SUPPORT STUDY MATERIAL

XI Physics

Support Material, Study Notes and VBQ

Syllabus

Unit No	Name	
I	Physical World and Measurement	
II	Kinematics	
III	Law of Motion	
IV	Work, Energy, Power	
V	Motion of System of Particles and Rigid Body	
VI	Gravitation	
VII	Properties of Bulk Matter	
VIII	Thermodynamics	
IX	Behaviour of Perfect Gas and Kinetic Theory of gases	
X	Oscillations and Waves	

MATHEMATICAL TOOLS

Physical constants:-

1. Mass of an electron (M_e) = 9.1×10^{-31}kg.
2. Mass of a proton (M_p) = 1.6725×10^{-27}kg.
3. Mass of a neutron (M_n) = 1.6746×10^{-27}kg.
4. Charge of an electron (e) = -1.6×10^{-19}c
5. Speed of light in vacuum (c) = 3×10^{8} m/sec.
6. Planck Constant (h) = 6.6256×10^{-34} J x sec .
7. Universal Gravitation constant (G) = 6.67×10^{-11} Nm^2/kg^2.
8. Avogadro Number (N_A) = 6.023×10^{23} mol^{-1}.
9. Boltzmann constant (K) = 1.38×10^{-23} J/K
10. Stefan Constant (σ) = 5.67×10^{-8} $W\,m^{-2}\,K^{-4}$.
11. Wien Displacement Constant (b) = 2.898×10^{-3} m K
12. Solar Constant (S) = 1.388×10^{3} $W\,m^{-2}$
13. Mass of the sun (M_S) = 2×10^{30}kg.
14. Mass of the earth (M_E) = 5.98×10^{24}kg
15. Radius of the earth (R_e) = 6400 Km. = 6.4×10^{6}m.
16. Density of earth 5.522×10^{3}kg/m^3.
17. Average angular velocity of the earth = 7.29×10^{-5}rad./sec
18. Average distance between the sun and earth = 1.5×10^{11}m.
19. Average distance between moon and the earth =3.84×10^{8}m.
20. Magnetic Moment of the earth = 6.4×10^{21} Amp. X m^2.

Conversion Coefficients

1. 1Light year = 9.46×10^{15}m.
2. 1 A.U. = 1.496×10^{11}m.
3. 1Å = 10^{-10}m.
4. 1 Pound = 0.4536kg = 453.6 gm
5. 1 Fermi = 10^{-15}m.
6. 1 C.S.L.= 1.4 x Mass of the sun.

7. 1 Shake = 10^{-8}sec
8. 1ev = 1.6 x10^{-19} Joule.
9. 1 Horse Power = 746 Watt.

Quadratic Equation

An equation of second degree is called a quadratic equation. It is of the form :-

$$ax^2 + bx + c = 0$$

The roots of a quadratic equation are

$$X = \frac{-b \pm (b^2 + 4ac)^{1/2}}{2a}$$

Binomial Theorem

If n is any integer, positive or negative or a fraction and x is any real number, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{\underline{|2}} + \ldots$$

If IxI <<1 , then $(1+x)^n = 1 + nx$.

Mensuration :-

1. Area of a circle = $\pi r^2 = \pi D^2/4$
2. Surface area of a sphere = $4\pi r^2 = \pi D^2$
3. Volume of a sphere = $4/3\ \pi r^3$
4. Surface area of a cylinder = $2\pi r(r+l)$
5. Volume of a cylinder = $\pi r^2 l$
6. Curved surface area of a cone = πrl
7. Volume of a cone = $1/3 \pi r^2 h$
8. Surface area of a cube = 6x$(\text{side})^2$
9. Volume of a cube = $(\text{side})^3$

Fundamental Trigonometric relations

$$\text{Cosec}\theta = \frac{}{\text{Sin}\theta}$$

$$\text{Sec}\theta = \frac{}{\text{Cos}\theta}$$

$\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{}{\text{Tan}\theta}$

$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

$1 + \tan^2\theta = \text{Sec}^2\theta$

$1 + \text{Cot}^2\theta = \text{Cosec}^2\theta$

Sin(A+B) = SinACosB + CosASinB

Cos (A + B) = CosACosB – SinASinB

Sin (A – B) = SinACosB – CosASinB

Cos (A – B) = CosACosB + SinASinB

$$\text{Tan}(A + B) = \frac{\text{TanA} + \text{TanB}}{-\text{TanATanB}}$$

Sin 2A = 2SinACosA

$\text{Cos } 2A = 2\text{Cos}^2 A - 1 = 1 - 2\text{Sin}^2 A = \text{Cos}^2 A - \text{Sin}^2 A$

Sin (A+B) + Sin(A-B) = 2 SinACosB

Cos(A+B) + Cos (A-B) = 2CosACosB

$$\text{CosC} + \text{CosD} = 2\text{Cos}\frac{(C+D)}{2}\text{Cos}\frac{(C-D)}{2})$$

Logarithms

$\text{Log}_a mn = \text{Log}_a m + \text{Log}_a n$

$\text{Log}_a \left(\frac{m}{n}\right) = \text{Log}_a m - \text{Log}_a n$

$\text{Log}_a m = \text{Log}_b m \times \text{Log}_a b$

$\text{Log}_{10} 10^3 = \text{Log}_{10} 1000 = 3$

$\text{Log}_a 1 = 0$

$\text{Log}_a a = 1$

Average Values

$\langle \text{Sin}\theta \rangle = 0$, $\langle \text{Cos}\theta \rangle = 0$

$\langle \text{Sin}^2\theta \rangle = ½$

$\langle \text{Cos}^2\theta \rangle = ½$

Approximate Values

If angle (θ) small then $\theta \longrightarrow 0$

$\text{Sin}\theta \cong \theta$

$\text{Cos}\theta \cong 1$

$\text{Tan}\theta \cong \theta$

Differential Formulae

1. Differentiation of a constant© is zero

$$\frac{dc}{dx} = 0$$

2. $\frac{\mathrm{d(cy)}}{\mathrm{dx}} = \mathrm{c}\frac{\mathrm{dy}}{\mathrm{dx}}$

3. $\frac{d(x^n)}{dx} = nx^{n-1}$

4. $\frac{\mathrm{d[f(x) \pm g(x)]}}{\mathrm{dx}} = \frac{\mathrm{df(x)}}{\mathrm{dx}} \pm \frac{\mathrm{dg(x)}}{\mathrm{dx}}$

5. $\frac{\mathrm{d\{f(x)g(x)\}}}{\mathrm{dx}} = \frac{\mathrm{f(x)dg(x)}}{\mathrm{dx}} + \frac{\mathrm{g(x)df(x)}}{\mathrm{dx}}$

6. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{\{g(x)\}^2}$

7. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

8. $\frac{de^x}{dx} = e^x$

9. $\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx}$

10. $\frac{d\log_e x}{dx} = \frac{}{x}$

11. $\frac{d(a^x)}{dx} = a^x \log_e a$

12. $\frac{d\log_a x}{dx} = \frac{}{x}\log_e a$

13. $\frac{d(\sin x)}{dx} = \cos x$

14. $\frac{d(\cos x)}{dx} = -\sin x$

15. $\frac{d(\tan x)}{dx} = sec^2\, x$

16. $\frac{d(\cot x)}{dx} = -cosec^2\, x$

17. $\frac{d(cosecx)}{dx} = -\mathrm{cosec}\, x\; cot\; x$

18. $\frac{d(secx)}{dx} = \sec x\; tan\; x$

Integral Formulae

1. $\int dx = x + c$ Where c = constant
2. $\int x^{n+1} dx = \frac{x^{n+1}}{n+1} + C$
3. $\int dx/x = \log_e x + c$
4. $\int \text{Sinx}\, dx = -\text{Cosx} + c$
5. $\int \text{Sin ax}\, dx = -\frac{\text{Cos ax}}{a}$
6. $\int \text{Cos x dx} = \text{Sin x} + c$
7. $\int \text{Sec}^2 x\, dx = \tan x + c$
8. $\int \text{Cosec}^2 x\, dx = -\text{Cot x} + c$
9. $\int \text{Sec x tan x dx} = \text{Sec x} + c$
10. $\int \text{Cosec x Cot x dx} = -\text{Cosec x} + c$
11. $\int e^x\, dx = e^x + c$

Physical World And Measurement

There are four fundamental forces which govern both macroscopic and microscopic phenomena. There are

(i) Gravitational force (iii) Electromagnetic force

(ii) Nuclear force (iv) Weak force

The relative strengths of these forces are

$$Fg :Fw:Fe:Fs=1:10^{25}:10^{36}:10^{38} .$$

All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.

(a) Fundamental quantities (b) Derived quantities.

The units of the fundamental quantities called fundamental units , and the units of derived quantities called derived units.

System of units:-

(a) MKS (b) CGS (c) FPS (d) SI

- *The dimensions of a physical quantity are the powers to which the fundamental quantities are raised to represent that physical quantity.*

- The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time, is called dimensional equation.
- According to this principle of homogeneity a physical equation will be dimensionally correct if the dimensions of all the terms in the all the terms occurring on both sides of the equation are the same.
- If any equation is dimensionally correct it is not necessary that must be mathematically correct too.
- There are three main uses of the dimensional analysis-

(a) To convert a unit of given physical quantities from one system of units to another system for which we use

$$n_2 = n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$$

(b) To check the correctness of a given physical relation.

(c) To derive a relationship between different physical quantities.

- Significant figures: - The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

For counting of the significant figure rule are as:

(i) All non- zero digits are significant figure.

(ii) All zero between two non-zero digits are significant figure.

(iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant. But such zeros are significant if they come from a measurement.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant.

(v) All zeros to the right of a decimal point are significant.

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. Single zero conventionally placed to the left of the decimal point is not significant.

(vii) The number of significant figures does not depend on the system of units.

- In addition or subtraction, the result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.

- In multiplication or division, the result should be reported to the same number of significant figures as that of the number with minimum of significant figures.
- Accuracy refers to the closeness of a measurement to the true value of the physical quantity and precision refers to the resolution or the limit to which the quantity is measured.
- Difference between measured value and true value of a quantity represents error of measurement.

  It gives an indication of the limits within which the true value may lie.

  Mean of n measurements

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \cdots \ldots + a_n}{n}$$

Absolute error (Δa) = a_{mean} - a_i Where a_i = measured value

It may be - positive, negative or zero.

(i) Mean absolute error

(ii) Relative error - it is the ratio of the mean absolute error to the true value.

$$\delta a = \text{I } \Delta a \text{ I} / a_{mean}$$

(iii) The relative error expressed in percent is called percentage error.

The error is communicated in different mathematical operations as detailed below:

(i) For $x = (a \pm b)$, $\Delta x = \pm (\Delta a + \Delta b)$

(ii) For $x = a \times b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

(iii) For $x = a/b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

(iv) For $x= a^n b^m /c^p$ $\Delta x/x = \pm (n\Delta a/a + m \Delta b/b + p\Delta c/c$

Very short answer type questions, (1 mark question)

Q1. State one law that holds good in all natural processes.

Ans. One such laws is the Newton's gravitation law, According to this law everybody in this nature are attracts with other body with a force of attraction which is directly proportional to the product of their masses and inversely proportionally To the square of the distance between them.

Q2: Among which type of elementary particles does the electromagnetic force act?

Ans : Electromagnetic force acts between on all electrically charged particles.

Q3. Name the forces having the longest and shortest range of operation.

Ans : longest range force is gravitational force and nuclear force is shortest range force.

Q4. If 'slap' times speed equals power, what will be the dimensional equation for 'slap'?

Ans . Slap x speed = power

Or slap = power/speed = $[MLT^{-2}]$

Q5. If the units of force and length each are doubled, then how many times the unit of energy would be affected?

Ans : Energy = Work done = Force x length

So when the units are doubled, then the unit of energy will increase four times.

Q6. Can a quantity has dimensions but still has no units?

Ans : No, a quantity having dimension must have some units of its measurement.

Q7. Justify L +L = L and L – L =L.

Ans: When we add or subtract a length from length we get length, So L +L =L AND L – L =L, justify.

Q8. Can there be a physical quantity that has no unit and no dimensions?

Ans : yes, like strain.

Q9. Given relative error in the measurement of length is 0.02, what is the percentage error?

Ans: percentage error = 2 %

Q10. If g is the acceleration due to gravity and λ is wavelength, then which physical quantity does represented by $\sqrt{g\lambda}$.

Ans. Speed or velocity.

Short answer type questions (2 marks)

Q1.If heat dissipated in a resistance can be determined from the relation:

$H = I^2Rt$ joule , If the maximum error in the measurement of current, resistance and time are 2% ,1% , and 1% respectively, What would be the maximum error in the dissipated heat?

Ans: % error in heat dissipated is ±6 %.

Q2. Name any three physical quantities having the same dimensions and also give their dimensions.

Ans : Any group of physical quantities, like work , energy and torque and their dimensions $[ML^2 T^{-2}]$.

Q3. In Van der Wall's equation $(P + a/V^2)(V – b) = RT$, Determine the dimensions of a and b.

Ans : $[a] = [ML^5 T^{-2}]$ and $[b] = [M^0L^3T^0]$.

Q4. Give the limitations of dimensional analysis.

Ans ……………………………

Q5. If $X= a+ bt^2$, where X is in meter and t is in second . find the unit of a and b?

Ans : unit of a is meter and unit of b is $m/sec^{2.}$

Q6. What is meant by significant figures ? State the rules for counting the number of significant figures in a measured quantity?

Ans. …………………………

Q7. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual relative errors.

Ans : For $x = a/b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

Q8. Deduce the dimensional formulae for the following physical quantities.

A) Gravitational constant.

B) Power

C) coefficient of viscosity

D) Surface tension.

Ans: (A) gravitational constant = $[M^{-1} L^3 T^{-2}]$,

B) Power = $[ML^2T^{-3}]$

C) Coefficient of viscosity = $[ML^{-1} T^{-1}]$

D) Surface tension = $[ML^0T^{-2}]$

Q9. Name the four basic forces in nature. Arrange them in the order of their increasing strengths.

Ans : (i) Gravitational force (ii) Electromagnetic force

(iii) nuclear force (iv) Weak force

The relative strengths of these forces are

$$F_g : F_w : F_e : F_s = 1 : 10^{25} : 10^{36} : 10^{38} .$$

Q10. Convert 1 Newton force in to Dyne.

Ans : $1N = 10^5$ Dyne.

Short answer type questions (3marks)

Q1. If E,M,J and G respectively denote energy, mass, angular momentum and gravitational constant, Calculate the dimensions of EJ^2/M^5G^2

Q2. The frequency ν of vibration of stretched string depends on its length L its mass per unit length m and the tension T in the string obtain dimensionally an expression for frequency ν.

Q3. What is meant by significant figures .State the rules for counting the number of significant figures in a measured quantity?

Q4. A physical quantity X is given by $X = A^2B^3/C\sqrt{D}$, If the percentage errors of measurement in A,B,C and D are 4%,2%,3% and 1% respectively, then calculate the % error in X.

Q5. If two resistors of resistance $R_1=(4 \pm 0.5)\Omega$ and $R_2=(16 \pm 0.5)\Omega$ are connected (1) In series and (2) Parallel . Find the equivalent resistance in each case with limits of % error.

Q6. The length of a rod measured in an experiment was found to be 2.48m, 2.46, 2.50m and 2.48m and 2.49m, Find the average length , the absolute error in each observation and % error.

Q7. A famous relation in physics relates moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of the light c. A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = m_o / (1 - v^2)^{1/2}$$

Guess where to put the missing c.

Q8. A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 4.2 kgm^2s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the units of time is ϒ sec. show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \Upsilon^2$ in terms of the new units.

Q9. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively, what are the dimensions of Y in MKS system?

Q10. In an experiment, on the measurement of g using a simple pendulum the time period was measured with an accuracy of 0.2 % while the length was measured with accuracy of 0.5%. Calculate the percentage error in the value of g.

Long answer question (5 marks)

Q1. Explain:

(i) Absolute error (iii) Mean absolute error

(ii) Relative error (iv) percentage error

(v) Random error

Q2. Convert:

(i) Gravitational constant (G) = 6.67×10^{-11} $Nm^2\ kg^{-2}$ to $cm^3\ g^{-1}\ s^{-2}$ (ii) The escape velocity v of a body depends on, the acceleration due to gravity 'g' of the planet and the radius R of the planet, Establish dimensionally for relation for the escape velocity.

Q3. Name the four basic forces in nature. Write a brief note of each, hence compare their strengths and ranges.

HOTs

Q1. What are the dimensions of $^1/u_0\epsilon_0$, where symbols have their usual meaning.

Ans : [$M^0\ L^2\ T^{-2}$]

Q2.What is the dimensions of $(^1/_2)\epsilon_0E^2$, Where E electric field and ϵ_0 permittivity of free space.

Ans : [$M^1\ L^{-1}\ T^{-2}$]

Q3. The pairs of physical quantities that have the same dimensions are:

(a) Reynolds's number and coefficient of friction,

(b) Curie and frequency of a light wave

(c) Latent heat and gravitational potential

(d) Planck's constant and torque.

Ans : (a), (b).

Q4. If L,C,R represent inductance , capacitance and resistance respectively, the combinations having dimensions of frequency are

(a) $^{1}/\sqrt{CL}$ (b) L/C (c) R/L (d) R/C

Ans : (a) and (c).

Q5. If the error in radius is 3%, what is error in volume of sphere?

(a) 3 % (b) 27 % (c) 9 % (d) 6 %

Ans : (c) 9%.

KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

$$\text{Displacement} \leq \text{Actual distance}$$

*Speed:- It is rate of change of distance covered by the body with respect to time.

$$\text{Speed} = \text{Distance travelled /time taken}$$

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ sec.^{2} and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,	For deceleration motion
$V = u + at$	$v = u - at$
$S = ut + {}^1/_2\, at^2$	$S = ut - {}^1/_2\, at^2$
$V^2 = u^2 + 2as$	$V^2 = u^2 - 2as$
$S_n = u + {}^a/_2(2n - 1)$	$S_n = u - {}^a/_2(2n - 1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When V_A and V_B are incident to each other at angle Θ

$$V_{AB} = (V^2_A + V^2_B - 2V_AV_B\text{Cos }\Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**
  (i) Only vectors of same nature can be added.
  (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And R = ($A^2 + B^2 + 2AB\ Cos\Theta)^{1/2}$

And $\tan\beta = B\ Sin\Theta / (A + B\ Cos\Theta)$,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

(iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(iv) Vector addition is associative,

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

(v) R is maximum if Θ = 0 and minimum if Θ = 180^0.

Subtraction of two vectors :-

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of B from A = vector addition of A and (-B),

$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$

Where R = [$A^2 + B^2 + 2AB\ Cos(180 - \Theta)\ ]^{1/2}$ and

$\tan\beta = B\ Sin(180 - \Theta) / [A + B\ Cos(180 - \Theta)]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector subtraction is not commutative $\overrightarrow{(A} - \vec{B}) \neq (\vec{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Here $A_x = A \cos\Theta$ and $A_y = A \sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \,.\, \vec{B} = AB \cos\Theta$$

(i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{i} \times \hat{j} = \hat{k}$ $\quad \hat{j} \times \hat{k} = \hat{I}$ $\quad \hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$ $\quad \hat{k} \times \hat{j} = -\hat{i}$ $\quad \hat{i} \times \hat{k} = -\hat{j}$

(iii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

$$V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$$

- Horizontal range

$$R = u^2 \,\mathrm{Sin}2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

- Flight time

$$T = 2u\,\mathrm{Sin}\Theta/g$$

- Height

$$H = u^2 \sin^2\Theta/2g$$

For maximum height $\Theta = 90^0$

$$H_{max.} = u^2/2g$$

Very Short answer type questions (1 marks)

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^0.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : $W = 2\pi/12 = \pi/6$ rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

Short Answer type question (2 marks)

Q1. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n-1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54$ m/sec^2.

Q3.what is relative velocity in one dimension, if V_A and V_B are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \sin 2\Theta / g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2 / g$$

and Height

$$H = u^2 \sin^2\Theta / 2g$$

For $\Theta = 45^0$

$$H = u^2 / 4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

And $R = (A^2 + B^2 + 2AB\cos\Theta)^{1/2}$

And $\tan\beta = B\sin\Theta / (A + B\cos\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r\,\omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max}/2$$

So $\quad h = R/2$

Or $\quad R = 2h$

Short answer questions (3 marks)

Q1. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0.}$

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{B} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

Long answer question (5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $u\mathrm{Cos}45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and (90 - Θ) ,

$$\text{then} \quad t_1 = 2u\,\text{Sin}\Theta/g$$

$$t_2 = 2u\,\text{Sin}(90 - \Theta)/g = 2u\,\text{Cos}\Theta/g$$

$$t_1 t_2 = 2R/g\ .$$

NEWTON'S LAWS OF MOTION

Newton' 1^{st} law or Law of Inertia

Every body continues to be in its state of rest or of uniform motion until and unless and until it is compelled by an external force to change its state of rest or of uniform motion.

Inertia

The property by virtue of which a body opposes any change in its state of rest or of uniform motion is known as inertia. Greater the mass of the body greater is the inertia. That is **mass is the measure of the inertia of the body**.

Numerical Application

If, $\vec{F} = 0$; $\vec{u}$ = constant

Physical Application

1. When a moving bus suddenly stops, passenger's head gets jerked in the forward direction.
2. When a stationery bus suddenly starts moving passenger's head gets jerked in the backward direction.
3. On hitting used mattress by a stick, dust particles come out of it.
4. In order to catch a moving bus safely we must run forward in the direction of motion of bus.
5. Whenever it is required to jump off a moving bus, we must always run for a short distance after jumping on road to prevent us from falling in the forward direction.

Key Concept

In the absence of external applied force velocity of body remains unchanged.

Newton' 2^{nd} law

Rate of change of momentum is directly proportional to the applied force and this change always takes place in the direction of the applied force.

$$\frac{d\vec{p}}{dt} \propto \vec{F}$$

or, $\frac{d\vec{p}}{dt} = \vec{F}$ (here proportionality constant is 1)

putting, $\vec{p} = m\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

or, $\vec{F} = \frac{dm\vec{v}}{dt}$

or, $\boxed{\vec{F} = \frac{md\vec{v}}{dt} + \frac{\vec{v}dm}{dt}}$

or, $\vec{F} = \frac{md\vec{v}}{dt}$ (if m is constant dm/dt = 0)

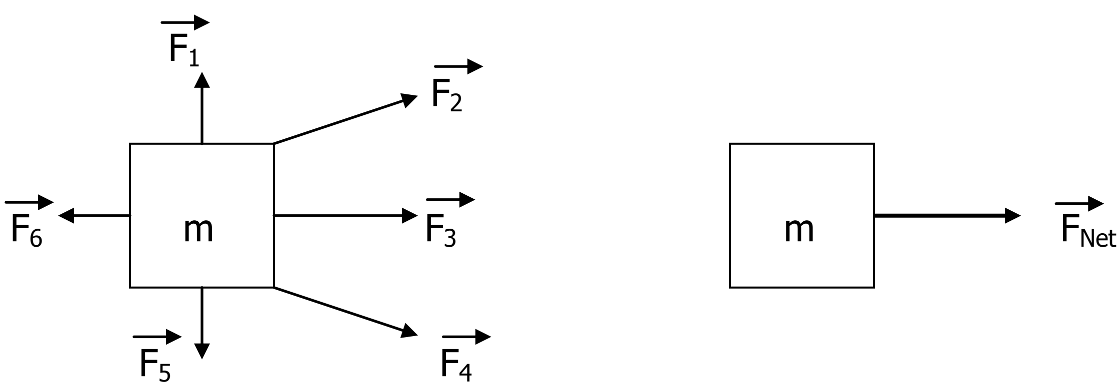
or, $\boxed{\vec{F} = m\vec{a}}$

Note :- Above result is not Newton's second law rather it is the conditional result obtained from it, under the condition when m = constant.

Numerical Application

$$\boxed{\vec{a} = \frac{\vec{F}_{Net}}{m}}$$

Where $\vec{F}_{Net}$ is the vector resultant of all the forces acting on the body.

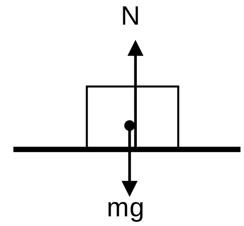


Where, $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6$

Physical Application

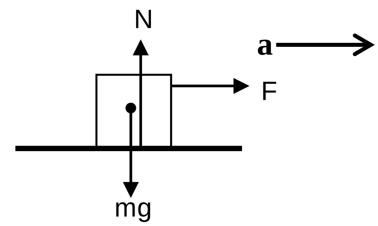
Horizontal Plane

i) Case - 1
Body kept on horizontal plane is at rest.



For vertical direction
$N = mg$(since body is at rest)

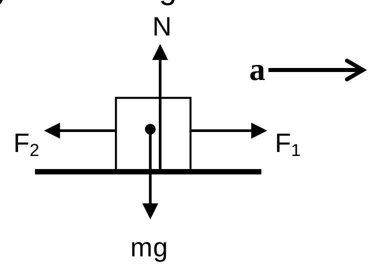
ii) Body kept on horizontal plane is accelerating horizontally under single horizontal force.



For vertical direction
$N = mg$ (since body is at rest)

For horizontal direction
$F = ma$

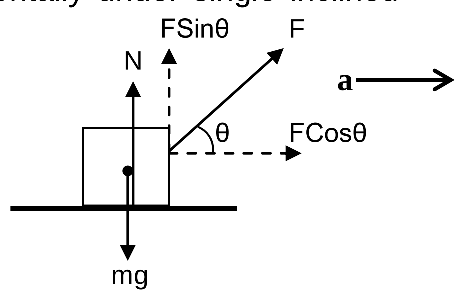
iii) Body kept on horizontal plane is accelerating horizontally towards right under two horizontal forces. ($F_1 > F_2$)



For vertical direction
$N = mg$ (since body is at rest)

For horizontal direction
$F_1 - F_2 = ma$

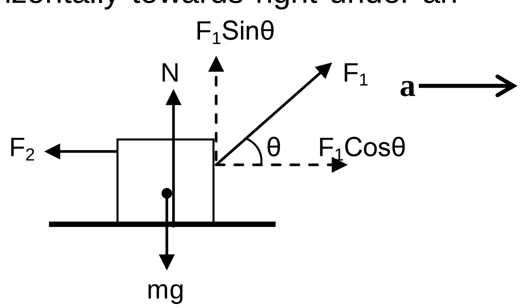
iv) Body kept on horizontal plane is accelerating horizontally under single inclined force



For vertical direction
$N + F\sin\theta = mg$ (since body is at rest)

For horizontal direction
$F\cos\theta = ma$

v) Body kept on horizontal plane is accelerating horizontally towards right under an inclined force and a horizontal force.



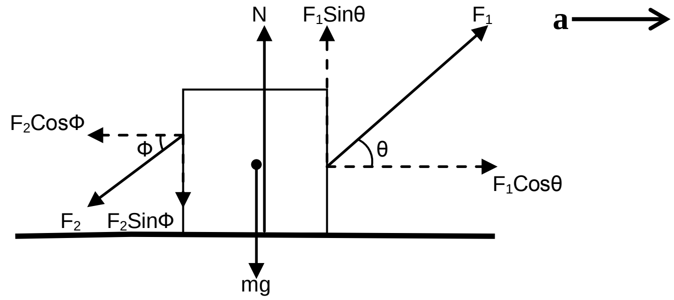
For vertical direction
$N + F_1\sin\theta = mg$ (since body is at rest)

For horizontal direction
$F_1\cos\theta - F_2 = ma$

vi) Body kept on horizontal plane is accelerating horizontally towards right under two inclined forces acting on opposite sides.

For vertical direction
$N + F_1 Sin\theta = mg + F_2\ Sin\Phi$
(since body is at rest)

For horizontal direction
$F_1 Cos\theta - F_2 Cos\Phi = ma$

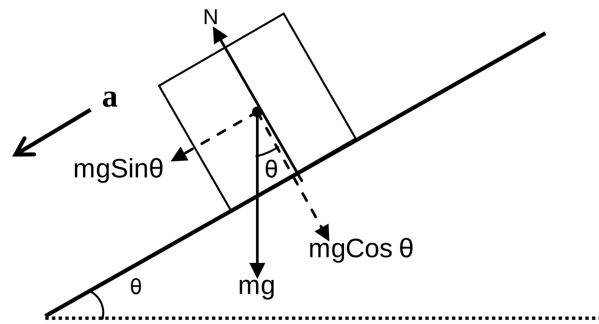


Inclined Plane

i) Case - 1
Body sliding freely on inclined plane.

Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

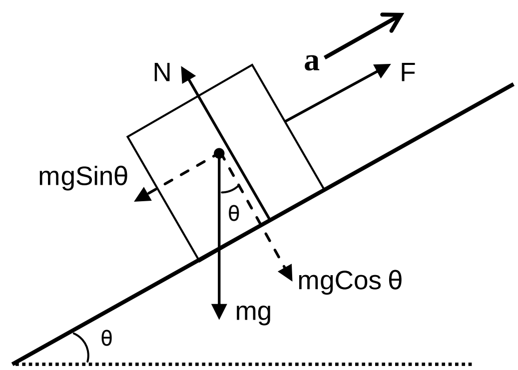
Parallel to the plane
$mgSin\theta = ma$



ii) Case - 2
Body pulled parallel to the inclined plane.

Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

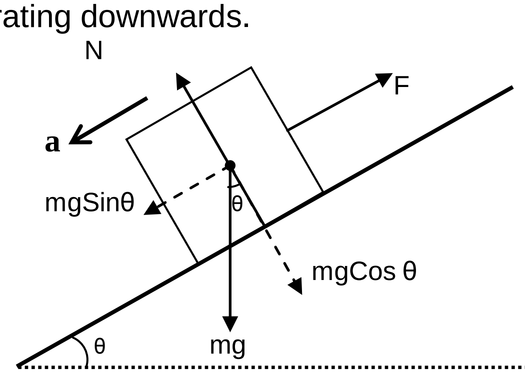
Parallel to the plane
$F - mgSin\theta = ma$



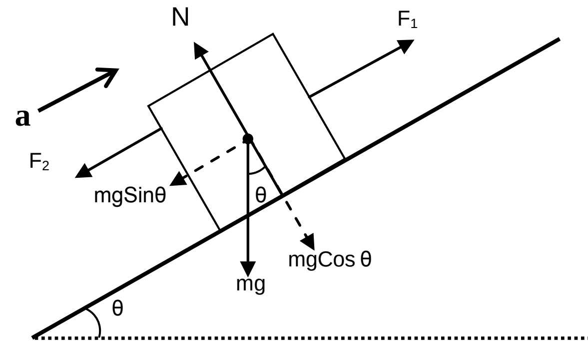
iii) Case - 3
Body pulled parallel to the inclined plane but accelerating downwards.

Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane
$mgSin\theta - F = ma$



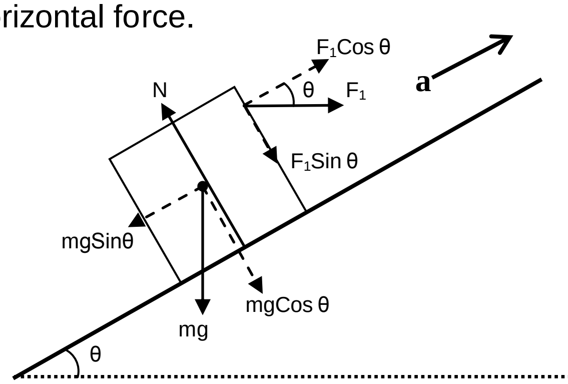
iv) Case - 4
Body accelerating up the incline under the effect of two forces acting parallel to the incline.



Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane
$F_1 - F_2 - mgSin\theta = ma$

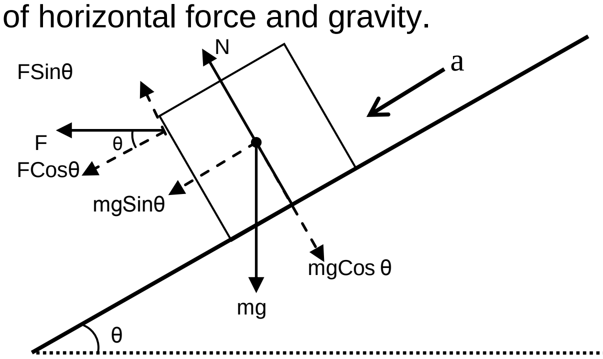
v) Case - 5
Body accelerating up the incline under the effect of horizontal force.



Perpendicular to the plane
$N = mgCos\theta + F_1Sin\theta$ (since body is at rest)

Parallel to the plane
$F_1Cos\theta - mgSin\theta = ma$

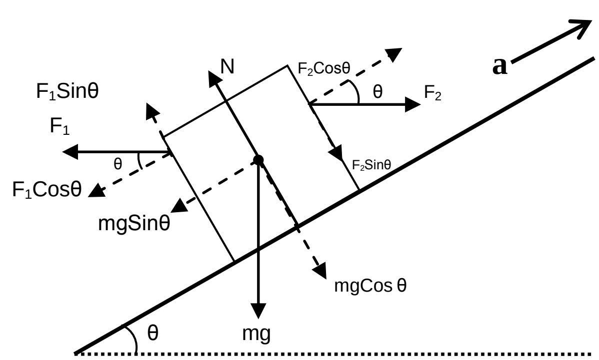
vi) Case - 6
Body accelerating down the incline under the effect of horizontal force and gravity.



Perpendicular to the plane
$N + FSin\theta = mgCos\theta$ (since body is at rest)

Parallel to the plane
$FCos\theta + mgSin\theta = ma$

vii) Case - 7
Body accelerating up the incline under the effect of two horizontal forces acting on opposite sides of a body and gravity.



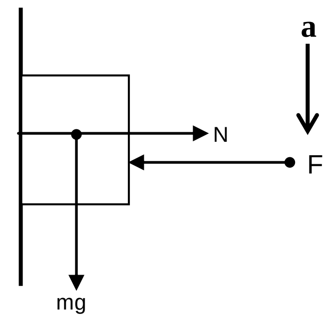
Perpendicular to the plane
$N + F_1Sin\theta = mgCos\theta + F_2Sin\theta$(since body is at rest)

Parallel to the plane
$F_2Cos\theta - F_1Cos\theta - mgSin\theta = ma$

Vertical Plane

i) Case - 1

Body pushed against the vertical plane by horizontal force and moving vertically downward.



For horizontal direction

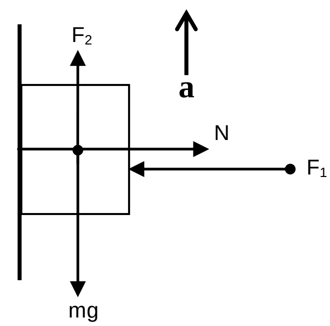
mg = ma (since body is at rest)

For vertical direction

F = N

ii) Case - 2

Body pushed against the vertical plane by horizontal force and pulled vertically upward.



For vertical direction

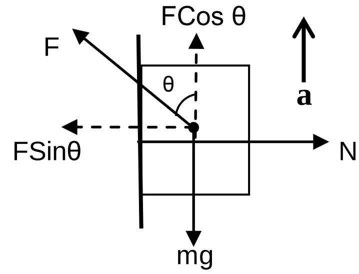
F_2 - mg = ma

For horizontal direction (since body is at rest)

N = F_1

iii) Case - 3

Body pushed against the vertical plane by inclined force and accelerates vertically upward.



For horizontal direction

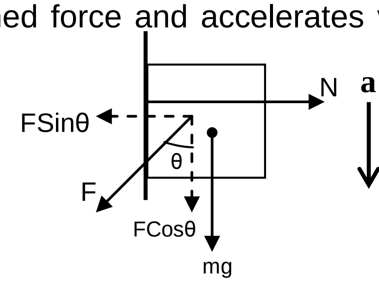
N = FSinθ (since body is at rest)

For vertical direction

FCosθ – mg = ma

iv) Case - 3

Body pushed against the vertical plane by inclined force and accelerates vertically downward.



For horizontal direction

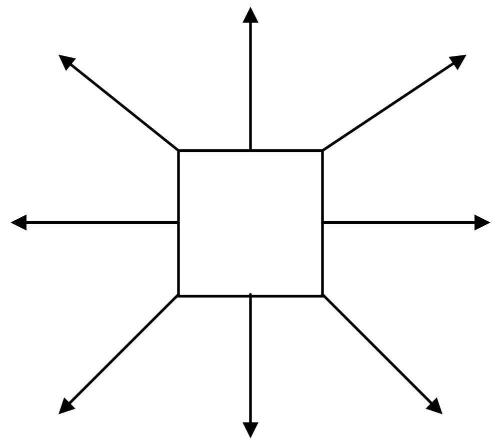
N = FSinθ (since body is at rest)

For vertical direction

FCosθ + mg = ma

Tension In A Light String

Force applied by any linear object such as string, rope, chain, rod etc. is known as it's tension. Since string is a highly flexible object so it can only pull the object and can never push. Hence tension of the string always acts away from the body to which it is attached irrespective of the direction.



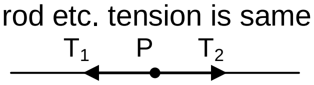
Tension of the string, being of pulling nature, always acts away from the body to which it is attached

Physical Application

i) Flexible wire holding the lamp pulls the lamp in upward direction and pulls the point of suspension in the downward direction.
ii) Rope holding the bucket in the well pulls the bucket in the upward direction and the pulley in the downward direction.
iii) Rope attached between the cattle and the peg pulls the cattle towards the peg and peg towards the cattle.
iv) When a block is pulled by the chain, the chain pulls the block in forward direction and the person holding the chain in reverse direction.

Key Point

In case of light string, rope, chain, rod etc. tension is same all along their lengths.



