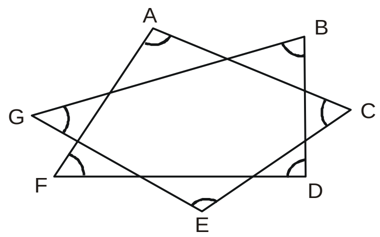
## SET - 1

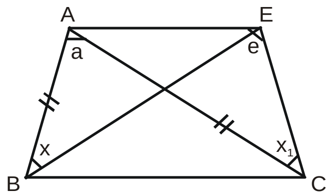
**1.** If all line segments are straight, in the given figure, then the sum of the angles at the corners marked in the diagram is :




(A) 360º (B) 450º (C*) 540º (D) 630º

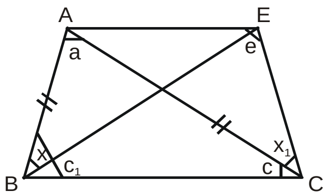
**Sol.** Sum of the angles of the seven triangles = 180º × 7 = 1260º.
The base angles of the triangle are the exterior angles of the seven-sided polygon.
Now their sum = 2 × 360º = 720º
$\therefore$ the sum of the angles at the vertices marked = 1260º – 720º = 540º.

**2.** In the adjoining figure, AB = AC, $x = x_1$. The value of e in terms of a is :




(A) e = 90º – a/2 (B*) e = 90º + a/2 (C) e = 180º – a (D) e = 2a

**Sol.**




As $x = x_1$ (given)

$\therefore$ AECB is a cyclic quad. (AE subtends eq. $\angle$s at B and C)

$c = c_1$ (base $\angle$s, isos. $\Delta$)

$c_1 = 180º - e$ (opp. $\angle$s cyclic quad.)

$$c_1 = \frac{180º - a}{2} \Rightarrow 180º - e = \frac{180º - a}{2}$$

$$180º - e = 90º - \frac{a}{2}$$

$$e = 90 + \frac{a}{2}.$$

**3.** If X + Y + Z = 30, (X,Y,Z > 0), then the value of (X – 2) (Y– 3) (Z – 4) will be :

(A) $\geq 1000$ (B) $\geq 800$ (C) $\geq 500$ (D) $\leq 343$

**Sol.** Let us consider the number X – 2, Y – 3, Z – 4.
Their AM is $\geq$ GM.

Therefore $\dfrac{[(X-2)+(Y-3)+(Z-4)]}{3} \geq [(X-2)(Y-3)(Z-4)]^{1/3}$

i.e.,$[(X-2)(Y-3)(Z-4)]^{1/3} \leq \dfrac{X+Y+Z-9}{3}$

i.e.,$[(X-2)(Y-3)(Z-4)]^{1/3} \leq \dfrac{30-9}{3} = 7$

On cubing both sides we get

$(X-2)(Y-3)(Z-4) \leq 7^3$

$\Rightarrow (X-2)(Y-3)(Z-4) \leq 343.$

**4.** The sum of the digits of the number $(4^{1000})(25^{1002})$ is :

(A) 7 (B) 8 (C) 11 (D) 13

**Sol.** $4^{1000} \times 25^{1002}$

$2^{2000} \times 5^{2004}$

$5^4(2 \times 5)^{2000}$

$625(10)^{2000}$

625 × 1000.......(2000 times)

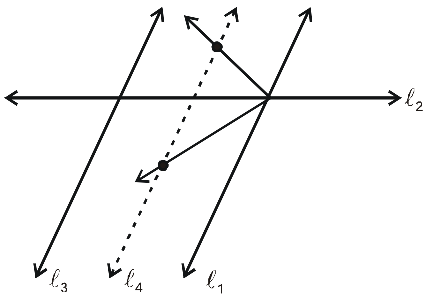
= 625000.........(2000 times)

Sum of digit = 6 + 2 + 5 = 13.

**5.** Line $\ell_2$ intersects $\ell_1$ and line $\ell_3$ is parallel to $\ell_1$. The three lines are distinct and lie in a plane. The number of points equidistant from all the three lines is :

(A) 0 (B) 1 (C*) 2 (D) 4

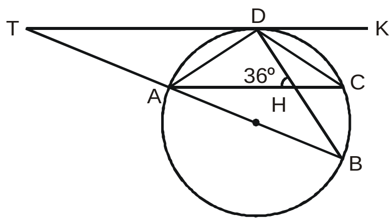
**Sol.**




Let draw line $\ell_4$ which is parallel to $\ell_1$ and $\ell_3$ and equidistant from $\ell_1$ and $\ell_3$

The point which are equidistant from $\ell_1$ and $\ell_1$ is lie an the angle bisector, which intersect $\ell_4$ at two points so there are two points which are eqidistant from all the three lines $\ell_1$, $\ell_2$ and $\ell_3$.

