# केन्द्रीय विद्यालय संगठन

# KENDRIYA VIDYALAYA SANGATHAN

# STUDY MATERIAL

# CLASS XI - MATHEMATICS

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KENDRIYA VIDYALAYA SANGATHAN,

REGIONAL OFFICE,

ERNAKULAM

# PREFACE

## “God is a child; and when he began to play, he cultivated Mathematics. It is the godliest of man’s games”

Good Education is defined as acquiring skills. There are many different ways to be educated and many subjects can be studied. A good education is one that teaches a student to think. Mathematics develops logic and skill of reasoning among students. Focus of this material is primarily to strengthen the mind to absorb the concepts and bring in the students the required self-confidence while learning the subject. Maths should be learnt with interest and it is made simple and approachable. The material is a supplement to the curriculum and arranged in a chronological manner as published in textbook. As per CBSE examination pattern Higher Order Thinking Skills questions with solutions can be found in each chapter. This will definitely facilitate students to approach examinations with ease and confidence. And finally, let the students remember that success is 1% inspiration and 99% perspiration. Hard work can never fail and will certainly help them reap rich rewards. Success will then become a habit for them.

## “Seeing much, suffering much, and studying much, are the three pillars of learning”

## “Learning is a treasure that will follow its owner everywhere”

## WE WISH ALL THE STUDENTS THE VERY BEST!!!

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## Unit wise Mark Distribution

| Units | | Marks |
|---|---|---|
| Sets function | | 29 |
| Algebra | | 37 |
| Co-ordinate Geometry | | 13 |
| Calculus | | 06 |
| Reasoning | | 03 |
| Statistics and Probability | | 12 |
| | Total | 100 |

## Chapter 1

# Sets

Concept: Representation of a set

Concepts:- Different types of sets – Subsets- Power sets – Universal set – Operations on sets – Compliment of a set – Practical Problems.

Text book questions

| | |
|---|---|
| Ex: 1 | Questions 3, 4,5 |
| Ex: 2 | Questions 1, 2 |
| Ex: 3 | Questions 4, 5,6*,7* |
| Ex: 4 | Questions 4, 6, 9 |
| Ex: 5 | Questions 4*, 5* |
| Misc.Ex: | Questions 8, 9, 11, 15**, 16** |
| Example | Question: 34** |

*Note:* * *Important*

** *Very Important*

**Extra/HOT questions**

1. Write the following sets in set builder form
   I) {1/4, 2/5, 3/6, 4/7, 5/8}
   II) { ...., -5, 0, 5, 10, ....}
   III) {-4, 4}
2. Let A, B and C are three sets then prove the following:
   i) $A - (A \cap B) = A - B$
   ii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
   iii) $A - (B \cap C) = (A - B) \cup (A - C)$
   iv) $A \cap (B - C) = (A \cap B) - (A \cap C)$
3. Draw Venn diagrams for the following sets:
   i) $(A - B)' \cap A$
   ii) $(A \cap B \cap C)'$
   iii) $(A \cap B)'\ if\ A \subset B$
   iv) $(A - B) \cap (A \cup B)$
   v) $(A \cap B)'\ if\ A\ and\ B\ are\ disjoint\ sets$

vi) $(A \cup B \cup C)'$

vii) $(A - B) \cup (A \cap B)$

4. In a survey of 100 students, the number of students studying the various languages were found to be English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, Number of no language 24. Find
   i) How many students were studying Hindi?
   ii) How many students were studying English and Hindi

   [Ans:18,3]
5. In a survey of 25 students it was found that 15 had taken Maths, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Maths and chemistry, 9 had taken Maths and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken:
   i) Only Chemistry
   ii) Only Maths
   iii) Only Physics
   iv) Physics and Chemistry but not Maths
   v) Maths and Physics but not Chemistry
   vi) Only one of the subject
   vii) At least one of the subjects
   viii) None of the subjects [Ans:5, 4, 2, 1, 6, 11, 23, 2]
6. Of the members of three athletic team in a certain school, 21 are in the Basketball Team, 26 in the Hockey team and 29 in the Football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three. How many members are there in all?

   [Ans:43]
7. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A & B, 10 read magazine B&C and 3 read all the three. Find:
   i) How many read none of the magazines?
   ii) How many read magazine C only?
   iii) How many read magazine A only?
   iv) How many read magazine B & C but not A ?

   [Ans:18,32,13,0]
8. Let A and B be two finites sets such that n(A) = m and n(B) = n. If the ratio of number of elements of power sets of A and B is 64 and

n(A) + n(B) = 32. Find the value of m and n.
[Ans:19, 23]

9. In a survey of 400 students of a school, 100 were listed as smokers and 150 as chewers of Gum, 75 were listed as both smokers and gum chewers. Find out how many students are neither smokers nor gum chewers. [Ans:225]
10. In a university out of 100 teachers, 15 like reading newspapers only, 12 like learning computers only and 8 like watching movies only on TV in the spare time. 40 like reading news papers and watching movies, 20 like learning computer and watching movies, 10 like reading news paper and learning computer, 65 like watching movies. Draw a Venn diagram and show the various portions and hence evaluate the numbers of teachers who:
    i) Like reading newspapers
    ii) Like learning computers
    iii) Did not like to do any of the things mentioned above. [62, 39, 1]

# Chapter 2:
# Relations and Functions

**Concept:**

Cartesian products of sets – equality of ordered pairs- triple product- relations- functions- domain- range- different types of functions- algebra of functions.

Notes:

- If (a,b) = (c,d) then a = c and b = d.
- AxB = { (x,y) / x ϵ A, y ϵ B }
- AxAxA = { (x,y,z) / x, y, z ϵ A }
- A relation R is a subset of the Cartesian product.
- A function is a relation with every element of first set has one only one image in second set.
- The set of all first elements of the ordered pairs in a function is called domain.
- The set of all second elements of the ordered pairs in a function is called the range.
- Second set itself is known as co-domain.

## Text book questions

| | |
|---|---|
| Ex: 2.1 | Questions: 1, $2^*$, $5^*$, $7^*$ |
| Ex: 2.2 | Questions: 1, 2, 6, $7^*$ |
| Ex:2.3 | Questions: $2^*$, $5^*$ |
| Misc. Ex: | Questions: $3^*$, 4, 6, 8, 11, 12 |
| Example | Question: $22^*$ |

## Extra/HOT questions

1. Find x and y if $(x^2-3x, y^2-5y) = (-2, -6)$.
2. Draw he graph of the following functions:
    a) Modulus function in [ -4, 4 ]
    b) Signum function in [-6, 6 ]
    c) Greatest integer function in [-3, 4 ]

3. Find the domain of the following functions:
   a) $f(x) = \frac{x^2-1}{x-1}$
   b) $f(x) = \frac{3x+1}{x^2-5x+6}$
   c) $f(x) = \frac{2x-3}{(x-1)(x+2)}$

4. Find the domain and range of the following functions:
   a) $f(x) = \frac{1}{9-x^2}$
   b) $f(x) = \sqrt{x^2 - 1}$
   c) $f(x) = \frac{1}{x^2+4}$
   d) $f(x) = \frac{|x|}{1+|x|}$
5. If $f(x) = x^2 + \frac{1}{x^2}$ then show that f(a) = f(1/a) and also evaluate f(3/2)-f(2/3)
6. Let R = {(x,y) / x, y ϵ N , x+2y = 13} then write R as an ordered pair and also find the domain and range.
7. Let A = {x / x is a natural number <12 } and R be a relation in A defined by (x,y) in R if x+y = 12, then write R.
8. A function f is defined on the set of natural numbers as

$$f(x) = \begin{cases} x^2 \; if \; 1 \leq x < 5 \\ x + 3 \; if \; 5 < x \leq 8 \\ \frac{x-3}{2} \; if \; 8 < x \leq 11 \end{cases}$$

Write the function in roster form and also find the domain and range of the function.

9. Let A = {1,2,3,4}, B = {-1, 0, 1} and C = {3, 4} then verify the following:
   a) A X (B U C) = (A X B) U (A X C)
   b) A X ( B-C ) = (A X B) – (A X C)
   c) $A \; X \; (B \cap C) = (A \; X \; B) \cap (A \; X \; C)$
10. If A = {-3, -2, 0, 2, 3} write the subset B of A X A such that first element of B is either -3 or +3.

## Chapter 3: Trigonometric functions

**Concept:**

Radian measure- relation between degree and radian- trigonometric functions- sign of trigonometric functions- trigonometric functions of sum and difference of two angles- trigonometric equations- sine formula- cosine formula- their applications.

Notes:

- If in a circle of radius r, an arc of length 'l' subtends an angle of $\theta$ radians then $l = r\theta$.
- Radian measure =( $\pi/180$) x degree measure.
- Sin (-x) = - sin x
- Cos (-x) = cos x
- Cos($2n\pi+x$) = cos x
- Sin ( $2n\pi+x$) = sin x
- Sin x = 0 gives x = $n\pi$ where n $\in$ **Z**
- Cos x = 0 gives x = $(2n+1)\pi/2$ where n $\in$ **Z**
- Refer text book for other formulas.

**Text book questions**

| | |
|---|---|
| Ex:3.1 | Questions: $1^*$, $2^*$, $3^*$, 6 |
| Ex:3.2 | Questions: 6, 7, 8, 9, 10 |
| Ex:3.3 | Questions: 5, 6, $7^*$, 11, $12^*$, $14^*$, $15^*$, 16, 18, $21^*$, $22^{**}$, $23^{**}$, $24^*$, $25^*$ |
| Ex:3.4 | Questions: 5, 6, 7, 8, 9** |
| Misc. Ex: | Questions: 2, 3, 5, 6, 7, $8^*$, $9^*$, $10^*$ |
| Examples: | Questions: $24^{**}$, $25^{**}$, $26^*$, $27^*$, $29^{**}$ |

**Supplementary text**

Ex:3.5 Questions: 1, 3, 6, 7, 10, 11, 13, $14^{**}$, $15^{**}$, $16^{**}$

Examples: Questions: $27^{**}$, $28^{*}$

**Extra/ HOT Questions**

1. The angles of a triangle are in A.P and the greatest angle is double the least. Express the angles in degrees and radians
2. Show that the equation cosec x=4ab/(a+ b)$^2$ (ab>0) is possible if a=b
3. Show that a) sin 150 cos120 + cos330 sin 660 = -1

   b)$\frac{\cos(90+x)\sec(-x)\tan(180-x)}{\sec(360-x)\sin(180+x)\cot(90=x)} = 1$
4. If tan x=$\frac{m}{m+1}$ $and \tan y = \frac{1}{2m+1}$, show that x+y = $45^0$
5. Show that the following:

   a) cos 10 cos 50 cos60 cos70=3/16

   b) sin 10 sin 50 sin60 sin70=$\sqrt{3}/16$

   c) cos 20 cos40cos60=1/8
6. If sin x sin y =1/4 and 3tanx=4tany then prove that sin(x+y)=7/16
7. Prove that $\frac{\sin 11x\ sinx+sin7x\ sin3x}{cos11x\ sinx+cos7x\ sin3x} = tan8x$
8. If m tan(x-30)= n tan(x+120) then show that $\frac{m-n}{2(m+n)} = \frac{1}{4}sec2x$
9. Solve the equation 4 sinx cosx + 2sinx +2cosx+1=0
10. Solve the triangle when c=3.4cm ,A=$25^0$,B=$85^0$

    [ans; a=1.53cm,b=3.6cm,C=$80^0$
11. Show that for any parallelogram, if a and b are the sides of two non parallel sides , x is the angle between these two sides and d is the length of the diagonal that has a common vertex with sides a and b ,then $d^2$= $a^2$+$b^2$+2ab cos x

# Chapter 4

## PRINCIPLE OF MATHEMATICAL INDUCTION

***INTRODUCTION***

To prove certain results or statements in Algebra, that are formulated in terms of n, where n is a natural number, we use a specific technique called principle of mathematical induction (P.M.I)

**Steps of P.M.I**

**Step I -** Let p(n): result or statement formulated in terms of n (given question)

**Step II** – Prove that P(1) is true

**Step III** – Assume that P(k) is true

**Step IV** – Using step III prove that P(k+1) is true

**Step V** - Thus P(1) is true and P(k+1) is true whenever P(k) is true.

Hence by P.M.I, P(n) is true for all natural numbers n

**Type I**

Eg: Ex 4.1

1) Prove that

$1+3+3^2+\ldots\ldots\ldots+3^{n-1} = \frac{3^n-1}{2}$

Solution:-

**Step I** : Let P(n): $1+3+3^2+\ldots\ldots\ldots+3^{n-1} = \frac{3^n-1}{2}$

**Step II**: P(1):

LHS = 1

RHS = $= \frac{3-1}{2} = \frac{2}{2} = 1$

LHS=RHS

Therefore p(1) is true.

