Sample Paper–2
Class 11, Mathematics

Time: 3 hours **Max. Marks 100**

General Instructions

1. All questions are compulsory.
2. Use of calculator is not permitted. However you may use log table, if required.
3. Q.No. 1 to 12 are of very short answer type questions, carrying 1 mark each.
4. Q.No.13 to 28 carries 4 marks each.
5. Q.No. 29 to 32 carries 6 marks each.

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1. What universal set (s) would you propose for each of the following:
   (i) The set of right triangles
   (ii) The set of isosceles triangles

2. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

3. Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {(x, y): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.

4. Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length
   (i) 10 cm (ii) 15 cm

5. Find the value of the trigonometric function sin 765°

6. Express the given complex number in the form $a + ib$: $(1 - i)^4$

7. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

8. Name the octants in which the following points lie: (–4, 2, –5), (–4, 2, 5),

9. Evaluate the Given limit:
   $$\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

10. Evaluate the Given limit:
   $$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$

11. 2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

12. A die is thrown. Describe the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

13. Show that $A \cap B = A \cap C$ need not imply $B = C$.

14. Find the range of each of the following functions: $f(x) = x^2 + 2$, x, is a real number.

15. Find the general solution of cosec $x = -2$

16. Prove the following by using the principle of mathematical induction for all $n \in N$: $n(n+1)(n+5)$ is a multiple of 3.

17. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

18. Solve the following inequalities and represent the solution graphically on number line: $3x - 7 > 2(x - 6), 6 - x > 11 - 2x$

19. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

20. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

21. Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$.

22. If a, b, c and d are in G.P. show that $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$.

23. A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

24. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the

longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

25. Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $x^4 (5 \sin x - 3 \cos x)$

27. State the converse and contrapositive of each of the following statements:
(i) p: A positive integer is prime only if it has no divisors other than 1 and itself.
(ii) q: I go to a beach whenever it is a sunny day.
(iii) r: If it is hot outside, then you feel thirsty.

28. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that
(a) you both enter the same sections?
(b) you both enter the different sections?

29. In a group of students 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

30. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\tan x = -\frac{4}{3}$, x in quadrant II

31. A farmer buys a used tractor for Rs12000. He pays Rs6000 cash and agrees to pay the balance in annual instalments of Rs500 plus 12% interest on the unpaid amount. How much will be the tractor cost him?

32. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the remaining two observations.

Solutions

1. **(i)** For the set of right triangles, the universal set can be the set of triangles or the set of polygons.
   (ii) For the set of isosceles triangles, the universal set can be the set of triangles or the set of polygons or the set of two-dimensional figures.

2. We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
   $\therefore n(A \times A) = n(A) \times n(A)$
   It is given that $n(A \times A) = 9$
   $\therefore n(A) \times n(A) = 9$
   $\Rightarrow n(A) = 3$
   The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.
   We know that $A \times A = \{(a, a): a \in A\}$. Therefore, –1, 0, and 1 are elements of A.
   Since $n(A) = 3$, it is clear that A = {–1, 0, 1}.
   The remaining elements of set A × A are (–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), and (1, 1)

3. The relation R from A to A is given as
   $R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$
   i.e., $R = \{(x, y): 3x = y, \text{ where } x, y \in A\}$
   $\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
   The domain of R is the set of all first elements of the ordered pairs in the relation.
   $\therefore$Domain of R = {1, 2, 3, 4}
   The whole set A is the codomainof the relation R.
   $\therefore$Codomain of R = A = {1, 2, 3, …, 14}
   The range of R is the set of all second elements of the ordered pairs in the relation.
   $\therefore$Range of R = {3, 6, 9, 12}

4. We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.
   It is given that $r = 75$ cm
   (i) Here, $l = 10$ cm
   $$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$
   (ii) Here, $l = 15$ cm
   $$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$
   (iii) Here, $l = 21$ cm
   $$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

5. It is known that the values of sin x repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin(2\times 360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$$

6.

