KEY CONCEPTS
(INVERSE TRIGONOMETRY FUNCTION)

GENERAL DEFINITION(S):

1. $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc. denote angles or real numbers whose sine is x, whose cosine is x and whose tangent is x, provided that the answers given are numerically smallest available. These are also written as arc sinx, arc cosx etc.

If there are two angles one positive & the other negative having same numerical value, then positive angle should be taken.

2. PRINCIPAL VALUES AND DOMAINS OF INVERSE CIRCULAR FUNCTIONS :

(i) $y = \sin^{-1}x$ where $-1 \le x \le 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = x$.

(ii) $y = \cos^{-1}x$ where $-1 \le x \le 1$; $0 \le y \le \pi$ and $\cos y = x$.

(iii) $y = \tan^{-1}x$ where $x \in R$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $\tan y = x$.

(iv) $y = \operatorname{cosec}^{-1}x$ where $x \le -1$ or $x \ge 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$ and $\operatorname{cosec} y = x$.

(v) $y = \sec^{-1}x$ where $x \le -1$ or $x \ge 1$; $0 \le y \le \pi$; $y \ne \frac{\pi}{2}$ and $\sec y = x$.

(vi) $y = \cot^{-1}x$ where $x \in R$, $0 < y < \pi$ and $\cot y = x$.

NOTE THAT : **(a)** 1st quadrant is common to all the inverse functions .
(b) 3rd quadrant is **not used** in inverse functions .
(c) 4th quadrant is used in the CLOCKWISE DIRECTION i.e. $-\frac{\pi}{2} \le y \le 0$.

3. PROPERTIES OF INVERSE CIRCULAR FUNCTIONS :

P–1 **(i)** $\sin(\sin^{-1}x) = x$, $-1 \le x \le 1$ **(ii)** $\cos(\cos^{-1}x) = x$, $-1 \le x \le 1$

(iii) $\tan(\tan^{-1}x) = x$, $x \in R$ **(iv)** $\sin^{-1}(\sin x) = x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(v) $\cos^{-1}(\cos x) = x$; $0 \le x \le \pi$ **(vi)** $\tan^{-1}(\tan x) = x$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

P–2 **(i)** $\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$; $x \le -1$, $x \ge 1$

(ii) $\sec^{-1}x = \cos^{-1}\frac{1}{x}$; $x \le -1$, $x \ge 1$

(iii) $\cot^{-1}x = \tan^{-1}\frac{1}{x}$; $x > 0$

$= \pi + \tan^{-1}\frac{1}{x}$; $x < 0$

P–3 **(i)** $\sin^{-1}(-x) = -\sin^{-1}x$, $-1 \le x \le 1$
(ii) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$
(iii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $-1 \le x \le 1$
(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$, $x \in R$

P–4 **(i)** $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ $-1 \le x \le 1$ **(ii)** $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ $x \in R$

(iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$ $|x| \ge 1$

P–5 $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy < 1$

$= \pi + \tan^{-1}\frac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy > 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ where $x > 0$, $y > 0$

P–6 **(i)** $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $(x^2+y^2) \le 1$

Note that : $x^2 + y^2 \le 1 \Rightarrow 0 \le \sin^{-1}x + \sin^{-1}y \le \frac{\pi}{2}$

(ii) $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $x^2 + y^2 > 1$

Note that : $x^2 + y^2 > 1 \Rightarrow \frac{\pi}{2} < \sin^{-1}x + \sin^{-1}y < \pi$

(iii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$

(iv) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right]$ where $x \ge 0$, $y \ge 0$

P–7 If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if, $x > 0, y > 0, z > 0$ & $xy + yz + zx < 1$

Note : (i) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ then $x + y + z = xyz$

(ii) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ then $xy + yz + zx = 1$

P–8 $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$

Note very carefully that :

$$\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1}x & \text{if } |x| \le 1 \\ \pi - 2\tan^{-1}x & \text{if } x > 1 \\ -\left(\pi + 2\tan^{-1}x\right) & \text{if } x < -1 \end{cases}$$

$$\cos^{-1}\frac{1-x^2}{1+x^2} = \begin{cases} 2\tan^{-1}x & \text{if } x \ge 0 \\ -2\tan^{-1}x & \text{if } x < 0 \end{cases}$$

$$\tan^{-1}\frac{2x}{1-x^2} = \begin{cases} 2\tan^{-1}x & \text{if } |x| < 1 \\ \pi + 2\tan^{-1}x & \text{if } x < -1 \\ -\left(\pi - 2\tan^{-1}x\right) & \text{if } x > 1 \end{cases}$$

REMEMBER THAT :

(i) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2} \Rightarrow x = y = z = 1$

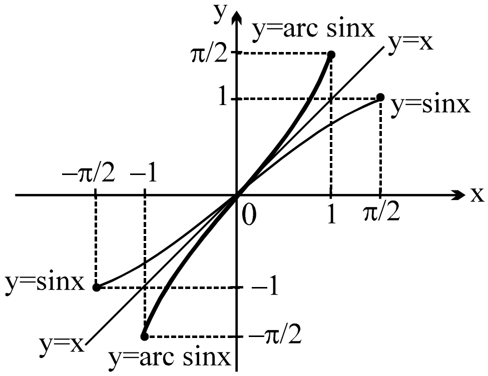
(ii) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi \Rightarrow x = y = z = -1$

