
Sample Paper-03
Mathematics
Class – XI

ANSWER

Section A

1. Solution:

Domain of is in the open interval $(-2, 2)$

2. Solution:

$$(A \cap B) = \{\emptyset\}$$

3. Solution

Max value is 2

4. Solution: Ellipse

5. Solution

Condition for colinearity is not satisfied here since

$$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$$

6. Solution:

$$b^2 + c^2 - 4ad > 0$$

Section B

7. Solution:

$$y = x^2 - x + 1$$

$$y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

$$f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

8. Solution:

Equation is $8y^2 + 24x - 40y + 134 = 0$

$= 4y^2 + 12x - 20y + 67 = 0$

This can be written as

$y^2 - 5y = -3x - \frac{67}{4}$

$(y - \frac{5}{2})^2 = -3x - \frac{67}{4} + \frac{25}{4} - 3(x + \frac{7}{2})$

Let $Y = y - \frac{5}{2}$

$X = x + \frac{7}{2}$

$Y^2 = -3X$

This is of the form $y^2 = -4ax$

Latus rectum is $= 3$

Vertex $(-\frac{7}{2}, \frac{5}{2})$

Axis $y = \frac{5}{2}$

Focus $(-\frac{7}{2} - \frac{3}{4}, \frac{5}{2})$

Directrix: referred to New axis: $X = a = \frac{3}{4}$

Directrix referred to Old axis: $\frac{3}{4} = x + \frac{7}{2}$

$x = \frac{3}{4} - \frac{7}{2}$

$x = -\frac{11}{4}$

9. Solution:

$\frac{7-4i}{3+2i} = \frac{7-4i}{3+2i} \times \frac{3-2i}{3-2i}$

$\frac{13-26i}{13} = 1-2i$

10. Solution

Either both factors are negative or both factors are positive to have this in equality. if $x < 2$

both factors are negative and if $x > 3$ both factors are positive. Hence the solution

is $x \in \{(-\infty, 2) \cup (3, \infty)\}$

11. Solution

$$\tan 5x = \cot 2x$$

$$\tan 5x = \tan\left(\frac{\pi}{2} - 2x\right)$$

$$5x = \left(\frac{\pi}{2} - 2x\right)$$

$$5x = n\pi + \left(\frac{\pi}{2} - 2x\right)$$

$$7x = n\pi + \frac{\pi}{2}$$

$$x = \frac{1}{7}\left(n\pi + \frac{\pi}{2}\right)$$

12. Solution

Total number of occurrence = $6 \times 6 = 36$

On each die there are 3 prime numbers $\{2, 3, 5\}$

Hence total number of favorable cases $3 \times 3 = 9$

Probability of getting a prime in each die = $\frac{9}{36} = \frac{1}{4}$

13. Solution:

$$\phi\left(\frac{\pi}{12}\right) = \sin 2\left(\frac{\pi}{12}\right)$$

$$= \sin \frac{\pi}{6}$$

$$= \frac{1}{2}$$

$$f(x) = \left(\frac{1}{2}\right)^3 - \frac{1}{2}$$

$$= \frac{1}{8} - \frac{1}{2}$$

$$= -\frac{3}{8}$$

14. Solution:

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \cdot \frac{1}{2m+1}} = 1 \end{aligned}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B + \tan A \tan B = 1$$

15. Solution:

$$f(\sqrt{3}) = -1$$

$$f(3) = 1$$

$$f(\sqrt{3}+1) = 1$$

16. Solution:

Use the inequality $AM \geq GM$

$$AM \text{ between } x, \frac{1}{x} = \frac{x + \frac{1}{x}}{2}$$

$$GM \text{ between } x, \frac{1}{x} = \sqrt{x \cdot \frac{1}{x}} = 1$$

$$\frac{x + \frac{1}{x}}{2} \geq 1$$

$$x + \frac{1}{x} \geq 2$$

$$\text{Since } -1 \leq \sin \theta \leq 1$$

$$\sin \theta = x + \frac{1}{x} \text{ is impossible}$$

17. Solution:

$$f(x) = \phi(x)$$

$$f(x) = 3x^2 + 1$$

$$\phi(x) = 7x - 1$$

$$3x^2 + 1 = 7x - 1$$

$$3x^2 - 7x + 2 = 0$$

$$(x - 2)(3x - 1) = 0$$

$$x = 2, x = \frac{1}{3}$$

Hence $f(x)$ and $\phi(x)$ are equal when the domain is in the set $\{\frac{1}{3}, 2\}$

18. Solution

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 \frac{x}{2})}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{2 \frac{x}{2}} \sin \frac{x}{2} \\
 &= \frac{1}{2} \cdot 1.0 \\
 &= 0
 \end{aligned}$$

