Sample Paper-04 (solved)
Mathematics
Class – XI

ANSWERS

Section A

1. Solution:

$$\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{7}\right)\right)$$

$$=\sin^{-1}\left(sin\left(\frac{\pi}{7}\right)\right)$$

$$=-\frac{\pi}{2}\le\frac{\pi}{7}\le\frac{\pi}{2}$$

$$=\frac{\pi}{7}$$

2. Solution: $(a,2a)$, $(a,-2a)$

3. Solution:

$$x+7=10$$
$$x=3$$
$$x+y=8$$
$$y=5$$

4. Solution:

$$\frac{3+2i}{1-i}=\frac{3+2i}{1-i}.\frac{1+i}{1-i}$$

$$=\frac{3+2i+3i+2i^2}{1-i^2}=\frac{1}{2}+\frac{5}{2}i$$

$$(1+2i)i-\frac{3+2i}{1-i}=(-2+i)-(\frac{1}{2}+\frac{5}{2}i)=-\frac{5}{2}-\frac{3}{2}i$$

5. Solution:

Domain= $[-1,1]$ Range= $[0,\pi]$

6. Solution :

$$0\le\cos^{-1}x\le\pi$$

$\sin$ *in this interval is positive and hence y is positive*

Section B

7. Solution:

$$\alpha+\beta=b$$
$$\alpha\beta=c$$
$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$
$$=b^2-2c$$

8. Solution:

$$(x+a)^n=P+Q$$
$$(x-a)^n=P-Q$$
$$(P+Q)(P-Q)=(x+a)(x-a)$$
$$P^2-Q^2=(x^2-a^2)^n$$

9. Solution:

Discriminant of Numerator $=9-24<0$ *and*
coefficient of x^2*is positive. Hence Numerator is always positive*
Hence dividing by the numerator on both sides of
the inequlity does not change the the sign of the inequality.
Hence we need only consider $\frac{1}{3x+4}<0$

$$x<\frac{-4}{3}$$
$$x\in(-\infty,-\frac{4}{3})$$

10. Solution:

$$\cot(A+15)-\tan(A-15)=\frac{\cos(A+15)}{\sin(A+15)}-\frac{\sin(A-15)}{\cos(A-15)}$$
$$=\frac{\cos(A+15)\cos(A-15)-\sin(A+15)\sin(A-15)}{\sin(A+15)\cos(A-15)}$$
$$=\frac{\cos 2A}{\frac{1}{2}(\sin 2A+\frac{1}{2})}$$
$$=\frac{2\cos 2A}{\sin 2A+\frac{1}{2}}$$
$$=\frac{4\cos 2A}{1+2\sin 2A}$$

11. Solution :

$4-x^2 \geq 0$

$x^2-4 \leq 0$

$Domain\ of\ x \in [-2,2]$

$y^2 = 4-x^2$

$x^2 = 4-y^2$

$x = \sqrt{4-y^2}$

$4-y^2 \geq 0$

$y^2-4 \leq 0$

$y \in [-2,2]$

$Also\ \ for\ all\ values\ of\ x \in [-2,2]$

$y = \sqrt{4-x^2} \geq 0$

$Range\ y \in [0,2]$

12. Solution :

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\cos\theta = \frac{1-4}{1+4} = \frac{-3}{5}$$

$$\sin\theta = \frac{2\tan\theta 2}{1+\tan^2\frac{\theta}{2}} = \frac{2.2}{1+4} = \frac{4}{5}$$

$$\frac{1}{2+\cos\theta+\sin\theta} = \frac{1}{2-\frac{3}{5}+\frac{4}{5}} = \frac{11}{5}$$

13. Solution:

$$\lim_{x\to 0}\frac{\sin 5x}{x+x^3} = \lim_{x\to 0}\frac{5\sin 5x}{5x(1+x^2)}$$

$$= \lim_{x\to 0}\frac{5\sin 5x}{5x}\lim_{x\to 0}\frac{1}{(1+x^2)}$$

$= 5.1.1$

$= 5$

14. Solution:

$$\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1$$

$$\cos 2x+\cos 4x=0$$

$$2\cos 3x cosx=0$$

$$Cos3x=0$$

$$x=\frac{\pi}{6}+\frac{\pi}{3}n$$

$$Cosx=0$$

$$x=\frac{\pi}{2}+\pi k=\frac{\pi}{6}+\frac{\pi}{3}n \quad n \text{ is integer}$$

15. Solution:

$$i^{30}+i^{40}+i^{60}=(i^4)^7.i^2+(i^4)^{10}+(i^4)^{15}$$

$$i^4=1=-1+1+1=1$$

16. Solution :

Substtuting the points $(0,0)$ *and* $(5,5)$*on the given line*

$$x+y-8=0$$

$$0+0-8=-8$$

$$5+5-8=2$$

Since the signs of the resulting numbers are different
the given points lie onopposite sides of the given line.