**6.** In the figure, AB is a diameter of the circle, TD is a tangent. If $\angle AHD = 36^\circ$, $\angle CDT$ is :



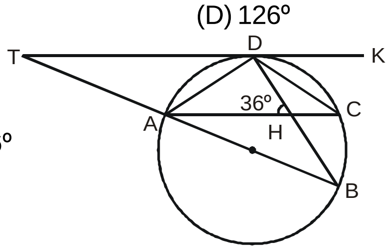

(A) 100º (B) 110º (C) 116º (D) 126º

**Sol.** $\angle ADB = 90^\circ$ ($\angle$in semi-circle)

$\angle DAH = 180^\circ - 90^\circ - 36^\circ = 54^\circ$

$\angle DAH = \angle CDK$ ($\angle$in alt. seg.) = 54º

$\Rightarrow$ $\angle CDT = 180^\circ - 54^\circ$ (adj $\angle$s on straight line) = 126º




**7.** The minimum value of $2^{\sin x} + 2^{\cos x}$ is :

(A) 1 (B) 2 (C) $2^{-\frac{1}{\sqrt{2}}}$ (D) $2^{1+\frac{1}{\sqrt{2}}}$

**Sol.** $2^{\sin x} + 2^{\cos x}$

As we know that sin x and cos x both lies between – 1 and + 1 but the values of sin x and cos x shows. Reverse pattern of each other so the sum will be minimum when both the values $2^{\sin x}$ and $2^{\cos x}$ are equal i.e. $2^{\sin x} = 2^{\cos x}$ i.e. sin x = cos x. i.e. x = $\pi/4$. So the minimum value will be

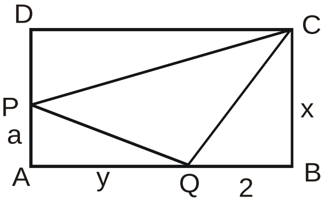
$2^{\sin \pi/4} + 2^{\cos \pi/4} = 2 \times 2^{\frac{1}{\sqrt{2}}} = 2^{\frac{1}{\sqrt{2}}+1}$

## SET - 2

**1.** In the figure, the quadrilateral ABCD is a rectangle, P lies on AD and Q and AB. The triangles PAQ, QBC and PCD all have the same area, and BQ = 2. The length of AQ, is :

(A) $3+\sqrt{5}$ (B) $2\sqrt{3}$ (C*) $\sqrt{5}+1$ (D) not uniquely determined

**Sol.**




ar $\Delta$BCQ = ar $\Delta$APQ

$\frac{1}{2}x \times 2 = \frac{1}{2}ay$

2x = ay

x = $\frac{ay}{2}$ ... (i)

ar$\Delta$BCQ = ar$\Delta$PDC

$\frac{1}{2} \times 2 \times x = \frac{1}{2}(x-a)(y+2)$

2x = xy + 2x – ay – 2a

xy = ay + 2a

$\frac{ay}{2} \times y = ay + 2a$ [Using (i)]

$\frac{y^2}{2} = y + 2$

$y^2 = 2y + 4$

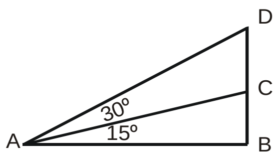
$y^2 - 2y - 4 = 0$

y = $\frac{2 \pm \sqrt{4+16}}{2}$

$= \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5} = 1 + \sqrt{5}$

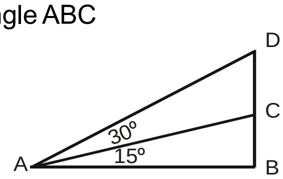
Ans. (C) because negative answer can't be possible.

**2.** In the figure, AB = x. The area of triangle ADC is (angle B = 90º)




(A) $\frac{1}{2}x^2 \sin 30^\circ$ (B) $\frac{1}{2}x^2 \tan 30^\circ$ (C) $\frac{1}{2}x^2 \tan 45^\circ$ (D) $\frac{1}{2}x^2(\tan 45^\circ - \tan 15^\circ)$

**Sol.** Area of triangle ADC = Area of triangle ABD – area of triangle ABC




$= \frac{x}{2} \times BD - \frac{x}{2} \times BC$

$= \frac{x}{2} \times (BD - BC)$

$= \frac{x}{2} x(x \tan 45^\circ - x \tan 15^\circ)$

$\therefore$ Area of triangle ADC = $\frac{1}{2} \times x^2(\tan 45^\circ - \tan 15^\circ)$.

**3.** In a single throw of 3 dice, the probability of not getting the same number on any 2 dice is

(A) 0.45 (B) 0.66 (C) 0.56 (D) 0.83 (E) None of these

**Sol.** P(not getting same number on 2 dice) = 1 – P(getting same number on 2 or 3 dice)

sample space for getting same number on 2 or 3 dice

= (1, 1, 1), (1, 1, 2), .....(1, 1, 6)

(2, 2, 1), (2, 2, 2).....(2, 2, 6)

.

.

.

(6, 6, 1), (6, 6, 2)......(6, 6, 6)

So total 36 cases.

