

Oscillations and Waves

- **Periodic Motion:** A motion which repeats itself over and over again after a regular interval of time.
- **Oscillatory Motion:** A motion in which a body moves back and forth repeatedly about a fixed point.
- **Periodic function:** A function that repeats its value at regular intervals of its argument is called periodic function. The following sine and cosine functions are periodic with period T.

$$f(t) = \sin \frac{2\pi t}{T} \quad \text{and} \quad g(t) = \cos \frac{2\pi t}{T}$$

These are called Harmonic Functions.

Note :- All Harmonic functions are periodic but all periodic functions are not harmonic.

One of the simplest periodic functions is given by

$$f(t) = A \cos \omega t \quad [\omega = 2\pi/T]$$

If the argument of this function ωt is increased by an integral multiple of 2π radians, the value of the function remains the same. The function $f(t)$ is then periodic and its period, T is given by

$$T = \frac{2\pi}{\omega}$$

Thus the function $f(t)$ is periodic with period T

$$f(t) = f(t + T)$$

Linear combination of sine and cosine functions

$$f(t) = A \sin \omega t + B \cos \omega t$$

A periodic function with same period T is given as

$$A = D \cos \phi \quad \text{and} \quad B = D \sin \phi$$

$$\therefore f(t) = D \sin (\omega t + \phi)$$

$$\therefore D = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1} \frac{B}{A}$$

- **Simple Harmonic Motion (SHM):** A particle is said to execute SHM if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from mean position and is always directed towards mean position.

Restoring Force \propto Displacement

$$F \propto x$$

$$F = -kx$$

Where 'k' is force constant.

- **Amplitude:** Maximum displacement of oscillating particle from its mean position.

$$x_{Max} = \pm A$$

- **Time Period:** Time taken to complete one oscillation.

- **Frequency:** $= \frac{1}{T}$. Unit of frequency is Hertz (Hz).

$$1 \text{ Hz} = 1 \text{ s}^{-1}$$

- **Angular Frequency:** $\omega = \frac{2\pi}{T} = 2\pi\nu$

$$\text{S.I unit } \omega = \text{rad s}^{-1}$$

- **Phase:**

1. The Phase of Vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.

It is denoted by ϕ .

2. **Initial phase or epoch:** The phase of particle corresponding to time $t = 0$.

It is denoted by ϕ_0 .

- **Displacement in SHM :**

$$x = A \cos (\omega t + \phi_0)$$

Where, x = Displacement,

A = Amplitude

ωt = Angular Frequency

ϕ_0 = Initial Phase.

Case 1: When Particle is at mean position $x = 0$

$$v = -\omega\sqrt{A^2 - 0^2} = -\omega A$$

$$v_{\max} = \omega A = \frac{2\pi}{T} A$$

Case 2: When Particle is at extreme position $x = \pm A$

$$v = -\omega\sqrt{A^2 - A^2} = 0$$

Acceleration

Case 3: When particle is at mean position $x = 0$,

$$\text{acceleration} = -\omega^2(0) = 0.$$

Case 4: When particle is at extreme position then

$$x = A \quad \text{acceleration} = -\omega^2 A$$

➤ **Formula Used :**

$$1. \quad x = A \cos(\omega t + \phi_0)$$

$$2. \quad v = \frac{dx}{dt} = -\omega\sqrt{A^2 - x^2}, \quad v_{\max} = \omega A.$$

$$3. \quad a = \frac{dv}{dt} = \omega^2 A \cos(\omega t + \phi_0) \\ = -\omega^2 x$$

$$a_{\max} = \omega^2 A$$

$$4. \quad \text{Restoring force } F = -kx = -m\omega^2 x$$

$$\text{Where } k = \text{force constant} \quad \& \quad \omega^2 = \frac{k}{m}$$

$$5. \quad \text{Angular freq. } \omega = 2\pi\nu = \frac{2\pi}{T}$$

$$6. \quad \text{Time Period } T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi \sqrt{\frac{x}{a}}$$

$$7. \quad \text{Time Period } T = 2\pi \sqrt{\frac{\text{Inertia Factor}}{\text{Spring Factor}}} = 2\pi \sqrt{\frac{m}{k}}$$

