

Then $e=\frac{c}{a} \Rightarrow c=ae=10\times\frac{4}{5}=8$

Now $c^2=a^2-b^2 \Leftrightarrow b^2=(a^2-c^2)=100-64=36$

$\therefore a^2=(10)^2=100$ & $b^2=36$

Hence the equation is $\frac{x^2}{36}+\frac{y^2}{100}=1$

Ans8. Clearly C = 12

Length of cat us rectum $=36 \Leftrightarrow \frac{2b^2}{a}=36$

$\Rightarrow b^2=18a$

Now $c^2=a^2+b^2 \Leftrightarrow a^2=c^2-b^2=144-18a$

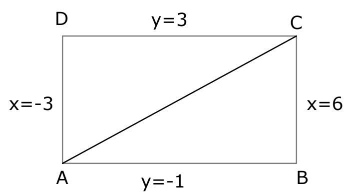
$a^2+18a-144=0$

$(a+24)(a-6)=0 \Leftrightarrow a=6$ $[\because a$ is non negative$]$

This $a^2=6^2=36$ & $b^2=108$

Hence, $\frac{x^2}{36}+\frac{y^2}{108}=1$

Ans9. Let ABCD be the given rectangle &
$AD=x=-3,\ BC=x=6,\ AB=y=-1$ & $CD=y=-3$



Then $A(-3,-1)$ & $c(6,3)$

So the equation of the circle with AC as diameter is given as

$(x+3)(x-6)\ +\ (y+1)(y-3)=0$

$\Rightarrow x^2+y^2-3x-2y-21=0$

Ans10. $x^2 = -8y$

& $x^2 = -4ay$

So, $4a = 8 \Leftrightarrow a = 2$

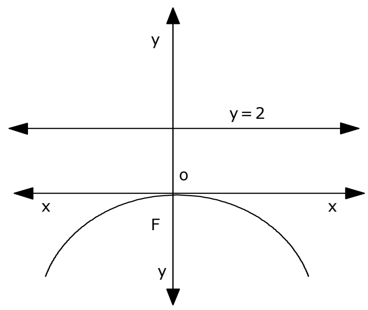
So it is case of downward parabola

So, foci is $F(0,-a)$ *ie* $F(0,-2)$

Its vertex is $0(0,0)$

So, $y = a = 2$

Its axis is y – axis, whose equation is $x = 0$ length of lotus centum $= 4a = 4\times 2 = 8$ units.



Ans11. $16x^2 + y^2 = 16$

Dividing by 16,

$$x^2 + \frac{y^2}{16} = 1$$

So $b^2 = 1$ & $a^2 = 16$ & $b = 1$ & $a = 4$

&

$$c = \sqrt{a^2 - b^2} = \sqrt{16-1}$$

$$= \sqrt{15}$$

Thus $a = 4$, $b = 1$ & $c = \sqrt{15}$

(i) Length of major axis $= 2a = 2\times 4 = 8$ units

Length of minor axis $= 2b = 2\times 1 = 2$ units

(ii) Coordinates of the vertices are

$A(-a,0)$ & $B(a,0)$ *ie* $A(-4,0)$ & $B(4,0)$

(iii) Coordinates of foci are

$F_1(-c,0)$ & $F_2(c,0)$ *ie* $F_1\left(-\sqrt{15},0\right)$ & $F_2\left(\sqrt{15,0}\right)$

(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

(v) Length of latus rectum $= \frac{2b^2}{a} = \frac{2}{4} = \frac{1}{2}$ units

Ans12. $25x^2 - 9y^2 = 225 \Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$

So, $a^2 = 9$ & $b^2 = 25$

& $c = \sqrt{a^2 + b^2} = \sqrt{9+25} = \sqrt{34}$

(i) Length of transverse axis $= 2a = 2\times3 = 6$ units

Length of conjugate axis $= 2b = 2\times5 = 10$ units

(ii) The coordinates of vertices are

$A(-a,0)$ & $B(a,0)$ *ie* $A(-3,0)$ & $B(3,0)$

(iii) The coordinates of foci are

$F_1,(-c,0)$ & $F_2(c,0)$ *ie* $F_1\left(-\sqrt{34},0\right)$ & $F_2\left(\sqrt{34},0\right)$

(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$

(v) Length of the lat us rectum $= \frac{2b^2}{a} = \frac{50}{3}$ units

TEST PAPER-02
CLASS - XI MATHEMATICS (Conic Section)

1. Find the equation of a circle with centre (b,a) & touching $x-$ axis? [1]

   $(A)\, x^2+y^2-2bx+2ay+b^2=0$ $(B)\, x^2+y^2+2bx-2ay+b^2=0$
   $(C)\, x^2+y^2-2bx-2ay+b^2=0$ (D) none of these

2. Find the lengths of axes of $3x^2-2y^2=6$? [1]

   $(A)\, 2\sqrt{2}\ \&\ 2\sqrt{5}$ units $(B)\, 2\sqrt{2}\ \&\ 2\sqrt{3}$ units
   $(C)\, 2\sqrt{5}\ \&\ 2\sqrt{2}$ units (D) none of these

3. Find the length of the latus rectum of $3x^2+2y^2=18$? [1]

   (A) 2 units (B) 3 units (C) 4 units (D) none of these

4. Find the length of the latus rectum of the parabola $3y^2=8x$ [1]

   $(A)\, \frac{4}{3}$ units $(B)\, \frac{8}{3}$ units $(C)\, \frac{2}{3}$ units (D) none of these

5. Show that the equation $6x^2+6y^2+24x-36y-18=0$ represents a circle. Also find its centre & radius. [4]

6. Find the equation of the parabola with focus at $F(5,0)$ & directrix is $x=-5$ [4]

7. Find the equation of the hyperbola with centre at the origin, length of the transverse axis 18 & one focus at (0,4) [4]

8. Find the equation of an ellipse whose vertices are $(0,\pm 13)$ & the foci are $(0,\pm 5)$ [4]

9. Find the equation of the ellipse whose foci are $(0,\pm 3)$ & length of whose major axis is 10 [4]

10. Find the equation of the hyperbola with centre at the origin, length of the transverse axis 8 & one focus at (0,6) [4]

11. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2=12y$ to the ends of its latus rectum. [6]

12. A man running in a race course notes that the sum of the distances of the two flag posts from him is always 12 m & the distance between the flag posts is 10 m. find the equation of the path traced by the man. [6]

TEST PAPER-02

CLASS - XI MATHEMATICS (Conic Section)

[ANSWERS]

Ans1. $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

Ans2. $2\sqrt{2}$ Units & $2\sqrt{3}$ units

Ans3. 4 units

Ans4. $\frac{8}{3}$ units

Ans5. $6x^2 + 6y^2 + 24x - 36y + 18 = 0$

So $x^2 + y^2 + 4x - 6y + 3 = 0$

Where, $2g = 4, 2f = -6 \& C = 3$

$\therefore g = 2, f = -3 \& C = 3$

Hence, centre of circle $= (-g, -f) = (-2, 3)$

&

Radius of circle $= \sqrt{4+9+9} = \sqrt{20}$

$= 2\sqrt{5}$ units

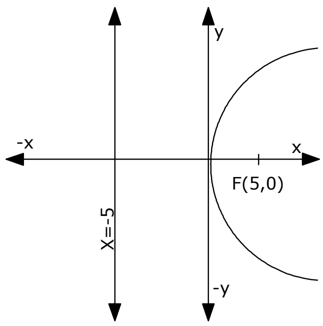
Ans6. Focus $F(5,0)$ lies to the right hand side of the origin

So, it is right hand parabola.

Let the required equation be

$y^2 = 4ax$ & $a = 5$

So, $y^2 = 20x.$



Ans7. Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 4.

length of the transverse axis $= 8 \Leftrightarrow 2a = 18$

$a = 9$

Also, $C^2 = (a^2 + b^2)$

$b^2 = c^2 - a^2 = 16 - 81 = -65$

So, $a^2 = 81$ & $b^2 = -65$

So, equation is $\frac{y^2}{81} + \frac{x^2}{65} = 1$

Ans8. Let the equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& a = 13

Let its foci be $(0, \pm c)$, then $c = 5$

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

So, $a^2 = 169$ & $b^2 = 144$

So, equation be $\frac{x^2}{144} + \frac{y^2}{169} = 1$

Ans9. Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Let $c^2 = a^2 - b^2$

Its foci are $(0, \pm c)$ & $c = 3$

Also, a = length of the semi- major axis = $\frac{1}{2} \times 10 = 5$

Now, $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 25 - 3 = 16.$

Then, $a^2 = 25$ & $b^2 = 16$

Hence the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1.$

Ans10. Let its equation by $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 6

& length of the transverse axis $= 8 \Rightarrow 2a = 8 \qquad \Rightarrow a = 4$

Also, $c^2 = a^2 + b^2 \Leftrightarrow b^2 = c^2 - a^2 \qquad \Rightarrow 36 - 16 = 20$

So, $a^2 = 16$ & $b^2 = 20$

Hence, the required equation is $\frac{y^2}{16} - \frac{x^2}{20} = 1$

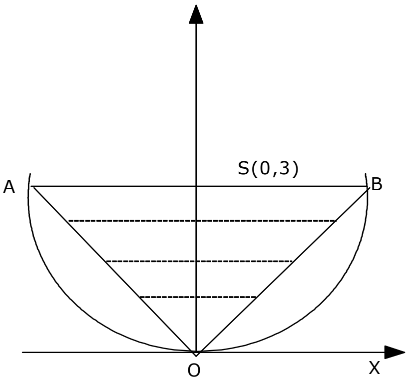
Ans11. The vertex of the parabola $x^2 = 12y$ ie $o(0,0)$.

0	0	1
6	3	1
-6	3	1

Comparing $x^2 = 12y$ with $x^2 = 4ay$, we get $a = 3$ the coordinates of its focus S are $(0,3)$.

Clearly, the ends of its latus rectum are :
$A(-2a, a)$ & $B(2a, a)$

Ie $A(-6,3)$ & $B(6,3)$



$\therefore$ area of $\Delta OBA = \frac{1}{2}$

$= \frac{1}{2}\left[1 \times (18+18)\right]$

$= 18$ units.

Ans12. We know that on ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (caked foci) is constant.

So, the path is ellipse.

Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2\left(1 - c^2\right)$

Clearly, $2a = 12$ & $2ae = 10$

$\Rightarrow a = b$ & $e = \frac{5}{6}$

$\Rightarrow b^2 = a^2\left(1 - e^2\right) = 36\left(1 - \frac{25}{36}\right)$

$\Rightarrow b^2 = 11$

Hence, the required equation is $\frac{x^2}{36} + \frac{y^2}{11} = 1$

TEST PAPER-03

CLASS - XI MATHEMATICS (Conic Section)

1. The equation $x^2+y^2-12x+8y-72=0$ represent a circle find its centre [1]
$(A)(-6,-4)$ $(B)(6,-4)$ $(C)(6,4)$ $(D)(-6,4)$

2. Find the equation of the parabola with focus $F(4,0)$ & directrix $x=-4$ [1]
$(A)\,y^2=32x$ $(B)\,y^2=-16x$ $(C)\,y^2=8x$ $(D)\,y^2=16x$

3. Find the coordinates of the foci of $\frac{x^2}{8}+\frac{y^2}{4}=1$ [1]
$(A)\,F_1(2,0)\,\&\,F_2(-2,0)$ $(B)\,F_1(-2,0)\,\&\,F_2(2,0)$
$(C)\,F_1(-2,0)\,\&\,F_2(-2,0)$ (D) none of these

4. Find the coordinates of the vertices of $x^2-y^2=1$ [1]
$(A)\,A(-1,0),B(-1,0)$ $(B)\,A(-1,0),B(1,0)$
$(C)\,A(1,0),B(-1,0)$ (D) none of these

5. Find the equation of the hyperbola whose foci are at $(0,\pm B)$ & the length of whose conjugate axis is $2\sqrt{11}$ [4]

6. Find the equation of the hyperbola whose vertices are $(0,\pm 3)$ & foci are $(0,\pm 8)$ [4]

7. Find the equation of the ellipse for which $e=\frac{4}{5}$ & whose vertices are $(0,\pm 10)$. [4]

8. Find the equation of the ellipse, the ends of whose major axis are $(\pm 7,0)$ & the ends of whose minor axis are $(0,\pm 2)$ [4]

9. Find the equation of the parabola with vertex at the origin & y+5 = 0 as its directrix. Also, find its focus [4]

10. Find the equation of a circle, the end points of one of whose diameters are $A(2,-3)\,\&\,B(-3,5)$. [4]

11. An equilateral triangle is inscribed in the parabola $y^2=4ax$ so that one angular point of the triangle is at the vertex of the parabola. Find the length of each side of the triangle. [6]

12. Find the equation of the hyperbola whose foci are at $(0,\pm\sqrt{10})$ & which passes through the points $(2,3)$ [6]

[ANSWERS]

Ans1. $(6,-4)$

Ans2. $y^2 = 16x$

Ans3. $F_1(-2,0)$ & $F_2(2,0)$

Ans4. $A(-1,0), B(1,0)$

Ans5. Let it equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Let it foci be $(0, \pm C)$

$\therefore C = 8$

Length of conjugate axis $= 2\sqrt{11}$

$\Rightarrow 2b = 2\sqrt{11} \Rightarrow b = \sqrt{11} \Rightarrow b^2 = 11$

Also, $C^2 = (a^2 + b^2) = (c^2 - b^2) = 64 - 11 = 53$

$a^2 = 53$

Hence, required equation is $\frac{y^2}{53} - \frac{x^2}{11} = 1$

Ans6. The vertices are $(0 \pm a)$

But it is given that the vertices are (0 ± 3)

$\therefore a = 3$

Let its foci be $(0, \pm c)$

But it is given that the foci are $(0, \pm 8)$

$\therefore c = 8$

Now $b^2 = (c^2 - a^2) = 8^2 - 3^2 = 64 - 9 = 55$

Then $a^2 = 3^2 = 9 \,\&\, b^2 = 55$

Hence the required equation is $\frac{y^2}{9} - \frac{x^2}{55} = 1$

Ans7. Its vertices are $(0, \pm a)$ & therefore a =10

Let $c^2 = \left(a^2 - b^2\right)$

Then, $e = \frac{c}{a} \Rightarrow c = ae = \left[10 \times \frac{4}{5}\right] = 8$

Now, $c^2 = \left(a^2 - b^2\right) \Rightarrow b^2 = \left(a^2 - c^2\right) = (100 - 64) = 36$

$\therefore a^2 = (10)^2 = 100 \,\&\, b^2 = 36$

Hence the required equation is $\frac{x^2}{36} + \frac{y^2}{100} = 1$

Ans8. Its vertices are $(\pm a, 0)$ & therefore, a = 5 ends of the minor axis are $c(0, -5) \,\&\, D(0, 5)$

$\therefore CD = 25$ ie length of minor axis = 25 units

$\therefore 2b = 25 \Rightarrow \frac{25}{2} = 12.5$

Now, $a = 5$ & $b = 12.5$ $\Rightarrow$ $a^2 = 25$ & $b^2 = 156.25$

Hence, the required equation $\frac{x2}{25} + \frac{y^2}{156.25} = 1$

Ans9. Let the vertex of the parabola be $o(0, 0)$

Now $y + 5 = 0 \Rightarrow y = -5$

Then the directrix is a line parallel

To the x axis at a distance of 5 unite below the x axis so the focus is $F(0, 5)$

Hence the equation of the parabola is

$x^2 = 4ay$ Where a = 5 i.e, $x^2 = 20y$

Ans10. Let the end points of one of whose diameters are (x_1, y_1) & (x_2, y_2) is given by

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Hence $x_1 = 2,\ \ y_1 = -3\ \ \&\ \ x_2 = -3,\ \ y_2 = 5$

$\therefore$ The required equation of the circle is

$(x-2)(x+3)+(y+3)(y-5)=0$

$\Rightarrow x^2 + y^2 + x - 2y - 21 = 0$

Ans11. Let ΔPQR be an equilateral triangle inscribed in the parabola $y^2 = 4ax$

Let QP = QP = QR = PR = C

Let ABC at the $x-$ axis at M.

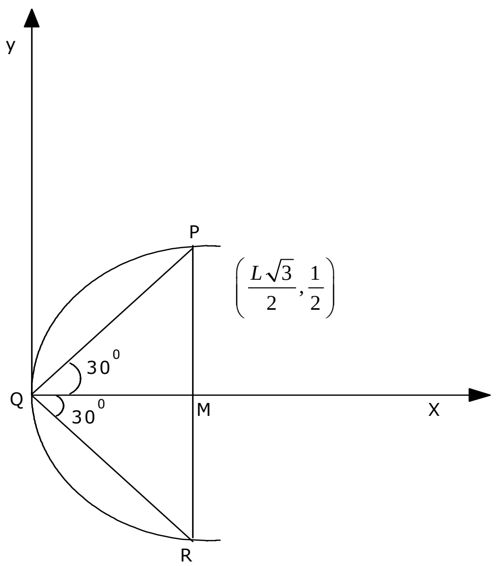
Then , $\angle\ \angle PQM = \angle RQWM = 30^\circ$

$\therefore \dfrac{QM}{QP} = \cos 30^\circ \Rightarrow QM = \angle \cos 30^\circ$

$\Rightarrow \dfrac{L\sqrt{3}}{2}$

$\Rightarrow \dfrac{PM}{QP} = \sin 30^\circ \Rightarrow PM = \angle \sin 30^\circ$

$\Rightarrow \dfrac{L}{2}$



$\therefore$ the coordinates of are $\left[\dfrac{L\sqrt{3}}{2}, \dfrac{L}{2}\right]$

Since P lies on the parabola $y^2 = 4ax,$ we have

$l^2 = 4a \times \dfrac{L\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$

Hence length of each side of the triangle is $8a\sqrt{3}$ units.

Ans12. Let it equation be $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1 \ldots\ldots(i)$

Let its foci be $(0, \pm C)$

But the foci are $\left(0, \pm\sqrt{10}\right)$

$\therefore C = \sqrt{10} \Leftrightarrow C^2 = 10 \Leftrightarrow \left(a^2 + b^2\right) = 10 \ldots\ldots(ii)$

Since (i) passes through (2,3), we have $\frac{9}{a^2} - \frac{4}{b^2} = 1$

Now $\frac{9}{a^2} - \frac{4}{b^2} = 1 \Leftrightarrow \frac{9}{a^2} - \frac{4}{\left(10 - a^2\right)} = 1 \ldots\ldots(iii)$

$\Rightarrow 9\left(10 - a^2\right) - 4a^2 = a^2\left(10 - a^2\right)$

$\Rightarrow a^2 - 23a^2 + 90 = 0$

$\Rightarrow \left(a^2 - 18\right)\left(a^2 - 5\right) = 0 \Leftrightarrow a^2 = 5$

[$\because a^2 = 18 \Rightarrow b^2 = -8$, which is not possible]

Then $a^2 = 5$ & $b^2 = 5$

Hence, the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1$,

i.e. $y^2 - x^2 = 5$

TEST PAPER-04

CLASS - XI MATHEMATICS (Conic Section)

1. Find the coordinates of the vertices of $x^2 - y^2 = 1$ [1]

$(A)\ A(-1,0)\ \&\ B(5,0)$ $(B)\ A(-5,0)\ \&\ B(-1,0)$

$(C)\ A(-1,0)\ \&\ B(-5,0)$ (D) none of these

2. Find the eccentricity of ellipse $4x^2 + 9y^2 = 1$ [1]

$(A)\ e = \frac{\sqrt{5}}{3}$ $(B)\ e = \frac{-\sqrt{5}}{3}$ $(C)\ e = \frac{\sqrt{3}}{5}$ $(D)\ e = \frac{3}{\sqrt{5}}$

3. Find the length of the latus rectum of $9x^2 + y^2 = 36$ [1]

$(A)\frac{1}{3}$ units $(B)\frac{1}{5}$ units $(C)1\frac{1}{3}$ units $(D)\frac{1}{6}$ units

4. Find the length of minor axis of $x^2 + 4y^2 = 100$ [1]

(A)10 units (B)12 units (C)14 units (D)8 units

5. Find the equation of ellipse whose vertices are $(0,\pm 13)$ & the foci are $(0,\pm 5)$ [4]

6. Find the equation of the hyperbola whose foci are $(\pm 5,0)$ & the transverse axis is of length 8. [4]

7. Find the equation of a circle, the end points of one of whose diameters are $A(-3,2)$ & $B(5,-3)$. [4]

8. If eccentricity is $\frac{1}{5}$ & foci are $(\pm 7,0)$ find the equation of an ellipse. [4]

9. Find the equation of the hyperbola where foci are $(\pm 5,0)$ & the transverse axis is of length 8. [4]

10. Find the length of axes & coordinates of the vertices of the hyperbola $\frac{x^2}{49} - \frac{y^2}{64} = 1$ [4]

11. Find the equation of the curve formed by the set of all these points the sum of whose distance from the points $A(4,0,0)$ & $B(-4,0,0)$ is 10 units. [6]

12. Find the equation of the hyperbola whose foci are at $(0,\pm\sqrt{10})$ & which passes through the point $(2,3)$. [6]

TEST PAPER-04

CLASS - XI MATHEMATICS (Conic Section)

[ANSWERS]

Ans1. $A(-1,0)\,\&\,B(5,0)$

Ans2. $e=\frac{\sqrt{5}}{3}$

Ans3. $1\frac{1}{3}\,units$

Ans4. $10units$

Ans5. Let the required equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=15.$

Its vertices are $(0\pm a)$ & therefore a = 13

Let its foci be $(0\pm C)$ then C = 5

$\therefore b^2=a^2-c^2=169-25=144$

This $b^2=144\,\&\,a^2=169$

Hence, the required equation is $\frac{x^2}{144}+\frac{y^2}{169}=1$

Ans6. Let the required equation be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Length of its Trans verse axis =2a

$\therefore 2a=8\Leftrightarrow a=4\Leftrightarrow a^2=16$

Let its foci be $(\pm C,0)$

Then C = 5

$\therefore b^2=\left(c^2-a^2\right)=5^2-4^2=9$

This $a^2=16\,\&\quad b^2=9$

Hence, the required equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$

Ans7. Let the equation be $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Hence $x_1=-3, y_1=2 \;\&\; x_2=5, y_2=-3$

So $(x+3)(x-5)+(y-2)(y+3)=0$

$$x^2-2x-15+y^2+y-6=0$$
$$x^2+y^2-2x+y-21=0$$

Ans8. Let the required equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Let its foci be $(\pm C,0)$, Then C =7

Also, $e=\frac{c}{a} \Leftrightarrow a=\frac{c}{e}=\frac{7}{\frac{1}{5}}=35$

Now $c^2=\left(a^2-b^2\right)$

$$b^2=a^2-c^2=(35)^2-49=1225-49=1176$$
$$\therefore a^2=1225 \;\&\; b^2=1176$$

Hence the required equation is $\frac{x^2}{1225}+\frac{y^2}{1176}=1$

Ans9. Let the required equation be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Length of its transverse axis $=2a$

$$\therefore 2a=8 \Leftrightarrow a=\frac{8}{2}=4$$

$$a^2=16$$

Let its foci be $(\pm C,0)$

Then C = 5

$$\therefore b^2=c^2-a^2=25-16=9$$

Hence the required equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$

Ans10. The equation of the given hyperbola is $\frac{x^2}{49}-\frac{y^2}{64}=1$

Comparing the given equation with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$ we get

$a^2=49\,\&\,b^2=64$

$\therefore C^2=\left(a^2+b^2\right)=49+64=113$

Length of transverse axis = $2a=2\times 7=14$ units

Length of conjugate axis = $2b=2\times 8=16$ units

The coordinators of the vertices are $A(-a,0)\,\&\,B(a-0)\,ie.A(-7,0)\,\&\,B(7,0)$

Ans11. Let $P(x,y,z)$ be an arbitrary point on the given curve

Then $PA+PB=10$

$\Rightarrow\sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$

$=\sqrt{(x+4)^2+y^2+z^2}=10-\sqrt{(x-4)^2+y^2+z^2}\,......(i)$

Squaring both sides

$\Rightarrow(x+4)^2+y^2+z^2=100-(x-4)^2+y^2+z^2-20\sqrt{(x-4)^2+y^2+z^2}$

$\Rightarrow 16x=100-20\sqrt{(x-4)^2+y^2+z^2}$

$\Rightarrow 5\sqrt{(x-4)^2+y^2+z^2}=25-4x$

$\Rightarrow 25\left[(x-4)^2+y^2+z^2\right]=625+16x^2-200x$

$\Rightarrow 9x^2+25y^2+25z^2-225=0$

Hence, the required equation of the curve is

$9x^2+25x^2+25z^2-225=0$

Ans12. Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1.......(i)$

Let its foci be $(0,\pm c)$

But, the foci are $\left(0, \pm\sqrt{10}\right)$

$\therefore C=\sqrt{10} \Leftrightarrow C^2=10$

& $a^2+b^2=10......(ii)$

Since (i) passes through $(2,3)$, we have

$$\frac{9}{a^2}-\frac{4}{b^2}=1$$

Now $\frac{9}{a^2}+\frac{4}{b^2}=1 \Leftrightarrow \frac{9}{a^2}-\frac{4}{\left(10-a^2\right)}=1$

$$\Rightarrow a^4-23a^2+90=0$$
$$\Rightarrow \left(a^2-18\right)\left(a^2-5\right)=0$$
$$\Rightarrow a^2=5$$

Then $a^2=5=b^2$

Hence, the required equation is $\frac{y^2}{5}-\frac{x^2}{5}=1$

i.e. $y^2-x^2=5$

TEST PAPER-05
CLASS - XI MATHEMATICS (Conic Section)

1. Find the centre of the circles $x^2+(y-1)^2=2$ [1]

(A) $(1,0)$ (B) $(0,1)$ (C) $(1,2)$ (D) None of these

2. Find the radius of circles $x^2+(y-1)^2=2$ [1]

$(A)\sqrt{2}$ $(B)2$ $(C)2\sqrt{2}$ (D) None of these

3. Find the length of latcus rectum of $x^2=-22y$ [1]

$(A)11$ $(B)-22$ $(C)22$ (D) None of these

4. Find the length of latcus rectum of $25x^2+4y^2=100$ [1]

$(A)\frac{3}{5}$units $(B)\frac{1}{5}$units $(C)\frac{8}{5}$units (D) None of these

5. Find the lengths of axes & length of lat us rectum of the hyperbola, $\frac{y^2}{9}-\frac{x^2}{16}=1$ [4]

6. Find the eccentricity of the hyperbola of $\frac{y^2}{9}-\frac{x^2}{16}=1$ [4]

7. Find the equation of the hyperbola with centre at the origin, length of the trans verse axis 6 & one focus at $(0,4)$ [4]

8. Find the equation of the ellipse, the ends of whose major axis are $(\pm 3,0)$ & at the ends of whose minor axis are $(0,\pm 4)$ [4]

9. Find the equation of the parabola with focus at $F(4,0)$ & directrix $x=-3$ [4]

10. If $y=2x$ is a chord of the circle $x^2+y^2-10x=0$, find the equation of the circle with this chord as a diameter [4]

11. Find the equation of the ellipse with centre at the origin, major axis on the y – axis & passing through the points $(3,2)$ & $(1,6)$ [6]

12. Prove that the standard equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [6]

Where a & b are the lengths of the semi major axis & the semi- major axis respectively & a > b.

