Sample Paper–5
Class 11, Mathematics

Time: 3 hours **Max. Marks 100**

General Instructions

1. All questions are compulsory.
2. Use of calculator is not permitted. However you may use log table, if required.
3. Q.No. 1 to 12 are of very short answer type questions, carrying 1 mark each.
4. Q.No.13 to 28 carries 4 marks each.
5. Q.No. 29 to 32 carries 6 marks each.

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1. Write the following intervals in set-builder form:
   (i) (–3, 0)
   (ii) [6, 12]

2. If A = {–1, 1}, find A × A × A.

3. If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm

5. Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

6. Express the given complex number in the form $a + ib$: i^{-39}

7. How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

8. Name the octants in which the following points lie: (1, 2, 3), (4, –2, 3),

9. Evaluate the Given limit: $\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$

10. Evaluate the Given limit: $\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$

11. Describe the sample space for the indicated experiment: A coin is tossed four times.

12. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:
    A: the sum is greater than 8, B: 2 occurs on either die
    C: The sum is at least 7 and a multiple of 3.
    Which pairs of these events are mutually exclusive?

13. Show that for any sets A and B: A = (A ∩ B) ∪ (A – B)

14. A function f is defined by $f(x) = 2x - 5$. Write down the values of
(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

15. Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

16. Prove the following by using the principle of mathematical induction for all $n \in N$: $3^{2n+2} - 8n - 9$ is divisible by 8.

17. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

18. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

19. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

20. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

21. Find a if the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

22. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$.

23. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m.

24. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

25. Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3,–2, 5).

26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin^n x$

27. Given below are two statements
p: *25 is a multiple of 5.*
q: 25 is a multiple of 8.

Write the compound statements connecting these two statements with "And" and "Or". In both cases check the validity of the compound statement.

28. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

29. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

30. Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

31. A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

32. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12.

Solution

1. **(i)** $(-3, 0) = \{x: x \in R, -3 < x < 0\}$
(ii) $[6, 12] = \{x: x \in R, 6 \leq x \leq 12\}$

2. It is known that for any non-empty set A, A × A × A is defined as
$A \times A \times A = \{(a, b, c): a, b, c \in A\}$
It is given that $A = \{-1, 1\}$
$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$
$(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

3. It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$
We know that the Cartesian product of two non-empty sets P and Q is defined as $P \times Q = \{(p, q): p \in P, q \in Q\}$
∴ A is the set of all first elements and B is the set of all second elements.
Thus, $A = \{a, b\}$ and $B = \{x, y\}$

4. We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then
$$\theta = \frac{l}{r}$$
Therefore, forr = 100 cm, l = 22 cm, we have
$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$
$$= \frac{126}{10} \text{ deg ree} = 12\frac{3}{5} \text{ deg ree} = 12°36' \quad [1° = 60']$$
Thus, the required angle is 12°36′.

5. $$\cot x = \frac{3}{4}$$
$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$
$$1 + \tan^2 x = \sec^2 x$$
$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$
$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$
$$\Rightarrow \frac{25}{9} = \sec^2 x$$
$$\Rightarrow \sec x = \pm\frac{5}{3}$$
Since x lies in the 3rd quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

6.

$$i^{-39} = i^{-4\times 9-3} = \left(i^4\right)^{-9} \cdot i^{-3}$$

$$= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right]$$

$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$

$$= \frac{-1}{i} \times \frac{i}{i}$$

$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

7. There are as many codes as there are ways of filling 4 vacant places □□□□ in succession by the first 10 letters of the English alphabet, keeping in mind that the repetition of letters is not allowed.
The first place can be filled in 10 different ways by any of the first 10 letters of the English alphabet following which, the second place can be filled in by any of the remaining letters in 9 different ways. The third place can be filled in by any of the remaining 8 letters in 8 different ways and the fourth place can be filled in by any of the remaining 7 letters in 7 different ways.
Therefore, by multiplication principle, the required numbers of ways in which 4 vacant places can be filled is $10 \times 9 \times 8 \times 7 = 5040$
Hence, 5040 four-letter codes can be formed using the first 10 letters of the English alphabet, if no letter is repeated.

8. The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.
The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, –2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

9. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}=\frac{(-1)^{10}+(-1)^5+1}{-1-1}=\frac{1-1+1}{-2}=-\frac{1}{2}$

10.

$\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Put $x + 1 = y$ so that $y \to 1$ as $x \to 0$.

