Sample Paper-05
Mathematics
Class – XI

ANSWERS

Section A

1. **Solution**

$x = a + ib$

$|x| + x = \sqrt{a^2 + b^2} + a + ib$

$\sqrt{a^2 + b^2} + a = 2$

$a^2 + b^2 = (2 - a)^2$

$b = 1$

$a^2 + 1 = 4 + a^2 - 4a$

$a = \frac{3}{4}$

$x = \frac{3}{4} + i$

2. **Solution**

$S = 1 + 3 + 5 + \cdots\cdots$

$S = \frac{n}{2}[2 + (n-1)(2)]$

$S = n^2$

3. **Solution**

First term = 5

Sum of first and second term = 14

Second term= 9

Common Difference= $9 - 5 = 4$

nth term = $5 + (n-1)4$

= $4n + 1$

4. **Solution**

Length of latus rectum of the ellipse = $\frac{2a^2}{b}$

5. **Solution**

$f(x+5)=5$

6. **Solution**

The number of weights that can be measured = number of subsets can be formed excluding the null set

$2^4-1=15$

Section B

7. **Solution**

When $f(x)=x^2$

$$f(x)=x^2$$
$$f'(x)=2x$$
$$f'(a+b)=2(a+b)$$
$$f'(a)=2a$$
$$f'(b)=2b$$
$$f'(a)+f'(b)=2(a+b)$$
$$=f'(a+b)$$

When $f(x)=x^3$

$$f(x)=x^3$$
$$f'(x)=3x^2$$
$$f'(a+b)=3(a+b)^2$$
$$f'(a)=3a^2$$
$$f'(b)=3b^2$$
$$f'(a)+f'(b)=3(a^2+b^2)$$
$$\neq f'(a+b)$$

8. **Solution**

$$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]=-p[p^2-3q]$$

9. **Solution**

Total number of 3 digit numbers with 0 in units place = 90

The digits that can go into tens place for the number to be divisible by 4 = $0,2,4,6,8$

100th place can be formed with any of the 9 digits excepting 0

Hence total number of 3 digits number divisible by 4 is $9\times5=45$

Probability= $\frac{45}{90}=\frac{1}{2}$

10. **Solution**

$$\tan(45+x)=\frac{1+\tan x}{1-\tan x}$$
$$=\frac{\cos x+\sin x}{\cos x-\sin x}$$
$$=\frac{\cos^2 x+\sin^2 x+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$
$$=\frac{1+\sin 2x}{\cos 2x}=\sec 2x+\tan 2x$$

11. **Solution**

$$\begin{aligned}
P(n) &= n(n+1)\\
P(1) &= 2\text{ , } even\\
P(k) &= k(k+1) \textit{ let this be true}\\
P(k+1) &= (k+1)(k+2)\\
&= k^2+3k+2\\
&= k^2+k+2k+2\\
&= k(k+1)+2(k+1)\ True
\end{aligned}$$

12. **Solution**

$$n[(A\cup B\cup C)]=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$$
$$n[(A\cup B\cup C)]=4000+2000+1000-400-400-400+200$$
$$n[(A\cup B\cup C)]=6000$$

13. **Solution.**

$$\frac{X^2}{k^2}+\frac{y^2}{\frac{k^2}{3}}=1$$

Latus rectum is = $\frac{\left(\frac{2k^2}{3}\right)}{k}$

= $\frac{2k}{3}$

$$e=\sqrt{\left(\frac{k^2-\frac{k^2}{3}}{k^2}\right)}$$
$$=\sqrt{\frac{2}{3}}$$
$$=\frac{\sqrt{6}}{3}$$

Coordinates of foci are $(ae, 0)$ and $(-ae, 0)$

Coordinates are $= (\frac{\sqrt{6}}{3}k, 0)$ and $(\frac{-\sqrt{6}}{3}k, 0)$

14. **Solution**

Slope of line AB joining the points $(-8, 0)$ and $(12, 0)$ = 0

Its midpoint = $(2, 0)$

Equation to the line perpendicular to AB and passing through $(2, 0)$ is $x = 2$

Slope of line AC joining the points $(-8, 0)$ and $(0, 8)$ = 1

Its midpoint = $(-4, 4)$

Equation to the line perpendicular to AC and passing through $(-4, 4)$ is $y = -x$

So the center of the circle will be the point of intersection of line AB and line AC. Center of circle at point $(2, -2)$

Radius = $\sqrt{(2-0)^2 + (-2-8)^2}$ = $\sqrt{104}$

Area = 104π

15. **Solution**

$$2S_1 = n[2a + (n-1)d]$$
$$2S_2 = 2n[2a + (2n-1)d]$$
$$2S_3 = 3n[2a + (3n-1)d]$$
$$\frac{2S_1}{n} = 2a + (n-1)d$$
$$\frac{2S_3}{3n} = 2a + (n-1)d$$
$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + d(n-1+3n-1)$$
$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + 2(2n-1)d$$
$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 2.\frac{2S_2}{2n}$$
$$\frac{2S_3}{3n} = \frac{2S_2}{2n} - \frac{2S_1}{n}$$
$$\frac{2S_3}{3n} = \frac{4S_2}{2n} - \frac{4S_1}{2n}$$
$$S_3 = 3(S_2 - S_1)$$

