### APPLICATION OF DERIVATIVES

- **Statements-1:** For the circle  $(x-1)^2 + (y-1)^2 = 1$ , the tangent at the point (1,0) is the x-axis. **Statements-2:** the derivative of a single valued function y = f(x) at x = a is the slope of the tangent drawn to the curve at x = a.
- 96. Statements-1: Both sin x, and cos x are decreasing functions in  $\left(\frac{\pi}{2}, \pi\right)$  [Good]

**Statements-2:** If a differentiable function decreases is an interval (a, b) then its derivative also decreases in (a, b).

- 97. Statements-1:  $e^{\pi} > \pi^{e}$  [Good]
  - **Statements-2:** The function  $x^{\frac{1}{x}}$  ( x > 0) has a local maximum at x = e
- 98. Statements-1: Conditions of LMVT fail in f(x) = |x 1| (x 1)Statements-2: |x - 1| is not differentiable at x = 1
- **99.** Let  $f(x) = \sum_{i=1}^{n} (x x_i)^2$ 
  - **Statement–1 :** Minimum value of f(x) occurs at x =  $\frac{\sum x_i}{n}$

**Statement–2:** Minimum of  $f(x) = ax^2 + bx + c$  (a > 0) occurs at x = -b/2a.

- 100. Statement-1:  $\alpha^{\beta} > \beta^{\alpha}$ , for  $2.91 < \alpha < \beta$ Statement-2:  $f(x) = \frac{\log_e x}{x}$  is a decreasing function for x > e.
- **Statement–1 :** Total number of critical points of  $f(x) = \max. \{1/2, \sin x, \cos x\} \pi \le x \le \pi$  are 5 **Statement–2 :** Total number of critical points of  $f(x) = \max\{1/2, x, \cos x\} \pi \le x \le \pi$  are 2
- 102. Let  $f(x) = 5p^2 + 4(x 1) x^2$ ,  $x \in R$  and p is a real constant Statement-1: If the maximum values of f(x) is 20, then p = -2. Statement-2: If the maximum value of f(x) is 20, then p = 2.
- 103. Let  $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$  and  $x \in [-1, 1]$ 
  - **Statements-1:** Range of f(x) is  $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ .

Statements-2: f(x) is an increasing function. **04.** Let  $f(x) = x^3$ 

104. Let  $f(x) = x^3$ Statements-1: x = 0, in the point of inflexion for f(x)

Statements-1: x = 0, in the point of inflexion for f(x)Statements-2: f''(x) < 0 for x < 0 and f''(x) > 0 for x > 0.

105. Suppose  $f(x) = \frac{x^2}{2} + \ell n x + 2 \cos x$ 

**Statements-1:** f is an increasing function.

**Statements-2:** derivative of f(x) with respect to x is always greater than zero.

- **106.** Let  $0 < x \le \frac{\pi}{2}$  and  $f(x) = \frac{\sin x}{x}$ 
  - **Statements-1:** The minimum value of f is  $\frac{2}{\pi}$ , attained at  $x = \frac{\pi}{2}$

Statements-2:  $0 < \sin x < x, \forall x \in \left[0, \frac{\pi}{2}\right]$ .

- 107. Statements-1: The equation  $x^2 = x \sin x + \cos x$  has only one solution. Statements-2: The derivative of the function  $x^2 - x \sin x - \cos x$  is  $x(2 - \cos x)$ .
- 108. Statement-1: Angle of intersects in between  $y = x^2$  and  $6y = 7 x^3$  at (1, 1) is  $\pi/4$

**Statement–2**: Angle of intersection between any two curve is angle between the tangents at the point of intersection.

109. Statement – 1: The curve  $y = x^{1/3}$  has a point of inflection at x = 0

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**Statement** -2: A point where y'' fails to exist can be a point of inflection

110. Let f(x) and g(x) are two positive and increasing function

**Statement – 1 :** If (f(x)) g(x) is decreasing then f(x) < 1

**Statement** – 2: If f(x) is decreasing then f'(x) < 0 and increasing then f'(x) > 0 for all x.

111. Statement – 1: If f(0) = 0,  $f'(x) = \ln (x + \sqrt{1 + x^2})$ , then f(x) is positive for all  $x \in R_0$ 

**Statements-2:** f(x) is increasing for x > 0 and decreasing for x < 0.

112. Statements-1: The two curves  $y^2 = 4x$  and  $x^2 + y^2 - 6x + 1 = 0$  at the point (1, 2) intersect orthogonally.

