Units and Measurements

- **Basic Trigonometric identities :**
  - $\sin(-x) = -\sin x$
  - $\cos(-x) = \cos x$
- **Trigonometric identities related to sum and difference of two angles:**
  - $\cos(x + y) = \cos x \cos y - \sin x \sin y$
  - $\cos(x - y) = \cos x \cos y + \sin x \sin y$
  - $\sin(x + y) = \sin x \cos y + \cos x \sin y$
  - $\sin(x - y) = \sin x \cos y - \cos x \sin y$
  - $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
  - $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- **Relations between trigonometric functions of an angle and the trigonometric function of twice *orthrice* that angle:**
  - $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$
  - $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1 + \tan^2 x}$
  - $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
  - $\sin 3x = 3\sin x - 4\sin^3 x$
  - $\cos 3x = 4\cos^3 x - 3\cos x$
  - $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$
- **Identities involving addition and subtraction of trigonometric functions:**
  - $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

- $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$
- $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
- $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

- **Some other important identities:**

  - $2\cos x\cos y = \cos(x+y) + \cos(x-y)$
  - $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$
  - $2\sin x\cos y = \sin(x+y) + \sin(x-y)$
  - $2\cos x\sin y = \sin(x+y) - \sin(x-y)$
  - $\cot(x+y) = \frac{\cot x\cot y - 1}{\cot y + \cot x}$
  - $\cot(x-y) = \frac{\cot x\cot y + 1}{\cot y - \cot x}$

Calculus

- Calculus is a way of calculating rates of changes, area, volumes, and surface areas .
- It is a tool that lets one do calculations with complicated curves, shapes, etc. that one would normally not be able to do with just algebra and geometry.

Differentiation

- It is the process of obtaining the derived function f′ from the function f, where f′x is the derivative of f at x.
- The derivatives of certain common functions are given in the table below:

Table of derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
sin x	cos x
cos x	−sin x
tan x	$\sec^2 x$
cot x	$-\mathrm{cosec}^2 x$
sec x	sec x tan x

cosec *x*	−(cosec *x*)(cot *x*)
ln *x*	$\frac{1}{x}$
e^x	e^x

Many other functions can be differentiated using the following rules of differentiation:

- If $h(x) = k\,f(x)$ for all x, where k is a constant, then $h'(x) = k\,f'(x)$.
- If $hx = f(x) + g(x)$ for all x, then $h'(x) = f'(x) + g'(x)$.
- The product rule: If $h(x) = f(x)g(x)$ for all x, then $h'x = f(x)g'x + f'(x)+g'(x)$.
- The reciprocal rule: If $h(x) = 1/f(x)$ and $f(x) \neq 0$ for all x, then:

$$h'(x) = -\frac{f'(x)}{(f(x))^2}$$

- The quotient rule: If $hx = f(x)/g(x)$ and $g(x) \neq 0$ for all x, then:

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

- The chain rule: If $h(x) = (f \circ g)(x) = f(g(x))$ for all x, then $h'(x) = f'(g(x))g'(x)$.

Integration:

- Integration is the process of finding an anti-derivative of a given function *f*.
- Such an anti-derivative may be called an indefinite integral of *f* and be denoted by $\int f(x)\,dx$.
- The term 'integration' is also used for any method of evaluating a definite integral.

$$\int_a^b f(x)dx$$

- The definite integral can be evaluated if an anti-derivative Φ of *f* can be found because then its value is $\Phi b - \Phi(a)$.

Application of differentiation :

- Rate of change of a variable (y) with respect to aother variable (x) is found using differentiation and is given by $r = \frac{dy}{dx}$
- For a curve drawn in an x-y plane, the slope of the tangent is given by $m = \frac{dy}{dx}$
- Maxima and minima of a function is found using derivatives:
  - Firstly, the values of x for which $\frac{dy}{dx} = 0$ are found . They are known as critical points.
  - The minimum value of y exists at the critical point where $\frac{d^2y}{dx^2} > 0$.
  - The maximum value of y exists at the critical point where $\frac{d^2y}{dx^2} < 0$.

