MODEL TEST PAPER – I

Time : 3 hours **Maximum Marks : 100**

General Instructions :

(i) All questions are compulsory.

(ii) Q. 1 to Q. 10 of Section A are of 1 mark each.

(iii) Q. 11 to Q. 22 of Section B are of 4 marks each.

(iv) Q. 23 to Q. 29 of Section C are of 6 marks each.

(v) There is no overall choice. However an internal choice has been provided in some questions.

SECTION A

1. A = {1, 2, 3, 4, 5, 6}, B = {2, 3, 5, 7, 9}

   U = {1, 2, 3, 4, ...10}, Write (A – B)´

2. Express $(1 - 2i)^{-2}$ in the standard form a + ib.

3. Find 20th term from end of the A.P. 3, 7, 11, ... 407.

4. Evaluate $5^2 + 6^2 + 7^2 + ... + 20^2$

5. Evaluate $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x}}{x}$

6. Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x + x^2} - 1}{x}$

7. A bag contains 9 red, 7 white and 4 black balls. If two balls are drawn at random, find the probability that both balls are red.

8. What is the probability that an ordinary year has 53 Sundays?

9. Write the contrapositive of the following statement :

   "it two lines are parallel, then they do not intersect in the same plane."

10. Check the validity of the compound statement "80 is a multiple of 5 and 4."

SECTION B

11. Find the derivative of $\frac{\sin x}{x}$ with respect to x from first principle.

OR

Find the derivative of $\frac{\sin x - x \cos x}{x \sin x + \cos x}$ with respect to x.

12. Two students Ajay and Aman appeared in an interview. The probability that Ajay will qualify the interview is 0.16 and that Aman will quality the interview is 0.12. The probability that both will qualify is 0.04. Find the probability that—

    (a) Both Ajay and Aman will not qualify.

    (b) Only Aman qualifies.

13. Find domain and range of the real function $f(x) = \frac{3}{1 - x^2}$

14. Let R be a relation in set A = {1, 2, 3, 4, 5, 6, 7} defined as R = {(a, b): a divides b, a ≠ b}. Write R in Roster form and hence write its domain and range.

OR

Draw graph of $f(x) = 2 + |x - 1|$.

15. Solve : $\sin^2 x - \cos x = \frac{1}{4}$.

16. Prove that $\cos 2\theta . \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 5\theta \sin \frac{5\theta}{2}$.

17. If x and y are any two distinct integers, then prove by mathematical induction that $x^n - y^n$ is divisible by $(x - y)$ $\forall n \in N$.

18. If $x + iy = (a + ib)^{1/3}$, then show that $\frac{a}{x} + \frac{b}{y} = 4\left(x^2 - y^2\right)$

OR

Find the square roots of the complex number 7 – 24i

19. Find the equation of the circle passing through points (1, –2) and (4, -3) and has its centre on the line $3x + 4y = 7$.

OR

The foci of a hyperbola coincide with of the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola, if its eccentricity is 2.

20. Find the coordinates of the point, at which yz plane divides the line segment joining points (4, 8, 10) and (6, 10, –8).

21. How many words can be made from the letters of the word 'Mathematics', in which all vowels are never together.

22. From a class of 20 students, 8 are to be chosen for an excusion party. There are two students who decide that either both of them will join or none of the two will join. In how many ways can they be choosen?

SECTION C

23. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had taken

    (i) atleast one of the three subjects,

    (ii) only one of the three subjects.

24. Prove that $\cos^3 A + \cos^3\left(\frac{2\pi}{3} + A\right) + \cos^3\left(\frac{4\pi}{3} + A\right) = \frac{3}{4}\cos 3A$.

25. Solve the following system of inequations graphically

$$x + 2y \le 40,\ 3x + y \ge 30,\ 4x + 3y \ge 60,\ x \ge 0,\ y \ge 0$$

OR

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

26. Find n, it the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left[\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right]^n$ is $\sqrt{6} : 1$.

27. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio $\left(3+2\sqrt{2}\right) : \left(3-2\sqrt{2}\right)$.

28. Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

29. Calculate mean and standard deviation for the following data

Age	***Number of persons***
20 – 30	3
30 – 40	51
40 – 50	122
50 – 60	141
60 – 70	130
70 – 80	51
80 – 90	2

OR

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that an observation 12 was misread as 8. Calculate correct mean and correct standard deviation.

