SHORT REVISION
Trigonometric Ratios & Identities

1. BASIC TRIGONOMETRIC IDENTITIES :

(a) $\sin^2\theta + \cos^2\theta = 1$; $-1 \le \sin\theta \le 1$;
$-1 \le \cos\theta \le 1 \quad \forall \quad \theta \in R$

(b) $\sec^2\theta - \tan^2\theta = 1$; $|\sec\theta| \ge 1 \quad \forall \quad \theta \in R$

(c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$; $|\operatorname{cosec}\theta| \ge 1 \quad \forall \quad \theta \in R$

2. IMPORTANT T′ RATIOS:

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in I$

(b) $\sin\frac{(2n+1)\pi}{2} = (-1)^n$ & $\cos\frac{(2n+1)\pi}{2} = 0$ where $n \in I$

(c) $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$;

$\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$;

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$; $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$

(d) $\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$; $\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$; $\tan\frac{\pi}{8} = \sqrt{2}-1$; $\tan\frac{3\pi}{8} = \sqrt{2}+1$

(e) $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$ & $\cos 36°$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

3. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

If θ is any angle, then $-\theta,\ 90 \pm \theta,\ 180 \pm \theta,\ 270 \pm \theta,\ 360 \pm \theta$ etc. are called **ALLIED ANGLES**.

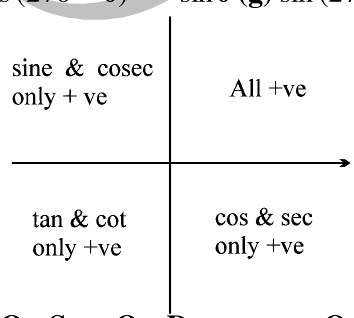
(a) $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

(b) $\sin(90°-\theta) = \cos\theta$; $\cos(90°-\theta) = \sin\theta$

(c) $\sin(90°+\theta) = \cos\theta$; $\cos(90°+\theta) = -\sin\theta$ **(d)** $\sin(180°-\theta) = \sin\theta$; $\cos(180°-\theta) = -\cos\theta$

(e) $\sin(180°+\theta) = -\sin\theta$; $\cos(180°+\theta) = -\cos\theta$

(f) $\sin(270°-\theta) = -\cos\theta$; $\cos(270°-\theta) = -\sin\theta$ **(g)** $\sin(270°+\theta) = -\cos\theta$; $\cos(270°+\theta) = \sin\theta$



4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES :

(a) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ **(b)** $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\,.\,\sin(A-B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\,.\,\cos(A-B)$

(e) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ **(f)** $\cot(A \pm B) = \frac{\cot A\cot B \mp 1}{\cot B \pm \cot A}$

5. FACTORISATION OF THE SUM OR DIFFERENCE OF TWO SINES OR COSINES :

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$ **(b)** $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$ **(d)** $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

6. TRANSFORMATION OF PRODUCTS INTO SUM OR DIFFERENCE OF SINES & COSINES :

(a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$ **(b)** $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$ **(d)** $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

7. MULTIPLE ANGLES AND HALF ANGLES :

(a) $\sin 2A = 2\sin A\cos A$; $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$;
$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$.
$2\cos^2 A = 1 + \cos 2A$, $2\sin^2 A = 1 - \cos 2A$; $\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$
$2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$.

(c) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$; $\tan\theta = \frac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}$

(d) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ **(e)** $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$ **(g)** $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

8. THREE ANGLES :

(a) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

NOTE IF : **(i)** $A+B+C = \pi$ then $\tan A + \tan B + \tan C = \tan A\tan B\tan C$
(ii) $A+B+C = \frac{\pi}{2}$ then $\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

(b) If $A + B + C = \pi$ then : **(i)** $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$
(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

9. MAXIMUM & MINIMUM VALUES OF TRIGONOMETRIC FUNCTIONS:

(a) Min. value of $a^2\tan^2\theta + b^2\cot^2\theta = 2ab$ where $\theta \in R$

(b) Max. and Min. value of $a\cos\theta + b\sin\theta$ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

(c) If $f(\theta) = a\cos(\alpha+\theta) + b\cos(\beta+\theta)$ where a, b, α and β are known quantities then
$-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)} \le f(\theta) \le \sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$

(d) If $\alpha,\beta \in \left(0,\frac{\pi}{2}\right)$ and $\alpha+\beta = \sigma$ (constant) then the maximum values of the expression
$\cos\alpha\cos\beta$, $\cos\alpha + \cos\beta$, $\sin\alpha + \sin\beta$ and $\sin\alpha\sin\beta$
occurs when $\alpha = \beta = \sigma/2$.

