AREA UNDER THE CURVES

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.

(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.

(C) **Statement – 1** is True, **Statement – 2** is False.

(D) **Statement – 1** is False, **Statement – 2** is True.

209. Let $|A_1|$ be the area bounded between the curves $y = |x|$ and $y = 1 - |x|$; $|A_2|$ be the area bounded between the curves $y = -|x|$ and $y = |x| - 1$.
Statement-1: $|A_1| = |A_2|$
Statement-2: Area of two similar parallelograms are equal.

210. **Statement-1:** Area bounded between the curves $y = |x - 3\pi|$ and $y = \cos^{-1}(\cos x)$ is $\pi^2/2$
Statement-2: $|x - 3\pi| = 3\pi - x$ for $5\pi/2 \le x \le 3\pi$
$\cos^{-1}(\cos x) = x - 2\pi$, $2\pi \le x \le 3\pi$

211. **Statement-1:** Area of the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ in the first quadrant is equal to π
Statement-2: Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = a^2$ is πab.

212. **Statement-1:** Area enclosed by the curve $|x| + |y| = 2$ is 8 units
Statement-2: $|x| + |y| = 2$ represents an square of side length $\sqrt{8}$ unit.

213. **Statement-1:** The area bounded by $y = x(x - 1)^2$, the y–axis and the line $y = 2$ is
$$\int_0^2 \left(x(x-2)^2 - 2\right) dx \text{ is equal to } \frac{10}{3}.$$
Statement-2: The curve $y = x(x - 1)^2$ is intersected by $y = 2$ at $x = 2$ only and for $0 < x < 2$, the curve $y = x(x - 1)^2$ lies below the line $y = 2$.

214. Let f be a non–zero odd function and $a > 0$.
Statement-1: $\int_{-a}^{a} f(x) = 0$. Because
Statement-2: Area bounded by $y = f(x)$, $x = a$, $x = -a$ and x–axis is zero.

215. **Statement-1:** The area of the curve $y = \sin^2 x$ from 0 to π will be more than that of the curve $y = \sin x$ from 0 to π.
Statement-2: $x^2 > x$ if $x > 1$.

216. **Statement-1:** The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is least if $k = 0$.
Statement-2: The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is $\sqrt{k^2 + 20}$.

217. **Statement-1:** The area of the ellipse $2x^2 + 3y^2 = 6$ will be more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

Statement-2: The length of the semi-major axis of ellipse $2x^2 + 3y^2 = 6$ is more than the radius of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

218. **Statement-1:** Area included between the parabolas $y = x^2/4a$ and the curve

$$y = \frac{8ab}{x^2+4a^2} \text{ is } \frac{a^2}{3}(6\pi - 4) \text{ sq. units.}$$

Statement-2: Both the curves are symmetrical about y-axis and required area is $\int_{x_1}^{x_2}(y_2 - y_1)\,dx$

219. **Statement-1:** The area of the region bounded by $y^2 = 4x$, $y = 2x$ is 1/3 sq. units.

Statement-2: The area of the region bounded by $y^2 = 4ax$, $y = mx$ is $\frac{8a^2}{3m^3}$ sq. units.

220. **Statement-1:** Area under the curve $y = \sin x$, above 'x' axis between two ordinates $x = 0$ & $x = 2\pi$ is 4 units.

Statement-2: $\int_0^{2\pi} \sin x\,dx = 4$

221. **Statement-1:** Area under the curve $y = [|\sin x| + |\cos x|]$, where [] denotes the greatest integer function. above 'x' axis and between the ordinates = 0 & $x = \pi$ is π units.
Statement-2: $f(x) = |\sin x| + |\cos x|$ is periodic with fundamental period $\pi/2$.

222. **Statement-1:** Area between $y = 2 - x^2$ & $y = -x$ is equal to $\int_{-1}^{2}(2 + x - x^2)\,dx$

Statement-2: When a region is determined by curves that intersect, the intersection points give the units of integration.

223. **Statement-1:** Area of the region bounded by the lines $2y = -x + 8$, x-axis and the lines $x = 3$ and $x = 5$ is 4 sq. units.

Statement-2: Area of the region bounded by the lines $x = a$, $x = b$, x-axis and the curve $y = f(x)$ is $\int_a^b f(x)\,dx$.

224. **Statement-1:** The area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$ is 27 sq. units.

Statement-2: The area bounded by the curve $y = f(x)$ the x-axis and $x = a$, $x = b$ is $\int_a^b f(x)\,dx$, where f is a continuous function defined on [a, b].

225. **Statement-1:** The area of the region $\left\{\begin{matrix}(x,y): & 0 \le y \le x^2+1, & \\ & 0 \le y \le x+1, & 0 \le x \le 2\end{matrix}\right\} = \frac{23}{3}$ sq. units.