8. P.E at displacement 'y' from mean position

$$E_P = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

9. K.E. at displacement 'y' from the mean position

$$E_K = \frac{1}{2}k(A^2 - y^2) = \frac{1}{2}m\omega^2(A^2 - y^2)$$

$$= \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

10. Total Energy at any point

$$E_T = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 mA^2 \nu^2$$

11. Spring Factor $K = F/y$

12. Period Of oscillation of a mass 'm' suspended from a massless spring of force constant 'k'

$$T = 2\pi \sqrt{\frac{m}{k}}$$

For two springs of spring factors k_1 and k_2 connected in parallel effective spring factor

$$k = k_1 + k_2 \quad \therefore T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

13. For two springs connected in series, effective spring factor 'k' is given as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{Or} \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Note:- When length of a spring is made 'n' times its spring factor becomes $\frac{1}{n}$ times and hence time period increases \sqrt{n} times.

14. When spring is cut into 'n' equal pieces, spring factor of each part becomes 'nk'.

$$T = 2\pi \sqrt{\frac{m}{nk}}$$

15. Oscillation of simple pendulum

$$T = 2\pi \sqrt{l/g}$$

$$\nu = \frac{1}{2\pi} \sqrt{g/l}$$

16. For a liquid of density ρ contained in a U-tube up to height 'h'

$$T = 2\pi\sqrt{h/g}$$

17. For a body dropped in a tunnel along the diameter of earth

$$T = 2\pi\sqrt{R/g}, \text{ where } R = \text{Radius of earth}$$

18. Resonance: If the frequency of driving force is equal to the natural frequency of the oscillator itself, the amplitude of oscillation is very large then such oscillations are called resonant oscillations and phenomenon is called resonance.

Waves

Angular wave number: It is phase change per unit distance.

i.e. $k = \frac{2\pi}{\lambda}$, S.I unit of k is radian per meter.

Relation between velocity, frequency and wavelength is given as :- $V = \nu\lambda$

Velocity of Transverse wave:-

(i) In solid molecules having modulus of rigidity ' η ' and density ' ρ ' is

$$V = \sqrt{\frac{\eta}{\rho}}$$

(ii) In string for mass per unit length ' m ' and tension ' T ' is $V = \sqrt{\frac{T}{m}}$

Velocity of longitudinal wave:-

(i) in solid $V = \sqrt{\frac{Y}{\rho}}$, Y = young's modulus

(ii) in liquid $V = \sqrt{\frac{K}{\rho}}$, K = bulk modulus

$$(iii) \quad \text{in gases } V = \sqrt{\frac{K}{\rho}}, \quad K = \text{bulk modulus}$$

According to Newton's formula: When sound travels in gas then changes take place

$$\text{in the medium are isothermal in nature. } V = \sqrt{\frac{P}{\rho}}$$

According to Laplace: When sound travels in gas then changes take place in the medium are adiabatic in nature.

$$V = \sqrt{\frac{P\gamma}{\rho}} \quad \text{Where } \gamma = \frac{C_p}{C_v}$$

Factors effecting velocity of sound :-

- (i) Pressure – No effect
- (ii) Density – $V \propto \frac{1}{\sqrt{\rho}}$ or $\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- (iii) Temp- $V \propto \sqrt{T}$ or $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

Effect of humidity :- sound travels faster in moist air

- (iv) Effect of wind –velocity of sound increasing along the direction of wind.