(e) If $\alpha,\beta \in \left(0,\frac{\pi}{2}\right)$ and $\alpha+\beta = \sigma$(constant) then the minimum values of the expression
$\sec\alpha + \sec\beta$, $\tan\alpha + \tan\beta$, $\operatorname{cosec}\alpha + \operatorname{cosec}\beta$ occurs when $\alpha = \beta = \sigma/2$.

(f) If A, B, C are the angles of a triangle then maximum value of
$\sin A + \sin B + \sin C$ and $\sin A\sin B\sin C$ occurs when $A = B = C = 60^0$

(g) In case a quadratic in $\sin\theta$ or $\cos\theta$ is given then the maximum or minimum values can be interpreted by making a perfect square.

10. Sum of sines or cosines of n angles,

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\,\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\,\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

EXERCISE–I

Q.1 Prove that $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha\cos\beta\cos(\alpha+\beta) = \sin^2\beta$

Q.2 Prove that $\cos 2\alpha = 2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$

Q.3 Prove that , $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.

Q.4 Prove that : (a) $\tan 20° . \tan 40° . \tan 60° . \tan 80° = 3$

(b) $\tan 9° - \tan 27° - \tan 63° + \tan 81° = 4$. (c) $\sin^4\frac{\pi}{16} + \sin^4\frac{3\pi}{16} + \sin^4\frac{5\pi}{16} + \sin^4\frac{7\pi}{16} = \frac{3}{2}$

Q.5 Calculate without using trigonometric tables :

(a) $\operatorname{cosec} 10° - \sqrt{3}\sec 10°$ (b) $4\cos 20° - \sqrt{3}\cot 20°$ (c) $\frac{2\cos 40° - \cos 20°}{\sin 20°}$

(d) $2\sqrt{2}\sin 10°\left[\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right]$ (e) $\cos^6\frac{\pi}{16} + \cos^6\frac{3\pi}{16} + \cos^6\frac{5\pi}{16} + \cos^6\frac{7\pi}{16}$

(f) $\tan 10° - \tan 50° + \tan 70°$

Q.6(a) If $X = \sin\left(\theta+\frac{7\pi}{12}\right) + \sin\left(\theta-\frac{\pi}{12}\right) + \sin\left(\theta+\frac{3\pi}{12}\right)$, $Y = \cos\left(\theta+\frac{7\pi}{12}\right) + \cos\left(\theta-\frac{\pi}{12}\right) + \cos\left(\theta+\frac{3\pi}{12}\right)$

then prove that $\frac{X}{Y} - \frac{Y}{X} = 2\tan 2\theta$.

(b) Prove that $\sin^2 12° + \sin^2 21° + \sin^2 39° + \sin^2 48° = 1 + \sin^2 9° + \sin^2 18°$.

Q.7 Show that : (a) $\cot 7\frac{1°}{2}$ or $\tan 82\frac{1°}{2} = \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

(b) $\tan 142\frac{1°}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$.

Q.8 If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, show that $\cos 2\theta = \frac{m+n}{2(m-n)}$.

Q.9 If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$, prove that $\frac{\sin y}{\sin x} = \frac{3+\sin^2 x}{1+3\sin^2 x}$.

Q.10 If $\cos(\alpha+\beta) = \frac{4}{5}$; $\sin(\alpha - \beta) = \frac{5}{13}$ & α, β lie between 0 & $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$.

Q.11 Prove that if the angles α & β satisfy the relation $\frac{\sin\beta}{\sin(2\alpha+\beta)} = \frac{n}{m}\left(|m|>|n|\right)$ then $\frac{1+\frac{\tan\beta}{\tan\alpha}}{m+n} = \frac{1-\tan\alpha\tan\beta}{m-n}$.