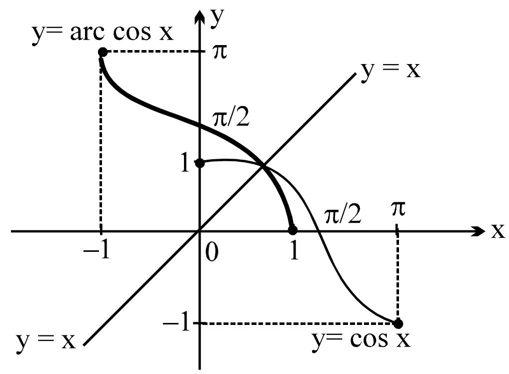
(iii) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$ and $\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

INVERSE TRIGONOMETRIC FUNCTIONS
SOME USEFUL GRAPHS

1. $y = \sin^{-1}x$, $|x| \le 1$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

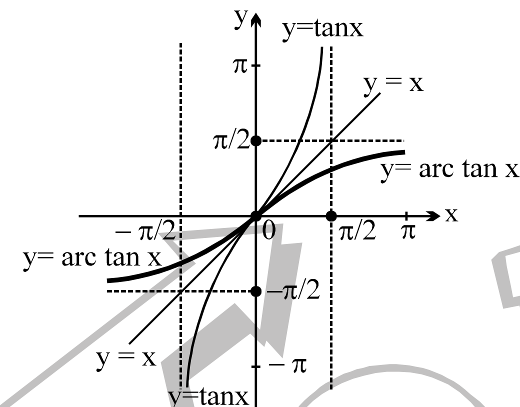
2. $y = \cos^{-1}x$, $|x| \le 1$, $y \in [0, \pi]$

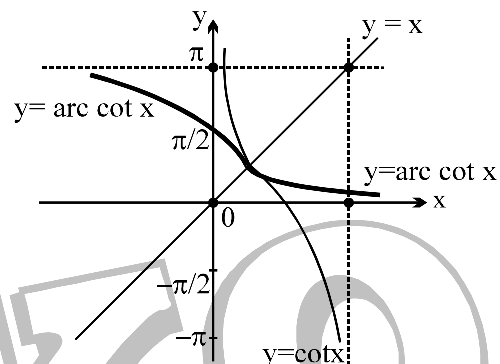




3. $y = \tan^{-1} x$, $x \in R$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

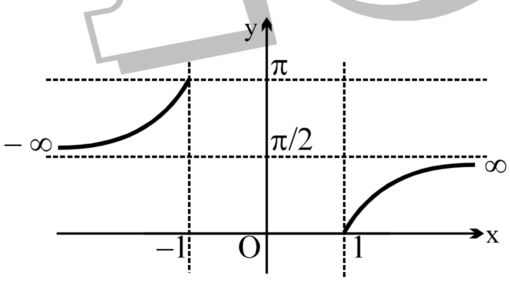
4. $y = \cot^{-1} x$, $x \in R$, $y \in (0, \pi)$

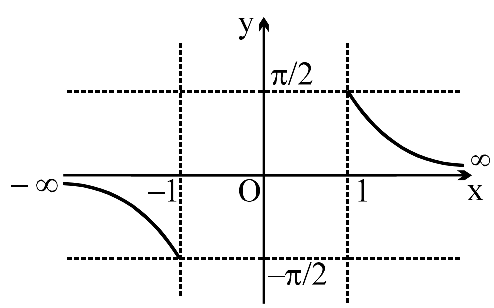




5. $y = \sec^{-1} x$, $|x| \geq 1$, $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

6. $y = \operatorname{cosec}^{-1} x$, $|x| \geq 1$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$





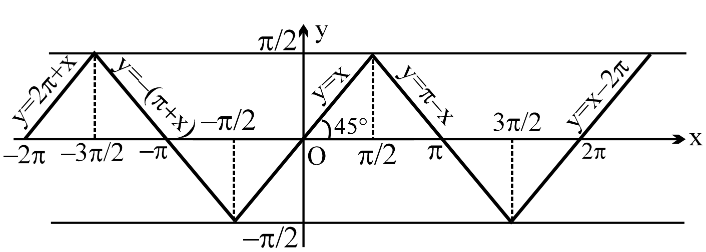
7. (a) $y = \sin^{-1}(\sin x)$, $x \in R$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

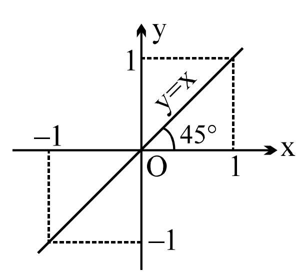
Periodic with period 2π

7.(b) $y = \sin(\sin^{-1} x)$,

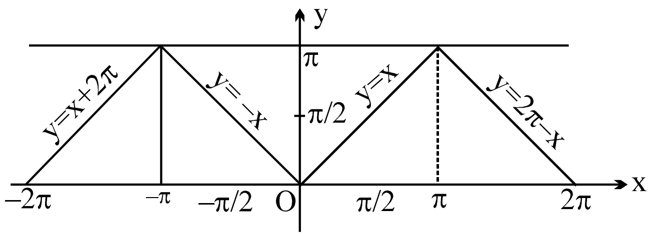
$= x$

$x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic

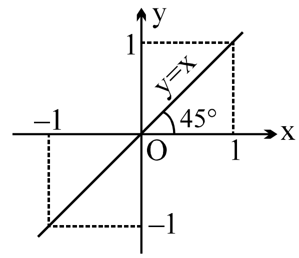




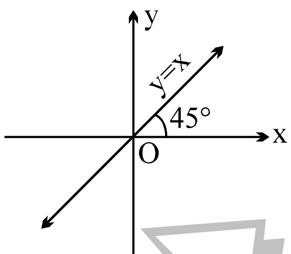
8. (a) $y = \cos^{-1}(\cos x)$, $x \in R$, $y \in [0, \pi]$, periodic with period 2π
$= x$