TEST PAPER-05

CLASS - XI MATHEMATICS (Conic Section)

[ANSWERS]

Ans1. $(0,1)$

Ans2. $\sqrt{2}$

Ans3. 22

Ans4. $\frac{8}{5}$ Units

Ans5. The given equation is $\frac{y^2}{9}-\frac{x^2}{16}=1$ means hyperbola

Comparing the given equation with $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we get

$a^2=9 \;\;\&\;\; b^2=16$

Length of transverse axis $=2a=2\times3=6$ units

Length of conjugate axis $=2b=2\times4=8$ units

The coordinates of the vertices are $A(0,-a) \;\;\&\;\; B(0,a)$ i.e $A(0,-3) \;\;\&\;\; B(0,3)$

Ans6. As in above question

$a=3 \;\;\&\;\; b=4$

&

$c^2=a^2+b^2=9+16=25$

So, c = 5

Then $e=\frac{c}{a}=\frac{5}{3}$

Ans7. Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Clearly c = 4

Length of transverse axis $=6 \Leftrightarrow 2a=6 \Leftrightarrow a=3.$

Also, $c^2=a^2+b^2 \Leftrightarrow b^2=c^2-a^2=4^2-3^2=16-9=7$

Then $a^2=3^2=9$ & $b^2=7$

Hence, the required equation is $\frac{y^2}{9}-\frac{x^2}{7}=1$

Ans8. Let the required equation be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Its vertices are $(\pm a,0)$ & $a=3$

Ends of minor axis are $C(0,-4)$ & $D(0,4)$

$\therefore CD=8$ ie length of the minor axis = 8 units

Now, $2b=8 \Leftrightarrow b=4$

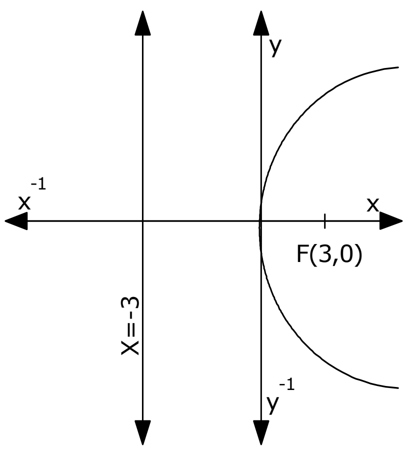
$\therefore a=3$ & $b=4$

Hence the required equation is $\frac{x^2}{9}+\frac{y^2}{16}=1$

Ans9. Focus $F(4,0)$ lies on the axis hand side of the origin so, it is a right handed parabola. Let the required equation be $y^2=4ax.$

Than, a = 4

Hence, the required equation is $y^2=16x$



Ans10. $y=2x$ & $x^2+y^2-10x=0$

Putting $y=2x$ in $x^2+y^2-10x=0$ we get

$5x^2-10x=0 \Leftrightarrow 5x(x-2)=0 \Leftrightarrow x=0$ or $x=2$

Now, $x=0 \Rightarrow y=0$ & $x=2 \Rightarrow y=4$

$\therefore$ the points of intersection of the given chord & the given circle are

$A(0,0)$ & $B(2,4)$

$\therefore$ the required equation of the circle with AB as diameter is

$(x-0)(x-2)+(y-0)(y-4)=0$

$\Rightarrow x^2+y^2-2x-4y=0$

Ans11. Let the required equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1......(i)$

Since $(3,2)$ lies on (i) we have $\frac{9}{b^2}+\frac{4}{a^2}=1......(ii)$

Also, since $(1,6)$ lies on (i), we have $\frac{1}{b^2}+\frac{36}{a^2}=1......(iii)$

Putting $\frac{1}{b^2}=u$ & $\frac{1}{a^2}=v$ these equations become:

$9u+4v=1......(iv)$ & $u+36v=1......(v)$

On multiplying (v) by 9 & subtracting (iv) from it we get

$320v=8 \Leftrightarrow v=\frac{8}{320}=\frac{1}{40} \Leftrightarrow \frac{1}{a^2}=\frac{1}{40} \Leftrightarrow a^2=40$

Putting $v=\frac{1}{40}$ in (v) we get

$u+\left[36\times\frac{1}{40}\right]=1 \Leftrightarrow u=\left[1-\frac{9}{10}\right]=\frac{1}{10} \Leftrightarrow \frac{1}{b^2}=\frac{1}{10} \Leftrightarrow b^2=10$

Then , $b^2=10$ & $a^2=40$

Hence the required equation is $\frac{x^2}{10}+\frac{y^2}{40}=1$

Ans12. Let the equation of the given curve be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & *let*

$P(x, y)$ be an arbitrary point on this curve

Then, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y^2=b^2\left[1-\frac{x^2}{a^2}\right]$

$\Rightarrow y^2=\frac{b^2\left[a^2-x^2\right]}{a^2}$......$(i)$

Also, let $\left(a^2-b^2\right)=c^2$......(ii)

Let $F_1(-c, 0)$ & $F_2(c, 0)$ be two fixed points on the x- axis, than

$PF_1=\sqrt{(x+c)^2+y^2}$

$=\sqrt{(x+c)^2+\frac{b^2\left(a^2-x^2\right)}{a^2}}$ using (i)

$=\sqrt{(x+c)^2+\frac{\left(a^2-c^2\right)\left(a^2-x2\right)}{a^2}}$ using (ii)

$=\sqrt{a^2+2cx+\frac{c^2x^2}{a^2}}$

$=\sqrt{\left[a+\frac{cx}{a}\right]^2}=\left[a+\frac{cx}{a}\right]$

Similarly, $PF_2=\left[a-\frac{cx}{a}\right]$

$\therefore PF_1+PF_2=\left[a+\frac{cx}{a}+a-\frac{cx}{a}\right]$

$\Rightarrow PF_1+PF_2=2a$

This shows that the given curve is an ellipse

Hence the equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

TEST PAPER-01

CLASS - XI MATHEMATICS (Introduction to 3D)

1. Name the octants in which the following lie. (5,2,3) [1]

2. Name the octants in which the following lie. (-5,4,3) [1]

3. Find the image of (-2,3,4) in the y z plane [1]

4. Find the image of (5,2,-7) in the xy plane [1]

5. Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR [4]

6. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1) [4]

7. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2,1) [4]

8. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle. [4]

9. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear. [4]

10. Show that coordinator of the centroid of triangle with vertices A($x_1y_1z_1$), B($x_2y_2z_2$), and C($x_3y_3z_3$) is $\left[\frac{x_1+y_1+z_1}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$ [4]

11. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent. [6]

12. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices. [6]

[ANSWERS]

Ans1. I

Ans2. II

Ans3. (2, 3, 4)

Ans4. (5, 2, 7)

Ans5. Suppose Q divides PR in the ratio $\lambda:1$. Then coordinator of Q are

$$\left(\frac{9\lambda+3}{\lambda+1},\frac{8\lambda+2}{\lambda+1},\frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5,\frac{8\lambda+2}{\lambda+1}=4,\frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$$\lambda=\frac{1}{2}.$$

So Q divides PR in the ratio $\frac{1}{2}:1$ or 1:2

Ans6. We know that Z- coordinate of every point on xy-plane is zero. So, let $P(x,y,o)$ be a point in xy-plane such that PA=PB=PC

Now, PA=PB

$\Rightarrow PA^2=PB^2$

$$\Rightarrow (x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)^2$$

$\Rightarrow 4x-6y=0$ or $2x-3y=0.......(i)$

$PB=PC$

$\Rightarrow PB^2=PC^2$

$$\Rightarrow (x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2+(0-1)^2$$

$\Rightarrow -6y+12=0 \Rightarrow y=2.........(ii)$

Putting $y=2$ in (i) we obtain $x=3$

Hence the required points (3,2,0).

Ans7. Let $P(x,y,z)$ be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA=PB

$\Rightarrow$ $PA^2=PB^2$

$$\Rightarrow \sqrt{(x-0)^2+(y-2)^2+(2-3)^2}=\sqrt{(x-2)^2+(y+2)^2+(z-1)^2}$$

$\Rightarrow 4x-8y-42+4=0$ or $x-2y-2+1=0$

Hence the required locus is $x-2y-2+1=0$

Ans8. We have

$$AB=\sqrt{(2-0)^2+(-1-1)^2(+3-2)^2}=\sqrt{4+4+1}=3$$

$$BC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{1+4+4}=3$$

And $CA=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{1+16+1}=3\sqrt{2}$

Clearly AB=BC and $AB^2+BC^2=AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

Ans9. Suppose the given points are collinear and C divides AB in the ratio $\lambda:1$.

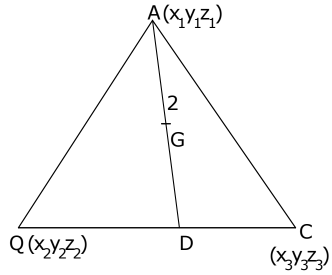
Then coordinates of C are

$$\left(\frac{\lambda-2}{\lambda+1},\frac{2\lambda+3}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$$

But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda=\frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

Ans10. Let D be the mid point of AC. Then



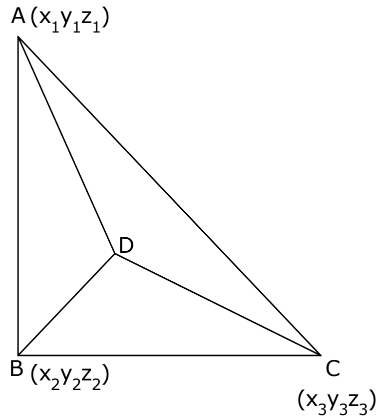
Coordinates of D are $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$.

Let G be the centroid of ΔABC. Then G, divides AD in the ratio 2:1. So coordinates of D are

$$\left(\frac{1.x_1+2\frac{(x_2+x_3)}{2}}{1+2},\frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2},\frac{1.z_1+2\left(\frac{z_2+z_3}{2}\right)}{1+2}\right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

Ans11. Let ABCD be tetrahedron such that the coordinates of its vertices are $A(x_1,y_1,z_1)$, $B(x_2,y_2,z_2)$, $C(x_3,y_3,z_3)$ and $D(x_4,y_4,z_4)$



The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

$$G_1\left[\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$$

$$G_2\left[\frac{x_1+x_2+x_4}{3},\frac{y_1+y_2+y_4}{3},\frac{z_1+z_2+z_4}{3}\right]$$

$$G_3\left[\frac{x_2+x_3+x_4}{3},\frac{y_2+y_3+y_4}{3},\frac{z_2+z_3+z_4}{3}\right]$$

$$G_4\left[\frac{x_4+x_3+x_1}{3},\frac{y_4+y_3+y_1}{3},\frac{z_4+z_3+z_1}{3}\right]$$

Now, coordinates of point G dividing DG_1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3},\frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3},\frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$$

Similarly the point dividing CG_2, AG_3 and BG_4 in the ratio 3:1 has the same coordinates.

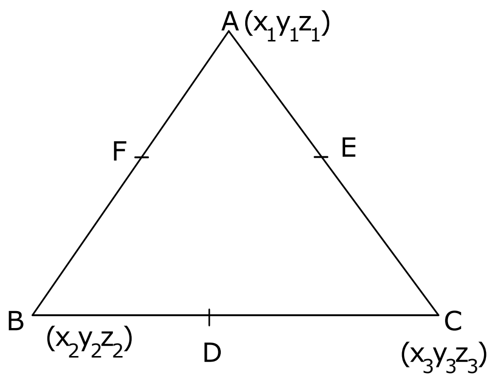
Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG_1, CG_2, AG_3 and BG_4.

Hence they are concurrent.

Ans12. Suppose vertices of ΔABC are $A(x_1y_1z_1)$, $B(x_2y_2z_2)$ and $C(x_3y_3z_3)$ respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and F(2,3,4)

$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$



$x_2+x_3=2......(i)$

$\frac{x_1+x_3}{2}=0$

$y_2+y_3=10......(ii)$

$\frac{y_1+y_3}{2}=4$

$z_2+z_3=2......(iii)$

$\frac{z_1+z_3}{2}=-2$

$x_1+x_3=0.......(iv)$

$\frac{x_1+x_2}{2}=2$

$y_1+y_2=8.......(v)$

$\frac{y_1+y_2}{2}=3$

$z_1+z_3=-4.......(vi)$

$\frac{z_1+z_2}{2}=4$

$x_1+x_2=4.......(vii)$

$y_1+y_2=6.......(viii)$

$z_1+z_2=8.......(ix)$

Adding eq. $(i),(iv),\&(vii)$

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3.......(\text{x})$

Subtracting eq. $(i), (iv), \& (vii)$ from (x) we get

$x_1 = 1, \quad x_2 = 3, \quad x_3 = -1$

Similarly, adding eq. $(ii), (v)$ and $(viii)$

$y_1 + y_2 + y_3 = 12......(\text{xi})$

Subtracting eq. $(ii), (v)$ and $(viii)$ from (xi)

$y_1 = 2, \quad y_2 = 4, \quad y_3 = 6$

Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1, \quad z_2 = 7, \quad z_3 = -5$

$\therefore$ Coordinates of vertices of Δ ABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

TEST PAPER-02

CLASS - XI MATHEMATICS (Introduction to 3D)

1. A point lie on X –axis what are co ordinate of the point [1]
2. Write the name of plane in which x axis and y - axis taken together. [1]
3. The point $(4,-3,-6)$ lie in which octants [1]
4. The point $(2,0,8)$ lie in which plane [1]
5. Prove by distance formula that the points $A(1,2,3)$, $B(-1,-1,-1)$ and $C(3,5,7)$ are collinear. [4]
6. Find the co ordinate of the point which divider the join of $P(2,-1,4)$ and $Q(4,3,2)$ in the ratio $2:5$ (i) internally (ii) externally [4]
7. Find the co ordinate of a point equidistant from the four points $0(0,0,0)$, $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$ [4]
8. Find the ratio in which the join the $A(2,1,5)$ and $B(3,4,3)$ is divided by the plane $2x+2y-2z=1$ Also find the co ordinate of the point of division [4]
9. Find the centroid of a triangle, mid points of whose sides are $(1,2,-3)$, $(3,0,1)$ and $(-1,1,-4)$ [4]
10. The mid points of the sides of a ΔABC are given by $(-2,3,5)$, $(4,-1,7)$ and $(6,5,3)$ find the co ordinate of A, B and C [4]
11. Let $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ be two points in space find co ordinate of point R which divides P and Q in the ratio $m_1:m_2$ by geometrically [6]
12. Show that the plane $ax+by+cz+d=0$ divides the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) in the ratio $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$ s [6]

TEST PAPER-02
CLASS - XI MATHEMATICS (Introduction to 3.D)

[ANSWERS]

Ans1. $(a,0,0)$

Ans2. XY Plane

Ans3. $VIII$

Ans4. XZ

Ans5. Distance

$$|AB| = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

Distance

$$|BC| = \sqrt{(3+1)^2 + (5+1)^2 + (7+1)^2} = \sqrt{16+36+64} = 2\sqrt{29}$$

Distance

$$|AC| = \sqrt{(3-1)^2 + (5-2)^2 + (7-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

$\therefore \ |BC| = |AB| + |Ac|$

$\therefore$ The paints A.B.C. are collinear.

Ans6. Let paint $R(x,y,z)$ be the required paint.

(i) For internal division

$$x = \frac{2\times4+5\times2}{2+5} = \frac{8+10}{7} = \frac{18}{7}$$

$$y = \frac{2\times3+5\times-1}{2+5} = \frac{6-5}{7} = \frac{1}{7}$$

$$z = \frac{2\times2+5\times4}{2+5} = \frac{4+20}{7} = \frac{24}{7}$$

$\therefore$ Required paint $R\left(\frac{18}{7}, \frac{1}{7}, \frac{24}{7}\right)$

(ii) For external division.

$$x = \frac{2\times4-5\times2}{2-5} = \frac{8-10}{-3} = \frac{-2}{-3} = \frac{2}{3}$$

$$y = \frac{2\times3-5\times-1}{2-5} = \frac{6+5}{-3} = \frac{11}{-3}$$

$$z = \frac{2\times2-5\times4}{2-5} = \frac{4-20}{-3} = \frac{-16}{-3} = \frac{16}{3}$$

$\therefore$ Required point $R\left(\frac{2}{3}, \frac{-11}{3}, \frac{16}{3}\right)$

Ans7. Let $P(x, y, z)$ be the required point

According to condition

$OP = PA = PB = PC$

Now $OP = PA$

$\Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (x-a)^2 + (y-0)^2 + (z-0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2ax + a^2 + y^2 + z^2$

$2ax = a^2$

$\therefore x = \frac{a}{2}$

Similarly $OP = PB$

$\Rightarrow y = \frac{b}{2}$

$A(x, y, z,)$ $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ D, E and F are mid points of side $BC, CA,$ and AB respectively,

Then $\frac{x_1 + x_2}{2} = -1$

$x_1 + x_2 = -2......(1)$

$\frac{y_1 + y_2}{2} = 1$

$y_1 + y_2 = 2......(2)$

$\frac{z_1 + z_2}{2} = -4$

$z_1 + z_2 = -8......(3)$

$\frac{x_2 + x_3}{2} = 1$

$x_2 + x_3 = 2......(4)$

$\frac{y_2 + y_3}{2} = 2$

$y2 + y3 = 4......(5)$

$\frac{z_2 + z_3}{2} = -3$

$z_2 + z_3 = -6......(6)$

$\frac{x_1 + x_3}{2} = 3$

$x_1 + x_3 = 6......(7)$

$$\frac{y_1+y_3}{2}=0$$

$y_1+y_3=0......(8)$

$$\frac{z_1+z_3}{2}=1$$

$z_1+z_3=2......(9)$

Adding eq (1),(4) and (7) we get

$2(x_1+x_2+x_3)=-2+2+6$

$x_1+x_2+x_3=3......(10)$

Adding eq. (2),(5) and (8)

$2(y_1+y_2+y_3)=6$

$y_1+y_2+y_3=3......(11)$

And $OP=PC$

$\Rightarrow z=\dfrac{c}{2}$

Hence co ordinate of $P\left(\dfrac{a}{2},\dfrac{b}{2},\dfrac{c}{2}\right)$

Ans8. Suppose plane $2x+2y-2z=1$ divides $A(2,1,5)$ and $B(3,4,5)$ in the ratio $\lambda:1$ at pain C

Then co ordinate of paint C

$$\left(\frac{3\lambda+2}{\lambda+1},\frac{4\lambda+1}{\lambda+1}\ \frac{3\lambda+5}{\lambda+1}\right)$$

$\because$ Point C lies on the plane $2x+2y-2z=1$

$\therefore$ Points C must satisfy the equation of plane

$$2\left(\frac{3\lambda+2}{\lambda+1}\right)+2\left(\frac{4\lambda+1}{\lambda+1}\right)-2\left(\frac{3\lambda+5}{\lambda+1}\right)=1$$

$\Rightarrow 8\lambda-4=\lambda+1$

$\Rightarrow \lambda=\dfrac{5}{7}$

$\therefore$ Required ratio 5:7

Ans9. Suppose co ordinate of vertices of $\Delta\, ABC$ are

Adding eq. (3), (6) and (9)

$2(z_1+z_2+z_3)=-8-6+2$

$z_1+z_2+z_3=-6......(12)$

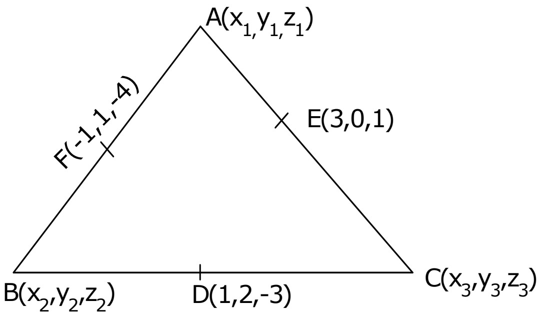
Co ordinate of centroid

$$x=\frac{x_1+x_2+x_3}{3}=\frac{3}{3}=1$$

$$y=\frac{y_1+y_2+y_3}{3}=\frac{3}{3}=1$$

$$z=\frac{z_1+z_2+z_3}{3}=\frac{-6}{3}=-2$$

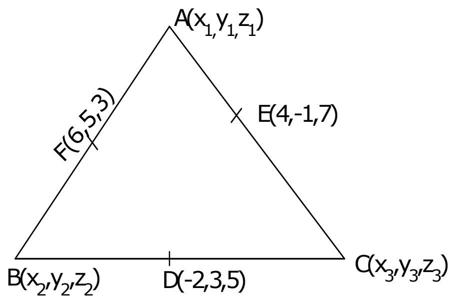
$(1,\ 1,-2)$



Ans10. Suppose co ordinate of point $A.B.C.$ are (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) respectively let D,E and F are mid points of side BC,CA and AB respectively

$$\therefore\ \frac{x_1+x_2}{2}=6$$

$$x_1+x_2=12......(1)$$



$$\frac{y_1+y_2}{2}=5$$

$$y_1+y_2=10......(2)$$

$$\frac{z_1+z_2}{2}=3$$

$$z_1+z_2=6......(3)$$

$$\frac{x_2+x_3}{2}=-2$$

$$x_2+x_3=-4......(4)$$

$$\frac{y_2+y_3}{2}=3$$

$$y_2+y_3=6......(5)$$

$$\frac{z_1+z_2}{2}=5$$

$$z_1+z_2=10......(6)$$

$$\frac{x_1+x_3}{2}=4$$

$$x_1+x_3=8......(7)$$

$$\frac{y_1+y_3}{2}=-1$$

$$y_1+y_3=-2......(8)$$

$$\frac{z_1 + z_3}{z} = 7$$

$z_1 + z_3 = 14......(9)$

Adding eq. (1), (4) and (7)

$2(x_1 + x_2 + x_3) = 12 - 4 + 8$

$x_1 + x_2 + x_3 = \frac{\cancel{16}}{\cancel{2}} = 8......(10)$

Similarly $\quad y_1 + y_2 + y_3 = 7......(11)$

$z_1 + z_2 + z_3 = 15......(12)$

Subtracting eq. (1), (4) and (7) from (10)

$x_3 = -4, \quad x_1 = 12, \quad x_2 = 0$

Now subtracting eq. (2), (5) and (8) from (11)

$y_3 = -3, \quad y_1 = 1, \quad y_2 = 9$

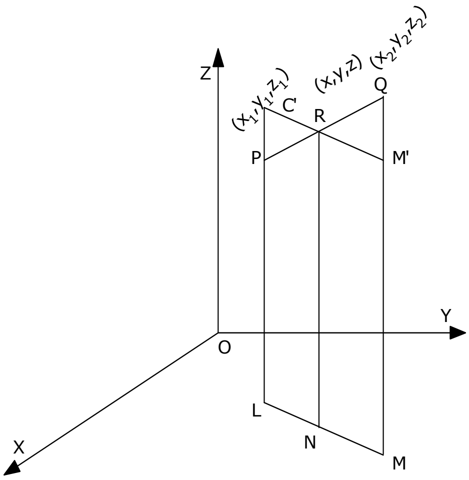
Similarly $\; z_3 = 9, \quad z_1 = 5, \quad z_2 = 1$