**Statements-2:** Two curves y = f(x) & y = g(x) intersect orthogonally at  $(x_1, y_1)$  if  $(f'(x_1), g'((x_1)) = -1$ .

113. Statements-1: If 27a + 9b + 3c + d = 0, then the equation  $4ax^3 + 3bx^2 + 2cx + d = 0$  has at least one real root lying between (0, 3)

**Statements-2:** If f(x) is continuous in [a, b], derivable in (a, b), then at least one point  $c \in (a, b)$  such that f'(c)=0.

114. Statements-1:  $f(x) = \{x\}$  has local minima at x = 1.

**Statements-2:** x = a will be local minima for y = f(x) provided  $\lim_{x \to a^{-}} f(x) > f(a)$  also

$$\lim_{x\to a^+} \ f(x) > f(a).$$

115. Statements-1:  $f(x) = \frac{1}{2} - x$ ;  $x < \frac{1}{2}$ 

 $=\left(\frac{1}{2}-x\right)^2$ ;  $x \ge \frac{1}{2}$ . Mean value theorem is applicable in the interval [0, 1].

**S-2:** For application of mean value theorem, f(x) must be continuous in [0, 1] and differentiable in (0, 1).

**116. Statements-1:** For some  $0 < x_1 < x_2 < \pi/2$ ,  $tan^{-1}x_2 - tan^{-1}x_1 < x_2 - x_1$ 

**Statements-2:** If  $f(x) > f(x_1) \Rightarrow x_2 > x_1$ 

function is always increasing

117. Statements-1: The graph of a continuous function y = f(x) has a cusp at point x = c if f''(x) has same sign on both sides of c.

**Statements-2:** The concavity at any point x = c depends upon f''(x). If f''(x) < 0 or f''(x) > 0 the function is either concave up or concave down.

**118.** Statements-1: If f be a function defined for all x such that  $|f(x) - f(y)| < (x - y)^2$  then f is constant

 $\textbf{Statements-2:} \ \ \text{If} \ \alpha(x) < \beta(x) < \gamma(x) \ \text{for all} \ x \ \text{and} \ \lim_{x \to a} \alpha(x) = \lim_{x \to a} \gamma(x) = L \ \Rightarrow \lim_{x \to a} \beta(x) = L$ 

**Statements-1:**  $f: R \to R$  be a function such that  $f(x) = x^3 + x^2 + 3x + \sin x$ . Then f is one-one.

**Statements-2:** f(x) is neither increasing nor decreasing.

**120.** Statements-1: If  $\alpha \& \beta$  are any two roots of equation  $e^x \cos x = 1$ , then the equation

 $e^x \sin x - 1 = 0$  has at least one root in  $(\alpha, \beta)$ **Statements-2:** f is continuous in  $[\alpha, \beta]$ . f is derivable in  $(\alpha, \beta)$ .  $f(\alpha) = f(\beta)$  then these exists

 $x \in (\alpha, \beta)$  such that f'(x) = 0

121. Statements-1: The minimum value of the expression  $x^2 + 2bx + c$  is  $c - b^2$ .

**Statements-2:** The first order derivative of the expression at x = -b is zero and second derivative is always positive.

122. Statements-1: Let  $\phi(x) = \sin(\cos x)$  in  $\left[0, \frac{\pi}{2}\right]$  then  $\phi(x)$  is decreasing in  $\left[0, \frac{\pi}{2}\right]$ 

**Statements-2:**  $\phi'(x) \le 0 \ \forall x \in \left[0, \frac{\pi}{2}\right]$ 

123. Statements-1: The function  $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$  is decreasing for every  $x \in (2, 3) \cup (-\infty, 1)$ 

**Statements-2:** f'(x) > 0 for the given values of x.

**Statements-1:** For the function  $f(x) = x^x$ , x = 1/e is a point of local minimum.

**Statements-2:** f'(x) changes its sign from –ve to positive in neighbourhood of x = 1/e.

- 125. Statements-1: Consider the function  $f(x) = (x^3 6x^2 + 12x 8) e^x$  is neither maximum nor minimum let x = 2Statements-2: f'(x) = 0, f''(x) = 0,  $f''(x) \neq 0$  at x = 2
- 126. Statements-1: Consider the function f(x)  $\frac{f(x_1 + x_2)}{2} < \frac{f(x_1) + f(x_2)}{2}$

