विघ्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

This is TYPE 1 Package please wait for Type 2

Subject : PHYSICS

Topic : SIMPLE HARMONIC MOTION

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Index

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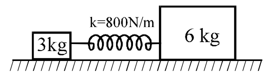
EXERCISE–I

Q.1 A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

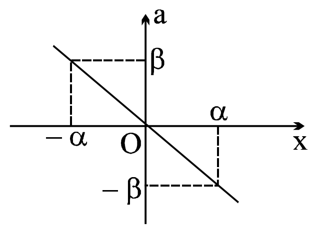
Q.2 The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6 cm and then released. Find



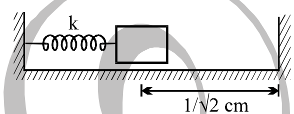
(a) time period (b) amplitude of 3 kg block (c) maximum momentum of 6 kg block

Q.3 A body undergoing SHM about the origin has its equation is given by $x = 0.2 \cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

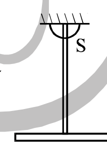
Q.4 The acceleration-displacement $(a – x)$ graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



Q.5 A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.



Q.6 Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of supension). Find the time period of such small oscillations.



Q.7 A force $f = – 10x + 2$ acts on a particle of mass 0.1 kg, where 'k' is in m and F in newton. If it is released from rest at $x = – 2$ m, find :
(a) amplitude ; (b) time period ; (c) equation of motion.

Q.8 Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by, $U = (x^2 – 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its time period.

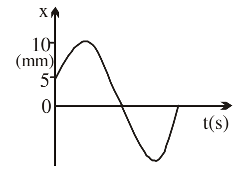
(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

Q.9 The resulting amplitude A' and the phase of the vibrations δ

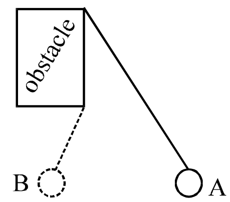
$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{4}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta)$ are _____ and _____ respectively.

Q.10 A body is executing SHM under the action of force whose maximum magnitude is 50N. Find the magnitude of force acting on the particle at the time when its energy is half kinetic and half potential.

Q.11 The figure shows the displacement - time graph of a particle executing SHM. If the time period of oscillation is 2s, then the equation of motion is given by x = __________.



Q.12 A simple pendulum has a time period T = 2 sec when it swings freely. The pendulum is hung as shown in figure, so that only one-fourth of its total length is free to swing to the left of obstacle. It is displaced to position A and released. How long does it take to swing to extreme displacement B and return to A? Assume that dispalcement angle is always small.



List of recommended questions from I.E. Irodov.

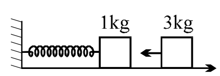
4.3, 4.17, 4.20, 4.21, 4.26, 4.27, 4.34, 4.38(a), 4.43, 4.45

EXERCISE–II

Q.1 A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

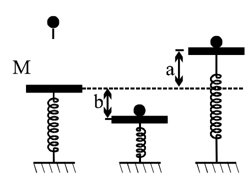
Q.2 The particle executing SHM in a straight line has velocities 8 m/s, 7 m/s, 4 m/s at three points distant one metre from each other. What will be the maximum velocity of the particle?

Q.3 One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to x-axis. A block of mass m = 1 kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is x = 10 + 3sin10t, t is in second and x in cm. Another block of mass M = 3kg, moving towards the origin with velocity 30cm/c collides with the block performing SHM at t = 0 and gets struck to it, calculate :



(i) new amplitude of oscillations.
(ii) new equation for position of the combined body.
(iii) loss of energy during collision. Neglect friction.

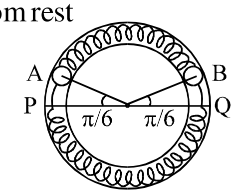
Q.4 A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales & depth 'b' below it.



(a) Find the constant of force of the spring.; (b) Find the oscillation frequency.
(c) What is the height above the initial level from which the mass m was dropped ?

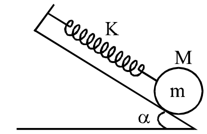
Q.5 Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in fig. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06m. Each spring has a natural length 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest

(a) Calculate the frequency of oscillation of the ball B.
(b) What is the total energy of the system.
(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.

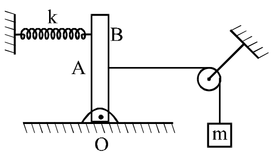


Q.6 An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass m. The piston and the cylinder have equal cross-sectional area A, atmospheric pressure is P_0 and when the piston is in equilibrium position. Show that the piston executes SHM and find the frequency of oscillation (system is completely isolated from the surrounding). $\gamma = C_p/C_v$. Height of the gas in equilibrium position is h.

