

ASSERTION AND REASON

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

- (A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2** is a correct explanation for **Statement – 1**.
 (B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2** is NOT a correct explanation for **Statement – 1**.
 (C) **Statement – 1** is True, **Statement – 2** is False.
 (D) **Statement – 1** is False, **Statement – 2** is True.

INDEFINITE & DEFINITE INEGGRATION

129. Let $F(x)$ be an indefinite integral of $\cos^2 x$.

Statement-1: The function $F(x)$ satisfies $F(x + \pi) = F(x) \forall$ real x

Statement-2: $\cos^2(x + \pi) = \cos^2 x$.

130. **Statement-1:** $\int |x| dx$ can not be found while $\int_{-1}^1 |x| dx$ can be found.

Statement-2: $|x|$ is not differentiable at $x = 0$.

131. **Statement-1:** $\int \left(\frac{1}{1+x^4} \right) dx = \tan^{-1}(x^2) + C$ **Statement-2:** $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

132. **Statement-1:** If y is a function of x such that $y(x-y)^2 = x$ then $\int \frac{dx}{x-3y} = \frac{1}{2} [\log(x-y)^2 - 1]$

Statement-2: $\int \frac{dx}{x-3y} = \log(x-3y) + c$

133. **Statement-1:** $f(x) = \operatorname{logsec} x - \frac{x^2}{2}$ **Statement-2:** $f(x)$ is periodic

134. **Statement-1:** $\int \frac{x^{9/2}}{\sqrt{1+x^{11}}} dx = \frac{2}{11} \ln |x^{11/2} + \sqrt{1+x^{11}}| + c$

Statement-2: $\int \frac{dx}{\sqrt{1+x^2}} = \ln |x + \sqrt{1+x^2}| + c$

135. **Statement-1:** $\int_0^{10} [\tan^{-1} x] dx = 10 - \tan 1$; where $[x] = \text{G.I.F.}$

Statement-2: $[\tan^{-1} x] = 0$ for $0 < x < \tan 1$ and $[\tan^{-1} x] = 1$ for $\tan 1 \leq x < 10$.

136. **Statement-1:** $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \frac{\pi}{4}$

Statement-2: $\int_0^a f(x) dx = \int_0^a f(a+x) dx$

$$\int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \int_0^{\pi/2} \frac{dx}{1+\cot^3 x} = \frac{\pi}{4}$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx.$$

137. **Statement-1:** $\int_0^{\pi} \sqrt{1-\sin^2 x} dx = 0$ **Statement-2:** $\int_0^{\pi} \cos x dx = 0$.

138. **Statement-1:** $\int e^x (\tan x + \sec^2 x) dx = e^x \tan x + c$

Statement-2 : $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c.$

139. **Statement-1** : If $f(x)$ satisfies the conditions of Rolle's theorem in $[\alpha, \beta]$, then $\int_{\alpha}^{\beta} f'(x) dx = \beta - \alpha$

Statement-2 : If $f(x)$ satisfies the conditions of Rolle's theorem in $[\alpha, \beta]$, then $\int_{\alpha}^{\beta} f'(x) dx = 0$

140. **Statement-1** : $\int_0^{4\pi} [|\sin x| + |\cos x|] dx$, where $[\cdot]$ denotes G.I.F. equals 8π .

Statement-2 : If $f(x) = |\sin x| + |\cos x|$, then $1 \leq f(x) \leq \sqrt{2}$.

141. Let $f(x)$ be a continuous function such that $\int_n^{n+1} f(x) dx = n^3$, $n \in I$

Statement-1 : $\int_{-3}^3 f(x) dx = 27$

Statement-2 : $\int_{-2}^2 f(x) dx = 27$

142. Let $I_n = \int_1^e (\ell nx)^n dx$, $n \in N$

Statement-I : $I_1, I_2, I_3 \dots$ is an increasing sequence.

Statement-II : ℓnx is an increasing function.

143. Let f be a periodic function of period 2. Let $g(x) = \int_0^x f(t) dt$ and $h(x) = g(x+2) - g(x)$.

Statement-1 : h is a periodic function.

Statement-2 : $g(x+2) - g(x) = g(2)$.

144. **Statement-1** : $\int \frac{e^x}{x} (1 + x \log x) dx = e^x \log x + c$

Statement-2 : $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c.$

145. **Statement-1** : If $I_1 = \int_x^1 \frac{dt}{1+t^2}$ and $I_2 = \int_1^{1/x} \frac{dt}{1+t^2}$, $x > 0$ then $I_1 = I_2$.

Statement-2 : $\int_{-2}^2 \min\{x - [x], -x - [-x]\} dx = 0$

146. **Statement-1** : $8 < \int_4^6 2x dx < 12$.

Statement-2 : If m is the smallest and M is the greatest value of a function $f(x)$ in an interval (a, b) , then the value of the integral $\int_a^b f(x) dx$ is such that for $a < b$, we have $M(b-a) \leq \int_a^b f(x) dx \leq m(b-a)$.

