UNITS AND MEASUREMENTS

Important Points:

1. Measurement of Length:

a) A meter scale is used for lengths from10^{-3}m to 10^{2}m.

b) A Vernier calipers is used for lengths to an accuracy of 10^{-4}m.

c) A screw gauge and a spherometer are used to measure lengths as less as to 10^{-5} m.

d) Large distances such as the distance of a planet or a star from the earth can be measured by parallax method.

1 Parsec = 3.08×10^{16} m

e) To measure a very small size like that of a molecule (10^{-8} m to 10^{-10}m) electron microscope can be used .Its resolution is about $0.6A^{0}$.

1 Fermi = 1 f = 10^{-15}m

2. Measurement of Mass:

The mass of atoms and molecules are expressed in the unified atomic mass unit (u).

1 Unified Atomic Mass Unit = 1.66×10^{-27}kg

3. Measurement of Time:

Cesium clock (or) atomic clock is based on the periodic vibrations of cesium atom.

These clocks are very accurate. A cesium atomic clock at National Physical Laboratory (NPL), New Delhi, is being used to maintain the Indian standard of time.

4. Fundamental Quantities:

A physical quantity which is independent of any other physical quantity is called fundamental quantity.

5. Derived Quantities:

Quantities that are derived from the fundamental quantities are called derived quantities.

6. Dimensions:

Dimensions are the powers to which the fundamental units are to be raised to get one unit of the physical quantity.

7. Dimensional Formula:

Dimensional formula is an expression showing the relation between fundamental and derived quantities.

8. Dimensional Constant:

Constants having dimensional formulae are called dimensional constants.

Ex:- Planck's constant, universal gravitational constant.

9. Dimensionless Quantities:

Quantities having no dimensions are called dimensionless quantities.

Ex: Angle, strain.

10. Numerical value of a physical quantity is inversely proportional to its unit.

$$N \propto \frac{1}{U} \Rightarrow \quad N_1 U_1 = N_2 U_2$$

11. Principle of Homogeneity:

Quantities having same dimensions can only be added or subtracted or equated.

12. Uses of Dimensional Formulae:

Dimensional formulae can be used-

a) To check the correctness of the formula or an equation.

b) To convert one system of units into another system.

c) To derive the relations among different physical quantities.

13. Limitations of Dimensional Methods:

1) These cannot be used for trigonometric, exponential and logarithmic functions.

2) These cannot be used to find proportionality constants.

3) If an equation is the sum or difference of two or more quantities, then these methods are not applicable.

4) If any side of the equation contains more than three variables, then these methods are not applicable.

14. Accuracy:

It is the closeness of the measured values to the true value.

15. Precision:

It is the closeness of the measured values with each other.

16. Errors:

The uncertainty in a measurement is called '**Errors**'. **Or**

It is difference between the measured and the true values of a physical quantity.

17. Types of Errors:

Errors in measurement can be broadly classified as

a) Systematic Errors   b) Random Errors

18. Systematic Errors:

a) Instrumental Errors: It arises from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument.

b) Imperfection in experimental technique or procedure.

19. Random Errors:

These errors are due to random and unpredictable fluctuations in experimental conduction, personal errors by the observer.

20. Least Count Errors:

It is the error associated with the resolution of the instrument.

21. Significant Figures:

Significant figures in a measurement are defined as the number of digits that are known reliably plus the uncertain digit.

22. Rules for determining the number of significant figures:

1. All non - zero digits are significant.

E.g. Number of SF in 9864, 9.864, 98.64 is 4.

2. All zeros occurring between two non-zero digits are significant.

E.g. Number of SF in 1.0605, 106.05; 1.0605 is 5.

3. All the zeros to the right of the decimal point but to the left of the first non zero digit are not significant.

E.g. Number of SF in 0.0203 is 3

4. All zeros to the right of the last non zero digit in a number after the decimal point are significant.

E.g. Number of SF in 0.020 is 2.

5. All zeros to the right of the last non zero digit in a number having no decimal point are not significant

E.g. Number of SF in 2030 is 3.

23. Rounding Off:

The process of omitting the non significant digits and retaining only the desired number of significant digits, incorporating the required modifications to the last significant digit is called 'Rounding Off The Number'.

24. Rules for rounding off Numbers:

1. The preceding digit is raised by 1 if the immediate insignificant digit to be dropped is more than 5.

E.g.: 4928 is rounded off to three significant figures as 4930.

2. The preceding digit is to be left unchanged if the immediate insignificant digit to be dropped is less than 5.

E.g. 4728 is rounded off to two significant figures as 4700.