MODEL TEST PAPER – I

Time : 3 hours **Maximum Marks : 100**

SOLUTIONS AND MARKING SCHEME

SECTION A

Note : For 1 mark questions in Section A, full marks are given if answer is correct (i.e. the last step of the solution). Here, solution is given for your help.

Marks

1. $A - B = \{1, 4, 6\}$

   $(A - B)^c = \{2, 3, 5, 7, 8, 9, 10\}$ ...(1)

2. $(1-2i)^{-2} = \dfrac{1}{(1-2i)^2}$

   $= \dfrac{1}{1+4i^2-4i} = \dfrac{1}{-3-4i} \times \dfrac{-3+4i}{-3+4i}$

   $= \dfrac{-3+4i}{9-16i^2}$

   $= \dfrac{-3}{25} + \dfrac{4}{25}i$ ...(1)

3. The given A.P. can be written in reverse order as 407, 403, 399, ...

   Now 20th term $= a + 19d$

   $= 407 + 19 \times (-4)$

   $= 407 - 76$

   $= 331$ ...(1)

4. $5^2 + 6^2 + 7^2 + \ldots.. + 20^2$

   $= \sum_{r=1}^{20} r^2 - \sum_{k=1}^{4} k^2$ $\because \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Marks

$$= \frac{20 \times 21 \times 41}{6} - \frac{4 \times 5 \times 9}{6}$$

$= 2870 - 30 = 2840$ ...(1)

5. $\lim_{x \to 0} \left(\frac{e^x - e^{-x}}{x} \right)$

$$= \lim_{x \to 0} \left(\frac{e^{2x} - 1}{e^x x} \right) \times \frac{2}{2}$$

$= 2$ $\quad \because \lim_{x \to 0} \frac{e^x - 1}{x} = 1$ ...(1)

6. $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$

$$= \lim_{x \to 0} \frac{x + x + x^2 - \cancel{1}}{x\left(\sqrt{1 + x + x^2} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x + 1}{\sqrt{1 + x + x^2} + 1} = \frac{1}{2}$$ ...(1)

7. Required Probability $= \frac{{}^9C_2}{{}^{20}C_2} = \frac{36}{190} = \frac{18}{95}$ ...(1)

8. 365 days = (7 × 52 + 1) days

After 52 weeks 1 day can be Sunday or Monday or .... Saturday. i.e., (7 cases)

P (53 Sundays) $= \frac{1}{7}$. ...(1)

9. If two lines intersect in same plane then they are not parallel. ...(1)

10. 5 and 4 both divide 80.

So, given statement is true. ...(1)

SECTION B

11. By definition,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$ ...(1)

Marks

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}\right)$$

$$= \lim_{h\to 0} \frac{x\sin(x+h) - (x+h)\sin x}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x[\sin(x+h) - \sin x] - h\sin x}{hx(x+h)}$$ ...(1)

$$= \lim_{h\to 0}\left[\frac{\cancel{x}\,\cancel{2}\cos\left(x+\frac{h}{2}\right).\sin\left(\frac{h}{2}\right)}{\cancel{x}(x+h)\frac{h}{2}\times\cancel{2}} - \frac{\sin x}{x(x+h)}\right]$$ ...(1)

$$= \frac{\cos x}{x} - \frac{\sin x}{x^2} = \frac{x\cos x - \sin x}{x^2}$$ ...(1)

OR

$$\frac{d}{dx}\left(\frac{\sin x - x\cos x}{x\sin x + \cos x}\right)$$

$$= \frac{(x \sin x + \cos x)\,(\cancel{\cos x} + x \sin x - \cancel{\cos x}) - (\sin x - x\cos x)\,(x\cos x + \cancel{\sin x} - \cancel{\sin x})}{(x \sin x + \cos x)^2}$$ ...(2)

$$= \frac{x^2 \sin^2 x + x\,\cancel{\sin x}\cos x - x\,\cancel{\sin x}\cos x + x^2\cos^2 x}{(x\sin x + \cos x)^2}$$ ...(1)

$$= \frac{x^2}{(x\sin x + \cos x)^2}$$ ...(1)

12. Let A = Event that Ajay will qualify.

B = Event that Aman will qualify.