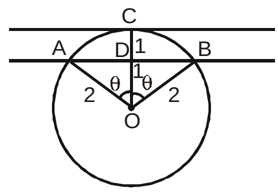
P (not getting same number on any two dice) $= 1 - \frac{36}{216}$

$= 1 - \frac{1}{6} = \frac{5}{6} = 0.83$

**4.** Two parallel lines are one unit apart. A circle of radius 2 touches one of the lines and cuts the other line. The area of the circular cap between the two parallel lines can be written in from of $\frac{a\pi}{3} - b\sqrt{3}$. The sum (a + b) of the two integers a and b equal to :

(A) 3 (B) 4 (C*) 5 (D) 6

**Sol.** In triangle AOD




$\cos\theta = \frac{1}{2}$

$\therefore \theta = 60^o$

$\angle AOB = 2\theta = 120^o$

area of segment ACB $= \frac{2\theta}{360^o}\pi r^2 - \frac{1}{2}r^2\sin 2\theta$

$= \frac{120^o}{360^o}\pi r^2 - \frac{1}{2}r^2\sin 120$

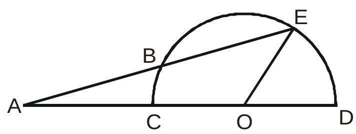
$\frac{4\pi}{3} - \sqrt{3} = \frac{a\pi}{3} - b\sqrt{3}$

a = 4

b = 1

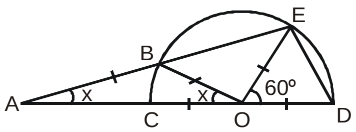
$\therefore$ a + b = 4 + 1 = 5.

**5.** In the figure, CD is the diameter of a semicircle CBED with centre O, and AB = OD. If $\angle EOD = 60^o$, then $\angle BAC$ is




(A) 15º (B*) 20º (C) 30º (D) 45º

**Sol.**




AB = OD = OE = OB = OC (radius)

$\angle BOC = \angle BAC = x$ (AB = OB)

$\angle OBE = x + x = 2x$ (Exterion angle of $\Delta ABO$)

$\angle OEB = \angle OBE = 2x$ (OB = OE)

$\angle EOD = x + 2x = 3x = 60^o$

x = 20º.

**6.** Let $\alpha$, $\beta$ be the roots of the equation $(x - a)(x - b) = c$, $c \neq 0$. Then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are :

(A) a, c (B) b, c (C*) a, b (D) a + c, b + c

**Sol.** $(x - a)(x - b) = c$

$x^2 - x(a + b) + ab - c = 0$

$\alpha + \beta = a + b$

$\alpha\beta = ab - c$

$ab = \alpha\beta + c$

$x^2 - x(\alpha + \beta) + \alpha\beta + c = 0$

$x^2 - x(\alpha + \beta) + ab = 0$

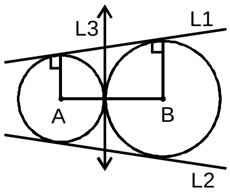
So, the roots are a and b.

Option (C) is the answer.

**7.** On a plane are two points A and B at a distance of 5 unit apart. The number of straight lines in this plane which are at distance of 2 units from A and 3 units from B, is :

(A) 1 (B) 2 (C*) 3 (D) 4

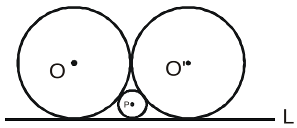
**Sol.**




Three straight lines L1, L2 & L3 can be drawn as shown in figure, which are at the distance 2 unit from A and 3 unit from B.

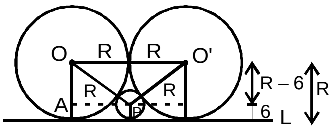
## SET - 3

**1.** Circles with centres O, O' and P each tangent of the line L and also mutually tangent. If the radii of circle O and circle O' are equal and the radius of the circle P is 6, then the radius of the larger circle is :




(A) 22 (B) 23 (C*) 24 (D) 25

**Sol.**




OP = R + 6, OA = R – 6 and AP = R.

$OP^2 = OA^2 + AP^2$

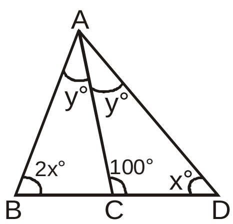
$(R + 6)^2 = (R - 6)^2 + R^2$

$R^2 + 12R + 36 = R^2 - 12R + 36 + R^2$

$24R = R^2$

So, R = 24 unit.

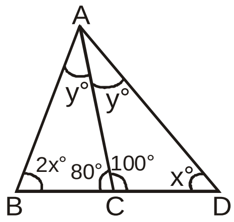
**2.** In the diagram B, C and D lie on a straight line, with $\angle ACD = 100°$, $\angle ADB = x°$, $\angle ABD = 2x°$ and $\angle DAC = \angle BAC = y°$. The value of (sin y°. tan y° + sec y°) equals :




(A*) 7/2 (B) 3 (C) 5/2 (D) 5

**Sol.**

$$\begin{array}{r} 80 = x + y \\ 100 = 2x + y \\ - \quad - \quad - \\ \hline -20 = -x \end{array}$$




x = 20

y = 60

sin y. tan y + sec y

= sin 60 × tan 60 + sec 60

$= \frac{\sqrt{3}}{2} \times \sqrt{3} + 2$

$= \frac{3}{2} + 2 = \frac{7}{2}$ .