Q.12 (a) If $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$, then find the greatest & least value of y .

(b) If $y = 1 + 2\sin x + 3\cos^2 x$, find the maximum & minimum values of y $\forall$ $x \in R$.

(c) If $y = 9\sec^2 x + 16\,\text{cosec}^2 x$, find the minimum value of $y \;\forall\; x \in R$.

(d) Prove that $3\cos\left(\theta+\frac{\pi}{3}\right) + 5\cos\theta + 3$ lies from -4 & 10 .

(e) Prove that $\left(2\sqrt{3}+4\right)\sin\theta + 4\cos\theta$ lies between $-2\left(2+\sqrt{5}\right)$ & $2\left(2+\sqrt{5}\right)$.

Q.13 If $A + B + C = \pi$, prove that $\sum\left(\frac{\tan A}{\tan B.\tan C}\right) = \sum(\tan A) - 2\sum(\cot A)$.

Q.14 If $\alpha + \beta = c$ where $\alpha, \beta > 0$ each lying between 0 and $\pi/2$ and c is a constant, find the maximum or minimum value of

(a) $\sin\alpha + \sin\beta$ (b) $\sin\alpha\sin\beta$

(c) $\tan\alpha + \tan\beta$ (d) $\text{cosec}\,\alpha + \text{cosec}\,\beta$

Q.15 Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n-sided regular polygon such that ;
$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$. Find the value of n.

Q.16 Prove that : $\text{cosec}\,\theta + \text{cosec}\,2\theta + \text{cosec}\,2^2\theta + \ldots\ldots + \text{cosec}\,2^{n-1}\theta = \cot(\theta/2) - \cot 2^{n-1}\theta$

Q.17 For all values of α, β, γ prove that;

$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\frac{\alpha+\beta}{2}.\cos\frac{\beta+\gamma}{2}.\cos\frac{\gamma+\alpha}{2}$.

Q.18 Show that $\frac{1+\sin A}{\cos A} + \frac{\cos B}{1-\sin B} = \frac{2\sin A - 2\sin B}{\sin(A-B)+\cos A - \cos B}$.

Q.19 If $\tan\beta = \frac{\tan\alpha + \tan\gamma}{1+\tan\alpha\,.\,\tan\gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha + \sin 2\gamma}{1+\sin 2\alpha\,.\,\sin 2\gamma}$.

Q.20 If $\alpha + \beta = \gamma$, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

Q.21 If $\alpha + \beta + \gamma = \frac{\pi}{2}$, show that $\frac{\left(1-\tan\frac{\alpha}{2}\right)\left(1-\tan\frac{\beta}{2}\right)\left(1-\tan\frac{\gamma}{2}\right)}{\left(1+\tan\frac{\alpha}{2}\right)\left(1+\tan\frac{\beta}{2}\right)\left(1+\tan\frac{\gamma}{2}\right)} = \frac{\sin\alpha + \sin\beta + \sin\gamma - 1}{\cos\alpha + \cos\beta + \cos\gamma}$.

Q.22 If $A + B + C = \pi$ and $\cot\theta = \cot A + \cot B + \cot C$, show that ,
$\sin(A-\theta)\,.\,\sin(B-\theta)\,.\,\sin(C-\theta) = \sin^3\theta$.

Q.23 If $P = \cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots\ldots + \cos\frac{17\pi}{19}$ and

$Q = \cos\frac{2\pi}{21} + \cos\frac{4\pi}{21} + \cos\frac{6\pi}{21} + \ldots\ldots\ldots + \cos\frac{20\pi}{21}$, then find $P - Q$.

Q.24 If A, B, C denote the angles of a triangle ABC then prove that the triangle is right angled if and only if $\sin 4A + \sin 4B + \sin 4C = 0$.

Q.25 Given that $(1 + \tan 1°)(1 + \tan 2°)\ldots\ldots(1 + \tan 45°) = 2^n$, find n.

EXERCISE–II

Q.1 If $\tan\alpha = p/q$ where $\alpha = 6\beta$, α being an acute angle, prove that;
$\frac{1}{2}(p\,\text{cosec}\,2\beta - q\sec 2\beta) = \sqrt{p^2+q^2}$.

