KENDRIYA VIDYALAYA SANGATHAN

NEW DELHI

RESOURCE MATERIAL FOR TEACHERS

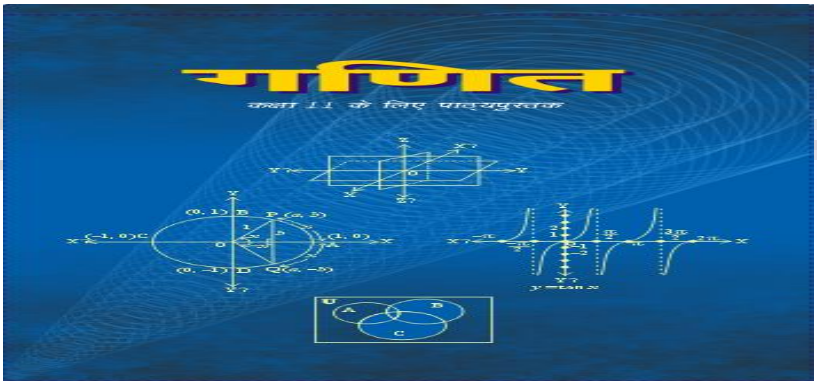
CLASS XI–MATHEMATICS

Workshop on

‘Developing resource material for teaching of Mathematics for classes IX-XII’

Venue: KendriyaVidyalayaSangathan, ZIET Mysore

(20th April to 25th April 2015)



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FOREWORD

There is an adage about Mathematics: "Mathematics is the Queen of all Sciences". This adage exemplifies the significance, scope and importance of mathematics in the realm of sciences. Being a 'Queen', as a subject, Mathematics deserves to be adored and admired by all. But unfortunately, this subject is perceived by the students as a most difficult subject. Not only in India, but across the globe, learning of the subject creates trepidation.

The perception about this subject being difficult in India is rather surprising as ours is a land of great mathematicians like Ramanujam, Bhaskara, Aryabatta et al. The origin and accomplishments of these great men should be a source of inspiration to both students and teachers alike. Yet, as the truth being otherwise, making concerted efforts to identify the reason for perceived fears, initiate suitable damage control and undertake remedial measures assume paramount importance. Kendriya Vidyalaya Sangathan, as a pace setting educational organization in the field of School Education which always strives to give best education to its students, thought it fit to take a pioneering step to empower teachers through teacher support materials. In-service education too strives to do the same. Yet, providing Teacher Resource Material in a compact format with word, audio and video inputs is indeed a novel one.

In the name of teacher resources the internet is abound with a lot of materials: books, audio and video presentations. Yet their validity and usability being debatable, a homemade product by in-house experts could be a solution. Hence, in response to KVS (HQ)'s letter dated 03.03.2015 on the subject "Developing Resource Material for Teaching of English and Maths", a six-day workshop was organized at ZIET, Mysore from 20-25 April 2015. The task allotted to ZIET, Mysore by the KVS is to prepare the resource materials for teachers of mathematics teaching classes IX to XII.

In the workshop, under the able coordinatorship of Mrs. V. Meenakshi, Assistant Commissioner, KVS, Ernakulam Region, four material production teams were constituted for preparing materials for classes IX, X, XI and XII separately. Mr. Ananathan, Principal, KV, No.1, Tambaram of Chennai Region headed Class XII Material Production team; Mr. Krishnamurthy, Principal, KV, Katkeshar of Hyderabad region headed Class XI Material Production Team; Mr. SibySebastian, Principal, KV, Bijapur of Bangalore Regionheaded Class X Material Production Team and Mr.Kovindu, Principal, KV, Ramavermapuram, Trissure of Ernakulam Region headed Class IX Material Production Team. Each team was aided ably by a group of five teachers of mathematics. After a thorough discussion among KVS faculty members and Mrs. Sarala, PGT (Maths), an invited faculty from Demonstration School, RIE, Mysore on the 'Reference Material Framework' on the first day, the teams broke up to complete their allotted work. Their tireless efforts which stretched beyond the prescribed office hours on all the six days helped complete the task of preparing four Teacher Resource Booklets – one each for classes IX, X, XI and XII in a time-bound manner.

Even a cursory glance of the index shall reveal the opt areas of support that the Resource Booklet strives to provide to the teachers of mathematics. The entire material production team deserves appreciation for the commendable work they did in a short period of six days. It is the earnest hope of KVS that the effective use of the Resource Materials will serve the purpose of real teacher empowerment which will result in better classroom teaching, enhanced student learning and above all creating in the minds of the students anabiding love for the subject of mathematics.

- Dr. E. Thirunavukkarasu
Deputy Commissioner & Course Director

PREFACE

"Our goals can only be reached through a vehicle of a plan, in which we must fervently believe, and upon which we must vigorously act. There is no other route to success"

KVS, Zonal Institute of Education and Training, Mysore organized a **6 Day Workshop on 'Developing Resource Material for teaching of Mathematics for Class XI'** from 20th April to 25th April 2015.

The Sponsored Five Post Graduate Teachers in Mathematics from Hyderabad Region were allotted two/ three topics from syllabus of Class XI to prepare Resource Material for teachers under the heads:

1. Expected Learning Outcomes.	2. Concept mapping in VUE portal.
3. Three levels of graded exercises.	4. Value based questions.
5. Error Analysis and remediation	6.Question Bank
7. Projects	8. Power point presentation.
9. Reference Web links.	10. Sample papers
11. Tips and Techniques to score better.	12. Entrance exam.

As per the given templates and instructions, each member elaborately prepared the Resource Material under Fourteen heads and presented it for review and suggestions and accordingly the package of resource materials for teachers were closely reviewed, modified and strengthened to give the qualitative final shape.

The participants shared their rich and potential inputs in the forms of varied experiences, skills and techniques in dealing with different concepts and content areas and contributed greatly to the collaborative learning and capacity building for teaching Mathematics with quality result in focus.

I would like to place on record my sincere appreciation to the Team Coordinator E.KrishnaMurthy, Principal, KV NFC Nagar, Hyderabad, Ms.M Rajeshwari, PGT(Maths) KV Picket, Mrs. G Shailaja, PGT (Maths) KV CRPF, Hyderabad, Mrs. G Sudha Reddy PGT (Maths) KV AFS Hakimpet, Mr.N.S.Subramanian,PGT (Maths) KV Gooty, Mrs.CH Hanumayamma PGT (Maths) KV Guntur, the Course Coordinator Mr.Arumugam, PGT (Phy) ZIET Mysore and the members of faculty for their wholehearted participation and contribution to this programme.

I express my sincere thanks to Dr.E.T.Arasu, Deputy Commissioner and Director KVS, ZIET, Mysore for giving me an opportunity to be a part of this programme and contribute at my best to the noble cause of strengthening Mathematics Education in particular and the School Education as a whole in general.

My best wishes to all Post Graduate Teachers in Mathematics for very focused classroom transactions using this Resource Material to bring in quality and quantity results in the Class XI Examinations.

Mrs.V. Meenakshi,
Assistant Commissioner,
KVS, Ernakulum Region

Guidelines to Teachers

The Resource Material has been designed to make learning Mathematics a delightful experience catering to every kind of learner. As the learners are introduced to a fascinating variety of tools, and participate in meaningful, fun filled activities their Mathematics competence will grow exponentially. Activities that cater to different learning styles such as problem solving, reasoning and proof, analytical, logical etc. are thoughtfully placed in the Resource Material.

1. **Expected Learning Outcomes:**

   In this section, the expected learning outcomes are enlisted chapter-wise and these are expected to realize among the students on completion of particular chapter. The teachers have to design their teaching program which includes mathematical activities, variety of tools and other mathematical tasks. Teachers may prepare their Power point presentations and use in their regular teaching in order to realise the desired outcomes.

2. **Concept mapping in VUE portal:**
   The concept mapping works under Visual Understanding Environment portal, which can be downloaded freely from "Google". A concept map is a type of graphic organizer used to help teachers/students organize and represent knowledge of a subject. Concept Maps begin with a main idea (or concept) and then branch out to show that main idea can be broken down into specific topics. The main idea and branches are usually enclosed in circles or boxes of any Geometrical figure, and relationship between concepts indicated by a connecting line linking new concepts. Each concept is embedded into the box, and those concepts in the form of power point presentation, word document, videos web links etc are uploaded in the same folder.

   Download VUE portal from google and click on this icon to view the content embedded

   How to use a concept Mapping?

   The teacher can use as a Teaching Aid for explaining the holistic view of the topic. It can be used as revision tool. Concept maps are a way to develop logical thinking and study skills by revealing connections and helping students see how individual ideas form a larger whole. These were developed to enhance meaningful learning in Mathematics. It enhances meta cognition (learning to learn, and thinking about knowledge). It helps in assessing learner understanding of learning objectives, concepts and the relationship among those concepts.

3. **Three levels of graded exercises**:

   In this section, selected questions collected from various reference books and are arranged in graded manner, in order the child attempt and learn mathematics in that order. Questions are given in three levels of nature easy, average and difficult respectively. These exercises facilitate the teacher to assign home/ practice works to the students as per their capabilities.

4. **Value based questions:**

   In this section, Value based questions are given in each chapter with an objective to make a student aware the moral values along with the value of problem solving. It is an endeavour to inculcate value system among the students and make them aware of social, moral values and cultural heritage of our great nation. It is expected that the students develop the values like friendliness, Honesty, Initiative, Compassion, Loyalty, Patience, Responsibility, Stability, Tactfulness and Tolerance along with problem solving skills and other applications.

5. **Error Analysis and remediation**:
   It has been observed that the students commit a few common errors. In order to overcome this issue, Teachers have listed, chapter wise, all possible common errors likely to be committed by the students and suitable measures to overcome those errors.

6. **Question Bank and Sample papers**:
   The questions were prepared chapter wise and kept in order for guiding the students suitably in their process of learning. Two sets of sample papers were also included for better understanding of the pattern of the Board Question Paper including weightage of marks.

7. **Power point presentation and Video clips**:
   As educators, our aim is to get students get energized and engaged in the hands-on learning process and video is clearly an instructional medium that is compelling and generates a much greater amount of interest and enjoyment than the more traditional printed material. Using sight and sound, video is the perfect medium for students who are auditory or visual learners. Video stimulates and engages students creating interest and maintaining that interest for longer periods of time, and it provides an innovative and effective means for educators to address and deliver the required curriculum content.
   PowerPoint is regarded as the most useful, accessible way to create and present visual aids. It is easy to create colorful, attractive designs using the standard templates and themes; easy to modify compared to other visual aids, such as charts, and easy to drag and drop slides to re-order presentation. It is easy to present and maintain eye contact with a large audience by simply advancing the slides with a keystroke, eliminating the need for handouts to follow the message.

   The Resource material contains Power Point Presentations of all lessons of Class IX and Video clips/links to Videos of concepts for clarity in understanding.

8. **Reference Web links:**

   What is EDMODO?

- Free, privacy, secure, social learning platform for teachers, students, parents, and schools.
- Provides teachers and students with a secure and easy way to post classroom materials, share links and videos, and access homework and school notices.
- Teachers and students can store and share all forms of digital content – blogs, links, pictures, video, documents, presentations, Assign and explain online, Attach and links, media, files, Organize content in Edmodo permanent Library, Create polls and quizzes, Grade online with rubric, Threaded discussions- prepare for online learning!
- Parent access.

9. **Tips and Techniques to score better**: This book includes tips and techniques for the students to score better. These tips will certainly help the teachers to guide their students for better achievements. The Tips and Techniques included in this book for better Teaching learning Process will certainly be handy for the teachers who use this book.

10. **Online Assessments:** Online Assessments are effective 21st century tools which empower the teachers to extend the class room beyond the four walls. In this technological era administering online assessments are very easy and immediate feedback is obtained. Free web portals such as Edmodo, Hot potato, Education Weekly etc. help teachers. This book includes an insight into these web portals. Online assessment is used primarily to measure cognitive abilities, demonstrating what has been learned after a particular educational event has occurred, such as the end of an instructional unit or chapter. Formative assessment is used to provide feedback during the learning process. In online assessment situations, objective questions are posed, and feedback is provided to the student either during or immediately after the assessment.
    http://www.halfbakedsoftware.com/hot_pot.php

Feedback:

The Post Graduate Teachers and Trained Graduate Teachers are requested to use this material in Classroom transaction and send feedback to Mrs.V.Meenakshi, Assistant Commissioner, Ernakulam Region. minakviswa@gmail.com

BEST WISHES!

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TEACHING OF MATHEMATICS

Introduction:

India is the land of Aryabhatta, Ramanujam and the like – great luminaries in the field of mathematics. Yet, this is one subject that our students dread the most. "It is a nightmarish experience learning this subject, and even the very thought of the subject sends jitters" is a common refrain of school-going students. Not only in India, a developing country, but also in other countries, be those come under the category of underdeveloped or developed, Mathematics is a subject of fear among our students. Both parents and students feel that class room teaching of the subject alone is not adequate for learning it effectively.

The Parent's worries:

The parents are worried lot. Getting a 'good' Mathematics teacher, whatever it means, is big problem. The ones they get in the 'market' are not of any big help to their children; yet they are left with no option but to depend upon either school teachers or coaches from outside. Class room teaching is woefully inadequate in enabling the children acquire confidence and interest in the subject. The quantum of individual attention paid to solving the problems of students in this subjects being almost nil, making them get through the examination is a challenge. "Something needs to be done to arrest the rut being set in Mathematics teaching" is the common prayer of parents.

Why do children consider mathematics a difficult subject?

When you ask the teachers of Mathematics as to why Mathematics is considered as a difficult subject, the answers you get are neither logical nor scientific. Here are a few samples:

The subject requires more of students study time than other subjects (why?)

Students fail to practice problems (what is the reason?)

The subject requires long hours of work, involving practice and drill (why is it so?)

This subject is different from other subjects (in what ways?) Though these answers might partially tell us the reason why the subject is detested by many, they fail to throw any light on the psychological prerequisites, if any, specially required for learning this subject.

What goes on in Mathematics classes?

A peep into Mathematics classes and a bit of observation of the ways in which Mathematics is taught by Mathematics teachers reveal a pattern which is as follows:

Introduction of the new topic: the teacher speaking in general terms for a few minutes about the topic on hand if it happens to be the beginning of the topic.

Working out problems on the black board either by the teacher himself or by calling out a student to do it: If the teacher does the problem on the board, one can see him doing it silently or lip-reading the steps involved in it. If the student does the problem on the board, either continuous interruption or silent observation of the teacher can be seen to be taking place in the class.

Occasional fielding of questions by the teacher on the problem area: When students raise doubts on the steps written how the steps have been arrived at etc., the teacher either clarifies the doubts or tells them to go through the steps again.

Leaving a large number of problems for the students to solve: Often after having solved one or two problems given under the exercise questions on the black board (at times those worked out problems happen to be given as model problems in the text book), teaches tend to leave a large number of remaining problems as home work to students.

What is wrong with the Existing Teaching Practices?

A critical analysis of what is wrong with the existing practices of Mathematics teaching is of prime importance. The analysis of commonly existing Mathematics teaching practices is given below:

Introduction of the Topic

Introduction of any new topic is done in not more than 5-10 minutes duration. This duration is not enough. You cannot throw light on the conceptual frame work of a topic integrating the related concepts learned in classes down below in a span of 5 to 10 minutes. The teacher cannot say much in such a short duration. What actually transpires in Mathematics classes in the name of introducing topic can be illustrated with an example: In the illustration, I have taken here on the topic 'quadratic equation' taught by a teacher which goes like this. "We are going to learn quadratic equations today. Any equation of the form $ax^2+bx+c=0$ in which $a\neq 0$, a and b are coefficients of x^2 and x respectively and c is a constant term is called a quadratic equation. Quadratic Equations have one of the three types of solutions –two different values for the variable x, same value for the variable x or no solution". This type of introduction with a bit of additional information added or otherwise is observed in many classes.

Obviously the introduction given by the teacher is insufficient. There are concepts learned in other classes which have vertical connection and relevance with the topic quadratic equation, namely algebraic equations, linear equations, factors, constants, coefficients, monomials, binomials and of course, algebraic expressions etc. Sparing 10 to 20 minutes to brush up the memory of the students in the topic is highly essential, if a teacher has to cater to the needs of the students of varying levels of understanding of the subject. Introduction given in a generalized manner without taking into account the students previous knowledge and current knowledge in a topic would serve no purpose.

Working out Problems:

Next the teacher picking up one or two problems randomly from the actual exercise for solving on the black board is a common practice observed in the Mathematics classes. Even selecting the worked out examples given in the text book for black board work is not uncommon among the teachers. While the majority of Mathematics teachers prefer to articulate the steps as they work out on the black board, the rest does not open their mouth while their written work is in progress. In the absence of any instruction, students copy down the black board work without listening much to what the teacher says is a common sight in Mathematics classes. The black board works with our teacher's explanation cannot be beneficial to the entire class. Leaving a few motivated students, who have developed interest in Mathematics through other sources of learning, the rest would tend to lose interest and accumulate doubts/ignorance over a period of time, if efforts are not made by the teacher to explain the steps as to why and how those steps occur in the way they are written on the black board. If the sequential relation and coherence among the steps in solving a problem is lost sight of, the entire subject matter would present a picture of mystery to students. This results in aversion to the subject and ultimately mathematics phobia. The brain develops a conditioned response to learning mathematics which I call, 'Mathematics Blindness' Anything related to number, order, sequence, logic and Mathematical operations becoming an anathema to the brain.

Handling students' questions:

The third aspect of teaching Mathematics, namely, how teachers handle questions posed by students, requires a closer examination. Questions, as a matter of fact, are not welcome in mathematics classes. They are perceived as speed breakers to smooth progress in the completion of syllabus. "After all how good a teacher is not important, but are you a teacher who completes the syllabus within the stipulated period of time is! Where is the time to explain each and everything? Even if you do so, there are not many takers. Parents have more faith in coaching classes than in our ability than to teach their children well. "Explanation likes this fly thick and fast the moment you talk about poor teaching of Mathematics. Even in the best of mathematics classes, there is no guarantee that the skill and the mental process of learning the subject, and the components of mastery learning are taken care of. Moving back and forth in elucidating the concepts of Mathematics is rarely done though it is an essential component to review and refresh the previous knowledge. When teachers proceed without this exercise, students stumble with many a doubt and asking that in the class for clarification is straddled with many a pitfall. Right choice of words for raising questions, receptivity of teacher and the possibility of getting answers from the teacher are all matters of speculation. Hence the students prefer the permissive coaching classes for seeking clarification for the doubts that arise in various levels of learning Mathematics. Yet, their hopes are dashed as coaching classes are as crowded as regular school classes and getting conceptual understanding of the subject becomes a real challenge.

The Home Assignments:

Teachers give large number of exercise problems for the students to solve at home. As seen earlier, doing one or two problems for the name sake does not help the students to acquire the insight required for solving problems at home on their own. The teacher solved problems are inadequate in number and variety, and the explanation, if any, given about problem solving in the classes is either incomprehensive or inadequate. As a result students get frustrated when they struggle with problems with answers not in sight. They lose interest when their woes are not taken care of.

Mathematics-the Queen of All sciences

Mathematics is one of the compulsory subjects of study up to class X and an optional one from class XI onwards. It is an important subject as it is considered as the 'Queen of all Sciences'. The abstract nature of Mathematics, precision and exactness being its hall mark, makes this subject appears as more difficult than other subjects. Even the simplest of concepts in it like numbers, addition, subtraction, multiplication, division prescribed for the primary classes warrant in-depth understanding and imagination and creative thinking on the part of the teacher for effective teaching. But do we have teachers who possess these qualities in our schools is a moot question.

Teaching Mathematics in the Primary Classes

Teachers are not having the subject specialization in Mathematics too are allotted this subject for teaching in Primary Classes. Concepts in mathematics up to class V level in schools, consists of basic operations such as addition, subtraction etc., that are taught in a routine manner. As a result, the student learns to do those basic operations following certain repetitive patterns oblivious of the "Why" aspect of those patterns. When they reach the middle level (Classes VI to VIII) and later on the secondary level (Classes IX to X), they understand that the 'patterns 'that they learn in the primary classes are not of much use and that they need to know 'something more'. The real problem to them is to know what is that 'something more'. It is the understanding of basic concepts which is more essential than knowing certain patterns of doing certain problems in Mathematics. But unfortunately, most students come to middle classes without learning anything about Mathematical concepts and how to use their conceptual understanding for solving problems. Mathematics learning, hence, becomes a big riddle by then, and the slow but steady process of developing disinterest in the subject sets in.

The Challenge in Middle and Secondary Classes:

The teachers of the middle and secondary classes have a challenge on hand: opening up the cognitive domain of students and then taking them to higher order mental abilities though sequential learning process. In other subjects, knowledge and understanding apart, memory play a vital role in scoring marks. Even without the former, with the latter (memory) alone, students can score marks in other subjects, where as in Mathematics, you are expected to do "problem solving" which is a higher order cognitive skill.

The Cognitive Domain:

The cognitive domain of the human brain is said to be responsible for thinking, understanding, imagination and creativity. The cognitive domain becomes a fully functional component of human brain after 10 or 11 years of age in children. This does not mean that this domain remains dormant and nonfunctional before this stage. In fact 'concept formation'-one of the difficult functional outputs of the cognitive domain –does take place even before 10 or 11 years of age. The lower order cognitive skills such as knowledge and understanding apart, the elementary level of skills of analysis, and simple problem solving skills are exhibited by primary class children. Systematic development of these skills is called for when the children reach the middle classes. Therefore, a thorough understanding of the process involved in problem solving, which has its genesis in concept learning is a must for teachers of Mathematics.

What Is Concept Learning?

A concept is an abstract idea, and mathematics is full of them. Concept learning involves acquiring a thorough comprehension and grasp of abstract ideas. Each concept in Mathematics has sub-components. For example, 'algebraic expressions' is a concept whose sub-components are 'algebra' (What?), 'expression' (What?), and 'algebraic expressions' (definition?). Besides these components, questions like what are numerical expressions, are algebraic expressions different from numeric expressions, what are algebraic equations, how do algebraic expressions differ from algebraic equations, and so on may needs to be answered to bring out clarity in learning the concept 'algebraic expressions'.

Knowledge Redundancy:

The information age we live in help us see information explosion taking place all around us. The newer learning taking place with geometric progression keeps replacing the current and past information, and hence knowledge is in constant state of flux. Processing information in order to add it to the existing corpus of knowledge is the need of the hour. Teachers, whose main business is transacting knowledge in class room, cannot remain isolated from information processing. They need to keep updating themselves; else they would become knowledge redundant.

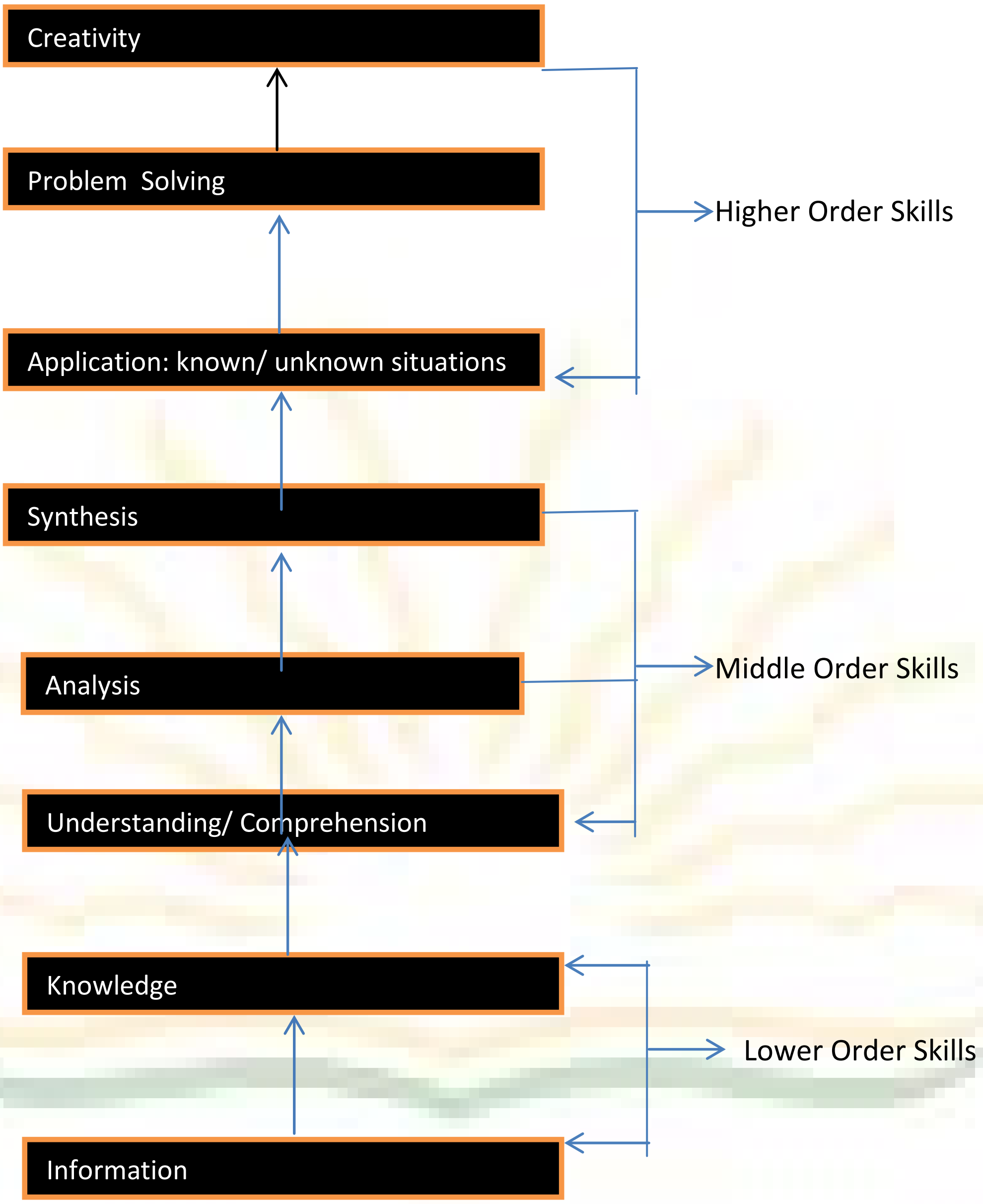
Class Room transactions Cognitive Skills:

Knowledge updated by the teachers is to be transacted in an effective manner, in capsules, in class room to facilitate students comprehending it. Students' language abilities and power of comprehension should be known to the teacher so as to select the best possible way of communicating knowledge with fosters comprehension. The real task of the teacher wanting to achieve total comprehension exists in analyzing and synthesizing knowledge. This is also acquired to develop application skills-in known situations to start with and progressively in unknown situations. Problem solving requires 'application skills', which are the by-product of analysis and synthesis. The skills of analyzing and synthesizing, and application of knowledge at known and unknown situations have an important component called 'thinking'. Thinking has two integral parts: divergent and convergent, while divergent thinking results in creativity, convergent in conversation.

The vertical Connectivity among the Cognitive Skills:

The skills in various levels of the cognitive domain do not function in isolation. There is a vertical connectivity among them, which can be presented by a flow chart as given under:

ORDERED SKILLS OF COGNITIVE DOMAIN: THE COGNITIVE LADDER



Information Processing:

Processing information may require special skills such as skimming and scanning. Yet 'information' is kept as the bottom as a lower order skill in view of the fact that the information processed as know ledge is readily made available in text books to study. Information processing is defined as Claude E. Shannon as the conversion of latent information into manifest information. Latent and manifest information are defined through the terms equivocation (remaining uncertainty, what value the sender has actually chosen), dissipation (uncertainty of the sender what the receiver has actually received) and transformation (saved effort of questioning- equivocation minus dissipation).

Knowledge:

Knowledge too has innumerable components yet the bookish knowledge is emphasized in class room teaching and hence its categorization as a lower order skill. Understanding of what is given in text is meant in a limited manner of 'Knowing what it is' rather than why and how. The Wikipedia, free encyclopedia, defines knowledge as information of which a person, organization or other entities aware. Knowledge is gained either by experience, learning and perception through association and reasoning. The term knowledge also means the confident understanding of a subject, potential with the ability to use it for specific purpose.

What is analysis?

There are many definitions given to the term **'analysis'. Some are given below:**

An investigation of the component parts whole and their relations in making up the whole.

A form of literary criticism in which the structure of a piece of writing is analyzed.

The use of closed- class words instead of infections: Ex: the father of a bride' instead of 'the bride's father'

In our article I use the term 'analysis' to refer to understanding the components that go into making something. For analogy, think of a TV set. The components are picture tube, condensers, resistances, speakers etc. are put together to make a composite whole called TV. Similarly any concepts in Mathematics consist of micro- concepts/ sub- concepts, the understanding, defining and elucidating of each micro- concept fall under the domain of analysis. In class VII, for example, the concept of 'rational number' is defined as follows:

"Any number that can be put in the form of p/q where p and q are integers and q not equal to zero is a rational number". Analysis of this concept includes the understanding and elucidation of

i) Why it is said "that can be put in the form of"?
ii) What does 'any number' mean?
iii) What are integers?
iv) Why 'q' should not be equal to zero?
v) Why and what for is this new set of numbers called rational numbers?
vi) What is 'rational' about these rational numbers?

A teacher attempting to teach the definition of rational numbers without throwing the light on the above questions and many more questions related to them is doing disservice to students wanting to learn Mathematics. Given a permissive and receptive atmosphere, students would come out with many questions as given above, the answers of which would be an appetizer for developing their analytical skills.

Synthesis:

The word 'synthesis' can be defined in many different ways. A few popular definitions are as follows:

- The art of putting different representations together and of grasping what is manifold in them in one act of knowledge.
- Synthesis is what first gives rise to knowledge, i.e. it is not analysis. It is an act of the imagination.
- Synthesis suggests the ability to put together separate ideas to form new wholes of a fabric, or establish new relationships.
- Synthesis involves putting ideas and knowledge in a new and unique form. This is where innovations truly takes place.
- The process of bringing pieces of an analysis together to make a whole.
- The process of building a new concept, solution, design for a purpose by putting parts together in a logical way.
- This is fifth level of Bloom's taxonomy and deals with the task of putting together parts to form a new whole. This might involve working with parts and putting them together in a creative new way or using old ideas to come up with new ones.

Synthesis is to be done for the purpose of establishing the Gestalt view that 'the whole is more than the sum total of its parts'. A suitable analogy can be assembling the components of a TV set and making it work.

Application:

A few definitions of the 'application' are given as follows:

The act of bringing something to bear; using it for a particular purpose: "he advocated the application of statistics to the problem"; "a novel application of electronics to medical diagnosis"

- A diligent effort; "it is a job requiring serious application"
- Utilizing knowledge acquired and processed by the mind for solving problems- both simple and complicated.

The skills associated with information, knowledge, comprehension, analysis and synthesis are to be acquired by students in order to go to the next level of the cognitive order called 'application'. The application of knowledge, skills, and attitudes has to be done in known situations to start with so that the students can progressively move on to unknown situations. Examples suitably selected can help them go through simple to complex situations and would guide them to acquire insight. This insight is a prime requisite for problem solving.

In school level Mathematics, the skills of analysis and synthesis and the insight learning that takes place as a result of the application of those skills would pave the way for solving exercise problems, which the teachers shy away from under the pretext of lack of time, and other priorities. The skills in the cognitive ladder are vertically connected, and the acquisition of those skills at each level requires the student's to allow their minds to think and assimilate ideas. This repetitive manner in which the sequential cognitive skills practiced would train the mind in Mathematical thinking which is otherwise called logical thinking.

Logical Thinking:

Logical thinking is defined as that thinking which is coherent and rational. Reasoning and abstract thought are synonymous with logical thinking. Logical thinking be in Mathematics or any other subject is required to establish the coherence of facts of matter and formation of logical patterns and sequences. Mind has the special ability to think and assimilate, and retain subject matter when presented in sequential manner. Mind grasps matter devoid of gaps quickly. Unanalyzed knowledge in its un-synthesized form poses difficulty in retaining it in long time memory, as concepts and its components do not function isolation. Hence, mind rejects fragmented information which lacks patterns.

Problem Solving:

The thought process involved in solving a problem is called problem - solving. Problem solving as a skill is developed crossing various other skills on its way. The skills lying down below 'problem solving' in the cognitive ladder can be compared to the floors of a building. You cannot reach the sixth floor without crossing the floors down below. Similarly when problem solving is attempted in classes with making explicit efforts to pass through the levels of knowledge, understanding, analysis, synthesis, and application, students fail miserably.

Often it is said that practice and drill in Mathematics would help learn the subject better. Hence again, by repeatedly working out problems, one has to 'memorize' the steps, but it does not guarantee success when problems are differently worded or twisted. Following the cognitive order-moving from information to problem solving steps in the classroom will help the students know the sequential mental processes involved in solving problems in Mathematics. As they practice these steps regularly it will boost their confidence in learning the subject. But classroom learning these days mostly concentrate on problem solving as a direct hit strategy. Working out the problems first without following the cognitive order is equivalent to putting the horse behind the cart, which will take the students nowhere. Even conscientious teachers tend to spend10-20% of their class time on concept teaching and 80-90% on working out the problems. This is totally incorrect. Concept learning and concept formation require knowledge comprehension, analysis and synthesis. After going through these steps, as the next stage, 'application' should be dealt with. At the end comes problem solving. The process of going through and gaining thorough grasp of knowledge, understanding, analysis, and synthesis warrants 80-90% of class room time, and hence just 10-20% class time is enough for problem solving.

The Cognitive Order Learning:

Failure to recognize orderly thinking, the basic quality of cognitive order, is the main reason for the difficulty faced by students in learning Mathematics. The earlier the teachers and students recognize the need for approaching Mathematics logically, the better it would be to create and sustain interest in the subject. The Mathematics classroom practices may, therefore, be fashioned by following the sequential steps given as under:

i) Teacher utilizing 5-10 minutes in the beginning of the class on asking questions on the knowledge, comprehension, analysis and synthesis part of the chapter on hand.
ii) After identifying the skill area in which students have problems, discussion to thrash out those problems should be taken up. Often the problems of students stem from lack of understanding of the basic concepts. It is essential therefore to keep doing 'concept recall' and 'concept clarification'. Comprehending basic concepts is an essential condition for moving on the further steps in the cognitive order, namely, analysis, synthesis etc.
iii) Then the points under application of skills down below may be taken up for discussion. Once the gray areas in application are cleared, the students may be instructed to do problem solving.
iv) If students falter in steps, the logical sequence of steps followed to solve any problem in the given chapter falling under the concepts learnt may be discussed again.
v) Effective questioning to draw out the conceptual understanding of the subject matter learnt by the students should be done at least every tenth minute in every period to ensure that they are actually with the teacher.
vi) Free-wheeling of ideas related to the subject by the students should be encouraged as it would help throw new light on the subject matter under study.
vii) Questions by the students, however silly they may seem, should be welcome in the class and the teacher should listen to them with patience and convincing answers given.
viii) After a full-fledged concept learning session following cognitive order learning, problem solving should be taken up where the students should be encouraged to work out the problems under the watchful eyes of the teacher.
ix) Vertical and horizontal connectivity of concepts in mathematics should always form an integral part of teaching learning, and students being thorough in the sequential conceptual elements be taken care of.

The Critical aspects of learning Mathematics:

Besides taking care of the above nine aspects of teaching, teachers desirous of making students love and do well in mathematics should need to pay heed to the following aspects as well:

A thorough comprehension of the domains - psychological, physical and practical- of effective learning of the subject by the students;

The process they have to follow scrupulously in acquiring skills for the mastery-level learning of the subject;

The role the teachers and parents have to play in fostering and sustaining students interest and enthusiasm so that they learn the subject with ease at class room and face the examination with confidence.

Let me sum up some of the benefits of Cognitive-order-Learning: This methodology- 'Cognitive-order-Learning' enables the students -

- To acquire subject learning competencies
- To develop problem solving skills
- To boost their confidence in the subject
- To widen their interest in the areas of mathematics
- To have and sustain self-directed and self-motivated activities in mathematical learning.
- To achieve mastery level learning of the subject.
- To help apply the skills acquired in Mathematics to other subjects.
- To utilize the cognitive domain to its full extent
- To remove examination phobia.

Conclusion:

The subject matter of Mathematics teachers is vast. An attempt has been made to give only the most rudimentary aspects of it. What requires as a clear understanding on the part of the teachers is that the subject, Mathematics, is neither difficult unconquerable. Yet, it is perceived to be so mostly owing to ineffective teaching, which jumps from knowledge to problem solving, leaving a vast territory of skills in between untouched. As said earlier, Mathematics being the queen of all science deserves an approach to teaching which is based on the sound scientific principles of human learning. In any class room, if students declare that they like Mathematics they enjoy learning it, and they have no difficulty in solving the problems, that class is said to have been blessed with a teacher teaching Mathematics the way it is deserves to be taught. That way surely is Cognitive-order-Learning approach with the thorough understanding of the critical aspects of learning Mathematics referred to above.

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Dy. Commissioner &
Director K.V. ZIET Mysore.