**Step III**: Assume that P(k) is true

i.e $1+3+3^2+\ldots\ldots\ldots+3^{k-1} = \frac{3^k-1}{2}$ ______(1)

**Step IV**: we have to prove that P(k+1) is true.

ie to prove that $1+3+3^2+\ldots\ldots\ldots+3^{k-1}+3 = \frac{3^{k+1}-1}{2}$

Proof

LHS $= (1+3+3^2+\ldots\ldots\ldots+3^{k-1}) +3$

$= \frac{3^k-1}{2} + 3^k$ from eq(1)

$= \frac{3^k-1+2.3^k}{2}$

$= \frac{3.3^k-1}{2} = \frac{3^{k+1}-1}{2}$ = RHS

Therefore P(k+1) is true

**Step V**: Thus P(1) is true and P(k+1) is true whenever P(k) is true. Hence by P.M.I, P(n) is true for all natural number n.

**Text book**

**Ex 4.1**

Q. 1,2, 3**(HOT), 4, 5*,6*,7,8,9,10*,11**,12,13**,14**,15,16**,17**, eg 1, eg 3

**Type 2**

Divisible / Multiple Questions like Q. 20**,21,22**,23 of Ex 4.1

eg 4, eg 6**(HOT)

Q 22. Prove that $3^{2n+2}-8n-9$ is divisible by 8 for all natural number n.

Solution

**Step I:** Let p(n): $3^{2n+2}-8n-9$ is divisible by 8

**Step II**: P(1): $3^4 - 8 - 9 = 81-17 = 64$ which is divisible by 8

Therefore p(1) is true

**Step III:** Assume that p(k) is true

i.e $3^{2k+2} - 8k-9 = 8m$; m is a natural number.

i.e $3^{2k}.9 = 8m+8k+9$

ie $3^{2k} = \frac{8m+8k+9}{9}$ ______(1)

**Step IV:** To prove that p(k+1) is true.

ie to prove that $3^{2k+4}$ -8(k+1) –9 is divisible by 8.

Proof: $3^{2k+4}$-8k-17 = $3^{2k}.3^4$-8k-17 =($\frac{8m+8k+9}{9}$) x $3^4$ – 8k-17(from eqn (1))

= (8m+8k+9)9-8k-17 = 72m+72k+81-8k-17 = 72m-64k+64 = 8[9m-8k+8] is divisible by 8.

**Step V**: Thus P(1) is true and P(k+1) is true whenever P(k) is true. hence by P.M.I, P(n) is true for all natural numbers n.

**Type III:** Problems based on Inequations

Ex 4.1 Q. 18,14, eg 7

(Q 18) Prove that 1+2+3+……+n< $\frac{(2n+1)^2}{8}$

**Step I** : Let P(n): 1+2+3+……+n< $\frac{(2n+1)^2}{8}$

**Step II**: P(1): 1< $\frac{9}{8}$ which is true, therefore p(1) is true.

**Step III**: Assume that P(k) is true.

ie 1+2+3+……+k< $\frac{(2k+1)^2}{8}$ _______(1)

**Step IV**: We have to prove that P(k+1) is true. ie to

prove that 1+2+3+…..+k+(k+1)< $\frac{(2k+3)^2}{8}$

Proof: Adding (k+1) on both sides of inequation (1)

1+2+3+…..+k+(k+1) < $\frac{(2k+1)^2}{8} + (k+1)$

$= \frac{(4k^2+4k+1)+8k+8}{8}$

$= \frac{4k^2+12k+9}{8}$

$= \frac{(2k+3)^2}{8}$

Therefore $1+2+3+\ldots..+k+(k+1) < \frac{(2k+3)^2}{8}$

P(k+1) is true.

**Step V**: Thus P(1) is true and P(k+1) is true whenever P(k) is true. Hence by P.M.I, P(n) is true for all natural number n.

**HOT/EXTRA QUESTIONS**

Prove by mathematical induction that for all natural numbers n.

1) $a^{2n-1} -1$ is divisible by a-1 (type II)
2) $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is an integer(HOT)
3) $\sin x + \sin 3x + \ldots\ldots + \sin (2n-1)x = \frac{\sin^2 nx}{\sin x}$ (HOT Type 1)
4) $3^{2n-1}+3^n+4$ is divisible by 2 (type II)
5) Let P(n): $n^2+n-19$ is prime, state whether P(4) is true or false
6) $2^{2n+3} \leq (n+3)!$ (type III)
7) What is the minimum value of natural number n for which $2^n<n!$ holds true?
8) $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 (type II)

**Answers**

5) false
7) 4

# Chapter 5
## COMPLEX NUMBERS AND QUADRATIC EQUATIONS

**INTRODUCTION**

$\sqrt{-36}$, $\sqrt{-25}$ etc do not have values in the system of real numbers.
So we need to extend the real numbers system to a larger system.
Let us denote $\sqrt{-1}$ by the symbol i.
ie $i^2 = -1$
A number of the form a+ib where a&b are real numbers is defined to be a complex number.
Eg 2+i3, -7+$\sqrt{2}$ i, $\sqrt{3}$ i, 4+1i, 5=5+0i, -7=-7+0i etc

For z = 2+i5, Re z =2 (real part)
and Im z = 5 (imaginary part)
Refer algebra of complex numbers of text book pg 98

1) **Addition of complex numbers**

$$(2+i3) + (-3+i2) = (2+-3) + i(3+2) = -1+5i$$

2) **Difference of complex numbers**

$$(2+i3)-(-3+i2) = (2+3) + i(3-2) = 5 + i$$

3) **Multiplication of two complex numbers**

$$\begin{aligned}(2+i3)(-3+i2) &= 2(-3+i2) + i3(-3+i2)\\ &= -6+4i-9i+6i^2\\ &= -6-5i-6 \qquad (i^2 = -1)\\ &= -12-5i\end{aligned}$$

4) **Division of complex numbers**

$$\begin{aligned}\frac{2+i3}{-3+i2} &= \frac{(2+i3)}{(-3+i2)} \times \frac{(-3-i2)}{(-3-i2)}\\ &= \frac{-6-4i-9i-6i^2}{(-3)^2-(i2)^2}\\ &= \frac{-6-13i+6}{9-(-1)\times 4}\\ &= \frac{-13i}{13} = -i\end{aligned}$$

5) **Equality of 2 complex numbers**

$a+ib = c+id$, iff $a=c$ & $b=d$

6) $a+ib = 0$, iff $a=0$ and $b=0$

Refer : the square roots of a negative real no & identities (text page 100,101)

**Formulas**

a) IF $Z=a+ib$ then modulus of Z ie $|Z| = (a^2+b^2)^{1/2}$

b) Conjugate of Z is a-ib

c) **Multiplicative inverse of a+ib** = $\dfrac{a}{(a^2+b^2)} - \dfrac{ib}{(a^2+b^2)}$

****d) Polar representation of a complex number**

$\boxed{a+ib = r(\cos\theta + i\sin\theta)}$

Where $r = |Z| = (a^2+b^2)^{1/2}$ and $\theta = \arg Z$(argument or amplitude of Z which has many different values but when $-\pi < \theta \leq \pi$, $\theta$ is called principal argument of Z.

**Trick method to find θ**

Step 1First find angle using the following

1) $\cos\theta = 1$ and $\sin\theta = 0$ then angle = 0
2) $\cos\theta = 0$ and $\sin\theta = 1$ then angle = $\pi/2$
3) $\sin\theta = \sqrt{3}/2$ and $\cos\theta = 1/2$ then angle = $\pi/3$
4) $\sin\theta = ½$ and $\cos\theta = \sqrt{3}/2$ then angle = $\pi/6$

Step 2: **To find θ**

1) If both sinθ and cosθ are positive then θ = angle (first quadrant)
2) If sinθ positive, cosθ negative then θ = π-angle (second quadrant)
3) If both sinθ and cosθ are negative the θ= π+angle (third quadrant)

4) If sinθ negative and cosθ positive then θ= 2π-angle (fourth quadrant)

   Or θ = - (angle) since $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$

5) If $\sin\theta = 0$ and $\cos\theta = -1$ then $\theta=\pi$

****e) Formula needed to find square root of a complex number**

$(a+b)^2 = (a-b)^2 + 4ab$

ie $[x^2 + y^2]^2 = [x^2 - y^2]^2 + 4x^2y^2$

**e) Powers of i**

**i)** $i^{4k} = 1$

**ii)** $i^{4k+1} = i$

**iii)** $i^{4k+2} = -1$

**iv)** $i^{4k+3} = -i$**, for any integer k**

**Examples:**

$i^1 = i, i^2 = -1, i^3 = -i$ and $i^4 = 1$&

$$i^{19} = i^{16} \times i^3 = 1 \times -i = -i$$

g) Solutions of quadratic equation $ax^2+bx + c = 0$ with real coefficients a,b,c and $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ , If $b^2 - 4ac \geq 0$

If $b^2 - 4ac < 0$ then $x = \frac{-b \pm \sqrt{4ac-b^2}\ i}{2a}$

Refer text page 102 the modulus and conjugate of a complex number properties given in the end. (i) to (v)
Ex 5.1
Q. 3*(1 mark), 8* (4 marks), 11**, 12**, 13**, 14**(4 Marks)

**Polar form (very important)**

**Ex 5.2**

Q 2**) Express Z = $-\sqrt{3}+i$ in the polar form and also write the modulus and the argument of Z

Solution Let $-\sqrt{3}+i = r(\cos\theta + i\sin\theta)$

Here $a = -\sqrt{3}$ , $b = 1$

$r = (a^2+b^2)^{1/2} = \sqrt{3+1} = \sqrt{4} = 2$

$-\sqrt{3}+i = 2\cos\theta + i \times 2\sin\theta$

Therefore $2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\cos\theta = -\sqrt{3}/2$ and $\sin\theta = ½$

Here $\cos\theta$ negative and $\sin\theta$ positive

Therefore $\theta = \pi - \pi/6 = 5\pi/6$ (see trick method given above)

Therefore polar form of $Z = -\sqrt{3}+i = 2(\text{Cos}5\pi/6 + i\text{Sin}5\pi/6)$

$|Z| = 2$ and argument of $Z = 5\pi/6$ and $-\sqrt{3} + i = 2(Cos5\pi/6 + iSin5\pi/6)$

**Ex 5.2**

Q (1 to 8)** Note: Q 1) $\theta = 4\pi/3$ or principal argument $\theta = 4\pi/3 - 2\pi = -2\pi/3$

Q 5) $\theta = 5\pi/4$ or principal argument $\theta = 5\pi/4 - 2\pi = -3\pi/4$

eg 7**, eg 8**

**Ex 5.3**

Q 1,8,9,10 (1 mark)

Misc examples (12 to 16)**

**Misc exercise**

Q 4**,5**,10**,11**,12**,13**,14**,15**,16**,17*,20**

Supplementary material

eg 12**

**Ex 5.4**

Q (1 to 6)**

**EXTRA/HOT QUESTIONS**

1** Find the square roots of the following complex numbers (4 marks)

i. $6 + 8i$
ii. $3 - 4i$
iii. $2 + 3i$ (HOT)
iv. $7 - 30\sqrt{2}i$
v. $\dfrac{3 + 4i}{3 - 4i}$ (HOT)

2** Convert the following complex numbers in the polar form

i. $3\sqrt{3} + 3i$
ii. $\dfrac{1 - i}{1 + i}$

iii. 1 + i

iv. $-1 + \sqrt{3}i$

v. -3 + 3i

vi. -2 –i

3. If a+ ib $=\frac{x+i}{x-i}$ where x is a real, prove that $a^2 + b^2 = 1$ and $b/a = 2x/(x^2-1)$ 4marks

4 Find the real and imaginary part of i. (1 mark)

5 Compute : $i + i^2 + i^3 + i^4$ (1 mark)

6 Solve the following quadratic equations (I mark)

i) $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

ii) $2x^2 + 5 = 0$

7 Find the complex conjugate and multiplicative inverse of (4 mark)

i) $(2 - 5i)^2$

ii) $\frac{2 + 3i}{3 - 7i}$

8 If $|Z| = 2$ and arg Z = π/4 then Z= ________. (1 mark)

**Answers**

1) i) $2\sqrt{2} + \sqrt{2}i$, $-2\sqrt{2} - \sqrt{2}i$

ii) 2 –i, -2 + i

iii) $\frac{\sqrt{\sqrt{13}+2}}{\sqrt{2}} + \frac{\sqrt{\sqrt{13}-2}\ i}{\sqrt{2}}$ , $\frac{\sqrt{\sqrt{13}+2}}{\sqrt{2}} + \frac{\sqrt{\sqrt{13}-2}\ i}{\sqrt{2}}$,

iv) $5 - 3\sqrt{2}\,i$ , $-5 + 3\sqrt{2}\ i$

v) 3/5 + 4/5 i, -3/5 -4/5 i

2) i) 6(cos π/6 + isinπ/6)

ii) cos (–π/2) + isin(-π/2)

iii) $\sqrt{2}$(cos π/4 + isinπ/4)

iv) 2(cos2π/3 + isin 2π/3)

iv) $3\sqrt{2}$ (cos 3π/4 + isin 3π/4)

vi) $2\sqrt{2}(\cos 5\pi/4 + i\sin 5\pi/4)$ or $2\sqrt{2}[\cos(-3\pi/4) + i\sin(-3\pi/4)]$

