SHORT REVISION

1. **DEFINITIONS:** **A VECTOR** may be described as a quantity having both magnitude & direction. A vector is generally represented by a directed line segment, say $\vec{AB}$. A is called the **initial point** & B is called the **terminal point**. The magnitude of vector $\vec{AB}$ is expressed by $|\vec{AB}|$.

ZERO VECTOR a vector of zero magnitude i.e.which has the same initial & terminal point, is called a **ZERO VECTOR**. It is denoted by O.

UNIT VECTOR a vector of unit magnitude in direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$ symbolically $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$.

EQUAL VECTORS two vectors are said to be equal if they have the same magnitude, direction & represent the same physical quantity.

COLLINEAR VECTORS two vectors are said to be collinear if their directed line segments are parallel disregards to their direction. Collinear vectors are also called **PARALLEL VECTORS**. If they have the same direction they are named as like vectors otherwise unlike vectors.

Simbolically, two non zero vectors $\vec{a}$ and $\vec{b}$ are collinear if and only if, $\vec{a}=K\vec{b}$, where $K \in R$

COPLANAR VECTORS a given number of vectors are called coplanar if their line segments are all parallel to the same plane. Note that **"TWO VECTORS ARE ALWAYS COPLANAR"**.

POSITION VECTOR let O be a fixed origin, then the position vector of a point P is the vector $\vec{OP}$. If $\vec{a}$ & $\vec{b}$ & position vectors of two point A and B, then ,

$\vec{AB} = \vec{b}-\vec{a}$ = pv of B − pv of A.

2. **VECTOR ADDITION :** ☞ If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\vec{OA}$ & $\vec{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\vec{OC}$, where OC is the diagonal of the parallelogram OACB.

☞ $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative) ☞ $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (associativity)

☞ $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$ ☞ $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$

3. **MULTIPLICATION OF VECTOR BY SCALARS :**

If $\vec{a}$ is a vector & m is a scalar, then $m\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$. This multiplication is called **SCALAR MULTIPLICATION**. If $\vec{a}$ & $\vec{b}$ are vectors & m, n are scalars, then:

$m(\vec{a})=(\vec{a})m=m\vec{a}$ $\quad$ $m(n\vec{a})=n(m\vec{a})=(mn)\vec{a}$

$(m+n)\vec{a}=m\vec{a}+n\vec{a}$ $\quad$ $m(\vec{a}+\vec{b})=m\vec{a}+m\vec{b}$

4. **SECTION FORMULA :**

If $\vec{a}$ & $\vec{b}$ are the position vectors of two points A & B then the p.v. of a point which divides AB in the ratio m : n is given by : $\vec{r}=\frac{n\vec{a}+m\vec{b}}{m+n}$. Note p.v. of mid point of AB = $\frac{\vec{a}+\vec{b}}{2}$.

5. **DIRECTION COSINES :**

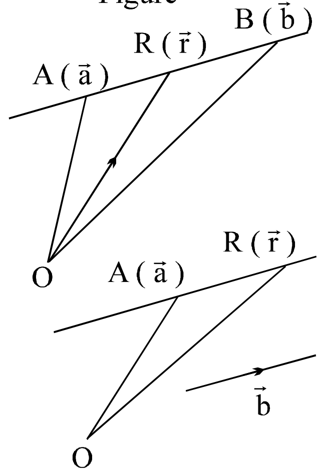
Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ the angles which this vector makes with the +ve directions OX,OY & OZ are called **DIRECTION ANGLES** & their cosines are called the **DIRECTION COSINES** .

$\cos\alpha=\frac{a_1}{|\vec{a}|}$, $\cos\beta=\frac{a_2}{|\vec{a}|}$, $\cos\Gamma=\frac{a_3}{|\vec{a}|}$. **Note that, $\cos^2\alpha+\cos^2\beta+\cos^2\Gamma=1$**

6. **VECTOR EQUATION OF A LINE :**

Parametric vector equation of a line passing through two point $A(\vec{a})$ & $B(\vec{b})$ is given by, $\vec{r}=\vec{a}+t(\vec{b}-\vec{a})$ where t is a parameter. If the line passes through the point $A(\vec{a})$ & is parallel to the vector $\vec{b}$ then its equation is, $\vec{r}=\vec{a}+t\vec{b}$

Figure



Note that the equations of the bisectors of the angles between the lines $\vec{r} = \vec{a} + \lambda\,\vec{b}$ & $\vec{r} = \vec{a} + \mu\,\vec{c}$ is :
$\vec{r} = \vec{a} + t\left(\hat{b}+\hat{c}\right)$ & $\vec{r} = \vec{a} + p\left(\hat{c}-\hat{b}\right)$.

7. **TEST OF COLLINEARITY :** Three points A,B,C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively are collinear, if & only if there exist scalars x, y, z not all zero simultaneously such that ; $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where $x + y + z = 0$.

8. **SCALAR PRODUCT OF TWO VECTORS :**

☞ $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta (0 \le \theta \le \pi)$,

note that if θ is acute then $\vec{a}.\vec{b} > 0$ & if θ is obtuse then $\vec{a}.\vec{b} < 0$

☞ $\vec{a}.\vec{a} = |\vec{a}|^2 = \vec{a}^2, \vec{a}.\vec{b} = \vec{b}.\vec{a}$ (commutative) ☞ $\vec{a}\,.\,(\vec{b} + \vec{c}) = \vec{a}\,.\vec{b} + \vec{a}\,.\vec{c}$ (distributive)

☞ $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ $(\vec{a} \ne 0 \quad \vec{b} \ne 0)$

☞ $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$; $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$ ☞ projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$.

Note: That vector component of $\vec{a}$ along $\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{\vec{b}^2}\right)\vec{b}$ and perpendicular to $\vec{b} = \vec{a} - \left(\dfrac{\vec{a}\cdot\vec{b}}{\vec{b}^2}\right)\vec{b}$.

☞ the angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ $0 \le \phi \le \pi$

☞ if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$|\vec{a}| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$, $|\vec{b}| = \sqrt{b_1^{\,2} + b_2^{\,2} + b_3^{\,2}}$

Note : (i) Maximum value of $\vec{a}\,.\,\vec{b} = |\,\vec{a}\,|\,|\,\vec{b}\,|$

(ii) Minimum values of $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{b} = -\,|\,\vec{a}\,|\,|\,\vec{b}\,|$

(iii) Any vector $\vec{a}$ can be written as , $\vec{a} = \left(\vec{a}\,.\,\hat{i}\right)\hat{i} + \left(\vec{a}\,.\,\hat{j}\right)\hat{j} + \left(\vec{a}\,.\,\hat{k}\right)\hat{k}$.

(iv) A vector in the direction of the bisector of the angle between the two vectors $\vec{a}\,\&\,\vec{b}$ is $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$. Hence bisector of the angle between the two vectors $\vec{a}\,\&\,\vec{b}$ is $\lambda\left(\hat{a} + \hat{b}\right)$, where $\lambda \in R^+$. Bisector of the exterior angle between $\vec{a}\,\&\,\vec{b}$ is $\lambda\left(\hat{a} - \hat{b}\right)$, $\lambda \in R^+$.

9. **VECTOR PRODUCT OF TWO VECTORS :**

(i) ☞ If $\vec{a}\,\&\,\vec{b}$ are two vectors & θ is the angle between them then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\vec{n}$, where $\vec{n}$ is the unit vector perpendicular to both $\vec{a}\,\&\,\vec{b}$ such that $\vec{a}\,,\vec{b}\,\&\,\vec{n}$ forms a right handed screw system .

(ii) ☞ Lagranges Identity : for any two vectors $\vec{a}$ & $\vec{b}$; $(\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

(iii) ☞ Formulation of vector product in terms of scalar product:

The vector product $\vec{a}\times\vec{b}$ is the vector $\vec{c}$, such that

(i) $|\vec{c}| = \sqrt{\vec{a}^2\vec{b}^2 - (\vec{a}\cdot\vec{b})^2}$ (ii) $\vec{c}\cdot\vec{a} = 0$; $\vec{c}\cdot\vec{b} = 0$ and

(iii) $\vec{a}, \vec{b}, \vec{c}$ form a right handed system

(iv) ☞ $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a}$ & $\vec{b}$ are parallel (collinear) $(\vec{a}\neq 0, \vec{b}\neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar.

☞ $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ (not commutative)

☞ $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})$ where m is a scalar .

☞ $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$ (distributive)

☞ $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ ☞ $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

(v) ☞ If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(vi) ☞ Geometrically $|\vec{a}\times\vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$.

