Sample Paper–1
Class 11, Mathematics

Time: 3 hours **Max. Marks 100**

General Instructions

1. All questions are compulsory.
2. Use of calculator is not permitted. However you may use log table, if required.
3. Q.No. 1 to 12 are of very short answer type questions, carrying 1 mark each.
4. Q.No.13 to 28 carries 4 marks each.
5. Q.No. 29 to 32 carries 6 marks each.

---

1. What universal set (s) would you propose for each of the following:
   (i) The set of right triangles
   (ii) The set of isosceles triangles

2. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

3. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by
   {(a, b): a, b ∈ A, b is exactly divisible by a}.
   (i) Write R in roster form
   (ii) Find the domain of R
   (iii) Find the range of R.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm

5. Find the value of the trigonometric function sin 765°

6. Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

7. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

8. Name the octants in which the following points lie: (1, 2, 3), (4, –2, 3),

9. Evaluate the Given limit:
   $$\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

10. Evaluate the Given limit:
    $$\lim_{x \to 4} \frac{4x + 3}{x - 2}$$

11. An experiment consists of recording boy-girl composition of families with 2 children.
    (i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?
    (ii) What is the sample space if we are interested in the number of girls in the family?

12. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:
A: the sum is greater than 8, B: 2 occurs on either die
C: The sum is at least 7 and a multiple of 3.
Which pairs of these events are mutually exclusive?

13. Show that for any sets A and B: $A = (A \cap B) \cup (A - B)$

14. Find the range of each of the following functions: $f(x) = x^2 + 2$, x, is a real number.

15. Prove that: $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

16. Prove the following by using the principle of mathematical induction for all $n \in N$:
$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+...+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

17. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

18. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

19. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

20. How many chords can be drawn through 21 points on a circle?

21. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

22. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

23. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m.

24. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

25. A point R with x-coordinate 4 lies on the line segment joining the points P (2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax^2 + \sin x)(p + q\cos x)$

27. Given below are two statements

p: 25 is a multiple of 5.

q: 25 is a multiple of 8.

Write the compound statements connecting these two statements with "And" and "Or". In both cases check the validity of the compound statement.

28. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

29. In a group of students 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

30. Prove that:

$$(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\frac{x-y}{2}$$

31. Find the sum of the following series up to n terms:

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$$

32. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12.

Solutions

1. **(i)** For the set of right triangles, the universal set can be the set of triangles or the set of polygons.
   (ii) For the set of isosceles triangles, the universal set can be the set of triangles or the set of polygons or the set of two-dimensional figures.

2. G = {7, 8} and H = {5, 4, 2}
   We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as
   P × Q = {(*p*, *q*): *p*∈ P, *q* ∈ Q}
   ∴G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
   H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

3. A = {1, 2, 3, 4, 6}, R = {(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}
   (i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
   (ii) Domain of R = {1, 2, 3, 4, 6}
   (iii) Range of R = {1, 2, 3, 4, 6}

4. We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ radian at the centre, then
   $$\theta = \frac{l}{r}$$
   Therefore, forr = 100 cm, l = 22 cm, we have
   $$\theta = \frac{22}{100}\text{ radian} = \frac{180}{\pi} \times \frac{22}{100}\text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100}\text{ degree}$$
   $$= \frac{126}{10}\text{ degree} = 12\frac{3}{5}\text{ degree} = 12°36' \qquad [1° = 60']$$
   Thus, the required angle is 12°36′.

5. It is known that the values of sin *x* repeat after an interval of 2π or 360°.
   $$\therefore \sin 765° = \sin(2 \times 360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$$

6.
   $$i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3}$$
   $$= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3$$
   $$= 1 \times i + 1 \times (-i) \qquad [i^4 = 1,\ i^3 = -i]$$
   $$= i + (-i)$$
   $$= 0$$

7. 3-digit numbers have to be formed using the digits 1 to 9.
Here, the order of the digits matters.
Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digit numbers $= {}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!}$

$= \frac{9\times 8\times 7\times 6!}{6!} = 9\times 8\times 7 = 504$

8. The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.
The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, –2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

9.

$$\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$$

At $z = 1$, the value of the given function takes the form $\frac{0}{0}$.

