विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता

सद्गुरु श्री रणछोड़दासजी महाराज

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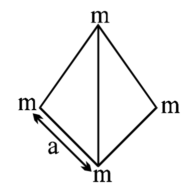
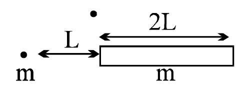
EXERCISE-I

Q.1 A remote sensing satellite is revolving in an orbit of radius x the equator of earth. Find the area on earth surface in which satellite can not send message.

Q.2 Four masses (each of m)are placed at the vertices of a regular pyramid (triangular base) of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '2a'.



Q.3 A small mass and a thin uniform rod each of mass ' m ' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



Q.4 An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:
(i) the initial speed of projection
(ii) the speed at half the maximum height.

Q.5 A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time________.

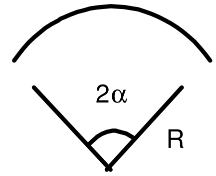
Q.6 A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5$J is equal to ________.

Q.7 A rocket starts vertically upwards with speed v_0. Show that its speed v at a height h is given by

$$v_0^2 - v^2 = \frac{(2gh)}{\left(1+\frac{h}{r}\right)}$$

where R is the radius of the earth. Hence deduce the maximum height reached by a rocket fired with speed equal to 90% of escape velocity.

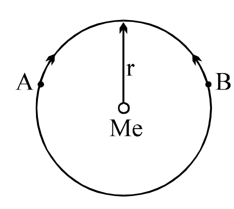
Q.8 Find the gravitational field strength and potential at the centre of arc of linear mass density λ subtending an angle 2α at the centre.



Q.9 A point P lies on the axis of a fixed ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be _______.

Q.10 Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

Q.11 Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the earth E but in opposite sense of rotation and therefore on a collision course (see figure).



(a) In terms of G, M_e, m and r find the total mechanical energy $E_A + E_B$ of the two satellite plus earth system before collision.

(b) If the collision is completely inelastic so that wreckage remains as one piece of tangled material (mass = 2m), find the total mechanical energy immediately after collision.

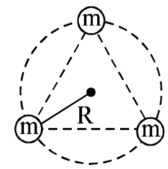
(c) Describe the subsequent motion of the wreckage.

Q.12 A particle is fired vertically from the surface of the earth with a velocity $k\upsilon_e$, where υ_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e. Calculate the height to which it will rise from the surface of the earth.

Q.13 A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is _______.

Q.14 Find the potential energy of a system of eight particles placed at the vertices of a cube of side L. Neglect the self energy of the particles.

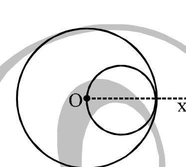
Q.15 A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find:



(i) the total P.E. of the system
(ii) the orbital speed of each moon such that they maintain this configuration.

Q.16 Two small dense stars rotate about their common centre of mass as a binary system with the period 1year for each. One star is of double the mass of the other and the mass of the lighter one is $\frac{1}{3}$ of the mass of the sun. Find the distance between the stars if distance between the earth & the sun is R.

Q.17 A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius r=R/2 whose centre is at x=R/2 as in fig. (a) Find gravitational field at points on the axis for x > R (ii) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.



Q.18 A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be
(a) then velocity in circular orbits.
(b) maximum distance between the two fragments before collision and
(c) magnitude of their relative velocity just before they collide.

Q.19 The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator barely provides the centripetal force needed for the rotation. (Why?)
(a) Show then that the corresponding shortest period of rotation is given by

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

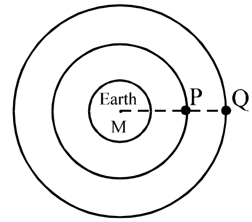
Where ρ is the density of the planet, assumed to be homogeneous.
(b) Evaluate the rotation period assuming a density of 3.0 gm/cm^3, typical of many planets, satellites, and asteroids. No such object is found to be spinning with a period shorter than found by this analysis.

Q.20 A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite.