(vii) ☞ Unit vector perpendicular to the plane of $\vec{a}$ & $\vec{b}$ is $\hat{n} = \pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

☞ A vector of magnitude 'r' & perpendicular to the palne of $\vec{a}$ & $\vec{b}$ is $\pm\dfrac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

☞ If θ is the angle between $\vec{a}$ & $\vec{b}$ then $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

(viii) **Vector area**

☞ If $\vec{a}, \vec{b}$ & $\vec{c}$ are the pv's of 3 points A, B & C then the vector area of triangle ABC = $\frac{1}{2}\left[\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right]$. The points A, B & C are collinear if $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 0$

☞ Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$ is given by $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$

10. SHORTEST DISTANCE BETWEEN TWO LINES :

If two lines in space intersect at a point, then obviously the shortest distance between them is zero. Lines which do not intersect & are also not parallel are called **SKEW LINES**. For Skew lines the direction of the shortest distance would be perpendicular to both the lines. The magnitude of the shortest distance vector would be equal to that of the projection of $\vec{AB}$ along the direction of the line of shortest distance, $\vec{LM}$ is parallel to $\vec{p}\times\vec{q}$ i.e. $\vec{LM} = \left|\text{Projection of } \vec{AB} \text{ on } \vec{LM}\right| = \left|\text{Projection of } \vec{AB} \text{ on } \vec{p}\times\vec{q}\right|$

$$= \left|\frac{\vec{AB}\,.\,(\vec{p}\times\vec{q})}{\vec{p}\times\vec{q}}\right| = \left|\frac{(\vec{b}-\vec{a})\,.\,(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

1. The two lines directed along $\vec{p}$ & $\vec{q}$ will intersect only if shortest distance = 0 i.e. $(\vec{b}-\vec{a}).(\vec{p}\times\vec{q}) = 0$ i.e. $(\vec{b}-\vec{a})$ lies in the plane containing $\vec{p}$ & $\vec{q}$. $\Rightarrow \left[(\vec{b}-\vec{a})\ \vec{p}\ \vec{q}\right] = 0$.

2. If two lines are given by $\vec{r}_1 = \vec{a}_1 + K\vec{b}$ & $\vec{r}_2 = \vec{a}_2 + K\vec{b}$ i.e. they are parallel then , $d = \left|\dfrac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$

11. SCALAR TRIPLE PRODUCT / BOX PRODUCT / MIXED PRODUCT :

☞ The scalar triple product of three vectors $\vec a, \vec b$ & $\vec c$ is defined as :

$\vec a \times \vec b . \vec c = |\vec a||\vec b||\vec c| \sin\theta \cos\phi$ where θ is the angle between $\vec a$ & $\vec b$ & ϕ is the angle between $\vec a \times \vec b$ & $\vec c$.

It is also defined as $[\vec a\ \vec b\ \vec c]$, spelled as box product .

☞ Scalar triple product geometrically represents the volume of the parallelopiped whose three couterminous edges are represented by $\vec a, \vec b$ & $\vec c$ i.e. $V = [\vec a\ \vec b\ \vec c]$

☞ In a scalar triple product the position of dot & cross can be interchanged i.e.

$\vec a . (\vec b \times \vec c) = (\vec a \times \vec b).\vec c$ OR $[\vec a\ \vec b\ \vec c] = [\vec b\ \vec c\ \vec a] = [\vec c\ \vec a\ \vec b]$

☞ $\vec a . (\vec b \times \vec c) = -\vec a.(\vec c \times \vec b)$ i.e. $[\vec a\ \vec b\ \vec c] = -[\vec a\ \vec c\ \vec b]$

☞ If $\vec a = a_1\hat i + a_2\hat j + a_3\hat k$; $\vec b = b_1\hat i + b_2\hat j + b_3\hat k$ & $\vec c = c_1\hat i + c_2\hat j + c_3\hat k$ then $[\vec a\, \vec b\, \vec c] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

In general , if $\vec a = a_1\vec l + a_2\vec m + a_3\vec n$; $\vec b = b_1\vec l + b_2\vec m + b_3\vec n$ & $\vec c = c_1\vec l + c_2\vec m + c_3\vec n$

then $\left[\vec a\, \vec b\, \vec c\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \left[\vec l\, \vec m\, \vec n\right]$; where $\vec \ell, \vec m$ & $\vec n$ are non coplanar vectors .

☞ If $\vec a, \vec b, \vec c$ are coplanar $\Leftrightarrow [\vec a\ \vec b\ \vec c] = 0$.

☞ Scalar product of three vectors, two of which are equal or parallel is 0 i.e. $[\vec a\, \vec b\, \vec c] = 0$,

Note : If $\vec a, \vec b, \vec c$ are non – coplanar then $[\vec a\ \vec b\ \vec c] > 0$ for right handed system & $[\vec a\ \vec b\ \vec c] < 0$ for left handed system .

☞ $[i\, j\, k] = 1$ ☞ $[K\vec a\ \vec b\ \vec c] = K[\vec a\ \vec b\ \vec c]$ ☞ $[(\vec a + \vec b)\ \vec c\ \vec d] = [\vec a\ \vec c\ \vec d] + [\vec b\ \vec c\ \vec d]$

☞ The volume of the tetrahedron OABC with O as origin & the pv's of A, B and C being $\vec a, \vec b$ & $\vec c$ respectively is given by $V = \frac{1}{6}[\vec a\ \vec b\ \vec c]$

☞ The positon vector of the centroid of a tetrahedron if the pv's of its angular vertices are $\vec a, \vec b, \vec c$ & $\vec d$ are given by $\frac{1}{4}[\vec a + \vec b + \vec c + \vec d]$.

Note that this is also the point of concurrency of the lines joining the vertices to the centroids of the opposite faces and is also called the centre of the tetrahedron. In case the tetrahedron is regular it is equidistant from the vertices and the four faces of the tetrahedron .

Remember that : $\left[\vec a - \vec b\quad \vec b - \vec c\quad \vec c - \vec a\right] = 0$ & $\left[\vec a + \vec b\quad \vec b + \vec c\quad \vec c + \vec a\right] = 2\left[\vec a\ \vec b\ \vec c\right]$.

***12. VECTOR TRIPLE PRODUCT :** Let $\vec a, \vec b, \vec c$ be any three vectors, then the expression $\vec a \times (\vec b \times \vec c)$ is a vector & is called a vector triple product .

GEOMETRICAL INTERPRETATION OF $\vec a \times (\vec b \times \vec c)$

Consider the expression $\vec a \times (\vec b \times \vec c)$ which itself is a vector, since it is a cross product of two vectors $\vec a$ & $(\vec b \times \vec c)$. Now $\vec a \times (\vec b \times \vec c)$ is a vector perpendicular to the plane containing $\vec a$ & $(\vec b \times \vec c)$ but $\vec b \times \vec c$ is a vector perpendicular to the plane $\vec b$ & $\vec c$, therefore $\vec a \times (\vec b \times \vec c)$ is a vector lies in the plane of $\vec b$ & $\vec c$ and perpendicular to $\vec a$. Hence we can express $\vec a \times (\vec b \times \vec c)$ in terms of $\vec b$ & $\vec c$ i.e. $\vec a \times (\vec b \times \vec c) = x\vec b + y\vec c$ where x & y are scalars .

☞ $\vec a \times (\vec b \times \vec c) = (\vec a . \vec c)\vec b - (\vec a . \vec b)\vec c$ ☞ $(\vec a \times \vec b) \times \vec c = (\vec a . \vec c)\vec b - (\vec b . \vec c)\vec a$

☞ $(\vec a \times \vec b) \times \vec c \neq \vec a \times (\vec b \times \vec c)$

13. LINEAR COMBINATIONS / Linearly Independence and Dependence of Vectors :

Given a finite set of vectors $\vec a, \vec b, \vec c, ...$ then the vector $\vec r = x\vec a + y\vec b + z\vec c + ...$ is called a linear combination of $\vec{a},\vec{b},\vec{c}$,...... for any x, y, z ...... $\in$ R. We have the following results :

(a) **FUNDAMENTAL THEOREM IN PLANE :** Let $\vec{a},\vec{b}$ be non zero , non collinear vectors . Then any vector $\vec{r}$ coplanar with $\vec{a},\vec{b}$ can be expressed uniquely as a linear combination of $\vec{a},\vec{b}$ i.e. There exist some unique $x,y \in R$ such that $x\vec{a} + y\vec{b}=\vec{r}$.

(b) **FUNDAMENTAL THEOREM IN SPACE :** Let $\vec{a},\vec{b},\vec{c}$ be non–zero, non–coplanar vectors in space. Then any vector $\vec{r}$, can be uniquily expressed as a linear combination of $\vec{a},\vec{b},\vec{c}$ i.e. There exist some unique $x,y \in R$ such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{r}$.

(c) If $\vec{x}_1,\vec{x}_2,......\vec{x}_n$ are n non zero vectors, & $k_1, k_2, ...k_n$ are n scalars & if the linear combination $k_1\vec{x}_1 + k_2\vec{x}_2 + ... k_n\vec{x}_n = 0 \Rightarrow k_1 = 0, k_2 = 0 ..... k_n = 0$ then we say that vectors $\vec{x}_1,\vec{x}_2,......\vec{x}_n$ are **LINEARLY INDEPENDENT VECTORS** .

(d) If $\vec{x}_1,\vec{x}_2,......\vec{x}_n$ are not **LINEARLY INDEPENDENT** then they are said to be **LINEARLY DEPENDENT** vectors . i.e. if $k_1\vec{x}_1 + k_2\vec{x}_2 + ... + k_n\vec{x}_n = 0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1,\vec{x}_2,......\vec{x}_n$ are said to be **LINEARLY DEPENDENT** .