Consider a point P on a light (massless) string. Let tensions on either side of it be T_1 and T_2 respectively and the string be accelerating towards left under these forces. Then for point P

$$T_1 - T_2 = ma$$

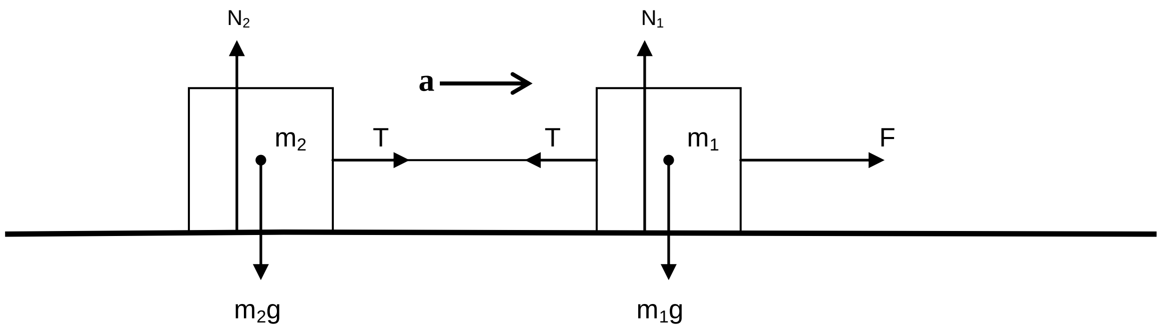
Since string is considered to be light mass m of point P is zero

or, $$T_1 - T_2 = 0$$

or, $$\mathbf{T_1 = T_2}$$

i) Case - 1

Two bodies connected by a string are placed on a smooth horizontal plane and pulled by a horizontal force.



For vertical equilibrium of m_1 and m_2

$N_1 = m_1g$ and $N_2 = m_2g$

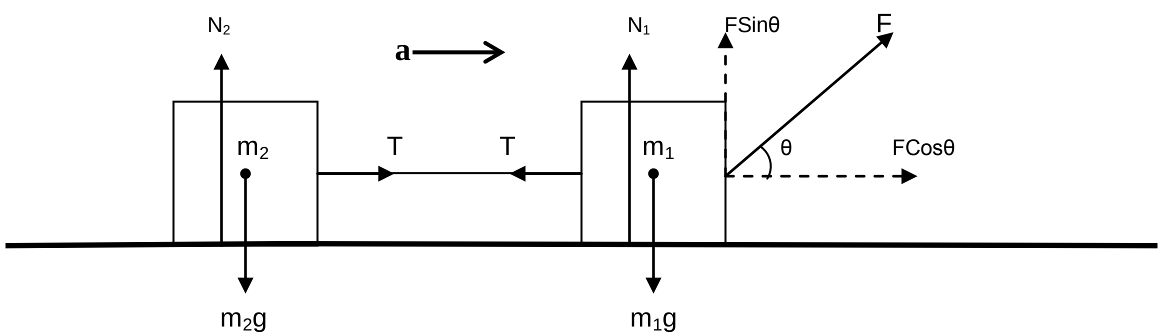
For horizontal acceleration of m_1 and m_2

$F - T = m_1a$ and $T = m_2a$

(Since both the bodies are connected to the same single string they have same acceleration)

ii) Case - 2

Two bodies connected by a horizontal string are placed on a smooth horizontal plane and pulled by a inclined force.



For vertical equilibrium of m_1 and m_2

$N_1 + FSin\theta = m_1g$ and $N_2 = m_2g$

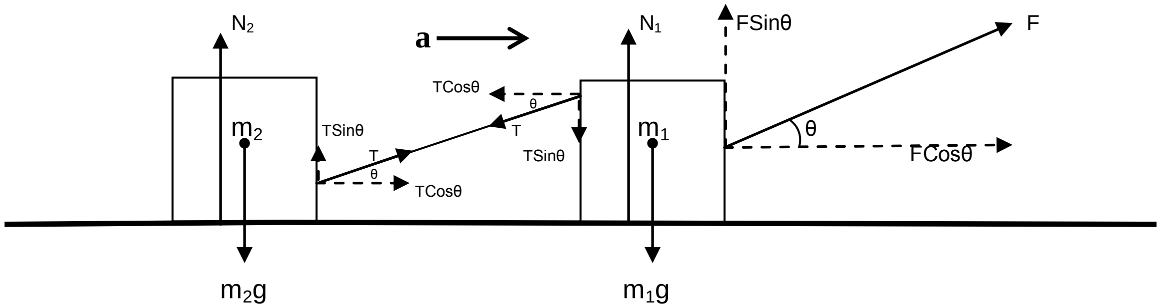
For horizontal acceleration of m_1 and m_2

$FCos\theta - T = m_1a$ and $T = m_2a$

(since both the bodies are connected to the same single string they have same accelerations)

iii) Case - 3

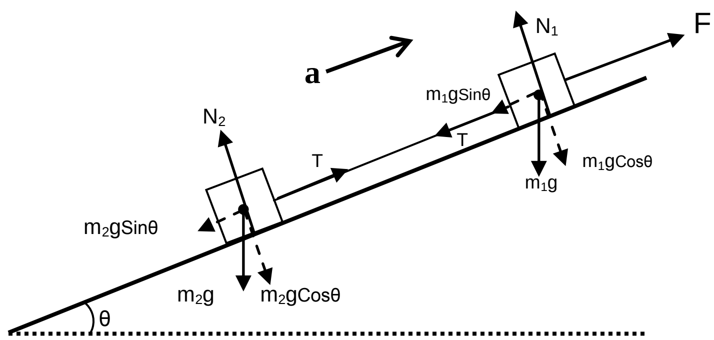
Two bodies connected by a inclined string are placed on a smooth horizontal plane and pulled by a inclined force.



For vertical equilibrium of m_1 and m_2
$\mathbf{N_1 + FSin\theta = m_1g + TSin\theta}$ **and** $\mathbf{N_2 + TSin\theta = m_2g}$

For horizontal acceleration of m_1 and m_2
$\mathbf{FCos\theta - TCos\theta = m_1a}$ **and** $\mathbf{TCos\theta = m_2a}$
(since both the bodies are connected to the same single string they have same accelerations)

iv) Case - 4
Two bodies connected by a string made to accelerate up the incline by applying force parallel to the incline.

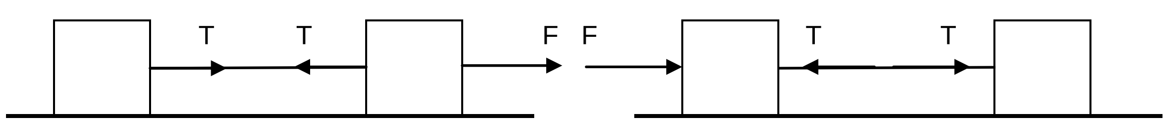


For equilibrium of m_1 and m_2 in the direction perpendicular to the plane
$\mathbf{N_1 = m_1gCos\theta}$ **and** $\mathbf{N_2 = m_2gCos\theta}$

For acceleration of m_1 and m_2 up the incline
$\mathbf{F - T - m_1gSin\theta = m_1a}$ **and** $\mathbf{T - m_2gSin\theta = m_2a}$

Tension of A light Rigid Rod

Force applied by rod is also known as its tension. Since rod is rigid, it cannot bend like string. Hence rod can pull as well as push. Tension of rod can be of pulling as well as pushing nature but one at a time. Tension of a rod attached to the body may be directed towards as well as away from the body.



Tension of rod is pulling both the blocks   Tension of rod is pushing both the blocks

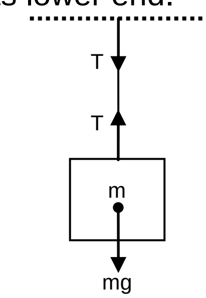
Physical Application

i) Pillars supporting the house **pushes** the house in the upward direction and **pushes** the ground in the downward direction.
ii) Wooden bars used in the chair **pushes** the ground in the downward direction and **pushes** the seating top in the upward direction.
iii) Parallel bars attached to the ice-cream trolley **pushes** the trolley in the forward direction and **pushes** the ice-cream vendor in the backward direction.(when the trolley is being pushed by the vendor)
iv) Rod holding the ceiling fan **pulls** the fan in the upward direction and **pulls** the hook attached to the ceiling in the downward direction.
v) Parallel rods attached between the cart and the bull **pulls** the cart in the forward direction and **pulls** the bull in the backward direction.

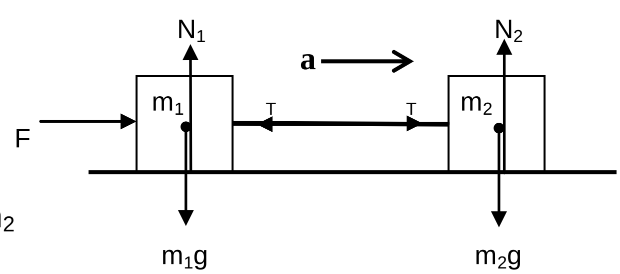
Different Cases of Light Rigid Rod

i) Case - 1
Rod attached from the ceiling and supporting the block attached to its lower end.
Since the block is at rest

T = mg



ii) Case - 2
Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pushing force.

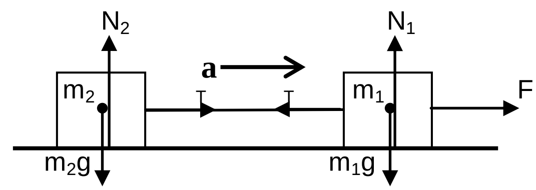


For vertical equilibrium of m_1 and m_2
$\mathbf{N_1 = m_1 g}$ **and** $\mathbf{N_2 = m_2 g}$

For horizontal acceleration of m_1 and m_2
$\mathbf{F - T = m_1 a}$ **and** $\mathbf{T = m_2 a}$

(Since both the bodies connected to the rod will have same acceleration)

iii) Case - 3
Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pulling force.



For vertical equilibrium of m_1 and m_2
$\mathbf{N_1 = m_1 g}$ **and** $\mathbf{N_2 = m_2 g}$

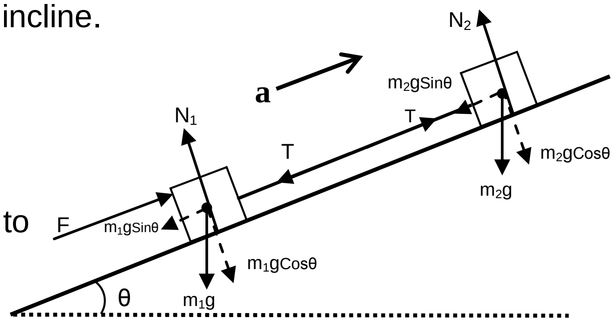
For horizontal acceleration of m_1 and m_2

$F - T = m_1a$ and $T = m_2a$

(Since both the bodies are connected to the same rod they have same acceleration)

iv) Case - 4

Rod is attached between two blocks placed on the incline plane and the blocks are accelerated by pushing parallel to the incline.



For vertical equilibrium of m_1 and m_2

$N_1 = m_1gCos\theta$ and $N_2 = m_2gCos\theta$

For acceleration of m_1 and m_2 parallel to the incline

$F - m_1gSin\theta - T = m_1a$,

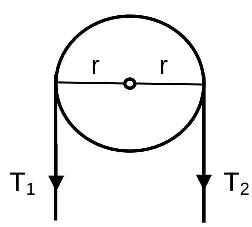
$T - m_2gSin\theta = m_2a$

Fixed Pulley

It is a simple machine in the form of a circular disc or rim supported by spokes having groove at its periphery. It is free to rotate about an axis passing through its center and perpendicular to its plane.

Key Point

In case of light pulley, tension in the rope on both the sides of the pulley is same (to be proved in the rotational mechanics)



Anticlockwise Torque - Clockwise Torque = Moment of Inertia x Angular acceleration

$$T_1 \times r - T_2 \times r = I\alpha$$

Since the pulley is light and hence considered to be massless, it's moment of inertia

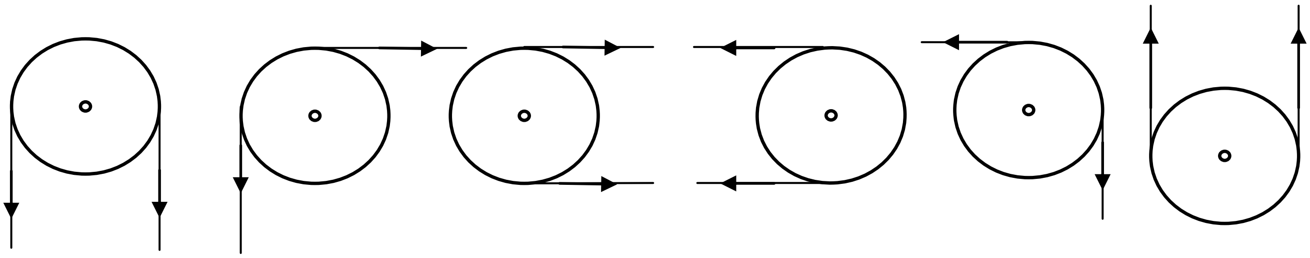
$$I = 0$$

or, $T_1 \times r - T_2 \times r = 0$

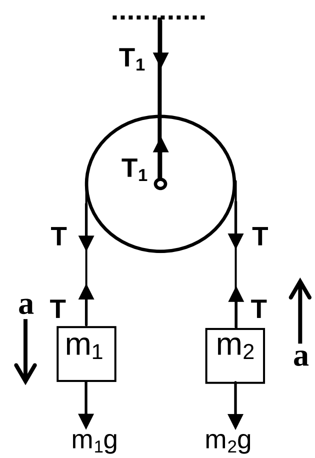
or, $T_1 \times r = T_2 \times r$

or, $\mathbf{T_1 = T_2}$

Different Cases of Fixed Pulley



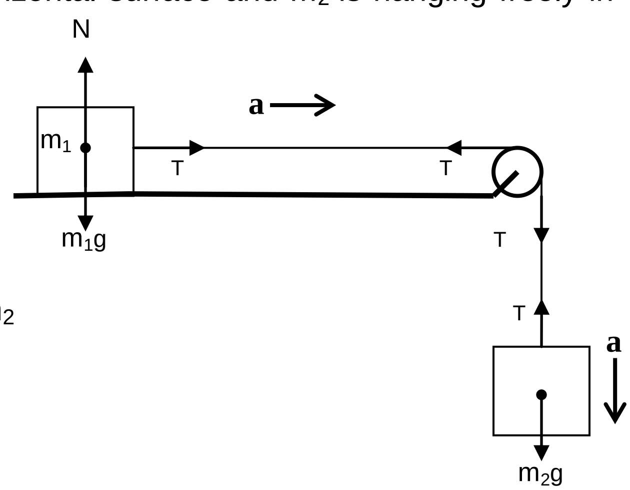
i) Case - 1
Two bodies of different masses ($m_1 > m_2$) are attached at two ends of a light string passing over a smooth light pulley



For vertical equilibrium of pulley
$\mathbf{T_1 = T + T = 2T}$

For vertical acceleration of m_1 and m_2
$\mathbf{m_1g - T = m_1a}$ and $\mathbf{T - m_2g = m_2a}$
m_1 accelerates downwards and m_2 accelerates upwards($m_1 > m_2$)

ii) Case - 2
Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on a horizontal surface and m_2 is hanging freely in air.

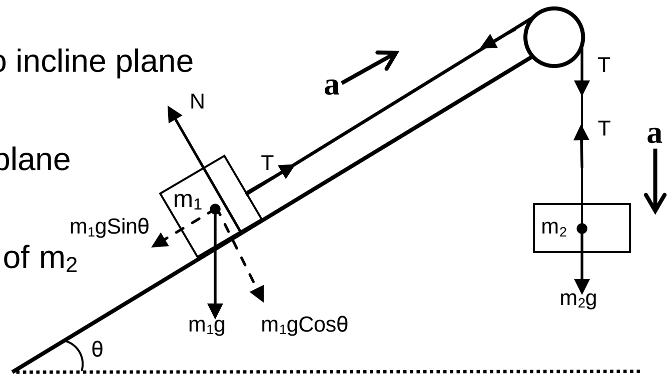


For vertical equilibrium m_1
$\mathbf{N = m_1g}$

For horizontal acceleration of m_1
$\mathbf{T = m_1a}$

For vertically downward acceleration of m_2
$\mathbf{m_2g - T = m_2a}$

iii) Case - 3
Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on an inclined surface and m_2 is hanging freely in air.



For equilibrium of m_1 perpendicular to incline plane
$N = m_1gCos\theta$

For acceleration of m_1 up the incline plane
$T - m_1gSin\theta = m_1a$

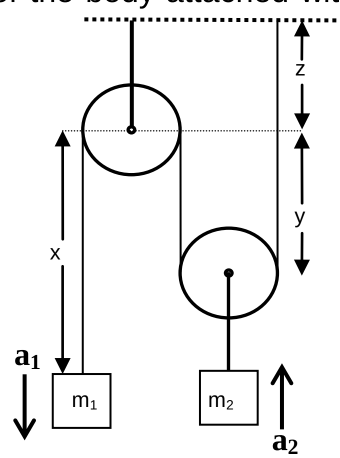
For vertically downward acceleration of m_2
$m_2g - T = m_2a$

Movable Pulley

The pulley which moves in itself is known as movable pulley.

Key Point

In case of light movable pulley, acceleration of a body (pulley) goes on decreasing on increasing the number of strings attached to it. That is the body attached with two ropes moves with half the acceleration of the body attached with single rope.



Length of the string is constant

$x + 2y + z = L$ (Constant)

Differentiating both sides with respect to t (Time)

$$\frac{dx}{dt} + 2\frac{dy}{dt} + \frac{dz}{dt} = \frac{dL}{dt}$$

or, $v_1 + 2v_2 + 0 = 0$ (z and L are constant)

or, $v_1 + 2v_2 = 0$

Again differentiating both sides with respect to t

$$\frac{dv_1}{dt} + 2\frac{dv_2}{dt} = 0$$

or, $a_1 + 2a_2 = 0$

or, $\mathbf{a_1 = - 2a_2}$

That is acceleration of $\mathbf{m_1}$ (body attached to a single string) is opposite and twice the acceleration of $\mathbf{m_2}$ (body attached to a double string)

Different Cases of Light Movable Pulley

i) Case - 1

Mass m_1 is attached at one end of the string and the other end is fixed to a rigid support. Mass m_2 is attached to the light movable pulley.

For vertical acceleration of m_1

$m_1g - T = m_12a$ (m_1 is connected to a single string)

For vertical acceleration of m_2

$T_2 - m_2g = m_2a$

(m_1 accelerates downwards and m_2 accelerates upwards since $m_1 > 2m_2$)

For the clamp holding the first pulley

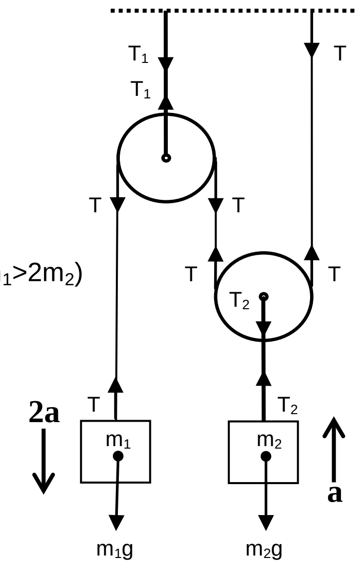
$T_1 = 2T$

For the clamp holding the movable pulley

$2T - T_2 = m_{pulley}a$

or, $2T - T_2 = 0$ (light pulley)

or, **$2T = T_2$**



ii) Case - 2

Mass m_1 is attached at one end of the string and placed on a smooth horizontal surface and the other end is fixed to a rigid support after passing through a light movable suspended pulley. Mass m_2 is attached to the light movable pulley.

For vertical equilibrium of m_1

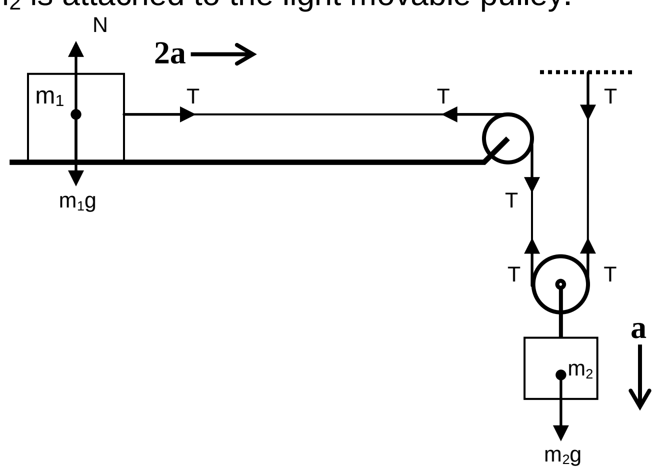
$N = m_1g$

For horizontal acceleration of m_1

$T = m_12a$

For vertical motion of m_2

$m_2g - 2T = m_2a$



iii) Case - 3

Mass m_1 is attached to the movable pulley and placed on a smooth horizontal surface. One end of the string is attached to the clamp holding the pulley fixed to the horizontal surface and from its other end mass m_2 suspended.

For vertical equilibrium of m_1

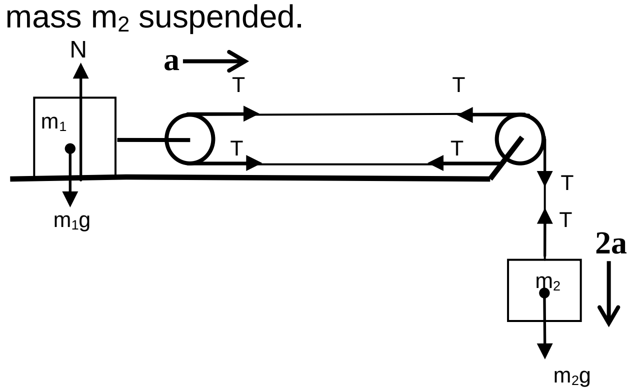
$N = m_1g$

For horizontal motion of m_1

$2T = m_1a$

For vertical motion of m_2

$m_2g - T = m_22a$



iv) Case - 4

Mass m_1 is attached to a movable pulley and placed on a smooth inclined surface. Mass m_2 is is suspended freely from a fixed light pulley.

For equilibrium of m_1 perpendicular to incline plane

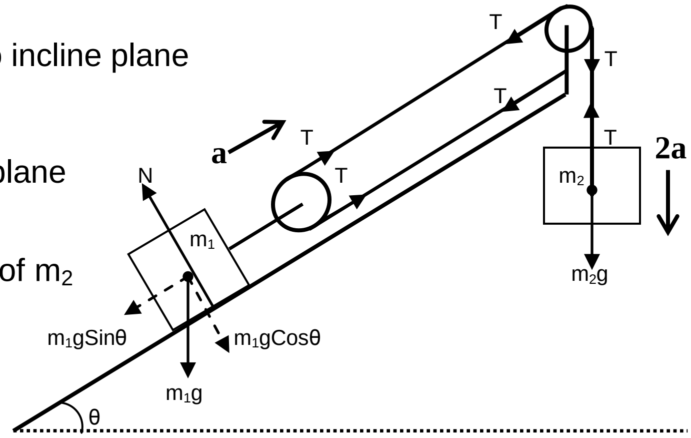
$\mathbf{N = m_1gCos\theta}$

For acceleration of m_1 up the incline plane

$\mathbf{2T - m_1gSin\theta = m_1a}$

For vertically downward acceleration of m_2

$\mathbf{m_2g - T = m_22a}$

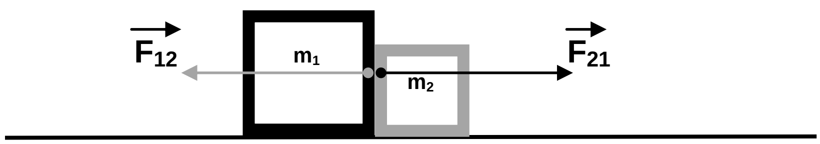


Newton' 3rd law or Law of Action and Reaction

Every action is opposed by an equal and opposite reaction.

or

For every action there is an equal and opposite reaction.



F_{12} is the force on the first body (m_1) due to second body (m_2)

F_{21} is the force on the second body (m_2) due to first body (m_1)

If $\vec{F}_{12}$ is action then $\vec{F}_{21}$ reaction and if $\vec{F}_{21}$ is action then $\vec{F}_{12}$ reaction

Numerical Application

Force on the first body due to second body (F_{12}) is equal and opposite to the force on the second body due to first body (F_{21}).

$$\vec{F}_{21} = -\vec{F}_{12}$$

Physical Application

i) When we push any block in the forward direction then block pushes us in the backward direction with an equal and opposite force.

ii) Horse pulls the rod attached to the cart in the forward direction and the tension of the rod pulls the cart in the backward direction.

iii) Earth pulls the body on its surface in vertically downward direction and the body pulls the earth with the same force in vertically upward direction.
iv) While walking we push the ground in the backward direction using static frictional force and the ground pushes us in the forward direction using static frictional force.
v) When a person sitting on the horse whips the horse and horse suddenly accelerates, the saddle on the back of the horse pushes the person in the forward direction using static frictional force and the person pushes the saddle in the backward direction using static frictional force.
Note – Normal reaction of the horizontal surface on the body is not the reaction of the weight of the body because weight of the body is the force with which earth attracts the body towards its center, hence its reaction must be the force with which body attracts earth towards it.

Linear Momentum

It is defined as the quantity of motion contained in the body. Mathematically it is given by the product of mass and velocity. It is a vector quantity represented by p.

$$\vec{p} = m\vec{v}$$

Principle Of Conservation Of Linear Momentum

It states that in the absence of any external applied force total momentum of a system remains conserved.

Proof-
We know that,

$$\vec{F} = m\vec{a}$$

or,
$$\vec{F} = \frac{md\vec{v}}{dt}$$

or,
$$\vec{F} = \frac{dm\vec{v}}{dt}$$

or,
$$\vec{F} = \frac{d\vec{p}}{dt}$$

if, $\vec{F} = 0$

$$\frac{d\vec{p}}{dt} = 0$$

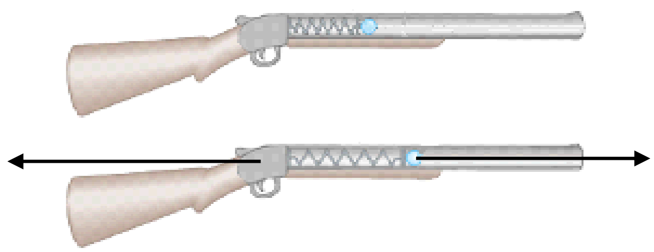
or, $\vec{p}$ = **Constant** (differentiation of constant is zero)

or,
$$\vec{p}_{initial} = \vec{p}_{final}$$

Physical Application

i) Recoil of gun – when bullet is fired in the forward direction gun recoils in the backward direction.
ii) When a person jumps on the boat from the shore of river, boat along with the person on it moves in the forward direction.
iii) When a person on the boat jumps forward on the shore of river, boat starts moving in the backward direction.
iv) In rocket propulsion fuel is ejected out in the downward direction due to which rocket is propelled up in vertically upward direction.

Different Cases of Conservation of Linear Momentum



Recoil of gun

Let mass of gun be m_g and that of bullet be m_b.
Initially both are at rest, hence their initial momentum is zero.

$$p_i = m_g u_g + m_b u_b = 0$$

Finally when bullet rushes out with velocity v_g, gun recoils with velocity v_b, hence their final momentum is

$$p_f = m_g v_g + m_b v_b$$

Since there is no external applied force, from the principal of conservation of linear momentum

$$p_f = p_f$$

or, $$m_g v_g + m_b v_b = 0$$

or, $$m_g v_g = -m_b v_b$$

or, $$\mathbf{v_g = -\frac{m_b v_b}{m_g}}$$

From above expression it must be clear that
1. Gun recoils opposite to the direction of motion of bullet.
2. Greater is the mass of mullet m_b or velocity of bullet v_b greater is the recoil of the gun.
3. Greater is the mass of gun m_g, smaller is the recoil of gun.

Impulse and Impulsive Force

Impulsive Force

The force which acts on a body for very short duration of time but is still capable of changing the position, velocity and direction of motion of the body up to large extent is known as impulsive force.

Example -

1. Force applied by foot on hitting a football.
2. Force applied by boxer on a punching bag.
3. Force applied by bat on a ball in hitting it to the boundary.
4. Force applied by a moving truck on a drum.

Note- Although impulsive force acts on a body for a very short duration of time yet its magnitude varies rapidly during that small duration.

Impulse

Impulse received by the body during an impact is defined as the product of average impulsive force and the short time duration for which it acts.

$$\mathbf{I = F_{avg} \times t}$$

Relation Between Impulse and Linear Momentum

Consider a body being acted upon by an impulsive force, this force changes its magnitude rapidly with the time. At any instant if impulsive force is F then elementary impulse imparted to the body in the elementary time dt is given by

$$dI = F \times dt$$

Hence total impulse imparted to the body from time t_1 to t_2 is

$$I = \int_{t_1}^{t_2} F dt$$

But from Newton's second law we know that

$$F = \frac{dp}{dt}$$

or, $$Fdt = dp$$

Therefore, $$I = \int_{p_1}^{p_2} dp$$

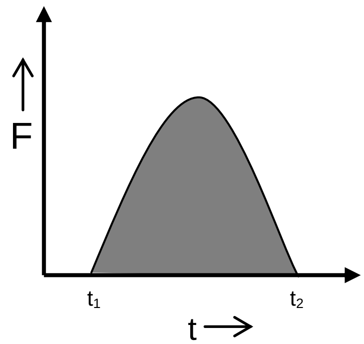
or, $$I = [p]_{p_1}^{p_2}$$

or, $$\mathbf{I = p_2 - p_1}$$

Hence impulse imparted to the body is equal to the change in its momentum.

Graph Between Impulsive Force and Time

With the time on x axis and impulsive force on y axis the graph of the following nature is obtained



Area enclosed under the impulsive force and time graph from t_1 to t_2 gives the impulse imparted to the body from time t_1 to t_2.

Physical Application

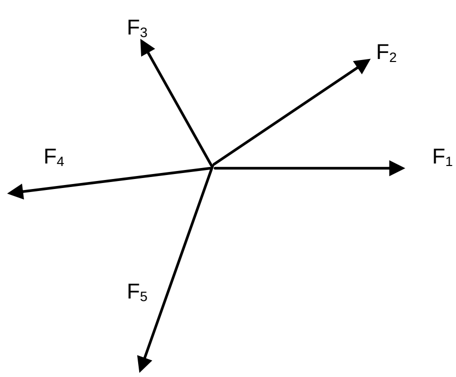
i) While catching a ball a player lowers his hand to save himself from getting hurt.
ii) Vehicles are provided with the shock absorbers to avoid jerks.
iii) Buffers are provided between the bogies of the train to avoid jerks.
iv) A person falling on a cemented floor receive more jerk as compared to that falling on a sandy floor.
v) Glass wares are wrapped in a straw or paper before packing.

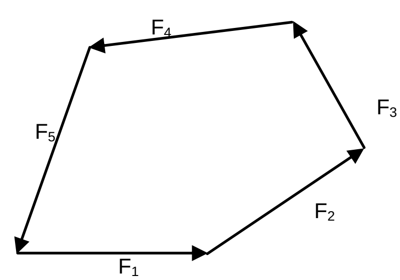
Equilibrium of Concurrent Forces

If the number of forces act at the same point, they are called concurrent forces. The condition or the given body to be in equilibrium under the number of forces acting on the body is that these forces should produce zero resultant.

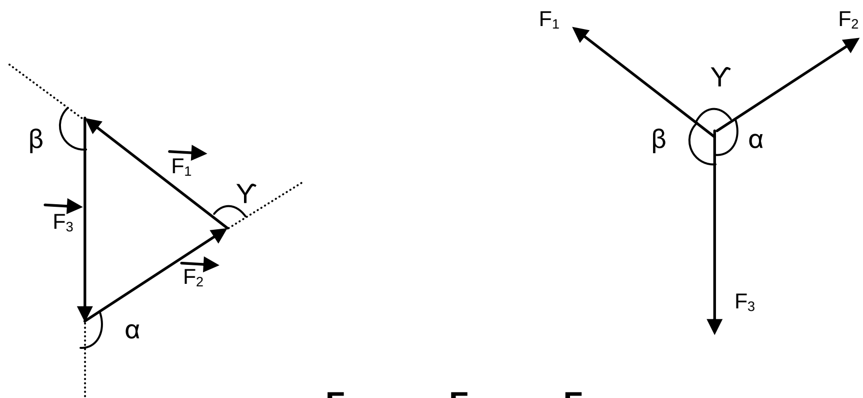
The resultant of the concurrent forces acting on a body will be zero if they can be represented completely by the sides of a closed polygon taken in order.

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = 0$$





Lami's Theorem – It states that the three forces acting at a point are in equilibrium if each force is proportional the sine of the angle between the other two forces.

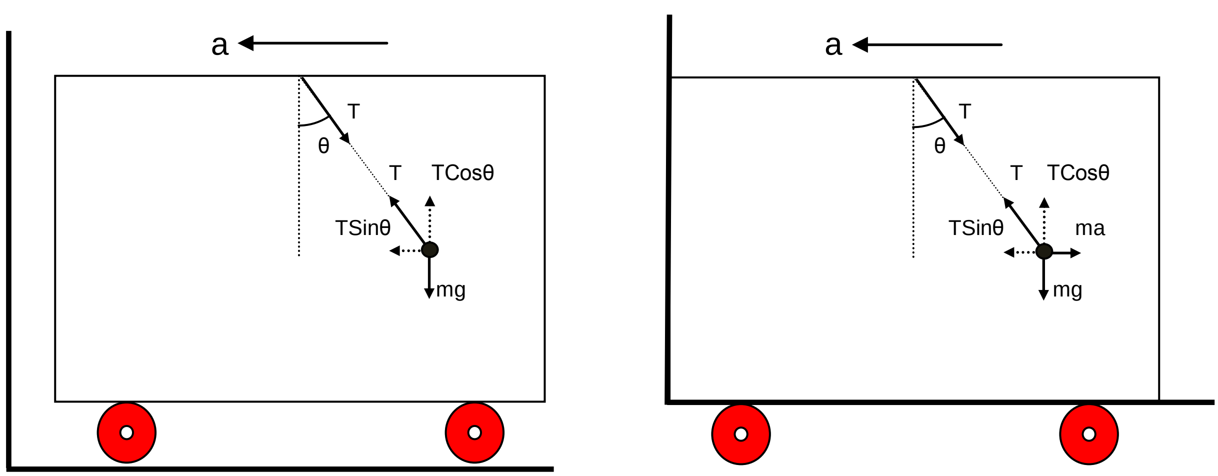


$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\Upsilon}$$

Inertial and Non-inertial Frame of Reference

Frame of reference is any frame with respect to which the body is analyzed. All the frames which are at rest or moving with a constant velocity are said to be inertial frame of reference. In such frame of reference all the three laws of Newton are applicable.

Any accelerated frame of reference is said to be non-inertial frame of reference. In such frames all the three laws of Newton are not applicable as such. In order to apply Newton's laws of motion in a non-inertial frame, along with all other forces a pseudo force F = ma must also be applied on the body opposite to the direction of acceleration of the frame.



Inertial Frame of Reference
(Frame outside the accelerated car)
For vertical equilibrium of body
TCosθ = mg
For horizontal acceleration of body, as the body is accelerated along with the car when observed from the external frame
TSinθ = ma a=0

Therefore, **Tanθ = a/g**

Inertial Frame of Reference
(Frame attached to the accelerated car)
For vertical equilibrium of body
TCosθ = mg
For horizontal equilibrium of the body, as the body is at rest when observed from the frame attached to the car
TSinθ = ma

Therefore, **Tanθ = a/g**

Since body is at rest when observed from the non-inertial frame attached to the accelerated car a pseudo force **F = ma** is applied on the body opposite to the acceleration of the car which balance the horizontal component of tension of the string TSinθ acting on the body.

Note- From which ever frame we may observe the situation, final result always comes out to be the same.

Reading of Spring Balance

Reading of a spring balance is equal to the tension in the spring of the balance but measured in kilogram.