4) 0,1

5) 0

6) i) $\sqrt{2}$ , 1

ii) $\sqrt{\frac{5}{2}}\, i$ , $-\sqrt{\frac{5}{2}}\, i$

7) i) $-21 + 10i$, $\frac{-21}{541} - \frac{10}{541} i$

ii) $-\frac{15}{58} - \frac{23}{58}i$, $\frac{3 - 7i}{2 + 3i}$

8) $\sqrt{2} + i\sqrt{2}$

# CHAPTER 6

# LINEAR INEQUALITIES

## Rules for solving inequalities

The following rules can be applied to any inequality

- Add or subtract the same number or expressions to both sides.
- Multiply or divide both sides by the same positive number.
- By multiplying or dividing with the same negative number, the inequality is reversed.
- $a<b$ implies $b>a$
- $a<b$ implies $-a>-b$
- $a<b$ implies $1/a > 1/b$
- $x^2 \leq a^2$ implies $x \leq a$ and $x \geq -a$

**Recall**

- $x = 0$ is y axis
- $y=0$ is x axis
- $x=k$ is a line parallel to y axis passing through $(k,0)$of x axis.
- $y=k$ is a line parallel to x axis and passing through $(0,k)$of y axis.

### Procedure to solve a linear inequality

- Simplify both sides by removing group symbols and collecting like terms.
- Remove fractions by multiplying both sides by an appropriate factor.
- Isolate all variable terms on one side and all constants on the other side.
- Make the coefficient of the variable 1 and get the solution (whenever coefficient of x is negative multiply through out by -1 so that the ineqality is reversed.

Eg $-2x \geq 5$

$2x \leq -5$

$x \leq -5/2$

$x \leq -2.5$ therefore solution is $(-\infty,-2.5]$

**Note:** If **$x\geq 0$, $y\geq 0$** is given in question every point in the shaded region in the **first quadrant** including the points on the line and the axis, represents the solution of the given system of ineqalities.

# Method to find Graphical Solution

- Draw lines corresponding to each equation treating it as equality
- Find the Feasible Region – intersection of all the inequalities

Steps to find Feasible region

Step 1: Take any point on the left (down) or right (up) of the line and substitute that in the given inequality.

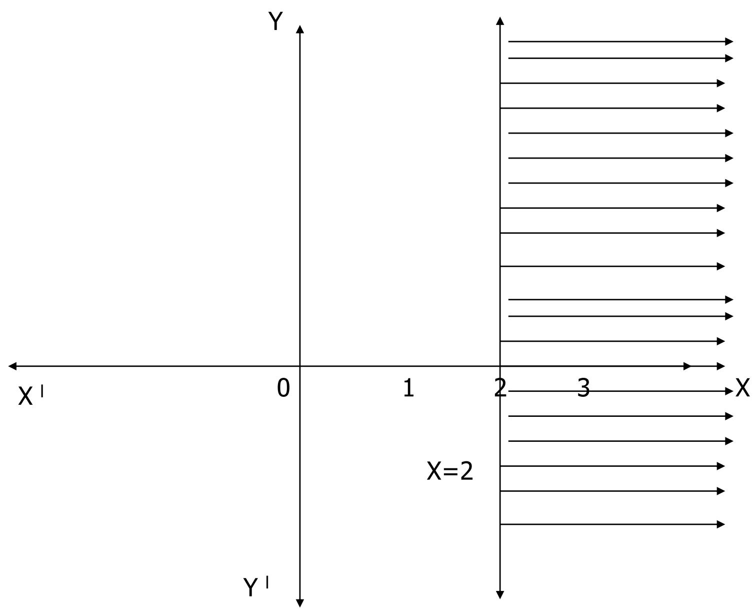
Step 2: If the point satisfies the inequality, the region containing that point is the required region. Otherwise opposite region is the required region.

Step 3: If inequality is of the type ≥ , ≤ then the points on the line are also included in the solution region, so draw dark lines.

Step 4: If inequality is of the type >, < then the points on the line are not included in the solution region, so draw dotted lines

Note: For the following in equation $x \geq 2$, substitute the point (0,0) (left side) in $x \geq 2$. Then we get $0 \geq 2$ which is false. Therefore right side is the required region.

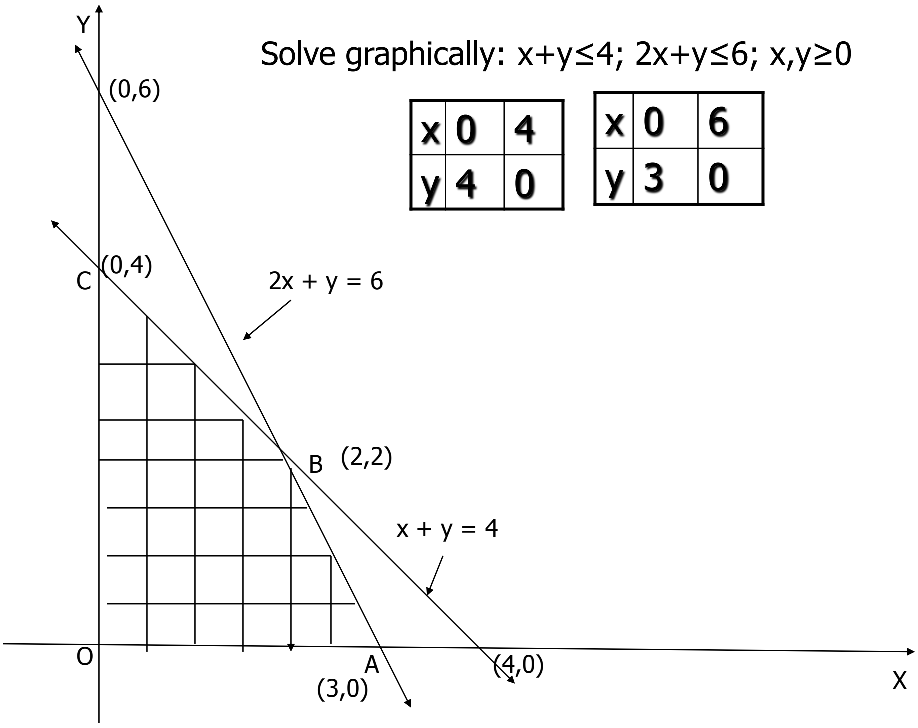
Shade the region corresponding to the inequation $x \geq 2$



Solve graphically: $x+y \le 4$; $2x+y \le 6$; $x, y \ge 0$

| | | |
|---|---|---|
| x | 0 | 4 |
| y | 4 | 0 |

| | | |
|---|---|---|
| x | 0 | 6 |
| y | 3 | 0 |




## Solve graphically

$x + y \le 400$

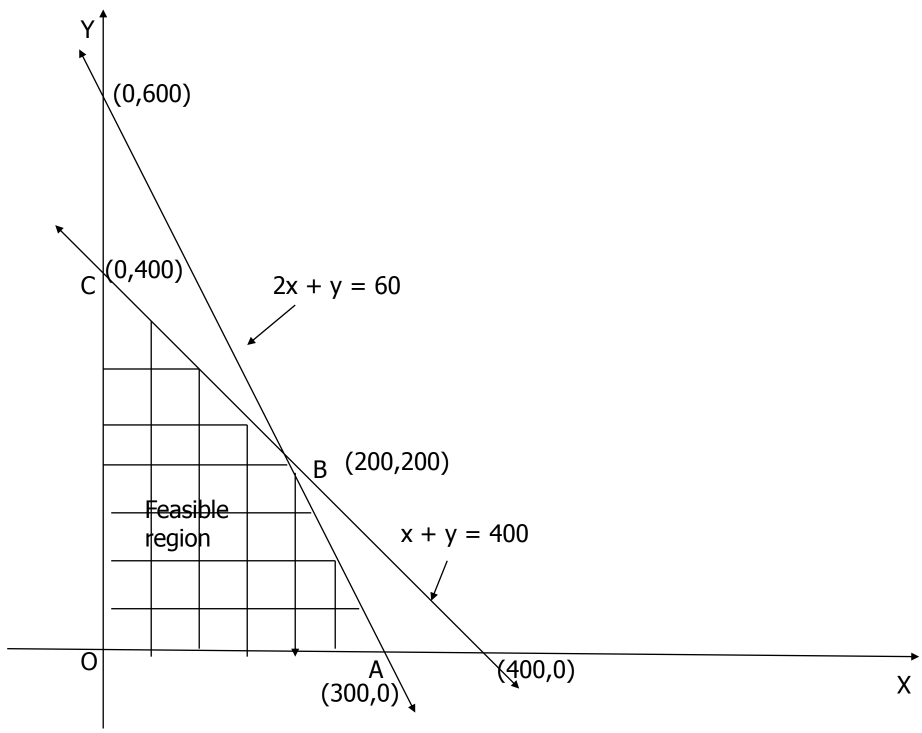
$2x + y \le 600$

Consider x+y=400

| | | |
|---|---|---|
| x | 0 | 400 |
| y | 400 | 0 |

## Consider 2x+y=600

| | | |
|---|---|---|
| X | 0 | 300 |
| y | 600 | 0 |




$x + y \leq 20$

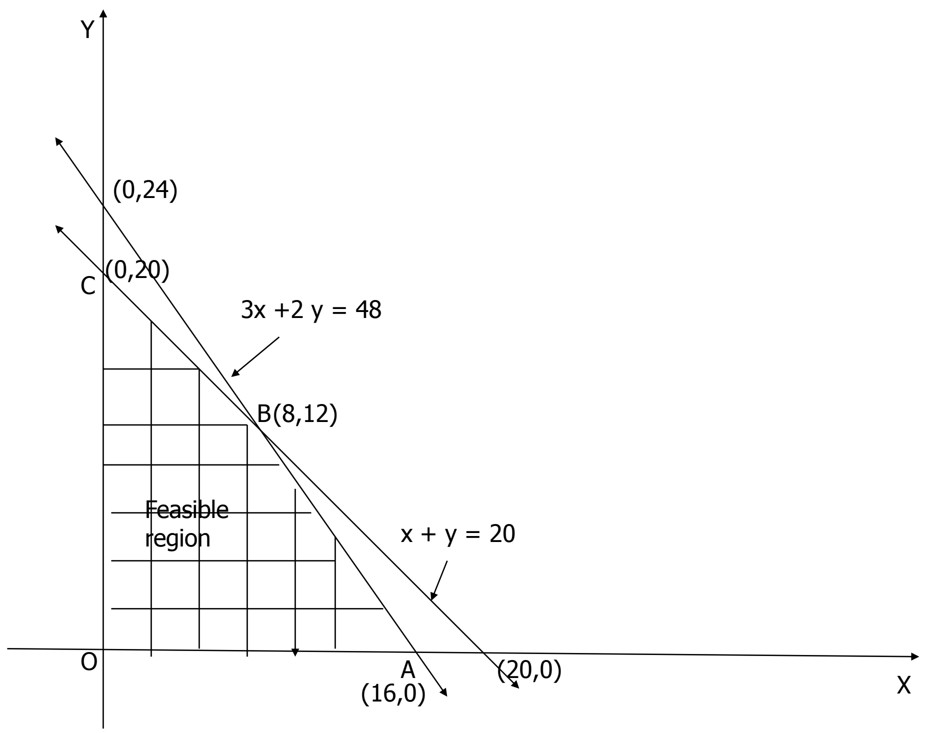
$360x + 240y \leq 5760$

## Consider x+y=20

| | | |
|---|---|---|
| x | 0 | 20 |
| y | 20 | 0 |

## Consider 3x+2y=48 (Dividing throughout by 120)

| | | |
|---|---|---|
| X | 0 | 16 |
| y | 24 | 0 |



Solve graphically:

