Chapter 9

SEQUENCES AND SERIES

Arithmetic progression (A.P)

Standard AP → a, a+d, a+2d……a+(n-1)d

$A_n = a + (n-1)\,d$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(a + an)$

Arithmetic mean A between the two numbers a and b is

$A = \frac{a+b}{2}$

If $A_1, A_2, \ldots A_n$ are n A.M between the two numbers a and b,

Then $d = \frac{b-a}{n+1}$

$A_1 = a + d = a + \frac{b-a}{n+1}$

$A_2 = a + 2d = a + 2\frac{b-a}{n+1}$

……

$A_n = a + nd = a + n\frac{b-a}{n+1}$

Geometric progression (G.P)

Standard GP → a, ar ,ar^2…….ar^{n-1}

$A_n = ar^{n-1}$

$S_n = \frac{a(r^n-1)}{r-1}$ or $\frac{a(1-r^n)}{1-r}$ if $r \neq 1$

$S_\infty = \frac{a}{1-r}$ if $|r| < 1$

If G is the GM between a and b, then $G = \sqrt{ab}$

If $G_1, G_2, \ldots G_n$ are n G.M between the two numbers a and b ,

then $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$

……

$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

Sum to n terms of special series

$S_n = 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

$S_n = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{\{n(n+1)\}^2}{4}$

TEXT BOOK QUESTIONS

* → Exercise 9.2 → Qns 5,7,8,11,14

* → Exercise 9.3 → Qns 2,3,5,11,16,17,19,21,23,25

* → Exercise 9.4 → Qns 3,4,5,6,7

* → Misc Exercise → Qns 3,4,5,10,12,14,18,21

** → Exercise 9.2 → Qns 9,10,12,13,15

** → Exercise 9.3→ Qns 12,13,14,15,18,22,26,27,28

** → Exercise 9.4 → Qns 1,2,8,9,10

** → Misc Exercise → Qns 19,22,23,24, 25,26

** → Examples 4,5,6,10,13,18,21

EXTRA/ HOT QUESTIONS

1. Which term of the sequence 25, $24\frac{1}{4}$, $23\frac{1}{2}$, $22\frac{3}{4}$,…..is the first negative term. (Ans.35)

2. How many terms are identical in the two AP.

2,4,6,…….up to 100 terms and 3,6,9…….. up to 80 terms (Ans.33)

3. solve for x : 1+4+7+……..+x = 590 (Ans.x=58)

4. Find the sum of all the three digit numbers which leaves the reminder 2 when divided by 5. (Ans.98910)

5. The digits of a three digit natural number are in AP and their sum is 15 .The number obtained by reversing the digits is 396 less than the original number. Find the number.

6. If p^{th}, q^{th}, and r^{th} terms of GP are in GP. Show that p,q,r are in AP

7. If a,b,c,d are in GP, then show that $a^2+ b^2$, $b^2+ c^2$, $c^2+ d^2$ are in GP

8. Evaluate $7^{\frac{1}{2}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{8}}$ ..... to infinite terms.

9. The common ratio of a GP is (- 4/5) and sum to infinity is (80/9). Find the first term. (Ans.7)

10. If S_1, S_2, S_3 are the sums of first n, 2n, 3n terms of a GP. Then Show that $s_1 (s_3-s_2) = (s_2-s_1)^2$

11. $\frac{1}{x+y}$, $\frac{1}{y+z}$, $\frac{1}{x+z}$ are in AP Show that y^2 , x^2 and z^2 are in AP .

12. Find the sum of $10^3 + 11^3 +.....+ 20^3$ (Ans.42075)

13. Find the n^{th} term and the sum of n terms of the series

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +$ .....

14. Find the sum of n terms of $1^3 + \frac{1^3+2^3}{2} + \frac{1^3+2^3+3^3}{3} +$ .....

15. If AM and GM of roots of a quadratic equation are 8 and 5 respectively, then write the quadratic equation. (Ans.x^2-16x+25=0)