8. (b) $y = \cos(\cos^{-1} x)$,
$= x$
$x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic



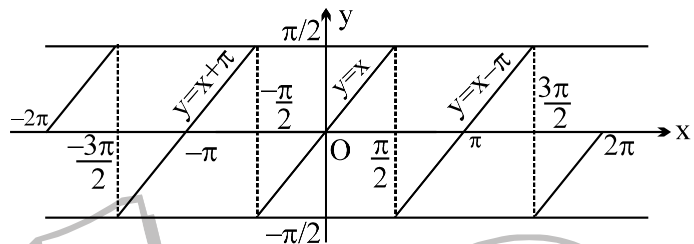
9. (a) $y = \tan(\tan^{-1} x)$, $x \in R$, $y \in R$, y is aperiodic
$= x$



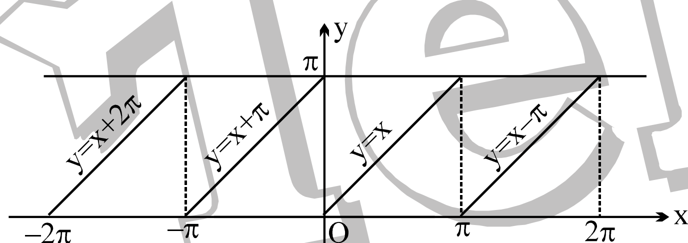
9. (b) $y = \tan^{-1}(\tan x)$,
$= x$

$x \in R - \left\{(2n-1)\frac{\pi}{2} \;\; n \in I\right\}$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

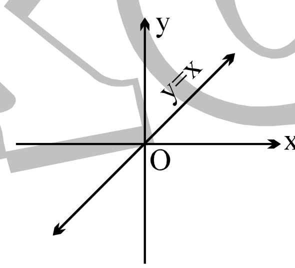
periodic with period π



10. (a) $y = \cot^{-1}(\cot x)$,
$= x$
$x \in R - \{n\pi\}$, $y \in (0, \pi)$, periodic with π



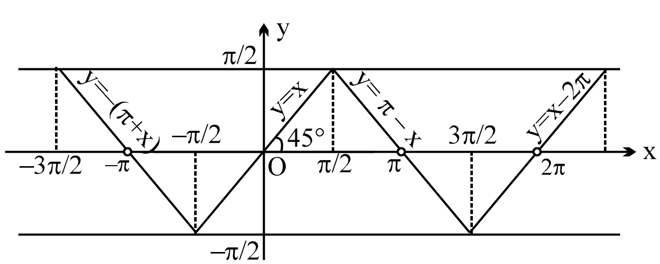
10. (b) $y = \cot(\cot^{-1} x)$,
$= x$
$x \in R$, $y \in R$, y is aperiodic



11. (a) $y = \mathrm{cosec}^{-1}(\mathrm{cosec}\, x)$,
$= x$

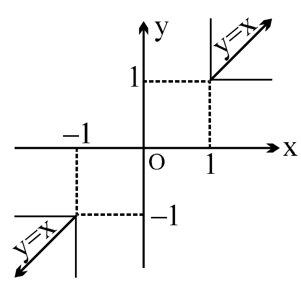
$x \in R - \{ n\pi , n \in I \}$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

y is periodic with period 2π



11. (b) $y = \mathrm{cosec}(\mathrm{cosec}^{-1} x)$,
$= x$

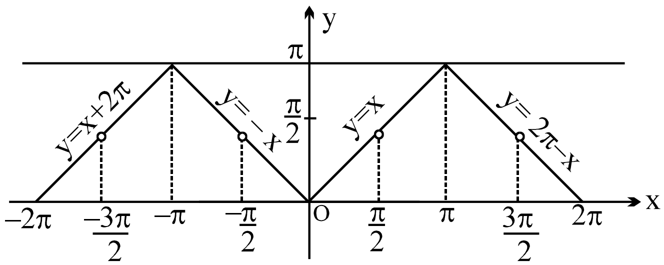
$|x| \geq 1$, $|y| \geq 1$, y is aperiodic

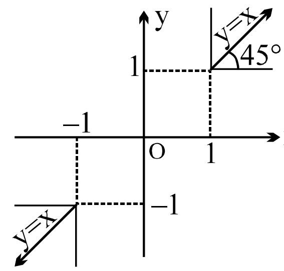


12. (a) $y = \sec^{-1}(\sec x)$,
$= x$
y is periodic with period 2π;

$x \in R - \left\{(2n-1)\frac{\pi}{2} \;\; n \in I\right\}$ $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

12. (b) $y = \sec(\sec^{-1} x)$,
$= x$
$|x| \geq 1$; $|y| \geq 1]$, y is aperiodic





EXERCISE–1

Q.1 Find the following

(i) $\tan\left[\cos^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right]$ (ii) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$ (iii) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(iv) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ (v) $\cos\left(\tan^{-1}\frac{3}{4}\right)$ (vi) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Q.2 Find the following :

(i) $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$ (ii) $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$ (iii) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ (iv) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

(v) $\sin\left[\cos^{-1}\frac{3}{5}\right]$ (vi) $\tan^{-1}\left(\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right)+\tan^{-1}\left(\frac{\tan\alpha}{4}\right)$ where $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$

Q.3 Prove that:

(a) $2\cos^{-1}\frac{3}{\sqrt{13}}+\cot^{-1}\frac{16}{63}+\frac{1}{2}\cos^{-1}\frac{7}{25}=\pi$ (b) $\cos^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(-\frac{7}{25}\right)+\sin^{-1}\frac{36}{325}=\pi$

(c) $\text{arc cos}\sqrt{\frac{2}{3}}-\text{arc cos}\frac{\sqrt{6}+1}{2\sqrt{3}}=\frac{\pi}{6}$

(d) Solve the inequality: $(\text{arc sec } x)^2-6(\text{arc sec } x)+8>0$

Q.4 Find the domain of definition the following functions.
(Read the symbols [*] and {*} as greatest integers and fractional part functions respectively.)