Wave equation:- if wave is travelling along + x-axis

- (i) $Y = A \sin (\omega t - kx)$, Where, $k = \frac{2\pi}{\lambda}$
- (ii) $Y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$
- (iii) $Y = A \sin \frac{2\pi}{T} (vt - x)$

If wave is travelling along –ve x- axis

- (iv) $Y = A \sin (\omega t + kx)$,Where , $k = \frac{2\pi}{\lambda}$
- (v) $Y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$
- (vi) $Y = A \sin \frac{2\pi}{T} (vt + x)$

Phase and phase difference

Phase is the argument of the sine or cosine function representing the wave.

$$\phi = 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

Relation between phase difference ($\Delta\phi$) and time interval (Δt) is $\Delta\phi = -\frac{2\pi}{T} \Delta t$

Relation between phase difference ($\Delta\phi$) and path difference (Δx) is $\Delta\phi = -\frac{2\pi}{\lambda} \Delta x$

Equation of stationary wave:-

$$(1) Y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \text{ (incident wave)}$$

$$Y_1 = \pm a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) \text{ (reflected wave)}$$

Stationary wave formed

$$Y = Y_1 + Y_2 = \pm 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

(2) For (+ve) sign antinodes are at $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$

And nodes at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$

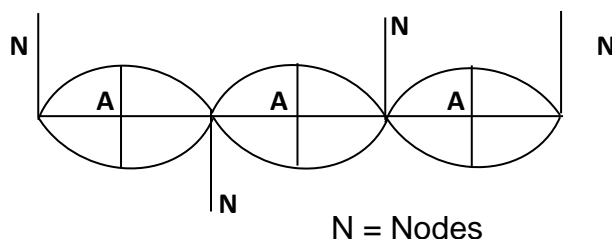
(3) For (-ve) sign antinodes are at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$

Nodes at $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$

(4) Distance between two successive nodes or antinodes are $\frac{\lambda}{2}$ and that between nodes and nearest antinodes is $\frac{\lambda}{4}$.

(5) Nodes-point of zero displacement-

Antinodes- point of maximum displacement-



A = Antinodes

Mode of vibration of strings:-

$$a) v = \frac{p}{2L} \sqrt{\frac{T}{m}} \text{ Where } T = \text{Tension}$$

M = mass per unit length

v = frequency, V = velocity of second, $p = 1, 2, 3, \dots$

$$b) \text{ When stretched string vibrates in } P \text{ loops } v_p = \frac{p}{2L} \sqrt{\frac{T}{m}} = p v$$

c) For string of diameter D and density ρ

$$v = \frac{1}{LD} \sqrt{\frac{T}{\pi \rho}}$$

d) Law of length $v \propto \frac{1}{L}, vL = \text{constant}$

ORGANPIPES

1. In an organ pipe closed at one end only odd harmonics are present

$$v_1 = \frac{V}{4L} \text{ (fundamental)}$$

$$v_2 = 3v \text{ (third harmonic or first overtone)}$$

$$v_3 = 5v$$

$$v_n = (2n-1) v$$

2. In an open organ pipe at both ends both odd and even harmonics are present.

$$v'_1 = \frac{V}{2L} = v' \text{ (first harmonic)}$$

$$v'_2 = 2v' \text{ (second harmonic or first overtone)}$$

$$v' = 3v$$

$$v' = (2n-1) v$$

3. Resonance tube: If L_1 and L_2 are the first and second resonance length with a tuning fork of frequency ' v ' then the speed of sound. $v = 4v(L_1 + 0.3D)$

Where D = internal diameter of resonance tube

$$v = 2v(L_2 - L_1)$$

$$\text{End correction} = 0.3D = \frac{L_2 - L_1}{2}$$

Beats formation

1. Beat frequency = No. of beats per second = Difference in frequency of two sources.

$$b = v_1 - v_2$$

$$2. v_2 = v_1 \pm b$$

3. If the prong of tuning fork is filed, its frequency increases. If the prong of a tuning fork is loaded with a little wax, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

Doppler effect in sound

1. If V , V_o , V_s , and V_m are the velocity of sound, observer, source and medium respectively, then the apparent frequency

$$v' = \frac{V + V_m - V_o}{V + V_m - V_s} \times v$$

2. If the medium is at rest ($v_m = 0$), then

$$v' = \frac{V - V_o}{V - V_s} \times v$$

3. All the velocities are taken positive with source to observer ($S \rightarrow O$) direction and negative in the opposite ($O \rightarrow S$) direction

(Questions)

(1 marks questions)

1. Which of the following relationships between the acceleration 'a' and the displacement 'x' of a particle involve simple harmonic motion?

