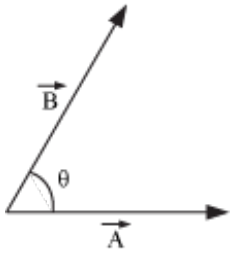
Work, energy, power

- Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\vec{A}.\vec{B} = AB\cos\theta$$



  - The result of the scalar product of two vectors is a scalar quantity.
  - When two vectors are parallel their scalar product is equal to the product of their magnitudes.
  - When two vectors are perpendicular their scalar product is equal to zero.
  - **Properties of Scalar Product of two vectors**
    - Scalar product of two vectors is commutative, i.e.,

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

    - Scalar product is distributive, i.e.,

$$\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

  - Scalar product of a vector with itself gives the square of its magnitude.
  - Dot Product of two vectors $\vec{A}$ and $\vec{B}$ in Cartesian Coordinates is

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Types of Collision

- Elastic collision – Those collisions in which both momentum and kinetic energy of the system are conserved.
- Inelastic collision – Those collisions in which momentum of the system is conserved, but kinetic energy is not conserved.

Characteristics of elastic collision

- total energy of the syatem is conserved
- linear momentum is conserved
- kinetic energy is conserved

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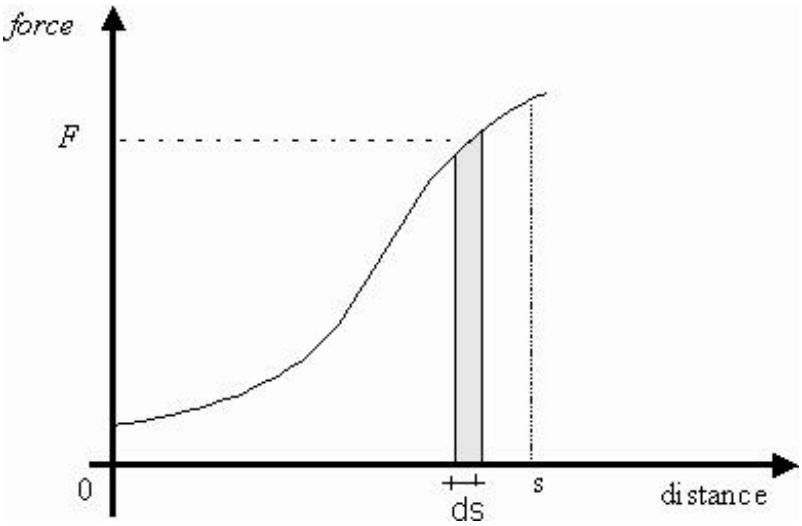
Work – energy theorem

- Work done by net force is equal to the change in kinetic energy of the body
- $W = \bar{F}.\bar{d} = Fd\ \cos\theta$ $Dotproduct, henceitisascalarquantity$
- No work is done if
    - displacement is zero
    - force is zero
    - force and displacement are mutually perpendicular i.e.,

$$\theta = \frac{\pi}{2} = 90°$$

- Kinetic energy, $K = \frac{1}{2}m\bar{v}.\bar{v} = \frac{1}{2}mv^2$

- **Work done by variable force**



- **Work done is the area subtended by the curve on the distance axis.**

$$W = \int_{x_i}^{x_f} F(x)\mathrm{d}x$$

- **Work – energy theorem for variable force**

$$\mathrm{d}K = F\mathrm{d}x$$
$$K_f - K_i = \int_{x_i}^{x_f} F\mathrm{d}x$$

- **Potential energy**
  - For constant force

    $V(h) = mgh$

  - For variable force,

$$\int_{x_i}^{x_f} F(x)\mathrm{d}x = -\int_{V_i}^{V_f} \mathrm{d}V = V_i - V_f$$

- Conservation of mechanical energy

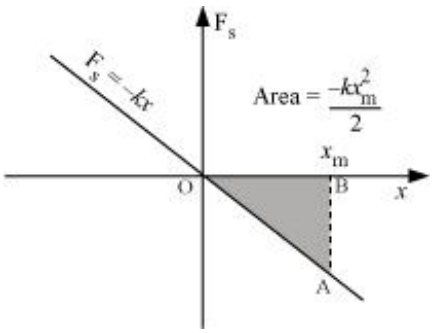
  $\Delta K + \Delta V = 0$

  For a body, $K_i + V(x_i) = K_f + V(x_f)$

- For a conservative force, work done on a closed path is zero.

Potential energy of a spring

- **Hook's Law :** $F_s = -kx$ (k = Spring constant; unit: N m^{-1})
  - For compression, $F_s \rightarrow$+ ve and $x \rightarrow$–ve
  - For expansion, $F_s \rightarrow$–ve and $x \rightarrow$+ve



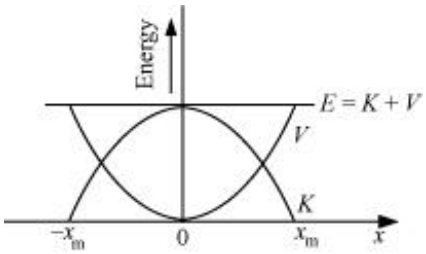
- Work done by a spring,

$$W_s = \int_0^{x_m} F_s \, dx = -\int_0^{x_m} kx \, dx = -\frac{kx_m^2}{2}$$

- Potential energy of the spring.

$$\text{P.E., } V(x) = \frac{kx^2}{2}$$

$$\text{Maximum speed, } v_m = \sqrt{\frac{k}{m}} x_m \text{ (at the equilibrium position)}$$



Different Forms of Energy

- Internal energy – The sum of kinetic and potential energies of all the molecules constituting the body is called internal energy.
- Heat energy – A body possesses heat energy due to the disorderly motion of its molecules.
- Chemical energy – A body possesses chemical energy because of chemical bonding of its atoms.
    - Exothermic reaction: Heat is released.
    - Endothermic reaction: Heat is absorbed.
- Electrical energy – It is the work done in order to move an electric charge from one point to another in an electric field.
- Nuclear energy – It is the energy released when a heavy nucleus $suchasU-235$ breaks up

into lighter nuclei on being bombarded by a slow neutron.

Power

- The rate of doing work is called power. The average power is given by,

$$P_{av} = \frac{W}{t}$$

- Instantaneous power **–** Limiting value of the average power of an agent in a small time interval, when the time interval approaches zero.

  If ΔW is work done in a small interval Δt, then instantaneous power is defined as

$$\mathrm{P} = \underset{\Delta t \to 0}{\mathrm{Lt}} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$