BRILLIANT PUBLIC SCHOOL, SITAMARHI

(Affiliated up to +2 level to C.B.S.E., New Delhi)

Class-XI

IIT-JEE Advanced Physics

Study Package

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STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS-XI

Chapters:

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P1. Unit Dimensions

Index:

EXERCISE

Q.1 If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:
(A) FT^2 (B) $F^{-1} A^2 T^{-1}$ (C) FA^2T (D) AT^2

Q.2 The dimensions $ML^{-1} T^{-2}$ can correspond to :
(A) moment of a force or torque (B) surface tension
(C) pressure (D) co-efficient of viscosity.
(useful relation are $\vec{\tau} = \vec{r} \times \vec{F}$, $S = F/l$, $F = 6\pi\eta r v$, where symbols have usual meaning)

Q.3 Which of the following can be a set of fundamental quantities
(A) length, velocity, time (B) momentum, mass, velocity
(C) force, mass, velocity (D) momentum, time, frequency

Q.4 Kinetic energy (K) depends upon momentum (p) and mass (m) of a body as $K \propto p^a m^b$
(A) a =1; b =1 (B) a =2; b = –1 (C) a =2; b=1 (D) a =1; b =2

Q.5 If area (A) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as.
(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

Q.6 The pressure of 10^6 dyne/cm^2 is equivalent to
(A) 10^5 N/m^2 (B) 10^6 N/m^2 (C) 10^7 N/m^2 (D) 10^8 N/m^2

Q.7 If 1 unit of mass = 4 kg; 1 unit of length = $\frac{1}{4}$ m and 1 unit of time = 5 sec, then 1 Joule = x units of energy in this system where x =
(A) 100 units (B) 0.01 units (C) 200 units (D) 0.02 units

Q.8 In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10 m. In this system, one unit of power will correspond to
(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) none of these

Q.9 In a book, the answer for a particular question is expressed as

$$b = \frac{ma}{k}\left[\sqrt{1+\frac{2kl}{ma}}\right]$$

here m represents mass, a represents accelerations , l represents length. The unit of b should be
(A) m/s (B) m/s^2 (C) meter (D) / sec.

Q.10 If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}$ Q, then P / Q is
(A) 1 (B) 3 / 2 (C) 2 (D) 4

Q.11 The resultant of two forces F_1 and F_2 is P. If F_2 is reversed, then resultant is Q. Then the value of $(P^2 + Q^2)$ in terms of F_1 and F_2 is
(A) $2(F_1^2 + F_2^2)$ (B) $F_1^2 + F_2^2$ (C) $(F_1 + F_2)^2$ (D) none of these

Q.12 A man moves towards 3 m north then 4 m towards east and finally 5m towards 37° south of west. His displacement from origin is

(A) $5\sqrt{2}$ m (B) 0 m (C) 1 m (D) 12 m

Q.13 Three forces P, Q & R are acting at a point in the plane . The angle between P & Q and Q & R are 150º & 120º respectively, then for equilibrium, forces P, Q & R are in the ratio

(A) 1 : 2 : 3 (B) 1 : 2 : $\sqrt{3}$ (C) 3 : 2 : 1 (D) $\sqrt{3}$: 2 : 1

Q.14 A man rows a boat with a speed of 18km/hr in northwest direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline.

(A) 9 km/hr (B) $18\frac{\sqrt{3}}{2}$ km/hr (C) 18 cos15°km/hr (D) 18 cos75° km/hr

Q.15 A bird moves from point $(1, -2, 3)$ to $(4, 2, 3)$. If the speed of the bird is 10 m/sec, then the velocity vector of the bird is :

(A) $5\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ (B) $5\left(4\hat{i}+2\hat{j}+3\hat{k}\right)$ (C) $0.6\hat{i}+0.8\hat{j}$ (D) $6\hat{i}+8\hat{j}$

Q.16 The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is

(A) 150° (B) 90° (C) 60° (D) 120°

Q.17 If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a}-\hat{b}|$ is

(A) 0 (B) 1 (C) 2 (D) 4

Q.18 For a particle moving in a straight line, the position of the particle at time (t) is given by

$$x = t^3 - 6t^2 + 3t + 7$$

what is the velocity of the particle when it's acceleration is zero ?

(A) – 9 ms^{-1} (B) – 12 ms^{-1} (C) 3 ms^{-1} (D) 42 ms^{-1}

Q.19 Use the approximation $(1 + x)^n \approx 1 + nx, |x| << 1$, to find approximate value for

(a) $\sqrt{99}$ (b) $\frac{1}{1.01}$ (c) $124^{1/3}$

Q.20 Use the small angle approximations to find approximate values for (a) sin 8° and (b) tan 5°

Q.21 A particle is in a uni-directional potential field where the potential energy (U) of a particle depends on the x-cordinate given by $U_x = k(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

Q.22 An enclosed ideal gas A has its pressure P as a function of its volume V as $P = P_0 - \alpha V^2$, where P_0 & α are constants . Find the physical dimensions of α .

Q.23 The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (l) [$k = \frac{\text{Force}}{\text{length}}$]. Find the relation among, (T), (m), (l) & (k) using dimensional method.

Q.24 The equation of state for a real gas at high temperature is given by $P = \frac{nRT}{V-b} - \frac{a}{T^{1/2}V(V+b)}$ where n, P, V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant . Find the dimensions of constant 'a' in the above equation.

Q.25 The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by $x = a \sin\omega t$ where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

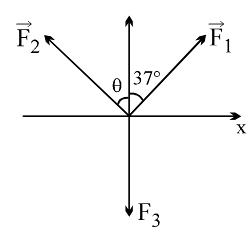
Q.26 If the velocity of light c, Gravitational constant G & Plank's constant h be chosen as fundamental units, find the dimension of mass, length & time in the new system.

Q.27 A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon
(a) Radius of orbit (R)
(b) Mass of planet (M)
(c) Universal gravitation constant (G)
Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

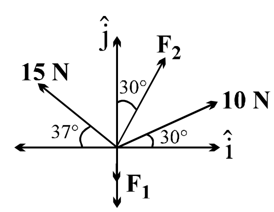
Q.28 Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

Q.29 When two forces of magnitude P and Q are perpendicular to each other, their resultant is of magnitude R. When they are at an angle of 180° to each other their resultant is of magnitude $\frac{R}{\sqrt{2}}$. Find the ratio of P and Q.

Q.30 A body acted upon by 3 given forces is under equilibrium.
(a) If $|\vec{F}_1| = 10$ Nt., $|\vec{F}_2| = 6$ Nt.
Find the values of $|\vec{F}_3|$ & angle (θ).
(b) Express $\vec{F}_2$ in unit vector form.



Q.31 If the four forces as shown are in equilibrium
Express $\vec{F}_1$ & $\vec{F}_2$ in unit vector form .



Q.32 A particle is acted upon by the forces $\vec{F}_1 = 2\hat{i} + a\hat{j} - 3\hat{k}$, $\vec{F}_2 = 5\hat{i} + c\hat{j} - b\hat{k}$, $\vec{F}_3 = b\hat{i} + 5\hat{j} - 7\hat{k}$, $\vec{F}_4 = c\hat{i} + 6\hat{j} - a\hat{k}$, . Find the values of the constants a, b, c in order that the particle will be in equilibrium.

Q.33 A plane body has perpendicular axes OX and OY marked on it and is acted on by following forces

5P in the direction OY

4P in the direction OX

10P in the direction OA where A is the point (3a, 4a)

15P in the direction AB where B is the point (– a, a)

Express each force in the unit vector form & calculate the magnitude & direction of sum of the vector of these forces.

Q.34 A particle moves along the space curve $\vec{r} = (t^2 + t)\hat{i} + (3t - 2)\hat{j} + (2t^3 - 4t^2)\hat{k}$.(t in sec, r in m) Find at time t = 2 the (a) velocity, (b) acceleration, (c) speed or magnitude of velocity and (d) magnitude of acceleration .

Q.35 A vector $\vec{A}$ of length 10 units makes an angle of 60° with a vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A} - \vec{B}$ & the angle it makes with vector $\vec{A}$.

Q.36 At time t the position vector of a particle of mass m = 3kg is given by $\vec{r} = 6t\,\hat{i} - t^3\hat{j} + \cos t\,\hat{k}$. Find the resultant force $\vec{F}(t)$, magnitude of its acceleration when $t = \frac{\pi}{2}$ & speed when $t = \pi$.

Q.37 Given that the position vector of a particle moving in x-y plane is given by $\vec{r} = (t^2 - 4)\hat{i} + (t - 4)\hat{j}$. Find

(a) Equation of trajectory of the particle

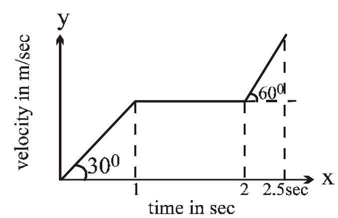
(b) Time when it crosses x-axis and y-axis

Q.38 The velocity time graph of a body moving in a straight line is shown. Find its

(a) instantaneous velocity at t = 1.5 sec.

(b) average acceleration from t = 1.5 sec. to t = 2.5 sec.

(c) draw its acceleration time graph from t = 0 to t = 2.5 sec



Q.39 The curvilinear motion of a particle is defined by v_x=50-16t and y=100-4t^2, where v_x is in metres per second, y is in metres and t is in seconds. It is also known that x=0 when t=0. Determine the velocity (**v**) and acceleration(**a**) when the position y=0 is reached.

Q.40 The force acting on a body moving in a straight line is given by $F = (3t^2 - 4t + 1)$ Newton where t is in sec. If mass of the body is 1kg and initially it was at rest at origin. Find

(a) displacement between time t = 0 and t = 2 sec.

(b) distance travelled between time t = 0 and t = 2 sec.

ANSWER KEY

EXERCISE

Q.1	D	Q.2	C	Q.3	C	Q.4	B
Q.5	B	Q.6	A	Q.7	A	Q.8	A
Q.9	C	Q.10	C	Q.11	A	Q.12	B
Q.13	D	Q.14	A	Q.15	D	Q.16	D
Q.17	B	Q.18	A	Q.19	(a) 9.95, (b) 0.99, (c) 4.986		
Q.20	0.14, 0.09	Q.21	L^{-1}, ML^2T^{-2}	Q.22	$ML^{-7}T^{-2}$		
Q.23	$T = a\sqrt{\dfrac{m}{k}}$	Q.24	$ML^5T^{-2}K^{1/2}$	Q.25	$\omega = K\sqrt{\dfrac{Gm}{r^3}}$		

Q.26 $[M] = [h^{1/2}\cdot c^{1/2}\cdot G^{-1/2}]$; $[L] = [h^{1/2}\cdot c^{-3/2}\cdot G^{1/2}]$; $[T] = [h^{1/2}\cdot c^{-5/2}\cdot G^{1/2}]$

Q.27 $v_0 = k\sqrt{\dfrac{GM}{R}}$ Q.28 (a) 180°, (b) 90°, (c) 0 Q.29 $2 \pm \sqrt{3}$

Q.30 (a) $|\vec{F}_3| = 8$ N, q = 90° (b) $\vec{F}_2 = -6\hat{i}$

Q.31 $\vec{F}_1 = -(12\sqrt{3}-1)\,\hat{j}$ & $\vec{F}_2 = (12-5\sqrt{3})\hat{i} + (12\sqrt{3}-15)\hat{j}$

Q.32 a = – 7, b = –3, c = – 4

Q.33 $5P\,\hat{j}, 4P\,\hat{i}, 6P\,\hat{i} + 8P\,\hat{j}, -12P\,\hat{i} - 9P\,\hat{j}, \sqrt{20}P, \tan^{-1}[-2]$ with the +ve x axis

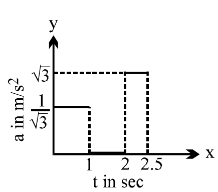
Q.34 (a) 5i + 3j + 8k, (b) 2i + 16k, (c) $7\sqrt{2}$, (d) $2\sqrt{65}$

Q.35 $2\sqrt{19}$;$\cos^{-1}\dfrac{7}{2\sqrt{19}}$

Q.36 $-18t\,\hat{j} - 3\cos t\,\hat{k}$; 3p; $3\sqrt{4+\pi^4}$

Q.37 (a) $y^2 + 8y + 12 = x$; (b) crosses x axis when t = 4 sec. , crosses y axis when t = $\underline{+}$ 2 sec.

Q.38 (a) $\dfrac{1}{\sqrt{3}}$ m/s , (b) $\dfrac{\sqrt{3}}{2}$ m/s^2 , (c)



Q.39 $\vec{v} = -30\hat{i} - 40\hat{j}$, $\vec{a} = -16\hat{i} - 8\hat{j}$ Q.40 (a) $\dfrac{2}{3}$ m, (b) t = 0, 1

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P2. Measurement

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ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity.

(1) indeterminate and (2) determinate (or systematic) errors.

1. Indeterminate errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of indeterminate error can be some what reduced by taking the average of measured values. Indeterminate errors have no fixed sign or size.
2. Determinate errors occur due to error in the procedure, or miscalibration of the intrument etc. Such errors have same size and sign for all the measurements. Such errors can be determined.

A measurement with relatively small indeterminate error is said to have high precision. A measurement with small indeterminate error and small determinate error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.

Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgment that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Indeterminate errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself. In general

$$\text{relative error} = \frac{\textit{absolute error in a measurement}}{\textit{size of the measurement}}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. ***Determinate errors*** **:** They have a known sign.

1. Suppose that a result R is calculated from the sum of two measured quantities A and B. We'll use a and b to represent the error in A and B respectively. r is the error in the result R. Then

   $(R + r) = (A + B) + (a + b)$

   The error in R is therefore : $r = a + b$.

   Similarly, when two quantities are subtracted, the determinate errors also get subtracted.

2. Suppose that a result R is calculated by multiplying two measured quantities A and B. Then $R = AB$.

   $(R + r) = (A + a)(B + b) = AB + aB + Ab + ab$

   $\Rightarrow \quad \frac{r}{R} = \frac{aB + bA}{AB} = \frac{a}{A} + \frac{b}{B}$. Thus when two quantities are multiplied, their relative determinate error add.

3. **Quotient rule :** When two quantities are divided, the relative determinate error of the quotient is the relative determinate error of the numerator minus the relative determinate error of the denominator. Thus if $R = \frac{A}{B}$ then

   $\frac{r}{R} = \frac{a}{A} - \frac{b}{B}$

4. **Power rule :** When a quantity Q is raised to a power, P, the relative determinate error in the result is P times the relative determinate error in Q.

   If $R = Q^P$, $\quad \frac{r}{R} = P \times \frac{q}{Q}$

   This also holds for negative powers,

5. The quotient rule is not applicable if the numerator and denominator are dependent on each other.

e.g if $R=\frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows

$\frac{1}{R}=\frac{1}{X}+\frac{1}{Y}$. Differentiating both the sides, we get

$$-\frac{dR}{R^2}=-\frac{dX}{X^2}-\frac{dY}{Y^2}. \qquad \text{Thus} \quad \frac{r}{R^2}=\frac{x}{X^2}+\frac{y}{Y^2} \quad \text{or} \quad \frac{r}{R}=\frac{R}{X}\times\frac{x}{X}+\frac{R}{Y}\times\frac{y}{Y}$$

B. ***Indeterminate error* :** They have unknown sign. Thus they are represented in the form A ± a.

Here we are only concerned with *limits of error*. We must assume a "worst-case" combination. In the case of substraction, A – B, the worst-case deviation of the answer occurs when the errors are either + a and – b or – a and + b. In either case, the maximum error will be (a + b)

1. ***Addition and subtraction rule* :** The absolute indeterminate errors add.
Thus if $R = A + B$, $r = a + b$
and if $R = A - B$, $r = a + b$

2. ***Product and quotient rule* :** The relative indeterminate errors add.

Thus if $R = AB$, $\frac{r}{R}=\frac{a}{A}+\frac{b}{B}$

and if $R=\frac{A}{B}$, then also $\frac{r}{R}=\frac{a}{A}+\frac{b}{B}$

3. ***Power rule* :** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

If $R = Q^P$, $\frac{r}{R}=P\times\frac{q}{Q}$

Examples

1. A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used is $S = (1/2)AT^2$. The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?
$S = 2 \pm 0.005$ meter.
$T = 4.2 \pm 0.2$ second.

Sol : We use capital letters for quantities, lower case for errors. Solve the equation for the result, a.

$A = 2S/T^2$. Its indeterminate-error equation is $\frac{a}{A}=2\frac{t}{T}+\frac{s}{S}$

Thus $A = 0.23 \pm 0.02\ m/s^2$.

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science :

1. Measured Values
2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

MEASURED VALUES

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position.For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

COMPUTED VALUE

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All non-zero digits (1-9) are to be counted as significant.
2. Zeros that have any non-zero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (ii) above are NOT be considered significant digits.

For example : 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros? The first three zeros would not be considered significant since they have no non-zero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the non-zero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example : 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated. But what about 1,000,000?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example : If I write the number above in this manner.

$$1.00 \times 10^6$$

I have indicated that the number has been recorded with three significant digits. On the other hand, if I write the same number as : 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant. So for example : 2.0040×10^4 will have five significant digits. This means that unit conversion will not change the number of significant digits. Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0171 (after rounding off)

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digited number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules? The only way is to express the answer in exponential notation so 2400 could be expressed as : 2.4×10^3

Rule for expressing the correct number of significant digits in an addition or substraction :

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation.Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)
The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.
Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)
The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e. having only one position to the right of its decimal). Notice we aren't really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits :

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.
2. Consider the digit to the right of the last reported digit.
3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682
The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is : 18.4

Another example : Round off 4.565 to three significant digits.
The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

EXPERIMENTS

(i) Measurement of length

The simplest method measuring the length of a straight line is by means of a meter scale. But there exists some limitation in the accuracy of the result:

(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier callipers (b) micrometer scales (screw gauge) are used.

VERNIER CALLIPERS:

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being either slightly longer and shorter than the divisions of the main scale.

Least count of Vernier Callipers

The least count or Vernier constant (v. c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

Vernier constant = 1 ms – 1 vs = $\left(1-\frac{N-1}{N}\right)$ ms = $\frac{1ms}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

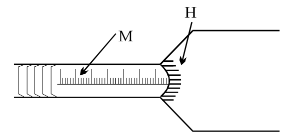
Zero error:

If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

SCREW GAUGE (OR MICROMETER SCREW)

In general vemier callipers can measure accurately upto 0.01 em and for greater accuracy micrometer screw devices e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tuming it axially. The instrument is provided with two scales:



(i) The main scale or pitch scale M graduated along the axis of the screw.

(ii) The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

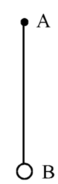
(a) **Pitch :** The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap =5 mm, then pitch = 0.5 mm

(b) **Least count :** In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration:, if the total cap division is 100, then least count = 0.5mm/100 = 0.005 mm

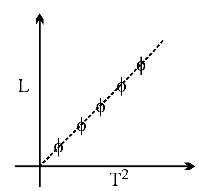
Zero Error : In a perfect instrument the zero of the heat scale coincides with the line of graduation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies above the line of graduation and versa. The corresponding corrections will be just opposite.

(ii) **Measurement of g using a simple pendulum**

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation. The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch. The time period (T) of a single oscillation is now calculated by division.



Observations are now taken by using different lengths for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear. g is given by $g = 4\pi^2 \dfrac{L}{T^2}$



The major errors in this experiment are

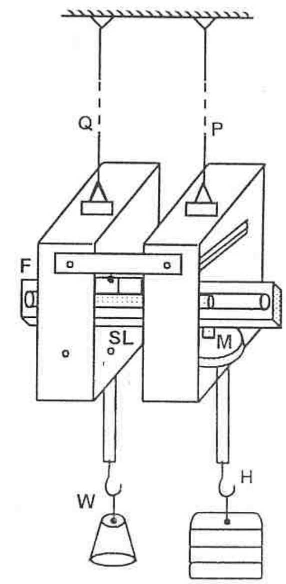
(a) **Systematic :** Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numerical estimate for the time period. However the practice is to ensure that the amplitude is small.

(b) **Statistical :** Errors arising from measurement of length and time.

$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging. The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line. The slope of this fit gives the correct value of L/T^2

(iii) Determination of Young's Modulus by Searle's Method

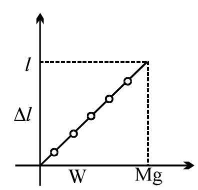


The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along .the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the side of the experimental wire.On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length *M* is obtained by

$$\Delta l = n \times \text{pitch} + f \times \text{least count}$$

The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the true increase in length of the wire corresponding to the given value of load. From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin. The slope of the line gives

$$\tan\theta = \frac{l}{W} = \frac{l}{Mg}$$



Now, stress $= \dfrac{Mg}{\pi r^2}$ and strain $= \dfrac{l}{L}$

Y = Stress/ strain $= \dfrac{MgL}{\pi r^2 l} = \dfrac{L}{\pi r^2 \tan\theta}$

With known values of initial length L, radius r of the experimental wire and $\tan\theta$, Young's modulus Y can be calculated.

(iv) Specific Heat of a liquid using a calorimeter:

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) through a heating coil immersed within the liquid for a known length of time (t). The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1)S_l(\theta_2 - \theta_0) + m_1 S_c(\theta_2 - \theta_0) = i.\ V.\ t$$

or, $(m_2 - m_1)S_l + m_1 S_c = i.\ V.\ t\,/(\theta_2 - \theta_0)$ ..... (1)

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown: one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate $m_1 S_c$.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

The direction of the current is reversed midway during the experiment to remove the effect of any differential contacts, radiation correction is introduced to take care of the second major source of systematic error.

Radiation correction: The temperature of the system is recorded for half the length of time t, i.e. t/2, where t is the time during which the current was switched on} after the current is switched off. The fall in temperature δ, during this interval is now added to the final temperature θ_2 to give the corrected final temperature:

$\theta'_2 = \theta_2 + \delta$

This temperature is used in the calculation of the specific heat, S_l.

Error analysis :

After correcting for systematic errors, equation (i) is used to estimate the remaining errors.

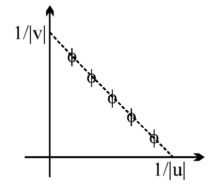
(v) Focal length of a concave mirror and a convex lens using the u-v method.

In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder. The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.

A plot of |u| versus |v| gives a rectangular hyperbola. A plot of $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.



The intercepts are equal to $\frac{1}{|f|}$, where f is the focal length.

Error : The systematic error in this experiment is mostly due to improper position of the object on the holder. This error maybe eliminated by reversing the holder (rotating the holder by 180° about the vertical) and then taking the readings again. Averages are then taken.

The equation for errors gives:

$$\left|\frac{\delta f}{f}\right| = \left|\frac{\delta u}{u}\right| + \left|\frac{\delta v}{v}\right| + \frac{|\delta u| + |\delta v|}{|u| + |v|}$$

The errors δu, δv correspond to the error in the measurement of u and v.

Index Error or Bench Error and its correction: In an experiment using an optical bench we are required to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distances between the indices with the help of the scale engraved on the bench. These distances are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error.

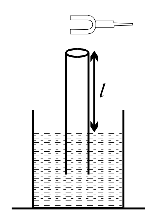
Index Error = Observed distance – actual distance and

Index Correction = Actual – observed distance

Note: Index correction whether positive or negative, is always added algebraically to the observed distance to get the corrected distance.

(vi) Speed of sound using resonance column

A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it resonates with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again until a second resonance length (l_2) is found (provided the tube is long)



Then, $l_2 - l_1 = \lambda / 2$, provided l_1, l_2 are resonance lengths for adjacent resonances.

∴ $\lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by

$C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected *for* the "end effect":

λ(fundamental) $= 4(l_1 + 0.3d)$, where d = diameter

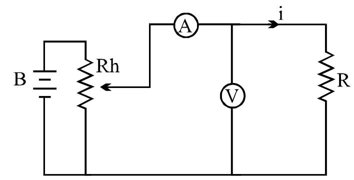
Errors : The major systematic errors introduced are due to end effects in (end correction) and also due to excessive humidity.

Random errors are given by

$$\frac{\delta C}{C} = \frac{\delta(l_2 - l_1)}{l_2 - l_1} = \frac{\delta l_2 + \delta l_1}{l_2 - l_1}$$

(vii) Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of V vs i is plotted and it is found to be linear (within errors).



The magnitude of R is determined by either

(a) taking the ratio $\frac{V}{i}$ and then

(b) fitting to a straight line: V = iR, and determining the slope R.

Errors :

Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimsed by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimised as much as possible.

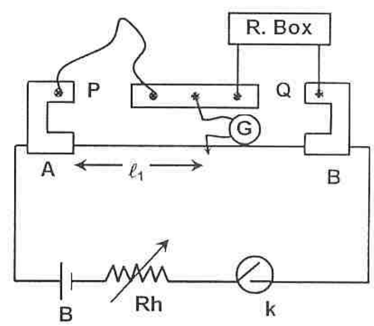
Error analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least counts of the instruments used.

(viii) Specific resistance of the material of a wire using a meter bridge :

A known length (l) of a wire is connected in one of the gaps (P) of a metre bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G).The balance length (l) is found by closing key k and momentarily connecting the galvanometer until it gives zero deflection (null point). Then,



$$\frac{P}{Q} = \frac{l}{100 - l} \qquad ....(1)$$

using the expression for the meter bridge at balance. Here, represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho\frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L}P \qquad ....(2)$

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors : The major systematic errors in this experiment are due to the heating effect, end corrections introduced due to shift of the zero of the scale at A and B, and stray resistances in P and Q, and errors due to non-uniformity of the meter bridge wire.

Error analysis : End corrections can be estimated by including known resistances P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \qquad ..... (2)$$, where α and β are the end corrections.

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \qquad \text{..... (3)}$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point,

$$\frac{P}{Q} = \frac{l + \alpha}{100 - l + \beta} = \frac{l_1'}{l_2'},$$

Errors due to non-uniformity of the meter bridge wire can be minimised by interchanging the resistances in the gaps P and Q.

$$\therefore \qquad \frac{\delta P}{P} = \left|\frac{\delta l_1'}{l_1'}\right| + \left|\frac{\delta l_2'}{l_2'}\right|$$

where, $\delta l'_1$ and $\delta l'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e. when the balance point is in the middle of the bridge. The error in ρ is

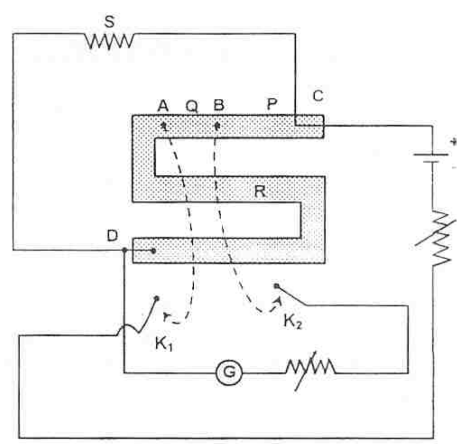
$$\frac{\delta P}{P} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$$

(ix) Measurement of unknown resistance using a P.O. Box

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm.

At balance, the unknown resistance

$$S = \left(\frac{P}{Q}\right)R \qquad \text{.....(1)}$$



The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_0$ (Ohm) and in the opposite direction when $R = R_0 + 1$ (ohm).

This implies that the unknown resistance, S lies between R_0 and $R_0 + 1$ (ohm). Now, the resistance in P and Q are made 100 Ω and 1000 Ω respectively, and the process is repeated.

Equation (1) is used to compute S.

The ratio P/Q is progressively made 1 : 10, and then 1 : 100. The resistance S can be accurately measured.

Errors : The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

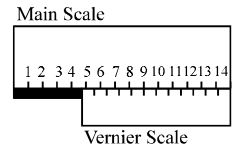
In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

EXERCISE

Q.1 In a Vernier Calipers (VC), N divisions of the main scale coincide with N + m divisions of the vernier scale. What is the value of m for which the instrument has minimum least count?

(A) 1 (B) N (C) Infinity (D) N/2

Q.2 Consider the vernier calipers as shown, the instrument has no zero error. What is the length of the rod shown, if 1 msd = 1mm? Use 7 msd = 8 vsd.

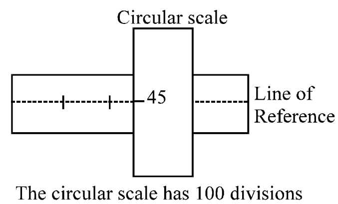


(A) 4.6 mm (B) 4.5 mm (C) 4.3 mm (D) none

Q.3 In a vernier calipers the main scale and the vernier scale are made up different materials. When the room temperature increases by ΔT°C, it is found the reading of the instrument remains the same. Earlier it was observed that the front edge of the wooden rod placed for measurement crossed the N^{th} main scale division and N + 2 msd coincided with the 2^{nd} vsd. Initially, 10 vsd coincided with 9 msd. If coefficient of linear expansion of the main scale is α_1 and that of the vernier scale is α_2 then what is the value of α_1 / α_2? (Ignore the expansion of the rod on heating)

(A) 1.8 / (N) (B) 1.8 /(N+2) (C) 1.8 / (N–2) (D) None

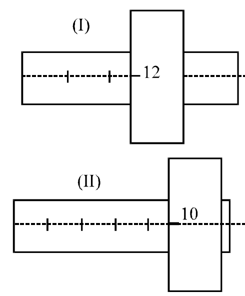
Q.4 Consider a screw gauge without any zero error. What will be the final reading corresponding to the final state as shown? It is given that the circular head translates P msd in N rotations. One msd is equal to 1mm.



(A) (P/N) (2 + 45/100) mm (B) (N/P) (2+45/N)mm

(C) P (2/N + 45/100)mm (D) $\left(2+\frac{45}{100}\times\frac{P}{N}\right)$mm

Q.5 A screw gauge has some zero error but its value is unknown. We have two identical rods. When the first rod is inserted in the screw, the state of the instrument is shown by diagram (I). When both the rods are inserted together in series then the state is shown by the diagram (II). What is the zero error of the instrument? 1 msd = 100 csd = 1mm



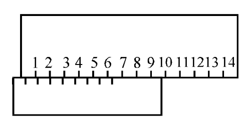
(A) –0.16 mm (B) +0.16 mm

(C) +0.14 mm (D) –0.14 mm

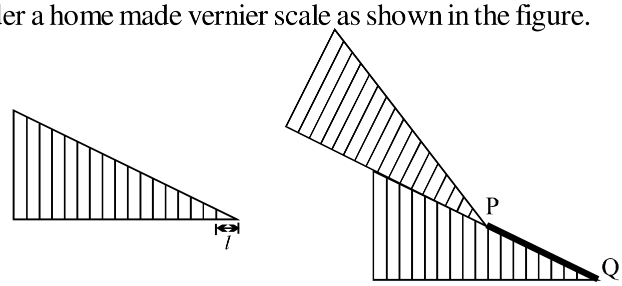
Q.6 The VC shown in the diagram has zero error in it (as you can see). It is given that 9 msd = 10 vsd.

(i) What is the magnitude of the zero error?

(ii) The observed reading of the length of a rod measured by this VC comes out to be 5.4 mm. If the vernier had been error free then ___msd would have coincided with ___ vsd.

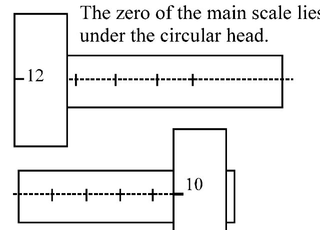


Q.7 Consider a home made vernier scale as shown in the figure.



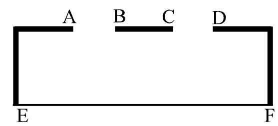
In this diagram, we are interested in measuring the length of the line PQ. If the angle of both the inclines is equal to θ then what is the least count of the instrument.

Q.8 The diagram shows the initial and the final state of SG, which has zero error in it. What can be the length of the object? 1 msd = 100 csd



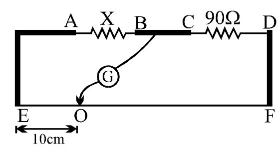
Q.9 In a meter bridge set up, which of the following should be the properties of the one meter long wire?
(A) High resistivity and low temperature coefficient
(B) Low resistivity and low temperature coefficient
(C) Low resistivity and high temperature coefficient
(D) High resistivity and high temperature coefficient

Q.10 Make the appropriate connections in the meter bridge set up shown. Resistance box is connected between _____. Unknown resistance is connected between______. Battery is connected between.



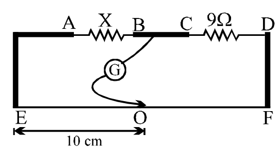
Options:
(A) AB (B) CD (C) EF (D) None

Q.11 Let the end error on the LHS and RHS be equal to one cm. For the balance point at O, find out the % tage error in the value of X? (If the end error is 1 cm from both sides then it means the corrected reading will become 10cm + 1cm from LHS and 90cm + 1cm from the RHS)



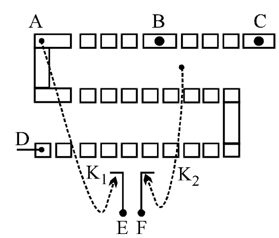
(A) 4.2% (B) 8.1% (C) 9.2% (D) None

Q.12 Consider the MB shown in the diagram, let the resistance X have temperature coefficient α_1 and the resistance from the RB have the temperature coefficient α_2. Let the reading of the meter scale be 10cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point? Neglect all the other changes in the bridge due to temperature rise.



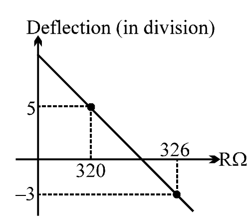
(A) $9(\alpha_1 - \alpha_2)\Delta T$ (B) $9(\alpha_1 + \alpha_2)\Delta T$ (C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$ (D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

Q.13 The diagram shows an incomplete sketch of a PO box. Battery is connected between________. The unknown resistance is connected between ________. The galvanometer is connected between_________. The key K_2 is connected between __________.



Options:
(A) CD (B) DA (C) CE (D) DF
(E) DE (F) BF (G) CF

Q.14 For a post office Box, the graph of galvanometer deflection versus R (resistance pulled out of RB) for the ratio 100 : 1 is given as shown. A careless student pulls out two non consecutive values R as shown in the figure. Find the value of unknown resistance.



(A) 3.2 ohm (B) 3.24 ohm
(C) 3.206 ohm (D) None

Q.15 When we operate a wheat stone bridge then in starting the key of the battery is closed first and the key of the G is closed later. When the circuit is to be closed then switches are released in the opposite order. Why?

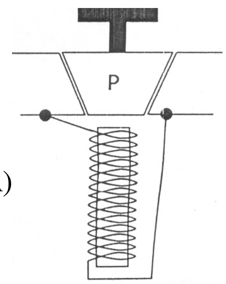
(A) Look at the diagram of the PO box, the switch is battery is always on the right hand hence it is easier to press it first.

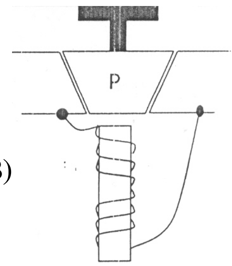
(B) This is done to avoid the damage of the galvanometer due to induced emf.

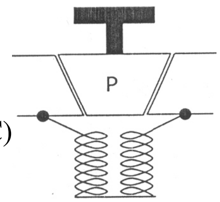
(C) If the G switch is pressed before the battery switch then large sparking takes place at the battery switch.

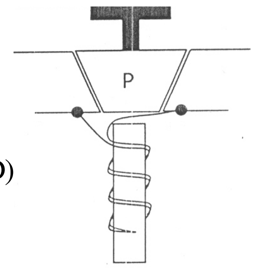
(D) While disconnecting if we open the battery switch before the G switch then we can observe induced current in the circuit till the G switch is not opened.

Q.16 Identify which of the following diagrams represent the internal construction of the coils wound in a resistance box or PQ box?

(A) 

(B) 

(C) 

(D) 

Q.17 Which of the following reading is most accurate

(A) 4.00 cm (B) 0.004 mm (C) 40.00 cm (D) 4.00 m

Q.18 The least count of a stop watch is 1/5 sec. The time of 20 oscillations of a pendulum is measured to be 25 sec. The minimum percentage error in the measurement of time will be

(A) 0.1% (B) 0.8% (C) 1.8% (D) 8%

Q.19 A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then

(A) n=10, m=0.5 cm (B) n=9, m=0.4 cm (C) n=10, m=0.8 cm (D) n=10 , m=0.2 cm

Q.20 Solve with due regard to significant digits

(i) $\sqrt{6.5-6.32}$ (ii) $\dfrac{2.91\times0.3842}{0.080}$

Q.21 A body travels uniformly a distance of (13.8 ± 0.2)m in time (4.0 ± 0.3) sec. Calculate its velocity.

Q.22 The main scale of a vernier calipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm and the fourth vernier division coincides with a scale division. Calculate the measured length of the cylinder.

Q.23 A short circuit occurs in a telephone cable having a resistance of $0.45\ \Omega m^{-1}$. The circuit is tested with a Wheatstone bridge. The two resistors in the ratio arms of the Wheatstone bridge network have values of 100Ω and 1110Ω respectively. A balance condition is found when the variable resistor has a value of 400Ω. Calculate the distance down the cable, where the short has occurred.

Q.24 5.74 gm of a substance occupies a volume of 1.2 cm^3. Calculate its density with due regard for significant figures.

Q.25 The time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The length of the pendulum is measured as $l = 10 \pm 0.1$cm and the time period as $T = 0.5 \pm 0.02$ s. Determine percentage error in the value of g.

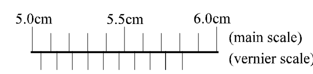
Q.26 A physical quantity P is related to four observables A, B, C and D as follows.

$$P = 4\pi^2 \frac{A^3 B^2}{\sqrt{C}\ D}$$

The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Determine the percentage error & absolute error in the quantity P. Value of P is calculated 3.763. Round off the result in scientific way.

Q.27 A glass prism of angle $A = 60^0$ gives minimum angle of deviation $\theta = 30^0$ with the max. error of 1^0 when a beam of parallel light passed through the prism during an experiment.
(i) Find the permissible error in the measurement of refractive index μ of the material of the prism.
(ii) Find the range of experimental value of refractive index 'μ'.

Q.28 In the given vernier calliper scale, the length of 1 main scale division is 1mm whereas the length of the vernier scale is 7.65 mm. Find the reading on the scale correct to significant digits as shown in the diagram.



ANSWER

Q.1	A	Q.2	B	Q.3	B	Q.4	D	Q.5	D	Q.6	(i) x = 0.6 msd, (ii) 6, 1st

Q.7 L.C = $l\left[\dfrac{1-\cos\theta}{\cos\theta}\right]$

Q.8 4 msd + 0.1 msd + 0.12 msd = 4.22 msd ; 4 msd + 0.1 msd + 1.12 msd = 5.22 msd & so on

Q.9 A Q.10 CD, AB, C Q.11 B Q.12 A Q.13 CE, CD, DF, BF

Q.14 B Q.15 B,C,D Q.16 D Q.17 C Q.18 B

Q.19 C Q.20 (i) 0.4 ; (ii) 14 Q.21 $v = (3.5 \pm 0.31)$ m/s

Q.22 3.07 cm Q.23 40 m Q.24 4.8 g/cm^3 Q.25 5%

Q.26 14%, 0.53, 3.76 Q.27 $5\pi/18\%$, $\sqrt{2}\left[1+\dfrac{\pi}{360}\right] > \mu > \sqrt{2}\left[1-\dfrac{\pi}{360}\right]$

Q.28 5.045 cm

STUDY PACKAGE

Target: IIT-JEE (<u>Advanced</u>)

SUBJECT: PHYSICS

<u>TOPIC: XI P3. Kinematics</u>

<u>Index:</u>

KEY CONCEPTS

THINGS TO REMEMBER :

1. $v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = v\frac{dv}{ds}$; $s = \int v\,dt$; $v = \int a\,dt$; $\frac{v^2}{2} = \int a\,ds$

where the symbols have their usual meaning .

2. The equations of motion for a body moving in straight line with uniform acceleration, are

(i) $v = u + at$ **(ii)** $s = \left(\frac{u+v}{2}\right)t = ut + \frac{at^2}{2} = vt - \frac{at^2}{2}$ **(iii)** $v^2 = u^2 + 2as$

(iv) $s_n = u + \frac{1}{2}a(2n-1)$ **(v)** $S = \left(\frac{v+u}{2}\right)t$

3. If a body is thrown vertically up with a velocity u in the uniform gravitational field then (neglecting air resistance) :

(i) Maximum height attained $H = \frac{u^2}{2g}$ **(ii)** Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$ **(iv)** Velocity of fall at the point of projection = u downwards

4. KINEMATIC GRAPH :

Slope of the displacement time graph at any particular time gives the magnitude of the instantaneous velocity at that particular time .

Slope of the v - t graph will give the magnitude of the instantaneous acceleration.

The area between the v - t graph , the time axis and the ordinates erected at the beginning & end of time interval considered will represent the total displacement of the body.

5. RELATIVE VELOCITY :

(a) Velocity of 'A' relative to 'B' is given by $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$.

$\vec{V}_{AB}$ refers to the velocity which 'A' appears to have as seen by B. The above idea of 1 dimensional relative motion can be extended to motion in 2 dimensions.

(b) Angular velocity of A relative to B i.e. ω_{AB} is given by

$$\omega_{AB} = \frac{\text{velocity of A relative to B in a direction perpendicular to AB}}{AB}$$

6. LEVEL GROUND PROJECTILE MOTION :

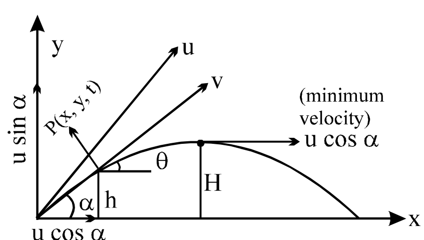
When a body is thrown obliquely (in a vertical plane) into the uniform gravitational field then the trajectory (actual path of motion) is a parabola. The horizontal component of velocity $u\cos\alpha$ remains unchanged where as vertical component decreases up to the maximum height and then increases .

(a) Time taken to reach the height point $t_H = \frac{u\sin\alpha}{g}$

(b) Maximum height $H = \frac{u^2\sin^2\alpha}{2g}$

(c) Total time of flight = $\frac{2u\sin\alpha}{g} = 2t_H$

(d) Horizontal range = $(u\cos\alpha).T = \frac{2}{g}(u\cos\alpha)(u\sin\alpha) = \frac{u^2\sin 2\alpha}{g}$



[Figure 1]

(e) $R_{max} = \frac{u^2}{g}$ if $\alpha = 45°$

Note that for a given velocity of projection & a given horizontal range there are in general two directions of projection which are complement of each other and are equally inclined to the direction of the maximum range.

(f) VELOCITY & DIRECTION OF MOTION AT A GIVEN TIME :

$$\left.\begin{aligned} V\cos\theta &= u\cos\alpha \\ V\sin\theta &= u\sin\alpha - gt \end{aligned}\right]$$ Squaring & adding these 2 equations we will get the velocity of the projectile. Dividing the velocities in y and x directions gives the direction of motion.

(g) VELOCITY & DIRECTION OF MOTION AT A GIVEN HEIGHT h :

$$\left.\begin{aligned} V^2\cos^2\theta &= u^2\cos^2\alpha \\ V^2\sin^2\theta &= u^2\sin^2\alpha - 2gh \end{aligned}\right] \text{ on adding } V^2 = u^2 - 2gh$$

(h) EQUATIONS OF MOTION IN VECTOR NOTATION :

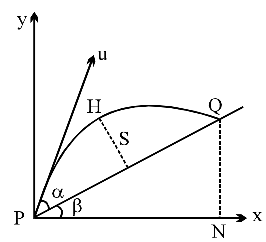
(i) $\vec{V} = \vec{u} + \vec{g}t$ **(ii)** $\vec{S} = \vec{u}t + \frac{1}{2}\vec{g}t^2$ **(iii)** $\vec{V}_{av} = \frac{\vec{S}}{t} = \vec{u} + \frac{1}{2}\vec{g}t$ ($\vec{V}_{av}$ = average velocity vector)

(i) EQUATION OF TRAJECTORY :

Oblique Projection (refer fig-1) $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} = x\tan\alpha\left(1 - \frac{x}{R}\right)$

Note that $\frac{dy}{dx}$ represent the direction of motion .

