विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

This is TYPE 1 Package please wait for Type 2

Subject : PHYSICS

Topic : ELECTROSTATICS

TEKO CLASSES .....the support

Index

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KEY CONCEPTS

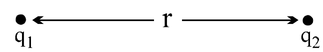
1. ELECTRIC CHARGE

Charge of a material body is that possesion (acquired or natural) due to which it strongly interacts with other material body. It can be postive or negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

2. COULOMB'S LAW : $F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$. In vector form $\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^3}\vec{r}$ where

ε_0 = permittivity of free space = $8.85 \times 10^{-12}\ N^{-1}\ m^{-2}\ c^2$ or F/m and
ε_r = Relative permittivity of the medium = Spec. Inductive Capacity = Dielectric Const.
ε_r = 1 for air (vacuum) = ∞ for metals $\varepsilon_0\varepsilon_r$ = Absolute permittivity of the medium

NOTE : The Law is applicable only for static and point charges. Only applicable to static charges as moving charges may result magnetic interaction also and only for point charges as if charges are extended, induction may change the charge distribution.



3. PRINCIPLE OF SUPER POSITION

Force on a point charge due to many charges is given by $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

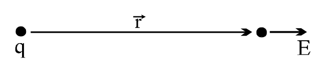
NOTE : The force due to one charge is not affected by the presence of other charges.

4. ELECTRIC FIELD, ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH (VECTOR QUANTITY)

"The physical field where a charged particle, irrespective of the fact whether it is in motion or at rest, experiences force is called an electric field". The direction of the field is the direction of the force experienced by a positively charged particle & the magnitude of the field (electric intensity) is the force experienced by the particle carrying unit charge $\vec{E} = \lim_{q\to 0}\frac{\vec{F}}{q}$ unit is NC^{-1}; S.I. unit is V/m here $\lim_{q\to 0}$ represents that this charge does not alter the magnitude of electric field. Due to charge induction on the source of electric field.

5. ELECTRIC FIELD DUE TO

(i) Point charge : $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\vec{r}$ (vector form)



Where $\vec{r}$ = vector drawn from the source charge to the point .

(ii) Continuous charge distribution $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r} = \int d\vec{E}$; $d\vec{E}$ = electric field due to an elementry charge . **Note** $E \neq \int dE$ because E is a vector quantity .

dq = λ dl (for line charge) = σ ds (for surface charge) = ρ dv (for volume charge) In general λ, σ & ρ are linear, surface and volume charge densities respectively.

(iii) Infinite line of charge $\vec{E} = \frac{2k\lambda}{r}$ where r = perpendicular distance of the point from the line charge .

(iv) Semi ∞ line of charge $\vec{E} = \frac{\sqrt{2}k\lambda}{r}$ as , $E_x = \frac{k\lambda}{r}$ & $E_y = \frac{k\lambda}{r}$ at a point above the end of wire at an angle 45° .

(v) Uniformly charged ring , $E_{centre} = 0$, $E_{axis} = \dfrac{kQx}{(x^2+R^2)^{3/2}}$

(vi) Electric field is maximum when $\dfrac{dE}{dx} = 0$ for a point on the axis of the ring. Here we get $x = R/\sqrt{2}$.

(vii) Infinite non conducting sheet of charge $\vec{E} = \dfrac{\sigma}{2\varepsilon_0}\hat{n}$ where

$\hat{n}$ = unit normal vector to the plane of sheet, where σ is surface charge density

(viii) ∞ charged conductor sheet having surface charge density σ on both surfaces $E = \sigma/\varepsilon_0$.

(ix) Just outside a conducting surface charged with a surface charge density σ, electric field is always given as $E = \sigma/\epsilon_0$.

(x) Uniformly charged solid sphere (Insulating material) $E_{out} = \dfrac{Q}{4\pi\varepsilon_0 r^2}$; $r \geq R$,

Behaves as a point charge situated at the centre for these points $E_{in} = \dfrac{Qr}{4\pi\varepsilon_0 R^3} = \dfrac{\rho r}{3\varepsilon_0}$;

$r \leq R$ where ρ = volume charge density

(xi) Uniformly charged spherical shell (conducting or non-donducting) or uniformly charged solid conducting sphere . $E_{out} = \dfrac{Q}{4\pi\varepsilon_0 r^2}$; $r \geq R$

Behaves as a point charge situated at the centre for these points $E_{in} = 0$; $r < R$

(xii) uniformly charged cylinder with a charge density ρ is -(radius of cylinder = R) for $r < R$

$E_m = \dfrac{\rho r}{2\epsilon_0}$; for $r > R$ $\quad E = \dfrac{\rho R^2}{2\epsilon_0 r}$

(xiii) Uniformly charged cylinderical shell with surface charge density σ is

for $r < R$ $E_m = 0$; for $r > R$ $E = \dfrac{\rho r}{\epsilon_0 r}$

6. ***ELECTRIC LINES OF FORCE (ELF)***

The line of force in an electric field is a hypothetical line, tangent to which at any point on it represents the direction of electric field at the given point.

Properties of (ELF) :

(i) Electric lines of forces never intersects .
(ii) ELF originates from positive charge or ∞ and terminate on a negative charge of infinity .
(iii) Preference of termination is towards a negative charge .
(iv) If an ELF is originated, it must require termination either at a negetive charge or at ∞ .
(v) Quantity of ELF originated or terminated from a charge or on a charge is proportional to the magnitude of charge.

7. ***ELECTROSTATIC EQUILIBRIUM***

Position where net force (or net torque) on a charge(or electric dipole) = 0

(i) **STABLE EQUILIBRIUM :** If charge is displaced by a small distance the charge comes (or tries to come back) to the equilibrium .

