fo/u fopkjr Hkh# tu] ugh vkjEHk dke]
foifr n[k NkM rjr e/;e eu dj ';keA
i#"k flg ldYi dj] lgr foifr vud]
^cuk^ u NkM /;; dk] j?kcj jk[k VdAA
jfpr% ekuo /ke' i.krk
/nx# Jh j.kNkMnkl th egkjkt

FUNCTIONS (ASSERTION AND REASON)

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) **Statement – 1** is False, **Statement – 2** is True.

1. Let $f(x) = \cos 3\pi x + \sin \sqrt{3}\pi x$.
Statement – 1 : f(x) is not a periodic function.
Statement – 2 : L.C.M. of rational and irrational does not exist

2. **Statement – 1:** If $f(x) = ax + b$ and the equation $f(x) = f^{-1}(x)$ is satisfied by every real value of x, then $a \in R$ and $b = -1$.
Statement – 2: If $f(x) = ax + b$ and the equation $f(x) = f^{-1}(x)$ is satisfied by every real value of x, then $a = -1$ and $b \in R$.

3. **Statements-1:** If $f(x) = x$ and $F(x) = \frac{x^2}{x}$, then $F(x) = f(x)$ always
Statements-2: At $x = 0$, F(x) is not defined.

4. **Statement–1 :** If $f(x) = \frac{1}{1-x}$, $x \neq 0, 1$, then the graph of the function $y = f(f(f(x)), x > 1$ is a straight line **Statement–2 :** $f(f(f(x))) = x$

5. Let $f(1 + x) = f(1 - x)$ and $f(4 + x) = f(4 - x)$
Statement–1 : f(x) is periodic with period 6
Statement–2 : 6 is not necessarily fundamental period of f(x)

6. **Statement–1 :** Period of the function $f(x) = \sqrt{1+\sin 2x} + e^{\{x\}}$ does not exist
Statement–2 : LCM of rational and irrational does not exist

7. **Statement–1 :** Domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$ is $(-\infty, 0)$ **Statement–2:** $|x| - x > 0$ for $x \in R^-$

8. **Statement–1 :** Range of $f(x) = \sqrt{4-x^2}$ is [0, 2]
Statement–2 : f(x) is increasing for $0 \le x \le 2$ and decreasing for $-2 \le x \le 0$.

9. Let $a, b \in R$, $a \neq b$ and let $f(x) = \frac{a+x}{b+x}$.
Statement–1 : f is a one–one function. **Statement–2 :** Range of f is $R - \{1\}$

10. **Statement–1 :** $\sin x + \cos(\pi x)$ is a non–periodic function.

Statement–2 : Least common multiple of the periods of sin x and cos (πx) is an irrational number.

11. **Statement–1:** The graph of f(x) is symmetrical about the line x = 1, then, f(1 + x) = f(1 – x).
Statement–2 : even functions are symmetric about the y-axis.

12. **Statement–1 :** Period of f(x) = sin $\frac{\pi x}{(n-1)!} + \cos\frac{\pi x}{n!}$ is 2(n)!
Statement–2 : Period of |cos x| + |sin x| + 3 is π.

13. **Statement–1 :** Number of solutions of tan(|$\tan^{-1}$x|) = cos|x| equals 2 **Statement–2** : ?

14. **Statement–1 :** Graph of an even function is symmetrical about y–axis
Statement–2 : If f(x) = cosx has x (+)ve solution then total number of solution of the above equation is 2n. (when f(x) is continuous even function).

15. If f is a polynomial function satisfying 2 + f(x).f(y) = f(x) + f(y) + f(xy) $\forall$ x, y$\in$R
Statement-1: f(2) = 5 which implies f(5) = 26
Statement-2: If f(x) is a polynomial of degree 'n' satisfying f(x) + f(1/x) = f(x). f(1/x), then f(x) = 1 x^n + 1

16. **Statement-1:** The range of the function $\sin^{-1}$ + $\cos^{-1}$x + $\tan^{-1}$x is [π/4, 3π/4]
Statement-2: $\sin^{-1}$x, $\cos^{-1}$x are defined for |x| $\le$ 1 and $\tan^{-1}$x is defined for all 'x'.

17. A function f(x) is defined as f(x) = $\begin{cases} 0 & \text{where x is rational} \\ 1 & \text{where x is irrational} \end{cases}$
Statement-1 : f(x) is discontinuous at xll x$\in$R
Statement-2 : In the neighbourhood of any rational number there are irrational numbers and in the vincity of any irrational number there are rational numbers.

18. Let f(x) = sin $\left(2\sqrt{3}\,\pi x\right) + \cos\left(3\sqrt{3}\,\pi x\right)$
Statement-1 : f(x) is a periodic function
Statement-2: LCM of two irrational numbers of two similar kind exists.

