विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

Subject : Mathematics
Topic : Sequence & Progression

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5. Que. from Compt. Exams
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Trigonometric Equation

1. Trigonometric Equation :

An equation involving one or more trigonometric ratios of an unknown angle is called a trigonometric equation.

2. Solution of Trigonometric Equation :

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, ...$

Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as :

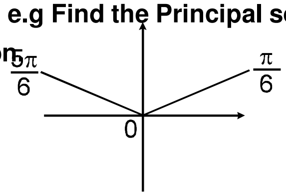
(i) Principal solution (ii) General solution.

2.1 Principal solutions:

The solutions of a trigonometric equation which lie in the interval $[0, 2\pi)$ are called **Principal solutions.**

e.g Find the Principal solutions of the equation $\sin x = \frac{1}{2}$.

Solution.



$\because$ $\sin x = \frac{1}{2}$

$\because$ there exists two values

i.e. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ which lie in $[0, 2\pi)$ and whose sine is $\frac{1}{2}$

$\therefore$ Principal solutions of the equation $\sin x = \frac{1}{2}$ are $\frac{\pi}{6}$, $\frac{5\pi}{6}$ **Ans.**

2.2 General Solution :

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called **General solution**.

General solution of some standard trigonometric equations are given below.

3. General Solution of Some Standard Trigonometric Equations :

(i) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

(ii) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

(iv) If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(v) If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(vi) If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$. [**Note:** α is called the principal angle]

Some Important deductions :

(i) $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

(ii) $\sin\theta = 1 \Rightarrow \theta = (4n + 1)\frac{\pi}{2}$, $n \in I$

(iii) $\sin\theta = -1 \Rightarrow \theta = (4n - 1)\frac{\pi}{2}$, $n \in I$

(iv) $\cos\theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$, $n \in I$

(v) $\cos\theta = 1 \Rightarrow \theta = 2n\pi$, $n \in I$

(vi) $\cos\theta = -1 \Rightarrow \theta = (2n + 1)\pi$, $n \in I$

(vii) $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

Solved Example # 1

Solve $\sin\theta = \frac{\sqrt{3}}{2}$.

Solution.

$\because$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin\theta = \sin\frac{\pi}{3}$

$\therefore$ $\boldsymbol{\theta = n\pi + (-1)^n \frac{\pi}{3}, n \in I}$ **Ans.**

Solved Example # 2

Solve $\boldsymbol{\sec 2\theta = -\frac{2}{\sqrt{3}}}$

Solution.

$\because$ $\sec 2\theta = -\frac{2}{\sqrt{3}}$

$\Rightarrow$ $\cos 2\theta = -\frac{\sqrt{3}}{2}$ $\Rightarrow$ $\cos 2\theta = \cos\frac{5\pi}{6}$

$\Rightarrow$ $2\theta = 2n\pi \pm \frac{5\pi}{6}, n \in I$

$\Rightarrow$ $\boldsymbol{\theta = n\pi \pm \frac{5\pi}{12}, n \in I}$ **Ans.**

Solved Example # 3

Solve $\boldsymbol{\tan\theta = 2}$

Solution.

$\because$ $\tan\theta = 2$ ....(i)

Let $2 = \tan\alpha$

$\Rightarrow$ $\tan\theta = \tan\alpha$

$\Rightarrow$ $\boldsymbol{\theta = n\pi + \alpha}$**, where** $\boldsymbol{\alpha = \tan^{-1}(2), n \in I}$

Self Practice Problems:

1. Solve $\cot\theta = -1$

2. Solve $\cos 3\theta = -\frac{1}{2}$

Ans. (1) $\theta = n\pi - \frac{\pi}{4}, n \in I$ (2) $\frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in I$

Solved Example # 4

Solve $\boldsymbol{\cos^2\theta = \frac{1}{2}}$

Solution.

$\because$ $\cos^2\theta = \frac{1}{2}$

$\Rightarrow$ $\cos^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow$ $\cos^2\theta = \cos^2\frac{\pi}{4}$

$\Rightarrow$ $\boldsymbol{\theta = n\pi \pm \frac{\pi}{4}, n \in I}$ **Ans.**

Solved Example # 5

Solve $\boldsymbol{4\tan^2\theta = 3\sec^2\theta}$

Solution.

$\because$ $4\tan^2\theta = 3\sec^2\theta$ ....(i)

For equation (i) to be defined $\theta \neq (2n+1)\frac{\pi}{2}, n \in I$

$\because$ equation (i) can be written as:

$\frac{4\sin^2\theta}{\cos^2\theta} = \frac{3}{\cos^2\theta}$ $\because$ $\theta \neq (2n+1)\frac{\pi}{2}, n \in I$

$\therefore$ $\cos^2\theta \neq 0$

$\Rightarrow$ $4\sin^2\theta = 3$

$\Rightarrow$ $\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \quad \sin^2\theta = \sin^2 \dfrac{\pi}{3}$

$\Rightarrow \quad \boldsymbol{\theta = n\pi \pm \dfrac{\pi}{3}, n \in I}$ **Ans.**

Self Practice Problems :

1. Solve $7\cos^2\theta + 3\sin^2\theta = 4$.

2. Solve $2\sin^2 x + \sin^2 2x = 2$

Ans. (1) $n\pi \pm \dfrac{\pi}{3}, n \in I$ (2) $(2n + 1)\dfrac{\pi}{2}, n \in I$ or $n\pi \pm \dfrac{\pi}{4}, n \in I$

Types of Trigonometric Equations :

Type -1

Trigonometric equations which can be solved by use of factorization.