$\therefore$ co ordinate of point $A, B.$ and C are

$A(12, 0, -4), \quad B(1, 9, -3)$, and $C(5, 1, 9)$

Ans11. Let co ordinate of Point R be (x, y, z) which divider line segment joining the point $P\ Q$ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ $\quad [By\ AA$ sinilsrity$]$



$$\therefore \quad \frac{PL'}{MQ'} = \frac{PR}{RQ}$$

$$\Rightarrow \frac{LL' - LP}{MQ - MM'} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{NR - LP}{MQ - NR} = \frac{m_1}{m_2} \qquad \left[\begin{array}{l} \because LL' = NR \\ \text{and } MM' = NR \end{array}\right]$$

$$\Rightarrow \frac{z - z_1}{z_2 - z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_1, z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly $\quad x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ and

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Ans12. Suppose the plane $ax+by+cz+d=0$ divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\lambda:1$

$$\therefore\ x=\frac{\lambda x_2+x_1}{\lambda+1},\quad y=\frac{\lambda y_2+y_1}{\lambda+1},\quad z=\frac{\lambda z_2+z_1}{\lambda+1}$$

$\because$ Plane $ax+by+cz+d=0$ Passing through (x, y, z)

$$\therefore\ Q\frac{(\lambda x_2+x_1)}{\lambda+1}+b\frac{(\lambda y_2+y_1)}{\lambda+1}+c\frac{(\lambda z_2+z_1)}{\lambda+1}+d=0$$

$$a(\lambda x_2+x_1)+b(\lambda y_2+y_1)+c(\lambda z_2+z_1)+d(\lambda+1)=0$$

$$\lambda(ax_2+by_2+cz_2+d)+(ax_1+by_1+cz_1+d)=0$$

$$\lambda=-\frac{(ax_1+by_1+cz_1+d)}{(ax_2+by_2+cz_2+d)}$$

Hence Proved

TEST PAPER-03
CLASS - XI MATHEMATICS (Introduction to 3.D)

1. A point is in the XZ plane. What is the value of y co-ordinates? [1]

2. What is the coordinator of XY plane [1]

3. The point $(-4,2,5)$ lie in which octants. [1]

4. The distance from origin to point (a,b,c) is: [1]

5. Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point $P(4,2,-6)$ and $Q(10,-16,6)$ [4]

6. Show that the point $P(1,2,3)$, $Q(-1,-2,-1)$, $R(2,3,2)$ and $S(4,7,6)$ taken in order form the vertices of a parallelogram. Do these form a rectangle? [4]

7. A point R with x co-ordinates 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$ find the co-ordinates of the point R [4]

8. If the points $P(1,0,-6)$, $Q(-3,P,q)$ and $R(-5,9,6)$ are collinear, find the values of P and q [4]

9. Three consecutive vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$ find forth vertex D [4]

10. If A and B be the points $(3,4,5)$ and $(-1,3,7)$ respectively. Find the eq. of the set points P such that $PA^2+PB^2=K^2$ where K is a constant [4]

11. Prove that the points $0(0,0,0)$, $A(2,0,0)$, $B(1,\sqrt{3},0)$, and $C\left(1,\frac{1}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron. [6]

12. If A and B are the points $(-2,2,3)$ and $(-1,4,-3)$ respectively, then find the locus of P such that 3|PA| = 2|PB| [6]

TEST PAPER-03

CLASS - XI MATHEMATICS (Introduction to 3.D)

[ANSWERS]

Ans1. Zero

Ans2. $(x, y, 0)$

Ans3. II

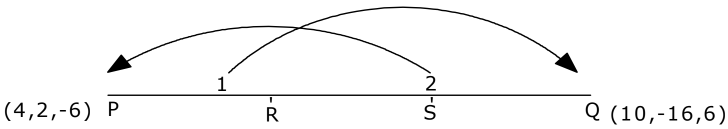
Ans4. $\sqrt{a^2+b^2+c^2}$

Ans5. Let R and S be the points of trisection of the segment PO. Then

$\therefore PR = RS = SQ$

$\Rightarrow 2PR = RQ$

$\Rightarrow \dfrac{PQ}{RQ} = \dfrac{1}{2}$



$\therefore$ R divides PQ in the ratio 1:2

$\therefore$ Co-ordinates of point $R\left[\dfrac{1(10)+2\times 4}{1+2}, \dfrac{1(-16)+2\times 2}{1+2}, \dfrac{1\times 6+2(-6)}{1+2}\right]$

$= R(6,-4,-2)$

Similarly $PS = 2SQ$

$\Rightarrow \dfrac{PS}{SQ} = \dfrac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ co-ordinates of point S $\left[\dfrac{2(10)+1(4)}{1+2}, \dfrac{2(-16)+1(2)}{1+2}, \dfrac{2(6)+1(-6)}{1+2}\right]$

$\therefore S(8,-10,2)$

Ans6. Mid point of PR is $\left(\dfrac{1+2}{2}, \dfrac{2+3}{2}, \dfrac{3+2}{2}\right)$

i.e. $\left(\dfrac{3}{2}, \dfrac{5}{2}, \dfrac{5}{2}\right)$

also mid point of QS is $\left(\dfrac{-1+4}{2}, \dfrac{-2+7}{2}, \dfrac{-1+6}{2}\right)$

i.e. $\left(\dfrac{3}{2}, \dfrac{5}{2}, \dfrac{5}{2}\right)$

Then PR and QS have same mid points.

$\therefore$ PR and QS bisect each other. It is a Parallelogram.

Now $PR=\sqrt{(2-1)^2+(3-2)^2+(2-3)^2}=\sqrt{3}$ and

$QS=\sqrt{(4+1)^2+(7+2)^2+(6+1)^2}=\sqrt{155}$

$\therefore PR\neq QS$ diagonals an not equal

$\therefore PQRS$ are not rectangle.

Ans7. Let the point. R divides the line segment joining the point P and Q in the ratio $\lambda:1$, Then co-ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1},\frac{-3}{\lambda+1},\frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$\Rightarrow \frac{8\lambda+2}{\lambda+1}=4$, $\lambda=\frac{1}{2}$

$\therefore$ co-ordinates of point R

$$\left[4,\frac{-3}{\frac{1}{2}+1},\frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \quad \text{i.e.}(4,-2,6)$$

Ans8. Given points

$P(1,0,-6)$, $Q(-3,P,q)$ and $R(-5,9,6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1},\frac{0+9K}{K+1},\frac{-6+6K}{K+1}\right)$

$Q(-3,P,q)$

$\frac{1-5K}{K+1}=-3$

$1-5K=-3K-3$

$-2K=-4$

$K=\frac{-4}{-2}$

$K=2$

$\therefore$ the value of P and q are 6 and 2.

Ans9. Given vertices of 11gm ABCD

$A(3,-1,2)$, $B(1,2,-4)$, $C(-1,1,2)$

Suppose co-or dine of forth vertex $D(x,y,z)$

Mid point of $AC\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$

$=(1,0,2)$

Mid point of $BD\left(\frac{x+1}{2},\frac{y+2}{2},\frac{-4+z}{2}\right)$

Mid point of AC = mid point of BD

$\frac{x+1}{2}=1\Rightarrow x=1$

$\frac{y+2}{2}=0\Rightarrow y=-2$

$\frac{-4+z}{2}=2\Rightarrow z=8$

Co-ordinates of point $D(1,-2,8)$

Ans10. Let co-ordinates of point P be (x,y,z)

$$PA^2=(x-3)^2+(y-4)^2+(z-5)^2$$
$$=x^2-6x+9+y^2-8y+16+z^2-10z+25$$
$$=x^2+y^2+z^2-6x-8y-10z+50$$

$$PB^2=(x+1)^2+(y-3)^2+(z-7)^2$$
$$=x^2+2x+1+y^2-6y+9+z^2-14+49$$
$$=x^2+y^2+z^2+2x-6y-14z+59$$

$$PA^2+PB^2=K^2$$
$$2\left(x^2+y^2+z^2\right)-4x-14y-24z+109=K^2$$
$$x^2+y^2+z^2-2x-7y-12z=\frac{K^2-109}{2}$$

Ans11. To prove O, A, B, C are vertices of regular tetrahedron.
We have to show that

|OA|=|OB|=|OC|=|AB|=|BC|=|CA|

|OA|= $\sqrt{(0-2)^2+0^2+0^2}=2$ unit

|OB|= $\sqrt{(0-1)^2+\left(0-\sqrt{3}\right)^2+0^2}=\sqrt{1+3}=\sqrt{4}=2$ unit

|OC|= $\sqrt{(0-1)^2+\left(0-\frac{1}{\sqrt{3}}\right)+\left(0-\frac{2\sqrt{2}}{3}\right)^2}$

$= \sqrt{1 + \frac{1}{3} + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2 \text{ unit}$

|AB|= $\sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2 + (10-0)^2} = \sqrt{1+3+0}$

$= \sqrt{4} = 2 \text{ unit}$

|BC|= $\sqrt{(1-1)^2 + \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 + \left(0 - \frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$

$= \sqrt{0 + \left(\frac{2}{\sqrt{3}}\right)^2 + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2 \text{ unit}$

|CA|= $\sqrt{(1-2)^2 + \left(\frac{1}{\sqrt{3}} - 0\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}} - 0\right)^2}$

$= \sqrt{1 + \frac{1}{3} + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2 \text{ unit}$

$\therefore$ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

$\therefore$ O, A, B, C are vertices of a regular tetrahedron.

Ans12. Given points $A(-2,2,3)$ and $B(-1,4,-3)$

Supper co-ordinates of point $P(x,y,z)$

|PA|= $\sqrt{(x+2)^2 + (y-2)^2 + (2-3)^2}$

|PA|= $\sqrt{x^2 + y^2 + z^2 + 4x - 4y - 6z + 17}$

|PB|= $\sqrt{(x+1)^2 + (y-4)^2 + (z+3)^2}$

|PB|= $\sqrt{x2 + y2 + z2 + 2x - 8y + 6z + 26}$

$\because$ 3|PA| = 2|PB|

9 PA^2=4 PB^2

$9\left(x^2 + y^2 + z^2 + 4x - 4y - 6z + 17\right) = 4\left(x^2 + y^2 + z^2 + 2x - 8y + 6z + 26\right)$

$5x^2 + 5y^2 + 5z^2 + 28x - 4y - 30z + 49 = 0$

TEST PAPER-01

CLASS - XI MATHEMATICS (Limits and Derivative)

1. Evaluate $\lim_{x\to 3}\left[\dfrac{x^2-9}{x-3}\right]$ [1]

2. Evaluate $\lim_{x\to 0}\dfrac{\sin 3x}{5x}$ [1]

3. Find derivative of 2^x [1]

4. Find derivative of $\sqrt{\sin 2x}$ [1]

5. Prove that $\lim_{x\to 0}\left(\dfrac{e^x-1}{x}\right)=1$ [4]

6. Evaluate $\lim_{x\to 1}\dfrac{(2x-3)\left(\sqrt{x}-1\right)}{\left(2x^2+x-3\right)}$ [4]

7. Evaluate $\lim_{x\to 0}\dfrac{x\tan 4x}{1-cas4x}$ [4]

8. It $y=\dfrac{(1-\tan x)}{(1+\tan x)}$. Show that [4]

   $\dfrac{dy}{dx}=\dfrac{-2}{(1+\sin 2x)}$

9. Differentiate $e^{\sqrt{\cot x}}$ [4]

10. Let $f(x)\begin{cases} a+bx, x<1 \\ 4, x=1 \\ b-ax, x>1 \end{cases}$ and if $\lim_{x\to 1} f(x)=f(1)$ [4]

    What are the possible value of a and b ?

11. Differentiate tan x from first principle. [6]

12. Differentiate $(x+4)^6$ From first principle. [6]

TEST PAPER-01

CLASS - XI MATHEMATICS (Limits and Derivative)

[ANSWERS]

Ans1
$$\lim_{x\to 3}\frac{x^2-9}{x-3} \qquad \frac{0}{0}\ form$$
$$\lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}=3+3=6$$

Ans2
$$\lim_{x\to 0}\frac{\sin 3x}{5x}$$
$$=\lim_{3x\to 0}\frac{\sin 3x}{3x}\times\frac{3}{5}$$
$$=1\times\frac{3}{5}=\frac{3}{5} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

Ans3 let $y=2^x$
$$\frac{dy}{dx}=\frac{d}{dx}2=2^x 10g2$$

Ans4
$$\frac{d}{dx}\sqrt{\sin 2x}=\frac{1}{2\sqrt{\sin 2x}}\frac{d}{dx}\sin 2x$$
$$=\frac{1}{\cancel{2}\sqrt{\sin 2x}}\times\cancel{2}\cos 2x$$
$$=\frac{\cos 2x}{\sqrt{\cos 2x}}$$

Ans5 We have
$$\lim_{x\to 0}\frac{e^x-1}{x}$$
$$\lim_{x\to 0}\left\{\frac{\left[\cancel{1}+x+\frac{x^2}{2^1.}+\frac{x^3}{3^1.}+---\right]-\cancel{1}}{x}\right\} \qquad \left[\because e^x=1+x+\frac{x^2}{2^1.}+---\right]$$
$$\lim_{x\to 0}\left\{\frac{x+\frac{x^2}{2^1.}+\frac{x^3}{3^1.}+---}{x}\right\}$$

$$=\lim_{x\to0}\cancel{x}\left\{\frac{1+\frac{x}{2^1}+\frac{x^2}{3^1}+---}{\cancel{x}}\right\}$$

$$=1+0=1$$

Ans6 $$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x^2+x-3)}$$

$$=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}\times\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$$

$$=\lim_{x\to1}\frac{(2x-3)(\cancel{x-1})}{(2x+3)(\cancel{x-1})(\sqrt{x}+1)}$$

$$=\lim_{x\to1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}=\frac{2\times1-3}{(2\times1+3)(\sqrt{1}+1)}$$

$$=\frac{-1}{10}$$

Ans7 $$\lim_{x\to0}\frac{x\tan4x}{1-cas4x}$$

$$=\lim_{x\to0}\frac{x\sin4x}{\cos4x\left[2\sin^2 2x\right]}$$

$$=\lim_{x\to0}\frac{2x\cancel{\sin}2x\cos2x}{\cos4x\left(2\sin^{\cancel{2}}2x\right)}$$

$$=\lim_{x\to0}\left(\frac{\cos2x}{\cos4x}.\frac{2x}{\sin2x}\times\frac{1}{2}\right)$$

$$=\frac{1}{2}\frac{\lim\limits_{2x\to0}\cos2x}{\lim\limits_{4x\to0}\cos4x}\times\lim_{2x\to0}\left(\frac{2x}{\sin2x}\right)=\frac{1}{2}\times1=\frac{1}{2}$$

Ans8 $$y=\frac{(1-\tan x)}{(1+\tan x)}$$

$$\frac{dy}{dx}=\frac{(1+\tan x)\frac{d}{dx}(1-\tan x)-(1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)\left(-\sec^2 x\right)-(1-\tan x)\sec^2 x}{(1+\tan x)^2}$$

$$=\frac{-\sec^2 x-\cancel{\tan x\sec^2} x-\sec^2+\cancel{\tan x\sec^2} x}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2}=\frac{-2}{\cos^2 x\left[1+\frac{sicx}{\cos x}\right]^2}$$

$$=\frac{-2}{\cancel{\cos^2} x\left[\frac{\cos x+\sin x}{\cancel{\cos^2} x}\right]^2}$$

$$=\frac{-2}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{-2}{1+\sin^2 x}$$

$$\therefore \frac{dy}{dx}=\frac{-2}{1+\sin 2x}$$ Hence proved

Ans9 let $y=e^{\sqrt{\cot x}}$

$$\frac{dy}{dx}=\frac{d}{dx}e^{\sqrt{\cot x}}=e^{\sqrt{\cot x}}\frac{d}{dx}\sqrt{\cot x}$$

$$=e^{\sqrt{\cot x}}\times\frac{1}{2\sqrt{\cot x}}.\frac{d}{dx}\cot x$$

$$=\frac{e^{\sqrt{\cot x}}}{2\sqrt{\cot x}}-\cos ec^2 x$$

$$=\frac{-\cos cec^2 e^{\sqrt{\cot x}}}{2\sqrt{\cot x}}$$

Ans10 Given f (1) =4

$$\because \lim_{x\to 1} f(x)=f(1)=4$$

$$\therefore \lim_{x\to 1} f(x) exist$$

$$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=4----(1)$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}(a+bx) \qquad \left[\begin{array}{l}\because\ for x<1\\ f(x)=a+bx\end{array}\right]$$

$= a+b$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}(b-ax) \qquad \left[\begin{array}{l}\because\ for x>1\\ f(x)=b-ax\end{array}\right]$$

$=b-a$

By eq. (1)

$a+b=b-a=4$

$a+b=4$
$b-a=4$
$\therefore a=0 \quad and \quad b=4$

Ans11 $f(x)=\tan x$

$f(x+h)=\tan(x+h)$

$$f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\sin(x+h)\cos x-\cos(x+h)\sin x}{h\cos(x+h)\cos x}$$

$$=\lim_{h\to 0}\frac{\sin[x+h-x]}{h\cos(x+h)\cos x} \qquad \begin{bmatrix}\because \sin(A-B)= \\ \sin A\cos B-\cos A\sin B\end{bmatrix}$$

$$=\lim_{h\to 0}\frac{\sinh}{h\cos(x+h)\cos x}$$

$$=\frac{\lim_{h\to 0}\frac{\sinh}{h}}{\lim_{h\to 0}\cos(x+h)\cos x}=\frac{1}{\cos x.\cos x} \qquad \left[\because \lim_{h\to 0}\frac{\sinh}{h}=1\right]$$

$$=\frac{1}{\cos^2 x}=\sec^2 x$$

Ans12 let $f(x)=(x+4)^6$

$f(x+h)=(x+h+4)^6$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h+4)^6-(x+4)^6}{h}$$

$$=\lim_{(x+h+4)\to(x+4)}\frac{(x+h+4)^6-(x+4)^6}{(x+h+4)-(x-4)}$$

$$=6(x+4)^{(6-1)} \qquad \left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=6(x+4)^5$$

TEST PAPER-02

CLASS - XI MATHEMATICS (Limits and Derivative)

1. Evaluate $\lim_{x\to 0}\frac{\sin^2 4x}{x^2}$ [1]

2. What is the value of $\lim_{x\to a}\left(\frac{x^2-a^n}{x-a}\right)$ [1]

3. Differentiate $\frac{2^x}{x}$ [1]

4. If $y=e^{\sin x}$ find $\frac{dy}{dx}$ [1]

5. If $y=\frac{1}{\sqrt{a^2-x^2}}$, find $\frac{dy}{dx}$ [4]

6. Differentiate $\sqrt{\frac{1-\tan x}{1+\tan x}}$ [4]

7. Differentiate $(i)\ \left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$ $(ii)\ \left(\frac{\sin x-1}{\sec x+1}\right)$ [4]

8. Evaluate $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-cox}{\left(x-\frac{\pi}{4}\right)}$ [4]

9. Evaluate $\lim_{x\to 0}\frac{(1+x)^6-1}{(1+x)^5-1}$ [4]

10. Evaluate $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ [4]

11. Find derivative of caser x by first principle [6]

12. Find the derivatives of the following fuchsias: [6]

$(i)\ \left(x-\frac{1}{x}\right)^3$ $\qquad(ii)\ \frac{(3x+1)\left(2\sqrt{x}-1\right)}{\sqrt{x}}$

TEST PAPER-02

CLASS - XI MATHEMATICS (Limits and Derivative)

[ANSWERS]

Ans1 $$\lim_{x\to 0}\frac{\sin^2 4x}{x^2 4^2}\times 4^2 = \lim_{4x\to 0}\left(\frac{\sin 4x}{4x}\right)^2\times 16$$
$$=1\times 16=16$$

Ans2 $$\lim_{x\to a}\frac{x^n-a^n}{x-a}=1$$

Ans3 $$\frac{d}{dx}\frac{2^x}{x}=\frac{x\frac{d}{dx}2^x-2^x\frac{d}{dx}x}{x^2}$$
$$=\frac{x\times 2^x 10g2-2^x\times 1}{x^2}$$
$$=2x\frac{[x+10g2-1]}{x^2}$$

Ans4 $$y=e^{\sin x}$$
$$\frac{dy}{dx}=\frac{d}{dx}e^{\sin x}$$
$$=e^{\sin x}\times\cos x=\cos xe^{\sin x}$$

Ans5 $$y=\frac{1}{\sqrt{a^2-x^2}}$$
$$put\ \left(a^2-x^2\right)=t$$
$$y=\frac{1}{\sqrt{t}}\text{ and } t=a^2-x^2$$
$$\frac{dy}{dt}=\frac{d}{dt}t^{\frac{-1}{2}}$$
$$=\frac{-1}{2}t^{\frac{-1}{2}-1}$$
$$=\frac{-1}{2}t\frac{-3}{2}$$

$$\frac{dt}{dx} = -2x$$

so.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-1}{\cancel{2}} t^{\frac{-3}{2}} \times \left(-\cancel{2}x\right) = x\ t^{\frac{-3}{2}}$$

$$= x\left(a^2 - x^2\right)\frac{-3}{2}$$

Ans6

$$\text{let } y = \sqrt{\frac{1 - \tan x}{1 + \tan x}}$$

$$\text{put } \frac{1 - \tan x}{1 + \tan x} = t$$

$$y = \sqrt{t} \quad \text{and } t = \frac{1 - \tan x}{1 + \tan x}$$

$$\frac{dy}{dt} = \frac{d}{dt} t^{\frac{1}{2}}$$

$$= \frac{1}{2} t^{\frac{1}{2} - 1} = \frac{1}{2} t^{\frac{-1}{2}}$$

$$= \frac{1}{2\sqrt{t}} = \frac{1}{2\sqrt{\frac{1 - \tan x}{1 + \tan x}}} = \frac{1}{2}\sqrt{\frac{1 + \tan x}{1 - \tan x}}$$

$$\frac{dt}{dx} = \frac{(1 + \tan x)\frac{d}{dx}(1 - \tan x) - (1 - \tan x)\frac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$$

$$= \frac{(1 + \tan x)\left(0 - \sec^2 x\right) - (1 - \tan x)\left(0 + \sec^2 x\right)}{(1 + \tan x)^2}$$

$$= \frac{\sec^2 x\left[-1 - \cancel{\tan x} - 1 + \cancel{\tan x}\right]}{(1 + \tan x)^2}$$

$$= \frac{-2\sec^2 x}{(1 + \tan x)^2}$$

$$\cancel{\frac{dy}{dx}} = \cancel{\frac{\frac{dy}{dt}}{\frac{dt}{dx}}} = \frac{1}{2}\sqrt{\cancel{\frac{1 + \tan x}{1 - \tan x}}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$=\frac{-\cancel{2}\sec^2 x}{(1+\tan x)^2}\times\frac{1}{\cancel{2}}\sqrt{\frac{1+\tan x}{1-\tan x}}$$

$$=\frac{-\sec^2 x}{(1+\tan x)^{\frac{3}{2}}(1-\tan x)^{\frac{1}{2}}}$$

Ans7(i) $$\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$$

$$=\frac{(\sin x-\cos x).\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x).\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$=\frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$$

$$=\frac{-\left[(\sin x-\cos x)^2+(\sin x+\cos x)^2\right]}{(\sin x-\cos x)^2}$$

$$=\frac{-2\left(\sin^2 x+\cos^2 x\right)}{\left(\sin^2 x+\cos^2 x-2\sin x\cos x\right)}$$

$$=\frac{-2}{(1-\sin 2x)}$$

(ii) $$\frac{d}{dx}\left[\frac{\sec x-1}{\sec x+1}\right]$$

$$=\frac{(\sec x+1).\frac{d}{dx}(\sec x-1)-(\sec x-1).\frac{d}{dx}(\sec x+1)}{(\sec x+1)^2}$$

$$=\frac{(\sec x+1)\sec x\tan x-(\sec x-1)\sec x\tan x}{(\sec x+1)^2}$$

$$=\frac{2\sec x\tan x}{(\sec x+1)^2}$$

Ans8 put $\left(x-\frac{\pi}{4}\right)=y,$ so that when $x\to\frac{\pi}{4}$ then $y\to 0$

$$\therefore \lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\left(x-\frac{\pi}{4}\right)}$$

$$=\lim_{y\to 0}\frac{\left[\sin\left(\frac{\pi}{4}+y\right)-\cos\left(\frac{\pi}{4}+y\right)\right]}{y} \qquad \left[\text{puthing }\left(x-\frac{\pi}{4}=y\right)\right]$$

$$= \lim_{y \to 0} \frac{\left[\left(\sin\frac{\pi}{4}\cos y + \cos\frac{\pi}{4}\sin y - \sin\frac{\pi}{4}\sin y\right)\right]}{y}$$

$$= \frac{2}{\sqrt{2}} \times \lim_{y \to 0}\left(\frac{\sin y}{y}\right) = \left(\sqrt{2} \times 1 = \sqrt{2}.\right)$$