7. PROJECTILE UP AN INCLINED PLANE :



(a) Total time of flight on the inclined plane

$$T = \frac{2u}{g}\,\frac{\sin(\alpha-\beta)}{\cos\beta}$$

(b) Range PQ on the inclined plane

$$PQ = \frac{2u^2}{g}\,\frac{\cos\alpha\,.\,\sin(\alpha-\beta)}{\cos^2\beta} = \frac{u^2}{g\cos^2\beta}\,[\sin(2\alpha-\beta) - \sin\beta]$$

(c) For Maxmimum range $2\alpha - \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\beta}{2}$

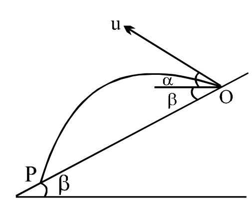
Hence the direction for maximum range bisects the angle between the vertical and the inclined plane .

(d) $R_{max} = \frac{u^2}{g(1+\sin\beta)}$

(e) Greatest distance of the projectile from the inclined plane ;

$S = \frac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$ when the projectile is at H, its velocity perpendicular to the plane is zero .

8. PROJECTILE DOWN AN INCLINED PLANE :



(a) Time of flight $= \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$

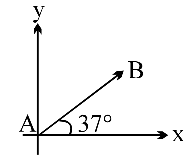
(b) Range OP $= \frac{2u^2\sin(\alpha+\beta)\,.\,\cos\alpha}{g\cos^2\beta}$

(c) Maximum range $= \frac{u^2}{g(1-\sin\beta)}$

(d) Angle of projection α for maximum range $= \frac{\pi}{4} - \frac{\beta}{2}$

EXERCISE – I

Q.1 A butterfly is flying with velocity $10\hat{i}+12\hat{j}$ m/s and wind is blowing along x axis with velocity u. If butterfly starts motion from A and after some time reaches point B, find the value of u.

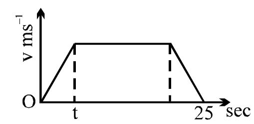


Q.2 Find the change in velocity of the tip of the minute hand (radius = 10 cm) of a clock in 45 minutes.

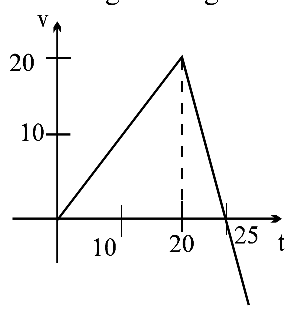
Q.3 A, B & C are three objects each moving with constant velocity. A's speed is 10 m/sec in a direction $\overrightarrow{PQ}$. The velocity of B relative to A is 6 m/sec at an angle of, $\cos^{-1}(15/24)$ to PQ. The velocity of C relative to B is 12 m/sec in a direction $\overrightarrow{QP}$, then find the magnitude of the velocity of C.

Q.4 Rain is falling vertically with a speed of 20 ms^{-1} relative to air. A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.

Q.5 The velocity-time graph of the particle moving along a straight line is shown. The rate of acceleration and deceleration is constant and it is equal to 5 ms^{-2}. If the average velocity during the motion is 20 ms^{-1}, then find the value of t.



Q.6 The fig. shows the v–t graph of a particle moving in straight line. Find the time when particle returns to the starting point.



Q.7 A particle is projected in the X-Y plane. 2 sec after projection the velocity of the particle makes an angle 45° with the X - axis. 4 sec after projection, it moves horizontally. Find the velocity of projection (use g = 10 ms^{-2}).

Q.8 A stone is dropped from a height h . Simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time when two stones cross each other.

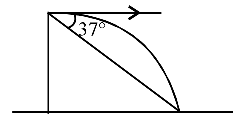
Q.9 A particle is projected upwards with a velocity of 100 m/sec at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking g = 10 m/sec^2.

Q.10 A balloon is ascending vertically with an acceleration of 0.2m/s^2. Two stones are dropped from it at an interval of 2 sec. Find the distance between them 1.5 sec after the second stone is released.(use g=9.8m/s^2)

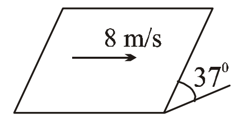
Q.11 A large number of bullets are fired in all direction with the same speed v. What is the maximum area on ground on which these bullets can spread?

Q.12 A boat starts from rest from one end of a bank of a river of width d flowing with velocity u. The boat is steered with constant acceleration a in a direction perpendicular to the bank. If point of start is origin, direction of bank is x axis and perpendicular to bank is y axis. Find the equation of trajectory of the boat .

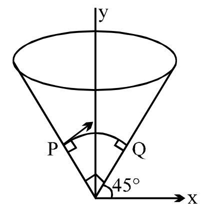
Q.13 A ball is thrown horizontally from a cliff such that it strikes ground after 5 sec. The line of sight from the point of projection to the point of hitting makes an angle of 37° with the horizontal. What is the initial velocity of projection.



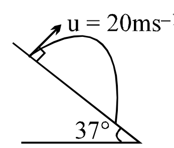
Q.14 A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8m/s. Find it's speed after 1 sec.



Q.15 A particle is projected from point P with velocity $5\sqrt{2}$ m/s perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. Find the time of the flight of the particle.

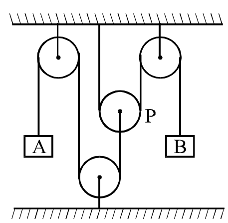


Q.16 Find range of projectile on the inclined plane which is projected perpendicular to the incline plane with velocity 20m/s as shown in figure.

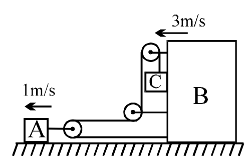


Q.17 Initial acceleration of a particle moving in a straight line is a_0 and initial velocity is zero. The acceleration reduces continuously to half in every t_0 seconds as $a = \frac{a_0}{2^{\frac{t}{t_0}}}$. Find the terminal velocity of the particle.

Q.18 Find the acceleration of movable pulley P and block B if acceleration of block A = 1 m/s^2 ↓.



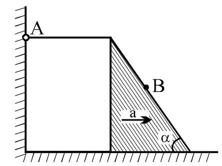
Q.19 The velocities of A and B are marked in the figure. Find the velocity of block C (assume that the pulleys are ideal and string inextensible).



Q.20 A particle is moving in x-y plane such that x = t + sin(t) meter , y = cos (t) meter. t is the time in sec. Find the length of the path taken by the particle from t = 0 to t = 2π sec.

Q.21 The speed of a particle when it is at its greatest height $\sqrt{2/5}$ is of its speed when it is at its half the maximum height. The angle of projection is ______ and the velocity vector angle at half the maximum height is ______ .

Q.22 A weightless inextensible rope on a stationary wedge forming angle α with the horizontal. One end of the rope is fixed to the wall at point A. A small load is attached to the rope at point B. The wedge starts moving to the right with a constant acceleration. Determine the acceleration a_1 of the load when it is still on the wedge.



Q.23 The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration = g/2. Under the same conditions of projection, find the horizontal range of the projectile.

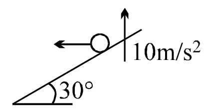
Q.24 A rocket is launched at an angle 53º to the horizontal with an initial speed of 100 ms^{-1} . It moves along its initial line of motion with an acceleration of 30 ms^{-2} for 3 seconds. At this time its engine falls & the rocket proceeds like a free body. Find :

(i) the maximum altitude reached by the rocket

(ii) total time of flight .

(iii) the horizontal range . [sin 53º = 4/5]

Q.25 A particle is thrown horizontally with relative velocity 10 m/s from an inclined plane, which is also moving with acceleration 10 m/s^2 vertically upward. Find the time after which it lands on the plane (g = 10 m/s^2)



List of recommended questions from I.E. Irodov.

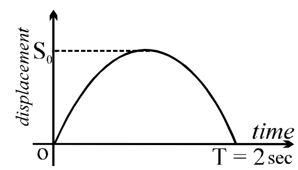
1.1, 1.4 to 1.8, 1.10, 1.11, 1.14, 1.15, 1.17, 1.18, 1.19, 1.21, 1.24, 1.26, 1.27, 1.31, 1.32, 1.33, 1.34(a)

EXERCISE–II

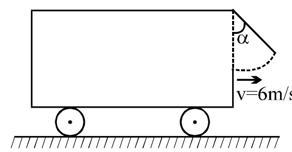
Q.1 A train takes 2 minutes to acquire its full speed 60kmph from rest and 1 minute to come to rest from the full speed. If somewhere in between two stations 1 km of the track be under repair and the limited speed on this part be fixed to 20kmph, find the late running of the train on account of this repair work, assuming otherwise normal at running of the train between the stations.

Q.2 A speeder in an automobile passes a stationary policeman who is hiding behind a bill board with a motorcycle. After a 2.0 sec delay (reaction time) the policeman accelerates to his maximum speed of 150 km/hr in 12 sec and catches the speeder 1.5 km beyond the billboard. Find the speed of speeder in km/hr.

Q.3 A particle is moving on a straight line. Its displacement from the initial position is plotted against time in the graph shown. What will be the velocity of the particle at 2/3 sec? Assume the graph to be a sine curve.



Q.4 A glass wind screen whose inclination with the vertical can be changed, is mounted on a cart as shown in figure. The cart moves uniformly along the horizontal path with a speed of 6 m/s. At what maximum angle α to the vertical can the wind screen be placed so that the rain drops falling vertically downwards with velocity 2 m/s, do not enter the cart?



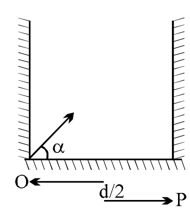
Q.5 An aeroplane is observed by two persons travelling at 60 km/hr in two vehicles moving in opposite directions on a straight road. To an observer in one vehicle the plane appears to cross the road track at right angles while to the observer in the other vehicle the angle appears to be 45°. At what angle does the plane actually cross the road track and what is its speed relative to the ground.

Q.6 How long will a plane take to fly around a square with side a with the wind blowing at a velocity u, in the two cases
(a) the direction of the wind coincides with one of the sides
(b) the direction of the wind coincides with one diagonal of the square. The velocity of the plane in still air is v > u.

Q.7 Two ships A and B originally at a distance d from each other depart at the same time from a straight coastline. Ship A moves along a straight line perpendicular to the shore while ship B constantly heads for ship A, having at each moment the same speed as the latter. After a sufficiently great interval of time the second ship will obviously follow the first one at a certain distance. Find the distance.

Q.8 The slopes of the wind-screen of two motorcars are $\beta_1 = 30^0$ and $\beta_2 = 15^0$ respectively. The first car is travelling with a velocity of v_1 horizontally. The second car is travelling with a velocity v_2 in the same direction. The hail stones are falling vertically. Both the drivers observe that the hail stones rebound vertically after elastic collision with the wind-screen. Find the ratio of v_1/v_2.

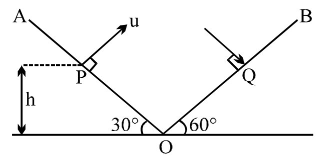
Q.9 A small ball is thrown between two vertical walls such that in the absence of the wall its range would have been 5d. The angle of projection is α. Given that all the collisions are perfectly elastic, find
(a) Maximum height attained by the ball.
(b) Total number of collisions before the ball comes back to the ground, and
(c) Point at which the ball falls finally. The walls are supposed to be very tall.



Q.10 A hunter is riding an elephant of height 4m moving in straight line with uniform speed of 2m/sec. A deer running with a speed V in front at a distance of $4\sqrt{5}$m moving perpendicular to the direction of motion of the elephant. If hunter can throw his spear with a speed of 10m/sec. relative to the elephant, then at what angle θ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed 'V' of the deer.

Q.11 A projectile is to be thrown horizontally from the top of a wall of height 1.7 m. Calculate the initial velocity of projection if it hits perpendicularly an incline of angle 37º which starts from the ground at the bottom of the wall. The line of greatest slope of incline lies in the plane of motion of projectile.

Q.12 Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m s^{-1} along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q, calculate

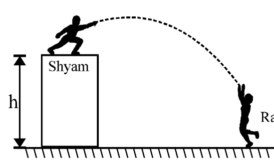


(a) velocity with which particle strikes the plane OB,
(b) time of flight,
(c) vertical height h of P from O,
(d) maximum height from O attained by the particle and
(e) distance PQ

Q.13 A particle is projected with a velocity $2\sqrt{ag}$ so that it just clears two walls of equal height 'a' which are at a distance '2a' apart. Show that the time of passing between the walls is $2\sqrt{a/g}$.

Q.14 A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height 2h above the ground. If at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocities of the bird and the stone, if the stone still hits the bird while descending.

Q.15 Two persons Ram and Shyam are throwing ball at each other as shown in the figure. The maximum horizontal distance from the building where Ram can stand and still throw a ball at Shyam is d_1. The maximum horizontal distance of Ram from the building where Shyam can throw a ball is d_2. If both of them can throw ball with a velocity of $\sqrt{2gk}$, find the ratio of d_1/d_2. Neglect the height of each person.



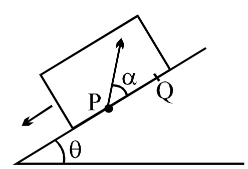
EXERCISE # III

Q.1 The motion of a body is given by the equation $\frac{d\,v(t)}{dt} = 6.0 - 3\,v(t)$; where v (t) is the speed in m/s & t in sec., if the body has v = 0 at t = 0 then
(A) the terminal speed is 2.0 m/s
(B) the magnitude of the initial acceleration is 6.0 m/s²
(C) the speed varies with time as $v(t) = 2\,(1 - e^{-3t})$ m/s
(D) the speed is 1.0 m/s when the acceleration is half the initial value. [JEE ’1995]

Q.2 Two guns, situated at the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find
(a) the time interval between the firings, and
(b) the coordinates of the point P. Take origin of the coordinates system at the foot of the hill right below the muzzle and trajectories in X-Y plane. [JEE'1996]

Q.3 The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants & x and y are respectively the horizontal & vertical distances of the projectile from the point of projection . The maximum height attained is ______ & the angle of projection from the horizontal is ______. [JEE '1997]

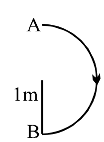
Q.4 A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of a box, a particle is projected inside the box. The initial speed of the particle with respect to box is u and the direction of projection makes an angle α with the bottom as shown in figure.



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected. [JEE'1998]

Q.5 A particle of mass 10^{-2} kg is moving slong the positive x-axis under the influence of a force $F(x) = \frac{-K}{2x^2}$ where $K = 10^{-2}$ N m^2 . At time t = 0 it is at x = 1.0 m & its velocity is v = 0 . Find :
(i) its velocity when it reaches x = 0.50 m
(ii) the time at which it reaches x = 0.25 m . [JEE '1998]

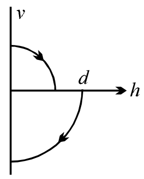
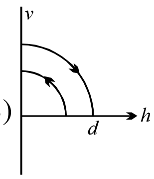
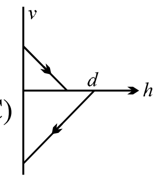
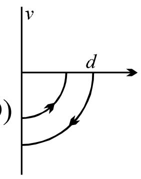
Q.6 In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is : [JEE '99]



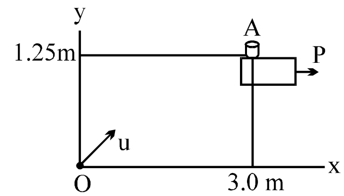
(A) 3.14 m/sec (B) 2.0 m/sec
(C) 1.0 m/sec (D) zero

Q.7 The co-ordinates of a particle moving in a plane are given by $x(t) = a\cos(\pi t)$ and $y(t) = b\sin(\pi t)$ where a, b (<a) & π are positive constants of appropriate dimensions.
(A) the path of the particle is an ellipse
(B) the velocity & acceleration of the particle are normal to each other at $t = \pi/(2\pi)$
(C) the acceleration of the particle is always directed towards a focus
(D) the distance travelled by the particle in time interval $t = 0$ o $t = \pi/(2\pi)$ is a . [JEE '1999]

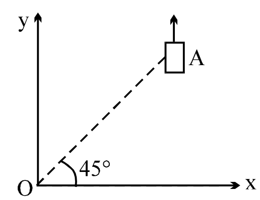
Q.8 A ball is dropped vertically from a height d above the ground it hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistances, its velocity v varies with the height h above the ground as [JEE'2000 (Scr)]

(A) 
(B) 
(C) 
(D) 

Q.9 An object A is kept fixed at the point x = 3 m and y = 1.25 m on a plank P raised above the ground. At time t = 0 the plank starts moving along the + x direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find u and the time after which the stone hits the object. Take g = 10 m/s^2. [JEE 2000]



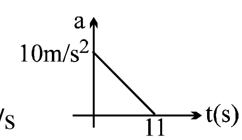
Q.10 On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$. [JEE 2002]

Q.11 A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be [JEE' 2004 (Scr)]

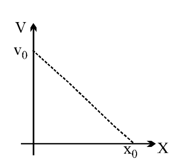


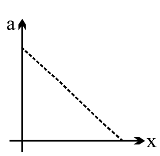
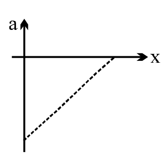
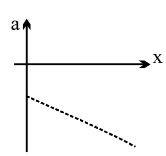
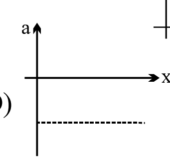
(A) 110 m/s (B) 55 m/s (C) 550 m/s (D) 660 m/s

Q.12 A small block slides without friction down an inclined plane starting from rest. Let Sn be the distance travelled from time t = n - 1 to t = n. Then $\frac{S_n}{S_{n+1}}$ is [JEE' 2004 (Scr)]

(A) $\frac{2n-1}{2n}$ (B) $\frac{2n+1}{2n-1}$ (C) $\frac{2n-1}{2n+1}$ (D) $\frac{2n}{2n+1}$

Q.13 The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be



(A) 
(B) 
(C) 
(D) 

[JEE' 2005 (Scr)]

ANSWER KEY
EXERCISE # I

Q.1 6 m/s Q.2 $\left(\frac{\pi\sqrt{2}}{3}\right)$ cm/min Q.3 5 m/sec Q.4 $\tan^{-1}(1/2)$

Q.5 5 s Q.6 36.2 sec. Q.7 $20\sqrt{5}$ Q.8 $\sqrt{\left(\frac{h}{8g}\right)}$

Q.9 20 sec Q.10 50m Q.11 $\frac{\pi v^4}{g^2}$ Q.12 $y = \frac{a x^2}{2u^2}$

Q.13 100/3 m/s Q.14 10 m/s Q.15 1 sec Q.16 75 m

Q.17 $\frac{a_0 t_0}{ln(2)}$ Q.18 $a_P = 1$ m/s$^2 \downarrow$, $a_B = 2$ m/s$^2 \uparrow$ Q.19 5 m/s

Q.20 8 m Q.21 60^0, $\tan^{-1}\left(\sqrt{3/2}\right)$ Q.22 $2a\sin(\alpha/2)$

Q.23 R + 2H Q.24 (i) 1503.2 m (ii) 35.54 sec (iii) 3970.56 m

Q.25 $\frac{1}{\sqrt{3}}$ sec

EXERCISE # II

Q.1 160 sec Q.2 122.7 km/hr Q.3 $\frac{S_0\pi}{4}$

Q.4 $2\tan^{-1}(1/3)$ Q.5 $\theta = \tan^{-1}2$, v = 134.16 km/h

Q.6 (a) $\frac{2a\left(v+\sqrt{v^2-u^2}\right)}{v^2-u^2}$, (b) $\frac{2\sqrt{2}a\left(\sqrt{2v^2-u^2}\right)}{v^2-u^2}$ Q.7 $\frac{d}{2}$ Q.8 3

Q.9 (a) $5d/4 \tan\alpha$, (b) 9, (c) point O Q.10 $\theta = 37^0$, v = 6m/s

Q.11 u = 3m/s Q.12 (a) 10 ms^{-1}, (b) 2 sec, (c) 5 m, (d) 16.25 m, (e) 20 m

Q.14 $\frac{2}{\sqrt{2}+1}$ Q.15 $\sqrt{\frac{k-h}{k+h}}$

EXERCISE # III

Q.1 A, B, C, D Q.2 (a) 1 sec, (b) ($5\sqrt{3}$ m, 5 m) Q.3 $\frac{a^2}{4b}$, $\tan^{-1}a$

Q.4 (a) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$, (b) $v = \frac{u\cos(\alpha+\theta)}{\cos\theta}$ Q.5 (i) $\vec{V} = -1\ \hat{i}$ m/s (ii) $t = \frac{\pi}{3} + \frac{\sqrt{3}}{4}$

Q.6 Q.7 A, B Q.8 A Q.9 u = 7.29 m/s, t = 1 sec

Q.10 (a) 45°, (b) 2 m/sec Q.11 B Q.12 C Q.13 B

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P4. General Physics

Index:

ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity. (1) random and (2) systematic errors.

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.
2. Systematic errors occur due to error in the procedure, or miscalibration of the intrument etc. Such errors have same size and sign for all the measurements. Such errors can be determined.

A measurement with relatively small random error is said to have high precision. A measurement with small random error and small systematic error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.

Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgment that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Random errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Least Count Error :– If the instrument has known least count, the absolute error is taken to be half of the least count unless otherwise stated.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself.

In general,

$$\text{relative error} = \frac{\textit{absolute error in a measurement}}{\textit{size of the measurement}}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. ***Systematic errors*** **:** They have a known sign.

1. Suppose that a result R is calculated from the sum of two measured quantities A and B. We'll use a and b to represent the error in A and B respectively. r is the error in the result R. Then

   $$(R + r) = (A + B) + (a + b)$$

   The error in R is therefore : $r = a + b$.

   Similarly, when two quantities are subtracted, the systematic errors also get subtracted.
2. Suppose that a result R is calculated by multiplying two measured quantities A and B. Then $R = AB$.

   $(R + r) = (A + a)(B + b) = AB + aB + Ab + ab$

   $\Rightarrow \frac{r}{R} = \frac{aB + bA}{AB} = \frac{a}{A} + \frac{b}{B}$. Thus when two quantities are multiplied, their relative systematic error add.
3. **Quotient rule :** When two quantities are divided, the relative systematic error of the quantient is the relative systematic error of the numerator minus the relative systematic error of the denominator. Thus if

   $R = \frac{A}{B}$ then $\frac{r}{R} = \frac{a}{A} - \frac{b}{B}$
4. **Power rule :** When a quantity Q is raised to a power, P, the relative systematic error in the result is P times the relative systematic error in Q.

   If $R = Q^P$, $\quad \frac{r}{R} = P \times \frac{q}{Q}$

   This also holds for negative powers,
5. The quotient rule is not applicable if the numerator and denominator are dependent on each other.

   e.g if $R = \frac{XY}{X + Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\frac{1}{R}=\frac{1}{X}+\frac{1}{Y}$. Differentiating both the sides, we get

$$-\frac{dR}{R^2}=-\frac{dX}{X^2}-\frac{dY}{Y^2}. \qquad \text{Thus} \quad \frac{r}{R^2}=\frac{x}{X^2}+\frac{y}{Y^2} \quad \text{or} \quad \frac{r}{R}=\frac{R}{X}\times\frac{x}{X}+\frac{R}{Y}\times\frac{y}{Y}$$

B. ***Random error*** **:** They have unknown sign. Thus they are represented in the form $A \pm a$.

Here we are only concerned with *limits of error*. We must assume a "worst-case" combination. In the case of substraction, A – B, the worst-case deviation of the answer occurs when the errors are either + a and – b or – a and + b. In either case, the maximum error will be (a + b).
For example in the experiment on finding the focal length of a convex lens, the object distance(u) is found by subtracting the positions of the object needle and the lens. If the optical bench has a least count of 1 mm, the error in each position will be 0.5 mm. So, the error in the value of u will be 1 mm.

1. ***Addition and subtraction rule*** **:** The absolute random errors add.
Thus if $R = A + B$, $\quad r = a + b$
and if $R = A - B$, $\quad r = a + b$

2. ***Product and quotient rule*** **:** The relative random errors add.

Thus if $R = AB$, $\quad \frac{r}{R} = \frac{a}{A}+\frac{b}{B}$

and if $R = \frac{A}{B}$, then also $\frac{r}{R} = \frac{a}{A}+\frac{b}{B}$

3. ***Power rule*** **:** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

If $R = Q^P$, $\frac{r}{R} = P\times\frac{q}{Q}$

Examples

1. A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used

is $S = (1/2)AT^2$. The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?
$S = 2 \pm 0.005$ meter.
$T = 4.2 \pm 0.2$ second.

Sol: We use capital letters for quantities, lower case for errors. Solve the equation for the result, a.
$A = 2S/T^2$. Its random-error equation is $\frac{a}{A}=2\frac{t}{T}+\frac{s}{S}$
Thus $\quad A = 0.23 \pm 0.02$ m/s^2.

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science :
1. Measured Values 2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

MEASURED VALUES

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position.For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

COMPUTED VALUE

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All non-zero digits (1-9) are to be counted as significant.
2. Zeros that have any non-zero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (ii) above are NOT be considered significant digits.

For example : 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros? The first three zeros would not be considered significant since they have no non-zero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the non-zero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example : 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated. But what about 1,000,000?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example : If I write the number above in this manner.

$$1.00 \times 10^6$$

I have indicated that the number has been recorded with three significant digits. On the other hand, if I write the same number as : 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant. So for example : 2.0040×10^4 will have five significant digits. This means that unit conversion will not change the number of significant digits. Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0171 (after rounding off)

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digited number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules? The only way is to express the answer in exponential notation so 2400 could be expressed as : 2.4×10^3

Rule for expressing the correct number of significant digits in an addition or substraction :

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation.Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)
The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.
Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)
The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e. having only one position to the right of its decimal). Notice we aren't really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits :

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.
2. Consider the digit to the right of the last reported digit.
3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682
The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is : 18.4

Another example : Round off 4.565 to three significant digits.
The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

EXPERIMENTS

(i) Measurement of length

The simplest method measuring the length of a straight line is by means of a meter scale. But there exists some limitation in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm
For greater accuracy devices like
(a) Vernier callipers (b) micrometer scales (screw gauge) are used.

VERNIER CALLIPERS:

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being either slightly longer and shorter than the divisions of the main scale.

Least count of Vernier Callipers

The least count or Vernier constant (v. c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

Vernier constant = 1 ms – 1 vs = $\left(1-\frac{N-1}{N}\right)$ ms = $\frac{1ms}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

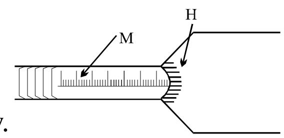
Zero error:

If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

SCREW GAUGE (OR MICROMETER SCREW)

In general vemier callipers can measure accurately upto 0.01 em and for greater accuracy micrometer screw devices e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tuming it axially. The instrument is provided with two scales:



(i) The main scale or pitch scale M graduated along the axis of the screw.

(ii) The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

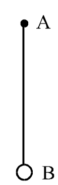
(a) **Pitch :** The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap =5 mm, then pitch = 0.5 mm

(b) **Least count :** In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration:, if the total cap division is 100, then least count = 0.5mm/100 = 0.005 mm

Zero Error : In a perfect instrument the zero of the heat scale coincides with the line of graduation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies above the line of graduation and versa. The corresponding corrections will be just opposite.

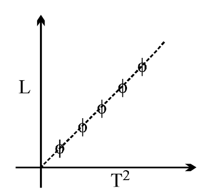
(ii) **Measurement of g using a simple pendulum**

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation. The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch. The time period (T) of a single oscillation is now calculated by division.



Observations are now taken by using different lengths for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear.

g is given by $g = 4\pi^2 \frac{L}{T^2}$



The major errors in this experiment are

(a) **Systematic :** Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numerical estimate for the time period. However the practice is to ensure that the amplitude is small.

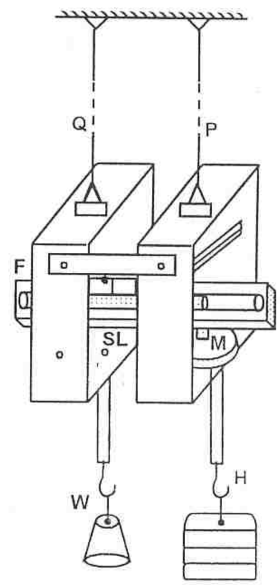
(b) **Statistical :** Errors arising from measurement of length and time.

$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging.

The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line. The slope of this fit gives the correct value of L/T^2

(iii) Determination of Young's Modulus by Searle's Method



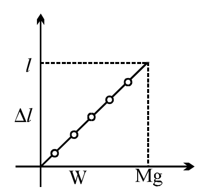
The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along .the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the side of the experimental wire.On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length *M* is obtained by

$$\Delta l = n \times \text{pitch} + f \times \text{least count}$$

The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the true increase in length of the wire corresponding to the given value of load.

From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin. The slope of the line gives

$$\tan\theta = \frac{l}{W} = \frac{l}{Mg}$$



Now, stress $= \dfrac{Mg}{\pi r^2}$ and strain $= \dfrac{l}{L}$

$$Y = \text{Stress/ strain} = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi r^2 \tan\theta}$$

With known values of initial length L, radius r of the experimental wire and $\tan\theta$, Young's modulus Y can be calculated.

(iv) Specific Heat of a liquid using a calorimeter:

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) through a heating coil immersed within the liquid for a known length of time (t). The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1)S_l(\theta_2 - \theta_0) + m_1 S_c(\theta_2 - \theta_0) = i.\ V.\ t$$

or, $\quad (m_2 - m_1)S_l + m_1 S_c = i.\ V.\ t/(\theta_2 - \theta_0) \quad$ ..... (1)

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown: one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate $m_1 S_c$.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

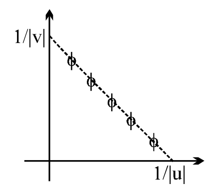
The direction of the current is reversed midway during the experiment to remove the effect of any differential contacts, radiation correction is introduced to take care of the second major source of systematic error.

Radiation correction: The temperature of the system is recorded for half the length of time t, i.e. t/2, where t is the time during which the current was switched on} after the current is switched off. The fall in temperature δ, during this interval is now added to the final temperature θ_2 to give the corrected final temperature: $\theta'_2 = \theta_2 + \delta$
This temperature is used in the calculation of the specific heat, S_l.

Error analysis :
After correcting for systematic errors, equation (i) is used to estimate the remaining errors.

(v) Focal length of a concave mirror and a convex lens using the u-v method.
In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder.
The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).
The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).
For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.

A plot of |u| versus |v| gives a rectangular hyperbola. A plot of $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.



The intercepts are equal to $\frac{1}{|f|}$, where f is the focal length.

Error : The systematic error in this experiment is mostly due to improper position of the object on the holder. This error maybe eliminated by reversing the holder (rotating the holder by 180° about the vertical) and then taking the readings again. Averages are then taken.

The equation for errors gives: $\frac{\delta f}{f^2} = \frac{\delta u}{u^2} + \frac{\delta v}{v^2}$

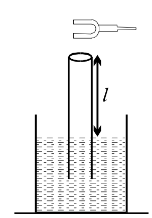
The errors δu, δv correspond to the error in the measurement of u and v.

Index Error or Bench Error and its correction: In an experiment using an optical bench we are required to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distances between the indices with the help of the scale engraved on the bench. These distances are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error.

Index Error = Observed distance – actual distance and
Index Correction = Actual – observed distance

Note: Index correction whether positive or negative, is always added algebraically to the observed distance to get the corrected distance.

(vi) Speed of sound using resonance column
A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it resonates with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again until a second resonance length (l_2) is found (provided the tube is long)



Then, $l_2 - l_1 = \lambda / 2$, provided l_1, l_2 are resonance lengths for adjacent resonances.
$\therefore \quad \lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by $C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected *for* the "end effect":

$\lambda(\text{fundamental}) = 4(l_1 + 0.3d)$, where d = diameter

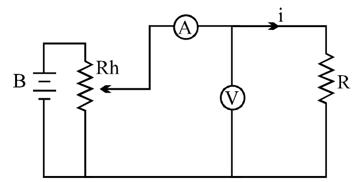
Errors : The major systematic errors introduced are due to end effects in (end correction) and also due to excessive humidity.

Random errors are given by

$$\frac{\delta C}{C} = \frac{\delta(l_2 - l_1)}{l_2 - l_1} = \frac{\delta l_2 + \delta l_1}{l_2 - l_1}$$

(vii) Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh



Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of V vs i is plotted and it is found to be linear (within errors). The magnitude of R is determined by either

(a) taking the ratio $\frac{V}{i}$ and then

(b) fitting to a straight line: V = iR, and determining the slope R.

Errors :

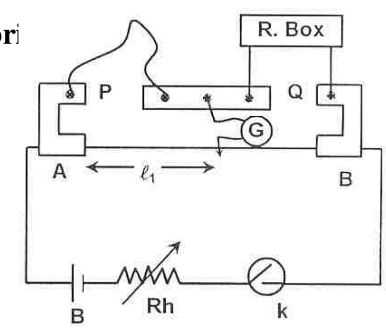
Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimsed by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimised as much as possible.

Error analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least counts of the instruments used.

(viii) Specific resistance of the material of a wire using a meter bri



A known length (l) of a wire is connected in one of the gaps (P) of a metre bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G).The balance length (l) is found by closing key k and momentarily connecting the galvanometer until it gives zero deflection (null point). Then,

$$\frac{P}{Q} = \frac{l}{100 - l} \qquad ....(1)$$

using the expression for the meter bridge at balance. Here, represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho\frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L}P \qquad ....(2)$

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors : The major systematic errors in this experiment are due to the heating effect, end corrections introduced due to shift of the zero of the scale at A and B, and stray resistances in P and Q, and errors due to non-uniformity of the meter bridge wire.

Error analysis : End corrections can be estimated by including known resistances P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \quad \text{..... (2),} \qquad \text{where } \alpha \text{ and } \beta \text{ are the end corrections.}$$

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \quad \text{..... (3)}$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point,

$$\frac{P}{Q} = \frac{l + \alpha}{100 - l + \beta} = \frac{l_1'}{l_2'},$$

Errors due to non-uniformity of the meter bridge wire can be minimised by interchanging the resistances in the gaps P and Q.

$$\therefore \quad \frac{\delta P}{P} = \left|\frac{\delta l_1'}{l_1'}\right| + \left|\frac{\delta l_2'}{l_2'}\right|$$

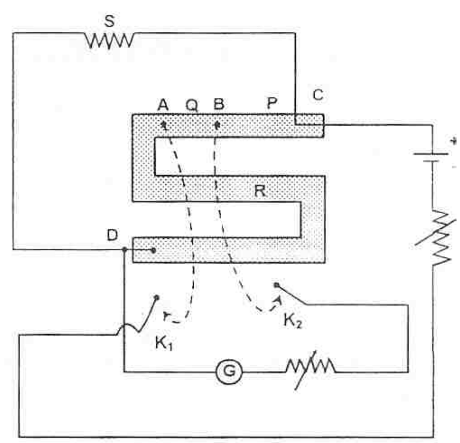
where, $\delta l'_1$ and $\delta l'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e. when the balance point is in the middle of the bridge. The error in ρ is

$$\frac{\delta P}{P} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$$

(ix) Measurement of unknown resistance using a P.O. Box

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm.



At balance, the unknown resistance

$$S = \left(\frac{P}{Q}\right) R \qquad \text{.....(1)}$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_0$ (Ohm) and in the opposite direction when $R = R_0 + 1$ (ohm).

This implies that the unknown resistance, S lies between R_0 and $R_0 + 1$ (ohm). Now, the resistance in P and Q are made 100 Ω and 1000 Ω respectively, and the process is repeated.

Equation (1) is used to compute S.

The ratio P/Q is progressively made 1:10, and then 1 :100. The resistance S can be accurately measured.

Errors : The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

EXERCISE I

Q.1 Number 1.6454(21) means

(A) 1.645421 (B) 1.6454×10^{21} (C) 1.6454 ± 21 (D) 1.6454 ± 0.0021

Q.2 A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then

(A) n=10, m=0.5 cm (B) n=9, m=0.4 cm (C) n=10, m=0.8 cm (D) n=10 , m=0.2 cm

Q.3 In a Vernier Calipers (VC), N divisions of the main scale coincide with N + m divisions of the vernier scale. What is the value of m for which the instrument has minimum least count?

(A) 1 (B) N (C) Infinity (D) N/2

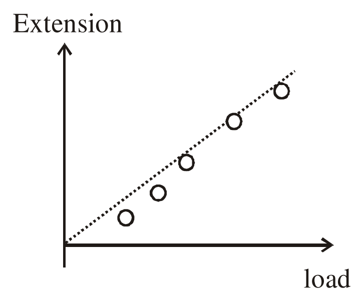
Q.4 In a vernier calipers the main scale and the vernier scale are made up different materials. When the room temperature increases by ΔT°C, it is found the reading of the instrument remains the same. Earlier it was observed that the front edge of the wooden rod placed for measurement crossed the N^{th} main scale division and N + 2 msd coincided with the 2^{nd} vsd. Initially, 10 vsd coincided with 9 msd. If coefficient of linear expansion of the main scale is α_1 and that of the vernier scale is α_2 then what is the value of α_1 / α_2? (Ignore the expansion of the rod on heating)

(A) 1.8 / (N) (B) 1.8 /(N+2) (C) 1.8 / (N–2) (D) None

Q.5 In the Searle's experiment, after every step of loading , why should we wait for two minutes before taking the readings? (More than one correct.)

(A) So that the wire can have its desired change in length.
(B) So that the wire can attain room temperature.
(C) So that vertical oscillations can get subsided.
(D) So that the wire has no change in its radius.

Q.6 A graph is plotted between extension and load. The first two readings do not fall on the straight line whereas almost all the readings taken there after fall on a straight line. What is the main reason behind it?

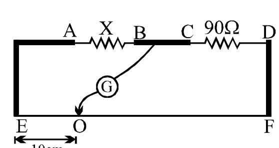


(A) The wire gets heated up during the course of the experiment.
(B) Initially the wire was not free from kinks.
(C) In the initial state the wire has less elasticity as compared to later stages.
(D) The wire has some plastic deformation in the later stages.

Q.7 In a meter bridge set up, which of the following should be the properties of the one meter long wire?

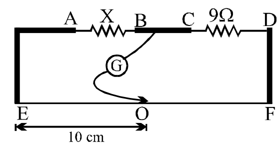
(A) High resistivity and low temperature coefficient
(B) Low resistivity and low temperature coefficient
(C) Low resistivity and high temperature coefficient
(D) High resistivity and high temperature coefficient

Q.8 Let the end error on the LHS and RHS be equal to one cm. For the balance point at O, find out the % tage error in the value of X? (If the end error is 1 cm from both sides then it means the corrected reading will become 10cm + 1cm from LHS and 90cm + 1cm from the RHS)



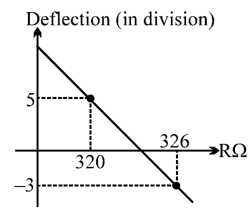
(A) 4.2% (B) 8.1% (C) 9.2% (D) None

Q.9 Consider the MB shown in the diagram, let the resistance X have temperature coefficient α_1 and the resistance from the RB have the temperature coefficient α_2. Let the reading of the meter scale be 10cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point? Neglect all the other changes in the bridge due to temperature rise.



(A) $9(\alpha_1 - \alpha_2)\Delta T$ (B) $9(\alpha_1 + \alpha_2)\Delta T$ (C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$ (D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

Q.10 For a post office Box, the graph of galvanometer deflection versus R (resistance pulled out of RB) for the ratio 100 : 1 is given as shown. A careless student pulls out two non consecutive values R as shown in the figure. Find the value of unknown resistance.



(A) 3.2 ohm (B) 3.24 ohm
(C) 3.206 ohm (D) None

Q.11 When we operate a wheat stone bridge then in starting the key of the battery is closed first and the key of the G is closed later. When the circuit is to be closed then switches are released in the opposite order. Why?

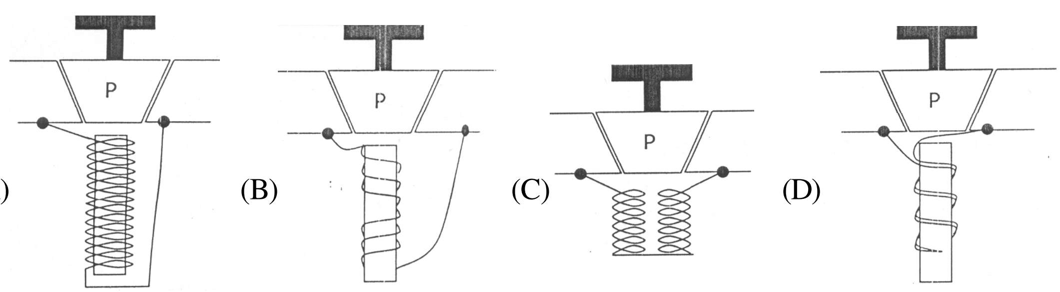
(A) Look at the diagram of the PO box, the switch is battery is always on the right hand hence it is easier to press it first.

(B) This is done to avoid the damage of the galvanometer due to induced emf.

(C) If the G switch is pressed before the battery switch then large sparking takes place at the battery switch.

(D) While disconnecting if we open the battery switch before the G switch then we can observe induced current in the circuit till the G switch is not opened.

Q.12 Identify which of the following diagrams represent the internal construction of the coils wound in a resistance box or PO box?



EXERCISE II

Q.1 How many significant figures are given in the following quantities ?

(A) 343 g (B) 2.20 (C) 1.103 N (D) 0.4142 s
(E) 0.0145 m (F) 1.0080 V (G) 9.1×10^4 km (H) 1.124×10^{-3} V

Q.2 Perform the following operations:

(A) 703 + 7 + 0.66 (B) 2.21×0.3

(C) 12.4×84 (D) $\frac{14.28}{0.714}$

Q.3 Solve with due regard to significant digits

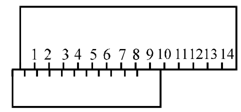
(i) $\sqrt{6.5-6.32}$ (ii) $\dfrac{2.91\times0.3842}{0.080}$

Q.4 The main scale of a vernier calipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm; and the fourth vernier division coincides with a scale division. Calculate the measured length of the cylinder.

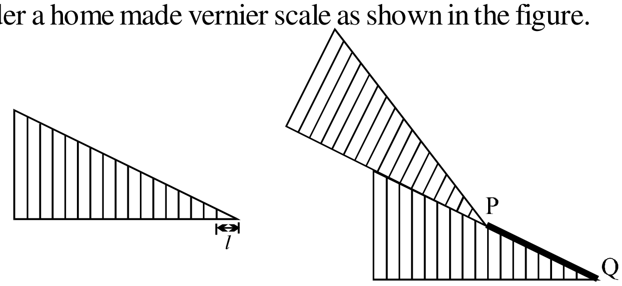
Q.5 The VC shown in the diagram has zero error in it (as you can see). It is given that 9 msd = 10 vsd.

(i) What is the magnitude of the zero error? (1msd = 1 mm)

(ii) The observed reading of the length of a rod measured by this VC comes out to be 5.4 mm. If the vernier had been error free then ___msd would have coincided with ___ vsd.



Q.6 Consider a home made vernier scale as shown in the figure.



In this diagram, we are interested in measuring the length of the line PQ. If both the inclines are identical and their angles are equal to θ then what is the least count of the instrument.

Q.7 The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

Q.8 The pitch of a screw gauge is 1 mm and there are 50 divisions on its cap. When nothing is put in between the studs, 44^{th} division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the main scale reads three divisions and the circular scale reads 26 divisions. Calculate the thickness of the plate.

Q.9 In a given optical bench, a needle of length 10 cm is used to estimate bench error. The object needle, image needle & lens holder have their reading as shown.

$x_0 = 1.1$ cm

$x_I = 0.8$ cm

$x_L = 10.9$ cm

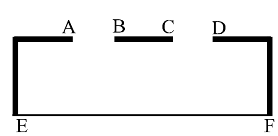
Estimate the bench errors which are present in image needle holder and object needle holder. Also find the focal length of the convex lens when.

$x_0 = 0.6$ cm

$x_I = 22.5$ cm

$x_L = 11.4$ cm

Q.10 Make the appropriate connections in the meter bridge set up shown. Resistance box is connected between _____. Unknown resistance is connected between______. Battery is connected between ______.



Q.11 A body travels uniformly a distance of (13.8 ± 0.2)m in time (4.0 ± 0.3) sec. Calculate its velocity.