Solved Example # 6

Solve $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$.

Solution.

$\because \quad (2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$\Rightarrow \quad (2\sin x - \cos x)(1 + \cos x) - \sin^2 x = 0$

$\Rightarrow \quad (2\sin x - \cos x)(1 + \cos x) - (1 - \cos x)(1 + \cos x) = 0$

$\Rightarrow \quad (1 + \cos x)(2\sin x - 1) = 0$

$\Rightarrow \quad 1 + \cos x = 0$ or $2\sin x - 1 = 0$

$\Rightarrow \quad \cos x = -1$ or $\sin x = \dfrac{1}{2}$

$\Rightarrow \quad x = (2n + 1)\pi, n \in I$ or $\sin x = \sin\dfrac{\pi}{6}$ $\Rightarrow x = n\pi + (-1)^n \dfrac{\pi}{6}, n \in I$

$\therefore$ Solution of given equation is

$\boldsymbol{(2n + 1)\pi, n \in I}$ or $\boldsymbol{n\pi + (-1)^n \dfrac{\pi}{6}, n \in I}$ **Ans.**

Self Practice Problems :

1. Solve $\cos^3 x + \cos^2 x - 4\cos^2\dfrac{x}{2} = 0$

2. Solve $\cot^2\theta + 3\text{cosec}\,\theta + 3 = 0$

Ans. (1) $(2n + 1)\pi, n \in I$

(2) $2n\pi - \dfrac{\pi}{2}, n \in I$ or $n\pi + (-1)^{n+1}\dfrac{\pi}{6}, n \in I$

Type - 2

Trigonometric equations which can be solved by reducing them in quadratic equations.

Solved Example # 7

Solve $2\cos^2 x + 4\cos x = 3\sin^2 x$

Solution.

$\because \quad 2\cos^2 x + 4\cos x - 3\sin^2 x = 0$

$\Rightarrow \quad 2\cos^2 x + 4\cos x - 3(1 - \cos^2 x) = 0$

$\Rightarrow \quad 5\cos^2 x + 4\cos x - 3 = 0$

$\Rightarrow \quad \left\{\cos x - \left(\dfrac{-2+\sqrt{19}}{5}\right)\right\}\left\{\cos x - \left(\dfrac{-2-\sqrt{19}}{5}\right)\right\} = 0$ ....(ii)

$\because \quad \cos x \in [-1, 1] \ \forall\ x \in R$

$\therefore \quad \cos x \neq \dfrac{-2-\sqrt{19}}{5}$

$\therefore$ equation (ii) will be true if

$\cos x = \dfrac{-2+\sqrt{19}}{5}$

$\Rightarrow \quad \cos x = \cos\alpha$, where $\cos\alpha = \dfrac{-2+\sqrt{19}}{5}$

$\Rightarrow \quad \boldsymbol{x = 2n\pi \pm \alpha}$ **where** $\boldsymbol{\alpha = \cos^{-1}\left(\dfrac{-2+\sqrt{19}}{5}\right), n \in I}$ **Ans.**

Self Practice Problems : **1.** Solve $\cos 2\theta - (\sqrt{2} + 1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$

2. Solve $4\cos\theta - 3\sec\theta = \tan\theta$

Ans. (1) $2n\pi \pm \frac{\pi}{3}, n \in I$ or $2n\pi \pm \frac{\pi}{4}, n \in I$

(2) $n\pi + (-1)^n \alpha$ where $\alpha = \sin^{-1}\left(\frac{-1-\sqrt{17}}{8}\right), n \in I$

or $n\pi + (-1)^n \beta$ where $\beta = \sin^{-1}\left(\frac{-1+\sqrt{17}}{8}\right), n \in I$

Type - 3

Trigonometric equations which can be solved by transforming a sum or difference of trigonometric ratios into their product.

Solved Example # 8

Solve $\cos 3x + \sin 2x - \sin 4x = 0$

Solution.

$\cos 3x + \sin 2x - \sin 4x = 0 \Rightarrow \cos 3x + 2\cos 3x.\sin(-x) = 0$

$\Rightarrow \cos 3x - 2\cos 3x.\sin x = 0 \Rightarrow \cos 3x\,(1 - 2\sin x) = 0$

$\Rightarrow \cos 3x = 0$ or $1 - 2\sin x = 0$

$\Rightarrow 3x = (2n+1)\frac{\pi}{2}, n \in I$ or $\sin x = \frac{1}{2}$

$\Rightarrow x = (2n+1)\frac{\pi}{6}, n \in I$ or $x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

$\therefore$ solution of given equation is

$(2n+1)\frac{\pi}{6}, n \in I$ or $n\pi + (-1)^n \frac{\pi}{6}, n \in I$ Ans.