**3.** $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} =$

(A) 1 (B) 2 (C) 3 (D) 4

**Sol.** $\frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1.$

**4.** The sides of a triangle with positive area have lengths 4, 6 and x. The sides of a second triangle with positive area have length 4, 6 and y. The smallest positive number that is not the possible value of |x – y|, is (x and y are integers)

(A) 2 (B) 4 (C) 6 (D*) 8

**Sol.** As we know sum of two side in always greater than third side

$\therefore$ 4 + 6 > x

10 > x

4 + 6 > y

10 > y

And we also know that the difference of two side in always less than the third side.

$\therefore$ 6 – 4 < x

2 < x

6 – 4 < y

2 < y

So we can say that

2 < x < 10 & 2 < y < 10

0 < |x – y| < 8.

So option (D) in not possible.

**5.** Container A of volume a, is half full. Container B of volume b, is one-third full. Container C of volume c, is empty. If the water in the containers is divided equally among the containers, then what part of the container C will be full ?

(A*) $\frac{(3a + 2b)}{18c}$ (B) $\frac{(2a + b)}{24c}$ (C) $\frac{(a + 4b)}{6c}$ (D) $\frac{(a + b)}{3c}$

**Sol.** Part of container C filled with water

$$= \frac{\text{amount of water in container A} + \text{amount of water in container B} + \text{amount of water in container C}}{3c} = \frac{\frac{a}{2} + \frac{b}{3} + 0}{3c} = \frac{3a + 2b}{18c}.$$

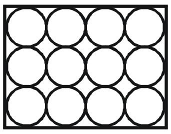
**6.** The number of common terms of the two sequences 17, 21, 25, ....., 417 and 16, 21, 26, .... 466 is :
(A) 21 (B) 19 (C) 20 (D) 91

**Sol.** 17, 21, 25, 29, 33, 37, 41, ....., 417
16, 21, 26, 31, 36, 41, ...., 466
We know that $t_n = a + (n - 1)d$
Now the common terms are 21, 41, 61, ....., 401
Set $t_n = 401$
$401 = 21 + (n - 1)20 \Rightarrow 401 - 21 = (n - 1)20$

$\Rightarrow 380 = (n - 1)\, 20 \Rightarrow \dfrac{380}{20} = n - 1$

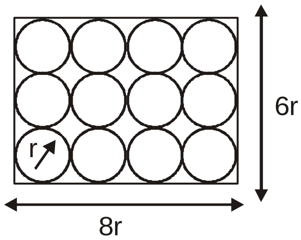
$\Rightarrow 19 = n - 1 \Rightarrow 20 = n.$

**7.** Each of the congruent circles shown is externally tangent to other circles and/or to the side(s) of the rectangle as shown. If each circle has circumference $16\pi$, then the length of a diagonal of the rectangle, is



(A*) 80 (B) 40 (C) 20 (D) 15

**Sol.**




diagonal = $\sqrt{(8r)^2 + (6r)^2} = 10r$
$2\pi r = 16\pi$
$r = 8$
Diagonal = 10 × 8 = 80.

## SET - 4

**1.** The GCD of two numbers is 13 and their product is 4732. The possible number of pairs is/are
(A) 0 (B) 1 (B) 2 (D) 3

**Sol.** Let the numbers are 13P and 13Q.
Product is $PQ \times 13^2 = 4732$.
$\therefore$ PQ = 28. As P and Q are co-prime
$\therefore$ P = 1, Q = 28 or P = 4, Q = 7.

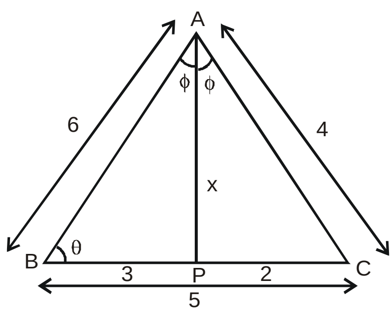
**2.** In the triangle ABC, AB = 6, BC = 5, CA = 4. AP bisects the angle A and P lies on BC. Then AP equals
(A) 3.75 (B) 3.1 (C) 2.9 (D) 4.2

**Sol.** As AP is angle bisector of $\angle BAC \Rightarrow BP = 3, PC = 2$

In $\Delta ABP$, $\dfrac{\sin\phi}{3} = \dfrac{\sin\theta}{x}$

In $\Delta ABC$, $\dfrac{\sin 2\phi}{5} = \dfrac{\sin\theta}{4}$

$\Rightarrow \dfrac{x\sin\phi}{12} = \dfrac{2\sin\phi\cos\phi}{5} \Rightarrow x = \dfrac{24}{5}\cos\phi$



$$\cos 2\phi = \frac{6^2+4^2-5^2}{2\times6\times4} = \frac{36+16-25}{48} = \frac{27}{48}$$