- (a) $a=0.7x$ (b) $a=-200x^2$ (c) $a = -10x$ (d) $a=100x^3$

Ans: - (c) represent SHM.

2. Can a motion be periodic and not oscillatory?

Ans: - Yes, for example, uniform circular motion is periodic but not oscillatory.

3. Can a motion be periodic and not simple harmonic? If your answer is yes, give an example and if not, explain why?

Ans:- Yes, when a ball is dropped from a height on a perfectly elastic surface, the motion is oscillatory but not simple harmonic as restoring force $F=mg=\text{constant}$ and not $F \propto -x$, which is an essential condition for S.H.M.

4. A girl is swinging in the sitting position. How will the period of the swing change if she stands up?

Ans:- The girl and the swing together constitute a pendulum of time period

$$T = 2\pi \sqrt{\frac{l}{g}}$$

As the girl stands up her centre of gravity is raised. The distance between the point of suspension and the centre of gravity decreases i.e. length 'l' decreases. Hence the time period 'T' decreases.

5. The maximum velocity of a particle, executing S.H.M with amplitude of 7mm is 4.4 m/s. What is the period of oscillation?

Ans: - $V_{\max} = \omega A = \frac{2\pi}{T} A$, $T = \frac{2\pi A}{V_{\max}} = \frac{2 \times 22 \times .007}{7 \times 4.4} = 0.01s$

6. Why the longitudinal wave are also called pressure waves?

Ans: - Longitudinal wave travel in a medium as series of alternate compressions and rare fractions i.e. they travel as variations in pressure and hence are called pressure waves.

7. How does the frequency of a tuning fork change, when the temperature is increased?

Ans: -As the temperature is increased, the length of the prong of a tuning fork increased. This increased the wavelength of a stationary waves set up in the tuning fork. As frequency,

$$v = \frac{1}{\lambda}, \text{ So the frequency of tuning fork decreases.}$$

8. An organ pipe emits a fundamental note of a frequency 128Hz. On blowing into it more strongly it produces the first overtone of the frequency 384Hz. What is the type of pipe –Closed or Open?

Ans: - The organ pipe must be closed organ pipe, because the frequency the first overtone is three times the fundamental frequency.

9. All harmonic are overtones but all overtones are not harmonic. How?

Ans: -The overtones with frequencies which are integral multiple of the fundamental frequency are called harmonics. Hence all harmonic are overtones. But overtones which are non-integrals multiples of the fundamental frequency are not harmonics.

10. What is the factor on which pitch of a sound depends?

Ans: - The pitch of a sound depends on its frequency.

(2 Marks questions)

1. At what points is the energy entirely kinetic and potential in S.H.M? What is the total distance travelled by a body executing S.H.M in a time equal to its time period, if its amplitude is A?

Ans. The energy is entirely kinetic at mean position i.e. at $y=0$. The energy is entirely potential at extreme positions i.e.

$$y = \pm A$$

$$\text{Total distance travelled in time period } T = 2A + 2A = 4A.$$

2. A simple pendulum consisting of an inextensible length 'l' and mass 'm' is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2 . Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards?

Ans. When the lift is stationary, $T = 2\pi \sqrt{\frac{l}{g}}$

When the lift accelerates upwards with an acceleration of 4.5 m/s^2

$$T' = 2\pi \sqrt{\frac{l}{g+4.5}}$$

Therefore, the time period decreases when the lift accelerates upwards.