Self Practice Problems :

1. Solve $\sin 7\theta = \sin 3\theta + \sin\theta$

2. Solve $5\sin x + 6\sin 2x + 5\sin 3x + \sin 4x = 0$

3. Solve $\cos\theta - \sin 3\theta = \cos 2\theta$

Ans. (1) $\frac{n\pi}{3}, n \in I$ or $\frac{n\pi}{2} \pm \frac{\pi}{12}, n \in I$

(2) $\frac{n\pi}{2}, n \in I$ or $2n\pi \pm \frac{2\pi}{3}, n \in I$

(3) $\frac{2n\pi}{3}, n \in I$ or $2n\pi - \frac{\pi}{2}, n \in I$ or $n\pi + \frac{\pi}{4}, n \in I$

Type - 4

Trigonometric equations which can be solved by transforming a product of trigonometric ratios into their sum or difference.

Solved Example # 9

Solve $\sin 5x.\cos 3x = \sin 6x.\cos 2x$

Solution.

$\because \sin 5x.\cos 3x = \sin 6x.\cos 2x \Rightarrow 2\sin 5x.\cos 3x = 2\sin 6x.\cos 2x$

$\Rightarrow \sin 8x + \sin 2x = \sin 8x + \sin 4x \Rightarrow \sin 4x - \sin 2x = 0$

$\Rightarrow 2\sin 2x.\cos 2x - \sin 2x = 0 \Rightarrow \sin 2x\,(2\cos 2x - 1) = 0$

$\Rightarrow \sin 2x = 0$ or $2\cos 2x - 1 = 0$

$\Rightarrow 2x = n\pi, n \in I$ or $\cos 2x = \frac{1}{2}$

$\Rightarrow x = \frac{n\pi}{2}, n \in I$ or $2x = 2n\pi \pm \frac{\pi}{3}, n \in I$

$\Rightarrow x = n\pi \pm \frac{\pi}{6}, n \in I$

$\therefore$ Solution of given equation is

$\frac{n\pi}{2}, n \in I$ or $n\pi \pm \frac{\pi}{6}, n \in I$ Ans.

Type - 5

Trigonometric Equations of the form $a\sin x + b\cos x = c$, where $a, b, c \in R$, can be solved by dividing both sides of the equation by $\sqrt{a^2 + b^2}$.

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Solved Example # 10

Solve $\sin x + \cos x = \sqrt{2}$

Solution.

$\because$ $\sin x + \cos x = \sqrt{2}$ ....(i)

Here $a = 1,\ b = 1.$

$\therefore$ divide both sides of equation (i) by $\sqrt{2}$, we get

$\sin x \cdot \dfrac{1}{\sqrt{2}} + \cos x \cdot \dfrac{1}{\sqrt{2}} = 1$

$\Rightarrow$ $\sin x \cdot \sin\dfrac{\pi}{4} + \cos x \cdot \cos\dfrac{\pi}{4} = 1$

$\Rightarrow$ $\cos\left(x - \dfrac{\pi}{4}\right) = 1$

$\Rightarrow$ $x - \dfrac{\pi}{4} = 2n\pi,\ n \in I$

$\Rightarrow$ $x = 2n\pi + \dfrac{\pi}{4},\ n \in I$

$\therefore$ Solution of given equation is

$\mathbf{2n\pi + \dfrac{\pi}{4},\ n \in I}$ **Ans.**

Note : Trigonometric equation of the form $a \sin x + b \cos x = c$ can also be solved by changing **sinx** and **cosx** into their **corresponding tangent of half the angle.**

Solved Example # 11

Solve $3\cos x + 4\sin x = 5$

Solution.

$\because$ $3\cos x + 4\sin x = 5$ ....(i)

$\because$ $\cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ & $\sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$\therefore$ equation (i) becomes

$\Rightarrow$ $3\left(\dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) = 5$ ....(ii)

Let $\tan\dfrac{x}{2} = t$

$\therefore$ equation (ii) becomes

$3\left(\dfrac{1 - t^2}{1 + t^2}\right) + 4\left(\dfrac{2t}{1 + t^2}\right) = 5$

$\Rightarrow$ $4t^2 - 4t + 1 = 0$

$\Rightarrow$ $(2t - 1)^2 = 0$

$\Rightarrow$ $t = \dfrac{1}{2}$ $\because$ $t = \tan\dfrac{x}{2}$

$\Rightarrow$ $\tan\dfrac{x}{2} = \dfrac{1}{2}$

$\Rightarrow$ $\tan\dfrac{x}{2} = \tan\alpha$, where $\tan\alpha = \dfrac{1}{2}$

$\Rightarrow$ $\dfrac{x}{2} = n\pi + \alpha$

$\Rightarrow$ $\mathbf{x = 2n\pi + 2\alpha}$ **where** $\mathbf{\alpha = \tan^{-1}\left(\dfrac{1}{2}\right),\ n \in I}$ **Ans.**

Self Practice Problems :

1. Solve $\sqrt{3}\cos x + \sin x = 2$

2. Solve $\sin x + \tan\frac{x}{2} = 0$

Ans. (1) $2n\pi + \frac{\pi}{6}$, $n \in I$ (2) $x = 2n\pi$, $n \in I$

Type - 6

Trigonometric equations of the form P(sinx ± cosx, sinx cosx) = 0, where p(y, z) is a polynomial, can be solved by using the substitution sinx ± cosx = t.

Solved Example # 12

Solve $\mathbf{\sin x + \cos x = 1 + \sin x.\cos x}$

Solution.

