EXERCISE–14

Part : (A) Only one correct option

1. An ellipse and a hyperbola have the same centre origin, the same foci and the minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then $\frac{1}{e_1^2} + \frac{1}{e_2^2} =$

(A) 1 (B) 2 (C) 4 (D) none

2. The line 5x + 12y = 9 touches the hyperbola $x^2 - 9y^2 = 9$ at the point

(A) (– 5, 4/3) (B) (5, – 4/3) (C) (3, – 1/2) (D) none of these

3. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ & the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide then the value of b^2 is :

(A) 4 (B) 9 (C) 16 (D) none

4. The tangents from $(1, 2\sqrt{2})$ to the hyperbola $16x^2 - 25y^2 = 400$ include between them an angle equal to:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

5. If $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola $xy = c^2$, the coordinates of orthocentre of the ΔPQR are

(A) (x_4, y_4) (B) $(x_4, -y_4)$ (C) $(-x_4, -x_4)$ (D) $(-x_4, -y_4)$

6. The asymptotes of the hyperbola xy = hx + ky are :

(A) x – k = 0 & y – h = 0 (B) x + h = 0 & y + k = 0
(C) x – k = 0 & y + h = 0 (D) x + k = 0 & y – h = 0

7. The combined equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ is

(A) $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$ (B) $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$
(C) $2x^2 + 5xy + 2y^2 = 0$ (D) none of these

8. If the hyperbolas, $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$ and $x^2 + 3xy + 2y^2 + 2x + 3y + c = 0$ are conjugate of each other, then the value of 'c' is equal to :

(A) – 2 (B) 4 (C) 0 (D) 1

9. P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT. ON is equal to :

(A) e^2 (B) a^2 (C) b^2 (D) b^2/a^2

10. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is :

(A) $(x^2 - y^2)^2 = 4c^2xy$ (B) $(x^2 + y^2)^2 = 2c^2xy$ (C) $(x^2 + y^2) = 4x^2xy$ (D) $(x^2 + y^2)^2 = 4c^2xy$

11. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to

(A) $-\frac{a^2}{b^2}$ (B) $-\frac{b^2}{a^2}$ (C) $-\frac{b^4}{a^4}$ (D) $-\frac{a^4}{b^4}$

12. The equations of the transverse and conjugate axes of a hyperbola are respectively x + 2y – 3 = 0, 2x – y + 4 = 0, and their respective lengths are $\sqrt{2}$ and $2/\sqrt{3}$. The equation of the hyperbola is

(A) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ (B) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$
(C) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ (D) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

13. The chord PQ of the rectangular hyperbola $xy = a^2$ meets the x-axis at A; C is the mid point of PQ & 'O' is the origin. Then the Δ ACO is :

(A) equilateral (B) isosceles (C) right angled (D) right isosceles.

14. The number those triangles that can be inscribed in the rectangular hyperbola $xy = c^2$ whose all sides touch the parabola $y^2 = 4ax$ is :

(A) 0 (B) 1 (C) 2 (D) Infinite

15. The number of points from where a pair of perpendicular tangents can be drawn to the hyperbola,

$x^2 \sec^2 \alpha - y^2 \operatorname{cosec}^2 \alpha = 1$, $\alpha \in (0, \pi/4)$, is :
(A) 0 (B) 1 (C) 2 (D) infinite

16. If hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through the focus of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then eccentricity of hyperbola is
(A) $\sqrt{2}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) None of these

17. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2 : 1, the equation of the hyperbola is :
(A) $4x^2 - 5y^2 = 4a^2$ (B) $4x^2 - 5y^2 = 5a^2$ (C) $5x^2 - 4y^2 = 4a^2$ (D) $5x^2 - 4y^2 = 5a^2$

18. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola satisfies
(A) $e > \sqrt{3}$ (B) $1 < e < 2\frac{2}{\sqrt{3}}$ (C) $e = \frac{2}{\sqrt{3}}$ (D) $e > \frac{2}{\sqrt{3}}$

19. If $x \cos \alpha + y \sin \alpha = p$, a variable chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2a^2} = 1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle whose radius is equal to
(A) $\sqrt{2}$ a (B) $\sqrt{3}$ a (C) 2 a (D) $\sqrt{5}$ a