17. Solution :

$$\tan^{-1}x=A$$

$$\tan A=x$$

$$\cot^{-1}x=B$$

$$\cot B=x$$

$$\tan(\frac{\pi}{2}-B)=x$$

$$\tan^{-1}x=\frac{\pi}{2}-B$$

$$\tan^{-1}x=A$$

$$A=\frac{\pi}{2}-B$$

$$A+B=\frac{\pi}{2}$$

$$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$$

18. Solution:

$11^{n+2}+12^{2n+1}$ *is divisible by* 133

$n=1$

$11^3+12^3=(11+12)(11^2-11.12+12^2)$

$=23.133$

Let it be true for k

$11^{k+2}+12^{2k+1}$ *is divisible by* 133

For $k=k+1$

$11^{k+3}+12^{2k+3}=11.11^{k+2}+12^2.12^{2k+1}$

$=11.11^{k+2}+144.12^{2k+1}$

$=11.11^{k+2}+133.12^{2k+1}+11.12^{2k+1}$

$=11.11^{k+2}+11.12^{2k+1}+133.12^{2k+1}$

Is divisible by 133*since* $11^{k+2}+12^{2k+1}$ *is divisible by* 133

19. Solution:

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$

$=n(U)-[n(A)+n(B)-n(A\cap B)]$

$800-[200+300-100]$

$=400$

Section C

20. Solution:

$$\frac{1}{\log_a b}=\log_b a$$

$$\frac{1}{\log_{2a} b}=\log_b 2a$$

$$\frac{1}{\log_{4a} b}=\log_b 4a$$

$$\frac{\log_b a+\log_b 4a}{2}=\frac{\log_b (2a)^2}{2}$$

$$=2\frac{\log_b 2a}{2}$$

$$=\log_b 2a$$

Thus , $\frac{1}{\log_{2a} b}$ *is , the , AM , between* $\frac{1}{\log_a b},\frac{1}{\log_{4a} b}$

21. Solution:

Probability of surviving = $\frac{9}{10}$

Required to find out the probability of 4 are safe or 5 are safe

Probability of 5 is safe = $\left(\frac{9}{10}\right)^5$

Probability of 4 is safe = ${}^5C_4\left(\frac{9}{10}\right)^4\frac{1}{10}$

Required Probability = $\left(\frac{9}{10}\right)^5+5\left(\frac{9}{10}\right)^4\frac{1}{10}$ = $\frac{45927}{5000}$

22. Solution:

$$T_{2r+1}={}^{40}C_{2r}$$
$$T_{r+2}={}^{40}C_{r+1}$$
$${}^{40}C_{2r}={}^{40}C_{r+1}$$
$$2r+r+1=40$$
$$3r=39$$
$$r=13$$

23. Solution :

$$y=\log_{10}x$$
$$x=10^y$$
$$\log_e x=y\log_e 10$$
$$y=\frac{\log_e x}{\log_e 10}$$
$$\frac{dy}{dx}=\left(\frac{1}{\log_e 10}\right)\frac{1}{x}$$

24. Solution:

There are 3 even numbers 2, 4, 6

So the units place, 10th places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2\times5p_4=720$

25. Solution:

Let the origin be shifted to (h,k)

$$x = x' + h$$
$$y = y' + k$$

Then

$$(x'+h)^2 + (y'+k)^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + y'^2 + x'(2h-4) + y'(2k+6) + h^2 + k^2 - 4h + 6k - 36 = 0$$
$$2h - 4 = 0$$
$$h = 2$$
$$2k + 6 = 0$$
$$k = -3$$
$$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$$
$$x'^2 + y'^2 + 13 - 62 = 0$$
$$x'^2 + y'^2 = 49$$

26. Solution:

$$\frac{2+4+12+14+11+x+y}{7} = 8$$
$$43 + x + y = 56$$
$$x + y = 13$$
$$\frac{2^2+4^2+12^2+14^2+11^2+x^2+y^2}{7} - (mean)^2 = 19$$
$$\frac{4+16+144+196+121+x^2+y^2}{7} - 64 = 19$$
$$\frac{481+x^2+y^2}{7} = 83$$
$$481 + x^2 + y^2 = 581$$
$$x^2 + y^2 = 100$$
$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$
$$169 + (x-y)^2 = 200$$
$$(x-y)^2 = 31$$
$$x - y = 5.57$$
$$x + y = 13$$
$$x = 9.285$$
$$y = 3.715$$