SHORT REVISION

SOLUTIONS OF TRIANGLE

I. **SINE FORMULA :** In any triangle ABC , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

II. **COSINE FORMULA :** **(i)** $\cos A = \frac{b^2+c^2-a^2}{2bc}$ or $a^2 = b^2 + c^2 - 2bc.\cos A$

(ii) $\cos B = \frac{c^2+a^2-b^2}{2ca}$ **(iii)** $\cos C = \frac{a^2+b^2-c^2}{2ab}$

III. **PROJECTION FORMULA :** **(i)** $a = b\cos C + c\cos B$ **(ii)** $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$

IV. **NAPIER'S ANALOGY – TANGENT RULE :** **(i)** $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

(ii) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$ **(iii)** $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

V. **TRIGONOMETRIC FUNCTIONS OF HALF ANGLES :**

(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$; $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

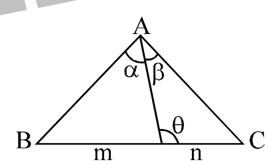
(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ where $s = \frac{a+b+c}{2}$ & Δ = area of triangle.

(iv) Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$.

VI. **M – N RULE :** In any triangle ,

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta = n\cot B - m\cot C$$



VII. $\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$ = area of triangle ABC.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Note that $R = \frac{abc}{4\Delta}$; Where R is the radius of circumcircle & Δ is area of triangle

VIII. Radius of the incircle 'r' is given by:

(a) $r = \frac{\Delta}{s}$ where $s = \frac{a+b+c}{2}$ **(b)** $r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$

(c) $r = \frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$ & so on **(d)** $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

IX. Radius of the Ex–circles r_1, r_2 & r_3 are given by :

(a) $r_1 = \frac{\Delta}{s-a}$; $r_2 = \frac{\Delta}{s-b}$; $r_3 = \frac{\Delta}{s-c}$ **(b)** $r_1 = s\tan\frac{A}{2}$; $r_2 = s\tan\frac{B}{2}$; $r_3 = s\tan\frac{C}{2}$

(c) $r_1 = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$ & so on **(d)** $r_1 = 4R\sin\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$;

$r_2 = 4R\sin\frac{B}{2}.\cos\frac{A}{2}.\cos\frac{C}{2}$; $r_3 = 4R\sin\frac{C}{2}.\cos\frac{A}{2}.\cos\frac{B}{2}$

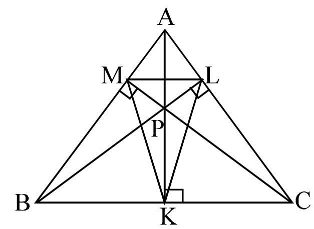
X. **LENGTH OF ANGLE BISECTOR & MEDIANS :**

If m_a and β_a are the lengths of a median and an angle bisector from the angle A then,

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2} \quad \text{and} \quad \beta_a = \frac{2bc\cos\frac{A}{2}}{b+c}$$

Note that $m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2)$

XI. ORTHOCENTRE AND PEDAL TRIANGLE :



The triangle KLM which is formed by joining the feet of the altitudes is called the pedal triangle.

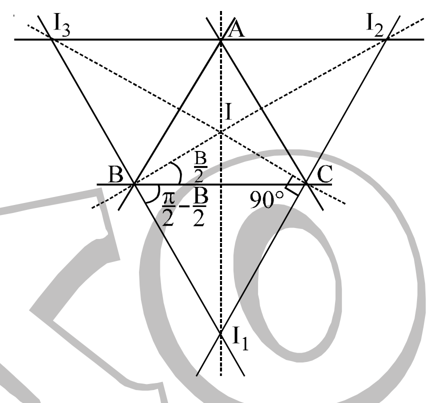
– the distances of the orthocentre from the angular points of the ΔABC are $2R\cos A$, $2R\cos B$ and $2R\cos C$

– the distances of P from sides are $2R\cos B\cos C$, $2R\cos C\cos A$ and $2R\cos A\cos B$

– the sides of the pedal triangle are $a\cos A$ $(= R\sin 2A)$, $b\cos B$ $(= R\sin 2B)$ and $c\cos C$ $(= R\sin 2C)$ and its angles are $\pi - 2A$, $\pi - 2B$ and $\pi - 2C$.