Q.12 Consider S = x cos (θ) for x = (2.0 ± 0.2) cm, θ = 53 ± 2 °. Find S.

Q.13 Two resistance R_1 and R_2 are connected in (i) series and (ii) parallel. What is the equivalent resistance with limit of possible percentage error in each case of $R_1 = 5.0 \pm 0.2\ \Omega$ and $R_2 = 10.0 \pm 0.1\ \Omega$.

Q.14 A short circuit occurs in a telephone cable having a resistance of $0.45\ \Omega m^{-1}$. The circuit is tested with a Wheatstone bridge. The two resistors in the ratio arms of the Wheatstone bridge network have values of 100Ω and 1110Ω respectively. A balance condition is found when the variable resistor has a value of 400Ω. Calculate the distance down the cable, where the short has occurred.

Q.15 5.74 gm of a substance occupies a volume of $1.2\ cm^3$. Calculate its density with due regard for significant figures.

Q.16 The time period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{l/g}$

The length of the pendulum is measured as $l = 10 \pm 0.1$cm and the time period as $T = 0.5 \pm 0.02$ s. Determine percentage error in the value of g.

Q.17 A physical quantity P is related to four observables A, B, C and D as $P = 4\pi^2 A^3 B^2 /(\sqrt{C}\, D)$

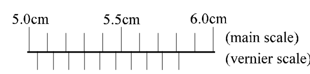
The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Determine the percentage error & absolute error in the quantity P. Value of P is calculated 3.763. Round off the result in scientific way.

Q.18 A glass prism of angle $A = 60°$ gives minimum angle of deviation $\theta = 30°$ with the max. error of 1^0 when a beam of parallel light passed through the prism during an experiment.

(i) Find the permissible error in the measurement of refractive index μ of the material of the prism.

(ii) Find the range of experimental value of refractive index 'μ'.

Q.19 In the given vernier calliper scale, the length of 1 main scale division is 1mm whereas the length of the vernier scale is 7.65 mm. Find the reading on the scale correct to significant digits as shown in the diagram.



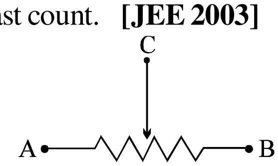
EXERCISE–III

Q.1 The edge of a cube is $a = 1.2 \times 10^{-2}$ m. Then its volume will be recorded as : **[JEE 2003]**

(A) $1.7 \times 10^{-6}\ m^3$ (B) $1.70 \times 10^{-6}\ m^3$ (C) $1.70 \times 10^{-7}\ m^3$ (D) $1.78 \times 10^{-6}\ m^3$

Q.2 In a vernier callipers, n divisions of its main scale match with (n + 1) divisions on its vernier scale. Each division of the main scale is a units. Using the vernier principle, calculate its least count. **[JEE 2003]**

Q.3 Schematic of a rheostat is shown below. Connect a battery to it so that the rheostat acts as a potential divider. Specify which are the output terminals. **[JEE 2003]**



Q.4 In the relation $P = \dfrac{\alpha}{\beta} e^{\frac{\alpha Z}{k\theta}}$

P is pressure, Z is distance ,k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be

(A) $[M^0L^2T^0]$ (B) $[M^1L^2T^1]$ (C) $[M^1L^0T^{-1}]$ (D) $[M^0L^2T^{-1}]$ **[JEE 2004]**

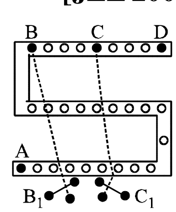
Q.5 A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is **[JEE 2004]**

(A) 1 (B) 2 (C) 3 (D) 4

Q.6 For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **[JEE 2004]**

(A) *B* and *C* (B) *C* and *D*

(C) *A* and *D* (D) B_1 and C_1



Q.7 In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. **[JEE 2004]**

Q.8 The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47^{th} division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures. **[JEE 2004]**

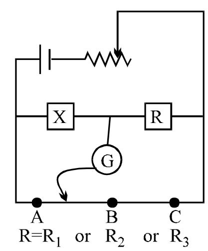
Q.9 Draw the circuit for experimental verification of Ohm's law using a source of variable D.C. voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter. **[JEE 2004]**

Q.10 In a resonance column method, resonance occurs at two successive level of l_1=30.7 cm and l_2 = 63.2 cm using a tuning fork of f = 512 Hz. What is the maximum error in measuring speed of sound using relations $v = f\lambda$ & $\lambda = 2(l_2 - l_1)$ **[JEE 2005]**

(A) 256 cm/sec (B) 92 cm/sec (C) 128 cm/sec (D) 102.4 cm/sec

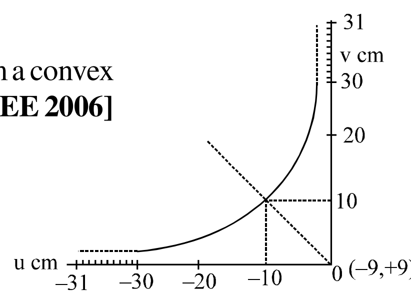
Q.11 The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. **[JEE 2005]**

Q.12 An unknown resistance X is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why? **[JEE 2005]**

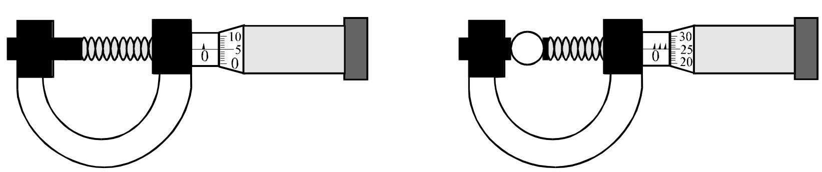


Q.13 Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is **[JEE 2006]**

(A) 0.50 ± 0.05 cm
(B) 0.50 ± 0.10 cm
(C) 5.00 ± 0.05 cm
(D) 5.00 ± 0.10 cm



Q.14 The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is **[JEE 2006]**



(A) 2.25 mm (B) 2.20 mm (C) 1.20 mm (D) 1.25 mm

Q.15 A student performs an experiment for determination of $g\left(=\dfrac{4\pi^2 l}{T^2}\right)$ $l \approx 1$m and he commits an error of Δl.

For the experiment takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1sec. For which of the following data, the measurement of g will be most accurate? **[JEE 2006]**

	Δl	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 sec	10	5 mm
(B)	5 mm	0.2 sec	20	5 mm
(C)	5 mm	0.1 sec	20	1 mm
(D)	1 mm	0.1 sec	50	1 mm

ANSWER KEY

EXERCISE I

Q.1	D	Q.2	C	Q.3	A	Q.4	B
Q.5	A,C	Q.6	B	Q.7	A	Q.8	B
Q.9	A	Q.10	B	Q.11	B, C, D	Q.12	D

EXERCISE II

Q.1 (A) 3, (B) 3, (C) 4, (D) 4, (E) 3, (F) 5, (G) 5, (H) 4

Q.2 (A) 711, (B) 0.7, (C) 1.0×10^3, (D) 20.0

Q.3 (i) 0.4, (ii) 14 Q.4 3.07 cm

Q.5 (i) x = –0.7 msd, (ii) 6, 1 Q.6 L.C. = $l\left[\dfrac{1-\cos\theta}{\cos\theta}\right]$

Q.7 2.84 mm Q.8 $R_t = 3.64$ mm

Q.9 5.5 ± 0.05 cm Q.10 CD, AB, EF

Q.11 $v = (3.4 \pm 0.31)$ m/s

Q.12 $S = (1.20 \pm 0.18)$ cm Q.13 $R_8 = 15\ \Omega \pm 2\%$, $R_p = 3.3\ \Omega \pm 7\%$

Q.14 40 m Q.15 4.8 g/cm^3 Q.16 9% Q.17 14%, 0.53, 3.76

Q.18 (i) 5π/18%, (ii) $\sqrt{2}\left[1+\dfrac{\pi}{360}\right] > \mu > \sqrt{2}\left[1-\dfrac{\pi}{360}\right]$ Q.19 5.045 cm

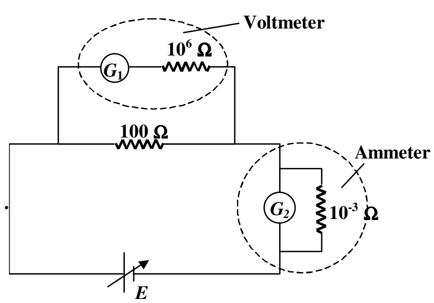
EXERCISE – III

Q.1 A Q.2 $\dfrac{a}{n+1}$ Q.3 [A,C] or [B,C] Q.4 A

Q.5 D Q.6 C Q.7 $\Delta Y = 0.0489$ Y = 10.758×10^9 N/m^2

Q.8 2.6 cm^2 (in two significant figures)

Q.9



Q.10 D Q.11 2.66 g/cm^3

Q.12 R_2 gives most accurate value

Q.13 C Q.14 C Q.15 D

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

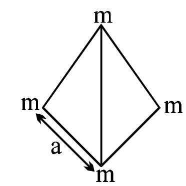
TOPIC: XI P5. Gravitation

Index:

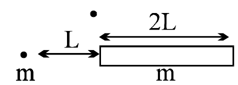
EXERCISE-I

Q.1 A remote sensing satellite is revolving in an orbit of radius x the equator of earth. Find the area on earth surface in which satellite can not send message.

Q.2 Four masses (each of m)are placed at the vertices of a regular pyramid (triangular base) of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '2a'.



Q.3 A small mass and a thin uniform rod each of mass ' m ' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



Q.4 An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:
(i) the initial speed of projection
(ii) the speed at half the maximum height.

Q.5 A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time________.

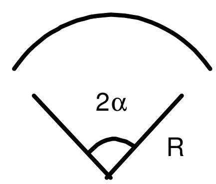
Q.6 A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5$J is equal to ________.

Q.7 A rocket starts vertically upwards with speed v_0. Show that its speed v at a height h is given by

$$v_0^2 - v^2 = \frac{(2gh)}{\left(1+\frac{h}{r}\right)}$$

where R is the radius of the earth. Hence deduce the maximum height reached by a rocket fired with speed equal to 90% of escape velocity.

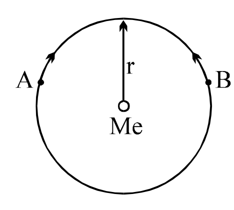
Q.8 Find the gravitational field strength and potential at the centre of arc of linear mass density λ subtending an angle 2α at the centre.



Q.9 A point P lies on the axis of a fixed ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be _______.

Q.10 Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

Q.11 Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the earth E but in opposite sense of rotation and therefore on a collision course (see figure).



(a) In terms of G, M_e, m and r find the total mechanical energy $E_A + E_B$ of the two satellite plus earth system before collision.

(b) If the collision is completely inelastic so that wreckage remains as one piece of tangled material (mass = 2m), find the total mechanical energy immediately after collision.

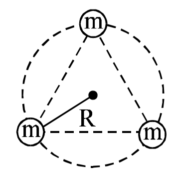
(c) Describe the subsequent motion of the wreckage.

Q.12 A particle is fired vertically from the surface of the earth with a velocity $k\upsilon_e$, where υ_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e. Calculate the height to which it will rise from the surface of the earth.

Q.13 A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is ________.

Q.14 Find the potential energy of a system of eight particles placed at the vertices of a cube of side L. Neglect the self energy of the particles.

Q.15 A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find:

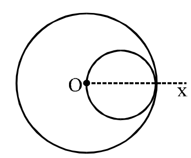


(i) the total P.E. of the system

(ii) the orbital speed of each moon such that they maintain this configuration.

Q.16 Two small dense stars rotate about their common centre of mass as a binary system with the period 1year for each. One star is of double the mass of the other and the mass of the lighter one is $\frac{1}{3}$ of the mass of the sun. Find the distance between the stars if distance between the earth & the sun is R.

Q.17 A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius r=R/2 whose centre is at x=R/2 as in fig. (a) Find gravitational field at points on the axis for x > R (ii) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.



Q.18 A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be

(a) then velocity in circular orbits.

(b) maximum distance between the two fragments before collision and

(c) magnitude of their relative velocity just before they collide.

Q.19 The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator barely provides the centripetal force needed for the rotation. (Why?)

(a) Show then that the corresponding shortest period of rotation is given by

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

Where ρ is the density of the planet, assumed to be homogeneous.

(b) Evaluate the rotation period assuming a density of 3.0 gm/cm^3, typical of many planets, satellites, and asteroids. No such object is found to be spinning with a period shorter than found by this analysis.

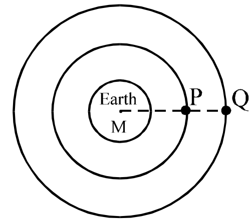
Q.20 A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite.

List of recommended questions from I.E. Irodov.

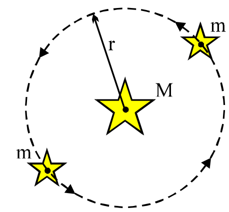
1.213, 1.216 to 1.220, 1.224, 1.226, 1.227, 1.229

EXERCISE-II

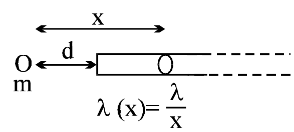
Q.1 A satellite P is revolving around the earth at a height h = radius of earth (R) above equator. Another satellite Q is at a height 2h revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this situation.



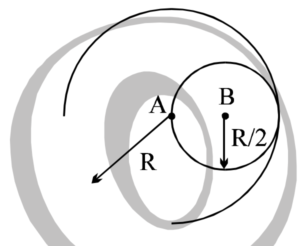
Q.2 A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit, see figure. Derive an expression for the period of revolution of the stars; the radius of the orbit is r.



Q.3 Find the gravitational force of interaction between the mass m and an infinite rod of varying mass density λ such that $\lambda(x) = \lambda/x$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in figure.



Q.4 Inside an isolated fixed sphere of radius R and uniform density r, there is a spherical cavity of radius R/2 such that the surface of the cavity passes through the centre of the sphere as in figure. A particle of mass m is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere.



Q.5 In a certain double star system the two stars rotate in circular orbits about their common centre of mass. The stars are spherical, they have same density ρ and their radii arc R and 2 R. Their centres are 5 R apart. Find the period T of stars in terms of ρ, R & G.

Q.6 A ring of radius R is made from a thin wire of radius r. If ρ is the density of the material of wire then what will be the gravitational force exerted by the ring on the material particle of mass m placed on the axis of ring at a distance x from its centre. Show that the force will be maximum when $x = R/\sqrt{2}$ and the maximum value of force will be given as

$$F_{max} = \frac{4\pi^2 G r^2 \rho m}{(3)^{3/2} R}$$

Q.7 In a particular double star system, two stars of mass 3.22×10^{30} kg each revolve about their common center of mass, 1.12×10^{11} m away.

(a) Calculate their common period of revolution, in years.

(b) Suppose that a meteoroid (small solid particle in space) passes through this centre of mass moving at right angles to the orbital plane of the stars. What must its speed be if it is to escape from the gravitational field of the double star?

Q.8 A man can jump over b=4m wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth = 6400 km.

Q.9 A launching pad with a spaceship is moving along a circular orbit of the moon , whose radius R is triple that of moon Rm. The ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity $\vec{v}_0$ and the launching pad then falls to the moon . Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.

Q.10 A small satellite revolves around a heavy planet in a circular orbit. At certain point in its orbit a sharp impulse acts on it and instantaneously increases its kinetic energy to 'k' (< 2) times without change in its direction of motion. Show that in its subsequent motion the ratio of its maximum and minimum distances from the planet is $\frac{k}{2-k}$, assuming the mass of the satellite is negligibly small as compared to that of the planet.

Q.11 A satellite of mass m is in an elliptical orbit around the earth of mass M (M>>m) The speed of the satellite at its nearest point to the earth (perigee) is $\sqrt{\frac{6GM}{5R}}$ where R=its closest distance to the earth. It is desired to transfer this satellite into a circular orbit around the earth of radius equal its largest distance from the earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

Q.12 A body is launched from the earth's surface a an angle $\alpha=30^{o}$ to the horizontal at a speed $v_0=\sqrt{\frac{1.5GM}{R}}$.

Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (ii) The radius of curvature of trajectory at its top point.

Q.13 Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to reach centre of earth if it is projected into the tunnel from surface of earth with speed needed for it to escape the gravitational field of earth.

EXERCISE-III

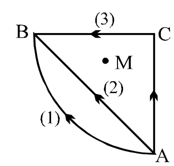
Q.1 If the distance between the earth and the sun were half its present value, the number of days in a year would have been [JEE' 96]
(A) 64.5 (B) 129 (C) 182.5 (D) 730

Q.2 Distance between the centres of two stars is 10 a. The masses of these stars are M and 16 M and their radii a and 2a respectively. A body of mass m is fired at night from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a. [JEE' 96]

Q.3 An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.) E_0. Its potential energy is [JEE' 97]
(A) $-E_0$ (B) $1.5\ E_0$ (C) $2\ E_0$ (D) E_0

Q.4 A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth surface. Assume that the spaceship and the astronaut fall on a straight line from the earth centre. The radius of the earth is 6400 km. [REE 98]

Q.5 In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1 , W_2, W_3 respectively then



(A) $W_1 = W_2 = W_3$ (B) $W_1 > W_2 > W_3$ (C) $W_1 = W_2 > W_3$ (D) $W_1 < W_2 < W_3$
[JEE' (Scr.) 2003]

Q.6 A body is projected vertically upwards from the bottom of a crater of moon of depth R/100 where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. [JEE' 2003]

Q.7 A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then [JEE 2006]
(A) $T_A > T_B$ (if $r_A > r_B$) (B) $T_A > T_B$ (if $m_A > m_B$)
(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$ (D) $T_A = T_B$

ANSWER KEY

EXERCISE-I

Q.1 $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$ Q.2 $-\frac{3Gm^2}{a}$ Q.3 $\frac{G\ m^2}{3\ L^2}$ Q.4 (i) $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, (ii) $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

Q.5 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from west to east

Q.6 1×10^5J Q.7 $\frac{81}{19}R$ Q.8 $\frac{2G\lambda}{R}(\sin\alpha), (-G\lambda\ 2\alpha)$ Q.9 $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$

Q.10 $h = \frac{\sqrt{5}-1}{2}R$ Q.11 (a) $-GmM_e/r$, (b) $-2GmM_e/r$ Q.12 $\frac{R_e k^2}{1-k^2}$

Q.13 $t = \frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$ Q.14 $\frac{-4GM^2}{L}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

Q.15 (i) $-\frac{3Gm}{R}\left(\frac{m}{\sqrt{3}}+M\right)$, (ii) $\sqrt{\frac{G}{R}\left(\frac{m}{\sqrt{3}}+M\right)}$ Q.16 R

Q.17 $\vec{g} = +\frac{\pi G\rho_0 R^3}{6}\left[\frac{1}{\left(x-\frac{R}{2}\right)^2}-\frac{8}{x^2}\right]\hat{i}$, $\vec{g} = -\frac{2\pi G\rho_0 R}{3}\hat{i}$ Q.18 (a) $\sqrt{\frac{GM}{r}}$; (b) $r\sqrt{2}$; (c) $\sqrt{\frac{2GM}{r}}$

Q.19 (b) 1.9 h Q.20 $2\times\sqrt{R^3/GM}$

EXERCISE-II

Q.1 $\frac{2\pi R^{3/2}(6\sqrt{6})}{\sqrt{GM}\,(2\sqrt{2}+3\sqrt{3})}$ Q.2 $\frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$ Q.3 $\frac{Gm\lambda}{2d^2}$ Q.4 $\sqrt{\frac{2}{3}\pi G\rho R^2}$

Q.5 $T = 5\sqrt{\frac{5\pi}{3G\rho}}$ Q.7 (a) $T=4\pi\sqrt{\frac{r^3}{Gm}}$, (b) $v = \sqrt{\frac{4Gm}{r}}$ Q.8 $\sqrt{6.4}$km

Q.9 $\cos\theta = \frac{3}{\sqrt{10}}$ Q.11 $\sqrt{\frac{GM}{R}}\left[\sqrt{\frac{2}{3}}-\sqrt{\frac{8}{15}}\right]$

Q.12 **(a)** $h=\left(\frac{\sqrt{7}}{2}+1\right)R$, **(b)** 1.13R Q.13 $T = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$

EXERCISE-III

Q.1 B Q.2 $v_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$ Q.3 C Q.4 $T = 3 \times 10^{-2}$ N

Q.5 A Q.6 h = 99R Q.7 D

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P6. Particle Dynamics

Index:

KEY CONCEPT

FORCE

1. There are, basically, five forces, which are commonly encountered in mechanics.

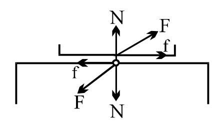
(i) **Weight :** Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

$$W = \frac{GMm}{R^2} = mg$$

(ii) **Contact Force :** When two bodies come in contact they exert forces on each other that is called contact forces.

(a) **Normal force (N) :** It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

(b) **Frictional force :** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.



(iii) **Tension :** The force exerted by the end of a taut string, rope or chain is called the tension. The direction of tension is to pull the body while that of normal reaction is to push the body.

(iv) **Spring force :** The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (units Nm^{-1}).

NEWTON'S LAWS

2. **Newton's First Law :** Every particle continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by the action of an applied force.

3. **Newton's Second Law :** $\vec{F}_{net} = m\vec{a}$

4. **Newton's Third Law :** Whenever two bodies interact they exert forces on each other which are equal in magnitude and opposite in direction. So whenever body A exerts a force F on body B, B exerts a force – F on A.

Inertial Reference Frame : A reference frame in which Newton's first law is valid is called an inertial reference frame. An inertial frame is either at rest or moving with uniform velocity.

Non-Inertial Frame : An accelerated frame of reference is called a non-inertial frame. Objects in non-inertial frames do not obey Newton's first law.

Pseudo Force : It is an imaginary force which is recognized only by a non-inertial observer to explain the physical situation according to Newton's law. The magnitude of this force F_P is equal to the product of the mass m of the object and acceleration a of the frame of reference. The direction of the force is opposite to the direction of acceleration.

$$F_P = -ma$$

The **force of friction** comes into action only when there is a relative motion between the two contact surfaces or when an attempt is made to have it.

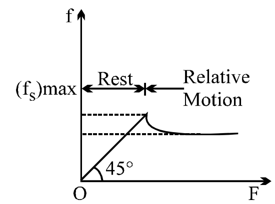
The force of friction on each body is in a direction opposite to its motion (existing or impending) relative to other body.

5. **Static friction :** The frictional force acting between any two surfaces at rest with respect to each other is called the force of static friction (f_s).

$$f_s \le \mu_s N$$

where μ_s is the static coefficient of friction.

6. **Kinetic friction :** The frictional force acting between surfaces in relative motion with respect to each other is called the force of kinetic friction or sliding friction (f_k).

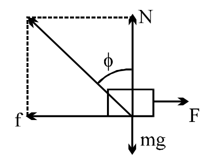


$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction.

$$\mu_s > \mu_k$$

Angle of friction (ϕ) : Mathematically, the angle of friction (ϕ) may be defined as the angle between the normal reaction N and the resultant of the maximum friction force f and the normal reaction.



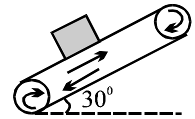
Thus $\tan\phi = \dfrac{f}{N}$

Since $f = \mu N$, therefore,

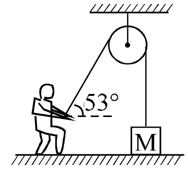
$$\tan\phi = \mu$$

(NEWTONS LAW FORCE & FRICTION)
EXERCISE–I

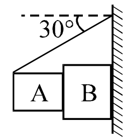
Q.1 A block of mass 1 kg is stationary with respect to a conveyor belt that is accelerating with 1 m/s^2 upwards at an angle of 30° as shown in figure. Determine force of friction on block and contact force between the block & bell.



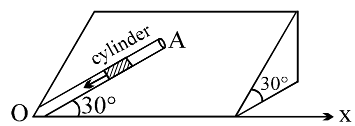
Q.2 A man of mass 63 kg is pulling a mass M by an inextensible light rope passing through a smooth and massless pulley as shown in figure. The coefficient of friction between the man and the ground is $\mu = 3/5$. Find the maximum value of M that can be pulled by the man without slipping on the ground.



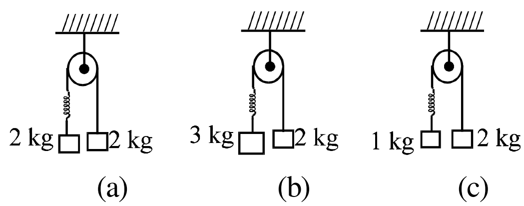
Q.3 Two blocks A and B of mass m 10 kg and 20 kg respectively are placed as shown in figure. Coefficient of friction between all the surfaces is 0.2. Then find tension in string and acceleration of block B. (g = 10 m/s^2)



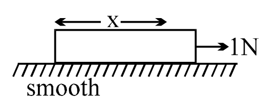
Q.4 An inclined plane makes an angle 30° with the horizontal. A groove OA = 5 m cut in the plane makes an angle 30° with OX. A short smooth cylinder is free to slide down the influence of gravity. Find the time taken by the cylinder to reach from A to O. (g = 10 m/s^2)



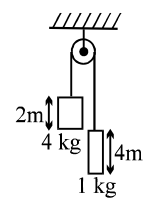
Q.5 Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown in figure. If x_1, x_2 and x_3 be the constan extensions in the spring in these three cases then find the ratio of their extensions.



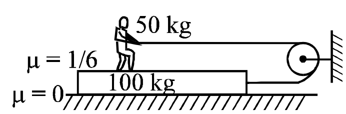
Q.6 A rope of length L has its mass per unit length λ varies according to the function $\lambda(x) = e^{x/L}$. The rope is pulled by a constant force of 1N on a smooth horizontal surface. Find the tension in the rope at $x = L/2$.



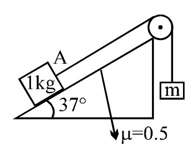
Q.7 In figure shown, both blocks are released from rest. Find the time to cross each other?



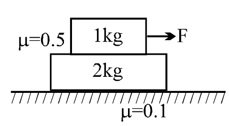
Q.8 A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.



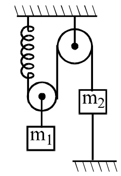
Q.9 In the figure, what should be mass m so that block A slide up with a constant velocity?



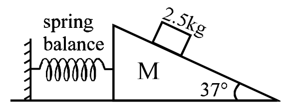
Q.10 What should be minimum value of F so that 2 kg slides on ground but 1 kg does not slide on it? [g = 10 m/sec^2]



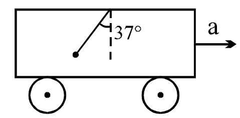
Q.11 In figure shown, pulleys are ideal $m_1 > 2\,m_2$. Initially the system is in equilibrium and string connecting m_2 to rigid support below is cut. Find the initial acceleration of m_2?



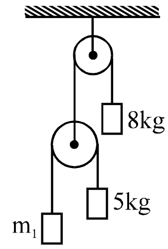
Q.12 Find the reading of spring balance as shown in figure. Assume that mass M is in equilibrium



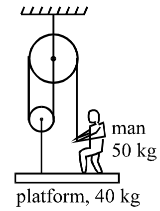
Q.13 At what acceleration of the trolley will the string makes an angle of 37° with vertical if a small mass is attached to bottom of string.



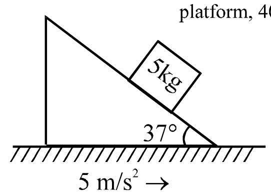
Q.14 At what value of m_1 will 8 kg mass be at rest.



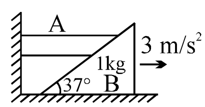
Q.15 What force must man exert on rope to keep platform in equilibrium?



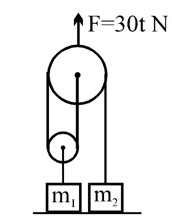
Q.16 Inclined plane is moved towards right with an acceleration of 5 ms^{-2} as shown in figure. Find force in newton which block of mass 5 kg exerts on the incline plane.



Q.17 Find force in newton which mass A exerts on mass B if B is moving towards right with 3 ms^{-2}. Also find mass of A.



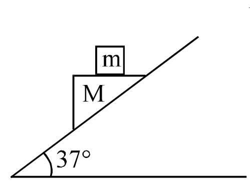
Q.18 Force F is applied on upper pulley. If F = 30t where t is time in second. Find the time when m_1 loses contact with floor.



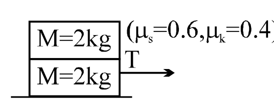
m_1=4kg, m_2=1kg

Q.19 A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has a $\mu = 0.5$. Plank rests on frictionless surface. Find the time when m_1 comes to rest w.r.t. plank.

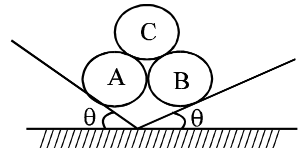
Q.20 Block M slides down on frictionless incline as shown. Find the minimum friction coefficient so that m does not slide with respect to M.



Q.21 The coefficient of static and kinetic friction between the two blocks and also between the lower block and the ground are $\mu_s = 0.6$ and $\mu_k = 0.4$. Find the value of tension T applied on the lower block at which the upper block begins to slip relative to lower block.



Q.22 Three identical rigid circular cylinders A, B and C are arranged on smooth inclined surfaces as shown in figure. Find the least value of θ that prevent the arrangement from collapse.



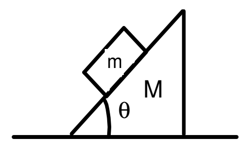
Q.23 Two men A and B of equal mass held on to the free ends of a massless rope which passes over a frictionless light pulley. Man A climbs up the rope with acceleration a relative to the rope while man B hangs on without climbing. Find the acceleration of the man B with respect to ground.

Q.24 A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.

Q.25 A body of mass M = 5kg rests on a horizontal plane having coefficient of fiction $\mu = 0.5$. At t = 0 a horizontal force F is applied that varies with time as F = 5t. Find the time instant t_0 at which motion starts and also find the distance of particle from starting point at t = 6 second.

EXERCISE–II

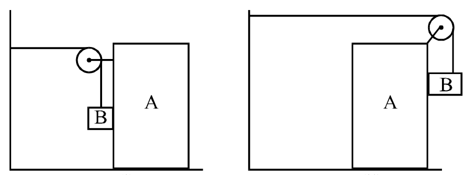
Q.1 A block of mass m lies on wedge of mass M as shown in figure. Answer following parts separately.



(a) With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

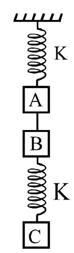
(b) Find the minimum friction coefficient required between wedge M and ground so that it does not move while block m slips down on it.

Q.2 A 20 kg block B is suspended from a cord attached to a 40 kg cart A. Find the ratio of the acceleration of the block in cases (i) & (ii) shown in figure immediately after the system is released from rest. (neglect friction)



Case (i) Case (ii)

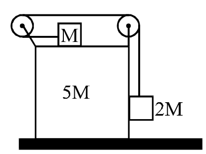
Q.3 The system shown adjacent is in equilibrium. Find the acceleration of the blocks A, B & C all of equal masses m at the instant when (Assume springs to be ideal)



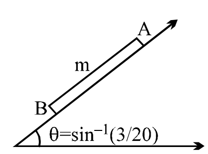
(a) The spring between ceiling & A is cut.
(b) The string (inextensible) between A & B is cut.
(c) The spring between B & C is cut.

Also find the tension in the string when the system is at rest and in the above 3 cases.

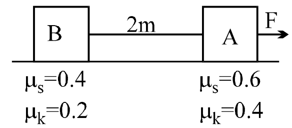
Q.4 In the system shown. Find the initial acceleration of the wedge of mass 5M. The pulleys are ideal and the cords are inextensible. (there is no friction anywhere).



Q.5 A plank of mass m is kept on a smooth inclined plane. A man of mass η times the mass of plank moves on the plank, starts from A, such that the plank is at rest, w.r.t. the inclined plane. If he reaches the other end B of the plank in t = 5sec. Then find the acceleration & the value of η, if the length of the plank is 50m.

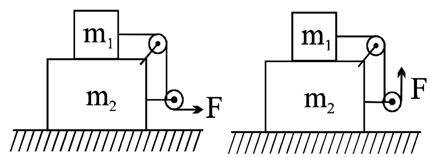


Q.6 Two horizontal blocks each of mass 1/2 kg are connected by a massless, inextensible string of length 2m and placed on a long horizontal table. The coefficient of static & kinetic friction are shown in the figure. Initially the blocks are at rest. If the leading block is pulled with a time dependent horizontal force $F = kt\,\hat{i}$ where k=1N/sec., determine



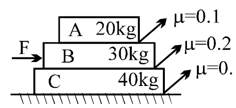
(a) The plots of acceleration of each block with time from t = 0 to t = 10sec.
(b) Velocity of blocks at t = 10sec.
(c) Distance transversed by the blocks in the time interval t = 0 to t = 10sec.
(d) If F stops acting at t = 10sec. find after how much further time would B collide with A.

Q.7 $m_1 = 20$kg, $m_2 = 30$kg. m_2 is on smooth surface. Surface between m_1 and m_2 has $\mu_s = 0.5$ and $\mu_k = 0.3$. Find the acceleration of m_1 and m_2 for the following cases
(a) (i) F = 160 N, (ii) F = 175 N ; (b) F = 160 N



(a) (b)

Q.8 A system of masses is shown in the figure with masses & co-efficients of friction indicated. Calculate :
(i) the maximum value of F for which there is no slipping anywhere .
(ii) the minimum value of F for which B slides on C.
(iii) the minimum value of F for which A slips on B.



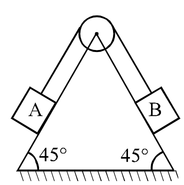
Q.9 A car begins to move at time t = 0 and then accelerates along a straight track with a speed given by $V(t) = 2t^2\ \text{ms}^{-1}$ for $0 \le t \le 2$
After the end of acceleration, the car continues to move at a constant speed. A small block initially at rest on the floor of the car begins to slip at t = 1sec. and stops slipping at t = 3 sec. Find the coefficient of static and kinetic friction between the block and the floor.

Q.10 A smooth right circular cone of semi vertical angle $\alpha = \tan^{-1}(5/12)$ is at rest on a horizontal plane. A rubber ring of mass 2.5kg which requires a force of 15N for an extension of 10cm is placed on the cone. Find the increase in the radius of the ring in equilibrium.

EXERCISE–III

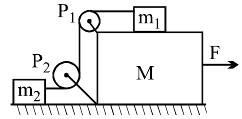
Q.1 A block of mass 0.1kg is held against a wall by applying a horizontal force of 5N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is
(A) 2.5N (B) 0.98N (C) 4.9N (D) 0.49N [JEE 1997]

Q.2 Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a massless inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the system of A and B is released from rest , find (i) the acceleration of A, (ii) tension in the string, (iii) the magnitude and the direction of friction acting on A. [JEE 1997]



Q.3 A spring of force constant k is cut into two pieces such that one piece such that one piece is double the length of the other. Then the long piece will have a force constant of
(A) (2/3) k (B) (3/2) k (C) 3k (D) 6k [JEE 1999]

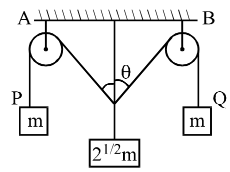
Q.4 In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The co-efficient of friction between M and ground is zero. The co-efficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless . The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2 . The string is perfectly vertical between P_1 and P_2.An external horizontal force F is applied to the mass M. Take $g = 10\ m/s^2$.



(a) Draw a free-body diagram for mass M, clearly showing all the forces.

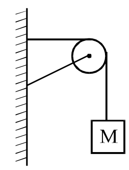
(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2 f_2$. Find f_1 and f_2 . Write down equations of motion of all the masses . Find F, tension in the string and accelerations of the masses. [JEE 2000]

Q.5 The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be [JEE (Scr) 2001]



(A) 0° (B) 30°
(C) 45° (D) 60°

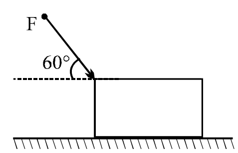
Q.6 A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given [JEE (Scr) 2001]



(A) $\sqrt{2}$ Mg (B) $\sqrt{2}$ mg

(C) $\sqrt{(M+m)^2+m^2}\ g$ (D) $\sqrt{(M+m)^2+M^2}\ g$

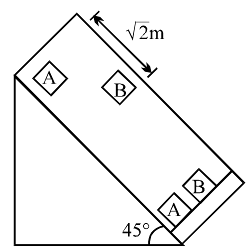
Q.7 A block of mass $\sqrt{3}$ kg is placed on a rough horizontal surface whose coefficient of friction is $1/2\sqrt{3}$ minimum value of force F (shown in figure) for which the block starts to slide on the surface. (g=10m/s^2)



(A) 20 N (B) $20\sqrt{3}$ N

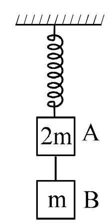
(C) $10\sqrt{3}$ N (D) None of these [JEE (Scr) 2003]

Q.8 Two blocks A and B of equal masses are released from an inclined plane of inclination 45° at t = 0. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially, the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line. [Take g = 10 m/s^2].



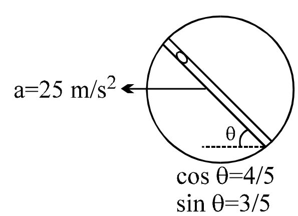
[JEE 2004]

Q.9 Two blocks A and B of masses 2m and m, respectively, are connected by a massless and inextensible string.The whole system is suspended by a massless spring as shown in the figure.The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively [JEE 2006]



(A) g, g (B) g, g/2
(C) g/2, g (D) g/2, g/2

Q.10 A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove in contact is μ = 2/5. The disc has an acceleration of 25 m/s^2. Find the acceleration of the block with respect to disc. [JEE 2006]

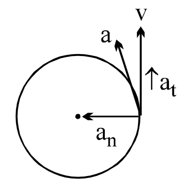


CIRCULAR MOTION & WORK POWER ENERGY

1. A body moving with constant speed in a circular path is continuously accelerated towards the centre of rotation. The magnitude of this normal acceleration is given by

$$a_n = \frac{v^2}{r} = \omega^2 r$$

where v is the constant speed ($v = \omega r$) and r is the radius of the circular path



Tangential area : $a_t = \frac{dv}{dt}$, $a = \sqrt{at^2 + a_n^2}$

2. **Radius of curvature :** $r = \frac{v^2}{a_n}$

3. According to Newton's second law, a body moving in a circular path with constant speed must be acted upon by an unbalanced force which is always directed towards the centre. This necessary unbalanced force is called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

4. Centrifugal force is a pseudo force which is observed an observer in rotating frame.

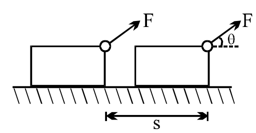
$$\vec{F}_{cf} = m\omega_{frame}^2 \vec{r}$$

Work (W) :

The **work W** done by a constant force F when its point of application undergoes a displacement s is defined as

$$W = F.s = Fs\cos\theta$$

where θ is the angle between F and s. Work is a scalar quantity and its SI units is N-m or joule (J).



Note: Only the component ($F\cos\theta$) of the force F which is along the displacement contributes to the work done.

If $F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $s = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

then $W = \vec{F}\cdot\vec{s} = F_x\Delta x + F_y\Delta y + F_z\Delta z$

5. **Work done by a Variable Force :** When the magnitude and direction of a force varies with position, The work done by such a force for an infinitesimal displacement ds is given by

$$dW = \vec{F}\cdot d\vec{s}$$

In terms of rectangular components,

$$W_{AB} = \int_{X_A}^{X_B} F_x dx + \int_{Y_A}^{Y_B} F_y dy + \int_{Z_A}^{Z_B} F_z dz$$

6. **Work Done by a Spring Force :** The work done by the spring force for a displacement from x_i to x_f is given by

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

7. **Work Energy theorem :**
Work done on a body can produce a change in its kinetic energy. Work is required to produce motion and it is also required to destroy motion.

$$W = \Delta K = K_f - K_i$$

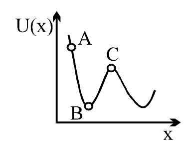
8. **Conservative Force :** The force which does work in complete independence of the path followed the body is called a conservative force. The gravitational force, spring force and electrostatic force are the examples of conservative forces.

9. **Non-Conservative Force :** The work done by a non-conservative force not only depends on the initial and final positions but also on the path followed. The common examples of such forces are : frictional force and drag force of fluids.

10. **Potential Energy :** The potential energy is defined only for conservative forces.

$$U_B - U_A = -\int_A^B F_c . ds$$

11. Conservative force : $F_c = -\frac{dU}{dx}$



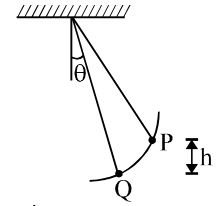
At equilibrium, $\frac{dU}{dx} = 0$

The point B is the position of stable equilibrium, because $\frac{d^2U}{dx^2} > 0$

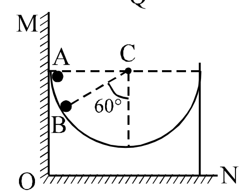
The point C is the position odf unstable equilibrium, because $\frac{d^2U}{dx^2} < 0$

(CIRCULAR MOTION & WORK POWER ENERGY)
EXERCISE–I

Q.1 The bob of a simple pendulum of length l is released from point P. What is the angle made by the net acceleration of the bob with the string at point Q.

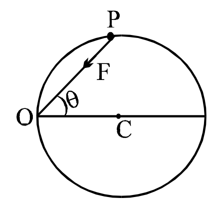


Q.2 A ball of mass 1 kg is released from position A inside a wedge with a hemispherical cut of radius 0.5 m as shown in the figure. Find the force exerted by the vertical wall OM on wedge, when the ball is in position B. (neglect friction everywhere). Take ($g = 10\ m/s^2$)



Q.3 A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of

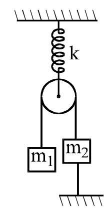
$$\frac{d^2\theta}{dt^2} \quad \& \quad \left(\frac{d\theta}{dt}\right)^2 .$$



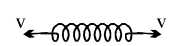
Q.4 A particle is moving in x direction, under the influence of force $F = \pi \sin \pi x$. Find the work done by another external agent in slowly moving a particle from $x = 0$ to $x = 0.5$ m.

Q.5 A particle moves in a circle of radius R with a constant speed v. Then, find the magnitude of average acceleration during a time interval $\frac{\pi R}{2v}$.

Q.6 In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring ($m_1 > m_2$).



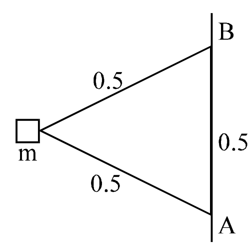
Q.7 A spring of mass m is pulled such that a given instant, velocity of both of its end is v in the opposite direction. Find the kinetic energy of the spring.



Q.8 A particle of mass 3 kg is rotating in a circle of radius 1 m such that the angle rotated by its radius is given by $\theta = 3\ (t + \sin t)$. Find the net force acting on the particle when $t = \pi/2$.

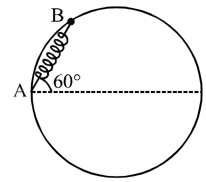
Q.9 For a particle rotating in a vertical circle with uniform speed, the maximum and minimum tension in the string are in the ratio 5 : 3. If the radius of vertical circle is 2m, then find the speed of revolving body.

Q.10 Two strings of length $l = 0.5$ m each are connected to a block of mass m = 2 kg at one end and their ends are attached to the point A and B 0.5 m apart on a vertical pole which rotates with a constant angular velocity $\omega = 7$ rad/sec. Find the ratio $\frac{T_1}{T_2}$ of tension in the upper string (T_1) and the lower string (T_2). [Use $g = 9.8\ m/s^2$]



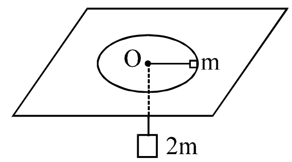
Q.11 A force $\vec{F} = -k(x\hat{i} + y\hat{j})$ [where k is a positive constant] acts on a particle moving in the x-y plane. Starting from origin, the particle is taken to (a, a) and then to $\left(a/\sqrt{2}, 0\right)$. Find the total work done by the force F on the particle.

Q.12 A bead of mass m is attached to one end of a spring of natural length $\sqrt{3}$ R and spring constant $k = \frac{(\sqrt{3}+1)\,mg}{R}$. The other end of the spring is fixed at point A on a smooth fixed vertical ring of radius R as shown in the figure. What is the normal reaction at B just after the bead is released?

