LIMITS, CONTINUITY & DIFFERENTIABILITY

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) **Statement – 1** is False, **Statement – 2** is True.

43. **Statements-1:** The set of all points where the function f(x) = $\begin{cases} 0, & x = 0 \\ \dfrac{x}{1+e^{1/x}}, & x \neq 0 \end{cases}$ is differentiable is $(-\infty, \infty)$.

Statements-2: $Lf'(0) = 1$, $Rf'(0) = 0$ and $f'(x) = \dfrac{1+e^{1/x} - x(e^{1/x} \times \frac{-1}{x^2}}{(1+e^{1/x})^2}$, which exists $\forall x \neq 0$.

44. **Statements-1:** f(x) = $\begin{cases} 3-x^2, & x > 2 \\ x^3+1, & x \le 2 \end{cases}$ then f(x) is differentiable at x = 1

Statements-2: A function y = f(x) is said to have a derivative if

$$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h}$$

45. Consider the function $f(x) = (|x| - |x-1|)^2$
Statement – 1: f(x) is continuous everywhere but not differentiable at x = 0 and 1.
Statement – 2: $f'(0^-) = 0$, $f'(0^+) = -4$, $f'(1^-) = 4$, $f'(1^+) = 0$.

46. **Statement – 1:** $\lim\limits_{x\to 0} \dfrac{e^{1/x}-1}{e^{1/x}+1}$ does not exist
Statement – 2: L.H.L. = 1 and R.H.L. = –1

47. **Statement–1 :** $\lim\limits_{x\to 0} \cos^{-1}(\cos^2 x)$ does not exist
Statement–2 : $\operatorname{cosec}^{-1}x$ is well defined for $|x| \ge 1$.

48. Let $f : [0, 2] \to [0, 2]$ be a continuous function
Statement–1 : f(x) = x for at least one $0 \le x \le 2$
Statement–2 : f(x) = – x for at least one $0 \le x \le 2$

49. Let h(x) = f(x) + g(x) and f′(a), g′(a) are finite and definite
Statement–1 : h(x) is continuous at x = 9 and hence $h(x) = x^2 + 1 \cos x|$ is continuous at x = 0
Statement–2 : h(x) is differentiable at x = a and hence $h(x) = x^2 + |\cos x|$ is differentiable at x = 0

50. **Statement–1** : $f(x) = e^{|x|}$ is non differentiable at x = 0.
Statement–2 : Left hand derivative of f(x) is – 1 and right hand derivative of f(x) is 1.

51. **Statement–2** : $\lim\limits_{x\to 0} [\cos x] = 0$, where [x] = G.I.F
Statement–2 : as $x \to 0$, cos x lies between 0 and 1.

52. **Statement–1** : $\lim\limits_{x\to\infty} \sec^{-1}\left(\dfrac{x}{x+1}\right)$ does not exist.
Statement–2 : $\sec^{-1} t$ is defined for those t, whose modulus value is more than or equal to 1.

53. Suppose [·] and {·} denotes the greatest integer function and fractional part function respectively. Let $f(x) = \{x\} + \sqrt{\{x\}}$.
Statement–1 : f is not differentiable at integrable points.
Statement–2 : f is not continuous at integral points.

54. **Statement–1** : $\lim\limits_{x\to 0} \dfrac{2^{1/x}}{1+2^{1/x}} = 1$. **Statement–2** : $\lim\limits_{x\to 0} \dfrac{\cos^{-1}(1-x)}{\sqrt{x}} = \sqrt{2}$.

55. **Statement–1** : The number of points of discontinuity of f(x) is all 0. Where $f(x) = \int_0^x t\sin\left(\frac{1}{t}\right)$.

Statement–2 : The function h(x) = max {- x, 1, x^2} $\forall$ b x $\in$ R, is not differnetiable at two values of x.

56. **Statement–1 :** If p, q, r all are positive , then $\lim_{x\to\infty}\left(1+\frac{1}{p+qx}\right)^{r+sx}$ is $e^{s/q}$

Statement–2 : $\lim_{x\to 0}(1+x)^{1/x}$ = e.