147. **Statement-1** : $\int e^{ax} \sin bx dx = \frac{e^{ax}}{A} (a \sin bx - b \cos bx) + c$ Then A is $\sqrt{a^2 + b^2}$

Statement-2 : $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx = e^x \tan x + c$

148. **Statement-1** : $\int \frac{d(x^2 + 1)}{\sqrt{\lambda^2 + 2}}$ is equal to $2\sqrt{x^2 + 2} + c$

Statement-2 : $\int \frac{x^{a/2}}{\sqrt{1+x^{11}}} dx$ is $\frac{2}{11} \ln |x + \sqrt{1+x^{11}}| + c$

149. Statement-1 : $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^3 x} dx$ is $\pi/12$ Statement-2 : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

150. Statement-1 : If f satisfies $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$ then $\int_{-5}^5 f(x) dx = 0$

Statement-2 : If f is an odd function then $\int_{-a}^a f(x) dx = 0$

151. Statement-1 : If $f(x)$ is an odd function of x then $\int_a^x f(t) dt$ is an even function of (n)

Statement-2 : If graph of $y = f(x)$ is symmetric about y -axis then $f(x)$ is always an even function.

152. Statement-1 : Area bounded by $y = \{x\}$, $\{x\}$ is fractional part of $x = 0, x = 2$ and x -axis is 1.

Statement-2 : Area bounded by $y = |\sin x|$, $x = 0, x = 2\pi$ is 2 sq. unit.

153. Statement-1: $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-2^2}} + \dots + \frac{1}{\sqrt{3n}} \right) = \frac{\pi}{3}$

Statement-2: $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x) dx$, symbols have their usual meaning.

154. Statement-1: If $I_n = \int \tan^n x dx$, then $5(I_4 + I_6) = \tan^5 x$.

Statement-2: If $I_n = \int \tan^n x dx$, then $\frac{\tan^{n-1} x}{n} - I_{n-2} = I_n, n \in \mathbb{N}$.

155. Statement-1: If $a > 0$ and $b^2 - 4ac < 0$, then the value of the integral $\int \frac{dx}{ax^2 + bx + c}$ will be of the type $\mu \tan^{-1} \left(\frac{x+A}{B} \right) + c$, where A, B, C, μ are constants.

Statement-2: If $a > 0, b^2 - 4ac < 0$ then $ax^2 + bx + c$ can be written as sum of two squares.

156. Statements-1: $\int \frac{x^2 - x + 1}{(x^2 + 1)^{3/2}} e^x dx = \frac{e^x}{\sqrt{x^2 + 1}} + c$ Statements-2: $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

157. Statements-1: $\int \frac{x^2 - 2}{(x^4 + 5x^2 + 4) \tan^{-1} \left(\frac{x^2 + 2}{x} \right)} dx = \log |\tan^{-1} (x + 2/x)| + c$

Statements-2: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

158. Statements-1: $\int \frac{\ln \frac{x}{e}}{(\ln x)^2} = \frac{x}{\ln x} + c$ Statements-2: $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$.

159. Statements-1: $\int \frac{1}{x^3 \sqrt{1+x^4}} dx = -\frac{1}{2} \sqrt{1 + \frac{1}{x^4}} + c$ Statements-2: For integration by parts we have to follow ILATE rule.

160. Statements-1: A function $F(x)$ is an antiderivative of a function $f(x)$ if $F'(x) = f(x)$

Statements-2: The functions $x^2 + 1, x^2 - \pi, x^2 + \sqrt{2}$ are all antiderivatives of the function $2x$.

161. Statements-1: $\int_a^b \frac{|x|}{x} dx = |b| - |a|, a < b$

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Statements-2: If $f(x)$ is a function continuous every where in the interval (a, b) except $x = c$ then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

162. **Statements-1:** $4 \leq \int_1^3 \sqrt{3+x^3} dx \leq 2\sqrt{30}$

Statements-2: m and M be the least and the maximum value of a continuous function

$y = f(x)$ in $[a, b]$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

163. **Statements-1:** $1 < \int_0^1 e^{x^2} dx < e$

Statements-2: if $f(x) \leq g(x) \leq h(x)$ in (a, b) then $\int_a^b f(x)dx \leq \int_a^b g(x)dx \leq \int_a^b h(x)dx$

164. **Statements-1:** $\left| \int_0^1 \sqrt{1+x^4} dx \right| < \sqrt{1.2}$

Statements-2: For any functions $f(x)$ and $g(x)$, integrable on the interval (a, b) , then

$$\left| \int_a^b f(x)g(x)dx \right| \leq \sqrt{\int_a^b f^2(x)dx} \sqrt{\int_a^b g^2(x)dx}$$

165. **Statements-1:** $\int_{-1}^1 \frac{1}{x^2} dx = -2$

Statements-2: If $F(x)$ is antiderivative of a continuous function (a, b) then $\int_a^b f(x)dx = F(b) - F(a)$

166. **Statements-1:** $\frac{\cos x}{(1 + \sin x)^2}$ can be integrated by substitution if $\sin x = t$.

Statements-2: All integrands are integrated by the method of substitution only.

167. **Statement-1 :** $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx = \int e^x \tan x + c$

Statement-2 : $\int e^x (f(x) + f'(x))dx = e^x f(x) + c$

168. **Statements-1:** $\int e^x (x+1) \cos^2(x.e^x) dx = \frac{1}{2} x.e^x + \frac{1}{4} \sin 2(x.e^x) + C$

Statements-2: $\int f(\phi(x)) \phi'(x) dx, \{\phi(x) = t\}$ equals $\int f(t) dt$.