– circumradii of the triangles PBC, PCA, PAB and ABC are equal .

XII EXCENTRAL TRIANGLE :



The triangle formed by joining the three excentres I_1, I_2 and I_3 of ΔABC is called the excentral or excentric triangle.

Note that :

– Incentre I of ΔABC is the orthocentre of the excentral $\Delta I_1I_2I_3$.

– ΔABC is the pedal triangle of the $\Delta I_1I_2I_3$.

– the sides of the excentral triangle are

$4R\cos\frac{A}{2}$, $4R\cos\frac{B}{2}$ and $4R\cos\frac{C}{2}$

and its angles are $\frac{\pi}{2} - \frac{A}{2}$, $\frac{\pi}{2} - \frac{B}{2}$ and $\frac{\pi}{2} - \frac{C}{2}$.

– $II_1 = 4R\sin\frac{A}{2}$; $II_2 = 4R\sin\frac{B}{2}$; $II_3 = 4R\sin\frac{C}{2}$.

XIII. THE DISTANCES BETWEEN THE SPECIAL POINTS :

(a) The distance between circumcentre and orthocentre is $= R\,.\sqrt{1 - 8\cos A\cos B\cos C}$

(b) The distance between circumcentre and incentre is $= \sqrt{R^2 - 2Rr}$

(c) The distance between incentre and orthocentre is $\sqrt{2r^2 - 4R^2\cos A\cos B\cos C}$

XIV. Perimeter (P) and area (A) of a regular polygon of n sides inscribed in a circle of radius r are given by

$$P = 2nr\sin\frac{\pi}{n} \qquad \text{and} \qquad A = \frac{1}{2}nr^2\sin\frac{2\pi}{n}$$

Perimeter and area of a regular polygon of n sides circumscribed about a given circle of radius r is given by

$$P = 2nr\tan\frac{\pi}{n} \qquad \text{and} \qquad A = nr^2\tan\frac{\pi}{n}$$

EXERCISE–I

With usual notations, prove that in a triangle ABC:

Q.1 $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} = 0$

Q.2 $a\cot A + b\cot B + c\cot C = 2(R + r)$

Q.3 $\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)} = \frac{3}{r}$

Q.4 $\frac{r_1 - r}{a} + \frac{r_2 - r}{b} = \frac{c}{r_3}$

Q.5 $\frac{abc}{s}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \Delta$

Q.6 $(r_1 + r_2)\tan\frac{C}{2} = (r_3 - r)\cot\frac{C}{2} = c$

Q.7 $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$

Q.8 $(r + r_1)\tan\frac{B-C}{2} + (r + r_2)\tan\frac{C-A}{2} + (r + r_3)\tan\frac{A-B}{2} = 0$

Q.9 $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

Q.10 $(r_3+r_1)(r_3+r_2)\sin C = 2r_3\sqrt{r_2r_3+r_3r_1+r_1r_2}$

Q.11 $\frac{1}{bc}+\frac{1}{ca}+\frac{1}{ab}=\frac{1}{2Rr}$

Q.12 $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{4R}{r^2s^2}$

Q.13 $\frac{bc-r_2r_3}{r_1}=\frac{ca-r_3r_1}{r_2}=\frac{ab-r_1r_2}{r_3}=r$

Q.14 $\left(\frac{1}{r}+\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)^2=\frac{4}{r}\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)$

Q.15 $Rr(\sin A+\sin B+\sin C)=\Delta$

Q.16 $2R\cos A = 2R + r - r_1$

Q.17 $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s^2}{\Delta}$

Q.18 $\cot A+\cot B+\cot C=\frac{a^2+b^2+c^2}{4\Delta}$

Q.19 Given a triangle ABC with sides a = 7, b = 8 and c = 5. If the value of the expression $\left(\sum\sin A\right)\left(\sum\cot\frac{A}{2}\right)$ can be expressed in the form $\frac{p}{q}$ where $p, q \in N$ and $\frac{p}{q}$ is in its lowest form find the value of $(p+q)$.

Q.20 If $r_1 = r + r_2 + r_3$ then prove that the triangle is a right angled triangle.