$$\begin{aligned}(1-i)^4 &= \left[(1-i)^2\right]^2\\ &= \left[1^2 + i^2 - 2i\right]^2\\ &= [1-1-2i]^2\\ &= (-2i)^2\\ &= (-2i)\times(-2i)\\ &= 4i^2 = -4 \qquad\qquad [i^2 = -1]\end{aligned}$$

7. Each signal requires the use of 2 flags.

There will be as many flags as there are ways of filling in 2 vacant places in succession by the given 5 flags of different colours.
The upper vacant place can be filled in 5 different ways by any one of the 5 flags following which, the lower vacant place can be filled in 4 different ways by any one of the remaining 4 different flags.
Thus, by multiplication principle, the number of different signals that can be generated is $5\times 4 = 20$

8. The x-coordinate, y-coordinate, and z-coordinate of point (–4, 2, –5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.
The x-coordinate, y-coordinate, and z-coordinate of point (–4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

9.

$$\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$$

At $z = 1$, the value of the given function takes the form $\frac{0}{0}$.

Put $z^{\frac{1}{6}} = x$ so that $z \to 1$ as $x \to 1$.

$$\text{Accordingly, } \lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=\lim_{x\to1}\frac{x^2-1}{x-1}$$

$$=\lim_{x\to1}\frac{x^2-1^2}{x-1}$$

$$=2.1^{2-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=2$$

$$\therefore \lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=2$$

10.

$$\lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$$

At $x = -2$, the value of the given function takes the form $\frac{0}{0}$.

$$\text{Now, } \lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to-2}\frac{\left(\frac{2+x}{2x}\right)}{x+2}$$

$$=\lim_{x\to-2}\frac{1}{2x}$$

$$=\frac{1}{2(-2)}=\frac{-1}{4}$$

11. Let us denote 2 boys and 2 girls in room X as B_1, B_2 and G_1, G_2 respectively. Let us denote 1 boy and 3 girls in room Y as B_3, and G_3, G_4, G_5 respectively.
Accordingly, the required sample space is given by
$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$

12. When a die is thrown, the sample space is given by S = {1, 2, 3, 4, 5, 6}.
Accordingly:
(i) A = {1, 2, 3, 4, 5, 6}
(ii) B = Φ
(iii) C = {3, 6}
(iv) D = {1, 2, 3}
(v) E = {6}
(vi) F = {3, 4, 5, 6}

A ∪ B = {1, 2, 3, 4, 5, 6}, A ∩ B = Φ
B ∪ C = {3, 6}, E ∩ F = {6}
D ∩ E =Φ, A – C = {1, 2, 4, 5}
D – E = {1, 2, 3}, $F' = \{1,2\}$, $E \cap F' = \phi$

13. Let A = {0, 1}, B = {0, 2, 3}, and C = {0, 4, 5}
Accordingly, $A \cap B = \{0\}$ and $A \cap C = \{0\}$
Here, $A \cap B = A \cap C = \{0\}$
However, $B \neq C$ [$2 \in B$ and $2 \notin C$]

14. $f(x) = x^2 + 2$, x, is a real number
The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f = [2, \infty)$
Alter:
Let x be any real number.
Accordingly,
$x^2 \geq 0$
$\Rightarrow x^2 + 2 \geq 0 + 2$
$\Rightarrow x^2 + 2 \geq 2$
$\Rightarrow f(x) \geq 2$
$\therefore$ Range of $f = [2, \infty)$

15. cosec $x = -2$
It is known that
$$\operatorname{cosec}\frac{\pi}{6} = 2$$
$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$
$$\text{i.e., } \operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$
Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.
$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$
$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$
$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$
Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

16. Let the given statement be P(n), i.e.,
P(n): n (n + 1) (n + 5), which is a multiple of 3.
It can be noted that P(n) is true for n = 1 since 1 (1 + 1) (1 + 5) = 12, which is a multiple of 3.
Let P(k) be true for some positive integer k, i.e.,

k (k + 1) (k + 5) is a multiple of 3.

