EXERCISE–10

Part : (A) Only one correct option

1. If (2, 0) is the vertex & y – axis the directrix of a parabola, then its focus is:
(A) (2, 0) (B) (– 2, 0) (C) (4, 0) (D) (– 4, 0)

2. A parabola is drawn with its focus at (3, 4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is:
(A) $x^2 - 6x - 8y + 25 = 0$ (B) $y^2 - 8x - 6y + 25 = 0$
(C) $x^2 - 6x + 8y - 25 = 0$ (D) $x^2 + 6x - 8y - 25 = 0$

3. The length of the chord of the parabola, $y^2 = 12x$ passing through the vertex & making an angle of 60º with the axis of x is:
(A) 8 (B) 4 (C) 16/3 (D) none

4. The length of the side of an equilateral triangle inscribed in the parabola, $y^2 = 4x$ so that one of its angular point is at the vertex is:
(A) $8\sqrt{3}$ (B) $6\sqrt{3}$ (C) $4\sqrt{3}$ (D) $2\sqrt{3}$

5. The circles on focal radii of a parabola as diameter touch:
(A) the tangent at the vertex (B) the axis (C) the directrix (D) none of these

6. The equation of the tangent to the parabola $y = (x - 3)^2$ parallel to the chord joining the points (3, 0) and (4, 1) is:
(A) $2x - 2y + 6 = 0$ (B) $2y - 2x + 6 = 0$ (C) $4y - 4x + 11 = 0$ (D) $4x - 4y = 11$

7. The angle between the tangents drawn from a point (– a, 2a) to $y^2 = 4ax$ is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

8. An equation of a tangent common to the parabolas $y^2 = 4x$ and $x^2 = 4y$ is
(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$ (C) $x + y + 1 = 0$ (D) $y = 0$

9. The line $4x - 7y + 10 = 0$ intersects the parabola, $y^2 = 4x$ at the points A & B. The co-ordinates of the point of intersection of the tangents drawn at the points A & B are:
(A) $\left(\frac{7}{2}, \frac{5}{2}\right)$ (B) $\left(-\frac{5}{2}, -\frac{7}{2}\right)$ (C) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (D) $\left(-\frac{7}{2}, -\frac{5}{2}\right)$

10. AP & BP are tangents to the parabola, $y^2 = 4x$ at A & B. If the chord AB passes through a fixed point (– 1, 1) then the equation of locus of P is
(A) $y = 2(x - 1)$ (B) $y = 2(x + 1)$ (C) $y = 2x$ (D) $y^2 = 2(x - 1)$

11. Equation of the normal to the parabola, $y^2 = 4ax$ at its point $(am^2, 2am)$ is:
(A) $y = -mx + 2am + am^3$ (B) $y = mx - 2am - am^3$ (C) $y = mx + 2am + am^3$ (D) none

12. At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes?
(A) (4, 4) (B) (9, 6) (C) (4, – 1) (D) (1, 2)

13. If on a given base, a triangle be described such that the sum of the tangents of the base angles is a constant, then the locus of the vertex is:
(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

14. A point moves such that the square of its distance from a straight line is equal to the difference between the square of its distance from the centre of a circle and the square of the radius of the circle. The locus of the point is:
(A) a straight line at right angles to the given line (B) a circle concentric with the given circle
(C) a parabola with its axis parallel to the given line (D) a parabola with its axis perpendicular to the given line.

15. P is any point on the parabola, $y^2 = 4ax$ whose vertex is A. PA is produced to meet the directrix in D & M is the foot of the perpendicular from P on the directrix. The angle subtended by MD at the focus is:
(A) $\pi/4$ (B) $\pi/3$ (C) $5\pi/12$ (D) $\pi/2$

16. If the distances of two points P & Q from the focus of a parabola $y^2 = 4ax$ are 4 & 9, then the distance of the point of intersection of tangents at P & Q from the focus is:
(A) 8 (B) 6 (C) 5 (D) 13

17. Tangents are drawn from the point (– 1, 2) on the parabola $y^2 = 4x$. The length of intercept made by these tangents on the line x = 2 is:
(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none of these

18. From the point (4, 6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$. The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4, 6) is:
(A) 8 (B) 4 (C) 2 (D) none of these

19. Locus of the intersection of the tangents at the ends of the normal chords of the parabola $y^2 = 4ax$ is
(A) $(2a + x)\,y^2 + 4a^3 = 0$ (B) $(2a + x) + y^2 = 0$
(C) $(2a + x)\,y^2 + 4a = 0$ (D) none of these