169. **Statements-1:** $\int \log x dx = x \log x - x + c$

Statements-2: $\int uv dx = u \int v dx + \int \left(\frac{du}{dx} \int v dx \right) dx$

170. **Statements-1:** $\int e^x \left(\frac{x^2 + 4x + 2}{x^2 + 4x + 4} \right) dx = \frac{e^x}{(x+2)^2} + C$ **Statements-2:** $\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$

171. **Statements-1:** $\int_{-1}^1 \frac{\sin x - x^2}{3 + |x|} = -2 \int_0^1 \frac{x^2}{3 + |x|}$ **Statements-2:** $\int_{-a}^a f(x) = dx = \int_0^a f(x)dx + \int_0^a f(-x)dx$

172. **Statements-1:** The value of $\int_0^1 \sqrt{(1+x)(1+x^3)} dx$ can not exceed $\sqrt{\frac{15}{8}}$

Statements-2: If $m \leq f(x) \leq M \forall x \in [a, b]$ then $m(b-a) \leq \int_a^b f(x)dx \leq (b-a)M$

173. **Statements-1:** $\int_0^{\pi/2} \frac{(\sin x)^{5/2}}{(\sin x)^{5/2} + (\cos x)^{5/2}} dx = \frac{\pi}{4}$ **Statements-2:** Area bounded by $y = 3x$ and $y = x^2$ is $= \frac{9}{2}$ sq. units

174. **Statements-1:** $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx = \log|10x + x^{10}| + c$ **Statements-2:** $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

175. **Statements-1:** $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx = \tan(xe^x) + c$ **Statements-2:** $\int \sec^2 x dx = \tan x + c$

176. **Statement-1 :** $f(x) = \int_1^x \frac{\ln t dt}{1+t+t^2} (x > 0)$, then $f(x) = -f\left(\frac{1}{x}\right)$

Statements-2: $f(x) = \int_1^x \frac{\ln t dt}{t+1}$, then $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{2} (\ln x)^2$

177. **Statement-1 :** $\int_{-1}^1 \frac{\sin x - x^2}{3-|x|} dx = \int_0^1 \frac{-2x^2}{3-|x|} dx$.

Statements-2: Since $\frac{\sin x}{3-|x|}$ is an odd function. So, that $\int_{-1}^1 \frac{\sin x}{3-|x|} dx = 0$.

178. **Statements-1 :** $\int_0^{n\pi+t} |\sin x| dx = (2n+1) - \cos t (0 \leq t \leq \pi)$

Statements-2: $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ and $\int_0^{na} f(x) dx = n \int_0^a f(x) dx$ if $f(a+x) = f(x)$

179. **Statements-1:** The value of the integral $\int_0^1 e^{x^2} dx$ belongs to $[0, 1]$

Statements-2: If m & M are the lower bound and the upper bounds of $f(x)$ over $[a, b]$ and f is integrable, then $m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$.

180. **Statements-1:** $\int_0^\infty [\cot^{-1} x] dx = \cot 1$, where $[\cdot]$ denotes greatest integer function.

Statements-2: $\int_a^b f(x) dx$ is defined only if $f(x)$ is continuous in (a, b) $[\cdot]$ function is discontinuous at all integers

181. **Statements-1:** $\int_{-4}^4 (\sqrt{1+x+x^2} - \sqrt{1-x+x^2}) dx = 0$ **Statements-2:** $\int_{-a}^a f(x) dx = 0$ if $f(x)$ is an odd function.

182. **Statements-1:** All continuous functions are integrable

Statements-2: If a function $y = f(x)$ is continuous on an interval $[a, b]$ then its definite integral over $[a, b]$ exists.

183. **Statements-1:** If $f(x)$ is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x) dx = 0$, then $f(x) = 0$ at least once in $[a, b]$

Statements-2: If f is continuous on $[a, b]$, then at some point c in $[a, b]$ $f(c) = \frac{1}{b-a} \int_a^b f(x) dx$

184. **Statements-1:** $\int_{-4}^4 |x+2| dx = 50$ **Statements-2:** $\int_0^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ where $C \in (A, B)$

185. **Statements-1:** $\int_{-2}^2 \log\left(\frac{1+x}{1-x}\right) dx = 0$ **Statements-2:** If f is an odd function $\int_{-a}^a f(x) dx = 0$
186. **Statement-1** If $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$ then $\int_0^{\infty} x^m e^{-ax} dx = \frac{m!}{a^{m+1}}$ **Statement-2:** $\frac{d^n}{dx^n}(e^{kx}) = k^n e^{kx}$ and $\frac{d^n}{dx^n}\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}$
187. **Statement-1:** $\int_0^{10} \{x - [x]\} dx = 5$ **Statements-2:** $\int_a^{na} f(x) dx = n \int_a^a f(x) dx$
188. **Statements-1:** $\int_0^{\pi} |\cos x| dx = 2$ **Statements-2:** $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ where $a < c < b$.
189. **Statements-1:** $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = \pi$ **Statements-2:** $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
190. **Statements-1:** $\int_0^{1000} e^{x-[x]} dx = 1000(e-1)$ **Statements-2:** $\int_0^n e^{x-[x]} dx = n \int_0^1 e^{x-[x]} dx$
191. **Statements-1:** $\int_0^{\pi} \frac{dx}{1+2^{\tan x}} = \frac{\pi}{2}$ **Statements-2:** $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

ANSWER

- | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 129. D | 130. B | 131. D | 132. C | 133. A | 134. A | 135. A | 136. C |
| 137. D | 138. A | 139. A | 140. D | 141. D | 142. D | 143. A | 144. A |
| 145. C | 146. A | 147. D | 148. C | 149. A | 150. A | 151. C | 152. C |
| 153. D | 154. C | 155. A | 156. C | 157. A | 158. A | 159. B | 160. B |
| 161. A | 162. A | 163. A | 164. A | 165. D | 166. C | 167. C | 168. A |
| 169. C | 170. A | 171. A | 172. A | 173. B | 174. A | 175. A | 176. D |
| 177. A | 178. A | 179. D | 180. A | 181. A | 182. B | 183. A | 184. A |
| 185. A | 186. A | 187. C | 188. A | 189. D | 190. A | 191. A | |