Q.21 If two times the square of the diameter of the circumcircle of a triangle is equal to the sum of the squares of its sides then prove that the triangle is right angled.

Q.22 In acute angled triangle ABC, a semicircle with radius r_a is constructed with its base on BC and tangent to the other two sides. r_b and r_c are defined similarly. If r is the radius of the incircle of triangle ABC then prove that, $\frac{2}{r}=\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}$.

Q.23 Given a right triangle with $\angle A = 90°$. Let M be the mid-point of BC. If the inradii of the triangle ABM and ACM are r_1 and r_2 then find the range of r_1/r_2.

Q.24 If the length of the perpendiculars from the vertices of a triangle A, B, C on the opposite sides are p_1, p_2, p_3 then prove that $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$.

Q.25 Prove that in a triangle $\frac{bc}{r_1}+\frac{ca}{r_2}+\frac{ab}{r_3}=2R\left[\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)-3\right]$.

EXERCISE–II

Q.1 With usual notation, if in a ΔABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$; then prove that, $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$.

Q.2 For any triangle ABC, if B = 3C, show that $\cos C=\sqrt{\frac{b+c}{4c}}$ & $\sin\frac{A}{2}=\frac{b-c}{2c}$.

Q.3 In a triangle ABC, BD is a median. If $l(BD)=\frac{\sqrt{3}}{4}\cdot l(AB)$ and $\angle DBC=\frac{\pi}{2}$. Determine the $\angle ABC$.

Q.4 ABCD is a trapezium such that AB, DC are parallel & BC is perpendicular to them. If angle ADB = θ, BC = p & CD = q, show that $AB=\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$.

Q.5 If sides a, b, c of the triangle ABC are in A.P., then prove that $\sin^2\frac{A}{2}\ \text{cosec}\,2A$; $\sin^2\frac{B}{2}\ \text{cosec}\,2B$; $\sin^2\frac{C}{2}\ \text{cosec}\,2C$ are in H.P.

Q.6 Find the angles of a triangle in which the altitude and a median drawn from the same vertex divide the angle at that vertex into 3 equal parts.

Q.7 In a triangle ABC, if $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ are in AP. Show that $\cos A, \cos B, \cos C$ are in AP.

Q.8 ABCD is a rhombus. The circumradii of Δ ABD and Δ ACD are 12.5 and 25 respectively. Find the area of rhombus.

Q.9 In a triangle ABC if $a^2 + b^2 = 101c^2$ then find the value of $\frac{\cot C}{\cot A + \cot B}$.

Q.10 The two adjacent sides of a cyclic quadrilateral are 2 & 5 and the angle between them is $60°$. If the area of the quadrilateral is $4\sqrt{3}$, find the remaining two sides.

Q.11 If I be the in–centre of the triangle ABC and x, y, z be the circum radii of the triangles IBC, ICA & IAB, show that $4R^3 - R(x^2 + y^2 + z^2) - xyz = 0$.

Q.12 Sides a, b, c of the triangle ABC are in H.P., then prove that $\operatorname{cosec} A(\operatorname{cosec} A + \cot A)$; $\operatorname{cosec} B(\operatorname{cosec} B + \cot B)$ & $\operatorname{cosec} C(\operatorname{cosec} C + \cot C)$ are in A.P.

Q.13 In a Δ ABC, (i) $\frac{a}{\cos A} = \frac{b}{\cos B}$ (ii) $2\sin A\cos B = \sin C$

(iii) $\tan^2\frac{A}{2} + 2\tan\frac{A}{2}\tan\frac{C}{2} - 1 = 0$, prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i).

Q.14 The sequence $a_1, a_2, a_3, \ldots\ldots$ is a geometric sequence.
The sequence $b_1, b_2, b_3, \ldots\ldots$ is a geometric sequence.

$b_1 = 1$; $b_2 = \sqrt[4]{7} - \sqrt[4]{28} + 1$; $a_1 = \sqrt[4]{28}$ and $\sum_{n=1}^{\infty}\frac{1}{a_n} = \sum_{n=1}^{\infty} b_n$

If the area of the triangle with sides lengths a_1, a_2 and a_3 can be expressed in the form of p/q where p and q are relatively prime, find $(p + q)$.