20. If the tangents & normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then:
(A) $x_1 = x_2$ (B) $x_1 = y_2$ (C) $y_1 = y_2$ (D) $x_2 = y_1$

21. Tangents are drawn from the points on the line $x - y + 3 = 0$ to parabola $y^2 = 8x$. Then all the chords of contact passes through a fixed point whose coordinates are:
(A) (3, 2) (B) (2, 4) (C) (3, 4) (D) (4, 1)

22. The distance between a tangent to the parabola $y^2 = 4Ax$ (A > 0) and the parallel normal with gradient 1 is:
(A) 4 A (B) $2\sqrt{2}$ A (C) 2 A (D) $\sqrt{2}$ A

23. A variable parabola of latus ractum ℓ, touches a fixed equal parabola, then axes of the two curves being

parallel. The locus of the vertex of the moving curve is a parabola, whole latus rectum is:
(A) ℓ (B) 2ℓ (C) 4ℓ (D) none

24. Length of the focal chord of the parabola $y^2 = 4ax$ at a distance p from the vertex is:
(A) $\frac{2a^2}{p}$ (B) $\frac{a^3}{p^2}$ (C) $\frac{4a^3}{p^2}$ (D) $\frac{p^2}{a}$

25. AB is a chord of the parabola $y^2 = 4ax$ with vertex at A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is
(A) a (B) 2a (C) 4a (D) 8a

26. The locus of the foot of the perpendiculars drawn from the vertex on a variable tangent to the parabola $y^2 = 4ax$ is:
(A) $x(x^2 + y^2) + ay^2 = 0$ (B) $y(x^2 + y^2) + ax^2 = 0$
(C) $x(x^2 - y^2) + ay^2 = 0$ (D) none of these

27. T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then:
(A) SL = 2 (TN) (B) 3 (SL) = 2 (TN) (C) SL = TN (D) 2 (SL) = 3 (TN)

28. The point of contact of the tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) and makes an angle θ with the axis of the parabola such that $\tan\theta > 2$ is
(A) (4/9, 2) (B) (36, 18) (C) (4, 6) (D) (1/4, 3/2)

29. If the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect at (16, 8) at an angle θ, then θ is equal to
(A) $\tan^{-1}\left(\frac{3}{5}\right)$ (B) $\tan^{-1}\left(\frac{4}{5}\right)$ (C) π (D) $\frac{\pi}{2}$

30. From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If θ_1 & θ_2 are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \frac{\pi}{4}$, then the locus of P is:
(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$ (C) $x - y - 1 = 0$ (D) $x + y + 1 = 0$

31. Locus of the point of intersection of the normals at the ends of parallel chords of gradient m of the parabola $y^2 = 4ax$ is:
(A) $2xm^2 - ym^3 = 4a(2 + m^2)$ (B) $2xm^2 + ym^3 = 4a(2 + m^2)$
(C) $2xm + ym^2 = 4a(2 + m)$ (D) $2xm^2 - ym^3 = 4a(2 - m^2)$

32. The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the intersection of those at (4a, – 4a) & (9a, – 6a) is:
(A) $5x - y + 115a = 0$ (B) $5x + y - 135a = 0$ (C) $5x - y - 115a = 0$ (D) $5x + y + 115 = 0$

33. The point(s) on the parabola $y^2 = 4x$ which are closest to the circle,
$x^2 + y^2 - 24y + 128 = 0$ is/are:
(A) (0, 0) (B) $\left(2, 2\sqrt{2}\right)$ (C) (4, 4) (D) none

34. If P_1Q_1 and P_2Q_2 are two focal chords of the parabola $y^2 = 4ax$, then the chords P_1P_2 and Q_1Q_2 intersect on the
(A) directrix (B) axis (C) tangent at the vertex (D) none of these

35. If $x + y = k$, is the normal to $y^2 = 12x$, then k is **[IIT - 2000]**
(A) 3 (B) 9 (C) –9 (D) – 3

36. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is **[IIT - 2001]**
(A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x + 3)$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x + 1)$

37. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord are: **[IIT - 2003]**
(A) {– 1, 1} (B) {– 2, 2} (C) {– 2, 1/2} (D) {2, – 1/2}