(i) $f(x)=\text{arc cos}\frac{2x}{1+x}$ (ii) $\sqrt{\cos(\sin x)}+\sin^{-1}\frac{1+x^2}{2x}$

(iii) $f(x)=\sin^{-1}\left(\frac{x-3}{2}\right)-\log_{10}(4-x)$

(iv) $f(x)=\frac{\sqrt{1-\sin x}}{\log_5(1-4x^2)}+\cos^{-1}(1-\{x\})$, where {x} is the fractional part of x .

(v) $f(x)=\sqrt{3-x}+\cos^{-1}\left(\frac{3-2x}{5}\right)+\log_6\left(2|x|-3\right)+\sin^{-1}\left(\log_2 x\right)$

(vi) $f(x)=\log_{10}\left(1-\log_7(x^2-5x+13)\right)+\cos^{-1}\left(\frac{3}{2+\sin\frac{9\pi x}{2}}\right)$

(vii) $f(x)=e^{\sin^{-1}\left(\frac{x}{2}\right)}+\tan^{-1}\left[\frac{x}{2}-1\right]+\ell n\left(\sqrt{x-[x]}\right)$

(viii) $f(x)=\sqrt{\sin(\cos x)}+\ln(-2\cos^2 x+3\cos x+1)+e^{\cos^{-1}\left(\frac{2\sin x+1}{2\sqrt{2\sin x}}\right)}$

Q.5 Find the domain and range of the following functions .
(Read the symbols [*] and {*} as greatest integers and fractional part functions respectively.)

(i) $f(x)=\cot^{-1}(2x-x^2)$ (ii) $f(x)=\sec^{-1}(\log_3\tan x+\log_{\tan x}3)$

(iii) $f(x) = \cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$ (iv) $f(x) = \tan^{-1}\left(\log_{\frac{4}{5}}\left(5x^2 - 8x + 4\right)\right)$

Q.6 Find the solution set of the equation, $3\cos^{-1} x = \sin^{-1}\left(\sqrt{1-x^2}\,(4x^2-1)\right)$.

Q.7 Prove that:

(a) $\sin^{-1}\cos(\sin^{-1} x) + \cos^{-1}\sin(\cos^{-1} x) = \frac{\pi}{2}$, $|x| \le 1$

(b) $2\tan^{-1}(\text{cosec}\tan^{-1}x - \tan\cot^{-1}x) = \tan^{-1}x$ $(x \ne 0)$

(c) $\tan^{-1}\left(\frac{2mn}{m^2-n^2}\right) + \tan^{-1}\left(\frac{2pq}{p^2-q^2}\right) = \tan^{-1}\left(\frac{2MN}{M^2-N^2}\right)$ where $M = mp - nq$, $N = np + mq$,

$\left|\frac{n}{m}\right| < 1$; $\left|\frac{q}{p}\right| < 1$ and $\left|\frac{N}{M}\right| < 1$

(d) $\tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z) = \cot(\cot^{-1} x + \cot^{-1} y + \cot^{-1} z)$

Q.8 Find the simplest value of, $\text{arc}\cos x + \text{arc}\cos\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right)$, $x \in \left(\frac{1}{2}, 1\right)$

Q.9 If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$ then prove that $\frac{x^2}{a^2} - \frac{2.xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$.

Q.10 If $\text{arc}\sin x + \text{arc}\sin y + \text{arc}\sin z = \pi$ then prove that : $(x, y, z > 0)$

(a) $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

(b) $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

Q.11 If $a > b > c > 0$ then find the value of : $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$.

Q.12 Solve the following equations / system of equations:

(a) $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$ (b) $\tan^{-1}\frac{1}{1+2x} + \tan^{-1}\frac{1}{1+4x} = \tan^{-1}\frac{2}{x^2}$

(c) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$ (d) $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}x = \frac{\pi}{4}$

(e) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$ (f) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ & $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$

(g) $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$ $(a > 0, b > 0)$.

Q.13 Let l_1 be the line $4x + 3y = 3$ and l_2 be the line $y = 8x$. L_1 is the line formed by reflecting l_1 across the line $y = x$ and L_2 is the line formed by reflecting l_2 across the x-axis. If θ is the acute angle between L_1 and L_2 such that $\tan\theta = \frac{a}{b}$, where a and b are coprime then find $(a + b)$.

Q.14 Let $y = \sin^{-1}(\sin 8) - \tan^{-1}(\tan 10) + \cos^{-1}(\cos 12) - \sec^{-1}(\sec 9) + \cot^{-1}(\cot 6) - \text{cosec}^{-1}(\text{cosec } 7)$. If y simplifies to $a\pi + b$ then find $(a - b)$.