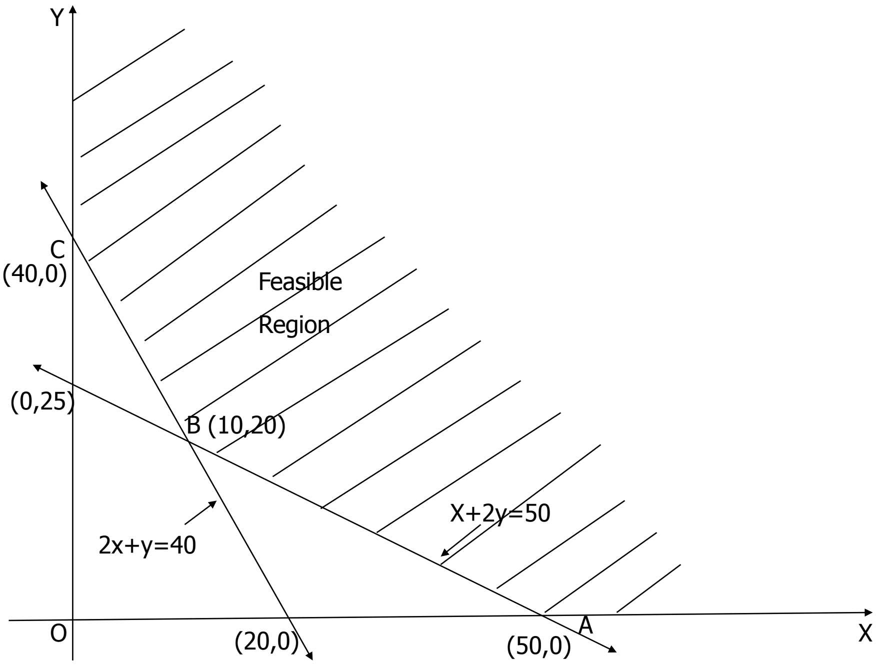
- $200X + 100Y \geq 4000$
- $X + 2Y \geq 50$

## Consider 2x+y=40

| | | |
|---|---|---|
| X | 0 | 20 |
| y | 40 | 0 |

## Consider x+2y=40

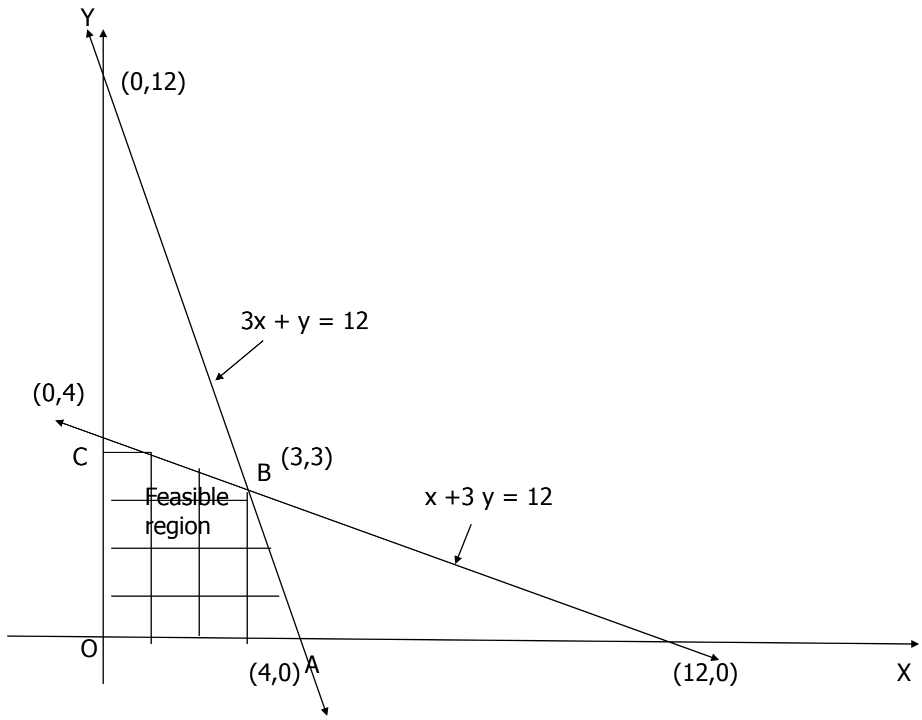
| | | |
|---|---|---|
| X | 0 | 50 |
| y | 25 | 0 |




**Solve graphically:**

$x+3y \leq 12$

$3x+y \leq 12$




**Solve graphically:**

$x/10 + y/20 \geq 14$

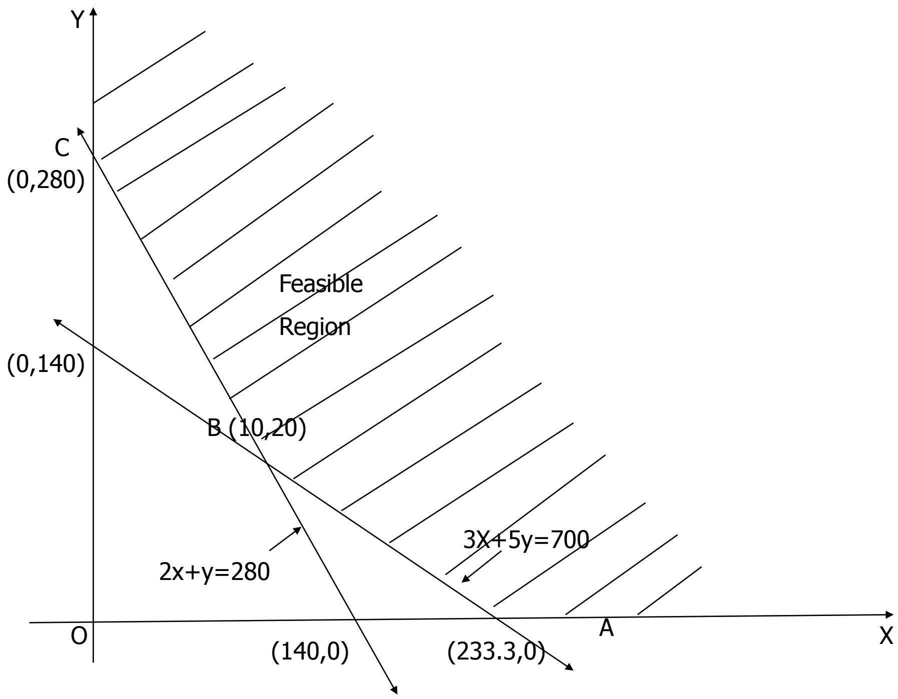
$3/50\, x + 1/10\, y \geq 14$

## Consider 2x+y=280

| | | |
|---|---|---|
| X | 0 | 140 |
| y | 280 | 0 |

## Consider 3x+5y=700

| | | |
|---|---|---|
| X | 0 | 233.3 |
| y | 140 | 0 |




**Ex 6.1**

1to 4 (1 mark)

11 to 15, 16**, 18**, 20**, 22**, 24**, 25**,26**

eg 15**

**Ex 6.3**

(8 to 15)**

eg 16**

eg17**

**Misc Ex**

4*, 5*,6*, 8*, 9*, 10*, (11 to 14)**

**EXTRA /HOT QUESTIONS**

1) Solve i) $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

ii) $\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$

iii) $\frac{2-3x}{5} < \frac{1-x}{3} < \frac{3+4x}{2}$

2) Solve graphically
   i) $2x+y \geq 3,\ x-2y \leq -1,\ x\geq0,\ y\geq0$
   ii) $x+4y \leq 4,\ 2x+3y \leq 6,\ x\geq0,\ y\geq0$
   iii) $x+y \geq 1,\ x \leq 5,\ y \leq 4,\ 2x + 3y \leq 12,\ x\geq0,\ y\geq0$
3) The water acidity in a pool is considered normal when the average pH reading of 3 daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal
4) In the first 4 papers each of hundred marks Ravi got 90, 75, 73, 85 marks. If he wants an average of greater than or equal to 75 marks and less than 80 marks, find the range of marks she should score in the fifth paper.
5) The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm find the minimum length of the shortest side.

**Answers**

1) i) $(4,\infty)$

ii) $(-\infty, 63/10]$

iii) $(1/4 , \infty)$

3 Between 7.77 and 8.67

4 More than or equal to 52 but less than 77

5 9 cm

# CHAPTER 7

## PERMUTATIONS (Arrangements) AND COMBINATIONS (selections)

In permutation **order is important**, since 27 & 72 are different numbers(arrangements). In combination order is not important.

- **Fundamental principle of counting (FPC)**

| Station A | m ways | Station B | n ways | Station C |
|---|---|---|---|---|

then by FPC there are mn ways to go from station A to station C

- The number of permutations of n different things taken r at a time, where repetition is not allowed is given by $\mathbf{{}^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1)}$ **where $0<r\leq n$.**

  eg $\quad {}^5P_2 = 5 \times 4 = 20$

  $\quad\quad {}^7P_3 = 7 \times 6 \times 5 = 210$

- Factorial notation: **$n! = 1 \times 2 \times 3 \times \ldots\ldots \times n$, where n is a natural number**

  eg $5! = 1 \times 2 \times 3 \times 4 \times 5$

  **we define $0! = 1$**

  also $n! = n(n-1)!$

  $\quad = n(n-1)(n-2)!$

- $\mathbf{{}^nP_r = \dfrac{n!}{(n-r)!}}$ **Where $0 \leq r \leq n$**

- Number of permutations of n different things, taken r at a time, where repetition is allowed is $\mathbf{n^r}$

- Number of permutations of n objects taken all at a time, where $P_1$ objects are of first kind, $P_2$ objects are of second kind…..$P_k$ objects are of the $k^{th}$ kind and rest, if any, are all different is $\mathbf{\dfrac{n!}{P_1!.P_2!\ldots P_k!}}$ **(eg 9)**

- The number of combinations of n different things taken r at a time is given by

  $\mathbf{{}^nC_r = \dfrac{n(n-1)(n-2)\ldots.(n-r+1)}{1.2.3\ldots\ldots r}}$, **where $0< r\leq n$**

  eg ${}^5C_3 = \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} = {}^5C_2$

- $^nC_r = {^nC_{n-r}}$
  eg $^5C_3 = {^5C_2}$
  $^7C_5 = {^7C_2}$
- $^nC_r = \dfrac{n!}{r!(n-r)!}$, **where $0 \le r \le n$.**
- $^nC_r = {^nC_s}$ **implies r = s or n = r+s** (eg 17* )   1 mark
- $^nC_n = {^nC_0} = 1$
- $^nC_1 = n$
  eg $^5C_1 = 5$
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

**Ex 7.1**

1, 2, 4

**Ex 7.2**

4* , 5* (1 mark)

eg 8* (1 mark), eg 11*, 12**,13**,14**,16**  (4 marks)

**Ex 7.3**

7*, 8*, 9**,10**,11**

Theorm 6 to prove (4 marks)*

eg 17* (1 mark) use direct formula n = 9+8 = 17 since $^nC_r = {^nC_s}$ implies r = s or n = r+s

eg 19**

**Ex 7.4**

2**,3*,5*,6*,7**,8*,9*

eg 21**, eg 23*(HOT), eg 24*

**Misc Ex**

1**,2**,3**,4*,5*,7**,10**,11**

**EXTRA/HOT QUESTIONS**

1) How many permutations can be made with letters of the word MATHEMATICS ? In how many of them vowels are together?
2) In how many ways can 9 examination papers be arranged so that the best and the worst papers are never together. (HOT)
3) How many numbers greater than 56000 can be formed by using the digits 4,5,6,7,8; no digit being repeated in any number.
4) Find the number of ways in which letters of the word ARRANGEMENT can be arranged so that the two A’s and two R’s do not occur together. (HOT)
5) If C(2n,3): C(n,3):: 11:1 find n.
6) If P(11,r) = P(12,r-1) find r.

7) Five books, one each in Physics, Chemistry, Mathematics, English and Hindi are to be arranged on a shelf. In how many ways can this be done?
8) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ find the values of n and r.
9) A box contains five red balls and six black balls. In how many ways can six balls be selected so that there are at least two balls of each color.
10) A group consist of 4 girls and 7 boys in how many ways can a committee of five members be selected if the committee has i) no girl
    ii) atleast 1 boy and 1 girl
    iii) atlest 3 girls.

