

# WORK, POWER AND ENERGY

## Important Points:

### 1. Dot Product:

a) Scalar product is defined as the product of the magnitudes of two vectors and the cosine of the angle between them. The dot product of two vectors  $\vec{a}$  and  $\vec{b}$  is given by

$$\vec{a} \cdot \vec{b} = ab \cos \theta$$

b) Dot product of unit vectors -

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

c) If  $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$  and  $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$$\text{Then } \vec{A} \cdot \vec{B} = A_1B_1 + A_2B_2 + A_3B_3$$

### 2. Work:

Work is said to be done by a force when a body undergoes displacement in the direction of force.

3. If a force  $F$  acts on a body at an angle  $\theta$  and displaces the body through a distance 'S' work done,  $W = \vec{F} \cdot \vec{S} = FS \cos \theta$

**Units:** S.I. - Joule

C.G.S- erg

$$1 \text{ Joule} = 10^7 \text{ ergs}$$

## 5. Power:

The rate of doing work is called Power.

$$\text{Power } (P) = \frac{\text{Work}}{\text{time}} = \frac{Fs}{t} = \frac{W}{t}$$

$$P = \frac{\overline{F} \cdot \overline{S}}{t} = \overline{F} \cdot \overline{V}$$

Power is a scalar.

### Unit of power:

Watt or J/sec

1 Horse Power H. P. = 746 watt

## 6. Energy:

The capacity of doing work is called energy. It is a scalar; its unit is same as that of work.

SI - Joule

C. G. S - erg

## 7. Potential Energy:

Energy possessed by a body by virtue of its position is known as potential energy.

$$\text{P. E.} = mgh$$

## 8. The restoring force on the spring per unit elongation is called force constant or spring constant

$$F = - Kx$$

(Negative sign indicates that the force is opposite to elongation)

9. The elastic P.E. stored in a compressed spring is

$$P.E. = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{1}{2} \cdot \frac{F^2}{K}$$

Where K is spring constant

10. **Kinetic Energy:**

The energy possessed by a body by virtue of its motion is called **Kinetic Energy**.

11. A flying bird, moving aeroplane, freely falling body, a body moving on an incline, and oscillating pendulum possess both P. E. and K. E.

12. **The relation between K.E. and Momentum:**  $KE = \frac{p^2}{2m}$

13. **Work - Energy Theorem:**

The work done by the resultant force acting on a body is equal to the change in its kinetic energy.

$$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

14. **Law of Conservation of Energy:**

Energy can neither be created nor destroyed. One form of energy can be converted into another form.

15. **Collision:**

The interaction among bodies involving exchange of momentum is called a collision.

16. **Elastic Collision:**

In elastic collision both momentum and kinetic energy are conserved.

17. **Inelastic Collision:**

In inelastic collision only momentum is conserved; kinetic energy is not conserved.

### 18. Law of Conservation Of Momentum:

In the absence of external force, total momentum before collision is equal to the total momentum after collision.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

19. Two bodies of masses  $m_1$  and  $m_2$  moving with velocities  $u_1$  and  $u_2$  undergo head on elastic collision, If  $v_1$  and  $v_2$  are their respective velocities after collision, then

i)  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

ii)  $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

iii)  $v_2 - v_1 = u_1 - u_2$

iv)  $v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2$

v)  $v_2 = \left( \frac{2m_1}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2$

### 20. Coefficient of Restitution:

$$e = \frac{\text{Relative velocity of separation after collision}}{\text{Relative velocity of approach before collision}} = \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

It depends on nature of colliding bodies.

21. For a perfectly elastic collision  $e = 1$

For a perfectly inelastic collision  $e = 0$

For other collisions 'e' lies between 0 and 1

## Very Short Answer Questions

1. State the conditions under which a force does not work.

A. A force does not work, when

- i) The displacement is zero,
- ii) The force is zero and
- iii) Force and displacement are perpendicular.

2. Define work, power and energy. State their S.I Units?

A. **Work:**

Work is said to be done by a force, when the body undergoes displacement in the direction of force.

Work done = Force  $\times$  Displacement

**Unit:** Joule

**Power:**

The time rate of doing work is called power.

$$Power = \frac{\text{Work done}}{\text{Time}} = \frac{W}{t}$$

**Unit:** watt

**Energy:**

The capacity of a body to do work is called energy.

