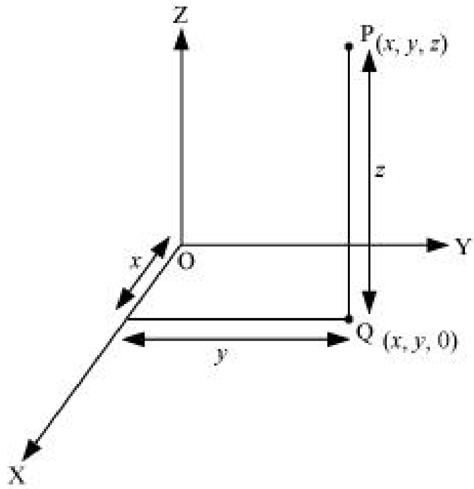
# Introduction to Three Dimensional Geometry

- **Three-dimensions coordinate planes**
  - The coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called $x$, $y$, and $z$-axes.
  - The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.
  - The three coordinate planes divide the space into eight parts known as octants.
  - In three-dimensional geometry, the coordinates of a point P are always written in the form of triplets i.e., $(x, y, z)$. Here, $x$, $y$, and $z$ are the distances from the YZ, ZX and XY-planes. Also, the coordinates of the origin are (0, 0, 0).




- The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in the eight octants.

| Octants → / Coordinates ↓ | I | II | III | IV | V | VI | VII | VIII |
|---|---|---|---|---|---|---|---|---|
| ***x*** | + | – | – | + | + | – | – | + |
| ***y*** | + | + | – | – | + | + | – | – |
| ***z*** | + | + | + | + | – | – | – | – |

**Example**: The point (–5, 6, –7) lies in the VI octant.

- In Coordinates of points lying on different axes:
  - Any point on the $x$-axis is of the form $(x, 0, 0)$
  - Any point on the $y$-axis is of the form $(0, y, 0)$
  - Any point on the $z$-axis is of the form $(0, 0, z)$
- Coordinates of points lying in different planes:
  - Coordinates of a point in the YZ-plane are of the form $(0, y, z)$
  - Coordinates of a point in the XY-plane are of the form $(x, y, 0)$
  - Coordinates of a point in the ZX-plane are of the form $(x, 0, z)$

**Example**: The points (–5, 6, 0), (0, –5, 6), (–5, 0, 6) lies in the XY-plane, YZ-plane and ZX-plane respectively.

- **distance formula**

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**Example:** Find the point(s), lying on the *z*-axis, whose distance from point (2, –1, 3) is 3 units.
**Solution:** Let the required point be (0, 0, *z*).
We know that the distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1) + (z_2 - z_1)^2}$.

Therefore,
$$\sqrt{(2-0)^2 + (-1-0)^2 + (3-z)^2} = 3$$

*On squaring both the sides, we get*

$$4 + 1 + 9 + z^2 - 6z = 9$$
$$\Rightarrow z^2 - 6z + 5 = 0$$
$$\Rightarrow z^2 - 5z - z + 5 = 0$$
$$\Rightarrow z(z-5) - 1(z-5) = 0$$
$$\Rightarrow z = 1, 5$$

Thus, the required points on the *z*-axis are (0, 0, 1) and (0, 0, 5).