(ii) **UNSTABLE EQUILIBRIUM :** If charge is displaced by a small distance the charge does not return to the equilibrium position.

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8. ***ELECTRIC POTENTIAL*** *(Scalar Quantity)*

"Work done by external agent to bring a unit positive charge(without accelaration) from infinity to a point in an electric field is called electric potential at that point" .

If $W_{\infty r}$ is the work done to bring a charge q (very small) from infinity to a point then potential at that point is $V = \frac{(W_{\infty r})_{ext}}{q}$; S.I. unit is volt (= 1 J/C)

9. ***POTENTIAL DIFFERENCE***

$$V_{AB} = V_A - V_B = \frac{(W_{BA})_{ext}}{q} \quad V_{AB} = \text{p.d. between point } A \,\&\, B .$$

W_{BA} = w.d. by external source to transfer a point charge q from B to A (Without acceleration).

10. ***ELECTRIC FIELD & ELECTRIC POINTENIAL***

$\vec{E} = -\text{grad } V = -\nabla V$ {read as gradient of V} $\text{grad} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$;

Used when EF varies in three dimensional coordinate system.

For finding potential difference between two points in electric field, we use –

$$V_A - V_B = -\int_A^B \vec{E}.\vec{dt}$$ if E is varying with distance

if E is constant & here d is the distance between points A and B.

11. ***POTENTIAL DUE TO***

(i) a point charge $V = \frac{Q}{4\pi\varepsilon_0 r}$ **(ii)** many charges $V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} + ...$

(iii) continuous charge distribution $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$

(iv) spherical shell (conducting or non conducting) or solid conducting sphere

$$V_{out} = \frac{Q}{4\pi\varepsilon_0 r} \;;\; (r \ge R) \;,\quad V_{in} = \frac{Q}{4\pi\varepsilon_0 R} \;;\; (r \le R)$$

(v) non conducting uniformly charged solid sphere :

$$V_{out} = \frac{Q}{4\pi\varepsilon_0 r} \;;\; (r \ge R) \;,\quad V_{in} = \frac{1}{2}\frac{Q(3R^2 - r^2)}{4\pi\varepsilon_0 R} \;;\; (r \le R)$$

12. ***EQUIPOTENTIAL SURFACE AND EQUIPOTENTIAL REGION***

In an electricfield the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field meet at right angles.

The region where E = 0, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

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13. ***MUTUAL POTENTIAL ENERGY OR INTERACTION ENERGY***

"The work to be done to integrate the charge system ."

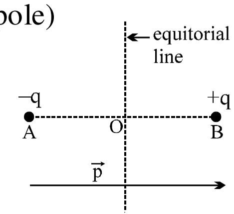
For 2 particle system $U_{mutual} = \dfrac{q_1q_2}{4\pi\varepsilon_0 r}$

For 3 particle system $U_{mutual} = \dfrac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \dfrac{q_2q_3}{4\pi\varepsilon_0 r_{23}} + \dfrac{q_3q_1}{4\pi\varepsilon_0 r_{31}}$

For n particles there will be $\dfrac{n\,(n-1)}{2}$ terms . Total energy of a system = $U_{self} + U_{mutual}$

14. P.E. of charge q in potential field U = qV. Interaction energy of a system of two charges $U = q_1V_2 = q_2V_1$.

15. ***ELECTRIC DIPOLE.*** O is mid point of line AB (centre of the dipole)



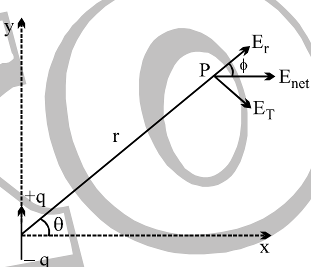
(a) on the axis (except points on line AB)

$\vec{E} = \dfrac{\vec{p}r}{2\pi\varepsilon_0[r^2 - (a^2/4)]^2} \approx \dfrac{\vec{p}}{2\pi\varepsilon_0 r^3}$ (if r << a)

$\vec{p} = q\vec{a}$ = Dipole moment ,

r = distance of the point from the centre of dipole

(b) on the equitorial ; $\vec{E} = \dfrac{\vec{p}}{4\pi\varepsilon_0[r^2 + (a^2/4)]^{3/2}} \approx -\dfrac{\vec{p}}{4\pi\varepsilon_0 r^3}$



(c) At a general point P(r, θ) in polar co-ordinate system is

Radial electric field $E_r = \dfrac{2kp\sin\theta}{r^3}$

Tangentral electric field $E_T = \dfrac{kp\cos\theta}{r^3}$

Net electric field at P is $E_{net} = \sqrt{E_r^2 + E_T^2} = \dfrac{kp}{r^3}\sqrt{1 + 3\sin^2\theta}$

Potential at point P is $V_P = \dfrac{kp\sin\theta}{r^2}$

Note : If θ is measured from axis of dipole. Then sinθ and cosθ will be interchanged.

(d) Dipole V = $\dfrac{P\theta}{4\pi\varepsilon_0 r^2} = \dfrac{\vec{p}.\vec{r}}{4\pi\varepsilon_0 r^3}$ $\vec{p} = q\vec{a}$ electric dipole moment . If θ is angle between $\vec{p}$ and reaches vector of the point.

(e) **Electric Dipole in uniform electric field :** torque $\vec{\tau} = \vec{p} x \vec{E}$; $\vec{F} = 0$.

Work done in rotation of dipole is w = PE (cos θ_1 – cos θ_2)

(f) P.E. of an electric dipole in electric field $U = -\vec{p}.\vec{E}$.

