Sample Paper–3
Class 11, Mathematics

Time: 3 hours **Max. Marks 100**

General Instructions

1. All questions are compulsory.
2. Use of calculator is not permitted. However you may use log table, if required.
3. Q.No. 1 to 12 are of very short answer type questions, carrying 1 mark each.
4. Q.No.13 to 28 carries 4 marks each.
5. Q.No. 29 to 32 carries 6 marks each.

1. Write the following as intervals:
   (i) $\{x: x \in R, -4 < x \leq 6\}$
   (ii) $\{x: x \in R, -12 < x < -10\}$

2. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

3. State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.
   If P = $\{m, n\}$ and Q = $\{n, m\}$, then P × Q = $\{(m, n), (n, m)\}$.

4. Find the radian measures corresponding to the following degree measures:
   (i) – 47° 30' (ii) 240°

5. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

6. Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

7. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

8. A point is in the XZ-plane. What can you say about its y-coordinate?

9. Evaluate the Given limit:
   $$\lim_{r\to1} \pi r^2$$

10. Evaluate the Given limit:
   $$\lim_{x\to4} \frac{4x+3}{x-2}$$

11. Describe the sample space for the indicated experiment: A die is thrown two times.

12. A die is thrown. Describe the following events:

(i) A: a number less than 7
(ii) B: a number greater than 7
(iii) C: a multiple of 3
(iv) D: a number less than 4
(v) E: an even number greater than 4
(vi) F: a number not less than 3

Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

13. Show that for any sets A and B: A ∪ (B – A) = (A ∪ B)

14. Find the domain and range of the following real function: $f(x) = \sqrt{9 - x^2}$

15. Prove that: (sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

16. Prove the following by using the principle of mathematical induction for all $n \in N$: $41^n - 14^n$ is a multiple of 27.

17. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.

18. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

19. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

20. How many chords can be drawn through 21 points on a circle?

21. Expand using Binomial Theorem $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$.

22. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

23. Find the direction in which a straight line must be drawn through the point (–1, 2) so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

24. A man running a racecourse notes that the sum of the distances from the two flag posts form him is always 10 m and the distance between the flag posts is 8 m. find the equation of the posts traced by the man.

25. A point R with x-coordinate 4 lies on the line segment joining the points P (2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sec x - 1}{\sec x + 1}$$

27. Check the validity of the statements given below by the method given against it.
   (i) p: The sum of an irrational number and a rational number is irrational (by contradiction method).
   (ii) q: If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).

28. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
   (i) the digits are repeated?
   (ii) the repetition of digits is not allowed?

29. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

30. Prove that:

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$$

31. A manufacturer reckons that the value of a machine, which costs him Rs 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

32. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	**Mathematics**	**Physics**	**Chemistry**
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solutions

1. **(i)** $\{x: x \in R, -4 < x \leq 6\} = (-4, 6]$
   (ii) $\{x: x \in R, -12 < x < -10\} = (-12, -10)$

2. G = {7, 8} and H = {5, 4, 2}
   We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as
   $P \times Q = \{(p, q): p \in P, q \in Q\}$
   $\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$
   $H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

3. False
   If $P = \{m, n\}$ and $Q = \{n, m\}$, then
   $P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$

4. (i) $-47° 30'$
   $-47° 30' = -47\frac{1}{2}$ degree [1° = 60']
   $= \frac{-95}{2}$ degree
   Since 180° = π radian
   $\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$
   $\therefore -47° 30' = \frac{-19}{72}\pi \text{ radian}$

   (ii) 240°
   We know that 180° = π radian
   $\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$

5. Number of revolutions made by the wheel in 1 minute = 360
   ∴Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$
   In one complete revolution, the wheel turns an angle of 2π radian.
   Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,
   12 π radian
   Thus, in one second, the wheel turns an angle of 12π radian.

