Class XI: Math
Chapter 13: Limits and Derivatives

Chapter Notes

Key-Concepts

1. The expected value of the function as dictated by the points to the left of **a** point defines the left hand limit of the function at that point. $\lim\limits_{x\to a^-} f(x)$ is the expected value of f at x =a given the values of f near x to the left of a

2. The expected value of the function as dictated by the points to the right of point **a** defines the right hand limit of the function at that point. $\lim\limits_{x\to a^+} f(x)$ is the expected value of f at x =a given the values of f near x to the left of a.

3. Let y = f(x) be a function. Suppose that a and L are numbers such that

as x gets closer and closer to a, f(x) gets closer and closer to L we say that the limit of f(x) at x =a is L i.e $\lim\limits_{x\to a} f(x) = L$.

4. Limit of a function at a point is the common value of the left and right hand limit, if they coincide. i.e $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$.

5. **Real life Examples of LHL and RHL**
(a) If a car starts from rest and accelerates to 60 kms/hr in 8 seconds, means initial speed of the car is 0 and reaches 60 at 8 seconds after the start.

On recording the speed of the car we can see that this sequence of numbers is approaching 60 in such a way that each member of the sequence is less than 60. This sequence illustrates the concept of approaching a number from the left of that number.

(b) Boiled Milk at 100 degrees is placed on a shelf; temperature goes on dropping till it reaches room temperature.

As time increases, temperature of milk, **t** approaches room temperature say 30°. This sequence illustrates the concept of approaching a number from the right of that number.

6. Let f and g be two functions such that both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists then

a) Limit of sum of two functions is sum of the limits of the functions, i.e.,

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

b) Limit of difference of two functions is difference of the limits of the functions i.e.,

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

c) Limit of product of two functions is product of the limits of the functions, i.e.,

$$\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$$

d) Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non zero), i.e.,

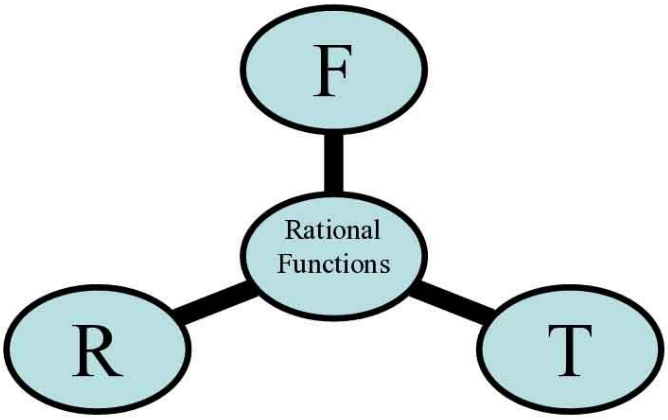
$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

7. For any positive integer n,

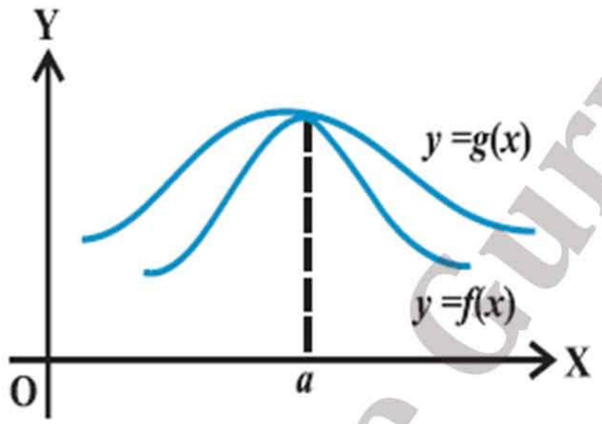
$$\lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$$