Q.2 Let $A_1, A_2, A_3 \ldots\ldots\ldots\ldots A_n$ are the vertices of a regular n sided polygon inscribed in a circle of radius R. If $(A_1A_2)^2 + (A_1A_3)^2 + \ldots\ldots\ldots + (A_1A_n)^2 = 14R^2$, find the number of sides in the polygon.

Q.3 Prove that: $\dfrac{\cos 3\theta + \cos 3\phi}{2\cos(\theta-\phi)-1} = (\cos\theta + \cos\phi)\cos(\theta+\phi) - (\sin\theta+\sin\phi)\sin(\theta+\phi)$

Q.4 Without using the surd value for $\sin 18^0$ or $\cos 36^0$, prove that $4\sin 36^0 \cos 18^0 = \sqrt{5}$

Q.5 Show that, $\dfrac{\sin x}{\cos 3x}+\dfrac{\sin 3x}{\cos 9x}+\dfrac{\sin 9x}{\cos 27x}=\dfrac{1}{2}(\tan 27x - \tan x)$

Q.6 Let $x_1 = \prod_{r=1}^{5}\cos\dfrac{r\pi}{11}$ and $x_2 = \sum_{r=1}^{5}\cos\dfrac{r\pi}{11}$, then show that

$x_1 \cdot x_2 = \dfrac{1}{64}\left(\operatorname{cosec}\dfrac{\pi}{22}-1\right)$, where Π denotes the continued product.

Q.7 If $\theta = \dfrac{2\pi}{7}$, prove that $\tan\theta\,.\,\tan 2\theta + \tan 2\theta\,.\,\tan 4\theta + \tan 4\theta\,.\,\tan\theta = -7$.

Q.8 For $0 < x < \dfrac{\pi}{4}$ prove that, $\dfrac{\cos x}{\sin^2 x(\cos x - \sin x)} > 8$.

Q.9 (a) If $\alpha = \dfrac{2\pi}{7}$ prove that, $\sin\alpha + \sin 2\alpha + \sin 4\alpha = \dfrac{\sqrt{7}}{2}$ (b) $\sin\dfrac{\pi}{7}\,.\,\sin\dfrac{2\pi}{7}\,.\,\sin\dfrac{3\pi}{7} = \dfrac{\sqrt{7}}{8}$

Q.10 Let $k = 1°$, then prove that $\sum_{n=0}^{88}\dfrac{1}{\cos nk\cdot\cos(n+1)k} = \dfrac{\cos k}{\sin^2 k}$

Q.11 Prove that the value of $\cos A + \cos B + \cos C$ lies between 1 & $\dfrac{3}{2}$ where $A + B + C = \pi$.

Q.12 If $\cos A = \tan B$, $\cos B = \tan C$ and $\cos C = \tan A$, then prove that $\sin A = \sin B = \sin C = 2\sin 18°$.

Q.13 Show that $\dfrac{3+\cos x}{\sin x}$ $\forall x \in R$ can not have any value between $-2\sqrt{2}$ and $2\sqrt{2}$. What inference can you draw about the values of $\dfrac{\sin x}{3+\cos x}$?

Q.14 If $(1 + \sin t)(1 + \cos t) = \dfrac{5}{4}$. Find the value of $(1 - \sin t)(1 - \cos t)$.

Q.15 Prove that from the equality $\dfrac{\sin^4\alpha}{a}+\dfrac{\cos^4\alpha}{b}=\dfrac{1}{a+b}$ follows the relation ; $\dfrac{\sin^8\alpha}{a^3}+\dfrac{\cos^8\alpha}{b^3}=\dfrac{1}{(a+b)^3}$.

Q.16 Prove that the triangle ABC is equilateral iff, $\cot A + \cot B + \cot C = \sqrt{3}$.

Q.17 Prove that the average of the numbers $n\sin n°$, $n = 2, 4, 6, \ldots\ldots, 180$, is $\cot 1°$.

Q.18 Prove that : $4\sin 27° = \left(5+\sqrt{5}\right)^{1/2} - \left(3-\sqrt{5}\right)^{1/2}$.

Q.19 If A+B+C = π; prove that $\tan^2\dfrac{A}{2}+\tan^2\dfrac{B}{2}+\tan^2\dfrac{C}{2} \ge 1$.