(g) Force on a dipole when placed in a non uniform electric field is $F = -\dfrac{d}{dx}\left(-\vec{P}.\vec{E}\right)\hat{i} = \vec{P}.\dfrac{d\vec{E}}{dx}\hat{i}$.

16. ***ELECTRIC FLUX***

(i) For uniform electric field; $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ = angle between $\vec{E}$ & area vector ($\vec{A}$). Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(ii) If $\vec{E}$ is not uniform throughout the area A , then $\phi = \int \vec{E}.d\vec{A}$

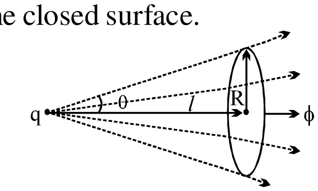
17. ***GAUSS'S LAW*** (Applicable only to closed surface) " Net flux emerging out of a closed surface is $\frac{q}{\varepsilon_0}$." $\phi = \oint \vec{E}d\vec{A} = \frac{q}{\varepsilon_0}$ q = net charge enclosed by the closed surface .

ϕ does not depend on the **(i)** shape and size of the closed surface
(ii) The charges located outside the closed surface.

CONCEPT OF SOLID ANGLE :

Flux of charge q having through the circle of radius R is

$$\phi = \frac{q/\epsilon_0}{4\pi} \text{ x } \Omega = \frac{q}{2\epsilon_0}(1-\cos\theta)$$



Solid angle of coneof half angle θ is $\Omega=2\pi(1-\cos\theta)$

18. Energy stored p.u. volume in an electric field = $\frac{\varepsilon_0 E^2}{2}$

19. Electric pressure due to its own charge on a surface having charged density σ is $P_{ele} = \frac{\sigma^2}{2\varepsilon_0}$.

20. Electric pressure on a charged surface with charged density σ due to external electric field is $P_{ele} = \sigma E_1$

IMPORTANT POINTS TO BE REMEMBERED

(i) Electric field is always perpendicular to a conducting surface (or any equipotential surface) . No tangential component on such surfaces .

(ii) Charge density at sharp points on a conductor is greater.

(iii) When a conductor is charged, the charge resides only on the surface.

(iv) For a conductor of any shape E (just outside) = $\frac{\sigma}{\varepsilon_0}$

(v) p.d. between two points in an electric field does not depend on the path joining them .

(vi) Potential at a point due to positive charge is positive & due to negative charge is negative.

(vii) Positive charge flows from higher to lower (i.e. in the direction of electric field) and negative charge from lower to higher (i.e. opposite to the electric field) potential .

(viii) When $\vec{p} || \vec{E}$ the dipole is in stable equilibrium

(ix) $\vec{p} || (-\vec{E})$ the dipole is in unstable equilibrium

(x) When a charged isolated conducting sphere is connected to an unchaged small conducting sphere then potential (and charge) remains almost same on the larger sphere while smaller is charged .

(xi) Self potential energy of a charged shell = $\frac{KQ^2}{2R}$.

(xii) Self potential energy of an insulating uniformly charged sphere = $\frac{3kQ^2}{5R}$.

(xiii) A spherically symmetric charge {i.e ρ depends only on r} behaves as if its charge is concentrated at its centre (for outside points).

(xiv) **Dielectric strength of material :** The minimum electric field required to ionise the medium or the maximum electric field which the medium can bear without breaking down.

EXERCISE # I

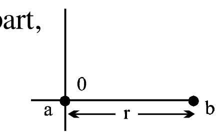
Q.1 A negative point charge 2q and a positive charge q are fixed at a distance l apart. Where should a positive test charge Q be placed on the line connecting the charge for it to be in equilibrium? What is the nature of the equilibrium with respect to longitudinal motions?

Q.2 Two particles A and B each carrying a charge Q are held fixed with a separation d between then A particle C having mass m ans charge q is kept at the midpoint of line AB. If it is displaced through a small distance x (x << d) perpendicular to AB,

(a) then find the time period of the oscillations of C.

(b) If in the above question C is displaced along AB, find the time period of the oscillations of C.

Q.3 Draw E – r graph for 0 < r < b, if two point charges a & b are located r distance apart, when

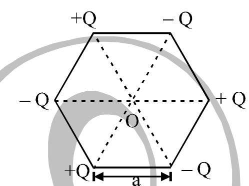


(i) both are + ve (ii) both are – ve

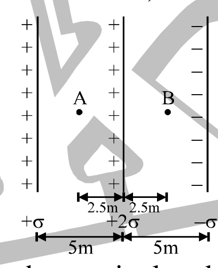
(iii) a is + ve and b is – ve (iv) a is – ve and b is + ve

Q.4 A charge + 10^{-9} C is located at the origin in free space & another charge Q at (2, 0, 0). If the X–component of the electric field at (3, 1, 1) is zero, calculate the value of Q. Is the Y–component zero at (3, 1, 1)?

Q.5 Six charges are placed at the vertices of a regular hexagon as shown in the figure. Find the electric field on the line passing through O and perpendicular to the plane of the figure as a function of distance x from point O. (assume x >> a)

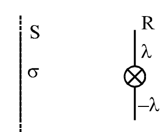


Q.6 The figure shows three infinite non-conducting plates of charge perpendicular to the plane of the paper with charge per unit area + σ, + 2σ and – σ. Find the ratio of the net electric field at that point A to that at point B.

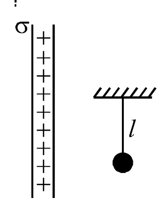


Q.7 A thin circular wire of radius r has a charge Q. If a point charge q is placed at the centre of the ring, then find the increase in tension in the wire.

Q.8 In the figure shown S is a large nonconducting sheet of uniform charge density σ. A rod R of length l and mass 'm' is parallel to the sheet and hinged at its mid point. The linear charge densities on the upper and lower half of the rod are shown in the figure. Find the angular acceleration of the rod just after it is released.