Que. from Compt. Exams

(Indefinite Integral)

1. $\int \frac{dx}{\cos(x-a)\cos(x-b)} =$
- (a) $\operatorname{cosec}(a-b) \log \frac{\sin(x-a)}{\sin(x-b)} + c$ (b) $\operatorname{cosec}(a-b) \log \frac{\cos(x-a)}{\cos(x-b)} + c$
- (c) $\operatorname{cosec}(a-b) \log \frac{\sin(x-b)}{\sin(x-a)} + c$ (d) $\operatorname{cosec}(a-b) \log \frac{\cos(x-b)}{\cos(x-a)} + c$
2. $\int \frac{dx}{\sqrt{x+a}\sqrt{x+b}} =$ [AISSE 1989]
- (a) $\frac{2}{3(b-a)} [(x+a)^{3/2} - (x+b)^{3/2}] + c$ (b) $\frac{2}{3(a-b)} [(x+a)^{3/2} - (x+b)^{3/2}] + c$
- (c) $\frac{2}{3(a-b)} [(x+a)^{3/2} + (x+b)^{3/2}] + c$ (d) None of these
3. $\int \frac{3 \cos x + 3 \sin x}{4 \sin x + 5 \cos x} dx =$ [EAMCET 1991]
- (a) $\frac{27}{41} x - \frac{3}{41} \log(4 \sin x + 5 \cos x)$ (b) $\frac{27}{41} x + \frac{3}{41} \log(4 \sin x + 5 \cos x)$
- (c) $\frac{27}{41} x - \frac{3}{41} \log(4 \sin x - 5 \cos x)$ (d) None of these
4. If $\int (\sin 2x + \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x-c) + a$, then the value of a and c is [Roorkee 1978]

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- (a) $c = \pi/4$ and $a = k$ (an arbitrary constant) (b) $c = -\pi/4$ and $a = \pi/2$
 (c) $c = \pi/2$ and a is an arbitrary constant (d) None of these

5. $\int \frac{x^3 - x - 2}{(1 - x^2)} dx =$ [AI CBSE 1985]

- (a) $\log\left(\frac{x+1}{x-1}\right) - \frac{x^2}{2} + c$ (b) $\log\left(\frac{x-1}{x+1}\right) + \frac{x^2}{2} + c$ (c) $\log\left(\frac{x+1}{x-1}\right) + \frac{x^2}{2} + c$ (d) $\log\left(\frac{x-1}{x+1}\right) - \frac{x^2}{2} + c$

6. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx =$ [IIT 1986]

- (a) $\sin 2x + c$ (b) $-\frac{1}{2} \sin 2x + c$ (c) $\frac{1}{2} \sin 2x + c$ (d) $-\sin 2x + c$

7. $\int \frac{x^2 dx}{(a + bx)^2} =$ [IIT 1979]

- (a) $\frac{1}{b^2} \left[x + \frac{2a}{b} \log(a + bx) - \frac{a^2}{b} \frac{1}{a + bx} \right]$ (b) $\frac{1}{b^2} \left[x - \frac{2a}{b} \log(a + bx) + \frac{a^2}{b} \frac{1}{a + bx} \right]$
 (c) $\frac{1}{b^2} \left[x + \frac{2a}{b} \log(a + bx) + \frac{a^2}{b} \frac{1}{a + bx} \right]$ (d) $\frac{1}{b^2} \left[x + \frac{a}{b} - \frac{2a}{b} \log(a + bx) - \frac{a^2}{b} \frac{1}{a + bx} \right]$

8. $\int \frac{dx}{(1 + x^2)\sqrt{p^2 + q^2(\tan^{-1} x)^2}} =$

- (a) $\frac{1}{q} \log[q \tan^{-1} x + \sqrt{p^2 + q^2(\tan^{-1} x)^2}] + c$ (b) $\log[q \tan^{-1} x + \sqrt{p^2 + q^2(\tan^{-1} x)^2}] + c$
 (c) $\frac{2}{3q} (p^2 + q^2 \tan^{-1} x)^{3/2} + c$ (d) None of these

9. $\int \frac{x^5}{\sqrt{1 + x^3}} dx =$ [IIT 1985]

- (a) $\frac{2}{9} (1 + x^3)^{3/2} + c$ (b) $\frac{2}{9} (1 + x^3)^{3/2} + \frac{2}{3} (1 + x^3)^{1/2} + c$
 (c) $\frac{2}{9} (1 + x^3)^{3/2} - \frac{2}{3} (1 + x^3)^{1/2} + c$ (d) None of these

10. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals [MP PET 2002]

- (a) $-\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (b) $\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (c) $\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (d) $-\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$

11. $\int \frac{a dx}{b + ce^x} =$ [MP PET 1988; BIT Ranchi 1979]

- (a) $\frac{a}{b} \log\left(\frac{e^x}{b + ce^x}\right) + c$ (b) $\frac{a}{b} \log\left(\frac{b + ce^x}{e^x}\right) + c$ (c) $\frac{b}{a} \log\left(\frac{e^x}{b + ce^x}\right) + c$ (d) $\frac{b}{a} \log\left(\frac{b + ce^x}{e^x}\right) + c$