3. Does the function $y = \sin^2 \omega t$ represent a periodic or a S.H.M? What is period of motion?

Ans. Displacement $y = \sin^2 \omega t$

$$\text{Velocity } v = \frac{dy}{dt} = 2 \sin \omega t \times \cos \omega t \times \omega$$

$$v = \omega \sin 2\omega t$$

$$\text{Acceleration } a = \frac{dv}{dt} = \omega \times \cos 2\omega t \times 2\omega$$

$$a = 2 \omega^2 \cos 2\omega t.$$

As the acceleration is not proportional to displacement y , the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω .

$$\therefore \text{Time Period } T = \frac{2\pi}{\text{Angular freq.}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

4. All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?

Ans. All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So these functions cannot be used to represent bounded motion like SHM.

5. A simple Harmonic Motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$. What is its time period?

Ans. $\frac{d^2x}{dt^2} = -\alpha x$ Or $a = -\alpha x$

$$T = 2\pi \sqrt{\frac{x}{a}} = 2\pi \sqrt{\frac{x}{\alpha x}} = \frac{2\pi}{\sqrt{\alpha}}$$

$$T = \frac{2\pi}{\sqrt{\alpha}}$$

6. The Length of a simple pendulum executing SHM is increased by 2.1%. What is the percentage increase in the time period of the pendulum of increased length?

Ans. Time Period, $T = 2\pi \sqrt{\frac{l}{g}}$ i.e. $T \propto \sqrt{l}$.

The percentage increase in time period is given by

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \frac{\Delta l}{l} \times 100 \text{ (for small variation)}$$

$$= \frac{1}{2} \times 2.1\%$$

$$= 1.05\%$$

7. A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from $x = A$ to $x = A/2$.

Ans. Displacement from mean position $= A - A/2 = A/2$.

When the motion starts from the positive extreme position, $y = A \cos \omega t$.

$$\therefore \frac{A}{2} = A \cos \frac{2\pi}{T} t.$$

$$\cos \frac{2\pi}{T} t = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\text{or } \frac{2\pi}{T} t = \frac{\pi}{3}$$

$$\therefore t = T/6$$

8. An open organ pipe produces a note of frequency $5/2$ Hz at 15°C , calculate the length of pipe. Velocity of sound at 0°C is 335 m/s.

Ans. Velocity of sound at 15°C

$$V = V_0 + 0.61 \times t = 335 + 0.61 \times 15 = 344.15 \text{ m/s. (Thermal coefficient of velocity of sound wave is } .61/^\circ\text{C)}$$

Fundamental frequency of an organ pipe

$$v = \frac{V}{4L}, \therefore L = \frac{V}{4v} = \frac{344.15}{4 \times 512} = 0.336 \text{ m}$$

9. An incident wave is represented by $Y(x, t) = 20 \sin(2x - 4t)$. Write the expression for reflected wave

- (i) From a rigid boundary
- (ii) From an open boundary.

Ans.(i) The wave reflected from a rigid boundary is

$$Y(x, t) = -20\sin(2x+4t)$$

(i)The wave reflected from an open boundary is

$$Y(x, t) = 20\sin(2x+4t)$$

Explain why

- (i) in a sound wave a displacement node is a pressure antinode and vice- versa
- (ii) The shape of pulse gets- distorted during propagation in a dispersive medium.

Ans. (i) At a displacement node the variations of pressure is maximum. Hence displacement node is the a pressure antinode and vice-versa.

(ii)When a pulse passes through a dispersive medium the wavelength of wave changes.

So, the shape of pulse changes i.e. it gets distorted.

