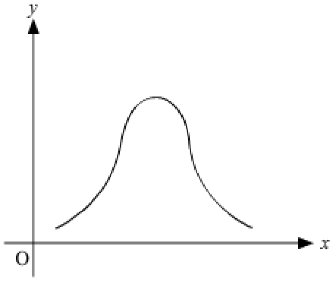
Statistics

Measure of Skewness

- In a symmetrical (normal) distribution, frequencies about the mean are symmetrically distributed. Mean, median and mode coincide in a symmetrical distribution, i.e., mean = median = mode.



- Skewness (S_k) is the measure of lack of symmetry. A distribution is skewed if mean ≠ median ≠ mode.

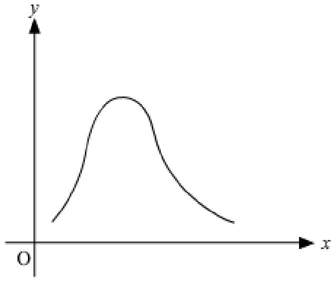
Quartiles are not at the same distance from the median.

The frequency curve drawn is stretched more to one side than to the other.

The value of skewness can be positive or negative, or it may not be defined.

- **Positively Skewed ($S_k > 0$)**

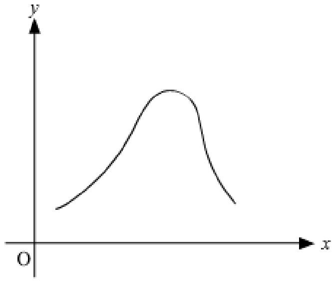
In a distribution, if the value of mean is maximum and that of mode is minimum, then the distribution is said be positively skewed. Median lies in between the mean and mode. The curve is more elongated towards the right.



For skewness to be positive, mean > median > mode.

- **Negatively Skewed ($S_k < 0$)**

In a distribution, if the value of mode is maximum and that of mean is minimum, then the distribution is said to be negatively skewed. The curve is more elongated towards the left.



For skewness to be negative, mean < median < mode.

- Measures of skewness tell us the direction and extent of asymmetry in a series. There are two measures of skewness. These are:

1. Absolute measure of skewness

2. Relative measure of skewness

- Mean deviation about mean $[\mathrm{M.D.}(\bar{x})]$:
  - **For ungrouped data:** $\mathrm{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$, where $\bar{x}$ is the mean given by $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

- Mean deviation about mean $[\mathrm{M.D.}(\bar{x})]$:
  - **For grouped data:** $\mathrm{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$, where $\bar{x}$ is the mean given by $\bar{x} = \frac{1}{\mathrm{N}}\sum_{i=1}^{n} f_i x_i$ and

Example:
Calculate mean deviation about mean for the following data:

Class	0 – 10	10 – 20	20 – 30	30 – 40
Frequency	21	19	49	11

Solution:
Here, assumed mean (*a*) = 25 and class size (*h*) = 10

Class	Frequency (f_i)	Mid-point (x_i)	$d_i = \frac{x_i - 25}{10}$	$f_i d_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0 – 10	21	5	–2	–42	15	315
10 – 20	19	15	–1	–19	5	95
20 – 30	49	25	0	0	5	245
30 – 40	11	35	1	11	15	165
Total	**100**			**–50**		**820**

Here, $\mathrm{N} = \sum_{i=1}^{4} f_i = 100$

Now,

Mean, $\bar{x} = a + \frac{1}{\mathrm{N}}\sum_{i=1}^{4} f_i d_i \times h = 25 + \frac{(-50)}{100} \times 10 = 25 - 5 = 20$

$\therefore \mathrm{M.D}\left(\bar{x}\right) = \frac{1}{\mathrm{N}}\sum_{i=1}^{4} f_i\left|x - \bar{x}\right| = \frac{1}{100} \times 820 = 8.2$

- **Example:**

Find the mean deviation about the median for the following data:
181, 29, 150, 270, 160, 16, 27, 180, 200

- **Solution:**

Here, the number of observations is 9 and these can be arranged in ascending order as
16, 27, 29, 150, 160, 180, 181, 200, 270

Median, M = $\left(\frac{9+1}{2}\right)^{\text{th}}$ observation or 5^{th} observation = 160

$$\therefore \mathrm{M.D.}(\mathrm{M}) = \frac{1}{9}\sum_{i=1}^{9}\left|x_i - \mathrm{M}\right|$$

$$= \frac{1}{9}\left(|16-160| + |27-160| + |29-160| + |150-160| + |160-160| + |180-160| + |181-160| + |200-160| + |270-160|\right)$$

$$= \frac{1}{9}(144 + 133 + 131 + 10 + 0 + 20 + 21 + 40 + 110)$$

$$= \frac{1}{9} \times 609$$

$$= 67.67$$

- The mean of the squares of the deviations from mean is called the variance and it is denoted by s^2.
- Variance of Data:
  - **For ungrouped data:** $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ (In direct method) or $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2$ (In shortcut method), where $\bar{x}$ is the mean.
- Standard deviation is the square root of variance and it is denoted by *s*. This means:

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

Example:
The mean and standard deviations of 50 observations were calculated as 30 and 4 respectively. Later, it was found that by mistake, 13 was taken instead of 18 for one observation during the calculation. Find the correct mean and the correct standard deviation.