12. $\int \sin \sqrt{x} dx =$ [Roorkee 1977]

- (a) $2[\sin \sqrt{x} - \cos \sqrt{x}] + c$ (b) $2[\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}] + c$
 (c) $2[\sin \sqrt{x} + \cos \sqrt{x}] + c$ (d) $2[\sin \sqrt{x} + \sqrt{x} \cos \sqrt{x}] + c$

13. $\int \frac{x^2}{(9 - x^2)^{3/2}} dx =$

- (a) $\frac{x}{\sqrt{9 - x^2}} - \sin^{-1} \frac{x}{3} + c$ (b) $\frac{x}{\sqrt{9 - x^2}} + \sin^{-1} \frac{x}{3} + c$ (c) $\sin^{-1} \frac{x}{3} - \frac{x}{\sqrt{9 - x^2}} + c$ (d) None of these

14. $\int x \sqrt{\frac{1 - x^2}{1 + x^2}} dx =$

- (a) $\frac{1}{2} [\sin^{-1} x^2 + \sqrt{1 - x^4}] + c$ (b) $\frac{1}{2} [\sin^{-1} x^2 + \sqrt{1 - x^2}] + c$
 (c) $\sin^{-1} x^2 + \sqrt{1 - x^4} + c$ (d) $\sin^{-1} x^2 + \sqrt{1 - x^2} + c$

15. If $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log(f(x)) + c$, then $f(x) =$
- (a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$ (b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$ (c) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$ (d) $\frac{1}{a^2 \cos^2 x - b^2 \sin^2 x}$
16. $\int \frac{dx}{4 \sin^2 x + 5 \cos^2 x} =$ [AISSE 1986]
- (a) $\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + c$ (b) $\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{\tan x}{\sqrt{5}} \right) + c$ (c) $\frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + c$ (d) None of these
17. $\int \frac{x^2 + 1}{x^4 - x^2 + 1} dx =$ [MP PET 1991]
- (a) $\tan^{-1} \left(\frac{1 + x^2}{x} \right) + c$ (b) $\cot^{-1} \left(\frac{1 + x^2}{x} \right) + c$ (c) $\tan^{-1} \left(\frac{x^2 - 1}{x} \right) + c$ (d) $\cot^{-1} \left(\frac{x^2 - 1}{x} \right) + c$
18. $\int (\log x)^2 \, dx =$ [IIT 1971, 77]
- (a) $x(\log x)^2 - 2x \log x - 2x + c$ (b) $x(\log x)^2 - 2x \log x - x + c$
 (c) $x(\log x)^2 - 2x \log x + 2x + c$ (d) $x(\log x)^2 - 2x \log x + x + c$
19. The value of $\int \frac{\sqrt{x^2 - a^2}}{x} dx$ will be [UPSEAT 1999]
- (a) $\sqrt{x^2 - a^2} - a \tan^{-1} \left[\frac{\sqrt{x^2 - a^2}}{a} \right]$ (b) $\sqrt{x^2 - a^2} + a \tan^{-1} \left[\frac{\sqrt{x^2 - a^2}}{a} \right]$
 (c) $\sqrt{x^2 - a^2} + a^2 \tan^{-1} [\sqrt{x^2 - a^2}]$ (d) $\tan^{-1} x / a + c$
20. $\int \tan^3 2x \sec 2x \, dx =$ [IIT 1977]
- (a) $\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + c$ (b) $\frac{1}{6} \sec^3 2x + \frac{1}{2} \sec 2x + c$
 (c) $\frac{1}{9} \sec^2 2x - \frac{1}{3} \sec 2x + c$ (d) None of these
21. $\int x \sin^{-1} x \, dx =$ [MP PET 1991]
- (a) $\left(\frac{x^2}{2} - \frac{1}{4} \right) \sin^{-1} x + \frac{x}{4} \sqrt{1 - x^2} + c$ (b) $\left(\frac{x^2}{2} + \frac{1}{4} \right) \sin^{-1} x + \frac{x}{4} \sqrt{1 - x^2} + c$
 (c) $\left(\frac{x^2}{2} - \frac{1}{4} \right) \sin^{-1} x - \frac{x}{4} \sqrt{1 - x^2} + c$ (d) $\left(\frac{x^2}{2} + \frac{1}{4} \right) \sin^{-1} x - \frac{x}{4} \sqrt{1 - x^2} + c$
22. $\int \sqrt{\frac{a-x}{x}} \, dx =$
- (a) $a \left[\sin^{-1} \sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}} \sqrt{\frac{a-x}{a}} \right] + c$ (b) $\sin^{-1} \frac{x}{a} + \frac{x}{a} \sqrt{a^2 - x^2} + c$
 (c) $a \left[\sin^{-1} \frac{x}{a} - \frac{x}{a} \sqrt{a^2 - x^2} \right] + c$ (d) $\sin^{-1} \frac{x}{a} - \frac{x}{a} \sqrt{a^2 - x^2} + c$
23. If $x \in \left(\frac{\pi}{4}, \frac{3\pi}{4} \right)$, then $\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x \, dx =$
- (a) $e^{\sin x} + c$ (b) $e^{\sin x - \cos x} + c$
 (c) $e^{\sin x + \cos x} + c$ (d) $e^{\cos x - \sin x} + c$
24. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log(9e^{2x} - 4) + C$, then A, B and C are [IIT 1990]
- (a) $A = \frac{3}{2}, B = \frac{36}{35}, C = \frac{3}{2} \log 3 + \text{constant}$
 (b) $A = \frac{3}{2}, B = \frac{35}{36}, C = \frac{3}{2} \log 3 + \text{constant}$