Q.15 Show that : $\sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(-\frac{19\pi}{8}\right)\right) = \frac{13\pi}{7}$

Q.16 Let $\alpha = \sin^{-1}\left(\frac{36}{85}\right)$, $\beta = \cos^{-1}\left(\frac{4}{5}\right)$ and $\gamma = \tan^{-1}\left(\frac{8}{15}\right)$, find $(\alpha + \beta + \gamma)$ and hence prove that

(i) $\sum \cot\alpha = \prod \cot\alpha$, (ii) $\sum \tan\alpha \cdot \tan\beta = 1$

Q.17 Prove that : $\sin\cot^{-1}\tan\cos^{-1}x = \sin\operatorname{cosec}^{-1}\cot\tan^{-1}x = x$ where $x \in (0,1]$

Q.18 If $\sin^2x + \sin^2y < 1$ for all $x, y \in R$ then prove that $\sin^{-1}(\tan x \cdot \tan y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Q.19 Find all the positive integral solutions of, $\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$.

Q.20 Let $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$ be a function defined $R \to \left(0, \frac{\pi}{2}\right]$ then find the complete set of real values of α for which $f(x)$ is onto.

EXERCISE–2

Q.1 Prove that: (a) $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$

(b) $\cos^{-1}\frac{\cos x + \cos y}{1 + \cos x \cos y} = 2\tan^{-1}\left(\tan\frac{x}{2} \cdot \tan\frac{y}{2}\right)$ (c) $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}} \cdot \tan\frac{x}{2}\right] = \cos^{-1}\left[\frac{b + a\cos x}{a + b\cos x}\right]$

Q.2 If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$ prove that $x^2 = \sin 2y$.

Q.3 If $u = \cot^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$ then prove that $\sin u = \tan^2\theta$.

Q.4 If $\alpha = 2\operatorname{arc}\tan\left(\frac{1+x}{1-x}\right)$ & $\beta = \operatorname{arc}\sin\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$, what the value of $\alpha + \beta$ will be if $x > 1$.

Q.5 If $x \in \left[-1, -\frac{1}{2}\right]$ then express the function $f(x) = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$ in the form of $a\cos^{-1}x + b\pi$, where a and b are rational numbers.

Q.6 Find the sum of the series:

(a) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + ... + \sin^{-1}\frac{\sqrt{n} - \sqrt{n-1}}{\sqrt{n(n+1)}} + ... \infty$

(b) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + ... + \tan^{-1}\frac{2^{n-1}}{1 + 2^{2n-1}} + ... \infty$

(c) $\cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21 + \cot^{-1}31 + ...$ to n terms.

(d) $\tan^{-1}\frac{1}{x^2 + x + 1} + \tan^{-1}\frac{1}{x^2 + 3x + 3} + \tan^{-1}\frac{1}{x^2 + 5x + 7} + \tan^{-1}\frac{1}{x^2 + 7x + 13}$ to n terms.

(e) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \tan^{-1}\frac{1}{32} + ... \infty$

Q.7 Solve the following

(a) $\cot^{-1}x + \cot^{-1}(n^2 - x + 1) = \cot^{-1}(n - 1)$

(b) $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a \quad a \ge 1;\ b \ge 1,\ a \ne b.$

(c) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

Q.8 Express $\frac{\beta^3}{2}\operatorname{cosec}^2\left[\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right] + \frac{\alpha^3}{2}\sec^2\left[\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right]$ as an integral polynomial in α & β.

Q.9 Find the integral values of K for which the system of equations ;

$$\begin{bmatrix} \operatorname{arc}\cos x + (\operatorname{arc}\sin y)^2 & = \frac{K\pi^2}{4} \\ (\operatorname{arc}\sin y)^2 \,.\, (\operatorname{arc}\cos x) & = \frac{\pi^4}{16} \end{bmatrix}$$ possesses solutions & find those solutions.

Q.10 If the value of $\lim_{n\to\infty}\sum_{k=2}^{n}\cos^{-1}\left(\frac{1+\sqrt{(k-1)k(k+1)(k+2)}}{k(k+1)}\right)$ is equal to $\frac{120\pi}{k}$, find the value of k.

Q.11 If $X = \operatorname{cosec} . \tan^{-1} . \cos . \cot^{-1} . \sec . \sin^{-1} a$ & $Y = \sec \cot^{-1} \sin \tan^{-1} \operatorname{cosec} \cos^{-1} a$; where $0 \le a \le 1$. Find the relation between X & Y. Express them in terms of 'a'.

Q.12 Find all values of *k* for which there is a triangle whose angles have measure $\tan^{-1}\left(\frac{1}{2}\right)$, $\tan^{-1}\left(\frac{1}{2}+k\right)$, and $\tan^{-1}\left(\frac{1}{2}+2k\right)$.

Q.13 Prove that the equation ,$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\,\pi^3$ has no roots for $\alpha < \frac{1}{32}$ and $\alpha > \frac{7}{8}$

Q.14 Solve the following inequalities :
(a) $\operatorname{arc}\cot^2 x - 5\operatorname{arc}\cot x + 6 > 0$ (b) $\operatorname{arc}\sin x > \operatorname{arc}\cos x$ (c) $\tan^2(\operatorname{arc}\sin x) > 1$

Q.15 Solve the following system of inequations
$4\operatorname{arc}\tan^2 x - 8\operatorname{arc}\tan x + 3 < 0$ & $4\operatorname{arc}\cot x - \operatorname{arc}\cot^2 x - 3 \ge 0$

Q.16 Consider the two equations in x ; (i) $\sin\left(\frac{\cos^{-1}x}{y}\right) = 1$ (ii) $\cos\left(\frac{\sin^{-1}x}{y}\right) = 0$

The sets $X_1, X_2 \subseteq [-1, 1]$; $Y_1, Y_2 \subseteq I - \{0\}$ are such that

X_1 : the solution set of equation (i) X_2 : the solution set of equation (ii)
Y_1 : the set of all integral values of y for which equation (i) possess a solution
Y_2 : the set of all integral values of y for which equation (ii) possess a solution
Let : C_1 be the correspondence : $X_1 \to Y_1$ such that $x\,C_1\,y$ for $x \in X_1$, $y \in Y_1$ & (x, y) satisfy (i).
C_2 be the correspondence : $X_2 \to Y_2$ such that $x\,C_2\,y$ for $x \in X_2$, $y \in Y_2$ & (x, y) satisfy (ii).
State with reasons if C_1 & C_2 are functions ? If yes, state whether they are bijjective or into?