6.

$$\begin{aligned} i^9 + i^{19} &= i^{4\times2+1} + i^{4\times4+3} \\ &= \left(i^4\right)^2 \cdot i + \left(i^4\right)^4 \cdot i^3 \\ &= 1\times i + 1\times(-i) \qquad \left[i^4 = 1,\ i^3 = -i\right] \\ &= i + (-i) \\ &= 0 \end{aligned}$$

7. There will be as many ways as there are ways of filling 3 vacant places □□□ in succession by the given six digits. In this case, the units place can be filled by 2 or 4 or 6 only i.e., the units place can be filled in 3 ways. The tens place can be filled by any of the 6 digits in 6 different ways and also the hundreds place can be filled by any of the 6 digits in 6 different ways, as the digits can be repeated.
Therefore, by multiplication principle, the required number of three digit even numbers is 3 × 6 × 6 = 108

8. If a point is in the XZ plane, then its *y*-coordinate is zero.

9. $\lim\limits_{r\to1} \pi r^2 = \pi\left(1\right)^2 = \pi$

10. $\lim\limits_{x\to4} \dfrac{4x+3}{x-2} = \dfrac{4(4)+3}{4-2} = \dfrac{16+3}{2} = \dfrac{19}{2}$

11. When a die is thrown, the possible outcomes are 1, 2, 3, 4, 5, or 6.
When a die is thrown two times, the sample space is given by S = {(*x*, *y*): *x*, *y* = 1, 2, 3, 4, 5, 6}
The number of elements in this sample space is 6 × 6 = 36, while the sample space is given by:
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

12. When a die is thrown, the sample space is given by S = {1, 2, 3, 4, 5, 6}.
Accordingly:
(i) A = {1, 2, 3, 4, 5, 6}
(ii) B = Φ
(iii) C = {3, 6}
(iv) D = {1, 2, 3}
(v) E = {6}
(vi) F = {3, 4, 5, 6}

A ∪ B = {1, 2, 3, 4, 5, 6}, A ∩ B = Φ
B ∪ C = {3, 6}, E ∩ F = {6}
D ∩ E =Φ, A – C = {1, 2, 4, 5}
D – E = {1, 2, 3}, $F' = \{1, 2\}, E \cap F' = \phi$

13. To prove: A ∪ (B – A) ⊂ A ∪ B
Let x ∈ A ∪ (B – A)
⇒ x ∈ A or x ∈ (B – A)
⇒ x ∈ A or (x ∈ B and x ∉ A)
⇒ (x ∈ A or x ∈ B) and (x ∈ A or x ∉ A)
⇒ x ∈ (A ∪ B)
∴ A ∪ (B – A) ⊂ (A ∪ B) … (3)
Next, we show that (A ∪ B) ⊂ A ∪ (B – A).
Let y ∈ A ∪ B
⇒ y ∈ A or y ∈ B
⇒ (y ∈ A or y ∈ B) and (y ∈ A or y ∉ A)
⇒ y ∈ A or (y ∈ B and y ∉ A)
⇒ y ∈ A ∪ (B – A)
∴ A ∪ B ⊂ A ∪ (B – A) … (4)
Hence, from (3) and (4), we obtain A ∪ (B – A) = A ∪B.

14. $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or [–3, 3].
For any value of x such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3.
∴The range of $f(x)$ is $\{x: 0 \le x \le 3\}$ or [0, 3].

15. L.H.S.
= (sin 3x + sin x) sin x + (cos 3x – cos x) cos x
$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$
$= \cos 3x \cos x + \sin 3x \sin x - \left(\cos^2 x - \sin^2 x\right)$
$= \cos(3x - x) - \cos 2x \qquad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$
$= \cos 2x - \cos 2x$
$= 0$
= RH.S.