$\because$ $\sin x + \cos x = 1 + \sin x.\cos x$ ....(i)

Let $\sin x + \cos x = t$

$\Rightarrow$ $\sin^2 x + \cos^2 x + 2\sin x.\cos x = t^2$

$\Rightarrow$ $\sin x.\cos x = \frac{t^2 - 1}{2}$

Now put $\sin x + \cos x = t$ and $\sin x.\cos x = \frac{t^2 - 1}{2}$ in (i), we get

$t = 1 + \frac{t^2 - 1}{2}$

$\Rightarrow$ $t^2 - 2t + 1 = 0$

$\Rightarrow$ $t = 1$ $\because$ $t = \sin x + \cos x$

$\Rightarrow$ $\sin x + \cos x = 1$ ....(ii)

divide both sides of equation (ii) by $\sqrt{2}$, we get

$\Rightarrow$ $\sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow$ $x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

(i) if we take positive sign, we get

$\mathbf{x = 2n\pi + \frac{\pi}{2}, n \in I}$ **Ans.**

(ii) if we take negative sign, we get

$\mathbf{x = 2n\pi, n \in I}$ **Ans.**

Self Practice Problems:

1. Solve $\sin 2x + 5\sin x + 1 + 5\cos x = 0$

2. Solve $3\cos x + 3\sin x + \sin 3x - \cos 3x = 0$

3. Solve $(1 - \sin 2x)(\cos x - \sin x) = 1 - 2\sin^2 x$.

Ans. (1) $n\pi - \frac{\pi}{4}$, $n \in I$ (2) $n\pi - \frac{\pi}{4}$, $n \in I$

(3) $2n\pi + \frac{\pi}{2}$, $n \in I$ or $2n\pi$, $n \in I$ or $n\pi + \frac{\pi}{4}$, $n \in I$

Type - 7

Trigonometric equations which can be solved by the use of boundness of the trigonometric ratios sinx and cosx.

Solved Example # 13

Solve $\mathbf{\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0}$

Solution.

$\because$ $\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$ ....(i)

$\Rightarrow$ $\sin x.\cos\frac{x}{4} - 2\sin^2 x + \cos x + \sin\frac{x}{4}.\cos x - 2\cos^2 x = 0$

$\Rightarrow$ $\left(\sin x.\cos\frac{x}{4} + \sin\frac{x}{4}.\cos x\right) - 2(\sin^2 x + \cos^2 x) + \cos x = 0$

$\Rightarrow$ $\sin\frac{5x}{4} + \cos x = 2$ ....(ii)

Now equation (ii) will be true if

$\sin\frac{5x}{4} = 1$ **and** $\cos x = 1$

$\Rightarrow \frac{5x}{4} = 2n\pi + \frac{\pi}{2}, n \in I$ **and** $x = 2m\pi, m \in I$

$\Rightarrow x = \frac{(8n+2)\pi}{5}, n \in I$ ....(iii) **and** $x = 2m\pi, m \in I$ ....(iv)

Now to find general solution of equation (i)

$\frac{(8n+2)\pi}{5} = 2m\pi$

$\Rightarrow 8n + 2 = 10m$

$\Rightarrow n = \frac{5m-1}{4}$

if	$m = 1$	then	$n = 1$
if	$m = 5$	then	$n = 6$
	....	....	....
	....	....	....
if	$m = 4p - 3, p \in I$	then	$n = 5p - 4, p \in I$

$\therefore$ general solution of given equation can be obtained by substituting either $m = 4p - 3$ in equation (iv) or $n = 5p - 4$ in equation (iii)

$\therefore$ general solution of equation (i) is
$(8p - 6)\pi, p \in I$ **Ans.**

Self Practice Problems :

1. Solve $\sin 3x + \cos 2x = -2$

2. Solve $\sqrt{3\sin 5x - \cos^2 x - 3} = 1 - \sin x$

Ans. (1) $(4p - 3)\frac{\pi}{2}, p \in I$ (2) $2m\pi + \frac{\pi}{2}, m \in I$

SHORT REVISION

TRIGONOMETRIC EQUATIONS & *INEQUATIONS*

THINGS TO REMEMBER :

1. If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

2. If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

3. If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

4. If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.

5. $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.

6. $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$. [Note : α is called the principal angle]

7. **TYPES OF TRIGONOMETRIC EQUATIONS :**

(a) Solutions of equations by factorising . Consider the equation ;
$(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$; $\cot x - \cos x = 1 - \cot x \cos x$

(b) Solutions of equations reducible to quadratic equations. Consider the equation :
$3\cos^2 x - 10\cos x + 3 = 0$ and $2\sin^2 x + \sqrt{3}\sin x + 1 = 0$

(c) Solving equations by introducing an Auxilliary argument . Consider the equation :
$\sin x + \cos x = \sqrt{2}$; $\sqrt{3}\cos x + \sin x = 2$; $\sec x - 1 = (\sqrt{2} - 1)\tan x$

(d) Solving equations by Transforming a sum of Trigonometric functions into a product.
Consider the example : $\cos 3x + \sin 2x - \sin 4x = 0$;
$\sin^2 x + \sin^2 2x + \sin^2 3x + \sin^2 4x = 2$; $\sin x + \sin 5x = \sin 2x + \sin 4x$

(e) Solving equations by transforming a product of trigonometric functions into a sum.