Accordingly, $\lim_{x\to 0}\frac{(x+1)^5-1}{x}=\lim_{y\to 1}\frac{y^5-1}{y-1}$

$=\lim_{y\to 1}\frac{y^5-1^5}{y-1}$

$=5.1^{5-1}$ $\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$

$=5$

$\therefore \lim_{x\to 0}\frac{(x+5)^5-1}{x}=5$

11. When a coin is tossed once, there are two possible outcomes: head (H) and tail (T).
When a coin is tossed four times, the total number of possible outcomes is $2^4 = 16$
Thus, when a coin is tossed four times, the sample space is given by:
S = {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

12. When a pair of dice is rolled, the sample space is given by

$S=\{(x,y):x,y=1,2,3,4,5,6\}$

$$=\left\{\begin{array}{llllll}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6)\\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6)\\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6)\\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6)\\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6)\\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{array}\right\}$$

Accordingly,

$A=\{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$B=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(1,2),(3,2),(4,2),(5,2),(6,2)\}$

$C=\{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

It is observed that

$A \cap B = \Phi$

$B \cap C = \Phi$

$C \cap A = \{(3,6),(4,5),(5,4),(6,3),(6,6)\} \neq \phi$

Hence, events A and B and events B and C are mutually exclusive.

13. To show: A = (A ∩ B) ∪ (A – B)
Let $x \in A$
We have to show that $x \in (A \cap B) \cup (A - B)$
Case I
$x \in A \cap B$
Then, $x \in (A \cap B) \subset (A \cup B) \cup (A - B)$
Case II
$x \notin A \cap B$
$\Rightarrow x \notin A$ or $x \notin B$
$\therefore x \notin B$ [$x \notin A$]
$\therefore x \notin A - B \subset (A \cup B) \cup (A - B)$
$\therefore A \subset (A \cap B) \cup (A - B)$ … (1)
It is clear that
$A \cap B \subset A$ and $(A - B) \subset A$
$\therefore (A \cap B) \cup (A - B) \subset A$ … (2)
From (1) and (2), we obtain
$A = (A \cap B) \cup (A - B)$

14. The given function is $f(x) = 2x - 5$.
Therefore,
(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

15. $\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

16. Let the given statement be P(n), i.e.,
P(n): $3^{2n+2} - 8n - 9$ is divisible by 8.

It can be observed that P(n) is true for n = 1 since $3^{2\times1+2} - 8\times1 - 9 = 64$, which is divisible by 8.
Let P(k) be true for some positive integer k, i.e.,
$3^{2k+2} - 8k - 9$ is divisible by 8.
$\therefore 3^{2k+2} - 8k - 9 = 8m$; where $m \in \mathbf{N}$ … (1)
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$$3^{2(k+1)+2} - 8(k+1) - 9$$
$$= 3^{2k+2} \cdot 3^2 - 8k - 8 - 9$$
$$= 3^2\left(3^{2k+2} - 8k - 9 + 8k + 9\right) - 8k - 17$$
$$= 3^2\left(3^{2k+2} - 8k - 9\right) + 3^2(8k+9) - 8k - 17$$
$$= 9.8m + 9(8k+9) - 8k - 17$$
$$= 9.8m + 72k + 81 - 8k - 17$$
$$= 9.8m + 64k + 64$$
$$= 8(9m + 8k + 8)$$
$$= 8r, \text{ where } r = (9m + 8k + 8) \text{ is a natural number}$$

Therefore, $3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8.

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

17. Let $\alpha = a + ib$ and $\beta = x + iy$
It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$
$$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$$
$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\frac{(x+iy) - (a+ib)}{1 - (a-ib)(x+iy)}\right|$$
$$= \left|\frac{(x-a) + i(y-b)}{1 - (ax + aiy - ibx + by)}\right|$$
$$= \left|\frac{(x-a) + i(y-b)}{(1 - ax - by) + i(bx - ay)}\right|$$
$$= \frac{|(x-a) + i(y-b)|}{|(1 - ax - by) + i(bx - ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$
$$= \frac{\sqrt{(x-a)^2 + (y-b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}$$
$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \quad [\text{Using } (1)]$$

$$= 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

18. Let x litres of 2% boric acid solution is required to be added.
Then, total mixture = (x + 640) litres
This resulting mixture is to be more than 4% but less than 6% boric acid.
∴2%x + 8% of 640 > 4% of (x + 640)
And, 2% x + 8% of 640 < 6% of (x + 640)
2%x + 8% of 640 > 4% of (x + 640)