19. Solution:

$$\begin{aligned}
 & 2 \sin^2 x + 14 \sin x \cos x + 50 \cos^2 x = 26 \\
 &= 2 \sin^2 x + 14 \sin x \cos x + 50 \cos^2 x = 26(\sin^2 x + \cos^2 x) \\
 &= -24 \sin^2 x + 14 \sin x \cos x + 24 \cos^2 x = 0 \\
 &= 24 \sin^2 x - 14 \sin x \cos x - 24 \cos^2 x = 0 \\
 &= 24 \tan^2 x - 14 \tan x - 24 = 0 \\
 &\tan x = \frac{14 \pm \sqrt{196 + 2304}}{48} \\
 &\tan x = \frac{14 \pm \sqrt{2500}}{48} \\
 &\tan x = \frac{14 \pm 50}{48} \\
 &\tan x = \frac{64}{48} ; \text{or} ; -\frac{36}{48} \\
 &\tan x = \frac{4}{3} \text{ or } -\frac{3}{4}
 \end{aligned}$$

Section C

20. Solution:

$$\begin{aligned}
 y &= \sin x \\
 y + \Delta y &= \sin(x + \Delta x) \\
 \Delta y &= \sin(x + \Delta x) - y \\
 \Delta y &= \sin(x + \Delta x) - \sin x
 \end{aligned}$$

$$\Delta y = 2 \cos \frac{2x + \Delta x}{2} \sin \frac{\Delta x}{2}$$

$$\frac{\Delta y}{\Delta x} = \frac{2 \cos \frac{2x + \Delta x}{2} \sin \frac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos \frac{2x + \Delta x}{2} \sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \cos x$$

$$\frac{dy}{dx} = \cos x$$

Note: As $\Delta x \rightarrow 0$; $\frac{\Delta x}{2}$ also $\rightarrow 0$

21. Solution:

The successive First order of difference is 4, 7, 10, 13, ... this is an AP.

The second order difference is (Difference of the first difference) 3, 3, 3, ...

Third order difference (Difference of second order differences) is all 0

n^{th} term

$$T_n = T_1 + (n-1)\Delta T_1 + \frac{(n-1)(n-2)}{2!} \Delta T_2 + \frac{(n-1)(n-2)(n-3)}{3!} \Delta T_3$$

$$= 12 + 4(n-1) + 3 \frac{(n-1)(n-2)}{2}$$

$$= \frac{3n^2 - n + 22}{2}$$

$$\text{Sum} = \frac{1}{2} (3 \Sigma n^2 - \Sigma n + 22n)$$

$$= \frac{1}{2} \left(3 \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n \right)$$

$$= \frac{1}{2} (n^3 + n^2 + 22n)$$

22. Solution:

Let the point A be (x_1, y_1) and B be (x_2, y_2)

Let the point C be a point be (x, y) on the circle

Then AC and BC are perpendicular

Product of Slopes of line AC and BC = -1

$$\frac{y - y_1}{x - x_1} \cdot \frac{y - y_2}{x - x_2} = -1$$

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

23. Solution

x_i	f_i	$f_i x_i$	$ x_i - 9 $	$f_i x_i - 9 $
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
$N = \sum f_i = 26$		$\sum f_i x_i = 234$		$f_i \sum x_i - 9 = 100$

$$\text{Mean} = \bar{X} = \frac{1}{N} (\sum f_i x_i) = \frac{234}{26} = 9$$

$$\text{Mean Deviation} = M.D = \frac{1}{N} (\sum f_i |x_i - 9|) = \frac{100}{26} = 3.84$$

24. Solution:

The odd digits 1, 3, 3, 1 can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits 2, 4, 2 can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!} \times \frac{3!}{2!} = 6 \times 3 = 18$ ways

25. Solution

Probability of one of them getting selected $P(E_1 \text{ or } E_2) = 1 - (\text{Probability of both getting selected} + \text{Probability of none getting selected})$

$$= 1 - [P(E_1 \cap E_2) + P(E_1' \cap E_2')]$$

$$= 1 - \left(\frac{1}{3} \times \frac{1}{5} + \frac{2}{3} \times \frac{4}{5} \right)$$

$$\begin{aligned}
 &= 1 - \left(\frac{1}{15} + \frac{8}{15} \right) \\
 &= 1 - \frac{9}{15} = \frac{6}{15} = \frac{2}{5}
 \end{aligned}$$

26. Solution

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and 3, E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$$a + (n-1)d = T_n$$

$$n = \frac{T_n}{d} - \frac{a}{d} + 1$$

In this case $\frac{a}{d} = 1$, Hence $n = \text{integer part of } \frac{T_n}{d}$

$$n(A) = \left[\frac{1000}{2} \right] = 500$$

$$n(B) = \left[\frac{1000}{3} \right] = 333$$

$$n(C) = \left[\frac{1000}{5} \right] = 200$$

$$n(D) = \left[\frac{1000}{2 \times 3} \right] = 166$$

$$n(E) = \left[\frac{1000}{2 \times 5} \right] = 100$$

$$n(F) = \left[\frac{1000}{3 \times 5} \right] = 66$$

$$n(G) = \left[\frac{1000}{2 \times 3 \times 5} \right] = 33$$

Numbers that are divisible by 2, 3, 5 are

$$\begin{aligned}
 n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \\
 &= 500 + 333 + 200 + 166 + 100 + 66 + 33 \\
 &= 734
 \end{aligned}$$

Numbers that are not divisible by 2, 3, 5 are

$$1000 - 734 = 266$$