∴k (k + 1) (k + 5) = 3m, where $m \in \mathbf{N}$ … (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$$(k+1)\{(k+1)+1\}\{(k+1)+5\}$$
$$=(k+1)(k+2)\{(k+5)+1\}$$
$$=(k+1)(k+2)(k+5)+(k+1)(k+2)$$
$$=\{k(k+1)(k+5)+2(k+1)(k+5)\}+(k+1)(k+2)$$
$$=3m+(k+1)\{2(k+5)+(k+2)\}$$
$$=3m+(k+1)\{2k+10+k+2\}$$
$$=3m+(k+1)(3k+12)$$
$$=3m+3(k+1)(k+4)$$
$$=3\{m+(k+1)(k+4)\}=3\times q, \text{ where } q=\{m+(k+1)(k+4)\} \text{ is some natural number}$$

Therefore, $(k+1)\{(k+1)+1\}\{(k+1)+5\}$ is a multiple of 3.

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

17. Let $z=\frac{1+2i}{1-3i}$, then

$$z=\frac{1+2i}{1-3i}\times\frac{1+3i}{1+3i}=\frac{1+3i+2i+6i^2}{1^2+3^2}=\frac{1+5i+6(-1)}{1+9}$$
$$=\frac{-5+5i}{10}=\frac{-5}{10}+\frac{5i}{10}=\frac{-1}{2}+\frac{1}{2}i$$

Let $z=r\cos\theta+ir\sin\theta$

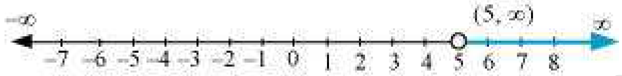
i.e., $r\cos\theta=\frac{-1}{2}$ and $r\sin\theta=\frac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=\left(\frac{-1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$
$$\Rightarrow r^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$
$$\Rightarrow r=\frac{1}{\sqrt{2}} \qquad [\text{Conventionally, } r>0]$$
$$\therefore \frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$$
$$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$
$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

18. $3x - 7 > 2(x - 6)$
$\Rightarrow 3x - 7 > 2x - 12$
$\Rightarrow 3x - 2x > -12 + 7$
$\Rightarrow x > -5 \dots (1)$
$6 - x > 11 - 2x$
$\Rightarrow -x + 2x > 11 - 6$
$\Rightarrow x > 5 \dots (2)$
From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(5, \infty)$. The solution of the given system of inequalities can be represented on number line as



19. 2 different vowels and 2 different consonants are to be selected from the English alphabet.
Since there are 5 vowels in the English alphabet, number of ways of selecting 2 different vowels from the alphabet = ${}^5C_2 = \frac{5!}{2!3!} = 10$
Since there are 21 consonants in the English alphabet, number of ways of selecting 2 different consonants from the alphabet $= {}^{21}C_2 = \frac{21!}{2!19!} = 210$
Therefore, number of combinations of 2 different vowels and 2 different consonants
$=10 \times 210 = 2100$
Each of these 2100 combinations has 4 letters, which can be arranged among themselves in 4! ways.
Therefore, required number of words = $2100 \times 4! = 50400$

20. Out of 17 players, 5 players are bowlers.
A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.
4 bowlers can be selected in ${}^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_7$ ways.
Thus, by multiplication principle, required number of ways of selecting cricket team $= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$

21. Firstly, the expression $(x + y)^4 + (x - y)^4$ is simplified by using Binomial Theorem.
This can be done as

$$(x+y)^4 = {}^4C_0x^4 + {}^4C_1x^3y + {}^4C_2x^2y^2 + {}^4C_3xy^3 + {}^4C_4y^4$$
$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$
$$(x-y)^4 = {}^4C_0x^4 - {}^4C_1x^3y + {}^4C_2x^2y^2 - {}^4C_3xy^3 + {}^4C_4y^4$$
$$= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$
$$\therefore (x+y)^4 + (x-y)^4 = 2\left(x^4 + 6x^2y^2 + y^4\right)$$

Putting $x = a^2$ and $y = \sqrt{a^2-1}$, we obtain

$$\left(a^2+\sqrt{a^2-1}\right)^4 + \left(a^2-\sqrt{a^2-1}\right)^4 = 2\left[\left(a^2\right)^4 + 6\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2 + \left(\sqrt{a^2-1}\right)^4\right]$$
$$= 2\left[a^8 + 6a^4\left(a^2-1\right) + \left(a^2-1\right)^2\right]$$
$$= 2\left[a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1\right]$$
$$= 2\left[a^8 + 6a^6 - 5a^4 - 2a^2 + 1\right]$$
$$= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$$

22. a, b, c, d are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc - cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= L.H.S.