Ans9 put $(1+x) = y$, so that when $x \to 0$ then $y \to 1$

$$\therefore \lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^5 - 1}$$

$$= \lim_{y \to 1}\left[\frac{y^6 - 1}{y^5 - 1}\right] = \frac{\lim_{y \to 1}\left(\frac{y^6-1}{y-1}\right)}{\lim_{y \to 1}\left(\frac{y^5-1}{y-1}\right)}$$

$$= \frac{\lim_{y \to 1}\left(\frac{y^6-1}{y-1}\right)}{\lim_{y \to 1}\left(\frac{y^5-1^5}{y-1}\right)} = \frac{6 \times 1^{(6-1)}}{5 \times 1^{(5-1)}}$$

$$= \frac{6 \times 1^5}{5 \times 1^4} = \frac{6}{5} \qquad \left[\because \lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}\right]$$

Ans10 $$\lim_{x \to a} \frac{\left(\sqrt{a+2x} - \sqrt{3x}\right)}{\left(\sqrt{3a+x} - 2\sqrt{x}\right)}$$

$$= \lim_{x \to a} \frac{\left(\sqrt{a+2x} - \sqrt{3x}\right)}{\left(\sqrt{3a+x} - 2\sqrt{x}\right)} \times \frac{\left(\sqrt{3a+x} + 2\sqrt{x}\right)}{\left(\sqrt{3a+x} + 2\sqrt{x}\right)} \times \frac{\left(\sqrt{a+2x} + \sqrt{3x}\right)}{\left(\sqrt{a+2x} + \sqrt{3x}\right)}$$

$$= \lim_{x \to a} \frac{\left[(a+2x) - 3x\right] \times \left(\sqrt{3a+x} + 2\sqrt{x}\right)}{\left[(3a+x) - 4x\right] \times \left(\sqrt{a+2x} + \sqrt{3x}\right)}$$

$$= \lim_{x \to a} \frac{\cancel{(a-x)} \times \left(\sqrt{3a} + x + 2\sqrt{x}\right)}{3\cancel{(a-x)} \times \left(\sqrt{a+2x} + \sqrt{3x}\right)}$$

$$= \lim_{x \to a} \frac{\left(\sqrt{3a+x} + 2\sqrt{x}\right)}{3\left[\sqrt{a+2x} + \sqrt{3x}\right]}$$

$$= \frac{\left(\sqrt{4a} + 2\sqrt{a}\right)}{3\left(\sqrt{3a} + \sqrt{3a}\right)} = \frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}} = \frac{\sqrt{2}}{3\sqrt{3}}$$

Ans11 proof let $f(x) = \text{cosec}\, x$

$$\text{By def, } f(x) = \underset{h\to 0}{Lt} \frac{f(x+h)-f(h)}{h}$$

$$= \underset{h\to 0}{Lt} \frac{\text{cosec}(x+h) - \text{cosec}\, x}{h}$$

$$= \underset{h\to 0}{Lt} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h} = \underset{h\to 0}{Lt} \frac{\sin x - \sin(x+h)}{h \sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt} \frac{2\cos\frac{x+x+h}{2}\sin\frac{x-x+h}{2}}{h\sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt} \frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{h\sin(x+h)\sin x}$$

$$= \frac{\underset{\frac{h}{2}\to 0}{Lt} \cos\left(x+\frac{h}{2}\right)}{\cos x . \underset{h\to 0}{L} + \sin(x+h)}, \underset{\frac{h}{2}\to 0}{Lt} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -\frac{\cos x}{\sin x . \sin x}.1 = -\cos ecx \cot x$$

Ans.12(i) $\text{let } f(x) = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x + \frac{1}{x}\left(x - \frac{1}{x}\right)$

$= x^3 - x^{-3} - 3x + 3x^{-1}.d.ff\ \ wr.t4, we$ get

$\text{f}(x) = 3\times x^2 - (-3)x^{-4} - 3\times 1 + 3\times(-1)x^{-2}$

$= 3x^2 + \frac{3}{x^4} - 3 - \frac{3}{x^2}.$

(ii) $\text{let } f(x) = \frac{(3x+1)(2\sqrt{x}-1)}{\sqrt{x}} = \frac{6x^{\frac{3}{2}} - 3x + 2\sqrt{x} - 1}{\sqrt{x}}$

$= 6x - 3x^{\frac{1}{2}} + 2 - x^{-\frac{1}{2}}, d : ff$ w.r.t. x.weget

$f(x) = 6\times 1 - 3\times\frac{1}{2}\times x^{-\frac{1}{2}} + 0 - \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= 6 - \frac{3}{2\sqrt{x}} + \frac{1}{2, x^{\frac{3}{2}}}.$

TEST PAPER-03

CLASS - XI MATHEMATICS (Limits and Derivative)

1. Evaluate $\lim_{x\to 1}\dfrac{x^{15}-1}{x^{10}-1}$ [1]

2. Differentiate $x\sin x$ with respect to x [1]

3. Evaluate $\lim_{x\to 1}\dfrac{x^2+1}{x+100}$ [1]

4. Evaluate $\lim_{x\to 0}\left[\cos cecx-\cot x\right]$ [1]

5. Find the derivative of $f(x)=1+x+x^2+--+x^{50}$ at $x=1$ [4]

6. Find the derivative of $\sin^2 x$ with respect to x using product rule [4]

7. Find the derivative of $\dfrac{x^5-\cos x}{\sin x}$. with respect to x [4]

8. Find $\lim_{x\to 0} f(x)$. when $f(x)=\begin{Bmatrix}\dfrac{|x|}{x}; x\neq 0\\ 0; x=0\end{Bmatrix}$ [4]

9. Find the derivative of the function [4]

   $f(x)=2x^2+3x-5$ at $x=-1$. Also show that $f'(0)+3f'(-1)=0$

10. Evaluate $\lim_{x\to 0}\dfrac{ax+x\cos x}{b\sin x}$ [4]

11. If $f(x)=\begin{Bmatrix}|x|+a; x<0\\ 0; x=0\\ |x|-a; x>0\end{Bmatrix}$ for what [6]

    Values of 'a' does $\lim_{x\to 0} f(x)$ exist

12. Find the derivative of sin $(x+1)$. with respect to x, from first principle. [6]

TEST PAPER-03

CLASS - XI MATHEMATICS (Limits and Derivative)

[ANSWERS]

Ans1 $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

$=\frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}}=\frac{15\times 1^{14}}{10\times 1^{9}}$

$=\frac{15}{10}=\frac{3}{2}$

Ans2 $\frac{d}{dx}x\sin x = x\cos x+\sin x.1$

$= x\cos x+\sin x$

Ans3 $\lim_{x\to 1}\frac{x^{2}+1}{x+100}=\frac{2}{101}$

Ans4 $\lim_{x\to 0}\left[\cos cec-\cot x\right]$

$=\lim_{x\to 0}\left[\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right]$

$=\lim_{x\to 0}\frac{1-\cos x}{\sin x}=\lim_{x\to 0}\frac{\cancel{2}\sin^{\cancel{2}}\frac{x}{\cancel{2}}}{\cancel{2}\sin\frac{x}{\cancel{2}}\cos\frac{x}{2}}$

$=\lim_{x\to 0}\tan\frac{x}{2}=0$

Ans5 $f\left(x\right)=1+x+x^{2}+---+x^{50}$

$f1\left(x\right)=\frac{d}{dx}\left(1+x+x^{2}+---+x^{50}\right)$

$=0+1+2x+3x^{2}+--+50x^{49}$

At $x=1$

$f^{1}\left(1\right)=1+2+3+--50$

$$=\frac{\cancel{50}^{25}(50+1)}{\cancel{2}}=25\times 51$$

$$\left[\begin{array}{l}1+2+3--+n \\ =\frac{n(n+1)}{2}\end{array}\right]$$

$=1305$

Ans6 let

$$y=\sin^2 x$$

$$y=\sin x\times\sin x$$

$$\frac{dy}{dx}=\frac{d}{dx}.\sin x\times\sin x$$

$$=\sin x\frac{d}{dx}\sin x+\sin x\frac{d}{dx}\sin x$$

$$=\sin x.\cos x+\sin x.\cos x$$

$$=2\sin x.\cos x=\sin 2x$$

Ans7 let

$$y=\frac{x^5-\cos x}{\sin x}$$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x5-\cos x}{\sin x}\right)$$

$$=\frac{\sin x\frac{d}{dx}\left(x^5-\cos x\right)-\left(x^5-\cos x\right)\frac{d}{dx}\sin x}{\sin^2 x}$$

$$=\frac{\sin x\left[5x^4+\sin x\right]-\left(x^5-\cos x\right)\cos x}{\sin^2 x}$$

$$=\frac{5x^4\sin x+\sin^2 x-x^5\cos x+\cos^2 x}{\sin^2 x}$$

$$=\frac{5x^4\sin x-x^5\cos x+1}{\sin^2 x}$$

Ans8 $$f(x)=\left\{\begin{array}{l}\frac{|x|}{x};x\neq 0 \\ 0;x=0\end{array}\right\}$$

We know that $|x|=\left\{\begin{array}{l}x;x\geq 0 \\ -x;x<0\end{array}\right\}$

$$\therefore\ f(x)=\left\{\begin{matrix}\frac{x}{x}=1.x>0\\ \frac{-x}{x}=-1.x<0\\ 0.x=0\end{matrix}\right\}$$

L.H.L. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0}-1=-1$

R.H.L. $\lim\limits_{x\to 0^+} d(x)=\lim\limits_{x\to 0}1=1$

L.H.L. $\neq$ R.H.L $\therefore \lim\limits_{x\to 0} f(x)$ does not exist

Ans9 $f(x)=2x^2+3x-5$

$f^1(x)=\frac{d}{dx}\left(2x^2+3x-5\right)$

$=4x+3$

At $x=-1$

$f^1(-1)4\times-1+3=-4+3=-1$

$f^1(0)=4\times0+3=3$

$f^1(0)+3f^1(-1)=3+3\times-1$

$=3-3=0$ Hence proved

Ans10 $\lim\limits_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

$=\lim\limits_{x\to 0}\frac{x[a+\cos x]}{x.b\frac{\sin x}{x}}$

$=\frac{\lim\limits_{x\to 0}(a+\cos x)}{\lim\limits_{x\to 0}b\frac{\sin x}{x}}$

$=\frac{a+1}{b\times1}=\frac{a+1}{b}$ $\left[\begin{matrix}\because \lim\limits_{x\to 0}\cos x=1\\ \lim\limits_{x\to 0}\frac{sicx}{x}=1\end{matrix}\right]$

Ans11 given $f(x)=\left\{\begin{matrix}|x|+a; x<0\\ 0; x=0\\ |x|-a; x>0\end{matrix}\right\}$

a=0

L.H.L. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} |x| + a$

$= \lim_{x\to 0} -x + a = a$

R.H.L. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} |x| - a$

$\because \lim_{x\to 0} f(x)$ exist

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$a = -a$

$2a = 0$

$a = 0$

At $a = 0 \lim_{x\to 0} f(x)$ exist

Ans12 let $f(x) = \sin(x+1)$

$$f(x+h) = \sin(x+h+1)$$

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left[\frac{x+h+1+x+1}{2}\right]\sin\left[\frac{x+h+1-x-1}{2}\right]}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left[x+1+\frac{h}{2}\right]\sin\frac{h}{2}}{h}$$

$$= \lim_{h\to 0} \cancel{2}\cos\left(x+1+\frac{h}{2}\right) \times \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\cancel{2}\frac{h}{2}}$$

$$= \cos(x+1) \times 1 = \cos(x+1)$$

TEST PAPER-04

CLASS - XI MATHEMATICS (Limits and Derivative)

1. Find $f^{1}(x)$ at $x=100$ [1]

   if $f(x)=99x$

2. Evaluate [1]

   $\lim\limits_{x\to -2}\frac{\tan \pi x}{x+2}$

3. Find derivative of $\sin^n x$ [1]

4. Find derivative of $1+x+x^2+x^3+---+x^{50}$ at $x=1$ [1]

5. Find derivative of $\tan x$ by first principle [4]

6. Evaluate $\lim\limits_{x\to 1}\frac{x+x^2+x^3+---+x^n-n}{(x-1)}$ [4]

7. Evaluate $\underset{x\to 4}{Lt}\frac{|4-x|}{x-4}$ (if it exist) [4]

8. For what integers m and n does both [4]

   $\underset{x\to 4}{Lt} f(x)$ and $\underset{x\to 1}{Lt} f(x)$ exist it

   $$f(x)=\begin{Bmatrix} mx^2+n; x<0 \\ nx+m; 0\le x\le 1 \\ nx^3+m; x>1 \end{Bmatrix}$$

9. Find derivative of $\frac{x^n-a^n}{x-a}$ [4]

10. If $y=\sqrt{x+}\frac{1}{\sqrt{x}}$. prove that $2x\frac{dy}{dx+y}=2\sqrt{x}$ [4]

11. Find the derivative of $\sin x+\cos x$ from first principle [6]

12. Find derivative of [6]

    $(i)\frac{x\sin x}{1+\cos x}(ii)(ax+b)(x+d)^2$

TEST PAPER-04

CLASS - XI MATHEMATICS (Limits and Derivative)

[ANSWERS]

Ans1 $f(x)=99x$

$f^{1}(x)=99$ at x=100

$f^{1}(x)=99$

Ans2 $\lim_{x\to -2}\frac{\tan \pi x}{x+2}$ $\frac{1}{0}$ *form*

$let\ x+2=y$

$x=y-2$

$\lim_{y\to 0}\frac{\tan \pi (y-2)}{y}$

$\lim_{y\to 0}\frac{-\tan \pi (2-y)}{y}=\lim_{y\to 0}\frac{\tan [2\pi -2y]}{y}$

$=\lim_{2y\to 0}\frac{+\tan 2y}{2y}\times 2$

$=1\times 2=2$

Ans3 $\frac{d}{dx}\sin^{n}x$

$=n\sin^{n-1}x\ \ \frac{d}{dx}\sin x$

$=n\sin^{n-1}x\ \cos x$

Ans4 $f(x)=1+x+x^{2}+x^{3}+---+x^{50}$

$f^{1}(x)=1+2x+3x^{2}+---+50x^{49}$

$at\ x=1$

$f^{1}(1)=1+2+3+---+50=\frac{50(50+1)}{2}$

$=25\times 51=1275$

Ans5 $let\ f(x) = \tan x$

$$f(x+h) = \tan(x+h)$$

$$f^{1}(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\tan(x+h-\tan x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos(x+h)\cos x\ h}$$

$$= \lim_{h\to 0}\frac{\sin[x+h-x]}{\cos(x+h)\cos x\ h}$$

$$= \frac{\lim_{h\to 0}\frac{\sinh}{h}}{\lim_{h\to 0}\cos(x+h)\cos x} = \frac{1}{\cos(x+0)\cos x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Ans6 $\lim_{x\to 1}\frac{x+x^2+x^3+---+x^n-1}{(x-1)}$

$$= \lim_{x\to 1}\frac{(x-1)+(x^2-1)+(x^3-1+--+(x^n-1))}{(x-1)}$$

$$= \lim_{x\to 1}\frac{\cancel{(x-1)}\left[1+(x+1)+(x^2+x+1)+----+x^{n-1}+x^{n-2}+----+1\right]}{\cancel{(x-1)}}$$

$$= 1+2+3+----+n = \frac{n(n+1)}{2}$$

Ans7 $\lim_{x\to 4}\frac{|4-x|}{x-4}$

$$L.H.L.\quad \lim_{x\to 4^-}\frac{-(4-x)}{x-4} = \lim_{x\to 4}\frac{-(\cancel{4-x})}{-(\cancel{4-x})} = 1$$

$R.H.L. \quad \lim_{x\to 4^+} \frac{4-x}{x-4} = \lim_{x\to 4} \frac{-(x-4)}{(x-4)} = -1$

$L.H.L. \neq R.H.L.$

$\therefore \lim_{x\to 4} \frac{|4-x|}{x-4}$ does not exist

Ans8 $\quad for \quad x = 0$

$L.H.L. \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0} mx^2 + n$

$= n$

$R.H.L. \quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} nx + m$

$= m$

$\because \lim_{x\to 0} f(x)$ exist

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$n = m$

For all real number $m = n \lim_{x\to 0} f(x)$ exist

For $x = 1$

$L.H.L. \quad \lim_{x\to 1^-} f(x) = \lim_{x\to 1} nx + m$

$= n + m$

$R.H.L. \quad \lim_{x\to 1} f(x) = \lim_{x\to 1} nx^3 + m$

$= n + m$

$\therefore \lim_{x\to 1} f(x) = \lim_{x\to 1^+} f(x)$

$m + n = m + n$

$\therefore$ all integral values of $m + n \lim_{x\to 1} f(x)$ exist

Ans9 $\quad \frac{d}{dx} \frac{x^n - a^n}{x - a}$

$$= \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$= \frac{(x-a)\left[nx^{n-1} - 0\right] - (x^n - a^n)[1-0]}{(x-a)^2}$$

$$=\frac{nx^{n-1}(x-a)-x^n+a^n}{(x-a)^2}$$

$$=\frac{nx^n-nax^{n-1}-x^n+a^n}{(x-a)^2}=\frac{x^n(n-1)-nax^{n-1}+a^n}{(x-a)^2}$$

Ans10 $y=\sqrt{x}+\frac{1}{\sqrt{x}}=x^{\frac{1}{2}}+x^{\frac{-1}{2}}$

Differentiating w.r.t. x we gill

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}+\left(\frac{-1}{2}\right)x^{\frac{-3}{2}}$$

$$=\frac{1}{2\sqrt{x}}-\frac{1}{2x^{\frac{3}{2}}}$$

$$2x\frac{dy}{dx}=\sqrt{x}-\frac{1}{\sqrt{x}}$$

$$2x\frac{dy}{dx}+y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)+\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$$

$2x\frac{dy}{dx}+y=2\sqrt{x}$ Hence proved

Ans11 $let\ \ f(x)=\sin x+\cos x$

$$f(x+h)=\sin(x+h)+\cos(x+h)$$

$$f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{[\sin(x+h)\cos(x+h)]-[\sin x+\cos x]}{h}$$

$$=\lim_{h\to 0}\frac{[\sin(x+h)-\sin x]+[\cos(x+h)-\cos x]}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left[\frac{x+h+x}{2}\right]\sin\frac{(x+h-x)}{2}-2\sin\frac{(x+h+x)}{2}\times\sin\left[\frac{x+h-x}{2}\right]}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}+\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \lim_{h\to 0} \cancel{2}\cos\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\cancel{2}\frac{h}{2}} + \lim_{h\to 0} -\cancel{2}\sin\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\cancel{2}\frac{h}{2}}$$

$$= \cos(x+0)\times 1 - \sin(0+x)\times 1$$

$$= \cos x - \sin x$$

Ans12 (i)

$$\frac{d}{dx}\frac{x\sin x}{1+\cos x}$$

$$= \frac{(1+\cos x)\frac{d}{dx}x\sin x - x\sin x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)\left[x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x\right] - x\sin x[0-\sin x]}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)[x\cos x + \sin x\times 1] + x\sin^2 x}{(1+\cos x)^2}$$

$$= \frac{x\cos x + x\cos^2 x + \sin x + \sin x\cos x + x\sin^2 x}{(1+\cos x)^2}$$

$$= \frac{x\left(\cos^2 x + \sin^2 x\right) + x\cos x + \sin x + \sin x\cos x}{(1+\cos x)^2}$$

$$= \frac{x + x\cos x + \sin x + \sin x\cos x}{(1+\cos x)^2}$$

(ii)

$$\frac{d}{dx}(ax+b)(cx+d)^2$$

$$= (ax+b)\frac{d}{dx}(cx+d)^2 + (cx+d)^2\frac{d}{dx}(ax+b)$$

$$= (ax+b)2(cx+d)\frac{d}{dx}(cx+d) + (cx+d)^2\times a$$

$$= 2(ax+b)(cx+d)\times c + a(cx+d)^2$$

$$= (cx+d)\left[2c(ax+b) + a(cx+d)\right]$$

$$= (cx+d)[2acx + 2bc + acx + ad]$$

$$= (cx+d)[3acx + 2abc + ad]$$

TEST PAPER-05

CLASS - XI MATHEMATICS (Limits and Derivative)

1. The value of $\lim_{h \to 0} \frac{e^{2h}-1}{h}$ [1]

2. Evaluate $\lim_{x \to 0} \frac{(1+x)^6-1}{(1+x)^2-1}$ [1]

3. $\lim_{x \to a} \frac{x^7+a^7}{x+a} = 7$ find the value of ‘a [1]

4. Differentiate $x^{-3}(5+3x)$ [1]

5. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$ [4]

6. Differentiate the function $y = \frac{(x+2)(3x-1)}{(2x+5)}$ with respect to x [4]

7. Find $\lim_{x \to 5} |x| - 5$ [4]

8. Find $\lim_{x \to 0} f(x)$ *and* $\lim_{x \to 1} f(x)$ *where* $f(x) = \begin{Bmatrix} 2x+3; x \le 0 \\ 3(x+1); (x>0) \end{Bmatrix}$ [4]

9. Find derivative of $\sec x$ by first principle [4]

10. Find derivative of $f(x) = \frac{4x+5\sin x}{3x+7\cos x}$ [4]

11. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$ [6]

12. Differentiate $(i)\left(\frac{a}{x^4}\right) - \frac{b}{x^2} + \cos x$ $(ii)(x+\cos x)(x-\tan x)$ [6]

TEST PAPER-05

CLASS - XI MATHEMATICS (Limits and Derivative)

[ANSWERS]

Ans1 $\lim_{2h\to 0}\frac{e^{2h}-1}{2h}\times 2$

$=1\times 2=2$

Ans2 $\lim_{x\to 0}\left[\frac{(1+x)^6-1}{(1+x)^2-1}\right]$

$let\ 1+x=y$

$x\to 0, y\to 1$

$\lim_{y\to 1}\frac{y^6-1}{y^2-1}=\lim_{y\to 1}\frac{\frac{y^6-1^6}{y-1}}{\frac{y^2-1^2}{y-1}}$

$=\frac{6\times 1^5}{2\times 1}=\frac{\cancel{6}}{\cancel{2}}=3$

Ans3 $\lim_{x\to a}\frac{x^7+a^7}{x+a}=7$

$=\frac{a^7+a^7}{a+a}=7$

$=\frac{\cancel{2}a^7}{\cancel{2}a}=7$

$=a^6=7$

$=a=\sqrt[6]{7}$

Ans4 $\frac{d}{dx}x^{-3}(5+3x)$

$=\frac{d}{dx}\left[5x^{-3}+3x^{-2}\right]$

$=5\times -3x^{-4}+3\times -2x^{-3}$

$=\frac{-15}{x^4}-\frac{6}{x^3}$

Ans5 $\lim_{x\to\frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$

$let\ \pi-2x=y$

$2x=\pi-y$

$x\to\frac{\pi}{2}, y\to 0$

$\lim_{y\to0}\frac{1+\cos(\pi-y)}{y^2}=\lim_{y\to0}\frac{1-\cos y}{y^2}$

$=\lim_{y\to0}\frac{2\sin^2\frac{y}{2}}{4\times\frac{y^2}{4}}$

$=\lim_{y\to0}\frac{1}{2}\times\frac{\sin^2\frac{y}{2}}{\left(\frac{y}{2}\right)^2}$

$=\frac{1}{2}\lim_{\frac{y}{2}\to0}\left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$

$=\frac{1}{2}\times1^2=\frac{1}{2}$

Ans6 $y=\frac{(x+2)(3x-1)}{(2x+5)}$

$\frac{dy}{dx}=\frac{d}{dx}\frac{(x+2)(3x-1)}{(2x+5)}$

$=\frac{(2x+5)\frac{d}{dx}(x+2)(3x-1)-(x+2)(3x-1)\frac{d}{dx}(2x+5)}{(2x+5)^2}$

$=\frac{(2x+5)\left[(x+2)\frac{d}{dx}(3x-1)+(3x-1)\frac{d}{dx}(x+2)\right]-(x+2)(3x-1)[2+0]}{(2x+5)^2}$

$=\frac{(2x+5)\left[(x+2)\times3+(3x-1)\times1\right]-2\left[3x^2+6x-x-2\right]}{(2x+5)^2}$