16. Let the given statement be P(n), i.e.,
P(n):$41^n - 14^n$is a multiple of 27.
It can be observed that P(n) is true for n = 1 since $41^1 - 14^1 = 27$, which is a multiple of 27.
Let P(k) be true for some positive integer k, i.e.,

$41^k - 14^k$ is a multiple of 27

$\therefore 41^k - 14^k = 27m$, where $m \in \mathbf{N}$ … (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$$41^{k+1} - 14^{k+1}$$
$$= 41^k \cdot 41 - 14^k \cdot 14$$
$$= 41\left(41^k - 14^k + 14^k\right) - 14^k \cdot 14$$
$$= 41\left(41^k - 14^k\right) + 41.14^k - 14^k \cdot 14$$
$$= 41.27m + 14^k\left(41 - 14\right)$$
$$= 41.27m + 27.14^k$$
$$= 27\left(41m - 14^k\right)$$
$$= 27 \times r, \text{ where } r = \left(41m - 14^k\right) \text{ is a natural number}$$

Therefore, $41^{k+1} - 14^{k+1}$ is a multiple of 27.

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

17.

$$|1 - i|^x = 2^x$$
$$\Rightarrow \left(\sqrt{1^2 + (-1)^2}\right)^x = 2^x$$
$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$
$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$
$$\Rightarrow \frac{x}{2} = x$$
$$\Rightarrow x = 2x$$
$$\Rightarrow 2x - x = 0$$
$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

18. Let x litres of water is required to be added.

Then, total mixture = (x + 1125) litres

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres.

This resulting mixture will contain more than 25% but less than 30% acid content.

$\therefore$30% of (1125 + x) > 45% of 1125

And, 25% of (1125 + x) < 45% of 1125

30% of (1125 + x) > 45% of 1125

$\Rightarrow \frac{30}{100}(1125+x) > \frac{45}{100} \times 1125$

$\Rightarrow 30(1125+x) > 45 \times 1125$

$\Rightarrow 30 \times 1125 + 30x > 45 \times 1125$

$\Rightarrow 30x > 45 \times 1125 - 30 \times 1125$

$\Rightarrow 30x > (45-30) \times 1125$

$\Rightarrow x > \frac{15 \times 1125}{30} = 562.5$

25% of (1125 + x) < 45% of 1125

$\Rightarrow \frac{25}{100}(1125+x) < \frac{45}{100} \times 1125$

$\Rightarrow 25(1125+x) > 45 \times 1125$

$\Rightarrow 25 \times 1125 + 25x > 45 \times 1125$

$\Rightarrow 25x > 45 \times 1125 - 25 \times 1125$

$\Rightarrow 25x > (45-25) \times 1125$

$\Rightarrow x > \frac{20 \times 1125}{25} = 900$

$\therefore 562.5 < x < 900$

Thus, the required number of litres of water that is to be added will have to be more than 562.5 but less than 900.

19. From the class of 25 students, 10 are to be chosen for an excursion party.
Since there are 3 students who decide that either all of them will join or none of them will join, there are two cases.
Case I: All the three students join.
Then, the remaining 7 students can be chosen from the remaining 22 students in ${}^{22}C_7$ ways.
Case II: None of the three students join.
Then, 10 students can be chosen from the remaining 22 students in ${}^{22}C_{10}$ ways.
Thus, required number of ways of choosing the excursion party is ${}^{22}C_7 + {}^{22}C_{10}$.

20. For drawing one chord on a circle, only 2 points are required.
To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.
Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.
Thus, required number of chords = ${}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$

21. Using Binomial Theorem, the given expression $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$ can be expanded as

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$= {}^4C_0\left(1+\frac{x}{2}\right)^4 - {}^4C_1\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + {}^4C_2\left(1+\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - {}^4C_3\left(1+\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{2}{x}\right)^4$$

$$=\left(1+\frac{x}{2}\right)^4 - 4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + 6\left(1+x+\frac{x^2}{4}\right)\left(\frac{4}{x^2}\right) - 4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right) + \frac{16}{x^4}$$

$$=\left(1+\frac{x}{2}\right)^4 - \frac{8}{x}\left(1+\frac{x}{2}\right)^3 + \frac{24}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} - \frac{16}{x^2} + \frac{16}{x^4}$$

$$=\left(1+\frac{x}{2}\right)^4 - \frac{8}{x}\left(1+\frac{x}{2}\right)^3 + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \qquad \ldots(1)$$