57. **Statement–1 :** For f(x) = $||x^2| - 4|x||$, the number of points of non differentiability is 3.
Statement–2 : A continuous function is always differentiable

58. **Statement–1 :** If f(x) = x (1 – logx) then for 0 < a < c < b
(a – b logc = b (1 – log b) – a (1 – loga)

Statement–2 : If f(x) is diff. (a , b) and cont. in [a, b] then for at least one a < c < b $f'(c) = \frac{f(b)-f(a)}{b-a}$

59. **Statement – 1 :** Let {x} denotes the fractional part of x. Then $\lim_{x\to 0}\frac{\tan\{x\}}{\{x\}}=1$

Statement – 2 : $\lim_{x\to 0}\frac{\tan x}{x}=1$

60. **Statement – 1 :** $\int_0^t \sin x\,dx$ = 1 – cost **Statement – 2 :** sinx is continuous in any closed interval [0, t]

61. **Statement – 1 :** $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]=0$ where [·] G.I.F. **Statement – 2 :** $\left[\lim_{x\to 0}\frac{\sin x}{x}\right]$ = 1

62. **Statement – 1 :** The function f(x) = $\frac{1}{x-4}$ is continuous at a point x = a $\neq$4.

Statement – 2 : For x = a, f(x) has a definite value and as x $\to$ a, f(x) has a limit which is also equal to its definite value of x = a $\neq$ 4.

63. **Statements-1:** $\lim_{x\to 0^+} x\sin\frac{1}{x}=1$ **Statements-2:** $\lim_{y\to\infty} y\sin\frac{1}{y}=1$

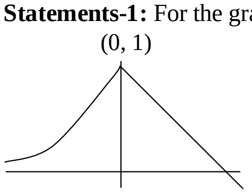
64. **Statements-1:** f(x) = $\lim_{n\to\infty}(\sin x)^{2n}$, then the set of points of discontinuities of f is {(2n + 1) π/2, n$\in$I}

Statements-2: Since -1 < sinx < 1, as n $\to\infty$, $(\sin x)^{2n}\to 0$, sinx = $\pm$ 1 $\Rightarrow \pm(1)^{2n}\to 1$, n $\to\infty$.

65. **Statements-1:** f(x) = $\lim_{n\to\infty}(\cos x)^{2n}$, then f is continuous everywhere in (-∞, ∞)

Statements-2: f(x) = cosx is continuous everywhere i.e., in (-∞, ∞)

66. **Statements-1:** For the graph of the function y = f(x) the valid statement is



f(x) is differentiable at x = 0
Statements-2: Lf ′(c) = R f ′(c), we say that f ′(c) exists and Lf ′(c) = Rf ′(c) = f ′(c).

67. **Statements-1:** $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]=1$

Statements-2: $\lim_{x\to a}$ f(g(x)) = f(L) where $\lim_{x\to a} g(x) = L$. Also function ‘f’ must be continuous at L.

68. **Statements-1:** f(x) = max (1, x^2, x^3) is differentiable $\forall$x$\in$R except x = -1, 1
Statements-2: Every continuous function is differentiable

69. **Statements-1:** $\lim\limits_{x\to\infty}\dfrac{\sin(2x+2)}{x}=2$

Statements-2: Since sinx has a range of [-1, 1] $\forall\ x\in R \Rightarrow \lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$

70. **Statements-1:** f(x) = $\begin{cases}\dfrac{|\sin x|}{x} & , \quad x>0\\ 1 & , \quad x=0\\ -\dfrac{|\sin x|}{x} & , \quad x<0\end{cases}$, is a continuous function at x = 0

Statements-2: If left hand limit = right hand limit & both the limits exists finitely then function can be made continuous.

71. **Statements-1:** f(x) = x|x| is differentiable at every point in its domain.
Statements-2: If f(x) is as a derivative at every point & g(x) has a derivative at every point in their domains, then h(x) = f(x).g(x) is differentiable at every point in its domain.

72. **Statements-1:** x = cosx for some x ∈ (0, π/2)
Statements-2: If f(x) is a continuous in an interval I and f(a) & f(b) are two values at a & b & c is any value in between f(a) & f(b), then there is some x in (a, b) where f(x) = c.

73. **Statements-1:** f : R → R and f(x) = $e^x - e^{-x}$ the range of f(x) is R
Statements-2: If f(x) is a continuous function in [a, b] then f(x) will take all values in between f(a) and f(b).

74. **Statements-1:** If a < b < c < d then (x – a) (x – c) – λ(x – b) (x – d) = 0 will have real for all λ ∈ R.
Statements-2: If f(x) is a function $f(x_1)\ f(x_2) < 0$ then f(x) = 0, for at least one $x \in (x_1, x_2)$.