Q.17 Given the functions $f(x) = e^{\cos^{-1}\left(\sin\left(x+\frac{\pi}{3}\right)\right)}$, $g(x) = \operatorname{cosec}^{-1}\left(\frac{4-2\cos x}{3}\right)$ & the function $h(x) = f(x)$ defined only for those values of x, which are common to the domains of the functions f(x) & g(x). Calculate the range of the function h(x).

Q.18 (a) If the functions $f(x) = \sin^{-1}\frac{2x}{1+x^2}$ & $g(x) = \cos^{-1}\frac{1-x^2}{1+x^2}$ are identical functions, then compute their domain & range .

(b) If the functions $f(x) = \sin^{-1}(3x - 4x^3)$ & $g(x) = 3\sin^{-1}x$ are equal functions, then compute the maximum range of x.

Q.19 Show that the roots r, s, and t of the cubic $x(x-2)(3x-7) = 2$, are real and positive. Also compute the value of $\tan^{-1}(r) + \tan^{-1}(s) + \tan^{-1}(t)$.

Q.20 Solve for x : $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right) < \pi - 3$.

EXERCISE–3

Q.1 The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is :

(A) zero (B) one (C) two (D) infinite [JEE '99, 2 (out of 200)]

Q.2 Using the principal values, express the following as a single angle :

$$3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \sin^{-1}\frac{142}{65\sqrt{5}}.$$ [REE '99, 6]

Q.3 Solve, $\sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c} = \sin^{-1}x$, where $a^2 + b^2 = c^2$, $c \neq 0$. [REE 2000(Mains), 3 out of 100]

Q.4 Solve the equation:

$$\cos^{-1}\left(\sqrt{6}x\right) + \cos^{-1}\left(3\sqrt{3}x^2\right) = \frac{\pi}{2}$$ [REE 2001 (Mains), 3 out of 100]

Q.5 If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - ....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x equals to [JEE 2001(screening)]

(A) 1/2 (B) 1 (C) – 1/2 (D) – 1

Q.6 Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$ [JEE 2002 (mains) 5]

Q.7 Domain of $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{4}, \frac{3}{4}\right)$ (C) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (D) $\left[-\frac{1}{4}, \frac{1}{2}\right]$

[JEE 2003 (Screening) 3]

Q.8 If $\sin\left(\cot^{-1}(x+1)\right) = \cos(\tan^{-1}x)$, then x =

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $\frac{9}{4}$

[JEE 2004 (Screening)]

INVERSE TRIGONOMETRY

EXERCISE–1

Q 1. **(i)** $\frac{1}{\sqrt{3}}$, **(ii)** 1, **(iii)** $\frac{5\pi}{6}$, **(iv)** $-\frac{\pi}{3}$, **(v)** $\frac{4}{5}$, **(vi)** $\frac{17}{6}$ **Q 2.** **(i)** $\frac{1}{2}$, **(ii)** -1, **(iii)** $-\frac{\pi}{4}$, **(iv)** $\frac{2\pi}{3}$, **(v)** $\frac{4}{5}$, **(vi)** α

Q.3 **(d)** $(-\infty, \sec 2) \cup [1, \infty)$

Q 4. **(i)** $-1/3 \le x \le 1$ **(ii)** $\{1, -1\}$ **(iii)** $1 \le x < 4$
(iv) $x \in (-1/2, 1/2), x \ne 0$ **(v)** $(3/2, 2]$
(vi) $\{7/3, 25/9\}$ **(vii)** $(-2, 2) - \{-1, 0, 1\}$ **(viii)** $\{x \mid x = 2n\pi + \frac{\pi}{6}, n \in I\}$

Q5. **(i)** D : $x \,\varepsilon\, R$ R : $[\pi/4, \pi)$
(ii) D: $x \in \left(n\pi, n\pi + \frac{\pi}{2}\right) - \left\{x \middle| x = n\pi + \frac{\pi}{4}\right\}$ $n \in I$; R : $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right] - \left[\frac{\pi}{2}\right]$
(iii) D : $x \in R$ R : $\left[0, \frac{\pi}{2}\right)$ **(iv)** D : $x \in R$ R : $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$

Q 6. $\left[\frac{\sqrt{3}}{2}, 1\right]$ **Q 8.** $\frac{\pi}{3}$ **Q.11** π

Q.12 **(a)** $x = \frac{1}{2}\sqrt{\frac{3}{7}}$ **(b)** $x = 3$ **(c)** $x = 0, \frac{1}{2}, -\frac{1}{2}$ **(d)** $x = \frac{3}{\sqrt{10}}$
(e) $x = 2 - \sqrt{3}$ or $\sqrt{3}$ **(f)** $x = \frac{1}{2}, y = 1$ **(g)** $x = \frac{a - b}{1 + ab}$