Consider the equation :

$\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x$; $8 \cos x \cos 2x \cos 4x = \dfrac{\sin 6x}{\sin x}$; $\sin 3\theta = 4 \sin\theta \sin 2\theta \sin 4\theta$

(f) Solving equations by a change of variable :

(i) Equations of the form of $a \cdot \sin x + b \cdot \cos x + d = 0$, where a, b & d are real numbers & $a, b \neq 0$ can be solved by changing $\sin x$ & $\cos x$ into their corresponding tangent of half the angle. Consider the equation $3 \cos x + 4 \sin x = 5$.

(ii) Many equations can be solved by introducing a new variable . eg. the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cdot \cos 2x$ changes to

$2(y+1)\left(y - \dfrac{1}{2}\right) = 0$ by substituting , $\sin 2x \cdot \cos 2x = y$.

(g) Solving equations with the use of the Boundness of the functions $\sin x$ & $\cos x$ or by making two perfect squares. Consider the equations :

$\sin x\left(\cos\dfrac{x}{4} - 2\sin x\right) + \left(1 + \sin\dfrac{x}{4} - 2\cos x\right) \cdot \cos x = 0$;

$\sin^2 x + 2\tan^2 x + \dfrac{4}{\sqrt{3}}\tan x - \sin x + \dfrac{11}{12} = 0$

8. TRIGONOMETRIC INEQUALITIES : There is no general rule to solve a Trigonometric inequations and the same rules of algebra are valid except the domain and range of trigonometric functions should be kept in mind.

Consider the examples : $\log_2\left(\sin\dfrac{x}{2}\right) < -1$; $\sin x\left(\cos x + \dfrac{1}{2}\right) \le 0$; $\sqrt{5 - 2\sin 2x} \ge 6\sin x - 1$

EXERCISE–I

Q.1 Solve the equation for x, $5^{\frac{1}{2}} + 5^{\frac{1}{2} + \log_5(\sin x)} = 15^{\frac{1}{2} + \log_{15} \cos x}$

Q.2 Find all the values of θ satisfying the equation; $\sin\theta + \sin 5\theta = \sin 3\theta$ such that $0 \le \theta \le \pi$.

Q.3 Find all value of θ, between 0 & π, which satisfy the equation; $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = 1/4$.

Q.4 Solve for x , the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

Q.5 Determine the smallest positive value of x which satisfy the equation, $\sqrt{1 + \sin 2x} - \sqrt{2}\cos 3x = 0$.

Q.6 $2\sin\left(3x + \dfrac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x \cdot \cos^2 2x}$

Q.7 Find the general solution of the trigonometric equation $3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$.

Q.8 Find all values of θ between $0°$ & $180°$ satisfying the equation; $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

Q.9 Find the solution set of the equation, $\log_{\frac{-x^2-6x}{10}}(\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}}(\sin 2x)$.

Q.10 Find the value of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$.

Q.11 Find the general solution of the equation, $\sin \pi x + \cos \pi x = 0$. Also find the sum of all solutions in $[0, 100]$.

Q.12 Find the least positive angle measured in degrees satisfying the equation
$\sin^3 x + \sin^3 2x + \sin^3 3x = (\sin x + \sin 2x + \sin 3x)^3$.

Q.13 Find the general values of θ for which the quadratic function

$(\sin\theta)\, x^2 + (2\cos\theta)x + \dfrac{\cos\theta + \sin\theta}{2}$ is the square of a linear function.

Q.14 Prove that the equations (a) $\sin x \cdot \sin 2x \cdot \sin 3x = 1$ (b) $\sin x \cdot \cos 4x \cdot \sin 5x = -1/2$ have no solution.

Q.15 Let $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$ for some real number k. Determine
(a) all real numbers k for which $f(x)$ is constant for all values of x.
(b) all real numbers k for which there exists a real number 'c' such that $f(c) = 0$.
(c) If $k = -0.7$, determine all solutions to the equation $f(x) = 0$.

Q.16 If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of **column-I** with the entries of **column-II**.

Column-I		**Column-II**	
(A)	$\sin\alpha + \sin\beta$	(P)	$\dfrac{2b}{a+c}$
(B)	$\sin\alpha \,.\, \sin\beta$	(Q)	$\dfrac{c-a}{c+a}$
(C)	$\tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2}$	(R)	$\dfrac{2bc}{a^2+b^2}$
(D)	$\tan\dfrac{\alpha}{2} \,.\, \tan\dfrac{\beta}{2} =$	(S)	$\dfrac{c^2-a^2}{a^2+b^2}$

Q.17 Find all the solutions of, $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$.

Q.18 Solve for x, $(-\pi \le x \le \pi)$ the equation; $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$.

Q.19 Solve the inequality $\sin 2x > \sqrt{2}\sin^2 x + (2 - \sqrt{2})\cos^2 x$.

Q.20 Find the set of values of 'a' for which the equation, $\sin^4 x + \cos^4 x + \sin 2x + a = 0$ possesses solutions. Also find the general solution for these values of 'a'.

Q.21 Solve: $\tan^2 2x + \cot^2 2x + 2\tan 2x + 2\cot 2x = 6$.

Q.22 Solve: $\tan^2 x \,.\, \tan^2 3x \,.\, \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$.