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NEW TRENDS IN ASSESSMENT

Introduction:

One of the main reasons for teachers to assess student learning is to obtain feedback that will guide teaching and assist in making modifications to lesson planning and delivery to ensure student progress. Assessment allows teachers to monitor progress, diagnose individual or group difficulties and adjust teaching practices. Assessment can support student motivation when students are provided with on-going information about their progress and with opportunities to set further goals for learning. Assessment is an interactive process between students and faculty that informs faculty how well their students are learning what they are teaching. The information is used by faculty to make changes in the learning environment, and is shared with students to assist them in improving their learning and study habits. This information is learner-centred, course based, frequently anonymous, and not graded.

Current trends in classroom Assessment:

The terms formative assessment and summative assessment are being redefined in education circles. Many teachers know formative assessment as the informal, daily type of assessment they use with students while learning is occurring. Summative assessment was the term used to "sum it all up," to indicate a final standing at the end of a unit or a course.

Current trends in assessment focus on judging student progress in three ways: Assessment for learning, assessment as learning and assessment of learning. Each assessment approach serves a different purpose.

Assessment for learning is especially useful for teachers as they develop, modify and differentiate teaching and learning activities. It is continuous and sustained throughout the learning process and indicates to students their progress and growth.

In assessment for learning, teachers monitor the progress made by each student in relation to the program of studies, outcomes and determine upcoming learning needs. Teachers ensure that learning outcomes are clear, detailed and ensure that they assess according to these outcomes.

They use a range of methods to gather and to provide students with descriptive feedback to further student learning. These methods may include checklists and written notes based on observations of students as they learn. The descriptive feedback gathered is used to inform planning for learning and to assist the teacher in differentiating instruction in order to meet the needs of all students.

The feedback may be shared in oral or written form with individual students or with the class as a whole. As the information gathered guides the planning process, it leads to the improvement of future student performance in relation to specific outcomes.

Assessment as learning focuses on fostering and supporting meta cognitive development in students as they learn to monitor and reflect upon their own learning and to use the information gathered to support and direct new learning.

It focuses on the role student's play in their learning. In this approach to assessment, students are viewed as the bridge between what they know and the unknown that is still to be learned. Their role is to assess critically both what and how they are learning. They learn to monitor their thinking and learning processes; to understand how they are acquiring and retaining new information or developing new skills and awareness; and how to make adjustments, adaptations and even changes when necessary.

For some students, being asked to reflect on their learning by using skills and strategies related to meta cognition (to think about thinking) might seem new and uncomfortable. They may need help to come to the realization that learning is a conscious process in which knowledge is constructed when the known, or previously acquired, encounters the new or unknown. This process often results in the restructuring or reintegration of what was previously learned.

Assessment of learning is cumulative in nature. It is used to confirm what students already know and what they can do in relation to the program of studies outcomes. Student progress is reported by way of a mark; e.g., a percentage or letter grade, a few times a year or a term. The report card is usually received by students, their parents/guardians as well as by school administrators.

Assessment of learning takes place at specific times in the instructional sequence, such as at the end of a series of lessons, at the end of a unit or at the end of the school year. Its purpose is to determine the degree of success students have had in attaining the program outcomes. Assessment of learning involves more than just quizzes and tests. It should allow students to move beyond recall to a demonstration of the complexities of their understanding and their ability to use the language.

Assessment of learning refers to strategies designed to confirm what students know, demonstrate whether or not they have met curriculum outcomes or the goals of their individualized programs, or to certify proficiency and make decisions about students' future program or placements.

<u>Teacher reflections Assessment procedures:</u>

It is important for a teacher to reflect on why and when students' progress is assessed.

The types of reflective questions that teachers can ask themselves when engaged in assessment **<u>for</u>** learning include:

► Am I observing in order to find out what my students know or are able to do?

► Does my assessment strategy allow student learning to be apparent? Are there elements I need to change in order to minimize anxiety or distractions that might get in the way of learning?

► Will I use the results of my observations to modify my instruction, either with a particular student or with a group of students, or the next time I teach this concept or skill to a new class?

► Will I share the results of my observations with the individual student so that the student and I can decide how to improve future performance?

► Will I share the results of my observations with the class in general (without identifying particular students) in order to provide some indicators as to where they can improve future performance?

The types of reflective questions that teachers can ask themselves when planning opportunities in support of assessment **<u>as</u>** learning include:

► Are the students familiar with the purpose of reflective tools, such as the one I am thinking of using? Will they be able to engage with the questions in a meaningful way?

► Have I provided/will I provide support for students in accordance with the various points mentioned in the reflective instrument; i.e., do I provide clear instructions, create a model, share a checklist, ensure that there are reference materials?

Teacher reflections: The types of reflective questions that teachers can ask themselves when

Planning opportunities in support of assessment **of** learning include:

► Am I using processes and assessment instruments that allow students to demonstrate fully their competence and skill?

► Do these assessments align with the manner in which students were taught the material?

► Do these assessments allow students to demonstrate their knowledge and skills as per the program of studies outcomes?

Student reflection assessment (Assessment as learning):

Students record their reflections by completing sentence starters such as

“Things that went well …”; “Things that got in my way …”; “Next time I will ….” Alternatively, they may check off various statements that apply to themselves or their performance on a checklist.

An overview of the different practices and variety of instruments that can be used and tailored to meet the needs of a specific assessment purpose.

Assessment for Learning	Assessment as Learning	Assessment of Learning
Informal observation / Formative assessments / Peer learning	Conferencing / Learning conversations / Peer assessment / Quizzes or Tests/ Self-assessment and Goal setting.	Performance Tasks / Projects Summative assessment Quizzes / Pen-paper tests Tests or Examinations. PSA OTBA

Formative Assessment is a process used by teachers and students as part of instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of core content. As assessment for learning, formative assessment practices provide students with clear learning targets, examples and models of strong and weak work, regular descriptive feedback, and the ability to self-assess, track learning, and set goal. Formative assessments are most effective when they are done frequently and the information is used to effect immediate adjustments in the day-to-day operations of the course.

Assessment is not formative unless something is “formed” as a result of interpreting evidence elicited. It informs teacher where the need/problem lies to focus on problem area. It helps teacher give specific feedback, provide relevant support and plan the next step. It helps student identify the problem areas, provides feedback and support. It helps to improve performance and provides opportunity to improve performance. Peer learning can be encouraged at all stages with variety of tools. Formative Assessment Strategies:

Tools for Formative Assessment	Techniques to check for understanding
One minute answer	A one-minute answer question is a focused question with a specific goal that can be answered within a minute or two.
Analogy prompt	A designated concept, principle, or process is like _______ because_____________________.
Think, pair, share / Turn to your partner	Students think individually, then pair (discuss with partner), then share with the class.
10-2 theory / 35-5 theory	10 minutes instruction and two minutes reflection/35 minutes instruction and 5 minutes reflection.
Self -assessment	A process in which students collect information about their own learning, analyze what it reveals about their progress toward intended learning goals or learning activity or at the end of the day.

Conclusion:

Teachers should continuously use a variety of tools understanding different learning styles and abilities and share the assessment criteria with the students. Allow peer and self-assessment. Share learning outcomes and assessment expectations with students. Incorporate student self-assessment and keep a record of their progress and Teachers keep records of student progress.

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TEACHING OF MATHEMATICS – MOVING FROM MATHPHOBIA TO MATHPHILIA

“Mathematics is for everyone and all can learn Mathematics”- NCF 2005

The Little Oxford Dictionary define phobia as fear or aversion. Psychology textbooks describe it as an abnormal fear. We hear of claustrophobia, acrophobia, nyctophobia, and anthropophobia.

The pioneers in the study of Mathematics anxiety, Richardson and Suinn (1972), defined Mathematics anxiety *in terms of* the (debilitating) effect of mathematics anxiety on performance*:* "***feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations***".

Is there such a thing as math phobia? To know the answer one needs to only teach mathematics particularly in the secondary and senior secondary classes. And the reality is that most school drop outs in the Board exam are due to failure in Mathematics. Studies indicate that students' anxiety about Mathematics increases between the sixth and twelfth grade.

With this reality check, this write-up aims to analyse the problem and by this parsing, redefine the teaching learning of Mathematics firmly grounded on foundations of success- for the student for the teacher, the society and the nation.

The suggestion that Mathematics anxiety threatens both performance and participation in Mathematics, together with the indications that Mathematics anxiety may be a fairly widespread phenomenon (e.g. Buxton, 1981), makes a discussion like this, concerning Mathematics anxiety in students, particularly the Board going students, of extreme importance.

Mathematics is termed as the queen of all sciences, having logical thinking as its crown and problem solving as its sceptre. Two essential elements which are necessary not just to master nuances of the numeral world but more importantly to have success in life in qualitative ways- these two are also the core life skills formulated by WHO for a healthy and successful life.

The question is, “Does the teaching of Mathematics in our classrooms realise any of these objectives? The huge population of children who balk at the very mention of the subject is an ever growing one as generation gives way to another.

The NFG Position paper on the teaching of Mathematics under the section **“Problems in Teaching and Learning of Mathematics”** states: <u>four problems</u> which we deem to be the core areas of concern:

1. ***A sense of fear and failure regarding mathematics among a majority of children*** (cumulative nature of mathematics, gender and social biases about math, use of language and more importantly symbolic language)

2. ***A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time*** *(*emphasises procedure and knowledge of formulas over understanding is bound to enhance anxiety*)*

3. ***Crude methods of assessment that encourage perception of mathematics as mechanical computation, and****(* only one right answer, sacrificing the process for the right solution, overemphasis on computation and absolute neglect for development of mathematical concepts)

4. ***Lack of teacher preparation and support in the teaching of mathematics***.(out dated methodology, depending on commercial guides due to insufficiency in conceptual clarity and understanding of the fundamentals of mathematics, inability to link formal mathematics with experiential learning , particularly in the secondary and senior secondary stages, incapacity to offer connections within mathematics or across subject areas to applications in the sciences)

Other problems are systemic in nature:

Compartmentalisation- Segregation of Primary, Secondary and Senior secondary

Curricular acceleration- The quantum and scope of the syllabus is much larger and wider with passing days

The NFG recommends four fold measures to ensure that all children learn Mathematics:

1. **Shifting the focus of mathematics education from achieving 'narrow' goals to 'higher' goals**, - whole range of processes here: formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communication. *Giving importance to these processes constitutes the difference between doing mathematics and swallowing mathematics, between mathematisation of thinking and memorising formulas, between trivial mathematics and important mathematics, between working towards the narrow aims and addressing the higher aims*

   The recommended methods are:

- ✓ cross curricular and integrated approaches within mathematics and across other disciplines,
- ✓ Simplifying mathematical communication
- ✓ multiplicity of approaches, procedures, solutions
- ✓ using the common man's mathematics or "folk algorithm"- basing problems on authentic real/ daily life contexts
- ✓ use of technology

2. **Engaging every student with a sense of success**, while at the same time offering conceptual challenges to the emerging mathematician- striving to reduce social barriers and gender stereotypes and focusing on active inclusion of all children in the teaching-learning of mathematics. Children with math phobia usually seem to have little confidence in themselves. They feel they are not good in math; they refrain from asking questions (little realizing that more than half the class is puzzled over the same Problem!); they are afraid to answer any question directed to them for fear of being labelled "dumb" or "stupid." Such fear or anxiety about math often begins during the Primary years and continues through life.

   Recommendations:

- ✓ Teacher need to model “problem –solving” particularly in the context of word problems. To work out diverse problems and build personal repertoire of problem solving skills and model them with enthusiasm and confidence.
- ✓ Move from simple step problem solving modes to increasingly complex and multi- step problem solving.
- ✓ Inculcate positive, persevering problem solving approaches- solve problems with them building rapport thus building their self-esteem and confidence.
- ✓ Use a “problem solving” bulletin board to bring problem solving as part of everyday learning activity
- ✓ In problem solving, arriving at the "correct answer" is not the most important step. More important is choosing the correct strategy for solving the problem. Eventhough there is only one correct answer, there will be more than a single correct strategy for solving a problem. When students are reassured of this fact, they will then be more willing to tackle new problems.

3. **Changing modes of assessment** to examine students’ Mathematisation abilities rather than procedural knowledge-

4. **Enriching teachers with a variety of Mathematical resources**. - The development of teacher knowledge is greatly enhanced by efforts within the wider educational community. Teachers need the support of others—particularly material, systems, and human and emotional support. While teachers can learn a great deal by working together with a group of supportive mathematics colleagues, professional development initiatives are often a necessary catalyst for major change. Activities like collaborative and strategic approaches, Mathematics Lab and experiments help in this aspect

Reflecting on and applying these thoughts to the KV context, what should the maths teachers need to do to ensure that all students learn Mathematics in the true sense of the word i.e. love it, think, learn and apply it.

Mathematics teachers need to move from emphasis on Computation to holistic Mathematical concept learning which will mathematise their thoughts and perspectives.

They need to be constantly conscious of and strive to promote a sense of achievement and comfort in learning of mathematics.

CONCLUSION

UNESCO’s The International Academy of Education in its paper-Effective Educational Practices Series on the topic” Effective Pedagogy in mathematics” by *Glenda Anthony and Margaret Walshaw postulates the following :*

1. **An Ethic of Care-**Caring classroom communities that are focused on mathematical goals help develop students' mathematical identities and proficiencies.
2. **Arranging For Learning-** Effective teachers provide students with opportunities to work both independently and collaboratively to make sense of ideas.
3. **Building on Students' Thinking-** Effective teachers plan mathematics learning experiences that enable students to build on their existing proficiencies, interests, and experiences.
4. **Worthwhile Mathematical Tasks-** Effective teachers understand that the tasks and examples they select influence how students come to view, develop, use, and make sense of mathematics
5. **Making Connections** Effective teachers support students increasing connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday experiences-
6. **Assessment for Learning-** Effective teachers use a range of assessment, practices to make students' thinking visible and to support students' learning.
7. **Mathematical Communication-** Effective teachers are able to facilitate classroom dialogue that is focused on mathematical argumentation
8. **Mathematical Language-** Effective teachers shape mathematical language by modelling appropriate terms and communicating their meaning in ways that students understand
9. **Tools And Representations-** Effective teachers carefully select tools and representations(number system itself, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories, textbooks and technology) to provide support for students' thinking
10. **Teacher Knowledge-**Teacher content knowledge, Teacher pedagogical content knowledge

The referred UNESCO paper can be downloaded from the websites of the IEA (http://www.iaoed.org) or of the IBE (http://www.ibe.unesco.org/ publications.htm)

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Qualities of a Successful Mathematics Teacher

A teacher who is attempting to teach without inspiring the pupil with a desire to learn is hammering on a cold iron ---Horace Mann

Not all students like mathematics, but a good mathematics teacher has the power to change that. A good mathematics teacher can help students who have traditionally struggled with mathematics begin to build confidence in their skills. Successful mathematics teachers have certain qualities that make them the experts they are. These are the teachers required by the society, because of their knowledge, style and handle on the subject; they know what really work for students.

A good mathematics teacher can be thought to need some qualities that are connected to his view of mathematics. This view consists of knowledge, beliefs, conceptions, attitudes and emotions.Beliefs and attitudes are formed on the basis of knowledge and emotions and they influence students' reactions to learn future Mathematics

- A good mathematics teacher should have sufficient knowledge and love of mathematics. He needs to have a profound understanding of basic mathematics and to be able to perceive connections between different concepts and fields.

- A teacher should have a sufficient knowledge of mathematics teaching and learning. He needs to understand children' thinking in order to be able to arrange meaningful learning situations. It is important that the teacher be aware of children' possible misconceptions. In addition, he needs to be able to use different strategies to promote children' conceptual understanding.

- A good mathematics teacher also needs additional pedagogical knowledge: the ability to arrange successful learning situations (for example, the ability to use group work in an effective way), knowledge of the context of teaching and knowledge of the goals of education.

- A good mathematics teacher's beliefs and conceptions should be as many-sided as possible and be based on a constructivist view of teaching and learning Mathematics.

- In the classroom, a talented mathematics teacher serves as a facilitator of learning, providing students with the knowledge and tools to solve problems and then encouraging students to solve them on their own. When students answer a problem incorrectly, he does not allow them to quit. He encourages students to figure out where they went wrong and to keep working at the problem until they get the correct answer, providing support and guidance where needed.

- A Good Mathematics teacher should have the ability to do quick error analysis, and must be able to concisely articulate what a student is doing wrong, so they can fix it. This is the trickiest part of being a good Mathematics Teacher. He should have ability to assign the home work that targeted what the student is learning in the classroom to minimize the mistakes committed and to have proper practice on the concepts taught.

- A successful Mathematics Teacher is seen as a leader in his classroom and in the school. His students respect him, not only for his knowledge of Mathematics, but for his overall attitude and actions. Students can tell that he respects them as well. He has control over the classroom, laying out clear rules and expectations for students to follow.

- A good mathematics teacher focuses less on the content being taught than the students being taught. A good mathematics teacher cares about his students and recognizes when a student needs some encouragement and addresses the problem to help the student refocus on the content.

- A Good mathematics teacher, in particular, possesses enormous amount of patience, because there are many different ways that students actually learn mathematics. And they learn at many different speeds. Math teachers are not frustrated by this attitude of students. He should have sufficient understanding Jean Piaget's theory on how youngsters create logic and number concepts.

- A Good mathematics teacher never lives in the past. He knows how to unlearn outmoded algorithms and outdated mathematical terms and re-learns new ones. He appreciates the change with all enthusiasm and welcomes it.

- He is approachable and explains, demonstrates new concepts/ problems in detail and creates fun. He commands respect and love by his subject knowledge and transaction skills.

- A good teacher sets high expectations for all his students. He expects that all students can and will achieve in his classroom. He doesn't give up on underachievers.

- A great teacher has clear, written-out objectives. Effective teacher has lesson plans that give students a clear idea of what they will be learning, what the assignments are and what the promoting policy is. Assignments have learning goals and give students ample opportunity to practice new skills. The teacher is consistent in grading and returns work in a timely manner.

- Successful teacher is prepared and organized. He is in his classrooms early and ready to teach. He presents lessons in a clear and structured way. His classes are organized in such a way as to minimize distractions.

- Successful teacher engages students and get them to look at issues in a variety of ways. He uses facts as a starting point, not an end point; he asks "why" questions, looks at all sides and encourages students to predict what will happen next. He asks questions frequently to make sure students are following along. He tries to engage the whole class, and he doesn't allow a few students to dominate the class. He keeps students motivated with varied, lively approaches.

- A good Mathematics teacher forms strong relationships with his students and show that he cares about them. He is warm, accessible, enthusiastic and caring. Teacher with these qualities is known to stay after school and make himself available to students and parents who need his services. He is involved in school-wide committees and activities and demonstrates a commitment to the school.
- A good mathematics teacher communicates frequently with parents. He reaches parents through conferences and frequent written reports home. He doesn't hesitate to pick up the telephone to call a parent if he is concerned about a student.

- There are five essential characteristics of effective mathematics lessons: the introduction, development of the concept or skill, guided practice, summary, and independent practice. There are many ways to implement these five characteristics, and specific instructional decisions will vary depending on the needs of the students. The successful mathematics teacher should have these characteristics in his regular teaching practice.
- In addition, every good Mathematics teacher has the positive values like Accuracy, Alertness, Courtesy, Empathy, Flexibility, Friendliness, Honesty, Initiative, Kindness, Loyalty, Patience, Responsibility, Stability, Tactfulness and Tolerance.

"The mathematics teacher is expected to have proficiency in the methodology '**cognitive – order –learning'** which enables the students to acquire subject learning competencies, to develop problem solving skills, to boost their confidence in the subject, to widen their interest in the areas of Mathematics, to have and sustain self-directed and self-motivated activities in mathematics learning, to achieve mastery level learning of the subject, to help apply the skills acquire d in learning mathematics to other subjects, to utilize the cognitive domain to its fullest extent and to remove examination phobia".

National Curriculum Framework – 2005 envisages that

- The main goal of mathematics teacher in teaching Mathematics should be Mathematisation (ability to think logically, formulate and handle abstractions) rather than 'knowledge' of mathematics (formal and mechanical procedures)
- The Mathematics teacher should have ability to teach Mathematics in such a way to enhance children' ability to think and reason, to visualize and handle abstractions, to formulate and solve problems. Access to quality mathematics education is the right of every child.

A balanced, comprehensive, and rigorous curriculum is a necessary component for student success in mathematics. A quality mathematics program which includes best mathematical tasks and models to assist teachers is essential in making sound instructional decisions that advance student learning.

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TEACHING STRATEGIES IN MATHEMATICS FOR EFFECTIVE LEARNING

Mathematics by virtue of its boundless practical applications and tasteful bid of its methods and results has long held a prominent place in human life. From the quick arithmetic that we do in our everyday lives to the onerous calculations of science and technology, Mathematics shapes and effects about every item around us.

But for many secondary and senior secondary students, Mathematics consists of facts in a vacuum, to be memorized because the teacher says so, and to be forgotten when the course of study is completed. In this common scenario, young learners often miss the chance to develop skills—specifically, reasoning skills—that can serve them for a lifetime.

In my 20+ years of mathematics teaching in schools across our country and in foreign lands, I have seen some truly remarkable changes in the way secondary school children perceive Mathematics and their ability to succeed in it depend upon the pedagogy.

Discovering approaches to make Mathematics exciting for students who are in the middle of the pack could have a profound effect on their futures. It would attract many students who are apprehensive in their own abilities into advanced careers. But it is going to require a fundamentally different approach to teaching mathematics from childhood through secondary school. Here are a few of the many possible ideas to begin that change.

Recreational Mathematics

Recreational inspiration consists of puzzles, games or contradictions. In addition to being selected for their specific motivational gain, these procedures must be brief and simple. An effective implementation of this procedure will allow students to complete the "recreation" without much effort. Using games and puzzles can make Mathematics classes very amusing, exciting and stimulating. Mathematical games provide opportunities for students to be dynamically involved in learning. Games allow students to experience success and satisfaction, thereby building their enthusiasm and self-confidence. But Mathematical games are not simply about fun and confidence building. Games help students to: understand Mathematical concepts, develop Mathematical skills, know mathematical facts, learn the language and vocabulary of Mathematics and develop ability in mental Mathematics.

Investigating Mathematics

Many teachers show students how to do some problems and then ask them to practice. Teachers can set students a challenge which hints them to discover and practice some new problems for themselves. The job for the teacher is to find the right challenges for students. The challenges need to be matched to the ability of the learners. The key point about investigations is that students are stimulated to make their own decisions about; where to start, how to deal with the challenge, what Mathematics they need to use, how they can communicate this Mathematics and how to describe what they have discovered. We can say that investigations are open because they leave many choices open to the student.

Creativity in Mathematics

Creativity is a word that is perhaps more easily associated with art, design and writing than it is with Mathematics, but this is wrong. Mathematics requires as much creativity in its teaching and learning as any other subject in the curriculum. It is important to remember that creative teaching and learning not only needs teachers to use creativity in planning inventive and thought provoking learning opportunities but must also encourage creative thinking and response from learners. A lesson in which the teachers' delivery and resources are creatively delivered but which fails to elicit creative thinking and response from students has not been fully successfully creative lesson

Problem solving is a key to Mathematics and this in itself presents an excellent way of encouraging creativity in your lessons. It is a common belief that a degree of rote learning is necessary before learners can engage in problem solving, but such an attitude may have the effect of pre-empting genuine creative thinking.

Group work

Research evidences has consistently shown that, regardless of the subject being studied learners working together in small groups tend to make greater progress in learning what is taught than when the same content is taught in other more didactic ways. Learners working collaboratively also appear more satisfied with their classes and have been shown to have greater recollection of learning. There are numerous ways in which you can arrange learners into groups in your class room. Informal groups' can be created by asking learners to turn to a neighbour and spend 2-3 minutes discussing a question you have posed. Such informal group can be arranged at any time in a class of any size to check on learners understanding, to provide an opportunity to apply new knowledge, or to provide change of pace within the lesson. A more formal arrangement can be made by the teacher establishing the groups. There are conflicting ideas for this but my personal preference is always for mixed ability group.

ICT in Mathematics Teaching and Learning

Appropriate use of ICT can enhance the teaching and learning of Mathematics in secondary and senior secondary level. ICT offers powerful opportunities for learners to explore Mathematical ideas, to generalize, explain results and analyse situations, and to receive fast and reliable, and non-judgmental, feedback. Their use needs careful planning – not just showing a power point presentation but also of activities that allow for off-computer Mathematical thinking as well as on-computer exploration. Decisions about when and how ICT should be used to help teach mathematical facts, skills or concepts should be based on whether or not the ICT supports effective teaching of the lesson objectives. The use of ICT should allow the teacher or learners to do something that would be more difficult without it, or to learn something more effectively or efficiently.

Theatre in Mathematics

The individuals who had the delight of being in front of an audience or performing in any capacity before an audience needn't be convinced about the magic of theatre. The world of theatre is one of the most important ways children learn about actions and implications, about customs and dogmas, about others and themselves. Students in every class room can claim the supremacy and potential of theatre today. We don't have to wait for costly tools and amenities. An occasion to create their own dramas based on what they learned in math and backing them in implementation improve the communication, leadership and motivation skills which will have a long lasting effect in their memory.

The National Focus Group Position Papers on all segments linking to education are an extraordinary repository of ideas, theory and procedure for teachers. The position paper devoted to Arts, music, dance and theatre clearly mentions why and how it may be integrated in the classroom and invokes what is called “Sensitivity Pyramid through Drama”. In cognizance with the NFG position papers theatre can work extremely well as micro level experimental innovative and creative math pedagogy. Theatre is an effective learning tool as it deals with action and imagination, understanding the concept being taught with a view to applying this understanding to real life situations.

Function dance performed by class XI students

Siby Sebastian, Principal
K V Bijapur (Karnataka)

TEACHING LEARNING MATHEMATICS WITH JOY

Every child is naturally motivated to learn and are capable of learning. Children construct knowledge by connecting the existing ideas with the new ideas. The teaching of Mathematics must enable them to examine and analyze their everyday experience. Mathematics is the pivot of analytical and rational thinking. It requires a constant practices to retain different methods, theories, proofs and reasons in memory. It can be done only through a systematic approach. The syllabus in the subject of Mathematics has undergone changes from time to time in accordance with the changing needs of the society. The curriculum at the secondary and senior secondary stage primarily aims at enhancing capacity of students to apply mathematics in solving day-to-day life problems and students should acquire the ability to solve problems.

The NCF-2005 (National curriculum frame work-2205) has elaborated on the insight of learning without burden to ensure that a child is not taken away from the joy of being young by de-linking school knowledge from everyday experience. One of the most important areas in this respect is regarding mathematics learning in schools. It is a common observations that a large number of students consider mathematics as a difficult subject when they enter secondary/senior secondary level. This is creating a phobia in the minds of students towards mathematics. This misconception makes the subject move abstract at that level. It is mainly due to wrong teaching practices which do not link the subject with their real life. It is very essential to know and make them understand that mathematics is very much related to real life, instead of teaching the subject in a mechanical manner where students are made to memorize formulae, theorems, proofs, algorithms etc. and apply these in solving problems.

It is in the hands of teachers to make mathematics teaching learning process of joyful experience for the learner. For this purpose a teacher has to make use of varieties of activities which involves student's participation in the development of concepts. Creating link between within the subjects and across the subjects motivates children and helps them to appreciate the subject. Mathematics has been projected as an abstract subject much to be feared by students of limited capabilities. The teachers can drive this phobia and make them understand the importance of the subject, that it has application in almost all walks of life and also through mathematics we can describe – understand and work with physical phenomena with utmost precision.

Teaching is a noble profession and the teachers are the one who has to protect novelty of this profession. The teachers' positive attitude and commitment towards the profession will certainly motivate a child to learn better mathematics need to be taught in an interactive manner by involving children in the teaching-learning process. The theories can be developed by asking questions and using examples and illustrations based on their daily life situation. This promotes independent thinking and problem solving skills in children. Teacher is a constant learner and a facilitator in the teaching-learning process. Gone are those days when teachers were to teach and children were there to listen and learn. In the present context- their knowledge and ability to transact the curriculum in the manner the children want to learn. For this the teachers have to improve the pedagogic skills. Every teacher will have to improve their teaching as well as evaluation techniques in order to ensure students to learn mathematics and love mathematics.

Let us think for a while and try to get a proper answer for the question "what makes mathematics so difficult and fearsome for many students?" It may be because of the subject itself or because of the person who teaches the subject. Ultimately, it is the teacher who has to make the subject interesting to learn and enable the child to understand the importance of it in one's life. So make use of latest technology support activities, real life situations and constructivist approach in the process of teaching learning. Introduce new concepts in a simple language, keeping in mind the language ability of children. Basic concepts have to be explained through attractive illustrations which connects them to their life outside the classroom.

Training programmes and workshops for creating appropriate leaning materials have been very helpful to the teachers in recent days in creating a classroom with difference. To meet the challenges of today and further it is necessary for a teacher to work with open-mind towards the learning situations. A resource material prepared by the experienced teachers would certainly help teachers to teach with ease, get a lot of ideas to transact and make the evaluation continuous and comprehensive. So teachers can make use of such opportunities and can bring name and fame to teaching profession.

Welcome! Let's enjoy the teaching profession with our students.

Sharada M
Teacher, DMS,
RIE Mysore

RESOURCES CHAPTER WISE

- **Expected Learning Outcomes**
- **Concept mapping in VUE portal**
- **Three levels of graded exercises including non-routine questions**
- **Value Based Questions**
- **Error Analysis and Remediation**
- **Question Bank**
- **Power point presentations**
- **Web Links**

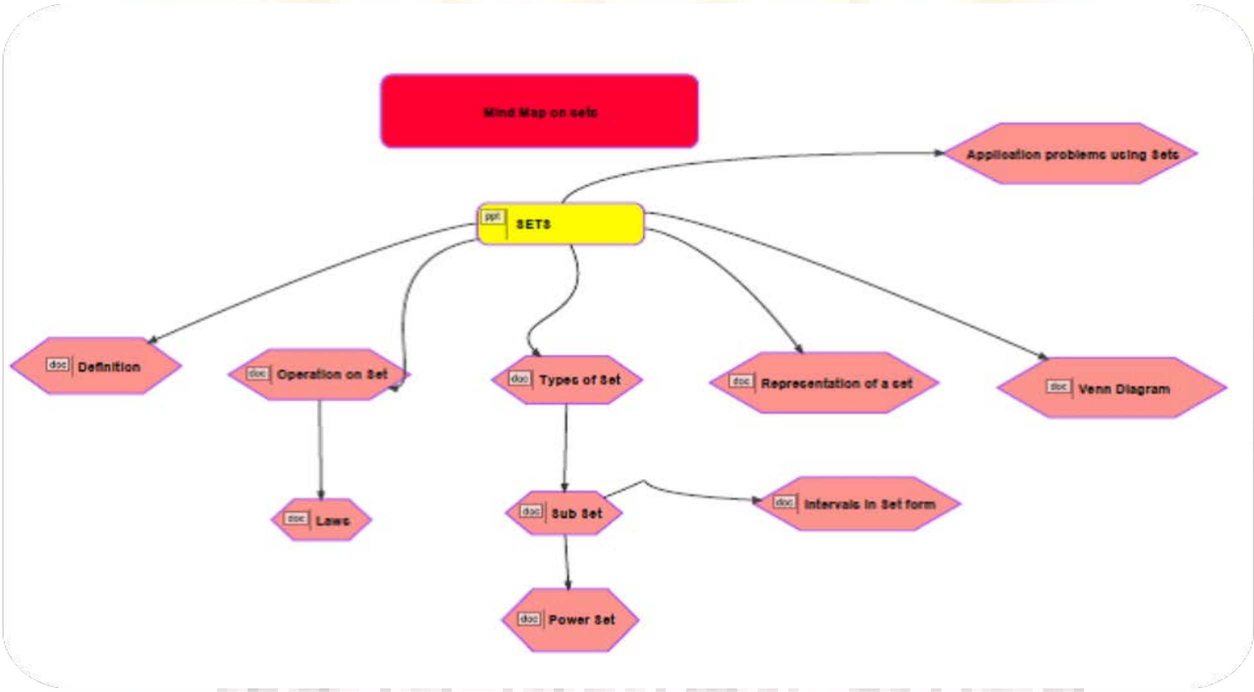
CHAPTER-1- SETS

LEARNING OUT COMES

Upon successful completion of Sets students should be able to:

1. Identify set, object and roster notation.
2. Determine if a given set is finite or infinite.
3. Determine if two are more sets are equal by examining their elements.
4. How to represent a set diagrammatically by using Venn diagram
5. Verification of laws by using Venn diagram.
6. Finding sub sets of a given set.
7. Understanding universal set, complement set.
8. Intersection, union and difference of two sets.
9. Using laws solving problems.
10. Solving problems on cardinality of sets.

CONCEPT MAPPING:



THREE LEVEL GRADED QUESTIONS:

LEVEL 1

1. $If A = \{1,2,3,4,5,6\} and B = \{2,4,6,8\} find i)\ A \cup B ii)\ A \cap B$.
2. Write the set A = { x : x is an integer, –1 < x < 4} in roster form
3. Write { x : –3 < x <7} as an interval.
4. Write all the possible subsets of A = {5,6}.
5. Show that $A \cap B = A \cap C need\ not\ imply B = C$.
6. If $n(\xi) = 50, n(A) = 30, n(A \cap B) = 12\ then\ find\ n(A - B)$.
7. Write the interval $[-9,4]$ in set builder form.
8. Given $A = \{x: x\ is\ a\ letter\ in\ the\ word\ ACCUMULATOR\}$. Express A in the roster form.
9. In a class of 50 students, 30 students like Mathematics, 25 like Science and 16 like both. Find the number of students who like i) either mathematics or science ii) neither Mathematics nor Science.
10. By using Venn diagram verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

LEVEL 2

1. If $\xi = \{1,2,3,\dots,12\}, A = \{x: 2x + 3 < 18\} and B = \{x: x^2 < 40\}$; write A,B in the roster form.
2. A,B are two finite sets such that n(A)= m and n(B)= k, then find the least value of $n(A \cup B)$.
3. If $A = \{2,3,5,7,8\}, B = \{1,5,9\} and A' = \{1,4,6,9\} find (A \cup B)'$.
4. Using properties of sets, prove that $A \cup (A \cap B) = A$.
5. If n(A - B) = 18, n(A ∪ B) = 70 and n(A ∩ B) = 25, then find n(B).
6. By using properties of sets, prove that $A - (A \cap B) = A - B$.
7. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?
8. A and B are two sets such that n(A-B)=14+x, n(B-A)=3x and $n(A \cap B) = x$. Draw a Venn diagram to illustrate this information. If n(A) = n(B), find i) the value of x ii) $n(A \cup B)$.

LEVEL 3

1. Two finite sets have m and k elements. If the total number of subjects of first set is 56 more than the total number of subjects of second set, then find the values of m and k.
2. Find the sets A,B,C such that $A \cap B, B \cap C\ and\ C \cap A\ are\ empty\ sets\ and\ A \cap B \cap C = \emptyset$.
3. Two sets A and B are such that $n(A \cup B) = 21, n(A' \cap B') = 9\ find n(\xi) where\ \xi\ is\ the\ universal\ set.$
4. For any two sets A, B prove that $P(A \cap B) = P(A) \cap P(B)$.
5. In an Examination 80% students passed Mathematics, 72% passed in Science and 13% failed in both the subjects. If 312 students passed in both the subjects, find the total number of students who appeared in the examination.
6. In a survey of 100 students, the number of students studying the various languages were found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find i) how many students were studying Hindi? ii) how many students were studying English and Hindi?

7. If A , B and C are three sets such that $A \cup B = C and A \cap B = \emptyset$ then show that A=C-B.

VALUE BASED QUESTION

1. In class XI of a certain school, 50 students eat burger and 42 students eat noodles in lunch time. If 24 students eat both burger and noodles, find the number students who eat i) burger only ii) Noodles only. Explain the importance of nutritious food over the junk food.

   Suggested values: Junk food often consumed more than requirement that increases weight which may cause many diseases such as sugar, B.P etc. Nutritious food keeps body healthy and fit. It increases working capacity and mind remains tension free.

2. In a survey of 100 students regarding watching T.V, it was found that 28 watch action movies , 30 watch comedy serials, 42 watch news channels , 8 watch action movies and comedy serials, 10 watch action movies and news channels , 5 watch comedy serials and news channels and 3 watch all the three programs. Draw a Venn diagram to illustrate this information and find i) how many watch news channels only? ii) How many do not watch any of the three programs?. According to you which T.V program is useful and why?

   Suggested values: News channels provide information about what is going on in our country at present and abroad. Comedy serials are interesting and enjoyable.

3. A college awarded 38 medals for Honesty, 15 for punctuality and 20 for obedience. If these medals were bagged by a total of 58 students and only 3 students got medals for all three values, how many students received medals for exactly two of the three values? Which value you prefer to be awarded most and why?

   Suggested values: I prefer honesty because corruption is the root cause of all problems of the country. Honest persons are always punctual, obedience and hard work.

4. There are 240 students in class XI of a school, 130 play cricket, 100 play football, 75 play volleyball, 30 of these play cricket and football, 25 play volleyball and cricket, 15 play football and volleyball. Also each student plays at least one of the three games. How many students play all the three games? Which news channel led the “Marks for Sports” campaign for promoting sports in India? Do you think such campaigns are important?