20. Two conics $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $x^2 = -\frac{1}{b}$ y intersect if
(A) $0 < b \le \frac{1}{2}$ (B) $0 < a < \frac{1}{2}$ (C) $a^2 < b^2$ (D) $a^2 > b^2$

21. Number of points on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from where mutually perpendicular tangents can be drawn to circle $x^2 + y^2 = a^2$ $(a > b)$ is
(A) 2 (B) 3 (C) infinite (D) 4

22. The normal to the rectangular hyperbola $xy = c^2$ at the point 't_1' meets the curve again at the point 't_2'. The value of $t_1^3 t_2$ is
(A) –1 (B) –|c| (C) |c| (D) 1

23. If the tangent and the normal to a rectangular hyperbola cut off intercepts x_1 and x_2 on one axis and y_1 and y_2 on the other axis, then
(A) $x_1y_1 + x_2y_2 = 0$ (B) $x_1y_2 + x_2y_1 = 0$ (C) $x_1x_2 + y_1y_2 = 0$ (D) none of these

24. If x = 9 is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is **[IIT - 1999]**
(A) $9x^2 - 8y^2 + 18x - 9 = 0$ (B) $9x^2 - 8y^2 + 18x + 9 = 0$
(C) $9x^2 - 8y^2 - 18x - 9 = 0$ (D) $9x^2 - 8y^2 + 18x + 9 = 0$

Part : (B) May have more than one options correct

25. The value of m for which y = mx + 6 is a tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{49} = 1$ is
(A) $\sqrt{\left(\frac{17}{20}\right)}$ (B) $-\sqrt{\left(\frac{17}{20}\right)}$ (C) $\sqrt{\left(\frac{20}{17}\right)}$ (D) $-\sqrt{\left(\frac{20}{17}\right)}$

26. If $(a \sec \theta, b \tan \theta)$ and $(a \sec\phi, b \tan \phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2} \tan\frac{\phi}{2}$ equals to
(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{e+1}{e-1}$

27. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is
(A) $y = \frac{3}{\sqrt{7}} x + \frac{15}{\sqrt{7}}$ (B) $y = 3\sqrt{\frac{2}{7}} x + \frac{15}{\sqrt{7}}$

(C) $y = 2\sqrt{\frac{3}{7}}\,x + 15\sqrt{7}$ (D) $y = 3\sqrt{\frac{2}{7}}\,x - \frac{15}{\sqrt{7}}$

28. The equation of a hyperbola with co-ordinate axes as principal axes, if the distances of one of its vertices from the foci are 3 & 1 can be :
(A) $3x^2 - y^2 = 3$ (B) $x^2 - 3y^2 + 3 = 0$ (C) $x^2 - 3y^2 - 3 = 0$ (D) none

29. If (5, 12) and (24, 7) are the foci of a conic passing through the origin then the eccentricity of conic is
(A) $\sqrt{386}/12$ (B) $\sqrt{386}/13$ (C) $\sqrt{386}/25$ (D) $\sqrt{386}/38$

30. If the normal at P to the rectangular hyperbola $x^2 - y^2 = 4$ meets the axes in G and g and C is the centre of the hyperbola, then
(A) PG = PC (B) Pg = PC (C) PG = Pg (D) Gg = PC

31. The tangent to the hyperbola, $x^2 - 3y^2 = 3$ at the point $(\sqrt{3}, 0)$ when associated with two asymptotes constitutes :
(A) isosceles triangle (B) an equilateral triangle
(C) a triangles whose area is $\sqrt{3}$ sq. units (D) a right isosceles triangle.

32. Which of the following equations in parametric form can represent a hyperbolic profile, where 't' is a parameter.
(A) $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ & $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$ (B) $\frac{tx}{a} - \frac{y}{b} + t = 0$ & $\frac{x}{a} + \frac{ty}{b} - 1 = 0$
(C) $x = e^t + e^{-t}$ & $y = e^t - e^{-t}$ (D) $x^2 - 6 = 2\cos t$ & $y^2 + 2 = 4\cos^2\frac{t}{2}$

33. If a hyperbola passes through the focii of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Its transverse and conjugate axes coincide respectively with the major and minor axes of the ellipse and if the product of eccentricities of hyperbola and ellipse is 1, then **[IIT - JEE]**
(A) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (B) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$
(C) focus of hyperbola is (5, 0) (D) focus of hyperbola is $(5\sqrt{3}, 0)$

EXERCISE–15

1. For the hyperbola $x^2/100 - y^2/25 = 1$, prove that
(i) eccentricity $= \sqrt{5}/2$
(ii) SA . S′A = 25, where S & S′ are the foci & A is the vertex .

2. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of their middle points is the curve, $y^2(x - a) = x^3$.

3. Find the asymptotes of the hyperbola $2x^2 - 3xy - 2y^2 + 3x - y + 8 = 0$. Also find the equation to the conjugate hyperbola & the equation of the principal axes of the curve .

4. Given the base of a triangle and the ratio of the tangent of half the base angles. Show that the vertex moves on a hyperbola whose foci are the extremities of the base.

5. If p_1 and p_2 are the perpendiculars from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on its asymptotes, then prove that, $\frac{1}{p_1 p_2} = \frac{1}{a^2} + \frac{1}{b^2}$.

6. If two points P & Q on the hyperbola $x^2/a^2 - y^2/b^2 = 1$ whose centre is C be such that CP is perpendicular to CQ & a < b, then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$.

7. If the normal at a point P to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ meets the x–axis at G, show that SG = e . SP, S being the focus of the hyperbola .

8. A transversal cuts the same branch of a hyperbola $x^2/a^2 - y^2/b^2 = 1$ in P, P′ and the asymptotes in Q, Q′. Prove that (i) PQ = P′Q′ & (ii) PQ′ = P′Q

9. If PSP′ & QSQ′ are two perpendicular focal chords of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ then prove that $\frac{1}{\ell(PS)\,.\,\ell(SP')} + \frac{1}{\ell(QS)\,.\,\ell(SQ')}$ is a constant .

10. A line through the origin meets the circle $x^2 + y^2 = a^2$ at P & the hyperbola $x^2 - y^2 = a^2$ at Q . Prove that the locus of the point of intersection of the tangent at P to the circle and the tangent at Q to the hyperbola is curve $a^4 (x^2 - a^2) + 4x^2y^4 = 0$.

11. Prove that the part of the tangent at any point of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ intercepted between the point of contact and the transverse axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point .

12. Let 'p' be the perpendicular distance from the centre C of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ to the tangent drawn at a point R on the hyperbola . If S & S′ are the two foci of the hyperbola, then show that $(RS + RS')^2 = 4a^2\left(1 + \frac{b^2}{p^2}\right)$.

13. Chords of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ are tangents to the circle drawn on the line joining the foci as diameter . Find the locus of the point of intersection of tangents at the extremities of the chords .

14. A point P divides the focal length of the hyperbola $9x^2 - 16y^2 = 144$ in the ratio S′P : PS = 2 : 3 where S & S′ are the foci of the hyperbola. Through P a straight line is drawn at an angle of 135° to the axis OX. Find the points of intersection of this line with the asymptotes of the hyperbola.

15. The angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45º. Show that the locus of the point P is a hyperbola. **[IIT - 1998]**

16. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of constant. **[IIT - 2005]**

Answers

EXERCISE–14

1. A	**2.** B	**3.** C	**4.** D	**5.** D
6. A	**7.** A	**8.** C	**9.** B	**10.** D
11. D	**12.** B	**13.** B	**14.** D	**15.** D
16. C	**17.** D	**18.** D	**19.** A	**20.** B
21. D	**22.** A	**23.** C	**24.** B	**25.** AB
26. BC	**27.** BD	**28.** AB	**29.** AD	**30.** ABC
31. BC	**32.** ACD		**33.** AC	

EXERCISE–15

3. $x - 2y + 1 = 0$; $2x + y + 1 = 0$; $2x^2 - 3xy - 2y^2 + 3x - y - 6 = 0$; $3x - y + 2 = 0$; $x + 3y = 0$

13. $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2 + b^2}$ **14.** $(-4, 3)$ & $\left(-\frac{4}{7}, -\frac{3}{7}\right)$

16. $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2 + y^2}{9}\right)^2$