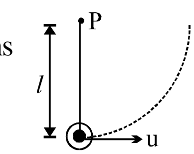


Q.13 Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2} m^2 upto a height of 10 m. If it is needed to deliver a volume 0.2 m^3 per second, find the power required. [Use g = 10 m/s^2]

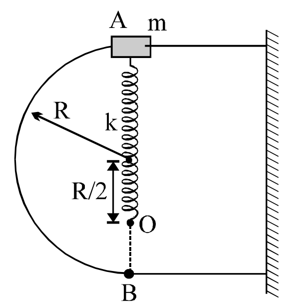
Q.14 A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



Q.15 Consider the shown arrangement when a is bob of mass 'm' is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity $\vec{u}$ having magnitude $\sqrt{3gl}$, find the minimum speed of the bob in subsequent motion.

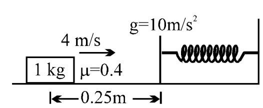


Q.16 A bead of mass m is tied at one end of a spring of spring constant $\frac{mg}{R}$ and unstretched length $\frac{R}{2}$ and other end to fixed point O. The smooth semicircular wire frame is fixed in vertical plane. Find the normal reaction between bead and wire just before it reaches the lowest point.



Q.17 A particle of mass m is hanging with the help of an elastic string of unstretched length a and force constant $\frac{mg}{a}$. The other end is fixed to a peg on vertical wall. String is given an additional extension of $2a$ in vertical downward direction by pulling the mass and released from rest. Find the maximum height reached by it during its subsequent motion above point of release. (Neglect interaction with peg if any)

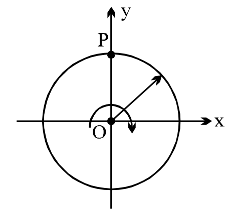
Q.18 A particle of mass 1 kg is given a horizontal velocity of 4 m/s along a horizontal surface, with which it has a coefficient of friction (both static and kinetic) of 0.4. The particle strikes a fixed ideal spring of force constant 6 N/m after travelling a distance of 0.25 m. Assume acceleration due to gravity is 10 m/s^2. Find the final displacement of the particle from its starting point.



Q.19 A point moves along a circle having a radius 20 cm with a constant tangential acceleration 5 cm/s^2. How much time is needed after motion begins for the normal acceleration of the point to be equal to tangential acceleration ?

Q.20 A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by U (r) = $2r^3$ Joule. If the body is moving in a circular orbit of 5m,then find its energy.

Q.21 A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $\left(6\hat{i}-8\hat{j}\right)$ m/s^2. find the angular acceleration of the ring & the angular velocity at that instant. Radius of the ring is 2m.



Q.22 A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At t = 0, it was subjected to a constant angular acceleration α and its angular speed increased to$(480/\pi)$ rpm in 2 sec. Particle then continues to move with attained speed. Calculate

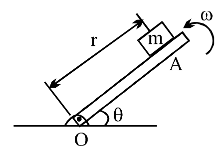
(a) angular acceleration of the particle,

(b) tangential velocity of the particle as a function of time.

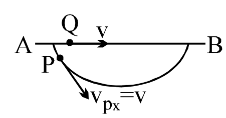
(c) acceleration of the particle at t = 0.5 second and at t = 3 second

(d) angular displacement at t = 3 second.

Q.23 The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity ω= 3 rad/sec. As it passes the position θ = 0, a small mass m is placed upon it at a radial distance r = 0.5 m. If the mass is observed to slip at θ = 37°, find the coefficient of friction between the mass & the member.



Q.24 A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at t = 0. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at t=0 along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Which bead reaches point B earlier?



Q.25 The blocks are of mass 2 kg shown is in equilibrium. At t = 0 right spring in fig (i) and right string in fig (ii) breaks. Find the ratio of instantaneous acceleration of blocks?

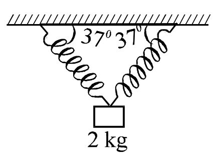


figure (i)

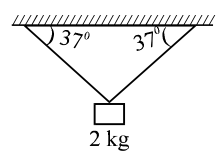


figure (ii)

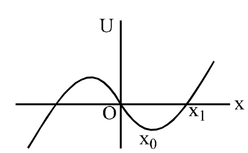
EXERCISE–II

Q.1 A particle is confined to move along the +x axis under the action of a force F(x) that is derivable from the potential U(x) =ax^3-bx.

(a) Find the expression for F(x)

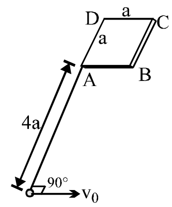
(b) When the total energy of the particle is zero, the particle can be trapped with in the interval x=0 to x=x_1. For this case find the values of x_1.

(c) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b.

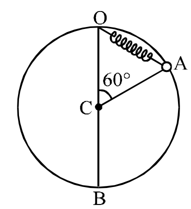


Q.2 A particle of mass 2kg is subjected to a two dimensional conservative force given by Fx =$-2x+2y$, Fy=$2x-y^2$. (x,y in m and F in N) If the particle has kinetic energy of (8/3) J at point (2,3), find the speed of the particle when it reaches (1,2).

Q.3 A square plate is firmly atached to a frictionless horizontal plane. One end of a taut cord is attached to point A of the plate and the other end is attached to a sphere of mass m. In the process, the cord gets wrapped around the plate. The sphere is given an initial velocity v_0 on the horizontal plane perpendicular to the cord which causes it to make a complete circuit of the plate and return to point A. Find the velocity of the sphere when it hits point A again after moving in a circuit on the horizontal plane. Also find the time taken by the sphere to complete the circuit.

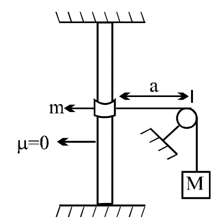


Q.4 A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle$ OCA = 60°, C being the centre of the ring. The natural length of the spring is also equal to r = 20cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.

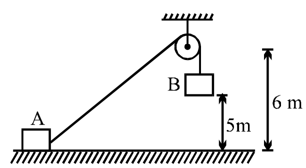


Q.5 A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M (> m). The ring is held on a level with the peg and released : Show that it first comes to rest after falling a distance:

$$\frac{2mMa}{M^2 - m^2}$$



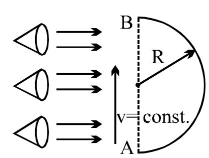
Q.6 A block of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6 m from the floor. A light inextensible string of length 16 m, connected with A passes over the pulley and another identical block B is hung from the string. Initial height of B is 5m from the floor as shown in Fig. When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.



(i) If at an instant string makes an angle θ with horizontal, calculate relation between velocity u of A and v of B.

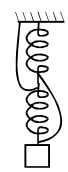
(ii) Calculate v when B strikes the floor.

Q.7 A small block can move in a straight horizontal linea along AB. Flash lights from one side projects its shadow on a vertical wall which has horizontal cross section as a circle. Find tangential & normal acceleration of shadow of the block on the wall as a function of time if the velocity of the block is constant (v).



Top View

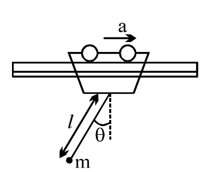
Q.8 In fig two identical springs, each with a relaxed length of 50cm and a spring constant of 500N/m, are connected by a short cord of length 10cm. The upper string is attached to the ceiling, a box that weighs 100N hangs from the lower spring. Two additional cords, each 85cm long , are also tied to the assembly; they are limp (i.e. slack).



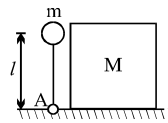
(a) If the short cord is cut, so that the box then hangs from the springs and the two longer cords, does the box move up or down?

(b) How far does the box move before coming to rest again?

Q.9 The small pendulum of mass m is suspended from a trolley that runs on a horizontal rail. The trolley and pendulum are initially at rest with θ = 0. If the trolley is given a constant acceleration a = g determine the maximum angle θ_{max} through which the pendulum swings. Also find the tension T in the cord in terms of θ.

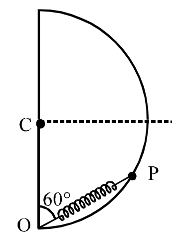


Q.10 A weightless rod of length l with a small load of mass m at the end is hinged at point A as shown in the figure and occupies a strictly vertical position, touching a body of mass M. A light jerk sets the system in motion. For what mass ratio M/m will the rod form an angle $\alpha = \pi/6$ with the horizontal at the moment of the separation from the body? What will be the velocity u of the body at this moment? Friction should be neglected.



EXERCISE–III

Q.1 A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length (3R/4) is attached to the lowest point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant K = mg/R. Consider the instant when the ring is released and



(i) draw the free body diagram of the ring.

(ii) determine the tangential acceleration of the ring and the normal reaction. [JEE 96]

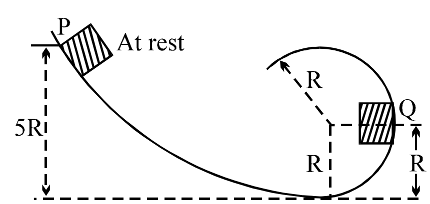
Q.2 Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1. [JEE 97]

Q.3 A small block of mass m slides along a smooth frictional track as shown in the fig. (i) If it starts from rest at P, what is is the resultant force acting on it at Q? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight. [REE 97]



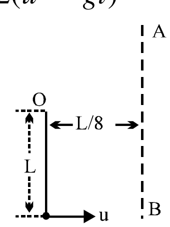
Q.4 A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a,0) and then parallel to the y-axis to the pint (a,a). The total work done by the force $\vec{F}$ on the particle is [JEE 98]

(A) $-2Ka^2$ (B) $2Ka^2$ (C) $-Ka^2$ (D) Ka^2

Q.5 A stone is tied to a string of length l is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity at it reaches a position where the string is horizontal is [JEE98]

(A) $\sqrt{(u^2 - 2gl)}$ (B) $\sqrt{2gl}$ (C) $\sqrt{(u^2 - gl)}$ (D) $\sqrt{2(u^2 - gl)}$

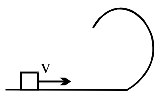
Q.6 A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance L/8 from O as shown. The object given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. [JEE'99, 10]

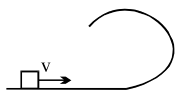


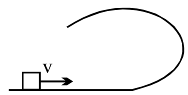
Q.7 A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end of A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and gravity is neglected, then the time after which the bead starts slipping is [JEE'2000]

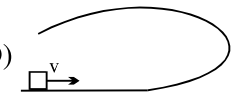
(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

Q.8 A small block is shot into each of the four tracks as shown below. Each of the tracks risks to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in [JEE(Scr)'2001]

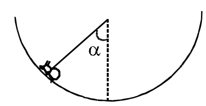
(A) 

(B) 

(C) 

(D) 

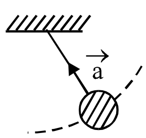
Q.9 An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by [JEE(Scr.)'2001]

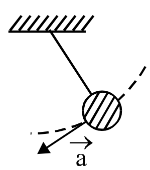


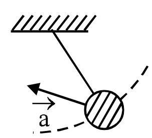
(A) cot $\alpha = 3$ (B) tan $\alpha = 3$ (C) sec $\alpha = 3$ (D) cosec $\alpha = 3$

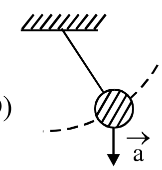
Q.10 A small ball of mass 2×10^{-3} Kg having a charge of 1 μc is suspended by a string of length 0.8m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. [JEE'2001]

Q.11 A simple pendulum is oscillating without damping. When the displacement of the bob is less that maximum, its acceleration vector $\vec{a}$ is correctly shown in [JEE (Scr.)'2002]

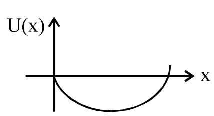
(A) 

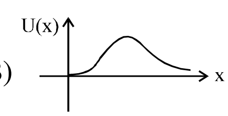
(B) 

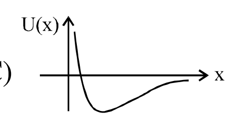
(C) 

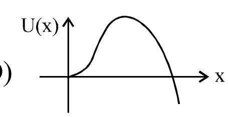
(D) 

Q.12 A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle x of the particle from the origin as $F(x) = -kx + ax^2$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy U (x) of the particle is [JEE (Scr.)'2002]

(A) 

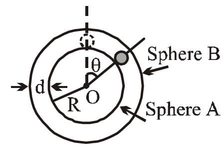
(B) 

(C) 

(D) 

Q.13 An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is [JEE (Scr.)'2002]
(A) 4 Mg/k (B) 2 Mg/k (C) Mg/k (D) Mg/2k

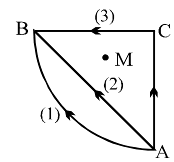
Q.14 A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). [JEE' 2002]



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

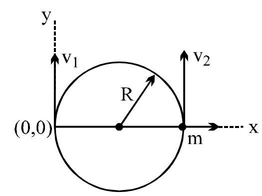
(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

Q.15 In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then [JEE (Scr.)'2003]

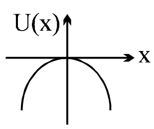


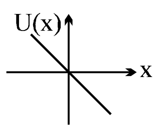
(A) $W_1 = W_2 = W_3$
(B) $W_1 = W_2 > W_3$
(C) $W_1 > W_2 > W_3$
(D) $W_1 < W_2 < W_3$

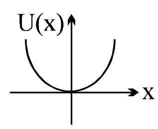
Q.16 A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time. [JEE' 2003]

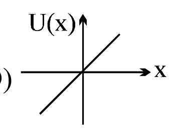


Q.17 A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function)

(A) 

(B) 

(C) 

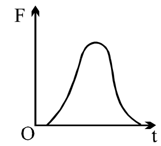
(D) 

[JEE' 2004(Scr)]

CENTRE OF MASS MOMENTUM & COLLISION

The action of force with respect to time is defined in terms of Impulse, that is,

$$\int F dt = mv_f - mv_i = \Delta p$$



Graphically, impulse is the area under the F-t graph

In the absence of a net external force, the momentum of a system is conserved.

i.e. $$\frac{dP}{dt} = F_{ext} = 0$$

$$p = p_1 + p_2 + \ldots\ldots\ldots + p_N = \text{constant}$$

1. **Collision** is a kind of interaction between two or more bodies which come in contact with each other for a very short time interval.
2. **Types of collision: Elastic and Inelastic**
   Collisions may be either elastic or inelastic. Linear momentum is conserved in both cases.

(i) A perfectly elastic collision is defined as one in which the total kinetic energy of the system is conserved.

(ii) In an inelastic collision, the total kinetic energy of the system changes.

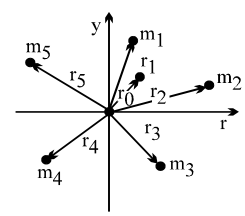
(iii) In a completely inelastic collision, the two bodies couple or stick togehter.

3. **Coefficient of Restitution :** It is defined as the ratio of the velocity of separation to the velocity of approach of the two colliding bodies.

$$e = \frac{\text{rel. velocity of separation}}{\text{rel. velocity of approach}}$$

For a perfectly elastic collision, $e = 1$
For an inelastic collision, $0 < e < 1$
For completely inelastic collision, $e = 0$
Note that the velocity of approach and the velocity of separation are always taken along the normal to the striking surface.

CENTRE OF MASS



1. **Discrete System :** The position vector of the centre of mass is

$$r_c = \frac{m_1 r_1 + m_2 r_2 + \ldots\ldots\ldots + m_n r_n}{m_1 + m_2 + \ldots\ldots\ldots m_n}$$

where $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ are the position vectors of masses m_1, m_2, ..., m_n respectively.
The components of the position vector of centre of mass are defined as

$$x_c = \frac{\sum m_i x_i}{M}; \qquad y_c = \frac{\sum m_i y_i}{M}; \qquad z_c = \frac{\sum m_i z_i}{M}$$

2. **Continuous system :** The centre of mass of a continuous body is defined as

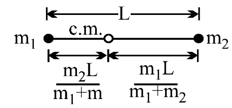
$$\vec{r}_c = \frac{1}{M}\int r\, dm$$

In the component form

$$x_c = \frac{1}{M}\int x\, dm; \qquad y_c = \frac{1}{M}\int y\, dm; \qquad z_c = \frac{1}{M}\int z\, dm$$

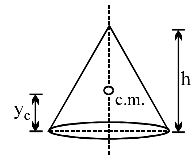
3. **Centre of Mass of Some Common Systems :**

(i) A system of two point masses.
The centre of mass lie closer to the heavier mass.



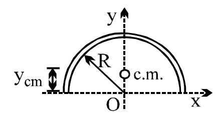
(ii) A circular cone

$y_c = \dfrac{h}{4}$



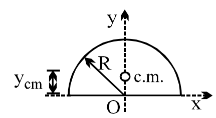
(iii) A semi-circular ring

$y_c = \dfrac{2R}{\pi}$; $x_c = 0$



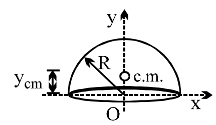
(iv) A semi-circular disc

$y_c = \dfrac{4R}{3\pi}$; $x_c = 0$



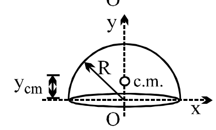
(v) A hemispherical shell

$y_c = \dfrac{R}{2}$; $x_c = 0$



(vi) A solid hemisphere

$y_c = \dfrac{3R}{8}$; $x_c = 0$



4. **Motion of the centre of mass :**

(i) **Velocity :** The instantaneous velocity of the centre of mass is defined as

$$v_c = \frac{\sum m_i v_i}{M}$$

(ii) **Acceleration :** The acceleration of the centre of mass is defined as

$$a_c = \frac{\sum m_i a_i}{M}$$

(iii) **Momentum :** The total momentum of a system of particles is

$$p = Mv_c$$

(iv) **Kinetic Energy :** The kinetic energy of a system of particles consisits of two parts.

$$K = K_c + K'$$

where $K_c = \frac{1}{2}Mv_c^2$, kinetic energy due to motion of c.m. relative to the fixed origin O,

and $K' = \sum \frac{1}{2} m_i v_i^2$, kinetic energy of the particles relative to the c.m.

Note that the term K' may involve translational, rotational or vibrational energies relative to the centre of mass.

5. **Newon's Laws of a system of particles :** The first and second laws of motion for a system of particles are modified as :

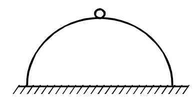
First law : The centre of mass of an isolated system is at rest or moves with constant velocity.

Second law : The net external force acting on a system of total of mass M is related to the acceleration of centre of mass of the system.

$$\sum \vec{F}_{ext} = M\,\vec{a}_{cm}$$

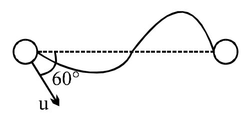
(CENTRE OF MASS MOMENTUM & COLLISION)
EXERCISE–I

Q.1 A hemisphere of radius R and of mass 4m is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the top of the hemisphere. Find the angular velocity of the particle relative to hemisphere at an angular displacement θ when velocity of hemisphere has become v.



Q.2 A man whose mass is m kg jumps vertically into air from a sitting position in which his centre of mass is at a height h_1 from the ground. When his feet are just about to leave the ground his centre of mass is h_2 from the ground and finally rises to h_3 when he is at the top of the jump. **(a)** What is the upward force exerted by the ground on him treating it as a constant? **(b)** Find work done by normal reaction from ground.

Q.3 In the figure shown, each tiny ball has mass m, and the string has length L. One of the ball is imparted a velocity u, in the position shown, in which the initial distance between the balls is $L/\sqrt{3}$. The motion of ball occurs on smooth horizontal plane. Find the impulse of the tension in the string when it becomes taut .

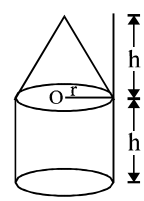


Q.4 Two trolleys A and B are free to move on a level frictionless track, and are initially stationary. A man on trolley A throws a bag of mass 10 kg with a horizontal velocity of 4 m/s with respect to himself on to trolley B of mass 100 kg. The combined mass of trolley A (excluding bag) and the man is 140 kg. Find the ratio of velocities of trolleys A and B, just after the bag lands on trolley B.

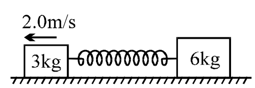
Q.5 A bob of mass m attached with a string of length l tied to a point on ceiling is released from a position when its string is horizontal. At the bottom most point of its motion, an identical mass m gently stuck to it. Find the angle from the vertical to which it rises.

Q.6 Two balls of equal masses are projected upward simultaneously, one from the ground with speed 50 m/s and other from a 40 m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.

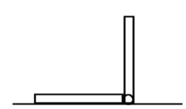
Q.7 Find the distance of centre of mass from O of a composite solid cone and solid cylinder made of same material.



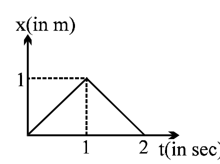
Q.8 Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring. Initially the spring is unstretched and the velocity of 2 m/s is imparted to 3 kg block as shown. Find the maximum velocity of 6 kg block during subsequent motion.



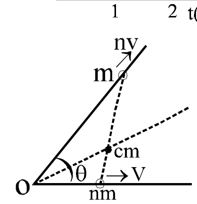
Q.9 Two planks each of mass m and length L are connected by a frictionless, massless hinge as shown in the figure. Initially the system is at rest on a level frictionless surface. The vertical plank falls anticlockwise and finaly comes to rest on the top of the horizontal plank. Find the displacement of the hinge till the two planks come in contact.



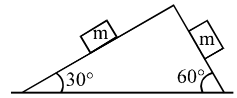
Q.10 2 bodies m_1 & m_2 of mass 1 and 2 kg respectively are moving along x-axis under the influence of mutual force only. The velocity of their centre of mass at a given instant is 2 m/s. The x coordinate of m_1 is plotted against time. Then plot the x coordinate of m_2 against time. (Both are initially located at origin)



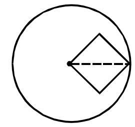
Q.11 Two masses, nm and m, start simultaneously from the intersection of two straight lines with velocities v and nv respectively. It is observed that the path of their centre of mass is a straight line bisecting the angle between the given straight lines. Find the magnitude of the velocity of centre of inertia.
(here θ = angle between the lines)



Q.12 Two blocks of equal masses m are released from the top of a smooth fixed wedge as shown in the figure. Find the magnitude of the acceleration of the centre of mass of the two blocks.



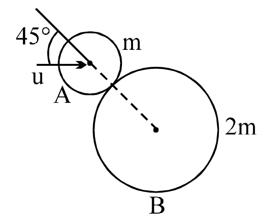
Q.13 From a uniform circular disc of radius R, a square is cut out with radius R as its diagonal. Find the centre of mass of remainder is at a distance.(from the centre)



Q.14 A sphere of mass m_1 in motion hits directly another sphere of mass m_2 at rest and sticks to it, the total kinetic energy after collision is 2/3 of their total K.E. before collision. Find the ratio of $m_1 : m_2$.

Q.15 Two bodies of same mass tied with an inelastic string of length l lie together. One of them is projected vertically upwards with velocity $\sqrt{6gl}$. Find the maximum height up to which the centre of mass of system of the two masses rises.

Q.16 Disc A of mass m collides with stationary disk B of mass 2m as shown in figure. Find the value of coefficient of restitution for which the two disks move in perpendicular direction after collision.

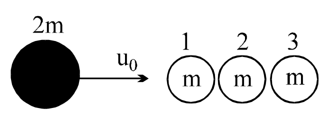


Q.17 A platform of mass m and a counter weight of mass (m + M) are connected by a light cord which passes over a smooth pulley. A man of mass M is standing on the platform which is at rest. If the man leaps vertically upwards with velocity u, find the distance through which the platform will descend. Show that when the man meets the platform again both are in their original positions.

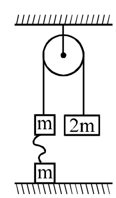
Q.18 The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

Q.19 After scaling a wall of 3 m height a man of weight W drops himself to the ground. If his body comes to a complete stop 0.15 sec. After his feet touch the ground, calculate the average impulsive force in the vertical direction exerted by ground on his feet. ($g = 9.8\ m/s^2$)

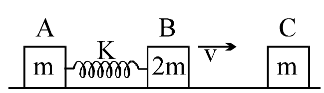
Q.20 A heavy ball of mass 2m moving with a velocity u_0 collides elastically head-on with a cradle of three identical balls each of mass m as shown in figure. Determine the velocity of each ball after collision.



Q.21 The Atwood machine in fig has a third mass attached to it by a limp string. After being released, the 2m mass falls a distance x before the limp string becomes taut. Thereafter both the mass on the left rise at the same speed. What is the final speed ? Assume that pulley is ideal.



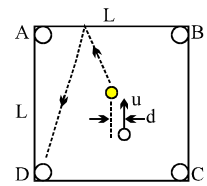
Q.22 Two blocks A and B of masses m and 2m respectively are connected by a spring of force constant k. The masses are moving to the right with uniform velocity v each, the heavier mass leading the lighter one. The spring in between them is of natural length during the motion. Block B collides with a third block C of mass m, at rest. The collision being completely inelastic. Calculate the maximum compression of the spring.



EXERCISE–II

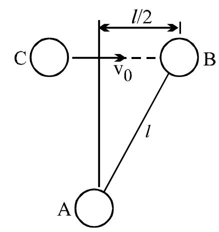
Q.1 A billiard table is 15 cm by 20 cm. A smooth ball of coefficient of restitution e = 4/9 is projected from a point on the shorter side so as to describe a rectangle and return to the point of projection after rebounding at each of the other three cushions. Find the position of the point and the direction of projection.

Q.2 In a game of Carom Board, the Queen (a wooden disc of radius 2 cm and mass 50 gm) is placed at the exact center of the horizontal board. The striker is a smooth plastic disc of radius 3 cm and mass 100 gm. The board is frictionless. The striker is given an initial velocity 'u' parallel to the sides BC or AD so that it hits the Queen inelastically with coefficient of restitution = 2/3. The impact parameter for the collision is 'd' (shown in the figure). The Queen rebounds from the edge AB of the board inelastically with same coefficient of restitution = 2/3 and enters the hole D following the dotted path shown. The side of the board is L.

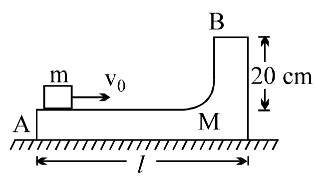


Find the value of impact parameter 'd' and the time which the Queen takes to enter hole D after collision with the striker.

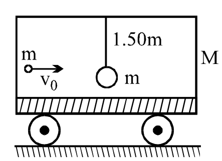
Q.3 Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible inelastic cord of length l and are at rest in the position shown when sphere B is struck directly by sphere C which is moving to the right with a velocity v_0. Knowing that the cord is taut when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, determine the velocity of each sphere immediately after impact.



Q.4 A wedge of mass M=2m rests on a smooth horizontal plane. A small block of mass m rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity $v_0 = 6\ ms^{-1}$. At highest point of its trajectory, the block collides with a particle of same mass m moving vertically downwards with velocity $v=2\ ms^{-1}$ and gets stuck with it. If the combined body lands at the end point A of body of mass M, calculate length l. Neglect friction ($g=10\ ms^{-2}$)



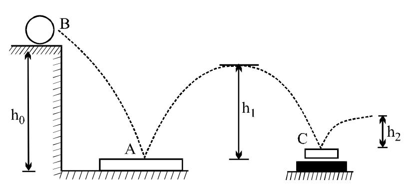
Q.5 A ball of mass = 1Kg is hung vertically by a thread of length $l = 1.50$ m. Upper end of the thread is attached to the ceiling of a trolley of mass M = 4 kg. Initially, trolley is stationary and it is free to move along horizontal rails without friction. A shell of mass m = 1 kg moving horizontally with velocity $v_0 = 6ms^{-1}$ collides with the ball and gets stuck with it. As a result, thread starts to deflect towards right. Calculate its maximum deflection with the vertical. ($g = 10m\ s^{-2}$)



Q.6 A 70g ball B droped from a height $h_0 = 9$ m reaches a height $h_2 = 0.25$m after bouncing twice from identical 210g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine

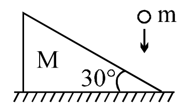
(a) the coefficient of resitution between the ball and the plates,

(b) the height h_1 of the ball's first bounce.



Q.7 A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

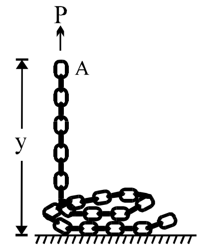
Q.8 A ball of mass m = 1 kg falling vertically with a velocity $v_0 = 2$ m/s strikes a wedge of mass M = 2kg kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is e = 1/2. Find the velocity of the wedge and the ball immediately after collision.



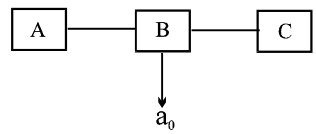
Q.9 A chain of length l and m lies in a pile on the floor. It its end A is raised vertically at a constant speed v_0, express in terms of the length y of chain which is off the floor at any given instant.

(a) the magnitude of the force P applied to end A.

(b) the reaction of the floor. (c) energy lost during the lifting of the chain.



Q.10 3 blocks of mass 1kg each kept on horizontal smooth ground are connected by 2 taut strings of length l as shown. B is pulled with constant acceleration a_0 in direction shown. Find the relative velocity of A & C just before striking.

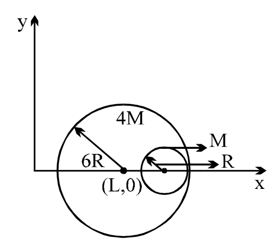


EXERCISE–III

Q.1 A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time t = 0. All colisions are completely inelastic, then
(i) the last block starts moving at $t = n(n-1)L/(2v)$
(ii) the last block starts moving at $t = (n-1)L/v$
(iii) the centre of mass of the system will have a final speed v/n
(iv) the centre of mass of the system will have a final speed v. [IIT 95]

Q.2 A small bucket of mass M (= 10^{-2}kg) is attached to a long cord of length L (= 5×10^{-2}m). The bucket is released from rest when the cord is in a horizontal position. In its lowest position the bucket scoops up m(=10^{-3}kg) of water, what is the height of the swing above the lowest position [REE 95]

Q.3 A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R. The masses of large and small spheres are 4M and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the large sphere when the smaller sphere reaches the other extreme position. [IIT 96]



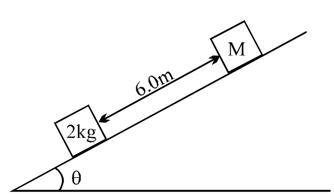
Q.4 A body of mass 5kg moves along the x axis with a velocity 2m/s. A second body of mass 10kg moves along the y axis with a velocity $\sqrt{3}$ m/s. They collide at the origin and stick together. Calculate
(i) the final velocity of the combined mass after collision
(ii) the amount of heat liberated in the collision. [REE 96]

Q.5 An isolated particle of mass m is moving in a horizontal plane (x-y) along the x-axis at a certain height above the ground. It suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later the smaller fragment is at y = + 15 cm. The larger fragment at this instant is at [IIT 97]
(A) y = – 5cm (B) y = + 20cm (C) y = + 5cm (D) y = – 20cm

Q.6 A cart is moving along +x direction with a velocity of 4m/s. A person in the cart throws a stone with a velocity of 6m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with the vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine
(a) the speed of the combined mass immediately after the collision with respect to an observer on the ground.
(b) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass. [IIT 97]

Q.7 A particle of mass m and velocity v collides elastically and obliquely with a stationary particle of mass m. Calculate the angle between the velocity vectors of the two particles after the collision. [REE 97]

Q.8 Two blocks of mass 2kg and M are at rest on an indiclined plane and are separated by a distance of 6.0m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10.0m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0m/s when it reaches its initial position. The other block M after the collision moves 0.5m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. [Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10m/s^2$] [IIT 99]



Q.9 Two trolleys A and B of equal masses M are moving in oppsite directions with velocities $\vec{v}$ and $-\vec{v}$ respectively on separate horizontal frictionless parallel tracks. When they start crossing each other, a ball of mass m is thrown from B to A and another of same mass is thrown from A to B with velocities normal to $\vec{v}$. The balls may be thrown in following two ways:

(i) balls from A to B and B to A are thrown simultaneously.

(ii) ball is thrown from A to B after the ball thrown from B reaches A.

Which procedure would lead to a larger change in the velocities of the trolleys? [REE 2000]

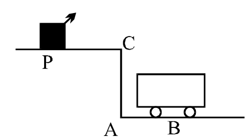
Q.10 A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to: [IIT (Scr) 2000]

(A) v (B) v^2 (C) v^3 (D) v^4

Q.11 Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time t = 0. They collide at time t_0. Their velocities become $\vec{v}_1^{\,'}$ and $\vec{v}_2^{\,'}$ at time $2t_0$ while still moving in air. The value of $\left|\left(m_1\vec{v}_1^{\,'} + m_2\vec{v}_2^{\,'}\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\right|$ is [IIT (Scr) 2001]

(A) zero (B) $(m_1 + m_2)gt_0$ (C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$

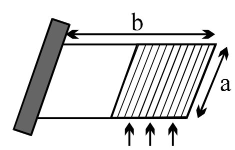
Q.12 A car P is moving with a uniform speed of $5(3^{1/2})$ m/s towards a carriage of mass 9 Kg at rest kept on the rails at a point B as shown in fig. The height AC is 120 m. Cannon balls of 1 Kg are fired from the car with an initial velocity 100 m/s at an angle 30^0 with the horizontal. The first canon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage. What will be the horizontal velocity of the carriage just after the second impact? [IIT 2001]



Q.13 Two block of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is : [IIT (Scr) 2002]

(A) 30 m/s (B) 20 m/s (C) 10 m/s (D) 5 m/s

Q.14 There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?
It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1m; g = 10 m/s^2.



[JEE 2006]

ANSWER KEY
(NEWTONS LAW FORCE & FRICTION)
EXERCISE – I

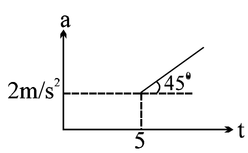
Q.1 contact force between the block and the belt is 10.5 N Q.2 35 kg
Q.3 306 N , 4.7 m/s^2
Q.4 2 sec Q.5 $x_2 > x_1 > x_3$ $x_1 : x_2 : x_3 : 15 : 18 : 10$ Q.6 $\dfrac{1}{\sqrt{e}+1}$
Q.7 1 sec
Q.8 $\dfrac{100}{3}$ N towards left Q.9 1 kg Q.10 3 N Q.11 $\left(\dfrac{m_1 - 2m_2}{2m_2}\right)g$
Q.12 12 N
Q.13 7.5 ms^{-2} Q.14 10/3 kg Q.15 300 N Q.16 55
Q.17 5N, 16/31 kg Q.18 2 sec Q.19 $\dfrac{4}{3}$ sec Q.20 $\dfrac{3}{4}$
Q.21 40 N Q.22 $\tan^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)$ Q.23 $\dfrac{a}{2}$ Q.24 1/2 sec Q.25 5 sec and $\dfrac{1}{6}$ m

EXERCISE – II

Q.1 (a) $a = g\cot\theta$, (b) $\mu_{min} = \dfrac{m\sin\theta\cos\theta}{m\cos^2\theta + M}$ Q.2 $\dfrac{3}{2\sqrt{2}}$

Q.3 (a) $a_A = \dfrac{3g\downarrow}{2} = a_B$; $a_C = 0$; T=mg/2; (b) $a_A = 2g\uparrow$, $a_B = 2g\downarrow$, $a_c = 0$, T=0; (c) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, $T = \dfrac{3mg}{2}$; T=2mg

Q.4 2g/23 Q.5 (a) $\eta = \dfrac{3}{5}$; (b) acceleration = 4 m/s^2

Q.6 (a) 

(b) 22.5 m/s ; (c) $\dfrac{275}{6}$ m; (d) $\sqrt{2}$ sec

Q.7 (a) (i) $a_1 = a_2 = 3.2$ m/s^2 , (ii) $a_1 = 5.75$ m/s^2 , $a_2 = 2m/s^2$; (b) $a_1 = 5$ m/s^2 , $a_2 = -10/3$ m/s^2

Q.8 (i)90N,(ii)112.5N(iii)150N Q.9 $\mu_s = 0.4$, $\mu_k = 0.3$

Q.10 $\Delta r = \dfrac{mg\cot\alpha}{4\pi^2 k}$, 1cm

EXERCISE – III

Q.1 B Q.2 (i) zero , (ii) can't be determined, (iii) can't be determined Q.3 B
Q.4 (b) a = 3/5 m/s^2 , T = 18 N, F = 60N Q.5 C Q.6 D Q.7 A
Q.8 11.313 m Q.9 C Q.10 10 m/s^2

(CIRCULAR MOTION & WORK POWER ENERGY)

EXERCISE – I

Q.1 $\tan^{-1}\left\{\frac{l\sin\theta}{2h}\right\}$ Q.2 $\frac{15\sqrt{3}}{2}$ N Q.3 2 tanθ Q.4 – 1 J

Q.5 $\frac{2\sqrt{2}\,v^2}{\pi R}$ Q.6 $\frac{2m_1^{\,2}g^2}{k}$ Q.7 $\frac{1}{6}mv^2$ Q.8 $9\sqrt{10}$ m/s²

Q.9 $4\sqrt{5}$ m/s Q.10 9 Q.11 $-\,ka^2/4$ Q.12 $\left(1-\sqrt{3}/2\right)mg$

Q.13 80 kW Q.14 $\sqrt{2g}$ rad/s Q.15 $\frac{1}{3}\sqrt{\frac{gl}{3}}$ Q.16 6mg

Q.17 9*a*/2 Q.18 $\frac{7}{12}$m Q.19 2 sec Q.20 625 J

Q.21 $-\,3\hat{k}$ rad/s², $-\,2\hat{k}$ rad/s

Q.22 (a) 2 rad/s², (b) 12+2t for $t \le 2$s, 16 for $t \ge 2$s, (c) $\sqrt{28565} \simeq 169$, 256 m/s² (d) 44 rad

Q.23 0.1875 Q.24 P Q.25 $\frac{25}{24}$

EXERCISE – II

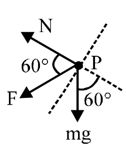
Q.1 $F = -3ax^2 + b$, $x = \sqrt{\frac{b}{a}}$, $KE_{max} = \frac{2b}{3\sqrt{3}}\sqrt{\frac{b}{a}}$ Q.2 2 m/s Q.3 $v = v_0$, $5\pi a/v_0$

Q.4 500N/m Q.6 $u = v\sec\theta$, $v = \frac{40}{\sqrt{41}}$ m/s

Q.7 $a_N = \frac{vR}{(2Rt - vt^2)}$, $a_t = \frac{R(vt-R)v^{1/2}}{(2Rt-vt^2)^{3/2}}$ Q.8 up, 10cm

Q.9 $\theta_{max}=\pi/2$, T=mg(3sinθ+3cosθ–2) Q.10 4, $\sqrt{gl/8}$

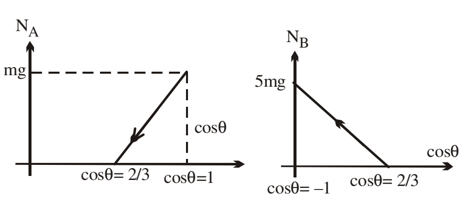
EXERCISE – III

Q.1 

, $a_t = 5\sqrt{3}\,g/8$, N=3mg/8 Q.2 (i) 36N, (ii) 11.66rad/sec ,(iii) 0.1m, 0.2m

Q.3 F=-8mgi-mgj, h=3R Q.4 C Q.5 D Q.6 $u = \sqrt{gL\left(\frac{3\sqrt{3}}{2}+2\right)}$ Q.7 A

Q.8 A Q.9 A Q.10 5.79 m/s Q.11 C Q.12 D Q.13 B

Q.14 (a) N=3mg cosθ – 2mg, (b) 

Q.15 A

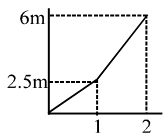
Q.16 $\vec{P}_{PM} = m\,\vec{v}_{PM} = -mv_2 \sin\omega t\,\hat{i} + m(v_2\cos\omega t - v_1)\,\hat{j}$ Q.17 A

(CENTRE OF MASS & MOMENTUM)

EXERCISE – I

Q.1 $\dfrac{5v}{R\cos\theta}$ Q.2 (a) $\dfrac{mg(h_3-h_2)}{(h_2-h_1)}$; (b) 0 Q.3 $\dfrac{[mu\sqrt{3}]}{4}$ Q.4 11/14

Q.5 $\cos^{-1}(3/4)$ Q.6 100 m Q.7 $\dfrac{5h}{16}$ Q.8 4/3 m/s

Q.9 L/4 Q.10 

Q.11 $\dfrac{2nv\cos(\theta/2)}{n+1}$ Q.12 $g/2$ Q.13 $\dfrac{R}{4\pi-2}$ Q.14 2 : 1

Q.15 l Q.16 $\dfrac{1}{2}$ Q.17 $\dfrac{Mu^2}{2g(M+2m)}$ Q.18 x = 6m Q.19 6.21 W

Q.20 $v_{\text{heavy ball}} = \dfrac{u_0}{27}$, $v_{\text{first ball}} = \dfrac{4u_0}{27}$, $v_{\text{second ball}} = \dfrac{4u_0}{9}$, $v_{\text{third ball}} = \dfrac{4u_0}{3}$ Q.21 $\sqrt{\dfrac{3gx}{8}}$

Q.22 $\sqrt{\dfrac{mv^2}{12k}}$

EXERCISE – II

Q.1 x = 3 units, $\tan\theta = 2/3$ Q.2 $5/\sqrt{17}$ cm, 153L/80u

Q.3 $v_c = -\dfrac{v_0}{15}$, $v_B = \dfrac{\sqrt{208}\,v_0}{15}$, $v_A = \dfrac{4\,v_0}{15}$ Q.4 40 cm

Q.5 37^0 Q.6 (a) 0.66, (b) 4 m

Q.7 impulse = $m(-3\hat{i}+4\hat{j})$, $e = \dfrac{9}{16}$ Q.8 $v_1 = \dfrac{1}{\sqrt{3}}$ m/s, $v_2 = \dfrac{2}{\sqrt{3}}$ m/s

Q.9 (a) $\dfrac{m}{l}(gy+v_0^2)$, (b) $mg\left(1-\dfrac{y}{l}\right)$, (c) $\dfrac{mv_0^{\,2}y}{2l}$ Q.10 $2\sqrt{2a_0 l}$

EXERCISE – III

Q.1 (i), (iii) Q.2 4.13×10^{-2}m Q.3 (L + 2R, 0) Q.4 4/3 m/s, 35/3 J

Q.5 A Q.6 2.5 m, 0, 319 m Q.7 90^o

Q.8 $e = \dfrac{5+\sqrt{3}}{8}$, $M = \dfrac{26}{\sqrt{3}}$ kg, Q.9 2 in case I Q.10 C Q.11 C

Q.12 $t_0 = 12$ sec, $v = \dfrac{100\sqrt{3}}{11}$ Q.13 C Q.14 10 m/s

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

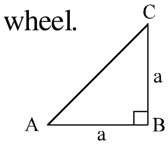
TOPIC: XI P7. Rotational dynamics

Index:

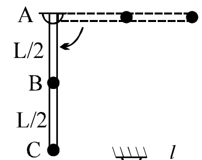
EXERCISE–I

Q.1 A wheel, of radius 1 m, is rolling purely on a flat, horizontal surface. It's centre is moving with a constant horizontal acceleration = 3 m/s^2. At a moment when the centre of the wheel has a velocity 3 m/s, then find the acceleration of a point 1/3 m vertically above the centre of the wheel.

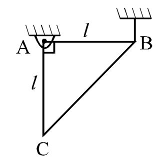
Q.2 A rigid body in shape of a triangle has $v_A = 5$ m/s $\downarrow$, $v_B = 10$ m/s $\downarrow$. Find velocity of point C.



Q.3 Two masses each of m are attached at mid point B & end point C of massless rod AC which is hinged at A. It is released from horizontal position as shown. Find the force at hinge A when rod becomes vertical

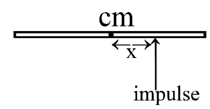


Q.4 An isosceles right triangular plate ABC of mass m is free to rotate in vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. Find the reaction at the support A.



