Exercise - 7

Part : (A) Only one correct option

1. If a, b, c > 0 & x, y, z $\in$ R, then the determinant $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$ =

(A) $a^x b^y c^z$ (B) $a^{-x} b^{-y} c^{-z}$ (C) $a^{2x} b^{2y} c^{2z}$ (D) zero

2. If a, b & c are non-zero real numbers, then D = $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ =

(A) abc (B) $a^2 b^2 c^2$ (C) bc + ca + ab (D) zero

3. The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix}$ =

(A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (B) 2 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (C) 3 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (D) none of these

4. The system of linear equations x + y – z = 6, x + 2y – 3z = 14 and 2x + 5y – λz = 9 ($\lambda \in$ R) has a unique solution if

(A) $\lambda = 8$ (B) $\lambda \neq 8$ (C) $\lambda = 7$ (D) $\lambda \neq 7$

5. If the system of equations x + 2y + 3z = 4, x + py + 2z = 3, x + 4y + μz = 3 has an infinite number of solutions, then:

(A) p = 2, μ = 3 (B) p = 2, μ = 4 (C) 3 p = 2 μ (D) none of these

6. Let f (θ) = $\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$ then $f\left(\frac{\pi}{6}\right)$ =

(A) 0 (B) 1 (C) 2 (D) none

7. The determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is:

(A) 0 (B) independent of θ
(C) independent of ϕ (D) independent of θ & ϕ both

8. Value of Δ = $\begin{vmatrix} \sin(2\alpha) & \sin(\alpha+\beta) & \sin(\alpha+\gamma) \\ \sin(\beta+\alpha) & \sin(2\beta) & \sin(\gamma+\beta) \\ \sin(\gamma+\alpha) & \sin(\gamma+\beta) & \sin(2\gamma) \end{vmatrix}$ is

(A) $\Delta = 0$ (B) $\Delta = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$
(C) $\Delta = 3/2$ (D) none of these

9. If a, b, c are complex number and z = $\begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ is

(A) purely real (B) purely imaginary (C) 0 (D) none of these

10. If A, B, C are angles of a triangle ABC, then $\begin{vmatrix} \sin\frac{A}{2} & \sin\frac{B}{2} & \sin\frac{C}{2} \\ \sin(A+B+C) & \sin\frac{B}{2} & \cos\frac{A}{2} \\ \cos\frac{(A+B+C)}{2} & \tan(A+B+C) & \sin\frac{C}{2} \end{vmatrix}$ is less than or equal to

(A) $\frac{3\sqrt{3}}{8}$ (B) $\frac{1}{8}$ (C) $2\sqrt{2}$ (D) 2

11. $\Delta = \begin{vmatrix} 1 & \frac{4\sin B}{b} & \cos A \\ 2a & 8\sin A & 1 \\ 3a & 12\sin A & \cos B \end{vmatrix}$ is (where a, b, c are the sides opposite to angles A, B, C respectively in a triangle)

(A) $\frac{1}{2}\cos 2A$ (B) 0 (C) $\frac{1}{2}\sin 2A$ (D) $\frac{1}{2}(\cos^2 A + \cos^2 B)$

12. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc\,(a+b+c)^3$ then the value of k is

(A) 1 (B) 2 (C) 0 (D) ab + bc + ac

13. Let m be a positive integer & $D_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$ $(0 \le r \le m)$, then the value of $\sum_{r=0}^{m} D_r$ is given by:

(A) 0 (B) $m^2 - 1$ (C) 2^m (D) $2^m \sin^2(2^m)$

14. If a, b, c, are real numbers, and $D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$ then D is

(A) purely real (B) purely imaginary (C) non real (D) integer

15. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then f(100) is equal to: **[IIT – 1999, 2]**

(A) 0 (B) 1 (C) 100 (D) – 100

Part : (B) May have more than one options correct

16. Let $\phi_1(x) = x + a_1$, $\phi_2(x) = x^2 + b_1x + b_2$ and $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \phi_1(x_1) & \phi_1(x_2) & \phi_1(x_3) \\ \phi_2(x_1) & \phi_2(x_2) & \phi_2(x_3) \end{vmatrix}$, then