38. The normal drawn at a point $(at_1^2, -2at_1)$ of the parabola $y^2 = 4ax$ meets it again in the point $(at_2^2, 2at_2)$, then **[IIT - 2003]**
(A) $t_2 = t_1 + \frac{2}{t_1}$ (B) $t_2 = t_1 - \frac{2}{t_1}$ (C) $t_2 = -t_1 + \frac{2}{t_1}$ (D) $t_2 - t_1 - \frac{2}{t_1}$

39. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is **[IIT - 2004]**
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

40. Let P be the point (1, 0) and Q a point of the locus $y^2 = 8x$. The locus of mid point of PQ is **[IIT - 2005]**
(A) $x^2 + 4y + 2 = 0$ (B) $x^2 - 4y + 2 = 0$ (C) $y^2 - 4x + 2 = 0$ (D) $y^2 + 4x + 2 = 0$

41. A parabola has its vertex and focus in the first quadrant and axis along the line y = x. If the distances of the vertex and focus from the origin are respectively $\sqrt{2}$ and $2\sqrt{2}$, then an equation of the parabola is **[IIT - 2006]**
(A) $(x + y)^2 = x - y + 2$ (B) $(x - y)^2 = x + y - 2$
(C) $(x - y)^2 = 8(x + y - 2)$ (D) $(x + y)^2 = 8(x - y + 2)$

Comprehension **[IIT - 2006]**

Let ABCD be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of the square ABCD. L is a line through A.

42. If P is a point on C_1 and Q in another point on C_2, $\dfrac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to **[IIT - 2006]**
(A) 0.75 (B) 1.25 (C) 1 (D) 0.5

43. A circle touch the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is **[IIT - 2006]**
(A) ellipse (B) hyperbola (C) parabola (D) parts of straight line

44. A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is **[IIT - 2006)]**
(A) $\frac{1}{2}$ sq. units (B) $\frac{2}{3}$ sq. units (C) 1 sq. units (D) 2 sq. units

Part : (B) May have more than one options correct

45. If one end of a focal chord of the parabola $y^2 = 4x$ is (1, 2), the other end lies on
(A) $x^2 y + 2 = 0$ (B) $xy + 2 = 0$ (C) $xy - 2 = 0$ (D) $x^2 + xy - y - 1 = 0$

46. The tangents at the extremities of a focal chord of a parabola
(A) are perpendicular (B) are parallel
(C) intersect on the directrix (D) intersect at the vertex

47. If from a variable point 'P' pair of perpendicular tangents PA and PB are drawn to any parabola then
(A) P lies on directrix of parabola (B) chord of contact AB passes through focus
(C) chord of contact AB passes through of fixed point
(D) P lies on director circle

48. A normal chord of the parabola subtending a right angle at the vertex makes an acute angle θ with the x – axis, then θ =
(A) arc tan 2 (B) arc sec $\sqrt{3}$ (C) arc cot $\sqrt{2}$ (D) $\frac{\pi}{2}$ – arc cot $\sqrt{2}$

49. Variable chords of the parabola $y^2 = 4ax$ subtend a right angle at the vertex. Then:
(A) locus of the feet of the perpendiculars from the vertex on these chords is a circle
(B) locus of the middle points of the chords is a parabola
(C) variable chords passes through a fixed point on the axis of the parabola (D) none of these

50. Two parabolas have the same focus. If their directrices are the x – axis & the y – axis respectively, then the slope of their common chord is:
(A) 1 (B) – 1 (C) 4/3 (D) 3/4

51. P is a point on the parabola $y^2 = 4ax$ $(a > 0)$ whose vertex is A. PA is produced to meet the directrix in D and M is the foot of the perpendicular from P on the directrix. If a circle is described on MD as a diameter then it intersects the x–axis at a point whose co–ordinates are:
(A) (– 3a, 0) (B) (– a, 0) (C) (– 2a, 0) (D) (a, 0)

EXERCISE–11

1. Find the vertex, axis, focus, directrix, latusrectum of the parabola $x^2 + 2y - 3x + 5 = 0$.

2. Find the set of values of α in the interval [π/2, 3π/2], for which the point (sinα, cosα) does not lie outside the parabola $2y^2 + x - 2 = 0$.

3. Two perpendicular chords are drawn from the origin 'O' to the parabola $y = x^2$, which meet the parabola at P and Q Rectangle POQR is completed. Find the locus of vertex R.

4. Find the equation of tangent & normal at the ends of the latus rectum of the parabola $y^2 = 4a(x - a)$.

5. Prove that the straight line $\ell x + my + n = 0$ touches the parabola $y^2 = 4ax$ if $\ell n = am^2$.

6. If tangent at P and Q to the parabola $y^2 = 4ax$ intersect at R then prove that mid point of R and M lies on the parabola, where M is the mid point of P and Q.