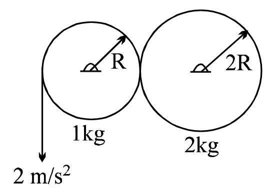
Q.5 A particle of mass m is projected with a velocity u at an angle of θ with horizontal. Find the intial angular momentum of the particle about the highest point of its trajectory.

Q.6 A uniform rod of length l is given an impulse at right angles to its length as shown. Find the distance of instantaneous centre of rotation from the centre of the rod.

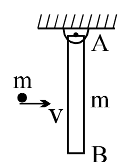


Q.7 A particle of mass 1 kg is moving with constant velocity of 10 m/s along the straight line $y = 7x + 4$. Find the angular momentum of the particle with respect to the point (3,4).

Q.8 Two discs A and B touch each other as in figure. A rope tightly wound on A is pulled down at 2 m/s^2. Find the friction force between A and B if slipping is absent



Q.9 A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle is brought to rest after the impact. Then find the impulsive reaction at A.



Q.10 A solid cylinder is released from rest from the top of an inclined plane of inclination 60° where friction coefficient varies with distance x as $\mu = \dfrac{2-3x}{\sqrt{3}}$. Find the distance travelled by the cylinder on incline before it starts slipping.

Q.11 Two men, each of mass 75 kg, stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned through by the disc with respect to the ground.

Q.12 A solid sphere of radius 3R, a solid disc of radius 2R and a ring of radius R (all are of mass m) roll down a rough inclined plane. Their acclerations are *a,b* and *c* respectively. Find the ratio of a/b and b/c.

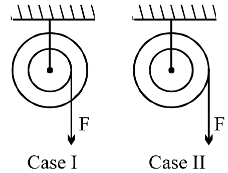
Q.13 A uniform disc of radius 1m and mass 2kg is mounted on an axle supported on fixed frictionless bearings. A light cord is wrapped around the rim of the disc and a mass of 1kg is tied to the free end. If it is released from rest, then find the tension in the cord.

Q.14 A uniform disc of mass m and radius R rotates about a fixed vertical axis passing through its centre with angular velocity ω. A particle of same mass m and having velocity 2ωR towards centre of the disc collides with the disc moving horizontally and sticks to its rim. Find

(a) the angular velocity of the disc. (b) the impulse on the particle due to disc.

(c) the impulse on the disc due to hinge.

Q.15 A person pulls along a rope wound up around a pulley with a constant force F for a time interval of t seconds. If a and b are the radii of the inner and the outer circumference ($a < b$), then find the ratio of work done by the person in the two cases shown in the figure is W_1/W_2.



Q.16 A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height $h = 4R/5$ above the centre line. If I is the impulse of the blow then find
(a) the minimum time after which the highest point B will touch the ground
(b) the displacement of the centre of mass during this internal.

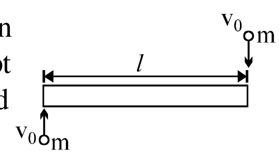
Q.17 A uniform ball of radius R rolls without slipping between two rails such that the horizontal distance is d between two contact points of the rail to the ball. If R=10cm, d=16cm and the angular velocity is 5rad/s then find the velocity of centre of mass of the ball.

Q.18 A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x-y plane) with its axis fixed along the y axis and free to rotate about its axis . The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and platform. Find the maximum torque acting on the cylinder during its motion.

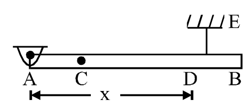
Q.19 The door of an automobile is open and perpendicular to the body. The automobile starts with an acceleration of 2 ft/sec^2, and the width of the door is 30 inches. Treat the door as a uniform rectangle, and neglect friction to find the speed of its outside edge as seen by the driver when the door closes.

Q.20 A bit of mud stuck to a bicycle's front wheel of radius r detaches and is flung horizontally forward when it is at the top of the wheel. The bicycle is moving forward at a speed v and it is rolling without slipping. Find the horizontal distance travelled by the mud after detaching from the wheel.

Q.21 On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length l, kept perpendicular to their velocity. The particles stick to the rod after the collision.Find the tension in rod during subsequent motion.



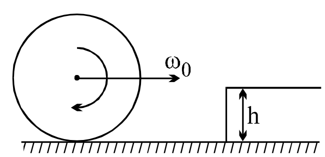
Q.22 A slender bar AB is supported in a horizontal position as in figure. At what distance x from the hinge A should the vertical string DE be attached to the bar in order that, when it is cut, there will be no immediate change in the reaction at A.



Q.23 A solid spherical ball which rests in equilibrium at the interior bottom of a fixed spherical globe is perfectly rough. the ball is struck a horizontal blow of such magnitude that the initial speed of its centre is v. Prove that, if v lies between $(10\,dg/7)^{1/2}$ and $(27\,dg/7)^{1/2}$, the ball will leave the globe, d being the difference between the radii of the ball and globe.

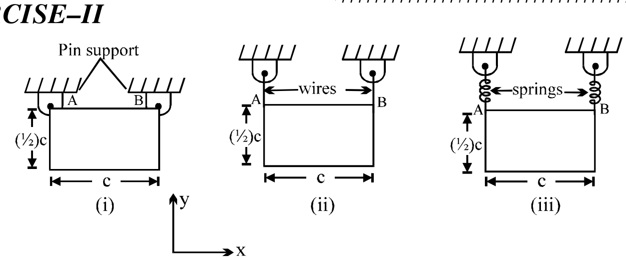
Q.24 A force of constant magnitude F starts acting on a uniform rod AB in gravity free space at the end A of the rod. The force always remains perpendicular to the rod, even as it moves. The mass of the rod is M and its length L. Then, find the value of the dot product $\vec{F}.\vec{a}_A$ at any later time(where $\vec{a}_A$ is acceleration of point A.)

Q.25 A solid uniform sphere of radius R and mass M rolls without slipping with angular velocity ω_0 when it encounters a step of height 0.4 R. Find the angular velocity immediately after inelastic impact with the rough step.



EXERCISE–II

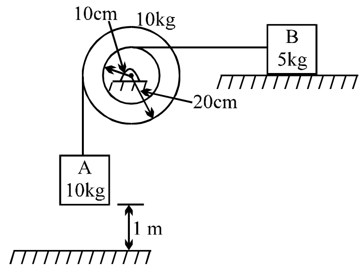
Q.1 A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;
(a) the angular acceleration of the plate.
(b) the acceleration of its mass center.



Q.2 The disk shown has weight 10 kg. Cylinder A and block B are attached to cords that are wrapped on the pulley as shown. The coefficient of kinetic friction between block B and the surface is 0.25. Knowing that the system is released from rest in the position shown, determine

(a) the velocity of cylinder A as it strikes the ground.

(b) the total distance that block B moves before coming to rest.

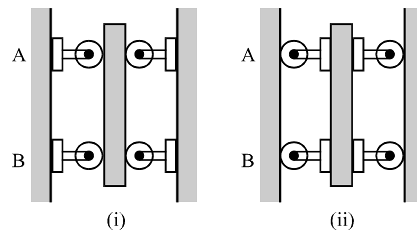


Q.3 A bar of mass m is held as shown between 4 disks, each of mass m' & radius $r = 75$ mm Determine the acceleration of the bar immediately after it has been released from rest, knowing that the normal forces exerted on the disks are sufficient to prevent any slipping and assuming that ;

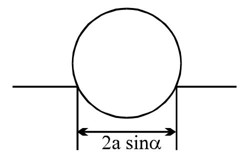
(a) $m = 5$ kg and $m' = 2$ kg .

(b) the mass m' of the disks is negligible.

(c) the mass m of the bar is negligible.

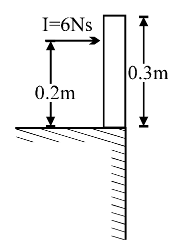


Q.4 Show that, if a uniform heavy right circular cylinder of radius a be rotated about its axis, and laid gently on two rough horizontal rails at the same level and distant $2a \sin\alpha$ apart so that the axis of the cylinder is parallel to the rails, the cylinder will remain in contact with both rails if the coefficient of friction $\mu < \tan \alpha$, but will initially rise on one rail if $\mu > \tan\alpha$.



Q.5 A diwali cracker known as **sudarashan chakra** works on the principle of thrust . Consider such a toy the centre of which is hinged.The initial mass of the toy is M_0 and radius is R . The toy is in the shape of a spiral the turns of which are very close (it can be assumed as a disc).The gases are ejected tangentially from the end of the toy with a constant velocity u relative to the toy. Find the angular velocity of the toy when mass remains half.

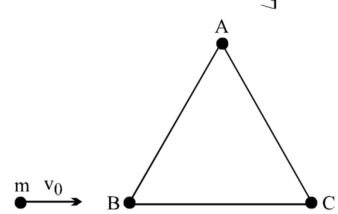
Q.6 A uniform thin rod with a mass $M = 0.60$ kg and a length of 0.30 m stands on the edge of a frictionless table as shown in the figure. The rod is struck , a horizontal impulse blow at a point 0.20 m above the table top, driving the rod directly off the table. Determine the orientation of the rod and the position of its C.M. 1 s after the blow is struck. ($g = 9.8$ m/s^2)



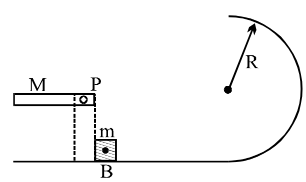
Q.7 Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact .

(i) Calculate the time required by the triangle ABC to complete half-revolution in its subsequent motion.

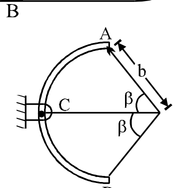
(ii) What is the net displacement of point B during this interval ?



Q.8 A rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure . First it is taken aside such that it becomes horizontal and then released . At the lowest point the rod hits the block B of mass m and stops . Find the ratio of masses such that the block B completes the circle. Neglect any friction.



Q.9 A uniform rod AB is bent in the shape of an arc of circle. Determine the angular acceleration of the rod immediately after it is released from rest and show that it is independent of β.

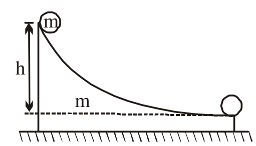


Q.10 Assume that the centre of mass of a girl crouching in a light swing has been raised to 1.2m. The girl has her centre of mass is 3.7m from the pivot of the swing while she is in the crouched position. The swing is released from rest and at the bottom of the arc the girl stands up instantaneously, thus raising her centre of mass 0.6m. Find the height of her centre of mass at the top of the arc.

Q.11 A uniform ball of radius R rolls without slipping between two rails such that the horizontal distance is d between the two contact points of the rail to the ball. (a) Show that at any instant, velocity of centre of mass is given as : $v_{cm} = \omega\sqrt{R^2 - \frac{d^2}{4}}$

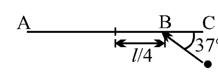
Discuss the above expression in the limits d=0 and d=2R. (b) For a uniform ball starting from rest and decending a vertical distance h while rolling without slipping down a ramp, $v_{cm} = \sqrt{\frac{10gh}{7}}$, if the ramp is replaced with two rails, show that : $v_{cm} = \sqrt{\dfrac{10gh}{5+\dfrac{2}{1-d^2/4R^2}}}$.

Q.12 A hollow sphere is released from the top of a movable wedge as shown in the figure. There is no friction between the wedge and the ground. There is sufficient friction between sphere and wedge to provide pure rolling of sphere. Find the velocity of centre of sphere w.r.t. ground just before it leaves the wedge horizontally.(Assume masses of the wedge and sphere are equal & h >> R the radius of sphere)



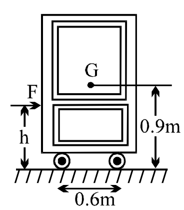
Q.13 A small ring of mass *m* is threaded on a horizontal smooth rod which is rotating about its end with constant angular velocity ω. The ring is initially located at the axis of rotation. When the distance of the ring from the axis becomes *r*, then find the power required to rotate the system with same angular velocity.

Q.14 A rod AC of length L and mass *m* is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass *m* moving with velocity *v* strikes rod at point B which is at a distance L/4 from mid point making angle 37^0 with the rod. The collision is elastic. After collision find



(a) the angular velocity of the rod.
(b) the distance which centre of the rod will travel in the time in which it makes half rotation.
(c) the impulse of the impact force.

Q.15 A 20 kg cabinet is mounted on small casters that allow it to move freely ($\mu = 0$) on the floor. If a 100 N force is applied as shown, determine
(a) the acceleration of the cabinet,
(b) the range of values of h for which the cabinet will not tip.



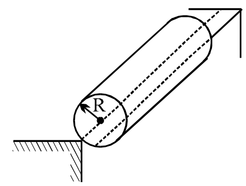
<u>List of recommended questions from I.E. Irodov.</u>
1.51 to 1.54, 1.58, 1.187, 1.188, 1.190, 1.192, 1.193, 1.199,
1.234, 1.237, 1.241, 1.245, 1.248, 1.249, 1.251, 1.255 to 1.264,
1.266, 1.270 to 1.279

EXERCISE–III

Q.1 Two point masses of 0.3kg and 0.7kg are fixed at the ends of a rod which is of length 1.4m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum is located at a distance of [JEE' 95]

(A) 0.42 from the mass of 0.3kg (B) 0.70 m from the mass of 0.7kg
(C) 0.98m from the mass of 0.3kg (D) 0.98m from the mass of 0.7kg

Q.2 A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shwon in figure. there is sufficient friction present at the edge so that a very small displacement cause the cylinder to roll of the edge without slipping. Determine



(a) the angle θ_C through which the cylinder rotates before it leaves contact with the edge,
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and
(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge. [JEE' 95]

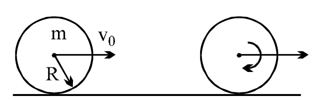
Q.3 A mass m moving with a constant velocity along a line parallel to the x axis, away from the origin. Its angular momentum with respect to the origin. [JEE' 97]

(A) is zero (B) remains constant
(C) goes on increasing (D) goes on decreasing

Q.4 A uniform disk of mass m and radius R is rolling up a rough inclined plane which makes an angle of 30° with the horizontal. If the coefficient of static and kinetic friction are each equal to μ and the only forces acting are gravitational and frictional, then the magnitude of the frictional force acting on the disk is ____ and its direction is ____ (write up or down) the inclined plane. [JEE' 97]

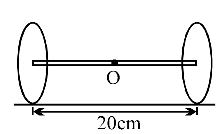
Q.5 A uniform disk of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t=0. After t_0 seconds it acquires a purely rolling motion as shown in figure.

(i) Calculate the velocity of the centre of mass of the disk at t_0.
(ii) Assuming the coefficient of friction to be μ calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0. [JEE' 97]



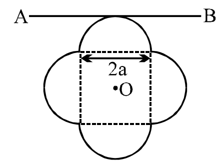
Q.6 Two thin circular disks of mass 2kg and radius 10cm each are joined by a rigid massless rod of length 20cm. The axis of the rod is along the perpendicular to the planes of the disks through their centre. The object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x-axis as the direction of motion of the truck and z -axis as the vertically upwards direction. If the truck has an acceleration of $9 m/s^2$ calculate :

(a) the force of friction on each disk
(b) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form of unit vectors in the x-y and z directions. [JEE' 97]



Q.7 A rod of weight w is supported by two parallel knife edges A and B is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A. The normal reaction on A is ___ and on B is ____ [JEE' 97]

Q.8 A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as in fig. The side of the square is 2a. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is 1.6Ma^2. The moment of inertia of the lamina about the tangent AB in the plane of lamina is ______ [JEE' 97]



Q.9 Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) I $\sin^2\theta$ (C) I$\cos^2\theta$ (D) I$\cos^2(\theta/2)$ [JEE' 98]

Q.10 The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that [JEE' 98]

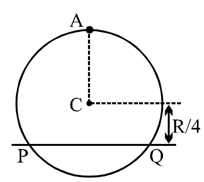
(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time

(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time

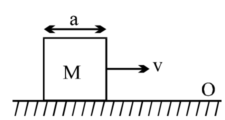
(C) the magnitude of $\vec{L}$ does not change with time

(D) $\vec{L}$ does not change with time

Q.11 A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the wedge of the disc as in fig. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position. [JEE' 98]



Q.12 A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is:



(A) 3v/4a (B) 3v/2a

(C) $\sqrt{3v}/\sqrt{2a}$ (D) zero [JEE' 99]

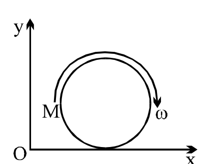
Q.13 A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angularspeeds are ω_A and ω_B, respectively. Then [JEE' 99]

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$ (C) $\omega_A = \omega$ (D) $\omega_B = \omega$

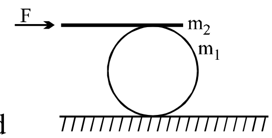
Q.14 A disc of mass M and radius R is rolling with angular speed w on a horizontal as shown. The magnitude of angular momentum of the disc about the origin O is: [JEE' 99]

(A) (1/2)MR$^2\omega$ (B) MR$^2\omega$

(C) (3/2)MR$^2\omega$ (D) 2MR$^2\omega$



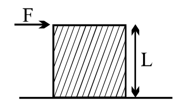
Q.15 A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find



(a) the accelerations of the plank and the center of mass of the cylinder, and

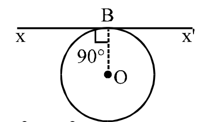
(b) the magnitudes and directions of frictional forces at contact points. [JEE' 99]

Q.16 A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of frictionis sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is: [JEE'(Scr)'2000]



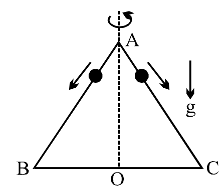
(A) infinitesimal (B) mg/4 (C) mg/2 (D) mg(1-μ)

Q.17 A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is: [JEE'(Scr)'2000]



(A) $\rho L^3/8\pi^2$ (B) $\rho L^3/16\pi^2$ (C) $5\rho L^3/16\pi^2$ (D) $3\rho L^3/8\pi^2$

Q.18 An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at AO. Then, the beads are released from rest simultaneously and allowed to slide down, one along AB and the other AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are: [JEE'(Scr)'2000]



(A) angula velocity and total energty (kinetic and potential)
(B) total angular momentum and total energy
(C) angular velocity and moment of inertia about the axis of rotaiton.
(D) total angular momentum and moment of inertia about the axis of rotation.

Q.19 A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

(a) Find the ratio m/M .

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision. [JEE' 2000]

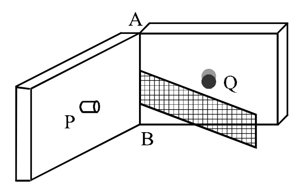
Q.20 Two heavy metallic plates are joined together at 90^0 to each other. A laminar sheet of mass 30 Kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 Kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N–s.

(a) Find the location of the centre of mass of the laminar sheet from AB.

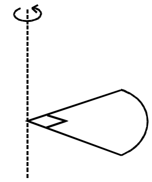
(b) At what angular velocity does the laminar sheet come back after the first impact ?

(c) After how many impacts, does the laminar sheet come to rest ?

[JEE' 2001]

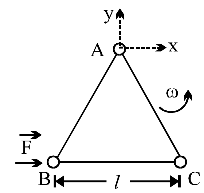


Q.21 One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is [JEE'(Scr)2001]



(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}\,MR^2$

Q.22 Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.



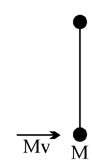
(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exterted by the hinge on the body, immediately after time T. [JEE' 2002]

Q.23 A particle is moving in a horizontal uniform circular motion. The angular momentum of the particle is conserved about the point : [JEE'(Scr)2003]

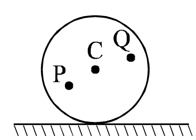
(A) Centre of the circle (B) Outside the circle
(C) Inside the circle (D) Point on circumference

Q.24 Two particles each of mass M are connected by a massless rod of length l. The rod is lying on the smooth sufrace. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be : [JEE'(Scr)2003]



(A) v/l (B) $2v/l$ (C) $v/2l$ (D) None

Q.25 A disc is rolling (without slipping) on a horizontal surface. C is its center and Q and P are two points equidistant from C. Let V_P, V_Q and V_C be the magnitude of velocities of points P, Q and C respectively, then [JEE' 2004 (Scr)]



(A) $V_Q > V_C > V_P$ (B) $V_Q < V_C < V_P$

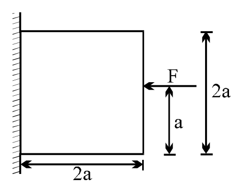
(C) $V_Q = V_P$, $V_C = \frac{1}{2}V_P$ (D) $V_Q < V_C > V_P$

Q.26 A child is standing with folded hands at the center of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is [JEE' 2004 (Scr)]

(A) 2K (B) $\frac{K}{2}$ (C) $\frac{K}{4}$ (D) 4K

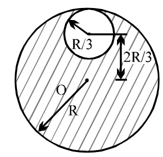
Q.27 A block of mass m is held fixed against a wall by a applying a horizontal force F. Which of the following option is incorrect:

(A) friction force = mg (B) F will not produce torque
(C) normal will not produce torque (D) normal reaction = F



[JEE'2005 (Scr)]

Q.28 A disc has mass 9m. A hole of radius $\frac{R}{3}$ is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is:



(A) $8\ mR^2$ (B) $4\ mR^2$ (C) $\frac{40}{9}\ mR^2$ (D) $\frac{37}{9}\ mR^2$

[JEE'2005 (Scr)]

Q.29 A particle moves in circular path with decreasing speed. Which of the following is correct

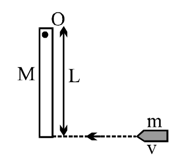
(A) $\vec{L}$ is constant (B) only direction of $\vec{L}$ is constant

(C) acceleration $\vec{a}$ is towards the centre (D) it will move in a spiral and finally reach the centre

[JEE'2005 (Scr)]

Q.30 A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O.

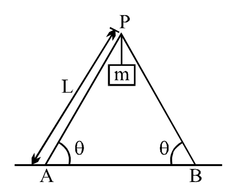
[JEE 2005]



Q.31 A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. [JEE 2005]

Q.32 Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.

[JEE 2005]

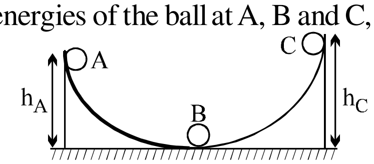


Q.33 A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(A) $\frac{2R}{\sqrt{15}}$ (B) $R\sqrt{\frac{2}{15}}$ (C) $\frac{4R}{\sqrt{15}}$ (D) $\frac{R}{4}$ [JEE 2006]

Q.34 A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and incline is μ. Then
(A) frictional force is always μmg cos θ.
(B) friction is a dissipative force
(C) by decreasing θ, frictional force decreases
(D) friction opposes translation and supports rotation. [JEE 2006]

Q.35 A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then
(A) $h_A > h_C$; $K_B > K_C$
(B) $h_A > h_C$; $K_C > K_A$
(C) $h_A = h_C$; $K_B = K_C$
(D) $h_A < h_C$; $K_B > K_C$



[JEE 2006]

ANSWER KEY

EXERCISE–I

Q.1 $5\ m/s^2$ Q.2 $5\sqrt{5}$ m/s Q.3 $\frac{28mg}{5}$ Q.4 2mg/3

Q.5 $\frac{mu^3 \sin^2\theta\cos\theta}{2g}$ Q.6 $\frac{l^2}{12x}$ Q.7 $21\sqrt{2}$ kg m^2/s, remains constant

Q.8 2 N Q.9 $\frac{mv}{4}$ Q.10 $\frac{1}{3}$ m Q.11 $\frac{4\pi}{5}$

Q.12 $\frac{a}{b} = \frac{15}{14}$ *and* $\frac{b}{c} = \frac{4}{3}$ Q.13 5 N

Q.14 (a) $\omega/3$, (b) $\frac{\sqrt{37}}{3}m\omega R$, (c) $\frac{\sqrt{37}}{3}m\omega R$ Q.15 a^2/b^2

Q.16 (a) $t = \frac{\pi R m}{2I}$; (b) $s = \frac{\pi R}{2}$ Q.17 0.3 m/s Q.18 $(1/2)MRA\,\omega^2$

Q.19 $\sqrt{15}$ ft/sec Q.20 $\sqrt{16rv^2/g}$ Q.21 $\frac{2mv_0^{\ 2}}{l}$

Q.22 $2l/3$ Q.24 $4F^2/M$ Q.25 $\frac{5\omega_0}{7}$

EXERCISE–II

Q.1 (i)(a) $\frac{1.2g}{c}$ (cw) (b) $-03.\left(\hat{i} + 2\hat{j}\right)g$ (ii) (a) $24g/17c$ (cw) (b) $12g/17 \downarrow$ (iii) (a) $2.4g/c$(cw) (b) $0.5g \downarrow$

Q.2 (a) $\sqrt{\frac{150}{13}}$ m/s, (b) $\frac{44}{13}$ m Q.3 (i) (a) $5g/9 \downarrow$ (b) $g \downarrow$ (c) 0 (ii) (a) $\frac{13g}{17} \downarrow$ (b) $g \downarrow$ (c) $\frac{2g}{3} \downarrow$

Q.5 $w = \frac{4u}{R}\left(\sqrt{2} - 1\right)$

Q.6 [10, –4.75 m] w.r.t. initial position of lower end of rod, $\frac{200}{3}$ rad with upward vertical

Q.7 (i) $t = \frac{6a\pi}{\sqrt{3}v_0}$; (ii) $s = \frac{a}{\sqrt{3}}\sqrt{1 + \left(2\pi + \sqrt{3}\right)^2}$ Q.8 $\frac{M}{m} = \sqrt{15}$ Q.9 g/2b

Q.10 2.3 m Q.12 $\sqrt{\frac{3}{7}gh}$ Q.13 $2mw^3r^2$

Q.14 (a) $\frac{72}{55}\frac{v}{\ell}$, (b) $\frac{\pi\ell}{3}$, (c) $\frac{24mV}{55}$ Q.15 (a) $5\ m/s^2 \rightarrow$, (b) $0.3 < h < 1.5$ m

EXERCISE–III

Q.1 C

Q.2 (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{4/7\ gR}$, (c) $K_T/K_R = 6$

Q.3 B

Q.4 $Mg\sin\theta/3$, up

Q.5 (i) $2v_0/3$, (ii) $t = v_0/3\mu g$, $W = \frac{1}{2}[3\mu mg^2t^2 - 2\mu m g t v_0]$ $(t < t_0)$, $W = -\frac{1}{6}mv_0^2$ $(t > t_0)$

Q.6. 6N, $-0.6\hat{j} \pm 0.6\hat{k}$

Q.7 $w(d-x)/d$, wx/d

Q.8 $4.8Ma^2$

Q.9 A

Q.10 A, B, C

Q.11 $v = \sqrt{5gR}$

Q.12 A

Q.13 C

Q.14 C

Q.15 $a_c = \frac{4F}{(3m_1+8m_2)}$, $a_p = \frac{8F}{(3m_1+8m_2)}$; $f_1 = \frac{3m_1F}{(3m_1+8m_2)}$, $f_2 = \frac{m_1F}{(3m_1+8m_2)}$

Q.16 C

Q.17 D

Q.18 B

Q.19 (a) $\frac{m}{M} = \frac{1}{4}$; (b) $x = \frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$

Q.20 (a) $l = 0.1$m; (b) $w' = 1$rad/s; (c) laminar sheet will never come to rest

Q.21 A

Q.22 (a) $\sqrt{3}\, m\,\omega^2 l$, (b) $F_x = F/4$, $F_y = \sqrt{3}\, m\,\omega^2 l$

Q.23 A

Q.24 A

Q.25 A

Q.26 B

Q.27 C

Q.28 B

Q.29 B

Q.30 $\omega = \frac{3mv}{(3m+M)L}$

Q.31 $a_{axis} = \frac{2g\sin\theta}{3}$

Q.32 $f = (M+m)\,g\frac{\cot\theta}{2}$

Q.33 A

Q.34 C,D

Q.35 A,B

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P8. Fluid Mechanics

Index:

KEY CONCEPTS

PART (A) - HYDROSTATICS

1. DENSITY AND RELATIVE DENSITY :

Density is mass per unit volume. Densities of solids and liquids are frequently compared with density of water and the relative density of a substance is defined as follows :

Relative Density with respect to water $= \dfrac{\text{density of the substance}}{\text{density of water}}$

$= \dfrac{\text{mass of any volume of substance}}{\text{mass of an equal volume of water}}$

(Also known as specific gravity of the substance) .

Note that relative density being a ratio of same type of physical quantities, it is a number only , without any physical dimension .

2. PRINCIPLE OF ARCHIMEDES :

The principle of Archimedes states that any body , totally or partially immersed in a fluid , experiences an upward force or thrust which is equal to the weight of fluid it displaces and acts vertically up through the C. G. of the displaced fluid . The term fluid covers liquids and gases .

3. LAW OF FLOTATION :

If a body floats in equilibrium in a fluid , its entire weight is supported by the upward thrust of the fluid . Hence , the weight of a floating body is equal to the weight of the fluid displaced by the body at the equilibrium state .

4. FLUID PRESSURE :

Pressure at any point in a fluid is defined as the normal force (or thrust) exerted by the liquid on the surface per unit area .

Pressure is measured in dyne cm^{-2} in C G S units and in N m^{-2} in SI units , (also known as pascal) .

[Note that pressure is a scalar quantity] .

When a plane surface is placed inside a liquid , the liquid exerts hydrostatic pressure on the surface , because of the weight of the liquid column above the surface . The total force exerted normally on the plane surface is called the thrust. The thrust over the surface is the vector sum of the thrusts over small area of the surface , over which the pressure can be considered to be uniform. Then :

(i) If the plane surface is horizontal, the pressure over the surface is uniform and the thrust = (area) × (the uniform pressure)

(ii) If the plane surface is rectangular with its plane vertical and a pair of sides horizontal , the thrust = (area) × (pressure at the centre of the area).

5. PRESSURE IN LIQUIDS :

The hydrostatics pressure 'p' at any point in a liquid varies directly.

(i) As the vertical height (h) of the point below the surface &

(ii) As the density (d) of the liquid.

It can be shown that $p = hdg$.

When a liquid is at rest, the pressure is same at all points at the same horizontal level . The pressure at a point in a liquid does not depend on either the shape of the vessel or the area of cross - section of the vessel.

6. **PASCAL 'S LAW :**

Pressure applied to a liquid (at rest) at one point is transmitted equally in all directions throughout the liquid . This is known as Pascal's Law .

The compressibility of all liquids is exceedingly small & for all practical purposes, liquids may be considered incompressible . Hence , the density is constant throughout the liquid .

Pressure at a point in a liquid is the same in all directions and is perpendicular to the surface upon which its acts .

7. **GUAGE PRESSURE AND TOTAL PRESSURE (ABSOLUTE PRESSURE) :**

The total pressure at any point in a liquid is the addition of :

(i) The pressure due to the liquid above the point. It is called guage pressure &

(ii) The atmospheric pressure acting on the surface of the liquid.

Thus total pressure = pressure due to liquid + atmospheric pressure

The total pressure is called absolute pressure.

8. **THRUST DUE TO PRESSURE :**

Total thrust on a horizontal surface immersed in a liquid = **(PRESSURE ON SURFACE)** $\times$ **(AREA OF SURFACE)**

Total thrust on a vertical surface immersed in a liquid =**(PRESSURE AT C.G. OF AREA)** $\times$ **(AREA OF SURFACE)**

PART (B) - HYDRODYNAMICS

1. **The study of fluids in flow is called HYDRODYNAMICS.**

2. **VELOCITY - FIELD :**

It is that space, where at every point in that space, there is a definite velocity. Then the space, where a fluid is in flow is a **VELOCITY - FIELD**.

3. **FLOW - LINE :**

In a velocity field is an imaginary line in that space , where the tangent to the line at any point on the line gives the direction of the velocity at that point. A flow line is also called a **VELOCITY - LINE** or a **STREAM - LINE** .

4. **TYPES OF FLOW OF A FLUID :**

Then the flow of a fluid can be classified as :

(i) A **STREAM-LINE FLOW** , the stream lines in the flow space remains steady as time progresses.

(ii) A **TRUBULENT-FLOW** , the stream lines in the flow space shift their positions as time progresses.

In a **STREAM-LINE FLOW** , a group of stream lines form a tubular volume of the flow space, the surface of which is tangential to the stream lines , forming the lateral boundary of that tubular volume. Such a tubular volume in the flow space is a **TUBE OF FLOW**.

(iii) A steady state flow is the flow in which the fluid properties at any point in the velocity field do not change with time.

5. **EQUATION OF CONTINUITY :**

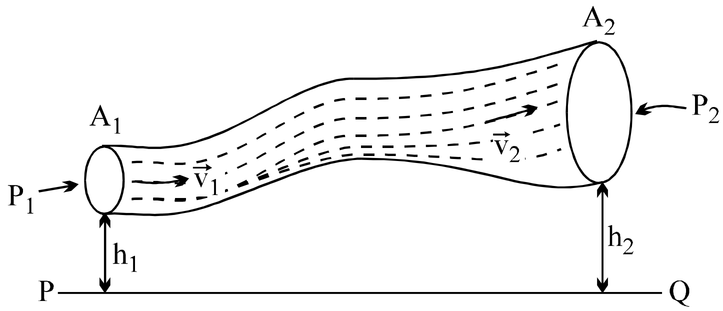
Equation of continuity states that for a steady state flow of a fluid in a pipe , the rate of mass flow across any cross section is constant .

$$\frac{dM}{dt} = \rho AV = \text{constant} .$$

If the fluid is incompressible density is constant at all points , hence , equation of continuity is $AV = \text{constant}$.

6. **BERNOULLI 'S EQUATION :**

Consider a tube of flow in the space of the stream line flow of a fluid, in a uniform gravitational field. The flow is steady state.



Then :

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

Generalising and removing suffixes $\frac{P}{\rho}+\frac{V_1^2}{2}+gz$ = constant .

This equation is called **BERNOULLI 'S EQUATION** for steady, non-viscous and incompressible fluid flow.

SURFACE TENSION

1. **Surface Tension :** Surface tension of a liquid is the normal force acting per unit length on either side of an imaginary line drawn the free surface of a liquid. The direction of this force is perpendicular to the line and tangential to the free surface of liquid.

$$T = \frac{F}{L}$$

Note: The surface tension of a liquid varies with temperature as well as dissolved impurities, etc. When soap mixed with water, the surface tension of water decrease.

2. **Surface Energy :** If the area of the liquid surface has to be increased work has to be done against the force of surface tension. The work done to form a film is stored as potential energy in the surface.

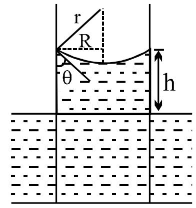
$$W = T\,\Delta A$$

3. **Excess Pressure :** Excess pressure inside a liquid drop

$$\Delta p = \frac{2T}{r}$$

For a soap bubble in air, there are two surfaces, and so,

$$\Delta p = 2 \times \frac{2T}{r} = \frac{4T}{r}$$



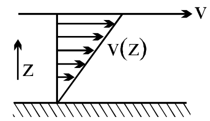
4. **Capillarity :** Water in the capillary rises to a height

$$h = \frac{2T}{rg\rho}$$

where r is the radius of meniscus, and $r = \frac{R}{\cos\theta}$ where θ is the angle of contact and thus $h = \frac{2T\cos\theta}{R\rho g}$

VISCOSITY

If a glass plate in contact with a water column of height h is moved with constant velocity v. Forces of viscosity appear between the solid surface and the layer in contact.



$$F = -\eta\, A \frac{dv}{dz}$$

where h is a consant called coefficient of viscosity, its cgs unit is poise.
Dimension is $ML^{1}T^{-1}$. The SI units of viscosity equal to 10 poise.

Stoke's Law and Terminal Velocity

When a sphere of radius r moves with a velocity v through a fluid of viscosity h, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta r v$$

If for a sphere viscous force become equal to the net weight acing downward, the velocity of the body become constant and is known as terminal velocity.

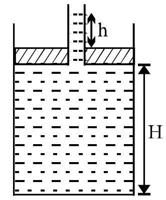
$$6\pi\eta r v_T = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$\Rightarrow \quad v_T = \frac{2}{9} r^2 \left\{\frac{\rho - \sigma}{\eta}\right\} g$$

EXERCISE # I

Q.1 A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

Q.2 A piston of mass M = 3kg and radius R = 4cm has a hole into which a thin pipe of radius r = 1cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750gm of water is now poured into the pipe so that the piston & pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in the pipe.



Q.3 A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enter water. Upto what depth will the ball go? How much time will it take to come again to the water surface? Neglect air resistance & velocity effects in water.

Q.4 Place a glass beaker, partially filled with water, in a sink. The beaker has a mass 390 gm and an interior volume of 500cm^3. You now start to fill the sink with water and you find, by experiment, that if the beaker is less than half full, it will float; but if it is more than half full, it remains on the bottom of the sink as the water rises to its rim. What is the density of the material of which the beaker is made?

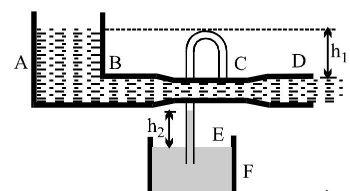
Q.5 Two spherical balls A and B made up of same material having masses 2m and m are released from rest. Ball B lies at a distance h below the water surface while A is at a height of 2h above water surface in the same vertical line, at the instant they are released.

(a) Obtain the position where they collide.

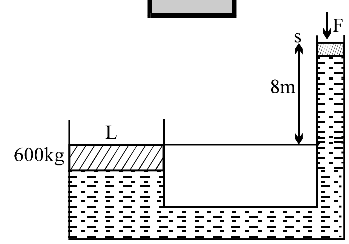
(b) If the bodies stick together due to collision, to what maximum height above water surface does the combined mass rise?

Specific gravity of the material of the balls is 2/3. Neglect viscosity and loss due to splash.

Q.6 Two very large open tanks A and F both contain the same liquid. A horizontal pipe BCD, having a constriction at C leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section at C is one half that at D and if D is at a distance h_1 below the level of liquid in A, to what height h_2 (in terms of h_1) will liquid rise in pipe E ?



Q.7 For the system shown in the figure, the cylinder on the left at L has a mass of 600kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25cm^2 and negligible weight. If the apparatus is filled with oil.(ρ = 0.75 gm/cm^3) Find the force F required to hold the system in equilibrium.

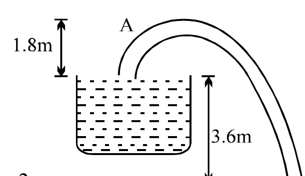


Q.8 A siphon has a uniform circular base of diameter $\dfrac{8}{\sqrt{\pi}}$ cm with its crest A 1.8 m above water level as in figure. Find

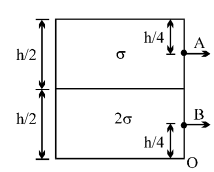
(a) velocity of flow

(b) discharge rate of the flow in m^3/sec.

(c) absolute pressure at the crest level A. [Use $P_0 = 10^5$ N/m^2 & g = 10m/s^2]



Q.9 A large tank is filled with two liquids of specific gravities 2σ and σ. Two holes are made on the wall of the tank as shown. Find the ratio of the distances from O of the points on the ground where the jets from holes A & B strike.

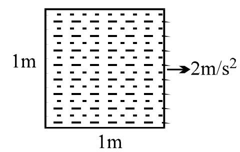


Q.10(a) A spherical tank of 1.2m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of $10\,m/s^2$. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

(b) The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water. Find the depth of the lake.

Q.11 A test tube of thin walls has some lead shots in it at its bottom and the system floats vertically in water, sinking by a length $l_o = 10\,cm$. A liquid of density less than that of water, is poured into the tube till the levels inside and outside the tube are even. If the tube now sinks to a length $l_o = 40\,cm$, the specific gravity of the liquid is ________ .

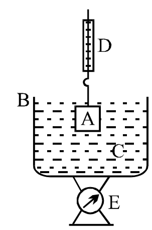
Q.12 An open cubical tank completely filled with water is kept on a horizontal surface. Its acceleration is then slowly increased to $2\,m/s^2$ as shown in the Fig. The side of the tank is 1m. Find the mass of water that would spill out of the tank.



Q.13 In air an object weighs 15N, when immersed completely in water the same object weighs 12N. When immersed in another liquid completely, it weighs 13N. Find
(a) the specific gravity of the object and
(b) the specific gravity of the other liquid.

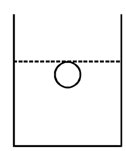
Q.14 Compute the work which must be performed to slowly pump the water out of a hemispherical reservoir of radius R = 0.6 m.

Q.15 Block A in figure hangs by a cord from spring balance D and is submerged in a liquid C contained in a beaker B. The mass of the beaker is 1kg & the mass of the liquid is 1.5 kg. The balance D reads 2.5 kg & balance E reads 7.5 kg.. The volume of block A is $0.003\,m^3$.
(i) What is the density of block & the liquid.
(ii) What will each balance read if block is pulled out of the liquid.



Q.16 A solid cube, with faces either vertical or horizontal, is floating in a liquid of density 6 g/cc. It has two third of its volume submerged. If enough water is added from the top so as to completely cover the cube, what fraction of its volume will remain immersed in the liquid?

Q.17 A ball is given velocity v_0 (greater than the terminal velocity v_T) in downward direction inside a highly viscous liquid placed inside a large container. The height of liquid in the container is H. The ball attains the terminal velocity just before striking at the bottom of the container. Draw graph between velocity of the ball and distance moved by the ball before getting terminal velocity.

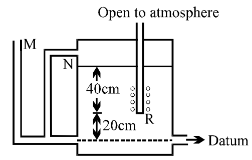


Q.18 Two arms of a U-tube have unequal diameters $d_1 = 1.0$ mm and $d_2 = 1.0$ cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.

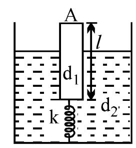
Q.19 A spherical ball of radius 1×10^{-4} m and density $10^4\,kg/m^3$ falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m².

Q.20 Calculate the rate of flow of glycerine of density 1.25×10^3 kg/m^3 through the conical section of a pipe if the radii of its ends are 0.1m & 0.04m and the pressure drop across its length is 10N/m^2.

Q.21 The tank in fig discharges water at constant rate for all water levels above the air inlet R. The height above datum to which water would rise in the manometer tubes M and N respectively are __________ & __________.



Q.22 A uniform cylindrical block of length l density d_1 and area of cross section A floats in a liquid of density d_2 contained in a vessel ($d_2 > d_1$). The bottom of the cylinder just rests on a spring of constant k. The other end of the spring is fixed to the bottom of the vessel. The weight that may be placed on top of the cylinder such that the cylinder is just submerged in the liquid is ____



Q.23 Find the speed of rotation of 1 m diameter tank, initially full of water such that water surface makes an angle of 45^0 with the horizontal at a radius of 30cm. What is the slope of the surface at the wall of the tank.

Q.24 A vertical uniform U tube open at both ends contains mercury. Water is poured in one limb until the level of mercury is depressed 2cm in that limb. What is the length of water column when this happens.

Q.25 An expansible balloon filled with air floats on the surface of a lake with 2/3 of its volume submerged. How deep must it be sunk in the water so that it is just in equilibrium neither sinking further nor rising ? It is assumed that the temperature of the water is constant & that the height of the water barometer is 9 meters.

List of recommended questions from I.E. Irodov.

Fluid Mechanics

1.315, 1.319 to 1.322, 1.324, 1.326, 1.327, 1.329

Surface Tension

2.161, 2.162, 2.167, 2.168, 2.169, 2.174, 2.176, 2.178, 2.181, 2.182

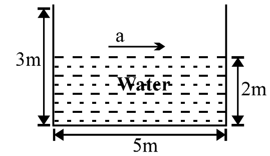
Viscosity

1.331, 1.332, 1.334, 1.336, 1.337, 1.339

EXERCISE # II

Q.1 A solid block of volume $V=10^{-3}m^3$ and density $d=800kg/m^3$ is tied to one end of a string, the other end of which is tied to the bottom of the vessel. The vessel contains 2 immiscible liquids of densities $\rho_1=1000kg/m^3$ and $\rho_2=1500kg/m^3$. The solid block is immersed with 2/5th of its volume in the liquid of higher density & 3/5th in the liquid of lower density. The vessel is placed in an elevator which is moving up with an acceleration of $a=g/2$. Find the tension in the string. $[g=10m/s^2]$

Q.2 An open rectangular tank 5m × 4m × 3m high containing water upto a height of 2m is accelerated horizontally along the longer side.