Note : atleast means $\geq$

**Answers**

1) 4989600, 120960
2) 282240 Hint (consider the best and the worst paper as one paper)
3) 90
4) 1678320
5) 6
6) 9
7) 120
8) n = 3, r = 2
9) 425
10) i) 21
    ii) 441
    iii) 91

# CHAPTER 8

## BINOMIAL THEOREM

Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + {}^nC_3a^{n-3}b^3 + \ldots\ldots + {}^nC_nb^n$$

Recall

1) ${}^nC_r = \dfrac{n!}{(n-r)!\, r!}$
2) ${}^nC_r = {}^nC_{n-r}$

   ${}^7C_4 = {}^7C_3 = \dfrac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

   ${}^8C_6 = {}^8C_2 = \dfrac{8 \times 7}{1 \times 2} = 28$
3) ${}^nC_n = {}^nC_0 = 1$
4) ${}^nC_1 = n$

**OBSERVATIONS/ FORMULAS**

1) The coefficients ${}^nC_r$ occurring in the binomial theorem are known as binomial coefficients.
2) There are (n+1) terms in the expansion of $(a+b)^n$, ie one more than the index.
3) The coefficient of the terms equidistant from the beginning and end are equal.
4) $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots\ldots + {}^nC_nx^n$. (By putting a = 1 and b = x in the expansion of $(a + b)^n$).
5) $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - {}^nC_3x^3 + \ldots\ldots + (-1)^n\, {}^nC_nx^n$ (By putting a = 1 and b = -x in the expansion of $(a + b)^n$).
6) $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n$ (By putting x = 1 in (4))
7) $0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots\ldots + (-1)^n\, {}^nC_n$. (By putting x = 1 in (5))

8**) (r + 1)$^{th}$ term in the binomial expansion for $(a+b)^n$ is called the general term which is given by

$$\boxed{\mathbf{T_{r+1} = {}^nC_r\, a^{n-r}\, b^r.}}$$

i.e to find 4$^{th}$ term = $T_4$, substitute r = 3.

9*) **Middle term** in the expansion of $(a+b)^n$

i) If **n is even**, middle term = $\left[\frac{n}{2}+1\right]^{th}$ term.

ii) If **n is odd**, then 2 middle terms are, $\left[\frac{n+1}{2}\right]^{th}$ term and $\left[\frac{n+1}{2}+1\right]^{th}$ term.

10*) To find the **term independent of x or the constant term,** find the coefficient of $x^0$.(ie put power of x = 0 and find r)

**Problems**

eg 4** (4 marks)

**Ex 8.1**

Q 2,4,7,9 (1 mark)

10*, 11*,12* (4 marks)

13**,14** (4 marks)

13**) Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer

Or

$3^{2n+2} - 8n - 9$ is divisible by 64

Solution: $9^{n+1} - 8n - 9 = (1+8)^{n+1} - 8n - 9$

$$= {}^{n+1}C_0 + {}^{n+1}C_1 8 + {}^{n+1}C_2 8^2 + {}^{n+1}C_3 8^3 + \ldots\ldots + {}^{n+1}C_{n+1} 8^{n+1} - 8n - 9$$

$$= 1 + 8n + 8 + 8^2\,[\,{}^{n+1}C_2 + {}^{n+1}C_3.8 + \ldots\ldots + 8^{n-1}] - 8n - 9$$

(since ${}^{n+1}C_0 = {}^{n+1}C_{n+1} = 1$, ${}^{n+1}C_1 = {}^{n+1}$,

$8^{n+1}/8^2 = 8^{n+1-2} = 8^{n-1}$)

$$= 8^2\,[\,{}^{n+1}C_2 + {}^{n+1}C_3.8 + \ldots\ldots + 8^{n-1}]$$ which is divisible by 64

**Problems**

eg 5*,6**,7* (4 marks)

eg 8**,9** (6 marks)

**Ex 8.2**

Q 2,3* (1 mark)

Q 7**,8**,9**,11**,12** (4 marks), 10** (6 marks)

eg 10**,11 (HOT),12 (HOT), 13(HOT), eg 15*,17** (4 marks)

**Misc ex**

Q 1** (6 mark),2,3(HOT), 8* (4 marks)

**Ex 8.2**

Q 10**(6 marks)

The coefficients of the $(r-1)^{th}$ , $r^{th}$ and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1: 3 : 5. Find n and r.

Solution

$T_{r+1} = {}^nC_r x^{n-r}$

$T_r = T_{(r-1)+1} = {}^nC_{r-1} x^{n-r+1}$

$T_{r-1} = T_{(r-2)+1} = {}^nC_{r-2}\, x^{n-r+2}$

Given ${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r :: 1 : 3 : 5$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3}$$

$$\frac{n!}{(n-r+2)!\,(r-2)!} \div \frac{n!}{(n-r+1)!\,(r-1)!} = \frac{1}{3}$$

$$\frac{(n-r+1)!}{(n-r+2)!} \times \frac{(r-1)!}{(r-2)!} = \frac{1}{3}$$

$$\frac{(n-r+1)!}{(n-r+1)!(n-r+2)} \times \frac{(r-2)!(r-1)}{(r-2)!} = \frac{1}{3}$$

$$\frac{r-1}{n-r+2} = \frac{1}{3}$$

$3r-3 = n-r+2$

$n-4r = -5$ ________(1)

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

simplify as above and get the equation $3n - 8r = -3$ ________(2)
solving (1) and (2) we get
$n = 7$ and $r = 3$.

**EXTRA/HOT QUESTIONS**

1) Using Binomial theorem show that $2^{3n} - 7n - 1$ or $8^n - 7n - 1$ is divisible by 49 where n is a natural number. (4 marks**)
2) Find the coefficient of $x^3$ in the equation of $(1+2x)^6 (1-x)^7$ (HOT)
3) Find n if the coefficient of $5^{th}$, $6^{th}$ & $7^{th}$ terms in the expansion of $(1+x)^n$ are in A.P.
4) If the coefficient of $x^{r-1}$, $x^r$, $x^{r+1}$ in the expansion of $(1+x)^n$ are in A.P. prove that $n^2 - (4r+1)n + 4r^2 - 2 = 0$. (HOT)
5) If $6^{th}$,$7^{th}$,$8^{th}$ & $9^{th}$ terms in the expansion of $(x+y)^n$ are respectively a,b,c &d then show that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$ (HOT)
6) Find the term independent of x in the expansion of $\left[3x^2 - \frac{1}{2x^3}\right]^{10}$ (4 marks*)
7) Using Binomial theorem show that $3^{3n} - 26n - 1$ is divisible by 676. (4 marks**)
8) The $3^{rd}$,$4^{th}$ & $5^{th}$ terms in the expansion of $(x+a)^n$ are 84, 280 & 560 respectively. Find the values of x, a and n. (6 marks**)
9) The coefficient of 3 consecutive terms in the expansion of $(1+x)^n$ are in the ratio 3 : 8 :14. Find n. (6 mark**)
10) Find the constant term in the expansion of $(x-1/x)^{14}$
11) Find the middle term(s) in the expansion of

    i) $\left[\frac{x}{a} - \frac{a}{x}\right]^{10}$ ii) $\left[2x - \frac{x^2}{4}\right]^9$

12) If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$
    Prove that $C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n = n.2^{n-1}$

**Answers**

2) -43
3) n = 7 or 14
6) 76545/8
8) x =1, a=2, n = 7
9) 10
10) -3432
11) i) -252
    ii) $\frac{-63}{32} x^{14}$

# Chapter 9

## SEQUENCES AND SERIES

**Arithmetic progression (A.P)**

Standard AP $\rightarrow$ a, a+d, a+2d.........a+(n-1)d

$A_n$ = a+ (n-1) d

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(a + an)$

Arithmetic mean A between the two numbers a and b is

$A = \frac{a+b}{2}$

If $A_1, A_2, \ldots A_n$ are n A.M between the two numbers a and b,

Then $d = \frac{b-a}{n+1}$

$A_1 = a + d = a + \frac{b-a}{n+1}$

$A_2 = a + 2d = a + 2\frac{b-a}{n+1}$

........................

$A_n = a + nd = a + n\frac{b-a}{n+1}$

**Geometric progression (G.P)**

Standard GP $\rightarrow$ a, ar ,$ar^2$.......$ar^{n-1}$

$A_n = ar^{n-1}$

$S_n = \frac{a(r^n-1)}{r-1}$ or $\frac{a(1-r^n)}{1-r}$ if $r \neq 1$

$S_\infty = \frac{a}{1-r}$ if $|r| < 1$

If G is the GM between a and b, then $G = \sqrt{ab}$

If $G_1, G_2, \ldots G_n$ are n G.M between the two numbers a and b ,

then $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$

...............................

$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

Sum to n terms of special series

$S_n = 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

$S_n = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{\{n(n+1)\}^2}{4}$

## TEXT BOOK QUESTIONS

* → Exercise 9.2 → Qns 5,7,8,11,14

* → Exercise 9.3 → Qns 2,3,5,11,16,17,19,21,23,25

* → Exercise 9.4 → Qns 3,4,5,6,7

* → Misc Exercise → Qns 3,4,5,10,12,14,18,21

** → Exercise 9.2 → Qns 9,10,12,13,15

** → Exercise 9.3→ Qns 12,13,14,15,18,22,26,27,28

** → Exercise 9.4 → Qns 1,2,8,9,10

** → Misc Exercise → Qns 19,22,23,24, 25,26

** → Examples 4,5,6,10,13,18,21

## EXTRA/ HOT QUESTIONS

1. Which term of the sequence 25, $24\frac{1}{4}$, $23\frac{1}{2}$, $22\frac{3}{4}$, .....is the first negative term. (Ans.35)

2. How many terms are identical in the two AP.

   2,4,6,.........up to 100 terms and 3,6,9.............. up to 80 terms (Ans.33)

3. solve for x : 1+4+7+..........+x = 590 (Ans.x=58)

4. Find the sum of all the three digit numbers which leaves the reminder 2 when divided by 5. (Ans.98910)

5. The digits of a three digit natural number are in AP and their sum is 15 .The number obtained by reversing the digits is 396 less than the original number. Find the number.

6. If $p^{th}$, $q^{th}$, and $r^{th}$ terms of GP are in GP. Show that p,q,r are in AP

7. If a,b,c,d are in GP, then show that $a^2+b^2$, $b^2+c^2$, $c^2+d^2$ are in GP

8. Evaluate $7^{\frac{1}{2}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{8}}$ ………………… to infinite terms.

9. The common ratio of a GP is (- 4/5) and sum to infinity is (80/9). Find the first term. (Ans.7)

10. If $S_1, S_2, S_3$ are the sums of first n, 2n, 3n terms of a GP. Then Show that $s_1(s_3-s_2) = (s_2-s_1)^2$

11. $\frac{1}{x+y}$ , $\frac{1}{y+z}$, $\frac{1}{x+z}$ are in AP Show that $y^2$ , $x^2$ and $z^2$ are in AP .

12. Find the sum of $10^3 + 11^3 + \ldots\ldots + 20^3$ (Ans.42075)

13. Find the $n^{th}$ term and the sum of n terms of the series

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots\ldots\ldots\ldots\ldots\ldots$

14. Find the sum of n terms of $1^3 + \frac{1^3+2^3}{2} + \frac{1^3+2^3+3^3}{3} + \ldots\ldots$

15. If AM and GM of roots of a quadratic equation are 8 and 5 respectively, then write the quadratic equation. (Ans.$x^2-16x+25=0$)

# Chapter 10

## STRAIGHT LINES

**SLOPE OF A LINE** : m = tanθ if θ is the angle of inclination.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ if $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the line.

SLOPE of a horizontal line is 0 and vertical line is not defined.

If $m_1$ and $m_2$ are slopes of $L_1$ and $L_2$ respectively.

$L_1 \parallel L_2 \rightarrow m_1 = m_2$

$L_1 \perp L_2 \rightarrow m_1 \times m_2 = -1$

Acute angle between $L_1$ and $L_2$

$\tan\theta = \left|\frac{m2 - m1}{1 + m1 \times m2}\right|$ $as\ 1 + m_1 m_2 \neq 0\ and$ the obtuse angle $\emptyset = 180 - \theta$.

**EQUATION OF STRAIGHT LINE**

x-axis → $y = 0$

y-axis → $x = 0$

|| to x-axis → $y = b$

|| to y-axis → $x = a$

Having slope m and making an intercept c on y-axis→ $y = mx + c$

Making intercepts a and b on the x-axis and y-axis → $\frac{x}{a} + \frac{y}{b} = 1$

passing through $(x_1, y_1)$ and $(x_2, y_2)$ → $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Having normal distance from orgin P and angle between the normal and positive x-axis $\omega$ → $x \cos\omega + y \sin\omega = P$.