Q.20 If A+B+C = π (A , B , C > 0), prove that $\sin\dfrac{A}{2}\,.\,\sin\dfrac{B}{2}\,.\,\sin\dfrac{C}{2} \le \dfrac{1}{8}$.

Q.21 Show that elliminating x & y from the equations, $\sin x + \sin y = a$; $\cos x + \cos y = b$ & $\tan x + \tan y = c$ gives $\dfrac{8ab}{\left(a^2+b^2\right)^2-4a^2} = c$.

Q.22 Determine the smallest positive value of x (in degrees) for which
$\tan(x + 100°) = \tan(x + 50°)\tan x\ \tan(x - 50°)$.

Q.23 Evaluate : $\sum_{n=1}^{n}\dfrac{\tan\dfrac{x}{2^n}}{2^{n-1}\cos\dfrac{x}{2^{n-1}}}$

Q.24 If $\alpha+\beta+\gamma=\pi$ & $\tan\left(\dfrac{\beta+\gamma-\alpha}{4}\right)\cdot\tan\left(\dfrac{\gamma+\alpha-\beta}{4}\right)\cdot\tan\left(\dfrac{\alpha+\beta-\gamma}{4}\right) = 1$, then prove that; $1 + \cos\alpha + \cos\beta + \cos\gamma = 0$.

Q.25 $\forall x \in R$, find the range of the function, $f(x) = \cos x\left(\sin x + \sqrt{\sin^2 x + \sin^2\alpha}\right)$; $\alpha \in [0, \pi]$

EXERCISE–III

Q.1 $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$ is true if and only if : [JEE '96, 1]
(A) $x + y \ne 0$ (B) $x = y,\ x \ne 0$ (C) $x = y$ (D) $x \ne 0,\ y \ne 0$

Q.2 (a) Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r\theta$, for every value of θ, then :
(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$

(C) $b_0 = -1,\ b_1 = n$ (D) $b_0 = 0,\ b_1 = n^2 - 3n + 3$

(b) Let $A_0 A_1 A_2 A_3 A_4 A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 & A_0A_4 is :

(A) $\frac{3}{4}$ (B) $3\sqrt{3}$ (C) 3 (D) $\frac{3\sqrt{3}}{2}$

(c) Which of the following number(s) is/are rational ? [JEE '98, 2 + 2 + 2 = 6 out of 200]

(A) $\sin 15°$ (B) $\cos 15°$ (C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

Q.3 For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ ....\ (1+\sec 2^n\theta)$ Then

(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$ [JEE '99,3]

Q.4(a) Let $f(\theta) = \sin\theta\ (\sin\theta + \sin 3\theta)$. Then $f(\theta)$: [JEE 2000 Screening. 1 out of 35]

(A) ≥ 0 only when $\theta \geq 0$ (B) ≤ 0 for all real θ

(C) ≥ 0 for all real θ (D) ≤ 0 only when $\theta \leq 0$.

(b) In any triangle ABC, prove that, $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\ \cot\frac{B}{2}\ \cot\frac{C}{2}$. [JEE 2000]

Q.5(a) Find the maximum and minimum values of $27^{\cos 2x} \cdot 81^{\sin 2x}$.

(b) Find the smallest positive values of x & y satisfying, $x - y = \frac{\pi}{4}$, $\cot x + \cot y = 2$. [REE 2000, 3]

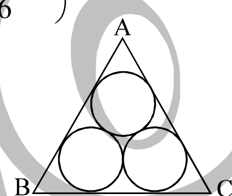
Q.6 If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$ then $\tan\alpha$ equals [JEE 2001 (Screening), 1 out of 35]

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$ (C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

Q.7 If θ and ϕ are acute angles satisfying $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then $\theta + \phi \in$ [JEE 2004 (Screening)]

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6}, \pi\right)$

Q.8 In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is



(A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$

(C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

[JEE 2005 (Screening)]

Q.9 Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$, then

(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_4 > t_3 > t_1 > t_2$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

[JEE 2006, 3]

ANSWER SHEET (*EXERCISE–I*)

Q 5. (a) 4 **(b)** −1 **(c)** $\sqrt{3}$ **(d)** 4 **(e)** $\frac{5}{4}$ **(f)** $\sqrt{3}$ **Q 10.** $\frac{56}{33}$

