Class XI: Math
Chapter 8: Binomial Theorem

Chapter Notes

Key Concepts

1. A binomial expression is an algebraic expression having two terms, for example (a+b),(a-b) etc.

2. The expansion of a binomial for any positive integral n is given by Binomial Theorem. The binomial theorem says that
$$(x+y)^n = x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + ---- + {}^nC_r\,x^{n-r}y^r + ----- + {}^nC_{n-1}xy^{n-1} + {}^nc_ny^n$$
In summation notation $(x+y)^n = \sum_{k=0}^{n} {}^nC_k\,x^{n-k}y^k$

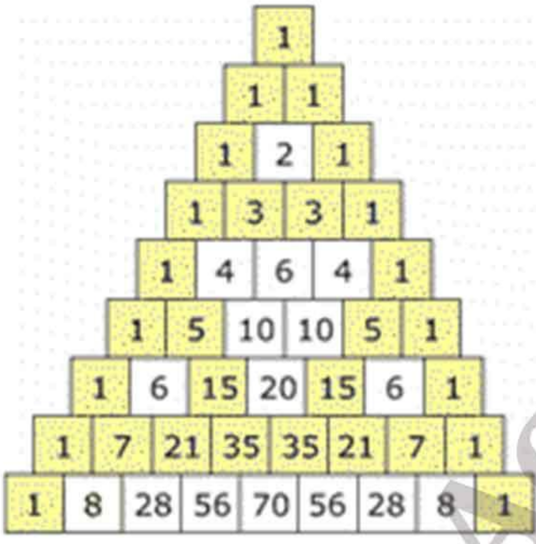
3. (i)In the binomial expansion of $(x+y)^n$ the number of terms is (n+1) i.e one more than the exponent.
(ii)The exponent of x goes on decreasing by unity and y increases by unity. Exponent of x is n in the first term, (n - 1) in the second term, and so on ending with zero in the last term.
(iii)The sum of the indices of x and y is always equal to the index of the expression.

4. The coefficients ${}^nC_r$, the number of combinations of n objects taken r at a time, occurring in the binomial theorem are known as binomial coefficients.

5. Binomial coefficients when arranged in the form given below is known as Pascals Triangle

Index	Coefficients
0	${}^0C_0$ (=1)
1	${}^1C_0$ (=1) ${}^1C_1$ (=1)
2	${}^2C_0$ (=1) ${}^2C_1$ (=2) ${}^2C_2$ (=1)
3	${}^3C_0$ (=1) ${}^3C_1$ (=3) ${}^3C_2$ (=3) ${}^3C_3$ (=1)
4	${}^4C_0$ (=1) ${}^4C_1$ (=4) ${}^4C_2$ (=6) ${}^4C_3$ (=4) ${}^4C_4$ (=1)
5	${}^5C_0$ (=1) ${}^5C_1$ (=5) ${}^5C_2$ (=10) ${}^5C_3$ (=10) ${}^5C_4$ (=5) ${}^5C_5$ (=1)

6. The array of numbers arranged in the form of triangle with 1 at the top vertex and running down the two slanting sides is known as Pascal's triangle, after the name of French mathematician Blaise Pascal. It is also known as Meru Prastara by Pingla.



7. Pascal's Triangle is a special triangle of numbers. It has an infinite number of rows. Pascal's Triangle is a storehouse of patterns.

8. In order to construct the elements of following rows, add the number directly above and to the left with the number directly above and to the right to find the new value. If either the number to the right or left is not present, substitute a zero in its place.

9. Using binomial theorem for non –negative index
$(x - y)^n = [x + (-y)]^n$

$$(x - y)^n = {}^nC_0x^n - {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 - {}^nC_3x^{n-3}y^3 + \ldots + (-1)^n\,{}^nC_ny^n$$

In summation notation $(x - y)^n = \sum_{k=0}^{n}(-1)^k\,{}^nC_kx^{n-k}y^k$

10. Binomial theorem can be used to expand trinomial by applying the binomial expansion twice.

11. General term in the expansion of $(x+y)^n$ is $T_{k+1} = {}^nC_k\,x^{n-k}\,y^k$

12. General term in the expansion of $(x-y)^n$ is $T_{k+1} = {}^nC_k\,(-1)^k\,x^{n-k}\,y^k$

13. If n is even , there is only one middle term in the expansion of $(x + y)^n$ and will be the $\left(\frac{n}{2}+1\right)^{th}$ term

14. If n is odd, there are two middle terms in the expansion of $(x + y)^n$ and they are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ term.

15. In the expansion $\left(x + \frac{1}{x}\right)^{2n}$, where $x \neq 0$, the middle term is $\left(\frac{2n+1+1}{2}\right)^{th}$ i.e., $(n + 1)^{th}$ term, as 2n is even

Summary

- , which is $(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots + {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n$.
- The coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.
- The general term of an expansion $(a + b)^n$ is $T_{r+1} = {}^nC_r\, a^{n-r}.\, b^r$.
- In the expansion $(a + b)^n$, if n is even, then he middle term is the $\left(\frac{n}{2}+1\right)^{th}$ term. If n is odd, then the middle term are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{th}$ terms.