(c) $A = -\frac{3}{2}, B = -\frac{35}{36}, C = -\frac{3}{2} \log 3 + \text{constant}$

(d) None of these

25. The value of $\int \sec^3 x \, dx$ will be [UPSEAT 1999]

(a) $\frac{1}{2} [\sec x \tan x + \log(\sec x + \tan x)]$

(b) $\frac{1}{3} [\sec x \tan x + \log(\sec x + \tan x)]$

(c) $\frac{1}{4} [\sec x \tan x + \log(\sec x + \tan x)]$

(d) $\frac{1}{8} [\sec x \tan x + \log(\sec x + \tan x)]$

26. $\int \frac{x-1}{(x+1)^3} e^x \, dx =$ [IIT 1983; MP PET 1990]

(a) $\frac{-e^x}{(x+1)^2} + c$ (b) $\frac{e^x}{(x+1)^2} + c$

(c) $\frac{e^x}{(x+1)^3} + c$ (d) $\frac{-e^x}{(x+1)^3} + c$

27. If $I = \int e^x \sin 2x \, dx$, then for what value of K , $KI = e^x (\sin 2x - 2 \cos 2x) + \text{constant}$ [MP PET 1992]

(a) 1 (b) 3
(c) 5 (d) 7

28. The value of $\int \frac{dx}{3-2x-x^2}$ will be [UPSEAT 1999]

(a) $\frac{1}{4} \log \left(\frac{3+x}{1-x} \right)$ (b) $\frac{1}{3} \log \left(\frac{3+x}{1-x} \right)$

(c) $\frac{1}{2} \log \left(\frac{3+x}{1-x} \right)$ (d) $\log \left(\frac{1-x}{3+x} \right)$

29. $\int x\sqrt{2x+3} \, dx =$ [AISSE 1985]

(a) $\frac{x}{3} (2x+3)^{3/2} - \frac{1}{15} (2x+3)^{5/2} + c$

(b) $\frac{x}{3} (2x+3)^{3/2} + \frac{1}{15} (2x+3)^{5/2} + c$

(c) $\frac{x}{2} (2x+3)^{3/2} + \frac{1}{6} (2x+3)^{5/2} + c$

(d) None of these

30. $\int \cos 2\theta \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta =$ [IIT 1994]

(a) $(\cos \theta - \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)$

(b) $(\cos \theta + \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)$

(c) $\frac{(\cos \theta - \sin \theta)^2}{2} \log \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right)$

(d) $\frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \log \sec 2\theta$

31. $\int \frac{x^2}{(x \sin x + \cos x)^2} dx =$ [MNR 1989; RPET 2000]

(a) $\frac{\sin x + \cos x}{x \sin x + \cos x}$ (b) $\frac{x \sin x - \cos x}{x \sin x + \cos x}$

(c) $\frac{\sin x - x \cos x}{x \sin x + \cos x}$ (d) None of these

32. If $u = \int e^{ax} \cos bx \, dx$ and $v = \int e^{ax} \sin bx \, dx$, then $(a^2 + b^2)(u^2 + v^2) =$

- (a) $2e^{ax}$ (b) $(a^2 + b^2)e^{2ax}$
(c) e^{2ax} (d) $(a^2 - b^2)e^{2ax}$

33. If $I_n = \int (\log x)^n \, dx$, then $I_n + nI_{n-1} =$

- (a) $x(\log x)^n$ (b) $(x \log x)^n$
(c) $(\log x)^{n-1}$ (d) $n(\log x)^n$

[Karnataka CET 2003]

34. $\int e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx =$ [Roorkee 1982]

- (a) $e^{x/2} \cos \frac{x}{2} + c$ (b) $\sqrt{2}e^{x/2} \cos \frac{x}{2} + c$
(c) $e^{x/2} \sin \frac{x}{2} + c$ (d) $\sqrt{2}e^{x/2} \sin \frac{x}{2} + c$

35. If $\int \frac{2x+3}{x^2-5x+6} dx = 9 \ln(x-3) - 7 \ln(x-2) + A$, then $A =$ [MP PET 1992]

- (a) $5 \ln(x-2) + \text{constant}$ (b) $-4 \ln(x-3) + \text{constant}$
(c) Constant (d) None of these

36. $\int \frac{dx}{2 + \cos x} =$

- (a) $2 \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right) + c$ (b) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right) + c$
(c) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right) + c$ (d) None of these

37. $\int \frac{x}{x^4 + x^2 + 1} dx$ equal to [MP PET 2004]

- (a) $\frac{1}{3} \tan^{-1}\left(\frac{2x^2+1}{3}\right)$ (b) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$
(c) $\frac{1}{\sqrt{3}} \tan^{-1}(2x^2+1)$ (d) None of these

38. $\int \frac{dx}{(\sin x + \sin 2x)} =$ [IIT 1984]

- (a) $\frac{1}{6} \log(1 - \cos x) + \frac{1}{2} \log(1 + \cos x) - \frac{2}{3} \log(1 + 2 \cos x)$
(b) $6 \log(1 - \cos x) + 2 \log(1 + \cos x) - \frac{2}{3} \log(1 + 2 \cos x)$
(c) $6 \log(1 - \cos x) + \frac{1}{2} \log(1 + \cos x) + \frac{2}{3} \log(1 + 2 \cos x)$
(d) None of these