Note : ☞ If $\vec{a} = 3i + 2j + 5k$ then $\vec{a}$ is expressed as a **LINEAR COMBINATION** of vectors $\hat{i}, \hat{j}, \hat{k}$. Also, $\vec{a}$, $\hat{i}, \hat{j}, \hat{k}$ form a linearly dependent set of vectors. In general , every set of four vectors is a linearly dependent system.

☞ $\hat{i}, \hat{j}, \hat{k}$ are **LINEARLY INDEPENDENT** set of vectors. For $K_1\hat{i} + K_2\hat{j} + K_3\hat{k} = 0 \Rightarrow K_1 = 0 = K_2 = K_3$.

☞ Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow \vec{a}$ is parallel to $\vec{b}$ i.e. $\vec{a} \times \vec{b} = 0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$. Conversely if $\vec{a} \times \vec{b} \neq 0$ then $\vec{a}$ & $\vec{b}$ are linearly independent .

☞ If three vectors $\vec{a},\vec{b},\vec{c}$ are linearly dependent, then they are coplanar i.e. $[\vec{a}, \vec{b}, \vec{c}] = 0$, conversely, if $[\vec{a}, \vec{b}, \vec{c}] \neq 0$, then the vectors are linearly independent.

14. COPLANARITY OF VECTORS :

Four points A, B, C, D with position vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z, w not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where, $x + y + z + w = 0$.

15. RECIPROCAL SYSTEM OF VECTORS :

If $\vec{a},\vec{b},\vec{c}$ & $\vec{a}',\vec{b}',\vec{c}'$ are two sets of non coplanar vectors such that $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$ then the two systems are called Reciprocal System of vectors.

Note : $\quad a' = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $b' = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $c' = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

16. EQUATION OF A PLANE :

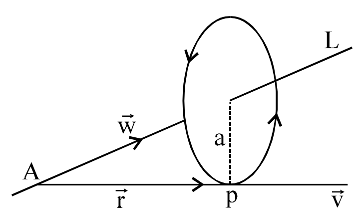
(a) The equation $(\vec{r} - \vec{r}_0).\vec{n} = 0$ represents a plane containing the point with p.v. $\vec{r}_0$ where $\vec{n}$ is a vector normal to the plane . $\vec{r}.\vec{n} = d$ is the general equation of a plane.

(b) Angle between the 2 planes is the angle between 2 normals drawn to the planes and the angle between a line and a plane is the compliment of the angle between the line and the normal to the plane.

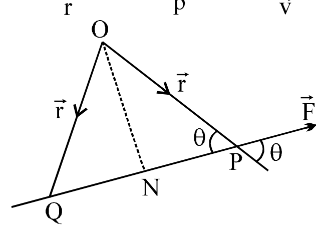
17. APPLICATION OF VECTORS :

(a) Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $\vec{W} = \vec{F}.\vec{s}$

(b) The tangential velocity $\vec{V}$ of a body moving in a circle is given by $\vec{V} = \vec{w} \times \vec{r}$ where $\vec{r}$ is the pv of the point P.



(c) The moment of $\vec{F}$ about 'O' is defined as $\vec{M} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the pv of P wrt 'O'. The direction of $\vec{M}$ is along the normal to the plane OPN such that $\vec{r}, \vec{F}$ & $\vec{M}$ form a right handed system.



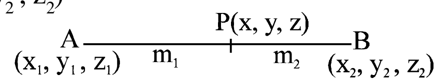
(d) Moment of the couple $= (\vec{r}_1 - \vec{r}_2) \times \vec{F}$ where $\vec{r}_1$ & $\vec{r}_2$ are pv's of the point of the application of the forces $\vec{F}$ & $-\vec{F}$.

3 -D COORDINATE GEOMETRY USEFUL RESULTS

A General :

(1) Distance (d) between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



(2) **Section Fomula**

$$x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2} \quad ; \quad y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2} \quad ; \quad z = \frac{m_2 z_1 + m_1 z_2}{m_1 + m_2}$$

(For external division take –ve sign)

Direction Cosine and direction ratio's of a line

(3) Direction cosine of a line has the same meaning as d.c's of a vector.

(a) Any three numbers a, b, c proportional to the direction cosines are called the direction ratios i.e.

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

same sign either +ve or –ve should be taken through out.

note that d.r's of a line joining x_1, y_1, z_1 and x_2, y_2, z_2 are proportional to $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$

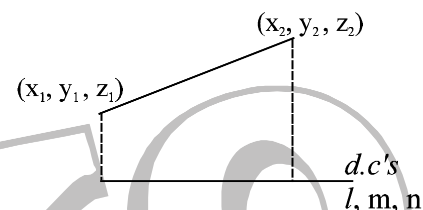
(b) If θ is the angle between the two lines whose d.c's are l_1, m_1, n_1 and l_2, m_2, n_2

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

hence if lines are perpendicular then $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

if lines are parallel then $\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}$

note that if three lines are coplanar then $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$



(4) Projection of the join of two points on a line with d.c's l, m, n are

$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

B PLANE: (i) General equation of degree one in x, y, z i.e. $ax + by + cz + d = 0$ represents a plane.

(ii) Equation of a plane passing through (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a, b, c are the direction ratios of the normal to the plane.

(iii) Equation of a plane if its intercepts on the co-ordinate axes are x_1, y_1, z_1 is

$$\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1.$$

(iv) Equation of a plane if the length of the perpendicular from the origin on the plane is p and d.c's of the perpendicular as l, m, , n is $l x + m y + n z = p$

(v) **Parallel and perpendicular planes** – Two planes

$a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are

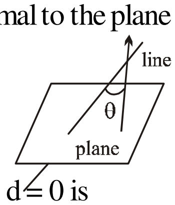
perpendicular if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ and

coincident if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{d_1}{d_2}$

(vi) Angle between a plane and a line is the compliment of the angle between the normal to the plane and the line . If $\left.\begin{matrix} \text{Line} : \vec{r} = \vec{a} + \lambda\vec{b} \\ \text{Plane} : \vec{r}.\vec{n} = d \end{matrix}\right]$ then $\cos(90 - \theta) = \sin\theta = \dfrac{\vec{b}.\vec{n}}{|\vec{b}|.|\vec{n}|}$.

where θ is the angle between the line and normal to the plane.



(vii) Length of the perpendicular from a point (x_1, y_1, z_1) to a plane $ax + by + cz + d = 0$ is

$$p = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

(viii) Distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\left|\dfrac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$

(ix) Planes bisecting the angle between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2 + b_2y + c_2z + d_2 = 0$ is given by

$$\left|\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right| = \pm \left|\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Of these two bisecting planes , one bisects the acute and the other obtuse angle between the given planes.

(x) Equation of a plane through the intersection of two planes P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$

C STRAIGHT LINE IN SPACE

(i) Equation of a line through A (x_1 , y_1 , z_1) and having direction cosines l ,m , n are $\dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n}$

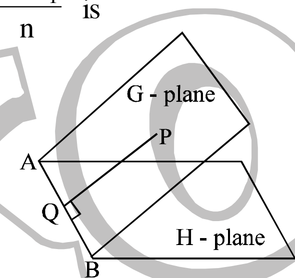
and the lines through (x_1 , y_1 , z_1) and (x_2 , y_2 , z_2) $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

(ii) Intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ together represent the unsymmetrical form of the straight line.

(iii) General equation of the plane containing the line $\dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n}$ is $A (x - x_1) + B(y - y_1) + c (z - z_1) = 0$ where $Al + bm + cn = 0$.

LINE OF GREATEST SLOPE

AB is the line of intersection of G-plane and H is the horizontal plane. Line of greatest slope on a given plane, drawn through a given point on the plane, is the line through the point 'P' perpendicular to the line of intersetion of the given plane with any horizontal plane.



EXERCISE–1

Q.1 If $\vec{a}$ & $\vec{b}$ are non collinear vectors such that, $\vec{p} = (x + 4y)\vec{a} + (2x + y + 1)\vec{b}$ & $\vec{q} = (y - 2x + 2)\vec{a} + (2x - 3y - 1)\vec{b}$, find x & y such that $3\vec{p} = 2\vec{q}$.

Q.2 (a) Show that the points $\vec{a} - 2\vec{b} + 3\vec{c}; 2\vec{a} + 3\vec{b} - 4\vec{c} \,\&\, - 7\vec{b} + 10\vec{c}$ are collinear .

(b) Prove that the points A = (1,2,3), B (3,4,7), C (–3,–2,–5) are collinear & find the ratio in which B divides AC.

Q.3 Points X & Y are taken on the sides QR & RS , respectively of a parallelogram PQRS, so that $\vec{QX} = 4\vec{XR}$ & $\vec{RY} = 4\vec{YS}$. The line XY cuts the line PR at Z . Prove that $\vec{PZ} = \left(\dfrac{21}{25}\right)\vec{PR}$.

Q.4 Find out whether the following pairs of lines are parallel, non-parallel & intersecting, or non-parallel & non-intersecting.