Q.9 A simple pendulum of length l and bob mass m is hanging in front of a large nonconducting sheet having surface charge density σ. If suddenly a charge +q is given to the bob & it is released from the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



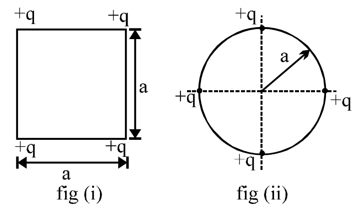
Q.10 A particle of mass m and charge – q moves along a diameter of a uniformly charged sphere of radius R and carrying a total charge + Q. Find the frequency of S.H.M. of the particle if the amplitude does not exceed R.

Q.11 A charge + Q is uniformly distributed over a thin ring with radius R. A negative point charge – Q and mass m starts from rest at a point far away from the centre of the ring and moves towards the centre. Find the velocity of this particle at the moment it passes through the centre of the ring.

Q.12 A spherical balloon of radius R charged uniformly on its surface with surface density σ. Find work done against electric forces in expanding it upto radius 2R.

Q.13 A point charge + q & mass 100 gm experiences a force of 100 N at a point at a distance 20 cm from a long infinite uniformly charged wire. If it is released find its speed when it is at a distance 40 cm from wire

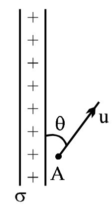
Q.14 Consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) to fig (ii).



Q.15 There are 27 drops of a conducting fluid. Each has radius r and they are charged to a potential V_0. They are then combined to form a bigger drop. Find its potential.

Q.16 Two identical particles of mass m carry charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a large distance with an initial speed V. Find the closest distance of approach.

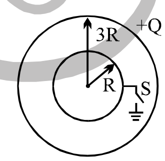
Q.17 A particle of mass m and negative charge q is thrown in a gravity free space with speed u from the point A on the large non conducting charged sheet with surface charge density σ, as shown in figure. Find the maximum distance from A on sheet where the particle can strike.



Q.18 Consider two concentric conducting spheres of radii a & b (b > a). Inside sphere has a positive charge q_1. What charge should be given to the outer sphere so that potential of the inner sphere becomes zero? How does the potential varies between the two spheres & outside ?

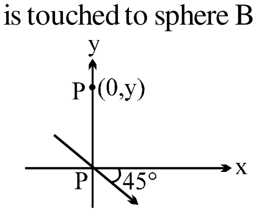
Q.19 Three charges 0.1 coulomb each are placed on the corners of an equilateral triangle of side 1 m. If the energy is supplied to this system at the rate of 1 kW, how much time would be required to move one of the charges onto the midpoint of the line joining the other two?

Q.20 Two thin conducting shells of radii R and 3R are shown in figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of switch S. Find the charge attained by the inner shell.



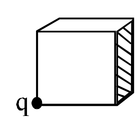
Q.21 Consider three identical metal spheres A, B and C. Spheres A carries charge + 6q and sphere B carries charge – 3q. Sphere C carries no charge. Spheres A and B are touched together and then separated. Sphere C is then touched to sphere A and separated from it. Finally the sphere C is touched to sphere B and separated from it. Find the final charge on the sphere C.

Q.22 A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point P (0, y).

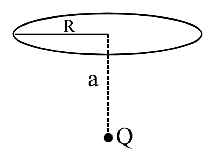


Q.23 Two point dipoles $p\hat{k}$ and $\frac{p}{2}\hat{k}$ are located at (0, 0, 0) and (1m, 0, 2m) respectively. Find the resultant electric field due to the two dipoles at the point (1m, 0, 0).

Q.24 The length of each side of a cubical closed surface is l. If charge q is situated on one of the vertices of the cube, then find the flux passing through shaded face of the cube.



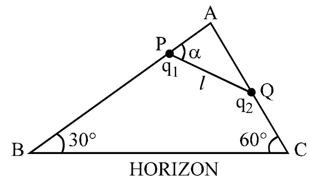
Q.25 A point charge Q is located on the axis of a disc of radius R at a distance a from the plane of the disc. If one fourth (1/4th) of the flux from the charge passes through the disc, then find the relation between a & R.



Q.26 A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

EXERCISE # II

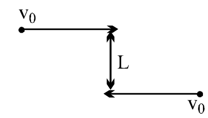
Q.1 A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown. Two bead of equal masses m each and carrying charges q_1 & q_2 are connected by a cord of length l & slide without friction on the wires. Considering the case when the beads are stationary, determine.



(a) The angle α. (b) The tension in the cord &

(c) The normal reaction on the beads. If the cord is now cut, what are the values of the charges for which the beads continue to remain stationary.

Q.2 A proton and an α-particle are projected with velocity $v_0 = \sqrt{\dfrac{ke^2}{ml}}$ each, when they are far away from each other, as shown. The distance between their initial velocities is L. Find their closest approach distance, mass of proton=m, charge=+e, mass of α-particle = 4m, charge = + 2e.

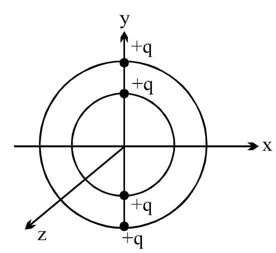


Q.3 A clock face has negative charges $-q, -2q, -3q, \ldots\ldots\ldots, -12q$ fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

Q.4 A circular ring of radius R with uniform positive charge density λ per unit length is fixed in the Y–Z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $\left(\sqrt{3}R,0,0\right)$ on the positive X-axis directly towards O, with initial velocity v . Find the smallest value of the speed v such that the particle does not return to P.

