Sample Paper-03 (solved)
Mathematics
Class – XI

ANSWER

Section A

1. **Solution:** Ellipse

2. **Solution**

Condition for colinearity is not satisfied here since

$$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$$

3. **Solution:**

$$b^2 + c^2 - 4ad > 0$$

4. **Solution:**

Domain of is in the open interval $(-2, 2)$

5. **Solution:**

$$(A \cap B) = \{\phi\}$$

6. **Solution**

Max value is 2

Section B

7. **Solution:**

$$\phi(\frac{\pi}{12}) = \sin 2.(\frac{\pi}{12})$$

$$= \sin\frac{\pi}{6}$$

$$= \frac{1}{2}$$

$$f(x) = (\frac{1}{2})^3 - \frac{1}{2}$$

$$= \frac{1}{8} - \frac{1}{2}$$

$$= -\frac{3}{8}$$

8. Solution:

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}.\frac{1}{2m+1}}\quad =1$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=1$$

$$\tan A+\tan B+\tan A\tan B=1$$

9. Solution:

$$f(\sqrt{3})=-1$$

$$f(3)=1$$

$$f(\sqrt{3+1})=1$$

10. Solution:

Use the inequality $AM\geq GM$

$$AM \text{ between } x,\frac{1}{x}=\frac{x+\frac{1}{x}}{2}$$

$$GM \text{ between } x,\frac{1}{x}=\sqrt{x.\frac{1}{x}}=1$$

$$\frac{x+\frac{1}{x}}{2}\geq 1$$

$$x+\frac{1}{x}\geq 2$$

$$Since\ \ -1\leq \sin\theta\leq 1$$

$$\sin\theta=x+\frac{1}{x} \text{ is impossible}$$

11. Solution:

$$f(x)=\phi(x)$$

$$f(x)=3x^2+1$$

$$\phi(x)=7x-1$$

$$3x^2+1=7x-1$$

$$3x^2-7x+2=0$$

$$(x-2)(3x-1)=0$$

$x = 2 \, , x = \frac{1}{3}$

$Hence\ f(x)\ and\ \phi(x)\ are\ equal\ when\ the\ domain\ is\ in\ the\ set\ \{\frac{1}{3}, 2\}$

12. Solution

$$\lim_{x\to 0} \frac{1-\cos x}{x}$$

$$= \lim_{x\to 0} \frac{1-(1-2\sin^2 \frac{x}{2})}{x}$$

$$= \lim_{x\to 0} \frac{\sin^2 \frac{x}{2}}{x}$$

$$= \lim_{x\to 0} \frac{\sin \frac{x}{2}}{2\frac{x}{2}} \sin \frac{x}{2}$$

$$= \frac{1}{2}.1.0$$

$$= 0$$

13. Solution:

$$2\sin^2 x + 14\sin x \cos x + 50\cos^2 x = 26$$

$$= 2\sin^2 x + 14\sin x \cos x + 50\cos^2 x = 26(\sin^2 x + \cos^2 x)$$

$$= -24\sin^2 x + 14\sin x \cos x + 24\cos^2 x = 0$$

$$= 24\sin^2 x - 14\sin x \cos x - 24\cos^2 x = 0$$

$$= 24\tan^2 x - 14\tan x - 24 = 0$$

$$\tan x = \frac{14 \pm \sqrt{196 + 2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$

$$\tan x = \frac{14 \pm 50}{48}$$

$$\tan x = \frac{64}{48} ; or ; -\frac{36}{48}$$

$$\tan x = \frac{4}{3}\ or\ -\frac{3}{4}$$

14. Solution:

$$y = x^2 - x + 1$$

$$y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

$$f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

15. Solution:

$$\textit{Equation is } 8y^2 + 24x - 40y + 134 = 0$$

$$= 4y^2 + 12x - 20y + 67 = 0$$

This can be written as

$$y^2 - 5y = -3x - \frac{67}{4}$$

$$\left(y - \frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \frac{25}{4} - 3\left(x + \frac{7}{2}\right)$$

$$\textit{Let } Y = y - \frac{5}{2}$$

$$X = x + \frac{7}{2}$$

$$Y^2 = -3X$$

This is of the form $y^2 = -4ax$

Latus rectum is $= 3$

$$\textit{Vertex}\left(-\frac{7}{2}, \frac{5}{2}\right)$$

$$\textit{Axis } y = \frac{5}{2}$$

$$\textit{Focus } \left(-\frac{7}{2} - \frac{3}{4}, \frac{5}{2}\right)$$

$$\textit{Directrix: referred to New axis: } X = a = \frac{3}{4}$$

$$\textit{Directrixreferred to Old axis: } \frac{3}{4} = x + \frac{7}{2}$$

$$x = \frac{3}{4} - \frac{7}{2}$$

$$x = -\frac{11}{4}$$

16. **Solution:**

$$\frac{7-4i}{3+2i}=\frac{7-4i}{3+2i}\times\frac{3-2i}{3+21}$$

$$\frac{13-26i}{13}=1-2i$$

17. **Solution**

Either both factors are negative or both factors are positive to have this in equality. if $x<2$ both factors are negative and if $x>3$ both factors are positive. Hence the solution is $x\in\{(-\infty,2)\cup(3,\infty)\}$