19. **Statements-1:** The domain of the function f(x) = $\cos^{-1}$x + $\tan^{-1}$x + $\sin^{-1}$x is [-1, 1]
Statements-2: $\sin^{-1}$x, $\cos^{-1}$x are defined for |x| $\le$ 1and $\tan^{-1}$x is defined for all x.

20. **Statement-1 :** The period of f(x) = = sin2x cos [2x] – cos2x sin [2x] is 1/2
Statements-2: The period of x – [x] is 1, where [·] denotes greatest integer function.

21. **Statements-1:** If the function f : R $\to$ R be such that f(x) = x – [x], where [·] denotes the greatest integer less than or equal to x, then f^{-1}(x) is equals to [x] + x
Statements-2: Function 'f ' is invertible iff is one-one and onto.

22. **Statements-1 :** Period of f(x) = sin 4π {x} + tan π [x] were, [·] & {·} denote we G.I.F. & fractional part respectively is 1.
Statements-2: A function f(x) is said to be periodic if there exist a positive number T independent of x such that f(T + x) = f(x). The smallest such positive value of T is called the period or fundamental period.

23. **Statements-1:** f(x) = $\frac{x+1}{x-1}$ is one-one function
Statements-2: $\frac{x+1}{x-1}$ is monotonically decreasing function and every decreasing function is one-one.

24. **Statements-1:** f(x) = sin2x (|sinx| - |cosx|) is periodic with fundamental period π/2
Statements-2: When two or more than two functions are given in subtraction or multiplication form we take the L.C.M. of fundamental periods of all the functions to find the period.

25. **Statements-1:** e^x = lnx has one solution.
Statements-2: If f(x) = x $\Rightarrow$ f(x) = f^{-1}(x) have a solution on y = x.

26. **Statements-1:** F(x) = x + sinx. G(x) = -x

H(x) = F(X) + G(x), is a periodic function.
Statements-2: If F(x) is a non-periodic function & g(x) is a non-periodic function then h(x) = f(x) ± g(x) will be a periodic function.

27. **Statements-1:** $f(x)=\begin{cases} x+1, & x\ge 0 \\ x-1, & x<0 \end{cases}$ is an odd function.
Statements-2: If y = f(x) is an odd function and x = 0 lies in the domain of f(x) then f(0) = 0

28. **Statements-1:** $f(x)=\begin{cases} x; & x\in Q \\ -x; & x\in Q^C \end{cases}$ is one to one and non-monotonic function.
Statements-2: Every one to one function is monotonic.

29. **Statement–1 :** Let f : [1, 2] ∪ [5, 6] → [1, 2] ∪ [5, 6] defined as $f(x)=\begin{cases} x+4, & x\in[1,2] \\ -x+7, & x\in[5,6] \end{cases}$ then the equation $f(x) = f^{-1}(x)$ has two solutions.
Statements-2: $f(x) = f^{-1}(x)$ has solutions only on y = x line.

30. **Statements-1:** The function $\frac{px+q}{rx+s}$ (ps – qr ≠ 0) cannot attain the value p/r.
Statements-2: The domain of the function g(y) = $\frac{q-sy}{ry-p}$ is all real except a/c.

31. **Statements-1:** The period of f(x) = sin [2] xcos [2x] – cos2x sin [2x] is 1/2
Statements-2: The period of x – [x] is 1.

32. **Statements-1:** If f is even function, g is odd function then $\frac{b}{g}$ (g ≠ 0) is an odd function.
Statements-2: If f(–x) = –f(x) for every x of its domain, then f(x) is called an odd function and if f(–x) = f(x) for every x of its domain, then f(x) is called an even function.

33. **Statements-1:** f : A → B and g : B → C are two function then $(gof)^{-1} = f^{-1} og^{-1}$.
Statements-2: f : A → B and g : B → C are bijections then f^{-1} & g^{-1} are also bijections.

34. **Statements-1:** The domain of the function $f(x)=\sqrt{\log_2 \sin x}$ is (4n + 1) $\frac{\pi}{2}$, n ∈ N.
Statements-2: Expression under even root should be ≥ 0

35. **Statements-1:** The function f : R → R given $f(x)=\log_a(x+\sqrt{x^2+1})$ a > 0, a ≠ 1 is invertible.
Statements-2: f is many one into.

36. **Statements-1:** φ(x) = sin (cos x) $x\in\left[0,\frac{\pi}{2}\right]$ is a one-one function.
Statements-2: $\phi'(x) \le \forall x \in \left[0,\frac{\pi}{2}\right]$

37. **Statements-1:** For the equation $kx^2 + (2 - k)x + 1 = 0$ k ∈ R – {0} exactly one root lie in (0, 1).
Statements-2: If $f(k_1)\, f(k_2) < 0$ (f(x) is a polynomial) then exactly one root of f(x) = 0 lie in (k_1, k_2).