3. If the immediate insignificant digit to be dropped is 5 then

a) If the preceding digit is even, it is to be unchanged and 5 is dropped

E.g. 4.728 is to be rounded off to two decimal places as 4.72.

b) If the preceding digit is odd, it is to be raised by 1.

E.g. 4.7158 is rounded off to two decimal places as 4.72.

25. Estimation of Errors:

a) Absolute error $= |\text{True value - }M\text{easured value}|$

$|\Delta a_i| = |a_{mean} - a_i|$ Absolute error is always positive.

b) Mean absolute Error $\left(|a_{mean}|\right)$ **:**

Mean absolute error $\Delta a_{mean} = \dfrac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots\ldots.. + |\Delta a_n|}{n}$

Mean absolute error is always positive.

c) Relative Error:

Relative error = $\dfrac{\Delta a_{mean}}{a_{mean}}$ Relative error has no units.

d) Percentage Error (δ_a) **:** $\delta_a = \left(\dfrac{\Delta a_{mean}}{a_{mean}} \times 100\right)\%$

26. Combination of Errors:

a) Error of a sum or a difference:

(i) If $x = a + b$

Maximum possible value of $\Delta x = \Delta a + \Delta b$

Relative error $\dfrac{\Delta x}{x} = \dfrac{\Delta a + \Delta b}{a + b}$

Percentage error, $\dfrac{\Delta x}{x}\% = \left[\left(\dfrac{\Delta a + \Delta b}{a + b}\right)100\right]\%$

(ii) Let $x = a - b$

Maximum possible value of $\Delta x = \Delta a + \Delta b$

Relative error $\dfrac{\Delta x}{x} = \dfrac{\Delta a + \Delta b}{a - b}$

Percentage error, $\dfrac{\Delta x}{x}\% = \left[\left(\dfrac{\Delta a + \Delta b}{a - b}\right)100\right]\%$

b) Errors of Multiplication or Division:

(i) Let $x = a\,b$

Maximum relative error, $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

Percentage error, $\frac{\Delta x}{x}\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)100\%$

(ii) Let $x = \frac{a}{b}$

Maximum relative error $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

Percentage error $\frac{\Delta x}{x}\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)100\%$

27. Let $x = \frac{a^p b^q}{c^r}$

Maximum relative error $\frac{\Delta x}{x} = p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta c}{c}$

Percentage error $\frac{\Delta x}{x}\% = \left(p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta c}{c}\right)100\%$

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Physical Quantity	Units	D.F.
Velocity (V)	$m\,s^{-1}$	LT^{-1}
Acceleration (A)	ms^{-2}	LT^{-2}
Linear momentum (P)	$Kg.ms^{-1}$	MLT^{-1}
Force (F)	N	MLT^{-2}
Impulse (J)	N.s	MLT^{-1}

Work (w)	Joule	ML^2T^{-2}
Angle (θ)	rad	Dimensionless
Angular velocity	rad s^{-1}	T^{-1}
Angular acceleration	rad s^{-2}	T^{-2}
Angular Momentum (L)	$Kg.m^2s^{-1}$	ML^2T^{-1}
Moment of inertia (I)	$Kg.m^2$	ML^2
Torque (τ)	N.m	ML^2T^{-2}
Power (P)	Watt	ML^2T^{-3}
Pressure (P)	Pascal	$ML^{-1}T^{-2}$
Gravitational Constant (G)	$N.m^2kg^{-2}$	$M^{-1}L^3T^{-2}$
Stress	Nm^{-2}	$ML^{-1}T^{-2}$
Strain	No Unit	Dimensionless
Elastic Constants	Pascal	$ML^{-1}T^{-2}$
Spring Constant (K)	N/m	MT^{-2}
Surface Tension (T)	N/m	MT^{-2}
Coefficient of Viscosity (η)	$\frac{N.s}{m^2}$	$ML^{-1}T^{-1}$
Expansion Coefficients	K^{-1}	K^{-1}
Universal Gas Constant (R)	J/mol/K	$ML^2T^{-2}\ mol^{-1}K^{-1}$
Boltzmann Constant (K)	J/K	$ML^2T^{-2}K^{-1}$
Specific Heat (C)	J/Kg/K	$L^2T^{-2}K^{-1}$

Latent Heat (L)	J/K	L^2T^{-2}
Thermal Conductivity (K)	W/m/K	$MLT^{-3}K^{-1}$
Stefan's Constant (a)	$\frac{W}{m^2 \times K^4}$	$MT^{-3}K^{-4}$

Very Short Answer Questions

1. Distinguish between accuracy and precision?

A:

Accuracy	Precision
1) It is the closeness of the measured value to the true value.	1) It is the closeness of the measured values of repeated measurements.
2) It depends on the minimization of errors in the measurement.	2) It depends on the least count of the measuring instrument.

