Sample Paper-04

Mathematics
Class – XI

ANSWERS

Section A

1. **Solution:**

$$\frac{3+2i}{1-i}=\frac{3+2i}{1-i}.\frac{1+i}{1-i}$$
$$=\frac{3+2i+3i+2i^2}{1-i^2}=\frac{1}{2}+\frac{5}{2}i$$
$$(1+2i)i-\frac{3+2i}{1-i}=(-2+i)-(\frac{1}{2}+\frac{5}{2}i)=-\frac{5}{2}-\frac{3}{2}i$$

2. **Solution:**

Domain= $[-1,1]$ Range= $[0,\pi]$

3. **Solution :**

$0\le\cos^{-1}x\le\pi$

sin in this interval is positive and hence y is positive

4. **Solution:**

$$\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{7}\right)\right)$$
$$=\sin^{-1}\left(sin\left(\frac{\pi}{7}\right)\right)$$
$$=-\frac{\pi}{2}\le\frac{\pi}{7}\le\frac{\pi}{2}$$
$$=\frac{\pi}{7}$$

5. **Solution:** $(a,2a)$,$(a,-2a)$

6. **Solution:**

$x+7=10$
$x=3$
$x+y=8$
$y=5$

Section B

7. **Solution:**

$\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1$

cos2x + cos4x = 0

2 cos3x cos x = 0

Cos3x = 0

$x=\frac{\pi}{6}+\frac{\pi}{3}n$

$Cosx=0$

$x=\frac{\pi}{2}+\pi k=\frac{\pi}{6}+\frac{\pi}{3}n \quad n \text{ is integer}$

8. **Solution:**

$i^{30}+i^{40}+i^{60}=(i^4)^7.i^2+(i^4)^{10}+(i^4)^{15}$

$i^4=1=-1+1+1=1$

9. **Solution:**

Substituting the points (0, 0) and (5, 5) on the given line

x + y – 8 = 0

0 + 0 – 8 = -8

5 + 5 – 8 = 2

Since the signs of the resulting numbers are different the given points lie on opposite sides of the given line.

10. **Solution :**

$\tan^{-1}x=A$

$\tan A=x$

$\cot^{-1}x=B$

$\cot B=x$

$\tan(\frac{\pi}{2}-B)=x$

$\tan^{-1}x=\frac{\pi}{2}-B$

$\tan^{-1}x=A$

$A=\frac{\pi}{2}-B$

$A+B=\frac{\pi}{2}$

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

11.**Solution:**

$11^{n+2}+12^{2n+1}$ is divisible by 133

n = 1

$11^3+12^3=(11+12)(11^2-11.12+12^2)$

=23.133

Let it be true for k

$11^{k+2}+12^{2k+1}$ is divisible by 133

For k = k + 1

$11^{k+3}+12^{2k+3}=11.11^{k+2}+12^2.12^{2k+1}$

$=11.11^{k+2}+144.12^{2k+1}$

$=11.11^{k+2}+133.12^{2k+1}+11.12^{2k+1}$

$=11.11^{k+2}+11.12^{2k+1}+133.12^{2k+1}$

Is divisible by 133 since $11^{k+2}+12^{2k+1}$ is divisible by 133

12.**Solution:**

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$

$=n(U)-[n(A)+n(B)-n(A\cap B)]$

$800-[200+300-100]$

$=400$

13.**Solution:** $\alpha+\beta=b$

$\alpha\beta=c$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=b^2-2c$

14.**Solution:**

$(x+a)^n=P+Q$

$(x-a)^n=P-Q$

$(P+Q)(P-Q)=(x+a)(x-a)$

$P^2-Q^2=(x^2-a^2)^n$

15.**Solution:**

Discriminant of numerator = 9 – 24 < and

Coefficient of x^2 is positive. Hence Numerator is always positive

Hence dividing by the numerator on both sides of

The equality does not change the sign of the inequality

Hence we need only consider $\frac{1}{3x+4}<0$

$x<\frac{-4}{3}$

$x\in(-\infty,-\frac{4}{3})$

16.**Solution:**

$$\cot(A+15)-\tan(A-15)=\frac{\cos(A+15)}{\sin(A+15)}-\frac{\sin(A-15)}{\cos(A-15)}$$
$$=\frac{\cos(A+15)\cos(A-15)-\sin(A+15)\sin(A-15)}{\sin(A+15)\cos(A-15)}$$
$$=\frac{\cos 2A}{\frac{1}{2}(\sin 2A+\frac{1}{2})}$$
$$=\frac{2\cos 2A}{\sin 2A+\frac{1}{2}}$$
$$=\frac{4\cos 2A}{1+2\sin 2A}$$