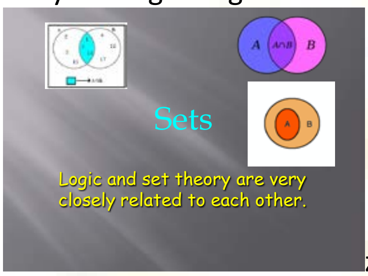
   Suggested values: NDTV led the campaign “Marks for Sports”, across India. Yes such campaign are important for promoting sports in India. Currently because of pressure to get marks in examinations, students do not focus much on sports.

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
SETS	
1.Confusion in using belongs to and subset	Belongs to be used between an element and a set. Subset is used between sets
2.sets are denoted by small letters	To insist that capital letters only denote a set And small letters for elements of a set
3.AU(B∩C) = (AUB) ∩ C	Associate and Distributive formulae should be well versed.
While solving practical problems on sets, students neglect n(A∩B) and in Venn diagram representation too.	Illustrations through Venn diagrams will help to overcome this problem.

QUESTION BANK

1. In a group of 800 people, 500 can speak Hindi and 320 can speak English.
   Find (i) How many can speak both Hindi and English? (ii) How many can speak Hindi only?
2. A survey shows that 84% of the Indians like grapes, whereas 45%like pineapple. What percentage of Indians like both grapes and pineapple?
3. In a survey of 100 peoples it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazine A and B, 10 read magazine A and C, 5 read Magazine B and C and 3 read all three magazines. Find.(i) How many read none of the three magazine?
   (ii) How many read magazine C only?
4. Let A, B be any two sets. Using properties of sets prove that, (i) $(A - B) \cup B = A \cup B$
   (ii) $(A \cup B) - A = B - A$
   [**Hint :** $A - B = A \cap B'$ and use distributive law.]
5. If $\mu=\{1,2,3,4,5,6,7,8,9\}$, $A = \{2,3,5,7,9\}$, $B = \{1,2,4,6\}$, verify (i) $(A \cup B)' = A' \cap B'$
   (ii) $B - A = B \cap A' = B - (A \cap B)$.
6. Which of the following are sets? Justify your answer. 1. The collection of all the months of a year beginning with letter M 2. The collection of difficult topics in Mathematics.



z

WEB LINK

1. SETS: www. mathxtc.com

CHAPTER-2- RELATIONS AND FUNCTIONS

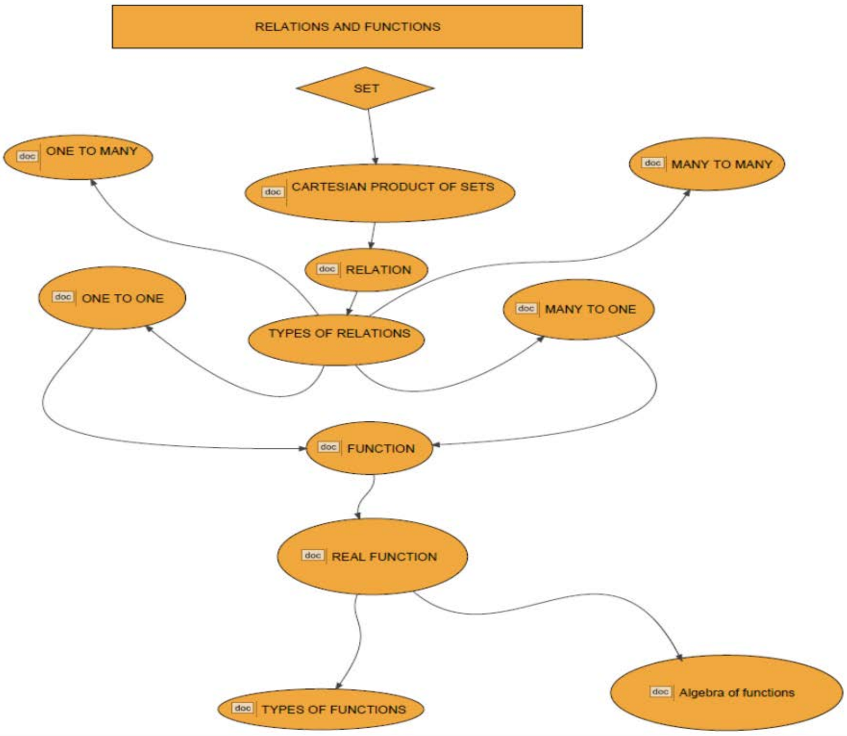
<u>LEARNING OUTCOMES</u>:

The students should be able to

- Define Cartesian product of sets
- Define a relation.
- Distinguish between different types of relations.
- Define a function and a real valued function.
- Differentiate between a relation and a function.
- Distinguish between different types of functions.
- Identify Domain, Co-Domain and range of various relations.
  Draw the graphs of identity function, greatest integer function, modulus function, Signum function, constant function.

CONCEPT MAPPING

GRADED LEVEL



QUESTIONS

LEVEL -1

1. Find a and b if (a – 1, b + 5) = (2, 3)
2. If A = {1, 3, 5}, B = {2,3}, find : a).A × B b).B × A
3. Let A = {1,2}, B = {2,3,4}, C = {4,5}, find
   a).A × (B U C)   b).A × (B ⋂ C)
4. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B
5. If A = {1,2,3,5} and B = {4,6,9},
   R = {(x, y) : |x – y| is odd, x ∈ A, y ∈ B} Write R in roster form
6. Which of the following relations are functions. Give reason.
   R = { (1,1), (2,2), (3,3), (4,4), (4,5)}
   R = { (2,1), (2,2), (2,3), (2,4)}
   R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}
7. Let f and g be two real valued functions, defined by, f(x) = x^2, g(x) = 3x +2, find :
   a. (f + g)(–2)
   b. (f – g)(1)
   c. (f. g)(–1)
   d. f (5)/g(5)
8. Find the domain of the real function, f(x)= -$|x|$.
9. Find the range of the following functions f(x) = $\sqrt{9-x^2}$
10. The Cartesian product A X A has 9 elements among which are found (-1,0) and (0, 1). Find the set A and the remaining elements of A X A.
11. If (x+1,y-5)= (6 , 9) find x and y.
12. If the set A has 3 elements and the set B = (3,4,5), then find the number of elements in (A X B).

LEVEL -2

1. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

2. Let R = { (x, y) : x, y ∈ N and y = 2x} be a relation on N. Find : 1.Domain, 2.Codomain , 3.Range
3. Find the domain and range of, $f(x) = |2x - 3| - 3$
4. Draw the graph of the Greatest Integer function.
5. Draw the graph of the Constant function, f: R →R; f(x) = 2, x ∈ R. Also find its domain and range.
6. Draw the graph of the identity function.
7. Draw the graph of the modulus function
8. Draw the graph of the signum function
9. Draw the graph of f(x) defined by

$$F(x) = \begin{cases} 1 - x, if x < 0 \\ 1, if x = 0 \\ x + 1, if x > 0 \end{cases}$$

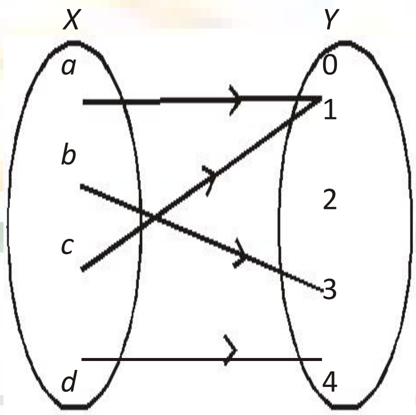
10. Which of the following relations are functions? Give reason.

a. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

b. R = { (2,1), (2,2), (2,3), (2,4)}

c R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

d.



VALUE BASED QUESTIONS

TOPIC: RELATIONS AND FUNCTIONS

1. A= {honest, violence}

B = {peace, prosperity, destruction, hatred}.

Write the set A X B, choose one element of A X B which you would like to have your values in life.

2. A = {May, April, August, June, July}

B = {15,28,29,30,31}

Write relation R given by R ={(a, b)∊ A X B }; ‘a’ month has ‘b’ number of days}.

Write a relation R connected with Independence Day. What values can you develop by celebrating the Independence Day?

3. Hard work and success of a student are inter related .i.e. one is a function of the other. A survey in a school for class XI is conducted and 80% of the students are found to be hard working and 70% successful. What advice you would like to give to the students who are hard working?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
1. confusion in the range and the co-domain	Insist range always a subset or a set equal to the co-domain
2. Confusion between the relation and the function.	Function is a particular case of the relation.

QUESTION BANK

TOPIC: RELATIONS AND FUNCTIONS

1. Find the domain of the following functions:
   a. F(x) = $|x|$ b. F(x) = $\sqrt{16 - x^2}$
2. Find the range of the following functions:
   a. F(x) = 2+3x, x∈ R, x> 0 b.. F(x) = x , x∈ R
3. Find the domain of the function f(x) = $\frac{x^2 + 3x + 5}{x^2 - 5x + 4}$
4. Find the domain and the range of the function $\sqrt{x-1}$
5. If f,g: R→ R be defined by f(x) = x+ 1 ,g(x) = 2x + 3 ,find
   a. f+ g b. f-g c. f. g d. $\frac{f}{g}$.
6. If $g(x) = \begin{cases} x+6, & x \le 1 \\ x-3, & x > a \end{cases}$ Then find g (-2), g(1).
7. Explain why, in each of the following relations, *y* is *not* a function of *x*.
   a) $X^2 + y^2 = 9$. b) $y = \begin{cases} x^2+3, & x \le 0 \\ x^2-1, & x \ge 0 \end{cases}$
8. If f is the subset of Z X Z defined by = {(a. b, a+b): a, b∈ Z }.Is f a function from Z toZ? Justify.
9. If f = {(1,1).(2,3),(0,-1),(-1,-3)}be a linear function from Z to Z, find f(x).
10. Draw the graph of the real function f : $R \rightarrow$ R defined by f(x) = x + 10.

PREREQUISITE KNOWLEDGE

SET : Set is a well defined collection of objects.

Eg. Set of all natural numbers.

WEB LINK

1. Relations and functions : http // www.taosschools.org

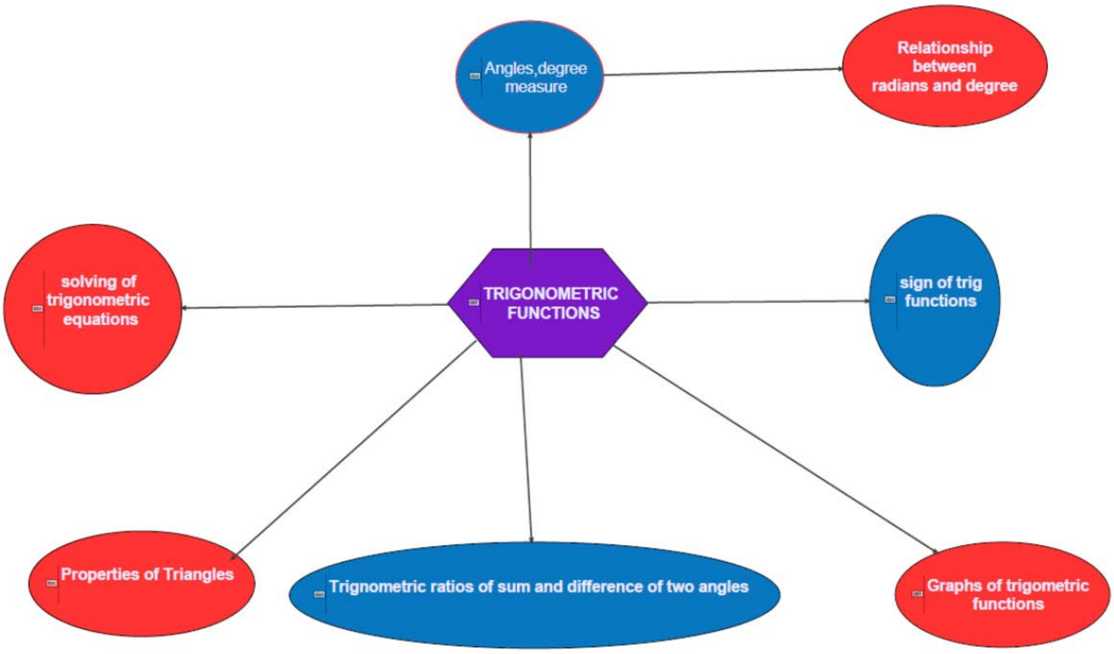
CHAPTER-3- TRIGONOMETRIC FUNCTIONS

LEARNING OUT COMES

Upon successful completion of Trigonometry, students should be able to:

1. Angles:
    i. Measure angles in degrees and radians and convert from one system to the other.
    ii. Tell the exact radian and degree measures of the special angles.
    iii. Use the formulas for the length of a circular arc and the area of a circular sector.
    iv. Find the angular and linear speed.
    v. Solve applied problems involving angles, arc length, area of sector, angular and linear speeds
2. Trigonometric circle
    i. State the definitions of sine and cosine of any angle on the unit circle .
    ii. Give the values of sine and cosine at the special angles on the unit circle.
    iii. Define tangent, cotangent, secant, and cosecant in terms of sine and cosine.
    iv. State the domains of the trigonometric functions.
    v. Determine which quadrants have positive and negative trigonometric values
    vi. Estimate the values of trigonometric functions of any angles using the trigonometric circle and the reference angles.
    vii. State and use the fundamental identities relating the trigonometric functions.
    viii. Verify that an equation is an identity by transforming one side into the other one.
    ix. State and use the definition of periodic functions.
    x. Graph sine and cosine functions using amplitude, period, and phase shifts.
    xi. Graph tangent, cotangent, secant, and cosecant functions
    xii. State for the trigonometric functions their domain, range, period, symmetries, (Vertical) asymptotes, *x*-intercepts, *y*-intercept, and whether the function is even or odd.
3. Analytical trigonometry
    (a) State and use various trigonometric identities: addition, difference, multiple angles, sub-multiple angles, product-to-sum, sum-to-product, etc.
    (b) Verify trigonometric identities and find counterexamples to false identities.
    (c) Define graph of trigonometric functions.
    (d) Solve trigonometric equations.
4. Applications of trigonometry
    (a) Geometry
        i. Use the laws of sines and cosines to solve non-right triangles.
        ii. State and use the formula of the area of a triangle given two sides and the angle between them.
        iii. Use the law of sines, law of cosines and the area formula to solve applied problems.
    (b) Complex numbers
        i. Use the definitions of the absolute value and of the conjugate of a complex number.
        ii. Perform basic arithmetic operations on complex numbers.
        iii. Determine the trigonometric form of a complex number.
        iv. Multiply and divide two complex numbers in trigonometric form.
        v. State and use De Mover’s formula.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL 1

1. Convert the radian measure into degree measure: $\frac{7\pi}{6}$.
2. Find the radius of the circle in which a central angle of 60^{c} intercepts an arc of 37.4 cm length (use $\pi = \frac{22}{7}$).
3. If $\tan x = \frac{-5}{12}$ and x lies in the second quadrant, find the values of other five trigonometric functions.
4. If $\sin x = \frac{3}{5}, \cos y = -\frac{12}{13}$ *and x and y both lie in the second quadrant* find the value of Sin (x+y).
5. Prove that $\sin(-690^{\circ})\cos(-300^{\circ}) + \cos(-750^{\circ})\sin(-240^{\circ}) = 1$.
6. Prove that :$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$.
7. Prove that :$\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x} = -\frac{\sin 2x}{\cos 10x}$.
8. Find the general solution of the equation : Cos 4x = Cos 2x.
9. Prove that $\cos 4x = 1 - 8sin^2 x cos^2 x$.
10. In any triangle ABC, prove that $a = bCosC + cCosB$.

LEVEL2

1. In a right angled triangle, the difference between two acute angles is $\frac{\pi}{18}$ in radian measure. Express the angles in degree.
2. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.
3. If x = $Sin^{14}\theta + Cos^{20}\theta$, then prove that $0 < x \leq 1$ *for all real* $\theta \in R$.

4. If $\tan 35° = a$ prove that $\frac{\tan 145° - \tan 125°}{1 + \tan 145° \tan 125°} = \frac{1-a^2}{2a}$.
5. Show that Cot x Cot 2x – Cot2x Cot3x – Cot3x Cot x = 1.
6. Prove that: $Sin^2x + Sin^2(x-y) - 2\ SinxCosySin(x-y) = \ Sin^2y$.
7. In any triangle ABC, prove that: $\frac{a+b}{c} = \frac{Cos\frac{A-B}{2}}{Sin\frac{C}{2}}$.
8. In any triangle ABC, prove that: $2(bcCosA + caCosB + abCosC) = a^2 + b^2 + c^2$.

LEVEL3

1. Find the value of $\tan\frac{\pi}{18}$.
2. Prove that :$\tan x \tan\left(\frac{\pi}{3} - x\right)\tan\left(\frac{\pi}{3} + x\right) = \ \tan 3x$.
3. Prove that :$Cos^2x + Cos^2\left(x + \frac{\pi}{3}\right) + Cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.
4. If $\tan x = \frac{3}{4}$ and x lies in the third quadrant , find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$.
5. Show that the value of the function $\frac{SinxCos\ 3x}{CosxSin\ 3x}$ where ever defined, never lies between 1/3 and 3.
6. Solve : 4 Sin x Cos x + 2 Sin x + 2 Cos x +1 =0.
7. If $\tan\frac{x+y}{2}$, $\tan z$, $\tan\frac{x-y}{2}$ *are in GP ,then show that* $Cosx = Cos\ y\ Cos\ 2z$.
8. If $0 < x < \frac{\pi}{2}$, *then prove that* $Cos(Sinx) > Sin\ (Cosx)$.
9. In any triangle ABC, prove that $(a-b)^2Cos^2\frac{C}{2} + (a+b)^2Sin^2\frac{C}{2} = c^2$.

VALUE BASED QUESTION

1. A wheel of a motor is rotating at 1200 r.p.m. If the radius of the wheel is 35 cm, what linear distance does a point of its rim transverse in 30 seconds? What steps should be taken to discourage reckless driving?

Suggested values: There should be fines and imprisonment for reckless driving. Conducting proper training of drivers to teach them about the risk associated.

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
Sin(A+B)=sinA +SinB	Insist on learning the formulae
Sin^2 A=$sinA^2$	Sin^2 A = (Sin A)(Sin A)
Cosx+cosy=2cos$\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$	In RHS, the coefficient should not be cancelled with 2 of the angle.

QUESTION BANK

1. Find the radian measure corresponding to

   (i) 5° 37' 30''; (ii) –37° 30´

2. Find the degree measure corresponding to (i) $\left(\frac{11}{16}\right)^c$; (ii) –4°

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

4. Find the value of $\tan \frac{19\pi}{3}$

5. Find the value of (i) $\sin(-1125°)$; (ii) $\cos(-2070°)$

6. Find the value of (i) $\tan 15°$; (ii) $\sin 75°$

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\cos A$

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$.

9. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

10. Express $2 \cos 4x \sin 2x$ as an algebraic sum of sines or cosines.

WEB LINK

Trigonometry: www.powershow.com

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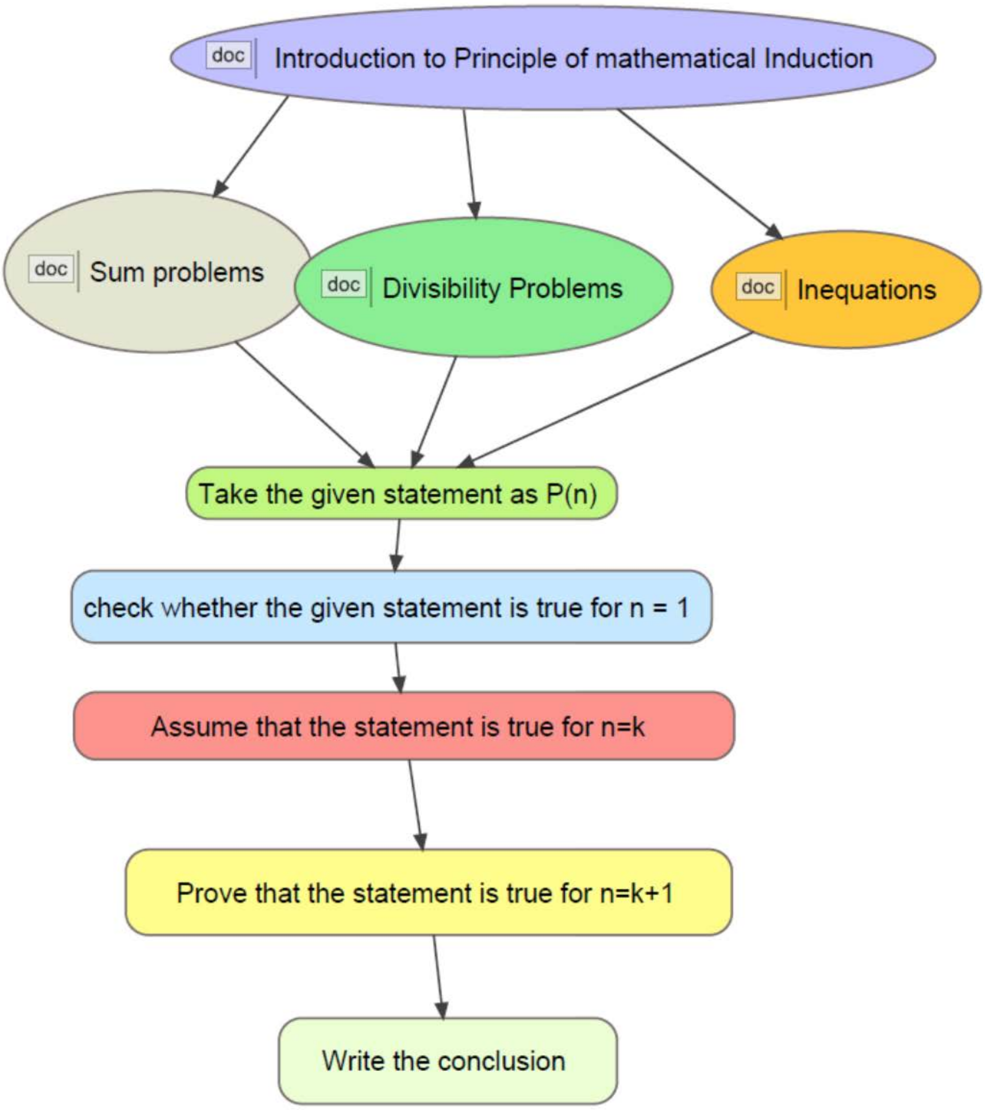
CHAPTER-4- PRINCIPLE OF MATHEMATICAL INDUCTION

LEARNING OUTCOMES :

The students should be able to

- Define the statement of PRINCIPLE OF MATHEMATICAL INDUCTION
- Define the statement P(n).
- Prove that P(1) is true for all n∈N.
- Prove that P(k+1) is true whenever P(k) is true.
- Conclude that P(n) is true for all n∈N.
- Applies the principle of PMI to various problems.
- understand the meaning of Principle of Mathematical Induction.
- understand clearly each steps involved in different type of induction.
- know how to use induction in daily lives.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL -1

Using the principle of mathematical induction, prove the following for all n $\in$ N:

1. $1+3+5+\text{---------} + (2n-1) = n^2$
2. $n^2 + n$ is an even natural number.
3. $3^n > n$

LEVEL -2

Using the principle of mathematical induction prove the following for all n $\in$N:

1. $3.6 + 6.9 + 9.12 + \dots\dots + 3n(3n + 3) = 3n(n + 1)(n + 2)$
2. $x^{2n-1} - 1$ is divisible by $x - 1$, $x \neq 1$
3. If x and y are any two distinct integers then $x^n - y^n$ is divisible by $(x - y)$
4. $7^n - 3^n$ is divisible by 4.

LEVEL – 3

Using the principle of mathematical induction prove the following for all n ∈N:

1. $7^n + 3.5^n - 5$ is divisible by 24.

2. $1^2 + 2^2 + 3^2 + \text{--------} + n^2 > \frac{n^3}{3}$

3. $(2n + 7) < (n + 3)^2$

4. $4^n + 15n - 1$ is divisible by 9

5. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9.

6. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{------} + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$

VALUE BASED QUESTIONS

TOPIC: PRINCIPLE OF MATHEMATICAL INDUCTION

1. The table denotes the cost of various items from different shops:

Shop no / cost per unit	Item A (In Rs)	Item C (In Rs)	Item C (In Rs)	Item D (In Rs)
1	20	21	23	26
2	30	32	31	29
3	40	41	40	41

If you have Rs.250/- to purchase all the four items from one shop, then which shop would it be? Why? Justify your answer. Do you use PMI to analyze your answer?

2. In a locality 50 children (of age between 10 to 14 yrs) can speak Hindi fluently, what can you say about the fluency in Hindi of all the other children (of age between 10 to 14 yrs) in the same locality? Does the principle of mathematical induction hold true in this case? What is the use of learning many languages?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
Not defining P(n)	Define P(n) positively and proceed.
Not testing for P(1)	It is must to test for P(1)
Errors in simplifying P(k+1) in the process of using P(k)	Enough practice should be given

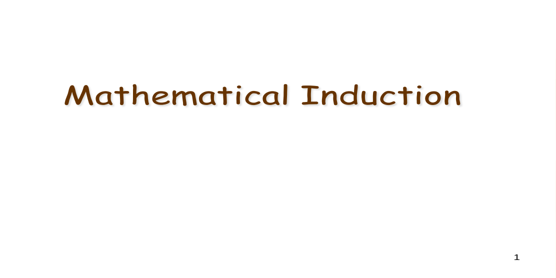
QUESTION BANK

TOPIC: PRINCIPLE OF MATHEMATICAL INDUCTION

Prove the following by the principle of mathematical induction for all n ∈ N:

1. $1^2 + 2^2 + 3^2 + \dots + n^2 = \{n (n + 1) (2n + 1)\}/ 6$
2. $1^3 + 2^3 + 3^3 + \dots + n^3 = n^2 (n + 1)^2 / 4$ for all positive integers n.
3. Prove that for any positive integer number n , $n^3 + 2n$ is divisible by 3

4. Prove that $3^n > n^2$ for $n = 1$, $n = 2$ and use the mathematical induction to prove that $3^n > n^2$ for n a positive integer greater than 2.
5. Prove that $n! > 2^n$ for n a positive integer greater than or equal to 4. (Note: n! is n factorial and is given by 1 * 2 * ...* (n-1)*n.)
6. Use mathematical induction to prove De mover's theorem $[R(\cos t + i \sin t)]^n = R^n(\cos nt + i \sin nt)$
7. $1+3+5+\cdots+2n-1 = n^2$;
8. $1^2+4^2+7^2+\cdots+(3n-2)^2 = (1/2)\,n(6n^2-3n-1)$;
9. Prove $3^n > 2^n$ for all natural numbers n.
10. $1.2.3 + 2.3.4 + \text{-----} + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
11. $10^{2n-1} + 1$ is divisible by 11.



WEB LINK

1. www.xpowerpoint.com

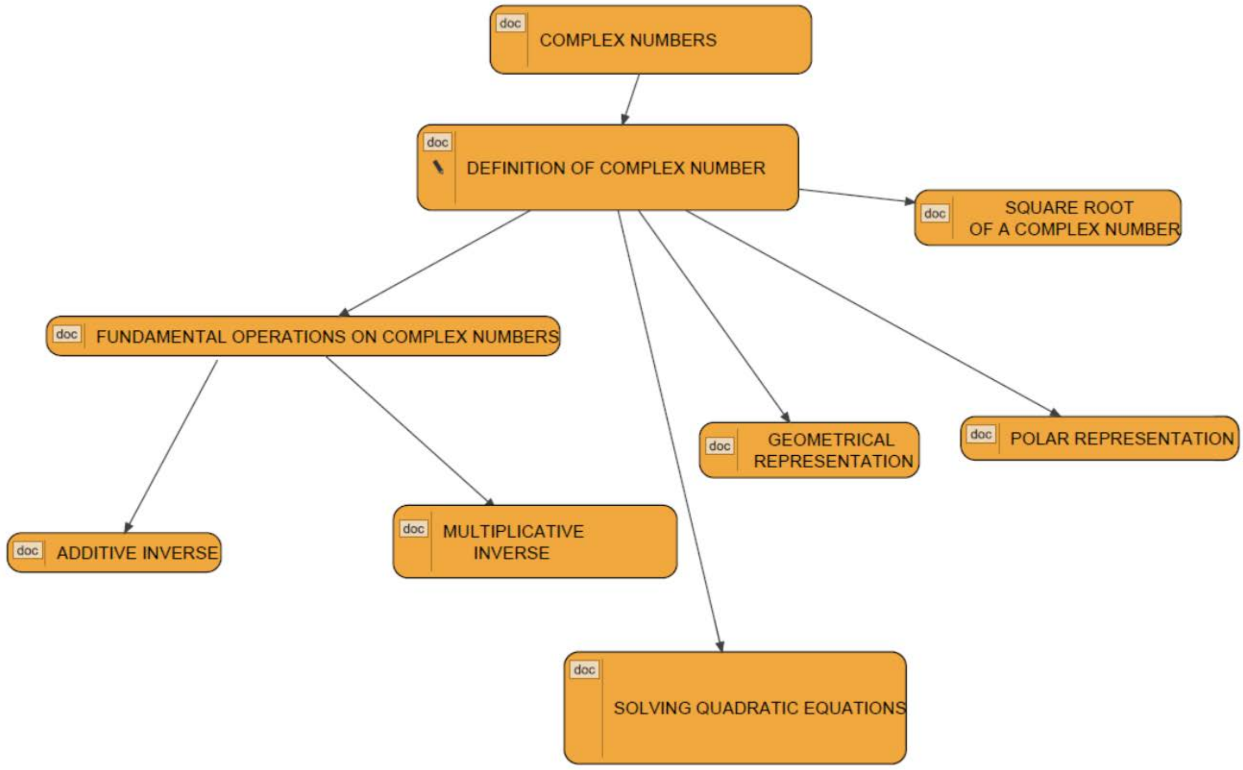
CHAPTER-5- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

LEARNING OUTCOMES:

The students should be able to

- Define a complex number as $z = a + ib$
- Represent a complex number geometrically on an Argand plane.
- Verify the properties of addition, subtraction, multiplication, division of complex numbers.
- Find the conjugate, modulus, inverse of a complex number.
- Represent the complex number in the polar form.
- Find the square root of the complex number.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL -1

1. Find values of x and y if, $(3x - 7) + 2iy = -5y + (5 + x)i$

2. Find the modulus of $z = 3 - 2i$

3. Find the multiplicative inverse of $(5 + 3i)$

4. Find the solution of the equation $x^2 + 5 = 0$ in complex numbers.

5. If z is a purely imaginary number and lies on the positive direction of y-axis, then what is the argument of z?

6. Express in the form of(a + ib): i^{-19}

7. Solve $21x^2 - 28x + 10 = 0$

8. Express in the form of (a + ib) : $(2+3i)^2$

9. Write the conjugate of (-3 -3i).

10. If x =(2+ 31i) and y = (2- 14i) ,then show that x+ y = y + x.

11. Simplify (-i) (3i) $\left(\frac{-i}{6}\right)^3$

LEVEL -2

1. For Complex numbers $z_1 = -1 + i$, $z_2 = 3 - 2i$ show that,
   $Im(z_1z_2) = Re(z_1) Im(z_2) + Im(z_3) Re(z_3)$
2. Convert the complex number (1- i) into polar form.
3. Find the modulus and argument of $z = 2 - 2i$
4. Solve the equation, $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$
5. Find the conjugate of each of the following (i)$(5+\sqrt{2}i)^3$ (ii)(6-3i)(2+5i)

LEVEL -3

1. Find the square root of –3 + 4i and verify your answer.

2. If x = –1 + i then find the value of $x^4 + 4x^3 + 4x^2 + 2$

3. Convert the complex number $z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in the polar form.

4. Find the real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

5. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

6. If $(x+iy)^3$ = u+ iv then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

7. If (a+ib)(c+id)(e+if)(g+ih) = A+iB, then show that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

8. Find the modulus and argument of the complex number $\frac{1+i}{1-i}$.

9. Express in the form of (a+ib), $\frac{(3+i\sqrt{5})((3-i\sqrt{5})}{(\sqrt{3}+i\sqrt{2})-(\sqrt{3}-i\sqrt{2})}$

10. If z_1, z_2 are complex numbers such that, $\left|\frac{z_1-3z_2}{3-z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1$ then find $|z_1|$

VALUE BASED QUESTIONS

TOPIC: COMPLEX NUMBERS AND QUADRATIC EQUATIONS

8. If honesty is represented by complex number $\propto$ and punctuality by complex number β, such that $|\beta|$ = 1, show that $\left|\frac{\beta-\propto}{1-\propto\beta}\right|$ is a perfect one. Do you want to score perfect one? Why?

9. If 1 is considered to be perfect, how many qualities are required for the expression $\left\{\frac{1+i}{1-i}\right\}^m$ to be perfect? Mention the 4 qualities.

10. A person is represented by a complex number z = x + iy. If a person is represented only by x then he is not sensitive towards environment and if a person is represented only by y then he is sensitive towards environment. If a person is related by the relation $\left|\frac{z-5i}{z+5i}\right|$ = 1, do you think that the person is Eco-friendly?

ERROR ANALYSIS

ERRORS	REMEDIATION
In z=a+ib, Im(z)=ib	Emphasis should be given on real and imaginary parts.
$\sqrt{-x}$ = - $\sqrt{x}$	Identification of real and complex numbers should be clear.
$\lvert z_1+z_2\rvert = \lvert z_1\rvert + \lvert z_2\rvert$	Explanation with counter example

QUESTION BANK

TOPIC: COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1. Convert the complex number z = 4(cos 60 – isin60) to rectangular form.
2. Convert the complex number $z = 2 - 2\sqrt{3}i$ to polar form
3. Find the multiplicative inverse of $\frac{1}{6+4i}$.
4. Find the conjugate of $\frac{2}{3+4i}$
5. Find the modulus of -2 -3i.
6. 6. Find the modulus and the argument of $\frac{1}{1+i}$.
7. Prove that the following complex numbers are purely real;

$$i)\left(\frac{2+3i}{3-4i}\right)\left(\frac{2-3i}{3+4i}\right) \quad ii)\left(\frac{3+2i}{2-3i}\right)+\left(\frac{3-2i}{2+3i}\right)$$

Q8. Find the square root of

$i) -15-8i \quad ii) 7-24i$

Q9. Solve the following quadratic equations:

$i) x^2-(3\sqrt{2}-2i)x-6\sqrt{2}i=0$

$ii) (2+i)x^2-(5-i)x+2(1-i)=0$

$iii) x^2-(5-i)x+(18+i)=0$

Q10. Show that a real value of x will satisfy the equation $\frac{1-ix}{1+ix}a=a-ib$ *if*

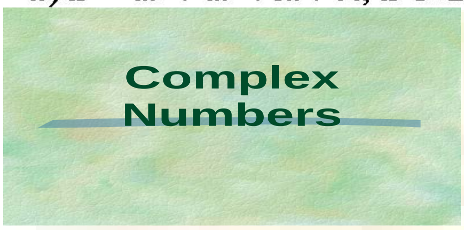
$a^2+b^2=1$, *where a, b are real.*

Q11. If $z=x+iy$ *and* $w=\frac{1-iz}{z-i}, |w|=1 \Rightarrow z$ *is purely real.*

Q12. Evaluate the following:

i) $2x^3+2x^2-7x+72$, when $x=\frac{3-5i}{2}$

ii) $x^4-4x^3+4x^2+8x+44$, $x=3+2i$



WEB LINK

1. COMPLEX NUMBERS :www.authorstream.com

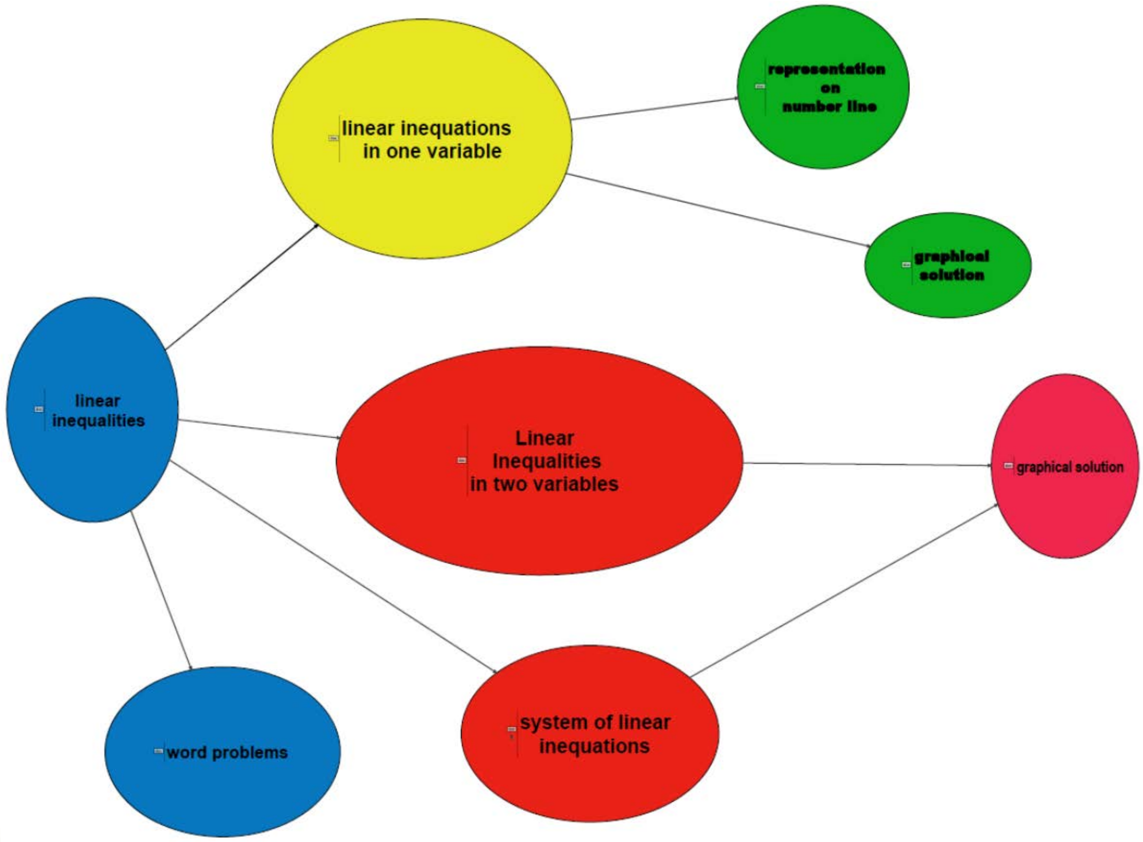
CHAPTER-6- LINEAR INEQUALITIES

Learning out comes

The students should be able to

1. Define an inequality.
2. differentiate in equations and equations
3. Identify various types of inequalities. (Numerical inequalities, Literal inequalities. Double inequalities, Slack inequities, linear inequalities in one variable, linear inequalities in two variables x and y, system of linear inequalities in two variables)
4. Solve the in equations Algebraically and graphically.
5. Apply their knowledge and understanding in solving the Application of inequalities in the present day situations

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL-1

1. Solve the inequality, $3x - 5 < x+7$, when i) x is a natural number ii) x is a whole number iii) x is an integer iv) x is a real number
2. Solve the inequality $|x+3| \geq 10.$
3. The cost and revenue functions of a product are given by $C(x)=20x+4000$ and $R(x)=60x+2000$ respectively, where x is the number of items produced and sold. How many items must be sold to realize some profit?