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) > \frac{4}{100}(x+640)$$

⇒ $2x + 5120 > 4x + 2560$
⇒ $5120 - 2560 > 4x - 2x$
⇒ $5120 - 2560 > 2x$
⇒ $2560 > 2x$
⇒ $1280 > x$
2% x + 8% of 640 < 6% of (x + 640)

$$\frac{2}{100}x + \frac{8}{100}(640) < \frac{6}{100}(x+640)$$

⇒ $2x + 5120 < 6x + 3840$
⇒ $5120 - 3840 < 6x - 2x$
⇒ $1280 < 4x$
⇒ $320 < x$
∴$320 < x < 1280$
Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

19. 5 men and 4 women are to be seated in a row such that the women occupy the even places.
The 5 men can be seated in 5! ways. For each arrangement, the 4 women can be seated only at the cross marked places (so that women occupy the even places).

$$M \times M \times M \times M \times M$$

Therefore, the women can be seated in 4! ways.
Thus, possible number of arrangements = 4! × 5! = 24 × 120 = 2880

20. A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.
3 boys can be selected from 5 boys in ${}^5C_3$ ways.
3 girls can be selected from 4 girls in ${}^4C_3$ ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$= \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{4 \times 3!}{3!}$

$= 10 \times 4 = 40$

21. It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^2 occurs in the $(r + 1)^{th}$ term in the expansion of $(3 + ax)^9$, we obtain

$T_{r+1} = {}^9C_r (3)^{9-r} (ax)^r = {}^9C_r (3)^{9-r} a^r x^r$

Comparing the indices of x in x^2 and in T_{r+1}, we obtain

$r = 2$

Thus, the coefficient of x^2 is

${}^9C_2 (3)^{9-2} a^2 = \frac{9!}{2!7!} (3)^7 a^2 = 36 (3)^7 a^2$

Assuming that x^3 occurs in the $(k + 1)^{th}$ term in the expansion of $(3 + ax)^9$, we obtain

$T_{k+1} = {}^9C_k (3)^{9-k} (ax)^k = {}^9C_k (3)^{9-k} a^k x^k$

Comparing the indices of x in x^3 and in T_{k+1}, we obtain

$k = 3$

Thus, the coefficient of x^3 is

${}^9C_3 (3)^{9-3} a^3 = \frac{9!}{3!6!} (3)^6 a^3 = 84 (3)^6 a^3$

It is given that the coefficients of x^2 and x^3 are the same.

$84 (3)^6 a^3 = 36 (3)^7 a^2$

$\Rightarrow 84a = 36 \times 3$

$\Rightarrow a = \frac{36 \times 3}{84} = \frac{104}{84}$

$\Rightarrow a = \frac{9}{7}$

Thus, the required value of a is $\frac{9}{7}$.

22. Let the two numbers be a and b.

G.M. $= \sqrt{ab}$

According to the given condition,

$a + b = 6\sqrt{ab}$ ...(1)

$\Rightarrow (a + b)^2 = 36(ab)$

Also,

$(a - b)^2 = (a + b)^2 - 4ab = 36ab - 4ab = 32ab$

$\Rightarrow a - b = \sqrt{32}\sqrt{ab}$

$= 4\sqrt{2}\sqrt{ab}$ ...(2)

Adding (1) and (2), we obtain

$$2a = \left(6+4\sqrt{2}\right)\sqrt{ab}$$

$$\Rightarrow a = \left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substituting the value of *a* in (1), we obtain

$$b = 6\sqrt{ab} - \left(3+2\sqrt{2}\right)\sqrt{ab}$$

$$\Rightarrow b = \left(3-2\sqrt{2}\right)\sqrt{ab}$$

$$\frac{a}{b} = \frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

23. The equations of the given lines are

$y = 3x + 1$ … (1)

$2y = x + 3$ … (2)

$y = mx + 4$ … (3)

Slope of line (1), $m_1 = 3$

Slope of line (2), $m_2 = \frac{1}{2}$

Slope of line (3), $m_3 = m$

It is given that lines (1) and (2) are equally inclined to line (3). This means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\therefore \left|\frac{m_1 - m_3}{1+m_1m_3}\right| = \left|\frac{m_2 - m_3}{1+m_2m_3}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{\frac{1}{2}-m}{1+\frac{1}{2}m}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{1-2m}{m+2}\right|$$

$$\Rightarrow \frac{3-m}{1+3m} = \pm\left(\frac{1-2m}{m+2}\right)$$

$$\Rightarrow \frac{3-m}{1+3m}=\frac{1-2m}{m+2} \text{ or } \frac{3-m}{1+3m}=-\left(\frac{1-2m}{m+2}\right)$$