Q.15 If p_1, p_2, p_3 are the altitudes of a triangle from the vertices A, B, C & Δ denotes the area of the triangle, prove that $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$.

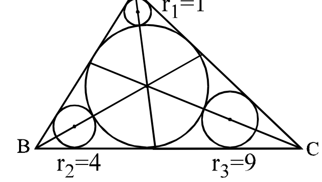
Q.16 The triangle ABC (with side lengths a, b, c as usual) satisfies
$\log a^2 = \log b^2 + \log c^2 - \log(2bc\cos A)$. What can you say about this triangle?

Q.17 With reference to a given circle, A_1 and B_1 are the areas of the inscribed and circumscribed regular polygons of n sides, A_2 and B_2 are corresponding quantities for regular polygons of 2n sides. Prove that
(1) A_2 is a geometric mean between A_1 and B_1.
(2) B_2 is a harmonic mean between A_2 and B_1.

Q.18 The sides of a triangle are consecutive integers n, $n + 1$ and $n + 2$ and the largest angle is twice the smallest angle. Find n.

Q.19 The triangle ABC is a right angled triangle, right angle at A. The ratio of the radius of the circle circumscribed to the radius of the circle escribed to the hypotenuse is, $\sqrt{2} : (\sqrt{3} + \sqrt{2})$. Find the acute angles B & C. Also find the ratio of the two sides of the triangle other than the hypotenuse.

Q.20 ABC is a triangle. Circles with radii as shown are drawn inside the triangle each touching two sides and the incircle. Find the radius of the incircle of the ΔABC.



Q.21 Line l is a tangent to a unit circle S at a point P. Point A and the circle S are on the same side of l, and the distance from A to l is 3. Two tangents from point A intersect line l at the point B and C respectively. Find the value of (PB)(PC).

Q.22 Let ABC be an acute triangle with orthocenter H. D, E, F are the feet of the perpendiculars from A, B, and C on the opposite sides. Also R is the circumradius of the triangle ABC.
Given $(AH)(BH)(CH) = 3$ and $(AH)^2 + (BH)^2 + (CH)^2 = 7$. Find

(a) the ratio $\dfrac{\prod \cos A}{\sum \cos^2 A}$, (b) the product (HD)(HE)(HF) (c) the value of R.

EXERCISE–III

Q.1 The radii r_1, r_2, r_3 of escribed circles of a triangle ABC are in harmonic progression. If its area is 24 sq. cm and its perimeter is 24 cm, find the lengths of its sides. [REE '99, 6]

Q.2(a) In a triangle ABC, Let $\angle C = \dfrac{\pi}{2}$. If 'r' is the inradius and 'R' is the circumradius of the triangle, then $2(r + R)$ is equal to:
(A) $a + b$ (B) $b + c$ (C) $c + a$ (D) $a + b + c$

(b) In a triangle ABC, $2\,ac\,\sin\dfrac{1}{2}(A - B + C) =$
(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$ (C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$
[JEE '2000 (Screening) 1 + 1]

Q.3 Let ABC be a triangle with incentre 'I' and inradius 'r'. Let D, E, F be the feet of the perpendiculars from I to the sides BC, CA & AB respectively. If r_1, r_2 & r_3 are the radii of circles inscribed in the quadrilaterals AFIE, BDIF & CEID respectively, prove that

$$\frac{r_1}{r-r_1}+\frac{r_2}{r-r_2}+\frac{r_3}{r-r_3} = \frac{r_1 r_2 r_3}{(r-r_1)(r-r_2)(r-r_3)}.$$ [JEE '2000, 7]

Q.4 If Δ is the area of a triangle with side lengths a, b, c, then show that: $\Delta \le \dfrac{1}{4}\sqrt{(a+b+c)abc}$
Also show that equality occurs in the above inequality if and only if $a = b = c$. [JEE ' 2001]

Q.5 Which of the following pieces of data does NOT uniquely determine an acute–angled triangle ABC (R being the radius of the circumcircle)?
(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R
[JEE ' 2002 (Scr), 3]