LEVEL-2

1. Solve $\frac{x-2}{x+5} > 2$
2. Solve $4x+3 \geq 2x+17$
3. Solve $1 \leq |x -| \leq 3$

LEVEL-3

1. Solve $|x+1| + |x| > 3$
2. Solve for x, $-5 \leq \frac{2-3x}{4} \leq 9$
3. Solve for x, $\frac{|x+3|+x}{x+2} > 1$

System of Linear inequalities in two variables are $ax+by+c \geq 0$, $ax+by+c \leq 0$

LEVEL-1

1. Show that the following system of linear inequalities has no solution
   $x+2y \leq 3, 3x+4y \geq 12, x \geq 0, y \geq 1$

2. Show that the solution set of the following system of linear inequalities is an unbounded region $2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$
3. Solve the following system of linear inequalities;
   $3x + 2y \geq 24, 3x + y \leq 15, x \geq 4$

EVEL-2

1. Solve the following system of linear inequalities; $\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$
2. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of Ph value for the third reading that will result in acidity level being normal.
3. A solution is to be kept 40^0c and 45^0c.What is the range of temperature in degree Fahrenheit, if the conversion formula is F=$\frac{9}{5}$ C+32?

LEVEL-3

1. In drilling world's deepest hole it was found that the temperature Tin degree Celsius, X km below the earth's surface was given by T=30+25(x-3),3$\leq x \leq 15$. At what depth will the temperature be between 155^0c and 205^0c.

2. A solution of 9% acid is to be diluted by adding 3%acid solution to it .The resulting mixture is to be more than 5% but less than 7%acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

QUESTIONS FOR SELF EVALUATION

1. $\frac{4}{X+1} \leq 3 \leq \frac{6}{X+1}, \ x > 0$
2. $\frac{|X-2|-1}{|X-2|+2} \leq 0$
3. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160cm, then show that breadth is greater than 20.

VALUE BASED QUESTIONS

LINEAR INEQUALITIES

1. A milk of 80% concentration is diluted at home by the seller by adding some water to it so that milk concentration is reduced between 65% to 87% If 640 liters of milk of 80% concentration is available, how much water has been added? Which value system the seller is lacking?
2. A man wants go on a country tour on a cycle to promote Eco – friendly atmosphere .He wants to tour on a cycle and for that he wants to decorate his cycle with three green pieces of cloth. He has a cloth of length 91 cm. He wants second length to be 3 cm larger than the shortest and the third length to be twice as long as the shortest. What are the possible lengths of the shortest cloth if third piece is to be at least 5cm larger than the second? Do you agree with his mission? What title you would like to give to the mission?
3. To pass in a subject, one must obtain an average of 33 out of 100 or higher to pass in the subject in five examinations. If a student's marks in the four subjects are 28, 31, 40 and 37, find the minimum marks a student must obtain to pass in the subject. A student obtained 42 marks

in the fifth subject by working hard and trying his best in the examination. Do you think student has passed in the examination? What value system does he posses?

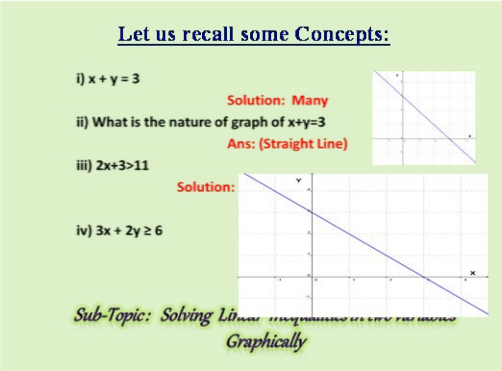
ERROR ANALYSIS

ERRORS	REMEDIATION
$\left(\frac{x-2}{x+3}\right) > 0 \ implies \ (x-2) > 0$	Concept should be explained with suitable examples.
While plotting the points like (2,0) and (0,3), students plot the first point on y-axis and the second on x-axis.	2D rectangular coordinate system should be made clear.
Shading the region when the line is passing through the origin	Checking of the shaded region by taking a point below the line or above the line.

QUESTION BANK

LINEAR INEQUALITIES

1. Solve the inequality $\frac{x-2}{x+5} > 2$
2. Solve $1 \leq |x-2| \leq 3$
3. Solve the system of inequalities: $\frac{x}{2x+1} \geq \frac{1}{4}, \frac{6x}{4x-1} < \frac{1}{2}$
4. Solve for x,$|x-3| \leq 5, |x| \geq 2.$
5. Solve the following system of equations graphically $2x + y \geq 4, x + y \leq 3, 2x - y \leq 6.$
6. If a young man rides his motor cycle at 25 km per hour, he has to spend 2Rs per km on petrol and if he rides it at 40 km per hour, the petrol cost rises to Rs5 per km. He has Rs100 to spend on petrol and wishes to find the maximum distance he can travel within 1 hour. Formulate the data in the form of in equations. Which mode of transport you will suggest to a student and why?
7. The marks obtained by a student in three tests are 50, 80, and 67. To obtain the average marks more than 70% in the fourth test, he restored to unfair means, which went unnoticed. What are the minimum marks he required? Which life value he is lacking and should improve upon?
8. Find the graphical solution of the system of in equations,
   $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$
9. Solve for real number x, $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$
10. A manufacturer has 600 litres of a 12%solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%.
11. A solution of 9%acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution , how many litres of 3% solution will have to be added?
12. Solve graphically: $x + y \leq 4, 3x + y \geq 3, x + 5y \geq 4, x \leq 3, y \leq 3, x \leq 0, y \leq 0.$
13. Solve the in equation $|7x - 2| \leq 11$ and represent it on the number line.



WEB LINK

Linear inequalities: mathxtc.com/…/numba Alg/files

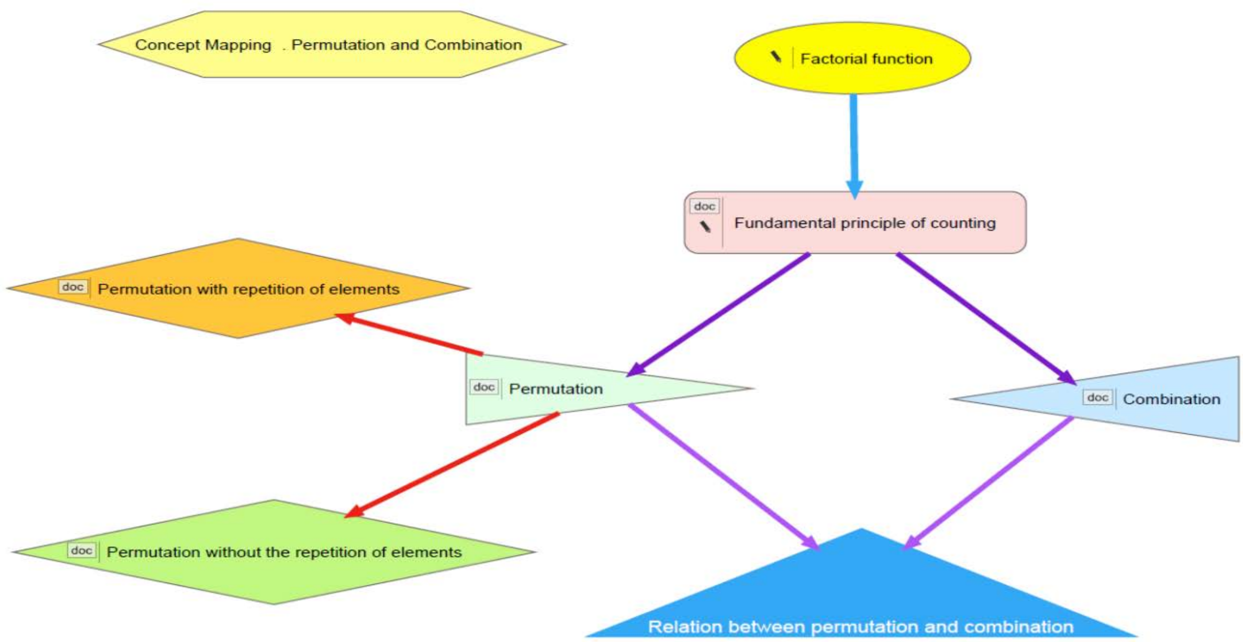
www.powershow.com/view/graphing linear inequalities

CHAPTER-7-PERMUTATIONS AND COMBINATIONS

LEARNING OUTCOMES

1. Students are able to solve the problems by using Fundamental principle of counting
2. students understand Permutation as an arrangement and apply their knowledge in solving problems
3. Students can apply permutations under restrictions in solving problem
4. Students are able to develop pascal's triangle
5. students can differentiate permutation and combination and can apply in solving problems

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

I.FUNDAMENTAL PRINCIPLE OF COUNTING

Level -1

1. Three tourists A, B & C arrive in a city where there are four hotels. In how many ways can they take up their accommodation, each at a different hotel?

2. A Coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Level-2

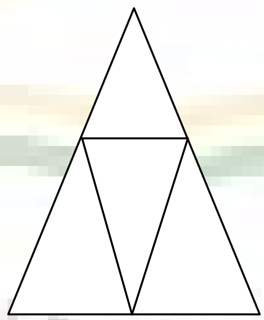
3. Given 5flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags one below the other?

4. In a class are 27 boys and 14 girls .the teacher wants to select 1boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection?

Level-3

5. How many numbers are there between 99 and 1000 having 7 in the unit's place? (ans 10x9=90)

6. In how many ways can this diagram be coloured subject to the following conditions i)each of the smaller triangle is tIo be painted with one of the three colours; red, blueor green ii)no two adjacent regions have the same colour. (ans: 3x2x2x2)



Note: PERMUTATIONS AS AN ARRANGEMENT

1. No OF PERMUTATIONS OF 'n' DIFFERENT THINGS TAKEN r AT A TIME, IS DENOTED BY P(n,r)

2. No OF PERMUTATIONS OF 'n' DIFFERENT THINGS TAKEN ALL AT A TIME IS P(n,n)

3. No OF PERMUTATIONS OF 'n' DIFFERENT THINGS TAKEN ALL AT A TIME, OUT OF WHICH PTHINGS ARE ALIKE OF ONE TYPE,q THINGS ARE ALIKE OF SECOND TYPE AND REST ARE ALL DIFFERENT IS$\frac{n!}{p!q!}$

4. No OF PERMUTATIONS OF 'n' DIFFERENT THINGS TAKEN r AT A TIME, WHEN EACH THING CAN BE REPEATED ANY NUMBER OF TIMES IS n^r

LEVEL-1

1. In how many different sequences can 7 questions be answered by an examinee out of 12 questions in the question paper.(ans ;p(12,7))
2. How many three digit numbers can be formed from the digits1,2,3,4,&5 assuming that repetition of the digits is not allowed (ans: p(5,3))
3. How many three letter words can be formed by the letters of the word NUMBER (ans: p(6,3))
4. Find the number of ways in which a chairman and a vice chairman can be chosen from amongst 12 persons assuming that one person cannot hold more than one position. (ans: p(12,2))
5. How many words with or without meaning, can be formed using all the letters of the EQUATION, Using each letter exactly once (ans: p(8,8))
6. Find the number of bijections from set A containing 7 elements onto itself ans: p(7,7))

LEVEL-2

1. Find the number of permutations of the letters of the word ALLAHABAD(A-4,L-2 OTHERS-3$ANS\frac{9!}{4!2!}$)
2. In how many ways 4 red,3yellow,and 2 green discs be arranged in a row if the discs of same colour are distinguishable(r-4,y-3,g-2) (ans: $\frac{9!}{4!2!3!}$)

LEVEL-3

1. In how many ways 3mathematics books,4history books,3 chemistry books and biology books can be arranged on a shelf so that all books of the same subject are together.(ans: 4!3!4!3!2!)
2. Find the number of 7-digit number can be formed using2,2,2,3,3,4,4.
3. Find the number of functions from set A containing 5 elements into a set B containing 4 elements (ans : $n^r = 4^5$)
4. In how many ways can different four digit numbers can be made by using the digits1,2,3,4,5,6? (ans : 6^4)

Note: PERMUTATIONS UNDER RESTRICTIONS

1. No. OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN r AT A TIME, WHEN A PARTICULAR THING IS ALWAYS INCLUDED IS r. P(n-1,r-1)
2. No. OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN r AT A TIME, WHEN p PARTICULAR THINGS ARE ALWAYS INCLUDED IS $\frac{r!}{(r-p)!}$P(n-p,r-p)
3. No. OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN r AT A TIME, WHEN p PARTICULAR THINGS ARE ALWAYS TOGETHER IS $P!\left(r-(p-1)\right)P(n-p,r-p)$

4. No. OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN r AT A TIME, WHEN p PARTICULAR THINGS ARE NOT TO BE TAKEN IS P(n-p,r)
5. No. OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN ALL AT A TIME, WHEN p PARTICULAR THINGS ALWAYS OCCUR TOGETHER IS $p!\,(n-p+1)!$
6. No OF PERMUTATIONS OF n DIFFERENT THINGS TAKEN ALL AT A TIME, WHEN m PARTICULAR THINGS NEVER OCCUR TOGETHER IS $n! - [m!\,(n-m+1)!]$

LEVEL-1

1. In how many ways can the letters of word' culprit' be arranged so that the vowels are never separate (Ans: n=7,p=2, $p!\,(n-p+1)!$ =2! 6!)
2. Number of arrangements of the letters of word PERTAINS taken 6 letters at a time so that

   a) r,t,s occur always (Ans: $\frac{r!}{(r-p)!}$ P(n-p,r-p)=$\frac{6!}{(6-3)!}$ P(8-3,6-3)=7200)

   b) r,t,s occur together(Ans: $P!\,\big(r-(p-1)\big)P(n-p,r-p)$ =3! $\big(6-(3-1)\big)P(8-3,6-3)$ =1400)

   c) p&t do not occur(Ans: P(n-p,r)=p(8-2,6)=720)

LEVEL-2

3. How many words with or without meaning,each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?
4. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S' s are together?

LEVEL-3

5. How many words with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated. if

   i) 4letters are used at a time

   ii) all letters are used at a time

   iii) all letters are used but first letter is a vowel

6. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

   i) do the words start with p ii) do all the vowels always occur together

   iii) do all the vowels never occur together iv) do the words begin with I and end in P ?

COMBINATIONS

LEVEL-1

1. If C(n,9)=C(n,8),find C(n,17).
2. C(n,r)+C(n,r-1)=C(n+1,r)
3. Find the number of ways of selecting 9 balls from 6 red balls,5 white balls and 5 blue balls if each selection consists of 3 balls of each colour
4. The English alphabet has 5 vowels and 21 consonants. How many words with two different consonants can be formed from the alphabet?

LEVEL-2

5. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme 20 families are to be chosen for assistance. In how many ways can the choice be made?
6. A boy has 3 library tickets and 8 books of his interest in the library. Of these8books, he does not want to borrow mathematics part-II, unless mathematics part-I is also borrowed. In how many ways can he choose the three books to be borrowed? (ans: 41)

LEVEL-3

7. There are 3 books on mathematics,4 on physics and 5 on English. How many different collections can be made such that each collection consists of
   i) One book of each subject
   ii) Atleast one book of each subject
   iii) Atleast one book of English
8. There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturer is to be formed. Find
   i) In how many ways committee can be formed
   ii) In how many ways a particular professor is included
   iii) In how many ways a particular lecturer is included
   v) In how many ways a particular lecturer is excluded

QUESTIONS FOR SELF EVALUTION

1. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT
2. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. Show that the number of ways in which the seating arrangements can be made is $\frac{11!}{5!6!}$ 9! 9!
3. A committee of 6 is to be chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women .In how different ways this can be done if two particular women refuse to serve on the committee.
4. In many ways can the letters of the word PERMUTATIONS be arranged
   i) Word start with ‘P’ and end with ‘S’
   ii) Vowels are all together
   iii) There are always 4 letters between P and S
5. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4?
6. A group consists of 4 girls and 7boys .In how many ways can a team of 5 members be selected if the team has
   i) No girls
   ii) At least one boy and one girl
   iii) At least three girls

VALUE BASED QUESTIONS

PERMUTATIONS AND COMBINATIONS

1. 5 Girls and 3 boys have been selected for President's Bravery awards. These 8 children have shown exemplary behavior in adverse conditions. During meeting with the president they have to stand for a photograph with president in a line. President has to be in the middle. In how many Ways photographs can be taken?

2. In how many ways can the letters of the word HONESTY be arranged? Do you like jumbled letters of the HONESTY? An honest person is always respected by the society. Do you agree? What are the consequences of dishonesty.

3. A team of 4 students is to be sent for a competition. 12 students offered their services for the same. But from past experience it was observed 5 students who had offered the services were not true to their work and they did mischief one time or the other are not to be selected. In how many ways can the five students be selected for a competition?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
A problem of permutation is confused with combination.	the key words namely arrangement, selection(choose) should be associated with permutations and combination respectively.
(x -y)! = x! - y!	The concept of factorial should be explained with suitable examples.
Confusion in the formulae of P(n,r) and C(n,r)	Student should be well versed with the formulae.

QUESTION BANK

PERMUTATIONS AND COMBINATIONS

1. Find n, if P(n,4):P(n-1,3)=9:1
2. Find n if, C(n,7)=C(n,17).
3. Find x if $\frac{1}{7!}+\frac{1}{8!}=\frac{x}{9!!}$
4. If C(n-1,r):C(n,r):C(n+1,r)=6:9:13, find n and r
5. How many numbers greater than a million can be formed by using the digits 4,6,0,6,7,4,6?
6. A polygon has 44 diagonals, find the number sides of a polygon.
7. How many words each of 3 consonants and 2 vowels can be formed from the letters of the word INVOLUTE?
8. How many different arrangements of the letters of the word INDEPENDENCE can be formed so that vowels always occur together?

9. In how many ways can 5 boys and 3 girls be seated in a row, so that no two girls sit together?
10. The different permutations of all the letters of the word EXAMINATION are listed as in a dictionary. If these permutations are considered as words, how many words are there before the first word starting with E?
11. How many words with or without meaning, each of 2 vowels and 3 consonants can be formed from the letter of the word EMPATHY? Do you agree that empathy is one value which everyone should acquire
12. From a class of 8 boys and 6 girls, 8 students are to be chosen for a competition including 4 boys and 4 girls. The two girls who won the prize last year should be included. In how many ways can selection be made?
13. There are 3 books on mathematics, 4 on physics and 5 on English. How many different collections can be made such that each collection consists of: match the following:

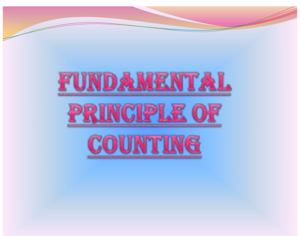
SNo	C1	C2
1	One book of each subject	i)3968
2	At least one book of each subject	ii)60
3	At least one book of English	Iii03255

2. Five boys and five girls form a line. Find the number of ways of making the seating arrangement under the following condition: (match the following)

S.No	C1	C2
1	Boys and girls alternate	$5!\,6!$
2	No two girls sit together	$10! - 5!\,6!$
3	All the girls sit together	$(5!)^2 \quad (5!)^2$
4	All the girls are never together	$2!\,5!\,5!$

3. There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturers is to be formed. Find:
(Match the following)

S.No	C1	C2
1	In how many ways a committee can be formed	C(10,2)XC(19,3)
2	In how many ways a particular professor is included	C(10,2)XC(19,3)
3	In how many ways a particular lecturer is included	C(9,1)XC(120,3)
4	In how many ways a particular lecturer is excluded	C(10,2)XC(20,3)



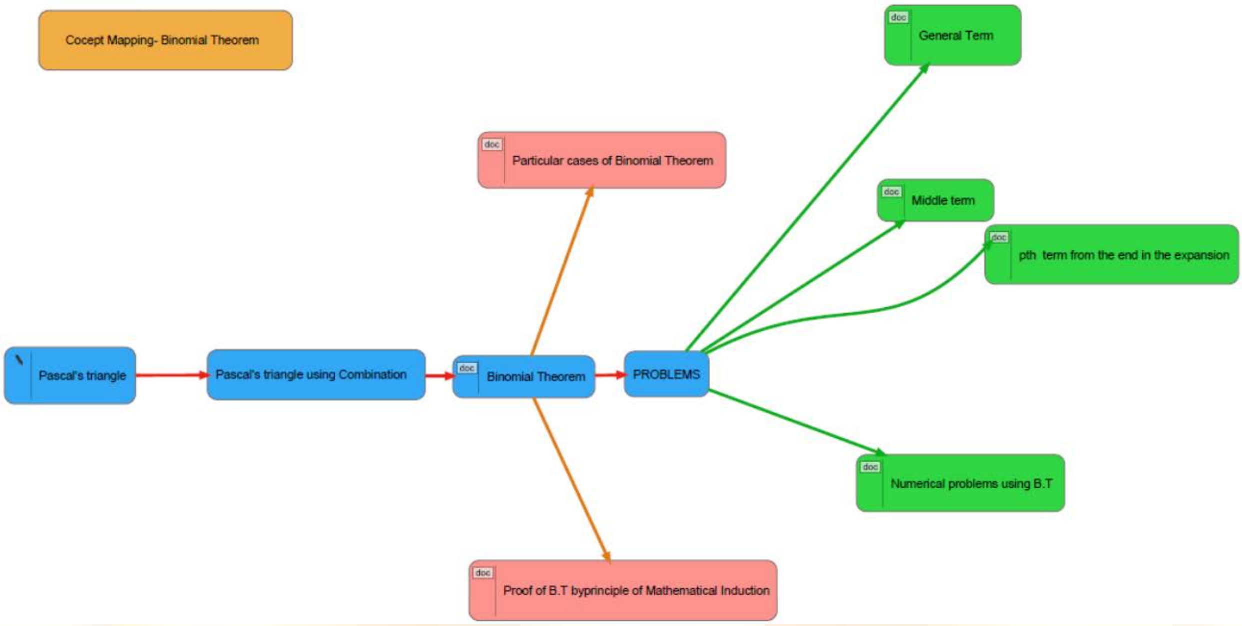
Permutations and combinations:www.xpowerpoint.com/ppt/permutation.html

CHAPTER-8-BINOMIAL THEOREM

LEARNING OUTCOMES

1. Students can apply Binomial theorem for any positive integer n
2. They are able to apply binomial theorem in evaluating the particular cases like $(1+x)^n$, $(X-Y)^n$,$(1-x)^n$ etc.
3. They are able to find the values of numbers like $(98)^5$ $(100)^7$ ETC and develop computational skills
4. Students can find General term, middle terms and independent terms of an expansion

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

BINOMIAL THEOREM

LEVEL-1

<u>USING BINOMIAL THEOREM</u>

1. Expand $(x^2 + \frac{3}{x})4$
2. Compute $(102)^5$
3. Indicate which number is larger $(1.1)^{1000}$ or 1000

LEVEL -2

1. Which of the following is larger?

   $99^{50} + 100^{50}$ or 101^{50}
2. Expand the following $(1-x+x^2)^4$

3. If Z = $(\frac{\sqrt{3}}{2}+\frac{i}{2})^5$ - $(\frac{\sqrt{3}}{2}-\frac{i}{2})^5$ then show that im(z)=0

LEVEL-3

1. Show that $2^{4n+4}-15n-16$,Where n$\in N$ is divisible by 225
2. The sum of the series $\sum_{r=0}^{10} C(20,r)$ is $2^{19}+\frac{C(20,10)}{2}$

GENERAL TERM AND MIDDLE TERM

LEVEL -1

1. Find the rth term in the expansion of $(x+\frac{1}{x})^{2r}$
2. Find the coefficient of x^{11} in the expansion of $(x^2+\frac{2}{x})^{15}$
3. Find the term independent of x in the expansion of $(\frac{\sqrt{x}}{\sqrt{3}}+\frac{\sqrt{3}}{2x^2})^{10}$

LEVEL-2

1. Find the middle term in the expansion of $(2ax-\frac{b}{x^2})^{12}$
2. Find the middle terms in the expansion of $(\frac{p}{x}+\frac{x}{p})^9$
3. If the term free from x in the expansion of $(\sqrt{x}-\frac{k}{x^2})^{10}$ is 405.Find the value of k.
4. Find the 4th term from an end in the expansion of= $(\frac{x^3}{2}+\frac{2}{x^2})^9$

LEVEL-3

1. Find the coefficient of x in the expansion of $(1-3x+7x^2)(1-x)^{16}$
2. If A and B are coefficients of x^n in the expansions of $(1+x)^n$ and $(1+x)^{2n-1}$ respectively, then show that $\frac{A}{B}$ = 2
3. Show that the middle term in the expansion of $(x-\frac{1}{x})^{2n}$ is $\frac{1X3X5X------(2n-1)}{n!}(-2)^n$

QUESTIONS FOR SELF EVALUATION

1. Determine whether the expansion of = $(x^2-\frac{2}{x})^{18}$ will contain a term containing x^{10}?
2. If the coefficient of second, third and fourth terms in the expansion of $(1+2x)^{2n}$ are in A.P. Show that $2n^2-9n+7=0$
3. Find the sixth term of the expansion $(y^{\frac{1}{2}}+x^{\frac{1}{3}})^n$, if the binomial coefficient of the third term from the end is 45.

VALUE BASED QUESTIONS

1. In a survey it was found that $(1.1)^{10000}$ number of people move on cycle (eco-friendly) and 1000 people use cars (need to be co friendly).Which is larger of the two? Which value system we should inculcate in public?

2. In a campaign on LOKPAL BILL $(102)^5$ people got registered through internet. How many people were in favor of LOKPAL BILL? Do you want to be a part of it? Which value system you want to acquire for the same?

3. The number of students who joined the "KEEP YOUR PLACE CLEAN" campaign are represented by $(9X - \frac{1}{3\sqrt{X}})^{18}$ find the number of students. Do you also wish to join the campaign?

ERROR ANALYSIS

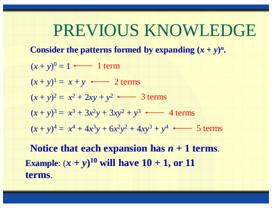
ERRORS	REMEDIAL MEASURES
The 3rd term of a binomial theorem using T_{r+1} is taken as T_3 in which r=3	$T_3 = T_{2+1}$ where r is 2.
Confusion between the term and coefficient of the term like the third term of the expansion $(x+y)^5$ and the coefficient of x^3	The third term of the expansion is $10x^3y^2$ and the coefficient of x^3 is $10y^2$

QUESTION BANK

BINOMIAL THEOREM

1. Evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$
2. If the coefficients of three successive terms in the expansion of $(1+x)^n$ be 45,120 and 210, find the value of n.
3. Find the sixth term in the expansion of $(\sqrt{y}+\sqrt[5]{x})^n$,if the binomial coefficient of the third term from end is 45.
4. Evaluate $(x^2-\sqrt{1-x^2})^4 + (x^2+\sqrt{1-x^2})^4$
5. Prove that $(2n)! = 2^n$ (n)! [1.3.5.....(2n – 1)]
6. If α=C (n,2), prove that C(α,2) = 3 C((n+1),4)
7. Determine whether the expansion of $(x^2-\frac{2}{x})^{18}$ will contain a term containing x^{10} ?
8. Find the middle terms in the expansion of $(3x-\frac{x^3}{6})^7$
9. Find the coefficients of x^5 , in the expansion of the product$(1+2x)^6\ (1-x)^7$.
10. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in AP, show that $2n^2 -9n +7 =0$

11. Using binomial theorem, prove that $2^{3n} - 7n - 1, n \in N$ is divisible by 49.
12. The coefficient of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio 1:7:42. Find n.



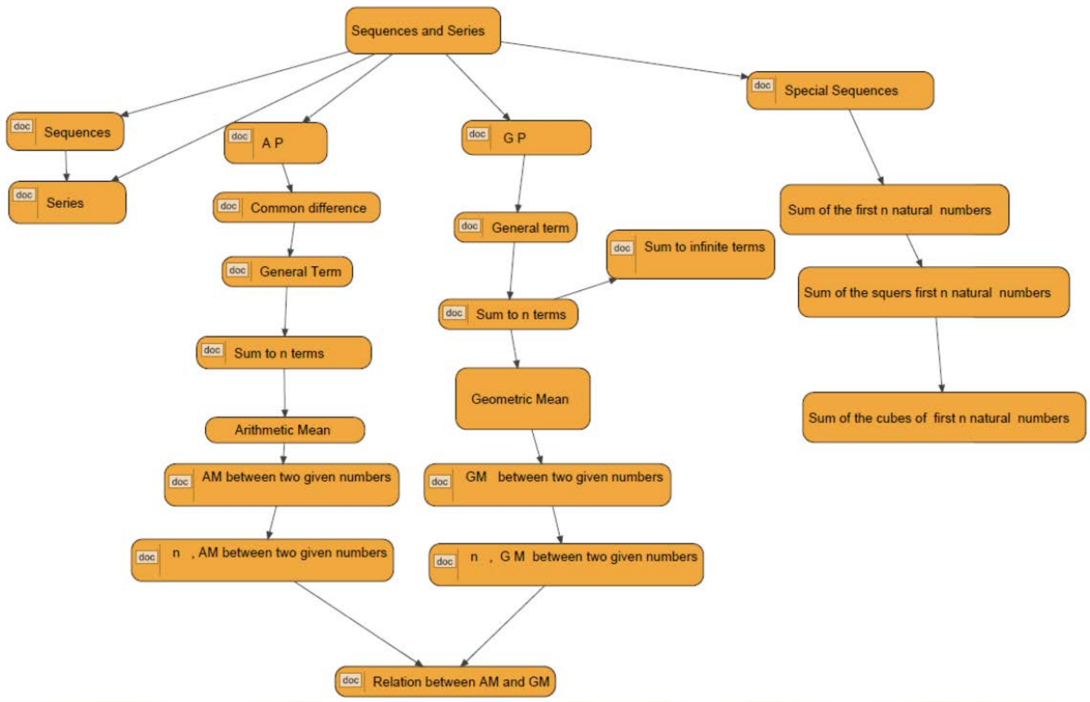
Binomial theorem: mathxtc.com/…/number Alg/files/the binomial theorem ppt

www.authorstream.com/presentation/...623028

CHAPTER-9-SEQUENCE AND SERIES
LEARNING OUT COMES OF SEQUENCE AND SERIES:
After studying this lesson, student will be able to :

- describe the concept of a sequence (progression);
- define an A.P. and cite examples;
- find common difference and general term of a A.P;
- find the fourth quantity of an A.P. given any three of the quantities a, d, n and t_n;
- calculate the common difference or any other term of the A.P. given any two terms of the A.P;
- derive the formula for the sum of ‘n’ terms of an A.P;
- calculate the fourth quantity of an A.P. given three of S, n, a and d;
- insert A.M. between two numbers;
- solve problems of daily life using concept of an A.P;
- state that a geometric progression is a sequence increasing or decreasing by a definite multiple of a non-zero number other than one;
- identify G.P.’s from a given set of progressions;
- find the common ratio and general term of a G.P;
- Calculate the fourth quantity of a G.P when any three of the quantities t_n, a, r and n are given.
- calculate the common ratio and any term when two of the terms of the G.P. are given;
- write progression when the general term is given;
- derive the formula for sum of n terms of a G.P;
- calculate the fourth quantity of a G.P. if any three of a, r, n and S are given;
- derive the formula for sum (S¥) of infinite number of terms of a G.P. when *r* <1;
- find the third quantity when any two of S_∞, a and r are given;
- convert recurring decimals to fractions using G.P;
- Insert G.M. between two numbers; and establish relationship between A.M. and G.M.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL1

1. Write the first four terms of the sequence defined by $a_n = 4n^2 + 3$.
2. Find the number of integers between 100 and 1000 that are divisible by 7.
3. Find the 15th term from the end of the sequence 7,10,13,…,130.
4. If the first term of an A.P is 2 and the sum of the first five terms is equal to one- fourth of the sum of the next five terms, find the sum of first 30 terms. Also find the 20th term.
5. If the 3rd , 6th and the last terms of a GP are 6,48 and 3072 respectively , find the first term and the number of terms in the GP.
6. Find the three numbers in G.P , whose sum is 19 and product is 216.
7. If pth , qth and rth terms of a G.P are a, b, c respectively. Show that $a^{q-r} b^{r-p} c^{p-q} = 1$.
8. Find the sum to n terms of the series : 1x2x3+2x3x4+3x4x5+…to n terms.
9. Find the sum of the series : $10^3 + 11^3 + \cdots + 20^3$.
10. Find the sum of the series : $1^2 + 3^2 + 5^2 + \cdots to\ n\ terms.$

LEVEL 2

1. If p times the pth term of an A.P is q times the qth term, then show that its (p+q)th term is Zero.
2. Which term of the sequence 25, $24\frac{1}{4}, 23\frac{1}{2}, 22\frac{3}{4}, \ldots is\ the\ first\ negative\ term$.
3. The ratio of the sums of m and n terms of an A.P is $m^2 : n^2$. Show that the ratio of mth and nth terms is (2m-1): (2n-1).
4. The sum of two numbers is 13/6. Even numbers of A.M are inserted between them and their sum exceeds their number by 1. Find the number of means inserted.
5. Three numbers whose sum is 21 are in A.P. If 2,2,14 are added to them respectively, the resulting numbers are in G.P. Find the numbers.

6. In a set of four numbers, the first three are in G.P and the last three are in A.P, with a common difference 5. If the first and the fourth numbers are equal, find the four numbers.
7. Find the sum to n terms of the sequence: 0.6+0.66+0.666+ … to n terms.
8. Find the sum to n terms of the sequence: 11+103+1005+10007+… to n terms.
9. Show that: $\frac{1x2^2+2x3^2+\cdots+nx(n+1)^2}{1^2x2+2^2x3+\cdots+n^2x(n+1)} = \frac{3n+5}{3n+1}$.

LEVEL3

1. If a,b c d are in increasing G.P. If the A.M between a and b is 6 and the A.M between c and d is 54, then find the A.M of a and d.
2. Let $t_1 = 7, t_2 = 7^7, t_3 = 7^{7^7}$ and so on. Find the digit at the tens' place of number t_{1000}.
3. If f is a function satisfying f(x+y)=f(x)+f(y) for all $x, y \in N\ such\ that\ f(1) = 3\ and\ \sum_{x=1}^{n} f(x) = 84$, then find the value of n.
4. The digits of a three digit natural number are in A.P and their sum is 15.The number obtained by reversing the digits is 396 less than the original number. Find the number.
5. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on the third day and so on. It took 8 days more to finish the work. Find the number of days in which the work was completed.
6. If the sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45. Find the series.
7. Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, prove that $y^3 + z^3 = 2xyz$.
8. If x,y,z are distinct positive numbers, prove that (x+y)(y+z)(z+x)>8xyz.Further if x+y+z=1, show that (1-x)(1-y)(1-z)>8xyz.
9. Find the sum to infinity of the series : $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \cdots$

Value Based Questions:

10. In rural area, a manufacturer of carpets produced 600 units in the third year and 700 units in the seventh year by an automatic machine. Assume that production increases by a fixed number of units every year, find i) the production in the first year ii) the total production in 7 years iii) the production in 10th year. Keeping in mind the rural area, justify the values to be promoted for the selection of manually operated, machines instead of automatic machine.
    Suggested values: Keeping "Save environment" and conversion of exhaustible resources in mind , the manually operated machine should be promoted so that energy could be saved.
11. A farmer buys a used tractor for Rs. 1200. He pays Rs. 6000 and agree to pay the balance in annual installments' of Rs.500 plus 12% interest on unpaid amount. How much will the tractor cost him? What steps are taken in India to promote lending to farmers?
    Suggested values : RBI gave the role of priority sector lending to the banks under which banks are stipulated to have fixed percentage of branches in remote areas and sectors such as agriculture.
12. An NGO wants to invest Rs. 300000 in a bond which pays 5% annual interest first year , 65 second year, 7% third year and so on. The interest received per year is to be utilized for the

education of poor people. If the investment is done for 10 days, find the total interest received by N.G.O. Which value is reflected in the question?