Again by using Binomial Theorem, we obtain

$$\left(1+\frac{x}{2}\right)^4 = {}^4C_0(1)^4 + {}^4C_1(1)^3\left(\frac{x}{2}\right) + {}^4C_2(1)^2\left(\frac{x}{2}\right)^2 + {}^4C_3(1)^1\left(\frac{x}{2}\right)^3 + {}^4C_4\left(\frac{x}{2}\right)^4$$

$$= 1 + 4\times\frac{x}{2} + 6\times\frac{x^2}{4} + 4\times\frac{x^3}{8} + \frac{x^4}{16}$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} \qquad \ldots(2)$$

$$\left(1+\frac{x}{2}\right)^3 = {}^3C_0(1)^3 + {}^3C_1(1)^2\left(\frac{x}{2}\right) + {}^3C_2(1)\left(\frac{x}{2}\right)^2 + {}^3C_3\left(\frac{x}{2}\right)^3$$

$$= 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8} \qquad \ldots(3)$$

From (1), (2), and (3), we obtain

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - \frac{8}{x}\left(1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8}\right) + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4}$$

$$= 1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{2} + \frac{x^4}{16} - \frac{8}{x} - 12 - 6x - x^2 + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4}$$

$$= \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5$$

22. It is given that A and G are A.M. and G.M. between two positive numbers. Let these two positive numbers be a and b.

$$\therefore AM = A = \frac{a+b}{2} \qquad \ldots(1)$$

$$GM = G = \sqrt{ab} \qquad \ldots(2)$$

From (1) and (2), we obtain

$a + b = 2A$ … (3)

$ab = G^2$ … (4)

Substituting the value of a and b from (3) and (4) in the identity $(a-b)^2 = (a+b)^2 - 4ab$, we obtain

$(a-b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$

$(a-b)^2 = 4(A+G)(A-G)$

$$(a-b) = 2\sqrt{(A+G)(A-G)} \qquad \ldots(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Substituting the value of *a* in (3), we obtain

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

23. Let $y = mx + c$ be the line through point (–1, 2).

Accordingly, $2 = m(-1) + c$.

$\Rightarrow 2 = -m + c$

$\Rightarrow c = m + 2$

$\therefore y = mx + m + 2 \ldots (1)$

The given line is

$x + y = 4 \ldots (2)$

On solving equations (1) and (2), we obtain

$$x = \frac{2-m}{m+1} \text{ and } y = \frac{5m+2}{m+1}$$

$\therefore \left(\frac{2-m}{m+1}, \frac{5m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2).

Since this point is at a distance of 3 units from point (– 1, 2), according to distance formula,

$$\sqrt{\left(\frac{2-m}{m+1}+1\right)^2 + \left(\frac{5m+2}{m+1}-2\right)^2} = 3$$

$$\Rightarrow \left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 = 3^2$$

$$\Rightarrow \frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$$

$$\Rightarrow \frac{1+m^2}{(m+1)^2} = 1$$

$$\Rightarrow 1 + m^2 = m^2 + 1 + 2m$$

$$\Rightarrow 2m = 0$$

$$\Rightarrow m = 0$$

Thus, the slope of the required line must be zero i.e., the line must be parallel to the *x*-axis.

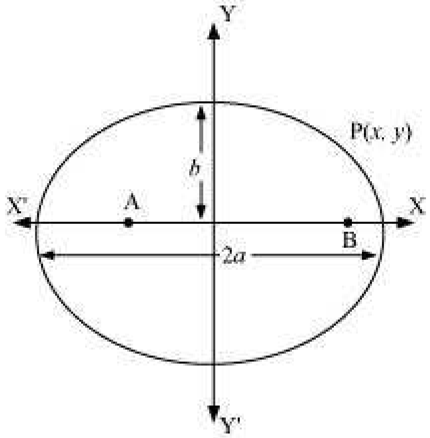
24. Let A and B be the positions of the two flag posts and P(*x*, *y*) be the position of the man.