Then P(A) = 0.16, P(B) = 0.12, P $(A \cap B)$ = 0.04 ...(1)

Now

(a) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Marks

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.16 + 0.12 - 0.04)$

$= 1 - 0.24 = 0.76$ ...(1½)

(b) $P(B \cap A^c) = P(B) - P(A \cap B)$

$= 0.12 - 0.04$

$= 0.08$ ...(1½)

13. $f(x) = \dfrac{3}{1-x^2}$

Clearly, f(x) is not defined for $x^2 = 1$ i.e., $x = \pm 1$

So, $D_f = R - \{-1, 1\}$ 2

For Range, Let $y = \dfrac{3}{1-x^2}$ then $y \neq 0$

$\Rightarrow \quad 1 - x^2 = \dfrac{3}{y}$

$\Rightarrow \quad x^2 = 1 - \dfrac{3}{y} = \dfrac{y-3}{y}$

$x = \pm\sqrt{\dfrac{y-3}{y}}$ ...(1)

for $x \in D_f$, $\quad \dfrac{y-3}{y} \geq 0$

$y - 3 \geq 0, y > 0$ or $y - 3 \leq 0, y < 0$

$y \geq 3, y > 0$ $\quad \Rightarrow y < 0$

$\Rightarrow y \geq 3.$

$\therefore R_f = (-\infty, 0) \cup [3, \infty)$ ...(1)

14. R = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 4) (2, 6), (3, 6)}...(2)

Domain = {1, 2, 3} ...(1)

Marks

Range = {2, 3, 4, 5, 6, 7} ...(1)

OR

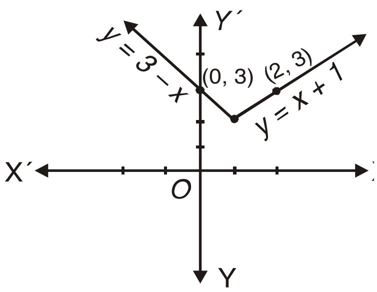
$f(x) = 2 + |x - 1|$

when $x \geq 1$, $f(x) = 2 + x - 1 = x + 1$

when $x < 1$, $f(x) = 2 + 1 - x = 3 - x$ ...(2)

x	1	2	0	–1	–2
y	2	3	3	4	5

...(1)



...(1)

15. $\sin^2 x - \cos x = \dfrac{1}{4}$

$\Rightarrow \quad 1 - \cos^2 x - \cos x = \dfrac{1}{4}$

$\Rightarrow \quad 4 - 4\cos^2 x - 4\cos x = 1$ ...(1)

$\Rightarrow \quad 4\cos^2 x + 4\cos x - 3 = 0$

$\Rightarrow \quad (2\cos x + 3)(2\cos x - 1) = 0$ ...(1)

$\Rightarrow \quad \cos x = -3/2, \qquad \cos x = 1/2 = \cos(\pi/3)$

Impossible $\qquad x = 2n\pi \pm \pi/3,\ n \in Z$ ...(2)

16. L.H.S. $= \cos 2\theta \cos\dfrac{\theta}{2} - \cos 3\theta \cos\dfrac{9\theta}{2}$

$$= \frac{1}{2}\left[2\cos 2\theta \cos\frac{\theta}{2} - 2\cos 3\theta \cos\frac{9\theta}{2}\right]$$ ...(1)

Marks

$$= \frac{1}{2}\left[\cos\left(2\theta + \frac{\theta}{2}\right) + \cos\left(2\theta - \frac{\theta}{2}\right) - \cos\left(3\theta + \frac{9\theta}{2}\right) - \cos\left(3\theta - \frac{9\theta}{2}\right)\right]$$ ...(1)

$$= \frac{1}{2}\left[\cos\frac{5\theta}{2} + \cancel{\cos\frac{3\theta}{2}} - \cos\frac{15\theta}{2} - \cancel{\cos\left(\frac{3\theta}{2}\right)}\right] \quad \because \cos(-\theta) = \cos\theta$$

$$= \frac{1}{2}\left[-2\sin\left(\frac{\frac{5\theta}{2} + \frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{5\theta}{2} - \frac{15\theta}{2}}{2}\right)\right]$$ ...(1)

$$= -\sin 5\theta . \sin\left(-\frac{5\theta}{2}\right)$$

$$= \sin(5\theta)\sin\left(\frac{5\theta}{2}\right) = \text{R.H.S.}$$ ...(1)

17. P(n) : $x^n - y^n$ is divisible by $(x - y)$

P(1) : $x - y$ is divisible by $(x - y)$.