Q 12. (a) $y_{max} = 11$; $y_{min} = 1$ **(b)** $y_{max} = \frac{13}{3}$; $y_{min} = -1$, (c) 49

Q14. (a) max = 2 sin (c/2), (b) max. = $\sin^2$ (c/2), (c) min = 2 tan (c/2), (d) min = 2 cosec (c/2)

Q 15. n = 7 **Q23.** 1 Q.25 $n = 23$

EXERCISE –II

Q.2 n = 7 **Q.13** $\left[-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$ **Q.14** $\frac{13}{4} - \sqrt{10}$ **Q.22** x = 30°

Q 23. $\frac{2}{\sin 2x} - \frac{1}{2^{n-1}\sin\frac{x}{2^{n-1}}}$ **Q.25** $-\sqrt{1+\sin^2\alpha} \leq y \leq \sqrt{1+\sin^2\alpha}$

EXERCISE–III

Q.1 B **Q.2 (a)** B, **(b)** C, **(c)** C **Q.3** A, B, C, D **Q.4 (a)** C

Q.5 (a) max. = 3^5 & min. = 3^{-5} ; (b) $x = \frac{5\pi}{12}$; $y = \frac{\pi}{6}$ **Q.6** C **Q.7** B

Q.8 B **Q.9** B

EXERCISE–IV (Objective)

Part : (A) Only one correct option

1. $\dfrac{\tan\left(x-\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$ when simplified reduces to:
(A) $\sin x \cos x$ (B) $-\sin^2 x$ (C) $-\sin x \cos x$ (D) $\sin^2 x$

2. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right]$ is equal to
(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha + \sin 6\alpha$

3. If tan A & tan B are the roots of the quadratic equation $x^2 - ax + b = 0$, then the value of $\sin^2(A + B)$.
(A) $\dfrac{a^2}{a^2+(1-b)^2}$ (B) $\dfrac{a^2}{a^2+b^2}$ (C) $\dfrac{a^2}{(b+c)^2}$ (D) $\dfrac{a^2}{b^2(1-a)^2}$

4. The value of $\log_2[\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)-\cos 2\alpha.\cos 2\beta]$:
(A) depends on α & β both (B) depends on α but not on β
(C) depends on β but not on α (D) independent of both α & β.

5. $\dfrac{\cos 20°+8\sin 70°\sin 50°\sin 10°}{\sin^2 80°}$ is equal to:
(A) 1 (B) 2 (C) 3/4 (D) none

6. If $\cos A = 3/4$, then the value of $16\cos^2(A/2) - 32\sin(A/2)\sin(5A/2)$ is
(A) – 4 (B) – 3 (C) 3 (D) 4

7. If $y = \cos^2(45º + x) + (\sin x - \cos x)^2$ then the maximum & minimum values of y are:
(A) 2 & 0 (B) 3 & 0 (C) 3 & 1 (D) none

8. The value of $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$ is equal to:
(A) 1/2 (B) 0 (C) 1 (D) none

9. The greatest and least value of $\log_{\sqrt{2}}\left(\sin x - \cos x + 3\sqrt{2}\right)$ are respectively:
(A) 2 & 1 (B) 5 & 3 (C) 7 & 5 (D) 9 & 7

10. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are
(A) $\frac{\pi}{3}$ & $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

11. $\dfrac{1}{\cos 290°} + \dfrac{1}{\sqrt{3}\sin 250°} =$
(A) $\dfrac{2\sqrt{3}}{3}$ (B) $\dfrac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

12. If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \dfrac{1}{\sin^2\alpha}}$ is equal to
(A) $1 + \cot\alpha$ (B) $-1 - \cot\alpha$ (C) $1 - \cot\alpha$ (D) $-1 + \cot\alpha$

13. If $x \in \left(\pi, \frac{3\pi}{2}\right)$ then $4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$ is always equal to
(A) 1 (B) 2 (C) – 2 (D) none of these

14. If $2\cos x + \sin x = 1$, then value of $7\cos x + 6\sin x$ is equal to
(A) 2 or 6 (B) 1 or 3 (C) 2 or 3 (D) none of these