Q.7 Find the angular frequency of the small oscillations of the sphere of mass M containing water of mass m. The spring has a constant K and cylinder executes pure rolling. What happens when the water in the cylinder freezes?



Q.8 A massless rod is hinged at O. A string carrying a mass *m* at one end is attached to point A on the rod so that OA = a. At another point B (OB= b) of the rod, a horizontal spring of force constant *k* is attached as shown. Find the period of small vertical oscillations of mass *m* around its equilibrium position. What can be the maximum amplitude of its oscillation so that its motion may remain simple harmonic.



EXERCISE–III

Q.1 State whether true or false
"Two simple harmonic motions are represented by the equations
$x_1 = 5\sin[2\pi t + \pi/4]$ and $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$ their amplitudes are in the ratio 1 : 2" [REE' 96]

Q.2 A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3Hz in a horizontal plane. The coefficient of static friction between block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface. [REE' 96]

Q.3 A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillations is a, then its time period T is

(A) proportional to $1/\sqrt{a}$
(B) independent of a
(C) proportional to $\sqrt{a}$
(D) proportional to $a^{3/2}$ [JEE' 98]

Q.4 A particle free to move along the x-axis has potential energy given by
$U(x) = k[1-\exp(-x^2)]$ for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then
(A) at point away from the origin, the particle is in unstable equilibrium.
(B) for any finite nonzero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic. [JEE' 99]

Q.5 Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^0, then

(A) the resultant amplitude is $(1+\sqrt{2})a$
(B) the phase of the resultant motion relative to the first is 90^0.
(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion.
(D) the resulting motion is not simple harmonic. [JEE' 99]

Q.6 The period of oscillation of simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α is given by [JEE' 2000]

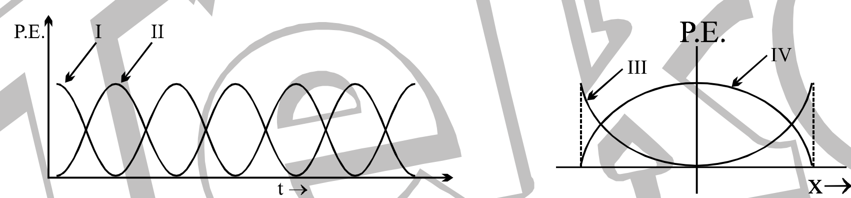
(A) $2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\dfrac{L}{g\sin\alpha}}$ (C) $2\pi\sqrt{\dfrac{L}{g}}$ (D) $2\pi\sqrt{\dfrac{L}{g\tan\alpha}}$

Q.7 A bob of mass M is attached to the lower end of a vertical string of length L and cross sectional area A. The Young's modulus of the material of the string is Y. If the bob executes SHM in the vertical direction, find the frequency of these oscillations. [REE' 2000]

Q.8 A particle executes simple harmonic motion between x = –A and x = +A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then [JEE (Scr)' 2001]
(A) $T_1 < T_2$ (B) $T_1 > T_2$ (C) $T_1 = T_2$ (D) $T_1 = 2T_2$

Q.9 A diatomic molecule has atoms of masses m_1 and m_2. The potential energy of the molecule for the interatomic separation r is given by $V(r) = -A + B(r - r_0)^2$, where r_0 is the equilibrium separation, and A and B are positive constants. The atoms are compressed towards each other from their equilibrium positions and released. What is the vibrational frequency of the molecule? [REE' 2001]

Q.10 A particle is executing SHM according to $y = a\cos\omega t$. Then which of the graphs represents variations of potential energy : [JEE (Scr)' 2003]



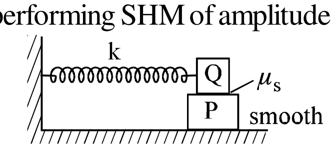
(A) (I) & (III) (B) (II) & (IV)
(C) (I) & (IV) (D) (II) & (III)

Q.11 Two masses m_1 and m_2 connected by a light spring of natural length l_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve x-axis pass through the origin at t = 0. At this position the string snaps. Position of mass m_1 at time t is given by the equation.

$$x_1(t) = v_0 t - A(1 - \cos\omega t)$$

Calculate :
(a) Position of the particle m_2 as a function of time.
(b) l_0 in terms of A. [JEE' 2003]

