

Class XI
Mathematics
Chapter Notes
Linear Inequalities

Definitions

1. Two real numbers or two algebraic expressions related by the symbol ' $<$ ', ' $>$ ', ' \leq ' or ' \geq ' form an inequality.
2. Inequalities containing ' $<$ ', or ' $>$ ' are called strict inequalities.
3. Inequalities containing ' \leq ' or ' \geq ' are called slack inequalities.
4. An inequality containing any two of ' $<$ ', ' $>$ ', ' \leq ' or ' \geq ' is called double inequality.
5. Solution of an inequality in one variable is the value of the variable which makes it a true statement
6. A linear expression in one variable involving the inequality symbol is linear inequality in one variable. General forms:
$$ax + b < 0 \quad (1)$$
$$ax + b > 0 \quad (2)$$
$$ax + b \leq 0 \quad (3)$$
$$ax + b \geq 0 \quad (4)$$
7. A linear inequality involving two variables is known as a linear inequality in two variables. General forms
$$ax + by < c \quad (5)$$
$$ax + by > c \quad (6)$$
$$ax + by \leq c \quad (7)$$
$$ax + by \geq c \quad (8)$$
$$ax^2 + bx + c \leq 0 \quad (9)$$
$$ax^2 + bx + c \geq 0 \quad (10)$$
7. The region containing all the solutions of an inequality is called the solution region.
8. The solution region of the system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

9. Quadratic inequality is quadratic polynomial with inequality sign.
Generic quadratic inequality is of the form $ax^2+bx+c > 0$

Concepts

1. If two real numbers are related by the symbols ' $<$ ', ' $>$ ', ' \leq ' or ' \geq ' then the inequality is a numerical inequality and in case of algebraic expressions it is literal inequality.
 $2 < 3$ is numerical inequality
 $5x + 2 \leq 7$ is literal inequality

- ## 2. Rules for simplifying the inequalities

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an equation.

If $a < b$ then $a + c < b + c$.

Rule 2: Both sides of an equation may be multiplied (or divided) by the same non – zero number.

If $a < b$ then $ac < bc$

Rule 3: Sign of inequality is reversed in case of multiplication (or division) by a negative number

If $a < b$ then $ak > bk$, where k is a negative number

Rule 4: Sign of inequality is reversed in case of taking the reciprocals

3. A linear inequality in one variable can be represented graphically as follows:

Representation of $x \leq 1$



Representation of $x \geq 1$

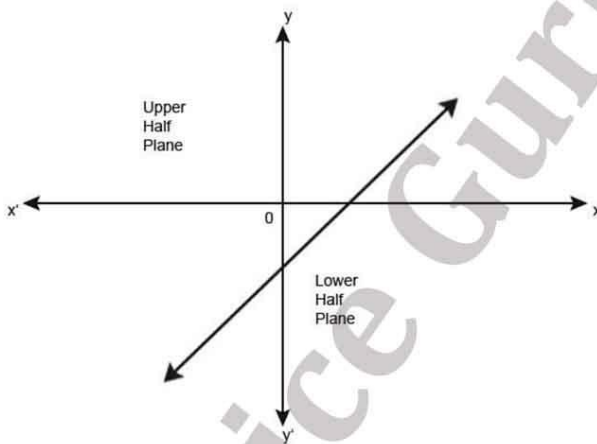
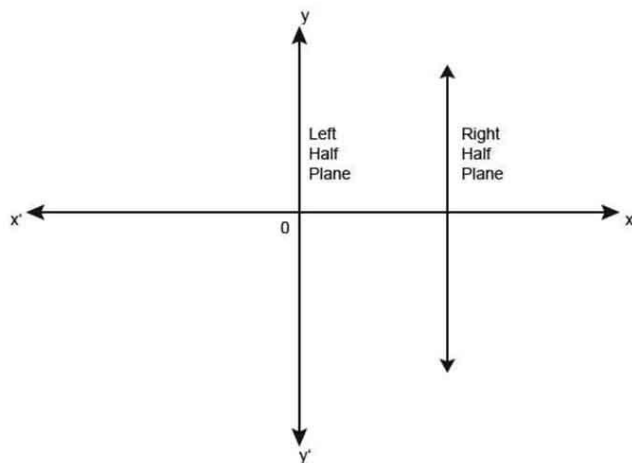


