CHAPTER 7

PERMUTATIONS (Arrangements) AND COMBINATIONS (selections)

In permutation **order is important**, since 27 & 72 are different numbers(arrangements). In combination order is not important.

- **Fundamental principle of counting (FPC)**

Station A — m ways — Station B — n ways — Station C

then by FPC there are mn ways to go from station A to station C

- The number of permutations of n different things taken r at a time, where repetition is not allowed is given by $\mathbf{{}^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1)}$ **where** $\mathbf{0<r\leq n}$.

  eg $\quad {}^5P_2 = 5 \times 4 = 20$

  $\quad\quad {}^7P_3 = 7 \times 6 \times 5 = 210$

- Factorial notation: **n! = 1 x 2 x 3 x …..x n, where n is a natural number**

  eg $5! = 1 \times 2 \times 3 \times 4 \times 5$

  we define 0! = 1

  also $n! = n(n-1)!$

  $\quad\quad = n(n-1)(n-2)!$

- $\mathbf{{}^nP_r = \dfrac{n!}{(n-r)!}}$ **Where** $\mathbf{0 \leq r \leq n}$

- Number of permutations of n different things, taken r at a time, where repetition is allowed is $\mathbf{n^r}$

- Number of permutations of n objects taken all at a time, where P_1 objects are of first kind, P_2 objects are of second kind…..P_k objects are of the k^{th} kind and rest, if any, are all different is $\mathbf{\dfrac{n!}{P_1!.P_2!\ldots P_k!}}$ **(eg 9)**

- The number of combinations of n different things taken r at a time is given by

  $\mathbf{{}^nC_r = \dfrac{n(n-1)(n-2)\ldots.(n-r+1)}{1.2.3\ldots\ldots r}}$, **where** $\mathbf{0< r\leq n}$

  eg ${}^5C_3 = \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} = {}^5C_2$

- **$^nC_r = {}^nC_{n-r}$**
  eg $^5C_3 = {}^5C_2$
  $^7C_5 = {}^7C_2$
- **$^nC_r = \dfrac{n!}{r!(n-r)!}$, where $0 \le r \le n$.**
- **$^nC_r = {}^nC_s$ implies $r = s$ or $n = r+s$** (eg 17*) 1 mark
- **$^nC_n = {}^nC_0 = 1$**
- **$^nC_1 = n$**
  eg $^5C_1 = 5$
- **$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$**

Ex 7.1

1, 2, 4

Ex 7.2

4* , 5* (1 mark)

eg 8* (1 mark), eg 11*, 12**,13**,14**,16** (4 marks)

Ex 7.3

7*, 8*, 9**,10**,11**

Theorm 6 to prove (4 marks)*

eg 17* (1 mark) use direct formula n = 9+8 = 17 since $^nC_r = {}^nC_s$ implies r = s or n = r+s

eg 19**

Ex 7.4

2**,3*,5*,6*,7**,8*,9*

eg 21**, eg 23*(HOT), eg 24*

Misc Ex

1**,2**,3**,4*,5*,7**,10**,11**

EXTRA/HOT QUESTIONS

1) How many permutations can be made with letters of the word MATHEMATICS ? In how many of them vowels are together?
2) In how many ways can 9 examination papers be arranged so that the best and the worst papers are never together. (HOT)
3) How many numbers greater than 56000 can be formed by using the digits 4,5,6,7,8; no digit being repeated in any number.
4) Find the number of ways in which letters of the word ARRANGEMENT can be arranged so that the two A's and two R's do not occur together. (HOT)
5) If C(2n,3): C(n,3):: 11:1 find n.
6) If P(11,r) = P(12,r-1) find r.

7) Five books, one each in Physics, Chemistry, Mathematics, English and Hindi are to be arranged on a shelf. In how many ways can this be done?
8) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ find the values of n and r.
9) A box contains five red balls and six black balls. In how many ways can six balls be selected so that there are at least two balls of each color.
10) A group consist of 4 girls and 7 boys in how many ways can a committee of five members be selected if the committee has i) no girl
    ii) atleast 1 boy and 1 girl
    iii) atlest 3 girls.

Note : atleast means $\geq$

Answers

1) 4989600, 120960
2) 282240 Hint (consider the best and the worst paper as one paper)
3) 90
4) 1678320
5) 6
6) 9
7) 120
8) n = 3, r = 2
9) 425
10) i) 21
    ii) 441
    iii) 91