8. Limit of polynomial function can be computed using substitution or Algebra of Limits.

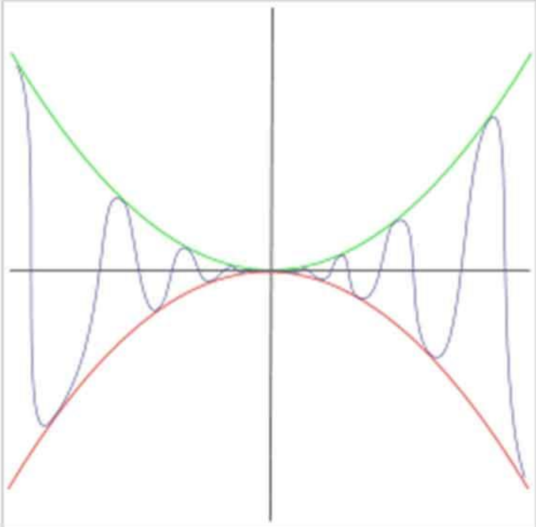
9. For computing the limit of a Rational Function when direct substitution fails then use factorisation , rationalization or the theorem .



10. Let f and g be two real valued functions with the same domain such that $f(x) \leq g(x)$ for all x in the domain of definition. For some a, if both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$.



11. Let f, g and h be real functions such that $f(x) \leq g(x) \leq h(x)$ for all x in the common domain of definition. For some real number a, if $\lim_{x\to a} f(x) = \ell = \lim_{x\to a} h(x)$, then $\lim_{x\to a} g(x) = \ell$.



An example of a squeezed function

12. Limit of trigonometric functions

i. $\lim\limits_{x\to 0}\sin x = 0$ ii $\lim\limits_{x\to 0}\cos x = 1$ iii $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$

iv $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = 0$

v $\lim\limits_{x\to 0}\dfrac{\tan x}{x} = 1$

13. Suppose f is a real valued function and a is a point in its domain of definition. The derivative of f at a is defined by

$$\lim_{x\to 0}\frac{f(a+h)-f(a)}{h}$$

Provided this limit exists and is finite. Derivative of f(x) at a is denoted by f′(a).

14. A function is differentiable in its domain if it is always possible to draw a unique tangent at every point on the curve.

15. Finding the derivative of a function using definition of derivative is known as the first principle of derivatives or ab –initio method.

16 Let f and g be two functions such that their derivates are defined in a common domain. Then

i. Derivative of sum of two functions is sum of the derivatives of the functions.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

ii. Derivative of difference of two functions is difference of the derivatives of the functions.

$$\frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

iii. Derivative of product of two functions is given by the following products rule.

$$\frac{d}{dx}\left[f(x).g(x)\right]=\frac{d}{dx}f(x).g(x)+f(x).\frac{d}{dx}g(x)$$

iv. Derivative of quotient of two functions is given by the following quotient rule (whenever the denominator is non – zero).

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\frac{d}{dx}f(x).g(x)-f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

17. Derivative of $f(x) = x^n$ is nx^{n-1} for any positive integer n.

18. Let $f(x) = a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + 2a_2 x + a_1$.

a_2x are all real numbers and $a_n \neq 0$. Then, the derivative functions is given by

$$\frac{df(x)}{dx}=na_n x^{n-1}+(n-1)a_{n-1}x^{n-2}+\ldots+2a_2x+a_1.$$

19. For a function f and a real number a, $\lim_{x\to a} f(x)$ and f(a) may not be same (In fact, one may be defined a d not the other one).

20. Standard Derivatives

f(x)	f′(x)
sin x	cos x
cos x	- sin x
tan x	sec^2x
cot x	- $cosec^2x$
sec x	sec x tan x
cosec x	- cosec x cot x
x^n	nx^{n-1}
c	0

21. The derivative is the instantaneous rate of change in terms of Physics and is the slope of the tangent at a point.

22 A function is not differentiable at the points where it is not defined or at the points where the unique tangent cannot be drawn.

23. $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$, y' are all different notations for the derivative w.r.t x