$=\frac{(2x+5)[3x+6+3x-1]-6x^2-12x+2x+4}{(2x+5)^2}$

$$=\frac{12x^2+30x+\cancel{10x}+25-6x^2-\cancel{10x}+4}{(2x+5)^2}$$

$$=\frac{6x^2+30x+29}{(2x+5)^2}$$

Ans7 $L.H.S. \lim_{x\to 5^-} f(x)$

$x=5-h$

$x\to 5, h\to 0$

$\lim_{h\to 0} f(5-h)$

$\lim_{h\to 0} |5-h|-5$

$=0$

$R.H.S. \lim_{x\to 5^+} f(x)$

$put\ x=5+h$

$x\to 5,\ h\to 0$

$\lim_{h\to 0} f(5+h)=\lim_{h\to 0}|5+h|-5$

$=0$

$R.H.S.=R.H.S.$

$\therefore \lim_{x\to 5^-} f(x)=\lim_{x\to 5^+} f(x)$

$\therefore \lim_{x\to 5} f(x)\,\mathrm{e}\,xist$

$\therefore \lim_{x\to 5} f(x)=0$

Ans8 $given\ f(x)=\begin{cases}2x+3, x\le 0\\ 3(x+1), x>0\end{cases}$

$for\ x=0$

$L.H.S. \lim_{x\to 0^-} f(x)=\lim_{x\to 0} 2x+3=3$

$R.H.S. \lim_{x\to 0^+} f(x)=\lim_{x\to 0} 3(x+1)=3$

$L.H.S.=R.H.S.$

$\therefore \lim_{x\to 0} f(x)\ exist$

$\therefore \lim_{x\to 0} f(x)=3$

$for\ x=1$

$L.H.S.$

$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1} 3(x+1)$

$= 3(1+1) = 6$

$R.H.S.$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1} 3(x+1)$

$= 3(1+1) = 6$

$L.H.S. = R.H.S.$

$\therefore \lim\limits_{x\to 1} f(x)$ *exist*

$\lim\limits_{x\to 1} f(x) = 6$

Ans9 $\quad$ *let* $f(x) = \sec x$

$f(x+h) = \sec(x+h)$

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\cos(s+h)} - \frac{1}{\cos x}}{h}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{\cos(x+h)\cos xh}$$

$$= \lim_{h\to 0} \frac{-2\sin\left[\frac{2x+h}{2}\right]\sin\left[\frac{-h}{2}\right]}{\cos(x+h)\cos x.h}$$

$$= \lim_{h\to 0} \frac{2\sin\left[\frac{2x+h}{2}\right]\sin\frac{h}{2}}{\cos(x+h)\cos xh} \qquad \left[\sin(-\theta = -\sin\theta)\right]$$

$$= \lim_{h\to 0} \frac{2\sin\frac{h}{2}}{2\frac{h}{2}} \times \frac{\lim\limits_{h\to 0} \sin\frac{(2x+h)}{2}}{\lim\limits_{h\to 0} \cos(x+h)\cos x}$$

$$= 1 \times \frac{\sin\left(\frac{2x+0}{2}\right)}{\cos(x+0)\cos x} = \frac{\sin x}{\cos x \ \cos x}$$

$$= \tan x \ \sec x$$

Ans10 $f(x)=\frac{4x+5\sin x}{3x+7\cos x}$

$$f^1(x)=\frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\times\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$

$$=\frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$

$$=\frac{\cancel{12x}+15x\cos x+28\cos x+35\cos^2 x-\cancel{12x}+28\sin x+15\sin x+35\sin^2 x}{(3x+7\cos x)^2}$$

$$=\frac{15x\cos x+35\left[\sin^2 x+\cos^2 x\right]+28+\cos x+43\sin x}{(3x+7\cos x)^2}$$

$$=\frac{15x\cos x+35+28\cos x+45\sin x}{(3x+7\cos x)^2}$$

Ans11 $\lim_{h\to 0}\frac{(a+h)^2\sin(a+x)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\frac{\left(a^2+2ah+h^2\right)\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{a^2\sin(a+h)+2ah\sin(a+h)+h^2\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{a^2\left[\sin(a+h)-\sin a\right]+2ah\sin(a+h)+h^2\sin(a+h)}{h}$$

$$=\lim_{h\to 0}\frac{a^2 2\cos\left[\frac{2a+h}{2}\right]\sin\frac{h}{2}}{2\frac{h}{2}}+\lim_{h\to 0}2a\sin(a+h)+\lim_{h\to 0}h\sin(a+h)$$

$$=a^2\cos\left[\frac{2a+0}{2}\right]\times 1+2a\sin[a+0]+0\times\sin a$$

$$=a^2\cos a+2a\sin a$$

Ans12 (i) $\frac{d}{dx}\left[\frac{a}{x^4}-\frac{b}{x^2}+\cos x\right]$

$$=\frac{d}{dx}ax^{-4}-\frac{d}{dx}bx^{-2}+\frac{d}{dx}\cos x$$

$$=a\left(-4x^{-5}\right)-b\left(-2x^{-3}\right)-\sin x$$

$$=\frac{-4a}{x^5}+\frac{2b}{x^3}-\sin x$$

(ii) $$\frac{d}{dx}(x+\cos x)(x-\tan x)$$

$$=(x+\cos x)\frac{d}{dx}(x-\tan x)+(x-\tan x)\frac{d}{dx}(x+\cos x)$$

$$=(x+\cos x)\left(1-\sec^2 x\right)+(x-\tan x)(1-\sin x)$$

$$=x-x\sec^2 x+\cos x-\cos x\sec^2 x+x-x\sin x-tacx+\tan x\sin x$$

$$=2x-x\sec^2 x+\cancel{\cos x}-\cancel{\cos x}-x\sin x-\tan x+\tan x\sin x$$

$$=2x-x\sec^2 x-x\sin x-\tan x+\tan x\sin x$$

TEST PAPER-01

CLASS - XI MATHEMATICS (Mathematical Reasoning)

1. Give three examples of sentences which are not statements. Give reasons for the answers. [4]

2. Write the negation of the following statements [4]

   (i) Chennai is the capital of Tamil Nadu.

   (ii) Every natural number is an integer.

3. Find the component statements of the following compound statements and check whether they are true or false. [4]

   (i) The number 3 is prime or it is odd.

4. Check whether the following pair of statements are negations of each other Give reasons for your answer. [4]

   (i) $x+y=y+x$ is true for every real numbers x and y.

   (ii) There exists real numbers x and y for which $x+y=y+x$.

5. Write the contra-positive and converse of the following statements. [4]

   (i) If x is a prime number, then x is odd.

   (ii) if the two lines are parallel, then they do not intersect in the same plane.

6. Show that the statement [4]

   P : "If x is a real number such that $x^3+4x=0$, then x is 0" is true by

   (i) direct method, (ii) method of contradiction, (iii) method of contra-positive

7. Given below are two statements [4]

   P : 25 is a multiple of 5.

   q: 25 is a multiple of 8

   Write the compound statements connecting these two statements with "and" and "OR". In both cases check the validity of the compound statement.

8. Write the following statement in five different ways, conveying the same meaning. [4]

   P : If a triangle is equiangular, then it is an obtuse angled triangle.

TEST PAPER-01

CLASS - XI MATHEMATICS (Mathematical Reasoning)

[ANSWERS]

Ans1. (i) The sentence "Rani is a beautiful girl" is not a statement. To some Rani may look beautiful and to other she may not look beautiful. We cannot say on logic whether or not this sentence is true.

(ii) The sentence 'shut the door' is not a statement. It is only an imperative sentence giving a direction to someone. There is no question of it being true or false.

(iii) The sentence 'yesterday was Friday' is not a statement. It is an ambiguous sentence which is true if spoken on Saturday and false if spoken on other days. Truth or false hood of the sentence depends on the time at which it is spoken and not on mathematical reasoning.

Ans2. (i) Chennai is not the capital of Tamil Nadu.

(ii) Every natural number is not an integer.

Ans3. The component statements of the given statement are

p: "The number 3 is prime"

q: "number 3 is odd"

There two have been connected by using the connective "or"

The given statement is true as both the statements are true.

Ans4. The given statements are

p : "$x + y = y + x$ is true for every real number x and y"

q : "There exists real numbers x and y for which $x + y = y + x$."

These statements are not negations of each other as they can be true at the same time. Infact, negation of p is

$\sim p$: "There are real numbers x and y for which $x + y \neq y + x$."

Note that p is always true whatever x and y may be and $\sim p$ is always false.

Ans5. If statement is $p \Rightarrow q$, then its contra-positive is $\sim q \Rightarrow \sim p$ and its converse is $q \Rightarrow p$.

(i) Contra-positive : "If x is not odd, then x is not a prime number."

Converse : "If x is odd, then x is a prime number."

(ii) Contra-positive : "If two lines interest in the same plane, then they are not parallel."

Converse: "If two lines do not intersect in the same plane, then they are parallel."

Ans6. Given statement is p: "If x is a real number such that $x^3 + 4x = 0$, then $x = 0$"

(i) Direct method: Let $x^3 + 4x = 0, x \in R$

$\Rightarrow x\left(x^2 + 4\right) = 0, x \in R \Rightarrow x = 0$ $\left(\because \text{if } x \in R \text{ then } x^2 + 4 \geq 4\right)$

Note that if the product of two numbers is zero then atleast one of them is surely zero.

Thus, we find that p is a true statement.

(ii) Method of contradiction.

Let x be a nonzero real number

$\Rightarrow x^2 > 0$ ($\because$ Square of a non- zero real number is always positive)

$\Rightarrow x^2 + 4 > 4$ $\Rightarrow x^2 + 4 \neq 0$

$\Rightarrow x\left(x^2 + 4\right) \neq 0$ $\left(\because x \neq 0 \text{ and } x^2 + 4 \neq 0\right)$

$\Rightarrow x^3 + 4x \neq 0$, which is a contradiction.

Hence, $x = 0$

(iii) Method of contra-positive:

Let q : "$x \in R$ and $x^3 + 4x = 0$"

r : "$x = 0$"

$\therefore$ Given statement p is $q \Rightarrow r$

Its contra-positive is $\sim r \Rightarrow \sim q$

i.e. "if x is a non- zero real number then $x^3 + 4x$ is also nonzero"

Now $x \neq 0, x \in R \quad \Rightarrow x^2 > 0 \quad \Rightarrow x^2 + 4 > 4 \quad \Rightarrow x^2 + 4 \neq 0$

$\Rightarrow x\left(x^2 + 4\right) \neq 0 \quad \Rightarrow x^3 + 4x \neq 0 \quad i.e. \sim r \Rightarrow \sim q.$

Thus the statement $\sim r \quad \Rightarrow \sim q.$ is always true

Hence, $q \Rightarrow r$ is always true

Note: Infact, 'Method of contradiction' is another form of 'contra-positive method' while proving an implication.

Ans7. Case I. Using the connective 'and', we obtain the compound statement "p and q".

i.e., "25 is a multiple of 5 and 8".

It is false statement as q is always false. $(\because$ 25 is not a multiple of 8$)$

Case II. Using the connective 'or', we obtain the compound statement "p or q".

i.e., "25 is a multiple of 5 or 8".

It is a true statement as p always true. $(\because$ 25 is a multiple of 5$)$

Ans8. Given statement is

"If a triangle is equiangular, then it is an obtuse angled triangle". Its five equivalents are as follows:

(i) "A triangle is equiangular only if it is an obtuse angled triangle".

(ii) "If a triangle is not obtuse angled triangle then it is not an equiangular triangle."

(iii) "equiangularity is a sufficient condition for triangle to be obtuse angled."

(iv) "A triangle being obtuse angled, is necessary condition for it to be equiangular".

(v) A triangle is obtuse is obtuse angled if it is equiangular.

TEST PAPER-01

CLASS - XI MATHEMATICS (Statistics)

1. In a test with a maximum sure 25, eleven students scored 3,9,5,3,12,10,17,4,7,19,21 marks respectively. Calculate the range. [1]
2. Coefficient of variation of two distributions is 70 and 75, and their standard deviations are 28 and 27 respectively what are their arithmetic mean? [1]
3. Write the formula for mean deviation. [1]
4. Write the formula for variance [1]
5. The mean of 2,7,4,6,8 and p is 7. Find the mean deviation about the median of these observations. [4]
6. Find the mean deviation about the mean for the following data! [4]

x_i	10	30	50	70	90
f_i	4	24	28	16	8

7. Find the mean, standard deviation and variance of the first n natural numbers. [4]
8. Find the mean variance and standard deviation for following data [4]
9. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations. [4]
10. Prove that the standard deviation is independent of any change of origin, but is dependent on the change of scale. [4]
11. Calculate the mean, variance and standard deviation of the following data: [6]

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

12. The mean and the standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who mistook one observation as 50 instead of 40. What are the correct mean and standard deviation? [6]

TEST PAPER-01

CLASS - XI MATHEMATICS (Statistics)

[ANSWERS]

Ans1. The nocks can be arranged in ascending order as 3,3,4,5,7,9,10,12,17,19,21.

Range = maximum value – minimum value

=21-3

= 18

Ans2. Given C.V (first distribution) = 70

Standard deviation = σ_1 = 28

C.V $\frac{\sigma 1}{\bar{x}1}\times 100$

$70=\frac{28}{\bar{x}1}\times 100$

= $\bar{x}=\frac{28}{70}\times 100$

$\bar{x}=40$

Similarly for second distribution

C.V $=\frac{\sigma_2}{x_2}\times 100$

$75=\frac{27}{\bar{x}_2}\times 100$

$\bar{x}_2=\frac{27}{75}\times 100$

$\bar{x}_2=36$

Ans3. $\text{MD}\left(\bar{x}\right)=\frac{\sum f_i\,|x_i-\bar{x}|}{\sum f_i}=\frac{1}{x}\sum f_i\,|x_i-\bar{x}|$

Ans4. Variance $\sigma^2=\frac{1}{n}\sum f_i\left(x_i-\bar{x}\right)^2$

Ans5. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers $\therefore n=6$

The near of these observations is 7

$\frac{2+7+4+6+8+p}{6}=7$

$=27+p=42$

$=p=15$

Arrange the observations in ascending order 2,4,6,7,8,15

$$\therefore \text{Medias (M)} = \frac{\frac{n}{2}\text{ th observation} + \left(\frac{n}{2}+1\right)\text{ th observation}}{2}$$

$$= \frac{\text{3rd observation + 4th observation}}{2}$$

$$= \frac{6+7}{2} = \frac{13}{2}$$

$= 6.5$

Calculate of mean deviation about Medias.

xi	xi-M	|xi-M|
2	-4.5	4.5
4	-2.5	2.5
6	-0.5	0.5
7	0.5	0.5
8	1.5	1.5
15	8.5	8.5
Total		18

$\therefore$ Media's deviation about median $= \frac{\cancel{18}^{3}}{\cancel{6}} = 3.$

Ans6. To calculate mean, we require $f_i xi$ values then for mean deviation, we require $|xi - \overline{x}|$ values and $f_i\,|xi - \overline{x}|$ values.

xi	f_i	$f_i xi$	$\|xi - \overline{x}\|$	$fi\,\|xi - \overline{x}\|$
10	4	4	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$n = \sum f_i = 80 \quad \sigma d \sum f_i xi = 4000$

$$\overline{x} = \frac{\sum f_i xi}{n} = \frac{4000}{80} = 50$$

Mean deviation about the mean

$$\text{MD}\left(\overline{x}\right) = \frac{\sum f_i\,|xi - \overline{x}|}{n} \qquad = \frac{1280}{80} = 16$$

Ans7. The given numbers are 1, 2, 3, ……, n

Mean $\bar{x} = \frac{\sum n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$

Variance $\sigma 2 = \frac{\sum xi^2}{n} - \bar{x}$

$= \frac{\sum n^2}{n} - \left(\frac{n+1}{2}\right)^2$

$= \frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4}$

$= (n+1)\left[\frac{2n+1}{6} - \frac{n+1}{4}\right]$

$= (n+1)\left(\frac{n-1}{12}\right) \qquad = \frac{n^2-1}{12}$

$\therefore$ Standard deviation $\sigma = \frac{\sqrt{n^2}}{12}$

Ans8.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Note: - 4th, 5th and 6th columns are filled in after calculating the mean.

xi	f_i	$f_i x_i$	$xi - \bar{x}$	$\left(xi - \bar{x}\right)^2$	$f_i x_i \left(xi - \bar{x}\right)^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	402			1374

Here $n = \sum f_i = 30, \quad \sum f_i x_i = 420$

$\therefore$ Mean $\bar{x} = \frac{\sum f_i x_i}{n} = \frac{420}{30} = 14$

$\therefore$ Variance $\sigma^2 = \frac{1}{n} \sum f_i \left(x_i - \bar{x}\right)^2$

$= \frac{1}{30} \times 1374$

$= 45.8$

$\therefore$ Standard deviation $\sigma = \sqrt{45.8}$

$= 6.77$

Ans9. Let $x_i, x_2 ...... x_6$ be the six given observations

Then $\bar{x} = 8$ and $\sigma = 4$

$\bar{x} = \frac{\sum x_i}{n} = 8 = \frac{x_1 + x_2 + ... + x_6}{6}$

$x_1 + x_2 + ... x_6 = 48$

Also $\sigma^2 \frac{\sum x_1^2}{n} - (\bar{x})^2$

$= 4^2 = \frac{x_1^2 + x_2^2 ...... + x_6^2}{6} - (8)^2$

$= x_1^2 + x_2^2 + ... x_6^2$

$= 6 \times (16 + 64) = 480$

As each observation is multiplied by 3, new observations are

$3x_1, 3x_2, ... 3x_6$

New near $\bar{X} = \frac{3x_1 + 3x_2 + ... 3x_6}{6}$

$= \frac{3(x_1 + x_2 + ... x_6)}{6}$

$= \frac{3 \times 48}{6}$

$= 24$

Let σ_1 be the new standard deviation, then

$\sigma_1^2 = \frac{(3x_1)^2 + (3x_2)^2 + ... + (3x_6)^2}{6} - (\bar{X})^2$

$= \frac{9(x_1^2 + x_2^2 + ... x_6^2)}{6} - (24)^2$

$= \frac{9 \times 480}{6} - 576$

$= 720 - 576$

$= 144$

$\sigma_1 = 12$

Ans10. Let us use the transformation $u = ax + b$ to change the scale and origin

Now $u = ax + b$

$= \sum u = \sum(ax+b) = a\sum x + b.n$

Also $\sigma u^2 = \frac{\sum(u-\bar{u})^2}{n} = \frac{\sum(ax+b-a\bar{x}-b)^2}{n}$

$= \frac{\sum a^2(x-\bar{x})^2}{n} = \frac{a^2\sum(x-\bar{x})^2}{n}$

$= a^2\sigma x^2$

$\therefore \quad \sigma^2 u = a2\sigma^2 u$

$= \sigma u = |a|\sigma x$

Both σu , σx are positive which shows that standard deviation is independent of choice of origin, but depends on the scale of nears cerement.

Ans11.

Classes	Frequency	Mid Point	$f_i xi$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
Total	50		3100		10050

Here $n = \sum f_i = 50, \sum f_i x_i = 3100$

$\therefore$ Mean $\bar{x} = \frac{\sum f_i x_i}{n} = \frac{3100}{50} = 62$

Variance $\sigma^2 = \frac{1}{n}\sum f_i (xi - \bar{x})^2$

$= \frac{1}{50} \times 10050$

$= 201$

Standard deviation $\sigma = \sqrt{201} = 14.18$

TEST PAPER-02
CLASS - XI MATHEMATICS (Statistics)

1. Find the median for the following data. [1]

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

2. Write the formula of mean deviation about the median [1]

3. Find the rang of the following series 6,7,10,12,13,4,8,12 [1]

4. Find the mean of the following data 3,6,11,12,18 [1]

5. Calculate the mean deviation about the mean for the following data [4]

Expenditure	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
persons	4	8	9	10	7	5	4	3

6. Find the mean deviation about the median for the following data [4]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of boys	8	10	10	16	4	2

7. An analysis of monthly wages point to workers in two firms A and B, belonging to the same industry, given the following result. [4]

	Firm A	Firm B
No of wages earns	586	648
Average monthly wages	Rs 5253	Rs 5253

8. Find the mean deviation about the median of the following frequency distribution [4]

Class	0-6	6-12	12-18	18-24	24-30
Frequency	8	10	12	9	5

9. Calculate the mean deviation from the median from the following data [4]

Salary per week(in Rs)	10-20	20-30	30-40	40-50	50-60	60-70
no. of workers	4	6	10	20	10	6

10. Let $x_1, x_2 ......x_n$ values of a variable Y and let 'a' be a non zero real number. Then prove that the variance of the observations $ay_1, ay_2 ......ay_n$ is $a^2 \operatorname{var}(Y)$. also, find their standard deviation. [4]

11. 200 candidates the mean and standard deviation was found to be 10 and 15 respectively. After that if was found that the scale 43 was misread as 34. Find the correct mean and correct S.D [6]

12. Find the mean deviation from the mean 6,7,10,12,13,4,8,20 [6]

TEST PAPER-02

CLASS - XI MATHEMATICS (Statistics)

[ANSWERS]

Ans1.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6
$c.f$	8	14	16	18	20	26

$n = 26.$ Median is the average of 13th and 14th item, both of which lie in the c.f 14

$\therefore x_i = 7$

$\therefore$ median $= \dfrac{\text{13observation} + \text{14th observation}}{2}$

$= \dfrac{7+7}{2} = 7$

Ans2. $MD.(M) = \dfrac{\sum f_i |x_i M|}{\sum f_i} = \dfrac{1}{n}\sum f_i |x_i - M|$

Ans3. Range = maximum value – minimum value
= 113-4
=9

Ans4. Mean = $\dfrac{\text{sun of observation}}{\text{Total no of observation}}$

$= \dfrac{50}{5} = 10$

Ans5

Expenditure	No. of persons f_i	Mid point x_i	$f_i x_i$	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	5	550	2750	192	960
600-700	4	650	2600	292	1168
700-800	3	750	2250	392	1176
	50		17900		7896

$n = \sum f_i = 50$

$\sum f_i x_i = 17900$

$\therefore$ mean = $\frac{1}{n} \sum f_i x_i = \frac{17900}{50} = 358$

$MD(\bar{x}) = \frac{1}{n} \sum fi \mid x_i - \bar{x} \mid$

$= \frac{7896}{50} = 157.92$

Ans6.

Marks	No. of boys	Cumulative Frequency	Mid points	$\mid x_i - M \mid$	$f_i \mid x_i - M \mid$
0-10	8	8	5	22	176
10-20	10	18	15	12	120
20-30	10	28	25	2	20
30-40	16	44	35	8	128
40-50	4	48	45	18	72
50-60	2	50	55	28	56
total	50				572

$\frac{n}{2}^{th}$ or 25^{th} item $= 20 - 30$, which is the median class.

Median $= l + \frac{\frac{n}{2} - c}{f} \times c = 20 + \frac{25-18}{10} \times 10$

= 27

$MD(M) = \frac{1}{n} \sum f_i \mid x_i - M \mid = \frac{572}{50} = 11.44$

Ans7. For firm A, number of workers = 586

Average monthly wage is Rs 5253

Total wages = Rs 5253×586

= Rs 3078258

For firm B, total wages = Rs 253×648

=Rs 3403944

Hence firm B pays out amount of monthly wages.

Ans8.

Class	Mid value	Frequency	$C.f$	$\lvert x_i - 14 \rvert$	$f_i \lvert x_i - 14 \rvert$
0-6	3	8	8	11	88
6-12	9	10	18	5	50
12-18	15	12	30	1	12
18-24	21	9	39	7	63
21-30	27	5	44	13	65
			$N = \sum f_i = 44$		$\sum f_i \lvert x_i - 14 \rvert = 278$

$N = 44 = \frac{N}{2}$

12-18 is the medias class

Medias = $l + \frac{\frac{N}{2} - F}{f} \times h$

$h = 6,\ l = 12,\ f = 12,\ F = 18$

$= 12 + \frac{22 - 18}{12} \times 6$

Medias $= 12 + \frac{4 \times 6}{12}$

$= 14$

Mean deviation about median = $\frac{1}{N} \sum f_i \lvert x_i - 14 \rvert$

$= \frac{278}{74} = 6.318$

Ans9.