Put $z^{\frac{1}{6}} = x$ so that $z \to 1$ as $x \to 1$.

$$\text{Accordingly, } \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1} = \lim_{x\to 1}\frac{x^2-1}{x-1}$$

$$= \lim_{x\to 1}\frac{x^2-1^2}{x-1}$$

$$= 2.1^{2-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= 2$$

$$\therefore \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1} = 2$$

10. $\lim_{x\to 4}\frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$

11. (i) When the order of the birth of a girl or a boy is considered, the sample space is given by S = {GG, GB, BG, BB}
(ii) Since the maximum number of children in each family is 2, a family can either have 2 girls or 1 girl or no girl. Hence, the required sample space is S = {0, 1, 2}

12. When a pair of dice is rolled, the sample space is given by

$S=\{(x,y):x,y=1,2,3,4,5,6\}$

$$=\left\{\begin{matrix}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6)\\(2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6)\\(3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6)\\(4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6)\\(5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6)\\(6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{matrix}\right\}$$

Accordingly,

$A=\{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$B=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(1,2),(3,2),(4,2),(5,2),(6,2)\}$

$C=\{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

It is observed that

$A \cap B = \Phi$

$B \cap C = \Phi$

$C\cap A=\{(3,6),(4,5),(5,4),(6,3),(6,6)\}\neq\phi$

Hence, events A and B and events B and C are mutually exclusive.

13. To show: $A = (A \cap B) \cup (A - B)$

Let $x \in A$

We have to show that $x \in (A \cap B) \cup (A - B)$

Case I

$x \in A \cap B$

Then, $x \in (A \cap B) \subset (A \cup B) \cup (A - B)$

Case II

$x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\therefore x \notin B$ $[x \notin A]$

$\therefore x \notin A - B \subset (A \cup B) \cup (A - B)$

$\therefore A \subset (A \cap B) \cup (A - B)$ … (1)

It is clear that

$A \cap B \subset A$ and $(A - B) \subset A$

$\therefore (A \cap B) \cup (A - B) \subset A$ … (2)

From (1) and (2), we obtain

$A = (A \cap B) \cup (A - B)$

14. $f(x) = x^2 + 2$, x, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

15. L.H.S.

= (sin $3x$ + sin x) sin x + (cos $3x$ – cos x) cos x

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= \cos 3x \cos x + \sin 3x \sin x - \left(\cos^2 x - \sin^2 x\right)$

$= \cos(3x - x) - \cos 2x \qquad \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$

$= \cos 2x - \cos 2x$

$= 0$

= RH.S.

16. Let the given statement be P(n), i.e.,

$P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

For n = 1, we have

$P(1): \frac{1}{3.5} = \frac{1}{3(2.1+3)} = \frac{1}{3.5}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$P(k): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \qquad \ldots (1)$

We shall now prove that P(k + 1) is true.

Consider

$$\left[\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}\right]+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$$

$$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)} \qquad [\text{Using }(1)]$$

$$=\frac{1}{(2k+3)}\left[\frac{k}{3}+\frac{1}{(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{3(2k+5)}\right]$$

$$=\frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$$

$$=\frac{(k+1)}{3\{2(k+1)+3\}}$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

17. Let $\alpha = a + ib$ and $\beta = x + iy$

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2+y^2}=1$$

$$\Rightarrow x^2+y^2=1 \qquad \ldots \text{(i)}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$=\frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$$

$$=\frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$=\frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$=\frac{\sqrt{\left(x^2+y^2\right)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2\left(x^2+y^2\right)+b^2\left(y^2+x^2\right)-2ax-2by}}$$

$$=\frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad \left[\text{Using } (1)\right]$$

$$=1$$

$$\therefore\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

18. Let x litres of 2% boric acid solution is required to be added.
Then, total mixture = $(x + 640)$ litres
This resulting mixture is to be more than 4% but less than 6% boric acid.
$\therefore 2\%x + 8\%$ of $640 > 4\%$ of $(x + 640)$
And, $2\%\ x + 8\%$ of $640 < 6\%$ of $(x + 640)$
$2\%x + 8\%$ of $640 > 4\%$ of $(x + 640)$