38. **Statements-1:** Domain of $f(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is {–1, 1}
Statements-2: $x+\frac{1}{x}\ge 2$ when x > 0 and $x+\frac{1}{x}\le -2$ when x < 0.

39. **Statements-1:** Range of f(x) = |x|(|x| + 2) + 3 is [3, ∞)
Statements-2: If a function f(x) is defined ∀ x ∈ R and for x ≥ 0 if a ≤ f(x) ≤ b and f(x) is even function than range of f(x) f(x) is [a, b].

40. **Statements-1:** Period of {x} = 1. **Statements-2:** Period of [x] = 1

41. **Statements-1:** Domain of f = ϕ. If $f(x) = \dfrac{1}{\sqrt{[x]-x}}$

Statements-2: $[x] \le x \ \forall \ x \in R$

42. **Statements-1:** The domain of the function $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is [–1, 1]

Statements-2: $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \le 1$ and $\tan^{-1}x$ is defined for all 'x'

ANSWER KEY

1. A	**2. D**	**3. A**	**4. C**	**5. A**	**6. A**	**7. A**
8. C	**9. B**	**10. C**	**11. A**	**12. C**	**13. B**	**14. A**
15. A	**16. A**	**17. A**	**18. A**	**19. A**	**20. A**	**21. D**
22. A	**23. A**	**24. A**	**25. D**	**26. C**	**27. D**	**28. C**
29. C	**30. A**	**31. A**	**32. A**	**33. D**	**34. A**	**35. C**
36. A	**37. C**	**38. A**	**39. A**	**40. A**	**41. A**	**42. A**

SOLUTIONS

4. $f(f(x)) = \dfrac{1}{1-f(x)} = \dfrac{1}{1-\dfrac{1}{1-x}} = \dfrac{x-1}{x}$

$\therefore$ $f(f(f(x))) = \dfrac{1}{1-f(f(x))} = \dfrac{1}{1-\dfrac{x-1}{x}} = x$ Ans. C

5. f(1 + x) = f(1 – x) ... (1)

f(4 + x) = f(4 – x) ... (2)

$x \to 1 - x$ in (1) $\Rightarrow$ f(1 – x) = f(x) ... (3)

$x \to 4 - x$ in (2) $\Rightarrow$ f(2 – x) f(8 – x) = f(x) ... (4)

(1) and (4) $\Rightarrow$ f(2 – x) = f(8 – x) .... (5)

Use $x \to x - x$ in (5), we get

f(x) = f(6 + x)

$\Rightarrow$ f(x) is periodic with period 6

Obviously 6 is not necessary the fundamental period. Ans. A

6. L.C.M. of {π, 1} does not exist

$\therefore$ (A) is the correct option.

7. (a)

Clearly both are true and statement – II is correct explantion of Statement – I .

8. (c)

$$f'(x) = \frac{-x}{\sqrt{4-x^2}}$$

$\therefore$ f(x) is increasing for $-2 \le x \le 0$ and decreasing for $0 \le x \le 2$.

9. Suppose a > b. Statement – II is true as $f'(x) = \dfrac{b-a}{(b+x)^2}$, which is always negative and hence monotonic in its continuous part. Also $\lim\limits_{x \to -b^+} f(x) = \infty$ and $\lim\limits_{x \to -b^-} f(x) = -\infty$. Moreover

$\lim_{x\to\infty} f(x) = 1+$ and $\lim_{x\to-\infty} f(x) = -1-$. Hence range of f is R – {1}.
F is obviously one–one as $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
However statement – II is not a correct reasoning for statement – I
Hence (b) is the correct answer.

10. Statement – I is true, as period of sin x and cos πx are 2π and 2 respectively whose L.C.M does not exist.
Obviously statement – II is false
Hence (c) is the correct answer.

11. Graph of f(x) is symmetric about the line x = 0 if f(- x) = f(x) i.e. if f(0 – x) = f(0 + x)
∴ Graph of y = f(x) is symmetric about x = 1, if f(1 + x) = f(1 – x).
Hence (a) is the correct answer.

12. Period of sin $\frac{\pi x}{(n-1)!} = 2(n-1)!$

Period of cos $\frac{\pi x}{n!} = 2(n)!$

⇒ Period of f(x) = L.C.M of 2(n – 1)! And 2(n)! = 2(n!)

Now, $f(x) = |\text{copsx}| + |\sin x| + 3 = \sqrt{1+|\sin 2x|} + 3$

∴ f(x) is periodic function with period = $\frac{\pi}{2}$.

Hence (c) is the correct answer.

13. $\tan(|\tan^{-1}x|) = |x|$, since $|\tan^{-1}x| = \tan^{-1}|x|$
Obviously cos|x| and |x| meets at exactly two points
∴ (B) is the correct option.