General form → $Ax + By + C = 0$

Distance of a point $(x_1, y_1)$ from a line $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

## TEXT BOOK QUESTIONS

* → Exercise 10.1 → Qns 5,8,9

* → Exercise 10.2 → Qns 7,8,9,10,11,16

* → Exercise 10.3 → Qns 3,4,5,7,8,9,10,12,16

* → MiscExercis→ Qns 1,6,7,8,9,12,14,15,23

** → Exercise 10.1 → Qns 11,13

** → Exercise 10.2 → Qns 12,13,15,18,20

** → Exercise 10.3 → Qns 13,14,17,18

** → Misc Exercise → Qns 3,4,11,18,19

** → Example → 2,3,13,14,15,17,19,20,23

Misc Example →23

## EXTRA/ HOT QUESTIONS

1. Find the equation of the line through (4,-5) and parallel to the line joining the points (3,7) & (-2,4).

   (Ans.3x-5y-37=0)

2. If A(1,4) , B(2,-3) and C(-1,-2) are the vertices of a triangle ABC . find
   a) The equation of the median through A
   b) The equation of the altitude through A
   c) The right bisector of side BC
3. Find the equation of the straight line which passes through (3,-2) and cuts off positive intercepts on the x axis and y axis which are in the ratio 4:3
4. Reduce the equation 3x-2y+4 =0 to intercept form. Hence find the length of the segment intercepted between the axes.
5. Find the image of the point (1,2) in the line x-3y+4 =0
6. If the image of the point (2,1) in a line is (4,3) .Find the equation of the line.

7. Find the equation of a line passing through the point (-3,7) and the point of intersection of the lines 2x-3y+5 = 0 and 4x+9y = 7.

   (Ans.8x+3y+3=0)

8. Find the equation of straight lines which are perpendicular to the line

12x+5y = 17 and at a distance of 2 units from the point (-4,1)

(ans. 5x-12y+6=0 & 5x-12y+58=0)

9.The points A(2,3) B(4,-1) & C(-1,2) are the vertices of a triangle. Find the length of perpendicular from A to BC and hence the area of ABC (Ans.$\frac{14}{\sqrt{34}}$ units & 7 sq.units)

10. Find the equation of straight line whose intercepts on the axes are thrice as long as those made by 2x +11y = 6

(Ans.2x+11y =18)

# Chapter 11

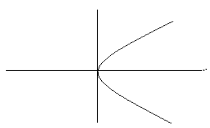
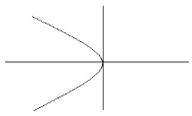
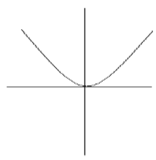
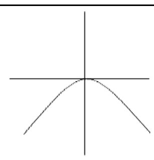
# CONIC SECTION

**CIRCLE:**

The equation of a circle with centre at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$

Equation of a circle with centre at origin and radius r is $x^2 + y^2 = r^2$

**PARABOLA**( Symmetric about its axis)

| | Right | Left | Upward | Downward |
|---|---|---|---|---|
| Equation | $y^2 = 4ax$ | $y^2 = -4ax$ | $x^2 = 4ay$ | $x^2 = -4ay$ |
| Axis | y = 0 | y= 0 | x= 0 | x= 0 |
| Figure |  |  |  |  |
| Focus | (a, 0) | (-a, 0) | (0, a) | (0, -a) |
| Vertex | (0,0) | (0,0) | (0,0) | (0,0) |
| Latus Rectum | 4a | 4a | 4a | 4a |
| Directrix | x = -a | x = a | y = -a | y =a |

**ELLIPSE** ( Symmetric about both the axis)

| | | |
|---|---|---|
| Equation | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ |
| Equation of the major axis | y=0 | x=0 |
| Length of major axis | 2a | 2a |
| Length of minor axis | 2b | 2b |
| Vertices | $(\pm$ a, 0) | (0,$\pm$ a ) |
| Foci | $(\pm$ c, 0) | (0,$\pm$ c ) |
| Eccentricity | $e = \frac{c}{a}$ | $e = \frac{c}{a}$ |
| Latus Rectum | $\frac{2b^2}{a}$ | $\frac{2b^2}{a}$ |

**HYPERBOLA**

| | | |
|---|---|---|
| Equation | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ |
| Equation of the transverse axis | y =0 | x =0 |
| Length of transverse axis | 2a | 2a |
| Length of conugate axis | 2b | 2b |
| Vertices | (± a, 0) | (0,± a ) |
| Foci | (± c, 0) | (0,± c ) |
| Eccentricity | $e = \frac{c}{a}$ | $e = \frac{c}{a}$ |
| Latus Rectum | $\frac{2b^2}{a}$ | $\frac{2b^2}{a}$ |

TEXT BOOK QUESTIONS

* →Exercise 11.1 → Qns 10,11

* → Exercise 11.2 → Qns 5,6,8

* → Exercise 11.3 → Qns 5,6,7,8,9,10

* →Exercise 11.4 → Qns 4,5,6

* →Example → 4,17,18,19

** →Exercise 11.1 → Qns 9,12,13,14

** → Exercise 11.2 → Qns 11,12

** → Exercise 11.3 → Qns 13 to Qns 20

** → Exercise 11.4 → Qns 10 to Qns 15

Extra Questions:

1.Find the centre and the radius of $3x^2 + 3y^2 + 6x - 4y - 1 = 0$

(ans : (-1, 2/3), 4/3)

2. Find the value of p so that $x^2 + y^2 + 8x + 10y + p = 0$, is the equation of the circle of radius 7 units. (ans : -8)

3. Find the equation of the circle when the end points of the diameter are

A ( -2 ,3), B ( 3, -5) ( ans: $x^2 + y^2 - x + 2y - 21 = 0$ )

4. Find the equation of the circle circumscribing the triangle formed by the straight lines: $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$

(ans: $x^2 + y^2 - 17x - 19y + 50 = 0$ )

5. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum. ( ans : ½ x 12 x 3 sq.units)

6. Find the equation of the ellipse with eccentricity ¾ , foci on y- axis , center at the origin and passes through the point ( 6, 4) ( ans: $16x^2 + 7y^2 = 688$)

7. Find the length of major axis and minor axis of $4x^2 + y^2 = 100$

8. Find the equation of the parabola with the centre at origin, length of transverse axis 6 units and a focus at( 0, 4). (ans: $7y^2 - 9x^2 = 63$)

9. The line $5x - y = 3$ is a tangent to a circle at a point ( 2, 7) and its centre is on the line $x + 2y = 19$. Find the equation of the circle ( ans: $x^2 + y^2 - 14x - 12y + 59 = 0$ )

10. Find equation of the circle which touches the y – axis at origin and whose radius is 3 units. ( ans: $x^2 + y^2 - 6x = 0$ )

# Chapter 12

## THREE DIMENSIONAL GEOMETRY

Any point on x – axis - > ( x, 0, 0)

Any point on y – axis - > ( 0, y, 0)

Any point →¡on z – axis - > ( 0, 0, z)

Any point on XY - plane →( x, y, 0)

Any point on YZ - plane → ( 0, y, z)

Any point on ZX - plane → ( x, 0,z)

Distance between two points P ( $x_1$ , $y_1$, $z_1$ ) and Q ( $x_2$ , $y_2$, $z_2$ ) is

|PQ| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

The co- ordinates of R which divides a line segment joining the points

P ( $x_1$ , $y_1$, $z_1$ ) and Q ( $x_2$ , $y_2$, $z_2$ )

Internally and externally in the ratio m : n *are* respectively

R $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$ and

S $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

The coordinates of the centroid of the triangle whose vertices are ( $x_1$ , $y_1$, $z_1$ ) , ( $x_2$ , $y_2$, $z_2$ ) and ( $x_3$ , $y_3$, $z_3$ ) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

## TEXT BOOK QUESTIONS

*→ Exercise 12 .2 -- 3, 4, 5

→ Example – 7, 8,9, 10,11,12,13

**→ Exercise 12 .3 -- 3, 4, 5

→ Misc Q 1 to Q 6

## Extra Questions:

1.Find the distance between ( -3, 4, -6) and its image in the XY – plane.

( ans : 12 units)

2.Find the points on the y- axis which are at a distance of 3 units from the point ( 2, 3, -1)

( ans : ( 0, 1, 0), ( 0, 5, 0) )

3.If A and B are the points ( 1, 2, 3 ) and ( -1, 4 , -3) respectively then find the locus of a point P such that $PA^2 - PB^2 = 2k^2$

( ans : $2x – 2y + 6z + 6 + k^2 = 0$)

4. If the points A ( 1, 0 , -6 ), B ( -3, p, q ) and C ( -5, 9, 6) are collinear, find the values of p and q.

( ans : p = 6 , q = 2 )

5. Two vertices of a triangle are ( 2, -6 ,4) , (4, -2, 3) and its centroid is $\left(\frac{8}{2}, -1, 3\right)$ , find the third vertex.

( ans : ( 2,5,2))

# Chapter 13

# LIMITS AND DERIVATIVES

**Some standard results on limits**

1) $\lim_{x \to c} f(x) = l$ iff $\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x) = l$
2) $\lim_{x \to a} k = k$ where k is a fixed real number.
3) $\lim_{x \to c} f(x) = f(c)$ where f(x) is a real polynomial in x.
4) $\lim_{x \to c} x^n = c^n$ for all n $\in$ N
5) $\lim_{x \to c} |x| = |c|$

**Algebra of limits**

Refer : NCERT Textbook page no: 292.

## Evaluation of algebraic limits

### Type I ( Direct substitution method)

Eg: Evaluate $\lim_{x \to 1} (x^3 - x^2 + 1) = 1^3 - 1^2 + 1 = 1$

Evaluate the following limits:

NCERT Textbook Exercise 13.1 Q 1,2,3,4,5,9,11,12.

**Additional Questions and HOT Questions**

** 1) $\lim_{x \to -1} (1 + x + x^2 + \cdots + x^{10})$ Ans: 1

** 2) $\lim_{x \to -1} (x^5 + 2x^8)^{60}$ Ans: 1

** 3) $\lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ Ans:2

### Type II (Factorisation method)

Eg : Evaluate : $\lim_{x \to 1} \frac{x^3 - 1}{x-1}$

$\lim_{x \to 1} \frac{x^3 - 1}{x-1}$ ($\frac{0}{0}$ form) $= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x \to 1} (x^2 + x + 1)$

$= 1^2 + 1 + 1 = 3$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 7,8 , Example 2.

**Evaluate**:

** 1) $\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$ ans: $\frac{-1}{10}$

** 2) $\lim_{x\to \sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}\,x-8}$ ans: $\frac{8}{5}$

** 3) $\lim_{x\to 2} \frac{x^3-4x^2+4x}{x^2-4}$ ans: 0

**<u>Type III</u>**

For any positive integer n, $\boldsymbol{\lim_{x\to a} \frac{x^n-a^n}{x-a} = n\,a^{n-1}}$**(** This is applicable for any rational number n and for a$>0$.

Evaluate : $\lim_{x\to 2} \frac{x^4-16}{x-2}$

**Ans**: $\lim_{x\to 2} \frac{x^4-16}{x-2} = \lim_{x\to 2} \frac{x^4-2^4}{x-2} = 4.\ 2^{4-1} = 4.2^3 = 32$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 10 , Example 3.

1) Evaluate : $\lim_{x\to 2} \frac{x^{10}-1024}{x^5-32}$ Ans: 64

2) If $\lim_{x\to -a} \frac{x^9+a^9}{x+a} = 9$ **,** find the value of a.