(i) $\vec{r}_1 = \hat{i} + \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$
$\vec{r}_2 = 2\hat{i} + \hat{j} + 3\hat{k} + \mu\left(-6\hat{i} + 4\hat{j} - 8\hat{k}\right)$

(ii) $\vec{r}_1 = \hat{i} - \hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$
$\vec{r}_2 = 2\hat{i} + 4\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$

(iii) $\vec{r}_1 = \hat{i} + \hat{k} + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$
$\vec{r}_2 = 2\hat{i} + 3\hat{j} + \mu\left(4\hat{i} - \hat{j} + \hat{k}\right)$

Q.5 Let OACB be paralelogram with O at the origin & OC a diagonal. Let D be the mid point of OA. Using vector method prove that BD & CO intersect in the same ratio. Determine this ratio.

Q.6 A line EF drawn parallel to the base BC of a Δ ABC meets AB & AC in F & E respectively. BE & CF meet in L. Use vectors to show that AL bisects BC.

Q.7 'O' is the origin of vectors and A is a fixed point on the circle of radius 'a' with centre O. The vector $\vec{OA}$

is denoted by $\vec{a}$. A variable point 'P' lies on the tangent at A & $\overrightarrow{OP} = \vec{r}$. Show that $\vec{a}.\vec{r} = |a|^2$. Hence if P ≡ (x,y) & A ≡ (x_1,y_1) deduce the equation of tangent at A to this circle.

Q.8 (a) By vector method prove that the quadrilateral whose diagonals bisect each other at right angles is a rhombous.

(b) By vector method prove that the right bisectors of the sides of a triangle are concurrent.

Q.9 The resultant of two vectors $\vec{a}$ & $\vec{b}$ is perpendicular to $\vec{a}$. If $|\vec{b}| = \sqrt{2}|\vec{a}|$ show that the resultant of $2\vec{a}$ & $\vec{b}$ is perpendicular to $\vec{b}$.

Q.10 $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A ≡ (x, y, z) ; B ≡ (y, – 2z, 3x) ; C ≡ (2z, 3x, – y) and D≡(1,–1, 2) respectively. If $|\vec{a}| = 2\sqrt{3}$; $(\vec{a}\,\hat{}\,\vec{b}) = (\vec{a}\,\hat{}\,\vec{c})$; $(\vec{a}\,\hat{}\,\vec{d}) = \dfrac{\pi}{2}$ and $(\vec{a}\,\hat{}\,\hat{j})$ is obtuse, then find x, y, z.

Q.11 If $\vec{r}$ and $\vec{s}$ are non zero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then show that the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

Q.12 Use vectors to prove that the diagonals of a trapezium having equal non parallel sides are equal & conversely.

Q.13(a) Find a unit vector $\hat{a}$ which makes an angle $(\pi/4)$ with axis of z & is such that $\hat{a} + i + j$ is a unit vector.

(b) Prove that $\left(\dfrac{\vec{a}}{\vec{a}^2} - \dfrac{\vec{b}}{\vec{b}^2}\right)^2 = \left(\dfrac{\vec{a} - \vec{b}}{|\vec{a}||\vec{b}|}\right)^2$

Q.14 Given four non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. The vectors $\vec{a}, \vec{b}$ & $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ & $\vec{c}$ and $(\widehat{\vec{a}\vec{b}}) = (\widehat{\vec{b}\vec{c}}) = \dfrac{\pi}{3}$, $(\widehat{\vec{d}\vec{a}}) = \alpha$, $(\widehat{\vec{d}\vec{b}}) = \beta$ then prove that $(\widehat{\vec{d}\,\vec{c}}) = \cos^{-1}(\cos\beta - \cos\alpha)$.

Q.15 (a) Use vectors to find the acute angle between the diagonals of a cube.

(b) Prove cosine & projection rule in a triangle by using dot product.

Q.16 In the plane of a triangle ABC, squares ACXY, BCWZ are described, in the order given, externally to the triangle on AC & BC respectively. Given that $\overrightarrow{CX} = \vec{b}$, $\overrightarrow{CA} = \vec{a}$, $\overrightarrow{CW} = \vec{x}$, $\overrightarrow{CB} = \vec{y}$. Prove that $\vec{a}.\vec{y} + \vec{x}.\vec{b} = 0$. Deduce that $\overrightarrow{AW}.\overrightarrow{BX} = 0$.

Q.17 A Δ OAB is right angled at O ; squares OALM & OBPQ are constructed on the sides OA and OB externally. Show that the lines AP & BL intersect on the altitude through 'O'.

Q.18 Given that $\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$; $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $(\vec{u}\cdot\vec{R} - 10)\hat{i} + (\vec{v}\cdot\vec{R} - 20)\hat{j} + (\vec{w}\cdot\vec{R} - 20)\hat{k} = 0$. Find the unknown vector $\vec{R}$.

Q.19 If O is origin of reference, point $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$; $D(\vec{a} + \vec{b})$; $E(\vec{b} + \vec{c})$; $F(\vec{c} + \vec{a})$; $G(\vec{a} + \vec{b} + \vec{c})$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then prove that these points are vertices of a cube having length of its edge equal to unity provided the matrix.

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orghogonal. Also find the length XY such that X is the point of intersection of CM and GP ; Y is the point of intersection of OQ and DN where P, Q, M, N are respectively the midpoint of sides CF, BD, GF and OB.

Q.20 (a) If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Deduce the Sine rule for a Δ ABC.

(b) Find the minimum area of the triangle whose vertices are A(–1, 1, 2); B(1, 2, 3) and C(t, 1, 1) where t is a real number.

Q.21 (a) Determine vector of magnitude 9 which is perpendicular to both the vectors :

$$4\hat{i} - \hat{j} + 3\hat{k} \;\;\&\;\; -2\hat{i} + \hat{j} - 2\hat{k}$$

(b) A triangle has vertices (1, 1, 1) ; (2, 2, 2), (1, 1, y) and has the area equal to $\csc\left(\dfrac{\pi}{4}\right)$ sq. units. Find the value of y.

Q.22 The internal bisectors of the angles of a triangle ABC meet the opposite sides in D, E, F ; use vectors to prove that the area of the triangle DEF is given by

$\dfrac{(2abc)\,\Delta}{(a+b)(b+c)(c+a)}$ where Δ is the area of the triangle.

Q.23 If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of the vertices of a cyclic quadrilateral ABCD prove that :

$$\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}\right|}{(\vec{b}-\vec{a})\cdot(\vec{d}-\vec{a})}+\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}\times\vec{b}\right|}{(\vec{b}-\vec{c})\cdot(\vec{d}-\vec{c})}=0$$

Q.24 The length of the edge of the regular tetrahedron D – ABC is 'a'. Point E and F are taken on the edges AD and BD respectively such that E divides $\vec{DA}$ and F divides $\vec{BD}$ in the ratio 2:1 each. Then find the area of triangle CEF.

Q.25 Let $\vec{a}=\sqrt{3}\,\hat{i}-\hat{j}$ and $\vec{b}=\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$ and $\vec{x}=\vec{a}+(q^2-3)\vec{b}$, $\vec{y}=-p\vec{a}+q\vec{b}$. If $\vec{x}\perp\vec{y}$, then express p as a function of q, say $p=f(q)$, $(p\neq 0\ \&\ q\neq 0)$ and find the intervals of monotonicity of $f(q)$.

EXERCISE–2

Q.1 $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$ are the vertices of the triangle ABC such that $\vec{a}=\frac{1}{2}(2\hat{i}-\vec{r}-7\hat{k})$; $\vec{b}=3\vec{r}+\hat{j}-4\hat{k}$; $\vec{c}=22\hat{i}-11\hat{j}-9\vec{r}$. A vector $\vec{p}=2\hat{j}-\hat{k}$ is such that $(\vec{r}+\vec{p})$ is parallel to $\hat{i}$ and $(\vec{r}-2\hat{i})$ is parallel to $\vec{p}$. Show that there exists a point $D(\vec{d})$ on the line AB with $\vec{d}=2t\,\hat{i}+(1-2t)\hat{j}+(t-4)\hat{k}$. Also find the shortest distance C from AB.

Q.2 The position vectors of the points A, B, C are respectively (1, 1, 1) ; (1, –1, 2) ; (0, 2, –1). Find a unit vector parallel to the plane determined by ABC & perpendicular to the vector (1, 0, 1) .

Q.3 Let $\begin{vmatrix}(a_1-a)^2 & (a_1-b)^2 & (a_1-c)^2\\(b_1-a)^2 & (b_1-b)^2 & (b_1-c)^2\\(c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2\end{vmatrix}=0$ and if the vectors $\vec{\alpha}=\hat{i}+a\hat{j}+a^2\hat{k}$; $\vec{\beta}=\hat{i}+b\hat{j}+b^2\hat{k}$; $\vec{\gamma}=\hat{i}+c\hat{j}+c^2\hat{k}$ are non coplanar, show that the vectors $\vec{\alpha}_1=\hat{i}+a_1\hat{j}+a_1^2\hat{k}$;$\vec{\beta}_1=\hat{i}+b_1\hat{j}+b_1^2\hat{k}$ and $\vec{\gamma}_1=\hat{i}+c_1\hat{j}+c_1^2\hat{k}$ are coplaner.