Suggested values : It shows the wisdom and foresightedness of the person who spends in the education of poor people. It helps in the uplift of the weaker sections of Society.

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
Confusion in definition of sequences and series.	Definition should be explained through counter examples.
Confusion in the number of terms of a GP	Concept should be explained through suitable examples.
Condition for sum to infinite terms of a GP (lrl<1)is neglected while solving problems.	Emphasis should be given on the value of lrl while solving the problems.
While writing the nth term in a special sequence, students generalize using the first term alone.	Students should verify for n=2,3 also while finding the nth term and sum to n terms. This can be rectified only practice.

QUESTION BANK

1. The 4^{th} term of an A.P is equal to 3 times the first term and the seventh term exceeds twice the 3^{rd} term by 1. Find the 1st term & the common difference?

2. In an A.P p^{th}, q^{th} and r^{th} terms are 'a','b' & 'c', prove that p(b-c) + q(c-a)+ r(a-b)= 0.

3. If a^2,b^2,c^2 are in A.P , prove that a/(b+c) , b/(c+a) , c/(a+b) are in A.P

4. If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b)=0$ are equal , prove that 1/a , 1/b ,1/c are in A.P.

5. The sum of n-terms of two arithmetic series are in the ratio (7n+1):(4n+27), find the ratio of their 11th terms?

6. The sum of first p,q,r terms of an A.P are a,b &c , prove that a(q-r)/p + b(r-p)/q + c(p-q)/r =0

7. The sum of three consecutive nos. in an A.P is 18 and their product is 192. Find the numbers?

8. The ratio of the 2nd to 7th of 'n' A.M's between -7 & 65 is 1:7, find 'n'?

9. Three numbers whose sum is 15 are in A.P. If 8, 6 & 4 be added to them

Problems for bright students

Q1.The sum of two numbers is six times their geometric mean. Show that the numbers are in the ratio ($3+2\sqrt{2})/(3—2\sqrt{2})$

Q2.Show that the ratio of the sum of first n- terms of a G.P to the sum of terms from (n +1)th to (2n)th term is $1/r^n$

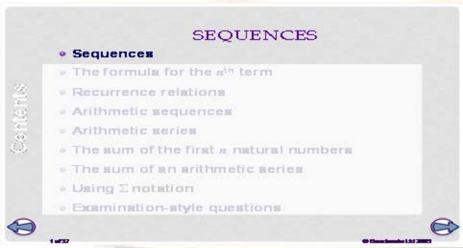
Q3.Show that $1 \times 2^2 + 2 \times 3^2 + \ldots\ldots\ldots + n(n+1)^2 \quad 3n+5$

$1^2 \times 2 + 2^2 \times 3 + \ldots\ldots\ldots + n^2 x(n+1) \quad 3n+1$

Q4.Sum the series : 1+ 5 +14 +30 +........upto n-terms

Q5. If S_1 ,S_2 & S_3 denote respectively the sum of first n- natural numbers, their squares & their Cubes then show that $9S_2^2 = S_3(1+8S_1)$

Q6. If a ,b , c are in G .P & the equations $ax^2 + 2bx + c = 0$ & $dx^2 + 2ex + f = 0$ have a common root then show that d/a , e/b , f/c are in A.P.



Sequences and series:www.xpowerpoint.com/ppt/sequences and series-html
www.pptsearch365.com/ppt-on-sequences and series.html
www.powershow.com/..../sequences _ and_ series_powerpoint_ppt

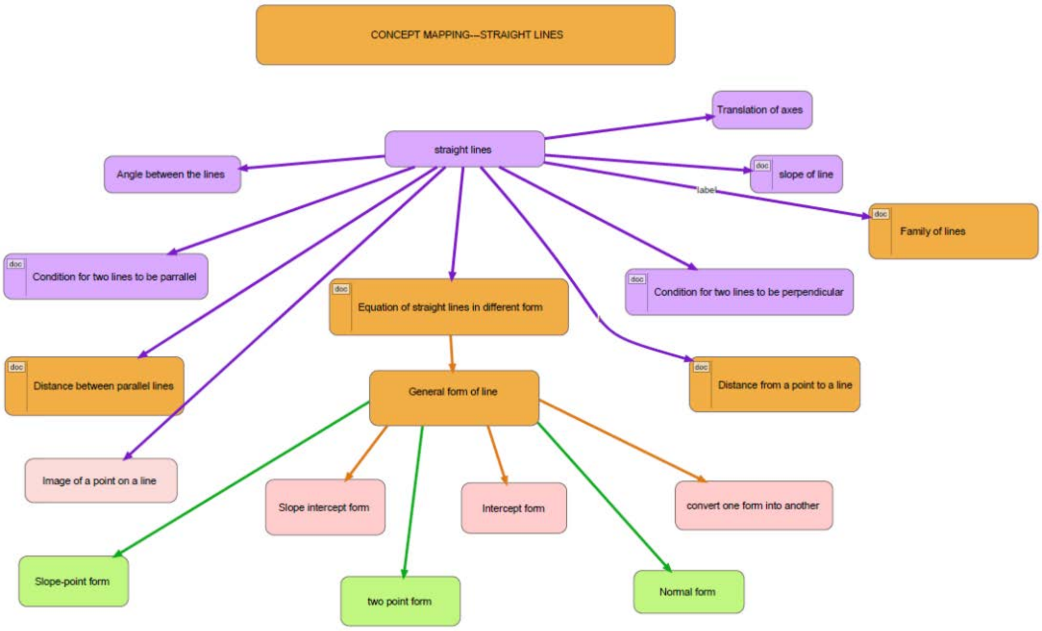
CHAPTER-10-STRAIGHT LINES

Expected learning outcomes:

1. Students are able to find out slope of a line when (i) Inclination was given,(ii)two points were given.
2. They are able to find out the given lines are perpendicular or parallel from the given slopes.
3. They are able to find out the angle between the lines when the slopes of the lines were given.
4. They are able to find out various forms of the line and conversions from one form to another form.
5. They are able to find out distance of a line from a given point not lying on the line.
6. They are able to find out distance between the parallel lines.
7. They are able to find out the coordinates of a point using translation of axes.

8. They are able to find out equation of the families of lines passing through the point of intersection of the given lines.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

Level : I

1. If the lines $2x + y - 3 = 0$, $5x + ky - 3 = 0$ and $3x - y - 2 = 0$ are concurrent, find the value of k.
2. Find the values of θ and p, if the equation $x \cos\theta + y \sin\theta = p$ is the normal form of the line $\sqrt{3}\,x + y + 2 = 0$.
3. In what ratio, the line joining (– 1, 1) and (5, 7) is divided by the line $x + y = 4$?
4. If sum of the perpendicular distances of a variable point P (x, y) from the lines $x + y - 5 = 0$ and $3x - 2y + 7 = 0$ is always 10. Show that P must move on a line.
5. The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K = 273 when F = 32 and that K = 373 when F = 212. Express K in terms of F and find the value of F, when K = 0.
6. Find angles between the lines $\sqrt{3}\,x + y = 1$ and $x + \sqrt{3}\,y = 1$.
7. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are 'a' and 'b', then show that $1/p = 1/a + 1/b$.
8. Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).
9. A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

10. Show that the path of a moving point such that its distances from two lines $3x-2y=5$ and $3x+2y=5$ are equal is a straight line.

LEVEL-2

1. The owner of a milk store finds that, he can sell 980 liters of milk each week at Rs 14/liter and 1220 liters of milk each week at Rs 16/liter. Assuming a linear relationship between selling price and demand, how many liters could he sell weekly at Rs 17/liter?
2. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.
3. If (1,2) and (3,8) are pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.
4. Three consecutive vertices of parallelogram are (–2, –1),(1,0) and (4,3), find the fourth vertex.
5. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6,q). Find p and q.
6. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
7. Find the equation of line with slope –1 and whose perpendicular distance from the origin is equal to 5.
8. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $3x - 4y + 1 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.
9. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y=2x=c$. Find c and remaining two vertices.
10. Find points on the line $x+y+3=0$ that are at a distance of $\sqrt{5}$ units from the line $x+2y=2=0$

VALUE BASED QUESTIONS

1. 10 students are participating in a debate on "SAVE WILD LIFE". Five students have to speak in favour and five students against these students are standing on two lines face to face whose equations are $5x-2y-1=0$ and $10x-4y+7=0$ for a debate. Are the students standing on parallel lines? During debate you favour which team?
2. One honest person and one dishonest person are standing on a ground represented by the points A(2,3) B(4,1). Two poles Pand Q are erected between AB such that AP:PB = 1:2 and AQ : QB = 2:1. Find the position of the two poles. Five students are asked to choose pole P or pole Q. Which pole do you think they are likely to choose? Which value system it depicts?
3. Line through the points (-2,6)and (4,8)is perpendicular to the line through the points (8,12) and (x,24).If value of x is positive then we conclude that the four persons standing at these points acquired positive value system and if value x is negative , then the four persons acquired negative value system and needs to advice for positive value system. What do you think, you need to advice them?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
The value of Area is calculated in –ve units	Modulus should be incorporated in finding areas.
While finding the length of the normal from $3x+4y+9=0$,then $p=-9/5$	Perpendicular distance from the origin to the line cannot be –ve.

QUESTION BANK

Q.1. The slope of a line is double of the slope of another line. If tangent of the angle between them is 1/3, find the slope of the lines.

Q.2. Find the equation of the line that has y intercept 4 and is parallel to the line 2x + 3y =7

Q.3. Find the equation of the line that has x intercept - 3 and is perpendicular to line 3x+5y =4.

Q.4. Prove that the lines 7x + 2y + 5 = 0 and 14x+ 4y- 8 = 0 are parallel to each other.

Q.5. Prove that the lines 3x- 2y + 5 = 0 and 4x + 6y+ 23 = 0 are perpendicular.

Q.6. A line perpendicular to the line segment joining the points (1,0) and (2,3) divides it in the ratio 1:n. Find the equation of the line.

Q.7. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Q.8. Find the equation of a line which passes through the point (3, - 2) and is inclined at 600 to the line √3x + y = 1.

Q.9. Find equation of the line passing through the point (2,2) and cutting off intercepts on the axes whose sum is 9.

Q10. A line such that its segment between the axis is bisected at the point (x_1, y_2). prove that the equation of the line is $\frac{x}{2x_1}+\frac{y}{2y_2}=1$

Q11. In the triangle ABC with vertices A (2, 3), B (4, 1) and C (1, 2), find the equation and length of altitude from the vertex A.

Q.12. Find equation of the line which is equidistant from parallel lines 9x + 6y -7 =0 and 3x + 2y +6=0

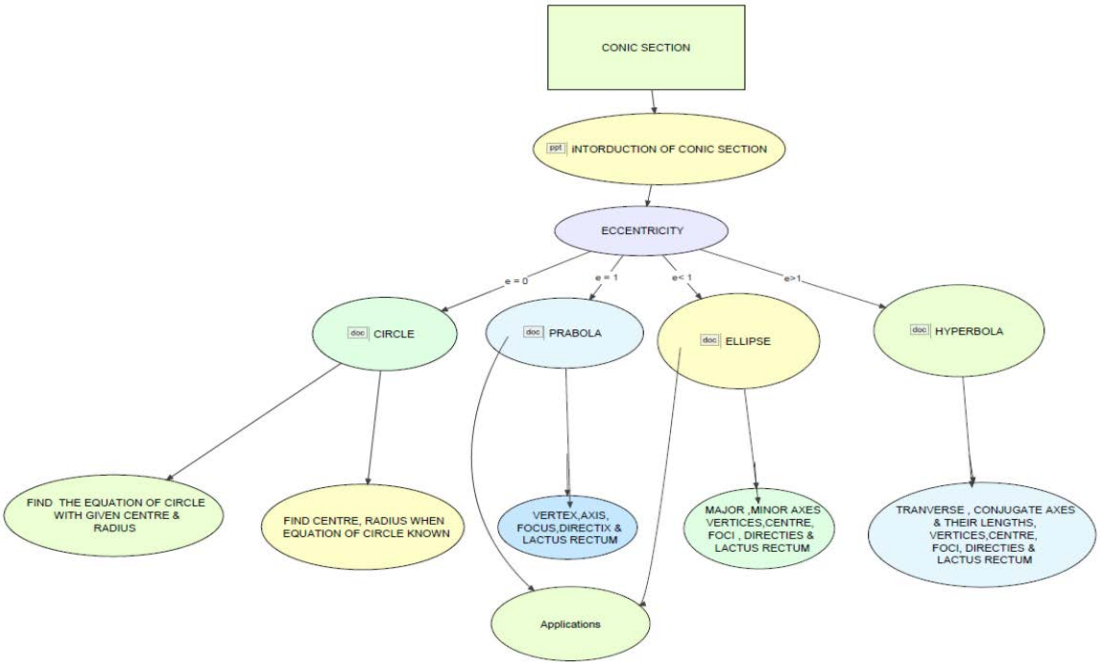
1. Straight lines: https://youtu.be/k5DWvLbNc-o, https://youtu.be/D81l4zRSofk., www.authorstream.com/…. Straight lines ppt entertainment

CHAPTER-11-CONIC SECTIONS

Expected learning out comes:

1. Students are able to find out radius & centre of a circle when the equation of a circle is given and vice versa.
2. They are able to find out focus, vertex and length of latus rectum of the parabola from its standard equation.
3. They are able to find out equation of parabola when focus or vertex was given.
4. They are able to find out from the standard equation of ellipse its foci, vertices, eccentricity, length of major axis, length of minor axis and length of latus rectum etc.
5. They are able to find out from the standard equation of hyperbola its foci, vertices, eccentricity, length of major axis, length of minor axis and length of latus rectum etc.
6. They are able to apply their knowledge and understanding in solving application problems of parabola and ellipse.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL-1

CIRCLES

1. Find an equation of the circle with Centre at (0,0) and radius r.
2. Find the equation of the circle with Centre (–3, 2) and radius 4.
3. Find the Centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$
4. Find the equation of the circle passing through the points (4,1) and (6,5) and whose Centre is on the line $4x + y = 16$.
5. Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.

PARABOLA

6. Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola $y^2 = 8x$.
7. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum of the parabola $x^2 = -12x$.
8. Find the equation of the parabola with vertex (0,0), passing through (5,2) and symmetric with respect to x-axis.
9. The focus of a parabolic mirror is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, find the diameter of the mirror.

11. A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its Centre, there is a deflection of 3 cm at the Centre and the deflected beam is in the shape of a parabola. How far from the Centre is the deflection 1 cm?

ELLIPSE

12. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

    i) $\frac{x^2}{25}+\frac{y^2}{16}=1$ ii) $4x^2+9y^2=36$.

13. Find the equation for the ellipse with Vertices ($\pm$6, 0), foci ($\pm$4, 0)
14. Find the equation for the ellipse with Length of major axis 26, foci ($\pm$5, 0)
15. Find the equation for the ellipse with Centre at (0, 0), major axis on the *y*-axis and passes through the points (3, 2) and(1,6).
16. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the Centre. Find the height of the arch at a point 1.5 m from one end.

HYPERBOLA

11. In each of the following, find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

    i) $\frac{y^2}{9}-\frac{x^2}{27}=1$ ii) $16x^2 - 9y^2 = 576$.

12. Find the equations of the hyperbola with vertices ($\pm$2, 0), and foci ($\pm$3, 0)
13. Find the equations of the hyperbola with foci ($\pm$5, 0), the transverse axis is of length 8.
14. Find the equations of the hyperbola with foci ($\pm$5, 0), the conjugate axis is of length 24
15. Find the equations of the hyperbola with vertices ($\pm$7, 0), $e=\frac{4}{3}$.

LEVEL-2

1. Find the Centre and radius of $3x^2 + 3y^2 + 6x - 4y - 1 = 0$
2. Find equation(s) of circle passing through points (1,1), (2,2) and whose radius is 1 unit.
3. One end of diameter of a circle $x^2+ y^2- 6x + 5y - 7 = 0$ is (7, –8). Find the coordinates of other end.
4. Find the coordinates of focus, and length of latus rectum of parabola$3y^2= 8x$.
5. Find the equation of the ellipse coordinates of whose foci are (±2, 0) and length of latus rectum is 10/3.
6. Find the equation of hyperbola with Centre at origin, transverse axis along x-axis, eccentricity 5 and sum of lengths of whose axes is 18.
7. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.
8. Show that the four points (7,5), (6, –2) (–1,–1) and (0,6) are concyclic.
9. One end of diameter of a circle $x^2 + y^2 - 6x + 5y - 7 = 0$ is (7, –8). Find the coordinates of other end.
10. Find equation of the circle which touches the y- axis at origin and whose radius is 3 units.

Value based questions

CONIC SECTIONS

1. A debate on National Integration was organized and the seating arrangement was done in the form of a parabola, represented by $y^2=12x$. The last two participants are sitting at corners which represent the end points of latus rectum of a parabola. What is the distance between these two participants? What do you think, they are going to discuss at debate on National Integration? Which value system they are trying to highlight?
2. A man is running on a path and notes the distance of two extreme flag posts on path is 10 metres. He notes that he can read the messages of value system "HONESTY" and "RESPECT FOR OTHERS" on the poles whichever side he moves, also the eccentricity of the path is 0.8.find the equation of the path traced by the man. Do you think these values system are necessary in life?
3. Three tables are placed at the points A(4,3),B(8,-3) and C(0,9).On one table, information about disadvantages of copying, on the second table references about "TOWARDS A CLEANER ENVIRONMENT" and on the third table views about "TOWARDS A JUST SOCIETY" were presented. What is your view about placement of tables? Are these along a circle or a straight line? What are your thoughts about a "JUST SOCIETY".

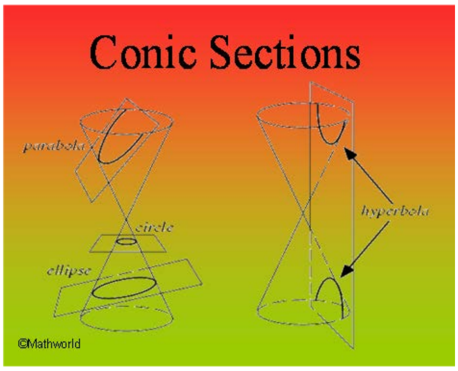
ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
For the equation, $3x^2+3y^2+6x+8y+5=0$, centre = (-3,-4)	Before writing the centre, the equation should be brought to the standard form.
While writing the coordinates of focus and vertices of a parabola, ellipse and hyperbola, x and y coordinates are interchanged.	Standard equations are to be identified properly from which the students can write the required coordinates correctly.

QUESTION BANK

1. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).
2. Find the equation of the circle passing through the points (2,3) and (–1,1) and whose centre is on the line $x – 3y – 11 = 0$.
3. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$
4. Find the equation of the ellipse with major axis along the x-axis and passing through the points (4, 3) and (-1, 4).
5. Find the equation of the ellipse, whose length of the major axis is 20 and foci are (0, ± 5).
6. Find the equation of the hyperbola with foci (0, ±3) and vertices (0, $\pm\sqrt{11}/2$)
7. Find the equation of the hyperbola with vertices (±7,0) and e = 4/3.
8. Find the equation of the hyperbola where foci are (0, ±12) and the length of the latus rectum is 36.
9. A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected is in the shape of a parabola. How far from the centre is the deflection 1 cm.

10. Find the equation of the ellipse whose foci are (4,3),(-4,3) and whose semi minor axis is 3.



1. Conic sectionshttps://youtu.be/5kM39HZDQ5s

.https://youtu.be/VbLRAXZ7zB8

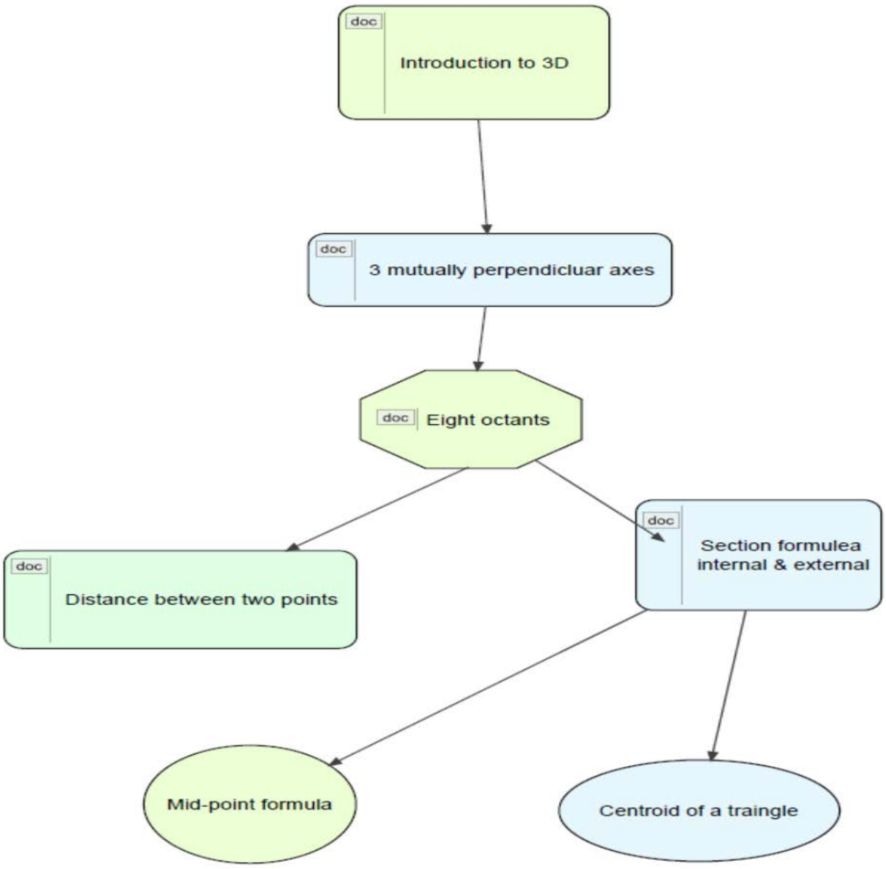
. https://youtu.be/9t8ENyXNs0U

CHAPTER-12-INTRODUCTION TO 3D GEOMETRY

Expected learning out comes:

1. Students are able to find out, where the given point lies in the space.
2. They are able to identify the eight octants and the three coordinate planes (xy, yz, zx).
3. They are able to solve the problems using distance formula.
4. They are able to solve the problems using section formula.
5. They are able to solve the collinearity problems using distance formula.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LEVEL1:

1. Find the distance between the points P(1, –3, 4) and Q (– 4, 1, 2).
2. Find the equation of the set of the points P such that its distances from the points A (3, 4, –5) and B (– 2, 1, 4) are equal.
3. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, – 6), respectively, find the coordinates of the point C.
4. Three vertices of a parallelogram ABCD are A(3, – 1, 2), B (1, 2, – 4) and C (– 1, 1, 2). Find the coordinates of the fourth vertex.
5. If A and B be the points (3, 4, 5) and (–1, 3, –7), respectively, find the equation of the set of points P such that $PA^2+PB^2=k^2$, where *k* is a constant.
6. Show that the points (4,-3,-1),(5,-7,6) and (3,1,-8) are collinear.
7. Show that the points A (1, 2, 3), B (–1, –2, –1), C (2, 3, 2) and D (4, 7, 6) are the vertices of a parallelogram ABCD, but it is not a rectangle.
8. Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).
9. Find the equation of the set of points P, the sum of whose distances from A (4,0, 0) and B (– 4, 0, 0) is equal to 10.
10. Find the coordinates of a point on y -axis which are at a distance of 5√2 from the point P(3,–2,5).

LEVEL 2

1. Find the distance between (-3,4,-6) and its image in the XY- plane.
2. Find the points on the y- axis which are at a distance of 3 units from the points (2,3,-1)
3. Find the coordinate of Point P which is five-sixth of the way from A(2,3,-4) to B(8,-3,2)
4. If the points A(1,0,-6) B(-3,p,q) and C(-5,9,6) are collinear. Find the values of 'p' and 'q'.
5. A point R with x–coordinate 4 lies on the line segment joining the points P(2,–3, 4) and Q(8,0, 10). Find the coordinates of the point R.

VALUE BASED QUESTION

1. Three students are standing in a park with sign boards "SAVE ENVIRONMENT", "DON'T LITTER" and "KEEP YOUR PLACE CLEAN". Their positions are marked by the points A(0,7,10),B(-1,6,6) and C(-4,9,6). The three are holding green coloured ribbon together. Does the ribbon form the sides of a right angled triangle? Do you feel the need to promote, what is written on sign boards?

2. Four students in traditional dresses represent four states of India, standing at the points represented by O(0,0,0) and A(x,0,0), B(0,y,0), C(0,0,z). Find the place, in terms of coordinates, where a girl representing "BHARATHMATA" be placed so that she is equidistant from four students. What message does it convey?

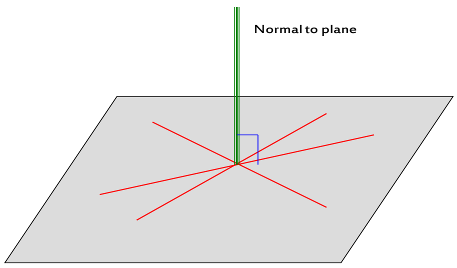
3. In a room, a globe is hanged in the middle of the room using a string. Globe keeps on rotating depicting the history of "STRUGGLE FOR INDEPENDENCE". If globe is represented by the point (3,4,5). What is its distance from one corner of a room, assumed as the origin? Do you remember in which year, the first war of independence was fought?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
(3,4,-5) lies in first octant	The concept of octants should be clear so that the child recognizes the correct octant.
The ratio -2:1 is taken as internal division.	The negative sign in the ratio indicates external division.

QUESTION BANK

1. Find the ratio in which the line joining the points (2, 4, 16) and (3, 5, -4) is divided by the plane 2x – 3y+ z + 6 = 0. Also find the co-ordinates of the point of division.
2. Show that the points (-1, -6, 10), (1, -3, 4), (-5, -1, 1) and (-7, -4, 7) are the vertices of a rhombus.
3. Three vertices of a parallelogram ABCD are A (3, -4, 7) B (5, 3, -2) and C (1, 2, -3). Find the fourth vertex D.
4. Find the coordinates of the points which trisect the line segment AB, given A (2, 1, -3) and B (5, -8, 3).
5. Find the coordinates of the point P which is five-sixth of the way from A(2, 3, –4) to B(8, –3, 2).
6. The x co-ordinate of a point is 9. Find its other co-ordinates if this point lies on the line joining the points (7, 2, 1) and (10, 5, 7).
7. Find lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).
8. Determine the point in XY plane which is equidistant from the points A (1, –1, 0) B(2, 1, 2) and C(3, 2, –1).
9. Find the locus of the point which is equidistant from the points A(0,2,3) and B(2,–2,1).
10. The Centroid of triangle ABC is at (1,1,1). If co-ordinates of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of points C.



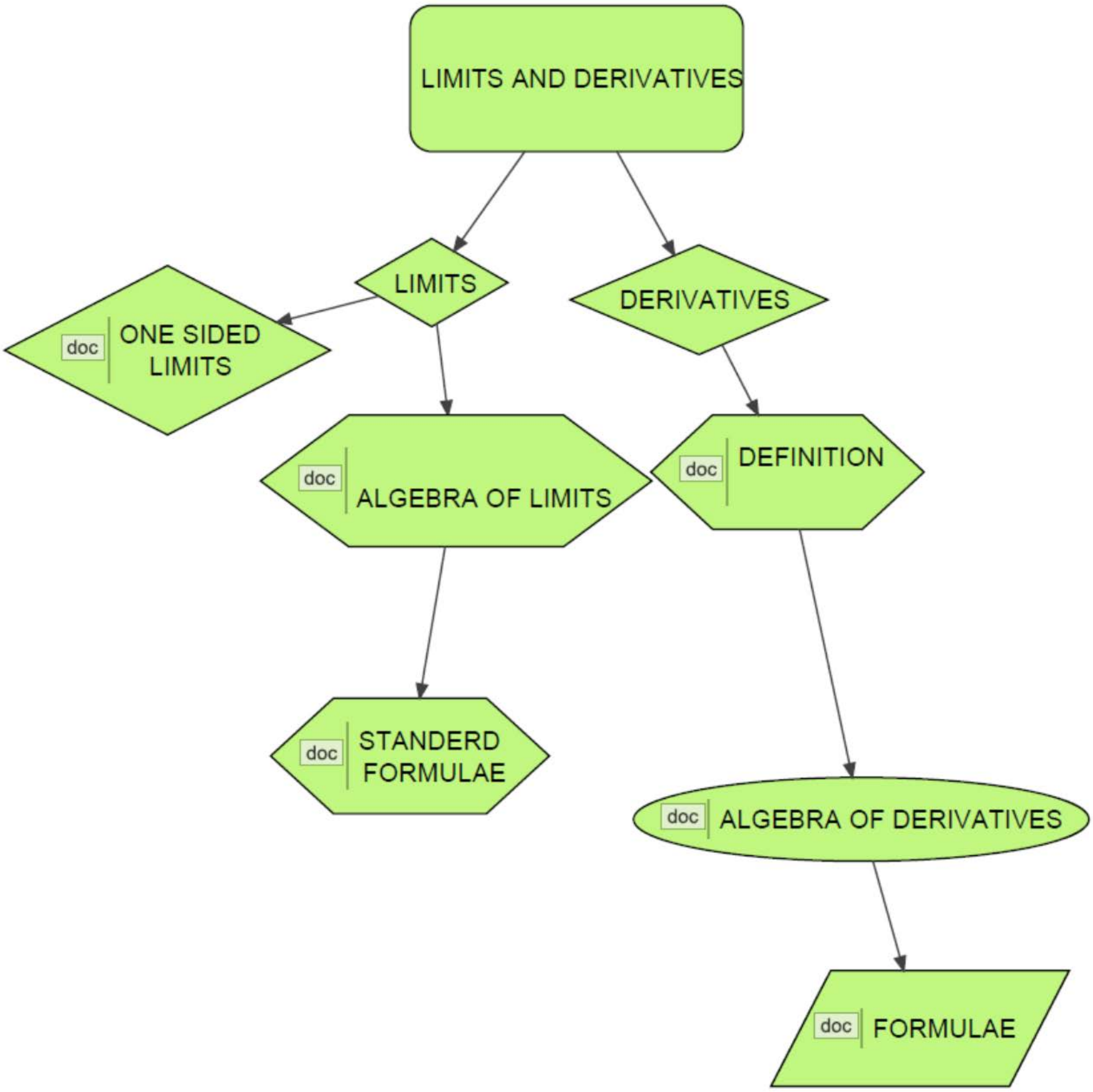
2. 3Dimensional geometry:
   https://youtu.be/IlUpB0LmlX0?list=PLauXkHsTK5c9t_VV8Rl5RHtNalGrx4Vt9
   : https://youtu.be/Z4XQmd-RNjU

CHAPTER-13-LIMITS AND DERIVATIVES

EXPECTED LEARNING OUTCOMES:

1. Students will imagine and observe the intuitive idea of limits through graphs.
2. Students will comprehend one sided limits both graphically and through the definition.
3. Students will apply the concept of one sided limits to find the existence of the limit.
4. They observe the nature of the function involving the limit.
5. They compare whether the function is a difference Quotient
6. After understanding the derivative, they apply limit of Difference Quotient to obtain the derivative.
7. They apply differentiation to find the slope of the tangent at any point on the curve.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

LHL and RHL

Level I

1. $Find\ \lim_{x\to 0} f(x)\ and\ \lim_{x\to 1} f(x)\ ,where\ f(x) = \begin{Bmatrix} 2x+3, x \le 0 \\ 3(x+1), x > 0 \end{Bmatrix}$

2. $If\ f(x) = \begin{Bmatrix} x, for\ x < 1 \\ 2\ for\ x = 1 \\ x+2, for\ x > 1 \end{Bmatrix}, then\ find\ \lim_{x\to 1} f(x)$

Level II

1. $Evaluate\ \lim_{x\to 0} f(x)\ ,where\ f(x) = \begin{Bmatrix} \frac{|x|}{x}, x \ne 0 \\ 0, x = 0 \end{Bmatrix}$

2. Let f(x) be a function, $f(x) = \begin{Bmatrix} \frac{6x}{|x|-2}, x \ne 0 \\ 0, \quad x = 0 \end{Bmatrix}, Evaluate : \lim_{x\to 0} f(x)$

Level III

1. $Suppose\ \ f(x) = \begin{Bmatrix} a+bx, x < 1 \\ 4, \quad x = 1 \\ b-ax, x > 1 \end{Bmatrix} and\ if\ \lim_{x\to 1} f(x) = f(1),\ what\ are\ the\ possible\ values\ of\ a\ and\ b?$

Standard limits

Level I

Evaluate (1 to 5)

1. $\lim_{x\to 1} \left[\frac{x^2+1}{x+100}\right]$

2. $\lim_{x\to 0} \left[\frac{3\ sinx}{x}\right]$

3. $\lim_{x\to 0} \left[\frac{e^{2x}-1}{x}\right]$

4. $\lim_{x\to 0} \left[\frac{2^x-1}{x}\right]$

5. $\lim_{x\to 0} \left[\frac{cosx}{1+sinx}\right]$

Level II

Evaluate (1 to 3)

1. $\lim_{x\to 1} \left[\frac{x^2-9x+20}{x^2-6x+5}\right]$

2. $\lim_{x\to 0} \left[\frac{cosec2x-cot2x}{x}\right]$

3. $\lim_{x\to 1} \left[\frac{\log_e x}{x-1}\right]$

4. If $\lim_{x\to(-a)} \left[\frac{x^7+a^7}{x+a}\right] = 7$, find the value of a?

Level III

1. $Evaluate\ \lim_{x\to \pi/2} \left[\frac{1+cos2x}{(\pi-2x)^2}\right]$

2. If $\lim_{x\to 1}\left[\frac{x^4-1}{x-1}\right] = \lim_{x\to k}\left[\frac{x^3-k^3}{x^2-k^2}\right]$, find all the values of k

3. Evaluate $\lim_{x\to 0}\left[\frac{10^x-5^x-2^x+1}{x^2}\right]$

4. $Evaluate$ $\lim_{x\to 0}\left[\frac{sinx-2sin3x+sin5x}{x^3}\right]$

DERIVATIVES

First Principles

Level I

Using first principles, find the derivative of

1. (x + 4)
2. x^3
3. e^{2x}

Level II

Using first principles, find the derivative of

1. Sin2x2. log(x+1)

Level III

Using first principles, find the derivative of

1. $\left(\frac{x+1}{x-1}\right)$2.$\sqrt{cosx}$

TECHNIQUES OF DERVIATIVES

Level I

Find the derivatives of the following

1. x^3+ e^x+2^x2. x^2cosx3. (x+1)(2x-5)

Level II

Find the derivatives of the following

1 $\frac{x^2+3}{x}$

2 (ax^2+sinx)(p+qcosx)

3 $\frac{(4x+5sinx)}{(3x+7cosx)}$

Level III

Find the derivatives of the following

1. $Sin^n x$
2. $\frac{x}{sin^n x}$
3. $(ax+b)^n$

ERROR ANALYSIS:

1. Mistakes while using LHL and RHL.	Clear explanation of LHL and RHL is a must.
2. $\frac{d(uv)}{dx} = \frac{d(u)}{dx}\frac{d(v)}{dx}$	To Learn product rule of differentiation thoroughly and to use it properly.
3. $\frac{d(u/v)}{dx} = (\frac{v'u \pm u'v}{v^2})$	To Learn quotient rule of differentiation thoroughly and to use it properly.
4. $(sinx)^2$=sinx^2	Explanation of sin^2x as sinx × sinx and sin$x^2 = \sin(x \times x)$ $in\ important$

QUESTION BANK

EEVALUATE :

1. $\lim_{X\to 2}\left(\frac{X^3-2X^2}{X^2-5X+6}\right)$

2. $\lim_{X\to 1}\left[\frac{X-2}{X^2-X}-\frac{1}{X^3-3X^2+2X}\right]$

3. $\lim_{x\to 0}\left[\frac{\sqrt{1+x}-1}{x}\right]$

4. $\lim_{x\to \frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}$

5. $\lim_{x\to 3-} \frac{x}{[x]}$

Differentiate the following with respect to x

6. $\frac{\sin x+\cos x}{\sin x-\cos x}$

7. $\frac{x^5-\cos x}{\sin x}$

8. $\frac{x}{1+\tan x}$

9. $x^2\cos x$

Differentiate the following using first principles

10. $\sqrt{\sin x}$

11. Cosec2x

12. Tanx

Introduction to Limits

Limits derivatives: www.powershow.com/view/277665-NmRiZ/limits........