∴ L.H.S. = R.H.S.

$$\therefore \left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right) = \left(ab+bc+cd\right)^2$$

23. Let the coordinates of point A be $(a, 0)$.

Draw a line (AL) perpendicular to the x-axis.

We know that angle of incidence is equal to angle of reflection. Hence, let

$\angle BAL = \angle CAL = \Phi$

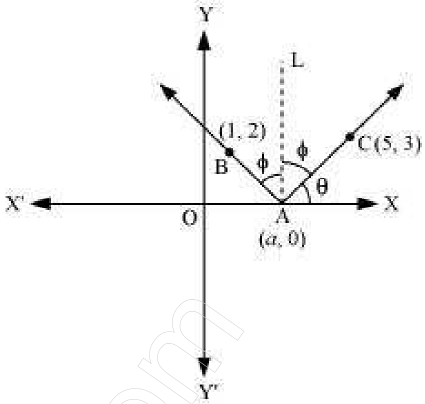
Let $\angle CAX = \theta$

$\therefore \angle OAB = 180° - (\theta + 2\Phi) = 180° - [\theta + 2(90° - \theta)]$

$= 180° - \theta - 180° + 2\theta$

$= \theta$

$\therefore \angle BAX = 180° - \theta$



Now, slope of line AC $= \dfrac{3-0}{5-a}$

$\Rightarrow \tan\theta = \dfrac{3}{5-a}$ ...(1)

Slope of line AB $= \dfrac{2-0}{1-a}$

$\Rightarrow \tan(180° - \theta) = \dfrac{2}{1-a}$

$\Rightarrow -\tan\theta = \dfrac{2}{1-a}$

$\Rightarrow \tan\theta = \dfrac{2}{a-1}$ ...(2)

From equations (1) and (2), we obtain

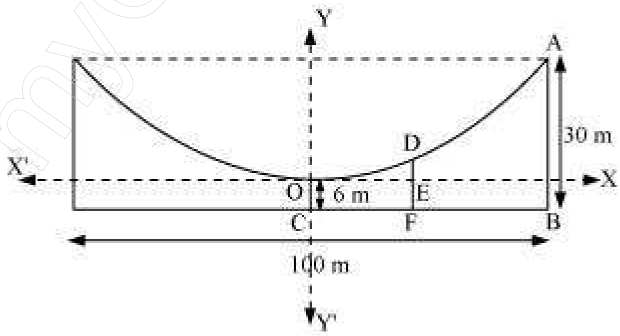
$\dfrac{3}{5-a} = \dfrac{2}{a-1}$

$\Rightarrow 3a - 3 = 10 - 2a$

$\Rightarrow a = \dfrac{13}{5}$

Thus, the coordinates of point A are $\left(\dfrac{13}{5}, 0\right)$.

24. The vertex is at the lowest point of the cable. The origin of the coordinate plane is taken as the vertex of the parabola, while its vertical axis is taken along the positive y-axis. This can be diagrammatically represented as



Here, AB and OC are the longest and the shortest wires, respectively, attached to the cable. DF is the supporting wire attached to the roadway, 18 m from the middle.

Here, AB = 30 m, OC = 6 m, and $BC = \dfrac{100}{2} = 50$ m.

The equation of the parabola is of the form $x^2 = 4ay$ (as it is opening upwards).

The coordinates of point A are (50, 30 – 6) = (50, 24).

Since A (50, 24) is a point on the parabola,

$(50)^2 = 4a(24)$

$\Rightarrow a = \frac{50\times 50}{4\times 24} = \frac{625}{24}$

∴Equation of the parabola, $x^2 = 4\times\frac{625}{24}\times y$ or $6x^2 = 625y$

The x-coordinate of point D is 18.