(3 Marks Questions)

1. The speed of longitudinal wave 'V' in a given medium of density ρ is given by the formula, use this formula to explain why the speed of sound in air.
 - (a) is independent at pressure
 - (b) increases with temperature and
 - (c) increases with humidity

2. Write any three characteristics of stationary waves.

Ans. (i) in stationary waves, the disturbance does not advance forward. The conditions of crest and trough merely appear and disappear in fixed position to be followed by opposite condition after every half time period. (ii) The distance between two successive nodes or antinodes is equal to half the wavelength. (iii) The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

3. Show that the speed of sound in air increased by .61m/s for every 1°C rise of temperature.

Ans. $V \propto \sqrt{T}$

$$\frac{V_t}{V_0} = \sqrt{\frac{t+273}{0+273}}$$

$$V_t = V_0 \left(1 + \frac{t}{273}\right)^{1/2} = V_0 \left(1 + \frac{1}{2} \cdot \frac{t}{273}\right)$$

$$V_t = V_0 + \frac{V_0 \times t}{546}$$

At, 0°C speed of sound in air is 332 m/s.

$$\therefore V_t - V_0 = \frac{332 \times t}{546}$$

When $t = 1^{\circ}\text{C}$, $V_t - V_0 = 0.61\text{m/s}$.

4. Find the ratio of velocity of sound in hydrogen gas $Y = \frac{7}{5}$ to that in helium gas

$Y = \frac{5}{3}$ at the same temperature. Given that molecular weight of hydrogen and helium are 2 and 4 respectively.

Ans. $V = \sqrt{\frac{\gamma RT}{M}}$

At constant temperature,

$$\frac{V_H}{V_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \frac{M_H}{M_{He}}} = \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{2}} = 1.68.$$

5. The equation of a plane progressive wave is, $y = 10\sin 2\pi(t - 0.005x)$ where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, $y = 10\sin 2\pi(t - 0.005x)$ (1)

Standard equation for harmonic wave is, $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (2)

Comparing eqn (1) & (2), $A = 10$, $\frac{1}{T} = 1$, $\frac{1}{\lambda} = 0.005$

(i) Amplitude $A = 10\text{cm}$

(ii) Frequency $\nu = \frac{1}{T} = 1\text{Hz}$

(iii) Wavelength $\lambda = \frac{1}{0.005} = 200\text{cm}$

(iv) Velocity $v = \nu \lambda = 1 \times 200 = 200\text{cm/s}$

6. Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. $Y = A \sin (2\pi\nu t + \phi_o)$

$$= 0.01 \sin (1200\pi t + \frac{\pi}{6})$$

7. The amplitude of oscillations of two similar pendulums similar in all respect are 2cm & 5cm respectively. Find the ratio of their energies of oscillations.

Ans. $\frac{E_1}{E_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{2}{5}\right)^2 = 4:25$

8. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

Ans. A periodic motion repeats after a definite time interval T. So,

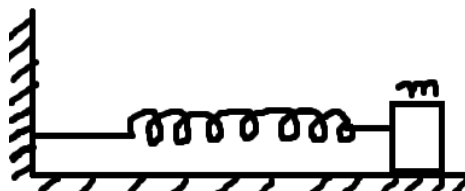
$$y(t) = y(t + T) = y(t + 2T) \text{ etc.}$$

9. A spring of force constant 1200N/m is mounted horizontal table. A mass of 3Kg is attached to the free end of the spring, pulled sideways to a distance of 2.0cm and released.

(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?



Ans. Here $k = 1200\text{N/m}$, $m = 3\text{Kg}$, $A = 2\text{cm} = 2 \times 10^{-2}\text{m}$

$$(i) \quad v = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2} \times \frac{1}{3.14} \sqrt{\frac{1200}{3}} = 3.2\text{s}^{-1}$$

$$(ii) \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20\text{s}^{-1}$$

$$\text{Maximum acceleration} = \omega^2 A = (20)^2 \times 2 \times 10^{-2} = 8\text{m/s}^2$$

$$(iii) \quad \text{Maximum speed} = \omega A = 20 \times 2 \times 10^{-2} = 0.40\text{m/s}$$

10. Which of the following function of time represent, (a) simple harmonic (b) periodic but not SHM and (c) non periodic ?

$$(i) \sin \omega t - \cos \omega t \quad (ii) \sin^3 \omega t \quad (iii) 3\cos\left(\frac{\pi}{2} - 2\omega t\right) \quad (iv) \exp(-\omega^2 t^2)$$

Ans. (i) $x(t) = \sin \omega t - \cos \omega t = \sqrt{2} \sin\left(\omega t - \frac{\pi}{2}\right)$, so the function is in SHM.

