

Chapter 13

LIMITS AND DERIVATIVES

Some standard results on limits

- 1) $\lim_{x \rightarrow c} f(x) = l$ iff $\lim_{x \rightarrow c-} f(x) = \lim_{x \rightarrow c+} f(x) = l$
- 2) $\lim_{x \rightarrow a} k = k$ where k is a fixed real number.
- 3) $\lim_{x \rightarrow c} f(x) = f(c)$ where $f(x)$ is a real polynomial in x .
- 4) $\lim_{x \rightarrow c} x^n = c^n$ for all $n \in \mathbb{N}$
- 5) $\lim_{x \rightarrow c} |x| = |c|$

Algebra of limits

Refer : NCERT Textbook page no: 292.

Evaluation of algebraic limits

Type I (Direct substitution method)

Eg: Evaluate $\lim_{x \rightarrow 1} (x^3 - x^2 + 1) = 1^3 - 1^2 + 1 = 1$

Evaluate the following limits:

NCERT Textbook Exercise 13.1 Q 1,2,3,4,5,9,11,12.

Additional Questions and HOT Questions

** 1) $\lim_{x \rightarrow -1} (1 + x + x^2 + \dots + x^{10})$ Ans: 1

** 2) $\lim_{x \rightarrow -1} (x^5 + 2x^8)^{60}$ Ans: 1

** 3) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ Ans:2

Type II (Factorisation method)

Eg : Evaluate : $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} \quad \left(\frac{0}{0} \text{ form} \right) &= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (x^2 + x + 1) \\ &= 1^2 + 1 + 1 = 3 \end{aligned}$$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 7,8 , Example 2.

Evaluate:

$$** \quad 1) \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} \quad \text{ans: } \frac{-1}{10}$$

$$** \quad 2) \lim_{x \rightarrow \sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}x-8} \quad \text{ans: } \frac{8}{5}$$

$$** \quad 3) \lim_{x \rightarrow 2} \frac{x^3-4x^2+4x}{x^2-4} \quad \text{ans: } 0$$

Type III

For any positive integer n, $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$ (This is applicable for any rational number n and for $a > 0$).

Evaluate : $\lim_{x \rightarrow 2} \frac{x^4-16}{x-2}$

$$\text{Ans: } \lim_{x \rightarrow 2} \frac{x^4-16}{x-2} = \lim_{x \rightarrow 2} \frac{x^4-2^4}{x-2} = 4 \cdot 2^{4-1} = 4 \cdot 2^3 = 32$$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 10 , Example 3.

$$1) \quad \text{Evaluate : } \lim_{x \rightarrow 2} \frac{x^{10}-1024}{x^5-32} \quad \text{Ans: } 64$$

$$2) \quad \text{If } \lim_{x \rightarrow -a} \frac{x^9+a^9}{x+a} = 9, \text{ find the value of } a.$$

Type IV

Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$

$$\text{Ans: } \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{1+x-1}$$

$$\text{Put } y = 1+x \text{ when } x \rightarrow 0, y \rightarrow 1$$

$$\therefore \text{ value} = \lim_{y \rightarrow 1} \frac{\sqrt{y}-1}{y-1}$$

$$= \lim_{y \rightarrow 1} \frac{y^{1/2} - 1^{1/2}}{y-1}$$

$$= \frac{1}{2}$$

Evaluate

1) NCERT Ex 13.1 Q 6

$$2) \lim_{x \rightarrow 0} \frac{(1-x)^n - 1}{x} \quad \text{ans} = -n$$

$$3) \lim_{x \rightarrow 0} \frac{(1+x)^6 - 1}{(1+x)^5 - 1} \quad \text{ans} = \frac{6}{5}$$

$$4) \lim_{x \rightarrow 0} \frac{(x+8)^{1/3} - 2}{x} \quad \text{ans} = \frac{1}{12}$$

Type V

Concepts : $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \cos x = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

NCERT Exercise 13.1 : Q 13, 14, 16, 18, 19, 20, 21

Evaluate : $\lim_{x \rightarrow 0} \frac{\tan 8x}{\sin 3x} \quad \text{ans: } \frac{8}{3}$

Evaluate : $\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \sin 3x} \quad \text{ans: } 1$