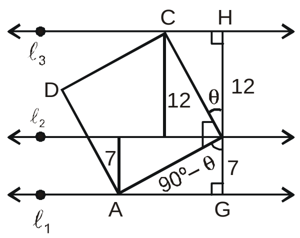
$$\cos^2\phi = \frac{1+\cos 2\phi}{2} = \frac{75}{96}$$

$$\therefore\ x = \left(\frac{24}{5}\right) \times \left(\frac{7.07}{8}\right) = 4.2$$

**3.** Three parallel lines $\ell_1$, $\ell_2$ and $\ell_3$ are drawn through the vertices A, B and C of a square ABCD. If the distance between $\ell_1$ and $\ell_2$ is 7 and between $\ell_2$ and $\ell_3$ is 12, then the area of the square ABCD is :

(A*) 193 (B) 169
(C) 196 (D) 225

**Sol.**




Let a be the side of square :
In triangle CHB

$\cos\theta = \frac{12}{a}$

In triangle ABG

$\cos(90-\theta) = \frac{7}{a}$ $\Rightarrow \sin\theta = \frac{7}{a}$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{144}{a^2} + \frac{49}{a^2} = 1$ $\Rightarrow a^2 = 193$

**4.** Given $x = 1 + a + a^2 + \ldots\ldots\ldots \infty$ and $y = 1 + b + b^2 + \ldots\ldots \infty$ where a and b are proper functions. $1 + ab + a^2b^2 + \ldots\ldots \infty$ equal to

(A) $\frac{xy}{x+y-1}$ (B) $\frac{x^2y^2}{x-y}$ (C) $\frac{x+y}{x-y}$ (D) $\frac{xy}{x^2-y^2}$

**Sol.** Here x is sum of an infinite series with first term as 1 and common ratio a

$\therefore\ x = \frac{1}{1-a} \Rightarrow x(1-a) = 1 \Rightarrow x - xa = 1 \Rightarrow a = \frac{x-1}{x}$

Similarly

$y = \frac{1}{1-b} \Rightarrow y(1-b) = 1 \Rightarrow y - yb = 1 \Rightarrow y - 1 = yb$

$\Rightarrow b = \frac{y-1}{y}$. Now $ab = \left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right) = \frac{xy-x-y+1}{xy}$

Now, $s = 1 + ab + a^2b^2 + \ldots\ldots \infty$

$$\therefore\ s = \frac{1}{1-ab} = \frac{1}{1-\left(\frac{xy-x-y+1}{xy}\right)} = \frac{xy}{xy-xy+x+y-1} = \frac{xy}{x+y-1}$$

**5.** The probability that a teacher will given a surprise test during a class meeting is $\frac{1}{5}$. If a student is absent on two days, then what is the probability that he will miss at least one test ?

(A) $\frac{9}{25}$ (B) $\frac{16}{25}$ (C) $\frac{4}{5}$ (D) $\frac{2}{5}$

**Sol.** P(at least one test) = 1 – P(no test)

$$= 1 - \underbrace{(1 - \frac{1}{5})}_{\text{day 1}} \times \underbrace{(1 - \frac{1}{5})}_{\text{day 2}} = 1 - \frac{16}{25} = \frac{9}{25}$$

**6.** In an army during a war, 4 men out of every 25, were wounded and 2 out of every 25 were killed. 38000 men returned unhurt. What was the original number of men in the army ?

(A) 50000 (B) 37500 (C) 28200 (D) 56550

**Sol.** Let the total men in army = x. Out of every 25, 4 are wounded.

$\therefore$ Number of men wounded out of x = $\frac{4x}{25}$

2 men were killed out of every 25.

$\therefore$ Number of men killed out of x = $\frac{2x}{25}$

$\therefore$ Total number of men = Number of men wounded + number of men killed + unhurt men

$$x = \frac{4x}{25} + \frac{2x}{25} + 38000$$

$\Rightarrow 19x = 950000$

$\therefore x = 50000.$

**7.** If x > y > 0 and 2 log(x – y) = logx + logy, then x/y equals :

(A) $3 + \sqrt{5}$ (B*) $\frac{3+\sqrt{5}}{2}$ only (C) $\frac{3-\sqrt{5}}{2}$ only (D) $\frac{3\pm\sqrt{5}}{2}$

**Sol.** 2 log (x–y) = logx + logy

$(x-y)^2 = xy$

$x^2 + y^2 - 3xy = 0$

$\left(\frac{x}{y}\right)^2 - \frac{3x}{y} + 1 = 0$ (divide by $y^2$)

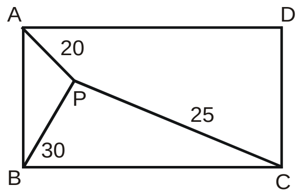
$\therefore \frac{x}{y} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

since x > y

$\therefore \frac{x}{y} > 1$

$\therefore \frac{x}{y} = \frac{3+\sqrt{5}}{2}$

**1.** A, B, C and D walk towards a point P taking the shortest path from the four vertices of a rectangle. After they reach the point P, which is inside the rectangle, it is found that the distance travelled by A is 20m, B is 30m, C is 25 m (as shown in the figure). Find the distance travelled by D.