**Unit:** Joule

**3. State the relation between kinetic energy and momentum of a body?**

A. Momentum  $P = mv$ .

$$\text{Kinetic energy } E = KE = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}mv^2 \times \left(\frac{m}{m}\right) = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$\therefore KE = \frac{P^2}{2m}$$

**4. State the sign of work done by a force in the following**

a) Work done by a man lifting a bucket out of a well by means of a rope tied to the bucket.

b) Work done by gravitational force in the above case

A: a) Work done is positive, because the bucket displaces in the direction of force.

b) Work done is negative because, the displacement is opposite to the gravitational force.

$$W = F.S \cos \theta = F.S \cos 180^\circ = -F.s$$

**5. State the sign of work done by a force in the following**

a) Work done by friction on a force in the following

b) Work done by gravitational force in the above case.

A: a) Work done is negative, because friction always opposite to the direction of motion.

b) The work done is positive.

**6. State the sign of work done by a force in the following.**

- a) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.**
- b) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.**

- A:**
- a) The work done is positive, because the applied force and displacement are in the same direction.
  - b) The work done is negative, because the direction of resistive force is opposite to the direction motion of the pendulum.

**7. State if each of the following statements is true or false. Give reasons for your answer.**

- a) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.**
- b) The work done by earth's gravitational force in keeping the moon in its orbit for its one revolution is zero.**

- A:**
- a) False, if external forces present, energy of the system changes.
  - b) True, because gravitational force is conservative.

**8. Which Physical Quantity Remains Constant.**

**(i) In Elastic Collision**

**(ii) in an Inelastic Collision**

- A.**
- (i) Elastic Collision:** Both momentum and kinetic energy remains constant.
  - (ii) Inelastic Collision:** Momentum alone remains constant.

9. A body freely falling from a certain height 'h' after striking a smooth floor rebounds and rises to a height  $\frac{h}{2}$ . What is the coefficient of restitution between the floor and the body?

A.  $h_1 = h$  and  $h_2 = \frac{h}{2}$

$$e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{\frac{h}{2}}{h}} = \frac{1}{\sqrt{2}}$$

10. What is the total displacement of a freely falling body, after successive rebounds from the ground, before it comes to stop? Assume that 'e' is the coefficient of restitution between the body and the ground.

- A. The total displacement of a freely falling body, after successive rebounds from the ground, before it comes to stop is 'h'.

### Short Answer Questions

1. What is potential energy? Derive an expression for the gravitational potential energy.

- A. **Potential Energy:**

The energy possessed by a body by virtue of its position or state is called potential energy.

#### Expression for Potential Energy:

Consider a body of mass 'm' initially at rest on the surface of the earth. The force acting on the body is its weight  $mg$ , vertically downwards. Force required to lift the body through a height  $h$  is  $mg$  upwards.

Work required in lifting the body.

$$W = \text{Force} \times \text{displacement} = mgh$$



The work done is stored as potential energy in the body P.E. =mgh

2. A lorry and a car moving with the same momentum are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in shorter time? Which will come to rest in less distance?

A. a) Momentum = Impulse **or**  $mv = Ft$

Since momentum of the body and breaking force are constant, both lorry and car come to rest in the same time.

b)  $Fs = \frac{p^2}{2m}$  or  $s \propto \frac{1}{m}$

Since the mass of the lorry is more than the mass of the car, lorry comes to rest in less distance.

3. Distinguish between conservative and non-conservative forces with one example each?

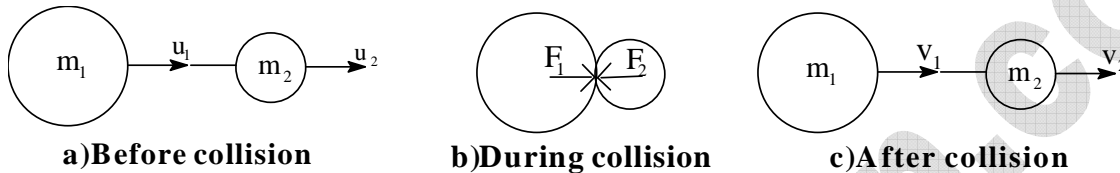
A.