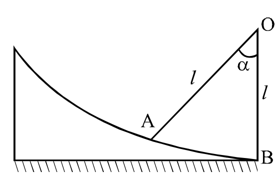
Q.5 2 small balls having the same mass & charge & located on the same vertical at heights h_1 & h_2 are thrown in the same direction along the horizontal at the same velocity v . The 1st ball touches the ground at a distance l from the initial vertical . At what height will the 2nd ball be at this instant ? The air drag & the charges induced should be neglected.

Q.6 Two concentric rings of radii r and 2r are placed with centre at origin. Two charges +q each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle 90° about Z-axis then it is again rotated by 90° about Y-axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by 90° about X-axis, find the total work required to perform all three steps.



Q.7 A positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R . A point mass having charge + q and mass m is fired towards the centre of the sphere with velocity v from a point at distance r (r > R) from the centre of the sphere. Find the minimum velocity v so that it can penetrate R/2 distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

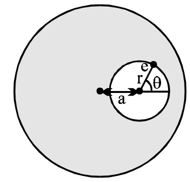
Q.8 An electrometer consists of vertical metal bar at the top of which is attached a thin rod which gets deflected from the bar under the action of an electric charge (fig.) . The reading are taken on a quadrant graduated in degrees . The length of the rod is l and its mass is m . What will be the charge when the rod of such an electrometer is deflected through an angle α . Make the following assumptions :



(a) the charge on the electrometer is equally distributed between the bar & the rod

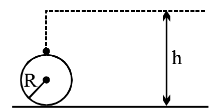
(b) the charges are concentrated at point A on the rod & at point B on the bar.

Q.9 A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centres of the sphere and the cavity is a . An electron e is kept inside the cavity at an angle $\theta = 45°$ as shown . How long will it take to touch the sphere again?



Q.10 Two identical balls of charges q_1 & q_2 initially have equal velocity of the same magnitude and direction. After a uniform electric field is applied for some time, the direction of the velocity of the first ball changes by 60º and the magnitude is reduced by half . The direction of the velocity of the second ball changes there by 90º. In what proportion will the velocity of the second ball changes ?

Q.11 Electrically charged drops of mercury fall from altitude h into a spherical metal vessel of radius R in the upper part of which there is a small opening. The mass of each drop is m & charge is Q. What is the number 'n' of last drop that can still enter the sphere. Given that the $(n + 1)^{th}$ drop just fails to enter the sphere.

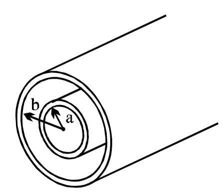


Q.12 Small identical balls with equal charges are fixed at vertices of regular 2004 - gon with side a. At a certain instant, one of the balls is released & a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each part.

Q.13 The electric field in a region is given by $\vec{E} = \frac{E_0 x}{l}\vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces x = 0, x = a, y = 0, y = a, z = 0 and z = a. Take $E_0 = 5 \times 10^3$N/C, l = 2cm and a = 1cm.

Q.14 2 small metallic balls of radii R_1 & R_2 are kept in vacuum at a large distance compared to the radii. Find the ratio between the charges on the 2 balls at which electrostatic energy of the system is minimum. What is the potential difference between the 2 balls? Total charge of balls is constant.

Q.15 Figure shows a section through two long thin concentric cylinders of radii a & b with a < b . The cylinders have equal and opposite charges per unit length λ . Find the electric field at a distance r from the axis for
(a) r < a (b) a < r < b (c) r > b

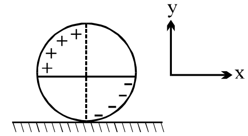


Q.16 A solid non conducting sphere of radius R has a non-uniform charge distribution of volume charge density, $\rho = \rho_0 \frac{r}{R}$, where ρ_0 is a constant and r is the distance from the centre of the sphere. Show that:

(a) the total charge on the sphere is $Q = \pi \rho_0 R^3$ and

(b) the electric field inside the sphere has a magnitude given by, $E = \frac{KQr^2}{R^4}$.

Q.17 A nonconducting ring of mass m and radius R is charged as shown. The charged density i.e. charge per unit length is λ. It is then placed on a rough nonconducting horizontal surface plane. At time t = 0, a uniform electric field $\vec{E} = E_0 i$ is switched on and the ring start rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.

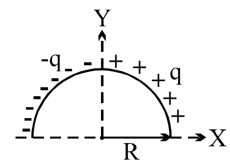


TEKO CLASSES, Director : SUHAG R. KARIYA (S. R. K. Sir) PH: (0755)- 32 00 000, 0 98930 58881 , BHOPAL, (M.P.)

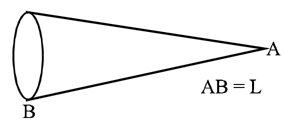
Q.18 Two spherical bobs of same mass & radius having equal charges are suspended from the same point by strings of same length. The bobs are immersed in a liquid of relative permittivity ε_r & density ρ_0. Find the density σ of the bob for which the angle of divergence of the strings to be the same in the air & in the liquid ?

Q.19 An electron beam after being accelerated from rest through a potential difference of 500 V in vacuum is allowed to impinge normally on a fixed surface. If the incident current is 100 μA, determine the force exerted on the surface assuming that it brings the electrons to rest. ($e = 1.6\times10^{-19}$ C ; $m = 9.0\times10^{-31}$ kg)

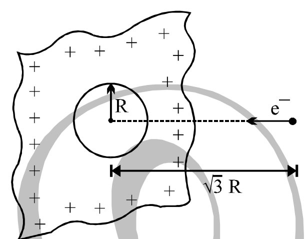
Q.20 Find the electric field at centre of semicircular ring shown in figure.