$$\Rightarrow \frac{2}{100}x+\frac{8}{100}(640)>\frac{4}{100}(x+640)$$

$\Rightarrow 2x + 5120 > 4x + 2560$
$\Rightarrow 5120 - 2560 > 4x - 2x$
$\Rightarrow 5120 - 2560 > 2x$
$\Rightarrow 2560 > 2x$
$\Rightarrow 1280 > x$
$2\%\ x + 8\%$ of $640 < 6\%$ of $(x + 640)$

$$\frac{2}{100}x+\frac{8}{100}(640)<\frac{6}{100}(x+640)$$

$\Rightarrow 2x + 5120 < 6x + 3840$
$\Rightarrow 5120 - 3840 < 6x - 2x$
$\Rightarrow 1280 < 4x$
$\Rightarrow 320 < x$
$\therefore 320 < x < 1280$
Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

19. 2 different vowels and 2 different consonants are to be selected from the English alphabet.
Since there are 5 vowels in the English alphabet, number of ways of selecting 2 different vowels from the alphabet = ${}^5C_2=\frac{5!}{2!3!}=10$
Since there are 21 consonants in the English alphabet, number of ways of selecting 2 different consonants from the alphabet $={}^{21}C_2=\frac{21!}{2!19!}=210$
Therefore, number of combinations of 2 different vowels and 2 different consonants $=10\times210=2100$
Each of these 2100 combinations has 4 letters, which can be arranged among themselves in 4! ways.
Therefore, required number of words = $2100 \times 4! = 50400$

20. For drawing one chord on a circle, only 2 points are required.
To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.
Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.
Thus, required number of chords = ${}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21\times 20}{2} = 210$

21. In order to prove that $(a - b)$ is a factor of $(a^n - b^n)$, it has to be proved that
$a^n - b^n = k(a - b)$, where k is some natural number
It can be written that, $a = a - b + b$

$$\therefore a^n = (a-b+b)^n = \left[(a-b)+b\right]^n$$
$$= {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + {}^nC_nb^n$$
$$= (a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + b^n$$
$$\Rightarrow a^n - b^n = (a-b)\left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$$
$$\Rightarrow a^n - b^n = k(a-b)$$
where, $k = \left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$ is a natural number

This shows that $(a - b)$ is a factor of $(a^n - b^n)$, where n is a positive integer.

22. Let the two numbers be a and b.
G.M. = $\sqrt{ab}$
According to the given condition,
$$a + b = 6\sqrt{ab} \quad \ldots(1)$$
$$\Rightarrow (a+b)^2 = 36(ab)$$
Also,
$$(a-b)^2 = (a+b)^2 - 4ab = 36ab - 4ab = 32ab$$
$$\Rightarrow a - b = \sqrt{32}\sqrt{ab}$$
$$= 4\sqrt{2}\sqrt{ab} \quad \ldots(2)$$
Adding (1) and (2), we obtain
$$2a = \left(6+4\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow a = \left(3+2\sqrt{2}\right)\sqrt{ab}$$
Substituting the value of a in (1), we obtain
$$b = 6\sqrt{ab} - \left(3+2\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow b = \left(3-2\sqrt{2}\right)\sqrt{ab}$$
$$\frac{a}{b} = \frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

23. The equations of the given lines are

$y = 3x + 1$ … (1)

$2y = x + 3$ … (2)

$y = mx + 4$ … (3)

Slope of line (1), $m_1 = 3$

Slope of line (2), $m_2 = \frac{1}{2}$

Slope of line (3), $m_3 = m$

It is given that lines (1) and (2) are equally inclined to line (3). This means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\therefore \left|\frac{m_1 - m_3}{1 + m_1 m_3}\right| = \left|\frac{m_2 - m_3}{1 + m_2 m_3}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{1-2m}{m+2}\right|$$

$$\Rightarrow \frac{3-m}{1+3m} = \pm\left(\frac{1-2m}{m+2}\right)$$

$$\Rightarrow \frac{3-m}{1+3m} = \frac{1-2m}{m+2} \text{ or } \frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$$