Conservative Force	Non - Conservative Force
1. A force is conservative if the work done by the force on a body along a closed path is zero.	1. A force is non-conservative if the work done by the force along any closed path is not zero.
2. Work done is independent of the path followed by the body.	2. Work done depends on the path by the body.
<b>Ex:</b> Gravitational force, Electrostatic force.	<b>Ex:</b> Frictional force, viscous force

4. Show that in the case of one dimensional elastic collision, the relative velocity of approach of two colliding bodies before collision is equal to the relative velocity of separation after collision?

**A. One Dimensional Elastic Collision:**

Consider two bodies of masses and moving with velocities and collide elastically. Let and be the velocities of the two bodies after collision.



From the law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{Or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \dots\dots\dots (1)$$

From the law of conservation kinetic energy,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\text{Or } m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \dots\dots\dots (2)$$

$$\text{From equations (2) and (1), } \frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$$u_1 - u_2 = v_2 - v_1$$

Hence the relative velocity of approach of two colliding bodies before collision is equal to the relative velocity of separation after collision.

**5. Show that two equal masses undergo oblique elastic collision will move at right angles after collision, if the second body initially at rest?**

A. A body collides with another body of equal mass at rest. The collision is oblique and perfectly elastic. The two bodies move at right angles after collision.

$$m\vec{u}_1 - m \times 0 = m\vec{v}_1 + m\vec{v}_2 \Rightarrow \vec{u}_1 = \vec{v}_1 + \vec{v}_2$$

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Rightarrow u_1^2 = v_1^2 + v_2^2$$

Since  $u_1 = \sqrt{v_1^2 + v_2^2}$ , the two bodies move at right angles after collision.

**6. Derive an expression for the height attained by a freely falling body after 'n' number of rebounds from the floor?**

A: Let a body be allowed to fall freely from a height h on to the floor. It strikes the floor with a velocity  $u_1$ , so that-

$$u_1 = \sqrt{2gh} \rightarrow (1).$$

Before and after collision, the velocities of the floor is  $u_2 = v_2 = 0$ .

Let  $v_1$  be the final velocity of the body after first collision.

$$\text{Now } e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v_1}{\sqrt{2gh} - 0}$$

$$\therefore v_1 = -e\sqrt{2gh} \rightarrow (2) \text{ (-ve sign indicates that the sphere rebounds)}$$

$$\therefore \text{The height } (h_1) \text{ attained by it after 1}^{\text{st}} \text{ rebound is } h_1 = \frac{v_1^2}{2g} = \frac{(e\sqrt{2gh})^2}{2g}$$

$$h_1 = e^2h \text{ Or } h_1 = (e^2)^1h \rightarrow (3)$$

Now for the second time, the body falls from this height  $h_1$  and strikes the ground with a velocity  $= \sqrt{2gh_1}$

$$\therefore e = \frac{0 - v_2}{\sqrt{2gh_1} - 0} \quad \therefore v_2 = -e\sqrt{2gh_1} = -e\sqrt{2ge^2h} = -e^2\sqrt{2gh} \rightarrow (4)$$

The maximum height attained by the body after second rebound

$$h_2 = \frac{v_2^2}{2g} = \frac{(-e^2\sqrt{2gh})^2}{2g} = e^4h \quad \therefore h_2 = (e^2)^2h \rightarrow (5)$$

Similarly maximum height attained after n rebounds is  $h_n = (e^2)^n h$

## 7. Explain the law of conservation of energy?

### A: Law of conservation of energy:

Energy can neither be created nor destroyed. But it can be converted from one form to another form. The total energy of a system remains constant.

When a body undergoes displacement  $\Delta x$  under the action of a conservative force  $F$ . The change in K.E. of the body is given by the work energy theorem,  $\Delta k = F(x)dx \rightarrow (1)$

As the force is conservative, the change in P.E  $= \Delta U = F(x)\Delta x \rightarrow (2)$

$$\therefore \Delta k = -\Delta U$$

$$\Delta k + \Delta U = 0$$

$$\Delta(k + U) = 0 \rightarrow (3)$$

The sum of K.E. and P.E. or mechanical energy of the body under the action of a conservative force is a constant.