7. Find the equation of normal to the parabola $x^2 = 4y$ at (9, 6).

8. Find the equation of the chord of $y^2 = 8x$ which is bisected at (2, – 3)

9. Find the locus of the mid-points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

10. Find the equation of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the point (6, 9).

11. Prove that the normals at the points, where the straight line $\ell x + my = 1$ meets the parabola $y^2 = 4ax$, meet on the normal at the point $\left(\dfrac{4am^2}{\ell^2}, \dfrac{4am}{\ell}\right)$ of the parabola.

12. If the normals at three points P, Q, and R on parabola $y^2 = 4ax$ meet in a point O and S be the focus, prove that SP. SQ . SR = a. SO^2.

13. Show that the locus of the point of intersection of the tangents to $y^2 = 4ax$ which intercept a constant length d on the directrix is $(y^2 - 4ax)(x + a)^2 = d^2x^2$.

14. Show that the distance between a tangent to the parabola $y^2 = 4ax$ and the parallel normal is $a\sec^2\theta \operatorname{cosec}\theta$, where θ is the inclination of the either with the axis of the parabola.

15. P and Q are the point of contact of the tangents drawn from a point R to the parabola $y^2 = 4ax$. If PQ be a normal to the parabola at P, prove that PR is bisected by the directrix.

16. A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of the parabola $y^2 = 4ax$. If PQ is the common chord of the circle and the parabola and $L_1 L_2$ is the latus rectum, then prove that the area of the trapezium PL_1L_2Q is $\left(\frac{2+\sqrt{2}}{2}\right)a^2$.

17. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissa are in A.P., then prove that slopes of the tangents at the 3 conormal points are in GP.

18. Prove that the length of the intercept on the normal at the point $(at^2, 2at)$ made by the circle which is described on the focal distance of the given point as diameter is $a\sqrt{1+t^2}$.

19. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius b; then axes being AB and a perpendicular diameter, prove that the locus of the focus of the parabola is $\frac{x^2}{b^2}+\frac{y^2}{b^2-a^2}=1$

20. PNP′ is a double ordinate of the parabola then prove that the locus of the point of intersection of the normal at P and the straight line through P′ parallel to the axis is the equal parabola $y^2 = 4a(x - 4a)$.

21. Find the locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other. **[IIT - 1997]**

22. Let C_1 and C_2 be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflections of P and Q, respectively, with respect to the line $y = x$. Prove that P_1 lies on C_2, Q_1 lies on C_1 and $PQ \geq \min\{PP_1, QQ_1\}$. Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P, Q) with P on C_1 and Q on C_2. **[IIT - 2000]**

23. Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_2 m_2 = \alpha$ is a part of the parabola itself then find α. **[IIT - 2003]**

Answers

EXERCISE–10

1. C **2.** A **3.** A **4.** A **5.** A **6.** D **7.** B

8. C **9.** C **10.** A **11.** A **12.** D **13.** B **14.** D

15. D **16.** B **17.** B **18.** C **19.** A **20.** C **21.** C

22. B **23.** B **24.** C **25.** C **26.** A **27.** C **28.** A

29. A **30.** C **31.** A **32.** B **33.** C **34.** A **35.** B

36. C **37.** A **38.** A **39.** B **40.** C **41.** C **42.** A

43. C **44.** C **45.** ABD **46.** AC **47.** ABCD

48. BD **49.** ABC **50.** AB **51.** AD

EXERCISE–11

1. vertex $\equiv \left(\frac{3}{2}, -\frac{29}{8}\right)$, focus $\left(\frac{3}{2}, -\frac{33}{8}\right)$

axis $x = 3$, directrix $y = -\frac{29}{3}$. Latus rectum = 2.

2. $\alpha \in [\pi/2, 5\pi/6] \cup [\pi, 3\pi/2]$ **3.** $y^2 = x - 2$

4. Tangent $y = x$, $y = -x$,
Normal $x + y = 4a$, $x - y = 4a$

7. $2x + 9y = 72$ **8.** $4x + 3y + 1 = 0$

9. $y^2 - 2ax + 8a^2 = 0$

10. $x^2 + y^2 + 18x - 28y + 27 = 0$

21. $x^2 - 2y + 12 = 0$ **23.** $\alpha = 2$