If $\frac{3-m}{1+3m} = \frac{1-2m}{m+2}$, then

$$(3-m)(m+2) = (1-2m)(1+3m)$$

$$\Rightarrow -m^2 + m + 6 = 1 + m - 6m^2$$

$$\Rightarrow 5m^2 + 5 = 0$$

$$\Rightarrow \left(m^2 + 1\right) = 0$$

$$\Rightarrow m = \sqrt{-1}, \text{ which is not real}$$

Hence, this case is not posible.

If $\frac{3-m}{1+3m}=-\left(\frac{1-2m}{m+2}\right)$, then

$$\Rightarrow(3-m)(m+2)=-(1-2m)(1+3m)$$

$$\Rightarrow -m^2+m+6=-\left(1+m-6m^2\right)$$

$$\Rightarrow 7m^2-2m-7=0$$

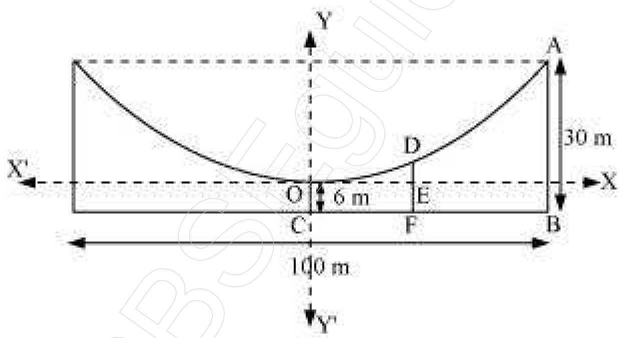
$$\Rightarrow m=\frac{2\pm\sqrt{4-4(7)(-7)}}{2(7)}$$

$$\Rightarrow m=\frac{2\pm 2\sqrt{1+49}}{14}$$

$$\Rightarrow m=\frac{1\pm 5\sqrt{2}}{7}$$

Thus, the required value of *m* is $\frac{1\pm 5\sqrt{2}}{7}$.

24. The vertex is at the lowest point of the cable. The origin of the coordinate plane is taken as the vertex of the parabola, while its vertical axis is taken along the positive *y*-axis. This can be diagrammatically represented as



Here, AB and OC are the longest and the shortest wires, respectively, attached to the cable.
DF is the supporting wire attached to the roadway, 18 m from the middle.

Here, AB = 30 m, OC = 6 m, and $\mathrm{BC}=\frac{100}{2}=50\text{ m}$.
The equation of the parabola is of the form $x^2 = 4ay$ (as it is opening upwards).
The coordinates of point A are (50, 30 – 6) = (50, 24).
Since A (50, 24) is a point on the parabola,

$$(50)^2=4a(24)$$

$$\Rightarrow a=\frac{50\times 50}{4\times 24}=\frac{625}{24}$$

∴Equation of the parabola, $x^2=4\times\frac{625}{24}\times y$ or $6x^2 = 625y$
The *x*-coordinate of point D is 18.
Hence, at $x = 18$,

$6(18)^2 = 625y$

$\Rightarrow y = \frac{6 \times 18 \times 18}{625}$

$\Rightarrow y = 3.11 \text{ (approx)}$

∴DE = 3.11 m

DF = DE + EF = 3.11 m + 6 m = 9.11 m

Thus, the length of the supporting wire attached to the roadway 18 m from the middle is approximately 9.11 m.

25. The coordinates of points P and Q are given as P (2, –3, 4) and Q (8, 0, 10).
Let R divide line segment PQ in the ratio *k*:1.
Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the *x*-coordinate of point R is 4.