This is true.

Hence P(1) is true. ...(1)

Let us assume that P(k) be true for some natural number k.

i.e., $x^k - y^k$ is divisible by $x - y$.

So, $x^k - y^k = t(x - y)$ where t is an integer. ...(1)

Now we want to prove that P(k + 1) is also true.

i.e., $x^{k+1} - y^{k+1}$ is divisible by $x - y$.

Now $x^{k+1} - y^{k+1}$

$= x . x^k - y . y^k$

$= x [t(x - y) + y^k] - y . y^k$ using (i).

$= tx (x - y) + (x - y) y^k$.

Marks

$= (x - y)(tx + y^k)$

$= (x - y) \, . \, m$ where $m = tx + y^k$ is an integer.

So, $x^{k+1} - y^{k+1}$ is divisible by $(x - y)$

i.e., P(k + 1) is true whenever P(k) is true.

Hence by P.M.I., P(n) is true $\forall \, n \in N$. ...(2)

18. $x + iy = (a + ib)^{1/3}$

$\Rightarrow \quad (x + iy)^3 = a + ib$

$\Rightarrow \quad x^3 + i^3 y^3 + 3xyi(x + iy) = a + ib$ ...(1)

$\Rightarrow \quad x^3 - iy^3 + 3x^2yi - 3xy^2 = a + ib$

$\Rightarrow \quad (x^3 - 3xy^2) + i(3x^2y - y^3) = a + ib$ ...(1)

Comparing real and imaginary parts,

$x(x^2 - 3y^2) = a$ and $y(3x^2 - y^2) = b$

$x^2 - 3y^2 = \frac{a}{x}$ (i) $\qquad 3x^2 - y^2 = \frac{b}{y}$ (ii) ...(1)

Adding (i) and (ii) we get.

$4(x^2 - y^2) = \frac{a}{x} + \frac{b}{y}$ ...(1)

OR

Let the square root of $7 - 24i$ be $x + iy$

Then $\sqrt{7 - 24i} = x + iy$

$\Rightarrow \quad 7 - 24i = x^2 - y^2 + 2xyi$ ...(1)

Comparing real and imaginary parts.

$x^2 - y^2 = 7$ (i), $\qquad xy = -12$ (ii) ...(1)

Marks

We know that

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$\Rightarrow \quad (x^2 + y^2)^2 = 49 + 4\ (144)$

$$x^2 + y^2 = 25 \qquad \text{(iii)}$$

Solving (i), (ii) we get $x = \pm 4$, $y = \pm 3$ ...(1)

From equation (ii) we conclude that $x = 4$, $y = -3$ and $x = -4$, $y = 3$.

Required square roots are,

$4 - 3i$ and $-4 + 3i$ ...(1)

19. Let the equation of circle be,

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{(i)}$$

$\because \quad (1, -2)$ and $(4, -3)$ lie on (i).

So, $\quad (1 - h)^2 + (-2 - k)^2 = r^2$

and $\quad (4 - h)^2 + (-3 - k)^2 = r^2$ ...(1)

So, equating value of r^2, we get.

$$1 + h^2 - 2h + 4 + k^2 + 4k = 16 + h^2 - 8h + 9 + k^2 + 6k$$

$\Rightarrow \quad 6h - 2k = 20$

$$3h - k = 10 \qquad \text{(ii)}$$

As centre lies on $3x + 4y = 7$

So, $\quad 3h + 4k = 7 \qquad$ (iii) ...(1)

Solving (ii) and (iii) we get

$$k = \frac{-3}{5}, h = \frac{47}{15}$$ ...(1)

So, $\quad r = \dfrac{\sqrt{1465}}{15} \qquad$ Put in (i)

Marks

Hence required equation is

$15x^2 + 15y^2 - 94x + 18y + 55 = 0$ ...(1)

OR

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a = 5, b = 3 \quad \Rightarrow \quad \sqrt{a^2 - b^2} = c = 4$

$\Rightarrow$ foci of ellipse is (±4, 0) ...(1)