Hence, at $x = 18$,

$6(18)^2 = 625y$

$\Rightarrow y = \frac{6\times 18\times 18}{625}$

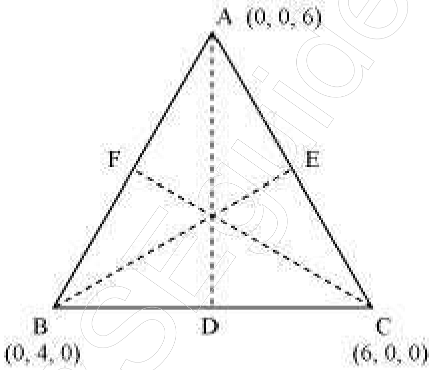
$\Rightarrow y = 3.11$ (approx)

∴DE = 3.11 m

DF = DE + EF = 3.11 m + 6 m = 9.11 m

Thus, the length of the supporting wire attached to the roadway 18 m from the middle is approximately 9.11 m.

25. Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

∴Coordinates of point D = $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ = (3, 2, 0)

$AD = \sqrt{(0-3)^2+(0-2)^2+(6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Since BE is the median, E is the mid-point of AC.

$\therefore$ Coordinates of point E $= \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = (3,0,3)$

$BE = \sqrt{(3-0)^2+(0-4)^2+(3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$

Since CF is the median, F is the mid-point of AB.

$\therefore$ Coordinates of point F $= \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0,2,3)$

Length of $CF = \sqrt{(6-0)^2+(0-2)^2+(0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7 .

26. Let $f(x) = x^4(5\sin x - 3\cos x)$

By product rule,

$$f'(x) = x^4 \frac{d}{dx}(5\sin x - 3\cos x) + (5\sin x - 3\cos x)\frac{d}{dx}(x^4)$$

$$= x^4\left[5\frac{d}{dx}(\sin x) - 3\frac{d}{dx}(\cos x)\right] + (5\sin x - 3\cos x)\frac{d}{dx}(x^4)$$

$$= x^4\left[5\cos x - 3(-\sin x)\right] + (5\sin x - 3\cos x)(4x^3)$$

$$= x^3\left[5x\cos x + 3x\sin x + 20\sin x - 12\cos x\right]$$

27. (i) Statement p can be written as follows.
If a positive integer is prime, then it has no divisors other than 1 and itself.
The converse of the statement is as follows.
If a positive integer has no divisors other than 1 and itself, then it is prime.
The contrapositive of the statement is as follows.
If positive integer has divisors other than 1 and itself, then it is not prime.

(ii) The given statement can be written as follows.
If it is a sunny day, then I go to a beach.
The converse of the statement is as follows.
If I go to a beach, then it is a sunny day.
The contrapositive of the statement is as follows.
If I do not go to a beach, then it is not a sunny day.

(iii) The converse of statement r is as follows.
If you feel thirsty, then it is hot outside.
The contrapositive of statement r is as follows.
If you do not feel thirsty, then it is not hot outside.

28. My friend and I are among the 100 students.

Total number of ways of selecting 2 students out of 100 students = ${}^{100}C_2$

(a) The two of us will enter the same section if both of us are among 40 students or among 60 students.

∴ Number of ways in which both of us enter the same section = ${}^{40}C_2 + {}^{60}C_2$

∴ Probability that both of us enter the same section

$$= \frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2} = \frac{\frac{40!}{2!\,38!} + \frac{60!}{2!\,58!}}{\frac{100!}{2!\,98!}} = \frac{(39\times 40) + (59\times 60)}{99\times 100} = \frac{17}{33}$$

(b) P(we enter different sections)

= 1 − P(we enter the same section)

$= 1 - \frac{17}{33} = \frac{16}{33}$

29. Let U be the set of all students in the group.
Let E be the set of all students who know English.
Let H be the set of all students who know Hindi.
$\therefore$ H ∪ E = U
Accordingly, $n(\text{H}) = 100$ and $n(\text{E}) = 50$
$n(\text{H} \cap \text{E}) = 25$
$n(\text{U}) = n(\text{H}) + n(\text{E}) - n(\text{H} \cap \text{E})$
$= 100 + 50 - 25$
$= 125$
Hence, there are 125 students in the group.