(A) Δ is independent of a_1 (B) Δ is independent of b_1 and b_2
(C) Δ is independent of x_1, x_2 and x_3 (D) none of these

17. If $\Delta = \begin{vmatrix} x & 2y-z & -z \\ y & 2x-z & -z \\ y & 2y-z & 2x-2y-z \end{vmatrix}$, then

(A) x – y is a factor of Δ (B) $(x-y)^2$ is a factor of Δ
(C) $(x-y)^3$ is a factor of Δ (D) Δ is independent of z

18. Let $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$, then

(A) Δ is independent of θ (B) Δ is indepedent of ϕ
(C) Δ is a constant (D) $\left.\frac{d\Delta}{d\theta}\right]_{\theta=\pi/2} = 0$

19. Let $\Delta = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then

(A) x + a is a factor of Δ (B) $(x+a)^2$ is a factor of Δ
(C) $(x+a)^3$ is a factor of Δ (D) $(x+a)^4$ is not a factor of Δ

20. Let $\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$, then

(A) $1 - x^3$ is a factor of Δ (B) $(1 - x^3)^2$ is factor of Δ
(C) $\Delta(x) = 0$ has 4 real roots (D) $\Delta'(1) = 0$

21. The determinent $\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix}$ is equal to zero if

(A) b, c, d are in A.P. (B) b, c, d are in G.P.
(C) b, c, d are in H.P. (D) α is a root of $ax^3 - bx^2 - 3cx - d = 0$

Exercise - 8

1. Using the properties of determinants, evalulate:

(i) $\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$. (ii) $\begin{vmatrix} \sqrt{13} + \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} + \sqrt{26} & 5 & \sqrt{10} \\ 3 + \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$.

2. Find the non – zero roots of the equation, $\Delta = \begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ ax + b & bx + c & c \end{vmatrix} = 0$.

3. Show that $\Delta = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2b^2c^2$

4. Prove that, $\begin{vmatrix} 2 & \alpha + \beta + \gamma + \delta & \alpha\beta + \gamma\delta \\ \alpha + \beta + \gamma + \delta & 2(\alpha + \beta)(\gamma + \delta) & \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) \\ \alpha\beta + \gamma\delta & \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) & 2\alpha\beta\gamma\delta \end{vmatrix} = 0$.

5. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha - \beta)^2 (\beta - \gamma)^2 (\gamma - \alpha)^2$.

6. Find the value of 'a' if the three equations,
$(a + 1)^3 x + (a + 2)^3 y = (a + 3)^3$; $(a + 1) x + (a + 2) y = (a + 3)$ & $x + y = 1$ are consistent.

7. Investigate for what values of λ, μ the simultaneous equations
$x + y + z = 6$; $x + 2y + 3z = 10$ & $x + 2y + \lambda z = \mu$ have;
(a) A unique solution
(b) An infinite number of solutions.
(c) No solution.

8. Find those values of c for which the equations:
$2x + 3y = 3$
$(c + 2)x + (c + 4)y = c + 6$
$(c + 2)^2 x + (c + 4)^2 y = (c + 6)^2$ are consistent.
Also solve above equations for these values of c.

9. Prove that $\Delta = \begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix} = (\alpha\beta' - \alpha'\beta)(\beta\gamma' - \beta'\gamma)(\gamma\alpha' - \gamma'\alpha)$

10. If $a^2 + b^2 + c^2 = 1$, then prove that $\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1 - \cos\phi) & ac(1 - \cos\phi) \\ ba(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1 - \cos\phi) \\ ca(1 - \cos\phi) & cb(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$
is independent of a, b, c

11. Show that the value of the determinant $\begin{vmatrix} \tan(A+P) & \tan(B+P) & \tan(C+P) \\ \tan(A+Q) & \tan(B+Q) & \tan(C+Q) \\ \tan(A+R) & \tan(B+R) & \tan(C+R) \end{vmatrix}$ vanishes for all values of
A, B, C, P, Q & R where A + B + C + P + Q + R = 0.

12. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

13. Show that, $\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix} = \sin(2x+2x^2)$.

14. If $\begin{vmatrix} \frac{1}{a+x} & \frac{1}{b+x} & \frac{1}{c+x} \\ \frac{1}{a+y} & \frac{1}{b+y} & \frac{1}{c+y} \\ \frac{1}{a+z} & \frac{1}{b+z} & \frac{1}{c+z} \end{vmatrix} = \frac{P}{Q}$ where Q is the product of the denominators, prove that $P = (a-b)(b-c)(c-a)(x-y)(y-z)(z-x)$

15. If A_1, B_1, C_1,................are respectively the cofactors of the elements a_1, b_1, c_1,...........of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then prove that

(i) $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} = a_1\Delta$.   (ii) $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \Delta^2$

16. Show that, $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} a^2 & c^2 & 2ac-b^2 \\ 2ab-c^2 & b^2 & a^2 \\ b^2 & 2bc-a^2 & c^2 \end{vmatrix}$

17. Using consistancy of equations, prove that if $bc+qr = ca+rp = ab+pq = -1$ then $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$.

18. Show that : $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix} = \sin(\alpha-\beta) + \sin(\beta-\gamma) + \sin(\gamma-\alpha)$.

19. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c \equiv (l_1x + m_1y + n_1)(l_2x + m_2y + n_2)$, then prove that $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

20. Find all the values of t for which the system of equations;

$(t-1)x + (3t+1)y + 2tz = 0$
$(t-1)x + (4t-2)y + (t+3)z = 0$
$2x + (3t+1)y + 3(t-1)z = 0$

has non trivial solutions and in this context find the ratios of x: y: z, when t has the smallest of these values.

21. Let a > 0, d > 0. Find the value of determinant

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}.$$ **[IIT – 1996, 5]**

22. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$ represents a straight line **[IIT – 2001, 6]**

Exercise - 7

1. D **2.** D **3.** B **4.** B **5.** D **6.** B **7.** B

8. A **9.** B **10.** B **11.** B **12.** B **13.** A **14.** A

15. A **16.** AB **17.** AB **18.** BD **19.** ABD **20.** ABD

21. BD

Exercise - 8

1. (i) 0 (ii) $5(3\sqrt{2}-5\sqrt{3})$ **2.** x = – 2 b/a

6. a = – 2

7. (a) $\lambda \ne 3$ (b) $\lambda = 3, \mu = 10$ (c) $\lambda = 3, \mu \ne 10$

8. for c = 0, x = – 3, y = 3; for c = – 10, $x = -\frac{1}{2}$, $y = \frac{4}{3}$

20. t = 0 or 3; x: y: z = 1: 1: 1

21. $\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$

Exercise - 9

Part : (A) Only one correct option

1. Let a, b, c, d, u, v be integers. If the system of equations ax + by = u, cx + dy = v has a unique solution in integers, then
(A) ad – bc = 1
(B) ad – bc = – 1
(C) ad – bc ≠ 0
(D) ad – bc need not be equal to ± 1

2. If AB = O for the matrices
$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\theta - \phi$ is
(A) an odd multiple of $\frac{\pi}{2}$
(B) an odd multiple of π
(C) an even multiple of $\frac{\pi}{2}$
(D) 0

3. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then value of X^n is
(A) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$
(B) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$
(C) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$
(D) none of these

4. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then
(A) $\alpha = \pm\frac{1}{\sqrt{2}}$
(B) $\beta = \pm\frac{1}{\sqrt{6}}$
(C) $\gamma = \pm\frac{1}{\sqrt{3}}$
(D) all of these

5. If A, B are two n × n non-singular matrices, then
(A) AB is non-singular
(B) AB is singular
(C) $(AB)^{-1} = A^{-1}B^{-1}$
(D) $(AB)^{-1}$ does not exist

6. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to
(A) det (A^{-1})
(B) det (B^{-1})
(C) det (A)
(D) det (B)

7. If A is a square matrix of order n × n and k is a scalar, then adj (kA) is equal to
(A) k adj A
(B) k^n adj A
(C) k^{n-1} adj A
(D) k^{n+1} adj A