(ii) $x(t) = \sin^3 \omega t = \frac{1}{4}(3\sin \omega t - \sin 3\omega t)$, represent two separate SHM motion but their combination does not represent SHM.

$$(iii) \quad x(t) = 3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left(2\omega t - \frac{\pi}{4}\right), \text{ represent SHM.}$$

$$(iv) \quad \exp(-\omega^2 t^2) = \text{non periodic.}$$

(5 Marks Questions)

1. (a) A light wave is reflected from a mirror. The incident & reflected wave superimpose to form stationary waves. But no nodes & antinodes are seen, why?

(b) A standing wave is represented by $y = 2A \sin Kx \cos \omega t$. If one of the component wave is $y_1 = A \sin(\omega t - Kx)$, what is the equation of the second component wave?

Ans. (a) As is known, the distance between two successive nodes or two successive antinodes is $\frac{\lambda}{2}$. The wavelength of visible light is of the order of 10^{-7}m . As such as a

small distance cannot be detected by the eye or by a ordinary optical instrument. Therefore, nodes and antinodes are not seen.

$$(b) \text{ As, } 2\sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$\begin{aligned} y &= 2A \sin Kx \cos \omega t \\ &= A \sin(Kx + \omega t) + A \sin(Kx - \omega t) \end{aligned}$$

According to superposition principle,

$$y = y_1 + y_2$$

$$\text{and } y_1 = A \sin(\omega t - Kx) = -A \sin(Kx - \omega t)$$

$$\begin{aligned} y_2 &= y - y_1 = 2A \sin Kx \cos \omega t + A \sin(Kx - \omega t) \\ &= A \sin(Kx + \omega t) + 2A \sin(Kx - \omega t) \\ &= A \sin(Kx + \omega t) - 2A \sin(\omega t - Kx) \end{aligned}$$

2. Discuss Newton's formula for velocity of sound in air. What correction was made to it by Laplace and why?

Ans. According to Newton the change in pressure & volume in air is an isothermal process. Therefore he calculated, $v = \sqrt{\frac{p}{\rho}}$ on substituting the require value he found, the velocity of sound was not in close agreement with the observation value. Then Laplace pointed out the error in Newton's formula. According to Laplace the change in pressure and volume is an adiabatic process. So he calculated the value of sound as, $v = \sqrt{\frac{\gamma p}{\rho}}$ on putting require value he found velocity of sound as 332m/s very closed to observed theory.

3. (a) What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing wave.

(b) Draw fundamental nodes of vibration of stationary wave in (i) closed pipe, (ii) in an open pipe.

4. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4.

5. Explain Doppler's effect of sound. Derive an expression for the apparent frequency where the source and observer are moving in the same direction with velocity V_s and V_o respectively, with source following the observer.

$$[\text{Ans. } = v' = \frac{v-v_o}{v-v_s} * v]$$

6. For a travelling harmonic wave, $y = 2\cos(10t - 0.008x + 0.35)$ where x & y are in cm and t in second. What is the phase difference between oscillatory motions at two points separated by a distance of (i) 4cm (ii) 0.5m (iii) $\frac{\lambda}{2}$ (iv) $\frac{3\lambda}{4}$?