So, foci of required hyperbola are (±4, 0)

Distance between foci = 2ae = 8

$\because \quad e = 2, \quad a = 2$ ...(1)

Using $b^2 = a^2 (e^2 - 1)$

$\Rightarrow \quad b^2 = 4 (4 - 1) = 12$ ...(1)

Hence equation of hyperbola is,

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$ ...(1)

20. Let yz plane divides the line joining A(4, 8, 10) and B(6, 10, –8) in the ratio λ : 1. So by section formula, the point of intersection is

$$R\left(\frac{6\lambda+4}{\lambda+1}, \frac{10\lambda+8}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1}\right)$$ ...(1)

Because this point lies on yz plane i.e., x = 0

So, $\dfrac{6\lambda+4}{\lambda+1} = 0$

$\Rightarrow \quad \lambda = -2/3.$ ...(1)

$\therefore$ Ratio = 2 : 3 externally.

$\therefore \quad$ R(0, 4, 46) ...(2)

Marks

21. 'MATHEMATICS'

Vowels in above word = A, A, E, I

Consonants in above word = M, M, T, T, C, S, H

Total arrangements of letters of above word

$$= \frac{11!}{2!\ 2!\ 2!} = \frac{10 \times 11 \times 9 \times \cancel{8} \times 7 \times 720}{\cancel{8}}$$

= 990 × 5040

= 4989600 ...(2)

Consider all the vowels as one letter. Now we have 8 letters, which can be arranged in $\frac{8!}{2!\ 2!}$ ways. Vowels can be arranged among themselves in $\frac{4!}{2!}$ ways. Total arrangements when all vowels are always together

$$= \frac{8!}{2!\ 2!} \times \frac{4!}{2!}$$

$$= \frac{\cancel{8} \times 7 \times 6 \times 120 \times 24}{\cancel{8}} = 1{,}20{,}960$$ ...(1)

The number of arrangements when all the vowels never come together

= 4989600 − 120960

= 4868640. ...(1)

22. **Case I :** If 2 particular students always join party then remaining 6 out of 18 can be choosen in ${}^{18}C_6$ ways. ...(1½)

Case II : If 2 particular students always do not join the excursion party then selection of 8 students out of 18 can be done in ${}^{18}C_8$ ways.

So, Required number of ways ...(1½)

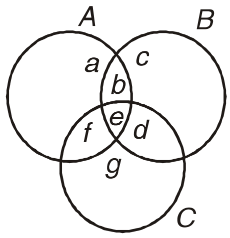
$$= {}^{18}C_6 + {}^{18}C_8$$

= 62322 ...(1)

Marks

SECTION C

23. Let A, B, C denote the sets of those students who take Maths, Physics, Chemistry respectively. ...(1)



...(1)

By given condition,

$$a + b + e + f = 15$$

$$b + c + e + d = 12, \; f + e + d + g = 11$$

$$e + f = 5, \; b + e = 9, \; e + d = 4, \; e = 3 \qquad ...(1)$$

Solving above equations, we obtain.

$$e = 3, \; d = 1, \; b = 6, \; f = 2, \; g = 5, \; c = 2, \; a = 4 \qquad ...(1)$$

(i) No. of students who had taken atleast one of the three subjects $= n\,(A \cup B \cup C)$

$$= a + b + c + d + e + f + g$$

$$= 23. \qquad ...(1)$$

(ii) No. of Students who had taken only one of the three subjects

$$= a + c + g$$

$$= 4 + 2 + 5 = 11 \qquad ...(1)$$

24. $\cos 3x = 4 \cos^3 x - 3 \cos x$

$\Rightarrow \quad 4 \cos^3 x = \cos 3x + 3 \cos x \qquad$ (i) $\qquad$ ...(1)

Using (i)

$$\cos^3 A = \frac{1}{4} \cos 3A + \frac{3}{4} \cos A$$

Marks

$$\cos^3\left(\frac{2\pi}{3}+A\right)=\frac{1}{4}\cos\left(3\left(\frac{2\pi}{3}+A\right)\right)+\frac{3}{4}\cos\left(\frac{2\pi}{3}+A\right)$$

$$\cos^3\left(\frac{4\pi}{3}+A\right)=\frac{1}{4}\cos(4\pi+3A)+\frac{3}{4}\cos\left(\frac{4\pi}{3}+A\right) \quad \text{...(1)}$$