Q.23 Find the set of values of x satisfying the equality

$\sin\left(x - \dfrac{\pi}{4}\right) - \cos\left(x + \dfrac{3\pi}{4}\right) = 1$ and the inequality $\dfrac{2\cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x}$.

Q.24 Let S be the set of all those solutions of the equation, $(1 + k)\cos x \cos(2x - \alpha) = (1 + k\cos 2x)\cos(x - \alpha)$ which are independent of k & α. Let H be the set of all such solutions which are dependent on k & α. Find the condition on k & α such that H is a non-empty set, state S. If a subset of H is $(0, \pi)$ in which $k = 0$, then find all the permissible values of α.

Q.25 Solve for x & y,
$$\begin{aligned} x\cos^3 y + 3x\cos y\sin^2 y &= 14 \\ x\sin^3 y + 3x\cos^2 y\sin y &= 13 \end{aligned}$$

Q.26 Find the value of α for which the three elements set $S = \{\sin\alpha, \sin 2\alpha, \sin 3\alpha\}$ is equal to the three element set $T = \{\cos\alpha, \cos 2\alpha, \cos 3\alpha\}$.

Q.27 Find all values of 'a' for which every root of the equation, $a\cos 2x + |a|\cos 4x + \cos 6x = 1$ is also a root of the equation, $\sin x\cos 2x = \sin 2x\cos 3x - \dfrac{1}{2}\sin 5x$, and conversely, every root of the second equation is also a root of the first equation.

Q.28 Solve the equations for 'x' given in **column-I** and match with the entries of **column-II**.

	Column-I		**Column-II**
(A)	$\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$	(P)	$n\pi \pm \frac{\pi}{3}$
(B)	$\sin 3\alpha = 4 \sin\alpha \sin(x+\alpha) \sin(x-\alpha)$ where α is a constant $\neq n\pi$.	(Q)	$n\pi + \frac{\pi}{4}$, $n \in I$
(C)	$\lvert 2\tan x - 1\rvert + \lvert 2\cot x - 1\rvert = 2$.	(R)	$\frac{n\pi}{4} + \frac{\pi}{8}$, $n \in I$
(D)	$\sin^{10}x + \cos^{10}x = \frac{29}{16}\cos^4 2x$.	(S)	$\frac{n\pi}{2} \pm \frac{\pi}{4}$

EXERCISE–II

Q.1 Solve the following system of equations for x and y [REE '2001(mains), 3]

$$5^{(\operatorname{cosec}^2 x - 3\sec^2 y)} = 1 \quad \text{and} \quad 2^{(2\operatorname{cosec} x + \sqrt{3}\lvert\sec y\rvert)} = 64.$$

Q.2 The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is
(A) 4 (B) 8 (C) 10 (D) 12 [JEE 2002 (Screening), 3]

Q.3 $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is
(A) 0 (B) 1 (C) 2 (D) 4 [JEE 2005 (Screening)]

Q.4 If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is
(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$ [JEE 2006, 3]

Q.5 The number of solutions of the pair of equations
$2\sin^2\theta - \cos 2\theta = 0$
$2\cos^2\theta - 3\sin\theta = 0$
in the interval $[0, 2\pi]$ is
(A) zero (B) one (C) two (D) four [JEE 2007, 3]

ANSWER EXERCISE–I

Q.1 $x = 2n\pi + \frac{\pi}{6}$, $n \in I$ **Q.2** $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ & π **Q.3** $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

Q.4 $\alpha - 2\pi$; $\alpha - \pi$, α, $\alpha + \pi$, where $\tan\alpha = \frac{2}{3}$ **Q.5** $x = \pi/16$

Q.6 $x = 2n\pi + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}$; $n \in I$ **Q.7** $x = 2n\pi + \frac{\pi}{12}$ **Q.8** 30°, 45°, 90°, 135°, 150°

Q.9 $x = -\frac{5\pi}{3}$ **Q.10** $\theta = 2n\pi$ **or** $2n\pi + \frac{\pi}{2}$; $n \in I$ **Q.11** $x = n - \frac{1}{4}$, $n \in I$; sum = 5025 **Q.12** 72°

Q.13 $2n\pi + \frac{\pi}{4}$ or $(2n+1)\pi - \tan^{-1}2$, $n \in I$ **Q.15** (a) $-\frac{3}{2}$; (b) $k \in \left[-1, -\frac{1}{2}\right]$; (c) $x = \frac{n\pi}{2} \pm \frac{\pi}{6}$

Q.16 (A) R; (B) S; (C) P; (D) Q **Q.17** $n\pi$; $n\pi + (-1)^n \frac{\pi}{10}$ **or** $n\pi + (-1)^n\left(-\frac{3\pi}{10}\right)$

Q.18 $\frac{\pm\pi}{3}, \frac{-\pi}{2}, \pm\pi$

Q.19 $n\pi + \frac{\pi}{8} < x < n\pi + \frac{\pi}{4}$

Q.20 $\frac{1}{2}\left[n\pi + (-1)^n \sin^{-1}\left(1-\sqrt{2a+3}\right)\right]$ where $n \in I$ and $a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$

Q.21 $x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{8}$ **or** $\frac{n\pi}{4} + (-1)^{n+1} \frac{\pi}{24}$