**<u>Type IV</u>**

Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

**Ans**: $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{1+x-1}$

Put y = 1+x when x→ 0 , y→ 1

∴ value = $\lim_{y\to 0} \frac{\sqrt{y}-1}{y-1}$

$= \lim_{y\to 0} \frac{y^{1/2} - 1^{1/2}}{y-1}$

$= \frac{1}{2}$

**Evaluate**

1) NCERT Ex 13.1 **Q** 6

2) $\lim_{x\to 0} \frac{(1-x)^n-1}{x}$ ans $= -$n

3) $\lim_{x\to 0} \frac{(1+x)^6-1}{(1+x)^5-1}$ ans $= \frac{6}{5}$

4) $\lim_{x\to 0} \frac{(x+8)^{1/3} - 2}{x}$ ans $= \frac{1}{12}$

## Type V

Concepts : $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \cos x = 1$

$\lim_{x\to 0} \frac{\tan x}{x} = 1$

NCERT Exercise 13.1 : Q 13, 14, 16, 18, 19, 20, 21

Evaluate : $\lim_{x\to 0} \frac{\tan 8x}{\sin 3x}$ ans: $\frac{8}{3}$

Evaluate : $\lim_{x\to 0} \frac{\sin 2x + 3x}{2x + \sin 3x}$ ans: 1

## Type VI

NCERT Exercise 13.1 : Q 17

Evaluate : $\lim_{x\to 0} \frac{1-\cos 5x}{1-\cos 6x}$

## Type VII

Evaluate : $\lim_{x\to \pi} \frac{\sin(\pi-x)}{\pi\,(\pi-x)}$

Let $x = \pi + h$

$$\therefore \lim_{x\to \pi} \frac{\sin(\pi-x)}{\pi\,(\pi-x)} = \lim_{h\to 0} \frac{\sin[\pi-(\pi+h)]}{\pi[\pi-(\pi+h)]}$$

$$= \frac{1}{\pi} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{\pi}$$

Textbook Q no: 22

Evaluate 1) $\lim_{x \to \pi/2} \frac{\tan 2x}{x - \frac{\pi}{2}}$ ans: 2

2) $\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{1-x}$ ans : $\frac{\pi}{2}$

3) $\lim_{x \to \pi} \frac{\sin x}{x-\pi}$ ans : (–1)

4) $\lim_{x \to \pi/2} \frac{1+\cos 2x}{(\pi-2x)^2}$ ans : ½

## Type VIII

Applying sin C $\pm$ sin D formulae

Evaluate 1) $\lim_{x \to 0} \frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}$

2) $\lim_{x \to a} \frac{\sin x-\sin a}{x-a}$ ans : cos a

## Type IX

$$\lim_{x\to c-} f(x) = \lim_{x\to c+} f(x) = l$$

** NCERT Exercise 13.1 -

1) Q 23

2) Q 24

3) Q 25

4) Q 26

** 5) If $f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases}$ , find $\lim_{x \to 2} f(x)$

6) If $f(x) = \begin{cases} x, & x > 0 \\ 1, & x = 0 \\ -x, & x < 0 \end{cases}$ , find $\lim_{x \to 0} f(x)$

## Type X

** NCERT Exercise 13.1 -

1) Q 28

2) Q 30

3) Q 32

4) Let $f(x) = \begin{cases} 4x-5, & x \le 2 \\ x-k, & x > 2 \end{cases}$ . Find k if $\lim_{x \to 2} f(x)$ exists. **Ans : k = –1**

## Type XI

**Exponential Limits**

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

NCERT Exercise 13.2 Q1 , Q2, Q4, Q6.

## Type XII

NCERT Exercise 13.2 Q3 , Q5

## Type XIII

$$\lim_{x \to 0} \frac{\log_e(1+x)}{x} = 1$$

NCERT Exercise 13.2 Q7 , Q8

# DIFFERENTIATION

## Type I

Find the derivatives of the following functions from first principle.

1) $x$

2) $\frac{1}{x}$

3) $-x$

4) $(-x)^{-1}$

5) $x^n$

6) NCERT Ex 13.2 Q4

** 7) sin x

** 8) cos x

** 9) tan x

** 10) sec x

** 11) cosec x

** 12) sin 2x

** 13) cos 3x

14) x sin x

15) $\sin^2 x$

16) sin(x+1)

17) Misc Example 19

**Type II**

Find the derivatives of the following

* 1) $3cot\ x + 5\ cosec\ x$

2) $5\sin x - 6\cos x + 7$

3) $2\tan x - 7\ cosec\ x$

4) $x^3 - 27$

5) $2x - \frac{3}{4}$

* 6) $x^2 + \sin x + \frac{1}{x^2}$

**Type III**

Ex 13.2 Q1

Ex 13.2 Q2

Ex 13.2 Q3

Ex 13.2 Q5

## Type IV

$$\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

NCERT Ex 13.2

* 1) Q7 (1),(2)

* 2) Q9 (2) ,(3), (4),(5)

* 3) Q11 (1)

* 4) Example 18

** 5) Misc Ex Q 15, Q22, Q25, Q29.

## Type V

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Misc Exercise Q 5,6,7,8,9,16,17,18,20,26.

## Additional questions and HOT Questions

1. Find the derivatives of the following functions

   a) $\dfrac{\sec x + \tan x}{\sec x - \tan x}$

   ** b) $(x^2 - 3x + 2)(x + 2)$

   ** c) $\dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

2. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$ show that $\frac{dy}{dx} = y$.

# Chapter 14

## MATHEMATICAL REASONING

Type-I

Concept: statements: A sentence which is either true or false, but not both

(1) N.C.E.R.T text book page 324
Question no.1(*) , 2(*),4(*),9(*),10(*)

Type-II

Negation of a statement: Denial of a statement is called negation of a statement.

(1) N.C.E.R.T text book page 329
Question no.1(*),2(*),3(*),4(*),5(*)

Type-III

Compound statement and component statement

A compound statement is a statement which is made up of two or more statements .Each statement is called a component statement.

(1) N.C.E.R.T page 327 example 4(*)

(2) N.C.E.R.T page 329 exercise 14.2 - question no.3(*)

Type-IV

Compound statement with 'and' or 'or'

(1) N.C.E.R.T page 330 example 6
(2) N.C.E.R.T page 333 example 8

Type-V

Quantifiers

Quantifiers are phrases like 'there exists' and 'for all' etc...

(1) N.C.E.R.T page 335 exercise 14.3 question no.2(*)

Type-VI

Inclusive or exclusive or

(1) N.C.E.R.T page 332 example 7
(2) N.C.E.R.T page 335 exercise 14.5 question no.4

Type-VII

Implications

(1) N.C.E.R.T page no.338 exercise 14.4 question no.1(**)
(2) N.C.E.R.T page no.345 Mis exercise 14.4 question no.7(**)
(3) Rewrite the following statement with if then in five different ways
If a number is a multiple of 9, then it is a multiple of 3.

Type-VIII

Contra positive and Converse statement

Contra positive statement of p=>q is ~ q implies~p

Converse of the statement p=>q is q=>p.

(1) N.C.E.R.T page 336 example 9(**)
(2) N.C.E.R.T page 337 example 10(**)
(3) N.C.E.R.T page 338 exercise 14.4 question no .2(**)
(4) N.C.E.R.T page 345 misc. exercise question no.2(**)

Type-IX

Validating statements

Direct method: By assuming that 'p' is true, prove that 'q' must be true.

Contra positive method: By assuming 'q' is false, prove that 'p' must be false.

Method of contradiction: Assume 'p' is not true. Then we arrive at some result which contradicts our assumption.

(1) N.C.E.R.T page 342 exercise 14.5 question no.1(**)
(2) N.C.E.R.T page 342 exercise 14.5 question no.3(**)
(3) N.C.E.R.T page 345 Mis exercise 14.5 question no.6(*)
(4) N.C.E.R.T page 340 example 13(**)
(5) N.C.E.R.T page 340 example 14(**)
(6) N.C.E.R.T page 340 example 15(**)

## Type-X

By giving a counter example we can disprove a given statement.

(1) N.C.E.R.T page 342 question no.4(**)
(2) N.C.E.R.T page 342 example 7(*)

## Type-XI

Validating of compound statement

(1) N.C.E.R.T mise exercise –question no.5(*)

# Chapter 15

# STATISTICS

Type-I

Concept: mean deviation for ungrouped data

M.D ($\bar{x}$) = $\frac{\sum|xi-\bar{x}\,|}{n}$ , M.D (M) = $\frac{\sum|xi-M\,|}{n}$

$\bar{x}$ = arithmetic mean M=median

(1) N.C.E.R.T page 351 example 1 (*)

(2) N.C.E.R.T page 351 example 2 (*)

(3) N.C.E.R.T page 352 example 3 (*)

(4) N.C.E.R.T page 360 exercise 15.1 question . 1 (*)

(5) N.C.E.R.T page 360 exercise 15.1question. 3 (*)

(6) N.C.E.R.T page 360 exercise 15.1 question. 4 (*)

Type-II

Concept: mean deviation for grouped data discrete distribution

M.D ($\bar{x}$) = $\frac{\sum fi|xi-\bar{x}\,|}{N}$ , M.D (M) = $\frac{\sum fi|xi-M\,|}{N}$ where N = $\sum fi$

(1)N.C.E.R.T page 353 example 4 (*)

(2) N.C.E.R.T page 354 example 5 (*)

(3) N.C.E.R.T page 360 exercise 15.1 question.5 (*)

(4) N.C.E.R.T page 360 exercise 15.1 question.6 (*)

(5) N.C.E.R.T page 360 exercise 15.1 question .7(*)

Type – III

Concept: mean deviation for grouped data continuous frequency distribution

M.D ($\bar{x}$ ) = $\frac{\sum fi|xi-\bar{x}\,|}{N}$ where$\bar{x}$ =A + $\frac{h}{N}\sum fi\,di$

M.D (M) = $\frac{\sum fi|xi-M|}{N}$ where M = $l+\left(\frac{\frac{N}{2}-C}{f}\right)$h

(1)N.C.E.R.T page 359 example 7(**)

(2) N.C.E.R.T page 361 exercise 15.1 question . 9(**)

(3) N.C.E.R.T page 361 exercise 15.1 question . 10(**)

(4) N.C.E.R.T page 361 exercise 15.1 question . 11(**)

Type-IV

Concept: variance and standard deviation for ungrouped data

$$\sigma^2 = \frac{\sum(xi-\bar{x})^2}{n}$$

(1)N.C.E.R.T page 364 example 8(*)

(2)N.C.E.R.T page 361 exercise 15.2 question .1(*)

(3) N.C.E.R.T page 361 exercise 15.2 question .2(*)

(4) N.C.E.R.T page 361 exercise 15.2 question .3(*)

Type-V

Concept: variance and standard deviation of a discrete frequency distribution

$$\sigma^2 = \frac{\sum fi(xi-\bar{x})^2}{N} \quad \text{or } \sigma^2 = \frac{\sum fixi^2}{N} - \left(\frac{\sum fixi}{N}\right)^2$$

(1)N.C.E.R.T page 365 example 9(*)

(2) N.C.E.R.T page 365 example 11(*)

(3) N.C.E.R.T page 371 exercise15.2 question. 4(*)

(4) N.C.E.R.T page 371 exercise 15.2 question.5(*)

Type-VI

Concept: variance and standard deviation of a continuous frequency distribution

$$\sigma^2 = \frac{\sum fixi^2}{N} - \left(\frac{\sum fixi}{N}\right)^2 \quad \text{or} \quad \sigma^2 = h^2\left[\frac{\sum fidi^2}{N} - \left(\frac{\sum fidi}{N}\right)^2\right] \text{ where di} = \frac{xi-A}{h}$$

(1) N.C.E.R.T page 370 example 12(**)

(2) N.C.E.R.T page 372 exercise 15.2 question .8(*)

(3) N.C.E.R.T page 372 exercise 15.2 question .9(*)

(4) N.C.E.R.T page 372 exercise 15.2 question .10(*)

Type-VII

Concept: coefficient of variation C.V = $\left(\frac{\sigma}{\bar{x}}\right)$ x 100

(1) N.C.E.R.T page 374 example 14(*)

(2) N.C.E.R.T page 376 exercise 15.3 question .5(*)

Type – VIII(HOT questions)

(1) N.C.E.R.T page 380 misc exercise questions 2,3,4,5(*)

## EXTRA AND HOT QUESTIONS

(1) Find the mean ,variance and standard deviation for the following data

| class | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|---|---|---|---|---|---|---|---|---|---|
| frequencies | 3 | 4 | 7 | 7 | 15 | 9 | 6 | 6 | 3 |

Ans: 56,422.33,20.65

(2) Find the mean variance and standard deviation using short cut method.

| Heights in cm | 70-75 | 75-80 | 80-85 | 85-90 | 90-95 | 95-100 | 100-105 | 105-110 | 110-115 |
|---|---|---|---|---|---|---|---|---|---|
| frequencies | 3 | 4 | 7 | 7 | 15 | 9 | 6 | 6 | 3 |

Ans: 93,105.52,10.27

(3) Calculate mean, variance and standard deviation for the following by short cut method.

| classes | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|---|---|---|---|---|---|---|---|
| frequency | 3 | 7 | 12 | 15 | 8 | 3 | 2 |

Ans: 62,201,$\sqrt{201}$

(4) Calculate the mean deviation from the median for the following data.

| classes | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|---|---|
| frequency | 4 | 6 | 10 | 20 | 10 | 6 | 4 |

Ans: Median=45

Mean deviation=11.33

## Chapter 16

## Probability

For important terms and definitions refer NCERT text book.