(A) 40m (B) 25m (C) $15\sqrt{3}$ (D) None of these

**Sol.** It can be obtained that $a^2 + x^2 = 20^2$ ...(i)

$b^2 + x^2 = 30^2$ ...(ii)

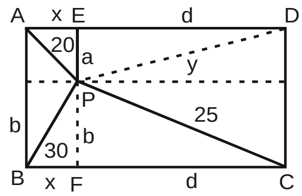
$b^2 + d^2 = 25^2$ ...(iii)

eq.(i) – eq.(ii) + eq.(iii)

$\Rightarrow a^2 + d^2 = 20^2 - 30^2 + 25^2$

But $a^2 + d^2 = y^2$ (from $\Delta$DEP)

y = DP = $\sqrt{20^2 - 30^2 + 25^2}$ = $5\sqrt{5}$.




**2.** If $x + \sqrt{x + \sqrt{x + \sqrt{x + .......}}} = 2$ then x equals :

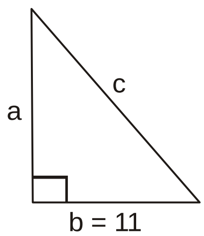
(A*) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) $2 \pm \sqrt{2}$ (D) $2 - \sqrt{3}$

**Sol.** $x + \sqrt{x + \sqrt{x + \sqrt{x + .......}}} = 2$ $\Rightarrow x + \sqrt{2} = 2$ $\Rightarrow x = 2 - \sqrt{2}$.

**3.** All the 3 sides of a right triangle are integers and one side has a length 11 units. Area of the triangle in square units lies between :

(A) 1 and 100 (B) 100 and 200 (C) 200 and 300 (D*) More than 300

**Sol.**




$c^2 = a^2 + b^2$

$c^2 - a^2 = 11^2$

$c^2 - a^2 = 121$

(c + a) (c – a) = 121 × 1

$$\begin{array}{l} c + a = 121 \\ \underline{c - a = 1} \\ 2c = 122 \end{array}$$

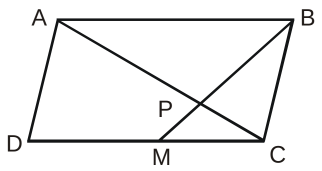
c = 61

a = 60

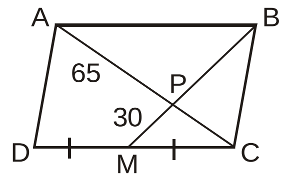
Area = $\frac{1}{2}$ab = $\frac{1}{2}$ × 60 × 11 = 330.

**4.** ABCD is a parallelogram, M is the midpoint of DC. If AP = 65 and PM = 30 then the largest possible integral value of AB is :

(A*) 124
(B) 120
(C) 119
(D) 118




**Sol.** $\Delta APB \sim \Delta\ CPM$




$$\frac{AP}{CP} = \frac{PB}{PM} = \frac{AB}{CM}$$

$$\frac{65}{CP} = \frac{PB}{30} = \frac{2CM}{CM}$$

PB = 60
In $\Delta$ APB
AB < 60 + 65
AB < 125
$\therefore$ AB = 124 is the largest integral value.

**5.** Six friends are sitting around a campfire. Each person in turn announces the total of the ages of the other five people. If 104, 105, 108, 114, 115 and 119 given the six sums of each group of five people. The age of the oldest person

(A*) is 29 (B) is 30 (C) is 31 (D) can not be found out

**Sol.** Let the six friend are $x_1, x_2, x_3, x_4, x_5, x_6$

$x_2 + x_3 + x_4 + x_5 + x_6 = 104$ ...(i)
$x_1 + x_3 + x_4 + x_5 + x_6 = 105$ ...(ii)
$x_1 + x_2 + x_4 + x_5 + x_6 = 108$ ...(iii)
$x_1 + x_2 + x_3 + x_5 + x_6 = 114$ ...(iv)
$x_1 + x_2 + x_3 + x_4 + x_6 = 115$ ...(v)
$x_1 + x_2 + x_3 + x_4 + x_5 = 119$ ...(vi)

Add all

$5(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = 665$
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 133$ – (vii)

$\therefore$ to get maximum age person
subtract (i) from (vii)
We get $x_1 = 29$.