Accordingly, PA + PB = 10.

We know that if a point moves in a plane in such a way that the sum of its distances from two fixed points is constant, then the path is an ellipse and this constant value is equal to the length of the major axis of the ellipse.

Therefore, the path described by the man is an ellipse where the length of the major axis is 10 m, while points A and B are the foci.

Taking the origin of the coordinate plane as the centre of the ellipse, while taking the major axis along the x-axis, the ellipse can be diagrammatically represented as



The equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis

Accordingly, $2a = 10 \Rightarrow a = 5$

Distance between the foci $(2c) = 8$

$\Rightarrow c = 4$

On using the relation $c=\sqrt{a^2-b^2}$, we obtain

$4=\sqrt{25-b^2}$

$\Rightarrow 16=25-b^2$

$\Rightarrow b^2=25-16=9$

$\Rightarrow b=3$

Thus, the equation of the path traced by the man is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

25. The coordinates of points P and Q are given as P (2, –3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1},\frac{k(0)-3}{k+1},\frac{k(10)+4}{k+1}\right)=\left(\frac{8k+2}{k+1},\frac{-3}{k+1},\frac{10k+4}{k+1}\right)$$

It is given that the x-coordinate of point R is 4.

$\therefore \frac{8k+2}{k+1}=4$

$\Rightarrow 8k+2=4k+4$

$\Rightarrow 4k=2$

$\Rightarrow k=\frac{1}{2}$

Therefore, the coordinates of point R are $\left(4,\frac{-3}{\frac{1}{2}+1},\frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)=(4,-2,6)$

26. Let $f(x)=\frac{\sec x-1}{\sec x+1}$

$$f(x)=\frac{\frac{1}{\cos x}-1}{\frac{1}{\cos x}+1}=\frac{1-\cos x}{1+\cos x}$$

By quotient rule,

$$f'(x)=\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{\sin x+\cos x\sin x+\sin x-\sin x\cos x}{(1+\cos x)^2}$$

$$=\frac{2\sin x}{(1+\cos x)^2}$$

$$=\frac{2\sin x}{\left(1+\frac{1}{\sec x}\right)^2}=\frac{2\sin x}{\frac{(\sec x+1)^2}{\sec^2 x}}$$

$$=\frac{2\sin x\sec^2 x}{(\sec x+1)^2}$$

$$=\frac{\frac{2\sin x}{\cos x}\sec x}{(\sec x+1)^2}$$

$$=\frac{2\sec x\tan x}{(\sec x+1)^2}$$

27. (i) The given statement is as follows.

p: the sum of an irrational number and a rational number is irrational.

Let us assume that the given statement, *p*, is false. That is, we assume that the sum of an irrational number and a rational number is rational.

Therefore, $\sqrt{a}+\frac{b}{c}=\frac{d}{e}$, where $\sqrt{a}$ is irrational and *b*, *c*, *d*, *e* are integers.

$\frac{d}{e}-\frac{b}{c}$ is a rational number and $\sqrt{a}$ is an irrational number.

This is a contradiction. Therefore, our assumption is wrong.

Therefore, the sum of an irrational number and a rational number is rational.

Thus, the given statement is true.

(ii) The given statement, *q*, is as follows.

If n is a real number with $n > 3$, then $n^2 > 9$.

Let us assume that n is a real number with $n > 3$, but $n^2 > 9$ is not true.

That is, $n^2 < 9$

Then, $n > 3$ and n is a real number.

Squaring both the sides, we obtain

$n^2 > (3)^2$

$\Rightarrow n^2 > 9$, which is a contradiction, since we have assumed that $n^2 < 9$.

Thus, the given statement is true. That is, if n is a real number with $n > 3$, then $n^2 > 9$.

