Sample Paper-05
Mathematics
Class – XI

Answers

Section A

1. Solution :

$$f(x)=a^x$$

$$f(y)=a^y$$

$$f(x).f(y)=a^x.a^y=a^{x+y}=f(x).f(y)$$

2. Solution :

When $x=0, y=1$ in both cases. Hence

$$(A\cap B)=\{0,1\}$$

3. Solution : 2^{pq}

4. Solution

They are parallel since

$$\begin{vmatrix} a & -b \\ \frac{a}{2} & \frac{-b}{2} \end{vmatrix}=0$$

Section B

5. Solution

Area of a triangle

$$\frac{1}{2}\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} = 9$$

6. Solution

$$x^2+y^2=25$$

7. Solution

When $f(x)=x^2$

$$f(x)=x^2$$
$$f'(x)=2x$$
$$f'(a+b)=2(a+b)$$

$$f'(a)=2a$$

$f'(b) = 2b$

$f'(a) + f'(b) = 2(a+b)$

$= f'(a+b)$

When $f(x) = x^3$

$f(x) = x^3$

$f'(x) = 3x^2$

$f'(a+b) = 3(a+b)^2$

$f'(a) = 3a^2$

$f'(b) = 3b^2$

$f'(a) + f'(b) = 3(a^2 + b^2)$

$\neq f'(a+b)$

8. Solution

$\alpha^3 + \beta^3 = (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta] = -p[p^2 - 3q]$

9. Solution

Total number of 3 digit numbers with 0 in units place = 90

The digits that can go into tens place for the number to be divisible by 4 = $0, 2, 4, 6, 8$

100th place can be formed with any of the 9 digits excepting 0

Hence total number of 3 digits number divisible by 4 is $9 \times 5 = 45$

Probability= $\frac{45}{90} = \frac{1}{2}$

10. Solution

$\tan(45+x) = \frac{1+\tan x}{1-\tan x}$

$= \frac{\cos x + \sin x}{\cos x - \sin x}$

$= \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{cos^2 x - \sin^2 x}$

$= \frac{1 + sin2x}{\cos 2x} = \sec 2x + \tan 2x$

11. Solution

P(n) = n(n+1)

P(1) = 2, even

P(k) = k(k+1) let this be true

P(k+1) = (k + 1)(k + 2)

= k^2 + 3k + 2

$$= k^2 + k + 2k + 2$$

$$= k(k+1) + 2(k+1)\ True$$

12. Solution

$$n[(A \cup B \cup C)] = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$n[(A \cup B \cup C)] = 4000 + 2000 + 1000 - 400 - 400 - 400 + 200$$
$$n[(A \cup B \cup C)] = 6000$$

Section C

13. Solution.

$$\frac{X^2}{k^2} + \frac{y^2}{\frac{k^2}{3}} = 1$$

Latus rectum is = $\dfrac{\left(\frac{2k^2}{3}\right)}{k}$

= $\dfrac{2k}{3}$

$$e = \sqrt{\left(\frac{k^2 - \frac{k^2}{3}}{k^2}\right)}$$
$$= \sqrt{\frac{2}{3}}$$
$$= \frac{\sqrt{6}}{3}$$

Coordinates of foci are $(ae, 0)$ and $(-ae, 0)$

Coordinates are = $(\frac{\sqrt{6}}{3}k, 0)$ and $(\frac{-\sqrt{6}}{3}k, 0)$

14. Solution

Slope of line AB joining the points $(-8,0)$ and $(12,0)$ = 0
Its midpoint = $(2,0)$
Equation to the line perpendicular to AB and passing through $(2,0)$ is $x = 2$
Slope of line AC joining the points $(-8,0)$ and $(0,8)$ = 1
Its midpoint = $(-4,4)$
Equation to the line perpendicular to AC and passing through $(-4,4)$ is $y = -x$
So the center of the circle will be the point of intersection of line AB and line AC. Center of circle at point (2, -2)
Radius = $\sqrt{(2-0)^2 + (-2-8)^2}$ = $\sqrt{104}$

Area $= 104\pi$

15. Solution

$$2S_1 = n[2a+(n-1)d]$$
$$2S_2 = 2n[2a+(2n-1)d]$$
$$2S_3 = 3n[2a+(3n-1)d]$$
$$\frac{2S_1}{n} = 2a+(n-1)d$$
$$\frac{2S_3}{3n} = 2a+(n-1)d$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 4a+d(n-1+3n-1)$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 4a+2(2n-1)d$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 2.\frac{2S_2}{2n}$$
$$\frac{2S_3}{3n} = \frac{2S_2}{2n}-\frac{2S_1}{n}$$
$$\frac{2S_3}{3n} = \frac{4S_2}{2n}-\frac{4S_1}{2n}$$
$$S_3 = 3(S_2-S_1)$$

16. Solution

$$f(x) = 3x^2-6x-11$$
$$f(x) = 3(x^2-2x-\frac{11}{3})$$
$$f(x) = 3(x^2-2x+1-1-\frac{11}{3})$$
$$f(x) = 3[(x-1)^2-\frac{11}{3}-1)$$
$$f(x) = 3[(x-1)^2-\frac{14}{3})$$
$$f(x) = 3(x-1)^2-14$$