14. (A)Since cos n is also even function. Therefore solution of cosx = f(x) is always sym. also out y–axis.

19. (a) Both A and R are obviously correct.

20. (a) f(x) = x [x]
f(x + 1) = x + 1 – ([x] + 1) = x – [x]
So, period of x – [x] is 1.
Let f(x) = sin (2x – [2x])

$$f\left(x+\frac{1}{2}\right) = \sin\left(2\left(x+\frac{1}{2}\right) - \left[2\left(x+\frac{1}{2}\right)\right]\right)$$

= sin (2x + 1 – [2x] – 1)
= sin (2x – [2x])
So, period is 1/2

21. f(1) = 1 – 1 = 0 f(0) = 0
∴ f is not one-one
∴ $f^{-1}(x)$ is not defined Ans. (D)

22. Clearly tan π[x] = 0 ∀ x∈R and period of sin 4 π {x} = 1. Ans. (A)

23. $f(x) = \frac{x+1}{x-1}$ $\qquad f'(x) = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2} < 0$

So f(x) is monotonically decreasing & every monotonic function is one-one.
So 'a' is correct.

24. f(x) = sin2x (|sinx| -|cosx|) is periodic with period π/2 because f(π/2 + x) = sin 2 (π/2 + x) (|sin (π/2 + x)| -|cos (π/2 + x)|)
= sin (π + 2x) (|cosx| - |sinx|) = -sin2x (|cosx| - |sinx|)
= sin2x (|sinx| - |cosx|)

Sometimes f(x + r) = f(x) where r is less than the L.C.M. of periods of all the function, but according to definition of periodicity, period must be least and positive, so 'r' is the fundamental period.
So 'f' is correct.

27. (D) If f(x) is an odd function, then $f(x) + f(-x) = 0 \ \forall \ x \in D_f$

28. (C) For one to one function if $x_1 \neq x_2$

$\Rightarrow f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in D_f$ $\sqrt{3} > 1$

but $f(\sqrt{3}) < f(1)$ and $3 > 1$

$f(5) > f(1)$ f(x) is one-to-one but non-monotonic

29. (C) $\left(\frac{3}{2}, \frac{11}{2}\right)$ and $\left(\frac{11}{2}, \frac{3}{2}\right)$ both lie on y = f(x) then they will also lie on $y = f^{-1}(x) \Rightarrow$ there are two solutions and they do not lie on y = x.

30. If we take $y = \frac{px+q}{rx+s}$ then $x = \frac{q-sx}{rx-p} \Rightarrow$ x does not exist if y = p/r

Thus statement-1 is correct and follows from statement-2 (A)

31. f(x) = sin(2x – [2x] $\qquad$ $f(x + 1/2) = \sin\left(2x+1-\left[2\left(x+\frac{1}{2}\right)\right]\right)$

= sin (2x + 1 – [2x] – 1] $\qquad$ = sin (2x – [2x].) i.e., period is 1/2.

f(x) = x – [x]

f(x + 1) = x + 1 – ([x] + 1) = x – [x]

i.e., period is 1. $\qquad$ (A)

32. (A) Let $h(x) = \frac{f(x)}{g(x)}$

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$$

$\therefore$ $h(x) = \frac{f}{g}$ is an odd function.

33. (D) Assertion : f : A $\rightarrow$ B, g : B $\rightarrow$ C are two functions then $(gof)^{-1} \neq f^{-1} og^{-1}$ (since functions need not posses inverses. Reason : Bijective functions are invertibles.

34. (A) for f(x) to be real $\log_2(\sin x) \geq 0$

$\Rightarrow \sin x \geq 2^o$ $\qquad$ $\Rightarrow \sin x = 1 \quad \Rightarrow \quad x = (4n + 1)\frac{\pi}{2}, n \in N.$

35. (C) f is injective since $x \neq y$ (x, y $\in$ R)

$\Rightarrow \log_a\left\{x+\sqrt{x^2+1}\right\} \neq \log_a\left\{y+\sqrt{y^2+1}\right\}$

$\Rightarrow f(x) \neq f(y)$

f is onto because $\log_a\left(x+\sqrt{x^2+1}\right) = y$ $\qquad$ $\Rightarrow x = \frac{a^y - a^{-y}}{2}$.

40. Since {x} = x – [x]

$\therefore$ {x + 1} = x + 1 – [x + 1]

= x + 1 – [x] – 1

= x – [x] = [x]

Period of [x] = 1 $\qquad$ Ans (A)

41. $f(x) = \frac{1}{\sqrt{[x]-x}}$ $[x] - x \neq 0$

$[x] \neq x \rightarrow [x] > x$ It is imposible or $[x] \leq x$
So the domain of f is ϕ
because reason $[x] \leq x$ Ans. (A)