28. **(i) When the digits are repeated**

Since four-digit numbers greater than 5000 are formed, the leftmost digit is either 7 or 5. The remaining 3 places can be filled by any of the digits 0, 1, 3, 5, or 7 as repetition of digits is allowed.

∴Total number of 4-digit numbers greater than 5000 = $2 \times 5 \times 5 \times 5 - 1$

$= 250 - 1 = 249$

[In this case, 5000 can not be counted; so 1 is subtracted]

A number is divisible by 5 if the digit at its units place is either 0 or 5.

∴Total number of 4-digit numbers greater than 5000 that are divisible by 5 = $2 \times 5 \times 5 \times 2 - 1 = 100 - 1 = 99$

Thus, the probability of forming a number divisible by 5 when the digits are repeated is $= \frac{99}{249} = \frac{33}{83}$.

(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7.

The remaining 3 places can be filled with any of the remaining 4 digits.

∴Total number of 4-digit numbers greater than 5000 = $2 \times 4 \times 3 \times 2$

$= 48$

When the digit at the thousands place is 5, the units place can be filled only with 0 and the tens and hundreds places can be filled with any two of the remaining 3 digits.

∴Here, number of 4-digit numbers starting with 5 and divisible by 5

$= 3 \times 2 = 6$

When the digit at the thousands place is 7, the units place can be filled in two ways (0 or 5) and the tens and hundreds places can be filled with any two of the remaining 3 digits.

∴ Here, number of 4-digit numbers starting with 7 and divisible by 5

$= 1 \times 2 \times 3 \times 2 = 12$

∴Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 6 + 12 = 18

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed is $\frac{18}{48} = \frac{3}{8}$.

29. Let C denote the set of people who like cricket, and

T denote the set of people who like tennis

∴ $n(C \cup T) = 65$, $n(C) = 40$, $n(C \cap T) = 10$

We know that:

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

∴ $65 = 40 + n(T) - 10$

⇒ $65 = 30 + n(T)$

⇒ $n(T) = 65 - 30 = 35$

Therefore, 35 people like tennis.

Now,

$(T - C) \cup (T \cap C) = T$

Also,

$(T - C) \cap (T \cap C) = \Phi$
$\therefore n(T) = n(T - C) + n(T \cap C)$
$\Rightarrow 35 = n(T - C) + 10$
$\Rightarrow n(T - C) = 35 - 10 = 25$
Thus, 25 people like only tennis.

30. L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$

$= 1 + 1 - 2[\cos(x - y)]$ $\qquad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= 2[1 - \cos(x - y)]$

$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right]$ $\qquad [\cos 2A = 1 - 2\sin^2 A]$

$= 4\sin^2\left(\frac{x - y}{2}\right) = \text{R.H.S.}$

31. Cost of machine = Rs 15625
Machine depreciates by 20% every year.

Therefore, its value after every year is 80% of the original cost i.e., $\frac{4}{5}$ of the original cost.

$\therefore$ Value at the end of 5 years = $\underbrace{15625 \times \frac{4}{5} \times \frac{4}{5} \times \ldots. \times \frac{4}{5}}_{5\text{times}}$ = 5 × 1024 = 5120
Thus, the value of the machine at the end of 5 years is Rs 5120.

32. Standard deviation of Mathematics = 12
Standard deviation of Physics = 15
Standard deviation of Chemistry = 20

The coefficient of variation (C.V.) is given by $\frac{\text{Standard deviation}}{\text{Mean}} \times 100$.

$\text{C.V.(in Mathematics)} = \frac{12}{42} \times 100 = 28.57$

$\text{C.V.(in Physics)} = \frac{15}{32} \times 100 = 46.87$

$\text{C.V.(in Chemistry)} = \frac{20}{40.9} \times 100 = 48.89$

The subject with greater C.V. is more variable than others.
Therefore, the highest variability in marks is in Chemistry and the lowest variability in marks is in Mathematics.