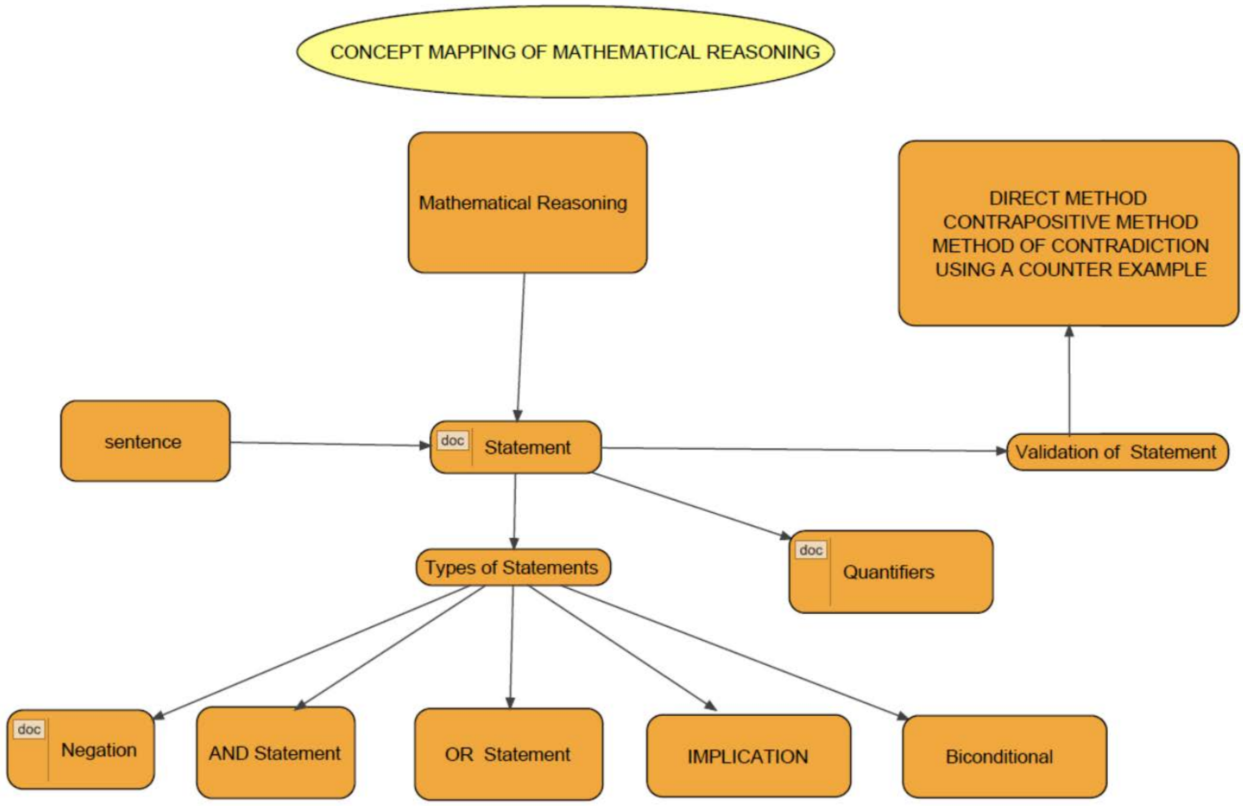
www.powershow.com/view4/57ec4b-NjNiM/limits

CHAPTER-14-MATHEMATICAL REASONING

Expected learning outcomes

1. Student will observe the difference between a sentence and a statement.
2. Student understands different types of statements.
3. They infer the validity of the statement through different methods.
4. The student applies the above concepts in the validity of many mathematical statements.
5. They will apply the above concepts in Boolean algebra and in digital electronics

CONCEPT MAPPING



GRADED LEVEL QUESTIONS
MATHEMATICAL REASONING
LEVEL I

1. Which of the following sentences are statements? Give reasons for your answer.
   (i) There are 35 days in a month.
   (ii) Mathematics is difficult.
   (iii) Answer this question.
   (iv) The product of (–1) and 8 is 8.

2. Write the negation of the following statements:
   (i) Chennai is the capital of Tamil Nadu.

(ii) All triangles are not equilateral triangle.
(iii) The number 2 is greater than 7.
(iv) Every natural number is an integer.

3. Find the component statements of the following compound statements and check whether they are true or false.
   (i) Number 3 is prime or it is odd.
   (ii) All integers are positive or negative.
   (iii) 100 is divisible by 3, 11 and 5.
4. Give three examples of sentences which are not statements. Give reasons for the answers.

LEVEL II

1. Write the contrapositive and converse of the following statements.
   (i) If x is a prime number, then x is odd.
   (ii) You cannot comprehend geometry if you do not know how to reason deductively.
   (iii) x is an even number implies that x is divisible by 4.
2. Write each of the following statements in the form "if-then"
   (i) You get a job implies that your credentials are good.
   (ii) A quadrilateral is a parallelogram if its diagonals bisect each other.
   (iii) To get an A+ in the class, it is necessary that you do all the exercises of the book.
3. Given statement in (a). Identify the statements given below as contra positive or converse of each other.
   (i) If you live in Delhi, then you have winter clothes.
   (ii) If you do not have winter clothes, then you do not live in Delhi.
   (iii) If you have winter clothes, then you live in Delhi.
4. By giving a counter example, show that the following statements are not true.
   (i) p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.
   (ii) q: The equation $x^2 – 1 = 0$ does not have a root lying between 0 and 2.

LEVEL III

1. Show that the statement *p:* "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by (i) direct method, (ii) method of contradiction, (iii) method of contra positive
2. Show that the following statement is true by the method of contra positive.
   p: *If x is an integer and x^2 is even, then x is also even.*
3. Show that $\sqrt{11}$ is an irrational number, using the method of Contradiction.

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
Confusion in sentence and statement	More examples should be given to clear the concept.
While writing the negation of a negation of a statement, one negation is neglected.	Concept should be explained through examples.
While writing the converse and contra positive of an implication statement, they interchange.	Concept should be explained through examples.
While proving the validity of the statement, the method specified is neglected.	Question should be read properly and sufficient practice should be given.

QUESTION BANK

1. Form the conjunction of the following simple statements:
   p: Dinesh is a boy
   q: Nagma is a girl
2. Express in English , the statement p→q, where
   p: it is raining today ,q: 2+3 > 4
3. Translate the following bi-conditional into symbolic form
   “ABC is an equilateral triangle if and only if it is equiangular”
4. Find whether the given statement is compound statement or not “2 is both an even number and a prime number”.
5. Write the negation of the given statement: x+y = y+x and 29 is a prime number.
6. Given the statement: “No rich man is happy”.
   Can you conclude that:
   (i) Happy people are not rich.
   (ii) Men who are not rich are happy.
   (iii) Some rich men are happy.

7. Rewrite the given statement in the form of conditional statement:
   “ When you sing , my ears hurt”.

Mathematical reasoning: www.powershow.com/view1/1 dcb42-ZDC IZ/mathematical

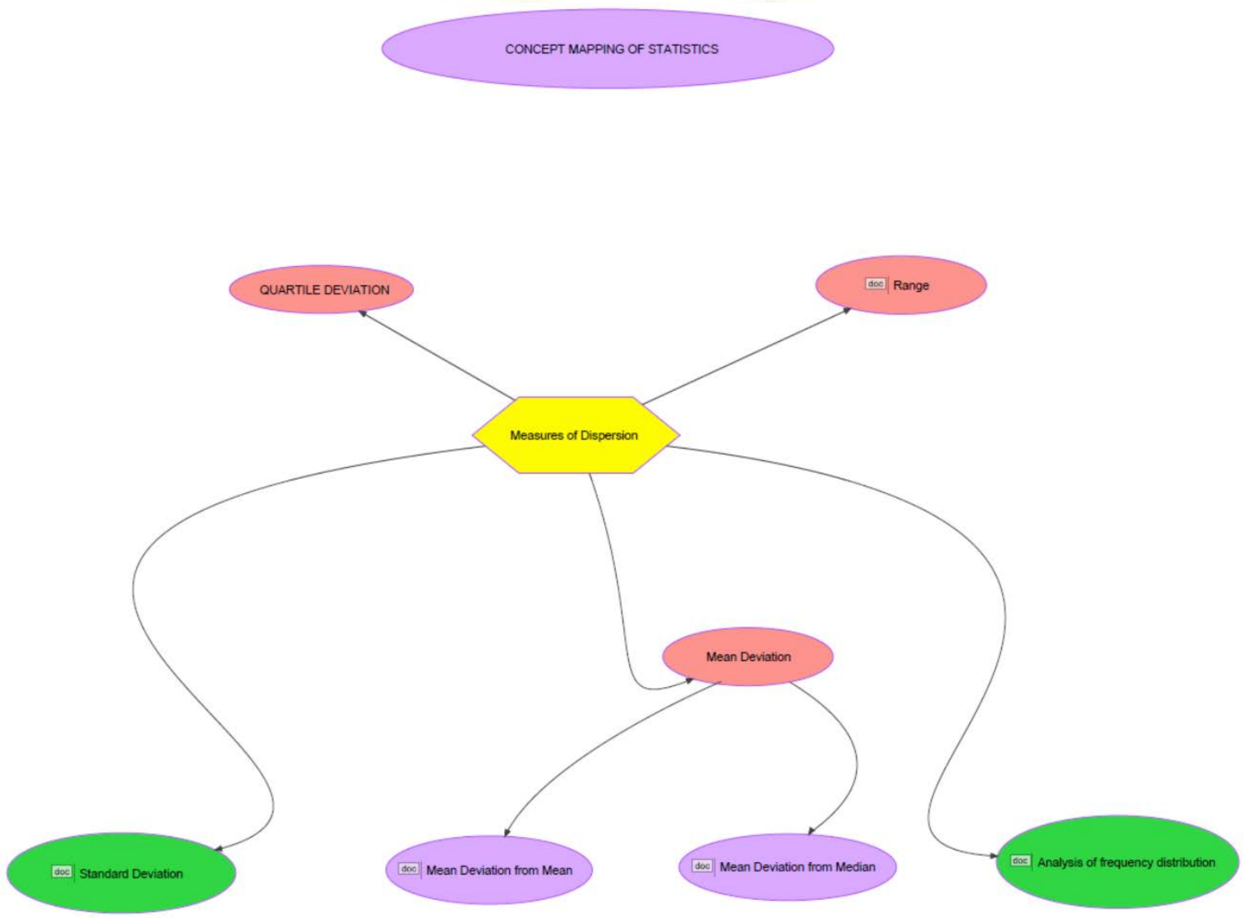
www.slideworld.com/ppt slides.aspx/mathematical reasoning

CHAPTER-15-STATISTICS

Expected learning outcomes

1. Students will observe the dispersion of the raw data through Range.
2. Students will comprehend that there are also other types of measures of dispersion.
3. Students will understand the formulae of mean deviation and standard deviation.
4. They will start calculating mean deviation and standard deviation.
5. They apply the above concepts in comparing the variability of 2 series

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

MEAN DEVIATION

LEVEL : I

1. Calculate the mean deviation from the mean for the following data: 13,17,16,14,11,13,10,16,11,18,12,17
2. Calculate the mean deviation about the median of the following data: 34,66,30,38,44,50,40,60,42,51

LEVEL : II

1. Calculate the mean deviation about mean from the following data

x_i :	03	09	17	23	27
f_i :	08	10	12	09	05

2. Calculate the mean deviation about median from the following data

x_i :	10	15	20	25	30	35	40	45
f_i :	07	03	08	05	06	08	04	09

LEVEL : III

1. Calculate the mean deviation from the mean of the following frequency distribution

Classes:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequencies:	02	03	08	14	08	03	02

2. Calculate the mean deviation from the median of the following frequency distribution

Wages per week in Rs:.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No: of workers :	04	06	10	20	10	06	04

3. Calculate the mean deviation from the median of the following data

Classes :	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Frequencies:	05	06	12	14	26	12	16	09

VARIANCE AND STANDARD DEVIATION

LEVEL : I

1. Find the variance and hence standard deviation from the following data:
   65,68,58,44,48,45,60,62,60,50
2. Find the variance and hence standard deviation of the following frequency distribution

x_i :	2	4	6	8	10	12	14	16
f_i :	4	4	5	15	8	5	4	5

Level: II

1. Using step-deviation method, find the standard deviation for the following frequency distribution

x_i:	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i:	1	5	12	22	17	9	4

2. Calculate the mean and standard deviation for the following frequency distribution (using step deviation method)

Marks:	20-30	30-40	40- 50	50-60	60-70	70-80	80-90
No: of students:	3	6	13	15	14	5	4

3. Calculate the mean , median and standard deviation of the following distribution

Class interval:	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70
Frequency :	2	3	8	12	16	5	2	3

Level: III

1. The variance of 20 observations is 5. If each observation is multiplied by 2, find the variance of the resulting observations.

2. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that 3 observations were in correct, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations were omitted.

3. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted

(ii) If it is replaced by 12

4. For the distribution $\Sigma(x-5) = 3$ and $\Sigma(x-5)^2 = 43$, whose total number of items is 18, Find the mean and standard deviation.

5. If each of the observations $p_1, p_2, p_3, \ldots p_n$ is increased by 'q' where 'q' is a negative or positive real number , show that variance remains unchanged.

COEFFICIENT OF VARIATION

LEVEL : I

1. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results.

	FIRM A	FIRM B
No of Workers	1000	1200
Average monthly wages (in Rupees)	2800	2800
Variance of distribution of wages	100	169

In Which firm, A or B is there greater variability in individual wages.

2. The following is the record of goals scored by team A in a foot ball session

No: of goals scored:	0	1	2	3	4
No: of matches :	1	9	7	5	3

For the team B, the mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

LEVEL : II

1. From the data given below, state which group is more variable 'A' or 'B'?

Marks:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
GROUP A	9	17	32	33	40	10	09
GROUP B :	10	20	30	25	43	15	07

2. Coefficient of variation of two distributions are 60 and70 and their standard deviations are 21 and 16 respectively, What are their arithmetic means.

LEVEL : III

1. The sum and sum of the squares corresponding to length x (in cm) and weight y(in gm) of 50 plant products are given below:
   $\Sigma x_i = 212$, $\Sigma x_i^2 = 902$, $\Sigma y_i = 261$, $\Sigma y_i^2 = 1457.6$
   Which is more varying, the length or weight?

2. If n = 10, mean = 12 and $\Sigma x_i^2 = 1530$, what is the coefficient of variation?

<u>VALUE BASED QUESTIONS- STATISTICS</u>

1. In a survey of 50 villages of a state, about the use of L.P.G. as a cooking made the following information about the families using L.P.G. was obtained

Number of families	0 - 10	10 - 20	20-30	30-40	40-50	50-60
Number of villages	6	8	14	10	4	2

Find the mean deviation about median for the following data. Do you think more awareness was needed for the villagers to use L.P.G. as a mode of cooking?

2. Circular sign boards are made with values to be acquired in life are made. Boards are of different diameters as:

Diameter of Board(in cm)	33- 36	37-40	41-46	45-48	49-52
Number of sign boards	15	17	21	22	25

Calculate the standard diameter and mean diameter of the sign boards. Which value you would like to see within maximum number of times?

ERROR ANALYSIS

ERRORS	REMEDIAL MEASURES
While finding the median, arranging the observations in an array is forgotten.	Explanation through practical situation namely arranging the students of the class, height wise to calculate the average height.
While calculating the Mean deviation, Modulus sign is forgotten	Explanation of $\sum(x_i - \bar{x}) = 0$ is suggested.
$(\sum x)^2 = \sum x^2$	Sum of the entries in the column headed with x^2 is $\sum x^2$ and square of $\sum x$ is $(\sum x)^2$

QUESTION BANK

Statistics

1. Find the mean deviation from the mean for the following data:

   11,13,4,7,8,6,15,14,3,19

2. Find the mean deviation from the median for the data:

   3,3,5,9,10,12,12,12,18,21,21

3. Find the variance and standard deviation for the following data:

   6,10,7,13,4,12,8,12

4. Calculate the standard deviation for the following distribution giving the age distribution of persons:

Age (in yrs) :	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No: of persons:	03	61	132	153	140	51	02

5. Find the mean and standard deviation for the following data:

x:	92	93	97	98	102	104	109
f :	03	02	03	02	06	03	03

6. Calculate the standard deviation and coefficient of variation for the following data:

x :	05	15	25	35	45	55
f :	12	18	27	20	17	06

7. Find the coefficient of variation for the following data:

Class:	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
f:	20	24	32	28	20	16	34	10	16

8. The mean and standard deviation of six observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

9. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

   (i) If wrong item is omitted (ii) If it is replaced by 12

Standard Deviation

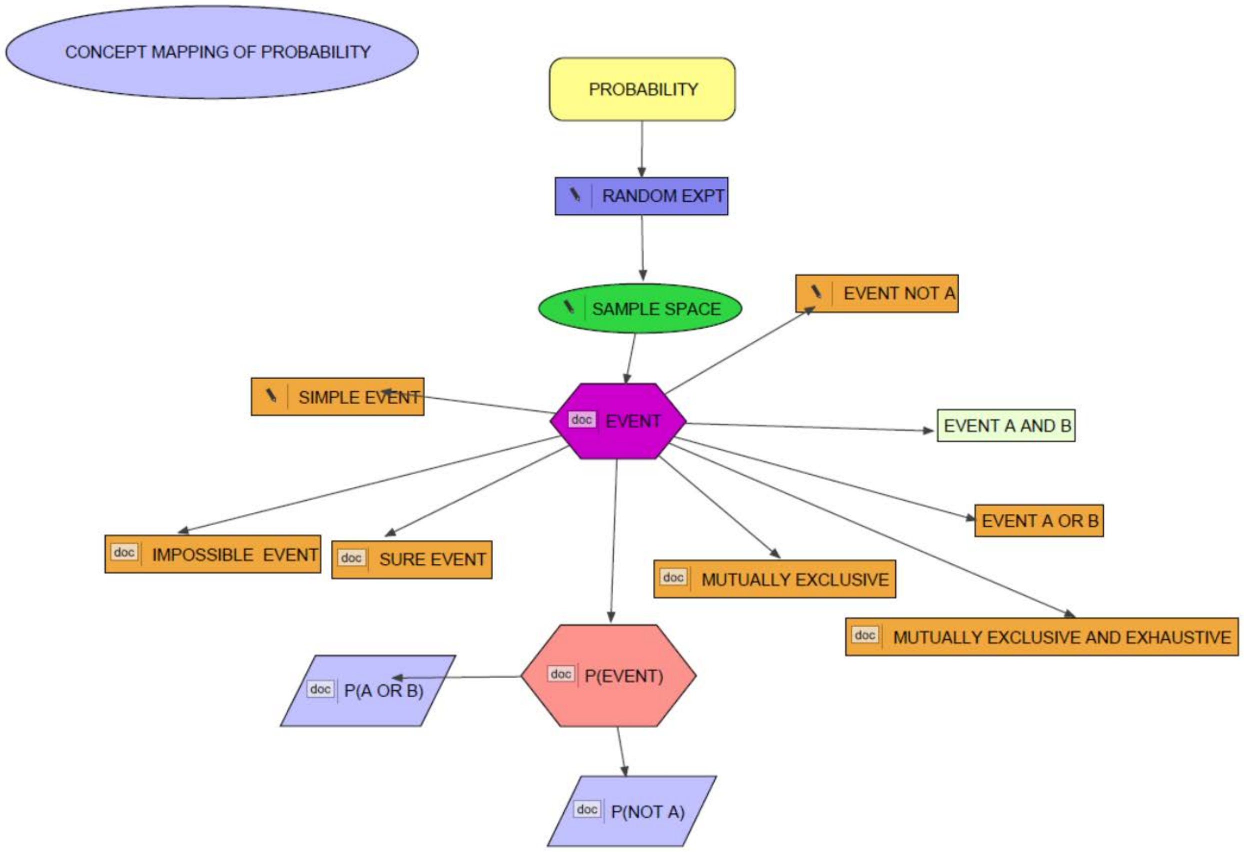
Statistics: www-psych.stanford.edu/-bigopp/statistics.ppt

CHAPTER-16-PROBABILITY

Expected learning outcomes

1 Students will observe the outcomes of the random experiment.
2 They will relate the set of all possible outcomes to a set ‘S’ the sample space.
3 They are able to relate event E of S as the subset of S
4. They understand the measure of uncertainty through P(E).
5. They will apply the concept of P(E) in many day to day situations.

CONCEPT MAPPING



GRADED LEVEL QUESTIONS

SAMPLE SPACE

Level I

1. A card is drawn from a pack of 52 cards. How many points are there in the sample space?
2. A coin is tossed thrice. Describe the sample space of this experiment?
3. A die is thrown twice. Describe the sample space of this experiment?

Level II

1. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space of this experiment.
2. The numbers 1, 2 ,3 and 4 are written separately on four slips of paper. The slips are put in the box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for this experiment.

Level III

1. Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.
2. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

EVENTS

Level I

1. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events.
   A: The sum is greater than 8,
   B: 2 occurs on either die,
   C: Sum is at least 7 and a multiple of 3.
   Which pair of these events are mutually exclusive?

2. In the experiment of rolling a die,
   A : getting a prime number
   B : getting an even number
   C : getting a number > 6
   D : getting a number which is a multiple of 5
   E: getting an odd number.
   Describe the following events
   i) B or E ii) B and C iii) Not A iv) D

VALUED BASED QUESTIONS:

A debate competition on the topic "WORK IS WORSHIP" is to be organized. Out of 10 outstanding students consisting of 6 boys and 4 girls. 3 students are to be selected. What is the probability of selecting one boy and two girls? Give your comments about the topic in 10 words approximately?

What is the probability that 2 letters chosen at random from the word VALUES are vowels?

In a room there are 9 persons out of whom 3 persons like to ride a cycle as it is ENVIRONMENT FRIENDLY as well as mode of exercise, 4 persons are promoters of "SAVE THE TIGER" campaign and 2 persons believe in "HONESTY IS THE BEST POLICY". A person is selected at random from the room. What is the probability a person related to "SAVE THE TIGER "campaign is chosen. Give your views about the same.

20 students appeared for an examination, out of which 3 students were caught copying. These three students are to be chosen for an advice and make them understand to acquire positive values in life. What is the probability of choosing the student who was caught copying? Which values in life they are lacking?

In a group of 25 persons, 10 persons are expert in delivering the values about "Empathy" and the remaining are experts in delivering the values about "Keep your surroundings clean". Five persons are to be chosen among these to deliver a seminar on importance of values in life. What is the probability that among the chosen two persons are expert in values about "Empathy"? Give your views about the acquiring values "Empathy".

ERROR ANALYSIS

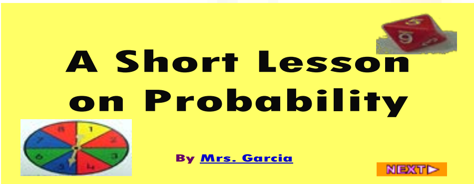
ERRORS	REMEDIAL MEASURES
Misunderstanding the question	To read the question properly (without any hurry) and understand it.
In calculating P(E)	Emphasize more, on the definition by giving good number of examples.

QUESTION BANK

Probability

1. If two cards are drawn from a well shuffled pack, what is the probability that at least one of the two is heart?
2. What is the probability that a leap year will have 53 Sundays?
3. What is the probability of getting a total of 10 in a single throw of two dice
4. Two dice are rolled simultaneously. What is the probability that the numbers on them are different
5. At random all the letters of the word "ARTICLE" are arranged in all possible ways. What is the probability that the arrangement begins with vowel and ends with a consonant?

6. The letters of the word ‘MISSISSIPI’ are arranged in a row at random. What is the probability that all S’s come together?
7. A five digit number without repetition is formed by the digits 1, 2, 3, 4, 5, 6, 7, 8. What is the probability that the number has even digits at both ends?
8. Three electric lamps are fitted in a room. 3 bulbs are chosen at random from 10 bulbs having 6 good bulbs. What is the probability that the room is lighted?
9. If $P(A\cup B) = 0.65$, $P(A\cap B) = 0.15$, then find $P(A^c) + P(B^c)$.
10. If $P(A) = 0.4$, $P(B)=0.5$, $P(C) = 0.6$, $P(A \cap B) = 0.2$, $P(B\cap C) = 0.3$, $P(C\cap A) = 0.25$, $P(A\cap B\cap C)=0.1$ then find $P(A\cup B\cup C)$.
11. A bag contains 5 black balls 4 white balls and 3 red balls. If a ball is selected at random what is the probability that it is a black or a red ball?
12. The probabilities of two events A and B are 0.25 and 0.40 respectively. The probability that both A and B occur is 0.15. What is the probability that neither A nor B occur?



Probability: www.powershow.com/view/1db38-NjlXo/probability

Web.stanford.edu/…./handouts/4 probablity.ppt

SAMPLE QUESTION PAPERS

Sample Paper-1

SESSION ENDING EXAMINATION

(Blue Print(Number of Questions are shown with in bracket)

Units	Topics	Chapters	VSA (1 mark)	SA (4 marks)	LA(6 marks)	Total	
I	Sets and Functions	Sets	1(1)		6(1)	7	29
		Relations and Functions		4(2)		8	
		Trigonometric Functions		4(2)*	6(1)	14	
II	Algebra	Mathematical Induction		4(1)		4	37
		Complex Numbers & Quadratic equations	1(1)	4(2)*		9	
		Linear Inequality			6(1)	6	
		Permutations and Combinations	1(1)		6(1)*	6	
		Binomial Theorem		4(1)*		4	
		Sequences and Series		4(2)*		8	
III	Coordinate Geometry	Straight Lines	1(1)	4(1)		5	13
		Conic Sections		4(1)		4	
		Three Dimensional Geometry		4(1)		4	
IV	Calculus	Limits and Derivatives			6(1)*	6	6
V	Mathematical Reasoning	Mathematical reasoning	1(3)				3
VI	Statistics and Probabilities	Statistics			6(1)	6	**12**
		Probability			6(1)	6	
Total			1(6)	4(13)	6(7)	100	100

SESSION ENDING EXAMINATION

CLASS XI **MATHEMATICS**

Time allowed: 3 hours *Maximum Marks: 100*

General Instructions:

(i) All questions are compulsory

(ii) The question paper consists of 26 questions divided into three sections A,B and C. Section A comprises of 6 questions of 1 mark each, section B comprises of 13 questions of four marks each and section C comprises of 7 questions of Six marks each

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required

SECTION-A (1 x 6= 6)

1. Draw venn diagram for $(A-B)^c$
2. How many words with or without meaning can be made from the letters of the word MONDAY, using each letter exactly once.
3. Show that the statement "For any real numbers a and b $a^2 = b^2$ implies that a = b" is not true by giving a counter example
4. "If the two lines are parallel, then they do not intersect in the same plane" write the contra positive
5. Identify whether **inclusive or** (or) **exclusive or** is used in the statement "A student who has taken computer science or Maths can apply for B.Sc., computer Science"
6. Write the intercepts of the line $5y - 2x + 4 = 0$

SECTION B (13 x 4 = 52)

7. Let f = {(1,1),(2,3),(0,-1),(-1,-3} be a function from **Z** to **Z** defined by f(x) = ax +b ,for some integers a and b. Determine a and b
8. IfA ={1,2,3},B={2,3,4,5},C={5,6}, Verify that A X (B∩C) = (AXB)∩(AXC)
9. Prove: $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$
10. Prove the following by using the principle of mathematical induction $\forall$ n$\in N$

$$1^2 + 3^2 + 5^2 + 7^2 + \; ----------- (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

11. Convert the complex number $z = \dfrac{i-1}{Cos\dfrac{\pi}{3} + iSin\dfrac{\pi}{3}}$ in to polar form.

12. If $Z_1 = 2 - i$, and $Z_2 = 1+i$ find $\left|\frac{z_1+z_2+1}{z_1-z_2-1}\right|$

OR Find the square root of -15-8i

13. Find the value of P so that the three lines 3x+ y -2 = 0 Px + 2y -3 =0 and 2x-y-3 =0 are concurrent
14. Find the term independent of x in the expansion $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$ (OR) Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.
15. Using section formula, show that the points A(2,-3, 4), B (-1, 2, 1) and C (0, 1/3, 2) are collinear.
16. Find the coordinates of the foci, the length of the major axis, the minor axis and the eccentricity and of the ellipse $36x^2 + 4y^2 =144$.
17. Find the sum of the sequence 5,55,555,5555,....to n terms
    (OR)

    Sum the series to n terms $1^2 + (1^2 + 2^2) + (1^2+ 2^2 +3^2) + (1^2 +2^2 +3^2 +4^2) + ...$
18. Solve sin2x –sin4x +sin6x =0 OR In any triangle ABC, Prove that a Sin(B-C)+b Sin(C-A) +c Sin(A-B) =0.
19. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b find the value of n.

Section C (7x6 =42)

20. A college awarded 38 medals in football 15 medals in basket ball and 20 medals in cricket. If these medals went to a total of 58 men and only three men got medal in all the three sports'. How many received medals in exactly two of the three sports? What attitude do you develop by taking part in games and sports?
21. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18 \cos^2 x - 1$
22. Solve following system of inequalities graphically
    $3x + 2y \le 150$, $x+4y \le 80$, $x \le 15$,$x \ge 0$, $y \ge 0$

    With existing constraints in your family, what values will you adopt to improve your studies?
23. In an examination, a question paper consists of 12 questions divided in to two parts that is, part A and Part B, containing 5 and 7 questions respectively. A student is required to attempt8 questions in all, selecting at least three from each part. In how many ways a student can select the questions? State any one value/life skill the examination system develops in you.
    OR

    a) How many 5 letter words containing 3 vowels and 2 consonants can be formed using the letters of the word EQUATION so that 3 vowels always occur together?

    b)In how many ways can the letters of the word PENCIL be arranged so that I is always next to L.
24. Two cards are drawn from well shuffled pack of 52 cards, find the probability of getting (i) all kings (ii) one king (iii) No king
25. Find the mean and standard deviation of the following data

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students	9	17	32	33	40	10	9

26. a) If $\lim_{x\to 1} f(x) = f(1)$ and f(x) = $\begin{cases} a + bx\ , x < 1 \\ 5 \qquad x = 1 \\ b - ax \quad x > 1 \end{cases}$, find the value of a and b

b)Find the derivative of 3 tan x +2 sin x - 5 sec x.

OR

a). Find the derivative of (x+3) (x-2) using first principle

b). Evaluate: $\lim_{x\to 2} \frac{x^5 - 32}{x-2}$

………

Sample paper 2

SESSION ENDING EXAMINATION

Blue Print(Number of Questions are shown with in bracket)

Units	**Topics**	**Chapters**	**VSA (1mark)**	**SA(4 marks)**	**LA(6 marks)**	**Total**	
I	Sets and Functions	Sets	1(1)		6(1)	7	29
		Relations and Functions		4(2)		8	
		Trigonometric Functions		4(2)*	6(1)	14	
II	Algebra	Mathematical Induction		4(1)		4	37
		Complex Numbers &Quadratic equations	1(1)	4(2)*		9	
		Linear Inequality			6(1)	6	
		Permutations and Combinations	1(1)		6(1)*	6	
		Binomial Theorem		4(1)*		4	
		Sequences and Series		4(2)*		8	
III	Coordinate Geometry	Straight Lines	1(1)	4(1)		5	13
		Conic Sections		4(1)		4	
		Three Dimensional Geometry		4(1)		4	
IV	Calculus	Limits and Derivatives			6(1)*	6	6
V	Mathematical Reasoning	Mathematical reasoning	1(3)				3
VI	Statistics and Probabilities	Statistics			6(1)	6	12
		Probability			6(1)	6	
Total			1(6)	4(13)	6(7)	100	100

SESSION ENDING EXAMINATION

CLASS : XI MM : 100

SUB : MATHEMATICS DURATION : 3Hrs

General Instructions:

1. All the questions are compulsory.
2. The question paper consists of 26 questions divided into 3 sections A,B and C. Section A consists of 6 questions of 1 mark each. Section B consists of 13 questions of 4 marks each. Section C consists of 7 questions of 6 marks each.
3. All the questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice, however internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION –A (1 X 6 = 6)

1. Write the power set of A, given A = {a, b, c }.
2. Find the slope of the line 5x + 3y – 2 = 0.
3. Write the contrapositive of the statement : `If x is a prime number, then x is odd".
4. Write the converse of the statement : " x is an even number implies that x is divisible by 4".
5. If C (n,7)= C (n,6), find C (n,2).
6. Write the negation of the statement p: $x^2 > x$, for every real number x.

SECTION – B (4 X 13 = 52)

7. Find the general solution of the following equation: $2\cos^2 x + 3 \sin x = 0$
8. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$ = tanx

   OR

   In any triangle ABC , prove that $\frac{a+b}{c} = \frac{\cos(B-C)/2/}{\sin C/2}$.
9. Using the principle of mathematical induction, prove that for every positive integer n, $7^n - 3^n$ is divisible by 4.
10. Find the square root of (-7 – 24i).

    OR

    If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4 (x^2 - y^2)$
11. Let A = { 9,10,11,12,13 } and f: A → N be defined by f(n) = the highest prime factor of n. List the elements of f and find the range of f.
12. If $f(x) = x^2$, g(x) = 2x + 1, be two real functions, find

    a) (f + g)(x) **b**) (f – g)(x) **c**) (f g)(x) **d**) $\left(\frac{f}{g}\right)(x)$
13. Convert $\frac{(1+7i)}{(2-i)^2}$ in the polar form
14. The sum of three numbers in geometric progression is 56. If we subtract 1,7,21 from these numbers in that order, we obtain an arithmetic progression. Find these numbers.

    0R

    If A.M and G.M of two positive numbers 'a' and 'b' are 10 and 8 respectively, find the numbers.

15. Find the equation of the circle with radius 5 whose centre lies on X-axis and pass through the point (2, 3).
16. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, -8) is divided by the YZ-plane.
17. Find the coefficient of x6y3 in the expansion of (x +2y)9

OR

Find the term independent of 'x' in the expansion of $\left[\frac{3}{2}x^2 - \frac{1}{3x}\right]^6$

18. Find the equation of a line perpendicular to the line: $x - 2y + 3 = 0$, and passing through the point (1, - 2).
19. Find the sum to n terms of the series $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots$

SECTION – C

20. a) If A = {3,5,7,9,11 }, B= {7,9.11,13 }, C = { 11,13,15}, find A ∩(B UC)

b) In a group of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football? Mention one advantage of playing games?

21. Solve the system of inequalities graphically

$x + 2y \le 10,\ x + y \ge 1,\quad x - y \le 0,$ $x \ge 0, y \ge 0$

22. Prove that $cos^2 x + cos^2\left(x + \frac{\pi}{3}\right) + cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.
23. a) In how many of the distinct permutations of the letters in MISSISSIPPI do the four 'I's not come together?

b) A committee of three persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done?

OR

a) Find the number of words with or without meaning which can be made using all the letters of the word " AGAIN". If these words are written as in a dictionary, what will be the 50th word?

b) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has no girl ?

24. Calculate Mean, Variance, and Standard deviation for the following distribution:

Classes	30 – 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	3	7	12	15	8	3	2

25. In a class of 60 students,30 opted for NCC,32 opted for NSS and 24 opted both NCC and NSS. If one of these students is selected at random, find the probability that:

(a) the student opted for NCC or NSS
(b) the student has opted neither for NCC nor NSS
(c) the student has opted for NSS but not NCC.
Mention two advantages of participating in NCC or NSS to develop a country.

26. (a) Find the derivative of $(ax^2 + \sin x)(p + q \cos x)$

(b) Evaluate $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

(OR)

a) Find the derivative of f(x) = $\frac{2x+3}{x-2}$ from the first principle.

b) Evaluate $\lim_{x\to \frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$

****

Sample paper 3

SESSION ENDING EXAMINATION

Blue Print (Number of Questions are shown with in bracket)

Units	**Topics**	**Chapters**	**VSA (1 mark)**	**SA (4 marks)**	**LA (6 marks)**	**Total**	
I	Sets and Functions	Sets	1(1)		6(1)	7	
		Relations and Functions		4(2)		8	29
		Trigonometric Functions		4(2)*	6(1)	14	
II	Algebra	Mathematical Induction		4(1)		4	
		Complex Numbers &Quadratic equations	1(1)	4(2)*		9	37
		Linear Inequality			6(1)	6	
		Permutations and Combinations	1(1)		6(1)*	6	
		Binomial Theorem		4(1)*		4	
		Sequences and Series		4(2)*		8	
III	Coordinate Geometry	Straight Lines	1(1)	4(1)		5	
		Conic Sections		4(1)		4	
		Three Dimensional Geometry		4(1)		4	13
IV	Calculus	Limits and Derivatives			6(1)*	6	6
V	Mathematical Reasoning	Mathematical reasoning	1(3)				3
VI	Statistics and Probabilities	Statistics			6(1)	6	
		Probability			6(1)	6	12
Total			1(6)	4(13)	6(7)	100	100

SESSION ENDING EXAMINATION

CLASS : XI **MM : 100**

SUB : MATHEMATICS **DURATION : 3Hrs.**

General Instructions:

1. *All the questions are compulsory.*
2. *The question paper consists of 26 questions divided into 3 sections A,B and C. Section A consists of 6 questions of 1 mark each. Section B consists of 13 questions of 4 marks each. Section C consists of 7 questions of 6 marks each.*
3. *All the questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice, however internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted.*

SECTION -A

1. If U={ 1,2,3,…,9}, A = {2,3,4} B= {3,4,5,6,} find (AUB)'.

2. Find the multiplicative inverse of 2 – 3i.

3. If the line $\frac{x}{4} - \frac{y}{7} = 1$ meets the X-axis in A and Y-axis in B then find the mid-point of AB.

4. Given below are two statements
   P:25 is a multiple of 5q: 25 is a multiple of 8
   Write the compound statement connecting these two statements with AND.
   Check the validity of the compound statement.