## Long Answer Questions

1. Develop the notions of work and kinetic energy and show that it leads work-energy theorem?

A. a) **Work:**

Work is said to be done by a force, when a body undergoes displacement in the direction of force.

$$\text{Work done} = \text{Force} \times \text{Displacement}$$

b) **Kinetic Energy:**

The energy possessed by a body by virtue of its motion is called Kinetic energy.

$$KE = \frac{1}{2}mv^2$$

**Work- Energy Theorem:**

The work done on a body by a resultant force is equal to the change in its kinetic energy.

**Proof:**

Consider a body of mass 'm' moving with an initial velocity 'u'. Let its velocity changes to 'v' due to a constant resultant force 'F'. Let 'a' be its uniform acceleration and 's' be its displacement.

$$v^2 - u^2 = 2as$$

Multiplying both sides with  $\frac{m}{2}$ ,  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = FS$

**Or** Final kinetic energy - Initial kinetic energy = Work done.

Hence work done by the resultant force is equal to the change in kinetic energy. This is known as work-energy theorem.

**2. What are Collisions? Explain the possible the types of Collisions. Develop the theory of one dimensional elastic collisions?**

**A. Collision:**

Collision is the interaction between two or more bodies that results in sudden change in momentum.

**Types of Collisions:**

(i) Elastic Collision

(ii) Inelastic collision

**Elastic Collision:**

The collision in which both momentum and kinetic energy are conserved is called elastic collision.

**Examples:**

Collision between hard billiard balls, Collision between fundamental particles etc.

**Inelastic Collisions:**

The collision in which, momentum alone is conserved, is called inelastic collision.

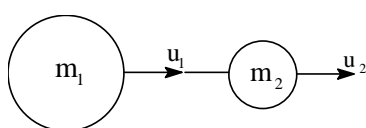
**Examples:**

Collision between any two vehicles.

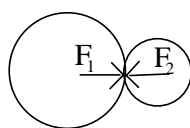
**One Dimensional Elastic Collision:**

Consider two bodies of masses  $m_1$  and  $m_2$  moving with velocities  $u_1$  and  $u_2$  collide elastically. Let  $u_1 > u_2$ . Let  $v_1$  and  $v_2$  be the velocities of the two bodies after

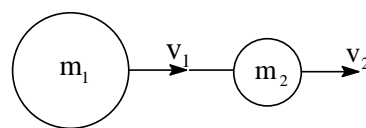
Collision.



**a) Before collision**



**b) During collision**



**c) After collision**

From the law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{Or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \dots\dots\dots (1)$$

From the law of conservation kinetic energy,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\text{Or } m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \dots\dots\dots (2)$$

$$\text{From equations (2) and (1), } \frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$$u_1 + u_2 = v_1 + v_2 \dots\dots\dots (3)$$

### Velocity of First Body:

$$\text{From equation (3), } v_2 = u_1 + v_1 - u_2 \dots\dots\dots (4)$$

Substituting equation (4) in equation (1) and simplifying, we get

$$v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2 \dots\dots\dots (5)$$

### Velocity of Second Body:

$$\text{Again, from equation (3), } v_1 = v_2 + u_2 - u_1$$

Substituting this in equation (1) and simplifying, we get

$$v_2 = \left( \frac{2m_1}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2 \dots\dots\dots (6)$$

If  $u_2 = 0$ , then from equations (5) and (6),

$$v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 \quad \text{and} \quad v_2 = \left( \frac{2m_1}{m_1 + m_2} \right) u_1$$

**3. State the law of conservation of energy and verify in the case of a freely falling body. What are the conditions under which the law of conservation of energy is applicable?**

**A. Statement:**

Energy can be neither created nor destroyed. One form of energy can be converted into another form.

**Verification:**

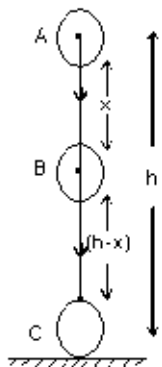
Consider a body of mass 'm' dropped from a height 'h' above the ground.