**Type- I**

Concept : sample space

(1)NCERT text book page 386 question no. 1 (*)

(2) NCERT text book page 386 question no. 2 (*)

(3) NCERT text book page 386 question no. 3 (*)

(4) NCERT text book page 386 question no. 4 (*)

(5) NCERT text book page 386 question no. 5 (*)

(6) NCERT text book page 386 question no. 11 (*)

(7) NCERT text book page 386 question no.12 (**)

**Type- II**

Concept : types of events

(1)NCERT text book page 393 question no. 2 (*)

(2) NCERT text book page 393 question no. 3 (*)

(3) NCERT text book page 393 question no. 1 (*)

(4) NCERT text book page 393 question no. 4 (**)

(5) NCERT text book page 392 example 7 (**)

**Type- III**

Concept : Algebra of events: AUB , A∩B , A but not B etc

(1) NCERT text book page 393 question no. 6 (**)

## EXTRA AND HOT QUESTIONS

(1) From a group of 2 men and 3 women 2 persons are selected . Describe the sample space of the experiment. If E is the event in which 1 man and 1 woman are selected. Then which are the cases favourable to E (Type-I*)

(2) Two dice are rolled. A is the event that the sum of the numbers shown on the two dice is 5.B is the event that at least one of the dice shows up a 3.Are the two events A and B.

(a) Mutually exclusive.

(b) Exhaustive (Type-II**)

(3) Two dice are thrown the events A , B, C are as follows

A: Getting an odd number on the first die.

B: Getting a total of 7 on the two dice.

C: Getting a total of greater than or equal to 8 on the two dice.

Describe the following events

(a) AUB

(b) A'

(c) B – C

(d) B∩C

## PROBABILITY OF AN EVENT

Important concepts

P(E) = $\frac{no\ of\ outcomes\ favourable\ to\ E}{total\ no\ of\ outcomes}$

If A and B are two mutually exclusive events P (AUB) = P(A) + P(B)

If A and B are any two events then P (AUB) = P(A) + P(B)-- P(A∩B)

P(not A) = 1—P(A)

**Type – I**

Concept: Probability of an event

(1) N.C.E.R.T text book page 404 question no.3(*)
(2) N.C.E.R.T text book page 404 question no.4(**)
(3) N.C.E.R.T text book page 404 question no.8(**)
(4) N.C.E.R.T text book page 404 question no.10(**)
(5) N.C.E.R.T text book page 403 example 14(**)
(6) N.C.E.R.T text book page 400 example 10(**)

## EXTRA AND HOT QUESTIONS

(7) Three identical dice are rolled . Find the probability that the same number will appear on each of them.

Ans: 1/36 (hot)

(8) Two dice are thrown simultaneously . Find the probability of getting a total of 9.

Ans: 1/9 (*)

(9) A bag contains 8 red ,3 white and 9 blue balls. Three balls are drawn at random from the bag. Determine the probability that none of the balls drawn is white .

Ans: 34/57 (**)

(10) In a single throw of 3 dice. Find the probability of not getting the same number on all the dice.

Ans: 35/36 (**)

(11) The letters of the word "SOCIETY " are placed at random in a row .What is the probability that the 3 vowels come together.

Ans: 1/7 (**)

(12) Find the probability that in an arrangement of the letters of the word "DAUGHTER" the letter D occupies the first place.

Ans: 1/8 (**)

(13) Find the probability that in a random arrangement of the letters of the word "INSTITUTION' the three T's are together.

Ans: P 1/110 (**)

## Type – II

$P(AUB) = P(A) + P(B)$ (mutually exclusive cases )

$P(AUB) = P(A) + P(B) - P(A \cap B)$

(1) N.C.E.R.T page 405 question no.14(**)

(2) N.C.E.R.T page 405 question no.15(*)

(3) N.C.E.R.T page 405 question no.16(**)

(4) N.C.E.R.T page 405 question no.17(**)

(5) N.C.E.R.T page 405 question no.18(**)

(6) N.C.E.R.T page 405 question no.19(**)

(7) N.C.E.R.T page 405 question no.20 (**)

(8) N.C.E.R.T page 409 misc exercise question no.3 (**)

(9) N.C.E.R.T page 401 example 11(**)

## EXTRA AND HOT QUESTIONS

(1) One card is drawn from a set of 17 cards numbered 1 to 17. Find the probability that the number is divisible by 3 or 7.
Ans: 7/17. (*)

(2) Two dice are thrown together. What is the probability that the sum of the numbers of the two faces is neither 9 nor 11.
Ans : 5/16 (*)

(3) Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.
Ans: 7/9 (**)

(4) Two cards are drawn from a well shuffled pack of 52 cards without replacement .Find the probability that neither a jack nor a card of spade is drawn.
Ans: 105/221 (**)

(5) If $P(AUB)=0.6$ and $P(A \cap B)=0.2$. Find $P(\bar{A}) + P(\bar{B})$
Ans: 1.2

(6) A and B are two mutually exclusive events if P(A) = 0.5 and P($\bar{B}$) =0.6.Find P(AUB)

**Type – III**

At least one, at most one cases

(1) N.C.E.R.T page 402 examples 12 (**) {hot}
(2) N.C.E.R.T page 407 example 15 (**)
(3) N.C.E.R.T page 408 misc exercise question .1 (**) {hot}
(4) N.C.E.R.T page 408 misc exercise question.2 (**)
(5) N.C.E.R.T page 409 misc exercise question 7 (**)
(6) N.C.E.R.T page 409 misc exercise question 9 (**)

## EXTRA AND HOT QUESTIONS

(1) Three coins are tossed once . Find the probability of getting

(a) Atmost 2 heads
(b) Atleast 2 heads
(c) Exactly 2 tails
(d) Atmost 2 tails
(e) 3 heads
(f) No heads

Ans: (a) 7/8 (b)1/2 (c)3/8 (d)7/8 (e)1/8 (f)1/8

(2) The probability that a student will get A,B,C or D grade are 0.4,0.35,0.15and 0.1 respectively. Find the probability that she will get

(a) B or C grade
(b) Atmost C grade

(3) In a single throw of 2 dice write the corresponding events and the probability of getting

(a) A total of 9
(b) Two ones
(c) Atleast one 6
(d) A sum of 9 or 11
(e) A sum of atleast 10
(f) A sum as a prime number

Ans: (a)1/9(b)1/36(c)11/36(d)1/6(e)1/6(f)5/12

## KENDRIYA VIDYALAYA SANGATHAN

**CLASS: XI** **SUBJECT: MATHEMATICS**

## SESSION ENDING EXAMINATION BLUE PRINT

| UNIT | S. NO | NAME OF CHAPTER | VSA | SA | LA | TOTAL | TOTAL OF UNIT |
|---|---|---|---|---|---|---|---|
| I | 1 | SETS | | 1 | 1 | 10(2) | 29(7) |
| | 2 | RELATIONS AND FUNCTIONS | 1 | 1 | - | 5(2) | |
| | 3 | TRIGONOMETRIC FUNCTIONS | - | 1+1* | 1* | 14(3) | |
| II | 4 | PRINCIPLE OF MATHEMATICAL INDUCTION | - | | 1* | 6(1) | 37(9) |
| | 5 | COMPLEX NUMBERS AND QUADRATIC EQUATIONS | - | 1* | - | 4(1) | |
| | 6 | LINEAR INEQUALITIES | | - | 1 | 6(1) | |
| | 7 | PERMUTRATIONS AND COMBINATIONS | - | 2 | - | 8(2) | |
| | 8 | BINOMIAL THEOREM | 1 | 2 | | 9(3) | |
| | 9 | SEQUENCES AND SERIES | | 1* | | 4(1) | |
| III | 10 | STRAIGHT LINES | 1 | 1* | - | 5(2) | 13(4) |
| | 11 | CONIC SECTIONS | | 1 | | 4(1) | |
| | 12 | INTRODUCTION TO THREE DIMENSIONAL GEAMETRY | - | 1 | - | 4(1) | |
| IV | 13 | LIMITS AND DERIVATIVES | | | 1 | 6(1) | 6(1) |
| V | 14 | MATHEMATICAL REASONING | 3 | - | - | 3(3) | 3(3) |
| VI | 15 | STATISTICS | - | - | 1 | 6(1) | 6(1) |
| VII | 16 | PROBABILITY | - | - | 1 | 6(1) | 6(1) |
| | | TOTAL | 6(6) | 48(13) | 42(7) | 100(26) | 100(26) |

1 * MEANS **Internal Choice**

2 **Number in Brackets indicates the number of questions**

3 **VSA (1 Mark) SA (4 Marks) LA (6 Marks)**

# KENDRIYA VIDYALAYA SANGATHAN, BANGALORE REGION

## SESSION ENDING EXAMINATION MODEL PAPER

**Class : XI** **Maximum Marks: 100**

**Subject: Mathematics** **Time Allotted: 3 Hrs.**

*General Instructions:*

1. *All questions are compulsory.*
2. *The question paper consists of 26 questions divided into 3 sections A, B and C. Section A comprises of 6questions of 1 mark each, section B comprises of 13 questions of 4 marks each and section C comprises of 7 questions of 6 marks each.*
3. *All questions in section A are to be answered in one word/one sentence/as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of section B and 2 questions of section C. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted.*

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## Section A

1. How many relations can be defined on the set A $= \{2,4,6,8\}$
2. Find the 4$^{th}$ term in the expansion of $(2-3y)^{12}$
3. Write the equation of the line which is equally inclined to both the axes and passing through the origin.
4. Determine whether this is a statement and give reason for your answer:

   The product of (-1) and 8 is 8.
5. Write the negation of the statement: *The number 2 is greater than 7.*
6. State whether the "*Or*" used in the following statement is "exclusive" or "inclusive": All integers are positive or negative.

## Section B

7. If A={3,6,9,12,15,18,21} ; B={4,8,12,16,20} ; C={2,4,6,8,10,12,14,16},find $A\cap(B-C) \,\&\, B\cup(A\cap C)$
8. Define the following functions and sketch the graphs of:

   a. Identity function b. constant function.
9. Prove that : $sinx + sin3x + sin5x + sin7x = 4\,cos\,x\;\;cos2x\,sin4x$

10. Solve the following: $2cos^2x + 3sinx = 0$

OR

In any triangle ABC, prove that $a\sin(B-C) + b\sin(C-A) + c\ sin\ (A-B) = 0$

11. Convert the following into polar form: $\frac{1+7i}{(2-i)^2}$.

OR

Find: $\sqrt{3+4i}$

12. How many words, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated if,
    a. 4 letters are used at a time?
    b. All the letters are use at a time?
    c. All the letters are used, but the first letter is a vowel?
13. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include at least 4 bowlers? **Comment on the importance of sports in school curriculum.**
14. The vertices of ΔPQR are P(2, 1), Q(-2, 3) and R(4, 5). Find the equation of the median through the vertex R.

OR

In the triangle ABC with vertices A(2, 3), B(4, -1) and C(1, 2), find the equation and length of the altitude from the vertex A.

15. Draw a rough sketch of the parabola represented by the equation $x^2 = -16y$. Also find the coordinates of its focus, equation of the directrix and length of the latus rectum.
16. Find the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) in the ratio (i) 2 : 3 internally, (ii) 2 : 3 externally.
17. Evaluate: $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$
18. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.
19. A person has 2 parents, 4 grandparents, 8 great-grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own. **What is your opinion on joint family system?**

**OR**

Find the sum to n terms of the series $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + $…………………

## Section C

20. A class has 175 students. The following is the description showing the number of students studying one or more of the following subjects in this class: Mathematics 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18.

    a. How many students are enrolled in at least one of the subjects?

    b. How many of the students are not studying any of the subjects? **Write any two life skills which can be achieved by learning Mathematics.**

21. Prove that: $cos^2 x + cos^2\left(x+\frac{\pi}{3}\right) + cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

OR

Find $\sin\frac{x}{2}$ , $\cos\frac{x}{2}$ $and \tan\frac{x}{2}$ if $\tan x = \frac{-4}{3}$ , $x\ lies\ in\ 2nd\ quadrant$

22. Solve the following system of inequalities graphically:

$$3x + 2y \leq 50; \quad x + 4y \leq 80; \quad x \leq 15; \quad y \geq 0; \quad x \geq 0$$

23. Find the mean, standard deviation and the coefficient of variation of the data given below: (use short cut method)

| Marks | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency | 2 | 1 | 12 | 29 | 25 | 12 | 10 | 4 | 5 |

24. Three dice are tossed once. Find the probability of getting:

    a. 1 triplet  b. 2 on two dice  c. sum of the numbers on the 3 dice is at the most 4

25. Prove the following using the Principal of Mathematical Induction for all $n \in \boldsymbol{N}$:

$$1^2 + 2^2 + 3^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

OR

Prove the following using the Principal of Mathematical Induction for all $n \in \boldsymbol{N}$: $3^{2n+2} - 8n - 9$ is divisible by 8.

26. (i) Find the derivative from first principles, of $\cos(x+1)$

(ii) Given $f(x) = \begin{cases} a+bx, x<1 \\ 4, x=1 \\ b-ax, x>1 \end{cases}$ . If $\lim_{x\to 1} f(x) = f(1)$ what are the possible values of $\boldsymbol{a}$ and $\boldsymbol{b}$ ?