$$\text{Ans. } y = 2\cos(10t - 0.008x + 0.35) \dots \dots \dots (i)$$

$$\text{We know, } y = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right) \dots \dots \dots (ii)$$

$$\text{From (i) \& (ii), } \frac{2\pi}{\lambda} = 0.008, \lambda = \frac{2\pi}{0.008} \text{ cm} = \frac{2\pi}{0.80} \text{ m.}$$

$$\text{Phase difference, } \Delta\phi = \frac{2\pi}{\lambda} * \text{path difference} = \frac{2\pi}{\lambda} * \Delta x.$$

$$(i) \quad \text{When } \Delta x = 4\text{cm}, \quad \Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 4 = 3.2\text{rad}.$$

$$(ii) \quad \text{When } \Delta x = 0.5\text{m}, \quad \Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 0.5 = 0.40\text{rad}.$$

$$(iii) \quad \text{When } \Delta x = \frac{\lambda}{2}, \quad \Delta\phi = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi\text{rad}.$$

$$(iv) \quad \text{When } \Delta x = \frac{3\lambda}{4}, \quad \Delta\phi = \frac{2\pi}{\lambda} * \frac{3\lambda}{4} = \frac{3\pi}{2}\text{rad}.$$

7. (i) A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).

Ans. (i) For the fundamental mode,

$$\lambda = 2L = 2 \times 100 = 200 \text{ cm} = 2\text{m}.$$

$$\text{Frequency } \nu = 2.53 \text{ kHz} = 2530 \text{ Hz}$$

$$\text{Speed of sound, } v = \nu\lambda = 2530 \times 2 = 5060 \text{ m/s}$$

$$= 5.06 \text{ km/s}$$

(ii) Length of pipe $L = 20 \text{ cm} = 0.2 \text{ m}$

$$\text{Speed of sound } v = 340 \text{ m/s}$$

Fundamental frequency of closed organ pipe

$$\nu = \frac{v}{4L} = \frac{340}{4 \times 0.2} = 425 \text{ Hz} \quad \text{sw can be excited}$$

Fundamental frequency of open organ pipe

$$\nu' = \frac{v}{2L} = \frac{340}{2 \times 0.2} = 850 \text{ Hz}$$

Hence source of frequency 430 Hz will not be in resonance with open organ pipe.

8. A train stands at a platform blowing a whistle of frequency 400 Hz in still air.

(i) What is the frequency of the whistle heard by a man running

(a) Towards the engine 10 m/s.

(b) Away from the engine at 10 m/s?

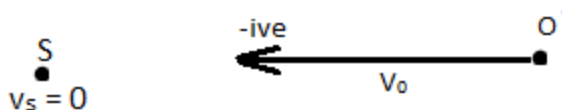
(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound received by the running man in each case?

Take speed of sound in still air = 340 m/s.

Ans.(i) (a) When the man runs towards the engine

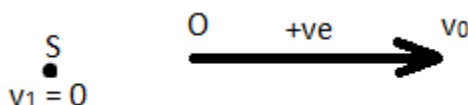
$$V_0 = -10 \text{ m/s}, \quad v_1 = 0$$



$$v' = \frac{v - v_0}{v - v_s} \times v = \frac{340 + 10}{340 - 0} \times 400 = \frac{350}{340} \times 400 = 411.8$$

(b) When the man runs away from the engine

$$V_0 = +10 \text{ m/s}, \quad v_s = 0$$



$$v'' = \frac{v - v_0}{v - v_s} \times v = \frac{340 - 10}{340 - 0} \times 400 = \frac{330}{340} \times 400 = 388.2 \text{ Hz}$$

(ii) (a) When the man runs towards the engine, relative velocity of sound

$$v' = v + v_0 = 340 + 10 = 350 \text{ m/s}$$

(b) When the man runs away from the engine, relative velocity of sound

$$v' = v - v_0 = 340 - 10 = 330 \text{ m/s}.$$

(iii) The wavelength of sound is not affected by the motion of the listener.

Its value is

$$\lambda = \frac{v}{\nu} = 340/400 = 0.85 \text{ m}$$

9. What is a spring factor? Derive the expression for resultant spring constant when two springs having constants k_1 and k_2 are connected in (i) parallel and (ii) in series.

10. Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation is equal to the average potential energy over the same period. At what distance from the mean position is the kinetic energy in simple harmonic oscillator equal potential energy?