Q.6 If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that

$$I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$$ [JEE 2003, Mains, 4 out of 60]

Q.7 The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. The ratio A : B : C is
(A) 3 : 5 : 2 (B) $1 : \sqrt{3} : 2$ (C) 3 : 2 : 1 (D) 1 : 2 : 3
[JEE 2004 (Screening)]

Q.8(a) In ΔABC, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is

(A) $(b-c)\sin\left(\dfrac{B-C}{2}\right) = a\cos\left(\dfrac{A}{2}\right)$ (B) $(b-c)\cos\left(\dfrac{A}{2}\right) = a\sin\left(\dfrac{B-C}{2}\right)$

(C) $(b+c)\sin\left(\frac{B+C}{2}\right)=a\cos\left(\frac{A}{2}\right)$ (D) $(b-c)\cos\left(\frac{A}{2}\right)=2a\sin\left(\frac{B+C}{2}\right)$

[JEE 2005 (Screening)]

(b) Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the points of contact.

[JEE 2005 (Mains), 2]

Q.9(a) Given an isosceles triangle, whose one angle is 120° and radius of its incircle is $\sqrt{3}$. Then the area of triangle in sq. units is

(A) $7 + 12\sqrt{3}$ (B) $12 - 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) 4π

[JEE 2006, 3]

(b) Internal bisector of $\angle A$ of a triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of ΔABC then

(A) AE is HM of b and c (B) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$

(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ (D) the triangle AEF is isosceles [JEE 2006, 5]

Q.10 Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

[JEE 2009, 5]

EXERCISE–I

Q.19 107 **Q.23** $\left(\frac{1}{2}, 2\right)$

EXERCISE–II

Q.3 120° **Q.6** π/6, π/3, π/2 **Q.8** 400 **Q.9** 50 **Q.10** 3 cms & 2 cms
Q.14 9 **Q.16** triangle is isosceles **Q.18** 4 **Q.19** $B=\frac{5\pi}{12}$; $C=\frac{\pi}{12}$; $\frac{b}{c}=2+\sqrt{3}$
Q.20 r = 11 **Q.21** 3 **Q.22** (a) $\frac{3}{14R}$, (b) $\frac{9}{8R^3}$, (c) $\frac{3}{2}$

EXERCISE–III

Q.1 6, 8, 10 cms **Q.2** **(a)** A, **(b)** B **Q.5** D **Q.7** D **Q.8** **(a)** B; **(b)** $\sqrt{5}$
Q.9 **(a)** C, **(b)** A, B, C, D **Q.10** 4

P. T. O.

Exercise - 1 (Objective Questions)

Part : (A) Only one correct option

1. In a triangle ABC, $(a + b + c)(b + c - a) = k.\, bc$, if :
(A) $k < 0$ (B) $k > 6$ (C) $0 < k < 4$ (D) $k > 4$

2. In a ΔABC, $A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ cm and ar (ΔABC) $= \frac{9\sqrt{3}}{2}$ cm^2. Then a is
(A) $6\sqrt{3}$ cm (B) 9 cm (C) 18 cm (D) none of these

3. If R denotes circumradius, then in ΔABC, $\frac{b^2 - c^2}{2aR}$ is equal to
(A) cos (B – C) (B) sin (B – C) (C) cos B – cos C (D) none of these

4. If the radius of the circumcircle of an isosceles triangle PQR is equal to PQ (= PR), then the angle P is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

5. In a Δ ABC, the value of $\frac{a\cos A + b\cos B + c\cos C}{a + b + c}$ is equal to:
(A) $\frac{r}{R}$ (B) $\frac{R}{2r}$ (C) $\frac{R}{r}$ (D) $\frac{2r}{R}$

6. In a right angled triangle R is equal to
(A) $\frac{s+r}{2}$ (B) $\frac{s-r}{2}$ (C) $s - r$ (D) $\frac{s+r}{a}$

7. In a ΔABC, the inradius and three exradii are r, r_1, r_2 and r_3 respectively. In usual notations the value of $r.\, r_1.\, r_2.\, r_3$ is equal to
(A) 2Δ (B) Δ^2 (C) $\frac{abc}{4R}$ (D) none of these