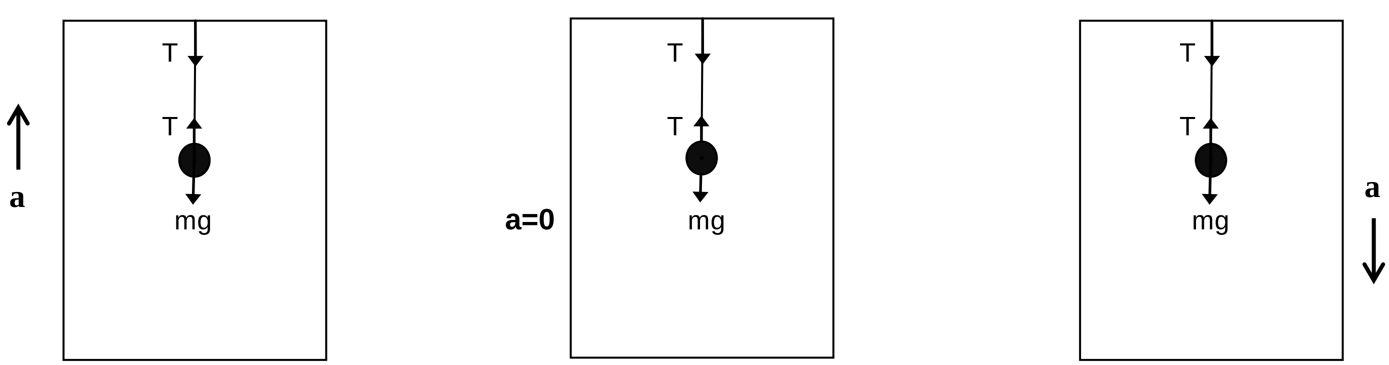
$$\textbf{Reading} = \frac{T}{g}\ \textbf{kgf}$$

Reading of Weighing Machine

Reading of a weighing machine is equal to the normal reaction applied by the machine but measured in kilogram.

$$\textbf{Reading} = \frac{N}{g}\ \textbf{kgf}$$

LIFT

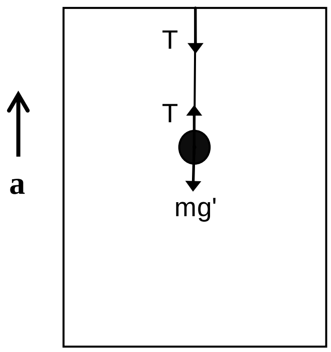


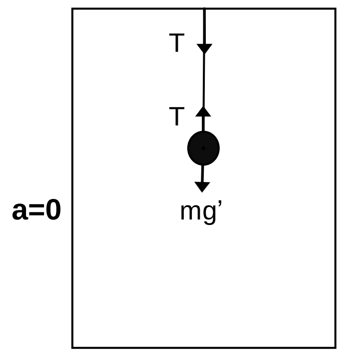
Observer Outside the Lift

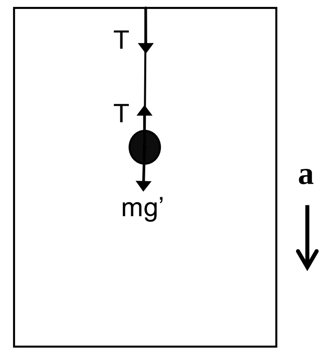
Lift Accelerating Vertically Up
Moving up with increasing velocity.
or
Moving down with decreasing velocity.
For vertical motion of body
$T - mg = ma$
or, $T = mg + ma$
or, $\mathbf{T = m(g + a)}$

Lift Accelerating Vertically Up
Moving up with constant velocity.
or
Moving down with constant velocity.
For vertical motion of body
$a=0$ $\mathbf{T = mg}$

Lift Accelerating Vertically Down
Moving up with decreasing velocity.
or
Moving down with increasing velocity.
For vertical motion of body
$mg - T = ma$
or, $T = mg - ma$
or, $\mathbf{T = m(g - a)}$







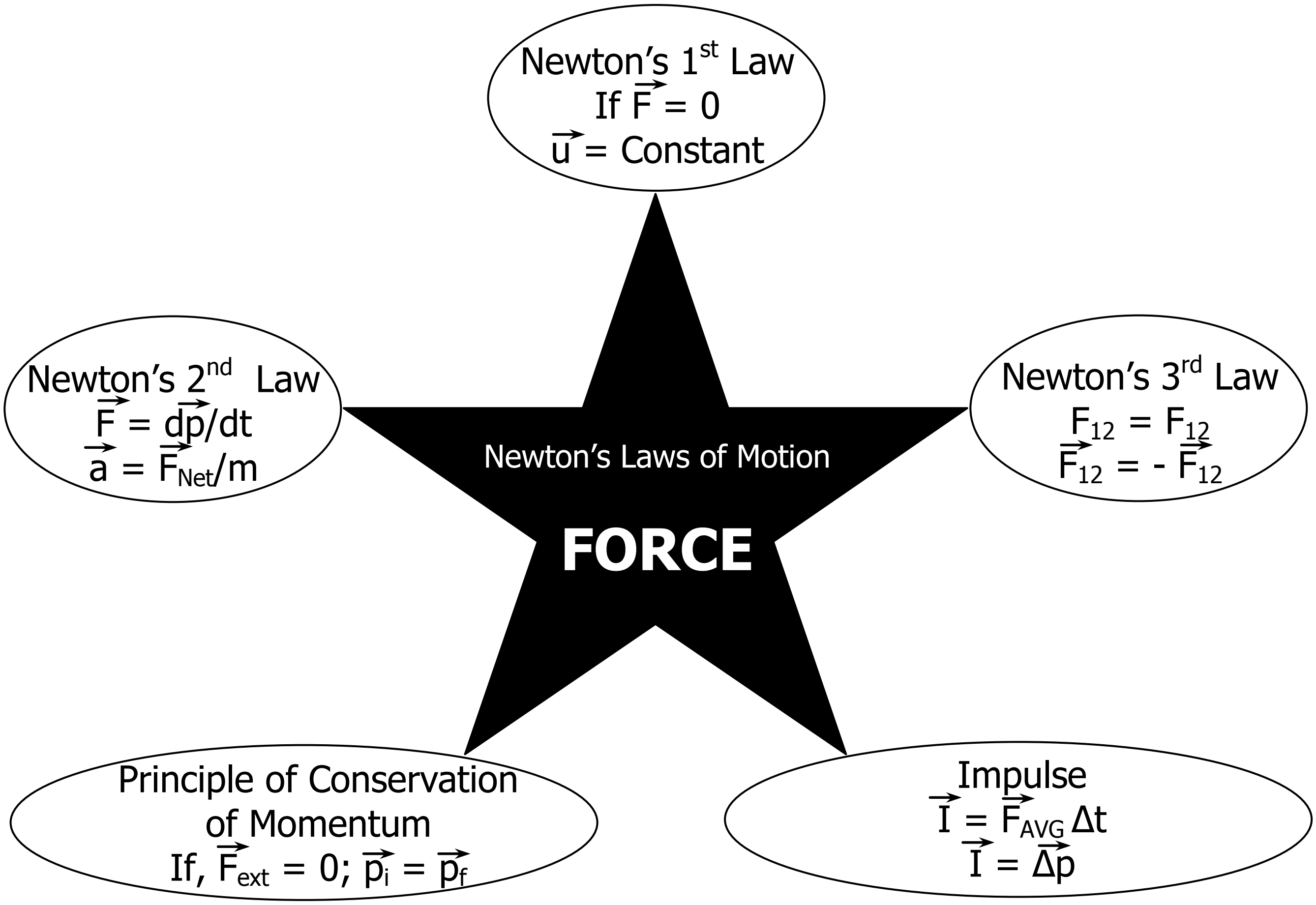
Observer Inside the Lift
(Body is at rest according to the observer inside the lift)

Lift Accelerating Vertically Up
Moving up with increasing velocity.
or
Moving down with decreasing velocity.
Since body is at rest
$\mathbf{T = mg'}$
but, $T = m(g + a)$
therefore, $\mathbf{g' = g + a}$
Where g' is apparent acceleration due to gravity inside the lift.

Lift Accelerating Vertically Up
Moving up with constant velocity.
or
Moving down with constant velocity.
Since body is at rest
$\mathbf{T = mg'}$
but, $T = mg$
therefore, $\mathbf{g' = g}$
Where g' is apparent acceleration due to gravity inside the lift.

Lift Accelerating Vertically Down
Moving up with decreasing velocity.
or
Moving down with increasing velocity.
Since body is at rest
$\mathbf{T = mg'}$
But, $\mathbf{T = m(g - a)}$
therefore, $\mathbf{g' = g - a}$
Where g' is apparent acceleration due to gravity inside the lift.

MEMORY MAP


Newton's 1st Law
If $\vec{F} = 0$
$\vec{u}$ = Constant
Newton's 2nd Law
$\vec{F} = d\vec{p}/dt$
$\vec{a} = \vec{F}_{Net}/m$
Newton's 3rd Law
$F_{12} = F_{12}$
$\vec{F}_{12} = - \vec{F}_{12}$
Newton's Laws of Motion
FORCE
Principle of Conservation of Momentum
If, $\vec{F}_{ext} = 0$; $\vec{p}_i = \vec{p}_f$
Impulse
$\vec{I} = \vec{F}_{AVG} \Delta t$
$\vec{I} = \Delta\vec{p}$

FRICTION

Friction - The property by virtue of which the relative motion between two surfaces in contact is opposed is known as friction.

Frictional Forces - Tangential forces developed between the two surfaces in contact, so as to oppose their relative motion are known as frictional forces or commonly friction.
Types of Frictional Forces - Frictional forces are of three types :-
1. Static frictional force
2. Kinetic frictional force
3. Rolling frictional force

Static Frictional Force - Frictional force acting between the two surfaces in contact which are relatively at rest, so as to oppose their relative motion, when they tend to move relatively under the effect of any external force is known as static frictional force. Static frictional force is a self adjusting force and its value lies between its minimum value up to its maximum value.

Minimum value of static frictional force - Minimum value of static frictional force is zero in the condition when the bodies are relatively at rest and no external force is acting to move them relatively.

$$f_{s(min)} = 0$$

Maximum value of static frictional force - Maximum value of static frictional force is $\mu_s N$ (where μ_s is the coefficient of static friction for the given pair of surface and N is the normal reaction acting between the two surfaces in contact) in the condition when the bodies are just about to move relatively under the effect of external applied force.

$$f_{s(max)} = \mu_s N$$

Therefore, $$f_{s(min)} \leq f_s \leq f_{s(max)}$$

or, $$\mathbf{0 \leq f_s \leq \mu_s N}$$

Kinetic Frictional Force - Frictional force acting between the two surfaces in contact which are moving relatively, so as to oppose their relative motion, is known as kinetic frictional force. It's magnitude is almost constant and is equal to $\mu_k N$ where μ_k is the coefficient of kinetic friction for the given pair of surface and N is the normal reaction acting between the two surfaces in contact. It is always less than maximum value of static frictional force.

$$\mathbf{f_k = \mu_k N}$$

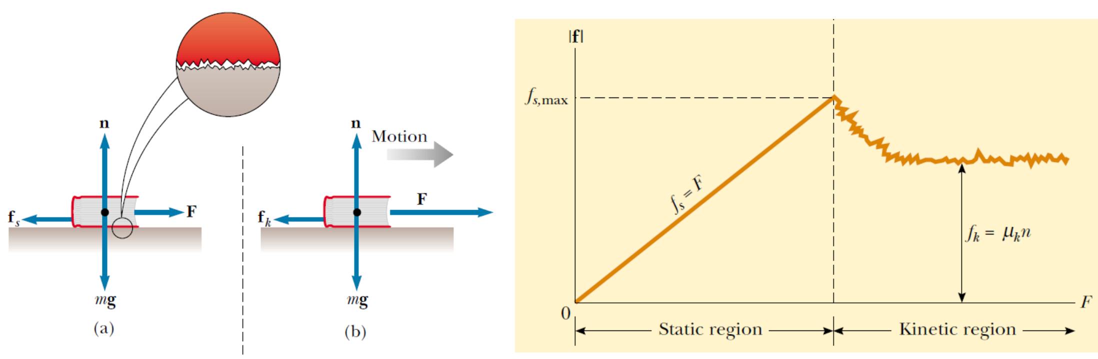
Since, $$f_k < f_{s(max)} = \mu_s N$$
Therefore, $$\mu_k N < \mu_s N$$
or, $$\mathbf{\mu_k < \mu_s}$$

Limiting Frictional Force – The maximum value of static frictional force is the maximum frictional force which can act between the two surfaces in contact and hence it is also known as limiting frictional force.

Laws of Limiting Frictional Force –
1. Static friction depends upon the nature of the surfaces in contact.
2. It comes into action only when any external force is applied to move the two bodies relatively, with their surfaces in contact.
3. Static friction opposes the impending motion.
4. It is a self adjusting force.
5. The limiting frictional force is independent of the area of contact between the two surfaces.

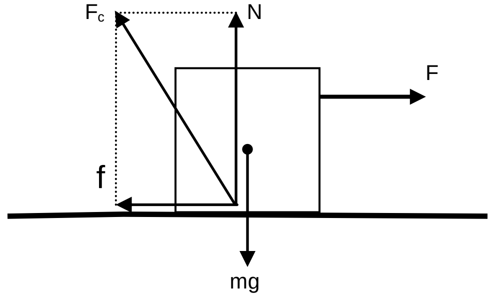
Cause of Friction

Old View - The surfaces which appear to be smooth as seen through our naked eyes are actually rough at the microscopic level. During contact, the projections of one surface penetrate into the depressions of other and vice versa. Due to which the two surfaces in contact form a saw tooth joint opposing their relative motion. When external force is applied so as to move them relatively this joint opposes their relative motion. As we go on increasing the external applied force the opposition of saw tooth joint also goes on increasing up to the maximum value known as limiting frictional force ($\mu_s N$) after which the joint suddenly breaks and the surfaces start moving relatively. After this the opposition offered by the saw tooth joint slightly decreases and comes to rest at almost constant value ($\mu_k N$)



Modern View – According to modern theory the cause of friction is the atomic and molecular forces of attraction between the two surfaces at their actual point of contact. When any body comes in contact with any other body then due to their roughness at the microscopic level they come in actual contact at several points. At these points the atoms and molecules come very close to each other and intermolecular force of attraction start acting between them which opposes their relative motion.

Contact Force - The forces acting between the two bodies due to the mutual contact of their surfaces are known as contact forces. The resultant of all the contact forces acting between the bodies is known as resultant contact force. Example friction (f) and normal reaction (N) are contact forces and their resultant (F_c) is the resultant is the resultant contact force.



$F_c = \sqrt{f^2 + N^2}$

Since maximum value of frictional force is Limiting frictional force ($\mu_s N$) Therefore maximum value of contact force is

$$F_{c(max)} = \sqrt{(\mu_s N)^2 + N^2}$$

or, $$F_{c(max)} = N\sqrt{\mu_s^2 + 1^2}$$

or, $$\mathbf{F_{c(max)} = N\sqrt{\mu_s^2 + 1}}$$

Angle of Friction – The angle between the resultant contact force (of normal reaction and friction) and the normal reaction is known as the angle of friction.

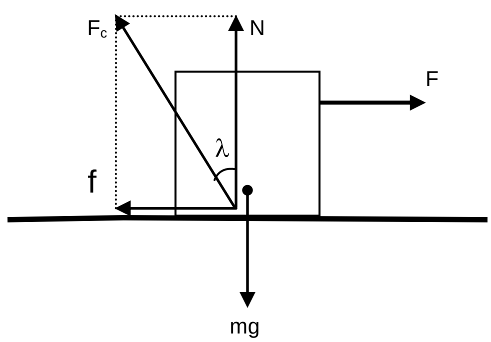
Tan $\lambda = \frac{f}{N}$

or, $\boldsymbol{\lambda = \mathrm{Tan}^{-1} \frac{f}{N}}$

or, $\lambda_{max} = \mathrm{Tan}^{-1} \frac{f_{max}}{N}$

or, $\lambda_{max} = \mathrm{Tan}^{-1} \frac{\mu_s N}{N}$

or, $\boldsymbol{\lambda_{max} = \mathrm{Tan}^{-1} \mu_s}$



Angle of Repose – The angle of the inclined plane at which a body placed on it just begins to slide is known as angle of repose.

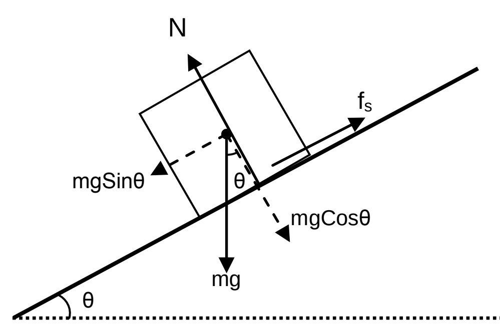
Perpendicular to the plane

N = mgCosθ (since body is at rest)

Parallel to the plane when body is at rest

$\mathbf{mgSin\theta = f_s}$

When body is just about to slide



$$mg\sin\theta = f_{s(max)} = \mu_s N = \mu_s mg\cos\theta$$

or, $$\tan\theta = \mu_s$$

or, $$\boldsymbol{\theta = \tan^{-1}\mu_s}$$

Note - Angle of repose is equal to the maximum value of angle of friction

Rolling Frictional Force - Frictional force which opposes the rolling of bodies (like cylinder, sphere, ring etc.) over any surface is called rolling frictional force. Rolling frictional force acting between any rolling body and the surface is almost constant and is given by $\mu_r N$. Where μ_r is coefficient of rolling friction and N is the normal reaction between the rolling body and the surface.

$$\mathbf{f_r = \mu_r N}$$

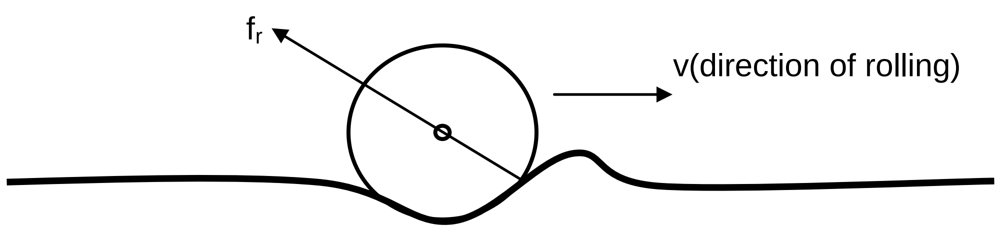
Note – Rolling frictional force is much smaller than maximum value of static and kinetic frictional force.

$$f_r << f_k < f_{s(max)}$$

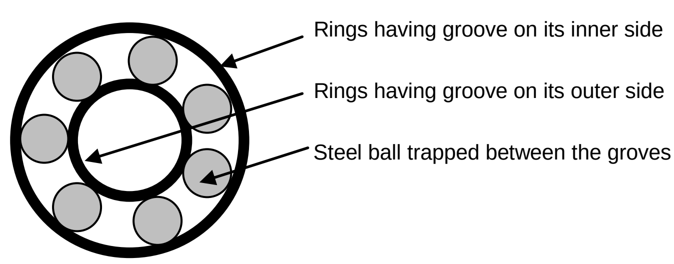
or, $$\mu_r N << \mu_k N < \mu_s N$$

or, $$\boldsymbol{\mu_r << \mu_k < \mu_s}$$

Cause of Rolling Friction – When any body rolls over any surface it causes a little depression and a small hump is created just ahead of it. The hump offers resistance to the motion of the rolling body, this resistance is rolling frictional force. Due to this reason only, hard surfaces like cemented floor offers less resistance as compared to soft sandy floor because hump created on a hard floor is much smaller as compared to the soft floor.



Need to Convert Kinetic Friction into Rolling Friction – Of all the frictional forces rolling frictional force is minimum. Hence in order to avoid the wear and tear of machinery it is required to convert kinetic frictional force into rolling frictional force and for this reason we make the use of ball-bearings.



Friction: A Necessary Evil – Although frictional force is a non-conservative force and causes lots of wastage of energy in the form of heat yet it is very useful to us in many ways. That is why it is considered as a necessary evil.

Advantages of Friction -
i) Friction is necessary in walking. Without friction it would have been impossible for us to walk.
ii) Friction is necessary for the movement of vehicles on the road. It is the static frictional force which makes the acceleration and retardation of vehicles possible on the road.
iii) Friction is helpful in tying knots in the ropes and strings.
iv) We are able to hold anything with our hands by the help of friction only.

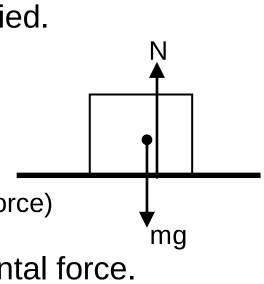
Disadvantages of Friction -
i) Friction causes wear and tear in the machinery parts.
ii) Kinetic friction wastes energy in the form of heat, light and sound.
iii) A part of fuel energy is consumed in overcoming the friction operating within the various parts of machinery.

Methods to Reduce Friction –
i) By polishing – Polishing makes the surface smooth by filling the space between the depressions and projections present in the surface of the bodies at microscopic level and there by reduces friction.
ii) By proper selection of material – Since friction depends upon the nature of material used hence it can be largely reduced by proper selection of materials.
iii) By lubricating – When oil or grease is placed between the two surfaces in contact, it prevents the surface from coming in actual contact with each other. This converts solid friction into liquid friction which is very small.

Physical Application

Horizontal Plane

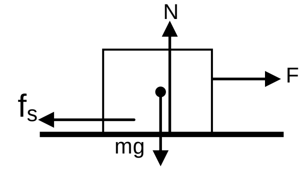
i) Body kept on horizontal plane is at rest and no force is applied.



For vertical equilibrium
$\mathbf{N = mg}$

$\mathbf{f_{friction} = 0}$ (friction is a opposing force and there is no external applied force)

ii) Body kept on horizontal plane is at rest under single horizontal force.



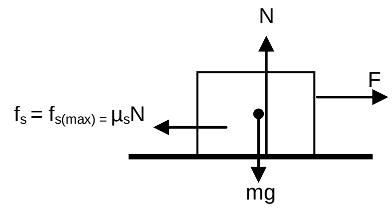
For vertical equilibrium
$\mathbf{N = mg}$ (since body is at rest)

For horizontal equilibrium (since body is at rest)
$\mathbf{F = f_s}$

iii) Body kept on horizontal plane is just about to move.

For vertical direction
$N = mg$ (since body is at rest)

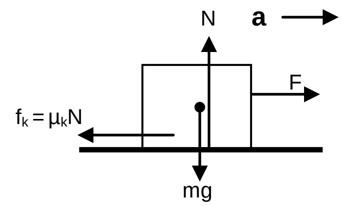
For horizontal direction (since body is just about to move)
$F = f_s = f_{s(max)} = \mu_s N$



iv) Body kept on horizontal plane is accelerating horizontally.

For vertical direction
$N = mg$ (since body is at rest)

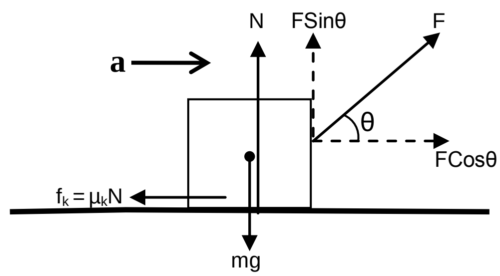
For horizontal direction
$F - f_k = ma$
or, **$F - \mu_k N = ma$**



v) Body kept on horizontal plane is accelerating horizontally towards right under single upward inclined force.

For vertical direction
$N + F\sin\theta = mg$ (since body is at rest)

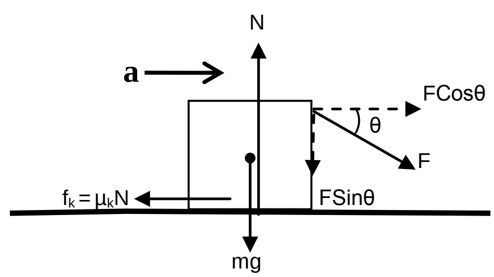
For horizontal direction
$F\cos\theta - f_k = ma$
or, **$F\cos\theta - \mu_k N = ma$**



vi) Body kept on horizontal plane is accelerating horizontally towards right under single downward inclined force.

For vertical direction
$N = F\sin\theta + mg$ (since body is at rest)

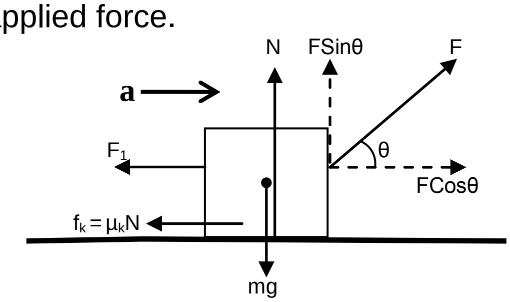
For horizontal direction
$F\cos\theta - f_k = ma$
or, **$F\cos\theta - \mu_k N = ma$**



vii) Body kept on horizontal plane is accelerating horizontally towards right under an inclined force and an opposing horizontally applied force.

For vertical direction
$N + F\sin\theta = mg$ (since body is at rest)

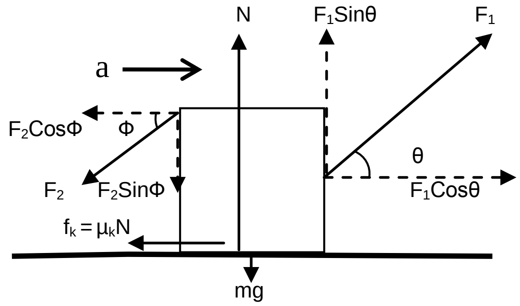
For horizontal direction
$F\cos\theta - F_1 - f_k = ma$
or, **$F\cos\theta - F_1 - \mu_k N = ma$**



vi) Body kept on horizontal plane is accelerating horizontally towards right under two inclined forces acting on opposite sides.

For vertical direction(since body is at rest)
$N + F_1 Sin\theta = mg + F_2\ Sin\Phi$

For horizontal direction
$F_1 Cos\theta - F_2 Cos\Phi - \mu_k N = ma$

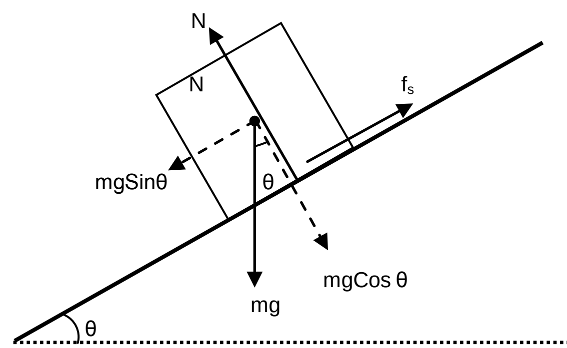


Inclined Plane

i) Case - 1
Body is at rest on inclined plane.

Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

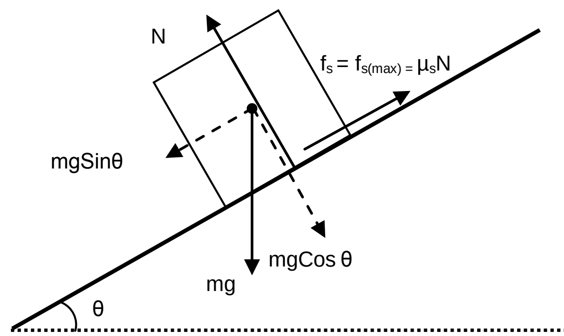
Parallel to the plane (since body is at rest)
$mgSin\theta = f_s$



ii) Case - 2
Body is just about to move on inclined plane.

Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

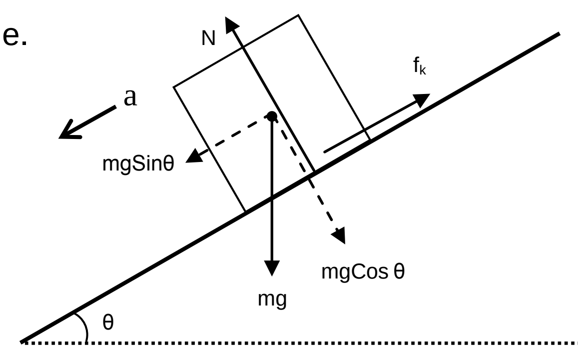
Parallel to the plane (since body is at rest)
$mgSin\theta = f_s = f_{s(max)} = \mu_s N$



iii) Case - 3
Body is accelerating downwards on inclined plane.

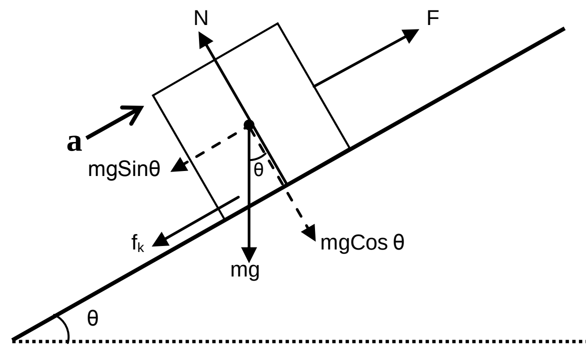
Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane
$mgSin\theta - f_k = ma$
or, **$mgSin\theta - \mu_k N = ma$**



iv) Case - 4

Body is accelerating up the incline under the effect of force acting parallel to the incline.



Perpendicular to the plane

N = mgCosθ (since body is at rest)

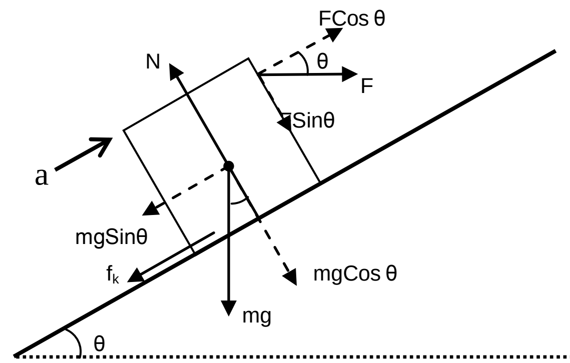
Parallel to the plane

F - f_k - mgSinθ = ma

or, **F - μ_kN - mgSinθ = ma**

v) Case - 5

Body accelerating up the incline under the effect of horizontal force.



Perpendicular to the plane

N = mgCosθ + FSinθ (since body is at rest)

Parallel to the plane

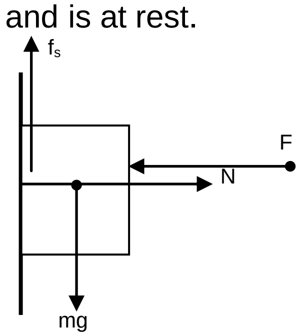
FCosθ - mgSinθ - f_k = ma

or, **FCosθ - mgSinθ - μ_kN ma**

Vertical Plane

i) Case - 1

Body pushed against the vertical plane by horizontal force and is at rest.



For horizontal direction (since body is at rest)

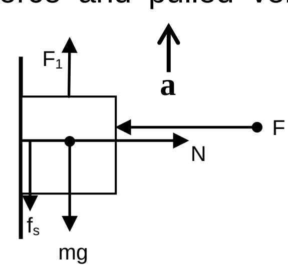
F = N

For vertical direction

mg = f_s

ii) Case - 2

Body pushed against the vertical plane by horizontal force and pulled vertically upward



For horizontal direction (since body is at rest)

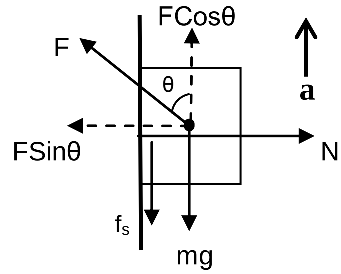
F = N

For vertical direction

F_1 - mg – f_s = ma

iii) Case - 3

Body pushed against the vertical plane by inclined force and accelerates vertically upward.



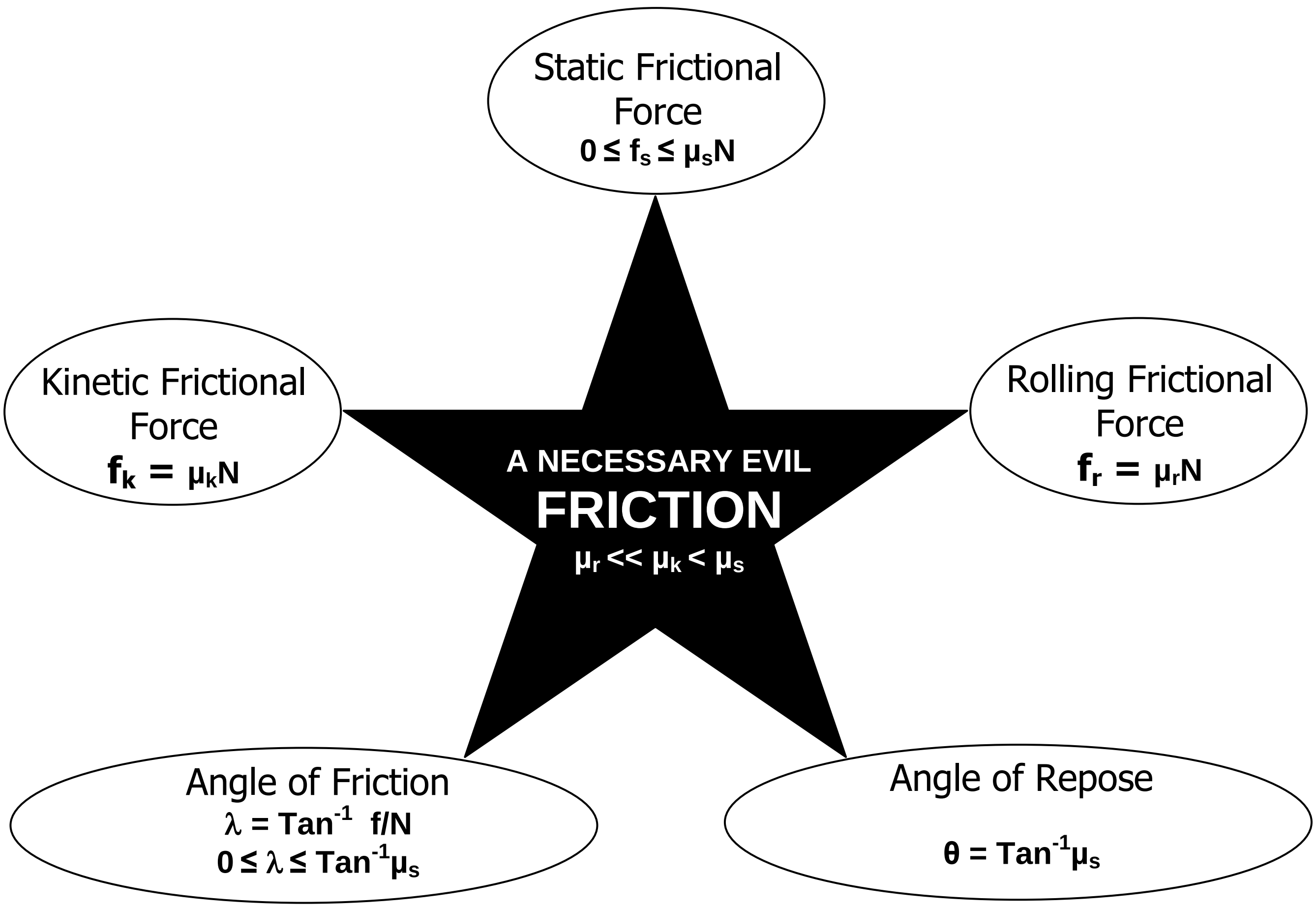
For horizontal direction

$N = F\sin\theta$ (since body is at rest)

For vertical direction

$F\cos\theta - mg - f_s = ma$

MEMORY MAP

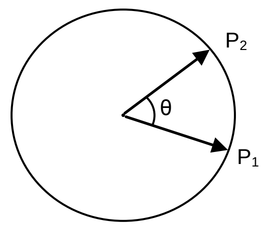

Static Frictional Force
$0 \le f_s \le \mu_s N$
Kinetic Frictional Force
$f_k = \mu_k N$
A NECESSARY EVIL
FRICTION
$\mu_r << \mu_k < \mu_s$
Rolling Frictional Force
$f_r = \mu_r N$
Angle of Friction
$\lambda = Tan^{-1}\ f/N$
$0 \le \lambda \le Tan^{-1}\mu_s$
Angle of Repose
$\theta = Tan^{-1}\mu_s$

CIRCULAR MOTION

Circular Motion – When a body moves such that it always remains at a fixed distance from a fixed point then its motion is said to be circular motion. The fixed distance is called the radius of the circular path and the fixed point is called the center of the circular path.

Uniform Circular Motion – Circular motion performed with a constant speed is known as uniform circular motion.

Angular Displacement – Angle swept by the radius vector of a particle moving on a circular path is known as angular displacement of the particle. Example :– angular displacement of the particle from P_1 to P_2 is θ.



Relation Between Angular Displacement and Linear Displacement –

Since,

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

$$\text{Anglular Displacement} = \frac{\text{arc } P_1P_2}{\text{radius}}$$

$$\boldsymbol{\theta = \frac{s}{r}}$$

Angular Velocity – Rate of change of angular displacement of a body with respect to time is known as angular displacement. It is represented by ω.

Average Angular Velocity – It is defined as the ratio of total angular displacement to total time taken.

$$\omega_{avg} = \frac{\text{Total Angular Displacement}}{\text{Total Time Taken}}$$

$$\boldsymbol{\omega_{avg} = \frac{\Delta\theta}{\Delta t}}$$

Instantaneous Angular Velocity – Angular velocity of a body at some particular instant of time is known as instantaneous angular velocity.

Or

Average angular velocity evaluated for very short duration of time is known as instantaneous angular velocity.

$$\omega = \lim_{\Delta t \to 0} \omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\boldsymbol{\omega = \frac{d\theta}{dt}}$$

Relation Between Angular Velocity and Linear Velocity

We know that angular velocity

$$\omega = \frac{d\theta}{dt}$$

Putting, $\theta = s/r$

$$\omega = \frac{d\,(s/r)}{dt}$$

or,

$$\omega = \frac{1}{r}\frac{ds}{dt}$$

or,

$$\boldsymbol{\omega = \frac{v}{r}}$$

or,

$$\mathbf{v = r\boldsymbol{\omega}}$$

Time Period of Uniform Circular Motion – Total time taken by the particle performing uniform circular motion to complete one full circular path is known as time period.

In one time period total angle rotated by the particle is 2π and time period is T. Hence angular velocity

$$\omega = \frac{2\pi}{T}$$

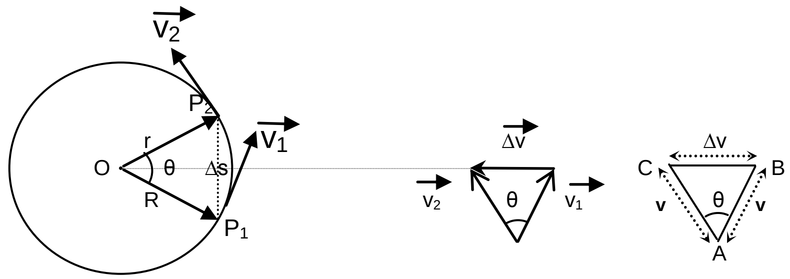
or,

$$\mathbf{T = \frac{2\pi}{\boldsymbol{\omega}}}$$

Frequency - Number of revolutions made by the particle moving on circular path in one second is known as frequency.

$$\mathbf{f = \frac{1}{T} = \frac{\boldsymbol{\omega}}{2\pi}}$$

Centripetal Acceleration – When a body performs uniform circular motion its speed remains constant but velocity continuously changes due to change of direction. Hence a body is continuously accelerated and the acceleration experienced by the body is known as centripetal acceleration (that is the acceleration directed towards the center).



Consider a particle performing uniform circular motion with speed v. When the particle changes its position from P_1 to P_2 its velocity changes from $\vec{v_1}$ to $\vec{v_2}$ due to change of direction. The change in velocity from P_1 to P_2 is $\vec{\Delta v}$ which is directed towards the center of the circular path according to triangle law of subtraction of vectors.

From figure ΔOP_1P_2 and ΔABC are similar, hence applying the condition of similarity

$$\frac{BC}{AB} = \frac{P_1 P_1}{O P_1}$$

or, $$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

or, $$\Delta v = \frac{v\Delta s}{r}$$

Dividing both sides by Δt, $$\frac{\Delta v}{\Delta t} = \frac{v\Delta s}{r\Delta t}$$

Taking limit $\Delta t \rightarrow 0$ both sides,

$$\lim_{\Delta t\to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t\to 0} \frac{\Delta \theta}{\Delta t}$$

or, $$\frac{dv}{dt} = \frac{vds}{dt}$$

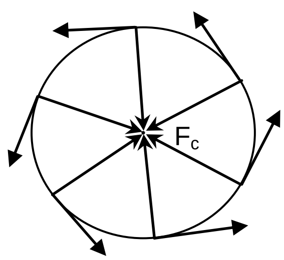
or, $$\boxed{\mathbf{a = \frac{v^2}{r}}}$$

Putting $v = r\omega$, $$\boxed{a = r\boldsymbol{\omega}^2}$$

Since the change of velocity is directed towards the center of the circular path, the acceleration responsible for the change in velocity is also directed towards center of circular path and hence it is known as centripetal acceleration.