Q.4 Given non zero number x_1, x_2, x_3 ; y_1, y_2, y_3 and z_1, z_2 and z_3 such that $x_i > 0$ and $y_i < 0$ for all i = 1, 2, 3.

(i) Can the given numbers satisfy

$$\begin{vmatrix}x_1 & x_2 & x_3\\y_1 & y_2 & y_3\\z_1 & z_2 & z_3\end{vmatrix}=0 \text{ and } \begin{cases}x_1x_2+y_1y_2+z_1z_2=0\\x_2x_3+y_2y_3+z_2z_3=0\\x_3x_1+y_3y_1+z_3z_1=0\end{cases}$$

(ii) If $P=(x_1, x_2, x_3)$; $Q\ (y_1, y_2, y_3)$ and O (0, 0, 0) can the triangle POQ be a right angled triangle?

Q.5 The pv's of the four angular points of a tetrahedron are : $A\left(\hat{j}+2\hat{k}\right)$; $B\left(3\hat{i}+\hat{k}\right)$; $C\left(4\hat{i}+3\hat{j}+6\hat{k}\right)$ & $D\left(2\hat{i}+3\hat{j}+2\hat{k}\right)$. Find :

(i) the perpendicular distance from A to the line BC. (ii) the volume of the tetrahedron ABCD.

(iii) the perpendicular distance from D to the plane ABC.

(iv) the shortest distance between the lines AB & CD.

Q.6 The length of an edge of a cube $ABCDA_1B_1C_1D_1$ is equal to unity. A point E taken on the edge $\vec{AA_1}$ is such that $\left|\vec{AE}\right|=\frac{1}{3}$. A point F is taken on the edge $\vec{BC}$ such that $\left|\vec{BF}\right|=\frac{1}{4}$. If O_1 is the centre of the cube, find the shortest distance of the vertex B_1 from the plane of the $\Delta\, O_1EF$.

Q.7 The vector $\vec{OP}=\hat{i}+2\hat{j}+2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

Q.8 Find the point R in which the line AB cuts the plane CDE where

$\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$, $\vec{c}=-4\hat{j}+4\hat{k}$, $\vec{d}=2\hat{i}-2\hat{j}+2\hat{k}$ & $\vec{e}=4\hat{i}+\hat{j}+2\hat{k}$.

Q.9 If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$; $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then show that the value of the scalar triple product $[\,n\vec{a}+\vec{b}\quad n\vec{b}+\vec{c}\quad n\vec{c}+\vec{a}\,]$ is $(n^3+1)\begin{vmatrix}\vec{a}\cdot\hat{i} & \vec{a}\cdot\hat{j} & \vec{a}\cdot\hat{k}\\\vec{b}\cdot\hat{i} & \vec{b}\cdot\hat{j} & \vec{b}\cdot\hat{k}\\\vec{c}\cdot\hat{i} & \vec{c}\cdot\hat{j} & \vec{c}\cdot\hat{k}\end{vmatrix}$

Q.10 Find the scalars α & β if $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$ & $(\vec{c}.\vec{c})\vec{a} = \vec{c}$ while $\vec{b}$ & $\vec{c}$ are non zero non collinear vectors.

Q.11 If the vectors $\vec{b}, \vec{c}, \vec{d}$ are not coplanar, then prove that the vector

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to $\vec{a}$.

Q.12 $\hat{a}$, $\hat{b}$, $\hat{c}$ are non–coplanar unit vectors. The angle between $\hat{b}$ & $\hat{c}$ is α, between $\hat{c}$ & $\hat{a}$ is β and between $\hat{a}$ & $\hat{b}$ is γ. If $A(\hat{a}\cos\alpha)$, $B(\hat{b}\cos\beta)$, $C(\hat{c}\cos\gamma)$, then show that in ΔABC,

$$\frac{\left|\hat{a} \times (\hat{b} \times \hat{c})\right|}{\sin A} = \frac{\left|\hat{b} \times (\hat{c} \times \hat{a})\right|}{\sin B} = \frac{\left|\hat{c} \times (\hat{a} \times \hat{b})\right|}{\sin C} = \frac{\prod \left|\hat{a} \times (\hat{b} \times \hat{c})\right|}{\left|\sum \sin\alpha \cos\beta \cos\gamma \, \hat{n}_1\right|}$$ where

$\hat{n}_1 = \dfrac{\hat{b} \times \hat{c}}{\left|\hat{b} \times \hat{c}\right|}$, $\hat{n}_2 = \dfrac{\hat{c} \times \hat{a}}{\left|\hat{c} \times \hat{a}\right|}$ & $\hat{n}_3 = \dfrac{\hat{a} \times \hat{b}}{\left|\hat{a} \times \hat{b}\right|}$.

Q.13 Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\,\vec{p}$, $\vec{b}.\vec{q} = 0$ & $(\vec{b})^2 = 1$, where μ is a scalar then prove that $\left|(\vec{a}.\vec{q})\vec{p} - (\vec{p}.\vec{q})\vec{a}\right| = \left|\vec{p}.\vec{q}\right|$.

Q.14 Show that $\vec{a} = \vec{p} \times (\vec{q} \times \vec{r})$; $\vec{b} = \vec{q} \times (\vec{r} \times \vec{p})$ & $\vec{c} = \vec{r} \times (\vec{p} \times \vec{q})$ represents the sides of a triangle. Further prove that a unit vector perpendicular to the plane of this triangle is

$$\pm \frac{\hat{n}_1 \tan(\vec{p} \wedge \vec{q}) + \hat{n}_2 \tan(\vec{q} \wedge \vec{r}) + \hat{n}_3 \tan(\vec{r} \wedge \vec{p})}{\left|\hat{n}_1 \tan(\vec{p} \wedge \vec{q}) + \hat{n}_2 \tan(\vec{q} \wedge \vec{r}) + \hat{n}_3 \tan(\vec{r} \wedge \vec{p})\right|}$$ where $\vec{a}, \vec{b}, \vec{c}, \vec{p}, \vec{q}$ are non zero vectors and

no two of $\vec{p}, \vec{q}, \vec{r}$ are mutually perpendicular & $\hat{n}_1 = \dfrac{\vec{p} \times \vec{q}}{|\vec{p} \times \vec{q}|}$; $\hat{n}_2 = \dfrac{\vec{q} \times \vec{r}}{|\vec{q} \times \vec{r}|}$ & $\hat{n}_3 = \dfrac{\vec{r} \times \vec{p}}{|\vec{r} \times \vec{p}|}$

Q.15 Given four points P_1, P_2, P_3 and P_4 on the coordinate plane with origin O which satisfy the condition

$$\overrightarrow{OP}_{n-1} + \overrightarrow{OP}_{n+1} = \frac{3}{2}\overrightarrow{OP}_n, \quad n = 2, 3$$

(i) If P_1, P_2 lie on the curve $xy = 1$, then prove that P_3 does not lie on the curve.

(ii) If P_1, P_2, P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 lies on this circle.

Q.16 Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value(s) of α, if any, such that $\left\{(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})\right\} \times (\vec{c} \times \vec{a}) = 0$. Find the vector product when $\alpha = 0$.

Q.17 Prove the result (Lagrange's identity) $(\vec{p} \times \vec{q}) \cdot (\vec{r} \times \vec{s}) = \begin{vmatrix} \vec{p}.\vec{r} & \vec{p}.\vec{s} \\ \vec{q}.\vec{r} & \vec{q}.\vec{s} \end{vmatrix}$ & use it to prove the following. Let (ab)denote the plane formed by the lines a,b. If (ab) is perpendicular to (cd) and (ac) is perpendicular to (bd) prove that (ad) is perpendicular to (bc).

Q.18 (a) If $p\vec{x} + (\vec{x} \times \vec{a}) = \vec{b}$; $(p \neq 0)$ prove that $\vec{x} = \dfrac{p^2\vec{b} + (\vec{b}.\vec{a})\vec{a} - p(\vec{b} \times \vec{a})}{p(p^2 + \vec{a}^2)}$.

(b) Solve the following equation for the vector $\vec{p}$; $\vec{p} \times \vec{a} + (\vec{p}.\vec{b})\vec{c} = \vec{b} \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left(\vec{p} \times \vec{a} + \dfrac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\vec{a} \cdot \vec{c}}\vec{c}\right)$ is perpendicular to $\vec{b} - \vec{c}$.

Q.19 Find a vector $\vec{v}$ which is coplanar with the vectors $\hat{i} + \hat{j} - 2\hat{k}$ & $\hat{i} - 2\hat{j} + \hat{k}$ and is orthogonal to the vector $-2\hat{i} + \hat{j} + \hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i} - \hat{j} + \hat{k}$ is equal to $6\sqrt{3}$.

Q.20 Consider the non zero vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ such that no three of which are coplanar then prove that $\vec{a}\left[\vec{b}\,\vec{c}\,\vec{d}\right] + \vec{c}\left[\vec{a}\,\vec{b}\,\vec{d}\right] = \vec{b}\left[\vec{a}\,\vec{c}\,\vec{d}\right] + \vec{d}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$. Hence prove that $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ represent the position vectors of the vertices of a plane quadrilateral if $\dfrac{\left[\vec{b}\,\vec{c}\,\vec{d}\right] + \left[\vec{a}\,\vec{b}\,\vec{d}\right]}{\left[\vec{a}\,\vec{c}\,\vec{d}\right] + \left[\vec{a}\,\vec{b}\,\vec{c}\right]} = 1$.