2. What are the different types of errors that can occur in a measurement?

A: The errors can be broadly classified as

a) Systematic Errors b) Random Errors c) Gross Errors.

3. How systematic errors be minimised or eliminated?

A: Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

4. Illustrate how the result of a measurement is to be reported indicating the error involved?

A: As every measurement contains errors, the result of a measurement is to be reported in such a manner to indicate the precision of measurement.

Suppose the length of an object is measured using a meter scale with a least count 0.1 cm .If the measured length is 62.5 cm, it is to be recorded as $(62.5 \pm 0.1)cm$ stating the limits of errors.

5. What are significant figures and what do they represent when reporting the result of a measurement?

A: The digits of a number (representing a measurement) that are definitely known plus one more digit (added at the end) that is estimated are called significant digits or significant figures.

In reporting the result of a measurement using significant figures clearly shows the precision of the instrument.

6. Distinguish between fundamental units and derived units?

A: **Fundamental Units:**

The units of fundamental quantities like length, mass, time etc. are called fundamental units.

Derived Units:

The units of derived quantities like velocity, force etc. is called as derived units.

7. Why do we have different units for the same physical quantity?

A: The order of magnitude of a measurable physical quantity differs significantly.

Ex. Inter-atomic distances are of the order of angstroms and interstellar distances are of the order of light years.

8. What is dimensional analysis?

A: Physical quantities having the same dimensions can only be added or subtracted. Dimensional analysis helps us in deducing certain relations among different physical quantities and checking the derivation, accuracy and dimensional consistency or homogeneity of various mathematical expressions.

9. How many orders of magnitude greater is the radius of the atom as compared to that of the nucleus?

A: The radius of a nucleus is of order of $10^{-15} m$ and that of the atom is nearly $10^{-10} m$. The radius of the atom is greater by 10^5 times as compared to that of the nucleus.

10. Express unified atomic mass unit in kg?

A: $1u = 1.66 \times 10^{-27} kg$

Short Answer Questions

1) The vernier scale of an instrument has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, then using this instrument what would be the minimum inaccuracy in the measurement of distance?

A. $Least\ count = \frac{1\ Main\ scale\ division}{Number\ of\ vernier\ scale\ divisions}$

Least count $= \frac{0.5 mm}{50} = 0.01$ mm.

Hence the minimum inaccuracy in the measurement is 0.01 mm.

2. In a system of units the unit of force is 100N, unit of length is 10m and the unit of time is 100s. What is the unit of mass in this system?

A: F = 100N, L=10m and T=100s

Force $F = MLT^{-2}$ Or $M = FL^{-1}T^2$

$\therefore M = (100\,N)(10\,m)^{-1}(100\,s)^2 = 10^5\ Kg$

3. The distance of a galaxy from Earth is of the order of $10^{25}\,m$. Calculate the order of magnitude of the time taken by light to reach us from the galaxy?

A. $d = 10^{25}\,m$ and $C = 3\times10^{8}\,ms^{-1}$

The time taken by light to reach us from that galaxy is

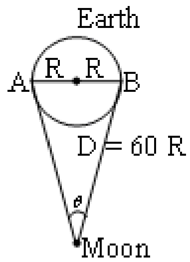
$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{10^{25}}{3\times10^{8}} = 3.3\times10^{16}\,s$$

The time taken is in the order of $10^{16}\,s$.

4. The Earth-Moon distance is about 60 Earth radius. What will be the approximate diameter of the Earth as seen from the Moon?

A. Let the radius of Earth be R, then the distance of Earth from the Moon is given by 60 R

Let θ be the angular diameter of Earth as seen from the Moon.



Diameter AB of earth, b = 2R

Distance of the earth as seen from the moon, $\theta = \frac{2R}{60R} = \frac{1}{30}\,rad$

$$\therefore \theta = \frac{1}{30}\times\frac{180^\circ}{\pi} = \frac{6^\circ}{\pi} = \frac{6}{3.142} = 1.9^\circ$$

5. Three measurements of the time for 20 oscillations of a pendulum are given as $t_1 = 39.6\,s$, $t_2 = 39.9\,s$ and $t_3 = 39.5\,s$. What is the precision in the measurements? What is the accuracy of the measurements?