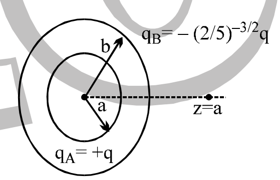
Q.21 A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to take a test charge q from infinity to apex A of cone. The slant length is L.



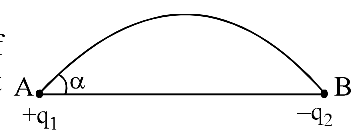
Q.22 An infinite dielectric sheet having charge density σ has a hole of radius R in it. An electron is released on the axis of the hole at a distance $\sqrt{3}R$ from the centre. What will be the velocity which it crosses the plane of sheet. (e = charge on electron and m = mass of electron)



Q.23 Two concentric rings, one of radius 'a' and the other of radius 'b' have the charges +q and $-(2/5)^{-3/2}$ q respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at z = a is in equilibrium.



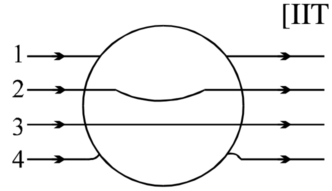
Q.24 Two charges $+q_1$ & $-q_2$ are placed at A and B respectively. A line of force emerges from q_1 at angle α with line AB. At what angle will it terminate at $-q_2$?



EXERCISE # III

Q.1 The magnitude of electric field $\vec{E}$ in the annular region of charged cylindrical capacitor
(A) Is same throughout
(B) Is higher near the outer cylinder than near the inner cylinder
(C) Varies as (1/r) where r is the distance from the axis
(D) Varies as ($1/r^2$) where r is the distance from the axis [IIT '96, 2]

Q.2 A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :



(A) 1 (B) 2
(C) 3 (D) 4 [IIT'96 , 2]

Q.3 A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E every where in space. The value of the line integral $\int_{\ell=\infty}^{\ell=0} -E.d\ell$ ($l = 0$ being centre of the ring) in volts is :
(A) + 2 (B) − 1 (C) − 2 (D) zero[JEE '97, 1]

Q.4 Select the correct alternative : [JEE '98 2 + 2 + 2 = 6]

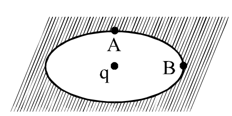
(i) A + ly charged thin metal ring of radius R is fixed in the xy–plane with its centre at the origin O . A – ly charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is:
(A) periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
(B) simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
(C) approximately simple harmonic, provided $z_0 << R$
(D) such that P crosses O & continues to move along the –ve z-axis towards $x = -\infty$

(ii) A charge +q is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0, ... \infty$ on the x-axis & a charge –q is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0, ... \infty$. Here x_0 is a +ve constant . Take the electric potential at a point due to a charge Q at a distance r from it to be $\frac{Q}{4\pi\epsilon_0 r}$. Then the potential at the origin due to the above system of charges is :
(A) 0 (B) $\frac{q}{8\pi\epsilon_0 x_0 \ell n2}$ (C) ∞ (D) $\frac{q\ell n2}{4\pi\epsilon_0 x_0}$

(iii) A non-conducting solid sphere of radius R is uniformly charged . The magnitude of the electric field due to the sphere at a distance r from its centre :
(A) increases as r increases, for $r < R$ (B) decreases as r increases, for $0 < r < \infty$
(C) decreases as r increases, for $R < r < \infty$ (D) is discontinuous at $r = R$.

Q.5 A conducting sphere S_1 of radius r is attached to an insulating handle . Another conducting sphere S_2 of radius R is mounted on an insulating stand . S_2 is initially uncharged . S_1 is given a charge Q, brought into contact with S_2 & removed, S_1 is recharged such that the charge on it is again Q & it is again brought into contact with S_2 & removed. This procedure is repeated n times.
(a) Find the electrostatic energy of S_2 after n such contacts with S_1.
(b) What is the limiting value of this energy as $n \to \infty$? [JEE '98, 7 + 1]

Q.6(i) An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the center of the cavity. The points A & B are on the cavity surface as shown in the figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity
(B) charge density at A = charge density at B
(C) potential at A = potential at B
(D) total electric field flux through the surface of the cavity is q/ε_0 . [JEE '99, 3]

(ii) A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m & positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

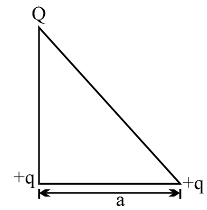
(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

[JEE '99, 5 + 5]

Q.7(a) The dimension of $\left(\frac{1}{2}\right)e_0 E^2$ (e_0 : permittivity of free space ; E : electric field) is :

(A) MLT^{-1} (B) ML^2T^{-2} (C) MLT^{-2} (D) ML^2T^{-1} (E) $ML^{-1}T^{-2}$

(b) Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to : [JEE 2000(Scr) 1 + 1]

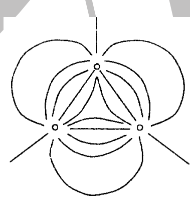
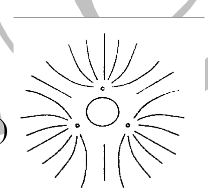
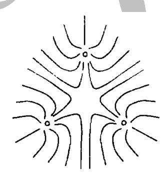
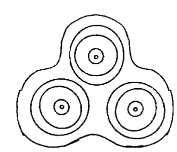


(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$

(C) $-2q$ (D) $+q$

(c) Four point charges $+8\ \mu C$, $-1\ \mu C$, $-1\ \mu C$ and $+8\ \mu C$, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge $+0.1\ \mu C$ moves along the $-x$ direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gratity free. (Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2) [JEE 2000, 10]

Q.8 Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in [JEE 2001 (Scr)]

(A)  (B)  (C)  (D) 

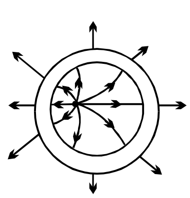
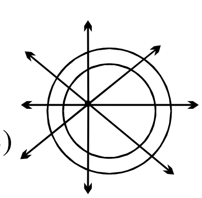
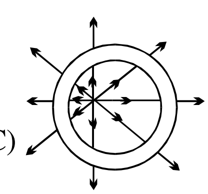
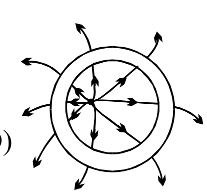
Q.9 A small ball of mass 2×10^{-3} Kg having a charge of $1\ \mu C$ is suspended by a string of length 0. 8m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so tht it can make complete revolution.