Q.21 The base vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are given in terms of base vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ as, $\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$; $\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$ & $\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$. If $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$, then express $\vec{F}$ in terms of

$\vec{a}_1, \vec{a}_2$ & $\vec{a}_3$.

Q.22 If $A(\vec{a})$; $B(\vec{b})$ & $C(\vec{c})$ are three non collinear points , then for any point $P(\vec{p})$ in the plane of the ΔABC, prove that ; (i) $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{p} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$

(ii) The vector $\vec{v}$ perpendicular to the plane of the triangle ABC drawn from the origin 'O' is given by $\vec{v} = \pm \dfrac{[\vec{a}\,\vec{b}\,\vec{c}]\,(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\Delta^2}$ where $\vec{\Delta}$ is the vector area of the triangle ABC.

Q.23 Given the points P (1, 1, –1), Q (1, 2, 0) and R (–2, 2, 2). Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$

(b) Equation of the plane in
(i) scalar dot product form (ii) parametric form (iii) cartesian form
(iv) if the plane through PQR cuts the coordinate axes at A, B, C then the area of the ΔABC

Q.24 Let $\vec{a}, \vec{b}$ & $\vec{c}$ be non coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$. Find scalars p , q & r in terms of θ.

Q.25 Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$.

$$\vec{x} + \vec{c} \times \vec{y} = \vec{a} \quad \text{and} \quad \vec{y} + \vec{c} \times \vec{x} = \vec{b} \quad \text{where } \vec{c} \text{ is a non zero vector.}$$

EXERCISE–3

Q.1 Find the angle between the two straight lines whose direction cosines l, m, n are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$.

Q.2 If two straight line having direction cosines l, m, n satisfy $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular, then show that $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.

Q.3 P is any point on the plane $lx + my + nz = p$. A point Q taken on the line OP (where O is the origin) such that OP. OQ = p^2. Show that the locus of Q is $p(lx + my + nz) = x^2 + y^2 + z^2$.

Q.4 Find the equation of the plane through the points (2, 2, 1), (1, –2, 3) and parallel to the x-axis.

Q.5 Through a point P (f, g, h), a plane is drawn at right angles to OP where 'O' is the origin, to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^5}{2fgh}$ where OP = r.

Q.6 The plane $lx + my = 0$ is rotated about its line of intersection with the plane z = 0 through an angle θ. Prove that the equation to the plane in new position is $lx + my \pm z\sqrt{l^2 + m^2}\ \tan\theta = 0$

Q.7 Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line $x + 1 = 2(y - 2) = z + 4$ and parallel to the plane $x + 5y + 4z = 0$.

Q.8 Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

Q.9 A variable plane is at a constant distance p from the origin and meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection.

Q.10 Find the distance of the point P (– 2, 3, – 4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

Q.11 Find the equation to the line passing through the point (1, –2, –3) and parallel to the line
$$2x + 3y - 3z + 2 = 0 = 3x - 4y + 2z - 4.$$

Q.12 Find the equation of the line passing through the point (4, –14, 4) and intersecting the line of intersection of the planes : $3x + 2y - z = 5$ and $x - 2y - 2z = -1$ at right angles.

Q.13 Let P = (1, 0, – 1) ; Q = (1, 1, 1) and R = (2, 1, 3) are three points.
(a) Find the area of the triangle having P, Q and R as its vertices.
(b) Give the equation of the plane through P, Q and R in the form $ax + by + cz = 1$.
(c) Where does the plane in part (b) intersect the y-axis.
(d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).

Q.14 Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersects the plane $2x + 2y + z + 6 = 0$.

Q.15 Feet of the perpendicular drawn from the point P (2, 3, –5) on the axes of coordinates are A, B and C. Find the equation of the plane passing through their feet and the area of ΔABC.

Q.16 Find the equations to the line which can be drawn from the point (2, –1, 3) perpendicular to the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{4}=\frac{y}{5}=\frac{z+3}{3}$ at right angles.

Q.17 Find the equation of the plane containing the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$.

Q.18 Find the value of p so that the lines $\frac{x+1}{-3}=\frac{y-p}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are in the same plane. For this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

Q.19 Find the equations to the line of greatest slope through the point (7, 2 , –1) in the plane $x - 2y + 3z = 0$ assuming that the axes are so placed that the plane $2x + 3y - 4z = 0$ is horizontal.

Q.20 Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be denoted by x, y and z sq. units respectively. Find the area of the triangle BCD.

Q.21 The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of 'r'.

Q.22 The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

Q.23 Find the foot and hence the length of the perpendicular from the point (5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{5-z}{5}$. Also find the equation of the plane in which the perpendicular and the given straight line lie.

Q.24 Find the equation of the line which is reflection of the line $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$.

Q.25 Find the equation of the plane containing the line $\frac{x-1}{2}=\frac{y}{3}=\frac{z}{2}$ and parallel to the line $\frac{x-3}{2}=\frac{y}{5}=\frac{z-2}{4}$. Find also the S.D. between the two lines.

EXERCISE–4

Q.1(a) Let $\vec{OA}=\vec{a}$, $\vec{OB}=10\vec{a}+2\vec{b}$ and $\vec{OC}=\vec{b}$ where O, A & C are non–collinear points . Let p denote the area of the quadrilateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If p = kq, then k = _______ .

(b) If $\vec{A}, \vec{B}$ & $\vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$, Prove that ;

$\left[\left(\vec{A}+\vec{B}\right)x\left(\vec{A}+\vec{C}\right)\right] x \left(\vec{B}x\vec{C}\right).\left(\vec{B}+\vec{C}\right)=0$ [JEE ' 97, 2 + 5]

Q.2(a) Vectors $\vec{x}, \vec{y}$ & $\vec{z}$ each of magnitude $\sqrt{2}$, make angles of 60^0 with each other. If $\vec{x}\times(\vec{y}\times\vec{z})=\vec{a}, \vec{y}\times(\vec{z}\times\vec{x})=\vec{b}$ and $\vec{x}\times\vec{y}=\vec{c}$ then find $\vec{x}, \vec{y}$ and $\vec{z}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

(b) The position vectors of the points P & Q are $5\hat{i}+7\hat{j}-2\hat{k}$ and $-3\hat{i}+3\hat{j}+6\hat{k}$ respectively. The vector $\vec{A}=3\hat{i}-\hat{j}+\hat{k}$ passes through the point P & the vector $\vec{B}=-3\hat{i}+2\hat{j}+4\hat{k}$ passes through the point Q. A third vector $2\hat{i}+7\hat{j}-5\hat{k}$ intersects vectors $\vec{A}$ & $\vec{B}$. Find the position vectors of the points of intersection. [REE ' 97, 6 + 6]

Q.3**(a)** Select the correct alternative(s)

(i) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors & $|\vec{c}|=\sqrt{3}$, then:

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

(ii) For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three?

(A) $\vec{u}.(\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}).\vec{u}$ (C) $\vec{v}.(\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v}).\vec{w}$

(iii) Which of the following expressions are meaningful ?

(A) $\vec{u}.(\vec{v} \times \vec{w})$ (B) $(\vec{u}.\vec{v}).\vec{w}$ (C) $(\vec{u}.\vec{v})\vec{w}$ (D) $\vec{u} \times (\vec{v}.\vec{w})$

(b) Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezeum lies on the line passing through the mid–points of the parallel sides.
(You may assume that the trapezeum is not a parallelogram.)

(c) For any two vectors $\vec{u}$ & $\vec{v}$, prove that [JEE ' 98 , 2 + 2 + 2 + 8 + 8]

(i) $(\vec{u}.\vec{v})^2 + |\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2$ & (ii) $(1+|\vec{u}|^2)(1+|\vec{v}|^2) = (1-\vec{u}.\vec{v})^2 + |\vec{u}+\vec{v}+(\vec{u}\times\vec{v})|^2$

Q.4(a) If $\vec{x}\times\vec{y}=\vec{a}$, $\vec{y}\times\vec{z}=\vec{b}$, $\vec{x}.\vec{b}=\gamma$, $\vec{x}.\vec{y}=1$ and $\vec{y}.\vec{z}=1$ then find $\vec{x}, \vec{y}$ & $\vec{z}$ in terms of $\vec{a}$, $\vec{b}$ and γ.

(b) Vectors $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ & $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are not coplanar. The position vectors of points A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$ respectively. Find the position vectors of a point P on the line AB & a point Q on the line CD such that $\vec{PQ}$ is perpendicular to $\vec{AB}$ and $\vec{CD}$ both.