Salary per Week (in Rs)	Mid value x_i	Frequency f_i	Cf	$\lvert d_i \rvert = x_i - 45$	$f \lvert d_i \rvert$
10-20	15	4	4	30	120
20-30	25	6	10	20	120
30-40	35	10	20	10	100
40-50	45	20	40	0	0
50-60	55	10	50	10	100
60-70	65	6	56	20	120
70-80	75	4	60	30	120
		$N = \sum f_i = 60$			$\sum f_i \lvert d_i \rvert = 680$

$N=60 \qquad =\frac{N}{2}=30$

40-50 is the median class

$l=40, \quad f=20, \quad h=10, \quad F=20$

Medias $=\dfrac{l-\frac{N}{2}-F}{f}\times h$

$=\dfrac{40+30-20}{20}\times 10=45$

Mean deviation = $\dfrac{\sum f_i\,|\,d_i\,|}{N}$ = $\dfrac{680}{60}=11.33$

Ans10. Let $v_1, v_2 ......v_n$ value of variables v such that $v_1=ay_i, 1, 2......n,$ then

$\overline{V}=\frac{1}{n}\sum_{i=1}^{n}v_i=\frac{1}{n}\sum_{i=1}^{n}(ayi)=a\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)=a\overline{y}$

$v_i-\overline{V}=ay_i-a\overline{y}$

$v_i-\overline{V}=a\left(y_i-\overline{Y}\right)$

$\left(v_i-\overline{V}\right)^2=a^2\left(y_i-\overline{Y}\right)^2$

$\sum_{i=1}^{n}\left(v_i-\overline{V}\right)^2=a^2\frac{1}{n}\sum_{i=1}^{n}\left(y_i-\overline{Y}\right)^2$

Var $(V)=a^2$ Var (Y)

$\sigma_u=\sqrt{\text{var}(v)}=\sqrt{a^2\ \text{var}(Y)}=|\,a\,|\sqrt{\text{var}(Y)}$

$=|\,a\,|\,\sigma_y$

Ans11. $n=200, \overline{X}=40, \sigma=\overline{15}$

$\overline{X}=\frac{1}{n}\sum x_i=\sum x_i=n\overline{X}=200\times 40=8000$

Corrected $\sum x_i$ = Incorrect $\sum x_i$ – (sum of incorrect +sum of correct value)

= 8000-34+43= 8009

$\therefore$ Corrected mean = $\dfrac{\text{corrected}\sum x_i}{n}=\dfrac{8009}{200}=40.045$

$\sigma=15$

$15^2=\frac{1}{200}\left(\sum x_i{}^2\right)-\left(\frac{1}{200}\sum x_i\right)^2$

$$225=\frac{1}{200}\left(\sum x_i^{\ 2}\right)-\left(\frac{8000}{200}\right)^2$$

$$225=\frac{1}{200}\times 1825=365000$$

Incorrect $\sum x_i^{\ 2}=365000$

Corrected $\sum x_i^{\ 2}$ = (incorrect $\sum x_i^{\ 2}$) – (sum of squares of incorrect values) + (sum of square of correct values)

$=365000-(34)^2+(43)^2=365693$

Corrected $\sigma=\sqrt{\frac{1}{n}\sum x_i^{\ 2}-\left(\frac{1}{n}\sum x_i\right)^2}=\sqrt{\frac{365693}{200}-\left(\frac{8009}{200}\right)^2}$

$\sqrt{1828.465-1603.602}=14.995$

Ans12. Let $\overline{X}$ be the mean

$$\overline{X}=\frac{6+7+10+12+13+4+8+20}{8}=10$$

x_i	$\lvert d_i \rvert = \lvert x_i-\overline{X} \rvert = \lvert x_i-10 \rvert$
6	4
7	3
10	0
12	2
13	3
4	6
8	2
20	10
Total	$\sum d_i$ = 30

$\sum d_i$ = 30 and n = 8

$$\therefore MD=\frac{1}{n}\sum \lvert d_i \rvert=\frac{30}{8}=3.75$$

$$\therefore MD=3.75$$

TEST PAPER-03
CLASS - XI MATHEMATICS (Statistics)

1. Find mean deviation about mean of the following observation 3, 9, 5, 3, 12, 10, 17, 4, 7, 19, 21 marks respectively. [1]

2. An analysis of monthly wager paid to be workers of two firms A and B belonging to the same industry gives the following results [1]

	Firm A	Firm B
No. of workers	1000	1200
Average monthly wager	Rs 2800	Rs 2800
Variance of distribution of wager	100	169

3. What are the median class of the observation [1]

Wager	10-20	20-30	30=40	40-50	50-60	60-70	70-80
No. of worker	4	6	10	20	10	6	4

4. Write limit ion of mean deviation. [1]

5. If each of the observation $x_1, x_2, ..., x_n$ is increased by 'a' where a is a negative or positive number show that the variance remains unchanged. [4]

6. Find the mean and variance of first n natural numbers. [4]

7. Find the mean deviation about median for the following data. [4]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of girls	6	8	14	16	4	2

8. Find mean deviation about for the following data [4]

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of student	2	3	8	14	8	3	2

9. Find the mean deviation about the median for the data 36, 72, 46, 42, 60, 45, 53, 46, 51, 49 [4]

10. Find the mean deviation about the mean [4]

x_i	5	10	15	20	25
f_i	7	4	6	3	5

11. Find the mean, variance, and standard deviation using short out method. [6]

Height in (cm)	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of girls	3	4	7	7	15	9	6	6	3

12. From the data given state which group in more C of D [6]

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group C	9	17	32	33	40	10	9
Group D	10	20	30	29	43	15	7

TEST PAPER-03

CLASS - XI MATHEMATICS (Statistics)

[ANSWERS]

Ans1. Mean $=\frac{110}{11}=10$

The deviations of the mean 10 are

$-7,-7,-6,-5,-3,-1,0,2,7,9,11$

Absolute values of deviations are 7,7,6,5,3,1,0,2,7,9,11

$\text{M.D.}\left(\bar{x}\right)=\frac{\sum|x_i-\bar{x}|}{n}$

$=\frac{58}{11}=5.27\left(approx\right)$

Ans2. The firm with greater variance will have more variability

$\therefore$ firm B has greater variability in individual wages.

Ans3. $\sum f_i=60$

$\frac{\sum f_i}{2}=\frac{60}{2}=30$

C.I	10-20	20-30	30-40	40-50	50-60	60-70	70-80
C.F	4	10	20	40	50	56	60

30 Falls on C.F 40 $\therefore$ median class 40-50

Ans4. In computation of mean deviation we are absolute values of deviations. Therefore it cannot be subjected to further algebraic treatment.

Ans5. Let $\bar{x}$ be the mean of $x_1, x_2, \ldots\ldots . x_n$ Then the variance is given by

$$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

If a is added to each observation

The new observation will be $y_i = x_i + a \ldots\ldots..(i)$

Let mean of new observation be $\bar{y}$, then

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right] = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$$

i.e $\bar{y} = \bar{x} + a \ldots\ldots..(ii)$

$\therefore$ The variance of new observation

$$\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i + a - \bar{x} - a\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 = \sigma_1^2$$

Ans6. The first n natural numbers are 1,2,3,…., n

Mean $\bar{x} = \dfrac{1+2+3+\ldots\ldots+n}{n} = \dfrac{n(n+1)}{2n} = \dfrac{n+1}{2}$

Variance = $\sigma^2 = \dfrac{\sum\left(x_i - \bar{x}\right)^2}{n} = \dfrac{1}{n}\left[\sum\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)\right]$

$$= \frac{\sum x_i^2}{n} - 2\bar{x}\frac{\sum x_i}{n} + n\frac{\bar{x}^2}{n}$$

$$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\sum x_i^2 = \frac{n(n+1)(2n+1)}{6}$$

Variance = $\dfrac{n(n+1)(2n+1)}{6n} - \left[\dfrac{n(n+1)}{2n}\right]^2$

$$=\frac{(n+1)(2n+1)}{6}-\frac{(n+1)^2}{4}$$

$$=(n+1)\left[\frac{(2n+1)}{6}-\frac{(n+1)}{4}\right]=\frac{(n+1)(n-1)}{12}=\frac{n^2-1}{12}$$

Ans7

Classes	Mid Value	f_i	$C.f$	$\lvert x_i-M \rvert$	$f_i \lvert x_i-M \rvert$
0-10	5	6	6	22.86	137.16
10-20	15	8	14	12.86	102.88
20-30	25	14	28	2.86	40.04
30-40	35	16	44	7.14	141.24
40-50	45	4	48	17.14	68.56
50-60	55	2	50	27.14	54.28
					517.16

$N=50 \quad \frac{N}{2}=25$

25 falls of C.F. 28

$\therefore$ class 20-30 is median class

$l=20, \quad f=14, \quad h=10$

Median $=l+\frac{\frac{n}{2}-c.f}{f}\times h$

$=20+\frac{25-14}{14}\times 10$

$=20+\frac{11}{14}\times 10=20+7.86$

$=27.86$

$\sum f_i \lvert x_i-M \rvert=517.16$

$\therefore$ Mean deviation about median $=\frac{517.16}{50} \quad =10.34$

Ans8. The assumed mean a = 45 and h= 10,

Marks Obtained	No. of students	Mid points	$d_i = \frac{x_i - 45}{10}$	fid_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10-20	2	15	-3	-6	30	60
20-30	3	25	-2	-6	20	60
30-40	8	35	-1	8	10	80
40-50	14	45	0	0	0	0
50-60	8	55	1	8	10	80
60-70	3	65	2	6	20	60
70-80	2	75	3	6	30	60
	40			0		400

$$\therefore \bar{x} = a + \frac{\sum_{i=1}^{7} f_i d_i}{N} \times h \qquad \Rightarrow 45 + \frac{0}{40} \times 10 = 45$$

And M.D $= \frac{1}{n} \sum_{i=1}^{7} f_i \lvert x_i - \bar{x} \rvert = \frac{400}{40} = 10$

Ans9. 36, 42, 45, 46, 49, 51, 53, 60, 72

The number of observations = 10

$\frac{10}{2} = 5^{th}$ Observation = 46

$\left(\frac{10}{2} + 1\right)^{th} = 6^{th}$ Observation = 49

Median = $\frac{1}{2}(46 + 49)$ = 47.5

$= \lvert 36 - 47.5 \rvert + \lvert 42 - 47.5 \rvert + \lvert 45 - 47.5 \rvert + \lvert 46 - 47.5 \rvert + \lvert 49 - 47.5 \rvert$
$+ \lvert 51 - 47.5 \rvert + \lvert 53 - 47.5 \rvert + \lvert 60 - 47.5 \rvert + \lvert 72 - 47.5 \rvert$

$= 11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5$

$= 70$

$\therefore$ MD $= \frac{70}{10} = 7$

Ans10.

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
Total	25	350		158

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{350}{25} = 14$$

$$\text{Mean deviation from the mean } = \frac{\sum f_i \lvert x_i - \bar{x} \rvert}{\sum f_i}$$

$$= \frac{158}{25} = 6.32$$

Ans11

Class	Mid Value	Frequency	$y_i = \frac{x_i - 92.5}{5}$	$f_i y_i$	y_i^2	$f_i y_i^2$
70-75	72.5	3	-4	-12	16	48
75-80	77.5	4	-4	-12	9	36
80-85	82.5	7	-2	-14	4	28
85-90	87.5	7	-1	-7	1	7
90-95	92.5	13	0	0	0	0
95-100	97.5	9	1	9	1	9
100-105	102.5	6	2	12	4	24
105-110	107.5	6	3	18	9	54
110-115	112.5	3	4	12	16	48
Total		60		6		254

$$y_i = \frac{x_i - 92.5}{5} = h = 5, A = 92.5$$

$$\bar{x} = A + \frac{\Sigma f_i y_i}{\Sigma f_i} \times h \quad = 92.5 + \frac{60}{60} \times 5 \quad = 93.0$$

Variance = $\sigma^2 = \frac{h^2}{N^2}\left[N \Sigma f y_i^2 - (\Sigma f_i y_i)^2\right]$

Standard deviation $\sigma = \sqrt{105.58} = 10.27$

Ans12.

Class	Mid point	$y_i = \frac{x_i - 45}{10}$	For Group C				For Group D		
			y_i^2	f_i	$f_i y_i$	$f_i y_i^2$	f_i	$f_i y_i$	$f_i y_i^2$
10-20	15	-3	9	9	-27	81	10	-30	90
20-30	25	-2	4	17	-34	68	20	-40	80
30-40	35	-1	1	32	-32	32	30	-30	30
40-50	45	0	0	33	0	0	25	0	0
50-60	55	1	1	40	40	40	43	43	43
60-70	65	2	4	10	20	40	15	30	160
70-80	75	3	9	9	27	81	7	21	189
Total				150	-6	342	150	-6	592

For group C:-

$$\bar{x} = A + \frac{\Sigma f_i y_i}{\Sigma f} \times h = 45 + \frac{-6}{150} \times 10$$

$$= 45 - 0.4 = 44.6$$

$$\sigma^2 = \frac{h^2}{N^2}\left[N \Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 342 - (-6)^2\right]$$

$$= \frac{1}{225}[51300 - 36]$$

$$= \frac{51264}{225} = 227.84$$

$$\sigma = 15.09$$

Coefficient of variation (C.V)

$$= \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{15.07}{44.6} \times 100 \quad = 33.83$$

For group D

$$\bar{x} = A + \frac{\sum f_i y_i}{\sum f} \times h$$

$$= 45 + \frac{-6}{100} \times 100$$

$$= 44.6$$

$$\sigma^2 = \frac{h^2}{N^2}\left[N \sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 592 - (-6)^2\right]$$

$$= \frac{1}{225}(88800 - 36)$$

$$= \frac{88764}{225}$$

$$= 394.50$$

$$\sigma = 19.86$$

Coefficient of variation $= \frac{\sigma}{x} \times 100$

$$= \frac{19.86}{44.6} \times 100$$

$$= 44.53$$

CV in group D is greater than CV in group C therefore group D is more variable than group C.

TEST PAPER-04

CLASS - XI MATHEMATICS (Statistics)

1. Find the mean deviation about the mean from the following data [1]

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

2. What do you understand by measures of dispersion [1]

3. Find the mean for the data [1]

x_i	5	10	15	20	25
f_i	7	4	6	3	5

4. Find the range 20, 28, 40, 12, 30, 15, 50 [1]

5. Find the mean deviation about median [4]

x_i	15	21	27	30	35
f_i	3	5	6	7	8

6. Find the mean deviation about mean from the following data. [4]

4, 7, 8, 9, 10, 12, 13, 17

7. [4]

Height in cm	95-105	105-115	115-125	125-135	135-145	145-155
No. of persons	9	13	26	30	12	10

8. Find the mean deviation about the medias from the data [4]

13, 16, 17, 14, 13, 11, 16, 10, 18, 11, 12, 17

9. Find the mean deviation about median for the following [4]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of girls	6	8	14	16	4	2

10. Find the mean and variance for the following 6, 7, 10, 12, 13, 4, 8, 12 [4]

11. The diameter of a semi circle (in mm) drawn in design are as following [6]

Diameter	33-36	37-40	41-44	45-48	49-52
No. of semi circles	15	17	21	22	25

calculate the standard deviation and mean diameter of the circle.

12. [6]

Classes	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Find the mean and variance of the following data.

TEST PAPER-04
CLASS - XI MATHEMATICS (Statistics)

[ANSWERS]

Ans1.

$$\bar{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

$$\text{MD} = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{12+20+10+2+8+5+13+4+4+6}{10} = \frac{84}{10} = 8.4$$

Ans2. Measures of dispersion are required to study the degree of scatterings of observations from the central value.

Ans3.

x_i	f_i	$f_i x_i$
5	7	35
10	4	40
15	6	90
20	3	60
25	5	125
Total	25	350

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{350}{25} = 14$$

Ans4. Range = max value – min value

= 50 – 12

= 38

Ans5.

x_i	f_i	$C.f$	$\lvert x_i - median \rvert$	$f_i \lvert x_i - median \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
		Total	32	148

$N = \sum f_i = 29$ which is odd, the median is $\left(\frac{n+1}{2}\right)^{th}$

i.e $\frac{29+1}{2} = 15^{th} = 30$

$\therefore$ median = 30

$\therefore$ Mean deviation from median = $\frac{\sum f_i x_i \,|x_i - 30|}{\sum f_i}$

$= \frac{148}{29} = 5.1$

Ans6. Mean $\bar{x} =$ 4, 7, 8, 9, 10, 12, 13, 17

$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8}$

$= \frac{80}{8} = 10$

$\sum |x_i - \bar{x}| = 6+3+2+1+0+2+3+7$

$= 24$

Mean deviation from near = $\frac{\sum |x_i - \bar{x}|}{n}$ $= \frac{24}{8} = 3$

Ans7

Classes	Mid value	$d_1 = \frac{x_i - 130}{10}$	f_i	$f_i d_i$	$\|x_1 - \bar{x}\|$	$f_i \|x_i - \bar{x}\|$
95-105	100	-3	9	-27	25.3	227.7
105-115	110	-2	13	-26	15.3	198.9
115-125	120	-1	26	-26	5.3	137.8
125-135	130	0	30	0	4.7	141.0
135-145	140	1	12	12	14.7	176.4
145-155	150	2	10	20	24.7	247.0
Total			100	-47		1128.8

Let the assumed mean be 130, then

$d_i = \frac{x_i - 130}{10}$

$\bar{x} = 130 + \frac{\sum f_i d_i}{\sum f_i} \times 10$

$= 130 + \frac{-47}{100} \times 10 \quad \Rightarrow 130 - 4.7 = 125.3$

Mean deviation from the mean = $\frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} \quad \Rightarrow \frac{1128.8}{100} = 11.288$

Ans8. 10, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18 (12 observations)

$\frac{12^{th}}{2}$ item $=6^{th}$ item $=13$

$\left(\frac{12}{2}+1\right)=7^{th}$ item $=14$

Median $=\frac{1}{2}(13+14)=13.5$

$\sum_{i=1}^{12}|x_1-x|=28$

MD = $\frac{28}{12}=2.33$

Ans9.

Classes	Mid value	f_i	$C.f$	$\lvert x_1-M \rvert$	$f_i \lvert x_i-M \rvert$
0-10	5	6	6	22.86	137.16
10-20	15	8	14	12.86	102.88
20-30	25	14	28	2.86	40.04
30-40	35	16	44	7.14	114.24
40-50	45	4	48	17.14	68.56
50-60	55	2	50	27.14	54.28
			Total		517.16

Now = 14, class corresponding to cumulative frequency 28 is 20-30

$\therefore l=20,\quad f=14,\quad h=10$

Meden $=\dfrac{l+\frac{N}{2}-C}{f}$

$=20+\frac{25-14}{14}\times 10$

$=20+\frac{11}{14}\times 10=20+7.86$

$=27.86$

$\sum f_i\,|x_i-M|=517.16$

Mean deviation about median $=\frac{517.16}{50}=10.34$

Ans10. Mean $\bar{x}=\frac{\sum x_i}{n}=\frac{6+7+10+12+13+4+8+12}{8}\quad=\frac{72}{8}=9$

x_i	$x_i - \overline{x}$	$\left(x_i - \overline{x}\right)^2$
6	6-9	$(-3)^2$
7	7-9	$(-2)^2$
10	10-9	1^2
12	12-9	3^2
13	13-9	4^2
4	4-9	$(-5)^2$
8	8-9	$(-1)^2$
12	12-9	$(-3)^2$

$$\Sigma\left(x_i - \overline{x}\right)^2 = 9+4+1+9+16+25+1+9 = 74$$

$$\text{Variance } = \frac{\Sigma\left(x_i - \overline{x}\right)^2}{\Sigma f_i} = \frac{74}{8} = 9.25$$

Ans11.

Class interval	Mid value x_i	Frequency f_i	$y_i = \frac{x_i - 42.5}{5}$	$f_i y_i$	y_i^2	$f_i y_i^2$
32.5-36.5	34.5	15	-2	-30	4	60
36.5-40.5	38.5	17	-1	-17	1	17
40.5-44.5	42.5	21	0	0	0	0
44.5-48.5	46.5	22	1	22	1	22
48.5-52.5	50.5	25	2	50	4	100
		100		25		199

The data is made continuous by making classes as above

$$y_i = \frac{x_i - A}{i} = \frac{x_i - 42.5}{4}$$

Where the assumed mean, A is assumed as 42.5 as $d_i = 4$

$\therefore$ mean $\overline{x} = A + i\overline{y}$

$$= 42.5 + 4 \times \frac{\Sigma f_i y_i}{\Sigma f_i}$$

$$= 42.5 + \frac{4 \times 25}{100} = 42.5 + 1 = 43.5$$

$$\text{Variance } \sigma^2 = \frac{i^2}{N^2}\left[N \Sigma f_i y_i^2 - \left(\Sigma f_i y_i\right)^2\right]$$

$$=\frac{16}{(100)^2}\left[100\times 99-(25)^2\right]$$

$$=\frac{16}{(100)^2}\times 19275$$

$$\sigma=\sqrt{\text{variance}}$$

$$\therefore \text{Standard deviation} == \frac{4}{100}\times\sqrt{19275}$$

$$=5.55$$

Ans12.

Classes	Mid value	Frequency	$y_i=\frac{x_i-25}{10}$	f_iy_i	$y_i^{\ 2}$	$f_iy_i^{\ 2}$
0-10	5	5	-2	-10	4	20
10-20	15	8	-1	-8	1	8
20-30	25	15	0	0	0	0
30-40	35	16	1	16	1	16
40-50	45	6	2	12	4	27
Total	50			10		68

$$y_i=\frac{x_i-25}{10}\quad\left[\because A=25 \text{ and } i=10\right]$$

$$\overline{x}=A+i\overline{y}=25+10\times\frac{\sum f_iy_i}{\sum f_i}$$

$$=25+10\times\frac{10}{50}$$

$$25+2=27$$

$$\text{Variance, } \sigma^2=\frac{i^2}{\sum f_i}\quad\left[\sum f_iy_i^{\ 2}-n\overline{y}^2\right]$$

$$=\frac{10^2}{50}\quad\left[68-50\times(0.2)^2\right]$$

$$=\frac{100}{50}\quad\left[68-50\times 0.04\right]$$

$$=2(68-2)$$

$$=2\times 66$$

$$=132$$

TEST PAPER-05

CLASS - XI MATHEMATICS (Statistics)

1. Find the range of the following 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5 [1]

2. Write the formula for coefficient of variance [1]

3. Find the mean 11, 14, 15, 17, 18 [1]

4. Name different methods of measuring dispersion [1]

5. Find the mean deviation about the median [4]

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

6. Find the mean of the following i) first 10 multiples of 3. [4]

7. Find the mean and variance of the data [4]

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

8. Find the mean and variance [4]

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

9. Find the mean and standard deviation using short out method [4]

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

10. Find the mean and variance for the following frequency [4]

Class	0-30	30-60	60-90	90-120	120-150	150-180	180
Frequency	2	3	5	10	3	5	2

11. The sum and the sum of squares corresponding to length x and weight y (both in can) of 50 plant products are given below:- [6]

$$\sum_{i=1}^{50} x_i = 212, \quad \sum_{i=1}^{50} x_i^2 = 902.8, \quad \sum_{i=1}^{50} y_i = 261$$

Which is more varying the length or weight?

12. The following is the record of goals scored by team C in football. [6]

No. of goals	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team O, mean number of goals scored per match was 2 with standard deviation 1.25 goals find which team may be considered

TEST PAPER-05

CLASS - XI MATHEMATICS (Statistics)

[ANSWERS]

Ans1. Range = max value – min value

=20 - 0

=20

Ans2. Coefficient of variance (C.V) = $\frac{\text{Standard Devuation}}{\text{Mean}} \times 100$

Ans3. Mean = $\frac{\text{Sun of observations}}{\text{Total no. of observations}}$

$= \frac{11+14+15+17+18}{5} = \frac{75}{5}$

$= 15$

Ans4. The different methods are:-

a) Range b)Quartile deviation c) Mean deviation d) Standard deviation.

Ans5

x_i	f_i	$C.f$	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
26	Total			84

Total frequencies = 26

$\therefore$ Median $= \frac{1}{2}\left[\frac{26^{th}}{2} value + \left(\frac{26}{2}+1\right)^h\right]$

$= \frac{1}{2}(13+14) values$

$= \frac{1}{2}(7+7) = 7$

MD = $\frac{\Sigma f_i (x_i - M)}{\Sigma f}$

$= \frac{84}{26} = \frac{42}{13} = 3.23$

Ans6. First 10 multiples of 3 are:-
3, 6, 9, 12, 15, 18, 21, 24, 27, 30

x_i	$y_i = \frac{x_i - 15}{3}$	y_i^2
3	-4	16
6	-3	9
9	-2	4
12	1	1
15	0	0
18	1	1
21	2	4
24	3	9
27	4	10
30	5	25
Total	5	85

Mean $= A + \frac{\sum y_i}{n} \times h$

$= 15 + \frac{5}{10} \times 3$

$= 15 + 1.5 = 16.5$

Ans7.

x_i	$y_i = \frac{x_i - 18}{2}$	f_i	$f_i y_i$	y_i^2	$f_i y_i^2$
6	-6	2	-12	36	72
10	-4	4	-16	16	64
14	-2	7	-14	4	28
18	0	12	0	0	0
24	3	8	24	9	72
28	5	4	20	25	100
30	6	3	18	36	108
	Total	40	20		444

$y = \frac{x_i - 18}{2}$ Where A = 18 and i = 2

Mean = $A + i\bar{y} = 18 + \frac{\sum f_i y_i}{\sum f_i} \times 2, y = \frac{20}{40} = \frac{1}{2}$

$18 + \frac{20}{40} \times 2 = 19$

Variance $\sigma^2 = \frac{i^2}{\sum f_i}\left[\sum f_i y_i^2 - n\bar{y}^2\right]$

$= \frac{2^2}{40}\left[444 - 40\times\frac{1}{4}\right] = \frac{4}{40}\times[444-10]$

$= \frac{434}{10} 43.4$

Ans8.