75. **Statements-1:** $\lim\limits_{x\to 0}\dfrac{1}{x^2}=\infty$

Statements-2: If $\lim\limits_{x\to a}\dfrac{1}{x^2}=\infty,$ then for every positive number G arbitrarily assign (however large) there exist a δ > 0 such that for all x ∈ (a – δ, a) ∪ (a, a + δ) f(x) – a > 0.

76. **Statements-1:** The maximum and the minimum values of the function $f(x)=\dfrac{e^x+e^{-x}}{2},\ -1\le x\le 3,$ exists.

Statements-2: If domain of a continuous function is in closed interval then its range is also in a closed interval.

77. **Statements-1:** For any function y = f(x) $\lim\limits_{x\to a}f(x)=f(a)$

Statements-2: If f(x) is a continuous function at x = a then $\lim\limits_{x\to a}f(x)=f(a)$

78. **Statements-1:** $\lim\limits_{n\to\infty}\dfrac{(\underline{|n})^{1/x}}{x}=\dfrac{1}{e}$

Statements-2: If y = f(x) is continuous in (a, b) then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{n}f\left(\dfrac{r}{n}\right)=\int\limits_a^b f(x)dx.$

79. **Statements-1:** If f is finitely derivable at c, then f is continuous at c.
Statements-2: If at x = c both LHD and RHD exist finitely but LHD ≠ RHD then f(x) is continuous at x = c.

80. **Statements-1:** If f(x + y) = f(x) + f(y), then f is either differentiable everywhere or not differentiable everywhere
Statements-2: Any function is either differentiable everywhere or not differentiable everywhere.

81. **Statements-1:** The function f(x) = $|x^3|$ is differentiable at x = 0
Statements-2: At x = 0 f′(x) = 0

82. **Statements-1:** : When |x| < 1 $\lim\limits_{n\to\infty}\dfrac{\log(x+2)-x^{2n}\cos x}{x^{2n}+1}=\log(x+2)$
Statements-2: For –1 < x < 1, as $n\to\infty, x^{2n}\to 0$.

83. **Statements-1:** : f(x) = $\dfrac{1}{x-[x]}$ is discontinuous for integral values of x. where [.] denotes greatest integer function. **Statements-2:** For integral values of x, f(x) is undefined.

84. **Statements-1:** : $f(x) = x^n \sin\left(\frac{1}{x}\right)$ is differentiable for all real values of x $(n \geq 2)$

Statements-2: for $n \geq 2$ right hand derivative = Left hand derivative (for all real values of x).

85. **Statements-1:** The function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is discontinuous at x = 0.

Statements-2: f(0) = 0.

86. **Statements-1:** The function f(x) defined by $\begin{cases} x & \text{for } x < 1 \\ 2-x & \text{for } 1 \leq x \leq 2 \\ -2+3x-x^2 & \text{for } x > 2 \end{cases}$ is differentiable at x = 2.

Statements-2: L.H.D. at x = 2 = R.H.D. at x = 2

87. **Statements-1:** $\lim_{x\to 0} \sec^{-1}\left[\frac{\sin x}{x}\right] = 0$ [.] denotes greatest integer function.

Statements-2: $\lim_{x\to 0} \sec^{-1}\left[\frac{\tan x}{x}\right] = 0$ [.] denotes greatest integer function.

88. **Statements-1:** $f(x) = \begin{cases} 2x+1 & x < 1 \\ x^2+x+1 & 1 \leq x < 2 \\ x^3-1 & x \geq 2 \end{cases}$ is continuous at $x = 1, 2$

Statements-2: $f'(1^-) = 2$ $f'(1^+) = 3$, $f'(1^-) = 5$ $f'(2^+) = 6$.

89. **Statements-1:** $\lim_{x\to 0} \frac{e^{1/x}}{x}$ does not exist

Statements-2: Right hand limit as $x \to 0$ does not exist

90. **Statements-1:** $\lim_{x\to 0} (1+3x)^{1/x} = e^3$ **Statements-2:** since $\lim_{x\to 0} (1 + x)^{1/x} = e$

91. **Statements-1:** sinx = 0 has atleast one roots between $(-\pi/2, \pi/2)$
Statements-2: Since sinx is continuous in $[-\pi/2, \pi/2]$ and $\sin(-\pi/2) = -1$, $\sin(\pi/2 = 1$ i.e. sinx has opposite sign is at $x = -\pi/2$, $x = \pi/2$, by intermediate theorem

92. **Statements-1:** Let $f(x) = \frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, x \neq 0$ = 0, x = 0 then f(x) has a jump discontinuity at x = 0.