Q.12 A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is

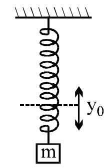


(A) kA (B) $\dfrac{kA}{2}$
(C) zero (D) $\mu_s mg$ [JEE' 2004]

Q.13 A simple pendulum has time period T_1. When the point of suspension moves vertically up according to the equation $y = kt^2$ where $k = 1\ m/s^2$ and 't' is time then the time period of the pendulum is T_2 then $\left(\frac{T_1}{T_2}\right)^2$ is [JEE' 2005 (Scr)]

(A) $\frac{5}{6}$ (B) $\frac{11}{10}$ (C) $\frac{6}{5}$ (D) $\frac{5}{4}$

Q.14 A small body attached to one end of a vertically hanging spring is performing SHM about it's mean position with angular frequency ω and amplitude a. If at a height y* from the mean position the body gets detached from the spring, calculate the value of y* so that the height H attained by the mass is maximum. The body does not interact with the spring during it's subsequent motion after detachment. ($a\omega^2 > g$). [JEE 2005]



Q.15 Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM
(A) for any value of A, B and C (except C = 0)
(B) if $A = -B;\ C = 2B$, amplitude $= \left|B\sqrt{2}\right|$
(C) if $A = B;\ C = 0$
(D) if $A = B;\ C = 2B$, amplitude $= |B|$ [JEE 2006]

Q.16 A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$ $l \approx 1$m and he commits an error of Δl. For The takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1sec. For which of the following data, the measurement of g will be most accurate?

	Δl	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 sec	10	5 mm
(B)	5 mm	0.2 sec	20	5 mm
(C)	5 mm	0.1 sec	20	1 mm
(D)	1 mm	0.1 sec	50	1 mm

[JEE 2006]

ANSWER KEY

EXERCISE–I

Q.1 $1/\sqrt{3}$ **Q.2** (a) $\frac{\pi}{10}$ sec, (b) 4 cm, (c) 2.40 kg m/s. **Q.3** 2 m/s **Q.4** $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

Q.5 100 Nm^{-1} **Q.6** $2\pi\sqrt{\frac{17L}{18g}}$ **Q.7** $\frac{11}{5}$ m, (b) $\frac{\pi}{5}$ sec., (c) $x = 0.2 - \frac{11}{5}\cos\omega t$

Q.8 (i) $x_0 = 2$m; (ii) $T = \sqrt{2}\,\pi$ sec.; (iii) $2\sqrt{3}$ **Q.9** $\frac{3\sqrt{5}\,A}{8}$, $\tan^{-1}\left(\frac{1}{2}\right)$

Q.10 $25\sqrt{2}$ N **Q.11** $x = 10\sin(\pi t + \pi/6)$ **Q.12** $\frac{3}{2}$ sec

EXERCISE–II

Q.1 $y = 0.1\sin(4t + \pi/4)$ **Q.2** $\sqrt{65}$ m/s **Q.3** 3cm, $x = 10 - 3\sin 5t$; $\Delta E = 0.135$J

Q.4 (a) $K = \frac{2mg}{b-a}$; (c) $\left(\frac{M+m}{m}\right)\frac{ab}{b-a}$, $\frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$

Q.5 $f = \frac{1}{\pi}$; $E = 4\pi^2 \times 10^{-5}$J; $v = 2\pi \times 10^{-2}$m/s

Q.6 $f = \frac{1}{2\pi}\sqrt{\frac{\gamma(P_0 + mg/A)A}{mh}}$ **Q.7** $\omega = \sqrt{\frac{K}{2M+m}}$; $\omega' = \sqrt{\frac{2K}{(4M+3m)}}$

Q.8 $(2\pi a/b)(m/k)^{1/2}$, a^2mg/b^2k

EXERCISE–III

Q.1 T **Q.2** 2cm **Q.3** A **Q.4** D **Q.5** A, C **Q.6** A **Q.7** $\frac{1}{2\pi}\sqrt{\frac{YA}{ML}}$

Q.8 A **Q.9** $\frac{1}{f} = 2\pi\sqrt{\frac{m_1m_2}{2B(m_1+m_2)}}$ **Q.10** A

Q.11 (a) $x_2 = v_0t + \frac{m_1}{m_2}A(1-\cos\omega t)$, (b) $l_0 = \left(\frac{m_1}{m_2}+1\right)A$ **Q.12** B **Q.13** C

Q.14 $y^* = \frac{mg}{k} = \frac{g}{\omega^2} < a$ **Q.15** A,B,D **Q.16** D