**At the point 'A':**

Initial velocity = 0

∴ Kinetic energy  $KE = \frac{1}{2}mv^2 = 0$

Potential energy  $PE = mgh$



∴ Total energy =  $PE + KE = mgh + 0 = mgh$  -----(1)

**At the point 'B':**

Let 'v' be the velocity of the body at 'B' and let  $AB = x$



Potential energy  $PE = mg(h - x) = mgh - mgx$

Using,  $v^2 - u^2 = 2as$  Or  $v^2 = 2gx$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx = mgx$$

Total energy =  $PE + KE = mgh - mgx + mgx = mgh$  -----(2)

**At the point 'C':**

Let 'v' be the velocity of the body at the ground.

Potential energy =  $mgh = mg(0) = 0$

Using,  $v^2 - u^2 = 2as$  Or  $v^2 = 2gh$

Kinetic energy  $KE = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$

Total energy =  $PE + KE = 0 + mgh = mgh$  ----- (3)

From equations (1), (2) and (3) it is clear that the total energy is constant. Thus the law of conservation of energy is verified.

## Problems

1. A test tube of mass 10 grams closed with a cork of mass 1 gram contains some ether. When the test tube is heated cork flies out under the pressure of the ether gas. The test tube is suspended horizontally by a weight less rigid bar of length 5cm. What is the minimum velocity with which the cork should fly out of the tube, so that test tube describing a full vertical circle about the point O. Neglect the mass of ether?

A  $m_1 = 10 \times 10^{-3} \text{ kg}, m_2 = 10^{-3} \text{ kg}$

$$m = (10 + 1)10^{-3} = 11 \times 10^{-3} \text{ kg}, l = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}, g = 10 \text{ m/s}^2$$

$$\text{Minimum velocity } (v) = \sqrt{5gl} = \sqrt{5 \times 10 \times 5 \times 10^{-2}}$$

$$\therefore v = 5\sqrt{10^{-1}} \text{ m/s}$$

2. A machine gun fires 360 bullets per minute and each bullet travels with a velocity of  $6000 \text{ ms}^{-1}$ . If the mass of each bullet is 5gm, find the power of the machine gun?

A  $\frac{N}{t} = 360 \text{ bullet / min} = \frac{360}{60} = 6 \text{ bullets / sec}$

$$v = 600 \text{ m/s}, m = 5 \text{ gm} = 5 \times 10^{-3} \text{ kg}$$

$$P = \frac{W}{t} = \frac{N \times \frac{1}{2}mv^2}{t}$$

$$P = \frac{6 \times 5 \times 10^{-1} \times 600 \times 600}{2} = 5400 \text{ W} = 5.4 \text{ K.W}$$

3. Find the useful power used in pumping  $3425 \text{ m}^3$  of water per hour from a well 8m deep to the surface, supposing 40% of the horse power during pumping is wasted. What is the horse power of the engine?

A.  $V = 3425 \text{ m}^3, t = 1 \text{ hour} = 60 \times 60 \text{ sec}$

$$h = 8\text{m}, 1 \text{ H.P} = 746 \text{ Walts}$$

$$\text{Mass}(m) = \text{volume} \times \text{density} = 3425 \times 10^3$$

$$\frac{60}{100} P = \frac{mgh}{t} = \frac{Vdgh}{t}$$

$$P = \frac{3425 \times 10^3 \times 9.8 \times 8 \times 100}{60 \times 60 \times 60 \times 746} = 1.66 \times 10^2 \text{ H.P}$$

4. A pump is required to lift 600kg of water per minute from a well 25m deep and to eject it with a speed of  $50\text{ms}^{-1}$ . Calculate the power required to perform the above task?

A.  $m = 600 \text{ kg/s}, h = 25\text{m}, v = 50\text{m/s}$

$$P = \frac{mgh + \frac{1}{2}mv^2}{t} = \frac{600 \times 10 \times 25 + \frac{1}{2} \times 600 \times (50)^2}{60} = 15.166 \text{ kw}$$

5. A block of mass 5kg initially at rest at the origin is acted on by a force along the X – Positive direction represented by  $F = (20 + 5x)\text{N}$ . Calculate the work done by the force during the displacement of the block from  $x = 0$  to  $x = 4\text{m}$ .