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**Statements-2:** f'(x) > 0, f''(x) > 0 where  $x_1 < x_2$ 

127. Consider the following function with regard to the function

 $f(x) = (x^3 - 6x^2 + 12x - 8) e^x$ 

**Statement-1:** f(x) is neither maximum nor minimum at x = 2

**Statement-2:** f'(x) = 0, f''(x) = 0,  $f'''(x) \neq 0$  at x = 2.

**Statements-1:** Equation  $f(x) = x^3 + 9x^2 + 2ax + a^2 + a + 1 = 0$  has at least one real negative root. 128.

Statements-2: Every equation of odd degree has at least one real root whose sign is opposite to that of its constant

#### ANSWER

95. B	96. C	97. A	98. D	99. A	100. A	101. A			
102. A	103. A	104. A	105. A	106. B	107. D	108. D			
109. A	110. A	111. A	112. D	113. A	114. A	115. D			
116. A	117. A	118. A	119. C	120. A	121. A	122. A			
123. C	124. A	125. A	126. A	127. A	128. A				

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- 1.  $\frac{d}{dx} \tan^{-1} \left[ \frac{\cos x \sin x}{\cos x + \sin x} \right] =$ [AISSE 1985, 87; DSSE 1982,84; MNR 1985; Karnataka CET 2002; RPET 2002, 03]
  - (a)  $\frac{1}{2(1+x^2)}$  (b)  $\frac{1}{1+x^2}$  (c) 1 (d) -1

- 2. If  $y = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2})$ , then  $\frac{dy}{dx} = \frac{1}{2}$ (a)  $\sqrt{x^2 + a^2}$  (b)  $\frac{1}{\sqrt{x^2 + a^2}}$  (c)  $2\sqrt{x^2 + a^2}$  (d)

- 3. If  $y = \cot^{-1}(\cos 2x)^{1/2}$ , then the value of  $\frac{dy}{dx}$  at  $x = \frac{\pi}{6}$  will be

  (b)  $\left(\frac{1}{2}\right)^{1/2}$  (c)  $(3)^{1/2}$  (d)  $(6)^{1/2}$

- If f(x + y) = f(x). f(y) for all x and y and f(5) = 2, f'(0) = 3, then f'(5) will be **[IIT 1981; Karnataka CET 2000; UPSEAT 2002;** MP PET 2002; AIEEE 2002]

- (c)
- (d)

If  $xe^{xy} = y + \sin^2 x$ , then at x = 0,  $\frac{dy}{dx} = 0$ 

[IIT 1996]

**6.** If  $u(x, y) = y \log x + x \log y$ , then

 $u_x u_y - u_x \log x - u_y \log y + \log x \log y =$  [EAMCET 2003]

- (d)
- 7. If  $y = f\left(\frac{2x-1}{x^2+1}\right)$  and  $f'(x) = \sin x^2$ , then  $\frac{dy}{dx} = [IIT 1982]$ 
  - (a)  $\frac{6x^2-2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

 $\frac{6x^2-2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$ 

- (c)  $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$
- (d)  $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$
- If  $x = \sec \theta \cos \theta$  and  $y = \sec^n \theta \cos^n \theta$ , then

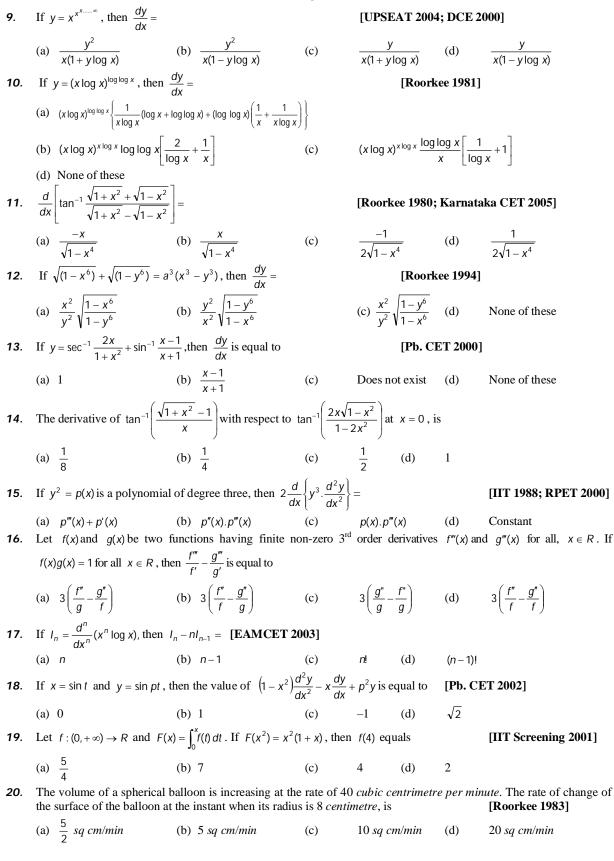
(a) 
$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