Statements-2: Since $\lim_{x\to 0^-} f(x) = 1$

and $\lim_{x\to 0^+} f(x) = 1$

93. **Statements-1:** The set of all points where the function

$f(x) = \begin{cases} 0, & x = 0 \\ \frac{x}{1+e^{1/x}}, & x \neq 0 \end{cases}$ is differentiable $(-\infty, \infty) - \{0\}$

Statements-2: $Lf'(0) = 1$, $Rf'(0) = 0$ is

$f'(x) = \frac{e^{1/x}+e^{1/x}}{(1+e^{1/x})^2}$. which exists $\forall x \neq 0$

94. **Statements-1:** $f(x) = \frac{[x]}{x}, x \neq 0$, where [] denotes greatest integer function, then f(x) is differentiable at x = 1

Statements-2: $L f'(1)$ $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{\frac{[x]}{|x|}-1}{x-1}$

$$= \lim_{x\to 1}\frac{\frac{0}{|x|}}{x-1} - 1 = \lim_{x\to 1^-}\frac{1}{x-1} = -\infty$$ $\therefore$ f′(1) does not exist.

ANSWER KEY

43. D	44. D	45. A	46. C	47. A	48. A	49. C
50. A	51. A	52. A	53. C	54. B	55. B	56. A
57. A	58. A	59. D	60. A	61. B	62. A	63. D
64. A	65. D	66. D	67. D	68. C	69. D	70. B
71. C	72. A	73. A	74. C	75. A	76. A	77. D
78. A	79. A	80. C	81. A	82. A	83. A	84. A
85. B	86. A	87. D	88. A	89. A	90. A	91. A
92. A	93. A	94. A				

SOLUTION

43. (D) Statement-1 is wrong Statement-2 is true.

47. Clearly $\cos^2 x < 1$ in the neighbourhood of the point x = 0 $\Rightarrow$ $\text{cosec}^{-1}(\cos^2 x)$ is well defined at x = 0 but not in the neightbourhood of the point x = 0 $\Rightarrow$ limit does not exist. Hence (A) is the correct option.

48. Clearly $0 \le f(0) \le 2$ and $0 \le f(2) \le 2$
As f(x) is continuous, f(x) attains all values between f(0) and f(2), and the graph will have no breaks. So graph will all the line y = x at are point x at least where $0 \le x \le 2$.

49. Since f′(a) and g′(a) are finite and definite $\Rightarrow$ h′(a) is also finite and definite
$\Rightarrow$ h(x) is differentiable at x = 0
$\Rightarrow$ h(x) is continuous at x = a.

50. (a) $e^{|x|} = \begin{cases} e^x, & x \ge 0 \\ e^{-x}, & x < 0 \end{cases}$ $\therefore$ L.H.D = – 1 R.H.D = 1.

51. (a) Clearly statement – I is true and statement – II is the correct explnation of statement – I.

52. Statement – II is true and correct reasoning for statement – I, because $\lim_{x\to\infty}\frac{x}{x+1} = 1-$.
Hence (a) is the correct answer.

53. Statement – II is false, as for any $n \in I$,
f(n +) = n, f(n –) = n – 1 + 1 = n, f(n) = n
However statement – I is true, as for any $n \in I$

$$f'(n+) = \lim_{h\to 0+}\frac{f(n+h)-f(n)}{h} = \lim_{h\to 0+}\frac{\sqrt{\{n+h\}}}{h} = \lim_{h\to 0+}\frac{1}{\sqrt{h}} = \infty.$$ Hence (c) is the correct answer.

54. $$\lim_{x\to 0}\frac{2^{1/x}}{1+2^{1/x}} = \lim_{x\to 0}\frac{1}{1+2^{-1/x}} = 1$$

$$\lim_{x\to 0}\frac{\cos^{-1}(1-x)}{\sqrt{x}} = \lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$ (let, $\cos^{-1}(1-x) = \theta \Rightarrow 1 - x = \cos\theta$)

$$\lim_{x\to 0^+}\frac{\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} = \sqrt{2}.$$ Hence (b) is the correct answer.