15. If cosec A + cot A = $\frac{11}{2}$, then tan A is
(A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{43}$

16. If $\cot\alpha + \tan\alpha = m$ and $\dfrac{1}{\cos\alpha} - \cos\alpha = n$, then
(A) $m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$ (B) $m(m^2n)^{1/3} - n(nm^2)^{1/3} = 1$
(C) $n(mn^2)^{1/3} - m(nm^2)^{1/3} = 1$ (D) $n(m^2n)^{1/3}\ m(mn^2)^{1/3} = 1$

17. The expression $\dfrac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to
(A) cos 2x (B) 2 cos x (C) $\cos^2 x$ (D) 1 + cos x

18. If $\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}$ and $\dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then tan A + tan B is equal to
(A) $\sqrt{3}/\sqrt{5}$ (B) $\sqrt{5}/\sqrt{3}$ (C) 1 (D) $(\sqrt{5}+\sqrt{3})/\sqrt{5}$

19. If $\sin 2\theta = k$, then the value of $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$ is equal to
(A) $\dfrac{1-k^2}{k}$ (B) $\dfrac{2-k^2}{k}$ (C) $k^2 + 1$ (D) $2 - k^2$

Part : (B) May have more than one options correct

20. Which of the following is correct ?

(A) $\sin 1° > \sin 1$ (B) $\sin 1° < \sin 1$ (C) $\cos 1° > \cos 1$ (D) $\cos 1° < \cos 1$

21. If $3 \sin \beta = \sin (2\alpha + \beta)$, then $\tan (\alpha + \beta) - 2 \tan \alpha$ is

(A) independent of α (B) independent of β

(C) dependent of both α and β (D) independent of α but dependent of β

22. It is known that $\sin \beta = \frac{4}{5}$ & $0 < \beta < \pi$ then the value of $\dfrac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos\frac{\pi}{6}}\cos(\alpha+\beta)}{\sin\alpha}$ is:

(A) independent of α for all β in $(0, \pi)$ (B) $\frac{5}{\sqrt{3}}$ for $\tan \beta > 0$

(C) $\frac{\sqrt{3}(7+24\cot\alpha)}{15}$ for $\tan \beta < 0$ (D) none

23. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha+\beta)\}$, then the length of the hypotenuse is:

(A) $2[1+\cos(\alpha-\beta)]$ (B) $2[1-\cos(\alpha+\beta)]$ (C) $4\cos^2\frac{\alpha-\beta}{2}$ (D) $4\sin^2\frac{\alpha+\beta}{2}$

24. If $x = \sec\phi - \tan\phi$ & $y = \operatorname{cosec}\phi + \cot\phi$ then:

(A) $x = \frac{y+1}{y-1}$ (B) $y = \frac{1+x}{1-x}$ (C) $x = \frac{y-1}{y+1}$ (D) $xy + x - y + 1 = 0$

25. $(a + 2) \sin \alpha + (2a - 1) \cos \alpha = (2a + 1)$ if $\tan \alpha =$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

26. If $\tan x = \frac{2b}{a-c}$, $(a \neq c)$

$y = a\cos^2x + 2b\sin x\cos x + c\sin^2x$

$z = a\sin^2x - 2b\sin x\cos x + c\cos^2x$, then

(A) $y = z$ (B) $y + z = a + c$ (C) $y - z = a - c$ (D) $y - z = (a-c)^2 + 4b^2$

27. $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$

(A) $2\tan^n\frac{A-B}{2}$ (B) $2\cot^n\frac{A-B}{2}$: n is even (C) 0 : n is odd (D) none

28. The equation $\sin^6x + \cos^6x = a^2$ has real solution if

(A) $a \in (-1, 1)$ (B) $a \in \left(-1, -\frac{1}{2}\right)$ (C) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $a \in \left(\frac{1}{2}, 1\right)$

EXERCISE–IV (Subjective)

1. The minute hand of a watch is 1.5 cm long. How far does its tip move in 50 minutes? (Use $\pi = 3.14$).

2. If the arcs of the same length in two circles subtend angles 75° and 120° at the centre, find the ratio of their radii.