(b) 
$$(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = x^2(y^2 + 4)$$

(c) 
$$(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = (y^2 + 4)$$

(d) None of these

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21.	A man of height 1.8 <i>metre</i> is moving away from a lamp post at the rate of 1.2 <i>m</i> /sec. If the height of the lamp post be 4.5 <i>metre</i> , then the rate at which the shadow of the man is lengthening is  [MP PET 1989]  (a) 0.4 <i>m</i> /sec  (b) 0.8 <i>m</i> /sec  (c) 1.2 <i>m</i> /sec  (d) None of these							
22.	The radius of the cylinder	• •			` ′			
	(a) $\frac{2}{3}R$	(b) $\sqrt{\frac{2}{3}}R$						
	3	13		4	14			
23.	The distance travelled $s$ (in <i>metre</i> ) by a particle in $t$ seconds is given by, $s = t^3 + 2t^2 + t$ . The speed of the particle after 1 second will be <b>[UPSEAT 2003]</b>							
	(a) 8 cm/sec (b) 6 cm/sec (c) 2 cm/sec (d) Nor							
24.								
	(a) $p = 2, q = -7$							
25.	At what points of the curve	e $y = \frac{2}{3}x^3 + \frac{1}{2}x^2$ , tangent	makes the e	qual angle w	vith axis	[UPSEAT 1999]		
	(a) $\left(\frac{1}{2}, \frac{5}{24}\right)$ and $\left(-1, -\frac{1}{6}\right)$	$\left(\frac{1}{2}, \frac{4}{9}\right)$ and $\left(-1, 0\right)$	$(c)\left(\frac{1}{3},\frac{1}{7}\right)$	and $\left(-3, \frac{1}{2}\right)$	(d)	$\left(\frac{1}{3}, \frac{4}{47}\right)$ and $\left(-1, -\frac{1}{3}\right)$		
26.	If the normal to the curve	y = f(x) at the point (3,4)	) makes an	angle $\frac{3\pi}{4}$ w	rith the positive	e x-axis then $f'(3)$ is equal		
	to							
	(a) -1	(b) $-\frac{3}{4}$	(c)	$\frac{4}{3}$ (d	l) 1			
27.	The point(s) on the curve	$v^3 + 3x^2 = 12y$ where the	tangent is v	vertical (para	llel to v-axis)	is (are)		
	The point(s) on the curve	y Tox 12y where the	tungent is t	orana (pana	1101 00 9 01115),	[IIT Screening 2002]		
	( 1 )	$\left( \sqrt{11} \right)$			( 1			
	(a) $\left(\pm \frac{4}{\sqrt{3}}, -2\right)$	(b) $\left(\pm \frac{\sqrt{11}}{3}, 1\right)$	(c)	(0,0) (d	$\left(\pm \frac{4}{\sqrt{3}}\right)$	2)		
28.	Let $f(x) = \int_0^x \frac{\cos t}{t} dt$ , $x > 0$ t	then $f(x)$ has						
	[Kurukshetra CEE 2002]							
	(a) Maxima when $n = -2, -4, -6,$			Maxima when $n = -1, -3, -5,$				
	(c) Minima when $n = 0, 2,$	,4,	(d)	Minima wh	nen $n = 1, 3, 5$			
29.	If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $min \ f(x) > max \ g(x)$ , then the relation between $b$ and $c$ is					relation between $b$ and $c$ is		
	[IIT Screening 2003] (a) No real value of $b$ and $c$ (b) $0 < c < b\sqrt{2}$							
		(d) $ c  >  b  \sqrt{2}$						
30	N characters of informatio		ne in hatch	nes of r char	acters each: th	e hatch processing time is		
00.	$\alpha + \beta x^2$ seconds; $\alpha$ and $\beta$		_			[MNR 1986]		
		_		_		[2:22:22:27:00]		
	(a) $\frac{\alpha}{\beta}$	(b) $\frac{\beta}{\alpha}$	(c)	$\sqrt{\frac{\alpha}{\beta}}$ (d	1) $\sqrt{\frac{\rho}{\alpha}}$			
31.	On the interval [0, 1], the f	function $v^{25}(1-v)^{75}$ takes	ite maximi	ım value at ti	he point	[IIT 1995]		
51.	(a) 0	(b) 1/2	(c)	1/3 (d		[H1 1775]		
32.	The function $f(x) = \int_{-1}^{x} t(e^{t} - 1)^{t}$	• •		,	,	[IIT 1999]		
	· ·							
	(a) 0	(b) 1		(c) 2	(d)	3		
33.	The maximum value of exp	-			[AMU	1999]		
	(a) exp(2)	(b) $\exp(2-\sqrt{3})$		exp(4) (d				
34.	If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ , where $a > 0$ attains its maximum and minimum at $p$ and $q$ respectively such that $p^2 = q$ , then $a$ equals [AIEEE 2003]							
				2 (1	. 1			
	(a) 3	(b) 1	(c)	2 (d	1) -			

