Class XI: Mathematics

Chapter 9: Sequence and Series

Chapter Notes

Top Definitions

1. A Sequence is an ordered list of numbers according to some rule. A sequence is denoted by $<a_n>_{n\geq1} = a_1, a_2, a_3, \ldots\ldots a_n$
2. The various numbers occurring in a sequence are called its terms.
3. A sequence containing finite number of terms is called a finite sequence. A finite sequence has last term.
4. A sequence which is not a finite sequence, i.e. containing infinite number of terms is called an infinite sequence. There is no last term in an infinite sequence.
5. A sequence is said to be an arithmetic progression if every term differs from the preceding term by a constant number. For example, sequence $a_1, a_2, a_3, \ldots a_n, \ldots$ is called an arithmetic sequence or an AP if $a_{n+1} = a_n + d$
   for all $n \in N$, where d is a constant called the common difference of AP.
6. A is the arithmetic mean of two numbers a and b if a,A,b forms an arithmetic progression.
7. A sequence is said to be a geometric progression or G.P., if the ratio of any tem to its preceding term is same throughout. Constant Ratio is common ratio denoted by r.

8. If three numbers are in GP, then the middle term is called the geometric mean of the other two.

Top Concepts

1. A sequence has a definite first member, second member, third member and so on.

2. The n^{th} term $<a_n>$ is called the general term of the sequence.

3. Fibonacci sequence 1, 1, 2, 3, 5, 8,.. … is generated by the recurrence relation given by

   $a_1 = a_2 = 1$

   $a_3 = a_1 + a_2 \ldots\ldots$

   $a_n = a_{n-2} + a_{n-1},\ n > 2$

4. A sequence is a function with domain the set of natural numbers or any of its subsets of the type {1, 2, 3, … k}.

5. The sum of the series is the number obtained by adding the terms.
6. General form of AP is *a*, *a* + *d*, *a* + 2*d*, ...a+(n-1)d. *a* is called the **first term** of the AP and *d* is called the **common difference** of the AP. d can be any real number.

7. If d>0 then AP is increasing if d< 0then AP is decreasing and d=0 then AP is constant.
8. For AP a , (a + d) , (a + 2d) , ... , $(\lambda - 2d)$, $(\lambda - d)$, λ with first term a and common difference d and last term λ general term is $\lambda-(n-1)d$.

9. Properties of Arithmetic Progression

i. If a constant is added to each term of an A.P., the resulting sequence is also an A.P.

ii. If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

iii. If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

iv. If each term of an A.P. is divided by a non – zero constant then the resulting sequence is also an A.P.

10. The arithmetic mean A of any two numbers a and b is given by

$$\frac{a+b}{2}$$

11. General Form of GP: a, ar, ar^2, ar^3, ... where a is the first term and r is the constant ratio r can take any non zero real number.

12. A sequence in geometric progression will remain in geometric progression if each of its terms is multiplied by a non zero constant.

13. A sequence obtained by the multiplying two GPs term by term results in a GP with common ratio the product of the common ratio of the two GPs.

14. The geometric mean (G.M.) of any two positive numbers a and b is given by $\sqrt{ab}$.

15. Let A and G be A.M. and G.M. of two given positive real numbers a and b, respectively, then $A \geq G$

$$\text{Where } A = \frac{a+b}{2}, \qquad \text{and } G = \sqrt{ab}$$

Top Formulae

1. n^{th} term or general term of the A.P. is $a_n = a + (n - 1)d$ where a is the first term, d is common difference.
2. General term of AP given its last term is $\lambda - (n - 1)d$
3. Let a, a + d, a + 2d, …, a + (n – 1)d be an A.P. Then

   $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}\left[a + \ell\right]$ where $\lambda = a + (n - 1)d$

4. Let $A_1, A_2, A_3, \ldots A_n$ be n numbers, between a and b such that a, $A_1, A_2, A_3, \ldots A_n$, b is an A.P. n numbers between a and b are as follows:

$$A_1 = a + d = a + \frac{b-a}{n+1}$$

$$A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$$

$$A_3 = a + 3d = a + \frac{3(b-a)}{n+1}$$

… … … …

… … … …

$$A_n = a + nd = a + \frac{n(b-a)}{n+1}$$

5. General term of GP is ar^{n-1} where a is the first termand r is the common ratio.
6. Sum to first n terms of GP $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

   (i)if r = 1, $S_n = a + a + a + \ldots + a$ (n terms) = na

(ii) If r<1 $S_n = \dfrac{q(1-r^n)}{1-r}$

(iii)If r>1 $S_n = \dfrac{a(r^n-1)}{r-1}$

7. Let $G_1, G_2, \ldots\ldots, G_n$ be n numbers between positive numbers a and b such that a, $G_1, G_2, G_3, \ldots\ldots G_n$, b is a G.P.

Thus $b = br^{n+1}$, or $r = \left(\dfrac{b}{a}\right)^{\frac{1}{n+1}}$

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}},\ G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}},\ G_3\ ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

8. The sum of first n natural Numbers is

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

9. Sum of squares of the first n natural numbers

$$1^2+2^2+3^2+\ldots\ldots..n^2= \frac{n(n+1)(2n+1)}{6}$$

10. Sum of cubes of first n natural numbers

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} = \frac{[n(n+1)]^2}{4}$$