Now L.H.S. of given result becomes

$$=\frac{1}{4}[\cos 3A+\cos(2\pi+3A)+\cos(4\pi+3A)]$$
$$+\frac{3}{4}\left[\cos A+\cos\left(\frac{2\pi}{3}+A\right)+\cos\left(\frac{4\pi}{3}+A\right)\right] \quad \text{...(1)}$$

$$=\frac{3}{4}\cos 3A+\frac{3}{4}\left[\cos A+2\cos(\pi+A)\cos\left(-\frac{\pi}{3}\right)\right] \quad \text{...(1)}$$

$$=\frac{3}{4}\cos 3A+\frac{3}{4}\left[\cos A-\cancel{2}\times\frac{1}{\cancel{2}}\cos A\right] \quad \text{...(1)}$$

$$=\frac{3}{4}\cos 3A=\text{R.H.S.} \quad \text{...(1)}$$

25. $x + 2y \le 40$ ...(i), $3x + y \ge 30$ ...(ii), $4x + 3y \ge 60$ ...(iii), $x, y \ge 0$.
...(1)

The corresponding equations are

$x + 2y = 40$

x	0	40
y	20	0

$3x + y = 30$

x	10	0
y	0	30

$4x + 3y = 60$

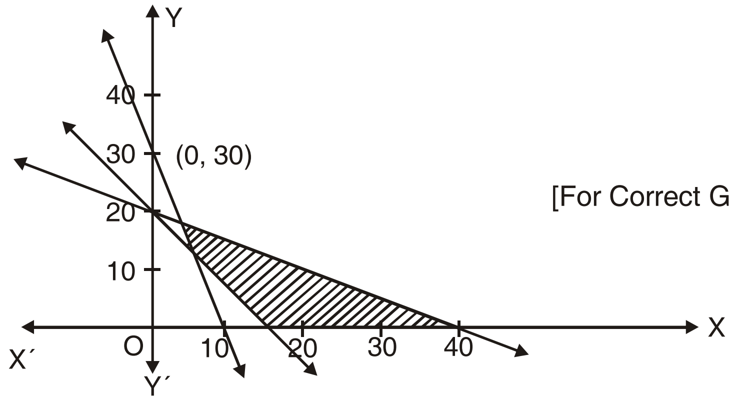
x	0	15
y	20	0

...(2)

Putting $x = 0 = y$ in (i), (ii), (iii) we get result True, false, false respectively. So, the shades will be made accordingly.

$x, y \ge 0$ shows I quadrant.

Marks



...(3)
[For Correct Graph]

Common shaded portion is required solution set.

OR

Quantity of 12% acid solution = 600 litres.

Quantity of acid $= 600 \times \frac{12}{100} = 72$ litres. ...(1)

Let x litres of 30% acid solution be mixed. Then according to given question. ...(1)

$$15\% \text{ of } (600 + x) < 72 + \frac{30}{100} \times x < 18\% \text{ of } (600 + x)$$ ...(1)

$\Rightarrow$ 15 (600 + x) < 7200 + 30x < 18 (600 + x)

9000 + 15x < 7200 + 30x, 7200 + 30x < 10800 + 18x

15x > 1800 , 12x < 3600

x > 120 , x < 300 ...(2)

So, 120 < x < 300

So, 30% acid solution must be between 120 litres and 300 litres. ...(1)

26. 5th term from beginning in $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$

$$= {}^nC_4 2^{\frac{n-4}{4}} . \left(\frac{1}{3}\right)$$ ...(1½)

Marks

5th term from the end in $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{2}}\right)^n$ is

$$= {}^nC_4\,(2)\,.\left(\frac{1}{3}\right)^{\frac{n-4}{4}} \qquad ...(1½)$$

According to given question,

$$\frac{{}^nC_4\,2^{\frac{n-4}{4}}.\left(\frac{1}{3}\right)}{{}^nC_4\,(2)\,.\left(\frac{1}{3}\right)^{\frac{n-4}{4}}} = \frac{\sqrt{6}}{1} \qquad ...(1½)$$

$$2^{\frac{n-4}{4}-1} \times 3^{\frac{n-4}{4}-1} = \sqrt{6}$$

$$6^{\frac{n-8}{4}} = 6^{1/2}$$

$$\Rightarrow \qquad \frac{n-8}{4} = \frac{1}{2} \qquad \Rightarrow \qquad n = 10 \qquad ...(1½)$$