Statement-2: The area bounded by the curves $y = f(x)$, x-axis ordinates $x = a$, $x = b$ is $\int_a^2 f(x)\,dx$

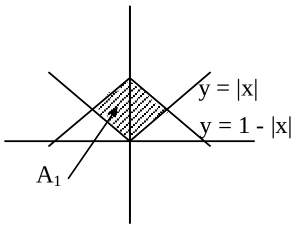
226. **Statement-1:** Area bounded by $y^2 = 4x$ and its latus rectum = 8/3
Statement-2: Area of the region bounded by $y^2 = 4ax$ and it is latus rectum $8a^2/3$

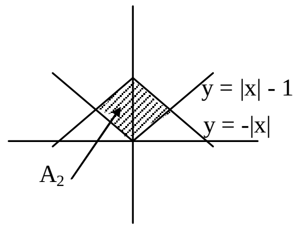
Answer Key

209. A	210. A	211. D	212. A	213. A	214. C	215. D	216. C
217. B	218. A	219. A	220. C	221. B	222. B	223. A	
224. A	225. D	226. A					

Details Solution

209. Clearly $|A_1| = |A_2|$



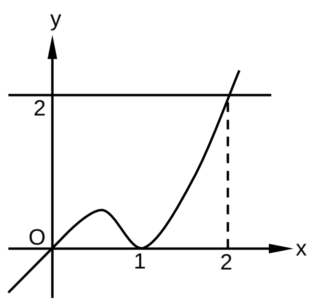


210. $\Delta = 2\int_{5\pi/2}^{3\pi}\left[(x-2\pi)-(3\pi-x)\right]dx = 2\int_{5\pi/2}^{3\pi}(2x-5\pi)dx = \pi^2/2.$

211. (d) Area of ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ in the first quadrant = $\frac{1}{4}\times\pi\times2\times1=\frac{\pi}{2}$.

212. (A) Clearly $|x| + |y| = 2$ represents a square of $\sqrt{8}$ units and area of square is equal to square of the side length.

213. Solving $y = x(x - 1)^2$ and $y = 2$, we get $x = 2$. Hence $y = x(x - 1)^2$ intersects the line $y = 2$ at $x = 2$ only.
Statement – II is true because of above and the graphs of $y = 2$ and $y = x(x - 1)^2$.
Statement – I is obviously true and it is because of statement – II.
Hence (a) is the correct answer.



214. Statement – I is true, as this is a property of definite integral.
As f is non–zero function, area bounded by given boundaries can not be zero.
Hence statement – II is false.
Hence (c) is the correct answer.

215. $\because \sin^2 x \le \sin x : \forall\, x \in (0, \pi)$
Therefore area of $y = \sin^2 x$ will be lesser from area of $y = \sin x$.
Statement – II is obviously true.
Hence (d) is the correct answer.

216. Let the line $y = kx + 2$ cuts $y = x^2 - 3$ at $x = \alpha$ and $\alpha = \beta$, area bounded by the curves = $\int_\alpha^\beta (y_1 - y_2) = \int_\alpha^\beta \left\{(kx+2)-(x^2-3)\right\}dx$

$\Rightarrow f(k) = \frac{\left(k^2+20\right)^{3/2}}{6}$

which clearly shows that statement –II is false but f(k) is least when $k = 0$.
Hence (c) is the correct answer.

217. Option (b) is correct.

The ellipse $\frac{x^2}{3}+\frac{y^2}{2}=1$ & the circles is $(x-1)^2 + (y+2)^2 = 1$.

$\Rightarrow$ Area of ellipse = $\pi\sqrt{3}\sqrt{2}=\sqrt{6}\pi$ and area of circle = $\pi \cdot (1)^2 = \pi$

$\Rightarrow$ The Statement-2 is true in this particular example. In general, this may not be true.

218. Required area = $2\left[\int_0^{2a}\frac{8a^3}{x^2+4a^2}dx-\int_0^{2a}\frac{x^2}{4a}dx\right]$

$= \frac{a^2}{3}(6\pi - 4)$

219. Req. area = $\int_0^{4a/m^2}\left(\sqrt{4ax}-mx\right)dx$

$= \frac{8a^2}{3m^3}$ sq. units

220. $\int_0^{2\pi}\sin x\,dx=\left[-\cos x\right]_0^{2\pi}$ = $[-\cos 2\pi - (-\cos(0))]$

$= [-1-(-1)] = 0$

So, c is correct.

221. $1 \le |\sin x| + |\cos x| \le \sqrt{2}$

So $[|\sin x| + |\cos x|] = 1$

So $\int_0^{\pi}1.dx=\pi$

'b' is correct.

223. Area = $\int_3^5\frac{8-x}{2}dx=\frac{1}{2}\left[8x-\frac{x^2}{2}\right]_3^5$ = 4 sq. units.

224. (A)

Required area

$\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx$ = 27 sq. units.

225. (D)

Required area is

$\int_0^1(x^2+1)dx+\int_1^2(x+1)dx=\frac{23}{6}$ sq. units.

226. area = ar (OAS)

$= \int_0^1 2\sqrt{x}\,dx$

$= 2\left[\frac{2}{3}.x^{3/2}\right]_0^1=\frac{4}{3}\times=\frac{4}{3}$

Whose area = $2\times\frac{4}{3}=\frac{8}{3}$ that is latus rectum by reason have latus rectum = $\frac{8a^2}{3}$ Ans. (A)