8. In a triangle if $r_1 > r_2 > r_3$, then
(A) $a > b > c$ (B) $a < b < c$ (C) $a > b$ and $b < c$ (D) $a < b$ and $b > c$

9. With usual notation in a Δ ABC $\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\left(\frac{1}{r_2} + \frac{1}{r_3}\right)\left(\frac{1}{r_3} + \frac{1}{r_1}\right) = \frac{KR^3}{a^2b^2c^2}$,
where 'K' has the value equal to:
(A) 1 (B) 16 (C) 64 (D) 128

10. The product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle is equal to:
(A) Δ (B) 2Δ (C) 3Δ (D) 4Δ

11. In a triangle ABC, right angled at B, the inradius is:
(A) $\frac{AB + BC - AC}{2}$ (B) $\frac{AB + AC - BC}{2}$ (C) $\frac{AB + BC + AC}{2}$ (D) None

12. The distance between the middle point of BC and the foot of the perpendicular from A is :
(A) $\frac{-a^2 + b^2 + c^2}{2a}$ (B) $\frac{b^2 - c^2}{2a}$ (C) $\frac{b^2 + c^2}{\sqrt{bc}}$ (D) none of these

13. In a triangle ABC, B = 60° and C = 45°. Let D divides BC internally in the ratio 1 : 3, then, $\frac{\sin\angle BAD}{\sin\angle CAD} =$
(A) $\sqrt{\frac{2}{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{3}$

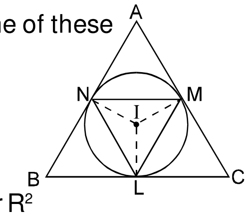
14. Let f, g, h be the lengths of the perpendiculars from the circumcentre of the Δ ABC on the sides a, b and c respectively. If $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \frac{abc}{fgh}$ then the value of λ is:
(A) 1/4 (B) 1/2 (C) 1 (D) 2

15. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3, 4 and 5 units. Then area of the triangle is equal to:

(A) $\frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$ (B) $\frac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ (C) $\frac{9\sqrt{3}(1+\sqrt{3})}{2\pi^2}$ (D) $\frac{9\sqrt{3}(\sqrt{3}-1)}{2\pi^2}$

16. If in a triangle ABC, the line joining the circumcentre and incentre is parallel to BC, then cos B + cos C is equal to:
(A) 0 (B) 1 (C) 2 (D) none of these

17. If the incircle of the Δ ABC touches its sides respectively at L, M and N and if x, y, z be the circumradii of the triangles MIN, NIL and LIM where I is the incentre then the product xyz is equal to:



(A) Rr^2 (B) rR^2 (C) $\frac{1}{2}Rr^2$ (D) $\frac{1}{2}rR^2$

18. If in a ΔABC, $\frac{r}{r_1} = \frac{1}{2}$, then the value of $\tan\frac{A}{2}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)$ is equal to :
(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None of these

19. In any ΔABC, then minimum value of $\frac{r_1 r_2 r_3}{r^3}$ is equal to
(A) 3 (B) 9 (C) 27 (D) None of these

20. In a acute angled triangle ABC, AP is the altitude. Circle drawn with AP as its diameter cuts the sides AB and AC at D and E respectively, then length DE is equal to
(A) $\frac{\Delta}{2R}$ (B) $\frac{\Delta}{3R}$ (C) $\frac{\Delta}{4R}$ (D) $\frac{\Delta}{R}$

21. AA_1, BB_1 and CC_1 are the medians of triangle ABC whose centroid is G. If the concyclic, then points A, C_1, G and B_1 are
(A) $2b^2 = a^2 + c^2$ (B) $2c^2 = a^2 + b^2$ (C) $2a^2 = b^2 + c^2$ (D) None of these

22. In a ΔABC, a, b, A are given and c_1, c_2 are two values of the third side c. The sum of the areas of two triangles with sides a, b, c_1 and a, b, c_2 is
(A) $\frac{1}{2}b^2 \sin 2A$ (B) $\frac{1}{2}a^2 \sin 2A$ (C) $b^2 \sin 2A$ (D) none of these