Centripetal Force – Force responsible for producing centripetal acceleration is known as centripetal force. Since centripetal acceleration is directed towards the center of the circular path the centripetal force is also directed towards the center of the circular path.

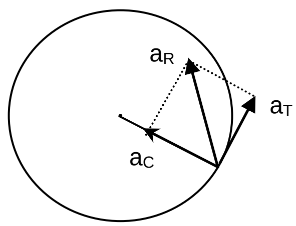
If a body is performing uniform circular motion with speed v and angular velocity ω on a circular path of radius r, then centripetal acceleration is given by



$$\boxed{F_c = \frac{mv^2}{r} = mr\omega^2}$$

Net Acceleration of a Body Performing Non-Uniform Circular Motion

When a body performs non-uniform circular motion its speed i.e. magnitude of instantaneous velocity varies with time due to which it experiences tangential acceleration a_T along with the centripetal acceleration a_C. Since both the accelerations act simultaneously on a body and are mutually perpendicular to each other, the resultant acceleration a_R is given by their vector sum.



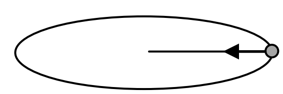
$$\vec{a_R} = \vec{a_T} + \vec{a_C}$$

$$a_R = \sqrt{a_T^2 + a_C^2}$$

Physical Application of Centripetal Force

i) Case - 1

Circular motion of a stone tied to a string.

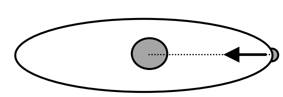


Centripetal force is provided by the tension of the string

$$F_c = \frac{mv^2}{r} = T$$

ii) Case - 2

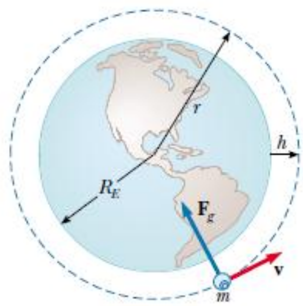
Circular motion of electron around the nucleus.



Centripetal force is provided by the electrostatic force of attraction between the positively charged nucleus and negatively charged electron

$$F_c = \frac{mv^2}{r} = F_E$$

iii) Case - 3
Circular motion of planets around sun or satellites around planet.



Centripetal force is provided by the gravitational force of attraction between the planet and sun

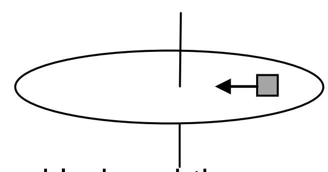
$$F_c = \frac{mv^2}{r} = F_g$$

iv) Case - 4
Circular motion of vehicles on a horizontal road.

Centripetal force is provided by the static frictional force between the road and the tyre of the vehicle.

$$F_c = \frac{mv^2}{r} = f_s$$

v) Case - 5
Circular motion of a block on rotating platform.



Centripetal force is provided by the static frictional force between the block and the platform.

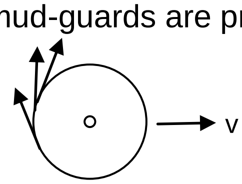
$$F_c = \frac{mv^2}{r} = f_s$$

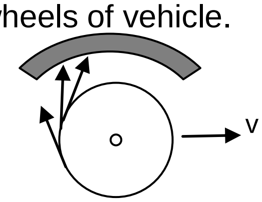
vi) Case - 6
Circular motion of mud particles sticking to the wheels of the vehicle.

Centripetal force is provided by the adhesive force of attraction between the mud particles and the tyres of the vehicle.

$$F_c = \frac{mv^2}{r} = F_{adhesive}$$

At very high speed when adhesive force is unable to provide necessary centripetal force, the mud particles fly off tangentially. In order to prevent the particles from staining our clothes, mud-guards are provided over the wheels of vehicle.

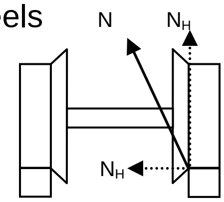




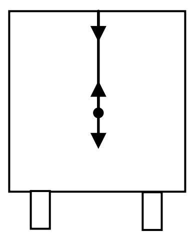
vii) Case - 7
Circular motion of a train on a horizontal track.

Centripetal force is provided by the horizontal component of the reaction force applied by the outer track on the inner projection of the outer wheels

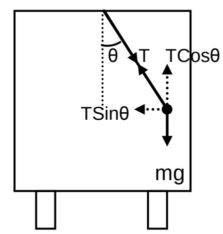
$$\mathbf{F_c = \frac{mv^2}{r} = N_{Horizontal}}$$



viii) Case - 8
Circular motion of a toy hanging from ceiling of vehicle.



Car moving with constant velocity on horizontal road



Car taking a turn with constant velocity on a horizontal road

Whenever car takes a turn, string holding the toy gets tilted outward such that the vertical component of the tension of string balances the weight of the body and the horizontal component of tension provides the necessary centripetal force.

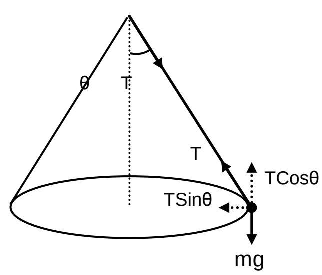
$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

Therefore,

$$\mathbf{\tan\theta = \frac{v^2}{rg}}$$

ix) Case - 9
Conical pendulum.



Whenever bob of a pendulum moves on a horizontal circle it’s string generates a cone. Such a pendulum is known as conical pendulum. The vertical component of the tension of the string balances the weight of the body and the horizontal component of tension provides the necessary centripetal force.

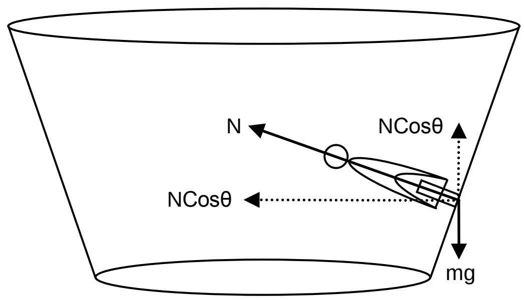
$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

Therefore, $$\mathbf{\tan\theta = \frac{v^2}{rg}}$$

x) Case - 10

Well of death.



In the well of death, the rider tries to push the wall due to its tangential velocity in the outward direction due to which wall applies normal reaction in the inward direction. The vertical component of the normal reaction balances the weight of the body and its horizontal component provides the necessary centripetal force.

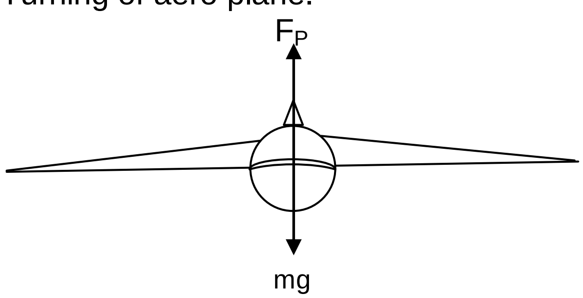
$$N\sin\theta = \frac{mv^2}{r}$$

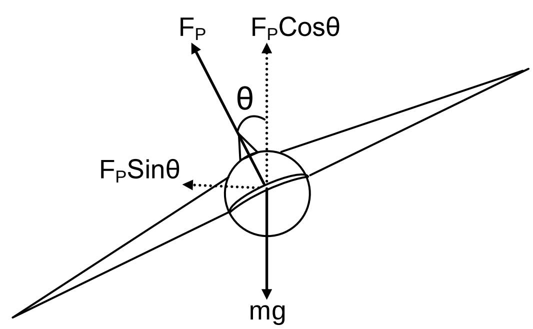
$$N\cos\theta = mg$$

Therefore, $$\mathbf{\tan\theta = \frac{v^2}{rg}}$$

xi) Case - 11

Turning of aero plane.





While taking a turn aero-plane tilts slightly inwards due to which it's pressure force also gets tilted inwards due to which it's pressure force also gets tilted inwards such that it's vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.

$$F_P \sin\theta = \frac{mv^2}{r}$$

$$F_P \cos\theta = mg$$

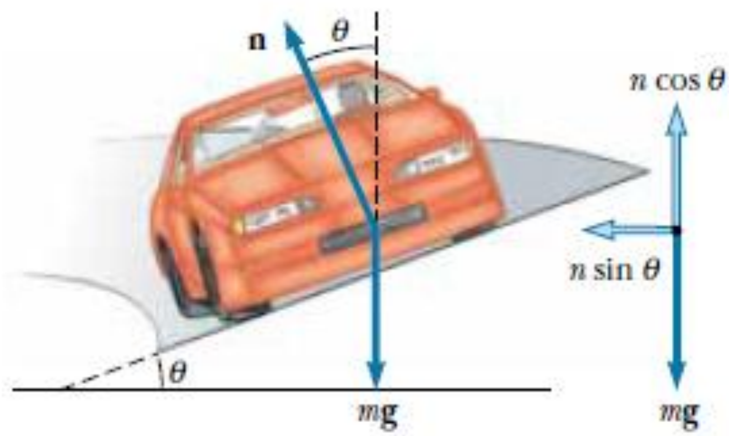
Therefore, $$\mathbf{\tan\theta = \frac{v^2}{rg}}$$

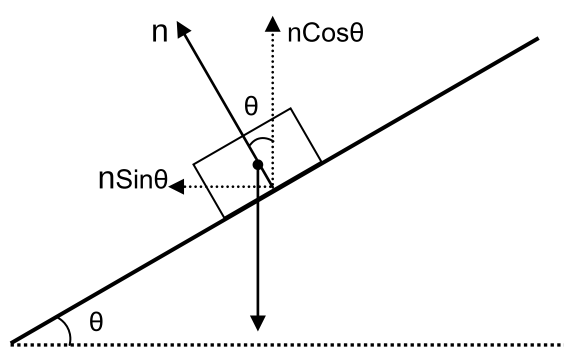
xi) Case - 11

Banking of Roads

In case of horizontal road necessary centripetal force mv^2/r is provided by static frictional force. When heavy vehicles move with high speed on a sharp turn (small radius) then all the factors contribute to huge centripetal force which if provided by the static frictional force may result in the fatal accident.

To prevent this roads are banked by lifting their outer edge. Due to this, normal reaction of road on the vehicle gets tilted inwards such that it's vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.





$$n\sin\theta = \frac{mv^2}{r}$$

$$n\cos\theta = mg$$

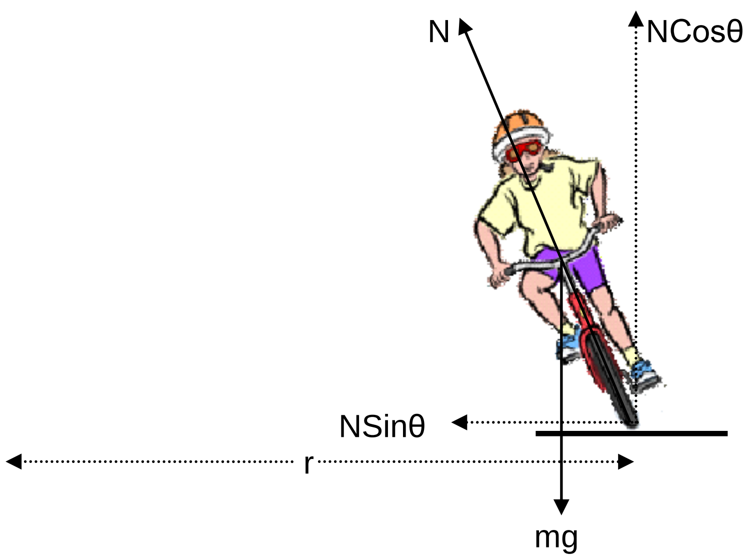
Therefore, $$\textbf{Tan}\boldsymbol{\theta} = \frac{\mathbf{v^2}}{\mathbf{rg}}$$

xii) Case - 12

Bending of Cyclist

In case of a cyclist moving on a horizontal circular track necessary centripetal force is provided by static frictional force acting parallel along the base. As this frictional force is not passing from the center of mass of the system it tends to rotate the cycle along with the cyclist and make it fall outward of the center of the circular path.

To prevent himself from falling, the cyclist leans the cycle inwards towards the center of the circle due to which the normal reaction of the surface of road on the cycle also leans inward such that that its vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.



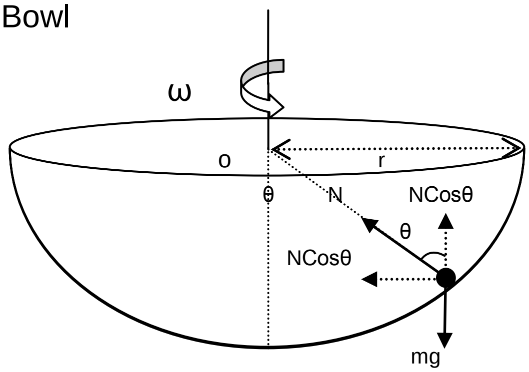
$$NSin\theta = \frac{mv^2}{r}$$

$$NCos\theta = mg$$

Therefore, $$\textbf{Tan}\boldsymbol{\theta} = \frac{\mathbf{v^2}}{\mathbf{rg}}$$

xiii) Case - 13

Motion of a Ball in a Bowl



When the bowl rotates with some angular velocity ω. The vertical component of the normal reaction of the bowl on the ball balances the weight of the body and its horizontal component provides the necessary centripetal force.

$$NSin\theta = \frac{mv^2}{r}$$

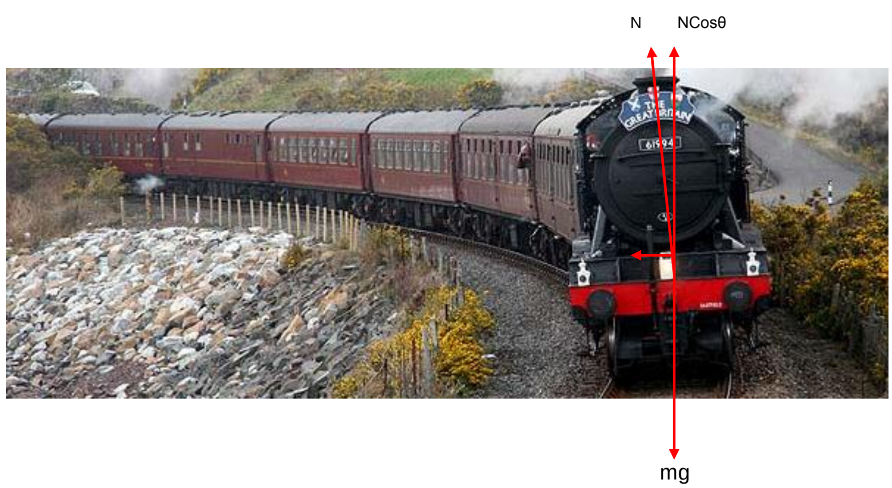
$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

xiv) Case - 14

Motion of a train on the banked tracks.

At the turns tracks are banked by slightly elevating the outer tracks with respect to the inner ones. This slightly tilts the train inwards towards the center of the circular path due to which the normal reaction of the tracks on the train also gets slightly tilted inwards such that the vertical component of the normal reaction balances the weight of the train and it’s horizontal component provides the necessary centripetal force.



$$NSin\theta = \frac{mv^2}{r}$$

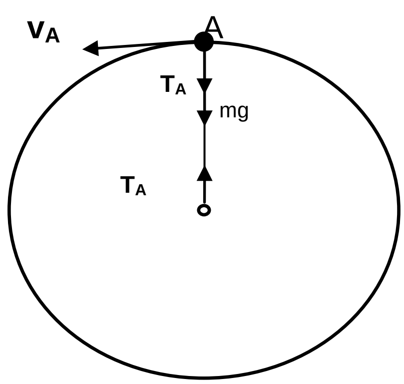
$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

Vertical Circular Motion

Whenever the plane of circular path of body is vertical its motion is said to be vertical circular motion.

Vertical Circular Motion of a Body Tied to a String



Consider a body of mass m tied to a string and performing vertical circular motion on a circular path of radius r. At the topmost point A of the body weight of the body mg and tension T_A both are acting in the vertically downward direction towards the center of the circular path and they together provide centripetal force.

$$T_A + mg = \frac{mv_A^2}{r}$$

<u>Critical velocity at the top most point</u>

As we go on decreasing the v_A , tension T_A also goes on decreasing and in the critical condition when v_A is minimum tension $T_A = 0$. The minimum value of v_A in this case is known as critical velocity $T_{A(Critical)}$ at the point A. From above

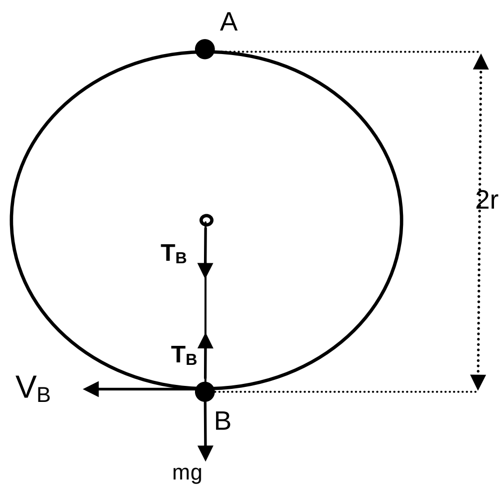
$$0 + mg = \frac{mv_{A(Critical)}^2}{r}$$

or, $$v_{A(Critical)}^2 = rg$$

or, $$\mathbf{v_{A(Critical)} = \sqrt{rg}}$$

If the velocity at point A is less than this critical velocity then the string will slag and the body in spite of moving on a circular path will tend to fall under gravity.

Critical velocity at the lower most point



Taking B as reference level of gravitational potential energy and applying energy conservation

$$E_A = E_B$$
$$P_A + K_A = P_B + K_B$$
$$mg2r + \frac{1}{2}mv_A^2 = mg0 + \frac{1}{2}mv_B^2$$

Putting, $v_A = \sqrt{rg}$

$$mg2r + \frac{1}{2}m(\sqrt{rg})^2 = 0 + \frac{1}{2}mv_B^2$$

or, $$4mgr + mgr = mv_B^2$$

or, $$5mgr = mv_B^2$$

or, $$v_B = \sqrt{5gr}$$

This is the minimum possible velocity at the lower most point for vertical circular motion known as critical velocity at point B.

$$\mathbf{v_{B(Critical)} = \sqrt{5gr}}$$

Tension at lowermost point in critical condition

For lowermost point B net force towards the center is centripetal force. Tension T_B acts towards the center of the circular path whereas weight mg acts away from it. Hence,

$$T_B - mg = \frac{mv_B^2}{r}$$

Putting, $v_B = \sqrt{5rg}$

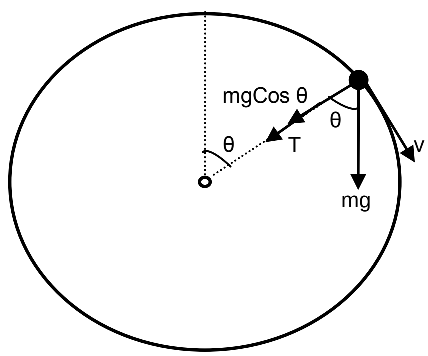
$$T_B - mg = \frac{m5gr}{r}$$

or, $$\mathbf{T_B = 6mg}$$

Hence in critical condition of vertical circular motion of a body tied to a string velocities at topmost and lowermost be $\sqrt{(rg)}$ and $\sqrt{(5rg)}$ respectively and tensions in the strings be 0 and 6mg respectively.

General Condition for Slagging of String in Vertical Circular Motion

For the body performing vertical circular motion tied to a string, slagging of string occurs in the upper half of the vertical circle. If at any instant string makes angle θ with vertical then applying net force towards center is equal to centripetal force, we have



$$T + mgCos\,\theta = \frac{mv^2}{r}$$

For slagging T = 0,

$$0 + mgCos\,\theta = \frac{mv^2}{r}$$

or, $$\mathbf{v = \sqrt{rgCos\,\theta}}$$

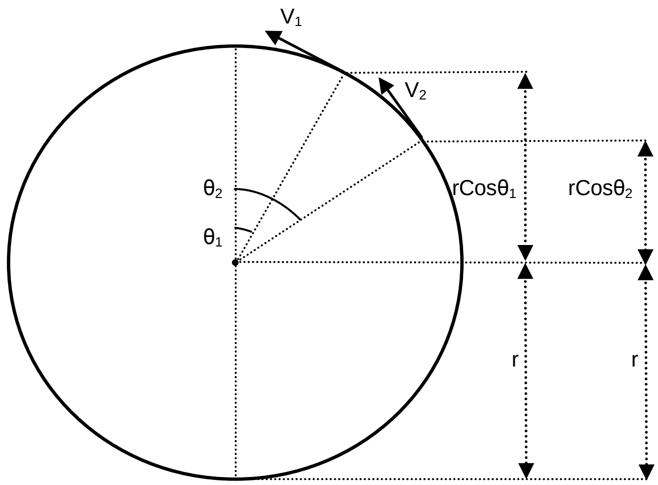
Case-1 At Topmost point θ = 0, therefore $v = \sqrt{rg}$

Case-2 At $\theta = 60^o$, therefore $v = \sqrt{rgCos60} = \sqrt{rg/2}$

Case-3 When string becomes horizontal that is at $\theta = 90^o$, $v = \sqrt{rgCos90} = 0$

Velocity Relationship at different Points of Vertical Circular Motion

Let initial and final velocities of the body performing vertical circular motion be v_1 and v_2 and the angle made by string with the vertical be θ_1 and θ_2. Taking lowermost point of vertical circular path as reference level and applying energy conservation,



$$E_1 = E_2$$

$$P_1 + K_1 = P_2 + K_2$$

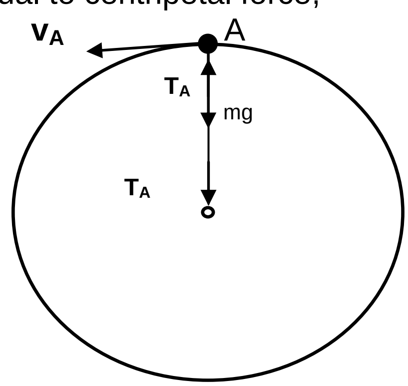
$$mg(r + rCos\theta_1) + \frac{1}{2}mv_1^2 = mg(r + rCos\theta_2) + \frac{1}{2}mv_2^2$$

or, $$mgr(Cos\theta_1 - Cos\theta_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

or, $$\mathbf{(v_2^2 - v_1^2) = 2gr(Cos\theta_1 - Cos\theta_2)}$$

Vertical Circular Motion of a Body Attached to a Rod

Since rod can never slag hence in the critical situation a body attached to the rod may reach the topmost position A of the vertical circular path with almost zero velocity. In this case its weight mg acts in vertically downward direction and tension of rod acts on the body in the vertically upward direction. Applying net force towards center is equal to centripetal force,



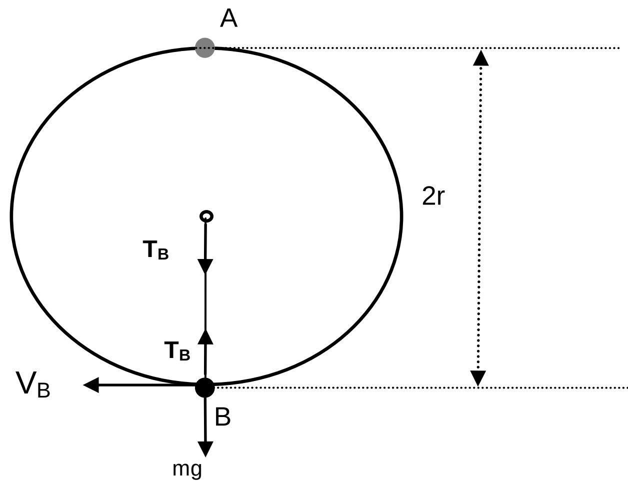
$$mg - T_A = \frac{mv_A^2}{r}$$

Putting $\mathbf{v_A = 0}$ (for critical condition)

$$mg - T_A = 0$$

or, $$\mathbf{T_A = mg}$$

Critical velocity and Tension at the lower most point



Taking B as reference level of gravitational potential energy and applying energy conservation

$$E_A = E_B$$
$$P_A + K_A = P_B + K_B$$
$$mg2r + \frac{1}{2}mv_A^2 = mg0 + \frac{1}{2}mv_B^2$$

Putting, $v_A = 0$(for critical condition)

$$mg2r + 0 = 0 + \frac{1}{2}mv_B^2$$

or, $$4mgr = mv_B^2$$

or, $$v_B = \sqrt{4rg}$$

This is the minimum possible velocity at the lower most point for vertical circular motion known as critical velocity at point B.

$$\mathbf{v_{B(Critical)} = \sqrt{4rg}}$$

Tension at lowermost point in critical condition

For lowermost point B applying net force towards center is equal to centripetal force. Tension T_B acts towards the center of the circular path whereas weight mg acts away from it in vertically downward direction. Hence,

$$T_B - mg = \frac{mv_B^2}{r}$$

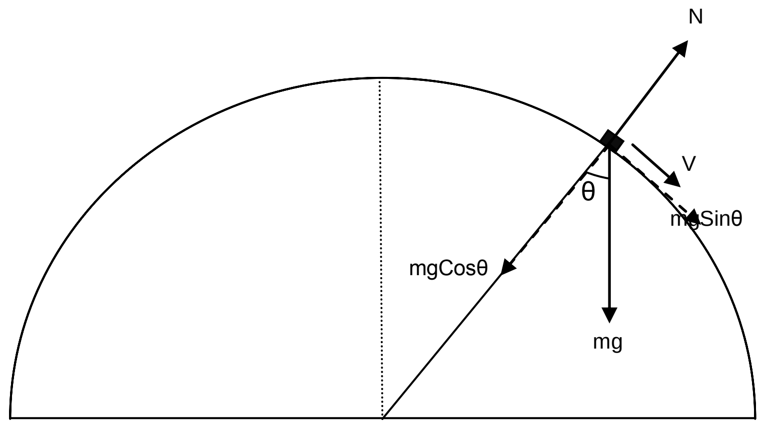
Putting, $v_B = \sqrt{4rg}$

$$T_B - mg = \frac{m4gr}{r}$$

or, $$\mathbf{T_B = 5mg}$$

Hence in critical condition of vertical circular motion of a body attached to the rod velocities at topmost and lowermost be 0 and $\sqrt{4rg}$ respectively and tensions in the rod be mg (pushing nature) and 5mg (pulling nature) respectively.

Motion of A Body Over Spherical Surface



A body of mass m is moving over the surface of the smooth sphere of radius r. At any instant when the radius of sphere passing through the body makes angle θ with the vertical the tangential velocity of the body is v. Since net force towards the center is centripetal force we have

$$mg\cos\theta - N = \frac{mv^2}{r}$$

or, $$\mathbf{N = mg\cos\theta - \frac{mv^2}{r}}$$

if v increases N decreases and when the body just loses contact with the sphere N = 0.

Putting N = 0,

$$0 = mg\cos\theta - \frac{mv^2}{r}$$

or, $$\frac{mv^2}{r} = mg\cos\theta$$

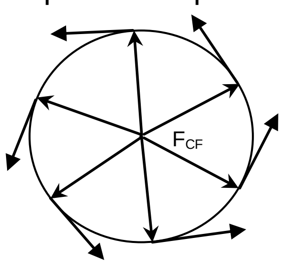
or, $$\mathbf{v = \sqrt{rg\cos\theta}}$$

This is the minimum velocity at which the body loses contact and it is the maximum velocity at which the body remains in contact with the surface.

CENTRIFUGAL FORCE

It is a pseudo force experienced by a body which is a part of the circular motion. It is a non-realistic force and comes into action only when the body is in a circular motion. Once the circular motion of the body stops, this force ceases to act. Its magnitude is exactly same as that of centripetal force but it acts opposite to the direction of the centripetal force that is in the radially outward direction.

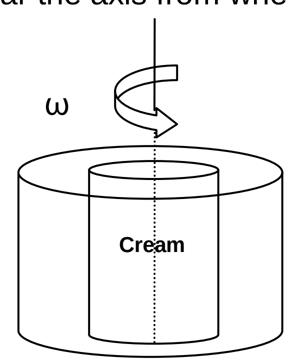
Frame of reference attached to a body moving on a circular path is a non-inertial frame since it an accelerated frame. So when ever any body is observed from this frame a pseudo force $F = ma = mv^2/r = mr\omega^2$ must be applied on the body opposite to the direction of acceleration along with the other forces. Since the acceleration of the frame in circular motion is centripetal acceleration $a = v^2/r$ directed towards the center of the circular path, the pseudo force applied on the bodies observed from this frame is $F = mv^2/r$ directed away from the center of the circular path. This pseudo force is termed as a centrifugal force.



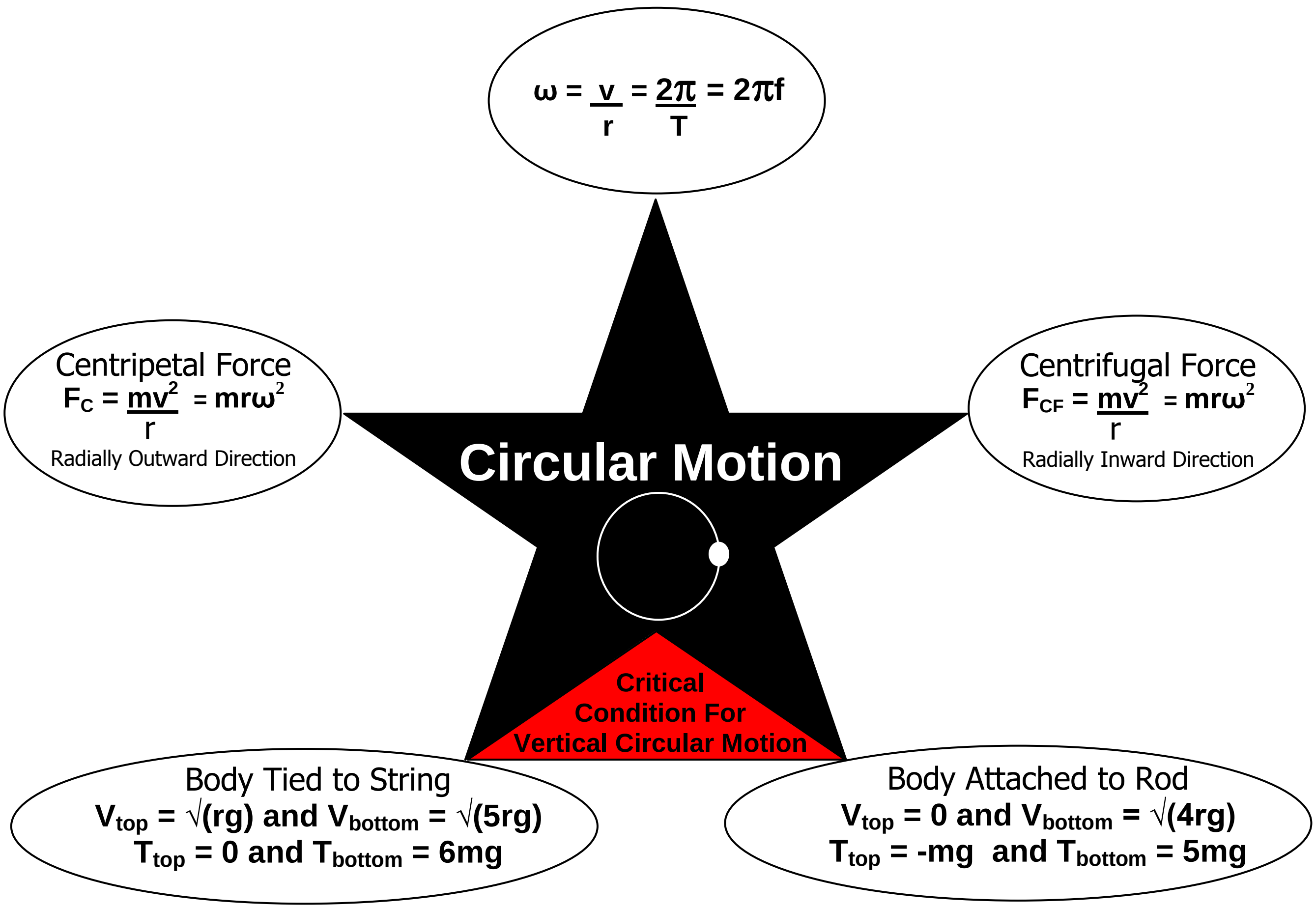
$$\boxed{\mathbf{F_{Centrifugal} = \frac{mv^2}{r} = mr\omega^2}}$$ (Directed in radially outward direction)

CENTRIFUGE

It is an apparatus used to separate cream from milk. It works on the principal of centrifugal force. It is a cylindrical vessel rotating with high angular velocity about its central axis. When this vessel contains milk and rotates with high angular velocity all the particles of milk start moving with the same angular velocity and start experiencing centrifugal force $F_{Centrifugal} = mr\omega^2$ in radially outward direction. Since centrifugal force is directly proportional to the mass of the particles, massive particles of milk on experiencing greater centrifugal force starts depositing on the outer edge of the vessel and lighter cream particles on experiencing smaller centrifugal force are collected near the axis from where they are separated apart.



MEMORY MAP



Critical Condition of Vertical Circular MOtion

Very Short Answer Type 1 Mark Questions

1. Is net force needed to keep a body moving with uniform velocity?
2. Is Newton's 2^{nd} law (F = ma) always valid. Give an example in support of your answer?
3. Action and reaction forces do not balance each other. Why?
4. Can a body remain in state of rest if more than one force is acting upon it?
5. Is the centripetal force acting on a body performing uniform circular motion always constant?
6. The string is holding the maximum possible weight that it could withstand. What will happen to the string if the body suspended by it starts moving on a horizontal circular path and the string starts generating a cone?
7. What is the reaction force of the weight of a book placed on the table?
8. What is the maximum acceleration of a vehicle on the horizontal road? Given that coefficient of static friction between the road and the tyres of the vehicle is μ.
9. Why guns are provided with the shoulder support?
10. While paddling a bicycle what are the types of friction acting on rear wheels and in which direction?

Answer

1. No.
2. It is valid in an inertial frame of reference. In non-inertial frame of reference (such as a car moving along a circular path), Newton's 2^{nd} law doesn't hold apparently.
3. Since they are acting on different bodies.
4. Yes, if all the forces acting on it are in equilibrium.
5. No, only its magnitude remains constant but its direction continuously goes on changing.
6. It will break because tension in the string increases as soon as the body starts moving.
7. The force with which the book attracts the earth towards it.
8. $a_{max} = fs(max)/m = \mu N/m = \mu mg/m = \mu g$.
9. So that the recoil of gun may be reduced by providing support to the gun by the shoulders.
10. Static friction in forward direction and rolling friction in backward direction.

Short Answer Type 2 Marks Questions

1. Explain why the water doesn't fall even at the top of the circle when the bucket full of water is upside down rotating in a vertical circle?
2. The displacement of a particle of mass 1kg is described by $s = 2t + 3t^2$. Find the force acting on particle? (F = 6N)
3. A particle of mass 0.3 kg is subjected to a force of F = -kx with k = 15 Nm^{-1}. What will be its initial acceleration if it is released from a point 10 cm away from the origin? ($a = -5\ ms^{-2}$)
4. Three forces F_1, F_2 and F_3 are acting on the particle of mass m which is stationary. If F_1 is removed, what will be the acceleration of particle? ($a = F_1/m$)

5. A spring balance is attached to the ceiling of a lift. When the lift is at rest spring balance reads 50 kg of a body hanging on it. What will be the reading of the balance if the lift moves :-

(i) Vertically downward with an acceleration of 5 ms^{-2}

(ii) Vertically upward with an acceleration of 5 ms^{-2}

(iii) Vertically upward with a constant velocity.

Take g = 10m/s^2. [(i) 25kgf,(ii) 75kgf, (iii) 50kgf]

6. Is larger surface area break on a bicycle wheel more effective than small surface area brake? Explain?

7. Calculate the impulse necessary to stop a 1500 kg car moving at a speed of 25ms^{-1}? (–37500 N-s)

8.Give the magnitude and directions of the net force acting on a rain drop falling freely with a constant speed of 5 m/s? (F_{net} = 0)

9. A block of mass .5kg rests on a smooth horizontal table. What steady force is required to give the block a velocity of 2 m/s in 4 s? (F= .25N)

10. Calculate the force required to move a train of 200 quintal up on an incline plane of 1 in 50 with an acceleration of 2 ms^{-2}. The force of friction per quintal is 0.5 N? (F = 44100N)

Short Answer Type 3 Marks Questions

1. A bullet of mass 0.02 kg is moving with a speed of 10m^{-1}s. It penetrates 10 cm of a wooden block before coming to rest. If the thickness of the target is reduced to 6 cm only find the KE of the bullet when it comes out? (**Ans :** 0.4 J)

2. A man pulls a lawn roller with a force of F. If he applies the force at some angle with the ground. Find the minimum force required to pull the roller if coefficient of static friction between the ground and the roller is μ?

3. A ball bounces to 80% of its original height. Calculate the change in momentum?

4. A pendulum bob of mass 0.1 kg is suspended by a string of 1 m long. The bob is displaced so that the string becomes horizontal and released. Find its kinetic energy when the string makes an angle of (i) 0°, (ii) 30°, (iii) 60° with the vertical?

5. The velocity of a particle moving along a circle of radius R depends on the distance covered s as F = 2αs where α is constant. Find the force acting on the particle as a function of s?

6. A block is projected horizontally on rough horizontal floor with initial velocity u. The coefficient of kinetic friction between the block and the floor is μ. Find the distance travelled by the body before coming to rest?

7. A locomotive of mass m starts moving so that its velocity v changes according to v = $\sqrt{(\alpha\ s)}$, where α is constant and s is distance covered. Find the force acting on the body after time t?

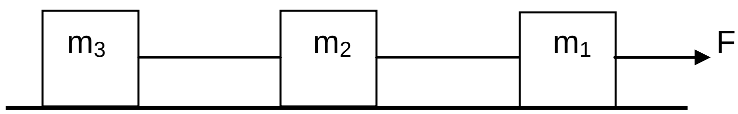
8. Derive an expression for the centripetal force?