8. Let A be a matrix of rank r. Then
(A) rank (A^T) = r
(B) rank (A^T) < r
(C) rank (A^T) > r
(D) none of these

9. If A = dig (2, –1, 3), B = dig (–1, 3, 2), then A^2B =
(A) dig (5, 4, 11)
(B) dig (–4, 3, 18)
(C) dig (3, 1, 8)
(D) B

10. If ω is a cube root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then A^{-1} =

(A) $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ (B) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ (D) $\frac{1}{2}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

11. If the system of equations ax + y + z = 0,. x + by + z = 0 and x + y + cz = 0, where a, b, c $\neq$ 1, has a non–trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is:
(A) –1 (B) 0 (C) 1 (D) None of these

12. If A is a square matrix of order 3, then the true statement is (where I is unit matrix).
(A) det (– A) = – det A (B) det A = 0
(C) det (A + I) = 1 + det A (D) det 2A = 2 det A

13. Which of the following is incorrect
(A) $A^2 - B^2 = (A + B)(A - B)$ (B) $(A^T)^T = A$
(C) $(AB)^n = A^nB^n$, where A, B commute (D) $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

14. The value of a for which system of equations, $a^3x + (a + 1)^3y + (a + 2)^3z = 0$, $ax + (a + 1)y + (a + 2)z = 0$, $x + y + z = 0$, has a non–zero solution is:
(A) –1 (B) 0 (C) 1 (D) none of these

15. If A $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ then AB is equal to
(A) B (B) 3B (C) B^3 (D) A + B

16. If A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies the equation $x^2 - (a + d)x + k = 0$, then
(A) k = bc (B) k = ad
(C) $k = a^2 + b^2 + c^2 + d^2$ (D) ad – bc

17. Let A = $\begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$, then A^{-1} exists if
(A) $x \neq 0$ (B) $\lambda \neq 0$ (C) $3x + \lambda \neq 0$, $\lambda \neq 0$ (D) $x \neq 0$, $\lambda \neq 0$

18. Identity the correct statement
(A) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is singular
(B) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is non singular
(C) If A^{-1} exists, $(\text{adj } A)^{-1}$ may or may not exist
(D) F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then F(x) . F(y) = F(x – y)

19. Matrix A = $\begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if x y z = 60 and 8x + 4y + 3z = 20, then A(adj A) is equal to
(A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (B) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$ (C) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (D) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

20. If P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^1 and x = $P^1Q^{2005}P$, then x is equal to **[IIT JEE - 2005]**
(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$
(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

Comprehension **[IIT JEE - 2006]**

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2, and U_3 are columns matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If U is 3×3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

21. The value of $|U|$ is **[IIT JEE - 2006]**
(A) 3 (B) –3 (C) 3/2 (D) 2

22. The sum of the elements of U^{-1} is **[IIT JEE - 2006]**
(A) –1 (B) 0 (C) 1 (D) 3

23. The value of $[3\ 2\ 0]\, U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is **[IIT JEE - 2006]**
(A) 5 (B) 5/2 (C) 4 (D) 3/2

Part : (B) May have more than one options correct

24. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is
(A) 2 if a = 6 (B) 2 if a = 1 (C) 1 if a = 2 (D) 1 if a = – 6

25. Which of the following statement is always true
(A) Adjoint of a symmetric matrix is a symmetric matrix
(B) Adjoint of a unit matrix is unit matrix
(C) A (adj A) = (adj A) A
(D) Adjoint of a diagonal matrix is diagonal matrix

26. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if
(A) $\alpha = 1/2$ (B) a, b, c are in A.P. (C) a, b, c are in G.P. (D) a, b, c are in H.P.

27. The singularity of matrix $\begin{bmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{bmatrix}$ depends upon which of the following parameter
(A) a (B) p (C) x (D) d

28. Which of the following statement is true
(A) Every skew symmetric matrix of odd order is non singular
(B) If determinant of a square matrix is nonzero, then it non singular
(C) Rank of a matrix is equal or higher than the order of the matrix
(D) Adjoint of a singular matrix is always singular

29. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then
(A) a + d = 0 (B) $k = -|A|$ (C) $k = |A|$ (D) none of these

30. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then
(A) $|A| = 2$ (B) A is non-singular
(C) Adj. $A = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$ (D) A is skew symmetric matrix

Exercise - 10

1. Find x so that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

2. If A and B are two square matrices such that AB = A & BA = B, prove that A & B are idempotent

3. If $f(x) = x^2 - 5x + 7$, find f (A) where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

4. Prove that the product of matrices

$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is the null matrix, when θ and ϕ differ by an odd multiple of $\pi/2$.