Q.13 57 **Q.14** 53 **Q 19.** $x = 1$; $y = 2$ & $x = 2$; $y = 7$ **Q.20** $\frac{1 \pm \sqrt{17}}{2}$

EXERCISE–2

Q 4. $-\pi$ **Q5.** $6\cos^2 x - \frac{9\pi}{2}$, so $a = 6$, $b = -\frac{9}{2}$

Q 6. **(a)** $\frac{\pi}{2}$ **(b)** $\frac{\pi}{4}$ **(c)** $\text{arc cot}\left[\frac{2n + 5}{n}\right]$ **(d)** arc tan $(x + n)$ – arc tan x **(e)** $\frac{\pi}{4}$

Q 7. **(a)** $x = n^2 - n + 1$ or $x = n$ **(b)** $x = ab$ **(c)** $x = \frac{4}{3}$ **Q 8.** $(\alpha^2 + \beta^2)(\alpha + \beta)$

Q 9. $K = 2$; $\cos\frac{\pi^2}{4}, 1$ & $\cos\frac{\pi^2}{4}, -1$ **Q 10.** 720 **Q.11** $X = Y = \sqrt{3 - a^2}$

Q 12. $k = \frac{11}{4}$ **Q 14.** **(a)** $(\cot 2, \infty) \cup (-\infty, \cot 3)$ **(b)** $\left[\frac{\sqrt{2}}{2}, 1\right)$ **(c)** $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$

Q15. $\left(\tan\frac{1}{2}, \cot 1\right]$ **Q16.** C_1 is a bijective function, C_2 is many to many correspondence, hence it is not a function

Q17. $[e^{\pi/6}, e^{\pi}]$ **Q 18.(a)** D : $[0, 1]$, R : $[0, \pi/2]$ **(b)** $-\frac{1}{2} \le x \le \frac{1}{2}$ **(c)** D : $[-1, 1]$, R : $[0, 2]$

Q.19 $\frac{3\pi}{4}$ **Q.20** $x \in (-1, 1)$

EXERCISE–3

Q.1 C **Q.2** π **Q.3** $x \in \{-1, 0, 1\}$ **Q.4** $x = \frac{1}{3}$ **Q.5** B **Q.7** D **Q.8** A

EXERCISE–4 (Inv. Trigono.)

Part : (A) Only one correct option

1. If $\cos^{-1}\lambda + \cos^{-1}\mu + \cos^{-1}v = 3\pi$ then $\lambda\mu + \mu v + v\lambda$ is equal to
(A) – 3 (B) 0 (C) 3 (D) – 1

2. Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) none of these

3. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is
(A) x = 2 (B) x = – 4 (C) x = 4 (D) none of these

4. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}]$, where $x \in \left(\frac{\pi}{2}, \pi\right)$ is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) $-\frac{\pi}{2}$

5. The set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x is
(A) {0} (B) (–2, 2) (C) R (D) none of these

6. $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$ is equal to
(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

7. $\cos^{-1}\left\{\frac{1}{2}x^2 + \sqrt{1-x^2}\cdot\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$ holds for
(A) $|x| \le 1$ (B) $x \in R$ (C) $0 \le x \le 1$ (D) $-1 \le x \le 0$

8. $\tan^{-1}a + \tan^{-1}b$, where a > 0, b > 0, ab > 1, is equal to
(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$
(C) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

9. The set of values of 'x' for which the formula $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ is true, is
(A) (– 1, 0) (B) [0, 1] (C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

10. The set of values of 'a' for which $x^2 + ax + \sin^{-1}(x^2 - 4x + 5) + \cos^{-1}(x^2 - 4x + 5) = 0$ has at least one solution is
(A) $(-\infty, -\sqrt{2\pi}] \cup [\sqrt{2\pi}, \infty)$ (B) $(-\infty, -\sqrt{2\pi}) \cup (\sqrt{2\pi}, \infty)$
(C) R (D) none of these

11. All possible values of p and q for which $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$ holds, is
(A) p = 1, q = $\frac{1}{2}$ (B) q > 1, p = $\frac{1}{2}$ (C) $0 \le p \le 1$, q = $\frac{1}{2}$ (D) none of these

12. If $[\cot^{-1}x] + [\cos^{-1}x] = 0$, where [.] denotes the greatest integer function, then complete set of values of 'x' is
(A) (cos1, 1] (B) (cot 1, cos 1) (C) (cot1, 1] (D) none of these

13. The complete solution set of the inequality $[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \le 0$, where [.] denotes greatest integer function, is
(A) $(-\infty, \cot 3]$ (B) [cot 3, cot 2] (C) $[\cot 3, \infty)$ (D) none of these

14. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$, $x \ne 0$ is equal to
(A) x (B) 2x (C) $\frac{2}{x}$ (D) $\frac{x}{2}$

15. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to
(A) 1/3 (B) 3 (C) 1 (D) – 1

16. If u = $\cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to

(A) $\sqrt{\tan\alpha}$ (B) $\sqrt{\cot\alpha}$ (C) $\tan\alpha$ (D) $\cot\alpha$

17. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, $\frac{\pi}{2} < x < \pi$, is:

(A) $\pi - \frac{x}{2}$ (B) $\frac{\pi}{2} + \frac{x}{2}$ (C) $\frac{x}{2}$ (D) $2\pi - \frac{x}{2}$

18. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$, is/are

(A) 0 (B) 1 (C) 2 (D) more than 2

19. The number of solutions of the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is

(A) 0 (B) 1 (C) 2 (D) 3

20. If $\tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+2.3} + \tan^{-1}\frac{1}{1+3.4} + \ldots\ldots + \tan^{-1}\frac{1}{1+n(n+1)} = \tan^{-1}\theta$, then θ is equal to