9. Find the maximum value of angle of friction and prove that it is equal to the angle of repose?

10. State and prove Lami's theorem?

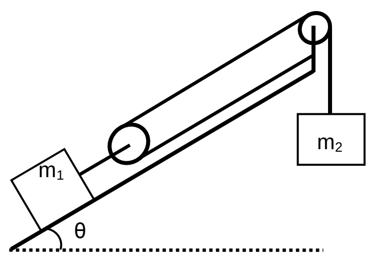
Long Answer Type 5 Marks Questions

1. Find the maximum and minimum velocity of a vehicle of mass m on a banked road of banking angle θ, if coefficient of static friction of the wheels of vehicle with the road is μ?
2. Find the maximum and minimum force applied parallel up the incline on a block of mass m placed on it if angle of inclination is θ and coefficient of static friction with the block is μ so that the block remains at rest?
3. Prove that in case of vertical circular motion circular motion of a body tied to a string velocities at topmost and lowermost point be √(rg) and √(5rg) respectively and tensions in the strings be 0 and 6mg respectively?
4. Find the maximum horizontal velocity that must be imparted to a body placed on the top of a smooth sphere of radius r so that it may not loose contact? If the same body is imparted half the velocity obtained in the first part then find the angular displacement of the body over the smooth sphere when it just loses contact with it?
5. Find the acceleration of the blocks and the tension in the strings?

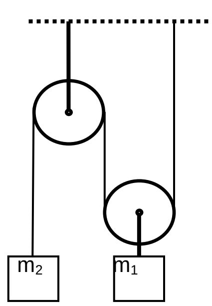


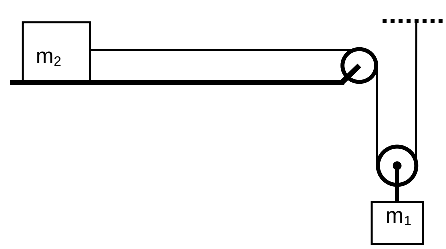
Some Intellectual Stuff

1. Find the acceleration of the blocks m_1 and m_2. All the surfaces are smooth and string and pulley are light? Also find the net force on the clamped pulley?



2. A body of mass m explodes into three fragments of with masses in the ratio 2:2:6. If the two similar masses move of perpendicular to each other with the speed of 10m/s each, find the velocity of the third particle and its direction relative to the two other bodies?
3. A mass of 5 kg is suspended by a rope of length 2m from the ceiling. A horizontal force of 50 N is applied at the mid point P of the rope? Calculate the angle that the rope makes with the vertical and the tension in the part of the rope between the point of suspension and point P?. Neglect the mass of the rope. (g = $10ms^{-2}$)
4. A body moving inside a smooth vertical circular track is imparted a velocity of √(4rg) at the lowermost point. Find its position where it just loses contact with the track?
5.

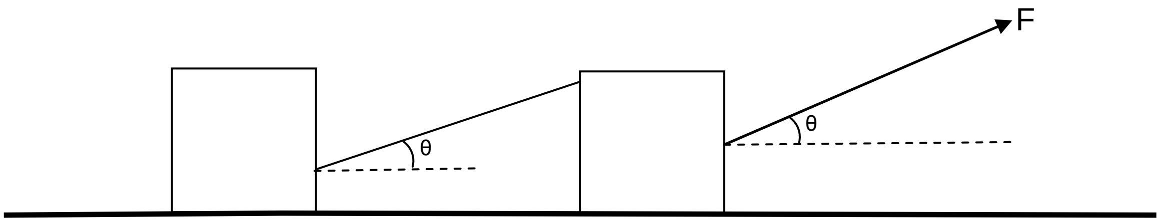




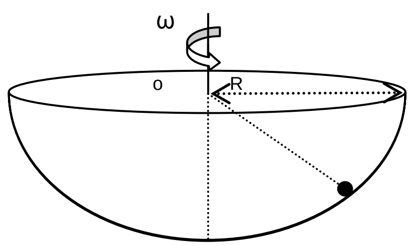
Find in both the cases

(i)Acceleration of the two blocks.(ii)Tension in the clamp holding the fixed pulley?

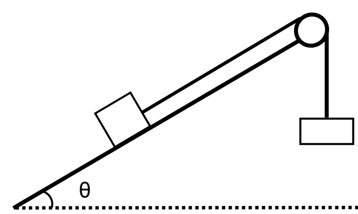
6. Mass of both the blocks is m and coefficient of kinetic friction with the ground is μ. Find the acceleration of the two blocks and tension in the string attached between the two blocks?



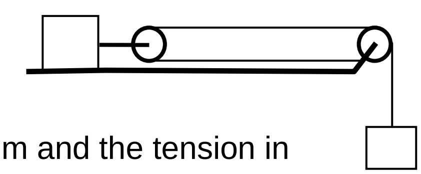
7. A small sphere of mass m is placed in a hemispherical bowl of radius R. Bowl is rotated with angular velocity ω. Find the angle made by the radius of the bowl passing through the sphere with the vertical when the sphere starts rotating with the bowl?



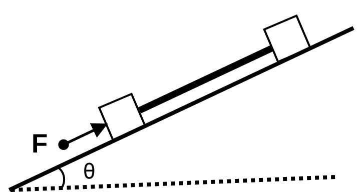
8. Mass of both the blocks is m find net force on the pulley?



9. Mass of both the blocks is m find acceleration of both the blocks and net force on the clamp holding the fixed pulley?



10. Mass of both the blocks is m find acceleration of the system and the tension in the rod?



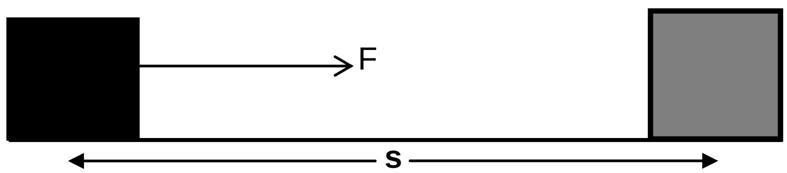
WORK ENERGY AND POWER

WORK

PHYSICAL DEFINITION

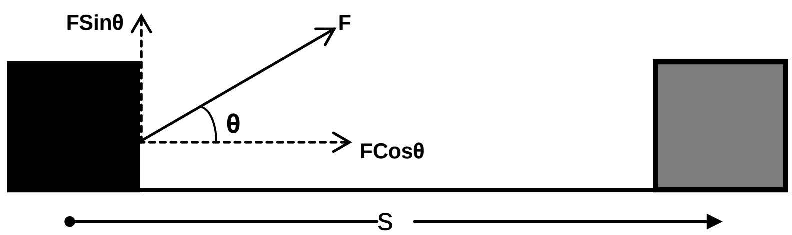
When the point of application of force moves in the direction of the applied force under its effect then work is said to be done.

MATHEMATICAL DEFINITION OF WORK



Work is defined as the product of force and displacement in the direction of force

$$\mathbf{W = F \times s}$$

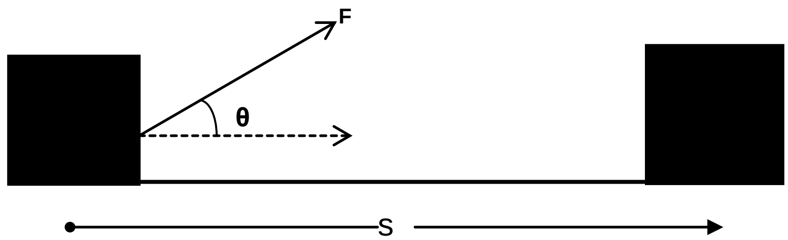


If force and displacement are not parallel to each other rather they are inclined at an angle, then in the evaluation of work component of force (F) in the direction of displacement (s) will be considered.

$$W = (F\cos\theta) \times s$$

or, $$\mathbf{W = Fs\cos\theta}$$

VECTOR DEFINITION OF WORK



Force and displacement both are vector quantities but their product, work is a scalar quantity, hence work must be scalar product or dot product of force and displacement vector.

$$\mathbf{W = \vec{F} . \vec{s}}$$

WORK DONE BY VARIABLE FORCE

Force varying with displacement

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

$$dW = \vec{F} . \vec{ds}$$

$$\mathbf{W = \int_{s_1}^{s_2} \vec{F} . \vec{ds}}$$

Force varying with time

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated.

$$dW = \vec{F} . \vec{ds}$$

Multiplying and dividing by dt,

$$dW = \frac{\vec{F} . \vec{ds}\ dt}{dt}$$

or,

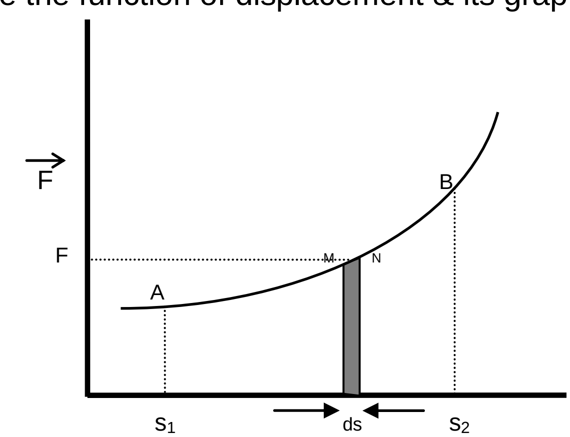
$$dW = \vec{F} . \vec{v}\ dt \quad (v=ds/dt)$$

Total work is obtained by integrating the elementary work from initial to final limits.

$$\mathbf{W = \int_{t_1}^{t_2} \vec{F} . \vec{v}\, dt}$$

WORK DONE BY VARIABLE FORCE FROM GRAPH

Let force be the function of displacement & its graph be as shown.



To find work done from s_1 to s_2 we consider two points M & N very close on the graph such that magnitude of force (F) is almost same at both the points. If elementary displacement from M to N is ds, then elementary work done from M to N is.

$$dW = F.ds$$
$$dW = \text{(length x breadth)of strip MNds}$$
$$dW = \text{Area of strip MNds}$$

Thus work done in any part of the graph is equal to area under that part. Hence total work done from s_1 to s_2 will be given by the area enclosed under the graph from s_1 to s_2.

$$\mathbf{W = Area\ (ABS_2S_1A)}$$

DIFFERENT CASES OF WORK DONE BY CONSTANT FORCE

Case i) Force and displacement are in same direction

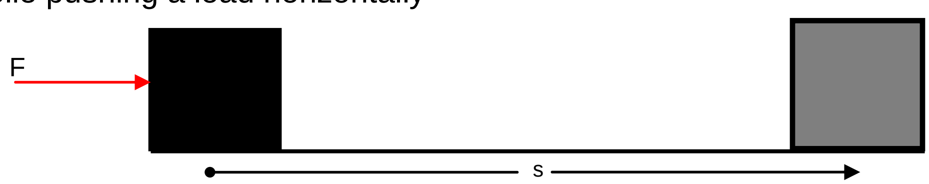
$\theta = 0$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 0$

or, $\mathbf{W = Fs}$

Ex - Coolie pushing a load horizontally



Case ii) Force and displacement are mutually perpendicular to each other

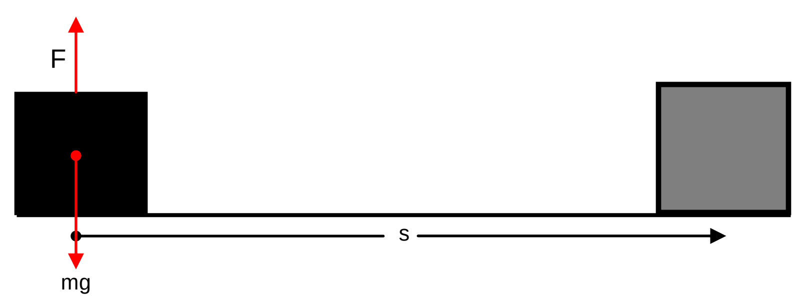
$\theta = 90$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 90$

or, $\mathbf{W = 0}$

Ex - coolie carrying a load on his head & moving horizontally with constant velocity. Then he applies force vertically to balance weight of body & its displacement is horizontal.



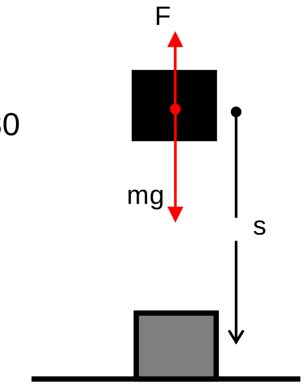
(3) Force & displacement are in opposite direction

$\theta = 180$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 180$

or, $\mathbf{W = - Fs}$



Ex - Coolie carrying a load on his head & moving vertically down with constant velocity. Then he applies force in vertically upward direction to balance the weight of body & its displacement is in vertically downward direction.

ENERGY

Capacity of doing work by a body is known as energy.

Note - Energy possessed by the body by virtue of any cause is equal to the total work done by the body when the cause responsible for energy becomes completely extinct.

TYPES OF ENERGIES

There are many types of energies like mechanical energy, electrical, magnetic, nuclear, solar, chemical etc.

MECHANICAL ENERGY

Energy possessed by the body by virtue of which it performs some mechanical work is known as mechanical energy.

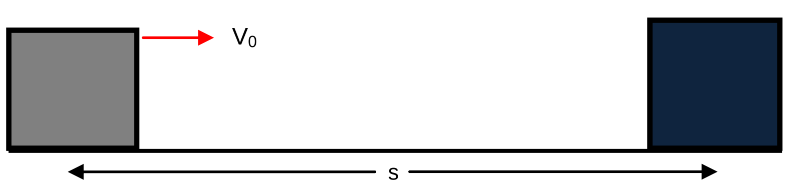
It is of basically two types-

(i) Kinetic energy

(ii) Potential energy

KINETIC ENERGY

Energy possessed by body due to virtue of its motion is known as the kinetic energy of the body. Kinetic energy possessed by moving body is equal to total work done by the body just before coming out to rest.



Consider a body of man (m) moving with velocity (v_o).After travelling through distance (s) it comes to rest.

$$u = v_o$$
$$v = 0$$
$$s = s$$

Applying, $v^2 = u^2 + 2as$

$$0 = v_0^2 + 2as$$

or, $2as = - v_0^2$

or, $a = \frac{-v_o^2}{2s}$

Hence force acting on the body,

$$F = ma$$
$$F_{on\ body} = - \frac{mv_o^2}{2s}$$

But from Newton's third law of action and reaction, force applied by body is equal and opposite to the force applied on body

$$F_{by\ body} = -F_{on\ body}$$
$$= +\frac{mv_o^2}{2s}$$

Therefore work done by body,

$$W = \vec{F}.\vec{s}$$

or, $W = \frac{mv_0^2}{2s}.s.Cos\ 0$

or, $W = \frac{1}{2} mv_o^2$

Thus K.E. stored in the body is,

$$\mathbf{K.E.= \frac{1}{2} mv_o^2}$$

KINETIC ENERGY IN TERMS OF MOMENTUM

K.E. of body moving with velocity v is

$$K.E. = \frac{1}{2} mv_o^2$$

Multiplying and dividing by m

$$K = \frac{1}{2}\frac{mv^2 \times m}{m}$$
$$= \frac{1}{2}\frac{m^2v^2}{m}$$

But, mv = p (linear momentum)

Therefore, $\mathbf{K = \frac{p^2}{2m}}$

POTENTIAL ENERGY

Energy possessed by the body by virtue of its position or state is known as potential energy. Example:- gravitational potential energy, elastic potential energy, electrostatic potential energy etc.

GRAVITATIONAL POTENTIAL ENERGY

Energy possessed by a body by virtue of its height above surface of earth is known as gravitational potential energy. It is equal to the work done by the body situated at some height in returning back slowly to the surface of earth.

Consider a body of mass m situated at height h above the surface of earth. Force applied by the body in vertically downward direction is

$$F = mg$$

Displacement of the body in coming back slowly to the surface of earth is

$$s = h$$

Hence work done by the body is

$$W = FsCos\theta$$

or, $$W = FsCos0$$

or, $$W = mgh$$

This work was stored in the body in the form of gravitational potential energy due to its position. Therefore

$$\mathbf{G.P.E = mgh}$$

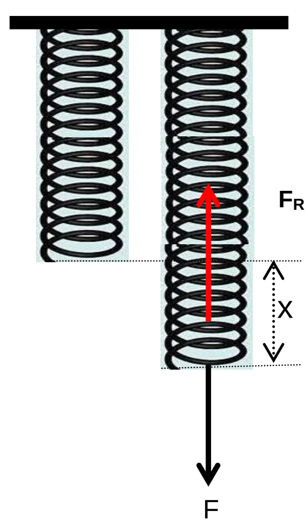
ELASTIC POTENTIAL ENERGY

Energy possessed by the spring by virtue of compression or expansion against elastic force in the spring is known as elastic potential energy.

Spring

It is a coiled structure made up of elastic material & is capable of applying restoring force & restoring torque when disturbed from its original state. When force (F) is applied at one end of the string, parallel to its length, keeping the other end fixed, then the spring expands (or contracts) & develops a restoring force (F_R) which balances the applied force in equilibrium.

On increasing applied force spring further expands in order to increase restoring force for balancing the applied force. Thus restoring force developed within the spring is directed proportional to the extension produced in the spring.



$$F_R \propto x$$

or, $\mathbf{F_R = kx}$ (k is known as spring constant or force constant)

If $x = 1$, $F_R = k$

Hence force constant of string may be defined as the restoring force developed within spring when its length is changed by unity.

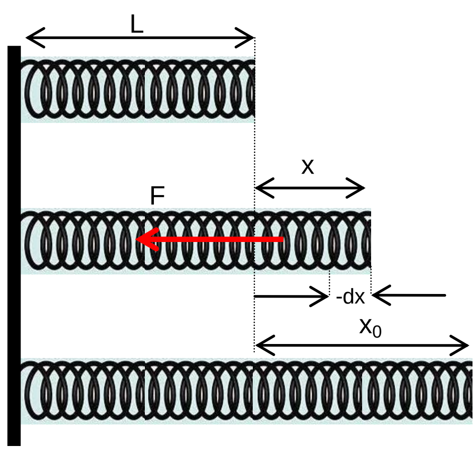
But in equilibrium, restoring force balances applied force.

$$\mathbf{F = F_R = k\,x}$$

If $x = 1$, $F = 1$

Hence force constant of string may also be defined as the force required to change its length by unity in equilibrium.

Mathematical Expression for Elastic Potential Energy



Consider a spring of natural length 'L' & spring constant 'k' its length is increased by x_o. Elastic potential energy of stretched spring will be equal to total work done by the spring in regaining its original length.

If in the process of regaining its natural length, at any instant extension in the spring was x then force applied by spring is

$$F = kx$$

If spring normalizes its length by elementary distance dx opposite to x under this force then work done by spring is

$$dW = F.\,(-dx)\,.\,\mathrm{Cos}0$$

(force applied by spring F and displacement –dx taken opposite to extension x are in same direction)

$$dW = -kxdx$$

Total work done by the spring in regaining its original length is obtained in integrating dW from x_0 to 0

$$W = \int_{x_0}^{0} -kxdx$$

or, $$W = -k[x^2/2]_{0}^{x_0}$$

or, $$W = -k\,(0^2/2 - x_0^2/2)$$

o r, $$W = -k\,(0 - x_0^2/2)$$

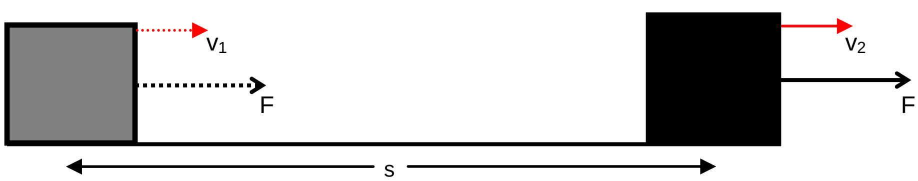
or, $$W = \frac{1}{2}kx_o^2$$

This work was stored in the body in the form of elastic potential energy.

$$\mathbf{E.P.E = \frac{1}{2}kx_o^2}$$

WORK ENERGY THEOREM

It states that total work done on the body is equal to the change in kinetic energy.(Provided body is confined to move horizontally and no dissipating forces are operating).



Consider a body of man m moving with initial velocity v_1. After travelling through displacement s its final velocity becomes v_2 under the effect of force F.

$$u = v_1$$
$$v = v_2$$
$$s = s$$

Applying, $$v^2 = u^2 + 2as$$
$$v_2^2 = v_1^2 + 2as$$

or, $$2as = v_2^2 - v_1^2$$

or, $$a = \frac{v_2^2 - v_1^2}{2s}$$

Hence external force acting on the body is

$$F = ma$$
$$F = m\frac{v_2^2 - v_1^2}{2s}$$

Therefore work done on body by external force

$$W = \vec{F}\,.\,\vec{s}$$

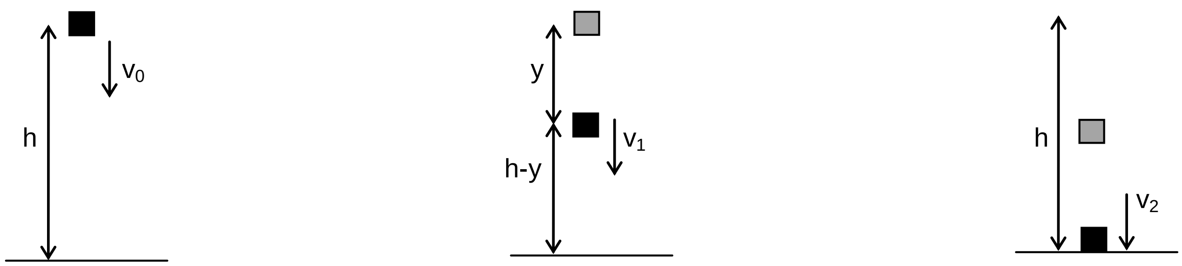
or, $$W = m\frac{v_2^2 - v_1^2}{2s}\,.\,s\,.\text{Cos } 0$$

(since force and displacement are in same direction) or, $W = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$

or, $\mathbf{W = K_2 – K_1}$

or, $\mathbf{W = \Delta K}$

PRINCIPLE OF CONSERVATION OF ENERGY



It states that energy can neither be creased neither be destroyed. It can only be converted from one form to another.
Consider a body of man m situated at height h & moving with velocity v_o. Its energy will be.

$$E_1 = P_1 + K_1$$

or, $\mathbf{E_1 = mgh + ½ mv_o^2}$

If the body falls under gravity through distance y, then it acquires velocity v_1 and its height becomes (h-y)

$u = v_o$
$s = y$
$a = g$
$v = v_1$
From $v^2 = u^2 + 2as$
$v_1^2 = v_o^2 + 2gy$
Energy of body in second situation
$E_2 = P_2 + K_2$
or, $E_2 = mg (h-y) + ½ mv^2$
or, $E_2 = mg (h-y) + ½ m (v_o^2 + 2gy)$
or, $E_2 = mgh - mgy + ½ mv_o^2 + mgy$

or, $\mathbf{E_2 = mgh + ½ mv_o^2}$

Now we consider the situation when body reaches ground with velocity v_2
$u = v_o$
$s = h$
$a = g$
$v = v_2$

From $v^2 = u^2 + 2as$

$$_2^2 = v_o^2 + 2gh$$

Energy of body in third situation

$$E_3 = P_3 + K_3$$

or, $E_3 = mg0 + ½\, mv_2^2$

or, $E_3 = 0 + ½\, m\,(v_o^2 + 2gh)$

or, $\mathbf{E_3 = ½\, mv_o^2 + mgh}$

From above it must be clear that $\mathbf{E_1 = E_2 = E_3}$. This proves the law of conservation of energy.

CONSERVATIVE FORCE

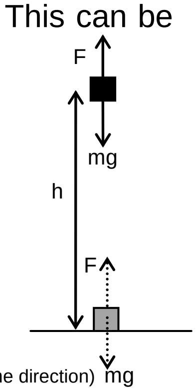
Forces are said to be conservative in nature if work done against the forces gets conversed in the body in form of potential energy. Example:- gravitational forces, elastic forces & all the central forces.

PROPERTIES OF CONSERVATIVE FORCES

1. Work done against these forces is conserved & gets stored in the body in the form of P.E.
2. Work done against these forces is never dissipated by being converted into non-usable forms of energy like heat, light, sound etc.
3. Work done against conservative forces is a state function & not path function i.e. Work done against it, depends only upon initial & final states of body & is independent of the path through which process has been carried out.
4. Work done against conservative forces is zero in a complete cycle.

TO PROVE WORK DONE AGAINST CONSERVATIVE FORCES IS A STATE FUNCTION

Consider a body of man m which is required to be lifted up to height h. This can be done in 2 ways.
(i) By directly lifting the body against gravity
(ii) By pushing the body up a smooth inclined plane.



Min force required to lift the body of mass m vertically is

$$F = mg$$

And displacement of body in lifting is

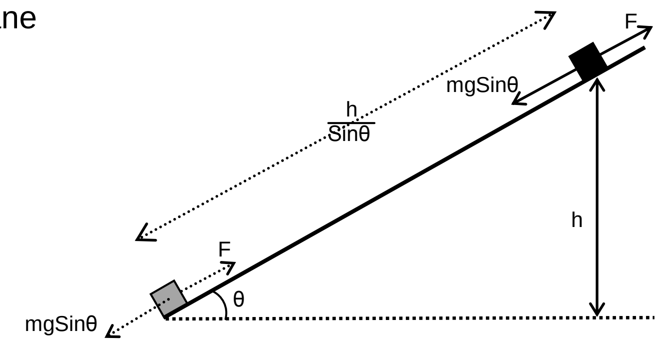
$$s = h$$

Hence work done in lifting is

$W_1 = FsCos0^o$ (since force and displacement are in same direction)

$$\mathbf{W_1 = mgh}$$

Now we consider the same body lifted through height h by pushing it up a smooth inclined plane



Min force required to push the body is

$$F = mgSin\theta$$

And displacement of body in lifting is

$$s = \frac{h}{Sin\theta}$$

Hence work done in pushing is

$$W_2 = FsCos0$$

or, $$W_2 = mgSin\theta \cdot \frac{h}{Sin\theta} \cdot 1$$

or, $$\mathbf{W_2 = mgh}$$

From above $W_1 = W_2$ we can say that in both the cases work done in lifting the body through height 'h' is same.

To Prove That Work Done Against Conservative Forces Is Zero In A Complete Cycle

Consider a body of man m which is lifted slowly through height h & then allowed to come back to the ground slowly through height h.

For work done is slowly lifting the body up,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical up displacement of the body is

$$s = h$$

Hence work done is

$$W = FsCos\theta$$

or, $W_I = FsCos0$ (since force and displacement are in same direction)

or, $\mathbf{W_I = mgh}$ (since force and displacement are in same direction)

For work done is slowly bringing the body down,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical down displacement of the body is

$$s = h$$

Hence work done is

or, $W_2 = FsCos180$(since force and displacement are in opposite direction)

or, $\mathbf{W_2 = - mgh}$

Hence total work done against conservative forces in a complete cycle is

$$W = W_1 + W_2$$

or, $W = (mgh) + (-mgh)$

or, $\mathbf{W = 0}$

NON-CONSERVATIVE FORCES

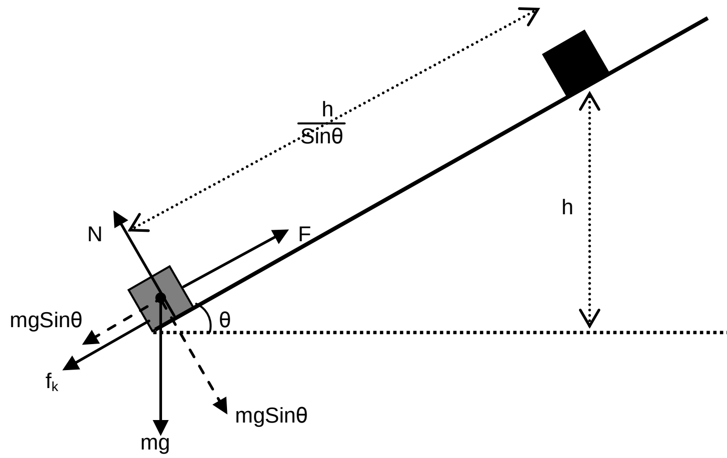
Non conservative forces are the forces, work done against which does not get conserved in the body in the form of potential energy.

PROPERTIES OF NON-CONSERVATIVE FORCES

1. Work done against these forces does not get conserved in the body in the form of P.E.
2. Work done against these forces is always dissipated by being converted into non usable forms of energy like heat, light, sound etc.
3. Work done against non-conservative force is a path function and not a state function.
4. Work done against non-conservative force in a complete cycle is not zero.

PROVE THAT WORK DONE AGAINST NON–CONSERVATIVE FORCES IS A PATH FUNCTION

Consider a body of mass (m) which is required to be lifted to height 'h' by pushing it up the rough incline of inclination.



Minimum force required to slide the body up the rough inclined plane having coefficient of kinetic friction μ with the body is

$$F = mgSin\theta + f_k$$

or, $$F = mgSin\theta + \mu N$$

or, $$F = mgSin\theta + \mu mgCos\theta$$

Displacement of the body over the incline in moving through height h is

$$s = \frac{h}{Sin\theta}$$

Hence work done in moving the body up the incline is

$$W = F.s.Cos0 \text{(since force and displacement are in opposite direction)}$$

or, $$W = (mgSin\theta + \mu mgCos\theta).\ \frac{h}{Sin\theta}.1$$

or, $$\mathbf{W = mgh + \frac{\mu mgh}{Tan\theta}}$$

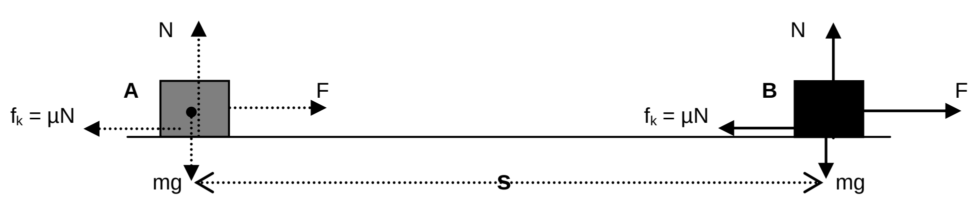
Similarly if we change the angle of inclination from θ to θ_1, then work done will be

$$\mathbf{W_1 = mgh + \frac{\mu mgh}{Tan\theta_1}}$$

This clearly shows that work done in both the cases is different & hence work done against non-conservative force in a path function and not a state function i.e. it not only depends upon initial & final states of body but also depends upon the path through which process has been carried out.

To Prove That Work Done Against Non-conservative Forces In A Complete Cycle Is Not Zero

Consider a body displaced slowly on a rough horizontal plane through displacement s from A to B.



Minimum force required to move the body is

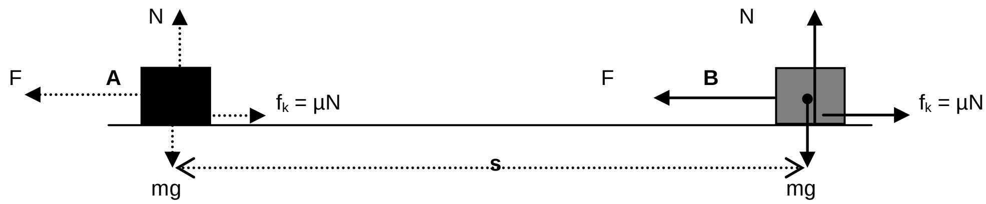
$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0 \text{(since force and displacement are in same direction)}$$

or, $$\mathbf{W = \mu mgs}$$

Now if the same body is returned back from B to A



Minimum force required to move the body is

$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0 \text{(since force and displacement are in same direction)}$$

or, $$\mathbf{W = \mu mgs}$$

Hence total work done in the complete process

$$\mathbf{W = W_1 + W_2 = 2\mu mgs}$$

Note - When body is returned from B to A friction reverse its direction.

POWER

Rate of doing work by a body with respect to time is known as power.

Average Power

It is defined as the ratio of total work done by the body to total time taken.

$$\mathbf{P_{avg} = \frac{Total\ work\ done}{Total\ time\ taken} = \frac{\Delta W}{\Delta t}}$$

Instantaneous Power

Power developed within the body at any particular instant of time is known as instantaneous power.

Or

Average power evaluated for very short duration of time is known as instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} P_{avg}$$

or, $$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$P_{inst} = \frac{dW}{dt}$$

or, $$P_{inst} = \frac{d\vec{F} . \vec{s}}{dt}$$

or, $$P_{inst} = \vec{F} . \frac{d\vec{s}}{dt}$$

or, $$\mathbf{P_{inst} = \vec{F} . \vec{v}}$$

EFFICIENCY

It is defined as the ratio of power output to power input.

Or

It is defined as the ratio of energy output to energy input.

Or

I It is defined as the ratio of work output to work input.

$$\eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}}$$

PERCENTAGE EFFICIENCY

Percentage Efficiency = Efficiency x 100

$$\textbf{Percentage Efficiency} = \eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}} \times 100$$

COLLISION

Collision between the two bodies is defined as mutual interaction of the bodies for a short interval of time due to which the energy and the momentum of the interacting bodies change.

Types of Collision

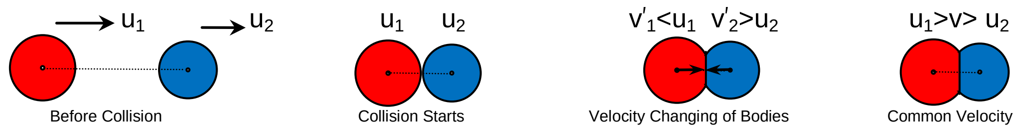
There are basically three types of collisions-

i) Elastic Collision – That is the collision between perfectly elastic bodies. In this type of collision, since only conservative forces are operating between the interacting bodies, both kinetic energy and momentum of the system remains constant.

ii) Inelastic Collision – That is the collision between perfectly inelastic or plastic bodies. After collision bodies stick together and move with some common velocity. In this type of collision only momentum is conserved. Kinetic energy is not conserved due to the presence of non-conservative forces between the interacting bodies.

iii) Partially Elastic or Partially Inelastic Collision – That is the collision between the partially elastic bodies. In this type of collision bodies do separate from each other after collision but due to the involvement of non-conservative inelastic forces kinetic energy of the system is not conserved and only momentum is conserved.

Collision In One Dimension – Analytical Treatment



Consider two bodies of masses m_1 and m_2 with their center of masses moving along the same straight line in same direction with initial velocities u_1 and u_2 with m_1 after m_2. Condition necessary for the collision is $u_1 > u_2$ due to which bodies start approaching towards each other with the velocity of approach $u_1 - u_2$. Collision starts as soon as the bodies come in contact. Due to its greater velocity and inertia m_1 continues to push m_2 in the forward direction whereas m_2 due to its small velocity and inertia pushes m_1 in the backward direction. Due to this pushing force involved between the two colliding bodies they get deformed at the point of contact and a part of their kinetic energy gets consumed in the deformation of the bodies. Also m_1 being pushed opposite to the direction of the motion goes on decreasing its velocity and m_2 being pushed in the direction of motion continues increasing its velocity. This process continues until both the bodies acquire the same common velocity v. Up to this stage there is maximum deformation in the bodies maximum part of their kinetic energy gets consumed in their deformation.

Elastic collision

In case of elastic collision bodies are perfectly elastic. Hence after their maximum deformation they have tendency to regain their original shapes, due to which they start pushing each other. Since m_2 is being pushed in the direction of motion its velocity goes on increasing and m_1 being pushed opposite to the direction of motion its velocity goes on decreasing. Thus condition necessary for separation i.e. $\mathbf{v_2 > v_1}$ is attained and the bodies get separated with velocity of separation $\mathbf{v_2 - v_1}$.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is again returned back to the system when the bodies regain their original shapes. Hence in such collision energy conservation can also be applied along with the momentum conservation.

Applying energy conservation

$$E_i = E_f$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ ....(i)}$$

Applying momentum conservation

$$p_i = p_f$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ....(ii)}$$

Dividing equation (i) by (ii)

$$u_1 + v_1 = v_2 + u_2$$

or, $$\mathbf{v_2 - v_1 = u_1 - u_2}$$

or, **Velocity of separation = Velocity of approach**

or, $$v_2 = v_1 + u_1 - u_2$$

Putting this in equation (i)

$$\mathbf{v_1 = \frac{(m_1-m_2)u_1}{(m_1+m_2)} + \frac{2m_2\,u_2}{(m_1+m_2)}}$$

Similarly we can prove

$$\mathbf{v_2 = \frac{(m_2-m_1)u_2}{(m_1+m_2)} + \frac{2m_1\,u_1}{(m_1+m_2)}}$$

Case 1- If the bodies are of same mass,

$$m_1 = m_2 = m$$

$$\mathbf{v_1 = u_2}$$

$$\mathbf{v_2 = u_1}$$

Hence in perfectly elastic collision between two bodies of same mass, the velocities interchange.ie. If a moving body elastically collides with a similar body at rest. Then the moving body comes at rest and the body at rest starts moving with the velocity of the moving body.

Case 2- If a huge body elastically collides with the small body,

$m_1 >> m_2$

m_2 will be neglected in comparison to m_1

$$v_1 = \frac{(m_1-0)u_1}{(m_1+0)} + \frac{2.0.\,u_2}{(m_1+0)}$$

$$\mathbf{v_1 = u_1}$$

and

$$v_2 = \frac{(0-m_1)u_2}{(m_1+0)} + \frac{2m_1\,u_1}{(m_1+0)}$$

$$\mathbf{v_2 = -u_2 + 2u_1}$$

If, $u_2 = 0$

$$\mathbf{v_2 = 2u_1}$$

Hence if a huge body elastically collides with a small body then there is almost no change in the velocity of the huge body but if the small body is initially at rest it gets thrown away with twice the velocity of the huge moving body.eg. collision of truck with a drum.

Case 3- If a small body elastically collides with a huge body,

$m_2 >> m_1$

m_1 will be neglected in comparison to m_2

$$v_1 = \frac{(0-m_2)u_1}{(0+m_2)} + \frac{2m_2\,u_2}{(0+m_2)}$$

or, $\mathbf{v_1 = -u_1 + 2u_2}$

If $u_2 = 0$

$$\mathbf{v_1 = -u_1}$$

and

$$v_2 = \frac{(m_2-0)u_2}{(0+m_2)} + \frac{2.0.u_1}{(0+m_2)}$$

$$\mathbf{v_2 = u_2}$$

Hence if a small body elastically collides with a huge body at rest then there is almost no change in the velocity of the huge body but if the huge body is initially at rest small body rebounds back with the same speed.eg. collision of a ball with a wall.