**6.** Let $f(x) = 2^x$. For which value of x is $f(x–2) = f(x) – 2$ ?

(A) 8/3 (B) 1.415 (C) 2 (D) $3 – \log_2 3$

**Sol.** $2^{x-2}, = 2^x – 2$. Let $y = 2^{x-2}$
$y = 4y – 2$
$3y = 2$
$y = 2/3$
$y = 2^{x-2} = 2/3$
$x = 2 + \log_2(2/3) = 2 + \log_2 2 – \log_2 3 = 3 – \log_2 3.$

**7.** The expression $2\sqrt{\frac{3}{2}+\sqrt{2}} - \left(\frac{3}{2}+\sqrt{2}\right)$ when simplified reduces to

(A) an irrational number
(B) a whole number
(C*) a rational which is not an integer
(D) a natural number

**Sol.** $2\sqrt{\frac{3}{2}+\sqrt{2}} - \left(\frac{3}{2}+\sqrt{2}\right)$

$$2\sqrt{\left(1+\frac{1}{\sqrt{2}}\right)^2} - \left(\frac{3}{2}+\sqrt{2}\right)$$

$2\left(1+\frac{1}{\sqrt{2}}\right) - \frac{3}{2} - \sqrt{2}$

$2 + \sqrt{2} - \frac{3}{2} - \sqrt{2}$

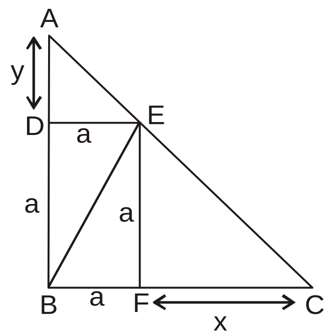
$= \frac{1}{2}$ So it is a rational.

## SET - 6

**1.** Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is :

(A*) $\frac{1}{4m}$ (B) $\frac{1}{2m+1}$ (C) m (D) $\frac{1}{8m^2}$

**Sol.** ar$\Delta$EFC = m($a^2$)




$\frac{1}{2} \times a \times x = ma^2$

x = 2am

$\Delta ADE \sim \Delta ABC$

$\frac{y}{y+a} = \frac{a}{a+2am}$

$\frac{y+a}{y} = 1 + 2m$

$1 + \frac{a}{y} = 1 + 2m$

$y = \frac{a}{2m}$

Req. ratio $= \frac{\frac{1}{2} \times a \times \frac{a}{2m}}{a^2} = \frac{1}{4m}$.

**2.** A and B can demolish a building in 3 days. B and C can do it in 6 days. C and A in 5 days. If A, B and C work together and C gets injured at the end of first day work and can not come back, then the total number of days to complete the work by A and B will be :

(A) $\frac{60}{13}$ (B*) $\frac{59}{20}$ (C) $\frac{20}{7}$ (D) 3

**Sol.** Work done by A and B in one day = $\frac{1}{3}$ part

Work done by B and C in one day = $\frac{1}{6}$ part

Work done by C and A in one day = $\frac{1}{5}$ part

Work done by A, B & C in 1 days = $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{5}\right)$

$= \frac{1}{2}\left(\frac{10+5+6}{30}\right)$

$= \frac{1}{2} \times \frac{21}{30} = \frac{7}{20}$ part

Work done by A, B and C in 1 day = $\frac{7}{20}$ part

Remaining work = $1 - \frac{7}{20} = \frac{13}{20}$ part.

Now, A & B can do complete work together in 3 days.

So, they can do $\frac{13}{20}$ part in

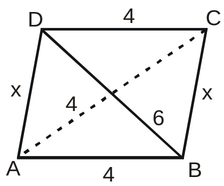
$= 3 \times \frac{13}{20} = \frac{39}{20}$ days

Total number of days to complete the work = $1 + \frac{39}{20} = \frac{59}{20}$ days.

**3.** In parallelogram ABCD, the length AB and CD are both 4, the length of diagonal AC = 4, and the length of diagonal BD = 6. The length AD equal to :

(A*) $\sqrt{10}$ (B) $\sqrt{12}$ (C) $\sqrt{15}$ (D) $\sqrt{20}$

**Sol.** ar$\Delta$ABC = ar $\Delta$ABD




$$\sqrt{\left(5+\frac{x}{2}\right)\left(1+\frac{x}{2}\right)\left(\frac{x}{2}-1\right)\left(5-\frac{x}{2}\right)} = \sqrt{\left(4+\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(4-\frac{x}{2}\right)}$$

$$\left(25-\frac{x^2}{4}\right)\left(\frac{x^2}{4}-1\right) = \frac{x^2}{4}\left(16-\frac{x^2}{4}\right)$$

$$\left(\frac{100-x^2}{4}\right)\left(\frac{x^2-4}{4}\right) = \frac{x^2}{4}\left(\frac{64-x^2}{4}\right)$$

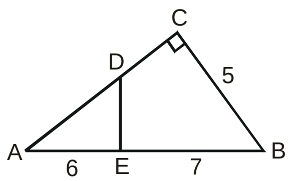
$100x^2 - 400 - x^4 + 4x^2 = 64x^2 - x^4$

$40x^2 = 400$

$x^2 = 10$

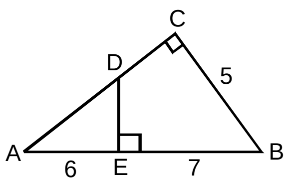
$x = \sqrt{10}$.