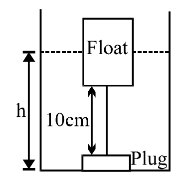


(a) Determine the maximum acceleration that can be given without spilling the water.

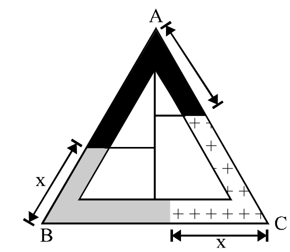
(b) Calculate the percentage of water split over, if this acceleration is increased by 20%.

(c) If initially, the tank is closed at the top and is accelerated horizontally by $9m/s^2$, find the gauge pressure at the bottom of the front and rear walls of the tank.

Q.3 A level controller is shown in the figure. It consists of a thin circular plug of diameter 10cm and a cylindrical float of diameter 20cm tied together with a light rigid rod of length 10cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3kg and the plug may be assumed as massless.

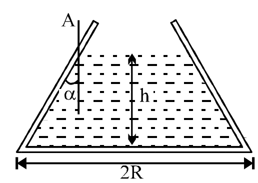


Q.4 A closed tube in the form of an equilateral triangle of side l contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find x in the figure if the densities of the liquids are in A.P.

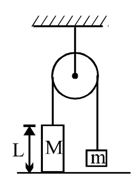


Q.5 A ship sailing from sea into a river sinks X mm and on discharging the cargo rises Y mm. On proceeding again into sea the ship rises by Z mm. Assuming ship sides to be vertical at water line, find the specific gravity of sea water.

Q.6 A conical vessel without a bottom stands on a table. A liquid is poured with the vessel & as soon as the level reaches h, the pressure of the liquid raises the vessel. The radius of the base of the vessel is R and half angle of the cone is α and the weight of the vessel is W. What is the density of the liquid ?



Q.7 As the arrangement shown in the fig is released the rod of mass M moves down into the water. Friction is negligible and the string is inextensible.



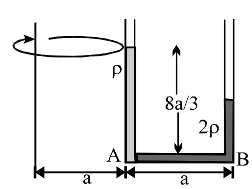
(a) Find the acceleration of the system w.r.t. the distance moved by each mass.

(b) Find the time required to completely immerse the rod into water

$$\text{if } \frac{m}{M} = \frac{\rho - \rho_{water}}{\rho} .$$

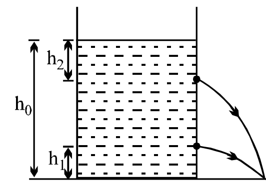
ρ = density of rod ; ρ_{water} = density of water

Q.8 The interface of two liquids of densities ρ and 2ρ respectively lies at the point A in a U tube at rest. The height of liquid column above A is 8a/3 where AB=a. The cross sectional area of the tube is S. With what angular velocity the tube must be whirled about a vertical axis at a distance 'a' such that the interface of the liquids shifts towards B by 2a/3.



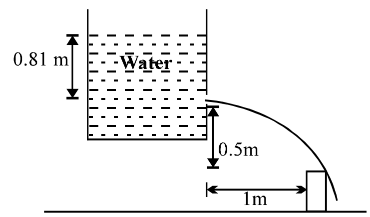
Q.9 A closed cylindrical tank 2m high & 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20.0 rad/s, how much of the bottom of the tank is uncovered? (The cylinder is rotated about vertical axis of symmetry passing through its length.)

Q.10 A cylinder of height H is filled with water to a height h_o ($h_o < H$), & is placed on a horizontal floor. Two small holes are punched at time t = 0 on the vertical line along the length of the cylinder, one at a height h_1 from the bottom & the other a depth h_2 below the level of water in the cylinder. Find the relation between h_1 & h_2 such that the instantaneous water jets emerging from the cylinder from the two holes will hit the ground at the same point.

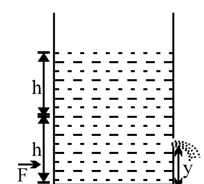


Q.11 A cylindrical tank with a height of h = 1m is filled with water up to its rim. What time is required to empty the tank through an orifice in its bottom? The cross sectional area of the orifice is (1/400)th of the tank. Find the time required for the same amount of water to flow out of the tank if the water level in the tank is maintained constant at a height of h = 1m from the orifice.

Q.12 For the arrangement shown in the figure. Find the time interval after which the water jet ceases to cross the wall.
Area of the tank = 0.5 m^2.
Area of the orifice = 1 cm^2.



Q.13 A cylindrical tank having cross-sectional area A = 0.5 m^2 is filled with two liquids of densities $\rho_1 = 900\ kgm^{-3}$ & $\rho_2 = 600\ kgm^{-3}$, to a height h = 60cm as shown in the figure. A small hole having area a = 5 cm^2 is made in right vertical wall at a height y=20cm from the bottom. Calculate



(i) velocity of efflux.
(ii) horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane.
(iii) minimum and maximum value of F to keep the cylinder at rest. The coefficient of friction between cylinder and the plane is $\mu = 0.01$.
(iv) velocity of the top most layer of the liquid column and also the velocity of the boundary separating the two liquids.

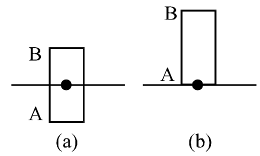
Q.14 A cylindrical wooden float whose base area S = 4000 cm^2 & the altitude H = 50 cm drifts on the water surface. Specific weight of wood d = 0.8 gf/cm^3.
(a) What work must be performed to take the float out of the water ?
(b) Compute the work to be performed to submerge completely the float into the water.

Q.15 A 10cm side cube weighing 5N is immersed in a liquid of relative density 0.8 contained in a rectangular tank of cross sectional area 15cm x 15cm. If the tank contained liquid to a height of 8cm before the immersion, determine the levels of the bottom of the cube and the liquid surface.

Q.16 A jug contains 15 glasses of orange juice. When you open the tap at the bottom it takes 12 sec to fill a glass with juice. If you leave the tap open. how long will it take to fill the remaining 14 glasses and thus empty the jug?

Q.17 An interstellar explorer discovers a remarkable planet made entirely of a uniform incompressible fluid on density ρ. The radius of the planet is R and the acceleration of gravity at its surface is g. What is the pressure at the center of the planet.

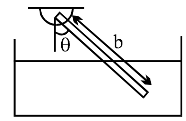
Q.18 A cylindrical rod of length $l = 2$m & density $\frac{\rho}{2}$ floats vertically in a liquid of density ρ as shown in Fig (a).



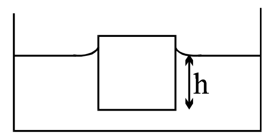
(a) Show that it performs SHM when pulled slightly up & released & find its time period. Neglect change in liquid level.

(b) Find the time taken by the rod to completely immerse when released from position shown in (b). Assume that it remains vertical throughout its motion. (take $g = \pi^2$ m/s^2)

Q.19 A uniform rod of length b capable of tuning about its end which is out of water, rests inclined to the vertical. If its specific gravity is 5/9, find the length immersed in water.



Q.20 A cube with a mass 'm' completely wettable by water floats on the surface of water. Each side of the cube is 'a'. What is the distance h between the lower face of cube and the surface of the water if surface tension is S. Take density of water as ρ_w. Take angle of contact m zero.

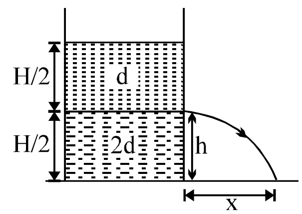


EXERCISE # III

Q.1 A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm², the water velocity is 1 m s^{-1} & the pressure is 2000 Pa. The pressure of water at another point where the cross sectional area is 5 cm², is ___________ pa.
[Density of water = 10^3 kg. m^{-3}] **[JEE '94 , 2]**

Q.2 A container of large uniform cross–sectional area A resting on a horizontal surface, holds two immiscible, non–viscous & incompressible liquids of densities d & 2d , each of height $\frac{H}{2}$ as shown in figure . The lower density liquid is open to the atmosphere having pressure P_0 .

(a) A homogeneous solid cylinder of length L $\left(L < \frac{H}{2}\right)$ cross–sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid–liquid interface with the length $\frac{L}{4}$ in the denser liquid. Determine :



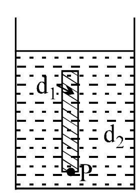
(i) The density D of the solid &
(ii) The total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored . A tiny hole of area s (s << A) is punched on the vertical side of the container at a height h $\left(h < \frac{H}{2}\right)$. Determine :

(i) The initial speed of efflux of the liquid at the hole ;
(ii) The horizontal distance x travelled by the liquid initially &
(iii) The height h_m at which the hold should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m . [Neglect the air resistance in these calculations]. **[JEE '95 , 10]**

Q.3 A cylindrical tank 1 m in radius rests on a platform 5 m high . Initially the tank is filled with water to a height of 5 m . A plug whose area is 10^{-4} m² is removed from an orifice on the side of the tank at the bottom . Calculate the following :
(i) initial speed with which the water flows from the orifice ;
(ii) initial speed with which the water strikes the ground &
(iii) time taken to empty the tank to half its original value . **[REE '95 , 5]**

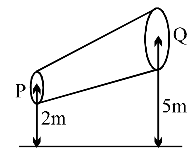
Q.4 A thin rod of length L & area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



[JEE '96 , 5]

Q.5 A large open top container of negligible mass & uniform cross–sectional area A has a small hole of cross–sectional area A/100 in its side wall near the bottom . The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0 . Assuming that the liquid starts flowing out horizontally through the hole at t = 0, calculate
(i) the acceleration of the container and
(ii) its velocity when 75 % of the liquid has drained out . **[JEE '97 , 5]**

Q.6 A nonviscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3}m^2 and 8×10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q.



[JEE '97]

Q.7 Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross–sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross–sectional area of the stream 0.15 m below the tap is **[JEE '98, 2]**

(A) 5.0×10^{-4} m^2 (B) 1.0×10^{-5} m^2 (C) 5.0×10^{-5} m^2 (D) 2.0×10^{-5} m^2

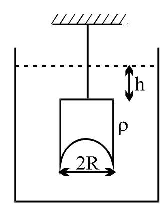
Q.8 A wooden stick of length l, and radius R and density ρ has a small metal piece of mass *m* (of negligible volume) attached to its one end. Find the minimum value for the mass *m* (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ (>ρ). **[JEE '99 , 10]**

Q.9 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to:

(A) $\dfrac{L}{\sqrt{2\pi}}$ (B) $2\pi L$ (C) L (D) $\dfrac{L}{2\pi}$

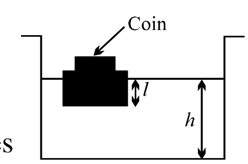
[JEE 2000 (Scr.)]

Q.10 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **[JEE 2001 (Scr.)]**



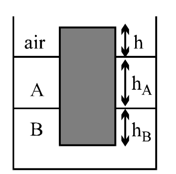
(A) Mg (B) Mg – v ρ g
(C) Mg + π R^{2}h ρ g (D) ρg (V + πR^{2}h)

Q.11 A wooden block, with a coin placed on its top, floats in water as shown in figure. The distances *l* and *h* are shown there. After some time the coin falls into the water. Then **[JEE 2002 (Scr.)]**



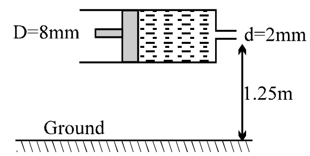
(A) *l* decreases and *h* increases (B) *l* increases and *h* decreases
(C) both *l* and *h* increase (D) both *l* and *h* decrease

Q.12 A uniform solid cylinder of density 0.8 gm/cm^3 floats in equilibrium in a combination of two non mixing liquids A and b with its axis vertical. The densities of the liquids A and B are 0.7 gm/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is h_A = 1.2 cm. The length of the part of the cylinder immersed in liquid B is h_B = 0.8 cm.



(a) Find the toal force exerted by liquid A on the cylinder.
(b) Find h, the length of the part of the cylinder in air.
(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. **[JEE 2002]**

Q.13 Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take g = 10 m/s2. **[JEE 2004]**



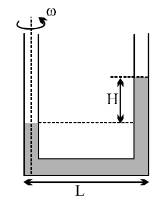
Q.14 A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[JEE 2004]**

Q.15 Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A}=0.1$ then v^2 is (where v is the velocity of water coming out of the hole)

(A) 48 (B) 51 (C) 50 (D) 51.5

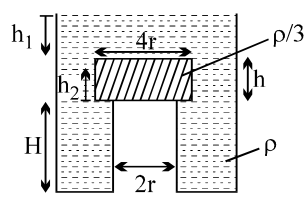
[JEE' 2005 (Scr)]

Q.16 A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L. **[JEE 2005]**



Comprehension –I

A wooden cylinder of diameter 4r, height h and density ρ/3 is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



Q.17 If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$ (C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$ **[JEE 2006]**

Q.18 Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$ (C) $\frac{2h}{3}$ (D) h **[JEE 2006]**

Q.19 If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position.
(B) for $h_2 = h/3$, cylinder again starts moving up
(C) for $h_2 = h/4$, cylinder again starts moving up
(D) for $h_2 = h/5$ cylinder again starts moving up **[JEE 2006]**

ANSWER KEY

EXERCISE # I

Q.1 $45^0, 9600\sqrt{2}$ (gauge)N/m^2

Q.2 $h = \frac{2m}{\pi}$, $H = \frac{11}{32\pi}$ m

Q.3 19.6 m, 4 sec

Q.4 2.79 gm/cc

Q.5 at the water surface, h/2

Q.6 $h_2 = 3\,h_1$

Q.7 37.5 N

Q.8 (a) $6\sqrt{2}$ m/s, (b) $9.6\sqrt{2} \times 10^{-3}$ M^3/sec, (c) 4.6×10^4 N/m^2

Q.9 $\sqrt{3} : \sqrt{2}$

Q.10 (a)$9600\sqrt{2}$, (b) nH

Q.11 0.75

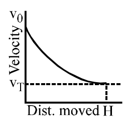
Q.12 100kg

Q.13 (a) 5, (b) 2/3

Q.14 101.8 Kgf–m

Q.15 (i) 2500 kg/m^3, $\frac{5000}{3}$ kg/m^3, (ii) $R_D = 7.5$ kg, $R_E = 2.5$ kg

Q.16 3/5

Q.17 

Q.18 2.5 cm

Q.19 20.4 m

Q.20 6.43×10^{-4} m^3/s

Q.21 20cm, 60cm

Q.22 $\ell(d_2 - d_1)\left(\frac{k}{d_2} + Ag\right)$

Q.23 $\omega = \frac{10}{\sqrt{3}}$ rad/s, $\tan\alpha = \frac{5}{3}$

Q.24 54.4 cm

Q.25 4.5m

EXERCISE # II

Q.1 6N

Q.2 4m/s^2, 10%, 0, 45kPa

Q.3 $h_1 = \frac{2(3+\pi)}{15\pi}$ =0.26 ; $h_2 = \frac{3+\pi}{10\pi}$ = 0.195

Q.4 x = 1/3

Q.5 $\frac{Y}{y-x+z}$

Q.6 $\rho = \frac{W}{\pi h^2 g\tan\alpha\left(R - \frac{1}{3}h\tan\alpha\right)}$

Q.7 (a) $\left(\frac{M-m}{M+m}\right)g - \frac{(M-m)gx}{(M+m)L}$ (b) $t = \frac{\pi}{2}\sqrt{\frac{L}{g}\left(\frac{M+m}{M-m}\right)}$

Q.8 $\sqrt{\frac{18g}{19a}}$

Q.9 $\frac{\pi}{80}$ m^2

Q.10 $h_1 = h_2$

Q.11 $80\sqrt{5}$ sec, $40\sqrt{5}$ sec

Q.12 431 sec

Q.13 (i) 4m/s, (ii) F = 7.2N, (iii) $F_{min} = 0$, $F_{max} = 52.2$ N, (iv) both 4×10^{-3} m/s

Q.14 (a) $\frac{d^2H^2S}{2\rho g} = 32\ Kgf - m$, (b) $\frac{1}{2}\ SH^2(1 - d)^2 = 2\ Kgf - m$

Q.15 $\frac{163}{36}$ cm, $\frac{388}{36}$ cm   Q.16 $t = \frac{12\sqrt{14}}{\sqrt{15} - \sqrt{14}}$ Q.17 $\frac{\rho g R}{2}$

Q.18 2 sec., 1 sec   Q.19 b/3   Q.20 $h = \frac{mg + 4sa}{\rho_w a^2 g}$

EXERCISE # III

Q.1 500 Pa

Q.2 (a)(i) $D = \frac{5}{4}d$, (ii) $p = P_0 + \frac{1}{4}(6H+L)dg$; (b)(i) $v = \sqrt{\frac{g}{2}(3H-4h)}$, (ii) $x = \sqrt{h(3H-4h)}$ (iii) $x_{max} = \frac{3}{4}H$

Q.3 (i) 10 m/s, (ii) 14.1 m/s, (iii) 2.5 hr   Q.4 $w = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$

Q.5 (i) 0.2 m/s^2, (ii) $\sqrt{2g\frac{m_0}{A\rho}}$ Q.6 + 29625 J/m^3, – 30000 J/m^3

Q.7 C

Q.8 $m_{min} = \pi r^2 l(\sqrt{\rho\sigma} - \rho)$; if tilted then it's axis should become vertical. C.M. should be lower than centre of bouyancy.   Q.9 A   Q.10 D

Q.11 D   Q.12 (a) 0, (b) h = 0.25 cm, (c) a= g/6 (upward)

Q.13 x = 2 m   Q.14 $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$ Q.15 C   Q.16 $H = \frac{L^2\omega^2}{2g}$

Q.17 C   Q.18 B   Q.19 A

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P9. Calorimetry and Heat Transfer

Index:

<u>*THERMAL EXPANSION*</u>

Definition of Heat :

Heat is a form of energy which is transferred between a system and its surrounding as a result of temperature difference only.

Thermal Expansion : Expansion due to increase in temperature.

1. Type of thermal expansion

	Coefficient of expansion	**For temperature change Δt change in**
(i) Linear	$\alpha = \lim\limits_{\Delta t \to 0} \frac{1}{l_0} \frac{\Delta l}{\Delta t}$	length $\Delta l = l_0\, \alpha\, \Delta t$
(ii) Superficial	$\beta = \lim\limits_{\Delta t \to 0} \frac{1}{A_0} \frac{\Delta A}{\Delta t}$	Area $\Delta A = A_0 \beta \Delta t$
(iii) Volume	$\gamma = \lim\limits_{\Delta t \to 0} \frac{1}{V_0} \frac{\Delta V}{\Delta t}$	volume $\Delta V = V_0 \gamma \Delta t$

(a) For isotropic solids $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ (let)
so $\beta = 2\alpha$ and $\gamma = 3\alpha$

(b) For anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$
Here α_1 , α_2 and α_3 are coefficient of linear expansion in X , Y and Z directions.

2. ***Variation in density :*** With increase of temperature volume increases so density decreases and vice-versa.

$$d = \frac{d_0}{(1 + \gamma \Delta t)}$$

For solids values of γ are generally small so we can write $d = d_0 (1 - \gamma \Delta t)$ (using bimomial expansion)

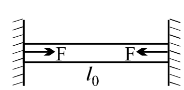
Note :

(i) γ for liquids are in order of 10^{-3}

(ii) For water density increases from 0 to 4°C so γ is –ve (0 to 4° C) and for 4° C to higher temperature γ is +ve. At 4° C density is maximum.

3. ***Thermal Stress :*** A rod of length l_0 is clamped between two fixed walls with distance l_0. If temperature is changed by amount Δt then

$$\text{stress} = \frac{F}{A} \quad \text{(area assumed to be constant)}$$



$$\text{strain} = \frac{\Delta l}{l_0}$$

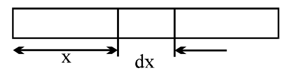
so, $$Y = \frac{F/A}{\Delta l / l_0} = \frac{F l_0}{A \Delta l} = \frac{F}{A \alpha \Delta t}$$

or $$F = Y A \alpha \Delta t$$

4. If α is not constant

(i) (α varies with distance) Let $\alpha = ax+b$

Total expansion = $\int expansion\ of\ length\ dx = \int_0^1 (ax+b)dx\,\Delta t$



(ii) (α varies with tempearture)
Let $\alpha = f(T)$

$$\Delta l = \int_{T_1}^{T_2} \alpha l_0\, dT$$

Caution : If α is in °C then put T_1 and T_2 in °C.
similarly if α is in K then put T_1 and T_2 in K.

CALORIMETRY

Quantity of heat transfered and specific heat

The amount of heat needed to incerase the temperature of 1 gm of water from 14.5° C to 15.5°C at STP is 1 calorie

$dQ = mcdT$

$Q = m\int_{T_1}^{T_2} C\,dT$ (be careful about unit of temperature, use units according to the given units of C)

Heat transfer in phase change

$Q = mL$ L = latent heat of substance in cal/ gm/ °C or in Kcal/ kg/ °C

$L_{ice} = 80$ cal/ gm for ice

$L_{steam} = 540$ cal/ gm

HEAT - TRANSFER

(A) Conduction : Due to vibration and collision of medium particles.

(i) Steady State : In this state heat absorption stops and temperature gradient throughout the rod becomes constant i.e. $\frac{dT}{dx}$ = constant.

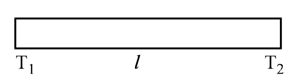
(ii) Before steady state : Temp of rod at any point changes

Note : If specific heat of any substance is zero, it can be considered always in steady state.

1. Ohm's law for Thermal Conduction in Steady State :

Let the two ends of rod of length l is maintained at temp T_1 and T_2($T_1 > T_2$)

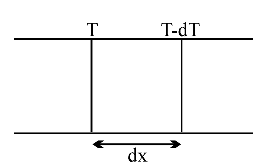
Thermal current $\frac{dQ}{dT} = \frac{T_1 - T_2}{R_{Th}}$



Where thermal resistance $R_{Th} = \frac{l}{KA}$

2. Differential form of Ohm's Law

$\frac{dQ}{dT} = KA\frac{dT}{dx}$ $\frac{dT}{dx}$ = temperature gradient



(B) ***Convection :*** Heat transfer due to movement of medium particles.

(C) ***Radiation:*** Every body radiates electromagnetic radiation of all possible wavelength at all temp>0 K.

1. ***Stefan's Law :*** Rate of heat emitted by a body at temp T K from per unit area $E = \sigma T^4$ J/sec/m^2

Radiation power $\frac{dQ}{dT} = P = \sigma A T^4$ watt

If a body is placed in a surrounding of temperature T_S

$$\frac{dQ}{dT} = \sigma A (T^4 - T_s^4)$$

valid only for black body

Emissivity or emmisive power $e = \frac{\text{heat from general body}}{\text{heat from black body}}$

If temp of body falls by dT in time dt

$$\frac{dT}{dt} = \frac{eA\sigma}{mS}(T^4 - T_s^4) \qquad \text{(dT/dt = rate of cooling)}$$

2. ***Newton's law of cooling***

If temp difference of body with surrounding is small i.e. $T = T_s$

then, $$\frac{dT}{dt} = \frac{4eA\sigma}{mS} T_s^3 (T - T_s)$$

so $$\frac{dT}{dt} \alpha\ (T - T_s)$$

3. ***Average form of Newtons law of cooling***

If a body cools from T_1 to T_2 in time δt

$$\frac{T_1 - T_2}{\delta t} = \frac{K}{mS}\left(\frac{T_1 + T_2}{2} - T_s\right) \qquad \text{(used generally in objective questions)}$$

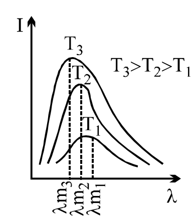
$$\frac{dT}{dt} = \frac{K}{mS}(T - T_s) \qquad \text{(for better results use this generally in subjective)}$$

4. ***Wein's black body radiation***

At every temperature (>0K) a body radiates energy radiations of all wavelengths. According to Wein's displacement law if the wavelength corresponding to maximum energy is $\lambda_{m.}$

then $\lambda_m T = b$ where b = is a constant (Wein's constant)

T = temperature of body



EXERCISE – I

Q.1 An aluminium container of mass 100 gm contains 200 gm of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice = 0.5 and L = 80 cal/gm, specific heat of Al = 0.2 cal/gm/°C)

Q.2 A U-tube filled with a liquid of volumetric coefficient of 10^{-5}/°C lies in a vertical plane. The height of liquid column in the left vertical limb is 100 cm. The liquid in the left vertical limb is maintained at a temperature = 0°C while the liquid in the right limb is maintained at a temperature = 100°C. Find the difference in levels in the two limbs.

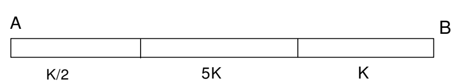
Q.3 A thin walled metal tank of surface area 5m^2 is filled with water tank and contains an immersion heater dissipating 1 kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25^{o}C. Find the temperature of the tank in the steady state

Q.4 A glass flask contains some mercury at room temperature. It is found that at different temperatures the volume of air inside the flask remains the same. If the volume of mercury in the flask is 300 cm^3, then find volume of the flask (given that coefficient of volume expansion of mercury and coefficient of linear expansion of glass are 1.8×10^{-4} (°C)$^{-1}$ and 9×10^{-6} (°C)$^{-1}$ respectively)

Q.5 A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, aporoximately. How much fast or slow will the clock run in 10^6 sec. ($\alpha_{invar}=1\times10^{-6}$/°C)

Q.6 A pan filled with hot food cools from 50.1 °C to 49.9 °C in 5 sec. How long will it take to cool from 40.1 °C to 39.9°C if room temperature is 30°C?

Q.7 A composite rod made of three rods of equal length and cross-section as shown in the fig. The thermal conductivities of the materials of the rods are K/2, 5K and K respectively. The end A and end B are at constant temperatures. All heat entering the face A goes out of the end B there being no loss of heat from the sides of the bar. Find the effective thermal conductivity of the bar

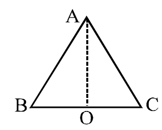


Q.8 An iron bar (Young's modulus = 10^{11} N/m^2 , $\alpha = 10^{-6}$ /°C) 1 m long and 10^{-3} m^2 in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

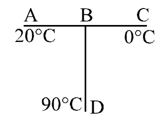
Q.9 A solid copper cube and sphere, both of same mass & emissivity are heated to same initial temperature and kept under identical conditions. What is the ratio of their initial rate of fall of temperature?

Q.10 A cylindrical rod with one end in a stream chamber and other end in ice cause melting of 0.1 gm of ice/sec. If the rod is replaced with another rod of half the length and double the radius of first and thermal conductivity of second rod is 1/4 that of first, find the rate of ice melting in gm/sec

Q.11 Three aluminium rods of equal length form an equilateral triangle ABC. Taking O (mid point of rod BC) as the origin. Find the increase in Y-coordinate of center of mass per unit change in temperature of the system. Assume the length of the each rod is 2m, and $\alpha_{al} = 4\sqrt{3}\times10^{-6}$ /°C



Q.12 Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB

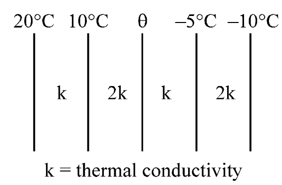


Q.13 If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3 L, determine the average coefficient of linear expansion of the composite rod.

Q.14 A volume of 120 ml of drink (half alcohol + half water by mass) originally at a temperature of 25°C is cooled by adding 20 gm ice at 0°C. If all the ice melts, find the final temperature of the drink. (density of drink = 0.833 gm/cc, specific heat of alcohol = 0.6 cal/gm/°C)

Q.15 A solid receives heat by radiation over its surface at the rate of 4 kW. The heat convection rate from the surface of solid to the surrounding is 5.2 kW, and heat is generated at a rate of 1.7 kW over the volume of the solid. The rate of change of the average temperature of the solid is 0.5°Cs^{-1}. Find the heat capacity of the solid.

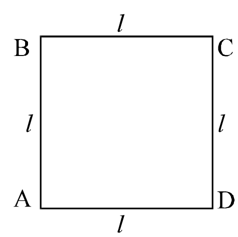
Q.16 The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.



Q.17 Two identical calorimeter A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal g^{-1} (C°)$^{-1}$ is dropped into A and a 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metals is 40°C. Find the specific heat of metal Y in cal g^{-1} (C°)$^{-1}$.

Q.18 Two spheres of same radius R have their densities in the ratio 8 : 1 and the ratio of their specific heats are 1 : 4. If by radiation their rates of fall of temperature are same, then find the ratio of their rates of losing heat.

Q.19 In the square frame of side l of metallic rods, the corners A and C are maintained at T_1 and T_2 respectively. The rate of heat flow from A to C is ω. If A and D are instead maintained T_1 & T_2 respectivley find, find the total rate of heat flow.



Q.20 A hot liquid contained in a container of negligible heat capacity loses temperature at rate 3 K/min, just before it begins to solidify. The temperature remains constant for 30 min. Find the ratio of specific heat capacity of liquid to specific latent heat of fusion is in K^{-1} (given that rate of losing heat is constant).

Q.21 A thermostatted chamber at small height h above earth's surface maintained at 30°C has a clock fitted in it with an uncompensated pendulum. The clock designer correctly designs it for height h, but for temperature of 20°C. If this chamber is taken to earth's surface, the clock in it would click correct time. Find the coefficient of linear expansion of material of pendulum. (earth's radius is R)

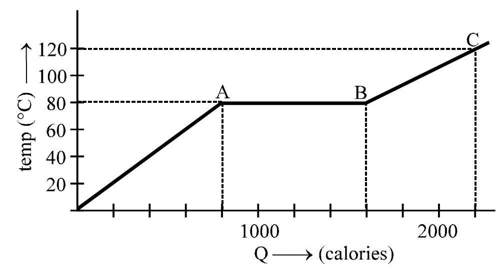
Q.22 The coefficient of volume expansion of mercury is 20 times the coefficient of linear expansion of glass. Find the volume of mercury that must be poured into a glass vessel of volume V so that the volume above mercury may remain constant at all temperature.

Q.23 Two 50 gm ice cubes are dropped into 250 gm of water into a glass. If the water was initially at a temperature of 25°C and the temperature of ice –15°C. Find the final temperature of water. (specific heat of ice = 0.5 cal/gm/°C and L = 80 cal/gm). Find final amount of water and ice.

Q.24 Water is heated from 10°C to 90°C in a residential hot water heater at a rate of 70 litre per minute. Natural gas with a density of 1.2 kg/m^3 is used in the heater, which has a transfer efficiency of 32%. Find the gas consumption rate in cubic meters per hour. (heat combustion for natural gas is 8400 kcal/kg)

Q.25 A metal rod A of 25cm lengths expands by 0.050cm. When its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40cm expands by 0.040 cm for the same rise in temperature. A third rod C of 50cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the lengths of each portion of the composite rod.

Q.26 A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph



(i) the mass of the substance ;
(ii) the specific latent heat of the melting process, and
(iii) the specific heat of the substance in the liquid state.

Q.27 One end of copper rod of uniform cross-section and of length 1.5 meters is in contact with melting ice and the other end with boiling water. At what point along its length should a temperature of 200°C be maintained, so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings.

Q.28 Two solids spheres are heated to the same temperature and allowed to cool under identical conditions. Compare: (i) initial rates of fall of temperature, and (ii) initial rates of loss of heat. Assume that all the surfaces have the same emissivity and ratios of their radii of, specific heats and densities are respectively $1 : \alpha$, $1 : \beta$, $1 : \gamma$.

Q.29 A vessel containing 100 gm water at 0°C is suspended in the middle of a room. In 15 minutes the temperature of the water rises by 2°C. When an equal amount of ice is placed in the vessel, it melts in 10 hours. Calculate the specific heat of fusion of ice.

Q.30 The maximum in the energy distribution spectrum of the sun is at 4753 Å and its temperature is 6050K. What will be the temperature of the star whose energy distribution shows a maximum at 9506 Å.

EXERCISE – II

Q.1 A copper calorimeter of mass 100 gm contains 200 gm of a mixture of ice and water. Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50°C. If the mass of the calorimeter and its contents is now 330 gm, what was the ratio of ice and water in the beginning? Neglect heat losses.
Given : Specific heat capacity of copper = 0.42×10^3 J $kg^{-1}K^{-1}$,
Specific heat capacity of water = 4.2×10^3 J $kg^{-1}K^{-1}$,
Specific heat of fusion of ice = 3.36×10^5 J kg^{-1}
Latent heat of condensation of steam = 22.5×10^5 Jkg^{-1}

Q.2 An isosceles triangle is formed with a rod of length l_1 and coefficient of linear expansion α_1 for the base and two thin rods each of length l_2 and coefficient of linear expansion α_2 for the two pieces, if the distance between the apex and the midpoint of the base remain unchanged as the temperatures varied show that $\frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

Q.3 A solid substance of mass 10 gm at – 10°C was heated to – 2°C (still in the solid state). The heat required was 64 calories. Another 880 calories was required to raise the temperature of the substance (now in the liquid state) to 1°C, while 900 calories was required to raise the temperature from –2°C to 3°C. Calculate the specific heat capacities of the substances in the solid and liquid state in calories per kilogram per kelvin. Show that the latent heat of fusion L is related to the melting point temperature t_m by L = 85400 + 200 t_m.

Q.4 A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If the entire mechanical work is used up in producing heat and the rate of raise in temperature of the block and the drill is 0.5 °C/s. Find
(a) the rate of working of the drill in watts, and
(b) the torque required to drive the drill.
Specific heat of steel = 0.1 and J = 4.2 J/cal. Use : $P = \tau\,\omega$

Q.5 A brass rod of mass m = 4.25 kg and a cross sectional area 5 cm^2 increases its length by 0.3 mm upon heating from 0°C. What amount of heat is spent for heating the rod? The coefficient of linear expansion for brass is 2×10^{-5}/K, its specific heat is 0.39 kJ/kg.K and the density of brass is 8.5×10^3 kg/m^3.

Q.6 A submarine made of steel weighing 10^9 g has to take 10^8 g of water in order to submerge when the temperature of the sea is 10°C. How much less water it will have to take in when the sea is at 15°C? (Coefficient of cubic expansion of sea water = 2×10^{-4}/°C, coefficient of linear expansion of steel = 1.2×10^{-5}/°C)

Q.7 A flow calorimeter is used to measure the specific heat of a liquid. Heat is added at a known rate to a stream of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow points of the liquid stream enables us to compute the specific heat of the liquid. A liquid of density 0.2 g/cm^3 flows through a calorimeter at the rate of 10 cm^3/s. Heat is added by means of a 250-W electric heating coil, and a temperature difference of 25°C is established in steady-state conditions between the inflow and the outflow points. Find the specific heat of the liquid.

Q.8 Toluene liquid of volume 300 cm^3 at 0°C is contained in a beaker an another quantity of toluene of volume 110 cm^3 at 100°C is in another beaker. (The combined volume is 410 cm^3). Determine the total volume of the mixture of the toluene liquids when they are mixed together. Given the coefficient of volume expansion $\gamma = 0.001/C$ and all forms of heat losses can be ignored. Also find the final temperature of the mixture.

Q.9 Ice at -20°C is filled upto height h = 10 cm in a uniform cylindrical vessel. Water at temperature θ°C is filled in another identical vessel upto the same height h= 10 cm. Now, water from second vessel is poured into first vessel and it is found that level of upper surface falls through $\Delta h = 0.5$ cm when thermal equilibrium is reached. Neglecting thermal capacity of vessels, change in density of water due to change in temperature and loss of heat due to radiation, calculate initial temperature θ of water.

Given, Density of water, $\rho_w = 1$ gm cm^{-3}
Density of ice, $\rho_i = 0.9$ gm/cm^3
Specific heat of water, $s_w = 1$ cal/gm ^{0}C
Specific heat of ice, $s_i = 0.5$ cal/gm^{0}C
Specific latent heat of ice, L = 80 cal/gm

Q.10 A composite body consists of two rectangular plates of the same dimensions but different thermal conductivities K_A and K_B. This body is used to transfer heat between two objects maintained at different temperatures. The composite body can be placed such that flow of heat takes place either parallel to the interface or perpendicular to it. Calculate the effective thermal conductivities $K_{||}$ and $K_{\perp}$ of the composite body for the parallel and perpendicular orientations. Which orientation will have more thermal conductivity?

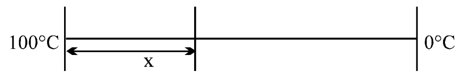
Q.11 Two identical thermally insulated vessels, each containing n mole of an ideal monatomic gas, are interconnected by a rod of length l and cross-sectional area A. Material of the rod has thermal conductivity K and its lateral surface is thermally insulated. If, at initial moment (t = 0), temperature of gas in two vessels is T_1 and T_2 ($< T_1$), neglecting thermal capacity of the rod, calculate difference between temperature of gas in two vessels as a function of time.

Q.12 A highly conducting solid cylinder of radius a and length l is surrounded by a co-axial layer of a material having thermal conductivity K and negligible heat capacity. Temperature of surrounding space (out side the layer) is T_0, which is higher than temperature of the cylinder. If heat capacity per unit volume of cylinder material is s and outer radius of the layer is b, calculate time required to increase temperature of the cylinder from T_1 to T_2. Assume end faces to be thermally insulated.

Q.13 A vertical brick duct(tube) is filled with cast iron. The lower end of the duct is maintained at a temperature T_1 which is greater than the melting point T_m of cast iron and the upper end at a temperature T_2 which is less than the temperature of the melting point of cast iron. It is given that the conductivity of liquid cast iron is equal to k times the conductivity of solid cast iron. Determine the fraction of the duct filled with molten metal.

Q.14 Water is filled in a non-conducting cylindrical vessel of uniform cross-sectional area. Height of water column is h_0 and temperature is 0°C. If the vessel is exposed to an atmosphere having constant temperature of – θ°C (< 0°C) at t = 0, calculate total height h of the column at time t.Assume thermal conductivity of ice to be equal to K.Density of water is ρ_ω and that of ice is ρ_i. Latent heat of fusion of ice is L.

Q.15 A lagged stick of cross section area 1 cm^2 and length 1 m is initially at a temperature of 0°C. It is then kept between 2 reservoirs of tempeature 100°C and 0°C. Specific heat capacity is 10 J/kg°C and linear mass density is 2 kg/m. Find



(a) temperature gradient along the rod in steady state.
(b) total heat absorbed by the rod to reach steady state.

Q.16 A cylindrical block of length 0.4 m an area of cross-section 0.04m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and the initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

Q.17 A copper calorimeter of negligible thermal capacity is filled with a liquid. The mass of the liquid equals 250 gm. A heating element of negligible thermal capacity is immersed in the liquid. It is found that the temperature of the calorimeter and its contents rises from 25°C to 30°C in 5 minutes when a current of 20.5 ampere is passed through it at potential difference of 5 volts. The liquid is thrown off and the heater is again switched on. It is now found that the temperature of the calorimeter alone is constantly maintained at 32°C when the current through the heater is 7A at the potential difference 6 volts. Calculate the specific heat capacity of the liquid. The temperature of the surroundings is 25°C.

Q.18 A solid copper sphere cools at the rate of 2.8°C per minute, when its temperature is 127°C. Find the rate at which another solid copper sphere of twice the radius lose its temperature at 327°C, if in both the cases, the room temperature is maintained at 27°C.

Q.19 A calorimeter contains 100 cm^3 of a liquid of density 0.88 g/cm^3 in which are immersed a thermometer and a small heating coil. The effective water equivalent of calorimeter, thermometer and heater may be taken to be 13 gm. Current of 2 A is passed through the coil. The potential difference across the coil is 6.3 V and the ultimate steady state temperature is 55°C. The current is increased so that the temperature rises slightly above 55°C, and then it is switched off. The calorimeter and the content are found to cool at the rate of 3.6°C/min.
(a) Find the specific heat of the liquid.
(b) The room temperature during the experiment was 10°C. If the room temperature rises to 26°C, find the current required to keep the liquid at 55°C. You may assume that Newton's law is obeyed and the resistance of the heater remains constant.

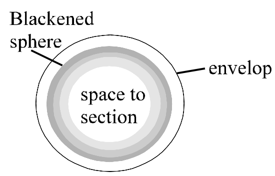
Q.20 End A of a rod AB of length L = 0.5 m and of uniform cross-sectional area is maintained at some constant temperature. The heat conductivity of the rod is k = 17 J/s-m°K. The other end B of this rod is radiating energy into vacuum and the wavelength with maximum energy density emitted from this end is λ_0 = 75000 Å. If the emissivity of the end B is e = 1, determine the temperature of the end A. Assuming that except the ends, the rod is thermally insulated.

Q.21 A wire of length 1.0 m and radius 10^{-3} m is carrying a heavy current and is assumed to radiate as a blackbody. At equilibrium temperature of wire is 900 K while that of the surroundings is 300 K. The resistivity of the material of the wire at 300 K is $\pi^2 \times 10^{-8}$ Ω-m and its temperature coefficient of resistance is 7.8×10^{-3} /°C. Find the current in the wire. [$\sigma \cong 5.68 \times 10^{-8}$ w/m^2K^4].

Q.22 The temperature distribution of solar radiation is more or less same as that of a black body whose maximum emission corresponds to the wavelength 0.483 μm. Find the rate of change of mass due to radiation. [Radius of Sun = 7.0×10^8 m]

Q.23 A black plane surface at a constant high temperature T_h, is parallel to another black plane surface at constant lower temperature T_l. Between the plates is vacuum. In order to reduce the heat flow due to radiation, a heat shield consisting of two thin black plates, thermally isolated from each other, it placed between the warm and the cold surfaces and parallel to these. After some time stationary conditions are obtained. By what factor η is the stationary heat flow reduced due to the presence of the heat shield? Neglect end effects due to the finite size of the surfaces.

Q.24 The shell of a space station is a blackened sphere in which a temperature T = 500K is maintained due to operation of appliances of the station. Find the temperature of the shell if the station is enveloped by a thin spherical black screen of nearly the same radius as the radius of the shell.



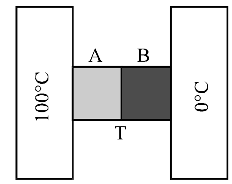
Q.25 A liquid takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C ? The temperature of surrounding is 20°C. Use exact method.

Q.26 Find the temperature of equilibrium of a perfectly black disc exposed normally to the Sun's ray on the surface of Earth. Imagine that it has a nonconducting backing so that it can radiate only to hemisphere of space. Assume temperature of surface of Sun = 6200 K, radius of sun = 6.9×10^8 m, distance between the Sun and the Earth = 1.5×10^{11} m. Stefan's constant = 5.7×10^{-8} W/m^2.K^4. What will be the temperature if both sides of the disc are radiate?