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	9	8
102	6	612	4	16	48
104	3	312	9	81	243
Total	22	2200	227		640

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2200}{22} = 100$

Variance $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{640}{22} = 29.09$

Ans9. Let $y_i = \frac{x_i - A}{i} = \frac{x^{2-64}}{1}$ $[\because i = 1$ and A is assumed to be 64 $]$

x_i	f_i	$y_i = x_i - 64$	$f_i y_i$	y_i^2	$f_i y_i^2$
60	2	-4	-8	16	32
61	1	-3	-3	9	9
62	12	-2	-24	4	48
63	29	-1	-29	1	29
64	25	0	0	0	0
65	12	1	12	1	12
66	10	2	20	4	40
67	4	3	12	9	36
68	5	4	20	16	80
	100		0	60	286

Mean = $A + i\bar{y} = 64 + \frac{\sum f_i y_i}{\sum f_i} = 64$

Variance $\sigma^2 \frac{1}{\sum f_i}\left[\sum f_i y_i^2 - n\bar{y}^2\right]$

$= \frac{1}{100}[286] = 2.86$

$\sigma = \sqrt{2.86} = 1.69$

Ans10

Classes	Mid value	Frequencies	$y_i = \frac{x_i - 105}{30}$	$f_i y_i$	y_i^2	$f_i y_i^2$
0-30	15	2	-3	-6	9	18
30-60	45	3	-2	-6	4	12
60-90	75	5	-1	-5	1	5
90-120	105	10	0	0	0	0
120-150	135	3	1	3	1	3
150-180	165	5	2	10	4	20
180-210	195	2	3	6	9	18
	Total	30		2		76

$y_i = \frac{x_i - A}{i} = \frac{x_i - 150}{30}$

$\bar{x_i} = A + i\bar{y}$

$105 + 30 \times \frac{\sum f_i y_i}{\sum f_i}$

$= 105 + 30 \times \frac{2}{30}$

$= 105 + 2 = 107$

Variance $= \sigma^2 = \frac{i^2}{\sum f_i}\left[\sum f_i y_i^2 - n\bar{y}^2\right]$

$= \frac{30^2}{30}\left[76 - 30 \times \frac{1}{225}\right]$

$= 30[76 - 0.13]$

$= 30 \times 75.87 = 2276$

Ans11. Length

$N = 50, \sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8, \therefore \bar{x} = \frac{\sum x_i}{N} = \frac{212}{50}$

$= 4.24cm$

Standard deviation $\sigma^2 = \frac{1}{N}\left[N\sum x_i^2 - \left(\sum x_i^2\right)\right]$

$\frac{1}{50}\sqrt{50\times 902.8-(212)^2}$

$\sigma = \sqrt{\frac{196}{50}} = \frac{14}{50} = 0.28$

Coefficient of variation $= \frac{\sigma}{x}\times 100 \;= \frac{0.28}{4.24}\times 100 = 6.6$

For weight

$N = 50, \sum_{i=1}^{15} y_i = 261, \sum_{i=1}^{50} y_i^2 = 1457.6$

Mean $\bar{x} = \frac{\sum y_i}{N} = \frac{261}{50} \quad = 5.22gm$

Standard Deviation $= \frac{1}{N}\sqrt{N\sum y_i^2 - (\sum y_i)^2}$

$= \frac{1}{50}\sqrt{50\times 1457.6-(261)^2}$

$\sqrt{\frac{4759}{50}} = 1.38$

$\therefore$ coefficient of variation (C.V) $= \frac{\sigma}{x}\times 100 = \frac{1.38}{5.22}\times 100 \;= 26.4$

Coefficient of variation of weight is more than that of length

Ans12.

No. of goals x_i	No. of match f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
0	1	0	0	0
1	9	1	9	9
2	7	4	14	28
3	5	9	15	45
4	3	16	12	48
Total	25		50	130

For Team C:-

Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{50}{25} = 2$

Standard deviation $= \sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$

$$=\frac{1}{25}\sqrt{25\times130-(50)^2}$$

$$=\frac{5}{25}\sqrt{130-100}$$

$$=\sqrt{\frac{30}{5}}=1.095$$

$\therefore$ coefficient of variation $=\frac{\sigma}{x}\times100$

$$=\frac{1.095}{2}\times100=54.75$$

For team D:-

Mean $\overline{x}$ = 2

S.D = $\sigma=1.25$

Coefficient of variation $=\frac{\sigma}{x}\times100$

$$=\frac{1.25}{2}\times100=62.5$$

Coefficient of variation of goals of team C is less than that of team D. therefore, team C is more cons is that than team D.

TEST PAPER-01

CLASS - XI MATHEMATICS (Probability)

1. Three coins are tossed simultaneously list the sample space for the event. [1]

2. Two dice are thrown simultaneously. Find the prob. of getting doublet. [1]

3. 20 cards are numbered from 1 to 20. One card is then drawn at random. What is the prob. of a prime no. [1]

4. If $\frac{3}{10}$ is the prob. that an event will happen, what is the prob. that it will not happen? [1]

5. If A and B are two mutually exclusive events such that P (A) $=\frac{1}{2}$ and P (B) $=\frac{1}{3}$ find P (A or B) [1]

6. A coin is tossed three times consider the following event A : No head appears, B : Exactly one head appears and C : At least two heads appears do they form a set of mutually exclusive and exhaustive events. [4]

7. A and B are events such that P (A) = 0.42, P (B) = 0.48, and P (A and B) = 0.16 determine (i) P (not A) (ii) P (not B) (iii) P (A or B) [4]

8. Find the prob. that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all king (ii) 3 kings (iii) at least 3 kings [4]

9. Three letters are dictated to three persons and an envelope is addressed to each of them, those letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the prob. that at least one letter is in its proper envelope. [6]

10. If 4 digit no. greater than 5,000 are randomly formed the digits 0,1,3,5 and 7 what is the prob. of forming a no. divisible by 5 when [6]
    (i). The digits are repeated (ii) The repetition of digits is not allowed.

TEST PAPER-01

CLASS - XI MATHEMATICS (Probability)

[ANSWERS]

Ans1. S = [HHH, HHT, HTH, THH, HTT, TTH, THT, TTT]

Ans2. $n=(s)=36$ [S be the sample space]

let E be the event of getting doublet

$$P(E)=\frac{6}{36} \qquad \left[\because E=\left((1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\right)\right]$$

$$=\frac{1}{6}$$

Ans3. Let S be the sample space and E be the event of prime no.

$$n(s)=\{1,2,3,.....20\}$$
$$n(E)=\{2,3,5,7,11,13,17,19\}$$
$$P(E)=\frac{n(E)}{n(S)}=\frac{8}{20}=\frac{2}{5}$$

Ans4. Let E be the event

$$P(E)=\frac{3}{10}$$
$$P(E)=1-P(E)$$
$$=1-\frac{3}{10}$$
$$=\frac{7}{10}$$

Ans5. $P(A\ or\ B)=P(A)+P(B)-p(A\cap B)$

$$=\frac{1}{2}+\frac{1}{3}-\phi \qquad \left[P(A\cap B)=\phi\right]$$

$$=\frac{5}{6}$$

Ans6. S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A = {TTT}, B = {HTT, THT, TTH}, C = {HHT, HTH, THH, HHH}

$A \cup B \cup C = S$

There fore A, B and C are exhaustive events.

Also $A \cap B = \phi, A \cap C = \phi, C \cap C = \phi$, disjoint i.e. they are mutually exclusive.

Ans7. $P(notA) = 1 - p(A) = 1 - 0.42 = 0.58$

$P(notB) = 1 - p(B) = 1 - 0.48 = 0.52$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$
$= 0.74$

Ans8. P (all king) $= \frac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{1}{7735}$

P (3 king) $= \frac{{}^4C_3 \times {}^{48}C_4}{{}^{52}C_7} = \frac{9}{1547}$

P (atleast 3 king) = p (3 king) + p (4 king)

$= \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$

Ans9. Let the tree letters be denoted by $A_1\ A_2$ and A_3 and three envelopes by $E_1\ E_2$ and E_3.

Total No. of ways to putting the letter into three envelopes is $3P_3$ = 6

No. of ways in which none of the letters is put into proper envelope = 2

Req. prob.

P (at least one letters is put into proper envelope) =1- P (none letters is put into proper envelopes)

$= 1 - \frac{2}{6}$

$= \frac{2}{3}$

Ans10. (i)

Thousand	H	T	U
5,7			

For a digit greatest then 5000 Thousand Place filled in 2 ways and remaining three place be filled in 5 ways

No. 40. can be formed = $2\times5\times5\times5=250$

ATQ

Thousand	H	T	U
5,7			0,5

If no. is divisible by 5

Unit place filled in 2 ways and thousand place also by 2 ways (5, 7)

No. formed $=2\times5\times5\times2=100$

Req. prob. $=\frac{100}{250}=\frac{2}{5}$

(ii) Digit not repeated

Thousand	H	T	U
5,7			

Thousand place filled in 2 ways

4 digit no. greater than 5 thousand $=2\times4\times3\times2=48$

Thousand	H	T	U
5			0
7			5,0

Favorable case $=\underbrace{1\times3\times2\times2}_{\text{7 at thousand place}} + \underbrace{1\times3\times2\times1}_{\text{5at thousand places}}$

$=12+6$

$=18$

Req. prob. $=\frac{18}{48}=\frac{3}{8}$

TEST PAPER-02

CLASS - XI MATHEMATICS (Probability)

1. If E and F are events such that $P(E)=\frac{1}{4}$, $P(F)=\frac{1}{2}$ and $P(E \text{ and } F)=\frac{1}{8}$ find P(not E and not F) [1]

2. A letter is chosen at random from the word 'ASSASSINATION'. Find the prob. that letter is a consonant. [1]

3. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that at it is a woman? [1]

4. 4 cards are drawn from a well snuffled deck of 52 cards what is the prob. of obtaining 3 diamonds and one spade. [1]

5. Describe the sample space. A coin is tossed and a die is thrown. [1]

6. From a group of 2 boys and 3 girls, two children are selected at random. Describes the sample space associated with [4]

   (i) E_1: both the selected children are boys

   (ii) E_2: at least one selected child is a boy

   (iii) E_3: one boy and one girl is selected

   (iv) E_4: both the selected children are girls

7. A book contains 100 pages. A page is chosen at random. What is the chance that the sum of the digit on the page is equal to9 [4]

8. A pack of 50 tickets numbered 1 to 50 is shuffled and the two tickets are drawn find the prob. [4]

   (i) Both the ticket drawn bear prime no.

   (ii) Neither of the tickets drawn bear prime no.

9. 20 cards are numbered from 1 to 20. One card is drawn at random what is the prob. that the no. on the card drawn is [6]

   (i) A prime no. (ii) An odd no. (iii) A multiple of 5 (iv) Not divisible by 3.

10. In a single throw of three dice, find the prob. of getting [6]

    (i) A total of 5 (ii) A total of at most 5.

TEST PAPER-02

CLASS - XI MATHEMATICS (Probability)

[ANSWERS]

Ans1. $P(E' \cap F') = P(E \cup F)'$

$= 1 - P(E \cup F)$ $\quad \left[\because P(E \cup F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}\right]$

$= 1 - \frac{5}{8}$

$= \frac{3}{8}$

Ans2. P (consonant) $= \frac{7}{13}$

Ans3. P (a woman member is selected) $= \frac{6}{10} = \frac{3}{5}$

Ans4. $\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4} = \frac{286}{20825}$ $\quad \left[\because \begin{array}{l} \text{3 Spades out of 13} \\ \text{and one ace out of 13} \end{array}\right]$

Ans5. {H1,H2,H3,H4,H5,H6,T1,T2,T3,T4,T5,T6}

Ans6. $S = \{B_1B_2,\ B_1G_1,\ B_1G_2,\ B_1G_3,\ B_2G_1,\ B_2G_2,\ B_2G_3,\ G_1G_2,\ G_1G_3,\ G_2G_3\}$

$E_1 = \{B_1B_2\}$

$E_2 = \{B_1B_2,\ B_1G_1,\ B_1G_2,\ B_1G_3,\ B_2G_1,\ B_2G_2,\ B_2G_3\}$

$E_3 = \{B_1G_1,\ B_1G_2,\ B_1G_3,\ B_2G_1,\ B_2G_2,\ B_2G_3\}$

$E_4 = \{G_1G_2,\ G_1G_3,\ G_2G_3\}$

Ans7. $E = \{9,\ 18,\ 27,\ 36,\ 45,\ 54,\ 63,\ 72,\ 81,\ 90\}$

$S = 100$

$P(E) = \frac{10}{100}$

$= \frac{1}{10}$

Ans8. Prime no. from 1 to 50 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

(i) Two ticket out of fifty can be drawn in $C(50,2)$

$$\text{P}\left(\text{both ticket bearing prime no.}\right)=\frac{{}^{15}C_2}{{}^{50}C_2}=\frac{3}{35}$$

(ii) P (neither of the tickets bear prime no.) $=\frac{{}^{35}C_2}{{}^{50}C_2}=\frac{17}{35}$

Ans9. Let S be the sample space

S={1,2,3,4,5,…………20}

Let E_1, E_2 and E_3, E_4 are the event of getting prime no., an odd no, multiple of 5 and not divisible by 3 respectively

$P(E_1)=\frac{8}{20}=\frac{2}{5},\qquad E_1=\{2,3,5,7,11,13,17,19\}$

$P(E_2)=\frac{10}{20}=\frac{1}{2},\qquad E_2=\{1,3,5,7,9,11,13,15,17,19\}$

$P(E_3)=\frac{4}{20}=\frac{1}{5},\qquad E_3=\{5,10,15,20\}$

$P(E_4)=\frac{14}{20}=\frac{7}{10},\qquad E_4=\{1,2,4,5,7,8,10,11,13,14,16,17,19,20\}$

Ans10. Let S be the sample space E_1 be the event of total of 5.

(i) $E_1=\left\{(1,1,3),(1,3,1),(3,1,1),(1,2,2),(2,1,2),(2,2,1)\right\}$

$S=6\times6\times6=216$

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{6}{216}=\frac{1}{26}$$

(ii) $E_2=\left\{(1,1,1),(1,1,2),(1,2,1),(2,1,1),(1,1,3),(1,3,1),(3,1,1),(1,2,2),(2,1,2),(2,2,1)\right\}$

$$P(E_2)=\frac{10}{216}=\frac{5}{108}$$

TEST PAPER-03

CLASS - XI MATHEMATICS (Probability)

1. We wish to choose one child of 2 boys and 3 girls. A coin is tossed. If it comes up heads, a boy is chosen, otherwise a girl is chosen. Describe the sample space. [1]

2. What is the chance that a leap year, selected at random, will contain 53 Sundays? [1]

3. If P (A) = 0.6, P(B) =0.4 and P ($A \cap B$) = 0, then the events are [1]

4. In general the prob. of an event lie between? [1]

5. A and B are two mutually exclusive events of an experiment. If P (not A) = 0.65, P ($A \cup B$) = 0.65 and P (B) = K, find K [1]

6. In a class XI of a school 40% of the students study mathematics and 30% study biology. 10% of the class study both mathematics and Biology. If a student is selected at random from the class, find the prob. that he will be studying mathematics or biology. [4]

7. A hockey match is played from 3 pm to 5 pm. A man arrives late for the match what is the prob. that he misses the only goal of the match which is scored at the 20^{th} mint of the match? [4]

8. In a single throw of two dice, find the prob. that neither a doublet nor a total of 10 will appear. [4]

9. The prob. that a person will get an electrification contract is $\frac{2}{5}$ and the prob. that he will not get a plumbing contract is $\frac{4}{7}$. If the prob. of getting at least one contract is $\frac{2}{3}$, what is the prob. that he will get both? [4]

10. In a town of 6000 people 1200, are over 50 yr. old and 2000 are females. It is known that 30% of the females are over 50 yr. what is the prob. that a randomly chosen individual from the town is either female or over 50 yr. [4]

TEST PAPER-03

CLASS - XI MATHEMATICS (Probability)

[ANSWERS]

Ans1. $\{HB_1,\ HB_2,\ TG_1,\ TG_2,\ TG_3\}$

Ans2. A leap year consists of 366 days and therefore 52 complete weeks and two days over. These two days may be (Sunday, Monday), (Monday, Tuesday),(Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), or (Saturday, Sunday)

P (a leap year has 53 Sunday) = $\frac{2}{7}$

Ans3. Exclusive and exhaustive

Ans4. 0 and 1.

Ans5. $P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = 1 - P(notA) + P(B)$

$0.65 = 1 - 0.65 + K$

$K = 0.30$

Ans6. $P(M) = \frac{40}{100}, P(B) = \frac{30}{100}$

$P(M \cap B) = \frac{10}{100}$

$P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$= \frac{40}{100} + \frac{30}{100} - \frac{10}{100} = 0.6$

Ans7. The man can arrive any time between 3 to 5 pm so that time = 2hr =120 mints

He see goal of he arrives within first 20 mint

P (he see the goal) $=\frac{20}{120}=\frac{1}{6}$

P (not see the goal) $=1-\frac{1}{6}=\frac{5}{6}$

Ans8. Let S be the sample space and E_1, E_2 are event of doublet, and event of getting a total of 10 respectively

$E_1=\{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

$E_2=\{(4,6)(5,5)(6,4)\}$

$n(S)=36$

$P(E_1)=\frac{6}{36}=\frac{1}{6}$

$P(E_2)=\frac{3}{36}=\frac{1}{12}$

$P(E_1 \cap E_2)=1$

$P(E_1 \cup E_2)=\frac{2}{9}$

$P(E_1' \cap E_2')=P(E_1 \cup E_2)'$

$=1-(E_1 \cup E_2)$

$=1-\frac{2}{9}=\frac{7}{9}$

Ans9. Let A = Event of getting an electrification contract

B = Event of getting a plumbing contract

$P(A)=\frac{2}{5}, \quad P(notB)=\frac{4}{7}$

$P(B)=1-\frac{4}{7}=\frac{3}{7}$

$P(A \cup B)=\frac{2}{3}$

Req. prop $=P(A \cup B)$

$=P(A)+P(B)-P(A \cap B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{5} + \frac{3}{7} - \frac{2}{3} = \frac{42 + 45 - 70}{105}$

$= \frac{17}{105}$

Ans10. A_1 : Event of person being a female

A_2 : Event of person being 50 yr. old

$n(A_1) = 2000, \quad n(A_2) = 1200$

$n(A_1 \cap A_2) = 30\% \text{ of } 2000 = \frac{30}{1\cancel{00}} \times 20\cancel{00}$

$= 600$

$n(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2)$

$2000 + 1200 - 600$

$= 2600$

$P(A_1 \cup A_2) = \frac{2600}{6000} = \frac{13}{30}$

TEST PAPER-04

CLASS - XI MATHEMATICS (Probability)

1. A box contains 1 white and 3 identical black balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment. [1]
2. Three coins are tossed once. Find the probability at most two heads. [1]
3. One card is drawn from a pack of 52 cards, find the probability that drawn card is either red or king. [1]
4. Five cards are drawn from a well shuffled pack of 52 cards. Find the probability that all the five cards are hearts. [1]
5. From a deck of 52 cards four cards are accidently dropped. Find the chance that the missing cards should be one from each other. [1]
6. In a class of 60 students 30 opted for NCC, 32 opted for NSS, 24 opted for both NCC and NSS. If one of these students is selected at random find the probability that [4]
   (i). The student opted for NCC or NSS
   (ii). The student has opted neither NCC nor NSS.
   (iii). The student has opted NSS but not NCC.
7. Two students Anil and Ashima appeared in an examination. The probability That Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. the probability that both will qualify the examination is 0.02 find the probability that [4]
   (a). Both Anil and Ashima will qualify the examination
   (b). At least one of them will not qualify the examination and
   (c). Only one of them will qualify the examination.
8. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students what is the probability that [4]
   (a) You both enter the same section?
   (b) You both enter the different section?
9. There are three mutually exclusive and exhaustive events E_1, E_2, and E_3. The odds are 8 : 3 against E_1 and 2 : 5 in favors of E_2 fined the odd against E_3. [4]
10. If an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both? [4]

[ANSWERS]

Ans1. S = {WB, BW, BB}

Ans2. S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

E = HHT, THH, HTH, HTT, THT, TTH, TTT

P (E) $= \frac{7}{8}$

Ans3. $p = \frac{26+2}{52}$

$= \frac{28}{52}$

$= \frac{7}{13}$

Ans4. $\frac{{}^{13}C_5}{{}^{52}C_5} = \frac{33}{66640}$

Ans5. $\frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4} = \frac{2197}{20825}$

Ans6. A student opted for NCC

B student opted for NSS

$P(A) = \frac{30}{60} = \frac{1}{2}$, $P(B) = \frac{32}{60} = \frac{8}{15}$

$P(A \cap B) = \frac{24}{60} = \frac{2}{5}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{8}{15} - \frac{2}{5}$

$= \frac{19}{30}$

(ii) $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - \frac{19}{30}$

$= \frac{11}{30}$

(iii) $P(B - A) = P(B) - P(A \cap B)$

$= \frac{8}{15} - \frac{2}{5} = \frac{2}{15}$

Ans7. Let E and F denote the event that Anil and Ashima will qualify the examination respectively P (E) = 0.05, P (F) = 0.10, P (E$\cap$ F) = 0.02

(a) $P(E' \cap F') = P(E \cup F)'$

$= 1 - P(E \cup F)$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - 0.13 = 0.87$

(b) (at least one of them will not quality) = 1-P(both of them will quality)

=1-0.02

= 0.98

(c) P (only one of them will quality) = $P(E \cap F') + P(E' \cap F)$

$= P(E) - P(E \cap F) + P(F) - P(E \cap F)$

$= 0.05 - 0.02 + 0.10 - 0.02$

$= 0.11$

Ans8. Two sections of 40 and 60 can be formed out of 100 in ${}^{100}C_{60}$ or ${}^{100}C_{40}$ ways

(a) P (both enter the same section) = $\frac{{}^{40}C_2}{{}^{100}C_2} + \frac{{}^{60}C_2}{{}^{100}C_2}$

$= \frac{17}{33}$

(b) req. probability = $\frac{{}^{40}C_1 \times {}^{60}C_1}{{}^{100}C_2}$

$= \frac{16}{33}$

Ans9. Odds against E_1 are 8:3

So odds in favors of E_1 are 3:8

$\therefore P(E_1) = \frac{3}{3+8} = \frac{3}{11}, \quad P(E_2) = \frac{2}{2+5} = \frac{2}{7}$

$P(E_1) + P(E_2) + P(E_3) = 1 \quad \left[\begin{array}{l} E_1, E_2, \text{ and } E_3 \text{ are mutually} \\ \text{exclusive and exnaustive} \end{array}\right]$

$= 1 - \frac{3}{11} - \frac{2}{7}$

$= \frac{34}{77}$

Odds against E_3 are

$= \frac{1 - P(E_3)}{P(E_3)}$

$= \frac{1 - \frac{34}{77}}{77} = \frac{43}{34}$

Ans10. A : student passes first examination

B : student passes Second examination

P (A) = 0.8, P (B) = 0.7

P (A$\cup$B) = 0.95

P (A$\cap$B) =?

P (A$\cup$B) = P (A) + P (B) - P (A$\cap$B)

0.95 = 0.8 + 0.7 - P (A$\cap$B)

0.55 = P (A$\cap$B).

TEST PAPER-05
CLASS - XI MATHEMATICS (Probability)

1. In a random sampling three items are selected from a lot. Each item is tested and classified as defective (D) or non – defective (H). Write the sample space. [1]

2. Let $S = \{W_1,\ W_2,\ W_3,\ W_4,\ W_5,\ W_6\}$ be sample space. Probability to outcome valid. [1]

W_1	W_2	W_3	W_4	W_5	W_6
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

3. The odds in favour of an event are 3:5, find the probability of occurrence of this event. [1]

4. What is the probability that an ordinary year has 53 Sundays? [1]

5. If odds against an event be 7:9, find the probability of non-occurrence of this event. [1]

6. One card is drawn from a well shuffled deck of 52 cards. If each out come is equally likely calculate the probability that the card will be. [4]
(i) a diamond (ii) Not an ace (iii) A black card (iv) Not a diamond.

7. In a lottery, a person chooses six different natural no. at random from 1 to 20 and if these six no. match with six no. already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [4]

8. From the employees of a company, 5 persons are elected to represent them in the managing committee of the company. [4]

S. No.	Person	Age.
1	Male	30
2	Male	33
3	Female	46
4	Female	28
5	Male	41

A person is selected at random from this group as a spoke person what is the probability the a spoke person will be either male or over 35 yr.

9. A die has two faces each with no. 1 three faces each with no. 2 and one face with no. 3 if the die is rolled once, determine [4]
(i) P (2) (ii) P (1 or 3) (iii) P(not 3)

10. Find the probability that in a random arrangement of the letters of the word UNIVERSITY the two I's come together. [4]

11. A bag contains 50 tickets no. 1,2,3,……,50 of which five are drawn at random and arranges in ascending order of magnitude $\left(x_1 < x_2 < x_3 < x_4 < x_5\right)$ find the probability that x_3 **= 30**

TEST PAPER-05

CLASS - XI MATHEMATICS (Probability)

[ANSWERS]

Ans1. S ={DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

Ans2. Yes , $\left[\because \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=1\right]$

Ans3. $P=\frac{3}{8}$

Ans4. $\frac{1}{7}$

Ans5. $1-\frac{9}{16}=\frac{16-9}{16}=\frac{7}{16}$

Ans6. (i) req. probability $=\frac{13}{52}=\frac{1}{4}$

(ii) req. probability $=1-\frac{4}{52}=1-\frac{1}{13}=\frac{12}{13}$

(iii) req. probability $=\frac{26}{52}=\frac{1}{2}$

(iv) req. probability $=1-\frac{1}{4}=\frac{3}{4}$

Ans7. Out of 20, a person can choose 6 natural no. In ${}^{20}C_6$ ways out of these there is only one choice which will match the six no. already by the committee

P (The person wins the prize) = $\frac{1}{{}^{20}C_6}$

$= \frac{1}{38760}$

Ans8. A: spoke person is a male

B: spoke person is over 35 yr.