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55. f(x) = $\int_0^x t\sin\left(\frac{1}{t}\right)dt$

$\therefore \quad f'(x) = x\sin\left(\frac{1}{x}\right)$

clearly, $f'(x)$ is a finite number at all x ∈ (0, π).

$\therefore$ f(x) is differentiable at all x ∈ (0, π).

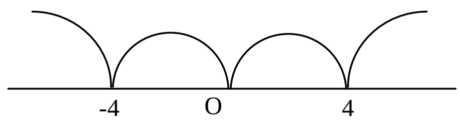
h(x) = $\begin{cases} x^2; & x \le -1 \\ 1; & -1 \le x \le 1 \\ x^2; & x \ge 1 \end{cases}$

from graph it is clear that h(x) is continuous at all x and it is not differentiable at x = - 1, 1.
Hence (b) is the correct answer.

56. (A) Required limit $\lim\limits_{x\to\infty} \frac{1}{ep+qx}$ (r + dx)

$e^{s/q}$

57. Graph of f(x) = $||x^2 - 4|x||$ is



So no of points of non–diff. is 3. Ans. : A

58. (A) f′(c) = $\frac{f(b)-f(a)}{b-a}$

– logc = $\frac{b(1-\log b)-a(1-\log a)}{b-a}$ ⇒ (a – b) logc = b(1 – logb) – a (1 – loga)

63. The **Statements-1:** is false sin as x → O^+, the function xsin $\frac{1}{x}$ = a qtyt. apro^n. zero) × (finite number between 0 & 1). Thus $\lim\limits_{x\to 0^+} x\sin\frac{1}{x} \to 0$.

The Statement-2 is true since it is equivalent to standard limit $\lim\limits_{x\to 0}\frac{\sin x = 1}{x}$

⇒ option (d) is correct.

64. Option (a) is correct. **Statements-1:** is the solution of Statement-2.

65. $\lim\limits_{n\to\infty} x^{2n} = \begin{matrix} 0 \\ 1 \end{matrix}\begin{cases} |x| & <1 \\ |x| & =1 \end{cases}$

f(x) = $\lim\limits_{n\to\infty}$ $(\cos x)^{2n}$ = $\begin{cases} 0, & \text{if } |\cos x| < 1 \\ 1 & \text{if } |\cos x| = 1 \end{cases}$

f(x) is continuous at all x, except for those values of x for which |cosx| = 1
⇒ x = nπ n∈I. Ans. (D)

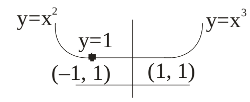
66. from Questions figure clearly
Ans. (D)

67. $\lim\limits_{x\to 0}\left[\frac{\sin x}{x}\right] = 0$

because sinx < x when x > 0

So $\frac{\sin x}{x} < 1$ for x > 0

So $\left[\frac{\sin x}{x}\right] = 0$ for x > 0 because $\frac{\sin x}{x}$ is odd function so it is correct for x < 0.



So, ‘d’ is correct.

68. The graph of max $(1, x^2, x^3)$ is as under clearly function is **NOT** differentiable at x = –1, 1.
Every continuous function is not necessarily differentiable.
So, ‘c’ is correct.

73. (A) $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$ and f(x) is continuous in R then f(x) will take all values in between (–∞, ∞)

74. (C) A quadratic polynomial is always continuous f(b).f(d) < 0 then there exist one value of x ∈ (b, d) at which f(x) = 0 if one root of a real equation is real then another real will also real. If f(x) is not continuous and $f(x_1).f(x_2) < 0$ then we cannot say that there is atleast one $x \in (x_1, x_2)$ at which f(x) = 0.

80. The **Statements-1:** is true. If f is differentiable at ‘c’ then f′(c) exists.

$\Rightarrow \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$ exists $\Rightarrow \lim_{h\to 0}\frac{f(c)+f(h)-f(c)}{h}$ exists

$= \lim_{h\to 0}\frac{f(h)}{h}$ exists. Now if p be some other point then $f'(0) = \lim_{h\to 0}\frac{f(p+h)-f(p)}{h} = \lim_{h\to 0}\frac{f(h)}{h}$
which exists.
Now any function is either differentiable nowhere or differentiable atleast one point, then it is differentiable for all x. Thus assertion is true.
The reason R is false since any function is either differentiable nowhere is differentiable at one point.