Application of integration :

- For a curve drawn in x-y plane, the area under the curve and x-axis is equal to the definite integral between the limits a and b and is given by:

$$S = \int_a^b f(x)\,dx$$

Units

- A unit is the chosen standard of measurement of quantity, which has the same nature as the quantity.

Systems of Units

- CGS System: Base units for length, mass and time in this system are centimeter, gram and second respectively.
- FPS System: Base units for length, mass and time in this system are foot, pound and second respectively.
- MKS System: Base units in this system are metre, kilogram and second.
- International System SI of Units: Based on seven base units; at present the internationally accepted system.

SI Base Quantities and Units

- Length – metre m
- Mass – kilogram kg
- Time – second s
- Electric current – ampere A
- Thermodynamic temperature – kelvin K
- Amount of substance – mole mol
- Luminous intensity – candela cd

Derived units

- These are units of the physical quantities which are derived from the seven basic fundamental units.

Rules for determining the number of significant figures:

- All non-zero digits are significant.
- All zeroes between two non-zero digits are significant.
- Zeroes preceding the first non-zero digits are not significant.
- Zeroes at the end or right of a number are significant, provided they are on the right side of the decimal point.
- If the number is less than 1, then the zero$s$ on the right of the decimal point and left of the first non-zero digit are not significant. (For example: In 0.00013, the underlined zeroes are not significant)

Rules for arithmetic operations with significant figures

- For addition and subtraction, the result cannot have more digits to the right of the decimal point than either of the original numbers.
- In multiplication or division, the final result should retain as many significant figures as there are in the measurement with the least significant figures.

Rules for rounding off the uncertain digits

- If the rightmost digit to be removed is more than 5, then the preceding number is increased by 1.
- If the rightmost digit to be removed is less than 5, then the preceding number is not changed.
- If the rightmost digit to be removed is 5, then the preceding number is not changed if it is an even number, but it is increased by one if it is an odd number.

Types of Error

- Systematic errors
  - Arise due to faulty instruments
  - Arise due to imperfect experimental procedure
  - Arise due to individual carelessness
- Random errors
  - Arise due to random and unpredictable fluctuations in experimental conditions
- Least-count errors
  - Associated with the resolution of the instrument

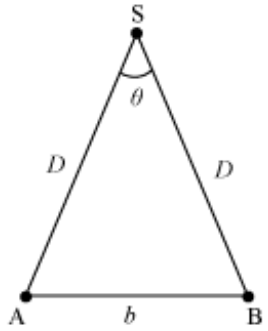
- Personal errors
  - Arise due to fault of an observer in taking reading, lack of proper setting of the apparatus etc.

Ways of Expressing an Error

- Absolute error: Magnitude of the difference between the actual value of the quantity and the individual measured value
- Relative error: Ratio of the mean absolute error to the value of the quantity being measured
  Relative error = $\frac{\Delta X}{X_{mean}}$

- Percentage error:
  Percentage error $= \frac{\Delta X}{X_{mean}} \times 100$

- **Measurement of length**
  - SI unit is metre m.
  - For measurement of large distances, parallax method is used.



$$D = \frac{b}{\theta}$$

- Units for expressing large distances are light year, Astronomical unit AU, and parsec.
  - 1 light year = 9.46 × 10^{15} m
  - 1 AU = 1.5 × 10^{11} m
  - 1 Parsec = 3.26 light years = 3.08 × 10^{16} m

- Units used to express small distances:
  - 1 micron $1mm$ = 10^{-6} m
  - 1 nanometre $1nm$ = 10^{-9} m
  - 1 angstrom ($1A^0$) = 10^{-10} m
  - 1 fermi $1fm$ = 10^{-15} m

Measurement of mass

- SI unit of mass – Kilogram
- While dealing with atoms and molecules, we use unified atomic mass unit $uoramu$ as standard unit.

  1 u = $1/12^{th}$ of the mass of C^{12} atom
  or, 1 u = 1.66 × 10^{-27} kg

- Masses of common objects can be measured by balance.
- Large masses can be measured by using gravitational method.
- Masses of sub-atomic particles can be measured by using mass spectrograph.
- Range of variation of mass – from 10^{-30} kg to 10^{55} kg