$\therefore \frac{8k+2}{k+1} = 4$

$\Rightarrow 8k+2 = 4k+4$

$\Rightarrow 4k = 2$

$\Rightarrow k = \frac{1}{2}$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

26. Let $f(x) = (ax^2 + \sin x)(p + q\cos x)$
By product rule,

$$f'(x) = (ax^2 + \sin x)\frac{d}{dx}(p + q\cos x) + (p + q\cos x)\frac{d}{dx}(ax^2 + \sin x)$$

$$= (ax^2 + \sin x)(-q\sin x) + (p + q\cos x)(2ax + \cos x)$$

$$= -q\sin x(ax^2 + \sin x) + (p + q\cos x)(2ax + \cos x)$$

27. The compound statement with ‘And’ is “25 is a multiple of 5 and 8”.
This is a false statement, since 25 is not a multiple of 8.
The compound statement with ‘Or’ is “25 is a multiple of 5 or 8”.
This is a true statement, since 25 is not a multiple of 8 but it is a multiple of 5.

28. Let E be the event in which the spokesperson will be a male and F be the event in which the spokesperson will be over 35 years of age.

Accordingly, P(E) = $\frac{3}{5}$ and P(F) = $\frac{2}{5}$

Since there is only one male who is over 35 years of age,

$P(E \cap F) = \frac{1}{5}$

We know that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\therefore P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$

Thus, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

29. Let U be the set of all students in the group.
Let E be the set of all students who know English.
Let H be the set of all students who know Hindi.
$\therefore$ H ∪ E = U
Accordingly, n(H) = 100 and n(E) = 50
$n(H \cap E)$ = 25
n(U) = n(H) + $n(E)$ – n(H ∩ E)
= 100 + 50 – 25
= 125
Hence, there are 125 students in the group.

30. L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$

$= 1 + 1 - 2[\cos(x - y)]$ $\qquad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= 2[1 - \cos(x - y)]$

$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right]$ $\qquad [\cos 2A = 1 - 2\sin^2 A]$

$= 4\sin^2\left(\frac{x - y}{2}\right) = \text{R.H.S.}$

31. The n^{th} term of the given series is $\frac{1^3 + 2^3 + 3^3 + ... + n^3}{1 + 3 + 5 + ... + (2n - 1)} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{1 + 3 + 5 + ... + (2n - 1)}$

Here, $1, 3, 5, \ldots (2n-1)$ is an A.P. with first term a, last term $(2n-1)$ and number of terms as n

$$\therefore 1+3+5+\ldots\ldots+(2n-1)=\frac{n}{2}\left[2\times1+(n-1)2\right]=n^2$$

$$\therefore a_n=\frac{n^2(n+1)^2}{4n^2}=\frac{(n+1)^2}{4}=\frac{1}{4}n^2+\frac{1}{2}n+\frac{1}{4}$$

$$\therefore S_n=\sum_{K=1}^{n}a_K=\sum_{K=1}^{n}\left(\frac{1}{4}K^2+\frac{1}{2}K+\frac{1}{4}\right)$$

$$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$$

$$=\frac{n\left[(n+1)(2n+1)+6(n+1)+6\right]}{24}$$

$$=\frac{n\left[2n^2+3n+1+6n+6+6\right]}{24}$$

$$=\frac{n\left(2n^2+9n+13\right)}{24}$$

32. **(i)** Number of observations (n) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{20}x_i$$

$$10=\frac{1}{20}\sum_{i=1}^{20}x_i$$

$$\Rightarrow \sum_{i=1}^{20}x_i=200$$

That is, incorrect sum of observations = 200

Correct sum of observations = 200 – 8 = 192

$$\therefore \text{Correct mean}=\frac{\text{Correct sum}}{19}=\frac{192}{19}=10.1$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2$$

$$= 2080 - 64$$

$$= 2016$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct}\sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{106.1 - 102.01}$$

$$= \sqrt{4.09}$$

$$= 2.02$$

(ii) When 8 is replaced by 12,
Incorrect sum of observations = 200
∴ Correct sum of observations = 200 – 8 + 12 = 204

$$\therefore \text{Correct mean} = \frac{\text{Correct sum}}{20} = \frac{204}{20} = 10.2$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2 + (12)^2$$

$$= 2080 - 64 + 144$$

$$= 2160$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct}\sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$= \sqrt{108 - 104.04}$$

$$= \sqrt{3.96}$$

$$= 1.98$$