Representation of $x > 1$



4. A linear inequality in two variables represents a half plane geometrically.

Types of half planes



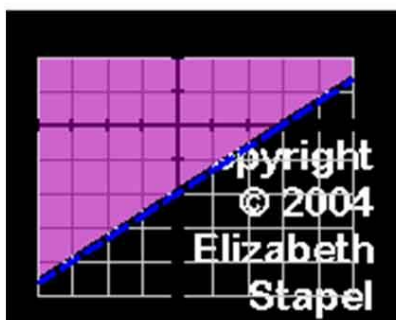
5. In order to identify the half plane represented by an inequality, take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisfies, then inequality represents the half plane and shade the region which contains the point, otherwise, the inequality represents the half plane which does not contain the point within it. For convenience, the point $(0, 0)$ is preferred.

6. If an inequality is of the type $ax + b \geq c$ or $ax + b \leq c$ i.e. slack inequality then the points on the line $ax + b = c$ are also included in the solution region



Solution of Slack Inequality

7. If an inequality is of the form $ax + by > c$ or $ax + by < c$, then the points on the line $ax + by = c$ are not to be included in the solution region.



Solution of Strict Inequality

8. To represent $x < a$ (or $x > a$) on a number line, put a circle on the number a and dark line to the left (or right) of the number a .

9. To represent $x \leq a$ (or $x \geq a$) on a number line, put a dark circle on the number a and dark the line to the left (or right) of the number x .

10. Steps to represent the linear inequality in two variables graphically

Step 1 Rewrite the inequality as linear equation, that is $ax+by = c$

step 2: Put $x=0$ to get y -intercept of the line i.e. $(0, c/b)$

Step 3: Put $y=0$ to get x intercept of the line i.e. $(c/a, 0)$

Step 4: Join the two points, each on x axis and y axis to get the graphical representation of the line.

Step5: Choose a point (x_1, y_1) in one of the planes i.e. either to the left or right or upper or lower half of the line, but not on the line.

Step 6 If (x_1, y_1) satisfies the given inequality. Then the required region is that particular half plane in which (x_1, y_1) lie.

On the other hand, if (x_1, y_1) does not satisfy the given inequality, then the required solution region is the half plane which does not contain (x_1, y_1)

11. Linear inequalities represent regions; regions common to the given inequalities will be the solution region.

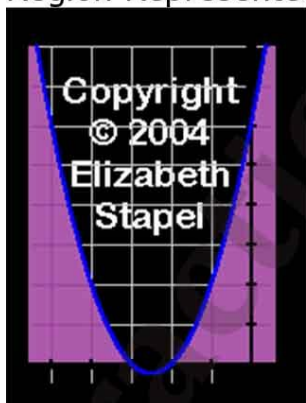
Like linear equations, there can be cases of overlapping of regions or no common regions for the given inequalities.

12. To solve a system of inequalities graphically

- Change the sign of equality to inequality and draw the graph of each line.
- Shade the region for each inequality.
- Common region to all the inequalities is the solution.

13. A linear inequality divides the plane into two half planes while a quadratic inequality is represented by a parabola which divides the plane into different regions.

Region Represented by the inequality $x^2 + 5x + 6 \geq 0$



14. Interval Notations:

Open Interval: The interval

which contains all the elements between a and b excluding a and b . In set

notations:

$$(a, b) = \{ x : a < x < b \}$$



Closed Interval :The interval which contains all the elements between a and b and also the end points a and b is called **closed interval**.

$$[a, b] = \{x : a \leq x \leq b\}$$



Semi open intervals:

$[a, b) = \{x : a \leq x < b\}$ includes all the elements from a to b including a and excluding b

$(a, b] = \{x : a < x \leq b\}$ includes all the elements from a to b excluding a and including b .