Q.22 $\frac{(2n+1)\pi}{4}$, $k\pi$, where $n, k \in I$

Q.23 $x = 2n\pi + \frac{3\pi}{4}$, $n \in I$

Q.24 (i) $|k \sin\alpha| \le 1$ (ii) $S = n\pi$, $n \in I$ (iii) $\alpha \in (-m\pi, 2\pi - m\pi)$ $m \in I$

Q.25 $x = \pm 5\sqrt{5}$ & $y = n\pi + \tan^{-1}\frac{1}{2}$ **Q.26** $\frac{n\pi}{2} + \frac{\pi}{8}$

Q.27 $a = 0$ or $a < -1$ **Q.28** (A) S; (B) P; (C) Q; (D) R

EXERCISE–II

Q.1 $x = n\pi + (-1)^n\frac{\pi}{6}$ and $y = m\pi \pm \frac{\pi}{6}$ where m & n are integers.

Q.2 B **Q.3** D **Q.4** A **Q.5** C

Exercise - 1 (Objective Questions)

Part : (A) Only one correct option

1. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is

(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (C) $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

2. All solutions of the equation, $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in:

(A) $2n\pi + \frac{2\pi}{3}$, $n \in I$ (B) $n\pi$ or $2m\pi \pm \frac{2\pi}{3}$ where $n, m \in I$

(C) $n\pi$ or $m\pi \pm \frac{\pi}{3}$ where $n, m \in I$ (D) $n\pi$ or $2m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

3. If $20\sin^2\theta + 21\cos\theta - 24 = 0$ & $\frac{7\pi}{4} < \theta < 2\pi$ then the values of $\cot\frac{\theta}{2}$ is:

(A) 3 (B) $\frac{\sqrt{15}}{3}$ (C) $-\frac{\sqrt{15}}{3}$ (D) -3

4. The general solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is:

(A) $2n\pi$; $n \in I$ (B) $n\pi$; $n \in I$ (C) $n\pi/3$; $n \in I$ (D) $2n\pi/3$; $n \in I$

5. A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in A.P. then the angle A, B, C are respectively.

(A) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (B) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$ (C) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (D) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

6. The maximum value of $3\sin x + 4\cos x$ is

(A) 3 (B) 4 (C) 5 (D) 7

7. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(A) $x^2 - 6x + 1 = 0$ (B) $6x^2 - x - 1 = 0$ (C) $6x^2 + x + 1 = 0$ (D) $x^2 - x + 6 = 0$

8. $\dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if:

(A) $\theta \in \left(0, \dfrac{\pi}{2}\right)$ (B) $\theta \in \left(\dfrac{\pi}{2}, \pi\right)$ (C) $\theta \in \left(\pi, \dfrac{3\pi}{2}\right)$ (D) $\theta \in \left(\dfrac{3\pi}{2}, 2\pi\right)$

9. The number of integral values of a for which the equation $\cos 2x + a\sin x = 2a - 7$ possesses a solution is
(A) 2 (B) 3 (C) 4 (D) 5

10. The principal solution set of the equation, $2\cos x = \sqrt{2 + 2\sin 2x}$ is

(A) $\left\{\dfrac{\pi}{8}, \dfrac{13\pi}{8}\right\}$ (B) $\left\{\dfrac{\pi}{4}, \dfrac{13\pi}{8}\right\}$ (C) $\left\{\dfrac{\pi}{4}, \dfrac{13\pi}{10}\right\}$ (D) $\left\{\dfrac{\pi}{8}, \dfrac{13\pi}{10}\right\}$

11. The number of all possible triplets (a_1, a_2, a_3) such that : $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x is
(A) 0 (B) 1 (C) 2 (D) infinite

12. If $2\tan^2 x - 5\sec x - 1 = 0$ has 7 different roots in $\left[0, \dfrac{n\pi}{2}\right]$, $n \in N$, then greatest value of n is
(A) 8 (B) 10 (C) 13 (D) 15

13. The solution of $|\cos x| = \cos x - 2\sin x$ is

(A) $x = n\pi, n \in I$ (B) $x = n\pi + \dfrac{\pi}{4}, n \in I$

(C) $x = n\pi + (-1)^n \dfrac{\pi}{4}, n \in I$ (D) $(2n+1)\pi + \dfrac{\pi}{4}, n \in I$

14. The arithmetic mean of the roots of the equation $4\cos^3 x - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval [0, 315] is equal to
(A) 49π (B) 50π (C) 51π (D) 100π

15. Number of solutions of the equation $\cos 6x + \tan^2 x + \cos 6x \,.\, \tan^2 x = 1$ in the interval $[0, 2\pi]$ is :
(A) 4 (B) 5 (C) 6 (D) 7

Part : (B) May have more than one options correct

16. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:
(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n \in I$ (B) $x = n\pi + (-1)^n(\pi/6)$, $n \in I$
(C) $x = n\pi + (-1)^n(\pi/3)$, $n \in I$ (D) $x = n\pi - (-1)^n(\pi/6)$, $n \in I$

17. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if
(A) $\cos 12x = \cos 14x$ (B) $\sin 13x = 0$
(C) $\sin x = 0$ (D) $\cos x = 0$

18. The equation $2\sin\dfrac{x}{2} \,.\, \cos^2 x + \sin^2 x = 2\sin\dfrac{x}{2} \,.\, \sin^2 x + \cos^2 x$ has a root for which