Inelastic collision

In case of inelastic collision bodies are perfectly inelastic. Hence after their maximum deformation they have no tendency to regain their original shapes, due to which they continue moving with the same common velocity.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is permanently consumed in the deformation of the bodies against non-conservative inelastic forces. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

Applying momentum conservation

$$p_i = p_f$$

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

or, $m_1u_1 + m_2u_2 = (m_1+m_2)v$

or, $$\mathbf{v = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)}}$$

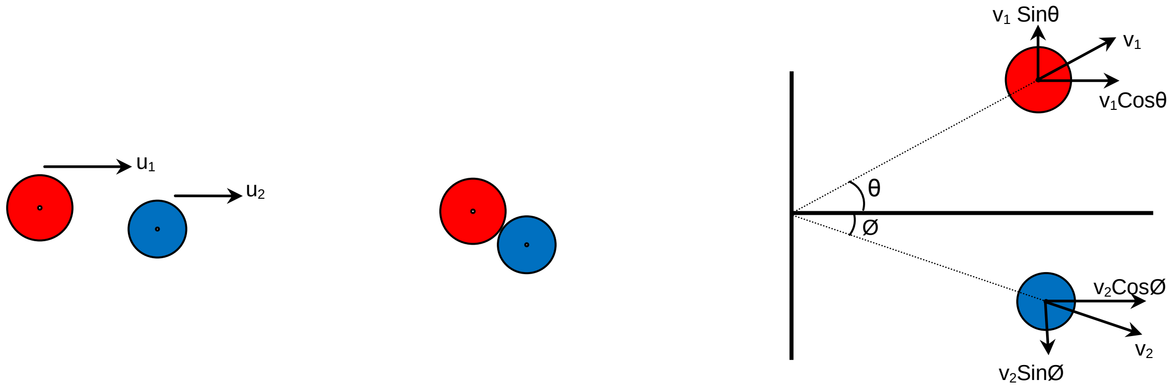
Partially Elastic or Partially Inelastic Collision

In this case bodies are partially elastic. Hence after their maximum deformation they have tendency to regain their original shapes but not as much as perfectly elastic bodies. Hence they do separate but their velocity of separation is not as much as in the case of perfectly elastic bodies i.e. their velocity of separation is less than the velocity of approach.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is only slightly returned back to the system. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

$$\mathbf{(v_2 - v_1) < (u_1 - u_2)}$$

Collision In Two Dimension – Oblique Collision



Before Collision | Collision Starts | After Collision

When the centers of mass of two bodies are not along the same straight line, the collision is said to be oblique. In such condition after collision bodies are deflected at some angle with the initial direction. In this type of collision momentum conservation is applied separately along x-axis and y-axis. If the collision is perfectly elastic energy conservation is also applied.

Let initial velocities of the masses m_1 and m_2 be u_1 and u_2 respectively along x-axis. After collision they are deflected at angles θ and Ø respectively from x-axis, on its either side of the x axis.

Applying momentum conservation along x-axis

$$p_f = p_i$$

$$\mathbf{m_1 v_1 \cos\theta + m_2 v_2 \cos\text{Ø} = m_1 u_1 + m_2 u_2}$$

Applying momentum conservation along y-axis

$$p_f = p_i$$

$$m_1 v_1 \sin\theta - m_2 v_2 \sin\text{Ø} = m_1 0 + m_2 0$$

or, $$m_1 v_1 \sin\theta - m_2 v_2 \sin\text{Ø} = 0$$

or, $$\mathbf{m_1 v_1 \sin\theta = m_2 v_2 \sin\text{Ø}}$$

In case of elastic collision applying energy conservation can also be applied

$$K_f = K_i$$

$$\mathbf{\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$$

Coefficient Of Restitution

It is defined as the ratio of velocity of separation to the velocity of approach.

$$\mathbf{e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}}$$

or, $$\mathbf{e = \frac{(v_2 - v_1)}{(u_1 - u_2)}}$$

Case-1 For perfectly elastic collision, velocity of separation is equal to velocity of approach, therefore

$$\mathbf{e = 1}$$

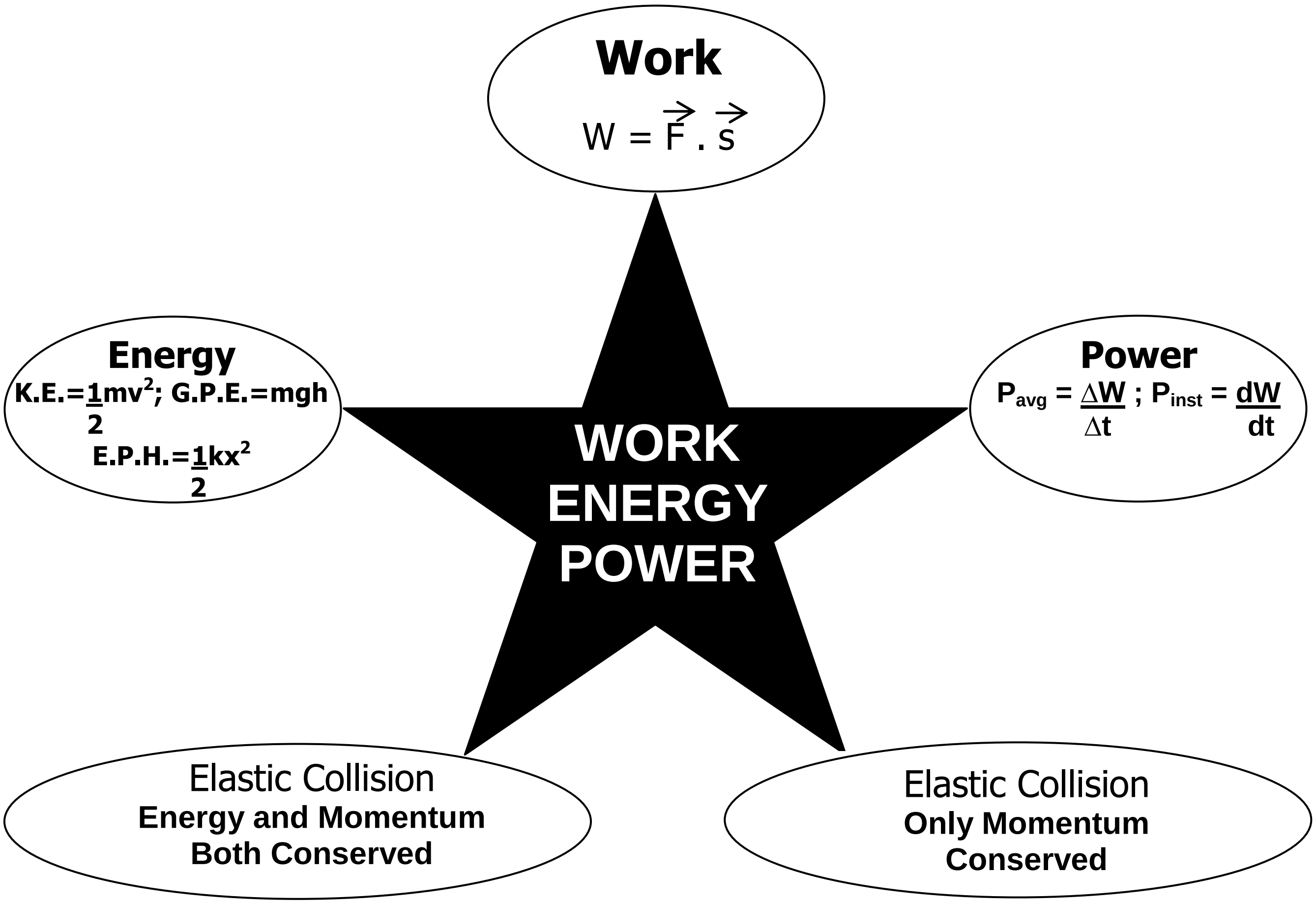
Case-2 For perfectly inelastic collision, velocity of separation is zero, therefore

$$\mathbf{e = 0}$$

Case-3 For partially elastic or partially inelastic collision, velocity of separation is less than velocity of approach, therefore

$$\mathbf{e < 1}$$

MEMORY MAP



Work
$W = \vec{F} \cdot \vec{s}$
Energy
K.E.=$\frac{1}{2}mv^2$; G.P.E.=mgh
E.P.H.=$\frac{1}{2}kx^2$
Power
$P_{avg} = \frac{\Delta W}{\Delta t}$; $P_{inst} = \frac{dW}{dt}$
WORK
ENERGY
POWER
Elastic Collision
Energy and Momentum
Both Conserved
Elastic Collision
Only Momentum
Conserved

Very Short Answer Type 1 Mark Questions

1. Define the conservative and non-conservative forces? Give example of each?

2. A light body and a heavy body have same linear momentum. Which one has greater K.E? (Ans: Lighter body has more K.E.)

3.If the momentum of the body is doubled by what percentage does its K.E changes? (300%)

4. A truck and a car are moving with the same K.E on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance? (Truck)

5. What happens to the P.E of a bubble when it rises up in water? (decrease)

6. Define spring constant of a spring?

7. What happens when a sphere collides head on elastically with a sphere of same mass initially at rest?

8. Derive an expression for K.E of a body of mass m moving with a velocity v by calculus method.

9. After bullet is fired, gun recoils. Compare the K.E. of bullet and the gun. (K.E. of bullet > K.E. of gun)

10. In which type of collision there is maximum loss of energy?

Very Short Answer Type 2 Marks Questions

1. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10% what is the speed with which the bob arrives the lowest point? (**Ans :** 6m/s)

2. Find the work done if a particle moves from position $\vec{r_1}$ = (4i + 3j + 6k)m to a position $\vec{r}_2$ = (14i = 13j = 16k) under the effect of force, $\vec{F}$ = (4i + 4j - 4k)N? (**Ans :** 40J)

3. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is stretched further through 0.1m calculate work done? (**Ans :** 4000 Nm–1, 60 J)

4. A pump on the ground floor of a building can pump up water to fill a tank of volume $30m^3$ in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump? The efficiency of the pump is 30%. (**Ans :** 43.556 kW)

5. Spring of a weighing machine is compressed by 1cm when a sand bag of mass 0.1 kg is dropped on it from a height 0.25m. From what height should the sand bag be dropped to cause a compression of 4cm? (**Ans :** 4m)

6. Show that in an elastic one dimensional collision the velocity of approach before collision is equal to velocity of separation after collision?

7. A spring is stretched by distance x by applying a force F. What will be the new force required to stretch the spring by 3x? Calculate the work done in increasing the extension?

8. Write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by length. Draw the graphs to show the variation of potential energy and force with elongation?

9. How does a perfectly inelastic collision differ from perfectly elastic collision? Two particles of mass m_1 and m_2 having velocities u_1 and u_2 respectively make a head on collision. Derive the relation for their final velocities?

10. In lifting a 10 kg weight to a height of 2m, 250 Joule of energy is spent. Calculate the acceleration with which it was raised?($g=10m/s^2$) (**Ans :** $2.5m/s^2$)

Short Answer Type 3 Marks Questions

1. An electrical water pump of 80% efficiency is used to lift water up to a height of 10m.Find mass of water which it could lift in 1hrour if the marked power was 500 watt?
2. A cycle is moving up the incline rising 1 in 100 with a const. velocity of 5m/sec. Find the instantaneous power developed by the cycle?
3. Find % change in K.E of body when its momentum is increased by 50%.
4. A light string passing over a light frictionless pulley is holding masses m and 2m at its either end. Find the velocity attained by the masses after 2 seconds.
5. Derive an expression for the centripetal force experienced by a body performing uniform circular motion.
6. Find the elevation of the outer tracks with respect to inner. So that the train could safely pass through the turn of radius 1km with a speed of 36km/hr. Separation between the tracks is 1.5m?
7. A block of mass m is placed over a smooth wedge of inclination θ. With what horizontal acceleration the wedge should be moved so that the block must remain stationery over it?
8. Involving friction prove that pulling is easier than pushing if both are done at the same angle.
9. In vertical circular motion if velocity at the lowermost point is $\sqrt{(6rg)}$ where find the tension in the string where speed is minimum. Given that mass of the block attached to it is m?
10. A bullet of mass m moving with velocity u penetrates a wooden block of mass M suspended through a string from rigid support and comes to rest inside it. If length of the string is L find the angular deflection of the string.

Long Answer Type 5 Marks Questions

1. What is conservative force? Show that work done against conservative forces is a state function and not a path function. Also show that work done against it in a complete cycle is zero?
2. A body of man 10 kg moving with the velocity of 10m/s impinges the horizontal spring of spring constant 100 Nm^{-1} fixed at one end. Find the maximum compression of the spring? Which type of mechanical energy conversion has occurred? How does the answer in the first part changes when the body is moving on a rough surface?
3. Two blocks of different masses are attached to the two ends of a light string passing over the frictionless and light pully. Prove that the potential energy of the bodies lost during the motion of the blocks is equal to the gain in their kinetic energies?

4. A locomotive of mass m starts moving so that its velocity v is changing according to the law v $\sqrt{(as)}$, where a is constant and s is distance covered. Find the total work done by all the forces acting the locomotive during the first t seconds after the beginning of motion?

5. Derive an expression for the elastic potential energy of the stretched spring of spring constant k. Find the % change in the elastic potential energy of spring if its length is increased by 10%?

Some Intellectual Stuff

1. A body of mass m is placed on a rough horizontal surface having coefficient of static friction μ with the body. Find the minimum force that must be applied on the body so that it may start moving? Find the work done by this force in the horizontal displacement s of the body?

2. Two blocks of same mass m are placed on a smooth horizontal surface with a spring of constant k attached between them. If one of the block is imparted a horizontal velocity v by an impulsive force, find the maximum compression of the spring?

3. A block of mass M is supported against a vertical wall by a spring of constant k. A bullet of mass m moving with horizontal velocity v_0 gets embedded in the block and pushes it against the wall. Find the maximum compression of the spring?

4. Prove that in case of oblique elastic collision of a moving body with a similar body at rest, the two bodies move off perpendicularly after collision?

5. A chain of length L and mass M rests over a sphere of radius R (L < R) with its one end fixed at the top of the sphere. Find the gravitational potential energy of the chain considering the center of the sphere as the zero level of the gravitational potential energy?

MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

CONCEPTS.

.Centre of mass of a body is a point where the entire mass of the body can be supposed to be concentrated

For a system of *n*-particles, the centre of mass is given by

$$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + \ldots\ldots + m_n\vec{r_n}}{m_1 + m_2 + m_3 + \ldots\ldots + m_n} = \frac{\sum_{i=1}^{i=n} m_i\vec{r_1}}{M}$$

.Torque τ *The turning effect of a force with respect to some axis, is called moment of force or torque due to the force.* Torque is measured as the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the axis of rotation.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

.Angular momentum ($\vec{L}$). It is the rotational analogue of linear momentum and is measured as the product of linear momentum and the perpendicular distance of its line of axis of rotation.

Mathematically: If $\vec{P}$ is linear momentum of the particle and $\vec{r}$ its position vector, then angular momentum of the particle, $\vec{L} = \vec{r} \times \vec{P}$

(a)In Cartesian coordinates : $L_Z = xp_y - yp_x$

(b)In polar coordinates : $L = r\ p\ sin\emptyset$,

Where $\emptyset$ is angle between the linear momentum vector $\vec{P}$ and the position of vector $\vec{r}$.

S.I unit of angular momentum is kg m^2s^{-1}.

Geometrically, angular momentum of a particle is equal to twice the product of mass of the particle and areal velocity of its radius vector about the given axis.

.Relation between torque and angular momentum:

(i)$\vec{\tau} = \frac{d\vec{L}}{dt}$ (ii) If the system consists of n-particles, then $\vec{\tau} = \frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt} + \frac{d\vec{L}_3}{dt} + \cdots + \frac{d\vec{L}_n}{dt}$.

.Law of conservation of angular momentum. If no external torque acts on a system, then the total angular momentum of the system always remain conserved.

Mathematically: $\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n = \vec{L}_{total} = a\ constant$

.**Moment of inertia(I).**the moment of inertia of a rigid body about a given axis of rotation is the sum of the products of masses of the various particles and squares of their respective perpendicular distances from the axis of rotation.

Mathematically: I=$m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$= $\sum_{i=1}^{i=n} m_i\, r_i^2$

SI unit of moment of inertia is kg m^2.

MI corresponding to mass of the body. However, it depends on shape & size of the body and also on position and configuration of the axis of rotation.

Radius of gyration (K).it is defined as the distance of a point from the axis of rotation at which, if whole mass of the body were concentrated, the moment of inertia of the body would be same as with the actual distribution of mass of the body.

Mathematically :K=$\frac{r_1^2+r_2^2+r_3^2+\cdots+r_n^2}{n}$= rms distance of particles from the axis of rotation.

SI unit of gyration is m. Note that the moment of inertia of a body about a given axis is equal to the product of mass of the body and squares of its radius of gyration about that axis i.e. I=Mk^2.

.**Theorem of perpendicular axes.** It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moment of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through the lamina.

Mathematically: $I_z = I_x + I_{y'}$

Where x & y-axes lie in the plane of the Lamina and z-axis is perpendicular to its plane and passes through the point of intersecting of x and y axes.

.Theorem of parallel axes. It states that the moment of inertia of a rigid body about any axis is equal to moment of inertia of the body about a parallel axis through its center of mass plus the product of mass of the body and the square of the perpendicular distance between the axes.

Mathematically: $I = I_c + M\,h^2$, where I_c is moment of inertia of the body about an axis through its centre of mass and h is the perpendicular distance between the two axes.

.**Moment of inertia** of a few bodies of regular shapes:

i. M.I. of a rod about an axis through its c.m. and perpendicular to rod, $I = \frac{1}{12}\,ML^2$

ii. M.I. of a circular ring about an axis through its centre and perpendicular to its plane, $I = MR^2$

iii. M.I. of a circular disc about an axis through its centre and perpendicular to its plane, $I = \frac{1}{2}\,MR^2$

iv. M.I. of a right circular solid cylinder about its symmetry axis, $I = \frac{1}{2}\,MR^2$

v. M.I. of a right circular hollow cylinder about its axis = MR^2

vi. M.I. of a solid sphere about its diameter, $I = \frac{2}{5}\,MR^2$

vii. M.I. of spherical shell about its diameter, $I = \frac{2}{3}\,MR^2$

.**Moment of inertia and angular momentum.** The moment of inertia of a rigid body about an axis is numerically equal to the angular momentum of the rigid body, when rotating with unit angular velocity about that axis.

Mathematically: $K.E\ of\ rotation = \frac{1}{2} I\ \omega^2$

.**Moment of inertia and kinetic energy of rotation.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the kinetic energy of rotation of the body, when rotation with unit angular velocity about that axis.

Mathematically:$K.E.of\ rotation = \frac{1}{2}\ I\omega^2$

.**Moment of inertia and torque.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce a unit angular acceleration in the body BOUT THE GIVEN AXIS.

MATHEMATICALLY: $\quad = I\alpha$

.**Law of conservation of angular momentum.** If no external torque acts on a system, the total angular momentum of the system remains unchanged.

Mathematically:

$I\omega = constant\ vector, i.e., in\ magnitude, I_1\omega_1 = I_2\omega_2,$

$provides\ no\ external\ torque\ acts\ on\ the\ system.$

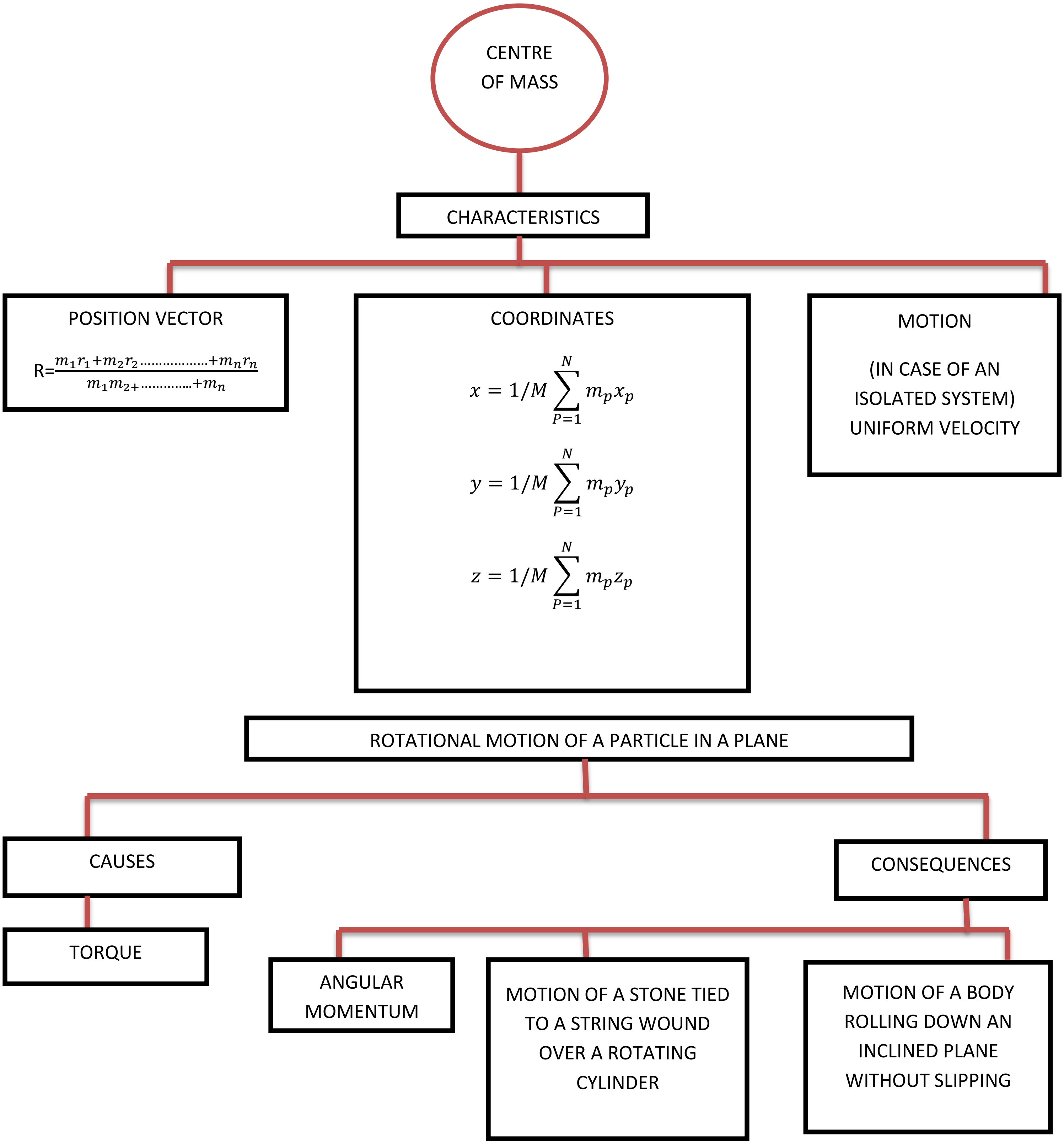
For translational equilibrium of a rigid body, $\vec{F}=\sum_i F_i=0$

For rotational equilibrium of a rigid body, $\vec{\tau} = \sum_i \vec{\tau}_i = 0$

1.The following table gives a summary of the **analogy** between various quantities describing linear motion and rotational motion.

s.no.	*Linear motion*	s.no.	*Rotational motion*
1.	**Distance/displacement (s)**	**1.**	**Angle or angular displacement (θ)**
2.	**Linear velocity, $\vartheta = \frac{ds}{dt}$**	**2.**	**Angular velocity, $\omega = \frac{d\theta}{dt}$**
3.	**Linear acceleration, $\alpha = \frac{dv}{dt} = \frac{d^2r}{dr^2}$**	**3.**	**Angular acceleration=$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dr^2}$**
4.	**Mass (m)**	**4.**	**Moment of inertia (I)**
5.	**Linear momentum, $p = m\ v$**	**5.**	**Angular momentum, $L = I\omega$**
6.	**Force, $F = m\ a$**	**6.**	**Torque, $\tau = Ia$**
7.	**Also, force $F = \frac{dp}{dt}$**	**7.**	**Also, torque, $\tau = \frac{dL}{dt}$**
8.	**Translational KE, $K_T = \frac{1}{2}mv^2$**	**8.**	**Rotational KE, $K_R = \frac{1}{2}I\omega^2$**
9.	**Work done, $W = Fs$**	**9.**	**Work done, $W = \tau\theta$**
	Power, $P = F\ v$		**Power, $P = \tau\omega$**

10.		**10.**	
11.	**Linear momentum of a system is conserved when no external force acts on the system.**	**11.**	**Angular momentum of a system is conserved when no external torque acts on the system**
12.	**Equation of translator motion** **i.** $\boldsymbol{v = u + at}$ **ii.** $\boldsymbol{s = ut + \frac{1}{2}at^2}$ **iii.** $\boldsymbol{v^2 - u^2 = 2as, where\ the\ symbols}$ ***have their usual meaning.***	**12.**	**Equations of rotational motion** **i.** $\boldsymbol{\omega_2 = \omega_1 + at}$ **ii.** $\boldsymbol{\theta = \omega_1 t + \frac{1}{2}at^2}$ **iii.** $\boldsymbol{\omega_2^2 - \omega_1^2 = 2a\theta,}$ $\boldsymbol{where\ the\ symbols}$ ***have their usual meaning.***


CENTRE OF MASS
CHARACTERISTICS
POSITION VECTOR
$R=\frac{m_1r_1+m_2r_2\ldots\ldots\ldots+m_nr_n}{m_1m_{2+}\ldots\ldots\ldots+m_n}$
COORDINATES
$x = 1/M \sum_{P=1}^{N} m_p x_p$
$y = 1/M \sum_{P=1}^{N} m_p y_p$
$z = 1/M \sum_{P=1}^{N} m_p z_p$
MOTION
(IN CASE OF AN ISOLATED SYSTEM) UNIFORM VELOCITY
ROTATIONAL MOTION OF A PARTICLE IN A PLANE
CAUSES
TORQUE
CONSEQUENCES
ANGULAR MOMENTUM
MOTION OF A STONE TIED TO A STRING WOUND OVER A ROTATING CYLINDER
MOTION OF A BODY ROLLING DOWN AN INCLINED PLANE WITHOUT SLIPPING

1 Marks Questions

1. If one of the particles is heavier than the other, to which will their centre of mass shift?

Answer:- The centre of mass will shift closer to the heavier particle.

2. Can centre of mass of a body coincide with geometrical centre of the body?

Answer:- Yes, when the body has a uniform mass density.

3.Which physical quantity is represented by a product of the moment of inertia and the angular velocity?

Answer:- Product of I and ω represents angular momentum(L=I ω).

4.What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ denote the adjacent sides of a parallelogram drawn from a point and the area of parallelogram is $\frac{1}{2}$AB.

Answer:- Area of parallelogram=$|\vec{A} \times \vec{B}|$ = AB$\sin\theta$ = $\frac{1}{2}$AB. (Given)

$\sin\theta = \frac{1}{2} = \sin 30^0$ or $\theta = 30^0$

5. Which component of linear momentum does not contribute to angular momentum?

Answer:- The radial component of linear momentum makes no contribution to angular momentum.

6.A disc of metal is melted and recast in the form of solid sphere. What will happen to the moment of inertia about a vertical axis passing through the centre ?

Answer:- Moment of inertia will decrease, because $I_d = \frac{1}{2}\,m\,r^2$ and $I_s = \frac{2}{5}\,m\,r^2$, the radius of sphere formed on recasting the disc will also decrease.

7. What is rotational analogue of mass of body?

Answer:- Rotational analogue of mass of a body is moment of inertia of the body.

8. What are factors on which moment of inertia depend upon?

Answer:- Moment of inertia of a body depends on position and orientation of the axis of rotation. It also depends on shape, size of the body and also on the distribution of mass of the body about the given axis.

9. Is radius of gyration of a body constant quantity?

Answer:- No, radius of gyration of a body depends on axis of rotation and also on distribution of mass of the body about the axis.

10. **Is the angular momentum of a system always conserved? If no, under what condition is it conserved?**

Answer:- No, angular momentum of a system is not always conserved. It is conserved only when no external torque acts on the system.

2 Marks Questions

1. Why is the handle of a screw made wide?

Answerwer:- Turning moment of a force= force × distance(r) from the axis of rotation. To produce a given turning moment, force required is smaller, when r is large. That's what happens when handle of a screw is made wide.

2. Can a body in translatory motion have angular momentum? Explain.

Answer:- Yes, a body in translatory motion shall have angular momentum, the fixed point about which angular momentum is taken lies on the line of motion of the body. This follows from $|L|$= r p $\sin\Theta$.

L=0, only when Ө $=0^0$or Ө$=180^0$.

3. A person is sitting in the compartment of a train moving with uniform velocity on a smooth track. How will the velocity of centre of mass of compartment change if the person begins to run in the compartment?

Answer:- We know that velocity of centre of mass of a system changes only when an external force acts on it. The person and the compartment form one system on which no external force is applied when the person begins to run. Therefore, there will be no change in velocity of centre of mass of the compartment.

4. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its K.E is halved, what happens to the angular momentum?

Answer:- $L = m\,v\,r$ and $v = r\,\omega = r(2\,\pi\,n)$

r = $\frac{v}{2\,\pi\,n}$ $\quad\therefore\quad$ $L = mv\left(\frac{v}{2\,\pi\,n}\right) = \frac{m\,v^2}{2\,\pi\,n}$

As,

K.E= $\frac{1}{2}\,mv^2$, therefore, $L = \frac{K.E}{\pi n}$

When K.E. is halved and frequency (n) is doubled, $L = \frac{K.E'}{\pi n'} = \frac{K.E/2}{\pi(2n)} = \frac{K.E}{4\pi n} = \frac{L}{4}$

i.e. angular momentum becomes one fourth.

5. An isolated particle of mass m is moving in a horizontal plane(x-y), along the x-axis at a certain height above the ground. It explodes suddenly into two fragments of masses m/4 and 3 m/4. An instant later, the smaller fragments is at y= +15 cm. What is the position of larger fragment at this instant?

Answer:- As isolated particle is moving along x-axis at a certain height above the ground, there is no motion along y-axis. Further, the explosion is under internal forces only. Therefore, centre of mass remains stationary along y-axis after collision. Let the co-ordinates of centre of mass be (x_{cm} , 0).

Now, $\quad y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2} = 0 \quad\therefore\quad m_1y_1 + m_2y_2 = 0$

Or $\quad y_2 = \frac{-m_1y_1}{m_2} = \frac{-m/4}{3m/4}\times 15 = -5\text{ cm}$

So, larger fragment will be at y= -5 ; along x-axis.

6. Why there are two propellers in a helicopter?

Answerwer:- If there were only one propeller in a helicopter then, due to conservation of angular momentum, the helicopter itself would have turned in the opposite direction.

7. A solid wooden sphere rolls down two different inclined planes of the same height but of different inclinations. (a) Will it reach the bottom with same

speed in each case ? (b) Will it take longer to roll down one inclined plane than other ? Explain.

Answer:- (a) Yes, because at the bottom depends only on height and not on slope.

(b) Yes, greater the inclination(θ), smaller will be time of decent, as $t \propto 1/\sin\theta$.

8. There is a stick half of which is wooden and half is of steel. It is pivoted at the wooden end and a force is applied at the steel end at right angles to its length. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is angular acceleration more and why?

Answer:- We know that torque, τ= Force × Distance = $I\,\alpha$ = constant

$$\therefore\ \alpha = \frac{\tau}{I} \quad \text{i.e}\ \ \alpha \propto \frac{1}{I}$$

Angular acc. (α) will be more, when I is small, for which lighter material(wood) should at larger distance from the axis of rotation I.e. when stick is pivoted at the steel end.

9. Using expressions for power in rotational motion, derive the relation $= I\,\alpha$, where letters have their usual meaning.

Answer:- We know that power in rotational motion, $P = \tau\,\omega$ ……….(i)

and K.E. of motion, E= $\frac{1}{2} I\,\omega^2$ ……….(ii)

As power= time rate of doing work in rotational motion, and work is stored in the body in the form of K.E.

$$\therefore \qquad P = \frac{d}{dt}\ (\text{K.E. of rotation})$$

$$= \frac{d}{dt}\left(\frac{1}{2}\,I\,\omega\right) = \frac{1}{2} I \times 2\omega\left(\frac{d\omega}{dt}\right)$$

$$P = I\,\omega\,\alpha$$

Using (i), $P = \tau\,\omega = I\,\omega\,\alpha$ or $\tau = I\,\alpha$, which is the required relation.

10. Calculate radius of gyration of a cylindrical rod of mass m and length L about an axis of rotation perpendicular to its length and passing through the centre.

Answer:- K=? , mass= m , length=L

Moment of inertia of the rod about an axis perpendicular to its length and passing through the centre is

$$I = \frac{mL^2}{12}$$

Also, $I = mK^2 \quad \therefore \; mK^2 = \frac{mL^2}{12} \quad$ or $\quad K = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}}$.

3 Marks Questions

1. Explain that torque is only due to transverse component of force. Radial component has nothing to do with torque.

2. Show that centre of mass of an isolated system moves with a uniform velocity along a straight line path.

3. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved ? Explain.

Ans:- Here, $L = I\,\omega =$ constant

K.E. of rotation, $K = \frac{1}{2}\, I\, \omega^2$

$$K = \frac{1}{21}\, I^2\, \omega^2 = \frac{L^2}{21}$$

As L is constant, $\therefore K \propto 1/I$

When moment of inertia(I) decreases, K.E. of rotation(K) increases. Thus K.E. of rotation is not conserved.

4. How will you distinguish between a hard boiled egg and a raw egg by spinning each on a table top?

Ans:- To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate shall be raw. This is because in a raw egg, liquid matter inside tries to get away from its axis of rotation. Therefore, its moment of inertia I increases. As $\tau = I\,\alpha =$ constant, therefore, α decreases i.e. raw egg will spin with smaller angular acceleration. The reverse is true for a hard boiled egg which will rotate more or less like a rigid body.

5.Equal torques are applied on a cylindrical and a hollow sphere. Both have same mass and radius. The cylinder rotates about its axis and the sphere rotates about one of its diameters. Which will acquire greater speed? Explain.

6.Locate the centre of mass of uniform triangular lamina and a uniform cone.

7. **A thin wheel can stay upright on its rim for a considerable length when rolled with a considerable velocity, while it falls from its upright position at the slightest disturbance when stationary. Give reason.**
Answer:- When the wheel is rolling upright, it has angular momentum in the horizontal direction i.e., along the axis of the wheel. Because the angular momentum is to remain conserved, the wheel does not fall from its upright position because that would change the direction of angular momentum. The wheel falls only when it loses its angular velocity due to friction.

8. Why is the speed of whirl wind in a tornado so high?
Answer:- In a whirl wind, the air from nearby region gets concentrated in a small space thereby decreasing the value of moment of inertia considerably. Since, **I** ω= constant, due to decrease in moment of inertia, the angular speed becomes quite high.

9. Explain the physical significance of moment of inertia and radius of gyration.

10. Obtain expression for K.E. of rolling motion.

5 Marks Questions

1. Define centre of mass. Obtain an expression for perpendicular of centre of mass of two particle system and generalise it for particle system.

2. Find expression for linear acceleration of a cylinder rolling down on a inclined plane.
A ring, a disc and a sphere all of them have same radius and same mass roll down

on inclined plane from the same heights. Which of these reaches the bottom (i) earliest (ii) latest ?

3. (i) Name the physical quantity corresponding to inertia in rotational motion. How is it calculated? Give its units.
(ii)Find expression for kinetic energy of a body.

4. State and prove the law of conservation of angular momentum. Give one illustration to explain it.

5. State parallel and perpendicular axis theorem.
Define an expression for moment of inertia of a disc R, mass M about an axis along its diameter.

TYPICAL PROBLEMS

1. A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system.

Ans:-

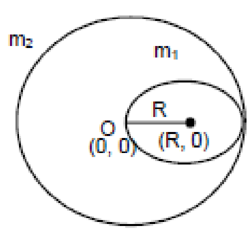
Let the centre of the bigger disc be the origin.

2R = Radius of bigger disc

R = Radius of smaller disc

$$m_1 = \pi R^2 \times T \times \rho$$

$m_2 = \pi(2R)^2 \times T \times \rho$, where T = Thickness of the two discs



ρ =Density of the two discs

∴ The position of the centre of mass

$$=\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}\ ,\frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

$$x_1 = R \qquad y_1 = 0$$

$$x_2 = 0 \qquad y_2 = 0$$

$$\left(\frac{\pi R^2T\rho R + 0}{\pi R^2T\rho + \pi(2R)^2T\rho}, \frac{0}{m^1 + m^2}\right) = \left(\frac{\pi R^2T\rho R}{5\pi R^2T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

2. Two blocks of masses 10 kg and 20 kg are placed on the x-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of centre of mass unchanged ?

Ans:- Two masses m_1 and m_2 are placed on the X-axis

m_1 = 10 kg , m_2 = 20kg

The first mass is displaced by a distance of 2 cm

$$\therefore \overline{X_{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$

$$\Rightarrow 0 = \frac{20 + 20x_2}{30}$$

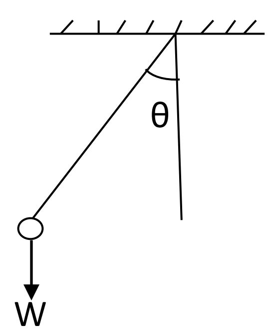
$$\Rightarrow 20 + 20x_2 = 0$$

$$\Rightarrow 20 = -20x_2$$

$$\Rightarrow x_2 = -1cm$$

$\therefore$ The 2nd mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

3. A simple of length $\boldsymbol{l}$ is pulled aside to make an angle $\boldsymbol{\theta}$ with the vertical. Find the magnitude of the torque of the weight $\boldsymbol{w}$ of the bob about the point of suspension. When is the torque zero ?



Ans:- A simple of pendulum of length l is suspended from a rigid support.

A bob of weight W is hanging on the other point.