39. If $\int \frac{2x+3}{(x-1)(x^2+1)} dx = \log_e \left\{ (x-1)^{\frac{5}{2}} (x^2+1)^a \right\} - \frac{1}{2} \tan^{-1} x + A$,

where A is any arbitrary constant, then the value of 'a' is

- (a) $5/4$ (b) $-5/3$
(c) $-5/6$ (d) $-5/4$

[MP PET 1998]

40. If $\int \frac{(2x^2+1) dx}{(x^2-4)(x^2-1)} = \log \left[\left(\frac{x+1}{x-1} \right)^a \left(\frac{x-2}{x+2} \right)^b \right] + C$, then the values of a and b are respectively [Roorkee 2000]

- (a) $1/2, 3/4$ (b) $-1, 3/2$
(c) $1, 3/2$ (d) $-1/2, 3/4$

(Definite Integral)

1. If I is the greatest of the definite integrals

$$I_1 = \int_0^1 e^{-x} \cos^2 x \, dx, \quad I_2 = \int_0^1 e^{-x^2} \cos^2 x \, dx$$

$$I_3 = \int_0^1 e^{-x^2} \, dx, \quad I_4 = \int_0^1 e^{-x^2/2} \, dx, \text{ then}$$

- (a) $I = I_1$ (b) $I = I_2$
(c) $I = I_3$ (d) $I = I_4$

2. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. The value of integral $\int_0^1 f(x)g(x) \, dx$ is equal to

[AIEEE 2003; DCE 2005]

- (a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$
(c) $\frac{1}{2}(e-3)$ (d) None of these

3. If $I_m = \int_1^x (\log x)^m \, dx$ satisfies the relation $I_m = k - II_{m-1}$, then

- (a) $k = e$ (b) $I = m$
(c) $k = \frac{1}{e}$ (d) None of these

4. Let f be a positive function. Let

$$I_1 = \int_{1-k}^k x f\{x(1-x)\} \, dx, \quad I_2 = \int_{1-k}^k f\{x(1-x)\} \, dx$$

when $2k-1 > 0$. Then I_1 / I_2 is [IIT 1997 Cancelled]

- (a) 2 (b) k
(c) $1/2$ (d) 1

5. If $\int_0^x f(t) \, dt = x + \int_x^1 t f(t) \, dt$, then the value of $f(1)$ is

[IIT 1998; AMU 2005]

- (a) $1/2$ (b) 0
(c) 1 (d) $-1/2$

6. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} \, dx$ is equal to [AMU 2000]

- (a) 1 (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$

7. If n is any integer, then $\int_0^\pi e^{\cos^2 x} \cos^3(2n+1)x \, dx =$

[IIT 1985; RPET 1995; UPSEAT 2001]

- (a) x (b) 1
(c) 0 (d) None of these

8. The value of the definite integral $\int_0^1 \frac{x \, dx}{x^3 + 16}$ lies in the interval $[a, b]$. The smallest such interval is

- (a) $\left[0, \frac{1}{17}\right]$ (b) $[0, 1]$
(c) $\left[0, \frac{1}{27}\right]$ (d) None of these

9. Let a, b, c be non-zero real numbers such that $\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c) \, dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c) \, dx$

Then the quadratic equation $ax^2 + bx + c = 0$ has

[IIT 1981; CEE 1993]

- (a) No root in $(0, 2)$
(b) At least one root in $(0, 2)$
(c) A double root in $(0, 2)$
(d) None of these

10. If $f(x) = \int_{-1}^x |t| \, dt$, $x \geq -1$, then [MNR 1994]

- (a) f and f' are continuous for $x+1 > 0$
 (b) f is continuous but f' is not continuous for $x+1 > 0$
 (c) f and f' are not continuous at $x=0$
 (d) f is continuous at $x=0$ but f' is not so

11. Let $g(x) = \int_0^x f(t) dt$ where $\frac{1}{2} \leq f(t) \leq 1, t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in (1, 2]$, then [IIT Screening 2000]

- (a) $-\frac{3}{2} \leq g(2) < \frac{1}{2}$ (b) $0 \leq g(2) < 2$
 (c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$ (d) $2 < g(2) < 4$

12. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is

- (a) π (b) $a\pi$
 (c) $\frac{\pi}{2}$ (d) 2π

[IIT Screening 2001; AIEEE 2005]

13. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$, and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is

- (a) 1 (b) -3
 (c) -1 (d) 2

[AIEEE 2004]

14. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be continuous functions, then the value of the integral

$$\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)] [g(x) - g(-x)] dx =$$

[IIT 1990; DCE 2000; MP PET 2001]

- (a) π (b) 1
 (c) -1 (d) 0

15. The numbers P, Q and R for which the function $f(x) = Pe^{2x} + Qe^x + Rx$ satisfies the conditions $f(0) = -1$, $f'(\log 2) = 31$ and

$$\int_0^{\log 4} [f(x) - Rx] dx = \frac{39}{2}$$
 are given by

- (a) $P=2, Q=-3, R=4$ (b) $P=-5, Q=2, R=3$
 (c) $P=5, Q=-2, R=3$ (d) $P=5, Q=-6, R=3$

16. $\left[\sum_{n=1}^{10} \int_{-2n-1}^{2n} \sin^{27} x dx \right] + \left[\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx \right]$ equals

- (a) 27^2 (b) -54
 (c) 36 (d) 0

[MP PET 2002]

17. Let $\int_0^1 f(x) dx = 1$, $\int_0^1 x f(x) dx = a$ and $\int_0^1 x^2 f(x) dx = a^2$, then the value of $\int_0^1 (x-a)^2 f(x) dx =$ [IIT 1990]