5. Write the Contrapositive of the statement;
   "If a number is divisible by 9, then it is divisible by 3".

6. By giving a counter example, show that the following statement not true
   If n is an odd integer, then n is prime

SECTION-B

7. Let f, g: R→R be defined, respectively by $f(x) = x + 1, g(x) = 2x - 3.$
   $Find\ f + g, f - g, (f.g)\ ,\ \frac{f}{g}$

8. In any triangle ABC, Prove that
   $a\,Sin(B - C) + bSin(C - A) + cSin(A - B)\ = 0$

   OR

   Solve $sin2x\ - sin4x\ + sin6x\ = 0$

9. Prove the following by using the principle of mathematical induction∀ n∈ N

$1^2+3^2+5^2+7^2+ \quad ------------(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

OR

For all n≥1,Prove that

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}-------+\frac{1}{n(n+1)}=\frac{n}{n+1}$

10. Find the square root of $-15-8i$

OR

If $(x+iy)^2 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

11. Prove that $\frac{sin5x-2sin3x+sinx}{cos5x-cosx}$ = tanx

12. Find the coefficient of x^6y^3 in the expansion of $(x+2y)^9$.

OR

Find the term independent of x in the expansion of $\left[\frac{3}{2}x^2-\frac{1}{3x}\right]^6$

13. Find the equation of the line through the intersection of $5x-3y=1$ and $2x+3y-23=0$and Perpendicular to the line $5x-3y-1=0$

$3x-4y-16=0$

14. A rod of length 12 cm moves with its ends always touching the coordinate axes, Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

15. Find the ratio in which the YZ-plane divides the line segment formed by joining the Points (-2,4,7) and (3,-5,8). Find the coordinates of the point on the YZ-plane.

16. The sum of the first 3 terms of a G.P is 13/12 and their product is -1. Find the common ratio and the terms

17. If A.M and G.M of two positive numbers a and b are 10 and 8 respectively, find the numbers.

(OR)

Find the sum to n terms of the series: 5+ 11+ 19+ 29+ 41 +...

18. Find the domain and Range of the function f(x)=$\sqrt{9-x^2}$

19. Convert the complex number $\frac{-16}{1+i\sqrt{3}}$ in to polar form

SECTION- C

20. In a group of children 45 play Football, out of which 30 play Football only, 28plays Hockey ,25 play Cricket ,out of which 11 play Cricket only, further 7 play Cricket and Football but not Hockey, 5 play Football and Hockey but not Cricket and 10 play Football and Cricket both. Represent the above information in a Venn diagram and find

i) How many children are there in the group?

ii) How many children play all the three games?

iii)How many children play Hockey only?

21. a) If Sinx = $\frac{3}{5}$ and Cosy = -$\frac{12}{13}$,where x and y both lie in second quadrant

Find the value of Sin (x – y).

b) Prove that $(Sin3x + Sinx)Sinx + (Cos3x - Cosx)Cosx = 0$

22. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements do

i) The words start with P.

ii) all the vowels always occur together

iii) The vowels begin with I and end in P.

(OR)

In an examination, a question paper consists of 12 questions divided in to two parts that is, part A and Part B, containing 5 and 7 questions respectively. A student is required to attempt8 questions in all, selecting at least three from each part. In how many ways a student can select the questions? State any one value/life skill the examination system develops in you.

23. a) Find the derivative of f(x) from the first principle f(x) = Sin(x+1)

b) Evaluate $\lim_{x\to 0}(\;cosecx - cotx)$

(OR)

a) Evaluate $\lim\limits_{x\to 2}\frac{x^4-2x^2}{x^2-5x+6}$

b) Find the derivative of the function: f(x) = $\frac{3x+5}{2x-1}$ from first principle.

24. Solve the following inequalities graphically

$3x + 2y \leq 24, x + 2y \leq 16 \;, x + y \leq 10, x \geq 0, y \geq 0.$

25. Calculate mean and standard deviation for the following data

Classes	0-30	30-60	60-90	90-120	120-150	150-80	180-210
Frequency	2	3	5	10	3	5	2

26. Two students Anil and Ayesha appeared in the examination. The probability that Anil will qualify the examination is0.05 and that Ayesha qualifies the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

a) Both Anil and Ayesha will not qualify the examination

b) At least one of them will not qualify the examination and

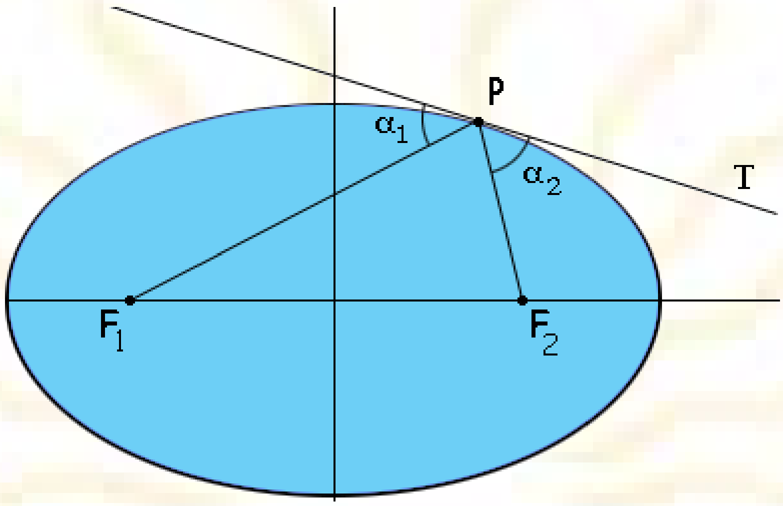
c) Any one of them will qualify the examination.

Examination system develops which type of moral values.

****

Lithotripsy - A Medical Application of the Ellipse

The ellipse is a very special and practical conic section. One important property of the ellipse is its reflective property. If you think of an ellipse as being made from a reflective material then a light ray emitted from one focus will reflect off the ellipse and pass through the second focus. This is also true not only for light rays, but also for other forms of energy, including shockwaves. Shockwaves generated at one focus will reflect off the ellipse and pass through the second focus. This characteristic, unique to the ellipse, has inspired a useful medical application. Medical specialists have used the ellipse to create a device that effectively treats kidney stones and gallstones. A *lithotripter* uses shockwaves to successfully shatter a painful kidney stone (or gallstone) into tiny pieces that can be easily passed by the body. This process is known as lithotripsy



As illustrated in the diagram above, when as energy ray reflects off a surface, the angle of incidence is equal to the angle of reflection.

$$\alpha_1 = \alpha_2$$

PROCEDURE

1. Constructing an ellipse by using circle compass.
2. Paste it on a decolam sheet and cut it.
3. Fix the foci.
4. On one focus fix a pen pointer.
5. On the other focus fix a reflector.

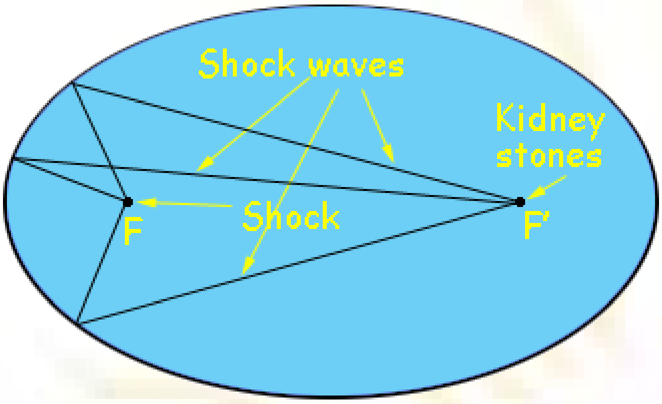
The amazing property of ellipse i.e whenever light is focused from one focus it must passes through the other is used to make Lithotripsy to work remarkably

Extracorporeal Shockwave Lithotripsy

Extracorporeal Shockwave Lithotripsy (ESWL) enables doctors to treat kidney and gall stones without open surgery. By using this alternative, risks associated with surgery are significantly reduced. There is a smaller possibility of infections and less recovery time is required than for a surgical procedure. The lithotripter is the instrument used in lithotripsy. The mathematical properties of an ellipse provide the basis for this medical invention.

The Foci

The lithotripter machine has a half ellipsoid shaped piece that rests opening to the patient's side. An ellipsoid is a three dimensional representation of an ellipse. In order for the lithotripter to work using the reflective property of the ellipse, the patient's stone must be at one focus point of the ellipsoid and the shockwave generator at the other focus. The patient is laid on the table and moved into position next to the lithotripter. Doctors use a fluoroscopic x-ray system to maintain a visual of the stone. This allows for accurate positioning of the stone as a focus. Because the stone is acting as one of the focus points, it is imperative that the stone be at precisely the right distance from the focus located on the lithotripter. This is essential in order for the shockwaves to be directed onto the stone.



The Cushion

The lithotripter also contains a coupling device. This is needed for the successful transmission of the rays through the body. A cushion, somewhat like a water balloon, wraps around the half ellipsoid. The cushion is filled with water and rests against the patient's side. The cushion is sealed to the patient's body using a silicone membrane. It is the water that allows the shockwaves to travel through the body's tissues safely because water and the soft tissue have the same density. The stone has a greater density and is shattered by the shockwaves, but the soft tissue suffers only minimal damage. Before the new lithotripters were made, patients would lay in a water bath to create the same effect.

Shockwaves

Electrohydraulic, piezoelectric, and electromagnetic energy systems use the focus of the ellipsoid to create the shockwaves needed to fracture the stone. The waves are generated at one focus and because of the elliptical shape, the waves are redirected onto the second focus, which is the stone itself. All of these waves cause the stone to crack and it eventually fragments into many tiny pieces that can then be easily passed by the body

PROJEC T-2

PARABOLIC CALCULATER.

With the help of parabola how to multiply numbers without actually multiplying it.

Procedure

1. Construct a parabola on a big graph paper.
2. Take a simple one$y = x^2$.
3. Paste it on a decolam sheet.
4. On Y-axis fix a moving screw.
5. Take any two screws one on left side and another on the right side on the graph.
6. Connect these two screws with the thread from the screw on y- axis.

   This will work as a calculator. The method is explained as follows:

   $y = x^2, i.e\ x^2 - y = 0. There\ are\ two\ roots\ x_1, x_2\ as\ we\ know$

   $the\ product\ of\ roots\ x_1 x_2 \ = -y$

PROJECT-3

Epic Tale of Ten Regular Polygons

The Objective

The problem I will be experimenting on is how the number of sides on a regular polygon affects the area of the polygon, in comparison to other polygons with the same fixed perimeter. I believe that the area of a regular polygon increases as the number of sides the polygon has increases.

Methods

In order to test this, I will calculate the area of the regular polygons with a number of sides ranging from three to twelve, but keeping the perimeter the same for all the polygons.

I choose 30 cm as my perimeter because a great majority of the numbers between three and twelve divide 30 evenly, or at least rationally.

With other perimeters, the length of the sides would be an irrational number, which can cause the calculations to be accurate, but not as accurate as possible.

Using trigonometric ratios, I found the area of the regular polygons, and then, in order to ensure that my calculations were not faulty, I have taken the help of the guide teacher for making sure that there were no errors in calculation.

I was then able to conclude that my work was accurate and I was then ready to analyze that data, and make a conclusion.

Results

I found that if the number of sides on a regular polygon increases, then the area of the polygon increases as well.

In addition, I was also correct in predicting that the polygon with the apothem had the largest area, because when using the formula for the area of a regular polygon, the only variable that could have affected the area on this situation was the apothem, so the longer the apothem, the larger the area.

Conclusions/Discussion

After analyzing my data, I made one significant observation not stated above. I noticed that as the number of sides increased, the more the polygon looked like a circle. I decided to do some additional experimentation, so I found the area of a circle that had a circumference of 30 cm. I found that the more sides on the polygon, the closer the area will be to that of a circle, but it will never have the same area as that of a circle. I think that if you even look at my experiment logically, it makes sense that the polygons with the most sides had the largest area, but my data proves this mathematically.

PROJECT-4

Displacement and Rotation of a Geometrical Figure

Objective:
To study between different points of a geometrical figure when it is displacement and/or rotated. Enhance familiarity with co-ordinate geometry.
Description:

1. A cut out of a geometrical figure such as a triangle is made and placed on a rectangular sheet of paper marked with X and Y-axis.
2. The co-ordinates of the vertices of the triangle and its centroid are noted.
3. The triangular cut out is displaced (along x-axis, along y-axis or along any other direction.)
4. The new co-ordinates of the vertices and the centroid are noted again.
5. The procedure is repeated, this time by rotating the triangle as well as displacing it. The new co-ordinate of vertices and centroid are noted again.
6. Using the distance formula, distance between the vertices of the triangle is obtained for the triangle in original position and in various displaced and noted positions.
7. Using the new co-ordinates of the vertices and the centroids, students will obtain the ratio in which the centroid divides the medians for various displaced and rotated positions of the triangles.

Result:
Students will verify that under any displacement and rotation of a triangle the displacement between verticals remain unchanged; also the centroid divides the medians in ratio 2:1 in all cases.

PYTHAGORAS THEOREM AND ITS EXTENSION

OBJECTIVE : To understand the Pythagoras theorem using geometrical representation by using areas of squares on each side of a right triangle, and extending it to three dimensional objects using volumes.

PYTHAGORAS THEOREM STATES that square on Hypotenuse of a right triangle is equal to sum of squares on remaining two sides.

1. **FOR A RIGHT TRIANGLE**

Description

1. Cut a triangle of sides 6cm, 8cm and 10cm
2. Cut squares equal to sides of triangle.
3. Divided each square into small squares of 1cm each

CALCULATIONS

1. The number of 1 cm squares in square drawn on Side of 6 cm were 36
2. The number of 1 cm squares in square drawn on Side of 8 cm were 64
3. Sum of square on these two sides = 64 + 36 = 100
4. The number of 1 cm squares in square drawn on Side of 10 cm (hypotenuse) were 100
5. Square on Hypotenuse of a right triangle is equal to sum of squares on remaining two sides. Hence Pythagoras Theorem is verified for a right triangle

FOR RIGHT CIRCULAR CYLINDER DESCRIPTION

1. Took right circular cylinders of radii 6cm, 8cm and 10cm.
2. Filled the two smaller cylinders (r = 6cm, 8cm) with sand.
3. Keep the cylinder with r = 10 cm empty.

METHOD

1. Poured the sand from cylinders with radii 6cm and 8cm into the biggest cylinder (r = 10cm).
2. We found that the bigger cylinder is completely filled with sand.
3. This shows volume of cylinder with radius 10cm = sum of volumes of the cylinders with volume 6cm and 8cm. Hence Pythagoras Theorem can be extended for right

CIRCULAR CYLINDERS

For Right Circular Cone Description

1. Took right circular cone of radii 6cm, 8cm and 10cm.
2. Filled the two smaller cones (r = 6cm, 8cm) with sand.
3. Keep the cone with r = 10cm empty.

METHOD:

1. Poured the sand from cone with radii 6cm and 8cm into the biggest cone (r = 10cm).
2. We find that the bigger cone is completely filled with sand.
3. This shows volume of cone with radius 10cm = sum of volumes of the cone with volume 6 cm and 8cm.
   Hence Pythagoras Theorem can be extended for right

CIRCULAR CONE: Similarly it can be done for cone.

LEARNING OBJECTIVE: Pythagoras theorem is true for not only for right triangles and can be extended for three dimensional figures such as cylinders and cones.

MATHEMATICS PROJECT ON PIE

Pie(π)

OBJECTIVE

To know about π the ratio of circumference of a circle to its diameter

Procedure

1. Collected information about n from teachers, books and internet
2. Collected work done by various mathematicians on n.
3. Written value of π up to 50 places of decimal.

What isπ

It is the sixteenth letter of Greek alphabets. Old Greek texts used it to represent number 80. Many mathematical, science and engineering formulae involveπwhich makes it one of the most important mathematical constants.

π = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

When was first πused in its present form?

It is believed that pie was first used to represent ratio of circumference of a circle to its diameter by William Jones in 1706 in his book **PALMORIORUM MATHESEOS**. Leonhard Euler a **SWISS** mathematician also used π in its present way.

What is value ofπ?

1. After the invention of wheel perhaps it was required to know the distance travelled by a wheel with particular diameter.
Initially it was found that the distance travelled by a wheel was a little more than three time the diameter.
Further research showed that distance travelled by wheel was approximately equal to 3 time diameter + %L = 11 of diameter. $(2r)^2$ 14.

2. AHMES – An Egyptian Mathematician calculated the value of2.56πas —— using area. This is good approximation of value of π

π = 3.14159265358979323846 2643383279 5828841971 693993751©58209749445923078l6 0628620899 8628034825 3421170679

3. Who gave n π= —? Archimedes said that ratio of area of a circle to that of square with side equal to circles diameter is close to22

11:14. Solving this we get the value of $\pi$is close to 22.

CALCULATION

area of circle _ 11: area of square 14

4. Chinese Contribution

(a) LIU HUI in 263 used regular polygons with increased number of3927 sides to approximate the value of π as_________1250355

(b) ZU CHONGZHIgave value of value of π as 3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 4428810975 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 4543266482 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 9171536436 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 5759591953 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 8912279381 8301194912 9833673362 4406566430 8602139494 6395224737 1907021798 6094370277 0539217176 2931767523 8467481846 7669405132 0005681271 4526356082 7785771342 7577896091 7363717872 1468440901 2249534301 4654958537 1050792279 6892589235 4201995611 2129021960 8640344181 5981362977 4771309960 5187072113 4999999837 2978049951 0597317328 1609631859 5024459455 3469083026 4252230825 3344685035 2619311881 7101000313 7838752886 5875332083 8142061717 7669147303 5982534904 2875546873 1159562863 8823537875 9375195778 1857780532 1712268066 1300192787 6611195909 2164201989

5. Indian Contribution62832

(a) ARYABHATA in 499 gave value of π as___________

(b) BRAHMAGUPTA in 640 gave value of π as V10

(c) S. RAMANUJAN gave value of n correct up to 8 places of decimal .

6. YASUMASA KANADA and his team in Tokyo calculated the value of n to 1.24 trillion decimal places.

Is π rational or irrational?

$\pi$is an irrational number was first proved by Johann Heinrich Lambert by showing that this continued fraction expansion holds tan(V) = ———–a?s^{2}3-x^{2}57- ■■

Then Lambert proved that if x £ 0 is rational, then the right-hand side of this expression must be irrational. Since tan (π/4) = 1, it follows that π/4 is irrational and therefore that $\pi$is irrational

Fibonacci Sequence in Plants

The Objective : The Fibonacci sequence is a sequence where the sum of two preceding numbers is equal to the next number in the sequence. 1,1, 2, 3, 5, 8, 13, 21, 34, 55 ... Research suggests the Fibonacci sequence is in plants as an evolutionary growth strategy. Cells grow on the tip of stems and as the stem grows, the cells grow down and out in spiraling patterns. This project had two goals. One, study how often Fibonacci numbers occur in plants. Two, compare Fibonacci numbers between plant families.

Methods/Materials

The materials were Asteraceae and Myrtaceae flowers, Pinaceae cones, a camera and color copier to record samples, and a plant identification book.

Visit plant stores, arboretum, florist, and gardens. Identify and sample Asteraceae and Myrtaceae flowers.

Collect Pinaceae cones. Record genus species and count flower petals. For cones, write genus species and count clockwise and counterclockwise spirals.

Count three of each. Copy or photograph samples and label with genus species.

Results

2/3 or 66% of Asteraceae flowers had a Fibonacci number of petals. 12/13 or 92% of the Myrtaceae flowers had a Fibonacci number of petals. 8/8 or 100% of the cones of the Pinaceae family had Fibonacci numbers of spirals.

Conclusions/Discussion

The Pinaceae family had consistent Fibonacci numbers. The number of spirals clockwise and counterclockwise were consecutive Fibonacci numbers on each cone. The Fibonacci numbers relate to an evolutionary strategy of compacting seeds efficiently. The Asteraceae family had the least Fibonacci numbers. In books, the aster family is recognized as the family with the most Fibonacci numbers. For example, Ian Stewart in Nature's Numbers says "In nearly all flowers, the number of petals is one of the numbers that occur in the sequence 1, 1, 3, 5, 8, 13, 21, 34, 55, 89. For instance lilies have 3 petals, buttercups have 5, delphiniums have 8, marigolds have 13, asters have 21, and most daisies (asters) have 34, 55, or 89." This is not true. Aster petals show some Fibonacci numbers but aren't consistent. 93% of Myrtaceae flower petals are Fibonacci, but most had 3 or 5 petals. This does not show conclusively that Fibonacci numbers occur in myrtles, it could just be a plant characteristic. Fibonacci numbers appear in plants, more often in the Pinaceae family and less in Asteraceae and Myrtaceae.

This project tests how often Fibonacci numbers appear in the flowers or cones of the Asteraceae, Mytraceae, or Pinaceae plant family.

MATH PROJECT FOR CLASS AIMS

To study about the nature of trigonometric functions and their graphs.

OBJECTIVES:

To draw the graphs of the following in [0, 2π] :

y = sin2x, y = sin 3x, y = 3 + sin 4x, y = 3cos2x, y = 3 + cos 3x, y = cos 4x, y = $\sin^2$2x, y = $\sin^2$ 3x, y = $\sin^2$ 4x, y = $\cos^2$ 2x, y = $\cos^2$3x, y = $\cos^2$ 4x, y =3 + $\sin^2$2x, y = 3 - $\sin^2$ 3x, y = 4$\sin^2$ 4x, y =4 + $\cos^2$ 2x, y = $\cos^2$3x, y = $\cos^2$ 4x

PROCEDURE:

1) **Find the range of the functions. i.e., max and min values of them**
   Example: -1<sinx< 1, 0 <$\sin^2$ 3x< 1
2) **Find the x coordinate of the point where it attains max and min values.**
   Example: sinx attains maximum value at $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $-\frac{\pi}{2}$ etc.,
3) **Find the points where the curve crosses the x axis and y axis.**
   Example: cosx crosses the x axis at $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $-\frac{\pi}{2}$ etc.,
4) **Plot these special points in the coordinate axes system.**
5) **Also find the increasing or decreasing nature of the function in a particular interval.**
   Example: cos 2x decreases from x= 0 to x = $\frac{\pi}{2}$, etc.
6) **Use above concepts to draw the graphs of the trigonometric functions.**

LEARNING OUTCOME: The children learn about the nature of the trigonometric functions in general and in particular they apply these concepts to draw the trigonometric functions.

Contents

1. **Aims of the project**
2. **Brief description of the project.**
3. **Procedure**
4. **Mathematics involved in it.Conclusion**

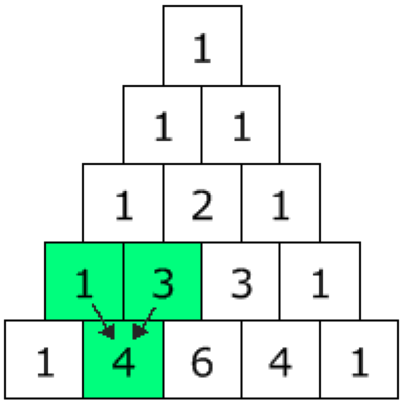
Aim of the project: To make the students acquainted with the triangular pattern formed by the Binomial co-efficients& the various properties associated with it.

Brief description of the project.

(A) **Pascal's Triangle.**
(B) **Pattern within the triangle.**
(C) **Pascal's Triangle in relation to probability.**
(D) **Pascal's Triangle in relation to combination.**

Pascal's Triangle

One of the most interesting Number Patterns is Pascal's Triangle (named after *Blaise Pascal*, a French Mathematician and Philosopher).

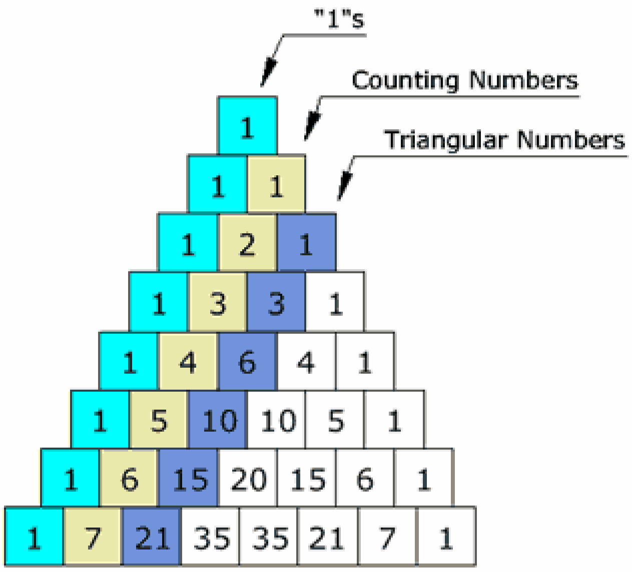


To build the famous triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

Each number is the two numbers above it added together (except for the edges, which are all "1").

(Here I have highlighted that **1+3 = 4)**

Patterns Within the Triangle



Diagonals

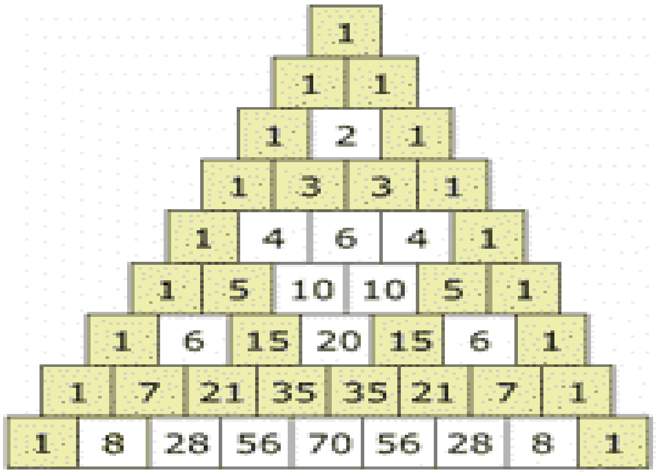
The first diagonal is, of course, just "1"s, and the next diagonal has the Counting Numbers (1,2,3, etc).

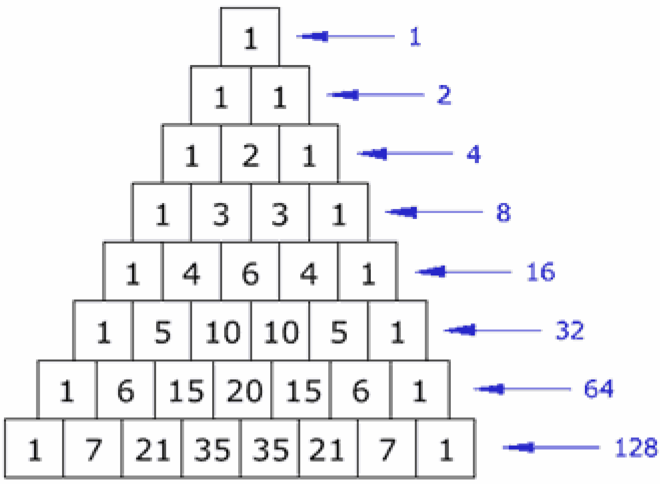
The third diagonal has the triangular numbers

(The fourth diagonal, not highlighted, has the tetrahedral numbers.)

Odds and Evens

If you color the Odd and Even numbers, you end up with a pattern the same as the Sierpinski Triangle





Horizontal Sums

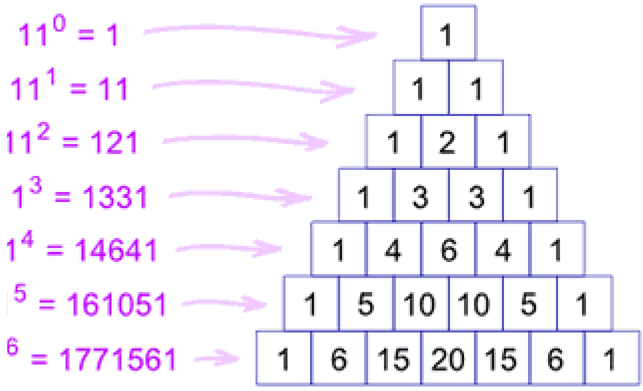
What do you notice about the horizontal sums?

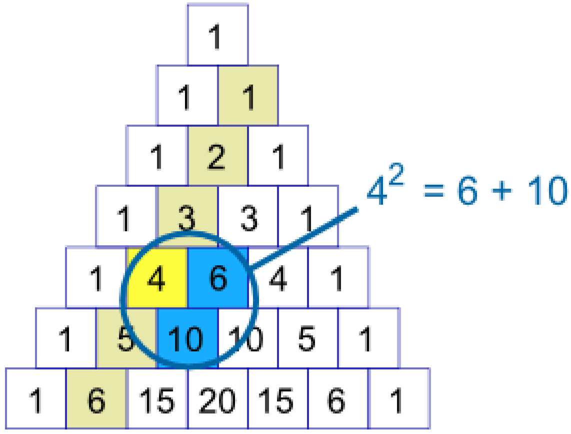
Is there a pattern? Isn't it amazing! It doubles each time (powers of 2).

Exponents of 11

Each line is also the powers (exponents) of 11:

- 11^0=1 (the first line is just a "1")
- 11^1=11 (the second line is "1" and "1")
- 11^2=121 (the third line is "1", "2", "1")
- etc!



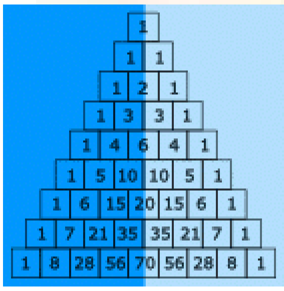


Squares

For the second diagonal, the square of a number is equal to the sum of the numbers next to it and below both of those.

Examples:

- $3^2 = 3 + 6 = 9$,
- $4^2 = 6 + 10 = 16$,
- $5^2 = 10 + 15 = 25$,& so on.
- (Hint: 4^2=6+10, 6=3+2+1, and 10=4+3+2+1)



Symmetrical

And the triangle is also symmetrical. The numbers on the left side have identical matching numbers on the right side, like a mirror image.

Using Pascal's Triangle

Heads and Tails

Pascal's Triangle can show you how many ways heads and tails can combine. This can then show you the probability of any combination.

For example, if you toss a coin three times, there is only one combination that will give you three heads (HHH), but there are three that will give two heads and one tail (HHT, HTH, THH), also three that give one head and two tails (HTT, THT, TTH) and one for all Tails (TTT). This is the pattern "1,3,3,1" in Pascal's Triangle.

Tosses	Possible Results (Grouped)	Pascal's Triangle
1	H T	1, 1
2	HH HT TH TT	1, 2, 1
3	HHH HHT, HTH, THH HTT, THT, TTH TTT	1, 3, 3, 1
4	HHHH HHHT, HHTH, HTHH, THHH HHTT, HTHT, HTTH, THHT, THTH, TTHH HTTT, THTT, TTHT, TTTH TTTT	1, 4, 6, 4, 1
	... etc ...	

Combinations

The triangle also shows you how many Combinations of objects are possible.

Example: You have 16 pool balls. How many different ways could you choose just 3 of them (ignoring the order that you select them)?

Answer: go down to the start of row 16 (the top row is 0), and then along 3 places (the first place is 0) and the value there is your answer, **560**.

Here is an extract at row 16:

```
      1    14    91    364  ...
   1    15    105   455   1365  ...
 1    16    120   5601820  4368  ...
```

A Formula for Any Entry in The Triangle

In fact there is a formula from Combinations for working out the value at any place in Pascal's triangle:

It is commonly called "n choose k" and written like this:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Notation: "n choose k" can also be written **C(n,k)**, $^{n}C_{k}$ or even $_{n}C_{k}$.

The "!" is "factorial" and means to multiply a series of descending natural numbers. Examples:

- 4! = 4 × 3 × 2 × 1 = 24
- 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040
- 1! = 1

So Pascal's Triangle could also be an **"n choose k"** triangle like this: (Note how the top row is **row zero** and also the leftmost column is zero)

$$
\begin{array}{ccccccccc}
 & & & & \binom{0}{0} & & & & \\
 & & & \binom{1}{0} & & \binom{1}{1} & & & \\
 & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & \\
 & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & \\
\binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4}
\end{array}
$$

Example: Row 4, term 2 in Pascal's Triangle is "6" ...

... let's see if the formula works:

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

Yes, it works! Try another value for yourself.

This can be very useful ... you can now work out any value in Pascal's Triangle **directly** (without calculating the whole triangle above it).

Polynomials

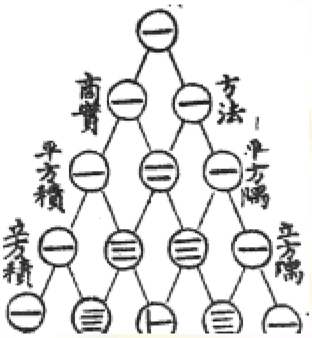
Pascal's Triangle can also show you the coefficients in binomial expansion:

Power	Binomial Expansion	Pascal's Triangle
2	$(x+1)^2 = \mathbf{1}x^2 + \mathbf{2}x + \mathbf{1}$	1, 2, 1
3	$(x+1)^3 = \mathbf{1}x^3 + \mathbf{3}x^2 + \mathbf{3}x + \mathbf{1}$	1, 3, 3, 1
4	$(x+1)^4 = \mathbf{1}x^4 + \mathbf{4}x^3 + \mathbf{6}x^2 + \mathbf{4}x + \mathbf{1}$	1, 4, 6, 4, 1
	... etc ...	

The First 15 Lines

For reference, I have included row 0 to 14 of Pascal's Triangle

```
                                          1
                                       1     1
                                    1     2     1
                                 1     3     3     1
                              1     4     6     4     1
                           1     5     10    10    5     1
                        1     6     15    20    15    6     1
                     1     7     21    35    35    21    7     1
                  1     8     28    56    70    56    28    8     1
               1     9     36    84    126   126   84    36    9     1
            1     10    45    120   210   252   210   120   45    10    1
         1     11    55    165   330   462   462   330   165   55    11    1
      1     12    66    220   495   792   924   792   495   220   66    12    1
   1     13    78    286   715   1287  1716  1716  1287  715   286   78    13    1
1     14    91    364   1001  2002  3003  3432  3003  2002  1001  364   91    14    1
```

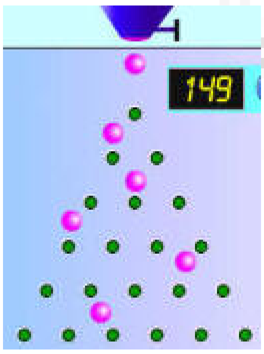


The Chinese Knew About It

This drawing is entitled "The Old Method Chart of the Seven Multiplying Squares"
It is from the front of Chu Shi-Chieh's book "*Ssu Yuan YüChien" (Precious Mirror of the Four Elements)*, written in **AD 1303** (over 700 years ago, and more than 300 years before Pascal!), and in the book it says the triangle was known about more than two centuries before that.

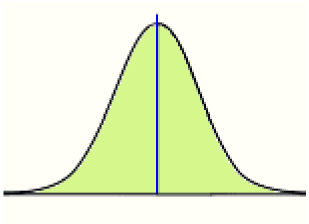
The Quincunx

An amazing little machine created by Sir Francis Galton is a Pascal's Triangle made out of pegs. It is called The Quincunx.



Balls are dropped onto the first peg and then bounce down to the bottom of the triangle where they collect in little bins.

At first it looks completely random (and it is), but then you find the balls pile up in a nice pattern: the Normal Distribution.



Conclusion:

By doing this project , the students could get acquaintance with:

(a) Various patterns.
(b) Concept of symmetry.
(c) Concept of combination.
(d) Concept of probability.
(e) Concept of binomial co-efficient

....