When the bob is at an angle θ with the vertical,

then total torque acting on the point of suspension = i = F × r

⇒ W r sin θ = W l sin θ

At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

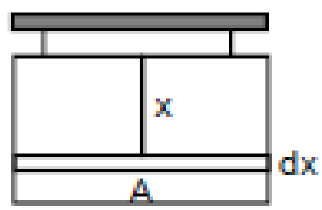
4. A square plate of mass 120 g and edge 5.0 cm rotates about one of edges. If it has a uniform angular acceleration of 0.2 rad/s^2, what torque acts on the plate ?

Ans:- A square plate of mass 120 gm and edge 5 cm rotates about one of the edge. Let take a small area of the square of width dx and length a which is at a distance x from the axis of

rotation.

Therefore mass of that small area

m/a^2 × a dx(m=mass of the square ; a= side of the plate)



$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)]_0^a$$
$$= ma^2/3$$

Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

= {(120 × 10^{-3} × 5^2 × 10^{-4})/3} 0.2

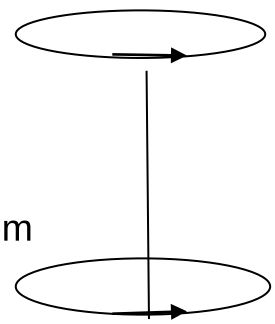
= 0.2 × 10^{-4} = 2 × 10^{-5} N-m.

5. A wheel of moment of inertia 0.10 kg-m^2 is rotating about a shaft at an angular speed of 160 rev/minute. A second wheel is set into rotation at 300 rev/minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200 rev/minute. Find the moment of

inertia of the second wheel.

Ans:- Wheel (1) has

I_1= 0.10 kg-m^2, ω_1 = 160 rev/min



Wheel (2) has
I_2= ? ; ω_2 = 300 rev/min
Given that after they are coupled, ω = 200 rev/min
Therefore if we take the two wheels to bean isolated system
Total external torque = 0
Therefore, $I_1\omega_1 + I_1\omega_2 = (I_1 + I_1)\omega$
⇒ 0.10 × 160 + I_2 × 300 = (0.10 + I_2) × 200
⇒ 5I_2 = 1 – 0.8
⇒ I_2 = 0.04 kg-m^2.

GRAVITATION

CONCEPTS

- **Kepler's law of planetry motion**

(a) Kepler's first law (law of orbit): Every planet revolves around the sun in an elliptical orbit with the sun is situated at one focus of the ellipse.

(b) Kepler's second law (law of area): The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time , i.e., the areal velocity of the planet around the sun is constant.

(c) Kepler's third law (law of period): The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semimajor axis of the elliptical orbit of the planet around the sun.

- Gravitation is the name given to the force of attraction acting between any two bodies of the universe.

- Newton's law of gravitation: It states that gravitational force of attraction acting between two point mass bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, i.e., $F=Gm_1m_2/r^2$, where G is the universal gravitational constant.

- Gravitational constant (G): It is equal to the force of attraction acting between two bodies each of unit mass, whose centres are placed unit distance apart. Value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula $G =[M^{-1}L^3T^{-2}]$. In SI unit, the value of $G =6.67 X 10^{-11} Nm^2kg^{-2}$.

- Gravity: It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth. Gravity is the measure of weight of the body. The weight of a body of mass m=mass X acceleration due to gravity=mg. The unit of weight of a body will be the same as those of force.

- **Acceleration due to gravity (g):** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is vector quantity. The value of g changes with height, depth, rotation of earth the value of g is zero at the centre of the earth. The value of g on the surface of earth is 9.8 ms^{-2}. The acceleration due to gravity (g) is related with gravitational constant (G) by the relaion, $g=GM/R^2$ where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity**:

(a) Effect of altitude, $g'=Gr^2/(R+h)^2$ and $g'=g(1-2h/R)$

The first is valid when h is comparable with R and the second relation is valid when h<<R.

The value of g decreases with increase in h.

(b) Effect of depth $g'=g(1-d/R)$

The acceleration due to gravity decreases with increase in depth d and becomes zero at the center of earth.

(c) Effect of rotation of earth: $g'=g-R\,\omega^2 cos^2\lambda$

The acceleration due to gravity on equator decreases on account of rotation of earth and increase with the increase in latitude of a place.

- Gravitational field: It is the space around a material body in which its gravitational pull can be experienced by other bodies. The strength of gravitational field at a point is the measure of gravitational intensity at that point. The intensity of gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field. The intensity of gravitational field at a point distance r from the center of the body of mass M is given by

$E=GM/r^2$=acceleration due to gravity.

- **Gravitational potential:** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point, V=work done(W)/test mass(m_0)= -GM/r. $V= \frac{W}{m_0}= -\frac{GM}{r}$

  Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, E= -dV/dr

  - **Gravitational potential energy of a body,** at a point in the gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point without acceleration.

    Gravitational potential energy U=gravitational potential X mass of body =$-\frac{GM}{r}$ X m.

  - **Inertial mass of a body** is defined as the force required to produce unit acceleration in the body.

    Gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity.

    Inertial mass of a body is identical to the gravitational mass of that body. The main difference is that the gravitational mass of a body is affected by the presence of other bodies near it. Whereas the inertial mass of a body remains unaffected by the presence of other bodies near it.

  - **Satellite:** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

(a) Orbital speed of satellite is the speed required to put the satellite into given orbit around earth.

- Time period of satellite(T): It is the time taken by satellite to complete one revolution around the earth. T=$\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

- Height of satellite above the earth surface:

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

- Total energy of satellite, E=P.E +K.E=$\frac{-GMm}{2(R+h)}$

Blinding energy of satellite = -E = GM m/(R+h)

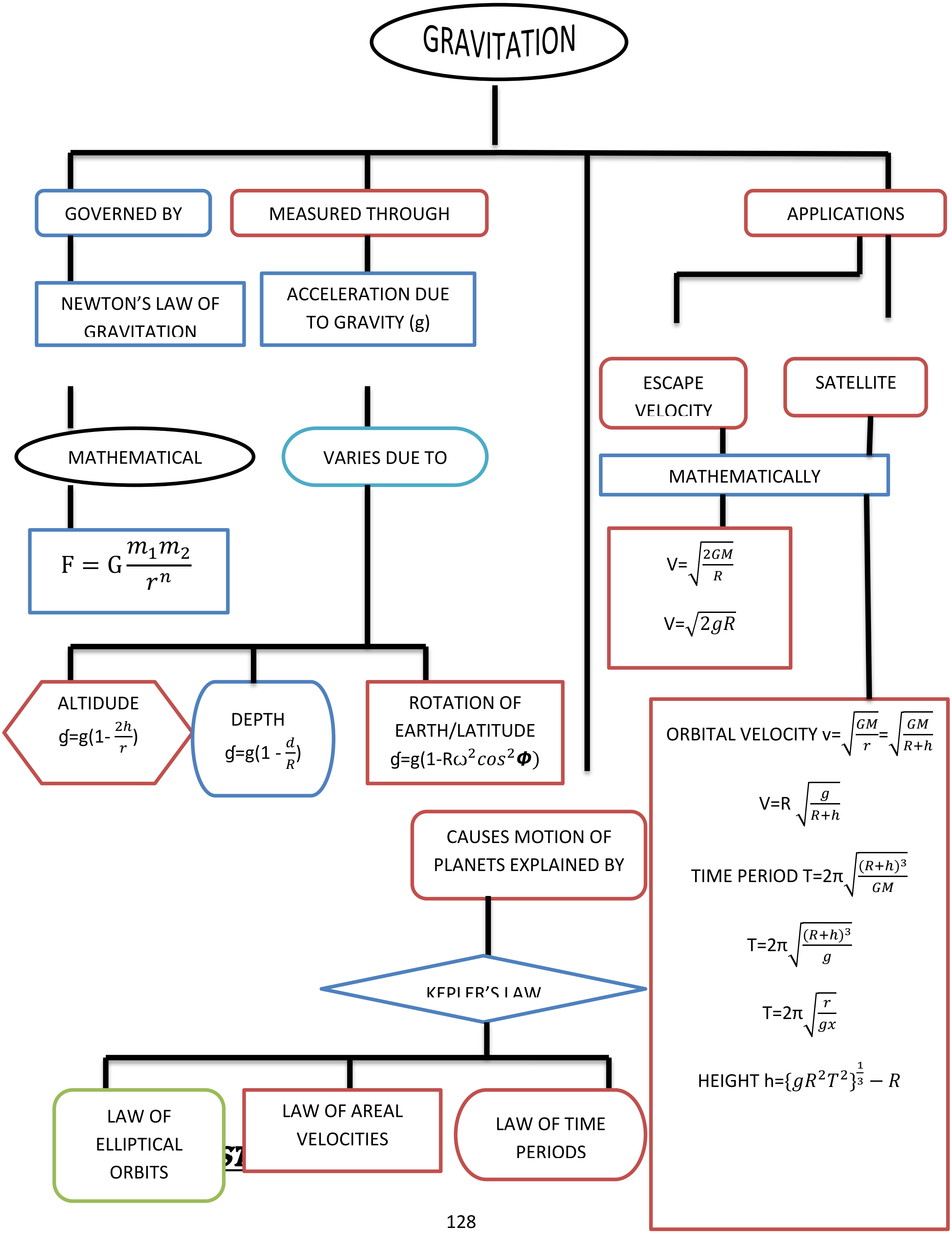
- Geostationary satellite: A satellite which revolves around the earth with the same angular speed in the same direction as is done by the earth around its axis is called geostationary or geosynchronous satellite. The height of geostationary satellite is = 36000 km and its orbital velocity = 3.1 km s^{-1}.

- Polar satellite: It is that satellite which revolves in polar orbit around earth ,i.e. , polar satellite passes through geographical north and south poles of earth once per orbit.

- Escape speed: The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth(or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape velocity v_e is given by, v_e =$\sqrt{\frac{2GM}{R}}$=$\sqrt{2gR}$. For earth, the value of escape speed is 11.2kms^{-1}.

- For a point close to the earth's surface , the escape speed and orbital speed are related as v_e =$\sqrt{2}\, v_o$

- Weightlessness: It is a situation in which the effective weight of the body becomes zero.


GRAVITATION
GOVERNED BY
MEASURED THROUGH
APPLICATIONS
NEWTON'S LAW OF GRAVITATION
ACCELERATION DUE TO GRAVITY (g)
ESCAPE VELOCITY
SATELLITE
MATHEMATICAL
VARIES DUE TO
MATHEMATICALLY
$F = G\frac{m_1 m_2}{r^n}$
$V=\sqrt{\frac{2GM}{R}}$
$V=\sqrt{2gR}$
ALTIDUDE
$ց=g(1-\frac{2h}{r})$
DEPTH
$ց=g(1-\frac{d}{R})$
ROTATION OF EARTH/LATITUDE
$ց=g(1-R\omega^2 cos^2\boldsymbol{\phi})$
ORBITAL VELOCITY $v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{R+h}}$
$V=R\sqrt{\frac{g}{R+h}}$
TIME PERIOD $T=2\pi\sqrt{\frac{(R+h)^3}{GM}}$
$T=2\pi\sqrt{\frac{(R+h)^3}{g}}$
$T=2\pi\sqrt{\frac{r}{gx}}$
HEIGHT $h=\{gR^2T^2\}^{\frac{1}{3}} - R$
CAUSES MOTION OF PLANETS EXPLAINED BY
KEPLER'S LAW
LAW OF ELLIPTICAL ORBITS
LAW OF AREAL VELOCITIES
LAW OF TIME PERIODS

Q1.When a stone of mass m is falling on the earth of mass M; find the acceleration of earth if any?

Ans. Force exerted by falling stone on earth, F=mg

Acceleration of earth=$\frac{F}{M}$=$\frac{mg}{M}$

Q2.Why G is called a universal constant?

Ans. It is so because the value of G is same for all the pairs of the bodies (big or small) situated anywhere in the universe.

Q3.According to Kepler's second law the radius vector to a planet from the sun sweeps out equal area in equal interval of time. The law is a consequence of which conservation law.

Ans. Law of Conservation of angular momentum.

Q4.What are the factors which determine ; Why some bodies in solar system have atmosphere and others don't have?

Ans. The ability of a body (planet) to hold the atmosphere depends on acceleration due to gravity.

Q5.What is the maximum value of gravitational potential energy and where?

Ans. The value of gravitational potential energy is negative and it increases as we move away from the earth and becomes maximum (zero) at infinity.

Q6.The gravitational potential energy of a body at a distance r from the center of earth is U. What is the weight of the body at that point?

Ans. U=$\frac{GMm}{r}$=($\frac{GM}{r^2}$) r m=g r m= (mg) r

Q7.A satellite revolving around earth loses height. How will its time period be changed?

Ans. Time period of satellite is given by; T=2$\pi\sqrt{\frac{(R+h)^3}{GM}}$. Therefore ,T will decrease, when h decreases.

Q8.Should the speed of two artificial satellites of the earth having different masses but the same orbital radius, be the same?

Ans.Yes it is so because the orbital speed of a satellite is independent of the mass of a satellite. Therefore the speeds of the artificial satellite will be of different masses but of the same orbital radius will be the same.

Q9.Can a pendulum vibrate in an artificial satellite?

Ans. No, this is because inside the satellite, there is no gravity ,i.e., g=0.

As t = 2π$\sqrt{l/g}$, hence, for g=0 , t = ∞ . Thus, the pendulum will not vibrate.

Q10.Why do different planets have different escape speed?

Ans. As, escape speed =$\sqrt{2GM/R}$, therefore its value are different for different planets which are of different masses and different sizes.

<u>2 MARKS QUESTIONS</u>

Q1.Show that weight of all body is zero at Centre of earth?

Ans. The value of acceleration due to gravity at a depth d below the surface of earth of radius R is given by ɠ=g(1-d/R).At the center of earth, (dept)d=R; so, ɠ =0.The weight of a body of mass m at the centre of earth =mg'=m x 0=0.

Q2.If a person goes to a height equal to radius of the earth from its surface. What would be his weight relative to that on the earth.

Ans. At the surface of the earth, weight W=mg=GM m/R^2.

At height h =R , weight W'=mg'=$\frac{GM\ m}{(R+h)^2}$=$\frac{GM\ m}{(R+R)^2}$ $\frac{W\prime}{W}$=$\frac{R^2}{(2R)^2}$ =$\frac{1}{4}$ W'=$\frac{W}{4}$

It means the weight would reduce to one-fourth of the weight on the surface of earth.

Q3.What will be the effect on the time period of a simple pendulum on taking to a mountain?

Ans. The time period of a pendulum, T=2π$\sqrt{l/g}$, i.e., T=∝1/$\sqrt{g}$.As the value of g is less at mountain than at plane, hence time period of simple pendulum will be more at mountain than at plane though the change will be very small.

Q4.A satellite is revolving around the earth, close to the surface of earth with a kinetic energy E. How much kinetic energy should be given to it so that it escapes from the surface of earth?

Ans. Let $v_{0,}v_e$ be the orbital and escape speeds of the satellite, then v_e=$\sqrt{2v_{0,}}$.

Energy in the given orbit,$E_1 = \frac{1}{2}mv_0^2 = E$

Energy for the escape speed,$E_2 = \frac{1}{2}m\, v_e^2 = \frac{1}{2}m\left(\sqrt{2}v_0^2\right) = 2E$

Energy required to be supplied =$E_2 - E_1 = E$.

Q5.A tennis ball and a cricket ball are to be projected out of gravitational field of the earth. Do we need different velocities to achieve so?

Ans. We require the same velocity for the two balls, while projecting them out of the gravitational field. It is so because, the value of escape velocity does not depend upon the mass of the body to be projected [i.e. , v_e=$\sqrt{2gR}$].

Q6.Suppose the gravitational force varies inversely as the nth power of the distance. Show that the time period of a planet in circular orbit of radius R around the sun will be proportional to $R^{(n+1)/2}$.

Ans. $\frac{GM\,m}{R^n} = mR(\frac{2\pi}{T})^2$

$$T^2 = \frac{R\ \times\ 4\pi^2 \times\ R^n}{GM} = \frac{4\pi^2 R^{(n+1)}}{GM}$$

$$T = \frac{2\pi}{\sqrt{GM}} . R^{(n+1)/2}$$

$$T \propto R^{(n+1)/2}$$

Q7.Draw graphs showing the variation of acceleration due to gravity with (a)height above the earth's surface, (b)depth below the Earth's surface.

Ans.(a)The variation of g with height h is related by relation g∝1/r^2 where r=R+h. Thus, the variation of g and r is a parabolic curve.

(b)The variation of g with depth is released by equation g'=g(1-d/R) i.e. g'∝ $(R - d)$ .Thus, the variation of g and d is a straight line.

Q8.Why does moon have no atmosphere?

Ans. Moon has no atmosphere because the value of acceleration due to gravity 'g' on surface of moon is small. Therefore, the value of escape speed on the surface of moon is small. The molecules of atmospheric gases on the surface of the moon have thermal speeds greater than the escape speed. That is why all the molecules of gases have escaped and there is no atmosphere on moon.

Q9.A rocket is fired with a speed v=2$\sqrt{gR}\, near$ the earth's surface and directed upwards. Find its speed in interstellar space.

Ans. Let v be the speed of rocket instellar space.

Using law of conservation of energy, we have$\frac{1}{2}m(2\sqrt{gR})^2$=$\frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$

$$=\frac{1}{2}m(\sqrt{2gR})^2 + \frac{1}{2}mv^2$$

$$v^2 = 4gR - 2gR$$

$$v = \sqrt{2gR}$$

3 marks questions

Q1.Explain how knowledge of g helps us to find (i) mass of earth and (ii)mean density of earth?

Q2. Obtain the expression for orbital velocity, time period, and altitude of a satellite.

Q3. What do you understand by 'Escape velocity'? Derive an expression for it in terms of parameters of given planet.

Q4. What do you understand by gravitational field, Intensity of gravitational field . Prove that gravitational intensity at a point is equal to the acceleration due to gravity at that point.

Q5.A mass M is broken into two parts of masses $m_1 and\, m_2$. How are $m_1\ and\, m_2$ related so that force of gravitational attraction between the two parts is maximum.

Ans. Let $m_1 = m,$ then $m_2 = M - m$. Gravitational force of attraction between them when placed distance r apart will be $= \frac{Gm(M-m)}{r^2}$.

Differentiating it w.r.t. m, we get

$$\frac{dF}{dm} = \frac{G}{r^2}\left[m\frac{d}{dm}(M-m) + (M-m)\frac{dm}{dm}\right] = \frac{G}{r^2}[m(-1) + M - m] = \frac{G}{r^2}(M-2m)$$

If F is maximum, then $\frac{dF}{dm} = 0$;

Then $\frac{G}{r^2}(M-2m) = 0$ or M=2m or m=$\frac{M}{2}$

Q6.Two particles of equal mass move in a circle of radius r under the action of their mutual gravitational attraction. Find the speed of each particle if its mass is m.

Ans. The two particles will move on a circular path if they always remain dramatically opposite so that the gravitation force on one particle due to other is directed along the radius. Taking into consideration the circulation of one particle we have

$$\frac{mv^2}{r} = \frac{Gmm}{(2r)^2} \text{ or } v = \sqrt{\frac{Gm}{4r}}$$

Q7.The magnitude of gravitational field at distances $r_1 \, and \, r_2$ from the centre of a uniform sphere of radius R and mass M are $I_1 and \, I_2$ respectively. Find the ratio of (I_1/I_2) if $r_1 > R \, and \, r_2 < R$.

Ans. When $r_1 > R,$ the point lies outside the sphere . Then sphere can be considered to be a point mass body whose whole mass can be supposed to be concentrated at its Centre. Then gravitational intensity at a point distance $r_1$from the Centre of the sphere will be,$I_1 = GM/r_1^2$

When $r_2 < R,$ the point P lies inside the sphere. The unit mass body placed at P, will experience gravitational pull due to sphere of radius r_2, whose mass is M'=$\frac{M\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi R^3} = \frac{Mr_2^3}{R^3}$.

Therefore, the gravitational intensity at P will be ,

$$I_2 = \frac{GMr_2^3}{R^3} \cdot \frac{1}{r_2^2} = \frac{GMr_2}{R^3}$$

$$\frac{I_1}{I_2} = \frac{GM}{r_1^2} \cdot \frac{R^3}{GMr_2} = \frac{R^3}{r_1^2 r_2}$$

Q8.Two bodies of masses $m_1 and \, m_2$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Find their relative velocity of approach at a separation distance r between them.

Ans. Let v_r be the relative velocity of approach of two bodies at a distance r apart. The reduced mass of the system of two particles is , $\mu = \frac{m_1 m_2}{m_1+m_2}$.

According to law of conservation of mechanical energy.

Decrease in potential energy = increase in K.E.

$$0-\left(-\frac{Gm_1m_2}{r}\right)=\frac{1}{2}\mu v_r^2 \quad \text{or} \quad \frac{Gm_1m_2}{r}=\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2 \quad \text{or} \quad v_r=\sqrt{\frac{2G(m_1+m_2)}{r}}$$

Q9.Since the moon is gravitationally attracted to the earth, why does it not simply crash on earth?

Ans. The moon is orbiting around the earth in a certain orbit with a certain period . The centripetal force required for the orbital motion is provided to the gravitational pull of earth. The moon can crash into the earth if its tangential velocity is reduced to zero. AS moon has tangential velocity while orbiting around earth, it simply falls around the earth rather than into it and hence cannot crash into the earth.

Q10.What are the conditions under which a rocket fired from earth, launches an artificial satellite of earth?

Ans. Following are the basic conditions: (i) The rocket must take the satellite to a suitable height above the surface of earth for ease of propulsion.

(ii)From the desired height, the satellite must be projected with a suitable speed, called orbital speed.

(iii)In the orbital path of satellite, the air resistance should be negligible so that its speed does not decrease and it does not burn due to the heat produced.

5 MARKS QUESTIONS

Q1.State Kepler's laws of planetary motion. Prove second Kepler's law using concept of conservation of angular motion.

Q2.State universal law of gravitation. What is the significance of this law. Find the expression for acceleration due to gravity.

Q3.Explain the variation of acceleration due to gravity with (I) altitude (ii) depth

Q4. Define gravitational potential energy. Derive the expression for gravitational potential energy. What is the maximum value of gravitational potential energy?

Q5.What is escape speed? Derive the expressions for it. Calculate escape speed for the Earth.

TYPICAL PROBLEMS

Q1.Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Ans. The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Consider the motion of one of the particles. The force on the particle is $F = \frac{Gm^2}{4R^2}$. If the speed is v, its acceleration is v^2/R.

Thus by Newton's Law,

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$V = \sqrt{\frac{Gm}{4R}}$$

Q2.A particle is fired vertically upward with a speed of 3.8km/s. Find the maximum height attained by the particle. Radius of earth=6400km and g at the surface=9.8m/s. Consider only earth's gravitation.

Ans. At the surface of the earth, the potential energy of the earth-particle system is $\frac{GMm}{R}$ with usual symbol. The kinetic energy is 1/2 mv^2 where $v_0 = 9.8km/s$. At the maximum height the kinetic energy is zero. If the maximum height reached is H, the potential energy of the earth-particle system at this instant is $-\frac{GMm}{R+H}$. Using conservation of energy , $-\frac{GMm}{R} + \frac{1mv^2}{2} = -\frac{GMm}{R+H}$

Writing GM=g$R^2 and$ dividing by m,

$$-gR+\frac{v_0^2}{2}=\frac{-gR^2}{R+H}$$

$$\frac{R^2}{R+H}=R-\frac{v_0^2}{2g}$$

$$R+H=\frac{R^2}{R-\frac{v_0^2}{2g}}$$

Putting the value of R,$v_0 and$ g on right side,

$$R+H=\frac{(6400\ km)^2}{6400-\frac{(9.8km/s)^2}{2\ X\ 9.8s^{-2}}}$$

$$=27300km$$

H = (27300 - 6400)km =20900km

3. Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the center.

Ans. A small section of rod is considered at 'x' distance mass of the element = (M/L). dx = dm

$dE_1=\frac{G(dm)X1}{(d^2+x^2)}\quad =2.\frac{G(dm)}{(d^2+x^2)}.\frac{d}{\sqrt{(d^2+x^2}}=\frac{2GMd\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$

Total gravitational field

E=$\int_0^{L/2}\frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$

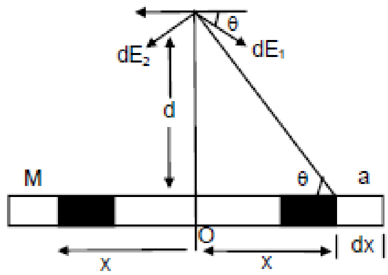
Integrating the above equation it can be found that,

$$E=\frac{2GM}{d\sqrt{L^2+4d^2}}$$

Resultant dE = 2 dE_1 sin θ

$$= 2 \times \frac{G(dm)}{(d^2+x^2)} \times \frac{d}{\sqrt{(d^2+x^2}} = \frac{2\times GM\times d\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$$

Total gravitational field



$$E=\int_0^{L/2} \frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$$

Integrating the above equation it can be found that,

$$E = \frac{2GM}{d\sqrt{L^2 + 4d^2}}$$

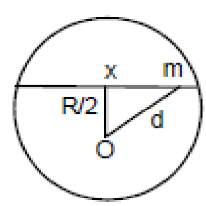
Q4.A tunnel is dug along a diameter of the earth. Find the force on a particle of mass m placed in the tunnel at a distance x from the centre.

Ans. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (\frac{1}{2})\sqrt{4x^2 + R^2}$$

M be the mass of the earth, M' the mass of the sphere of radius d/2.

Then M = (4/3) π$R^3\rho$



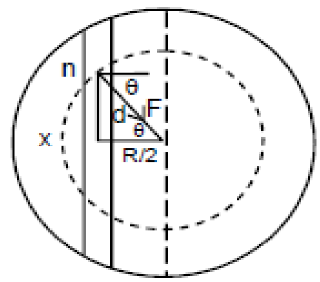
M' = (4/3)πd^3

Or $\frac{M'}{M} = \frac{d^3}{R^3}$

 Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

So, Normal force exerted by the wall = F cosθ



$$\frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2}$$

Therefore I think normal force does not depend on x.

Q5. (a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation.

(b)If the satellite is directly above the north pole at some instant , find the time it takes to come over equatorial plane. Mass of the earth=$6X10^{24}kg$

Ans.(a) Angular speed f earth & the satellite will be same

$$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$$

Or

$$\frac{1}{24\,X\,3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

Or $12|3600 = 3.14\sqrt{\frac{(R+h)^3}{gR^2}}$

Or $\frac{(R+h)^2}{gR^2} = \frac{(12\,X\,3600)^2}{(3.14)^2}$

Or $\frac{(6400+h)^3 X\ 10^9}{9.8X(6400)^2 X10^6} = \frac{(12X3600)^2}{(3.14)^2}$

Or $\frac{(6400+h)^3 X10^9}{6272X10^9} = 432 \times 10^4$

Or $(6400 + h)^3 = 6272 \times 432 \times 10^4$

Or $6400 + h = (6272 \times 432 \times 10^4)^{1/3}$

Or $h = (6272 \times 432 \times 10^4)^{\frac{1}{3}} - 6400$

$= 42300\ m.$

(b)Time taken from north pole to equator = (1/2) t

$$= \left(\frac{1}{2}\right) X6.28 \sqrt{\frac{(43200 + 6400)^3}{10\ X\ (6400)^2\ X\ 10^6}} = 3.14 \sqrt{\frac{(497)^3 X\ 10^6}{(64)^2\ X\ 10^{11}}}$$

$$= 3.14 \sqrt{\frac{497\ X\ 497\ X\ 497}{64\ X\ 64\ X\ 10^5}} = 6\ hour.$$

MECHANICS OF SOLID AND FLUID

- **Deforming force**:- A force acting on a body which produces change in its shape of body instead of its state of rest or uniform motion of the body.
- **Elasticity:**-The property of matter by virtue which it regains its original shape and size, when the deforming forces are removed is called elasticity.
- **Plasticity**:- The inability of a body to return to its original shape and size, when the deforming forces are removed is called plasticity.
- **Hooke's law**:- when a wire is loaded within elastic limit, the extension produced in wire is directly proportional to the load applied.

  OR

  Within elastic limit stress α strain

  $$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

- **Stress** :- Restoring force set up per unit area when deforming force acts on the body

  $$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

  S.I Unit of stress = N/m^2 or Pascal (Pa)

  Dimensional formula = $M^a L^b T^c$

Types of stress:-

Stress →
- Normal stress →
  - Tensile stress(When there is an increase in dimension of the body along the direction of force)
  - Compression stress(when there is decrease in dimension)
- Tangential stress (When deforming force acts tangential to the surface of body)

Strain:- The ratio of change in dimension to the original dimension is called strain

It has no unit

Types of strain:-

- Longitudinal strain= $\frac{\text{Change in Length}}{\text{Original Length}}$
- Volumetric strain = $\frac{\text{Change in Volume}}{\text{Original Volume}}$
- Sharing Strain = φ =$\frac{\Delta L}{L}$, Where ΔL =displacement of the face on which force is applied and L is the height of the face

Hooke's L aw:- Within elastic limit, stress α strain

$$\frac{\text{Strss}}{\text{Strain}} = \text{Constant (Modulus of Elasticity)}$$

Modulus of elasticity are of 3 types.

(1) **Young's Modulus (Y)** = $\frac{Normal\ stress}{Longitudinal\ Strain}$

(2) **Bulk Modulus (K)** = $\frac{Normal\ stress}{Volumetric\ Strain}$

(3) **Modulus of rigidity modulus (η)** =$\frac{Tangential\ stress}{Shearing\ \ Strain}$

- **Compressibility** : the reciprocal of bulk modulus of a material is called its compressibility

  Compressibility = 1/K

  Stress – Strain- diagram
- **Proportionality limit(P)** – The stress at the limit of proportionality point P is known as proportionality limit
- **Elastic limit** - the maximum stress which can be applied to a wire so that on unloading it return to its original length is called the elastic limit
- **Yield point(Y)-** The stress, beyond which the length of the wire increase virtually for no increase in the stress
- **Plastic region**- the region of stress- strain graph between the elastic limit and the breaking point is called the plastic region.

- **Fracture point or Breaking point(B)-** the value of stress corresponding to which the wire breaks is called breaking point
- **Work done in stretching a wire per unit volume/energy sored per unit volume of specimen**

  = ½ x stress x strain
- **Elastic after effect:-** The delay in regaining the original state by a body after the removal of the deforming force is called elastic after effect.
- **Elastic fatigue**:- the loss in strength of a material caused due to repeated alternating strains to which the material is subjected.
- **Poisson's ratio(ϭ)** :- The ratio of lateral strain to longitudinal strain is called

$$\text{Poisons ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

- Relation between Y,K,¶, ϭ
  1. Y=3K(1-2 ϭ)
  2. Y=2¶(1+ ϭ)
  3. $ϭ = \frac{3k-2¶}{2¶+6k}$
  4. $\frac{9}{\gamma}$ = 1/K +3/¶
- Applications of elasticity

1. Metallic part of machinery is never subjected to a stress beyond the elastic limit of material.
2. Metallic rope used in cranes to lift heavy weight are decided on the elastic limit of material
3. In designing beam to support load (in construction of roofs and bridges)
4. Preference of hollow shaft than solid shaft
5. Calculating the maximum height of a mountain

MECHANICS OF FLUID

- **Pressure** :The force/threat acting per unit area is called pressure

  S.I Unit of pressure is N/M^2 or pascal (Pa)

Dimensional formula ($ML^{-1}T^{-2}$)

- **Pascal's law:**- Pressure applied to an enclosed fluid is transmitted to all part of the fluid and to the wall of the container.
- **Application of Pascal's law:-**

  (1) Hydraulic lift, presses etc.

  (2) Hydraulic brakes
- **Pressure exerted by liquid column**:- $P = h\rho g$, where h= depth of liquid, ρ=density , g=acc$_n$. due to gravity.
- **Variation of pressure with depth**: $P = P_a + h\rho g$, where P_a =atmospheric pressure
- **Atmospheric pressure**:- The pressure exerted by atmosphere is called atmospheric pressure.

  At sea level, atmospheric pressure= 0.76m of Hg column

  Mathematically $1 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^{-2}$
- **Archimedes' principle**:- It states that when a body is immersed completely or partly in a fluid, it loses in weight equal to the weight of the fluid displaced by it.

  Mathematically: Apparent weight = True weight – $V\rho g$

  Where V is volume of fluid displaced, ρ is its density.
- **Viscosity**:- It is the property of liquid (or gases) due to which a backward dragging force acts tangentially between two layers of liquid when there is relative motion between them.
- **Newton's formula for Viscous force**:- the viscous force between two liquid layer each of area A and having a velocity gradient dv/dx is

  $\mathbf{F = \eta A (dv/dx)}$, where **η** is coefficient of viscosity
- **Coefficient of viscosity**:- It is define as the tangential viscous force which maintains a unit velocity gradient between two parallel layers each of unit area

  S.I unit of coefficient of viscosity is poiseuille or pascal-second

- **Poiseuille's equation**:- when a liquid of coefficient of viscosity flows through a tube of length 'l' and radius r, then the volume of liquid following out per second is given

$$V = \pi P r^4 / 8\,\eta l \;,$$

Where P is the difference of pressure between the two ends of the tube.

- **Stoke's law**: The backward dragging force acing on a small sphere of radius r falling with uniform velocity *v* through a medium of coefficient of viscosity is given by

$$F = 6\pi\,\eta r v$$

- **Terminal velocity**:- It is the maximum constant velocity acquired by the body while falling freely in a viscous medium

  The terminal velocity v of a spherical body of radius r and density ϭ while falling freely in a viscous medium of viscosity ¶ , density is given by

$$V = \frac{2}{q}\ \frac{r2}{\eta}\,(\sigma - \rho)g$$

- **Stream line**:- It is the path, straight or curved, the tangent at any point to which given the direction of the flow of liquid at that point
- **Tube of flow**:- A tube of flow is a bundle of stream lines having the same velocity of fluid elements over any cross section perpendicular to the direction of flow
- **Stream line flow**:- the flow of the liquid in which each molecule of the liquid passing through a point travels along the same path and with the some velocity as the preceeding molecule passing through the same point
- **Laminar flow**:- the flow of liquid, in which velocity of the layer varies from maximum at the axis to minimum for the layer in contact with the wall of the tube is called laminar flow.
- **Turbulent flow**:- It is the flow of liquid in which a liquid moves with a velocity greater than its critical velocity. The motion of the particles of liquid becomes disorderly or irregular.

- **Critical velocity**:- It is that velocity of liquid flow, upto which the flow of liquid is streamlined and above which its flow becomes turbulent. Critical velocity of a liquid (V_c) flowing through a tube is given by

$$\mathbf{V_c = K\eta / \rho\, r}$$

Where **ρ** is the density of liquid following through a tube of radius r and **η** the coefficient of viscosity of liquid

- **Reynold's umber**:- It is a pure number which determines the nature of flow of liquid through a pipe

Quantitatiively Renold's number N = **ρ** D V_c/ **η**

Where **η** is coefficient of viscosity of liquid , **ρ** is density of liquid D is the diameter of the tube, V_c is critical velocity

For stream line flow, Reynold's number <2000

For turbulent flow, Reynold's number > 3000

For uncertain flow, 2000<Reynold's number<3000

- **Theorem of continuity** : If there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant

Mathematically $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$

It is called the equation of continuity

For in compressible liquid $\rho_1 = \rho_2$ Therefore the equation continuity becomes

$$a_1 v_1 = a_2 v_2$$

Bernoulli's theorem:- It states that for an in compressible non-viscous liquid in steady flow, the total energy i.e. pressure energy, potential energy and kinetic energy remains constant its flow.

Mathematically $\frac{P}{\rho} + gh + \frac{1}{2} v^2$ = Constant

$$\frac{P}{\rho g} + h + \frac{v2}{2g} = \text{Constant}$$

The term $\frac{P}{\rho g}$, h and $\frac{v2}{2g}$ are called pressure head, gravitational head and velocity head respectively.

- **Application of Bernoull's theorem**
  - (i) Working of Bunsen burner
  - (ii) Lift of an air foil
  - (iii) Spinning of ball (Magnus effect)
  - (iv) Sprayer
  - (v) Ping pong ball in air jet.

- **Toricelli's theorem/speed of efflux**:-It states that the velocity of efflux i.e. the velocity with which the liquid flows out of an orifice (i.e. a narrow hole) is equal to that which is freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

  Quantitatively velocity of efflus

$$V = \sqrt{2gh}$$

Venturimeter:- It is a device use to measure the rate of flow of liquid. Venturimeter consists of a wide tube having a constriction in the middle. If a_1 and a_2 are the areas of cross section of the wide end and the threat, p_1 and p_2 are the pressure of liquid, then velocity of the liquid entering at the wide end is given by $V_1=a_2 \sqrt{2(P_1-P_2)\ \rho(a_1^2-a_2^2)}$

- **Surface tension (T)** :- It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contract so as to occupy a minimum surface area

  Mathematically $T = F/l$

  S.I Unit is : Nm^{-1} Dimensional formula : ML^0T^{-2}

  Surface Energy : The potential energy per unit area of the surface film is called the surface energy.

$$\text{Surface energy} = \frac{Work\ done\ in\ increasing\ the\ surface\ area}{Increase\ in\ area}$$

Surface tension is numerally equal to surface energy

- **Excess of pressure inside a drop and double**:- There is excess of pressure on concave side of a curved surface
  1. Excess of pressure inside a liquid drop = $2T/R$
  2. Excess of pressure inside a liquid bubble = $4T/R$
  3. Excess of pressure inside an air bubble = $2T/R$, Where T is the surface tension , R = radius of liquid drop
- **Angle of contact**:- The angle which the tangent to the free surface of the liquid at the point of contact makes with the wall of the containing vessel, is called the angle of contact

  For liquid having convex meniscus, the angle of contact is obtuse and for having concave meniscus, the angle of contact is acute.
- **Capillary tube**:- A tube of very fine bore is called capillary tube
- **Capillarity**:-The rise or fall of liquid inside a capillary tube when it is dipped in it is called capillarity
- **Ascent formula**:- when a capillary tube of radius 'r' is dipped in a liquid of density *s* and surface tension T, the liquid rises or depresses through a height,

$$H = 2T\cos\theta \ /r \rho \ g$$

  There will be rise a liquid when angle of contact θ is acute. There will be fall in liquid when angle of contact θ is obtuse.