81. For x > 0, $f(x) = x^3 \Rightarrow f'(x) = 3x^2 \Rightarrow f'(0) = 0$
for x < 0, $f(x) = -x^3 \Rightarrow f'(x) = -3x^2 \Rightarrow f'(0) = 0$. (A)

82. (a) Both **Statements-1:** and Statement-2 are true and Statement-2 is the correct explanation of **Statements-1:** .

83. (a) For all integral values of x, x – [x] = 0 $\Rightarrow f(x) = \frac{1}{0}$, which is not defined.

∴ **Statements-1:** and Statement-2 both are true and Statement-2 is the correct explanation of **Statements-1:** .

84. (a) $f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{h^n \sin\left(\frac{1}{h}\right)-0}{h}$

$= \lim_{h\to 0} h^{n-1}\sin\left(\frac{1}{h}\right)$ (n ≥ 2) = 0 finite number = 0

Hence **Statements-1:** and Statement-2 both are true and Statement-2 is the correct explanation of **Statements-1:** .

85. (B) $\lim_{x\to 0^-} f(x) = -1$

$\lim_{x\to 0^+} f(x) = 1$ L.H.L. at x = 0, ≠ R.H.L. at x = 0.

86. (A) L.H.D. at x = 2

$= \left\{\frac{d}{dx}(2-x)\right\}_{x=2} = -1$ R.H.D. at x = 2

$= \left\{\frac{d}{dx}(-2+3x-x^2)\right\}_{x=2} = -1$

89. $\lim_{x\to 0}+\frac{e^{1/x}}{x} = \lim_{x\to 0^+}\frac{1+\frac{1}{x}+\frac{1}{2!}\frac{1}{x^2}+\ldots}{x}$

$= \lim_{x\to 0^+}\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{2x^3!}+\ldots\right) = \infty$ (infinits)

$\therefore \lim\limits_{x\to 0^+} \frac{e^{1/x}}{x}$ does not exist **(Ans. A)**

90. $\lim\limits_{x\to 0}(1+3x)^{1/x} \lim\limits_{x\to 0}\left[\left(1+3x^{1/3x}\right)\right]^3$

$= e^3$

because $\lim\limits_{x\to 0}\left(1+x\right)^{1/x} = e$ **Ans. (A)**

91. f(x) = sinx continuous in [-π/2, π/2] by intermediate value theorem

f(-π/2) = sin (-π/2) = -1

$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$ $f\left(-\frac{\pi}{2}\right)$ and $f\left(\frac{\pi}{2}\right)$ are of opposite sign is

∴ by intermediate value theorem,∃ a point

c∈ [-π/2, π/2] such that f(x) = 0

∃s a point x∈ [-π/2, π/2] such that f(x) = 0

i.e., sinx = 0

thus sinx = 0 has at least one root between $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ Ans. (A)

92. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0}\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}} \lim\limits_{x\to 0}\frac{1-e^{-2/x}}{1+e^{-2/x}}$

$\lim\limits_{x\to 0^+} f(x) = 1$ $\lim\limits_{x\to 0^-} f(x) = 1$

x = 0, f(0) = 0 Hence f(x) is discontinuous at x = 0 then Ans. (A)

93. Lf′(0) = $\lim\limits_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0^-} \frac{\frac{x}{1+e^{1/x}}-0}{x}$

Rf′(0) = $\lim\limits_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0^+} \frac{\frac{x}{1+e^{1/x}}-0}{x}$

$= \lim\limits_{x\to 0^+} \frac{1}{1+e^{1/x}} = 0$

L f′(0) ≠ R f′(0) so it is differentiable in (-∞, ∞) – {0}

f′(x) = $\frac{1+e^{1/x}+e^{1/x}}{(1+e^{1/x})^2}$ $\forall x\neq 0$ Ans. (A)

94. Rf′(1) = $\lim\limits_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = x \to 1^+ \frac{\frac{[x]}{|x|}-1}{x-1}$

$= \lim\limits_{x\to 1^+} \frac{\frac{1}{|x|}-1}{x-1} = \lim\limits_{x\to 1^+} \frac{1-x}{x(x-1)} = x \to 1^+ -\frac{1}{x} = -1$

Lf′(1) = ∞ then f′(1) does not exist. then Ans. (A)

For 38 Years Que. of IIT-JEE (Advanced)

& **14 Years Que. of AIEEE (JEE Main)**

we have already distributed a book