Q.5(a) Let $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$ & $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|$, $|\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30º, then $\left|(\vec{a}\times\vec{b})\times\vec{c}\right| =$

(A) 2/3 (B) 3/2 (C) 2 (D) 3

(b) Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}=$

(A) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (B) $\frac{1}{\sqrt{3}}\left(-\hat{i}-\hat{j}-\hat{k}\right)$ (C) $\frac{1}{\sqrt{5}}\left(\hat{i}-2\hat{j}\right)$ (D) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}-\hat{k}\right)$

(c) Let $\vec{a}$ & $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a}.\vec{b})\vec{b}$ & $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is :

(A) $|\vec{u}|$ (B) $|\vec{u}| + |\vec{u}.\vec{a}|$ (C) $|\vec{u}| + |\vec{u}.\vec{b}|$ (D) $\vec{u} + \vec{u}.(\vec{a}+\vec{b})$

(d) Let $\vec{u}$ & $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$, then prove that $\left|(\vec{u} \times \vec{v}).\vec{w}\right| \le \frac{1}{2}$ and the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

Q.6(a) An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\vec{OA}=\vec{a}$ & $\vec{OB}=\vec{b}$, then calculate $\vec{OC}$ in terms of $\vec{a}$ & $\vec{b}$.

(b) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of,
$\left|(\vec{a}.\vec{d})(\vec{b}\times\vec{c}) + (\vec{b}.\vec{d})(\vec{c}\times\vec{a}) + (\vec{c}.\vec{d})(\vec{a}\times\vec{b})\right|$ independent of $\vec{d}$. [REE '99, 6 + 6]

Q.7**(a)** Select the correct alternative :

(i) If the vectors $\vec{a}, \vec{b}$ & $\vec{c}$ form the sides BC, CA & AB respectively of a triangle ABC, then

(A) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$ (B) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

(C) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ (D) $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 0$

(ii) Let the vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$. Let P_1 & P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ & $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is :

(A) 0 (B) π/4 (C) π/3 (4) π/2

(iii) If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit coplanar vectors, then the scalar triple product

$\left[2\vec{a}-\vec{b} \quad 2\vec{b}-\vec{c} \quad 2\vec{c}-\vec{a}\right] =$ [JEE ,2000 (Screening) 1 + 1 + 1 out of 35]

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

(b) Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent.
[JEE '2000 (Mains) 10 out of 100]

Q.8. (i) If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$, find a unit vector normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$.

(ii) Given that vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other, find vector $\vec{\upsilon}$ in terms of $\vec{a}$ & $\vec{b}$ satisfying the equations, $\vec{\upsilon}.\vec{a} = 0$, $\vec{\upsilon}.\vec{b} = 1$ and $\left[\vec{\upsilon}, \vec{a}, \vec{b}\right] = 1$

(iii) $\vec{a}$, $\vec{b}$ & $\vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}(\vec{b} + \vec{c})$. Find angle between vectors $\vec{a}$ & $\vec{b}$ given that vectors $\vec{b}$ & $\vec{c}$ are non-parallel.

(iv) A particle is placed at a corner P of a cube of side 1 meter. Forces of magnitudes 2, 3 and 5 kg weight act on the particle along the diagonals of the faces passing through the point P. Find the moment of these forces about the corner opposite to P. [REE '2000 (Mains) 3 + 3 + 3 + 3 out of 100]

Q.9(a) The diagonals of a parallelogram are given by vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$. Determine its sides and also the area.

(b) Find the value of λ such that a, b, c are all non-zero and
$(-4\hat{i} + 5\hat{j})a + (3\hat{i} - 3\hat{j} + \hat{k})b + (\hat{i} + \hat{j} + 3\hat{k})c = \lambda(a\hat{i} + b\hat{j} + c\hat{k})$ [REE '2001 (Mains) 3 + 3]

Q.10(a) Find the vector $\vec{r}$ which is perpendicular to $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) + 8 = 0$.

(b) Two vertices of a triangle are at $-\hat{i} + 3\hat{j}$ and $2\hat{i} + 5\hat{j}$ and its orthocentre is at $\hat{i} + 2\hat{j}$. Find the position vector of third vertex. [REE '2001 (Mains) 3 + 3]

Q.11 (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does NOT exceed

(A) 4 (B) 9 (C) 8 (D) 6

(b) Let $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

(A) only x (B) only y (C) NEITHER x NOR y (D) both x and y

[JEE '2001 (Screening) 1 + 1 out of 35]

Q.12(a) Show by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

(b) Find 3–dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying
$\vec{v}_1 \cdot \vec{v}_1 = 4,\quad \vec{v}_1 \cdot \vec{v}_2 = -2,\quad \vec{v}_1 \cdot \vec{v}_3 = 6,\quad \vec{v}_2 \cdot \vec{v}_2 = 2,\quad \vec{v}_2 \cdot \vec{v}_3 = -5,\quad \vec{v}_3 \cdot \vec{v}_3 = 29.$

(c) Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}$, $t \in [0, 1]$, where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. [JEE '2001 (Mains) 5 + 5 + 5 out of 100]

Q.13(a) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 45^0 (B) 60^0 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

(b) Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is [JEE 2002(Screening), 3 + 3]

(A) –1 (B) $\sqrt{10} + \sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

Q.14 Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r, c_r, where r = 1, 2, 3, are non-negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V < L^3$. [JEE 2002(Mains), 5]

Q.15 If $\vec{a} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{b} = \hat{j} + a\hat{k}$, $\vec{c} = a\hat{i} + \hat{k}$, then find the value of 'a' for which volume of parallelopiped formed by three vectors as coterminous edges, is minimum, is

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\pm\frac{1}{\sqrt{3}}$ (D) none [JEE 2003(Scr.), 3]

Q.16(i) Find the equation of the plane passing through the points (2, 1, 0) , (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. [JEE 2003, 4 out of 60]

Q.17 If $\vec{u}$, $\vec{v}$, $\vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}$, $\vec{y}$, $\vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $\left[\vec{x}\times\vec{y}\;\; \vec{y}\times\vec{z}\;\; \vec{z}\times\vec{x}\right] = \frac{1}{16}\left[\vec{u}\;\vec{v}\;\vec{w}\right]^2 \sec^2\frac{\alpha}{2}\;\sec^2\frac{\beta}{2}\;\sec^2\frac{\gamma}{2}$.
[JEE 2003, 4 out of 60]

Q.18(a) If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k =

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) 0 (D) – 1

(b) A unit vector in the plane of the vectors $2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$

(A) $\frac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (B) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (C) $\frac{2\hat{i}-5\hat{k}}{\sqrt{29}}$ (D) $\frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

(c) If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$, then $\vec{b}$ = [JEE 2004 (screening)]

(A) $\hat{i}$ (B) $\hat{i}-\hat{j}+\hat{k}$ (C) $2\hat{j}-\hat{k}$ (D) $2\hat{i}$

Q.19(a) Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four distinct vectors satisfying $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$. Show that $\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{d} \neq \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{d}$.

(b) T is a parallelopiped in which A, B, C and D are vertices of one face. And the face just above it has corresponding vertices A', B', C', D'. T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A., B., C., D. in S. The volume of parallelopiped S is reduced to 90% of T. Prove that locus of A. is a plane.

(c) Let P be the plane passing through (1, 1, 1) and parallel to the lines L_1 and L_2 having direction ratios 1, 0, –1 and –1, 1, 0 respectively. If A, B and C are the points at which P intersects the coordinate axes, find the volume of the tetrahedron whose vertices are A, B, C and the origin.

Q.20(a) If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,
$\vec{c}_1 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_2 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}_1\cdot\vec{c}}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a} - \frac{\vec{b}\cdot\vec{c}}{|\vec{b}|^2}\vec{b}_1$
then the set of orthogonal vectors is

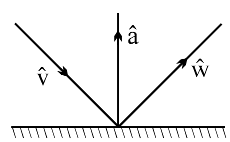
(A) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (B) $(\vec{a}, \vec{b}_1, \vec{c}_2)$ (C) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (D) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

(b) A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = k$, then the value of k is

(A) 3 (B) 1 (C) 1/3 (D) 9 [JEE 2005 (Screening), 3]

(c) Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, – 1).

(d) Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.