Type VI

NCERT Exercise 13.1 : Q 17

Evaluate : $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{1 - \cos 6x}$

Type VII

Evaluate : $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Let $x = \pi + h$

$$\begin{aligned} \therefore \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} &= \lim_{h \rightarrow 0} \frac{\sin[\pi - (\pi + h)]}{\pi[\pi - (\pi + h)]} \\ &= \frac{1}{\pi} \lim_{h \rightarrow 0} \frac{\sin h}{h} = \frac{1}{\pi} \end{aligned}$$

Textbook Q no: 22

Evaluate 1) $\lim_{x \rightarrow \pi/2} \frac{\tan 2x}{x - \frac{\pi}{2}}$ ans: 2

2) $\lim_{x \rightarrow 1} \frac{\cos \frac{\pi}{2} x}{1-x}$ ans : $\frac{\pi}{2}$

3) $\lim_{x \rightarrow \pi} \frac{\sin x}{x-\pi}$ ans : (-1)

4) $\lim_{x \rightarrow \pi/2} \frac{1+\cos 2x}{(\pi-2x)^2}$ ans : $\frac{1}{2}$

Type VIII

Applying $\sin C \pm \sin D$ formulae

Evaluate 1) $\lim_{x \rightarrow 0} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$

2) $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x-a}$ ans : $\cos a$

Type IX

$$\lim_{x \rightarrow c-} f(x) = \lim_{x \rightarrow c+} f(x) = l$$

** NCERT Exercise 13.1 -

1) Q 23

2) Q 24

3) Q 25

4) Q 26

** 5) If $f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases}$, find $\lim_{x \rightarrow 2} f(x)$

6) If $f(x) = \begin{cases} x, & x > 0 \\ 1, & x = 0 \\ -x, & x < 0 \end{cases}$, find $\lim_{x \rightarrow 0} f(x)$

Type X

** NCERT Exercise 13.1 -

1) Q 28

2) Q 30

3) Q 32

4) Let $f(x) = \begin{cases} 4x - 5, & x \leq 2 \\ x - k, & x > 2 \end{cases}$. Find k if $\lim_{x \rightarrow 2} f(x)$ exists. **Ans : k = -1**

Type XI

Exponential Limits

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

NCERT Exercise 13.2 Q1 , Q2, Q4, Q6.

Type XII

NCERT Exercise 13.2 Q3 , Q5

Type XIII

$$\lim_{x \rightarrow 0} \frac{\log_e(1+x)}{x} = 1$$

NCERT Exercise 13.2 Q7 , Q8

DIFFERENTIATION

Type I

Find the derivatives of the following functions from first principle.

1) x

2) $\frac{1}{x}$

3) $-x$

4) $(-x)^{-1}$

5) x^n

6) NCERT Ex 13.2 Q4

** 7) $\sin x$

** 8) $\cos x$

** 9) $\tan x$

** 10) $\sec x$

** 11) $\operatorname{cosec} x$

** 12) $\sin 2x$

** 13) $\cos 3x$

14) $x \sin x$

15) $\sin^2 x$

16) $\sin(x+1)$

17) Misc Example 19

Type II

Find the derivatives of the following

* 1) $3 \cot x + 5 \operatorname{cosec} x$

2) $5 \sin x - 6 \cos x + 7$

3) $2 \tan x - 7 \operatorname{cosec} x$

4) $x^3 - 27$

5) $2x - \frac{3}{4}$

* 6) $x^2 + \sin x + \frac{1}{x^2}$

Type III

Ex 13.2 Q1

Ex 13.2 Q2

Ex 13.2 Q3

Ex 13.2 Q5

Type IV

$$\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

NCERT Ex 13.2

* 1) Q7 (1),(2)

* 2) Q9 (2),(3),(4),(5)

* 3) Q11 (1)

* 4) Example 18

** 5) Misc Ex Q 15, Q22, Q25, Q29.

Type V

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Misc Exercise Q 5,6,7,8,9,16,17,18,20,26.

Additional questions and HOT Questions

1. Find the derivatives of the following functions

a) $\frac{\sec x + \tan x}{\sec x - \tan x}$

** b) $(x^2 - 3x + 2)(x + 2)$

** c) $\frac{\sin x - x \cos x}{x \sin x + \cos x}$

2. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, show that $\frac{dy}{dx} = y$.