$P(A) = \frac{3}{5}$

$P(B) = \frac{2}{5}$

$P(A \cap B) = \frac{1}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$

$= \frac{4}{5}$

Ans9. A: getting a face with no. 1

B: getting a face with no. 2

C: getting a face with no. 3

$P(A) = \frac{2}{6} = \frac{1}{3}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(C) = \frac{1}{6}$

(i) $P(2) = \frac{1}{3}$

(ii) $P(1 \text{ or } 3) = P(1) + P(3)$

$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

(iii) $P(not 3) = 1 - \frac{1}{6} = \frac{5}{6}$

Ans10. Total no. of words which can be formed by the letters of the word UNIVERSITY is $\frac{10!}{2!}$ regarding 2I'S as one letter no. of ways of arrangement in which both I'S are together = 9!

Req. probability $= \frac{9!}{\frac{10!}{2!}} = \frac{1}{5}$

Ans11. Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways

Since $x_1 < x_2 < x_3 < x_4 < x_5$

And $x_3 = 30$

$x_1 < x_2 < 30$

i.e x_1 and x_2 should come from tickets no to 1 to 29 and this may happen in ${}^{29}C_2$ ways.

Remaining two i.e. $x_4, x_5 > 30$ should Come from 20 tickets no. from 31 to 50 in ${}^{20}C_2$ ways

Favorable case = ${}^{29}C_2 \times {}^{20}C_2$

Req. probability $= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$

$= \frac{551}{15134}$

MODEL TEST PAPER No. 1
HALF YEARLY EXAM
CLASS – XI (MATHS)

Time : 3 hrs. **M.M. 100**

General Instructions :

1. *All questions are compulsory.*
2. *The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section Comprises of 07 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, interval choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculations is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q.1 Find the power set of $\{a,b,c\}$

Q.2 For any non empty set A & B show that (A-B) UB = AUB

Q.3 If $R = \{(x.y)\ x=2y+1,\ 3 \le y < 6,\ y \in N\}$ find domain & range.

Q.4 Express $\dfrac{3-7i}{2+5i}$ in the form of a+ib and find its congugate.

Q.5 Find the value of Sin 15°

Q.6 Find the middle term of $\left(\dfrac{x}{a}+\dfrac{a}{x}\right)^{10}$

Q.7 Find the solution set of the inequality 2x+5<17 where $x \in \{1,2,5,6,7,8,9,10\}$ and draw its graph on the number line.

Q.8 If $n_{c_{10}} = n_{c_3}$ find n & also n_{c_2}

Q.9 How many terms of the sequence 2,8, 16........... must be taken to make the sum 2186?

Q.10 Reduce the equation of line 2x-3y+4=0 to slope intercept form. Find its Y intercept & slope.

SECTION – B

Q.11 Out of 120 students who secured Ist class marks in Maths or in English, 75 students obtained Ist class in Maths and 15 in English. How many got Ist class marks in English only?

Q.12 Find domain and range of

$f(x) = \sqrt{1-x^2}$ from $R \to R$

or

If the function f : $R \to R$ is defined by $f(x) = \begin{cases} 3x-1 & x < -1 \\ x^2-3 & -1 \le x \le 3 \\ 5 & x > 3 \end{cases}$

find f(o), f(-1), f(3) & f(5)

Q.13 Solve the equation

Cos3x + Sin2x = 0

Q.14 Prove that $\cos\alpha + \cos\beta + \cos\lambda + \cos(\alpha+\beta+\lambda)$

$$= 4\, Cos\left(\frac{\alpha+\beta}{2}\right) Cos\left(\frac{\beta+\lambda}{2}\right) Cos\left(\frac{\lambda+\alpha}{2}\right)$$

Q.15 Prove that $\tan\left(\frac{\overline{\Lambda}}{4}+\vartheta\right)+\tan\left(\frac{\overline{\Lambda}}{4}-\vartheta\right)=2Sec\ 2\vartheta$

Q.16 Using principle of mathematical induction prove that

$1^2 + 3^2 + 5^2 + ...+ (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \times n \in N$

Q.17 Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 $\cancel{\forall} n \in N$

Q.18 Prove that there is no term involving x^5 in the expansion of $\left(x^3+\frac{5}{x^2}\right)^7$

Q.19 The sum of first p, q, r terms of an AP are a,b,c respectively show that

$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Q.20 How many different words can be formed with the letter of the word \`BHARAT'? How many of them.

i) will not have B, H together?

ii) will begin with B and end with T?

Q.21 In a school there are 15 teachers (including physical education teacher) and 5 captains of different games. The Principal wants to form a game committee consisting of physical education teacher, 2 other teachers and 2 captains. In how many ways can he do it?

Q.22 Find the equation of line through the inter-section of lines 3x+4y-7=0 and x-y+z=0 and which is parallel to the line 5x-y+11=0

or

Find the equation of the line which passes through the intersection of the lines 3x-2y-2=0 and 2x-y-1=0 and which is at a distance of $\sqrt{2}$ units form the point (1,0).

SECTION – C

Q.23 There are 200 individuals with a skin disorder 120 had been exposed to chemical A, 50 to chemical B and 30 to both chemicals A & B find the number of individuals exposed to

i) chemical A but not chemical B

ii) chemical B but not chemical A

iii) chemical A or chemical B

Q.24 Prove that the

Sin10 Sin50 Sin60 Sin70 = $\frac{\sqrt{3}}{16}$ or

$$Cos^2\frac{\overline{\Lambda}}{8}+Cos^2\frac{3\overline{\Lambda}}{8}+Cos^2\frac{5\overline{\Lambda}}{8}+Cos^2\frac{7\overline{\Lambda}}{8}=2$$

Q.25 Let Z_1 and Z_2 be the two complex numbers such that

$|Z_1+Z_2| = |Z_1-Z_2|$ prove that arg (Z_1) – arg(Z_2) = $\overline{\Lambda}/2$

Q.26 Solve the system of equation graphically

$$2x+y\geq 2, \quad x-y\leq 3, \quad x\geq 0, \quad y\geq 0$$

Q.27 Find n, if the ratio of the fifth term from beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Q.28 Find the sum to n terms of the series 0.5 + 0.55 + 0.555+.........to n terms

Q.29 If p is the length of the perpendicular segment from the origin on the line whose intercepts on the axes are a & b, show that.

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ or

Find the equation of the lines which passes through the point (-3,2) make an angle of 45° with the line 3x-4y+2=0

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MODEL TEST PAPER No. 2
HALF YEARLY EXAM
CLASS – XI (MATHS)

Time : 3 hrs. **M.M. 100**

General Instructions :

1. *All questions are compulsory.*
2. *The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section Comprises of 07 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, interval choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculations is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q.1 Find the subsets of {2,4}

Q.2 Find x and y if (x+1, 4) = (5, 2x-y)

Q.3 The Cartesian product A x A has 9 elements among which are found (-1,0) and (0,1) Find the set A and the remaining elements of A x A.

Q.4 Find domain of f(x) = $\sqrt{9-x^2}$

Q.5 Find radian measure of 520°

Q.6 2(x-3) < x + 8 solve for x, represent on number line.

Q.7 Prove that $4_{C_3} = 4_{C_1}$

Q.8 A(-1,2) and B(2,-3) Find slope of line AB.

Q.9 Find multiplicative inverse of $\sqrt{5}-3i$ in standard form.

Q.10 2cosx cosy = ...

Q.11 In a school there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teach Physics?

Q.12 Let A = {1,2,3.......................14} Define a relation R from A to A by

R = {(x,y) : 3x-y = 0 where x, y $\in$ A}

Write down its domain, co-domain and range.

Q.13 Show that

tan 3x tan 2x tan x = tan 3x – tan 2x – tan x

or

Prove that $$\frac{\tan\left(\frac{\Pi}{4}+x\right)}{\tan\left(\frac{\Pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Q.14 Draw the graph of modulus function.

Q.15 Prove that

$$2Cos\frac{\Pi}{13}\cdot Cos\frac{9\Pi}{13}+Cos\frac{3\Pi}{13}+Cos\frac{5\Pi}{13}=0$$

Q.16 Prove that $2.7^n + 3.5^n - 5$ is divisible by 24.

Q.17 Find modulus and argument of

Z= $-1 - i\sqrt{3}$

Q.18 The longest side of a triangle is 3 times the shortest side and the third side is 2cm shorter than the longest side. If the perimeter of the triangle is at least 61cm, find the minimum length of the shortest side.

Q.19 How many 3 digit even numbers can be made using the digits 1,2,3,4,6,7 if no digit is repeated?

Q.20 Expand $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Q.21 How many terms of the A.P.

$-6, \frac{-11}{2}, -5, \ldots\ldots\ldots\ldots\ldots$ are needed to give the sum -25?

Q.22 The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is –1. Find the common ratio and the terms.

Q.23 If P is the length of perpendicular from the origin to the line whose intercept on the axes are a and b. Then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Q.24 Solve the following system of inequalities graphically.

$x-2y\leq 3,\ 3x+4y\geq 12,\ x\geq 0,\ y\geq 1$

Q.25 A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

i) exactly 3 girls

ii) atleast 3 girls

iii) at most 3 girls

Q.26 The co-efficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n.

Q.27 In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B. 12 people liked products C and A. 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Q.28 Find the sum of n terms of the sequence 8, 88, 888, 8888,.....................

Q.29 i) Solve $2\cos^2 x + 3\sin x = 0$

ii) Find Principal and general solution of $\tan x = \sqrt{3}$

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CBSE MIXED TEST PAPER-01

(Unit Test)

CLASS - XI MATHEMATICS

[Time : 1.00 hrs.] **[M. M.:** 20]

General Instructions:-

All questions are compulsory.

Q1. (a) Write the set in roster form: 5 marks

A = {x : x is a prime number which is divisor of 603

(b) Write the subsets of {a, b}

(c) Find the union of A = {a, e, I, ou, u} B = {a, b, c}

(d) If A x B = {(a, x), (a, y), (b, x), (b, y)} find A and B.

(e) Find the domain and range.

(2,1) (4,2) (6,3) (8, 4) (10,5), (11,6), (13,9)}

Q2. If f(x) = x^2 find $\frac{f(1.2)-f(1)}{(1.2-1)}$ 2 marks

Q3. Write the relation R = {(x,x^2) : is prime number less then 103 in roster forms. 2 marks

Q4. If x and y are two sets such that n(x) = 17, n(y) = 23 and $n(x \cup y) = 35 \, findn\left(x \cap y\right)$ 3 marks

Q5. If u = {1,2,3,4……. 103, A = {2,4,6,8,10) and B = {2,3,5,7,9} verify 3 marks

$(A \cup B)` = A` \cap B` (ii)(A \cap B)` = A` \cup B`$

Q6. In a survey it was found that 21 people liked product A, 26 liked products B and 29 liked product C. If 15 people liked product A and B , 11 people liked products C and A, 14 people liked product B and C and 8 liked all the three products'. Find how many liked product C only A only.

CBSE MIXED TEST PAPER-04

CLASS - XI MATHEMATICS

[Time : 3.00 hrs.] **[M. M.:** 90]

General Instructions:-

1. All questions are compulsory.
2. The question paper consists of 27 questions divided into 3 sections, A, B and C.
3. Section A comprises of 10 questions of 1 mark each, Section B comprises of 11 questions of 4 marks each and Section C comprises of 6 questions of 6 marks each.

SECTION A

Q1. Find the equation of straight line which makes an angle 60^0 with the positive x-axis and cuts of an intercept – 3 on the y axis.

Q2. Evaluate: sin 130^0 cos 110^0 + cos 130^0 sin 110^0.

Q3. If A and B are 2 sets each than n (A) = 31, n(B) = 26 and n $\left(A \cup B\right)$ = 51. Find n $\left(A \cap B\right)$.

Q4. Let f = {(1,1),(2,3),(0,-1),(-1,-3) be a linear function from Z into Z. Find f(x).

Q5. Find the real numbers x and y if (x + iy) (3-2i) is the conjugate of 12-5i.

Q6. Show that the graph of the solutions of the inequality $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ on number line.

Q7. If ${}^nC_2 = {}^nC_8$, find the value of ${}^nC_2$.

Q8. If $2Cos\theta = a + \frac{1}{a}$, find the value of 2 cos 2θ.

Q9. Find the ratio in which the line segment joining the points (4,8,10) and B(6,10,8) is divided by the YZ-plane.

Q10. Evaluate $x \xrightarrow{\lim} 0 \frac{(1+x)^n - 1}{x}$

SECTION B

Q11. A and B are two sets such that n (A-B) = 14 + x, n (B-A) = 3x and n $\left(A \cap B\right)$ = x. Draw a venn diagram to illustrate this information and if n (A) = n(B) then find the value of x.

Q12. Find the domain and range of f(x) = $\frac{5}{3-x^2}$.

Q13. Using the principle of mathematical induction prove that $(x^{2n} - 1)$ is divisible by (x-1) where $x \neq 1$.

Q14. If $(x+iy)^{\frac{1}{3}} = a+ib$, prove that $\frac{x}{a}+\frac{y}{b} = 4(a^2-b^2)$.

Q15. Represent $\frac{1-3i}{1+2i}$ in the polar form.

Q16. Solve the following system of linear inequations graphically.

$x + 2y \leq 10$

$x + y \leq 6$

$x \leq 4$

$x \geq 0$ and $y \geq 0$.

Q17. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?

Q18. Find the angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

Q19. Find the equation of the curve formed by the set of all points, the sum of whose distances from the points A(4,0,0) and B(-4, 0, 0) is 10 units.

Q20. Find the derivative of cot x from the first principle.

Q21. Evaluate $x \xrightarrow{\lim} 0 \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

SECTION C

Q22. Prove that:

$$\frac{\cos 8A \cos 5A - \cos 12A \cos 9A}{Sin 8A \cos 5A + \cos 12A \sin 9A} = \tan 4A$$

Q23. Solve: $\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta+\frac{2\pi}{3}\right) = 3$.

Q24. How many four letter words can be formed using the letter of the word 'INEFFECTIVE'?

Q25. Find the coefficient of x^{32} and x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Q26. Reduce the equation $\sqrt{3}x + y + 2 = 0$ to the normal form.

Q27. Differentiate the following functions w.r.tx

(a) (x Sin x + Cos x) (x Cos x – Sin x)

(b) $\frac{\sec x - 1}{\sec x + 1}$

CBSE MIXED TEST PAPER-05

CLASS - XI MATHEMATICS

[Time : 3.00 hrs.] **[M. M.:** 100]

General Instructions:-

1. All questions are compulsory.
2. The question paper consists of **29** questions divided into **3** sections, A, B and C.
   Section A contains **10** questions of **1** mark each.
   Section B contains **12** questions of **4** marks each.
   Section C contains **7** questions of **6** marks each.
3. There is no overall choice. However an internal choice in **four** questions of **four** marks each and **two** questions of **six** marks each has been provided.
4. Use of calculator is not permitted.

SECTION A

Q1. If A = {-3, -1, 0,1,2}, then write the number of subset of A.

Q2. If (2a + b, 3b – 4) = (11, -7), find the values of a & b.

Q3. If $x = \frac{1}{2}$, find the value of tan 2x

Q4. Write the principle solution of the equation $\sin x = \frac{\sqrt{3}}{2}$.

Q5. Find the value of $3\sqrt{-16}\sqrt{-25}$.

Q6. Solve the following inequality

$2x + 3 \geq x + 1$

Q7. State fundamental principle of counting.

Q8. Write the next term of the following sequence:

0, 2, 12, 20.

Q9. Find slope of the line passing through the point (3,-2) and (-1,4).

Q10. Write the equation of a circle whose centre is (-1,2) and radius 3 units.

SECTION – B

Q11. Let A and B be two sets such that n(A) = 20, n $\left(A \cup B\right) = 42, n(A \cap B) = 4$ Find n(B), n(A-B) and n(B-A).

Q12. If the real functions f and g are defined by f(x) = x+ 1 and g (x) = 2x – 3, x $\in$ R, find the following:-

(i) f + g

(ii) f – g

(iii) fg

(iv) $\dfrac{f}{g}$

Or

Determine the domain and range of the relation R defined by R = {(x, x + 5): x $\in$ {0,1,2,3,4,}} Depict this relation by arrow diagram.

Q13. Prove that: $\dfrac{Sin(B-C)}{\cos B \cos C} + \dfrac{Sin(C-A)}{\cos C.\cos A} + \dfrac{Sin(A-B)}{\cos A.\cos B} = 0$

Q14. Prove that:-

$$\frac{\cos 9x - \cos 5x}{Sin17x - Sin3x} = -\frac{Sin2x}{\cos 10x}$$

Or

$$\frac{Sin5x - 2Sin3x + Sinx}{\cos 5x - \cos x} = \tan x$$

Q15. Prove the following by the principle of mathematical induction for all $n \in N$

$$1^3+2^3+3^3+--+n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Q16. Express the following expression in the form of a + ib

$$\frac{(3+i\sqrt{5})(\sqrt{3}-i\sqrt{5})}{(\sqrt{3}+\sqrt{2i})-(\sqrt{3}-i\sqrt{2})}$$

Q17. Solve the following system of inequalities graphically:

$3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$.

Q18. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Q19. Find the sum of the sequence

7,77,777,7777…….. to n terms.

Q20. Find the angle between the lines.

$y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$

Q21. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a & b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Q22. Find the co-ordinates of the foci, the vertices, the lengths of major and minor and the eccentricity of the ellipse $9x^2 + 4y^2 = 36$

Or

Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ the ends of its latus rectum.

SECTION-C

Q23. If survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B & C and 8 liked all the three products. Find how many liked product C only.

Q24. If cos A = m cos B, then prove that $\cot\left(\frac{A+B}{2}\right) = \frac{m+1}{m-1}\tan\left(\frac{B-A}{2}\right)$

Q25. Convert the complex number $\frac{-16}{1+i\sqrt{3}}$ in polar form.

Q26. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in dictionary, what will be the 49th word.

Or

A group consist of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) No girls (ii) atleast one boy and one girl.

Q27. (i) Find the middle term in the expansion of $\left(x-\frac{1}{2y}\right)^{10}$

(ii) Find the term which is independent of x in the expansion of: $\left(x^2+\frac{1}{x}\right)^9$

Q28. (i) Find equation of the line through the point (0,2) making an angle $\frac{2\pi}{3}$ with the positive x-axis.

Also find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

(ii) Reduce the equation $\sqrt{3}x+y-8=0$ into normal form. Find the value of p & w, where p is length of perpendicular from origin to the given line w is inclination of perpendicular.

Q29. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an Arithmetic Progression. Find the numbers.

Or

Find the sum of the following series up to n terms: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+......$

CBSE MIXED TEST PAPER-06

CLASS - XI MATHEMATICS

[Time : 1.5 hrs.] **[M. M.:** 40]

General Instructions:-

1. Total numbers of questions are 12.

   Q. No. 1-4 are of 1 mark each.

   Q. No. 5-10 are of 4 marks each.

   Q. No. 11. 12 are of 6 marks each.
2. All questions are compulsory. Internal choices are provided in TWO questions.
3. Write correct question number.

Q1. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60^0, what is A'?

Q2. What is the value of [-2.38] and [1.5] where [x] is the greatest integer les than or equal to x?

Q3. Find the principal solution of cos x = $\frac{1}{2}$.

Q4. If $\sin \theta = \frac{-1}{\sqrt{2}}$ and $\tan \theta = 1$, find the quadrant in which Q lies.

Q5. If U = {1,2,3,4,5,6,7,8,9}

A = {2,4,6,8}

B = {2,3,5,7}

Verify that

$(i)(A \cup B)' = A' \cap B'$

$(ii)(A \cap B)' = A' \cup B'$

Q6. Let A = {1,2,3,4} Let R be the relation on A defined by R = {(a,b): a∈ A, b∈ A a divides b}

Find

(i) Relation R in roaster form

(ii) Domain of R

(iii)Range of R.

Q7. (i) Which of the following relations are functions? If it is a function determine its domain and range

(a) {2,1), (3,1), (4,2)}

(b) {(1,3),(1,5),(2,5)}

(ii) Given f(x) $\begin{cases} 3x-8 \ for\ x & \le 5 \\ 7 \qquad for\ x & > 5 \end{cases}$

What is the value of the function at x = 3 and x = 7?

Q8. If f, g: R → R be defined respectively by

F(x) = x + 1, g(x) = 2x – 3

find f+ g, f-g, f.g., $\frac{f}{g}$.

Q9. Prove that

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x.$$

Q10. If tan x = -$\frac{4}{3}$, x is the quadrant II, find $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ **Or**

Find the value of $\tan\frac{\pi}{8}$.

Q11. Find the general solution of the following equation

sin x + sin 3x + sin 5x = 0. **Or**

Prove that $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

Q12. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all three subjects. Find the number of students that had taken

(i) Only Chemistry

(ii) Physics and Chemistry but not Mathematics

(iii) At least one of the three subjects

(iv) None of the three subjects.

CBSE MIXED TEST PAPER-07

CLASS - XI MATHEMATICS

[Time : 1.5 hrs.] **[M. M.:** 40]

Note: (1) All questions are compulsory.

(2) This question paper consists of 12 questions divided into three sections.

SECTION A (Q.1-4) – 1 marks each.

SECTION B (Q.5-10) – 4 marks each.

SECTION C (Q.11-12) – 6 marks each.

SECTION – A

Q1. Write the set $\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{6}{7}\right\}$ in the set builder form.

Q2. Find the domain and range of R if

R = {(x, y): x + 2y d < 6, x, y∊ N}

Q3. Write all the subsets of {- 1, 0, 1}

Q4. If f(x) = x^2 find $\frac{f(1.1)-f(1)}{1.1-1}$.

SECTION – B

Q5. Find the domain and range of the real function f defined by f(x) = $\sqrt{9-x}$.

OR

If A = {-1, 0, 2, 3} and B = {1, 4, 5, 8, 9, 10} and f = {(x, y): y = x^2 + 1}

List the elements of f.

Q6. Let f(x) = x2 and g(x) = 2x + 1 be two real functions then find

(f + g) (x), (f – g) (x), (f. g) (x), $\frac{f}{g}$ (x)

Q7. Prove that:-

$$\cos\left(\frac{\pi}{4}-A\right).\cos\left(\frac{\pi}{4}-B\right)-\sin\left(\frac{\pi}{4}-A\right)$$

$$\sin=\left(\frac{\pi}{4}-B\right)\sin(A+B)$$

Q8. In a group of 50 people, 35 speak Hindi and 25 speak English and Hindi both and all speak atleast one of the two languages find

(i) How many speak English only?

(ii) How many speak English?

Solve the trigonometric equations Sin 2x + cos x = 0

Q10. Prove that

$$(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4.\cos^2\left(\frac{x-y}{2}\right)$$

OR

$$\cos^2 x\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$$

Q11. If $\sin x = \frac{1}{4}$, x lies in 2nd quadrant find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

Q12. Prove by using principle of mathematical Induction for all ϵ N

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}.$$

CBSE TEST PAPER-08
Class – XI Mathematics

Time :-1.5 Hrs. **M.M. 40**

General Instructions :
1. All questions are compulsory.
2. Q. No. 1 to 4 carry 1 mark each, Q. No. 5 to 10 carry 4 marks each and Q. No. 11 to 12 carry 6 marks each.

Q1. If A {a, b} [1]

Find. P (A)

Q2. Find the value of tan $\frac{13\pi}{12}$. [1]

Q3. Find domain of the function [1]

F (x) = $\sqrt{x^2 - 9}$

Q4. Write the solution set of the equation $2x^2 + 3x - 2 = 0$ in roster form. [1]

OR

Write the following set in set builder form.

{1, 2, 3, 6, 9, 18}

Q5. Given U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 5, 7} [4]

B = {2, 4, 6, 8, 10} Verify $(A \cup B)' = A' \cap B'$

Q6. Let A = {1, 2, 3, 4, 5, 6, 7, 8} Let R be the relation on A defined by : [4]

{(a, b) : a, b $\in$ A, b is the highest prime factor of a}

(i) Write R in roster from

(ii) Find domain and range of R

Q7. Prove that : [4]

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

OR

Prove that :

$$\tan 4A = \frac{4\tan A(1-\tan^2 A)}{1-6\tan^2 A+\tan^4 A}$$

Q8. Solve sin x + sin 3x sin 5x = 0 [4]

OR

Solve 2 $\sin^2$ x + 3cos x = 0

Q9. Using properties of sets show that [4]

(a) $A \cup (A \cap B) = A$

(b) $A \cup B = A \cap B$ implies A = B

Q10. Let A = {1,2}, B = {2, 3, 4} C = {4, 5, 6} Find [4]

(a) A x $(B \cap C)$

(b) $(A \text{ x B}) \cap (A \text{ x C})$

Is A x $(B \cap C) = (A \text{ x B}) \cap (A \text{ x C})$?

Q11. If tan x = - $\frac{4}{3}$, $\frac{\pi}{2} <$ x $< \pi$. [6]

Find the value of

$Sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

Q12. In a survey of 75 students in a school, 30 students like to play Cricket, 32 like to play Hockey, 28 like to play Football, 8 like to play both Cricket and Football, 12 both Cricket and Hockey, 10 both Hockey and Football, 5 like to play all three games.

(a) Find how many students like exactly one game?

(b) Find how many students like to play neither cricket nor Hockey nor football.