[JEE 2001]

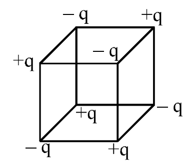
Q.10 Two equal point charges are fixed at x = − a and x = + a on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(A) x (B) x^2 (C) x^3 (D) 1/x [JEE 2002 (Scr), 3]

Q.11 A point charge 'q' is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct ? [JEE'2003 (scr)]

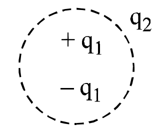
(A)  (B)  (C)  (D) 

Q.12 Charges +q and –q are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance. [JEE 2003]



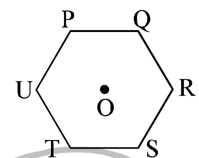
Q.13 A charge +Q is fixed at the origin of the co-ordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.
(a) calculate the K.E. of the dipole when it reaches to a point (d, 0)
(b) calculate the force on the charge +Q at this moment. [JEE 2003]

Q.14 Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to [JEE 2004 (SCR)]



(A) q_2 (B) only the positive charges
(C) all the charges (D) $+q_1$ and $-q_1$

Q.15 Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charges is possible for P, Q, R, S, T and U respectively? [JEE 2004 (SCR)]

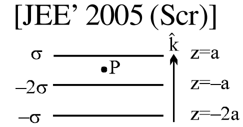


(A) +, -, +, -, -, + (B) +, -, +, -, +, – (C) +, +, -, +, -, – (D) –, +, +, –, +, –

Q.16 Two uniformly charged infinitely large planar sheet S_1 and S_2 are held in air parallel to each other with separation d between them. The sheets have charge distribution per unit area σ_1 and σ_2 (Cm^{-2}), respectively, with $\sigma_1 > \sigma_2$. Find the work done by the electric field on a point charge Q that moves from from S_1 towards S_2 along a line of length a (a < d) making an angle $\pi/4$ with the normal to the sheets. Assume that the charge Q does not affect the charge distributions of the sheets. [JEE 2004]

Q.17 Three large parallel plates have uniform surface charge densities as shown in the figure. What is the electric field at P. [JEE' 2005 (Scr)]

(A) $-\frac{4\sigma}{\epsilon_0}\hat{k}$ (B) $\frac{4\sigma}{\epsilon_0}\hat{k}$ (C) $-\frac{2\sigma}{\epsilon_0}\hat{k}$ (D) $\frac{2\sigma}{\epsilon_0}\hat{k}$



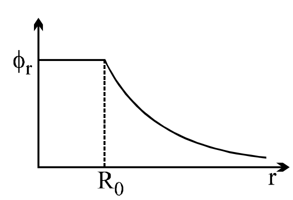
Q.18 Which of the following groups do not have same dimensions [JEE' 2005 (Scr)]
(A) Young's modulus, pressure, stress (B) work, heat, energy
(C) electromotive force, potential difference, voltage
(D) electric dipole, electric flux, electric field

Q.19 A conducting liquid bubble of radius a and thickness t (t <<a) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. [JEE 2005]

Q.20 The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in the adjacent figure, and given as

$$\phi_r = \frac{q}{4\pi\epsilon_o r} \quad (r \geq R_o)$$

$$\phi_r = \frac{q}{4\pi\epsilon_o R_o} \quad (r \leq R_o)$$

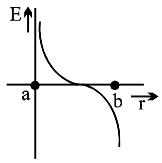
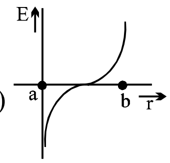
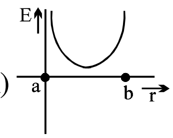
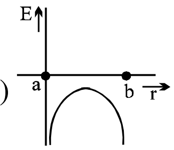


Which of the following option(s) is/are correct?
(A) For spherical region $r \leq R_o$, total electrostatic energy stored is zero.
(B) Within $r = 2R_o$, total charge is q.
(C) There will be no charge anywhere except at $r = R_o$.
(D) Electric field is discontinuous at $r = R_o$. [JEE 2006]

ANSWER KEY

EXERCISE # I

Q.1 $a = l(1+\sqrt{2})$, the equilibrium will be stable Q.2 (a) $\sqrt{\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}}$ (b) $\sqrt{\dfrac{m\pi^3\varepsilon_0 d^3}{2Qq}}$

Q.3 (i)  (ii)  (iii)  (iv) 

Q.4 $-\left[\dfrac{3}{11}\right]^{3/2} 3\times10^{-9}$ C Q.5 0 Q.6 0 Q.7 $\dfrac{qQ}{8\pi^2\varepsilon_0 r^2}$

Q.8 $\dfrac{3\sigma\lambda}{2m\epsilon_0}$ Q.9 $2\tan^{-1}\left(\dfrac{\sigma q_0}{2\varepsilon_0 mg}\right)$ Q.10 $\dfrac{1}{2\pi}\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$ Q.11 $\sqrt{\dfrac{2kQ^2}{mR}}$