**4.** In the figure C is a right angle, DE $\perp$ AB, A E = 6, EB = 7 and BC = 5. The area of the quadrilateral EBCD is




(A) 27.5 (B) 25 (C*) 22.5 (D) 20

**Sol.** AC = $\sqrt{13^2 - 5^2}$ = 12




$\Delta$AED ~ $\Delta$ACB (By AA)

$$\frac{AE}{AC} = \frac{ED}{CB}$$

$$\frac{6}{12} = \frac{ED}{5}$$

ED = 2.5

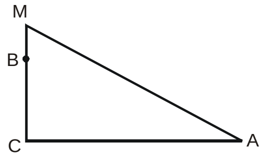
ar of quadrilateral EBCD = area $\Delta$ABC – area $\Delta$AED

$\frac{1}{2} \times 5 \times 12 - \frac{1}{2} \times 6 \times 2.5$

$= 30 - 7.5$

$= 22.5$

**5.** In the right triangle shown the sum of the distances BM and MA is equal to the distances BC and CA. If MB = x, CB = h and CA = d, then x equals.




(A*) $\frac{hd}{2h+d}$ (B) $d - h$ (C) $h + d - \sqrt{2d}$ (D) $\sqrt{h^2 + d^2 - h}$

**Sol.** ATQ. $x + y = h + d$

$y = h + d - x$

$y^2 = (x + h)^2 + d^2$

$(h + d - x)^2 = (x + h)^2 + d^2$

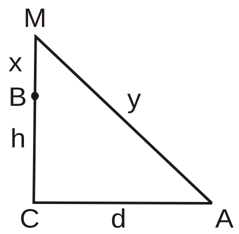
$h^2 + d^2 + x^2 + 2hd - 2dx - 2hx = x^2 + h^2 + 2xh + d^2$

$2hd - 2dx - 2xh = 2xh$

$hd = xh + dx + xh$

$hd = x(2h + d)$

$x = \frac{hd}{2h+d}$.



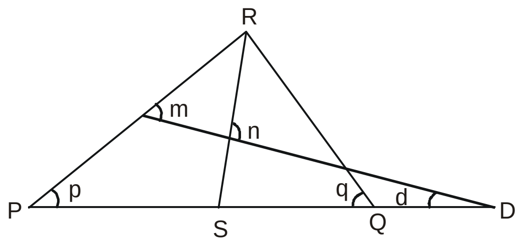

**6.** Given triangle PQR with RS bisecting $\angle R$, PQ extended to D and $\angle n$ a right angle, then

(A) $\angle m = \frac{1}{2}(\angle p - \angle q)$

(B) $\angle m = \frac{1}{2}(\angle p + \angle q)$

(C) $\angle d = \frac{1}{2}(\angle q + \angle p)$

(D) $\angle d = \frac{1}{2}\angle m$




**Sol.** $x + m = n = 90°$

In $\Delta PSR$

$\angle RSQ = x + p$

In $\Delta RSQ$

$x + x + p + q = 180°$

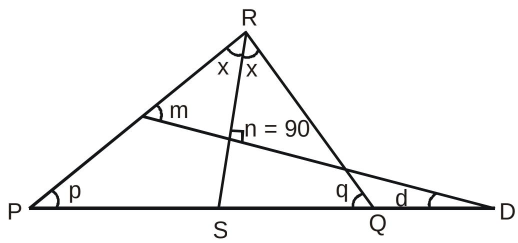
$2x + p + q = 180°$

$2(90° - m) + p + q = 180°$

$180° - 2m + p + q = 180°$

$2m = p + q$

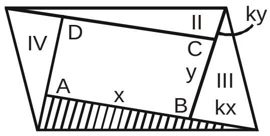
$m = \frac{p+q}{2}$.




**7.** The sides of rectangle are all produced in order, in such a way that the length of each side is increased by 'k' times itself. The area of the new quadrilateral formed becomes $2\frac{1}{2}$ times the area of the original rectangle. The value of 'k' is :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C*) $\frac{1}{2}$ (D) $\frac{2}{3}$

**Sol.**




The new figure obtained is made up of the original rectangle and four additional triangles, I, II, III, IV, as marked in the figure in which, I = II and III = IV. If x and y are the sides of the original rectangle, the sides of triangle I or II are x(1 + k), ky and of triangle III or IV are y(1 + k), kx.

Areas of I + II + III + IV $= 2[\frac{1}{2}x\ (1 + k)\ ky + \frac{1}{2}y(1 + k)kx]$

$= k(1 + k)xy + k(1 + k)xy = 2k(1 + k)xy.$

The area of the new quadrilateral = $2k(1 + k)xy + xy = \frac{5}{2}xy$ (given).

So, $2k(1 + k) + 1 = \frac{5}{2}$ giving $k(k + 1) = \frac{3}{4}$,

i.e. $4k^2 + 4k - 3 = 0, (2k + 3)(2k - 1) = 0$

i.e., $k = \frac{1}{2}$ or $-\frac{3}{2}$ which is impossible. Hence the required $k = \frac{1}{2}$.