- (a) 0 (b) a^2
 (c) $a^2 - 1$ (d) $a^2 - 2a + 2$

18. Given that $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$, then the value of $\int_0^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)}$ is

- (a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$
 (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

[Karnataka CET 1993]

19. If $I(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $I(m, n)$ in terms of $I(m+1, n-1)$ is

[IIT Screening 2003]

- (a) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$
 (b) $\frac{n}{m+1} I(m+1, n-1)$

- (c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
- (d) $\frac{m}{n+1} l(m+1, n-1)$
20. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5} =$ [AIEEE 2003]
- (a) $\frac{1}{30}$ (b) Zero
- (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
21. If $\int_0^t x f(x) dx = \frac{2}{5} t^5$, $t > 0$, then $f\left(\frac{4}{25}\right) =$ [IIT Screening 2004]
- (a) $\frac{2}{5}$ (b) $\frac{5}{2}$
- (c) $-\frac{2}{5}$ (d) None of these
22. For which of the following values of m , the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$ [IIT 1999]
- (a) -4 (b) -2
- (c) 2 (d) 4
23. Area enclosed between the curve $y^2(2a-x) = x^3$ and line $x = 2a$ above x -axis is [MP PET 2001]
- (a) πa^2 (b) $\frac{3\pi a^2}{2}$
- (c) $2\pi a^2$ (d) $3\pi a^2$
24. What is the area bounded by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ is [DCE 1999]
- (a) 0 (b) $\frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9 \sin^{-1}\left(\frac{1}{3}\right)$
- (c) 16π (d) None of these
25. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is [IIT Screening 2002]
- (a) 1 (b) 2
- (c) $2\sqrt{2}$ (d) 4
26. The volume of spherical cap of height h cut off from a sphere of radius a is equal to [UPSEAT 2004]
- (a) $\frac{\pi}{3} h^2(3a-h)$ (b) $\pi(a-h)(2a^2-h^2-ah)$
- (c) $\frac{4\pi}{3} h^3$ (d) None of these
27. If for a real number y , $[y]$ is the greatest integer less than or equal to y , then the value of the integral $\int_{\pi/2}^{3\pi/2} [2 \sin x] dx$ is [IIT 1999]
- (a) $-\pi$ (b) 0
- (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$
28. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x) dx = \frac{2A}{\pi}$, then the constants A and B are respectively [IIT 1995]
- (a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$
- (c) $\frac{4}{\pi}$ and 0 (d) 0 and $-\frac{4}{\pi}$
29. If $I_n = \int_0^\infty e^{-x} x^{n-1} dx$, then $\int_0^\infty e^{-\lambda x} x^{n-1} dx =$
- (a) λI_n (b) $\frac{1}{\lambda} I_n$

- (c) $\frac{I_n}{\lambda^n}$ (d) $\lambda^n I_n$
30. $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\lim_{n \rightarrow \infty} n[I_n + I_{n-2}]$ equals [AIEEE 2002]
- (a) $1/2$ (b) 1
(c) ∞ (d) 0
31. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln |x||$ is [AIEEE 2002]
- (a) 4 sq. unit (b) 6 sq. unit
(c) 10 sq. unit (d) None of these
32. $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin x} \, dx$, ($n \in \mathbb{N}$) equals [Kurukshetra CEE 1998]
- (a) $n\pi$ (b) $(2n+1)\frac{\pi}{2}$
(c) π (d) 0
33. If $\int_0^1 e^{x^2} (x - \alpha) \, dx = 0$, then [MNR 1994; Pb. CET 2001; UPSEAT 2000]
- (a) $1 < \alpha < 2$ (b) $\alpha < 0$
(c) $0 < \alpha < 1$ (d) None of these
34. $\int_{\pi}^{10\pi} |\sin x| \, dx$ is [AIEEE 2002]
- (a) 20 (b) 8
(c) 10 (d) 18
35. $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} \, dx$ is [AIEEE 2002]
- (a) $\pi^2/4$ (b) π^2
(c) 0 (d) $\pi/2$
36. On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^x (6 \cos t - 2 \sin t) \, dt =$
- (a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$
(c) Does not exist (d) None of these
37. If $I_1 = \int_0^1 2^{x^2} \, dx$, $I_2 = \int_0^1 2^{x^3} \, dx$, $I_3 = \int_1^2 2^{x^2} \, dx$, $I_4 = \int_1^2 2^{x^3} \, dx$, then [AIEEE 2005]
- (a) $I_3 = I_4$ (b) $I_3 > I_4$
(c) $I_2 > I_1$ (d) $I_1 > I_2$
38. If $2f(x) - 3f\left(\frac{1}{x}\right) = x$, then $\int_1^2 f(x) \, dx$ is equal to [J & K 2005]
- (a) $\frac{3}{5} \ln 2$ (b) $\frac{-3}{5} (1 + \ln 2)$
(c) $\frac{-3}{5} \ln 2$ (d) None of these
39. If $\int_a^b x^3 \, dx = 0$ and $\int_a^b x^2 \, dx = \frac{2}{3}$, then the value of a and b will be respectively [AMU 2005]
- (a) 1, 1 (b) -1, -1
(c) 1, -1 (d) -1, 1
40. The sine and cosine curves intersect infinitely many times giving bounded regions of equal areas. The area of one of such region is [DCE 2005]
- (a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $3\sqrt{2}$ (d) $4\sqrt{2}$