27. Let the numbers be a and b.

So, $\quad a + b = 6\sqrt{ab}$ ...(1)

$$a^2 + b^2 + 2ab = 36\ ab$$

$$\left(\frac{a}{b}\right)^2 - 34\left(\frac{a}{b}\right) + 1 = 0 \qquad ...(1½)$$

$$\Rightarrow \qquad \frac{a}{b} = \frac{34 \pm \sqrt{1156 - 4 \times 1 \times 1}}{2}$$

$$= \frac{34 \pm 24\sqrt{2}}{2} = 17 \pm 12\sqrt{2} \qquad ...(1½)$$

So, $\quad \frac{a}{b} = \frac{17+12\sqrt{2}}{1}$ taking +ve sign

$$= \frac{\left(3+2\sqrt{2}\right)^2}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

Marks

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

So, $a : b = (3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$ ...(2)

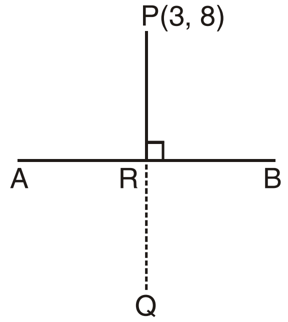
28. Slope of given line = –1/3

Slope of line PQ = 3

Equation of line PQ is

$$y - 8 = 3(x - 3)$$

$\Rightarrow$ $y = 3x - 1$ ...(1)



...(1)

Solving equations of AB and PQ we get coordinates of R (foot of perpendicular)

So, R (1, 2) ...(2)

Let Q (x', y') be image of P.

then as R is mid point of PQ. We have,

$$\frac{x'+3}{2} = 1 \quad \text{and} \quad \frac{y'+8}{2} = 2$$

$\Rightarrow$ $x' = -1$ $y' = -4$

$\therefore$ Q (–1, –4) ...(2)

Marks

29.

C.I.	x (mid values)	f	$u = \frac{x-A}{i}$	fu	fu^2
20-30	25	3	–3	–9	27
30-40	35	51	–2	–102	204
40-50	45	122	–1	–122	122
50-60	55 A	141	0	0	0
60-70	65	130	1	130	130
70-80	75	51	2	102	204
80-90	85	2	3	6	18
		$\Sigma f = 500$		**$\Sigma fu = 5$**	**$\Sigma fu^2 = 705$**

...(2)

$$\bar{x} = A + \frac{\Sigma fu}{\Sigma f} \times i$$

$$= 55 + \frac{5}{500} \times 10 = 55.1 \qquad ...(1)$$

$$\text{S.D.} = \sigma = i \times \sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{1}{N}\Sigma fu\right)^2} \qquad ...(1)$$

$$= 10\sqrt{\frac{1}{500}(705) - \left(\frac{5}{500}\right)^2}$$

$$= 10\sqrt{1.41 - 0.0001} = \sqrt{1.4099} \times 10$$

$$= 11.874 \qquad ...(2)$$

OR

$N = 20, \quad \bar{x} = 10, \quad \sigma = 2$

Using $\bar{x} = \frac{\Sigma x}{N}$ ...(1)

$\Rightarrow$ Incorrect $\Sigma x = 10 \times 20 = 200$

Correct $\Sigma x = 200 + 12 - 8 = 204$

Marks

$$\text{Correct Mean} = \frac{204}{20} = 10.2 \qquad \text{...(1½)}$$

Using $$\sigma^2 = \frac{1}{N}\Sigma x^2 - (\bar{x})^2 \qquad \text{...(1)}$$

$$4 = \frac{1}{20}\Sigma x^2 - (10)^2$$

$\Rightarrow$ Incorrect $\Sigma x^2 = 2080$

Correct $\Sigma x^2 = 2080 + (12)^2 - (8)^2$

$= 2160$ ...(1)

Correct S.D.

$$= \sqrt{\frac{1}{N}\Sigma x^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{1}{20}(2160) - (10.2)^2}$$

$$= \sqrt{108 - 104.04} = \sqrt{3.96} = 1.99. \qquad \text{...(1½)}$$