(A) $\frac{n}{n+2}$ (B) $\frac{n}{n+1}$ (C) $\frac{n+1}{n}$ (D) $\frac{1}{n}$

21. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of 'n' is:

(A) 1 (B) 5 (C) 9 (D) none of these

22. The number of real solutions of (x, y) where, $|y| = \sin x$, $y = \cos^{-1}(\cos x)$, $-2\pi \le x \le 2\pi$, is:

(A) 2 (B) 1 (C) 3 (D) 4

23. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to

(A) 3/4 (B) – 3/4 (C) 1/16 (D) 1/4

Part : (B) May have more than one options correct

24. α, β and γ are three angles given by

$\alpha = 2\tan^{-1}(\sqrt{2}-1)$, $\beta = 3\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\frac{1}{3}$. Then

(A) $\alpha > \beta$ (B) $\beta > \gamma$ (C) $\alpha < \gamma$ (D) $\alpha > \gamma$

25. $\cos^{-1}x = \tan^{-1}x$ then

(A) $x^2 = \left(\frac{\sqrt{5}-1}{2}\right)$ (B) $x^2 = \left(\frac{\sqrt{5}+1}{2}\right)$ (C) $\sin(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$ (D) $\tan(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

26. For the equation $2x = \tan(2\tan^{-1}a) + 2\tan(\tan^{-1}a + \tan^{-1}a^3)$, which of the following is invalid?

(A) $a^2x + 2a = x$ (B) $a^2 + 2ax + 1 = 0$ (C) $a \ne 0$ (D) $a \ne -1, 1$

27. The sum $\sum_{n=1}^{\infty}\tan^{-1}\frac{4n}{n^4-2n^2+2}$ is equal to:

(A) $\tan^{-1}2 + \tan^{-1}3$ (B) $4\tan^{-1}1$ (C) $\pi/2$ (D) $\sec^{-1}\left(-\sqrt{2}\right)$

28. If the numerical value of $\tan(\cos^{-1}(4/5) + \tan^{-1}(2/3))$ is a/b then

(A) a + b = 23 (B) a – b = 11 (C) 3b = a + 1 (D) 2a = 3b

29. If α satisfies the inequation $x^2 - x - 2 > 0$, then a value exists for

(A) $\sin^{-1}\alpha$ (B) $\cos^{-1}\alpha$ (C) $\sec^{-1}\alpha$ (D) $\text{cosec}^{-1}\alpha$

30. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right\}$ then:

(A) $f\left(\frac{2}{3}\right) = \frac{\pi}{3}$ (B) $f\left(\frac{2}{3}\right) = 2\cos^{-1}\frac{2}{3} - \frac{\pi}{3}$

(C) $f\left(\frac{1}{3}\right) = \frac{\pi}{3}$ (D) $f\left(\frac{1}{3}\right) = 2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$ m

EXERCISE–8

1. Find the value of the following :

(i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\tan\left[\cos^{-1}\frac{1}{2} + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

(iii) $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$

2. Solve the equation : $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$

3. Solve the equation : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

4. Solve the following equations :

(i) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $(x > 0)$

(ii) $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

5. Find the value of $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, if $x > y > 1$.

6. If $x = \sin(2\tan^{-1}2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ then find the relation between x and y .

7. If arc sinx + arc siny + arc sinz = π then prove that:(x, y, z > 0)

(i) $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

(ii) $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

8. Solve the following equations :

(i) $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ $a \ge 1$; $b \ge 1$, $a \ne b$.

(ii) $\sin^{-1}\sqrt{\frac{x}{1+x}} - \sin^{-1}\frac{x-1}{x+1} = \sin^{-1}\frac{1}{\sqrt{1+x}}$

(iii) Solve for x, if $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

9. If $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$. What the value of $\alpha + \beta$ will be if $x > 1$?

10. If X = cosec $\tan^{-1}$ cos $\cot^{-1}$ sec $\sin^{-1}$ a & Y = sec $\cot^{-1}$ sin $\tan^{-1}$ cosec $\cos^{-1}$ a; where $0 \le a \le 1$. Find the relation between X & Y. Express them in terms of 'a'.

11. Solve the following inequalities:

(i) $\cos^{-1}x > \cos^{-1}x^2$ (ii) $\sin^{-1}x > \cos^{-1}x$

(iii) $\tan^{-1}x > \cot^{-1}x$. (iv) $\sin^{-1}(\sin 5) > x^2 - 4x$.

(v) $\tan^2(\text{arc}\sin x) > 1$ (vi) $\text{arccot}^2 x - 5\,\text{arccot}\,x + 6 > 0$

(vii) $\tan^{-1}2x \ge 2\tan^{-1}x$

12. Find the sum of each of the following series :

(i) $\cot^{-1}\frac{31}{12} + \cos^{-1}\frac{139}{12} + \cot^{-1}\frac{319}{12} + \ldots + \cot^{-1}\left(3n^2 - \frac{5}{12}\right)$.

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots.. + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots\ldots\ldots\ldots\ldots\ldots \infty$

13. Prove that the equation, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$.

14. (i) Find all positive integral solutions of the equation, $\tan^{-1}x + \cot^{-1}y = \tan^{-1}3$.

(ii) If 'k' be a positive integer, then show that the equation:
$\tan^{-1}x + \tan^{-1}y = \tan^{-1}k$ has no non–zero integral solution.