Q.12 $-\dfrac{\pi\sigma^2R^3}{\varepsilon_0}$ Q.13 $20\sqrt{\ln 2}$ Q.14 $-\dfrac{kq^2}{a}\left(3-\sqrt{2}\right)$ Q.15 $9V_0$

Q.16 $\dfrac{Q^2}{m\pi\epsilon_0 V^2}$ Q.17 $\dfrac{2\epsilon_0 u^2 m}{q\sigma}$ Q.18 (i) $q_2 = -\dfrac{b}{a}q_1$; (ii) $\left[\begin{array}{lr} V_r=\dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{a}\right) \quad ; & a\le r\le b \\ V_b=\dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{b}-\dfrac{1}{a}\right) \quad ; & r=b \\ V_r=\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{r}+\dfrac{q_2}{r}\right) \quad ; & r\ge b \end{array}\right.$

Q.19 1.8×10^5 sec Q.20 $-Q/3$ Q.21 1.125 q Q.22 $\dfrac{kP}{\sqrt{2}\,y^3}(-\hat{i}-2\hat{j})$

Q.23 $-\dfrac{7}{8}kp\,\hat{k}$ Q.24 $\dfrac{q}{24\epsilon_0}$ Q.25 $a=\dfrac{R}{\sqrt{3}}$ Q.26 $\dfrac{Q}{2\varepsilon_0}$

EXERCISE # II

Q.1 (a) 60º (b) $mg+\dfrac{kq_1q_2}{\ell^2}$ (c) $\sqrt{3}$ mg , mg . q_1 & q_2 should have unlike charges for the beads to remain stationaly & $q_1q_2 = -mg\,l^2/k$

Q.2 $\left(\dfrac{5+\sqrt{89}}{8}\right)L$ Q.3 9.30 Q.4 $\sqrt{\dfrac{\lambda q}{2\varepsilon_0 m}}$ Q.5 $H_2 = h_1 + h_2 - g\left(\dfrac{\ell}{v}\right)^2$

Q.6 $W_{\text{first step}} = \left(\dfrac{8}{3}-\dfrac{4}{\sqrt{5}}\right)\dfrac{Kq^2}{r}$, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$ Q.7 $\left[\dfrac{2KQq}{mR}\left(\dfrac{r-R}{r}+\dfrac{3}{8}\right)\right]^{1/2}$

Q.8 $q = 4l\sqrt{4\pi\varepsilon_0 mg\sin\left(\dfrac{\alpha}{2}\right)}\,\sin\dfrac{\alpha}{2}$ Q.9 $\sqrt{\dfrac{6\sqrt{2}mr\epsilon_0}{e\rho a}}$ Q.10 $\dfrac{v}{\sqrt{3}}$

Q.11 $n=\dfrac{4\pi\varepsilon_0 mg(h-R)R}{q^2}$ Q.12 $\sqrt{4\pi\varepsilon_0 Ka}$ Q.13 2.2×10^{-12}C Q.14 $\dfrac{Q_1}{Q_2}=\dfrac{R_1}{R_2}$

Q.15 $0, \dfrac{2K\lambda}{r}, 0$ Q.17 $\lambda R E_0 \hat{i}$ Q.18 $\sigma = \dfrac{\varepsilon_r \rho_0}{\varepsilon_r - 1}$ Q.19 7.5×10^{-9} N

Q.20 $-\dfrac{4kq}{\pi R^2}\hat{i}$ Q.21 $\dfrac{Qq}{2\pi\epsilon_0 L}$ Q.22 $v = \sqrt{\dfrac{\sigma e R}{m\varepsilon_0}}$ Q.23 2

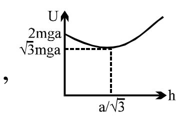
Q.24 $b = 2\sin^{-1}\left(\sin\dfrac{\alpha}{2}\sqrt{\dfrac{q_1}{q_2}}\right)$

EXERCISE # III

Q.1 C Q.2 D Q.3 A Q.4 (i) A, C, (ii) D, (iii) A, C

Q.5 (a) $U_2 = \dfrac{a^2Q^2}{8\pi\epsilon_0 R}\left(\dfrac{1-a^n}{1-a}\right)^2$ where $a = \dfrac{R}{r+R}$, (b) $U_2\,(n \to \infty) = \dfrac{RQ^2}{8\pi\epsilon_0 r^2}$

Q.6 (i) C, (ii) (a) $H = \dfrac{4a}{3}$, (b) $U = mg\left[2\sqrt{h^2+a^2}-h\right]$ equilibrium at $h = \dfrac{a}{\sqrt{3}}$,



Q.7 (a) E, (b) B, (c) $v_0 = 3$ m/s ; K.E. at the origin $= \left(27-10\sqrt{6}\right) \times 10^{-4}$ J approx. 2.5×10^{-4} J

Q.8 C Q.9 5.86 m/s Q.10 B Q.11 A

Q.12 $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}\cdot\dfrac{4}{\sqrt{6}}\left[3\sqrt{3}-3\sqrt{6}-\sqrt{2}\right]$

Q.13 (a) $K.E = \dfrac{P}{4\pi\varepsilon_0}\dfrac{Q}{d^2}$, (b) $\dfrac{QP}{2\pi\varepsilon_0 d^3}$ along positive x-axis Q.14 C

Q.15 -, +, +, -, +, - Q.16 $\dfrac{(\sigma_1-\sigma_2)Qa}{2\sqrt{2}\,\varepsilon_0}$ Q.17 C Q.18 D Q.19 $V' = \left(\dfrac{a}{3t}\right)^{1/3}.V$

Q.20 A,B,C,D