3. Sketch the following graphs :

(i) $y = 3 \sin 2x$ (ii) $y = 2 \tan x$ (iii) $y = \sin \frac{x}{2}$

4. Prove that $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1$.

5. Prove that $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$.

6. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$.

7. prove that $\left\{\dfrac{1-\cot^2\left(\frac{\alpha-\pi}{4}\right)}{1+\cot^2\left(\frac{\alpha-\pi}{4}\right)} + \cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2} = \operatorname{cosec} 4\alpha$.

8. Prove that, $\sin 3x . \sin^3 x + \cos 3x . \cos^3 x = \cos^3 2x$.

9. If $\tan\alpha = \frac{p}{q}$ where $\alpha = 6\beta$, α being an acute angle, prove that; $\frac{1}{2}(p\operatorname{cosec} 2\beta - q\sec 2\beta) = \sqrt{p^2+q^2}$.

10. If $\tan\beta = \frac{\tan\alpha+\tan\gamma}{1+\tan\alpha.\tan\gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha+\sin 2\gamma}{1+\sin 2\alpha.\sin 2\gamma}$.

11. Show that: (i) $\cot 7\frac{1°}{2}$ or $\tan 82\frac{1°}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

(ii) $\tan 142\frac{1°}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$. (iii) $4\sin 27° = (5+\sqrt{5})^{1/2} - (3-\sqrt{5})^{1/2}$

12. Prove that, $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.

13. If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = \frac{-3}{2}$, prove that
$\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$.

14. Prove that from the equality $\frac{\sin^4\alpha}{a} + \frac{\cos^4\alpha}{b} = \frac{1}{a+b}$ follows the relation $\frac{\sin^8\alpha}{a^3} + \frac{\cos^8\alpha}{b^3} = \frac{1}{(a+b)^3}$

15. Prove that: $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \operatorname{cosec}2^2\theta + \ldots + \operatorname{cosec}2^{n-1}\theta = \cot(\theta/2) - \cot 2^{n-1}\theta$. Hence or otherwise prove that $\operatorname{cosec}\frac{4\pi}{15} + \operatorname{cosec}\frac{8\pi}{15} + \operatorname{cosec}\frac{16\pi}{15} + \operatorname{cosec}\frac{32\pi}{15} = 0$

16. Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n–sided regular polygon such that; $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$. Find the value of n.

17. If $A + B + C = \pi$, then prove that

(i) $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1$ (ii) $\sin\frac{A}{2} . \sin\frac{B}{2} . \sin\frac{C}{2} \leq \frac{1}{8}$.

(iii) $\cos A + \cos B + \cos C \leq \frac{3}{2}$

18. If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$, $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$. Show that $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

19. If $P_n = \cos^n\theta + \sin^n\theta$ and $Q_n = \cos^n\theta - \sin^n\theta$, then show that
$P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\, P_{n-4}$
$Q_n - Q_{n-2} = -\sin^2\theta\cos^2\theta\, Q_{n-4}$ and hence show that
$P_4 = 1 - 2\sin^2\theta\cos^2\theta$, $Q_4 = \cos^2\theta - \sin^2\theta$

20. If $\sin(\theta+\alpha) = a$ & $\sin(\theta+\beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of
$\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta)$

21. If $A + B + C = \pi$, prove that
$\tan B\tan C + \tan C\tan A + \tan A\tan B = 1 + \sec A . \sec B . \sec C$.

22. If $\tan^2\alpha + 2\tan\alpha . \tan 2\beta = \tan^2\beta + 2\tan\beta . \tan 2\alpha$, then prove that each side is equal to 1 or $\tan\alpha = \pm\tan\beta$.

Answers

EXERCISE–IV

1. D **2.** B **3.** A **4.** D **5.** B **6.** C **7.** B
8. A **9.** B **10.** B **11.** B **12.** B **13.** B **14.** A
15. C **16.** A **17.** B **18.** D **19.** B **20.** BC
21. AB **22.** BC **23.** AC **24.** BCD **25.** BD **26.** BC
27. BC **28.** BD

EXERCISE–V

1. 7.85 cm **2.** $r_1 : r_2 = 8 : 5$

6. $\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$ and $\cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$

16. n = 7 **20.** $1 - 2a^2 - 2b^2$