If $\frac{3-m}{1+3m}=\frac{1-2m}{m+2}$, then

$$(3-m)(m+2)=(1-2m)(1+3m)$$
$$\Rightarrow -m^2+m+6=1+m-6m^2$$
$$\Rightarrow 5m^2+5=0$$
$$\Rightarrow (m^2+1)=0$$
$$\Rightarrow m=\sqrt{-1}, \text{ which is not real}$$

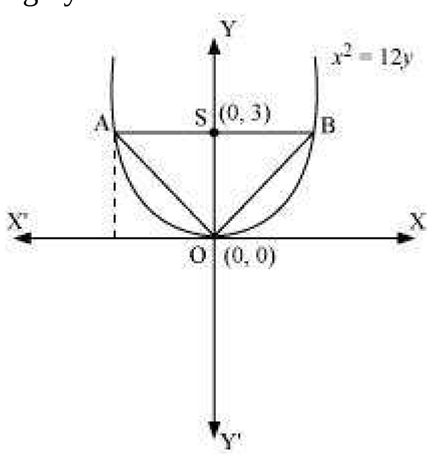
Hence, this case is not posible.

If $\frac{3-m}{1+3m}=-\left(\frac{1-2m}{m+2}\right)$, then

$$\Rightarrow (3-m)(m+2)=-(1-2m)(1+3m)$$
$$\Rightarrow -m^2+m+6=-(1+m-6m^2)$$
$$\Rightarrow 7m^2-2m-7=0$$
$$\Rightarrow m=\frac{2\pm\sqrt{4-4(7)(-7)}}{2(7)}$$
$$\Rightarrow m=\frac{2\pm2\sqrt{1+49}}{14}$$
$$\Rightarrow m=\frac{1\pm5\sqrt{2}}{7}$$

Thus, the required value of m is $\frac{1\pm5\sqrt{2}}{7}$.

24. The given parabola is $x^2 = 12y$.
On comparing this equation with $x^2 = 4ay$, we obtain $4a = 12 \Rightarrow a = 3$
$\therefore$The coordinates of foci are S $(0, a)$ = S $(0, 3)$
Let AB be the latus rectum of the given parabola.
The given parabola can be roughly drawn as



At $y = 3$, $x^2 = 12\ (3) \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

$\therefore$The coordinates of A are (–6, 3), while the coordinates of B are (6, 3).

Therefore, the vertices of ΔOAB are O (0, 0), A (–6, 3), and B (6, 3).

$$\text{Area of } \Delta\text{OAB} = \frac{1}{2}\left|0(3-3)+(-6)(3-0)+6(0-3)\right| \text{ unit}^2$$
$$= \frac{1}{2}\left|(-6)(3)+6(-3)\right| \text{ unit}^2$$
$$= \frac{1}{2}\left|-18-18\right| \text{ unit}^2$$
$$= \frac{1}{2}\left|-36\right| \text{ unit}^2$$
$$= \frac{1}{2}\times 36 \text{ unit}^2$$
$$= 18 \text{ unit}^2$$

Thus, the required area of the triangle is 18 unit^2.

25. If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, –2, 5).

Accordingly, $\text{AP} = 5\sqrt{2}$

$\therefore \text{AP}^2 = 50$

$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$

$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$

$\Rightarrow b^2 + 4b - 12 = 0$

$\Rightarrow b^2 + 6b - 2b - 12 = 0$

$\Rightarrow (b+6)(b-2) = 0$

$\Rightarrow b = -6 \text{ or } 2$

Thus, the coordinates of the required points are (0, 2, 0) and (0, –6, 0).

26. Let $y = \sin^n x$.

Accordingly, for $n = 1$, $y = \sin x$.

$\therefore \frac{dy}{dx} = \cos x, \text{ i.e., } \frac{d}{dx}\sin x = \cos x$

For $n = 2$, $y = \sin^2 x$.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin x \sin x)$

$$= (\sin x)' \sin x + \sin x (\sin x)' \qquad [\text{By Leibnitz product rule}]$$
$$= \cos x \sin x + \sin x \cos x$$
$$= 2\sin x \cos x \qquad \ldots(1)$$

For $n = 3$, $y = \sin^3 x$.

