Oscillations

- **Periodic motion** $\rightarrow$ Motion which repeats itself after regular intervals of time
- **Oscillatory motion** $\rightarrow$A body in oscillatory motion moves to and fro about its mean position in a fixed time interval.

- **Period (*T*)**: It is the interval of time after which a motion is repeated. Its unit is seconds s.

- **Time period** $\rightarrow$ Time required for one complete oscillation

$$T = \frac{1}{\nu}$$

Where, n $\rightarrow$ Frequency

- **Frequency** : Number of oscillations in one second

The unit is Hertz.

- An oscillatory motion is said to be simple harmonic, when the displacement (x) of the particle from origin varies with time given as,

$$x(t) = A\cos(\omega t + \phi)$$

- Displacement is sinusoidal function of time.
- Displacement – A continuous function of time for SHM

- Non-harmonic oscillation is a combination of two or more harmonic oscillation.

- SHM is defined as the projection of uniform circular motion on the diameter of a circle of reference.
- Amplitude – Maximum displacement on either side of the mean position

  - **Displacement** $\rightarrow$ It is indicated by sinusoidal trigonometric function.

    $x = A\sin wt$ and $w = 2\pi f$

    $x = A\cos wt$

  - **Velocity** $\rightarrow$If $x = A\sin(\omega t \pm f)$, then $v = \frac{dx}{dt} = \omega A\cos(\omega t \pm \phi)$.

$$v = \omega A\sqrt{1 - \sin^2(\omega t + \phi)}$$

$$= \omega A\sqrt{1 - \left(\frac{x^2}{A^2}\right)} = \omega\sqrt{A^2 - x^2}$$

- **Acceleration**→ $a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t \pm \phi) = -\omega^2 x$
- **Time period of a pendulum**→ $a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t \pm \phi) = -\omega^2 x$
- ***l* is the length of the pendulum.**
- **Restoring force**→It is the force that is responsible for maintaining SHM.

$F = -kx$

Here, *k* is the force constant.

- A particle of mass *m* oscillating under the influence of Hooke's law of restoring force given by $F = -kx$ exhibits simple harmonic motion with

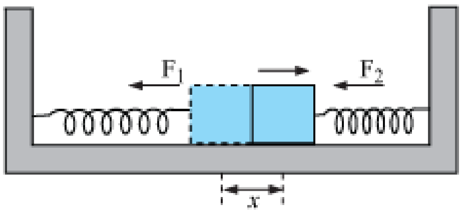
$$\omega = \sqrt{\frac{k}{m}} \text{ and}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$

- The maximum velocity of the particle in SHM is at mean position and it is given by $v_{\max} = \pm a\omega$.
- The minimum velocity of the particle in SHM is at extreme position and it is 0.
- At mean position, the particle has minimum acceleration and its magnitude is 0.
- At extreme position, the particle has maximum value of acceleration and its magnitude is $\omega^2 a$.
- The frequency of SHM is given by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.
- The period of SHM is given by $T = \frac{2\pi}{\sqrt{\frac{a}{x}}} = \frac{2\pi}{\sqrt{\text{Acceleration per unit displacement}}}$.
- The physical quantity that describes the state of oscillation is known as the phase of SHM.
- The physical quantity that describes the state of oscillation of the particle performing SHM at the beginning of the motion is called the epoch of SHM.

Energy in Simple Harmonic Motion

- Potential energy $= \frac{1}{2}m\omega^2 x^2 \text{ or } \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$
- Kinetic energy $= \frac{1}{2}m\omega^2 (A^2 - x^2) \text{ or } \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$
- Total energy $= \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t + \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t = \frac{1}{2}m\omega^2 A^2$

- When the spring is deformed, it is subjected to a restoring force [$F(x)$], which is proportional to the displacement, $x\, in the opposite direction$.



$F(x) \propto -x$

$\therefore F(x) = -kx \ldots(i)$

- angular frequency of the oscillations

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

- Time period (T) of the oscillations,

$$T = \frac{2\pi}{\omega}$$

$$\boxed{\therefore T = 2\pi\sqrt{\frac{m}{k}}}$$

- A simple pendulum is a heavy point mass suspended by a weightless, inextensible, flexible string attached to a rigid support from where it moves freely.
- The periodic motion of a simple pendulum for small displacements is simple harmonic.
- Time period of simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

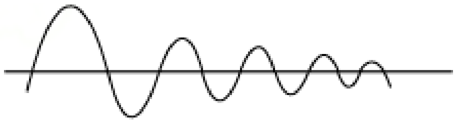
Laws of simple pendulum:

- The time period of the pendulum is directly proportional to the square root of its length.
- The time period of the pendulum is inversely proportional to the square root of the acceleration due to gravity of the place.
- The time period of the pendulum is independent of the mass of the bob.
- The time period of the pendulum does not depend upon its amplitude of oscillations.

Seconds Pendulum

- It is a simple pendulum that has a time period equal to 2 seconds.

- **Damped oscillation** → When the motion of an oscillator is reduced by an external force



Damped oscillation

Angular frequency of the damped oscillation

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Where, b is a damping constant

- Damping force (F_d) depends on the nature of the surrounding medium; it is proportional to the velocity (v) of the bob, and acts opposite to the direction of velocity.

$$F_d \propto -v$$
$$\therefore F_d = -bv$$

- **Forced oscillation** $\rightarrow$ When an external agency maintains an undamped oscillation by compensating for the loss of energy, it is called forced oscillation. The external force is a sinusoidal force.
- The expression for the external force is given by $F = F_m \sin(\omega_d t)$
- Here, F_m is amplitude of external force and ω_d is driving frequency
- The displacement of the natural oscillation dies out according to $x(t) = A\cos(\omega_d t + \Phi)$.
- The Amplitude, A, is the function of the forced frequency (ω_d) and the natural frequency, ω and is given by $A = \frac{F_m}{\{m^2(\omega^2 - \omega_d^2) + \omega_d^2 b^2\}^{\frac{1}{2}}}$
- Cases of damping:
- Case1 :Small damping; driving frequency far from natural frequency
- $\omega_d b << m(\omega^2 - \omega_d^2) \therefore A = \frac{F_m}{m(\omega^2 - \omega_d^2)}$
- Case 2: Driving frequency close to natural frequency ω_d is very close to ω. $m(\omega^2 - \omega_d^2) << \omega_d b \therefore A = \frac{F_0}{\omega_d b}$
- **Resonance:** The phenomenon of increase in amplitude when the frequency of the driving force is close to the natural frequency of the oscillator is called resonance.

$$\omega' \approx \omega_0$$

- The principle behind the phenomenon of resonance finds application in stethoscopes and in the tuners of radio sets.
- Resonance is used to increase the intensity of sound in musical instruments and to analyse musical instruments.
- The unknown frequency of a vibrating tuning fork can be determined using resonance.
- In string instruments, sound is produced by the vibration of strings.
- Sitar, veena, guitar and tanpura are examples of string instruments.
- In wind instruments, sound is produced by the vibration of air columns.

- Flute, bugle, bassoon and harmonium are examples of wind instruments.
- In percussion instruments, sound is produced by setting vibrations in a stretched membrane.
- Mridangam, tabla and drums are some examples of percussion instruments.