16. **Solution**

$$f(x)=3x^2-6x-11$$

$$f(x)=3(x^2-2x-\frac{11}{3})$$

$$f(x)=3(x^2-2x+1-1-\frac{11}{3})$$

$$f(x)=3[(x-1)^2-\frac{11}{3}-1)$$

$$f(x)=3[(x-1)^2-\frac{14}{3})$$

$$f(x)=3(x-1)^2-14$$

$f(x)$ will attain minimum when $x=1$

Minimum value of $f(x)=-14$

17. **Solution**

$$f(x)=\frac{a^x}{a^x+\sqrt{a}}$$

$$f(1-x)=\frac{a^{1-x}}{a^{1-x}+\sqrt{a}}$$

$$f(x)+f(1-x)=\frac{a^x}{a^x+\sqrt{a}}+\frac{a^{1-x}}{a^{1-x}+\sqrt{a}}$$

$$=1$$

18. **Solution**

$$\frac{\tan 2x\tan x}{\tan 2x-\tan x}=\frac{\frac{2\tan x}{1-\tan^2 x}\tan x}{\frac{2\tan x}{1-\tan^2 x}-\tan x}$$

$$=\frac{2\tan^2 x}{\tan x+\tan^2 x}$$

$$=\frac{2\tan x}{1+\tan^2 x}$$

$$=\sin 2x$$

19. **Solution**

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!)}{(n+2)!-(n+1)!}=\lim_{n\to\infty}\frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)!}$$

$$=\lim_{n\to\infty}\frac{(n+3)}{((n+1)}$$

$$\lim_{n\to\infty}\frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$
$$=1$$

Section C

20. **Solution**

$$\frac{dy}{dx}=\sin^n x.\{-\sin nx).(n)\}+\cos nx.\{n.\sin^{n-1}x.\cos x\}$$
$$\frac{dy}{dx}=n\sin^{n-1}x(\cos nx.\cos x-\sin x.\sin nx)$$
$$\frac{dy}{dx}=n\sin^{n-1}[\cos(n+1)x]$$

21. **Solution**

$$(a-b)^2=5ab$$
$$a^2+b^2-2ab=5ab$$
$$a^2+b^2=7ab$$
$$(a+b)^2=9ab$$
$$a+b=3\sqrt{ab}$$
$$\frac{1}{3}(a+b)=\sqrt{ab}$$
$$\log\left(\frac{1}{3}(a+b)\right)=\frac{1}{2}(\log a+\log b)$$

22. **Solution**

First term = $\frac{1}{10}$

Second term= $\frac{7}{10^2}$

Third term= $\frac{7^2}{10^3}$

r = $\frac{7}{10}$

This is a GP

Sum to infinity= $\dfrac{\frac{1}{10}}{1-\frac{7}{10}} = \dfrac{1}{3}$

23. **Solution**

Mean= 14= $\dfrac{8+12+13+15+22+14}{6}$

x_i	$x_i - Mean$	$(x_i - Mean)^2$
8	-6	36
12	-2	4
13	-1	1
15	1	1
22	8	64
14	0	0
		$\Sigma(x_i - Mean)^2 = 106$

Variance= $\dfrac{1}{n}\Sigma(x_i - Mean)^2 = \dfrac{106}{6} = 17.66$

SD= $\sqrt{Variance} = \sqrt{17.66} = 4.2$

24. **Solution**

$Let\ \dfrac{a}{y} = x$

$by = \dfrac{ab}{x}$

$f(1+\dfrac{a}{y})^{by} = f\left[(1+x)^{\frac{1}{x}}\right]^{ab}$

25. **Solution:**

Probability of all the three hitting the target= $\dfrac{4}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{2}{5}$

Probability of A alone missing the target = $\dfrac{1}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{1}{10}$

Probability of B alone missing the target = $\dfrac{4}{5}.\dfrac{1}{4}.\dfrac{2}{3} = \dfrac{2}{15}$

Probability of C alone missing the target = $\frac{4}{5}.\frac{3}{4}.\frac{1}{3}=\frac{1}{5}$

The probability that the target being hit at least two= $\frac{2}{5}+\frac{1}{10}+\frac{2}{15}+\frac{1}{5}=\frac{5}{6}$

26. **Solution**

Let T_{r+1} be the term that is independent of x

Then

$$T_{r+1}={}^9C_r(ax^2)^r(-\frac{b}{x})^{9-r}$$
$$2r+(r-9)=0$$
$$r=3$$

4th term is independent of x

$$T_4={}^9C_3(a)^3(-b)^6$$

$$={}^9C_3(a)^3(b)^6$$