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\sin x \sin^2 x\right)$$

$$= (\sin x)' \sin^2 x + \sin x \left(\sin^2 x\right)' \qquad \text{[By Leibnitz product rule]}$$

$$= \cos x \sin^2 x + \sin x (2 \sin x \cos x) \qquad \text{[Using (1)]}$$

$$= \cos x \sin^2 x + 2 \sin^2 x \cos x$$

$$= 3 \sin^2 x \cos x$$

We assert that $\frac{d}{dx}\left(\sin^n x\right) = n \sin^{(n-1)} x \cos x$

Let our assertion be true for $n = k$.

i.e., $\frac{d}{dx}\left(\sin^k x\right) = k \sin^{(k-1)} x \cos x \qquad ...(2)$

Consider

$$\frac{d}{dx}\left(\sin^{k+1} x\right) = \frac{d}{dx}\left(\sin x \sin^k x\right)$$

$$= (\sin x)' \sin^k x + \sin x \left(\sin^k x\right)' \qquad \text{[By Leibnitz product rule]}$$

$$= \cos x \sin^k x + \sin x \left(k \sin^{(k-1)} x \cos x\right) \qquad \text{[Using (2)]}$$

$$= \cos x \sin^k x + k \sin^k x \cos x$$

$$= (k+1) \sin^k x \cos x$$

Thus, our assertion is true for $n = k + 1$.

Hence, by mathematical induction, $\frac{d}{dx}\left(\sin^n x\right) = n \sin^{(n-1)} x \cos x$

27. The compound statement with 'And' is "25 is a multiple of 5 and 8".
This is a false statement, since 25 is not a multiple of 8.
The compound statement with 'Or' is "25 is a multiple of 5 or 8".
This is a true statement, since 25 is not a multiple of 8 but it is a multiple of 5.

28. Let E be the event in which the spokesperson will be a male and F be the event in which the spokesperson will be over 35 years of age.

Accordingly, P(E) = $\frac{3}{5}$ and P(F) = $\frac{2}{5}$

Since there is only one male who is over 35 years of age,

$$P(E \cap F) = \frac{1}{5}$$

We know that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$\therefore P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$$

Thus, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

29. Let C denote the set of people who like coffee, and
T denote the set of people who like tea
$n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$
We know that:
$n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$\therefore 70 = 37 + 52 - n(C \cap T)$
$\Rightarrow 70 = 89 - n(C \cap T)$
$\Rightarrow n(C \cap T) = 89 - 70 = 19$
Thus, 19 people like both coffee and tea.

30. It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$.

$\therefore$L.H.S. = $\sin x + \sin 3x + \sin 5x + \sin 7x$

$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$

$= 2\sin\left(\frac{x+5x}{2}\right)\cdot\cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$

$= 2\sin 3x\cos(-2x) + 2\sin 5x\ \cos(-2x)$

$= 2\sin 3x\cos 2x + 2\sin 5x\cos 2x$

$= 2\cos 2x[\sin 3x + \sin 5x]$

$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cdot\cos\left(\frac{3x-5x}{2}\right)\right]$

$= 2\cos 2x[2\sin 4x\cdot\cos(-x)]$

$= 4\cos 2x\sin 4x\cos x$ = R.H.S.

31. It is given that the man deposited Rs 10000 in a bank at the rate of 5% simple interest annually.

$\therefore$ Interest in first year $= \frac{5}{100} \times \text{Rs } 10000 = \text{Rs } 500$

$\therefore$Amount in 15th year = Rs $10000 + \underbrace{500 + 500 + ... + 500}_{14 \text{times}}$
= Rs 10000 + 14 × Rs 500
= Rs 10000 + Rs 7000
= Rs 17000

Amount after 20 years = $\text{Rs } 10000 + \underbrace{500 + 500 + ... + 500}_{20 \text{times}}$
= Rs 10000 + 20 × Rs 500
= Rs 10000 + Rs 10000
= Rs 20000

32. **(i)** Number of observations (n) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200$$

That is, incorrect sum of observations = 200

Correct sum of observations = 200 – 8 = 192

$\therefore$ Correct mean $= \dfrac{\text{Correct sum}}{19} = \dfrac{192}{19} = 10.1$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2$$

$$= 2080 - 64$$

$$= 2016$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct}\sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{106.1 - 102.01}$$

$$= \sqrt{4.09}$$

$$= 2.02$$

(ii) When 8 is replaced by 12,
Incorrect sum of observations = 200
∴ Correct sum of observations = 200 – 8 + 12 = 204

$$\therefore \text{Correct mean} = \frac{\text{Correct sum}}{20} = \frac{204}{20} = 10.2$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2 + (12)^2$$

$$= 2080 - 64 + 144$$

$$= 2160$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct}\sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$= \sqrt{108 - 104.04}$$

$$= \sqrt{3.96}$$

$$= 1.98$$

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