A. $t_{mean} = \frac{t_1 + t_2 + t_3}{3} = \frac{39.6 + 39.9 + 39.5}{3} = 39.66 \approx 39.7\,s$

Thus the instrument is precise up to 0.1s.

As 39.6 sec is closer to the mean value, it is more accurate.

6. A calorie is a unit of heat or energy and it equals about 4.2 J where $1J = 1kg\ m^2 s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals βm, the unit of time is γs. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-1}\gamma^2$ in terms of the new units?

A: 1Calorie = 4.2J = 4.2 Kgm^2S^{-2}

If 1 Calorie = n_2 new units

$$n_2 = 4.2\left[\frac{kg}{\alpha kg}\right]\left[\frac{m}{\beta m}\right]^2\left[\frac{s}{\gamma s}\right]^{-2} = 4.2\alpha^{-1}\beta^{-2}\gamma^2 \text{ new units}$$

$\therefore$ 1 Calorie $= 4.2\alpha^{-1}\beta^{-2}\gamma^2$

7. A new unit of length is chosen so that the speed of light in vacuum is 1 unit. If light takes 8 min and 20 s to cover this distance, what is the distance between the Sun and Earth in terms of the new unit?

A: The time taken by light to travel from the sun to the earth is = 500 s

The distance between the sun and earth in the new system,

(S) = Ct = 1$\times$ 500 = 500 New units.

8. A student measures the thickness of a human hair using a microscope of magnification 100. He makes 20 observations and finds that the average thickness (as viewed in the microscope) is 3.5 mm. What is the estimate of the thickness of hair?

A. Magnification of microscope = 100

Average thickness of human hair observed through microscope over

20 observation = 3.5 mm $= 3.5\times10^{-3} m$

According to the definition of magnification,

Magnification $= \frac{observed\ thickness}{Real\ thickness}$

$$100 = \frac{3.5\times10^{-3}}{Real\ thickness}$$

$$\therefore Real\ thickness = \frac{3.5\times10^{-3}}{100} = 35\,\mu m$$

9. A physical quantity X is related to four measureable quantities a, b, c and d as follows: $X = a^2\ b^3\ c^{5/2}\ d^{-2}$. The percentage error in the measurement of a,b,c and d are 1%,2%,3% and 4%. What is the percentage error in X?

A: $X = a^2\ b^3\ c^{5/2}\ d^{-2}$

The maximum percentage error in X is

$$\frac{\Delta X}{X}\times100 = 2\left(\frac{\Delta a}{a}\times100\right)+3\left(\frac{\Delta b}{b}\times100\right)+\frac{5}{2}\left(\frac{\Delta c}{c}\times100\right)+2\left(\frac{\Delta d}{d}\times100\right)$$

$$\therefore \frac{\Delta X}{X}\times100 = 2(1\%)+3(2\%)+\frac{5}{2}(3\%)+2(4\%) = 23.5\%$$

10. The velocity of a body is given by $v = At^2 + Bt + C$. If v and t are expressed in SI, what are the units of A, B and C?

A. Velocity $v = At^2 + Bt + C$

Dimensional formula of velocity $v = LT^{-1}$

From the principle of homogeneity,

$$LT^{-1} = AT^2 = BT = C$$

$\therefore$ The unit of $A = LT^{-3}$ is metre / sec^3

The unit of $B = LT^{-2}$ is metre / sec^2

The unit of $C = LT^{-1}$ is metre / sec

Problems

1) In the expression $P = E L^2 m^{-5} G^{-2}$ the quantities E, l, m and G denote energy angular momentum, mass and gravitational constant respectively. Show that P is a dimensionless quantity?

A: $P = E L^2 m^{-5} G^{-2}$

D.F of $E \to ML^2T^{-2}$

D.F. of $l \to ML^2T^{-1}$

D.F. of $m \to M$

D.F. of $G \to M^{-1}L^3T^{-2}$

Now D.F. of $P = \left(ML^2T^{-2}\right)\left(ML^2T^{-1}\right)^2 M^{-5}\left(M^{-1}L^3T^{-2}\right)^{-2}$

$= M^{(1+2-5+2)} L^{(2+4-6)} T^{(-2-2+4)} = M^0 L^0 T^0$

Hence 'P' is a dimensionless quantity

2) If the velocity of light c, Plank's constant h and the gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of dimensions of these quantities?