PART- C : MATHEMATICS

61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is :

(1) 219 (2) 256 (3) 275 (4) 510

61. (1)

Set A has 4 elements

Set B has 2 elements

$\therefore$ Number of elements in set $(A \times B) = 4 \times 2 = 8$

$\therefore$ Total number of subsets of $(A \times B) = 2^8 = 256$

Number of subsets having 0 elements = $8_{C_0} = 1$

Number of subsets having 1 element each = $8_{C_1} = 8$

Number of subsets having 2 elements each = $8_{C_2} = \dfrac{8!}{2!6!} = \dfrac{8 \times 7}{2} = 28$

$\therefore$ Number of subsets having at least 3 elements

$= 256 - 1 - 8 - 28 = 256 - 37 = 219$

62. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\dfrac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) straight line parallel to x–axis (2) straight line parallel to y–axis

(3) circle of radius 2 (4) circle of radius $\sqrt{2}$

62. (3)

$$\left|\frac{z_1 - 2z_2}{2 - z_1\overline{z}_2}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\ \overline{z}_2|^2$$

$$\Rightarrow (z_1 - 2z_2)(\overline{z}_1 - 2\overline{z}_2) = (2 - z_1\ \overline{z}_2)(2 - \overline{z}_1\ z_2)$$

$$\Rightarrow z_1\ \overline{z}_1 + 4\ z_2\ \overline{z}_2 = 4 + z_1\ \overline{z}_1\ z_2\ \overline{z}_2$$

$$\Rightarrow 4 + |z_1|^2\,|z_2|^2 - 4\,|z_2|^2 - |z_1|^2 = 0$$

$$\Rightarrow (|z_2|^2 - 1) \cdot (|z_2|^2 - 4) = 0$$

But $|z_2| \neq 1$, $\therefore$ $|z_2| = 2$

Hence, z lies on a circle of radius 2 centered at origin.

63. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to :

(1) 6 (2) −6 (3) 3 (4) −3

63. (3)

$x^2 - 6x - 2 = 0$

$\Rightarrow x^{10} - 6x^9 - 2x^8 = 0$

'α', β roots $\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ ...(1)

& $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$ ...(2)

(1) − (2)

$\alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3$

64. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to :

(1) (2, −1) (2) (−2, 1) (3) (2, 1) (4) (−2, −1)

64. (4)

$A A^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equation $a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ ...(1)

$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$ ...(2)

& $a^2 + 4 + b^2 = 0 \Rightarrow a^2 + b^2 = 5$ ...(3)

Solving $a = -2, b = -1$

65. The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non−trivial solution,

(1) is an empty set (2) is a singleton

(3) contains two elements (4) contains more than two elements

65. (3)

$(2-\lambda)\,x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

Non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$(2-\lambda)\,\{3\lambda + \lambda^2 - 4\} + 2\cdot\{-2\lambda + 2\} + (4 - 3 - \lambda) = 0$

$\Rightarrow (6\lambda + 2\lambda^2 - 8 - 3\lambda^2 - \lambda^3 + 4\lambda) - 4\lambda + 4 + 1 - \lambda = 0$

$\Rightarrow -\lambda^3 - \lambda^2 - 5\lambda + 3 = 0$

$x^3 - \lambda^2 + 2\lambda^2 - 2\lambda - 3\lambda + 3 = 0$

$\lambda^2(\lambda - 1) + 2\lambda(\lambda - 1) - 3(\lambda - 1) = 0$

$(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$(\lambda - 1)(\lambda + 3)(\lambda - 1) = 0$

$\lambda = 1, 1, -3$

66. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(1) 216 (2) 192 (3) 120 (4) 72

66. (2)

Case – 1

Any 5 – digit number > 6000 is all 5-digits number

Total number > 6000 using 5 – digits = 5! = 120

Case – 2

Using 4 – digits

___	___	___	___
↓	↓	↓	↓
Can be 6, 7 or 8	4 ways	3 ways	2 ways

i.e. 3 ways

Total number = $3 \times 4 \times 3 \times 2 = 72$

Total ways = 120 + 72 = 192

67. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is :

(1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-1\right)$ (4) $\frac{1}{2}\left(2^{50}+1\right)$

67. (1)

$$\left(1-2\sqrt{x}\right)^{50} = {}^{50}C_0 - {}^{50}C_1\left(2\sqrt{x}\right)^1 + {}^{50}C_2\left(2\sqrt{x}\right)^2 - {}^{50}C_3\left(2\sqrt{x}\right)^3 + {}^{50}C_4\left(2\sqrt{x}\right)^4 \ldots\ldots$$

So, sum of coefficient Integral powers of x

$$S = {}^{50}C_0 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + \ldots\ldots + {}^{50}C_{50}.2^{50}$$

Now, $(1+x)^{50} = 1 + {}^{50}C_1x + {}^{50}C_2x^2 + {}^{50}C_3x^3 + {}^{50}C_4x^4 + \ldots\ldots + {}^{50}C_{50}x^{50}$

Put x = 2, –2

$$3^{50} = 1 + {}^{50}C_1.2 + {}^{50}C_2.2^2 + {}^{50}C_3.2^3 + {}^{50}C_4.2^4 + \ldots\ldots + {}^{50}C_{50}.2^{20} \qquad \ldots\ldots (1)$$

$$1 = 1 - {}^{50}C_1.2 + {}^{50}C_2.2^2 - {}^{50}C_3.2^3 + {}^{50}C_4 2^4 - \ldots\ldots + {}^{50}C_{50}.2^{50} \qquad \ldots\ldots (2)$$

(1) + (2)

$$3^{50} + 1 = 2\left[1 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + \ldots\ldots + {}^{50}C_{50}.2^{50}\right]$$

$$\therefore \quad \frac{3^{50}+1}{2} = 1 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + \ldots. + {}^{50}C_{50}.2^{50}$$

68. If m is the A.M. of two distinct real numbers ℓ and n (ℓ , n > 1) and G1, G2 and G3 are three geometric means between ℓ and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals,

(1) $4\ell^2mn$ (2) $4\ell m^2n$ (3) $4\ell mn^2$ (4) $4\ell^2m^2n^2$

68. (2)

Given m is A.M. between ℓ & n

$\Rightarrow 2m = \ell + n \qquad \ldots(1)$

Given ℓ, G_1, G_2, G_3, n in G.P.

$$r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}} \Rightarrow r^4 = \frac{n}{\ell}$$

$\therefore\ G_1 = \ell,\ G_2 = \ell r^2,\ G_3 = \ell r^3$

So, $G_1^4 + 2G_2^4 + G_3^4 = \ell^4 r^4 [1 + 2r^4 + r^8]$

$$= \ell^4 \cdot \left(\frac{n}{\ell}\right)\left[1 + 2\left(\frac{n}{\ell}\right) + \left(\frac{n}{\ell}\right)^2\right]$$

$$= n\ell^3 \left[1 + \frac{n}{\ell}\right]^2 = n\ell^3 \frac{(n+\ell)^2}{\ell^2}$$

$$= n\ell\ (2m)^2 = 4\ell\ m^2 n$$

69. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \ldots$ is

(1) 71 (2) 96 (3) 142 (4) 192

69. (2)

$$T_n = \frac{1^3 + 2^3 + \ldots + n^3}{1 + 3 + 5 + \ldots + (2n-1)} = \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}[1 + (2n-1)]} = \frac{(n+1)^2}{4}$$

$$\therefore \sum_{n=1}^{9} T_n = \frac{1}{4}\sum_{n=1}^{9}(n+1)^2 = \frac{1}{4}[1^2 + 2^2 + \ldots + 10^2 - 1^2]$$

$$= \frac{1}{4}\left[\frac{10(10+1)(2\times 10+1)}{6} - 1\right] = 96$$

70. $\lim_{x\to 0} \frac{(1-\cos\ 2x)\ (3\ +\ \cos\ x)}{x\ \tan\ 4x}$ is equal to :

(1) 4 (2) 3 (3) 2 (4) $\frac{1}{2}$

70. (3)

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

$$\lim_{x\to 0}\frac{2\sin^2 x(3+\cos x)}{x\tan 4x}$$

$$\lim_{x\to 0}\frac{2\frac{\sin^2 x}{x^2}(3+\cos x)}{4\left(\frac{x\tan 4x}{4x^2}\right)} = 2$$

71. If the function, $g(x) = \begin{cases} k\sqrt{x+1} & , \ 0 \le x \le 3 \\ mx+2 & , \ 3 < x \le 5 \end{cases}$ is differentiable, then the value of $k + m$ is :

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

(27) VIDYALANKAR : JEE-MAIN 2015 : Paper and Solution

71. (1)

$$g(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ mx+2 & 3 < x \le 5 \end{cases}$$

$$\lim_{x \to 3^-} g(x) = \lim_{x \to 3^+} g(x) = g(3)$$

$2k = 3m + 2 = 2k$ …….. (1)

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & 0 \le x \le 3 \\ m & 3 < x \le 5 \end{cases}$$

$$\text{L.H.D at}_{x=3} = \lim_{x \to 3^-} g'(x) = \frac{k}{4}$$

$$\text{R.H.D at}_{x=3} = \lim_{x \to 3^+} g'(x) = m$$

L.H.D. R.H.D.

$\frac{k}{4} = m$ …….. (2)

From (i) & (ii)

$m = \frac{2}{5}, k = \frac{8}{5}$

$k + m = 2$

72. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) does not meet the curve again
(2) meets the curve again in the second quadrant
(3) meets the curve again in the third quadrant
(4) meets the curve again in the fourth quadrant

72. (4)

$$x^2 + 2xy - 3y^2 = 0$$

$$2x + 2\left(x\frac{dy}{dx} + y\right) - 6y\frac{dy}{dx} = 0$$

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$(2x - 6y)\frac{dy}{dx} + (2x + 2y) = 0$$

$$\frac{dy}{dx} = \frac{-(x+y)}{(x-3y)}$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 1$$

Slope of normal is -1.

Equation of normal is $y - 1 = -1(x - 1)$

$$x + y = 2$$

$$y = 2 - x$$

$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$

$x^2 + 4x - 2x^2 - 3(4 + x^2 - 4x) = 0$

$x^2 + 4x - 2x^2 - 12 - 3x^2 + 12x = 0$

$-4x^2 + 16x - 12 = 0$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

$x = 1, y = 1$

$x = 3, y = -1$

73. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to :

(1) – 8 (2) – 4 (3) 0 (4) 4

73. (3)

$$\lim_{x\to 0}\left(1+\frac{f(x)}{x^2}\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = 2$$

$\therefore\ f(x) = ax^4 + bx^3 + 2x^2 + 0x + 0$

$f'(x) = 4ax^3 + 3bx^2 + 4x$

$f'(1) = 4a + 3b + 4 = 0$ .... (1)

$f'(2) = 32a + 12b + 8 = 0$

$\Rightarrow 8a + 3b + 2 = 0$ .... (2)

Solving (1) and (2), we get $a = \frac{1}{2}$, $b = -2$

$$\therefore\ f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

$$f(2) = 8 - 16 + 8 = 0$$

74. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals :

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $\left(x^4+1\right)^{\frac{1}{4}} + c$ (3) $-\left(x^4+1\right)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

74. (4)

$$\int \frac{dx}{x^2.x^3\left(1+\frac{1}{x^4}\right)^{3/4}}$$

$$= \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}; \qquad \text{Let, } 1+\frac{1}{x^4}=t \qquad \Rightarrow \qquad -\frac{4}{x^5}dx = dt$$

$$= \frac{-1}{4}\int \frac{dt}{t^{3/4}} = \frac{-1}{4}\left[\frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}\right]+c = \frac{-1}{4}\left[\frac{t^{1/4}}{1/4}\right]+c$$

$$= -\left(1+\frac{1}{x^4}\right)^{1/4}+c = -\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$$

75. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$ is equal to :

(1) 2 (2) 4 (3) 1 (4) 6

75. (3)

$$\int_2^4 \frac{\log x^2}{\log x^2+\log(6-x)^2}dx$$

$$I = \int_2^4 \frac{\log x}{\log x+\log(6-x)}dx \qquad \text{........ (i)}$$

$f(a + b - x) = f(x)$

$$I = \int_2^4 \frac{\log(6-x)}{\log(6-x)+\log x}dx \qquad \text{........ (ii)}$$

(i) + (ii)

$$2I = \int_2^4 \frac{\log x+\log(6-x)}{\log(6-x)+\log x}dx$$

$$2I = \int_2^4 dx$$

$2I = 4 - 2$

$2I = 2$

$I = 1$

76. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is :

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

76. (4)

$y^2 \le 2x$

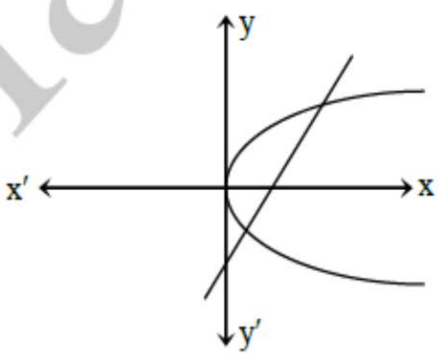
$\& \; y \ge 4x - 1$

By solving $y^2 = 2x$

$\& \quad y = 4x - 1$

$y = 1, \frac{-1}{2}$

$$A = \int_{\frac{-1}{2}}^{1} \left[\left(\frac{y+1}{4} \right) - \left(\frac{y^2}{2} \right) \right] dy = \frac{9}{32}$$



77. Let y(x) be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \ge 1)$. Then y(e) is equal to :

(1) e (2) 0 (3) 2 (4) 2e

77. (3)

$(x \ln x)\frac{dy}{dx} + y = 2x \log x, \qquad (x \ge 1)$

put x = 1 then y = 0

Now equation can be written as

$$\frac{dy}{dx} + \frac{y}{x \log x} = 2$$

$$\text{I.F.} = e^{\int \frac{1}{x \ln x} dx} = \ln x$$

Solution of differential equation is

$$y.\ln x = c + \int 2.\ln x dx$$

$y.\ln x = c + 2 [x \ln x - x]$

Given at x = 1, y = 0

$0 = c + 2. (-1)$

$\Rightarrow c = 2$

$y.\ln x = 2 + 2 [x\ln x - x]$

Put x = e

$y = 2 + 2[e - e]$

$y = 2$

78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is :

(1) 901 (2) 861 (3) 820 (4) 780

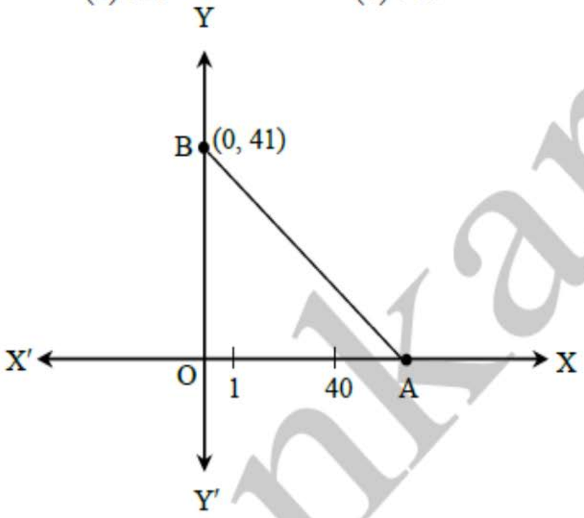
78. (4)

$y - 0 = -1(x - 41)$

$x + y = 41$

$39 + 38 + 37 + ... + 1 = 39\frac{(39+1)}{2}$

$= 780$



79. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \varepsilon R$, is a :

(1) straight line parallel to x–axis. (2) straight line parallel to y–axis

(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

79. (3)

$(2x - 3y + 4) + k(x - 2y + 3) = 0$ is family of lines passing through (1, 2). By congruency of triangles, we can prove that mirror image (h, k) and the point (2, 3) will be equidistant from (1, 2)

$\therefore$ (h, k) lies on a circle of radius $= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$

80. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 1 (2) 2 (3) 3 (4) 4

80. (3)

$x^2 + y^2 - 4x - 6y - 12 = 0$

$\therefore$ $C_1 = (2, 3)$ and $r_1 = \sqrt{4+9+12} = 5$

$x^2 + y^2 + 6x + 18y + 26 = 0$

$\therefore$ $C_2 = (-3, -9)$ and $r_2 = \sqrt{9+81-26} = 8$

$d(C_1, C_2) = \sqrt{(5)^2 + (12)^2} = 13$

$|r_1 + r_2| = 8 + 5 = 13$

$\therefore$ $d(C_1, C_2) = r_1 + r_2$

$\therefore$ Number of common tangents = 3

81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is :

(1) $\frac{27}{4}$ (2) 18 (3) $\frac{27}{2}$ (4) 27

81. (4)

$a = 3$

$b = \sqrt{5}$

$\frac{b^2}{a} = \frac{5}{3}$

$\therefore$ One of the end points of a latus rectum $= \left(2, \frac{5}{3}\right)$

$\therefore$ Equation of the tangent at $\left(2, \frac{5}{3}\right)$ is

$$\frac{x \times 2}{9} + \frac{y}{3} \times \frac{5}{5} = 1 \Rightarrow \frac{x}{9/2} + \frac{y}{3} = 1$$

Area of the rhombus formed by tangents $= \frac{1}{2} \times \frac{9}{2} \times 3 \times 4 = 27$ sq. units

82. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. It the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

(1) $x^2 = y$ (2) $y^2 = x$ (3) $y^2 = 2x$ (4) $x^2 = 2y$

82. (4)

Let $Q = (4t, 2t^2)$

and $O = (0, 0)$

$\therefore$ $P = \left(\frac{4t}{4}, \frac{2t^2}{4}\right)$

$\therefore$ $x = t$,

$y = \frac{t^2}{2} \Rightarrow 2y = x^2$

83. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is :

(1) $2\sqrt{14}$ (2) 8 (3) $3\sqrt{21}$ (4) 13

83. (4)

Let $x = 3r + 2$

$y = 4r - 1$

$z = 12r + 2$

$\therefore$ $3r + 2 - 4r + 1 + 12r + 2 = 16$

$\Rightarrow r = 1$

$\therefore$ $(x, y, z) = (5, 3, 14)$

Required distance $= \sqrt{4^2 + 3^2 + 12^2} = 13$

84. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is :

(1) $2x + 6y + 12z = 13$ (2) $x + 3y + 6z = -7$

(3) $x + 3y + 6z = 7$ (4) $2x + 6y + 12z = -13$

84. (3)

Put z = 0 in first two planes

$\therefore \; 2x - 5y = 3$

and $x + y = 5$

$\Rightarrow \; x = 4, y = 1$, when $z = 0$

Let $x + 3y + 6z = k$ be a plane parallel to given plane.

$\therefore \; 4 + 3 + 0 = k \Rightarrow k = 7$

$\therefore \; x + 3y + 6z = 7$ is required plane.

85. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non–zero vectors such that no two of them are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

85. (1)

$$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$$

$$\Rightarrow -\left\{(\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}\right\} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$$

$$\Rightarrow (\vec{c}\cdot\vec{a})\vec{b} - (\vec{c}\cdot\vec{b})\vec{a} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$$

$$\Rightarrow \text{Equate, } \; -(\vec{c}\cdot\vec{b}) = \frac{1}{3}|\vec{b}|\,|\vec{c}|$$

$$\Rightarrow \cos\theta = -\frac{1}{3}$$

$$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$$

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$ (3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

86. (1)

$n(S) = 3^{12}$

$n(E) = {}^{12}C_3 \cdot 2^9$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{12}C_3 \cdot 2^9}{3^{12}} = \frac{220.2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$$

Note : According to the question, all given options are wrong.

87. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

(1) 16.8 (2) 16.0 (3) 15.8 (4) 14.0

87. (4)

$$\frac{\sum x_i}{16} = 16 \Rightarrow \sum x_i = 256$$

$$\frac{(\sum x_i) - 16 + 3 + 4 + 5}{18} = \frac{252}{18} = 14$$

88. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is :

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) 2 : 3

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88. (1)

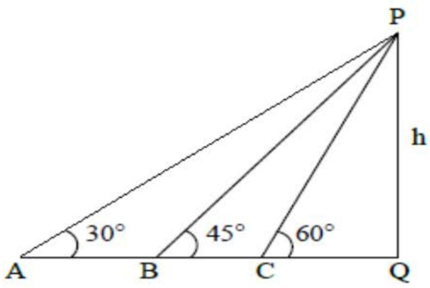
$$\frac{h}{AQ} = \frac{1}{\sqrt{3}}$$

$\Rightarrow AQ = \sqrt{3}h$

Similarly, $BQ = h$

$$CQ = \frac{h}{\sqrt{3}}$$

$$\therefore \frac{AB}{BC} = \frac{AQ - BQ}{BQ - CQ} = \frac{(\sqrt{3}-1)h}{\left(h - \frac{h}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{1}$$



89. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x - x^3}{1 - 3x^2}$ (2) $\frac{3x + x^3}{1 - 3x^2}$ (3) $\frac{3x - x^3}{1 + 3x^2}$ (4) $\frac{3x + x^3}{1 + 3x^2}$

89. (1)

$$\tan^{-1}(y) = \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - \frac{x.2x}{1-x^2}}\right)$$

$$\Rightarrow \tan^{-1}(y) = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) \Rightarrow y = \frac{3x - x^3}{1 - 3x^2}$$

90. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$ (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

90. (4)

$\sim [\sim s \vee (\sim r \wedge s)]$

$= \sim(\sim s) \wedge \sim (\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= s \wedge r \vee F$

$= s \wedge r$

TIPS AND TECHNIQUES

TOPIC : SETS

1. Any subset of A X A is a relation on A .

   If n(A) = p, n(B) = q, then n(A X B) = pq

   Total number of subsets of (A X B) = 2^{pq}.

   Hence 2^{pq} different relations are possible from A to B.

2. Cardinal number of $\emptyset$ is 0 .

3. If the Cardinal number of a set is n then number of its subsets is 2^n

TOPIC : RELATIONS AND FUNCTIONS

1. Any linear expression represents a function.

2. Range of 'f'is $\subseteq$ co domain of 'f'

3 f(x) = c is the only function which is both odd and even.

4.To check for a given function to be one – one, if we find two numbers x_1 & x_2 for which

$f(x_1) = f(x_2)$, $x_1 \neq x_2$ then we can say that the given function is not one – one without checking it.

Ex: $f(x) = x + \frac{1}{x} + 10$,is it one – one?

Soln: $\because$ $f(2) = f(\frac{1}{2})$, $\therefore$ it is not one – one.

*TOPIC : TRIGONOMETRIC F*UNCTIONS

1. When no unit is mentioned with an angle, it is always understood to be in radian.

2. The following steps may be followed for drawing the graphs of trigonometric functions

   a) Simplify the given function to a single T-ratio if possible.

   b) Find the period: i). sinx,cosx,secx, and cosecx are periodic functions with period 2π.

      ii).tanx,cotx are periodic functions with period π.

      iii).sin(ax+b),cos(ax+b),sec(ax+b),coscex(ax+b) are periodic functions with period $\frac{2\pi}{|a|}$.

      iv). tan(ax+b),cot(ax+b) are periodic functions with period $\frac{\pi}{|a|}$.

3. Prepare a chart between suitable x and y and select suitable scales on two axes.

4. Plot various points (x,y) and join them by free hand.

5. While solving a trigonometric equation, squaring the equation at any stage should be avoided.

6. Never cancel terms containing unknown terms on the two sides, which are in product,it may lead to loss of solutions.

First principle of mathematical induction

The proposition P(n) involving a natural number n is assumed to be true for all $n \in N$, follows the following three steps;

STEP-1 (VERIFICATION STEP)

Actual verification of the proposition P (n) for the starting value of n=1

STEP-2 (INDUCTION STEP)

Assuming that p (n) is true for n=k, k≥1, prove that it is also true for n=k+1

STEP-3

GENERALISATION STEP

Combining the above two steps lead to the conclusion that p(n) is true for all integers $n \in N$

Second principle of mathematical induction

Sometimes, the first PMI does not suffice. In such cases we use the extended principles as below

STEP-1 (VERIFICATION STEP)

We verify that P (n) is true for n=1 and n=2 both

STEP-2 (INDUCTION STEP)

Assuming that p (n) is true for n=k and n=k+1, k≥1, and prove that P (n) is true for n=k+2

STEP-3

GENERALISATION STEP

Combining the above two steps lead to the conclusion that p (n) is true for all integers $\forall n \in N$.

NOTE; the second principle of PMI is useful to prove recurrence relations which involve three successive terms

Ex: $p\,T_{n+1} = q\,T_n + r\,T_{n+1}$ ((If $a_1=1$, $a_2=5$ and $a_{n+2}=5a_{n+1} - 6\,a_n$, n≥1. Show by using PMI $a_n=3^n - 2^n$.

TOPIC : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 .Argument of z = 0 is not defined.

2..Argument of a complex number z = a + ib and the properties

Re(z)	Im(z)	Quadrant	Principal argument
a > 0	b > 0	1st	θ
a < 0	b > 0	2nd	$\pi - \theta$
a < 0	b < 0	3rd	$-\pi + \theta$
a > 0	b < 0	4th	- θ

Where $\theta = \tan^{-1}\frac{b}{a}$

3.If z lies in 2nd quadrant ,then conjugate of z lies in the 3rd quadrant.

If z lies in 1st quadrant ,then conjugate of z lies in the 4th quadrant.

LINEAR INEQUALITIES

1. To solve an inequality we can add (or subtract) the same quantity to (from) both sides without changing the sign of inequality.
2. Multiply (or divide) both sides by the same positive quantity without changing the sign of inequality. However ,if both sides of inequality are multiplied(or divided)by the same negative quantity the sign of inequality is reversed
3. If a,b∈ R and b≠0, then
   i) ab>0 or $\frac{a}{b} > 0$ implies a and b are of same sign
   ii) ab<0 or $\frac{a}{b} < 0$ implies a and b are of opposite sign
4. If a is any positive real number, i.e. a>o

$$|x| < a \text{ if and only if } -a < x < a$$

$$|x| \leq a \text{ if and only if } -a \leq x \leq a$$

$$|x| > a \text{ if and only if } x < -a \text{ or } x < a$$

$$|x| \geq a \text{ if and only if } x \leq -a \text{ or } x \geq a$$

PERMUTATIONS AND COMBINATIONS

1) 1! =1 AND 0! =1

2) Factorial of negative numbers or proper fractions is not defined.

3) CIRCULAR PERMUTATIONS

<u>Arrangements round a circular table</u>

When clockwise and anticlockwise arrangements are considered different then number of circular permutations of n different things taken all at a time is (n-1)!

Arrangements of flowers in a garland

When clockwise and anticlockwise arrangements are considered same, then number of circular permutations of

n different things taken all at a time is (n-1)!/2

NUMBER OF CIRCULAR PERMUTATIONS OF n DIFFERENT THINGS

Taken r at a time: i). p (n,r)/ r when clockwise is different from anti clockwise arrangement

ii). p(n,r)/ 2r when clockwise& anticlockwise are considered same

4. DIVISION INTO GROUPS

a) The number of ways in which (m+n) different things can be divided into two groups containing m and n things(m+n) respectively are C(m+n,m). C(n,n)
b) If m=n and group order is not important then $2n!/(2)!(n!)^2$
c) If m=n and groups are distinct then $2n! / (n!)^2$

5. m parallel lines in a plane are intersected by another family of n parallel lines. The total number of parallelograms so formed is$\left(\frac{mn(m-1)(n-1)}{4}\right)$

6. The number of diagonals of n-sided polygon is C(n,2)-n =$\frac{n(n-3)}{2}$ for n≥3

7. The number of rectangles in (m x n) board is C(m+1,2)x C(n+1,2)

8. The number of squares in n x n board is $n(n+1)(2n+1)$/6

BINOMIAL THEOREM

1. $(X+Y)^n + (x+y)^n$ =2(sum of terms at odd places) = 2($C_0 X^n + C_2 X^{n-2} y^2$+-----)

   Last term $C_n\ y^n$ if n is even and $C_{n-1} XY^{n-1}$ if n is odd

2. $(X+Y)^n - (x+y)^n$ =2(sum of terms at even places)

   Last term $C_{n-1} X y^{n-1}$ if n is even and $Cn_{-1}Y^n$ if n is odd'

3. P^{TH} term from end = $(n+2-p)^{th}$ term from the beginning (the coefficient of third term from beginning is c (n, 2) so the coefficient of third term from end is c (n, n-2)

4. The number of terms in the expansion of $(X+Y+Z)^n$ = $\frac{n(n+1)}{2}$

TOPIC : SEQUENCES AND SERIES

1. $1+ 3+ 5+ \text{------}+ (2n - 1) = n^2$
2. To find the n^{th} term of the series: $T_n = S_n - S_{n-1}$,where

   T_n = nth term

   S_n = sum of n terms

   S_{n-1} = sum of (n-1) terms

3. If A and G is AM and GM of two numbers respectively then they are $A +\sqrt{A^2 - G^2}$ and

   $A - \sqrt{A^2 - G^2}$

TOPIC : straight lines

1.Slope of a line Ax + By + C = 0 can be directly found as $m = \frac{-A}{B}$ or convert the line in the form y = mx + c.

2.From the concept of family of straight lines the concept of the pair of straight lines can be developed easily.

3.To convert a line Ax + By + C = 0 into any other form , the coefficient of x should always be one.

4.Any line parallel to the line Ax + By + C = 0 is Ax + By + K = 0.

5. Any line perpendicular to the line Ax + By + C = 0 is Bx – Ay + K = 0

TOPIC : CONIC SECTIONS

1.If 2 tangents are parallel to each other,then the distance between given parallel lines gives the diameter of the circle drawn in between the two tangents.

2.If a is the radius of the circle then (-a,-a) be the centre of the circle and the perpendicular distance from the centre to the given line gives the radius of the circle.

3.The directrix equation for the parabola can be remembered as :

   for $Y^2 = 4ax$ is x+a =0

   for $Y^2 = -4ax$ is x-a =0

4. Clay model can be used to demonstrate the 3 topics on conic section which can be taught effectively.

5.The path in a race course is based on the concept of ellipse.

TOPIC : INTRODUCTION TO 3- DIMENSIONAL GEOMETRY

1.If a point R divides a line AB externally then the point R lies outside the given line.

2 Negative sign in the ratio indicates external division.

3. The distance formula in 3-D is extension of the distance formula of 2-D and so is the case with the section formula.

4. The signs of the 8 octants are as given below.

Octants/ Coordinates	I	II	III	IV	V	VI	VII	VIII
x,y	+,+	-,+	-,-	+,-	+,+	-,+	-,-	+,-
z	+	+	+	+	-	-	-	-
(x,y,z)	(+,+,+)	(-,+,+)	(-,-,+)	(+,-,+)	(+,+,-)	(-,+,-)	(-,-,-)	(+,-,-)

LIMITS AND DERIVATIVES

1. To work any problem on limits at ease,

(a) Trigonometric formulae
(b) $\sum n$, $\sum n^2$ and $\sum n^3$
(c) Sum to first 'n' terms of an AP and GP
(d) Types of functions namely polynomial function, modulus function, rational fn etc are to be revised.

2. Standard limits on exponential functions and logarithmic function can be given

3. Limits at infinity and infinite limits can also be discussed.

4. L'Hospital rule can also be introduced to work problems on limits.

TOPIC :MATHEMATICAL REASONING

1. The converse of the statement($p \rightarrow q$) is ($q \rightarrow p$)

2. The contrapositive of the statement($p \rightarrow q$) is ($\sim q \rightarrow \sim p$)

3. The concept of reasoning can be made easier by using Boolean algebra.

STATISTICS

1. Use logarithm tables namely square tables, square root tables etc to evaluate the measures of dispersion.

2. Comparison of 2 standard deviations will suffice when the 2 means are the same in Coefficient of variance.

3. Step deviation method is always preferred while calculating Variance or Standard Deviation.(For discrete and continuous frequency distribution table.)

PROBABILITY

4. The concepts of permutations and combinations like
   (a) P(n,r)
   (b) C(n,r)
   (c) The number of permutations of 'n' objects where p1 objects are of one kind,p2 are of second kind,……pk are of kth kind and the rest, if any, are of different kind is

$$\frac{n!}{p_1\,!\,p_2\,!\,\ldots.\,p_k!}$$

   can be linked in solving the problems of probability.

5. The concepts of sets namely
   (a) $n(A\cup B) = n(A) + n(B) - n(A \cap B)$
   (b) $n(A\cup B\cup C)=n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$
   (c) DeMorgan's Laws :
   $(A\cup B)' = A' \cap B'$, $(A\cap B)'=A'\cup B'$
   can also be linked in solving the problems of probability.

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ANNEXURES:

1. Class XI mind maps Topic wise arranged and kept in the folder, the name of the folder is class XI mind map.
2. All the power point presentation chapter wise arranged and kept in a folder, the name of the folder is ppts.
3. All the word documents and power point presentations which were embedded in VUE file chapter wise kept in the separate folder; the name of the folder is Class XI mind map.

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ANNEXURE LIST:

S. No	Name of the topic	Nature of file attached	Name of the embedded files	Type
01	Sets	VUE	1. Definition (2) 2. Definition 3. Interval 4. Laws 5. Operation on sets. 6. power set 7. Set representation 8. Set 9. Sub set 10. Types of set 11. Venn Diagram	Word Document Word Document Word Document Word Document Word Document Word Document Word Document PPT Word Document Word Document Word Document
		PPT	Sets	PPT
02	Relations and Functions	VUE	1. Algebra of function 2. Cartesian Product 3. Defn. of real function 4. Function definition 5. Graph of functions 6. many to many 7. many to one 9. one to many 10. Relation example 11. Types of relation	Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document
		PPT	a. Relations and functions b. Exponential functions. c. Functions Domain Range	PPT PPT PPT
03	Trigonometric Functions	VUE	1. angles 2.Graphs of trig functions 3. Measurement of angles 4. Properties of triangles 5. Sign of trigonometric function 6. Sign of trigonometric function 7. Solution 8. Sum and difference of angles 9. Trigonometry ppt	Word Document PPT Word Document Word Document Word Document Word Document Word Document Word Document PPT
		PPT	1. Trigonometric Functions 2. Trigonometry in 3D	PPT PPT
04	Principle of Mathematical Induction	VUE	1. Divisibility 2. Example of MI 3. Inequality 4. Sum	Word Document Word Document Word Document Word Document
		PPT	Principle of Mathematical Induction	PPT
05	Complex Numbers and Quadratic Equations	VUE	1. Additive inverse 2. Argand plane 3. Complex no. definition 4. Modulus 5. Multiplicative inverse of a Complex no. 6. Need of Complex numbers 7. Polar form 8. Properties 9. Quadratic equations 10. Square root of a complex no	Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document
		PPT	Complex Numbers and Quadratic Equations	PPT

06	**Linear Inequalities**	**VUE**	**1. Def of inequalities.** **2. Graphical solution in one variable** **3. Graphical solution in two variables** **4. Graphical solution** **5. Inequalities problem** **6. Linear inequalities** **7. Linear inequality in two variables** **8. Number line in one variable** **9. System of linear in-equations** **10. Word problem**	**Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document**
07	**Permutations and Combinations**	**VUE**	**1. Definition of combination** **2. Definition of permutation** **3. Fundamental principle** **4. Permutation** **5. With restriction**	**Word Document** **Word Document** **Word Document** **Word Document** **Word Document**
		PPT	**Permutations and Combinations**	**PPT**
08	**Binomial Theorem**	**VUE**	**1. Definition of binomial.** **2. General term.** **3. Middle term** **4. Numerical problem** **5. Proof by MI** **6. r^{th} term from end** **7. Special Cases**	**Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document**
		PPT	**Binomial Theorem**	**PPT**
09	**Sequence and Series**	**VUE**	**1. A.P Def** **2. AM** **3. Am of n terms** **4. Sequence and Series** **5. Definition GP** **6. Definition of series** **7. Definition of sequence** **8. General term of AP** **9. General term of GP** **10. Geometric mean** **11. infinite terms sum** **12. nth term, GM** **13. Relation between AM, GM** **14. Special sequence.** **15. Sum to n terms of GP.** **16. Sum to n terms** **17. Tips in AP**	**Word Document** **Word Document** **Word Document** PPT **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document**
		PPT	**Sequence and Series**	**PPT**
10	**Straight Lines**	**VUE**	**1. Angle between the lines** **2. Condition for parallel** **3. Condition for perpendicular** **4. Distance formula** **5. Family of lines** **6. Forms of lines** **7. Intercept form** **8. Normal form** **9. Shifting the origin** **10. Slope** **11. Slope-intercept form** **12. Two-point form**	**Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document** **Word Document**
		PPT	**1. Straight Lines** **2. Translations**	**PPT** **PPT**

11	Conic Sections	VUE	1. Circles 2. Conic sections 3. Ellipse 4. Hyperbola 5. Parabola	Word Document PPT Word Document Word Document Word Document
		PPT	1. What are conic section	PPT
12	Introduction to Three-Dimensional Geometry	VUE	1. Distance formula 2. Introduction 3. Octants 4. Points on coordinate axes 5. Section formula	Word Document Word Document Word Document Word Document Word Document
		PPT	Introduction to Three-Dimensional Geometry	PPT
13.	Limits and Derivatives	VUE	1. Algebra of Derivatives 2. Algebra of limits 3. Definition of Derivatives 4. Formulae 5. One sided limit 6. Standard limits	Word Document Word Document Word Document Word Document Word Document Word Document
		PPT	1. Limits 2. Limits and Derivatives	PPT PPT
14	Mathematical reasoning	VUE	1. Negation of a statement 2. Quantifiers 3. Statement	Word Document Word Document Word Document
		PPT	Mathematical reasoning	PPT
15	Statics	VUE	1. CV 2. Formula 3. From mean 4. From median 5. Range 6. Variance	Word Document Word Document Word Document Word Document Word Document Word Document
		PPT	Standard Deviation	PPT
16	Probability	VUE	1. Event – MR 2. Impossible event 3. MU ex and Exhaustive event 4. Mu ex event 5. Probability A or B 6. Probability not A 7. Probability of an event 8. Sure event	Word Document Word Document Word Document Word Document Word Document Word Document Word Document Word Document
		PPT	Probability	PPT