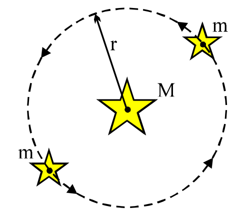
<u>List of recommended questions from I.E. Irodov.</u>
1.213, 1.216 to 1.220, 1.224, 1.226, 1.227, 1.229

EXERCISE-II

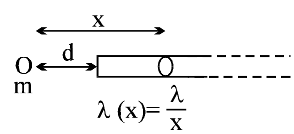
Q.1 A satellite P is revolving around the earth at a height h = radius of earth (R) above equator. Another satellite Q is at a height 2h revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this situation.



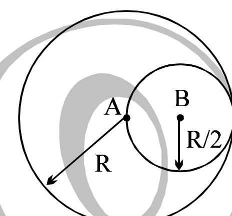
Q.2 A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit, see figure. Derive an expression for the period of revolution of the stars; the radius of the orbit is r.



Q.3 Find the gravitational force of interaction between the mass m and an infinite rod of varying mass density λ such that $\lambda(x) = \lambda/x$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in figure.



Q.4 Inside an isolated fixed sphere of radius R and uniform density r, there is a spherical cavity of radius R/2 such that the surface of the cavity passes through the centre of the sphere as in figure. A particle of mass m is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere.



Q.5 In a certain double star system the two stars rotate in circular orbits about their common centre of mass. The stars are spherical, they have same density ρ and their radii arc R and 2 R. Their centres are 5 R apart. Find the period T of stars in terms of ρ, R & G.

Q.6 A ring of radius R is made from a thin wire of radius r. If ρ is the density of the material of wire then what will be the gravitational force exerted by the ring on the material particle of mass m placed on the axis of ring at a distance x from its centre. Show that the force will be maximum when $x = R/\sqrt{2}$ and the maximum value of force will be given as

$$F_{max} = \frac{4\pi^2 G r^2 \rho m}{(3)^{3/2} R}$$

Q.7 In a particular double star system, two stars of mass 3.22×10^{30} kg each revolve about their common center of mass, 1.12×10^{11} m away.

(a) Calculate their common period of revolution, in years.

(b) Suppose that a meteoroid (small solid particle in space) passes through this centre of mass moving at right angles to the orbital plane of the stars. What must its speed be if it is to escape from the gravitational field of the double star?

Q.8 A man can jump over b=4m wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth = 6400 km.

Q.9 A launching pad with a spaceship is moving along a circular orbit of the moon , whose radius R is triple that of moon Rm. The ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity $\vec{v}_0$ and the launching pad then falls to the moon . Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.

Q.10 A small satellite revolves around a heavy planet in a circular orbit. At certain point in its orbit a sharp impulse acts on it and instantaneously increases its kinetic energy to 'k' (< 2) times without change in its direction of motion. Show that in its subsequent motion the ratio of its maximum and minimum distances from the planet is $\frac{k}{2-k}$, assuming the mass of the satellite is negligibly small as compared to that of the planet.

Q.11 A satellite of mass m is in an elliptical orbit around the earth of mass M (M>>m) The speed of the satellite at its nearest point to the earth (perigee) is $\sqrt{\frac{6GM}{5R}}$ where R=its closest distance to the earth. It is desired to transfer this satellite into a circular orbit around the earth of radius equal its largest distance from the earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

Q.12 A body is launched from the earth's surface a an angle $\alpha=30^o$ to the horizontal at a speed $v_0=\sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (ii) The radius of curvature of trajectory at its top point.

Q.13 Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to reach centre of earth if it is projected into the tunnel from surface of earth with speed needed for it to escape the gravitational field of earth.