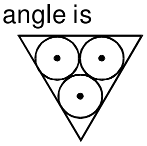
23. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle, then 2(r + R) is equal to **[IIT - 2000]**
(A) a + b – c (B) b + c (C) c + a (D) a + b + c

24. Which of the following pieces of data does NOT uniquely determine an acute - angled triangle ABC (R being the radius of the circumcircle)? **[IIT - 2002]**
(A) a , sin A, sin B (B) a, b, c (C) a, sin B, R (D) a, sin A, R

25. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is
(A) $\sqrt{3} : (2 + \sqrt{3})$ (B) 1 : 6 (C) $1 : 2 + \sqrt{3}$ (D) 2 : 3 **[IIT - 2003]**

26. The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the angle of the triangle are in the ratio **[IIT - 2004]**
(A) 1 : 3 : 5 (B) 2 : 3 : 4 (C) 3 : 2 : 1 (D) 1 : 2 : 3

27. In an equilateral triangle, 3 coincs of radii 1 unit each are kept so that they touche each other and also the sides of the triangle. Area of the triangle is **[IIT - 2005]**



(A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$ (C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

28. If P is a point on C_1 and Q is a point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ equals
(A) 1/2 (B) 3/4 (C) 5/6 (D) 7/8

29. A circle C touches a line L and circle C_1 externally. If C and C_1 are on the same side of the line L, then locus of the centre of circle C is
(A) an ellipse (B) a circle (C) a parabola (D) a hyperbola

30. Let ℓ be a line through A and parallel to BD. A point S moves such that its distance from the line BD and the vertex A are equal. If the locus of S meets AC in A_1, and ℓ in A_2 and A_3, then area of $\Delta A_1 A_2 A_3$ is
(A) 0.5 $(unit)^2$ (B) 0.75 $(unit)^2$ (C) 1 $(unit)^2$ (D) (2/3) $(unit)^2$

Part : (B) May have more than one options correct

31. In a ΔABC, following relations hold good. In which case(s) the triangle is a right angled triangle?
(A) $r_2 + r_3 = r_1 - r$ (B) $a^2 + b^2 + c^2 = 8R^2$ (C) $r_1 = s$ (D) $2R = r_1 - r$

32. In a triangle ABC, with usual notations the length of the bisector of angle A is :
(A) $\dfrac{2bc\cos\frac{A}{2}}{b+c}$ (B) $\dfrac{2bc\sin\frac{A}{2}}{b+c}$ (C) $\dfrac{abc\,\mathrm{cosec}\,\frac{A}{2}}{2R(b+c)}$ (D) $\dfrac{2\Delta}{b+c}\cdot\mathrm{cosec}\,\dfrac{A}{2}$

33. AD, BE and CF are the perpendiculars from the angular points of a Δ ABC upon the opposite sides, then :
(A) $\dfrac{\text{Perimeter of }\Delta DEF}{\text{Perimeter of }\Delta ABC} = \dfrac{r}{R}$ (B) Area of ΔDEF = $2\Delta\cos A\cos B\cos C$
(C) Area of ΔAEF = $\Delta\cos^2 A$ (D) Circum radius of ΔDEF = $\dfrac{R}{2}$

34. The product of the distances of the incentre from the angular points of a Δ ABC is:
(A) $4R^2r$ (B) $4Rr^2$ (C) $\dfrac{(abc)R}{s}$ (D) $\dfrac{(abc)r}{s}$

35. In a triangle ABC, points D and E are taken on side BC such that BD = DE = EC. If angle ADE = angle AED = θ, then:
(A) $\tan\theta = 3\tan B$ (B) $3\tan\theta = \tan C$
(C) $\dfrac{6\tan\theta}{\tan^2\theta - 9} = \tan A$ (D) angle B = angle C

36. With usual notation, in a Δ ABC the value of $\Pi(r_1 - r)$ can be simplified as:
(A) $abc\,\Pi\tan\dfrac{A}{2}$ (B) $4rR^2$ (C) $\dfrac{(abc)^2}{R(a+b+c)^2}$ (D) $4Rr^2$

Exercise - 2 (Subjective Questions)

1. If in a triangle ABC, $\dfrac{\cos A + 2\cos C}{\cos A + 2\cos B} = \dfrac{\sin B}{\sin C}$, prove that the triangle ABC is either isosceles or right angled.