A. Dimensions of, $c = LT^{-1}$; $G = M^{-1}L^3T^{-2}$ and $h = ML^2T^{-1}$

(a) $M^2 = \frac{hc}{G} \Rightarrow M = \sqrt{\frac{hc}{G}}$

(b) $L^2 = \frac{G\,h}{c^3} \Rightarrow L = \sqrt{\frac{Gh}{c^3}}$

(c) $T^2 = \frac{G\,h}{c^5} \Rightarrow T = \sqrt{\frac{G\,h}{c^5}}$

3) An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's Third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional analysis, that $T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$, **where k is a dimensionless constant and g is acceleration due to gravity?**

A: $T^2 \alpha\, r^3 \Rightarrow T \alpha\, r^{3/2}$

T is function of g and $R \Rightarrow T\ \alpha\ g^x R^y$

$$\therefore \left[L^0 M^0 T^1\right] = \left[L^{3/2} M^0 T^0\right]\left[L^1 M^0 T^{-2}\right]^x \left[L^1 M^0 T^0\right]^y \Rightarrow T \propto r^{3/2} g^x . R^y$$

Comparing the powers, $x = -\frac{1}{2}$ and $y = -1$

$$\therefore T = kr^{3/2} g^{-1/2} R^{-1} = \frac{k}{R}\sqrt{\frac{r^3}{g}}$$

4) State the number of significant figures in the following:

(a) 6729 (b) 0.024 (c) 0.08240 (d) 6.032 (e) 4.57×10^8

A: (a) 4 (b) 2 (c) 4 (d) 4 (e) 3

5) A stick has a length of 12.132 cm and another has a length of 12.4 cm. If the two sticks are placed end to end what is the total length? If the two sticks are placed side by side, what is the difference in their lengths?

A. (a) Total length $L = (12.132 + 12.4)cm$

Or $L = (12.13 + 12.4)cm$

$\therefore L = 12.53cm$

(b) Difference in the length $L = 12.4 - 12.132$

Or $L = 12.4 - 12.13$

$\therefore L = 0.3cm$

6) Each side of a cube is measured to be 7.203 m. What is (i) The total surface area and

(ii) The volume of the cube, to appropriate significant figures?

A: Side of the cube = 7.203 m

The number of significant figures in the measured length is 4. The calculated area and the volume should be rounded off to 4 significant figures.

Surface area of the cube $=6(7.203)^2\, m^2 = 311.299254\, m^2 = 311.3\, m^2$

Volume of the cube $=(7.203)^3\, m^3 = 373.714754\, m^3 = 373.7\, m^3$

7) The measured mass and volume of a body are 2.42 g and 4.7 cm^3 respectively with possible errors 0.01 g and 0.1 cm^3. Find the maximum error in density.

A: Density $=\dfrac{mass}{volume}$

Maximum percentage error in density $=\dfrac{\Delta m}{m}\times 100+\dfrac{\Delta \upsilon}{\upsilon}\times 100=\left(\dfrac{0.01}{2.42}\times 100+\dfrac{0.1}{4.7}\times 100\right)=2\%$

8) The error in measurement of radius of a sphere is 1%. What is the error in the measurement of volume?

A. We know that, $V \propto r^3$

$$\therefore \frac{\Delta V}{V}\times 100 = 3\frac{\Delta r}{r}\times 100 = 3(1) = 3\%$$

9) The percentage error in the mass and speed are 2% and 3% respectively. What is the maximum error in kinetic energy calculated using these quantities?

A. $K=\dfrac{1}{2}m\upsilon^2$

$$\frac{\Delta K}{K}\times 100 = \left(\frac{\Delta m}{m}\times 100\right)+2\left(\frac{\Delta \upsilon}{\upsilon}\times 100\right)$$

$$\therefore \frac{\Delta K}{K}\times 100 = 2+2(3) = 8\%$$

10) One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). If the size of the hydrogen molecule is about what is the ratio of molar volume to the atomic volume of a mole of hydrogen?

A: 1 mole of hydrogen contains 6.023×10^{23} atoms.

Molar volume $= 22.4\,lit = 22.4\times 10^{-3} m^3 \left(\because 1\,lit = 10^{-3} m^3\right)$

Atomic volume $= \frac{4}{3}\pi r^3 \times 6.023\times 10^{23}$

But, $r = 0.5\overset{0}{A} = 0.5\times 10^{-10} m$

$$\text{Required ratio} = \frac{22.4\times 10^{-3}}{\frac{4}{3}\times\pi\times\left(0.5\times 10^{-10}\right)^3\times 6.023\times 10^{23}} = 7\times 10^4$$