EXERCISE-III

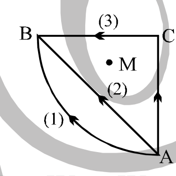
Q.1 If the distance between the earth and the sun were half its present value, the number of days in a year would have been [JEE' 96]
(A) 64.5 (B) 129 (C) 182.5 (D) 730

Q.2 Distance between the centres of two stars is 10 a. The masses of these stars are M and 16 M and their radii a and 2a respectively. A body of mass m is fired at night from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a. [JEE' 96]

Q.3 An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.) E_0. Its potential energy is [JEE' 97]
(A) $-E_0$ (B) $1.5\ E_0$ (C) $2\ E_0$ (D) E_0

Q.4 A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth surface. Assume that the spaceship and the astronaut fall on a straight line from the earth centre. The radius of the earth is 6400 km. [REE 98]

Q.5 In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1 , W_2, W_3 respectively then



(A) $W_1 = W_2 = W_3$ (B) $W_1 > W_2 > W_3$ (C) $W_1 = W_2 > W_3$ (D) $W_1 < W_2 < W_3$
[JEE' (Scr.) 2003]

Q.6 A body is projected vertically upwards from the bottom of a crater of moon of depth R/100 where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. [JEE' 2003]

Q.7 A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then [JEE 2006]
(A) $T_A > T_B$ (if $r_A > r_B$) (B) $T_A > T_B$ (if $m_A > m_B$)
(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$ (D) $T_A = T_B$

ANSWER KEY

EXERCISE-I

Q.1 $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$ Q.2 $-\frac{3Gm^2}{a}$ Q.3 $\frac{G\ m^2}{3\ L^2}$ Q.4 (i) $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, (ii) $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

Q.5 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from west to east

Q.6 1×10^5J Q.7 $\frac{81}{19}R$ Q.8 $\frac{2G\lambda}{R}(\sin\alpha), (-G\lambda\ 2\alpha)$ Q.9 $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$

Q.10 $h=\frac{\sqrt{5}-1}{2}R$ Q.11 (a) $-GmM_e/r$, (b) $-2GmM_e/r$ Q.12 $\frac{R_e k^2}{1-k^2}$

Q.13 $t=\frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$ Q.14 $\frac{-4GM^2}{L}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

Q.15 (i) $-\frac{3Gm}{R}\left(\frac{m}{\sqrt{3}}+M\right)$, (ii) $\sqrt{\frac{G}{R}\left(\frac{m}{\sqrt{3}}+M\right)}$ Q.16 R

Q.17 $\vec{g}=+\frac{\pi G\rho_0 R^3}{6}\left[\frac{1}{\left(x-\frac{R}{2}\right)^2}-\frac{8}{x^2}\right]\hat{i}$, $\vec{g}=-\frac{2\pi G\rho_0 R}{3}\hat{i}$ Q.18 (a) $\sqrt{\frac{GM}{r}}$; (b) $r\sqrt{2}$; (c) $\sqrt{\frac{2GM}{r}}$

Q.19 (b) 1.9 h Q.20 $2\times\sqrt{R^3/GM}$

EXERCISE-II

Q.1 $\frac{2\pi R^{3/2}(6\sqrt{6})}{\sqrt{GM}\,(2\sqrt{2}+3\sqrt{3})}$ Q.2 $\frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$ Q.3 $\frac{Gm\lambda}{2d^2}$ Q.4 $\sqrt{\frac{2}{3}\pi G\rho R^2}$

Q.5 $T=5\sqrt{\frac{5\pi}{3G\rho}}$ Q.7 (a) $T=4\pi\sqrt{\frac{r^3}{Gm}}$, (b) $v=\sqrt{\frac{4Gm}{r}}$ Q.8 $\sqrt{6.4}$km

Q.9 $\cos\theta=\frac{3}{\sqrt{10}}$ Q.11 $\sqrt{\frac{GM}{R}}\left[\sqrt{\frac{2}{3}}-\sqrt{\frac{8}{15}}\right]$

Q.12 **(a)** $h=\left(\frac{\sqrt{7}}{2}+1\right)R$, **(b)** 1.13R Q.13 $T=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$

EXERCISE-III

Q.1 B Q.2 $v_{min}=\frac{3}{2}\sqrt{\frac{5GM}{a}}$ Q.3 C Q.4 $T=3\times10^{-2}$ N

Q.5 A Q.6 h = 99R Q.7 D