2. In a triangle ABC, if $a\tan A + b\tan B = (a + b)\tan\left(\dfrac{A+B}{2}\right)$, prove that triangle is isosceles.

3. If $\left(1-\dfrac{r_1}{r_2}\right)\left(1-\dfrac{r_1}{r_3}\right) = 2$ then prove that the triangle is the right triangle.

4. In a Δ ABC, $\angle C = 60°$ & $\angle A = 75°$. If D is a point on AC such that the area of the Δ BAD is $\sqrt{3}$ times the area of the Δ BCD, find the $\angle$ ABD.

5. The radii r_1, r_2, r_3 of escribed circles of a triangle ABC are in harmonic progression. If its area is 24 sq. cm and its perimeter is 24 cm, find the lengths of its sides.

6. ABC is a triangle. D is the middle point of BC. If AD is perpendicular to AC, then prove that $\cos A.\cos C = \dfrac{2(c^2-a^2)}{3ac}$.

7. Two circles, of radii a and b, cut each other at an angle θ. Prove that the length of the common chord is $\dfrac{2ab\sin\theta}{\sqrt{a^2+b^2+2ab\cos\theta}}$.

8. In the triangle ABC, lines OA, OB and OC are drawn so that the angles OAB, OBC and OCA are each equal to ω, prove that
(i) $\cot\omega = \cot A + \cot B + \cot C$
(ii) $\mathrm{cosec}^2\omega = \mathrm{cosec}^2 A + \mathrm{cosec}^2 B + \mathrm{cosec}^2 C$

9. In a plane of the given triangle ABC with sides a, b, c the points A′, B′, C′ are taken so that the Δ A′BC, Δ AB′C and Δ ABC′ are equilateral triangles with their circum radii R_a, R_b, R_c; in–radii r_a, r_b, r_c & ex–radii r_a', r_b' & r_c' respectively. Prove that;
(i) $\Pi r_a : \Pi R_a : \Pi r_a' = 1 : 8 : 27$ (ii) $r_1 r_2 r_3 = \dfrac{\left[\sum(3R_a + 6r_a + 2r_a')\right]^3}{648\sqrt{3}}\Pi\tan\dfrac{A}{2}$

10. The triangle ABC is a right angled triangle, right angle at A. The ratio of the radius of the circle

circumscribed to the radius of the circle escribed to the hypotenuse is, $\sqrt{2} : (\sqrt{3} + \sqrt{2})$. Find the acute angles B & C. Also find the ratio of the two sides of the triangle other than the hypotenuse.

11. The triangle ABC is a right angled triangle, right angle at A. The ratio of the radius of the circle circumscribed to the radius of the circle escribed to the hypotenuse is, $\sqrt{2} : (\sqrt{3} + \sqrt{2})$. Find the acute angles B & C. Also find the ratio of the two sides of the triangle other than the hypotenuse.

12. If the circumcentre of the Δ ABC lies on its incircle then prove that,

$$\cos A + \cos B + \cos C = \sqrt{2}$$

13. Three circles, whose radii area a, b and c, touch one another externally and the tangents at their points of contact meet in a point; prove that the distance of this point from either of their points of contacts is $\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}$.

Answers

EXERCISE # 1

1. C **2.** B **3.** B **4.** D **5.** A **6.** B **7.** B

8. A **9.** C **10.** B **11.** A **12.** B **13.** C **14.** A

15. A **16.** B **17.** C **18.** B **19.** C **20.** D **21.** C

22. A **23.** A **24.** D **25.** A **26.** A **27.** B **28.** B

29. C **30.** C **31.** ABCD **32.** ACD **33.** ABCD

34. BD **35.** ACD **36.** ACD

EXERCISE # 2

4. $\angle ABD = 30°$ **5.** 6, 8, 10 cms

10. $B = \frac{5\pi}{12}$, $C = \frac{\pi}{12}$, $\frac{b}{c} = 2+\sqrt{3}$

11. $B = \frac{5\pi}{12}$, $C = \frac{\pi}{12}$, $\frac{b}{c} = 2+\sqrt{3}$