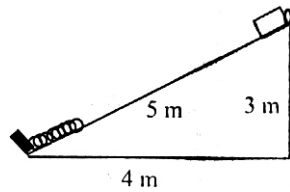
A.  $m = 5\text{kg}, u = 0, F = (20 + 5x) \text{ N}$

$x = 0$  to  $x = 4\text{m}$

$$W = \int_{x=0}^{x=4} F \cdot dx = \int_{x=0}^{x=4} (20x + 5x) dx = 20[x]_0^4 + 5\left(\frac{x^2}{2}\right)_0^4$$

$W = 120 \text{ J}$ .

6. A block of mass 5kg is sliding down a smooth inclined plane as shown. The spring arranged near the bottom of the inclined plane has a force constant 600 N/m. Find the compression in the spring at the moment the velocity of the block is maximum?



**A:**  $m = 5\text{kg}$ ,  $K = 600\text{ N/m}$ ;  $\sin \theta = \frac{3}{5}$

$$F = mg \sin \theta$$

$$F = 5 \times 10 \times \frac{3}{5} = 30\text{N} ; K = \frac{F}{x}$$

$$x = \frac{F}{K} = \frac{30}{600} = 0.05\text{m}$$

$$\therefore x = 5\text{cm}$$

7. A force  $F = -\frac{K}{x^2}$  ( $x \neq 0$ ) acts on a particle along the X-axis. Find the work done by the force in displacing the particle from  $x = +a$  to  $x = +2a$ . Take K as a positive constant?

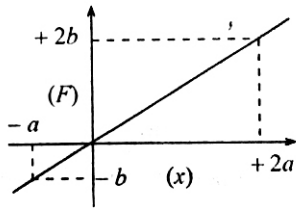
**A:**  $F = \frac{-K}{x^2}$

$$x = +a \text{ to } x = +2a$$

$$W = \int_{x=a}^{x=2a} F \cdot dx = \int_{x=a}^{x=2a} \frac{-K}{x^2} dx = -K \left[ \frac{x^{-2+1}}{-2+1} \right] = K \left[ \frac{1}{x} \right]_a^{2a} = K \left[ \frac{1}{2a} - \frac{1}{a} \right] = K \left[ \frac{a-2a}{2a \times a} \right]$$

$$\therefore W = \frac{-K}{2a}$$

8. A force  $F$  acting on a particle varies with the position  $x$  as shown in the graph. Find the work done by the force in displacing the particle from  $x = -a$  to  $x = +2a$ ?



**A:**  $x = -a$  to  $x = +2a$

$$\text{Average force} = \frac{-b + 2b}{2} = \frac{b}{2}$$

$$W = \int_{-a}^{2a} F \cdot dx = \int_{-a}^{2a} \frac{b}{2} \cdot dx$$

$$W = \frac{b}{2} [x]_{-a}^{2a} = \frac{b}{2} [2a + a] = \frac{3ab}{2}$$

9. From a height 20m above a horizontal floor, a ball is thrown down with initial velocity 20m/s. After striking the floor, the ball bounces to the same height from which it was thrown. Find the coefficient of restitution for the collision between the ball the ball and the floor? ( $g = 10 \text{ m/s}^2$ )

**A:**  $u_1 = \sqrt{u^2 + 2gh}$ ,  $u = 20 \text{ m/s}$ ,  $g = 10 \text{ m/s}^2$ ,  $h = 20 \text{ m}$

$$u_2 = \sqrt{(20)^2 + 2 \times 10 \times 20} = \sqrt{800}$$

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = \sqrt{400}$$

$$e = \frac{v_2 - u_1}{u_1 - u_2} = \frac{0 - \sqrt{400}}{\sqrt{800} - 0} \text{ (-ve sign indicates it rebounds)}$$

$$\therefore e = \frac{1}{\sqrt{2}}$$

10. A ball falls from a height of 10m on to a hard horizontal floor and repeatedly bounces. If the coefficient of restitution is  $\frac{1}{\sqrt{2}}$ . What is the total distance travelled by the ball before it ceases to rebound?

A.  $h = 10m, e = \frac{1}{\sqrt{2}}$

$$d = h \left( \frac{1+e^2}{1-e^2} \right) = 10 \left[ \frac{1+1/2}{1-1/2} \right] = 30m$$