Solution:
It is given that, number of observations (n) = 50
Incorrect mean, $\bar{x} = 30$
Incorrect standard deviation (s) = 4
We know that $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
i.e. $30 = \frac{1}{50}\sum_{i=1}^{50} x_i$ or $\sum_{i=1}^{50} x_i = 1500$
Incorrect sum of observations = 1500
∴Correct sum of observations = 1500 – 13 + 18 = 1505
∴ Correct mean = $\frac{1505}{50} = 30.1$
Now, standard deviation, $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$
$\Rightarrow \sigma = 4 = \sqrt{\frac{1}{50} \times \text{Incorrect} \sum_{i=1}^{n} x_i^2 - 900}$
$\Rightarrow 16 = \frac{1}{50} \times \text{Incorrect} \sum_{i=1}^{n} x_i^2 - (30)2$
$\Rightarrow \text{Incorrect} \sum_{i=1}^{n} x_i^2 = 916 \times 50 = 45800$
Now, correct $\sum_{i=1}^{n} x_i^2$ = Incorrect $\sum_{i=1}^{n} x_i^2 - (13)^2 + (18)^2 = 45800 - 169 + 324 = 45955$
Correct standard deviation $= \sqrt{\frac{\text{Correct } \Sigma x_i^2}{n} - (\text{Correct mean})^2}$
$= \sqrt{\frac{45955}{50} - (30.1)^2}$
$= \sqrt{919.1 - 906.01}$
$= \sqrt{13.09} = 3.62$

- Variance of Data:

  For discrete frequency distribution: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$ (In direct method)
  or $\sigma^2 = \frac{1}{N^2}\left[N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2\right]$ (In shortcut method), where $\bar{x}$ is the mean and $N = \sum_{i=1}^{n} f_i$

- Standard deviation is the square root of variance and it is denoted by σ.

  $\text{Standard Deviation} = \sqrt{\text{Variance}}$

- Variance of Data:
  - **For continuous frequency distribution:** $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$ or $\sigma^2 = \frac{1}{N^2}\left[N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2\right]$ (Indirect method) or

    $\sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2\right]$ (In shortcut method), where

  x_i = class marks of the class intervals, $\bar{x}$ = mean, $N = \sum_{i=1}^{n} f_i$, h = width of the class intervals, $y_i = \frac{x_i - A}{h}$, where A is the assumed mean.

- Standard deviation is the square root of variance and it is denoted by s. This means:

  $\text{Standard Deviation} = \sqrt{\text{Variance}}$

  Example:
  Find the variance and standard deviation for the following data.

Class	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	4	3	1	7

  Solution:
  Let assumed mean, A = 35
  Here, h = 10, N = 20
  We obtain the following table from the given data.

Class	Frequency (f_i)	Mid-points (x_i)	$y_i = \frac{x_i - 35}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
10 – 20	5	15	–2	4	–10	20
20 – 30	4	25	–1	1	–4	4
30 – 40	3	35	0	0	0	0
40 – 50	1	45	1	1	1	1
50 – 60	7	55	2	4	14	28
Total	**N = 20**					

- The measure of variability, which is independent of units, is called the coefficient of variation. The coefficient of variation (C.V.) is defined as

  $\text{C.V} = \frac{\sigma}{x} \times 100, \overline{x} \neq 0$

  Where, σ and $\overline{x}$ are standard deviation and mean of the data respectively.

- For comparing the variability or dispersion of two series, we first calculate the C.Vs of each series. The series having higher C.V. is said to be more variable than the other and the series having lower C.V. is said to be more consistent than the other.

- For two series with equal mean values, the series with greater standard deviation (or variance) is more variable or dispersed than the other. Also, the series with lower value of standard deviation (or variance) is said to be more consistent or less scattered than the other.

Example:
Which series, I or II, is more consistent?

	Series I	Series II
Mean	3100	3100
Variance	121	169

Solution:
Standard deviation of series I, $\sigma_1 = \sqrt{121} = 11$
Standard deviation of series II, $\sigma_2 = \sqrt{169} = 13$
Since the mean of both the series is the same, the series with lower standard deviation will be more consistent.
Thus, series I will be more consistent.