(A) $\sin 2x = 1$ (B) $\sin 2x = -1$ (C) $\cos x = \dfrac{1}{2}$ (D) $\cos 2x = -\dfrac{1}{2}$

19. $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$ if
(A) $\tan x = 3$ (B) $\tan x = -1$
(C) $x = n\pi + \pi/4$, $n \in I$ (D) $x = n\pi + \tan^{-1}(-3)$, $n \in I$

20. $\sin^2 x - \cos 2x = 2 - \sin 2x$ if
(A) $x = n\pi/2$, $n \in I$ (B) $x = n\pi - \pi/2$, $n \in I$
(C) $x = (2n+1)\pi/2$, $n \in I$ (D) $x = n\pi + (-1)^n \sin^{-1}(2/3)$, $n \in I$

Exercise - 2 (Subjective Questions)

1. Solve $\cot\theta = \tan 8\theta$

2. Solve $\cot\left(\frac{x}{2}\right) - \operatorname{cosec}\left(\frac{x}{2}\right) = \cot x$

3. Solve $\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0.$

4. Solve $\cos 2\theta + 3\cos\theta = 0.$

5. Solve the equation: $\sin 6x = \sin 4x - \sin 2x$.

6. Solve: $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.

7. Solve $4\sin x \,.\, \sin 2x \,.\, \sin 4x = \sin 3x$.

8. Solve $\sin^2 n\theta - \sin^2(n-1)\theta = \sin^2\theta$, where n is constant and $n \neq 0, 1$

9. Solve $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

10. Solve: $\sin^3 x \cos 3x + \cos^3 x \sin 3x + 0.375 = 0$

11. Solve the equation, $\dfrac{\sin^3 \frac{x}{2} - \cos^3 \frac{x}{2}}{2 + \sin x} = \dfrac{\cos x}{3}$.

12. Solve the equation: $\sin 5x = 16\sin^5 x$.

13. If $\tan\theta + \sin\phi = \frac{3}{2}$ & $\tan^2\theta + \cos^2\phi = \frac{7}{4}$ then find the general value of θ & ϕ.

14. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

15. Find the general solution of $\sec 4\theta - \sec 2\theta = 2$.

16. Solve the equation $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$.

17. Solve for x: $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x \,.\, \cos^2 2x}$.

18. Solve the equation for $0 \le \theta \le 2\pi$; $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$.

19. Solve: $\tan^2 x \,.\, \tan^2 3x \,.\, \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$.

20. Find the values of x, between 0 & 2π, satisfying the equation; $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$.

21. Solve: $\cos\frac{2x}{3}\ \cos 6x = -1$.

22. Solve the equation, $\sin^2 4x + \cos^2 x = 2\sin 4x \cos^4 x$.

Answers

EXERCISE # 1

1. D **2.** B **3.** D **4.** C **5.** B **6.** C **7.** B

8. B **9.** D **10.** A **11.** D **12.** D **13.** D **14.** C

15. D **16.** AD **17.** ABC **18.** ABCD **19.** CD

20. BC

EXERCISE # 2

1. $\left(n+\frac{1}{2}\right)\frac{\pi}{9}, n \in I$

2. $x = 4n\pi \pm \frac{2\pi}{3}, n \in I$

3. $\theta = n\pi - \frac{\pi}{3}, n \in I$ or $n\pi - \frac{\pi}{6}, n \in I$

4. $2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right), n \in I$

5. $\frac{n\pi}{4}, n \in I$ or $n\pi \pm \frac{\pi}{6}, n \in I$

6. $2n\pi, n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}, n \in I$

7. $x = n\pi, n \in I$ **or** $\frac{n\pi}{3} \pm \frac{\pi}{9}, n \in I$

8. $m\pi, m \in I$ or $\frac{m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{2}\right)\frac{\pi}{n}, m \in I$

9. $\left(n+\frac{1}{3}\right)\frac{\pi}{3}, n \in I$

10. $x = \frac{n\pi}{4} + (-1)^{n+1}\frac{\pi}{24}, \ n \in I$

11. $x = (4n+1)\frac{\pi}{2}, n \in I$

12. $x = n\pi$ **;** $x = n\pi \pm \frac{\pi}{6}, n \in I$

13. $\theta = n\pi + \frac{\pi}{4}, \phi = n\pi + (-1)^n\frac{\pi}{6}, n \in I$

14. $\alpha - 2\pi;\ \alpha - \pi,\ \alpha,\ \alpha + \pi$, where $\tan\alpha = \frac{2}{3}$

15. $\frac{2n\pi}{5} \pm \frac{\pi}{10}$ or $2n\pi \pm \frac{\pi}{2}, n \in I$

16. $x = (2n+1)\pi, , n \in I$ or $2n\pi \pm \frac{\pi}{3}, n \in I$

17. $(24\ell + 1)\frac{\pi}{12}, \ell \in I$ or $x = (24k-7)\frac{\pi}{12}, k \in I$

18. $\theta = \frac{7\pi}{12}, \frac{19\pi}{12}$

19. $\frac{(2n+1)\pi}{4}, k\pi$, where $n, k \in I$

20. $\frac{\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{13\pi}{7}$

21. ϕ

22. $x = (2n+1)\frac{\pi}{2}, n \in I$