30. Here, x is in quadrant II.
i.e., $\frac{\pi}{2} < x < \pi$
$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$
Therefore, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.
It is given that $\tan x = -\frac{4}{3}$.
$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$
$\therefore \cos^2 x = \frac{9}{25}$
$\Rightarrow \cos x = \pm\frac{3}{5}$
As x is in quadrant II, $\cos x$ is negative.
$\therefore \cos x = \frac{-3}{5}$
Now, $\cos x = 2\cos^2\frac{x}{2} - 1$
$\Rightarrow \frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$
$\Rightarrow 2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2\frac{x}{2}=\frac{2}{5}$

$\Rightarrow \cos^2\frac{x}{2}=\frac{1}{5}$

$\Rightarrow \cos\frac{x}{2}=\frac{1}{\sqrt{5}}$ $\qquad\left[\because \cos\frac{x}{2} \text{ is positive}\right]$

$\therefore \cos\frac{x}{2}=\frac{\sqrt{5}}{5}$

$\sin^2\frac{x}{2}+\cos^2\frac{x}{2}=1$

$\Rightarrow \sin^2\frac{x}{2}+\left(\frac{1}{\sqrt{5}}\right)^2=1$

$\Rightarrow \sin^2\frac{x}{2}=1-\frac{1}{5}=\frac{4}{5}$

$\Rightarrow \sin\frac{x}{2}=\frac{2}{\sqrt{5}}$ $\qquad\left[\because \sin\frac{x}{2} \text{ is positive}\right]$

$\therefore \sin\frac{x}{2}=\frac{2\sqrt{5}}{5}$

$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)}=2$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2.

31. It is given that the farmer pays Rs 6000 in cash.
Therefore, unpaid amount = Rs 12000 – Rs 6000 = Rs 6000
According to the given condition, the interest paid annually is
12% of 6000, 12% of 5500, 12% of 5000, …, 12% of 500
Thus, total interest to be paid = 12% of 6000 + 12% of 5500 + 12% of 5000 + … + 12% of 500
= 12% of (6000 + 5500 + 5000 + … + 500)
= 12% of (500 + 1000 + 1500 + … + 6000)
Now, the series 500, 1000, 1500 … 6000 is an A.P. with both the first term and common difference equal to 500.
Let the number of terms of the A.P. be n.
$\therefore 6000 = 500 + (n-1)500$
$\Rightarrow 1 + (n-1) = 12$
$\Rightarrow n = 12$
$\therefore$Sum of the A.P $=\frac{12}{2}\left[2(500)+(12-1)(500)\right]=6\left[1000+5500\right]=6(6500)=39000$
Thus, total interest to be paid = 12% of (500 + 1000 + 1500 + … + 6000)

= 12% of 39000 = Rs 4680

Thus, cost of tractor = (Rs 12000 + Rs 4680) = Rs 16680

32. Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y.

$$\text{Mean, } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 56 = 42 + x + y$$

$$\Rightarrow x + y = 14 \qquad \ldots(1)$$

$$\text{Variance} = 16 = \frac{1}{n}\sum_{i=1}^{7}\left(x_i - \bar{x}\right)^2$$

$$16 = \frac{1}{7}\left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2\times 8(x+y) + 2\times(8)^2\right]$$

$$16 = \frac{1}{7}\left[36+16+4+16+36+x^2+y^2-16(14)+2(64)\right]$$

...[Using (1)]

$$16 = \frac{1}{7}\left[108+x^2+y^2-224+128\right]$$

$$16 = \frac{1}{7}\left[12+x^2+y^2\right]$$

$$\Rightarrow x^2 + y^2 = 112 - 12 = 100$$

$$x^2 + y^2 = 100 \qquad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

$2xy = 196 – 100$

$\Rightarrow 2xy = 96$ … (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 – 2xy = 100 – 96$

$\Rightarrow (x – y)^2 = 4$

$\Rightarrow x – y = \pm 2$ … (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x – y = 2$

$x = 6$ and $y = 8$ when $x – y = – 2$

Thus, the remaining observations are 6 and 8.