Sample Paper-05
Mathematics
Class – XI

Time allowed: 3 hours   **M. M: 100**
General Instructions:
(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section A

1. Identify a function $f(x)$ so that $f(x).f(y) = f(x+y)$

2. If $A = \{(x, y) : y = a^x, x \in R\}$ and $B = \{(x, y) : y = a^{-x}, x \in R\}$ then what is $(A \cap B)$

3. If R is a relation from a set A containing p elements to a set B containing q elements the find the number of subsets of $A \times B$

4. Check whether the given lines are parallel or perpendicular.

    $ax - by + c = 0 \quad and \quad \dfrac{ax}{2} - \dfrac{by}{2} + d = 0$

Section B

5. Find the area of the triangle whose vertices are (2,0),(5,3),(2,6)

6. Write the equation of a circle with center (0,0) and radius 5

7. Prove that $f'(a+b) = f'(a) + f'(b)$ when $f(x) = x^2$ and when $f(x) = x^3$

8. If α, β are the roots of the equation $x^2 + px + q = 0$ Find $\alpha^3 + \beta^3$.

9. A positive 3 digit number has its units digit zero. Find the probability that the number is divisible by 4.

10. Prove that $\tan(45 + x) = \sec 2x + \tan 2x$

11. Prove by mathematical induction that $n(n+1)$ is even

12. Find $n[(A \cup B \cup C)]$ if $n(A) = 4000$ $n(B) = 2000$ $n(C) = 1000$ and

    $n(A \cap B) = n(B \cap C) = n(A \cap C) = 400$, $n(A \cap B \cap C) = 200$

Section C

13. Find the latus rectum, eccentricity and coordinates of the foci of the ellipse $x^2 + 3y^2 = k^2$

14. Find the area of the circle passing through the points $(-8,0),(0,8),(12,0)$

15. If S_1, S_2, S_3 are the sums of $n, 2n, 3n$ terms respectively of an AP prove that $S_3 = 3(S_2 - S_1)$

16. Find the least value of $f(x)$ if $f(x) = 3x^2 - 6x - 11$

17. Find $f(x) + f(1-x)$ if $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$

18. Prove that $\dfrac{\tan 2x \tan x}{\tan 2x - \tan x} = \sin 2x$

19. Find the limit $\displaystyle\lim_{n\to\infty} \dfrac{(n+2)! + (n+1)!)}{(n+2)! - (n+1)!}$

20. Evaluate $x^3 + x^2 - 4x + 13$ when $x = 1 + i$ and when $x = 1 - i$

21. Prove that the roots of the equation $(x - \alpha)(x - \beta) = k^2$ is always real

22. If the roots of the equation $lx^2 + nx + n = 0$ are in the ratio $p : q$ then prove that

$$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$$

23. Find $\displaystyle\lim_{x\to\pi}(\pi - x)\tan\frac{x}{2}$

Section D

24. If a, b, c are 3 consecutive integers prove that $(a-i)(a+i)(c+i)(c-i) = b^4 + 1$

25. Prove that $\dfrac{(1+i)^n}{(1-i)^{n-2}} = 2\,i^{n-1}$

26. Determine the coordinates of a point which is equidistant from the point $(1,2)\,and\,(3,4)$ and the

shortest distance from the line joining the points $(1,2)\,and\,(3,4)$ to the required point is $\sqrt{2}$