[JEE 2005 (Mains), 2 + 4 out of 60]

Q.21(a) A plane passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from the point (1, 2, 2) is

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

(b) Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is [JEE 2006,3 marks each]

(A) $4\hat{i}-\hat{j}+4\hat{k}$ (B) $3\hat{i}+\hat{j}-3\hat{k}$ (C) $2\hat{i}+\hat{j}-2\hat{k}$ (D) $4\hat{i}+\hat{j}-4\hat{k}$

(c) Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$ [JEE 2006, 5]

(d) **Match the following**

(i) Two rays in the first quadrant $x+y=|a|$ and $ax-y=1$ intersects each other in the interval $a\in(a_0,\infty)$, the value of a_0 is — (A) 2

(ii) Point (α,β,γ) lies on the plane $x+y+z=2$. Let $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\hat{k}\times(\hat{k}\times\vec{a})=0$, then $\gamma=$ — (B) 4/3

(iii) $\left|\int_0^1(1-y^2)\,dy\right|+\left|\int_1^0(y^2-1)\,dy\right|$ — (C) $\left|\int_0^1\sqrt{1-x}\,dx\right|+\left|\int_{-1}^0\sqrt{1+x}\,dx\right|$

(iv) If $\sin A\sin B\sin C+\cos A\cos B=1$, then the value of $\sin C=$ — (D) 1 [JEE 2006, 6]

(e) **Match the following**

(i) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right)=t$, then $\tan t=$ — (A) 0

(ii) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1=\frac{a}{b+c}$, $\cos\theta_2=\frac{b}{a+c}$, $\cos\theta_3=\frac{c}{a+b}$, then $\tan^2\frac{\theta_1}{2}+\tan^2\frac{\theta_3}{2}=$ — (B) 1

(iii) A line is perpendicular to $x+2y+2z=0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is — (C) $\frac{\sqrt{5}}{3}$

(D) 2/3 [JEE 2006, 6]

ANSWER KEY

EXERCISE–1

Q.1 $x = 2$, $y = -1$ Q.2 **(b)** externally in the ratio 1 : 3

Q.4 **(i)** parallel **(ii)** the lines intersect at the point p.v. $-2\hat{i} + 2\hat{j}$ **(iii)** lines are skew

Q.5 2 : 1 Q.7 $xx_1 + yy_1 = a^2$ Q.10 $x = 2, y = -2, z = -2$

Q.13 **(a)** $\frac{-1}{2}i - \frac{1}{2}j + \frac{1}{\sqrt{2}}k$ Q.15 (a) $\arccos\frac{1}{3}$ Q.18 $-\hat{i} + 2\hat{j} + 5\hat{k}$

Q.19 $\frac{\sqrt{11}}{3}$ Q.20 **(b)** $\frac{\sqrt{3}}{2}$ Q.21 (a) $\pm 3(\hat{i} - 2\hat{j} - 2\hat{k})$, (b) $y = 3$ or $y = -1$ Q.24 $\frac{5a^2}{12\sqrt{3}}$ sq. units

Q.25 $p = \frac{q(q^3 - 3)}{4}$; decreasing in $q \in (-1, 1)$, $q \neq 0$

EXERCISE–2

Q.1 $2\sqrt{17}$ Q.2 $\pm\frac{1}{3\sqrt{3}}(\hat{i} + 5\hat{j} - \hat{k})$ Q.4 NO, NO

Q.5 **(i)** $\frac{6}{7}\sqrt{14}$ **(ii)** 6 **(iii)** $\frac{3}{5}\sqrt{10}$ **(iv)** $\sqrt{6}$ Q.6 $\frac{11}{\sqrt{170}}$ Q.7 $\frac{4}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$

Q.8 p.v. of $\vec{R} = r = 3i + 3k$ Q.10 $\alpha = n\pi + \frac{(-1)^n \pi}{2}, n \in I \,\&\, \beta = 1$

Q.16 $\alpha = 2/3$; if $\alpha = 0$ then vector product is $-60\left(2\hat{i} + \hat{k}\right)$

Q.18 **(b)** $\left\{\vec{p} = \frac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{(\vec{a}\cdot\vec{c})(\vec{a}\cdot\vec{b})}\left(\vec{a} + \vec{c} \times \vec{b}\right) + \frac{(\vec{b}\cdot\vec{c})\vec{b}}{(\vec{a}\cdot\vec{b})} - \frac{(\vec{b}\cdot\vec{b})\vec{c}}{(\vec{a}\cdot\vec{b})}\right\}$ Q.19 $9\left(-\hat{j} + \hat{k}\right)$

Q.21 $F = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

Q.23 (a) $2\hat{i} - 3\hat{j} + 3\hat{k}$, (b) (i) -4, (ii) $\hat{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(\hat{j} + \hat{k}) + \mu(-3\hat{i} + \hat{j} + 3\hat{k})$, (iii) -4, (iv) $\frac{4\sqrt{22}}{9}$

Q.24 $p = -\frac{1}{\sqrt{1 + 2\cos\theta}}$; $q = \frac{2\cos\theta}{\sqrt{1 + 2\cos\theta}}$; $r = -\frac{1}{\sqrt{1 + 2\cos\theta}}$

or $p = \frac{1}{\sqrt{1 + 2\cos\theta}}$; $q = -\frac{2\cos\theta}{\sqrt{1 + 2\cos\theta}}$; $r = \frac{1}{\sqrt{1 + 2\cos\theta}}$

Q.25 $\vec{x} = \frac{\vec{a} + (\vec{c}\cdot\vec{a})\vec{c} + \vec{b} \times \vec{c}}{1 + \vec{c}^2}$, $y = \frac{\vec{b} + (\vec{c}\cdot\vec{b})\vec{c} + \vec{a} \times \vec{c}}{1 + \vec{c}^2}$

EXERCISE–3

Q.1 $\theta = 90^0$ Q.4 $y + 2z = 4$ Q.7 $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$

Q.8 $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ or $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ Q.9 $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$ Q.10 $\frac{17}{2}$

Q.11 $\frac{x-1}{6}=\frac{y+2}{13}=\frac{z+3}{17}$ Q.12 $\frac{x-4}{3}=\frac{y+14}{10}=\frac{z-4}{4}$

Q.13 (a) $\frac{3}{2}$; (b) $\frac{2x}{3}+\frac{2y}{3}-\frac{z}{3}=1$; (c) $\left(0, \frac{3}{2}, 0\right)$; (d) x = 2t + 2 ; y = 2t + 1 and z = – t + 3

Q.14 (1, –2, – 4) Q.15 $\frac{x}{2}+\frac{y}{3}+\frac{z}{-5}=1$, Area = $\frac{19}{2}$ sq. units Q.16 $\frac{x-2}{11}=\frac{y+1}{-10}=\frac{z-3}{2}$

Q.17 $2x + 3y + z + 4 = 0$ Q.18 p = 3, (2, 1, –3) ; $x + y + z = 0$

Q.19 $\frac{x-7}{22}=\frac{y-2}{5}=\frac{z+1}{-4}$ Q.20 $\sqrt{(x^2+y^2+z^2)}$ Q.21 $\frac{2}{3}$

Q.22 $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$; $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

Q.23 (9, 13, 15) ; 14 ; $9x - 4y - z = 14$ Q.24 $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$

Q.25 $x - 2y + 2z - 1 = 0$; 2 units

EXERCISE–4

Q.1 (a) 6

Q.2 (a) $\vec{x}=\vec{a}\times\vec{c}$; $\vec{y}=\vec{b}\times\vec{c}$; $\vec{z}=\vec{b}+\vec{a}\times\vec{c}$ or $\vec{b}\times\vec{c}-\vec{a}$ (b) (2, 8, –3) ; (0, 1, 2)

Q.3 (a) (i) D (ii) C (iii) A, C

Q.4 (a) $x=\dfrac{\frac{\vec{a}\times\vec{b}}{\gamma}-\vec{a}\times\frac{\vec{a}\times\vec{b}}{\gamma}}{\left(\frac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$; $y=\dfrac{\vec{a}\times\vec{b}}{\gamma}$; $z=\dfrac{\frac{\vec{a}\times\vec{b}}{\gamma}+\vec{b}\times\frac{\vec{a}\times\vec{b}}{\gamma}}{\left(\frac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$ (b) P ≡ (3, 8, 3) & Q ≡ (–3, –7, 6)

Q.5 (a) B (b) A (c) A, C Q.6 (a) $\vec{c}=-\sqrt{3}\,\vec{a}+2\vec{b}$ (b) $\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$

Q.7 (a) (i) B (ii) A (iii) A

Q.8 (i) $\pm\hat{i}$; (ii) $\frac{\vec{b}}{\vec{b}^2}+\frac{\vec{a}x\vec{b}}{(\vec{a}\times\vec{b})^2}$; (iii) $\frac{2\pi}{3}$; (iv) $|\vec{M}|=\sqrt{7}$

Q.9 (a) $\frac{1}{2}(5\hat{i}-\hat{j}-7\hat{k}), \frac{1}{2}(-\hat{i}+7\hat{j}-5\hat{k})$; $\frac{1}{2}\sqrt{1274}$ sq. units (b) $\lambda = 0,\ \lambda = -2 \pm \sqrt{29}$

Q.10 (a) $\vec{r}=-13\hat{i}+11\hat{j}+7\hat{k}$; (b) $\frac{5}{7}\hat{i}+\frac{17}{7}\hat{j}$

Q.11 (a) B (b) C Q.12 (b) $\vec{v}_1=2\hat{i},\ \vec{v}_2=-\hat{i}\pm\hat{j},\ \vec{v}_3=3\hat{i}\pm2\hat{j}\pm4\hat{k}$

Q.13 (a) B ; (b) C Q.15 D Q.16 (i) $x + y - 2z = 3$; (ii) (6, 5, –2)

Q.18 (a) B, (b) B, (c) A Q.19 (c) 9/2 cubic units

Q.20 (a) B, (b) D ; (c) $2x - y + z - 3 = 0$ and $62x + 29y + 19z - 105 = 0$, (d) $\hat{w}=\hat{v}-2(\hat{a}\cdot\hat{v})\,\hat{a}$

Q.21 **(a)** D; **(b)** A; **(c)** B, D; **(d)** (i) D, (ii) A, (iii) B, C, (iv) D; **(e)** (i) B, (ii) D, (iii) C