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**35.** The function 
$$f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$$
 is **[IIT 1995]**

- (a) Increasing on  $[0, \infty)$
- (b) Decreasing on  $[0, \infty)$
- (c) Decreasing on  $\left[0, \frac{\pi}{e}\right]$  and increasing on  $\left[\frac{\pi}{e}, \infty\right]$
- (d) Increasing on  $\left[0, \frac{\pi}{e}\right]$  and decreasing on  $\left[\frac{\pi}{e}, \infty\right]$
- The function  $f(x) = \sin^4 x + \cos^4 x$  increases, if

#### [HT 1999; Pb. CET 2001]

(a) 
$$0 < x < \frac{\pi}{8}$$

(a) 
$$0 < x < \frac{\pi}{8}$$
 (b)  $\frac{\pi}{4} < x < \frac{3\pi}{8}$ 

- (c)  $\frac{3\pi}{8} < x < \frac{5\pi}{8}$  (d)  $\frac{5\pi}{8} < x < \frac{3\pi}{4}$
- 37. Let  $h(x) = f(x) (f(x))^2 + (f(x))^3$  for every real number x. Then
  - (a) h is increasing whenever f is increasing
- (c) h is decreasing whenever f is decreasing
- **38.** In [0, 1] Lagrange's mean value theorem is NOT applicable to
- h is increasing whenever f is decreasing Nothing can be said in general [IIT Screening 2003]

(a) 
$$f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \ge \frac{1}{2} \end{cases}$$
 (b)  $f(x) = \begin{cases} \frac{\sin x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$ 

- (d) f(x) = |x|
- **39.** If the function  $f(x) = x^3 6ax^2 + 5x$  satisfies the conditions of Lagrange's mean value theorem for the interval [1, 2] and the tangent to the curve y = f(x) at  $x = \frac{7}{4}$  is parallel to the chord that joins the points of intersection of the curve with the ordinates x = 1 and x = 2. Then the value of a is

[MP PET 1998]

- (a)  $\frac{35}{16}$

- (c)  $\frac{7}{16}$  (d)  $\frac{5}{16}$
- **40.** Let  $f(x) = \begin{cases} x^{\alpha} \ln x, & x > 0 \\ 0, & x = 0 \end{cases}$ , Rolle's theorem is applicable to f for  $x \in [0,1]$ , if  $\alpha = (0,1)$

[IIT Screening 2004]

- (a) -2
- (b) -1
- (c)

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1	d	2	a	3	a	4	С	5	С
6	С	7	d	8	a	9	b	10	а
11	a	12	С	13	С	14	b	15	С
16	b	17	d	18	a	19	С	20	С
21	b	22	b	23	a	24	a	25	a
26	d	27	d	28	b,d	29	d	30	С
31	d	32	b,d	33	С	34	С	35	b
36	b	37	a,c	38	a	39	b	40	d

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