18. **Solution**

$$\tan 5x=\cot 2x$$

$$\tan 5x=\tan(\frac{\pi}{2}-2x)$$

$$5x=(\frac{\pi}{2}-2x)$$

$$5x=n\pi+(\frac{\pi}{2}-2x)$$

$$7x=n\pi+\frac{\pi}{2}$$

$$x=\frac{1}{7}(n\pi+\frac{\pi}{2})$$

19. **Solution**

Total number of occurrence = $6\times6=36$

On each die there are 3 prime numbers $\{2,3,5\}$

Hence total number of favorable cases $3\times3=9$

Probability of getting a prime in each die = $\frac{9}{36}=\frac{1}{4}$

Section C

20. **Solution:**

The odd digits $1,3,3,1$ can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits $2,4,2$ can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!}\times\frac{3!}{2!}=6\times3=18$ ways

21. **Solution**

Probability of one of them getting selected $P(E_1 or E_2)$ = 1- (Probability of both getting selected +

Probability of none getting selected)

$= 1-[P(E_1 \cap E_2)+P(E_1' \cap E_2')]$

$= 1-(\frac{1}{3}\times\frac{1}{5}+\frac{2}{3}\times\frac{4}{5})$

$= 1-(\frac{1}{15}+\frac{8}{15})$

$= 1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$

22. Solution

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and3 , E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$a+(n-1)d = T_n$

$n = \frac{T_n}{d}-\frac{a}{d}+1$

In this case $\frac{a}{d}=1$, Hence $n = integer\ \ part\ of\ \frac{T_n}{d}$

$n(A)=[\frac{1000}{2}]=500$

$n(B)=[\frac{1000}{3}]=333$

$n(C)=[\frac{1000}{5}]=200$

$n(D)=[\frac{1000}{2\times3}]=166$

$n(E)=[\frac{1000}{2\times5}]=100$

$n(F)=[\frac{1000}{3\times5}]=66$

$n(G)=[\frac{1000}{2\times3\times5}]=33$

Numbers that are divisible by 2, 3, 5 are

$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cup B)-n(A\cup C)-n(B\cup C)+n(A\cap B\cap C)$

$=500+333+200+1666+100+66+33$

$=734$

Numbers that are not divisible by 2, 3, 5 are

$1000-734=266$

23. **Solution:**

$y=\sin x$

$y+\Delta y=\sin(x+\Delta x)$

$\Delta y=\sin(x+\Delta x)-y$

$\Delta y=\sin(x+\Delta x)-\sin x$

$$\Delta y=2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$

$$\frac{\Delta y}{\Delta x}=\frac{2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x}=\frac{\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\cos x$$

$$\frac{dy}{dx}=\cos x$$

Note: As $\Delta x\to 0\,;\frac{\Delta x}{2}\ also\to 0$

24. **Solution**:

The successive First order of difference is $4,7,10,13,\ldots$ this is an AP.

The second order difference is(Difference of the first difference) $3,3,3,\ldots$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n=T_1+(n-1)\Delta T_1+\frac{(n-1)(n-2)}{2!}\Delta T_2+\frac{(n-1)(n-2)(n-3)}{3!}\Delta T_3$$

$$=\ 12+4(n-1)+3\frac{(n-1)(n-2)}{2}$$

$$=\frac{3n^2-n+22}{2}$$

$$\text{Sum} = \frac{1}{2}\left(3\,\Sigma n^2 - \Sigma n + 22n\right)$$

$$= \frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n\right)$$

$$= \frac{1}{2}\left(n^3 + n^2 + 22n\right)$$

25. Solution:

Let the point A be(x_1, y_1) *and B be* (x_2, y_2)

Let the point C be a point be(x, y)*on the circle*

Then AC and BC are perpendicular

Product of Solpes of line AC and BC $= -1$

$$\frac{y-y_1}{x-x_1}\cdot\frac{y-y_2}{x-x_2} = -1$$

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

26. Solution

x_i	f_i	$f_i x_i$	$\lvert x_i - 9\rvert$	$f_i \lvert x_i - 9\rvert$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N = \Sigma f_i = 26$	$\Sigma f_i\ x_i = 234$		$f_i \Sigma \lvert x_i - 9\rvert = 100$

$$Mean = \overline{X} = \frac{1}{N}(\Sigma f_i\ x_i) = \frac{234}{26} = 9$$

$$MeanDeviation = M.D = \frac{1}{N}(\Sigma f_i \lvert x_i - 9\rvert) = \frac{100}{26} = 3.84$$