EXERCISE – III

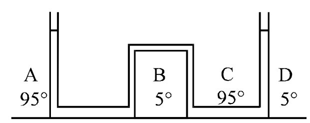
Q.1 The temperature of 100 gm of water is to be raised from 24º C to 90º C by adding steam to it. Calculate the mass of the steam required for this purpose. [JEE '96]

Q.2 Two metal cubes A & B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A & B are 300 W/mº C and 200 W/mº C respectively. After steady state is reached the temperature T of the interface will be ______. [JEE' 96]



Q.3 A double pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05m thick stagnant air space. In the steady state, the room glass interface and the glass outdoor interface are at constant temperatures of 27^0C and 0^0C respectively. Calculate the rate of heat flow through the window pane. Also find the temperatures of other interfaces. Given thermal conductivities of glass and air as 0.8 and 0.08 W $m^{-1}K^{-1}$ respectively. [JEE'97]

Q.4 The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B & C are 49 cm each. The two outer columns A & D are open to the atmosphere. A & C are maintained at a temperature of 95º C while the columns B & D are maintained at 5º C. The height of the liquid in A & D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. [JEE '97]



Q.5 A spherical black body with a radius of 12 cm radiates 450 W power at 500 K . If the radius were halved and the temperature doubled, the power radiated in watt would be :
(A) 225 (B) 450 (C) 900 (D) 1800

Q.6 Earth receives 1400 W/m^2 of solar power . If all the solar energy falling on a lens of area 0.2 m^2 is focussed on to a block of ice of mass 280 grams, the time taken to melt the ice will be ______ minutes. (Latent heat of fusion of ice = 3.3 x 10^5 J/kg) [JEE '97]

Q.7 A solid body X of heat capacity C is kept in an atmosphere whose temperature is T_A = 300K. At time t = 0, the temperature of X is T_0 = 400K. It cools according to Newton's law of cooling. At time t_1 its temperature is found to be 350K. At this time t_1, the body X is connected to a larger body Y at atmospheric temperature T_A, through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$. [JEE' 98]

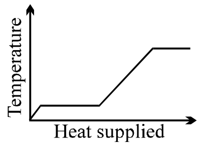
Q.8 A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant b = 2.88 × 10^6 nm K. Then [JEE' 98]
(A) $U_1 = 0$ (B) $U_3 = 0$ (C) $U_1 > U_2$ (D) $U_2 > U_1$

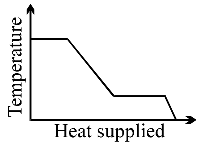
Q.9 A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is :

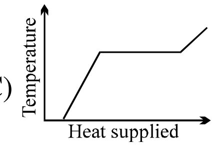
(A) proportional at ΔT (B) inversely proportional to ΔT [JEE' 99]

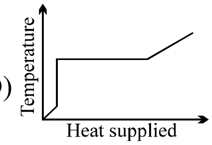
(C) proportional to $|\alpha_B - \alpha_C|$ (D) inversely proportional to $|\alpha_B - \alpha_C|$

Q.10 A block of ice at – 10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively? [JEE (Scr) 2000]

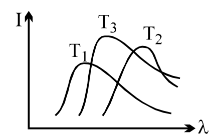
(A) 

(B) 

(C) 

(D) 

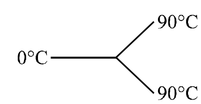
Q.11 The plots of intensity versus wavelength for three black bodies at temperature T_1 , T_2 and T_3 respectively are as shown. Their temperatures are such that [JEE (Scr) 2000]



(A) $T_1 > T_2 > T_3$ (B) $T_1 > T_3 > T_2$

(C) $T_2 > T_3 > T_1$ (C) $T_3 > T_2 > T_1$

Q.12 Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be [JEE(Scr)2001]



(A) 45°C (B) 60°C (C) 30°C (D) 20°C

Q.13 An ideal black body at room temperature is thrown into a furnace. It is observed that

(A) initially it is the darkest body and at later times the brightest.

(B) it the darkest body at all times

(C) it cannot be distinguished at all times.

(D) initially it is the darkest body and at later times it cannot be distinguished. [JEE(Scr)2002]

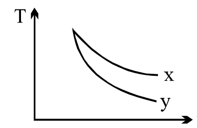
Q.14 An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according the empirical relations = A + BT, where A = 100 cal/kg-K and B = 2×10^{-2} cal/kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion for water = 8×10^4 cal/kg. Specific heat of water = 10^3 cal/kg-K) [JEE' 2001]

Q.15 Two rods one of aluminium of length l_1 having coefficient of linear expansion α_a, and other steel of length l_2 having coefficient of linear expansion α_s are joined end to end. The expansion in both the rods is same on variation of temperature. Then the value of $\frac{l_1}{l_1 + l_2}$ is [JEE' (Scr) 2003]

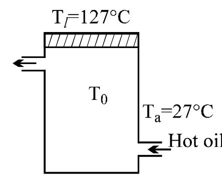
(A) $\frac{\alpha_s}{\alpha_a + \alpha_s}$ (B) $\frac{\alpha_s}{\alpha_a - \alpha_s}$ (C) $\frac{\alpha_a + \alpha_s}{\alpha_s}$ (D) None of these

Q.16 2 kg ice at – 20°C is mixed with 5 kg water at 20°C. Then final amount of water in the mixture would be; Given specific heat of ice = 0.5cal/g°C, specific heat of water = 1 cal/g°C, Latent heat of fusion of ice = 80 cal/g. [JEE' (Scr) 2003]
(A) 6 kg (B) 5 kg (C) 4 kg (D) 2 kg

Q.17 If emissivity of bodies X and Y are e_x and e_y and absorptive power are A_x and A_y then [JEE' (Scr) 2003]
(A) $e_y > e_x$; $A_y > A_x$ (B) $e_y < e_x$; $A_y < A_x$
(C) $e_y > e_x$; $A_y < A_x$ (D) $e_y = e_x$; $A_y = A_x$



Q.18 Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/(m-°C-sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid in steady state is at T_l = 127°. If the ambient temperature T_a = 27°C. Calculate

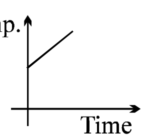


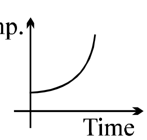
(a) rate of heat loss per unit area due to radiation from the lid.
(b) temperature of the oil. (Given $\sigma = \frac{17}{3} \times 10^{-8}$) [JEE' 2003]

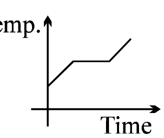
Q.19 Three discs A, B, and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively.
(a) Q_A is maximum (B) Q_B is maximum [JEE' 2004 (Scr.)]
(C) Q_C is maximum (D) $Q_A = Q_B = Q_C$

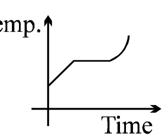
Q.20 Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0° C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in the two cases respectively. The ratio q_2/q_1 is
(A) 1/2 (B) 2/1 (C) 4/1 (D) 1/4 [JEE' 2004 (Scr.)]

Q.21 Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the variation of temperature with time?

(A)


(B)


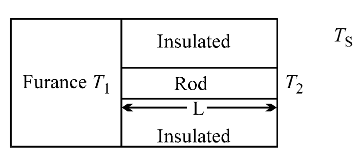
(C)


(D)


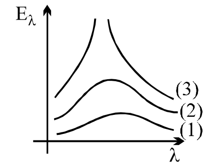
[JEE' 2004 (Scr.)]

Q.22 A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l, showing all the steps. [JEE 2004]

Q.23 One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$ where $\Delta T << T_S$, T_S being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_S)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. [JEE 2004]



Q.24 Three graphs marked as 1, 2, 3 representing the variation of maximum emissive power and wavelength of radiation of the sun, a welding arc and a tungsten filament. Which of the following combination is correct



(A) 1-bulb, 2 $\rightarrow$ welding arc, 3 $\rightarrow$ sun
(B) 2-bulb, 3 $\rightarrow$ welding arc, 1 $\rightarrow$ sun
(C) 3-bulb, 1 $\rightarrow$ welding arc, 2 $\rightarrow$ sun
(D) 2-bulb, 1 $\rightarrow$ welding arc, 3 $\rightarrow$ sun [JEE' 2005 (Scr)]

Q.25 In which of the following phenomenon heat convection does not take place
(A) land and sea breeze
(B) boiling of water
(C) heating of glass surface due to filament of the bulb
(D) air around the furance [JEE' 2005 (Scr)]

Q.26 2 litre water at 27°C is heated by a 1 kW heater in an open container. On an average heat is lost to surroundings at the rate 160 J/s. The time required for the temperature to reach 77°C is
(A) 8 min 20 sec (B) 10 min (C) 7 min (D) 14 min
[JEE' 2005 (Scr)]

Q.27 A spherical body of area A, and emissivity e = 0.6 is kept inside a black body. What is the rate at which energy is radiated per second at temperature T
(A) $0.6\,\sigma AT^4$ (B) $0.4\,\sigma AT^4$ (C) $0.8\,\sigma AT^4$ (D) $1.0\,\sigma AT^4$
[JEE' 2005 (Scr)]

Q.28 1 calorie is the heat required to increased the temperature of 1 gm of water by 1°C from
(A) 13.5°C to 14.5°C at 76 mm of Hg (B) 14.5°C to 15.5°C at 760 mm of Hg
(C) 0°C to 1°C at 760 mm of Hg (D) 3°C to 4°C to 760 mm of Hg
[JEE' 2005 (Scr)]

Q.29 In a dark room with ambient temperature T_0, a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room. Assuming that there is no change in the ambient temperature of the room, which of the following statement(s) is/are correct?
(A) The quantity of radiation absorbed by the black body in unit time will increase.
(B) Since emissivity = absorptivity, hence the quantity of radiation emitted by black body in unit time will increase.
(C) Black body radiates more energy in unit time in the visible spectrum.
(D) The reflected energy in unit time by the black body remains same. [JEE 2006]

Q.30 In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Then, find the final temperature of the mixture.
Given, L_{fusion} = 80 cal/g = 336 J/g, $L_{vaporization}$ = 540 cal/g = 2268 J/g,
S_{ice} = 2100 J/kg K = 0.5 cal/gK and S_{water} = 4200 J/kg K = 1 cal /gK [JEE 2006]

ANSWER KEY

EXERCISE –1

Q.1 25.5°C   Q.2 0.1 cm   Q.3 65°C   Q.4 2000 cm^3

Q.5 5 sec slow   Q.6 10 sec   Q.7 15K/16   Q.8 10, 000 N

Q.9 $\left(\frac{6}{\pi}\right)^{1/3}$ Q.10 0.2   Q.11 4×10^{-6} m/°C

Q.12 7/2   Q.13 5α/3   Q.14 4°C   Q.15 1000 J $(C°)^{-1}$

Q.16 5°C   Q.17 27/85   Q.18 2 : 1   Q.19 (4/3) ω

Q.20 1/90   Q.21 h/5R   Q.22 3V / 20

Q.23 0 °C, 125/4 g ice, 1275/4 g water

Q.24 104.2   Q.25 10cm , 40cm   Q.26 (i)0.02kg,(ii) 40,000calkg^{-1},(iii)750calk$g^{-1}K^{-1}$

Q.27 10.34 cm   Q.28 αβγ : 1 ; 1 : α^2 Q.29 80 k cal/kg   Q.30 3025 K

EXERCISE – II

Q.1 1 : 1.26   Q.3 800 cal kg^{-1} K^{-1}, 1000 cal kg^{-1} K^{-1}

Q.4 (a) 37.8 J/s (Watts), (b) 2.005 N-m   Q.5 25 kJ   Q.6 9.02×10^5 gm

Q.7 5000 J/°C kg   Q.8 decrease by 0.75 cm^3, 25°C   Q.9 45^0C

Q.10 $\kappa_{11} > K_\perp$, $K_\parallel = \frac{K_A + K_B}{2}$, $K_\perp = \frac{2K_AK_B}{K_A + K_B}$ Q.11 $(T_1 - T_2)e^{-\left(\frac{4KAt}{3nR\ell}\right)}$

Q.12 $\frac{a^2s}{2K}\log_e\left(\frac{b}{a}\right)\log_e\left(\frac{T_0 - T_1}{T_0 - T_2}\right)$ Q.13 $\frac{l_1}{l} = \frac{k(T_1 - T_m)}{k(T_1 - T_m) + (T_m - T_2)}$

Q.14 $h_0 + \left(1 - \frac{\rho_i}{\rho_w}\right)\sqrt{\frac{2k_i\theta t}{\rho_i L_f}}$ Q.15 (a) –100 °C/m, (b) 1000 J   Q.16 166.3 sec

Q.17 21000 $Jkg^{-1}K^{-1}$ Q.18 9.72^0C/min   Q.19 (a)0.42 cal/gm°C, (b) 1.74A

Q.20 $T_A = 423$ K   Q.21 36 A   Q.22 $\frac{dm}{dt} = 5.06 \times 10^9$ kg/s

Q.23 $\eta = 3$ Q.24 $T'' = \sqrt[4]{2} \times 500 = 600$ K

Q.25 10 minutes   Q.26 $T_0 = 420$ K, $T_0 = 353.6$ K

EXERCISE – III

Q.1 12 gm   Q.2 60° C   Q.3 41.53 Watt; 26.48 ^{0}C ; 0.55^0C

Q.4 2×10^{-4} C   Q.5 D   Q.6 5.5 min

Q.7 $k = \frac{\log_e 2}{t_1}$; $T = 300 + 50 \exp.\left[-\left\{\frac{KA}{LC} + \frac{\log_e 2}{t_1}\right\}2t_1\right]$

Q.8 D   Q.9 B, D   Q.10 A   Q.11 B   Q.12 B   Q.13 D   Q.14 0.5 kg

Q.15 A   Q.16 A   Q.17 A   Q.18 (a) 595 watt/m^2, (b) $T_0 \approx 420$ K   Q.19 B

Q.20 D   Q.21 C   Q.22 $\gamma_l = 2\alpha_s$ Q.23 $\frac{K}{4e\sigma LT_S^3 + K}$ Q.24 A

Q.25 C   Q.26 A   Q.27 A   Q.28 B   Q.29 A,B,C,D   Q.30 273 K

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P10. Kinetic Theory of Gases and Thermodynamics

Index:

KEY CONCEPTS

Kinetic Theory Of Gases

1. Assumption of kinetic theory of gases

(1) A gas consist of particles called molecules which move randomly in all directions.
(2) These molecules obey Newton's law of motion.
(3) Size of molecule negligible in comparison to average separation between the molecules.
(4) The forces on molecule are negligible except at the time of collision.
(5) All collision between molecules or between molecules and wall are pefectly elastic. Time of collision is very small.
(6) For large number of molecules the density and distribution of molecules with different velocities are independent of position, direction and time.

2. Pressure of an ideal gas

$$\boxed{P = \frac{1}{3}\rho\,\overline{v}^2 = \frac{1}{3}\rho\,\overline{v}^2_{rms}}$$

Here $\overline{v}$ = mean square speed

v_{rms} = root mean square speed
ρ = density of gas

$$P = \frac{2}{3}\left(\frac{1}{2}\rho v^2_{rms}\right)$$

$$P = \frac{2}{3}E$$

$$E = \frac{3}{2}P$$

So total K.E. $\boxed{K = \frac{3}{2}PV}$

3. R.M.S. velocity – depends on tempearture only for any gas.

$$v_{rms} = \frac{\sqrt{v_1^2+v_2^2+v_3^2+.........+v_n^2}}{n}$$

$$P = \frac{1}{3}\rho v^2_{rms}$$

$$\boxed{v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}}$$

4. Most Probable velocity – velocity which maximum number of molecules may have

$$\boxed{v_{mp} = \sqrt{\frac{2RT}{M}}}$$

5. Average velocity

$$v_{avg} = \frac{\vec{v}_1+\vec{v}_2+........+\vec{v}_n}{n} = 0$$

6. Average speed

$$\boxed{v_{avg} = \frac{|\vec{v}_1| + |\vec{v}_2| + |\vec{v}_3| + \ldots\ldots + |\vec{v}_n|.}{n} = \sqrt{\frac{8RT}{\pi M}}}$$

7. Ideal gas equation

PV= nRT (container form of gas law/ pressure volume form)

$P = \left(\frac{\rho}{M}\right)RT$ (open atmosphere / pressure density form)

8. Graham's law of diffusion :–

When two gases at the same pressure and tempearture are allowed to diffuse into each other the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas

$r \propto v_{rms}$ where r = rate of diffusion

so, $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}}$

9. Degree of Freedom (f) – No. of ways in which a gas molecule can distribute its energy

10. Law of equipartition of energy : – Energy in each degree of freedom = 1/2 KT joules

If degree of freedom is f. Energy = $\frac{f}{2}$ KT joules.

$$U = \frac{f}{2} KTnN_A = \frac{f}{2} nRT$$

11. Degree of freedom(f) in different gas molecules

Molecules	Translational	Rotational
Monoatomic	3	0
Diatomic	3	2
Polyatomic	3	2 (linear molecule) 3 (non-linear molecule)

Translational energy for all type of molecules = $\frac{3}{2}$**(nRT)**

Law of Thermodynamics

1. Zeroth law of thermodynamics :– If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium.Then B and C are also in thermal equilibrium.

2. First law of Thermodynamics:– Energy conservation for gaseous system.

Heat supplied to the gas = Increment in internal energy + work done by the gas.

$\Delta Q = \Delta U + \Delta W$ — ΔQ is +ve for heat supplied

in differential form $dQ = dU + dW$ — ΔQ is –ve for heat rejected and $dQ = nCdT$ C = molar specific heat

$C = C_P$ (constant pressure) ; $C = C_V$ = (constant volume)

$$dU = \frac{f}{2}\, nRdT$$

$$dW = \int_{v_1}^{v_2} P\, dv$$ (P = pressure of the gas of which work is to be calculated)

ΔW = +ve for work done by gas (in expansion of gas)

ΔW = –ve for work done on the gas (in contraction of gas)

Molar specific heat for a given process $C = \frac{f}{2} R + \frac{R\,PdV}{PdV + VdP} = C_V + \frac{R\,PdV}{PdV + VdP}$

Process	**C**	**Monoatomic**	**Diatomic**	**Polyatomic**
V= constant	$C_V = (f/2)R$	(3/2)R	(5/2)R	3R
P = constant	$C_P = \frac{f+2}{2}R$	(5/2)R	(7/2)R	4R

Mayor's Relation $C_P = C_V + R$

Note :– C of a gas depends on the process of that gas, which can be infinite in types.

Ratio of specific heat :– $\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$ $\xrightarrow{\text{monoatomic}}$ 5/3 = 1.67 ; $\xrightarrow{\text{diatomic}}$ 7/5 = 1.4 ; $\xrightarrow{\text{polyatomic}}$ 4/3 = 1.33

and $f = \frac{2}{\gamma - 1}$

$C_V = \frac{R}{\gamma - 1}$; $C_P = \frac{\gamma R}{\gamma - 1}$

Isochoric Process (V= constant)

$dV = 0 \Rightarrow dW = 0$

By FLT $dQ = dU = nC_V\, dT$

$$\boxed{Q = \int_{T_1}^{T_2} nC_v\, dT = nC_v(T_2 - T_1)}$$

* Be careful if $\Delta V = 0$ then not necessarily an Isochoric Process.

Isobaric Process (P = constant)

$dP = 0$

By FLT $dQ = dU + dW$

$n_{C_p}(T_2 - T_1) = (\frac{f}{2})nR(T_2 - T_1) + nR(T_2 - T_1)$

$$\boxed{W = nR(T_2 - T_1)}$$

* If $\Delta P = 0$ then not necessarily an Isobaric Process.

Isothermal Process (T = constant)

$dT = 0$, $dU = 0$

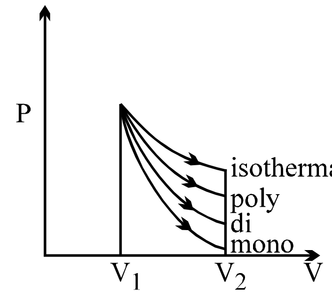
$$Q = W = (nRT)\int_{v_1}^{v_2} dV/V$$

$$\boxed{W = nRT\, ln\frac{V_2}{V_1} = nRT\, ln\frac{P_1}{P_2}}$$ $\left(\frac{V_2}{V_1} = \frac{P_1}{P_2} = \text{compression ratio}\right)$

Adiabatic Process dQ = 0 but if $\Delta Q = 0$, it is not necessaserily adibatic.

$dW = -dU$ By FLT

$$\boxed{W = \int_{T_1}^{T_2} \frac{nRdT}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}}$$



Work done is least for monoatomic gas

So $PdV + VdP = (\gamma-1)$ ....(ii)

For Adiabatic Process PV^{γ} = constant

$$\left|\frac{dP}{dV}\right|_{adiabatic} = \gamma\left|\frac{dP}{dV}\right|_{isothermal}$$

Polytropic process

PV^n = constant

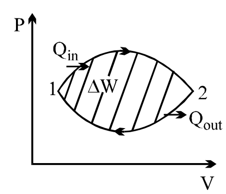
$$\boxed{P = \frac{K}{V^n} \Rightarrow \frac{dP}{dV} = n\left|\frac{K}{V^{n+1}}\right|}\ ; \qquad C = \frac{R}{\gamma-1} + \frac{R}{1-n}$$

So C is constant for polytropic process

Efficiency of a cyclic process

$\Delta U = 0$

so $\Delta Q = \Delta W$

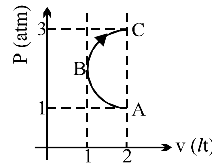


Efficiency $\eta = \frac{\text{work done by gas}}{\text{heat input}}$

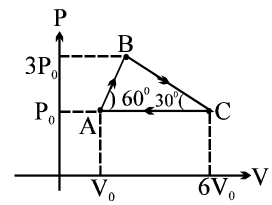
$$\eta = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

EXERCISE – I

Q.1 In the P-V diagram shown in figure, ABC is a semicircle. Find the workdone in the process ABC.



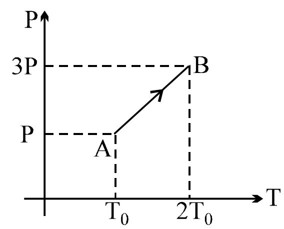
Q.2 Two moles of an ideal monoatomic gas undergone a cyclic process ABCA as shown in figure. Find the ratio of temperatures at B and A .



Q.3 The average degrees of freedom per molecules for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. Find the heat absorbed by the gas .

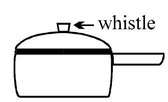
Q.4 1 mole of an ideal gas at initial temperature T was cooled isochorically till the gas pressure decreased n times. Then by an isobaric process, the gas was restored to the initial temperature T. Find the net heat absorbed by the gas in the whole process.

Q.5 Pressure versus temperature graph of an ideal gas is shown. Density of gas at point A is ρ_0. Find the density of gas at B.

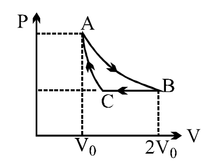


Q.6 PV-diagram of a monoatomic ideal gas is a straight line passing through origin. Find the molar heat capacity in the process.

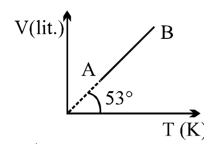
Q.7 An empty pressure cooker of volume 10 litres contains air at atmospheric pressure 10^5 Pa and temperature of 27°C. It contains a whistle which has area of 0.1 cm^2 and weight of 100 gm. What should be the temperature of air inside so that the whistle is just lifted up?



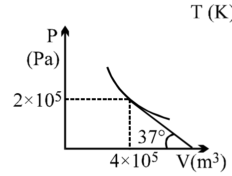
Q.8 In a cycle ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle
(Given : $T_A = T_B = 400$ K, $\gamma = 1.5$)



Q.9 V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.

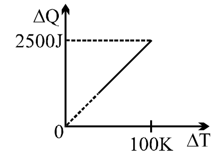


Q.10 P-V graph for an ideal gas undergoing polytropic process PV^m = constant is shown here.Find the value of m.



Q.11 Air at temperature of 400 K and atmospheric pressure is filled in a balloon of volume 1 m^3. If surrounding air is at temperature of 300 K, find the ratio of Buoyant force on balloon and weight of air inside

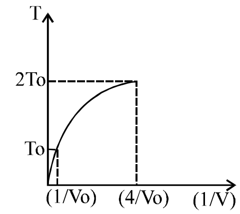
Q.12 One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.



Q.13 Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

Q.14 A gas is undergoing an adiabatic process. At a certain stage A, the values of volume and temperature $\equiv (V_0, T_0)$ and the magnitude of the slope of V-T curve is m. Find the value of C_P and C_V.

Q.15 Figure shows a parabolic graph between T and $\frac{1}{V}$ for a mixture of a gas undergoing an adiabatic process. What is the ratio of V_{rms} and speed of sound in the mixture?



Q.16 The height of mercury is a faulty barometer is 75 cm and the tube above mercury having air is 10 cm long. The correct barometer reading is 76 cm. If the faulty barometer reads 74 cm, find the true barometer reading.

Q.17 A piston divides a closed gas cylinder into two parts. Initially the piston is kept pressed such that one part has a pressure P and volume 5V and the other part has pressure 8P and volume V. The piston is now left free. Find the new pressures and volumes for the adiabatic and isothermal processes. For this gas $\gamma = 1.5$.

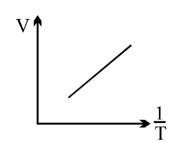
Q.18 A closed vessel of volume V_0 contains oxygen at a pressure P_0 and temperature T_0. Another closed vessel of the same volume V_0 contains helium at a pressure of P_0 and temperature $T_0/2$. Find the ratio of the masses of oxygen to the helium.

Q.19 A gas undergoes a process in which the pressure and volume are related by VP^n = constant. Find the bulk modulus of the gas.

Q.20 An ideal gas has a molar heat capacity C_V at constant volume. Find the molar heat capacity of this gas as a function of volume, if the gas undergoes the process : $T = T_0e^{\alpha V}$.

Q.21 A standing wave of frequency 1000 Hz in a column of methane at 27°C produces nodes which are 20.4 cm apart. Find the ratio of heat capacity of methane at constant pressure to that at constant volume (Take gas constant, $R = 8.31\ J \cdot K^{-1}mol^{-1}$)

Q.22 One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.



Q.23 One mole of an ideal gas is compressed from 0.5 lit to 0.25 lit. During the compression, 23.04×10^2 J of work is done on the gas and heat is removed to keep the temperature of the gas constant at all times. Find the temperature of the gas. (Take universal gas constant $R = 8.31\ J\ mol^{-1}K^{-1}$)

Q.24 A mixture of 4 gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume 300°K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules. (R = Universal gas constant)

Q.25 The pressure of an ideal gas changes with volumes as $P = aV$ where 'a' is a constant. One moles of this gas is expanded to 3 times its original volume V_0. Find
(i) the heat transferred in the process.
(ii) the heat capacity of the gas.

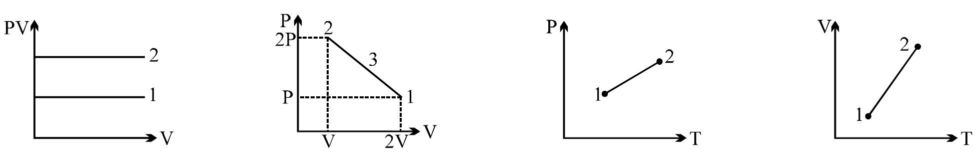
Q.26 If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K. Determine the change is the internal energy of the gas.

Q.27 70 calorie of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 40°C to 45°C. Find the amount of heat required to raise the temperature of the same gas through the same range at constant volume ($R = 2$ cal/mol-K)

Q.28 The volume of one mole of an ideal gas with specific heat ratio γ is varied according to the law $V = \frac{a}{T^2}$, where a is a constant. Find the amount of heat obtained by the gas in this process if the gas temperature is increased by ΔT.

Q.29 Find the molecular mass of a gas if the specific heats of the gas are C_P=0.2 cal/gm°C and C_V = 0.15 cal/gm°C. [Take $R = 2$ cal/mole°C]

Q.30 Examine the following plots and predict whether in (i) $P_1 < P_2$ and $T_1 > T_2$, in (ii) $T_1 = T_2 < T_3$, in (iii) $V_1 > V_2$, in (iv) $P_1 > P_2$ or otherwise.



(i) (ii) (iii) (iv)

List of recommended questions from I.E. Irodov.

2.1 to 2.7, 2.10 to 2.13, 2.17, 2.27, 2.29 to 2.35, 2.37 to 2.40, 2.43, 2.46, 2.48, 2.49, 2.63 to 2.73, 2.116, 2.120, 2.122, 2.127

EXERCISE – II

Q.1 A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the faulty barometer reads 69 cm and 70 cm respectively.

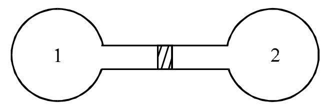
(i) What is the total length of the barometer tube?

(ii) What is the true reading when the faulty barometer reads 69.5 cm ?

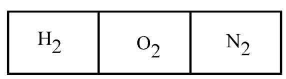
(iii) What is the faulty barometer reading when the true barometer reads 74 cm?

Q.2 Two bulbs of equal volume joined by a narrow tube of negligible volume contain hydrogen at 0°C and one atmospheric pressure. What is the pressure of the gas when one of the bulbs is immersed in steam at 100°C and the other in liquid oxygen at −190°C ? The volume of each bulb is 10^{-3}m^3 and density of hydrogen is 0.09 kg/m^3 at 0°C and at 1 atmosphere. What mass of hydrogen passes along the connecting tube?

Q.3 Two spherical flasks having a volume V_0 = 1.0 L each containing air are connected by a tube of diameter d = 6 mm and length l = 1 m. A small droplet of mercury contained in the tube is at its middle at 0°C. By what distance do the mercury droplets move if the flask 1 is heated by 2°C while flask 2 is cooled by 2°C. Ignore any expansion of flask wall.

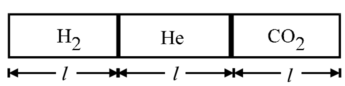


Q.4 A vessel of volume V = 30l is seperated into three equal parts by stationary semipermeable thin membranes as shown in the Figure. The left , middle and right parts are filled with m_{H_2} = 30g of hydorgen, m_{O_2} =160g of oxygen, and m_{N_2} = 70g of nitrogen respectively. The left partition lets through only hydrogen, while the right partition lets through hydrogen and nitrogen. What will be the pressure in each part of the vessel after the equilibrium has been set in if the vessel is kept at a constant temperature T = 300K?

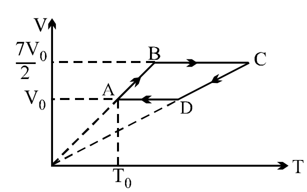


Q.5 A freely moving piston divides a vertical cylinder, closed at both ends, into two parts each containing 1 mole of air. In equilibrium, at T = 300 K, volume of the upper part is η = 4 times greater than the lower p part. At what temperature will the ratio of these volumes be equal to η' = 2?

Q.6 A non-conducting cylindrical vessel of length 3l is placed horizontally and is divided into three parts by two easily moving piston having low thermal conductivity as shown in figure. These parts contain H_2, He and CO_2 gas at initial temperatures θ_1 = 372°C, θ_2 = – 15°C and θ_3 = 157°C respectively. If initial length and pressure of each part are l and P_0 respectively, calculate final pressure and length of each part. Use : $\gamma_{CO_2} = 7/5$

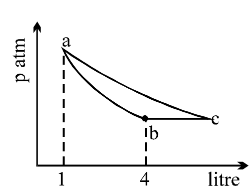


Q.7 A sample of an ideal non linear tri-atomic gas has a pressure P_0 and temperature T_0 taken through the cycle as shown starting from A. Pressure for process C → D is 3 times P_0. Calculate heat absorbed in the cycle and work done.



Q.8 RMS velocity of molecules of a di-atomic gas is to be increased to 1.5 times. Calculate ratio of initial volume to final volume, if it is done.
(i) Adiabatically ; (ii) Isobarically ; (iii) Calculate, also ratio of work done by gas during these processes.

Q.9 Figure shows three processes for an ideal gas. The temperature at 'a' is 600 K, pressure 16 atm and volume 1 litre. The volume at 'b' is 4 litre. Out of the two process ab and ac, one is adiabatic and he other is isothermal. The ratio of specific heats of the gas is 1.5. Answer the following :



(i) Which of ab and ac processes is adiabatic. Why?

(ii) Compute the pressure of the gas at b and c.

(iii) Compute the temperature at b and c.

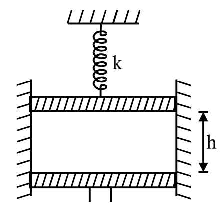
(iv) Compute the volume at c.

Q.10 Two vessels A and B both containing an ideal diatomic gas are connected together by a narrow tube of negligible volume fitted with a valve. A contains 5 mole of the gas at temperature 35°C and pressure $1.6 \times 10^5\ Nm^{-2}$, while B contains 2 moles of gas at temperature 17°C and pressure $8.3 \times 10^4\ Nm^{-2}$. The valve between the two vessel is opened to allow the contents to mix and achieve an equilibrium temperature of 27°C.

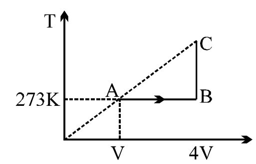
(i) Find the final pressure and the amount of heat transferred to the surrounding.

(ii) If the vessels along with the tube are perfectly insulated, calculate the final temperature and pressure.

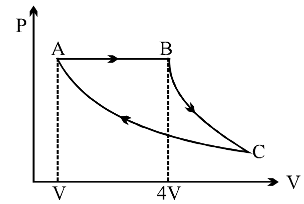
Q.11 An ideal gas at NTP is enclosed in a adiabatic vertical cylinder having area of cross section $A = 27\ cm^2$, between two light movable pistons as shown in the figure. Spring with force constant k = 3700 N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.



Q.12 At a temperature of $T_0 = 273°K$, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals Q = 27.7 kJ. Determine the ratio of molar specific heat capacities.

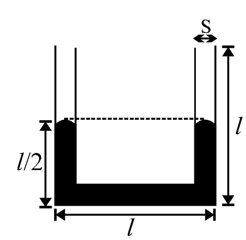


Q.13 A fixed mass of a gas is taken through a process $A \to B \to C \to A$. Here $A \to B$ is isobaric. $B \to C$ is adiabatic and $C \to A$ is isothermal. Find efficiency of the process. (take $\gamma = 1.5$)

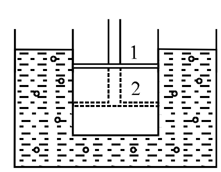


Q.14 A vessel of volume 30 litre is separated into three equal parts by stationary semipermeable membrane. The left, middle and right parts are filled with 30 gms of hydrogen, 160 gms of oxygen and 70 gms of nitrogen respectively. The left partition lets through only hydrogen while the right partition lets through hydrogen and nitrogen. If the temperature in all is 300 K find the ratio of pressure in the three compartments.

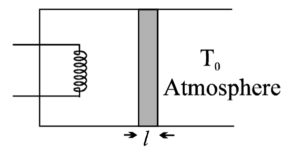
Q.15 A thin U–tube sealed at one end consists of three bends of length $l = 250$mm each, forming right angles. The vertical parts of the tube are filled with mercury to half the height as shown in the figure. All of mercury can be displaced from the tube by heating slowly the gas in the sealed end of the tube, which is separated from the atmospheric air by mercury. Determine the work A done by the gas thereby if the atmospheric pressure is $p_0 = 10^5$Pa, the density of mercury is $\rho_{mer} = 13.6 \times 10^3 kg/m^3$, and the cross-sectional area of the tube is $S = 1cm^2$.



Q.16 A cylinder containing a gas is closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position 1 to position 2. The piston is held at position 2 until the gas is again at 0°C and then slowly raised back to position 1. Represent the whole process on P-V diagram. If m = 100 gm of ice are melted during the cycle, how much work is done on the gas. Latent heat of ice = 80 cal/gm.



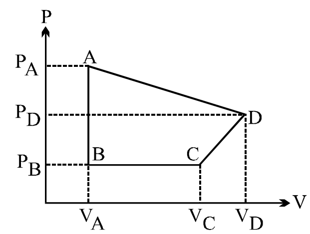
Q.17 An adiabatic vessel containing n moles of a ideal diatomic gas is fitted with a light conducting piston. The cross-sectional area, thickness and thermal conductivity of piston are A, l and K respectively. The other side of the piston is open to atmosphere of temperature T_0. Heat is supplied to the gas by means of an electric heater at a small constant rate q. Initial temperature of gas is T_0.



(a) Find the temperature of the gas as a function of time,
(b) Find the maximum temperature of the gas and
(c) What is the ratio of the maximum volume to the minimum volume?

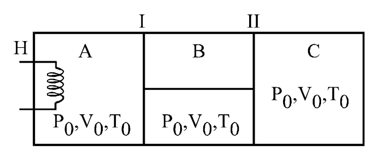
Q.18 A parallel beam of particles of mass m moving with velocities v impinges on a wall at an angle θ to its normal. The number of particles per unit volume in the beam is n. If the collision of particles with the wall is elastic, then find the pressure exerted by this beam on the wall.

Q.19 For the thermodynamic process shown in the figure.
$P_A = 1 \times 10^5$ Pa; $P_B = 0.3 \times 10^5$ Pa
$P_D = 0.6 \times 10^5$ Pa; $V_A = 0.20$ litre
$V_D = 1.30$ litre.



(a) Find the work performed by the system along path AD.
(b) In the total work done by the system along the path ADC is 85J find the volume at point C.
(c) How much work is performed by the system along the path CDA ?

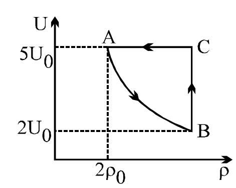
Q.20 The figure shows an insulated cylinder divided into three parts A, B and C. Pistons I and II are connected by a rigid rod and can move without friction inside the cylinder. Piston I is perfectly conducting while piston II is perfectly insulating. The initial state of the gas ($\gamma = 1.5$) present in each compartment A, B and C is as shown. Now, compartment A is slowly given heat through a heater H such that the final volume of C becomes $\frac{4V_0}{9}$. Assume the gas to be ideal and find.



(a) Final pressures in each compartment A, B and C
(b) Final temperatures in each compartment A, B and C
(c) Heat supplied by the heater
(d) Work done by gas in A and B.
(e) Heat flowing across piston I.

Q.21 How many atoms do the molecules of a gas consist of if γ increases 1.20 times when the vibrational degrees of freedom are "frozen" ? Assume that molecules are non linear.

Q.22 Figure shows the variation of the internal energy U with the density ρ of one mole of ideal monoatomic gas for a thermodynamic cycle ABCA. Here process AB is a part of rectangular hyperbola.



(a) Draw the P-V diagram for the above process.
(b) Find the net amount of heat absorbed by the system for the cyclic process.
(c) Find the work done in the process AB.

Q.23 An ideal monoatomic gas undergoes a process where its pressure is inversely proportional to its temperature.
(i) Calculate the specific heat for the process.
(ii) Find the work done by two moles of gas if the temperature changes from T_1 to T_2.

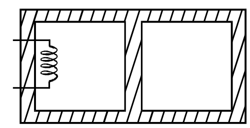
Q.24 An ideal diatomic gas undergoes a process in which its internal energy relates to the volume as

$$U = a\sqrt{V}$$, where α is a constant.

(a) Find the work performed by the gas and the amount of heat to be transferred to this gas to increase its internal energy by 100 J.

(b) Find the molar specific heat of the gas for this process.

Q.25 Two rectangular boxes shown in figure has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a monoatomic ideal gas ($\gamma = 5/3$) at a pressure p_0 volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partitions are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands, pushing the partition until the final pressure in both chambers becomes $243\ P_0/32$. Determine

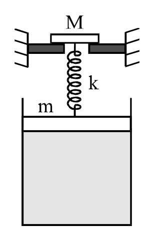


(i) the final temperature of the gas in each chamber and

(ii) the work-done by the gas in the right chamber.

Q.26 An adiabatic cylinder of length $2l$ and cross-sectional area A is closed at both ends. A freely moving non-conducting this piston divides the cylinder in two parts. The piston is connected with right end by a spring having force constat K and natural length l. Left part of the cylinder contains one mole of helium and right part contains 0.5 mole of each of helium and oxygen. If initial pressure of gas in each part is P_0, calculate heat supplied by the heating coil, connected to left part, to compress the spring through half of its natural length.

Q.27 0.01 moles of an ideal diatomic gas is enclosed in an adiabatic cylinder of cross-sectional area $A = 10^{-4} m^2$. In the arrangement shown, a block of mass $M = 0.8$ kg is placed on a horizontal support, and another block of mass $m = 1$ kg is suspended from a spring of stiffness constant $k = 16$ N/m. Initially, the spring is relaxed and the volume of the gas is $V = 1.4 \times 10^{-4}\ m^3$.



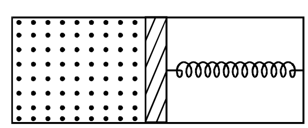
(a) Find the initial pressure of the gas.

(b) If block m is gently pushed down and released it oscillates harmonically, find its angular frequency of oscillation.

(c) When the gas in the cylinder is heated up the piston starts moving up and the spring gets compressed so that the block M is just lifted up. Determine the heat supplied.

Take atmospheric pressure $P_0 = 10^5\ Nm^{-2}$, $g = 10 m/s^2$.

Q.28 A thermally insulated vessel is divided into two parts by a heat-insulating piston which can move in the vessel without the friction. The left part of the vessel contains one mole of an ideal monatomic gas, & the right part is empty. The piston is connected to the right wall of the vessel through a spring whose length in free state is equal to the length of the vessel as shown in the figure. Determine the heat capacity C of the system, neglecting the heat capacities of the vessel, piston and spring.



EXERCIES – III

Q.1 The kinetic energy, due to translational motion, of most of the molecules of an ideal gas at absolute temperature T is ______ . [REE '94, 1]

Q.2 A vessel of volume 2×10^{-2} m^3 contains a mixture of hydrogen and helium at 47°C temperature and 4.15×10^5 N/m^2 pressure. The mass of the mixture is 10^{-2} kg. Calculate the masses of hydrogen and helium in the given mixture. [REE '94, 4]

Q.3 There are two vessels . Each of them contains one mole of a mono-atomic ideal gas . Initial volume of the gas in each vessel is 8.3×10^{-3} m^3 at 27°C . Equal amount of heat is supplied to each vessel . In one of the vessels, the volume of the gas is doubled isothermally, whereas the volume of the gas is held constant in the second vessel . The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system . [REE '94, 6]

Q.4 An ideal gas with pressure P, volume V & temperature T is expanded isothermally to a volume 2 V and a final pressure P_i . If the same gas is expanded adiabatically to a volume 2V, the final pressure is P_a. The ratio of the specific heats for the gas is 1.67 . The ratio P_a/P_i is ______ . [JEE '94, 2]

Q.5 An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding works involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively.
(i) Find the value of W_4. (ii) What is the efficiency of the cycle ? [JEE '94, 6]

Q.6 A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C & pressure of 1×10^5 Nm^{-2} . The total mass of the mixture is 28 gm. If the gram molecular weights of neon and argon are 20 & 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. [Universal gas constant R = 8.314 J/mol K] [JEE '94, 6]

Q.7 A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma = Cp/Cv = 5/3$ & another gas B $\gamma = 7/5$ with at a certain temperature T. The gram molecular weights of the gases A & B are 4 & 32 respectively. The gases A & B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation; $PV^{19/13}$ = const. in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T = 300K.

(c) If T is raised by 1 K from 300K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to 1/5 its initial volume V . Find the change in its adiabatic compressibility in terms of the given quantities. [JEE '95]

Q.8 The pressure in a monoatomic gas increases linearly from 4×10^5 N m^{-2} to 8×10^5 N m^{-2} when its volume increases from 0.2 m^3 to 0.5 m^3. Calculate the following: [REE '95, 5]
(a) work done by the gas, (b) increase in the internal energy, (c) amount of heat supplied,
(d) molar heat capacity of the gas.