$f(x)$ will attain minimum when $x=1$

Minimum value of $f(x) = -14$

17. Solution

$$f(x) = \frac{a^x}{a^x+\sqrt{a}}$$
$$f(1-x) = \frac{a^{1-x}}{a^{1-x}+\sqrt{a}}$$

$$f(x)+f(1-x)=\frac{a^x}{a^x+\sqrt{a}}+\frac{a^{1-x}}{a^{1-x}+\sqrt{a}} = 1$$

18. Solution

$$\frac{\tan 2x\tan x}{\tan 2x-\tan x}=\frac{\frac{2\tan x}{1-\tan^2 x}\tan x}{\frac{2\tan x}{1-\tan^2 x}-\tan x}$$

$$=\frac{2\tan^2 x}{\tan x+\tan^2 x}$$

$$=\frac{2\tan x}{1+\tan^2 x}$$

$$=\sin 2x$$

19. Solution

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!)}{(n+2)!-(n+1)!}=\lim_{n\to\infty}\frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)!}$$

$$=\lim_{n\to\infty}\frac{(n+3)}{((n+1)}$$

$$\lim_{n\to\infty}\frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$=1$$

20.Solution

Form a quadratic equation whose roots are

$1+i \quad and \quad 1-i$

The equation is

$x^2-2x+2=0$

The given expression

$x^3+x^2-4x+13=x(x^2-2x+2)+3(x^2-2x+2)+7$

$x^3+x^2-4x+13=x(0)+(0)+7$

$x^3+x^2-4x+13=7$

21. Solution

$x^2-(\alpha+\beta)x+\alpha\beta-k^2=0$

Discriminant of the above quadratic is

$\{(\alpha+\beta)\}^2-4(\alpha\beta-k^2)=(\alpha-\beta)^2+k^2$ is always positive and hence the roots are real.

22. Solution

Let the roots be

$p\alpha \quad and \quad q\beta$

Then

$$p\alpha+q\alpha=-\frac{n}{l}\ldots(1)$$

$$pq\alpha^2=\frac{n}{l}$$

$$\alpha=\frac{\sqrt{n}}{\sqrt{l}}\times\frac{1}{\sqrt{pq}}\ldots(2)$$

Hence substituting equation 2 in equation 1

$$(p+q)\frac{\sqrt{n}}{\sqrt{l}}\times\frac{1}{\sqrt{pq}}+\frac{n}{l}=0$$

On simplifying ,

$$\frac{\sqrt{p}}{\sqrt{q}}+\frac{\sqrt{q}}{\sqrt{p}}+\frac{\sqrt{n}}{\sqrt{l}}=0$$

23. Solution

$$\lim_{x\to\pi}(\pi-x)\tan\frac{x}{2}=\lim_{x\to\pi}\frac{2(\pi-x)}{2}\cot\frac{\pi-x}{2}$$

$$=\lim_{x\to\pi}\frac{2(\pi-x)}{2}\frac{\cos\frac{\pi-x}{2}}{\sin\frac{\pi-x}{2}}$$

$$=\lim_{x\to\pi}2\frac{\frac{\cos\frac{\pi-x}{2}}{\sin\frac{\pi-x}{2}}}{\frac{\pi-x}{2}}$$

$$=\lim_{\frac{x-\pi}{2}\to 0}2\frac{\frac{\cos\frac{\pi-x}{2}}{\sin\frac{\pi-x}{2}}}{\frac{\pi-x}{2}}$$

$$= \lim_{\frac{x-\pi}{2}\to 0} 2\frac{\cos\frac{x-\pi}{2}}{\frac{\sin\frac{x-\pi}{2}}{\frac{x-\pi}{2}}} = 2 \quad \textbf{since the limit of} \quad \frac{\sin\frac{\mathbf{x-\pi}}{\mathbf{2}}}{\frac{\mathbf{x-\pi}}{\mathbf{2}}} \quad = \quad \mathbf{1}$$

Section D

24. Solution

Let

$a = x-1$

$b = x$

$c = x+1$

Then

$(x-1-i)((x-1+i)(x+1+i)(x+1-i) = \{(x-1)^2 - i^2\}\{(x+1)^2 - i^2\}$

$= \{(x-1)^2 + 1\}\{(x+1)^2 + 1\}$

$= \{(x-1)(x+1)\}^2 + (x-1)^2 + (x+1)^2 + 1$

$= (x^2-1)^2 + (x-1)^2 + (x+1)^2 + 1$

$= x^4 + 1$

$= b^4 + 1$

25. Solution

Multiply both Numerator and denominator with $(1-i)^2$ Then

$$\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^n(1-i)^2}{(1-i)^n}$$

multiplying both Numerator & denominator with $(1+i)^n$

$$= \frac{(1+i)^n(-2i)(1+i)^n}{(1-i)^n(1+i)^n}$$

Simplifying

$$= \frac{\{(1+i)^2\}^n(-2i)}{(1-i^2)^n}$$

On expanding and simplifying

$$=\frac{2^n i^n(-2)i}{2^n}$$

$$=-2i^{n+1}$$

$$=\frac{2(i)^{n+1}}{i^2}=2i^{n-1}$$

26. Solution

Let the point be $A(1,2) and B(3,4)$

The mid-point of the line joining A and B is $C(2,3)$

Slope of line AB = $\frac{4-2}{3-1}=1$

Let the required point be $D(\alpha,\beta)$

Then D must be a point on the line perpendicular to the line AB and passing through point C

$\therefore \quad Slope \quad of \quad CD=-1$

$Equation \quad of \quad CD$
$y-3=-1(x-2)$
$x+y=5$
$Equation \quad of \quad AB$
$y-2=1(x-1)$
$x-y+1=0$

The point $D(\alpha,\beta)$ must satisfy the equation

$x+y=5$

$\therefore \alpha+\beta=5\ldots(1)$

The perpendicular distance from (α,β) to AB is

$$\frac{\alpha-\beta+1}{\sqrt{2}}=\sqrt{2}$$

$\alpha-\beta=1\ldots(2)$

Solving equations 1 and 2

$\alpha=3,\beta=2$