5. Given $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. If $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Then for what values of y, $F(x + y) = F(x)\,F(y)$.

6. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law $A^t A = I$.

7. Compute A^{-1} for the following matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Hence solve the system of equations;
$-x + 2y + 5z = 2$; $2x - 3y + z = 15$ & $-x + y + z = -3$

8. Show that $\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

9. Gaurav purchases 3 pens, 2 bags and 1 instrument box and pays Rs. 41. From the same shop Dheeraj purchases 2 pens, 1 bag and 2 instrument boxes and pays Rs. 29, while Ankur purchases 2 pens, 2 bags and 2 instrument boxes and pays Rs. 44. Translate the problem into a system of equations. Solve the system of equations by matrix method and hence find the cost of 1 pen, 1 bag and 1 instrument box.

10. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that $A^2 - 4A - 5I = O$.

(a) using A^{-1} **(b)** without using A^{-1}

11. Having given equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ where x, y, z are not all zero, prove that $a^2 + b^2 + c^2 + 2abc - 1 = 0$.

12. Consider the system of linear equations in x, y, z:
$(\sin 3\theta)\, x - y + z = 0$
$(\cos 2\theta)\, x + 4y + 3z = 0$
$2x + 7y + 7z = 0$
Find the values of θ for which this system has non – trivial solution.

13. Solve the following systems of linear equations by using the principle of matrix.

(i) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

(ii) $x + y + z = 9$
$2x + 5y + 7z = 52$
$2x + y - z = 0$

14. Compute A^{-1}, if $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$.

15. Find the rank of the following matrices:

(i) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

16. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations.
$x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$.

17. If $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, where a, b, c are real positive numbers, $abc = 1$ and $A^T A = 1$, then find the value of $a^3 + b^3 + c^3$. **[IIT JEE - 2003, 2]**

18. If M is 3×3 matrix M has its det.(M) = 1 and $MM^T = I$. Prove that del $(M - I) = 0$. **[IIT JEE - 2004, 2]**

19. If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$ $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and AX = U has infinitely many solution. Prove that BX = V has no unique solution, also prove that if afd $\neq$ 0, then BX = V has no solution. **[IIT JEE - 2004, 4]**

Exercise - 9

1. C **2.** A **3.** D **4.** D **5.** A **6.** C **7.** C

8. A **9.** B **10.** B **11.** C **12.** A **13.** A **14.** A

15. B **16.** D **17.** C **18.** B **19.** C **20.** A **21.** A

22. B **23.** A **24.** ABD **25.** ABCD **26.** AB

27. CD **28.** BD **29.** AC **30.** BC

Exercise - 10

1. $-\dfrac{9}{8}$ **3.** f(A) = 0 **5.** $y \,\varepsilon\, R$

6. $x = \pm\dfrac{1}{\sqrt{2}}$, $y = \pm\dfrac{1}{\sqrt{6}}$, $z = \pm\dfrac{1}{\sqrt{3}}$

7. $A^{-1} = -\dfrac{1}{7}\begin{bmatrix} -4 & 3 & 17 \\ -3 & 4 & 11 \\ -1 & -1 & -1 \end{bmatrix}$ & x = 2, y = –3, z = 2

9. Rs. 2, Rs. 15 & Rs. 5

12. $\theta = n\pi,\ n\pi + (-1)^n \dfrac{\pi}{6}$; $n \in I$

13. (i) x = 2; y = 2; z = 2 (ii) x = 1; y = 3; z = 5

14. x = 1; y = 2; z = 3

15. (i) 2 (ii) 3 (iii) 2 (iv) 2

16. x = 3; y = – 2; z = – 1 **17.** 4