17.**Solution:**

$4-x^2\geq 0$

$x^2-4\leq 0$

Domain of $x\in[-2,2]$

$y^2=4-x^2$

$x^2=4-y^2$

$x=\sqrt{4-y^2}$

$4-y^2\geq 0$

$y^2-4\leq 0$

$y\in[-2,2]$

Also for all values of $x\in[-2,2]$

$y=\sqrt{4-x^2}\geq 0$

Range $y\in[0,2]$

18.**Solution:**

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\cos\theta = \frac{1-4}{1+4} = \frac{-3}{5}$$

$$\sin\theta = \frac{2\tan\theta 2}{1+\tan^2\frac{\theta}{2}} = \frac{2.2}{1+4} = \frac{4}{5}$$

$$\frac{1}{2+\cos\theta+\sin\theta} = \frac{1}{2-\frac{3}{5}+\frac{4}{5}} = \frac{11}{5}$$

19.**Solution:**

$$\lim_{x\to 0}\frac{\sin 5x}{x+x^3} = \lim_{x\to 0}\frac{5\sin 5x}{5x(1+x^2)}$$

$$= \lim_{x\to 0}\frac{5\sin 5x}{5x}\lim_{x\to 0}\frac{1}{(1+x^2)}$$

$$= 5.1.1$$

$$= 5$$

Section C

20.**Solution :**

$$y = \log_{10} x$$

$$x = 10^y$$

$$\log_e x = y\log_e 10$$

$$y = \frac{\log_e x}{\log_e 10}$$

$$\frac{dy}{dx} = \left(\frac{1}{\log_e 10}\right)\frac{1}{x}$$

21.**Solution:**

There are 3 even numbers 2, 4, 6

So the units place, 10^{th} places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2 \times 5p_4 = 720$

22.**Solution:**

Let the origin be shifted to (h,k)

$x = x' + h$

$y = y' + k$

Then

$(x' + h)^2 + (y' + k)^2 - 4(x' + h) + 6(y' + k) = 36$

$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x' + h) + 6(y' + k) = 36$

$x'^2 + y'^2 + x'(2h - 4) + y'(2k + 6) + h^2 + k^2 - 4h + 6k - 36 = 0$

$2h - 4 = 0$

$h = 2$

$2k + 6 = 0$

$k = -3$

$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$

$x'^2 + y'^2 + 13 - 62 = 0$

$x'^2 + y'^2 = 49$

23. **Solution:**

$$\frac{2 + 4 + 12 + 14 + 11 + x + y}{7} = 8$$

$43 + x + y = 56$

$x + y = 13$

$$\frac{2^2 + 4^2 + 12^2 + 14^2 + 11^2 + x^2 + y^2}{7} - (mean)^2 = 19$$

$$\frac{4 + 16 + 144 + 196 + 121 + x^2 + y^2}{7} - 64 = 19$$

$$\frac{481 + x^2 + y^2}{7} = 83$$

$481 + x^2 + y^2 = 581$

$x^2 + y^2 = 100$

$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$169 + (x - y)^2 = 200$

$(x - y)^2 = 31$

$x - y = 5.57$

$x + y = 13$

$x = 9.285$

$y = 3.715$

24. **Solution:**

$$\frac{1}{\log_a b} = \log_b a$$

$$\frac{1}{\log_{2a} b} = \log_b 2a$$

$$\frac{1}{\log_{4a} b} = \log_b 4a$$

$$\frac{\log_b a + \log_b 4a}{2} = \frac{\log_b (2a)^2}{2}$$

$$= 2\frac{\log_b 2a}{2}$$

$$= \log_b 2a$$

Thus, $\frac{1}{\log_{2a} b}$ is, the, AM, between $\frac{1}{\log_a b}, \frac{1}{\log_{4a} b}$

25.**Solution:**

Probability of surviving = $\frac{9}{10}$

Required to find out the probability of 4 are safe or 5 are safe

Probability of 5 is safe = $\left(\frac{9}{10}\right)^5$

Probability of 4 is safe = ${}^5C_4\left(\frac{9}{10}\right)^4\frac{1}{10}$

Required Probability = $\left(\frac{9}{10}\right)^5 + 5\left(\frac{9}{10}\right)^4\frac{1}{10} = \frac{45927}{5000}$

26.**Solution:**

$T_{2r+1} = {}^{40}C_{2r}$

$T_{r+2} = {}^{40}C_{r+1}$

${}^{40}C_{2r} = {}^{40}C_{r+1}$

$2r + r + 1 = 40$

$3r = 39$

$r = 13$