Q.9 The temperature of an ideal gas is increased from 120 K to 480 K . If at 120 K the root-mean-square velocity of the gas molecules is v, at 480 K it becomes : [JEE '96, 2]
(A) 4v (B) 2v (C) v/2 (D) v/4

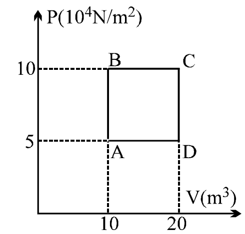
Q.10 At 27^{0}C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate : (i) the final temperature of the gas , (ii) change in its internal energy & (iii) the work done by the gas during the process . [JEE '96, 5]

Q.11 There is a soap bubble of radius 2.4×10^{-4} m in air cylinder which is originally at the pressure of 10^5 Nm^{-2}. The air in the cylinder is now compressed isothermally until the radius of the bubble is halved. Calculate now the pressure of air in the cylinder. The surface tension of the soap film is 0.08 Nm^{-1}. [REE '96, 5]

Q.12 A vertical hollow cylinder contains an ideal gas. The gas is enclosed by a 5kg movable piston with an area of cross-section 5×10^{-3} m^2. Now, the gas is slowly heated from 300 K to 350K and the piston rises by 0.1 m. The piston is now clamped at this position and the gas is cooled back to 300 K. Find the difference between the heat energy added during heating process & energy lost during the cooling process.
[1 atm pressure = 10^5 N m^{-2}] [REE '96, 5]

Q.13 The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J & 484 m/s respectively. The corresponding values at 600K are nearly (assuming ideal gas behaviour)
(A) 12.42×10^{-21} J, 968 m/s (B) 8.78×10^{-21} J, 684 m/s
(C) 6.21×10^{-21} J, 968 m/s (D) 12.42×10^{-21} J, 684 m/s [JEE '97, 1]

Q.14 A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as in figure. Given, molecular mass of Helium = 4



(i) what is the temperature of Helium in each of the states A, B, C & D ?
(ii) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.
(iii) How much is the heat involved in each of the processes ABC ADC. [JEE '97, 5]

Q.15 The average translational kinetic energy of a molecule in a gas becomes equal to 1 eV at a temperature ______ . [REE '97, 1]

Q.16 Two moles of an ideal monoatomic gas are confined within a cylinder by a massless & frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is, initially in its relaxed state. Now the gas is heated by an electric heated, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant & the heat supplied by the heater. [REE '97, 5]

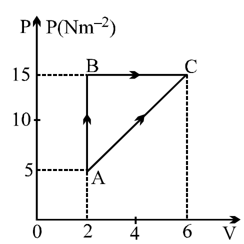
Q.17 Two vessels A & B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 litres of gas at 300 K and 2×10^5 Nm^{-2} pressure while vessel B has 4 litres of gas at 350 K & 4×10^5 N m^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure & temperature. Calculate the new pressure & temperature. [REE '97, 5]

Q.18 One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A . The process $A \rightarrow B$ is an adiabatic compression. $B \rightarrow C$ is isobaric expansion. $C \rightarrow D$ an adiabatic expansion $\rightarrow$ A is isochoric. The volume ratios are $V_a/V_b = 16$ and $V_c/V_b = 2$ & the temperature at A is $T_A = 300^0$K. Calculate the temperature of the gas at the points B & D and find the efficiency of the cycle.
[$(16^{0.4} = 3.03)$ $(1/8)^{0.4} = 0.435$] [JEE '97, 5]

Q.19 The average translational kinetic energy of O_2 (molar mass 32) molecules at a particluar temperature is 0.048 eV. The translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is
(A) 0.0015 (B) 0.003 (C) 0.048 (D) 0.768 [JEE '97, 3]

Q.20 Select the correct alternative . [JEE '97, 3]
A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of:
(A) P/8 (B) P (C) 2P (D) 8P

Q.21 In the given figure an ideal gas changes its state from state A to state C by two paths ABC and AC. (a) Find the path along which work done is the least. (b) The internal energy of gas at A is 10J and amount of heat supplied to change its state to C through the path AC is 200J. Calculate the internal energy at C. (c) The internal energy of gas at state B is 20J. Find the amount of heat supplied to the gas from A to B. [REE '98]



Q.22 Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas is A is 30K, then rise in temperature of the gas in B is
(A)30K (B) 18K (C) 50K (D) 42K [JEE' 98]

Q.23 Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The change in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then [JEE' 98]
(A) $4m_A=9m_B$ (B) $2m_A=3m_B$ (C) $3m_A=2m_B$ (D) $9m_A=4m_B$

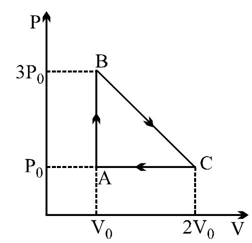
Q.24 A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300K. The ratio of the average rotational kinetic energy per O_2 molecule the that per N_2 molecule is [JEE' 98]
(A) 1:1 (B) 1:2 (C) 2:1 (D) depend on the moment of inertia of two molecules.

Q.25 Let v_{av}, v_{rms} and v_p respectively denote mean speed, root mean square speed and the most probable speed of the molecule in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m then :
(A) no molecule can have speed greater than $\sqrt{2}\, v_{rms}$ [JEE'98]
(B) no molecule can have speed less than $v_p/\sqrt{2}$
(C) $v_p<v_{av}<v_{rms}$ (D) the average kinetic energy of a molecule is $3/4\, mv_p^2$

Q.26 A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :
(A) 2P/3 (B) P (C) 3P/2 (D) 2P [JEE'98]

Q.27 During the melting of a slab of ice at 273K at atmospheric pressure:
(A) positive work is done by the ice-water system on the atmosphere.
(B) positive work is done on the ice-water system by the atmosphere
(C) the internal energy of the ice-water system increases
(D) the internal energy of ice-water system decreases. [JEE' 98]

Q.28 One mole of an ideal monoatomic gas is taken round the cyclic process ABCA as shown in figure, calculate
(a) the work done by the gas
(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB.
(c) the net heat absorbed by the gas in the path BC
(d) the maximum temperature attained by the gas during the cycle. [JEE' 98]



Q.29 The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300K is
(A) √(2/7) (B) √(1/7) (C) (√3)/5 (D) (√6)/5 [JEE' 99]

Q.30 A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
(A) 4 RT (B) 15 RT (C) 9 RT (D) 11 RT [JEE' 99]

Q.31 Two moles of an ideal monoatomic gas, initially at pressure p_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2. (a) Sketch the complete process on a p-V diagram. (b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Given answers in terms of p_1, V_1, V_2, Q and R] [JEE' 99]

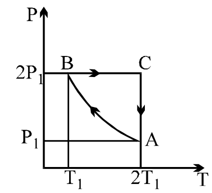
Q.32 A gas has molar heat capacity C = 37.35 J $mole^{-1}K^{-1}$ in the process PT = constant. Find the number of degrees of freedom of molecules in the gas. [REE' 99]

Q.33 A weightless piston divides a thermally insulated cylinder into two parts of volumes V and 3V. 2 moles of an ideal gas at pressure P = 2 atmosphere are confined to the part with volume V = 1 litre. The remainder of the cylinder is evacuated. The piston is now released and the gas expands to fill the entire space of the cylinder. The piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. The ratio of the specific heat of the gas γ = 1.5.
[REE' 99]

Q.34 Two moles of an ideal monatomic gas is taken through a cycle ABCA as shown in the P-T diagram . During the process AB, pressure and temperature of the gas vary such that PT = constant. If T_1 = 300 K, calculate:

(a) the work done on the gas in the process AB and

(b) the heat absorbed or released by the gas in each of the processes.Give answers in terms of the gas constant R. [JEE' 2000]



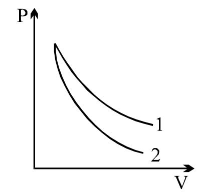
Q.35 One mole of an ideal gas is heated isobarically from the freezing point to the boiling point of water each under normal pressure. Find out the work done by the gas and the change in its internal energy. The amount of heat involved is 1kJ. [REE' 2000]

Q.36 A vertical cylinder of cross-sectional area $0.1m^2$ closed at both ends is fitted with a fricitonless piston of mass M dividing the cylinder into two parts. Each part contains one mole of an ideal gas in equilibrium at 300K. The volume of the upper part is $0.1m^3$ and that of the lower part is $0.05m^3$. What force must be applied to the piston so that the volumes of the two parts remain unchanged when the temperature is increased to 500K? [REE' 2000]

Q.37 P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to

(A) He and O_2 (B) O_2 and He

(C) He and Ar (D) O_2 and N_2



[JEE' 2001]

Q.38 In a given process on an ideal gas, dW = 0 and dQ < 0. then for the gas

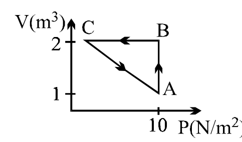
(A) the temperature will decrease. (B) the volume will increase

(C) the pressure will remain constant (D) the temperature will increase [JEE' 2001]

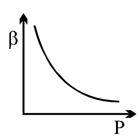
Q.39 An ideal gas is taken through the cycle A $\to$ B $\to$ C $\to$ A, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process C $\to$ A is [JEE(Scr)2002]

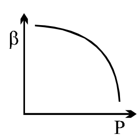
(A) –5J (B) –10 J

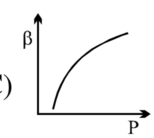
(C) –15 J (D) –20 J

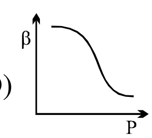


Q.40 Which of the following graphs correctly represents the variation of $\beta = -(dV/dP)/V$ with P for an ideal gas at constant temperature? [JEE (Scr)2002]

(A) 

(B) 

(C) 

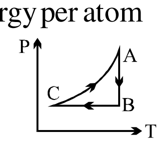
(D) 

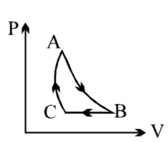
Q.41 A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of $100\ N/m^2$. During an observation time of 1 second, an atom travelling with the root mean square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = 25/3$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K. [JEE'2002]

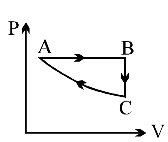
(a) Evaluate the temperature of the gas ; (b) Evaluate the average kinetic energy per atom

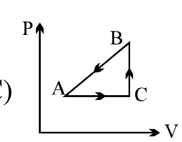
(c) Evaluate the total mass of helium gas in the box.

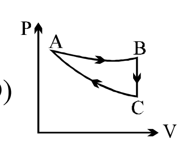
Q.42 In the figure AC represent Adiabatic process. The corresponding PV graph is [JEE (Scr) 2003]



(A) 

(B) 

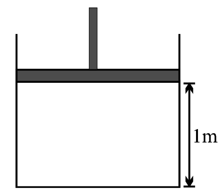
(C) 

(D) 

Q.43 An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. [JEE 2003]

Q.44 An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then [JEE' 2004 (Scr)]
(A) $P_3 > P_1$, $W > 0$ (B) $P_3 < P_1$, $W < 0$
(C) $P_3 > P_1$, $W < 0$ (D) $P_3 = P_1$, $W = 0$

Q.45 The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction. [JEE' 2004]

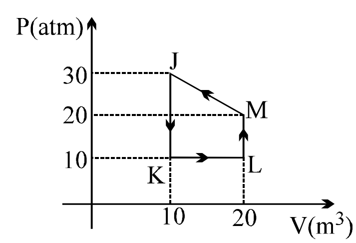


Q.46 An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100Ω resistor carrying current 1A for 5 minutes supplies heat to the gas. The change in internal energy of the gas is
(A) 10 KJ (B) 20 KJ (C) 30 KJ (D) 0 KJ
[JEE' 2005 (Scr)]

Q.47 When the pressure is changed from $p_1 = 1.01 \times 10^5$ Pa to $p_2 = 1.165 \times 10^5$ a then the volume changes by 10%. The bulk modulus is
(A) 1.55×10^5Pa (B) 0.0015×10^5Pa (C) 0.015×10^5Pa (D) none of these
[JEE' 2005 (Scr)]

Q.48 A cylinder of mass 1 kg is given heat of 20000 J at atmospheric pressure. If initially temperature of cylinder is 20°C, find
(a) final temperature of the cylinder
(b) work done by the cylinder.
(c) change in internal energy of the cylinder.
(Given that specific heat of cylinder = 400 J kg^{-1} °C^{-1}, Coefficient of volume expansion =9×10^{-5} °C^{-1}, Atmospheric pressure = 10^5 N/m^2 and density of cylinder = 9000 kg/m^3) [JEE 2005]

Q.49 Match the following for the given process :



Column 1		Column 2
(A) Process J → K	(P)	w > 0
(B) Process K → L	(Q)	w < 0
(C) Process L → M	(R)	Q > 0
(D) Process M → J	(S)	Q < 0

[JEE 2006]

ANSWER KEY

EXERCISE – I

Q.1 $\pi/2$atm-lt Q.2 27 : 4 Q.3 100 J Q.4 $RT\left[1-\frac{1}{n}\right]$

Q.5 $\frac{3}{2}\rho_0$ Q.6 2 R Q.7 327 °C Q.8 $1-\frac{3\left(1-\frac{1}{2^{1/3}}\right)}{ln 2}$

Q.9 1.25×10^4 N/m^2 Q.10 1.5 Q.11 4/3 Q.12 1.5

Q.13 5R Q.14 $\frac{mRT_0}{V_0}\left(1+\frac{T_0 m}{V_0}\right)R$ Q.15 $\sqrt{2}$

Q.16 74.9 cm Q.17 1.84P, 10V/3, 8V/3 (adiabatic), 13P/6, 30V/13, 48V/13 (isothermal)

Q.18 4 : 1 Q.19 P/n Q.20 $C_V + \frac{R}{\alpha V}$ Q.21 16/15

Q.22 $\frac{R}{2}$ Q.23 400 K Q.24 3600 R

Q.25 (i) $\left(\frac{\gamma+1}{\gamma-1}\right) 4aV_0^2$, (ii) $\left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$ Q.26 12600 J Q.27 50 calorie

Q.28 $R\Delta T\left(\frac{3-2\gamma}{\gamma-1}\right)$

Q.29 the molar mass of the gas is 40 gm, the number of degrees of freedom of the gas molecules is 6

Q.30 (i) $P_1 < P_2$, $T_1 < T_2$; (ii) $T_1 = T_2 < T_3$; (iii) $V_2 > V_1$; (iv) $P_1 > P_2$

EXERCISE – II

Q.1 (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm Q.2 0.497 atm, 0.0572 gm Q.3 0.263

Q.4 $p_1 = p_{H_2} \simeq 1.25 \times 10^6$ Pa ; $p_2 = p_{H_2} + p_{O_2} + p_{N_2} \simeq 2.8125 \times 10^6$ Pa ; $p_3 = p_{H_2} + p_{N_2} \simeq 1.5625 \times 10^6$ Pa

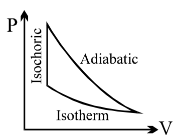
Q.5 750 K Q.6 $P = \frac{13}{12}P_0$, $l_1 = 0.6\, l$, $l_2 = 1.5\, l$, $l_3 = 0.9\, l$

Q.7 $31P_0V_0$; $-5P_0V_0$ Q.8 (i) 7.594, (ii) 4/9, (iii) – 2.5

Q.9 (ii) $P_b = P_c = 2$ atm, (iii) $T_b = 300$ K, $T_c = 600$ K, (iv) $V_c = 8$ litre

Q.10 (i) 1.263×10^5 Nm^{-2} ; 415 J, (ii) 302.8 K ; 1.275×10^5 Nm^{-2} Q.11 1.6 m, 364 K

Q.12 1.63 Q.13 $\frac{3-2\,ln 2}{3}$ Q.14 4 : 9 : 5 Q.15 7.71 J

Q.16 8000 cal. 

Q.17 (a) $\frac{L}{kA}\left(q-qe^{-\frac{kAt}{nLC_P}}\right)+T_0$, (b) $T_0+\frac{qL}{kA}$, (c) $\frac{qL}{kAT_0}+1$

Q.18 $2\,mnv^2\cos^2\theta$

Q.19 (a) $W_{AD} = 88$ J, (b) $V_C = 1.223$ litre , (c) $W_{CDA} = -85$ J

Q.20 (a) Final pressure in A $= \frac{27}{8}\ P_0 =$ Final pressure in C, Final pressure in B $= \frac{21}{4}P_0$

(b) Final temperature in A (and B) $= \frac{21}{4}T_0$, Final temperature in C $= \frac{3}{2}T_0$,

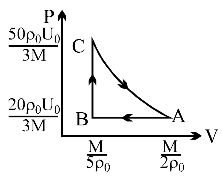
(c) $18\ P_0V_0$,

(d) work done by gas in A $= +\ P_0V_0$, work done by gas in B = 0,

(e) $\frac{17}{2}P_0V_0$

Q.21 four

Q.22 (a)



, (b) $Q = \left(\frac{10}{3}ln2.5 - 2\right)U_0$, (c) $-2U_0$

Q.23 $\frac{7R}{2M}$, $4R(T_2 - T_1)$

Q.24 (a) 80 J, 180 J, (b) 4.5 R

Q.25 $T_1 = (207/16)\ T_0$; $T_2 = \frac{9}{4}T_0$, $-\frac{15}{8}P_0V_0$

Q.26 $\frac{5}{4}Kl^2 + \frac{1}{2}\ (13\sqrt{2} - 7)\ P_0Al$

Q.27 (a) 2×10^5 N/m^2 ; (b) 6 rad/s, (c) 75 J

Q.28 C = 2 R

EXERCISE – III

Q1. K T

Q2. $m_H = 2.5 \times 10^{-3}$ kg , $m_{He} = 7.5 \times 10^{-3}$ kg

Q3. 369.3 K, 2.462×10^5 Pa

Q4. $1/2^{0.67}$

Q5. (i) 765 J, (ii) 10.83%

Q6. 23.928 g ; 4.072 g

Q7. $n_B = 2$; 401 ms^{-1} ; 0.167 % ; – 0.0248 V/T

Q8. (a) 1.8×10^5J ; (b) 4.8×10^5J ; (c) 6.6×10^5J ; (d) 17J/mol-K

Q9. B

Q10. (i) 189 K, (ii) –2767 J, (iii) 2767 J

Q11. 8.08 x 10^5 Pa

Q 12. 55 J

Q13. D

Q14. (i) $T_A = 120.33$ K, $T_B = 240.66$ K, $T_C = 481.32$ K, $T_D = 240.66$ K, (ii) No,
(iii) $\Delta Q_{ABC} = 3.25 \times 10^6$ J ; $\Delta Q_{ADC} = 2.75 \times 10^6$ J

Q15. 7730 K

Q16. 2000N/m, 1295J

Q17. 3.3×10^5 N/m^2, 338.71 K

Q18. $T_B = 909$ K, $T_D = 791$ K, $\eta = 61.4$ %

Q19. C

Q20. C

Q21. AC, 170 J, 10 J

Q22. D

Q23. C

Q24. A

Q25. C, D

Q26. B

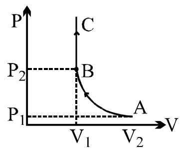
Q27. B,C

Q28. (a) P_oV_o, (b) $5/2\ P_oV_o$, $3P_oV_o$, (c) $1/2\ P_oV_o$, (d) $T_{max} = 25/8\ P_oV_o/R$

Q29. C

Q30. D

Q31. (a)



(b) $W = 3/2P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$; $\Delta U = 3/2\ P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3}-1\right]+Q$, Final $T = \frac{Q}{3R}+\frac{P_1V_2}{2R}\left(\frac{V_1}{V_2}\right)^{5/3}$

Q32. 5

Q33. 400 J, $2\ T_0$

Q34. (a) 1200R, (b) $Q_{AB} = -2100R$, $Q_{BC} = 1500R$, $Q_{CA} = 1200\ R\ ln2$

Q35. W = 830 J, U=170 J

Q36. 1660 N

Q37. B

Q38. A

Q.39 A

Q.40 A

Q.41 160 K, 3.3×10^{-21} J, 0.3 gm

Q 42. A

Q.43 $\Delta T = \frac{mv_0^2}{3R}$

Q.44 C

Q.45 $T_3 = 400\left(\frac{4}{3}\right)^{0.4}$ K

Q.46 C

Q.47 A

Q.48 (a) $T_{final} = 70°C$, (b) 0.05 J, (c) 19999.95 J

Q.49 (A) $\rightarrow$ S; (B) $\rightarrow$ P and R; (C) $\rightarrow$ R; (D) $\rightarrow$ Q and S

STUDY PACKAGE

Target: IIT-JEE (Advanced)

SUBJECT: PHYSICS

TOPIC: XI P11. Mechanical Waves

Index:

KEY CONCEPTS

1. **Wave Equation :**

(i) The equation for a progressive wave travelling in the positive x-direction is

$$y = \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

where y is the displacemnet at point x, at time t, A is the amplitude, T is the period and λ is the wavelength.

The frequency is $\frac{1}{T}$ and the velocity of the wave is $\frac{\lambda}{T}$.

(ii) The equation for a stationary wave is

$$y = \left(2A\cos\frac{2\pi x}{\lambda}\right)\sin\frac{2\pi t}{T}$$

(iii) Pitch, loudness and quality are the characteristics of a musical note. Pitch depends on the frequency. Loudness depends on intensity and quality depends on the waveform of the constituent overtones.

(iv) Resonance occurs when the forcing frequency is equal to the natural frequency of a vibrating body.

(v) Velocity of propagation of sound in a gas = $\sqrt{\frac{\gamma P}{D}}$, where D is the density of the gas and γ is the ratio of specific heats.

2. **Vibrating air columns :**

(i) In a pipe of length L closed at one end, the funamental note has a frequency $f_1 = \frac{v}{4L}$, where v is the velocity of sound in air.

(ii) The first overtone $f_2 = \frac{v}{L} = 2f_1$

3. **Propagation of sound in solids :**

(i) The velocity of propagation of a longitudinal wave in a rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

(ii) The velocity of propagation of a transverse wave in a streched string

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension in the string and m is the mass per unit length of the string.

(iii) In a sonometer wire of length L and mass per unit length m under tension T vibrating in n loops

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

(iv) Propagation of sound in gases

Laplace formula $v = \sqrt{\dfrac{\gamma P}{\rho}}$

where γ is the ratio of specific heats, P is the pressure and ρ is the density.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$

4. **Doppler Effects :**

(i) When a source of sound moves with a velocity v_s in a certain direction, the wavelength decreases in front of the source and increases behind the source.

$$\lambda' \text{ (in front)} = \frac{v - v_s}{f_s} \; ; \; f' = \frac{v}{\lambda'} = \frac{v}{v - v_s} f_s$$

$$\lambda'' \text{(behind)} = \frac{v + v_s}{f_s} \; ; \; f'' = \frac{v}{\lambda''} = \frac{v}{v + v_s} f_s$$

Here v is the velocity of sound in air.

(ii) The apparent frequency $= \dfrac{v - v_0}{v} f_s$

(a) When the source is moving towards the observer and the observer is moving away from the source, the apparent frequency

$$f_a = \frac{v - v_0}{v - v_s} f_s \qquad v_0 \longleftarrow o \quad v_s \longleftarrow s$$

(b) When the source and the observer are moving towards each other.

$$f_a = \frac{v + v_0}{v - v_s} f_s \qquad o \longrightarrow v_0 \quad v_s \longleftarrow s$$

(c) When the source and observer are moving away from each other,

$$f_a = \frac{v - v_0}{v + v_s} f_s \qquad v_0 \longleftarrow o \quad s \longrightarrow v_s$$

(d) When the source is moving away from the observer and the observer is moving towards the source

$$f_a = \frac{v + v_0}{v + v_s} f_s \qquad o \longrightarrow v_0 \quad s \longrightarrow v_s$$

Here all velocities are relation to the medium.

5. **Loudness of sound :**

The loudness level B of sound is expressed in decibels,

$$B = 10 \log \frac{I}{I_0}$$

where I is the intensity, I_0 is a reference intensity.

6. **Beats :**

When two tuning forks of close but different frequencies f_1 and f_2 are vibrating simultaneously at nearby places, a listener observes a fluctuation in the intensity of sound, called beats. The number of beats heard per second is $f_1 - f_2$.

EXERCISE–I

Q.1 Two stationary sources A and B are sounding notes of frequency 680 Hz. An observer moves from A to B with a constant velocity u. If the speed of sound is 340 ms^{-1}, what must be the value of u so that he hears 10 beats per second?

Q.2 Find the intensity of sound wave whose frequency is 250 Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is 1 kg/m^3, bulk modulus of elasticity of the medium is 400 N/m^2.

Q.3 Two strings A and B with $\mu = 2$ kg/m and $\mu = 8$ kg/m respectively are joined in series and kept on a horizontal table with both the ends fixed. The tension in the string is 200 N. If a pulse of amplitude 1 cm travels in A towards the junction, then find the amplitude of reflected and transmitted pulse.

Q.4 A parabolic pulse given by equation y (in cm) $= 0.3 - 0.1(x - 5t)^2$ $(y \geq 0)$ x in meter and t in second travelling in a uniform string. The pulse passes through a boundary beyond which its velocity becomes 2.5 m/s. What will be the amplitude of pulse in this medium after transmission?

Q.5 A car moving towards a vertical wall sounds a horn. The driver hears that the sound of the horn reflected from the cliff has a pitch half-octave higher than the actual sound. Find the ratio of the velocity of the car and the velocity of sound.

Q.6 The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2 :1. Find the ratio of the length of pipe to the length of string.

Q.7 A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork.

Q.8 Tuning fork A when sounded with a tuning fork B of frequency 480 Hz gives 5 beats per second. When the prongs of A are loaded with wax, it gives 3 beats per second. Find the original frequency of A.

Q.9 The loudness level at a distance R from a long linear source of sound is found to be 40dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level & amplitude at a point located at a distance '10R' from the source.

Q.10 A sonometer wires resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by M, the wire resonates with the same tuning fork forming three antinodes for the same position of bridges. Find the value of M.

Q.11 A car is moving towards a huge wall with a speed = c/10 , where c = speed of sound in still air. A wind is also blowing parallel to the velocity of the car in the same direction and with the same speed. If the car sounds a horn of frequency f, then what is the frequency of the reflected sound of the horn heared by driver of the car?

Q.12 A 40 cm long wire having a mass 3.2 gm and area of c.s. 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode. It vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire.

Q.13 A steel rod having a length of 1 m is fastened at its middle. Assuming young's modulus to be 2×10^{11} Pa, and density to be 8 gm/cm^3 find the fundamental frequency of the longitudinal vibration and frequency of first overtone.

Q.14 Two identical sounds A and B reach a point in the same phase. The resultant sound is C. The loudness of C is *n* dB higher than the loudness of A. Find the value of *n*.

Q.15 Sound of wavelength λ passes through a Quincke's tube, which is adjusted to give a maximum intensity I_0. Find the distance through the sliding tube should be moved to give an intensity $I_0/2$.

Q.16 In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level act as a closed organ pipe. A vibrating tuning fork is held above the open end, first and the second resonances occur when the water level is 24.1 cm and 74.1 cm repsectively below the open end. Find the diameter of the tube.[Hint : end correction is 0.3 d]

Q.17 In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is *c*. Find the velocity of sound in the gas.

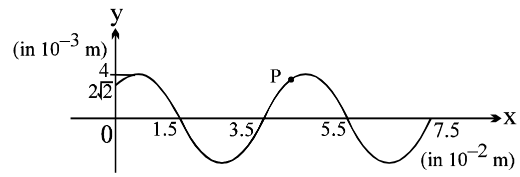
Q.18 A fixed source of sound emitting a certain frequency appears as f_a when the observer is approaching the source with speed v and frequency f_r when the observer recedes from the source with the same speed. Find the frequency of the source.

Q.19 A, B and C are three tuning forks. Frequency of A is 350Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax is put on A beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

Q.20 An open organ pipe filled with air has a fundamental frequency 500Hz. The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbondioxide to be 330 and 264 m/s respectively.

EXERCISE–II

Q.1 The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.



(a) Find the direction in which the wave is moving
(b) the equation of the wave
(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.

Q.2 A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

Q.3 A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At t = 0 the pulse is entirely located between x = 0 and x = 1 m. Draw a graph of the transverse velocity of particle of string versus time at x =+1m.

Q.4 A steel wire 8×10^{-4} m in diameter is fixed to a support at one end and is wrapped round a cylindrical tuning peg 5 mm in diameter at the other end. The length of the wire between the peg and the support is 0.06 m. The wire is initially kept taut but without any tension. What will be the fundamental frequency of vibration of the wire if it is tightened by giving the peg a quarter of a turn?
Density of steel = 7800 kg/m^3, Y of steel = 20×10^{10} N/m^2.

Q.5 The displacement of the medium in a sound wave is given by the equation ; $y_1 = A\cos(ax + bt)$ where A, a & b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave.
(a) what are the wavelength & frequency of the incident wave.
(b) write the equation for the reflected wave.
(c) in the resultant wave formed after reflection, find the maximum & minimum values of the particle speeds in the medium.

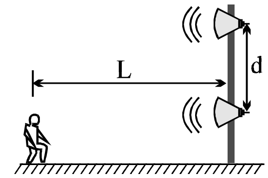
Q.6 In a stationary wave pattern that forms as a result of reflection of waves from an obstacle the ratio of the amplitude at an antinode and a node is β=1.5. What percentage of the energy passes across the obstacle?

Q.7(a) A standing wave in second overtone is maintained in a open organ pipe of length l. The distance between consecutive displacement node and pressure node is ________ .
(b) Two consecutive overtones produced by a narrow air column closed at one end and open at the other are 750Hz and 1050Hz. Then the fundamental frequency from the column is _______.
(c) A standing wave of frequency 1100Hz in a column of methane at 20^{0}C produces nodes that are 20 cm apart. What is the ratio of the heat capacity at constant pressure to that at constant volume.

Q.8 A string, 25cm long, having a mass of 0.25 gm/cm, is under tension. A pipe closed at one end is 40cm long. When the string is set vibrating in its first overtone, and the air in the pipe in its fundamental frequency, 8 beats/sec are heard. It is observed that decreasing the tension in the string, decreases the beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string.

Q.9 A metal rod of length l = 100 cm is clamped at two points. Distance of each clamp from nearer end is a=30cm. If density and Young's modulus of elasticity of rod material are ρ = 9000 kg m^{-3} and Y = 144 GPa respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

Q.10 Two speakers are driven by the same oscillator with frequency of 200 Hz. They are located 4 m apart on a vertical pole. A man walks straight towards the lower speaker in a direction perpendicular to the pole, as shown in figure.

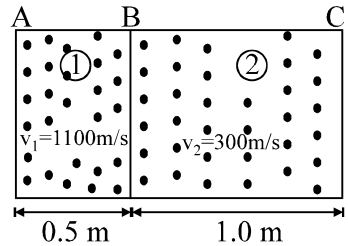


(a) How many times will he hear a minimum in sound intensity, and

(b) how far is he from the pole at these moments?

Take the speed of sound to be 330 m/s, and ignore any sound reflections coming off the ground.

Q.11 A cylinder ABC consists of two chambers 1 and 2 which contains two different gases. The wall C is rigid but the walls A and B are thin diaphragms. A vibrating tuning fork approaches the wall A with velocity u = 30 m/s and air columns in chamber 1 and 2 vibrates with minimum frequency such that there is node (displacement) at B and antinode (displacement) at A. Find



(i) the fundamental frequency of air column.

(ii) Find the frequency of tuning fork.

Assume velocity of sound in the first and second chamber be 1100 m/s and 300 m/s respectively. Velocity of sound in air 330 m/s.

Q.12 A source emits sound waves of frequency 1000 Hz. The source moves to the right with a speed of 32 m/s relative to ground. On the right a reflecting surface moves towards left with a speed of 64 m/s relative to the ground. The speed of sound in air is 332 m/s. Find

(a) the wavelength of sound in air by source

(b) the number of waves arriving per second which meet the reflecting surface.

(c) the speed of reflected waves.

(d) the wavelength of reflected waves.

Q.13 A supersonic jet plane moves parallel to the ground at speed v = 0.75 mach (1 mach = speed of sound). The frequency of its engine sound is $\nu_0 = 2$ kHz and the height of the jat plane is h = 1.5 km. At some instant an observer on the ground hears a sound of frequency $\nu = 2\,\nu_0$, Find the instant prior to the instant of hearing when the sound wave received by the observer was emitted by the jet plane. Velocity of sound wave in the condition of observer = 340 m/s.

Q.14 A train of length l is moving with a constant speed v along a circular track of radius R, The engine of the train emits a whistle of frequency f. Find the frequency heard by a guard at the rear end of the train.

Q.15 A bullet travels horizontally at 660 m/s at a height of 5 m from a man. How far is the bullet from the man when he hears its whistle? Velocity of sound in air = 340 m/s.

EXERCISE–III

Q.1 A metallic rod of length 1 m is rigidly clamped at its mid–point . Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the mid–point . The amplitude of an antinode is 2×10^{-6}m . Write the equation of motion at a point 2 cm from the mid-point and those of the constituent waves in the rod. [Young's modulus = 2×10^{11} Nm^{-2}, density = 8000 Kg m^{-3}].
[JEE '94, 6]

Q.2 A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane . Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle . [JEE '96, 3]

Q.3 Select the correct alternative : [JEE '96, $2 \times 2 = 4$]

(i) The extension in a string, obeying Hooke's law is x . The speed of wave in the stretched string is v. If the extension in the string is increased to 1.5 x , the speed of wave will be
(A) 1.22 v (B) 0.61 v (C) 1.50 v (D) 0.75 v

(ii) An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe . The fundamental frequency of the open pipe is :
(A) 200 Hz (B) 300 Hz (C) 240 Hz (D) 480 Hz

Q.4 A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is : [JEE '97, 1]
(A) 409 (B) 429 (C) 517 (D) 500

Q.5 The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz . Find the lengths of the pipes. [JEE '97, 5]

Q.6 A place progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m & initial phase zero propagates along the (–ve) x–direction with a velocity of 300 m/s . At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is ____ & the corresponding amplitude difference is ____ m. [JEE '97, 2]

Q.7 A band playing music at a frequency f is moving towards a wall at a speed v_b . A motorist is following the band with a speed v_m . If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist . [JEE '97, 5]

Q.8 A travelling in a stretched string is described by the equation $y = A \sin(kx - \omega t)$. The maximum particle velocity is : [JEE '97, 1]
(A) $A\omega$ (B) ω/k (C) $d\omega/dk$ (D) x/t

Q.9 Select the correct alternative(s). [JEE '98, 2 + 2 + 2]

(i) The (x , y) co–ordinates of the corners of a square plate are (0, 0) (L, 0) (L, L) & (0, L). The edges of the plate are clamped & transverse standing waves are set up in it . If u (x, y) denotes t he displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is/are : (a = positive constant)

(A) $a \cos\left(\frac{\pi x}{2L}\right) \cos\left(\frac{\pi y}{2L}\right)$ (B) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

(C) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right)$ (D) $a \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

(ii) A string of length 0.4 m & mass 10^{-2} kg is tightly clamped at its ends . The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt . The minimum value of Δt which allows constructive interference between successive pulses is :
(A) 0.05 s (B) 0.10 s (C) 0.20 s (D) 0.40 s

(iii) A transverse sinusoidal wave of amplitude a, wavelength λ & frequency f is travelling on a stretched string . The maximum speed of any point on the string is $\frac{v}{10}$, where v is speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\ ms^{-1}$, then λ & f are given by :

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-2}$ m (C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

Q.10 The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz . The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe & ΔP_0 the maximum amplitude of pressure variation.
(i) Find the length L of the air column. [JEE '98, 2 + 2 + 2 + 2]
(ii) What is the amplitude of pressure variation at the middle of the column ?
(iii) What are the maximum & minimum pressures at the open end of the pipe .
(iv) What are the maximum & minimum pressures at the closed end of the pipe ?

Q.11 In hydrogen spectrum the wvaelength of H_α line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is, [JEE '99, 2]
(A) 2×10^8 m/s (B) 2×10^7 m/s (C) 2×10^6 m/s (D) 2×10^5 m/s

Q.12 A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2kg. The wire PQR is under a tension of 80N. A sinusoidal wave–pulse of amplitude 3.5cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave–pulse. Calculate
(a) the time taken by the wave–pulse to reach the other end R of the wire, and
(b) the amplitude of the reflected and transmitted wave–pulses after the incident wave–pulse crosses the joint Q. [JEE "99, 4 + 6]

Q.13 As a wave progagates :
(A) the wave intensity remains constant for a plane wave
(B) the wave intensity decreases as the inverse of the distance from the sounce for a spherical wave
(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(D) total power of the sherical wave over the spherical survace centered at the source remains constant at all times. [JEE '99, 3]

Q.14 $y(x, t) = 0.8/[(4x + 5t)^2 + 5]$ represents a moving pulse, where x & y are in meter and t in second . Then:
(A) pulse is moving in +x direction (B) in 2s it will travel a distance of 2.5 m
(C) its maximum displacement is 0.16 m (D) it is a symmetric pulse. [JEE '99, 3]

Q.15 In a wave motion $y = a \sin(kx - \omega t)$, y can represent :
(A) electric field (B) magnetic field (C) displacement (D) pressure [JEE '99, 3]

Q.16 Standing waves can be produced : [JEE '99, 3]
(A) on a string clamped at both the ends
(B) on a string clamped at one end and free at the other
(C) when incident wave gets reflected from a wall
(D) when two identical waves with a phase difference of p αre moving in same direction

Q.17 A train moves towards a stationary observer with speed 34m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17m/s, the frequency registered is f_2. If the speed of sound is 340m/s then the ratio f_1/f_2 is [JEE 2000 (Scr), 1]
(A) 18/19 (B) 1/2 (C) 2 (D) 19/18

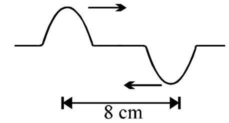
Q.18 Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate container kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by
(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$ (C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$ [JEE 2000 (Scr)]

Q.19 Two vibrating strings of the same material but lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension . Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1/f_2 is given by
(A) 2 (B) 4 (C) 8 (D) 1 [JEE 2000 (Scr), 1]

Q.20 A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain heights in the pipe . Find the heights of water level (from the bottom of the pipe) at which resonances occur . Neglect end correction . Now, the pipe is filled to a height H (~ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurence of first two resonances . Speed of sound in air is 340 m/s and $g = 10\ m/s^2$. [JEE 2000, 10]

Q.21 The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then [JEE 2001 (Scr)]
(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

Q.22 Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be



(A) zero (B) purely kinetic
(C) purely potential (D) partly kinetic and partly potential
[JEE 2001 (Scr)]

Q.23 A boat is travelling in a river with a speed of 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream ?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/sec in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver. (Temperature of the air and water = 20°C ; Density of river water = 10^3 Kg/m^3; Bulk modulus of the water = 2.088×10^9 Pa; Gas constant R = 8.31 J/mol-K; Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4) [JEE 2001, 5 + 5]

Q.24 A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is [JEE 2002 (Scr), 3]

(A) 242/252 (B) 2 (C) 5/6 (D) 11/6

Q.25 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is [JEE 2002 (Scr), 3]

(A) 25 kg (B) 5 kg (C) 12.5 kg (D) 1/25 kg

Q.26 Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of malar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B. [JEE 2002, 3 + 2]

Q.27 A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats then his velocity is [JEE 2003 (Scr)]

(A) 22 m/s (B) 24 m/s (C) 20 m/s (D) 18 m/s

Q.28 A cylindrical tube when sounded with a tuning fork gives, first resonance when length of air column is 0.1 and gives second resonance when the length of air column is 0.35 m. Then end correction is

(A) 0.025 m (B) 0.020 m (C) 0.018 m (D) 0.012 m

[JEE 2003 (Scr)]

Q.29 A stringe between x = 0 and x = l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. [JEE 2003]

Q.30 A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length, 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. [JEE 2003]

Q.31 A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is [JEE' 2004 (Scr)]

(A) $\frac{L}{3}$ (B) $\frac{4L}{3}$ (C) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (D) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

Q.32 A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500m/s and in air it is 300m/s. The frequency of sound recorded by an observer who is standing in air is
(A) 200 Hz (B) 3000 Hz (C) 120 Hz (D) 600 Hz [JEE' 2004 (Scr)]

Q.33 A string fixed at both ends is in resonance in its 2nd harmonic with a tuning fork of frequency f_1. Now its one end becomes free. If the frequency of the tuning fork is increased slowly from f_1 then again a resonance is obtained when the frequency is f_2. If in this case the string vibrates in nth harmonic then

(A) $n = 3, f_2 = \frac{3}{4} f_1$ (B) $n = 3, f_2 = \frac{5}{4} f_1$ (C) $n = 5, f_2 = \frac{5}{4} f_1$ (D) $n = 5, f_2 = \frac{3}{4} f_1$
[JEE' 2005 (Scr)]

Q.34 In a resonance column method, resonance occurs at two successive level of l_1=30.7 cm and l_2 = 63.2 cm using a tuning fork of f = 512 Hz. What is the maximum error in measuring speed of sound using relations $v = f\lambda$ & $\lambda = 2(l_2 - l_1)$
(A) 256 cm/sec (B) 92 cm/sec (C) 128 cm/sec (D) 102.4 cm/sec
[JEE' 2005 (Scr)]

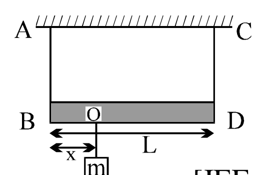
Q.35 A whistling train approaches a junction. An observer standing at junction observers the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed sound = 300 m/s). [JEE 2005]

Q.36 A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform.
[JEE 2005]

Q.37 A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length of BO is

(A) $\frac{L}{5}$ (B) $\frac{L}{4}$

(C) $\frac{4L}{5}$ (D) $\frac{3L}{4}$



[JEE 2006]

Comprehension –I

Q.38 Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along x-axis. Find the number of times intensity is maximum in time interval of 1 sec.

(A) 4 (B) 6 (C) 8 (D) 10 [JEE 2006]

Q.39 Find wave velocity of louder sound

(A) 100 m/s (B) 192 m/s (C) 200 m/s (D) 96 m/s [JEE 2006]

Q.40 At x = 0 how many times the value of $y_1 + y_2$ is zero in one second?

(A) 100 (B) 46 (C) 192 (D) 96 [JEE 2006]

ANSWER KEY

EXERCISE–I

Q.1 $2.5\ \text{ms}^{-1}$ Q.2 $\dfrac{\pi^2\times10^{-9}}{4}\ \text{W/m}^2$ Q.3 $A_r = -\dfrac{1}{3}$ cm, $A_t = \dfrac{2}{3}$ cm Q.4 0.2 cm

Q.5 1 : 5 Q.6 1 : 1 Q.7 1% Q.8 485 Hz

Q.9 30 dB, $10\sqrt{10}\ \mu$m Q.10 25 kg Q.11 11f / 9

Q.12 $1\times10^{9}\ \text{Nm}^2$ Q.13 2.5 kHz, 7.5 kHz Q.14 6 Q.15 $\lambda/8$

Q.16 3 cm Q.17 2c/3 Q.18 $\dfrac{f_r+f_a}{2}$ Q.19 345, 341 or 349 Hz

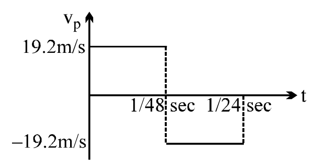
Q.20 33 cm and 13.2 cm

EXERCISE–II

Q.1 (a) negative x ; (b) $y = 4\times10^{-3}\sin 100\pi\left(3t + 0.5x + \dfrac{1}{400}\right)$ (x , y in meter) ; (c) $72\pi^2\times10^{-5}$ J

Q.2 $\dfrac{\pi}{\sqrt{2}\omega}$

Q.3



Q.4 10800 Hz

Q.5 (a) $2\pi/a$, $b/2\pi$, (b) $y_2 = \pm\,0.8\,A\cos(ax - bt)$, (c) max.=1.8 b A, min. = 0,

Q.6 96 % Q.7 (a) $l/6$; (b) 150 Hz; (c)1.28 Q.8 67.6 N

Q.9 10kHz, 30kHz Q.10 (a) 2 ; (b) 9.28 m and 1.99 m

Q.11 1650 Hz, 1500 Hz

Q.12 (a) 0.3 m, (b) 1320, (c) 332 m/s, (d) 0.2 m Q.13 5.9 sec Q.14 f Q.15 9.7 m

EXERCISE–III

Q.1 $y = 2\times10^{-6}\sin(0.1\pi)\cos(25000\,\pi t + \theta)$, for $\theta = 0$: $y_1 = 10^{-6}\sin(5\pi x - 25000\pi t)$, $y_2 = 10^{-6}\sin(5\pi x + 25000\,\pi t)$

Q.2 $f_{max} = 484$ Hz , $f_{min} = 403.3$ Hz Q.3 (i) A, (ii) A

Q.4 D Q.5 $L_c = 0.75$ m ; $L_o = 0.99$ m or 1.006 m Q.6 π rad, 0 m

Q.7 $\dfrac{2v_b(v+v_m)f}{v^2 - v_b^{\,2}}$ Q.8 A Q.9 (i) B, C (ii) B, (iii) A,C

Q.10 (i) $L = \frac{15}{16}$ m, (ii) $\frac{\Delta P_0}{\sqrt{2}}$, (iii) $P_{max} = P_{min} = P_0$, (iv) $P_{max} = P_0 + \Delta P_0$, $P_{min} = P_0 - \Delta P_0$

Q.11 B

Q.12 (a) Time = 140 ms, (b) $A_r = \frac{V_2 - V_1}{V_2 + V_1} A_i = 1.5$ cm ; $A_t = \frac{2V_2}{V_1 + V_2} A_i = 2$ cm

Q.13 A,C,D Q.14 B, C,D Q.15 A, B, C Q.16 A, B, C Q.17 D

Q.18 B Q.19 D Q.20 h = 3.2, 2.4, 1.6, 0.8, 0; $v = 5 \times 10^{-3} \sqrt{5H}$; $\Delta t = 80 (4-2\sqrt{3})$

Q.21 C Q.22 B Q.23 (a) 100696 Hz (b) 103038 Hz Q.24 B Q.25 A

Q.26 (a) 2.116, (b) $\frac{3}{4}$ Q.27 A Q.28 A Q.29 $E = \frac{A^2\pi^2 T}{4l}$ Q.30 336 m/s

Q.31 C Q.32 D Q.33 C Q.34 D Q.35 $V_s = 30$ m/s

Q.36 $y = (10 \text{ cm}) \sin(30\,t \pm \frac{3}{2} x + \phi)$ Q.37 A Q.38 A

Q.39 C Q.40 A