<u>Thermal expansion and calorimetry</u>

- Heat- it is a form of energy, which produce in us the sensation of warmth
- Temperature:- The degree of hotness or coldness of a body is called temperature
- Thermometer- It is a device used to measure the temperature of a body
- Scales of temperature:- there are four scales of temperature. Given below is scales of temp with lower and upper fixed point

  <u>Temperature scales</u> <u>Lower fixed point (Melting point office)</u> <u>Upper fixed point (Boiling point of water)</u>

1. Celsius	0^0C	100^0C
2. Fahrenheit	32^0F	212^0F
3. Reamur	0^0R	80^oR
4. Kelvin	273K	373K

- **Relation between the various temperature scales**

  If C,F, R and K are temperature of a body on Celsius, Fahrenheit , Reumer and Kelvin scale, then

  $$\mathbf{C/5 = F\text{-}32/9 = R/4 = K\ \text{-}273\ /5}$$

- **Thermal expansion**:- all solid expands on heating. There are three types of expansion.

  (1) Liner expansion- When a solid rod of initial length 'l' is heated through a temperature ΔT then its new length $\mathbf{l' = l(1+\alpha\Delta T)}$, where α is called coefficient of liner expansion

  (2) Superficial expansion- when a solid of initial surface area A is heated through temperature then its new Area is $\mathbf{A' = A\,(1 + \beta\Delta T)}$, where β is coefficient of superficial expansion

  (3) Cubical expansion- when a solid of initial volume V is heated through a temperature ΔT then its new volume is $\mathbf{V' = V\,(1 + \Upsilon\Delta T)}$,where ϒ is the coefficient of cubical expansion.

- Relation between α ,β and ϒ

  $$\mathbf{\alpha = \beta/2 = \Upsilon/3}$$

- In case of liquid $\mathbf{\Upsilon_r = \Upsilon_a + \Upsilon_g}$

  Where Υ_r=Coefficient of real expansion of a liquid

  Υ_a= Coefficient of apparent expansion of liquid

  Υ_g= Coefficient of cubical expansion of the vessel

  Thermal capacity = It is the amount of heat required to raise its temperature through one degree

- **Water equivalent** :- It is the mass of water which absorbs or emits the same amount of heat as is done by the body for the same rise or fall in temperature. It is represented by W = mc

- **Specific heat** :- It is the amount of heat required to raise the temperature of unit mass of substance through unit degree Celsius

$$C = \Delta Q / m\Delta T$$

- **Latent heat** :- It is define as the quantity of heat required to change the unit mass of the substance from its one state completely to another state at constant temperature

  Mathematically Q = ML

- *Types of Latent heat* → Latent heat of fusion (L_f) ; Latent heat of vaporization (L_v)

- **Calorimeter** :- Device used for measuring heat
- **Principle of calorimetry** :- Heat loss by hot body = Heat gain by cold body
- **Transfer of heat** :- there are three modes by which heat transfer takes place

  (1) **Conduction**:- It is the process by which heat is transmitted from one point to another through a substance in the direction of fall of temperature without the actual motion of the particles of the substance. When two opposite faces of a slab, each of cross section A and separated by a distance d are maintained at temperature T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t

  $Q = K A (T_1 - T_2)t/d$ Where K is coefficient of thermal conductivity of the mater

- **Coefficient of thermal conductivity**:- It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept at one degree difference of temperature

  S.I unit of coefficient of thermal conductivity : $J\ S^{-1}m^{-1}K^{-1}$ or $W\ m^{-1}K^{-1}$

  (2) **Convection**:- It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance.

  (3) **Radiation**:- It is the process by which heat is transmitted from one place to another without heating the intervening medium

- **Newton's laws of cooling**:- It states that the rate of loss of heat or rate of cooling of a body is directly proportional to the temperature difference between the body and the surrounding, provided the temperature difference is small

  Mathematically $-dQ/dt = K(T-T_0)$
- **Perfect black body**:- It is a body which absorbs heat radiations of all the wavelengths, which fall on it and emits the full radiation spectrum on being heated.
- **Stefan's law:-** It states that the total amount of heat energy radiated per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the substance of the body

  Mathematically $E \ \alpha \ T^4$

  $E = \Sigma T^4$ Where is called Stefan's constant

  It's value is $5.67 \times 10^{-8}\ JS^{-1}m^{-2}k^{-4}$

Wein's displacement law:- According to this law, the wavelength λ_m of maximum intensity of emission of black body radiation is inversel y proportional to absolute temperature (T) of black body.

$$\lambda_m \ \alpha \ \frac{1}{T}$$

$\lambda_m T = b$ where b is wien's constant

Questions with **(mark) are HOTs Question

<u>1 MARK QUESTIONS</u>

Q.1 A wire is stretched by a force such that its length becomes double. How will the Young's modulus of the wire be affected?

Ans. Young's modulus remains the same.

Q.2 How does the Young's modulus change with rise in temperature?

Ans. Young's modulus of a material decreases with rise in temperature.

Q.3 Which of the three modulus of elasticity – Y, K and η is possible in all the three states of matter (solid, liquid and gas)?

Ans. Bulk modulus (K)

Q.4 The Young's modulus of steel is much more than that for rubber. For the same longitudinal strain, which one will have greater stress?

Ans. Stress= Y X longitudinal strain. So steel will have greater stress.

Q.5 Which of the two forces – deforming or restoring is responsible for elastic behavior of substance?

Ans. Restoring force.

Q.6. Which mode of transfer of heat is the quickest?

Ans. Radiation.

** Q. 7 A boat carrying a number of large stones is floating in a water tank. What will happen to the level of water if the stones are unloaded into the water?

Ans. The level of water will fall because the volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

Q.8. A rain drop of radius r falls in air with a terminal velocity v. What is the terminal velocity of a rain drop of radius 3r ?

Ans. $v = \frac{2r^2(\varsigma-\sigma)g}{9\eta}$ $\quad v \propto r^2$

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2 \quad \rightarrow \quad v_2 = \left(\frac{3r}{r}\right) v_1^2 = 9v_1$$

**Q. 9 When air is blown in between two balls suspended close to each other , they are attracted towards each other. Why?

Ans. On blowing air between the two balls, the air velocity increases, decreasing pressure. The pressure on the outer side of the ball being more will exert forces on the balls, so they move towards each other.

Q.10. Why does air bubble in water goes up?

Ans. The terminal velocity , $v = \frac{2r^2(\varsigma-\sigma)g}{9\eta}$ As the density of air ς is less than density of water σ, the terminal velocity is negative. For this reason air bubbles moves upward.

2 MARKS QUESTIONS

Q.11 Steel is more elastic than rubber. Explain.

Ans. Consider two wire, one of steel and another of rubber having equal length L and cross sectional area A . When subjected to same deforming force F, the extension produce in steel is l_S and in rubber is l_R such that $l_R > l_S$.

Then $Y_s = \frac{FL}{Al_s}$ and $Y_r = \frac{FL}{Al_r}$

$\frac{Y_s}{Y_r} = \frac{l_r}{l_s}$

As $l_s < l_r \rightarrow Y_s > Y_r$

Hence steel is more elastic.

Q.12. A wire stretches by a certain amount under a load. If the load and radius are both increased to four times, find the stretch caused in the wire.

Ans. For a wire of radius r stretched under a force F,

$Y = \frac{FL}{\pi r^2 L}$ or $l = \frac{FL}{\pi r^2 Y}$

Let l' be the extension when both the load and the radius are increased to four times, Then, $l' = \frac{4F \; X \; L}{\pi(4r)^2 L} = \frac{FL}{4\pi r^2 Y} = \frac{l}{4}$

Q. 13. Calculate the percentage increase in the length of a wire of diameter 2mm stretched by a force of 1kg F. Young's modulus of the material of wire is 15 X $10^{10}Nm^{-2}$.

Ans. F = 1 Kg F = 9.8N Y = 15 X $10^{10}Nm^{-2}$ r = $\frac{2}{2}$ = 1mm = 10^{-3}m

Cross section of wire , $\pi r^2 = \pi \; X \; (10^{-3})^2 = \pi \; X \; 10^{-6} m^2$

Now $Y = \frac{FL}{al}$

$$\frac{l}{L} = \frac{F}{aY} = \frac{9.8}{\pi \; X \; 10^{-6} \; X \; 15 \; X \; 10^{10}} = 2.1 \; X \; 10^{-5}$$

Percentage increase = 2.1 X 10^{-5} X 100 = 0.0021%

Q. 14. The pressure of a medium is changed from 1.01 X 10^5 pa to 1.165 X 10^5 pa and changed in volume is 10% keeping temperature constant. Find the bulk modulus of the medium.

Ans. Here $\Delta p = 1.165 \; X \; 10^5 - 1.01 \; X \; 10^5 = 0.155 \; X \; 10^5$pa

$$\frac{\Delta V}{V} = 10\ \% = 0.1$$

Now $K = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{0.155 \; X \; 10^5}{0.1} = 1.55 \; X \; 10^5$ pa

Q.15. 27 identical drops of water are falling down vertically in air each with a terminal velocity of 0.15m/s. If they combine to form a single bigger drop, what will be its terminal velocity?

Ans. Let r = radius of each drop, v = 0.15 m/s

Now $v = \frac{2r2(\varsigma - \sigma)g}{9\eta}$ -------------------------------------(1)

Let R be the radius of the big drop.

Volume of big drop = Volume of 27 small drops

$$\frac{4}{3}\pi R^3 = 27 \text{ X } \frac{4}{3}\pi r^3$$

R = 3r

Let v_1 be the terminal velocity of bigger drop

$$v_1 = \frac{2R^2(\varsigma-\sigma)g}{9\eta} \text{ ------------------------------------(2)}$$

$$\frac{v_1}{v} = \frac{R^2}{r^2} = 9$$

v_1 = 9v = 9 X 0.15 = 1.35 m/s

Q.16. Water flows through a horizontal pipe line of varying cross section at the rate of $0.2m^3s^{-1}$. Calculate the velocity of water at a point where the area of cross section of the pipe is $0.02m^2$.

Ans. Rate of flow = av

$$v = \frac{Rate\ of\ flow}{a}$$

Rate of flow = $0.2m^3s^{-1}$ a = $0.02m^2$

$$v = \frac{0.2m^s s^{-1}}{0.02\mathrm{m}^2} = 10 \text{ ms}^{-1}$$

Q. 17. A cylinder of height 20m is completely filled with water. Find the efflux water (in m s-1) through a small hole on the side wall of the cylinder near its bottom. Given g = 10m/s.

Ans Here h = 20m , g = 10 m/s

Velocity of efflux , v = $\sqrt{2gh}$ = $\sqrt{2 \; X \; 10 \; X \; 20}$ = 20m/s

**Q.18. At what common temperature would a block of wood and a block of metal appear equally cold or equally hot when touched?

Ans. When touched an object appear cold if heat flows from our hand to the object. On the other hand it appears hot, if heat flows from the object towards our hand. Therefore a block of wood and block of metal will appear equally cold or equally hot if there is no exchange of heat between hand and the block. So the two blocks will appear equally cold or equally hot if they are at the same temperature as that of our hands i.e. the temperature of our body.

Q.19. A piece of chalk immersed into water emits bubbles in all directions. Why?

Ans. A piece of chalk has extremely narrow capillaries. As it is immersed in water, water rises due to capillary action. The air present in the capillaries in the chalk is forced out by the rising water. As a result bubbles are emitted from the chalk in all the directions.

3 MARKS

Q. 20. Water at a pressure of 4×10^4 Nm-2 flows at $2ms^{-1}$ through a pipe of $0.02m^2$ cross sectional area which reduces to $0.01m^2$. What is the pressure in the smaller cross section of the pipe?

Ans. $$a_1v_1 = a_2v_2$$

$$v_2 = \frac{a_1v_1}{v_2} = \frac{0.02 \ X \ 2}{0.01} = 4m/s$$

Again $$\frac{P_1}{\varsigma} + \frac{1}{2}v_1^2 = \frac{P_2}{\varsigma} + \frac{1}{2}v_2^2$$

$$P_1 = P_2 - \frac{1}{2}\varsigma\,(v_1^2 - v_2^2)$$

$$P_1 = 3.4 \times 10^4 Nm^{-2}$$

Q.21. What is surface tension and surface energy? Derive the relation between surface tension and surface energy.

Q.22. Derive equation of continuity for steady and irrotational flow of a perfectly mobile and incompressible fluid. What conclusion is drawn from it?

Q.23 What is Stoke's law? Derive the relation by the method of dimension.

Q.24. A piece of iron of mass 0.1 kg is kept inside a furnace, till it attains the temperature of the furnace. The hot piece of iron is dropped into a calorimeter containing 0.24 Kg of water at 20℃. The mixture attains an equilibrium temperature of 60℃. Find the temperature of the furnace. Given water equivalent of calorimeter = 0.01 kg and specific heat of iron = 470 J Kg^{-1} $K^{-1.}$

Ans. Let θ_1 be the temperature of the furnace i.e of the piece of iron.

Heat lost by the piece of iron $Q = M_1C_1 (\theta_1 - \theta)$

Here $M_1 = 0.1$ Kg $C_1 = 470$ J Kg^{-1} K^{-1} $\theta = 60℃$

$Q = 0.1 \text{ X } 470 (\theta_1 - 60) = 47 (\theta_1 - 60)$ ------------------------- (1)

Heat gain by water and the calorimeter , $Q = (M_2 + w) C_2 (\theta - \theta_2)$

$M_2 = 0.24$ Kg w = 0.01 Kg $\theta_2 = 20℃$ C_2 = Specific heat of water = 4200 J Kg^{-1} K^{-1}

$Q = (0.24 + 0.01) \times 4200 \text{ X } (60 – 20) = 42000$ -------------------------(2)

From (1) and (2) $47 (\theta_1 - 60) = 42000$

$\theta_1 = 953.62$ ℃

**Q. 25. Calculate the energy spent in spraying a drop of mercury of 1 cm radius into 10^6 droplets all of same size. Surface tension of mercury is 35×10^{-3} Nm^{-1}.

Ans. $T = 35 \times 10^{-3}$ Nm^{-1} $R = 1$ cm

Let r be the radius of each small drop, when the original drop is spitted into 10^6 small drops.

Then $10^6 \text{ X } \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$$r = 10^{-2} R$$

$$r = 10^{-2} \text{ X } 1 = 10^{-2} \text{ cm}$$

Initial surface area of the original drop = $4\pi R^2$ = 4π X 1^2 = $4\pi cm^2$

Final surface area of the 10^6 small drops = 10^6 X $4\pi r^2$ = 10^6 X 4π X $(10^{-2})^2$ = $400\,\pi$ cm^2

Therefore increase in surface area = $400\,\pi - 4\pi$ = $396\,\pi cm^2$ = 396π X $10^{-4}m^2$

Therefore energy spent = T X increase in surface area = 35 X 10^{-3} X 396π X 10^{-4} = 4.354 X 10^{-3}N

Q.26. A liquid takes 10 minutes to cool from 70℃ to 50℃. How much time will it take to cool from 60℃ to 40℃ ? The temperature of the surrounding is 20℃.

Ans. 1^{st} case $\theta_1 = 70^0C$ $\theta_2 = 50^0C$ $\theta_0 = 20℃$ t = 10 minutes

Using $\frac{\theta_1 - \theta_2}{t} = k\,(\frac{\theta_{1+\theta_2}}{2} - \theta_0)$, we get

$$\frac{20}{10} = k\,(60 - 20) = 40\,k$$

$$K = \frac{1}{20}$$

For 2^{nd} case $\theta_1 = 60^0C$ $\theta_2 = 40^0C$ $\theta_0 = 20℃$ t = ?

Using $\frac{\theta_1 - \theta_2}{t} = k\,(\frac{\theta_{1+\theta_2}}{2} - \theta_0)$, we get

$$\frac{20}{t} = \frac{1}{20}\,(50 - 20) = \frac{3}{2}$$

$$t = \frac{40}{3} = 13.33 \text{ minutes}$$

**Q. 28. A slab of stone of area $0.36 m^2$ and thickness of 0.1m is exposed to the lower surface of steam at 100℃. A block of ice at 0℃ rest on the upper surface of the slab. In one hour 4.8 Kg of ice is melted. Calculate the thermal conductivity of stone.

Ans. Here $A = 0.36 m^2$, d = 0.1m , $T_1 - T_2 = 100 - 0 = 100$℃ t = 1hr = 3600 sec

Mass of ice melted M = 4.8 Kg

We know Latent heat of ice $L = 336 \times 10^3 \, J \, Kg^{-1}$

Heat required to melt the ice $Q = ML = 4.8 \times 336 \times 10^3 = 1.613 \times 10^6$ K

Now $Q = \frac{KA(T_1 - T_2)t}{d}$

$$1.613 \times 10^6 = \frac{K \times 0.36 \times 100 \times 3600}{0.1}$$

$K = 1.245 \, wm^{-10}C^{-1}$

<u>5 MARKS</u>

Q. 28. Define capillarity and angle of contact. Derive an expression for the ascent of liquid inside a capillary tube where it is dipped in a liquid.

Q. 29. Show that there is always excess of pressure on the concave side of the meniscus of a liquid. Obtain the expression for the excess of pressure inside (i) a liquid drop (ii) liquid bubble.

Q. 30. State and prove the Bernoulli's principle. Give two practical application of it.

Q.31. Define terminal velocity. Show that the terminal velocity v of a sphere of radius r, density ς falling vertically through a viscous fluid of density **σ** and coefficient of viscosity η is given by

$$v = \frac{2(\varsigma - \sigma)r^2 g}{\eta}$$

Q. 32. State and explain Hooke's law. A wire is fixed at one end and is subjected to increasing load at the other end. Draw a curve between stress and strain. With the help of the curve, explain the term elastic limit, yield point, breaking point and permanent set. How this curve does may be used to distinguish between ductile and brittle substances.

THERMODYNAMICS

- **Thermal Equilibrium**:- Two systems are said to be in thermal equilibrium with each other if they have the same temperature.
- **Thermo dynamical system**:- An assembly of large numbers of particles having same temperature, pressure etc is called thermo dynamical system.
- **Thermodynamic variables** :- The variables which determine the thermodynamic behavior of a system are called thermodynamic variables
- **Zeroth law of Thermodynamics** :- IT states that if two system A and B are in thermal equilibrium with a third system C , then the two system A and B are also in thermal equilibrium with each other.
- **Thermodynamic Process** :- A thermodynamic process is said to be taking place , if the thermodynamic variable of the system change with time.
- **Types of thermodynamic Process**:-

  (1) *Isothermal process* – process taking place at constant temperature.

  (2) *Adiabatic process* – process where there is no exchange of heat.

  (3) *Isochoric process* – process taking place at constant volume

  (4) *Isobaric process* –Process taking place at constant Pressure.

  (5) *Cyclic process*:- Process where the system returns to its original state.
- **Equation of state** : A relation between pressure, volume and temperature for a system is called its equation of state .
- **Indicator diagram (P-V diagram**) :- The graphical representation of the state of a system with the help of two thermodynamical variables is called indicator diagram of the system.
- **Internal energy of a gas** :- It is the sum of kinetic energy and the intermolecular potential energy of the molecules of the gas. Internal energy is a function of temperature.
- **First law of Thermodynamics** :- It states that if an amount of heat dQ I added to a system , a part of heat is used in increasing its internal energy while the remaining part of heat may be used up as the external work done dW by the system.

Mathematically $dQ=dU+dW$

$$dQ=dU+PdV.$$

- **Work done during expansion / compression of gas**:- When the volume of gas changes from V_1 to V_2 , the work done is given by W= $\oint_{V1}^{V2} \mathrm{PdV}$ = Area under the P -V diagram.

- **Thermodynamical operations are**

(1) **Isothermal process** : A thermodynamic process that takes place at constant temperature is called an isothermal process.

- Equation of state for isothermal process : PV = constant.
- Work done during an isothermal process

$$W_{iso} = RT \log_e \frac{V2}{V1} = 2.303\, RT \log_e \frac{V2}{V1}$$

(2) **Adiabatic process** : A thermodynamic process that takes place in such a manner that no heat enters or leaves the system is called adiabatic process

➔ Equation of state for adiabatic process

(i) PV^{γ} = constant (ii) $TV^{\gamma-1}$ =constant (iii) $\frac{P^{\gamma-1}}{T^{\gamma}}$ =constant

➔ Work done during adiabatic change

$$W_{adia} = \frac{R(T1-T2)}{(\gamma-1)}$$

- **Reversible process** :- It is a process in which the system can be retraced to its original state by reversing the condiditions.
- **Irreversible process**:- It is a process in which the system cannot be retraced to its original state by reversing the conditions.

- **Second law of thermodynamics:**

- ➔ **Kelvin's statement of second law** – It is impossible to derive a continous supply of work by cooling a body to a temperature lower than that of the coldest of its surrounding.
- ➔ **Clausius statement of second law** – It is impossible for a self –acting machine unaided by any external agency to transfer heat from a body to another body at higher temperature.

- **Heat Engine** – a heat engine is a device for converting heat energy continuously into a mechanical work.
  - ➔ Component of heat engine- (i) source of heat (ii) Sink (iii) Working substance
- **Efficiency of heat Engine** :-It is defined as the ratio of the external work obtained to the amount of heat energy absorbed from the heat source.

  Mathematically

$$\eta = \frac{W}{Q1} = \frac{Q1 - Q2}{Q1} = 1 - \frac{Q2}{Q1}$$

- **Carnot's heat Engine** :- it is an ideal heat Engine which is based on carnot's reversible cycle.

  Efficiency of carnot's heat Engine

$$\eta = 1 - \frac{Q2}{Q1} = 1 - \frac{T2}{T1}$$

- **Refrigerator or Heat pump**:- it is heat engine working backward.
- **Co-efficient of performance** : It is the ratio of heat absorbed from cold body to the work done by the refrigerator.

Mathematically $\beta = \frac{Q2}{W} = \frac{Q2}{Q1-Q2} = \frac{T2}{T1-T2}$

All Questions with **(mark) are HOTs Question

Q1 Which Thermodynamical variable is defined by the first law of thermodynamics? 1

Ans: Internal energy.

Q2 What is the amount of work done in the Cyclic process? 1

Ans:It is numerically equal to the area of the cyclic process.

Q3 Out of the parameters- temperature, pressure,work and volume, which parameter does not

Characterize the thermodynamics state of matter? 1

Ans: Work

Q4 What is the nature of P-V diagram for isobaric and isochoric process? 1

Ans: The P-V diagram for an isobaric process is a straight line parrel to the volume axis while that

For an isochoric process is a straight line parallel to pressure axis.

Q5 On what factors does the efficiency of Carnot engine depends? 1

Ans: Temperature of the source of heat and sink.

** Q6 Can we increase the temperature of gas without supplying heat to it? 1

Ans: Yes, the temperature of gas can be by compressing the gas under Adiabatic condition.

Q7 Why does the gas get heated on compression? 1

Ans: Because the work done in compressing the gas increases the internal energy of the gas.

Q8 Which thermodynamic variable is defined by Zeroth law of thermodynamics? 1

Ans: Temperature

Q9 Can the whole of work be converted into heat? 1

Ans: Yes ,Through friction.

Q10 In a Carnot engine, temperature of the sink is increased. What will happen to its efficiency? 1

Ans: We know $\eta = 1 - \frac{T2}{T1}$

On increasing the temperature of the sink (T_2), the efficiency of the Carnot engine will decrease

**Q11 If hot air rises , why is it cooler at the top of mountain than near the sea level ? 2

Ans: Since atmospheric pressure decreases with height, pressure at the top of the mountain is lesser. When the hot air rises up,it suffer adiabatic expansion at the top of the mountain.For an adiabatic change,first law of thermodynamics may be express as

$$dU + dW = 0 \qquad (dQ = 0)$$

$$dW = -dU$$

Therefore work done by the air in rising up ($dW = +ve$) result in decrease in the internal

Energy of the air ($dU = -ve$) and hence a fall in the temperature.

Q12 What happen to the internal energy of a gas during (i) isothermal expansion (ii) adiabatic Expansion? 2

Ans: In isothermal expansion ,temperature remains constant.Therefore internal energy which is a function of temperature will remain constant.

(ii)for adiabatic change $dQ = 0$ and hence first law of thermodynamics becomes

$$0 = dU + dW$$

$$dW = - dU$$

During expansion, work is done by the gas i.e. dW is positive. Hence ,dU must be negative.

Thus ,in an adiabatic expansion , the internal energy of the system will decrease.

Q13.Air pressure in a car increases during driving. Explain Why? 2

Ans: During driving as a result of the friction between the tyre and road ,the temperature of

The tyre and the air inside it increases. Since volume of the tyre does not change, due to increase in temperature ,pressure of the increases (due to pressure law).

Q14 The efficiency of a heat engine cannot be 100%. Explain why ? 2

Ans: The efficiency of heat engine $\eta = 1 - \frac{T2}{T1}$

The efficiency will be 100% or 1, if $T_2 = 0$ K.

Since the temperature of 0 K cannot be reached, a heat engine cannot have 100% efficiency.

Q15 In an effort to cool a kitchen during summer, the refrigerator door is left open and the kitchen door and windows are closed. Will it make the room cooler ?

Ans: The refrigerator draws some heat from the air in front of it. The compressor has to do some

Mechanical work to draw heat from the air at lower temperature. The heat drawn from the air together with the work done by the compressor in drawing it, is rejected by the refrigerator with the help of the radiator provided at the back to the air. IT follows that in each cycle, the amount of heat rejected to the air at the back of the refrigerator will be greater than that is drawn from the air in front of it. Therefore temperature of the room will increase and make hotter.

Q16 Why cannot the Carnot's engine be realised in practice?

Ans: Because of the following reasons

(i) The main difficulty is that the cylinder should come in contact with the source,sink and stand again and again over a complete cycle which is very difficult to achieve in practice.

(ii) The working substance should be an ideal gas however no gas fulfils the ideal gas behaviour.

(iii) A cylinder with a perfectly frictionless piston cannot be realised

Q17 A slab of ice at 273K and at atmospheric pressure melt.(a) What is the nature of work done on

The ice water system by the atmosphere?(b)What happen to the internal energy of the ice- Water system?

Ans: (a) The volume of the ice decreases on melting. Hence the work done by the atmosphere on The ice – water system is positive in nature.

(b) Since heat is absorbed by the ice during melting, the internal energy of the ice-water system increases.

Q18 Why is the conversion of heat into work not possible without a sink at lower temperature? 2

Ans:For converting heat energy into work continuosly, a part of the heat energy absorbed from the source has to be rejected.The heat energy can be rejected only if there is a body whose

Temperature is less than that of the source. This body at lower temperature is called sink.

** Q19 Can water be boiled without heating ? 2

Ans:Yes, water can be boil without heating. This is done by increasing the pressure on the surface of water inside a closed insulated vessel. By doing so, the boiling point of the water decreases to the room temperature and hence starts boiling.

Q20 What are the limitations of the first law of thermodynamics ? 2

Ans: The limitations are --- (i) It does not tells us the directions of heat transfer

(ii) it does not tell us how much of the heat is converted into work.

(iii)it does not tell us under what conditions heat is converted into work.

**Q21 Calculate the fall in temperature when a gas initially at 72^0C is expanded suddenly to eight times its original volume. Given γ= 5/3. 3

Ans: Let V_1 = x cc V_2 =8x cc

T_1 =273+72 =345 K ¥ = 5/3 , T_2 = ?

Using the relation $T_1 V_1^{¥-1} = T_2 V_2^{¥-1}$

Therefore $T_2 = T_1 (V_1/V_2)^{¥-1}$

$=345x(1/8)^{2/3}$

Taking log of both sides, we get

Log T_2 = log 345 – 2/3 log 8

=2.5378 – 2/3(0.9031)

= 2.5378 -0.6020 = 1.9358

Or T_2 =86.26 K

Therefore the fall in temperature = 345 -86.26 258.74 K

Q22 A Carnot engine whose source temperature is at 400K takes 100 Kcal of heat at this temperature in each cycle and gives 70 Kcal to the sink. Calculate (i) the temperature of the sink

(ii) the efficiency of the engine.

Ans: Here T_1 =400K , Q_1 = 100 Kcal , Q_2 = 70 Kcal

T_2 = ? ,¶ = ?

(i) $Q_1/Q_2 = T_1/T_2$

Or $T_2 = (Q_2/Q_1)T_1$

Or T_2 =70/100 x400

Or T_2 = 280 K

(ii) $\eta= 1- T_2/T_1$

= 1 – 280/400

$= 1- 0.7 = 0.3$

Or % of $\eta = 0.3 \times 100 = 30\ \%$

Q23 If at 50^oC and 75 cm of mercury pressure, a definite mass of gas is compressed (i)slowly

(iii) suddenly, then what will be the final pressure and temperature of the gas in each case, if the final volume is one fourth of the initial volume? Given $\gamma = 1.5$

Ans:(I) When the gas is compressed slowly, the change is isothermal.

Therefore $P_2 V_2 = P_1 V_1$

$P_2 = P_1V_1/V_2$

$= (75x\ V_1/V_1)x4 = 300$ cm of mercury

Temperature remains constant at 50^oC

(ii)When the gas is compressed suddenly, the change is adiabatic

As per $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

$P_2 = P_1(V_1/V_2)^{\gamma}$

$= 75 \times (4)^{1.5} = 600$ cm of Hg

Also $T_2 V_2^{¥-1} = T_1 V_1^{¥-1}$

$T_2 = T_1 (V_1/V_2)^{¥-1} = 323x(4)^{(1.5-1)} = 646K$

$= 646-273 = 373\ ^oC$

Q24 Two engines A and B have their sources at 400K and 350 K and sink at350K and 300K

Respectively. Which engine is more efficient and by how much? 3

Ans: For engine A $T_1 = 400K$, $T_2 = 350$ K

Efficiency $\eta_A = 1 - T_2/T_1$

$= 1-350/400 = 1/8$

% of $\eta_A = 1/8 \times 100 = 12.5\%$

For Engine B $T_1 = 350$ K , $T_2 = 300$ K

Efficiency $\eta_B = 1 - T_2/T_1$

$= 1 - 300/350 = 1/7$

% of $\eta_B = 1/7 \times 100 = 14.3\%$

Since $\eta_B > \eta_A$ so engine A is much more efficient than engine B by (14.3% - 12.5%) = 1.8%

** Q25 Assuming a domestic refrigerator as a reversible heat engine working between melting point

Of ice and the room temperature at 27^oC, calculate the energy in joule that must be supplied to freeze 1Kg of water at 0^oC. 3

Ans: Here $T_1 = 27+273 = 300K$, $T_2 = 0 + 273 = 273$

Mass of water to be freezed , M = 1 Kg = 1000g

Amount of heat that should be removed to freeze the water

$Q_2 = ML = 1000X\ 80$ cal

$= 1000X80\ X4.2 = 3.36 \times 10^5$ J

Now $Q_1 = (T_1/T_2)X\ Q_2 = (300/273)X3.36x10^5 = 3.692\ x10^5$ J

Therefore energy supplied to freeze the water

$W = Q_1 - Q_2 = 3.693x10^5 - 3.36\ x10^5$

$= 3.32\ x10^5$ J

** Q26 A refrigerator freezes 5Kg of water at 0^oC into ice at 0^oc in a time interval of 20 minutes. Assume that the room temperature is 20^oC, calculate the minimum power needed to accomplish it.

Ans : Amount of heat required to convert water into ice at 0^oC,

$Q_2 = mL = (5Kg)x(80)Kcal/Kg$

$= 400$ Kcal

Now $T_1 = 20^oC = 273+20 = 293K$

$T_2 = 0^oc$ $0+273 = 273$ K

We know that $Q_2/W = T_2/(T_1 - T_2)$

Or $W = Q_2 \times (T_1 - T_2)/T_2$

$= 400x(293- 273)/273$

$= 29.3$ Kcal $= 29.3x4.2 \times 10^3 J$

$=123x10^3$ J

Time t = 20 min =20x60 =1200s

Power needed $P = W/t = 123x10^3/1200$

$=102.5$ W

**Q27 The temperature T_1 and T_2 of two heat reserviour in an ideal carnot engine are 1500^oC and

500^oC. Which of this increasing the temperature T_1 by 100^oC or decreasing T_2 by 100^oC would result in greater improvement of the efficiency of the engine?

Ans: Using ¶ $=1 - T_2/T_1 = (T_1 - T_2)/T_1$

(1)increasing T_1 by 100^oC ¶$_1$ = (1600 -500)/(1600 + 273)

= 1100/1873 = 59 %

(ii) Decreasing T_2 by 100^0C ¶$_2$ = 1500 –(500-100)/(1500 +273)

=1100/1773 = 67%

Therefore decreasing T_2 by 100^oC results in greater improvement of efficiency.

Q28 State the first law of thermodynamics and discussed the application of this law to the boiling process. 3

Q29 What is thermodynamic system ? Prove that work done by thermodynamic system is equal to the area under P-V diagram. 3

30 Prove that $C_p - C_v = R$, for an ideal gas . 3

Q31 What is isothermal process / State two essential conditions for such a process to takes place. Show analytically that the work by one mole of an ideal gas during volume expansion from V_1 V_2 at temperature T is given by
$W = RT \log_e V_2/V_1$ 5

Q32 Define an adiabatic process. State two essential conditions for such a process to takes place.Derive an expression for adiabatic process to takes place. 5

Q33 Discuss the four steps of Carnot's cycle and show that the efficiency is given by ¶ = $1 - T_2/ T_1$, Where T_1 and T_2 are the temperature of the source and sink respectively. 5

Q34 Describe the working of refrigerator as heat pump. Derive the expression of its coefficient of performance. If the door of a refrigerator is kept open for a long time ,will it make the room warm or cool ? 5

Q35 What is the need of introducing the second law of thermodynamics ? State the Kelvin –Planck and Claussius statement of second law of thermodynamics and show that both the statement are equivalent. 5

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KINETIC THEORY OF GASES

Boyle's Law: *At constant temperature volume of given mass of gas is inversely proportional to its pressure.*

$$V \,\alpha\, \frac{1}{P} \text{ or } PV = \text{constant}$$

Charle's Law: *At constant pressure volume of a given mass of gas is directly proportional to its absolute temperature.*

$$V \,\alpha\, T \text{ or } \frac{V}{T} = \text{constant}$$

*For 1° rise in temp.

$$V_t = Vo(1 + \frac{t}{273.15})$$

Gay Lussac's Law: At constant volume, pressure of a given mass of gas is directly proportional to its absolute temp.

$$\frac{P}{T} = \text{constant.}$$

For 1^0C rise in temperature $P_t = P_o\,(1 + \frac{t}{273.15})$

Ideal Gas Equation: for n mole of gas

PV=nRT,

for 1 mole , PV=RT

Universal gas constant: R = 8.31 J $mol^{-1}K^{-1}$

Boltzmann constant: $k_B = \frac{R}{N_A}$**where k_B = Boltzmann constant, N_A = Avogadro's no.**

Ideal gas: A gas which obeys gas law strictly is an ideal or perfect gas. The molecules of such a gas are of point size and there is no force of attraction between them.

Assumptions of Kinetic Theory of Gases

1. All gases consist of molecules which are rigid, elastic spheres identical in all respect for a given gas.

2. The size of a molecule is negligible as compared with the average distance between two molecules.

3. During the random motion, the molecules collide with one another and with the wall of the vessel.The collisions are almost instantaneous.

4. The molecular density remains uniform throughout the gas.

5. The collisions are perfectly elastic in nature and there are no forces of attraction or repulsion between them.

Pressure exerted by gas:

$$P = \frac{1}{3}.\frac{M}{V}\overline{v^2} = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}mn\overline{v^2}$$

Where: n=no. of molecules per unit volume.

m=mass of each molecule.

$\overline{v^2}$ =mean of square speed.

V = Volume

M = mass of gas

Average Kinetic energy of a gas: If M is molecular mass and V is molecular volume and m is mass of each molecule. Then

1. Mean K.E per mole of a gas,

$$E = \frac{1}{2}M\overline{v^2} = \frac{3}{2}PV = \frac{3}{2}RT = \frac{3}{2}K_B N_A T$$

2. Mean K.E per molecule of a gas,

$$\vec{E} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}k_B T$$

3. K.E of 1gram of gas,

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{RT}{M_0}$$ M_0 gram molecular weight

Avogadro Law: Equal volume of all gases under similar condition of temp. and pressure contain equal number of molecules.

Avogadro Number:

$$N_A = 6.0225 \times 10^{23} \text{ mol}^{-1}$$

Graham's Law of diffusion:

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

r = rate of diffusion

ρ = density

Delton's law of partial pressure: Total pressure exerted by a mixture of non-reacting gases occupying a given volume is equal to the sum of partial pressures which gas would exert if it alone occupied the same volume at given temp.

$$P = P_1 + P_2 + P_3 + \dots\dots$$

Average Speed :- $\bar{V} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$

$$\bar{v} = \sqrt{\frac{8k_bT}{\pi m}} \qquad = \sqrt{\frac{8RT}{\pi M_0}}$$

Root mean square:

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

Most probable speed:

$$V_{mp} = \sqrt{\frac{2K_BT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

Relation between :$\bar{v}$, $\overline{v_{rms}}$&v_{mp}

$$\bar{v} = 0.92v_{rms}, v_{mp} = 0.816v_{rms}$$

$$v_{rms} : \bar{v} : v_{mp} = 1.73 : 1.6 : 1.41$$

Therefore: $v_{rms} > \bar{v} > v_{mp}$

Degree of freedom:

f = 3N-k

where , f = no. of degree of freedom.

N = no. of of atoms in a molecule. k = no. of independent relation between the atoms.

1. Monoatomic gas – 2 degree of freedom.
2. Diatomic gas – 5 degree of freedom.

Law of equipartion of energy: For any thermodynamical system in thermal equilibrium, the energy of the system is equally divided amongst its various degree of freedom and energy associated with each degree of freedom corresponding to each molecule is $\frac{1}{2}$ K_B T , where K_B is the Boltzmann's constant and T is absolute temperature.

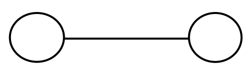
- The law of equipartition of energy holds good for all degrees of freedom whether translational , rotational or vibrational.
- A monoatomic gas molecule has only translational kinetic energy

$$E_t = 1/2mV_x^2 + 1/2mV_y^2 + 1/2mV_z^2 = 3/2K_BT$$

So a monoatomic gas molecule has only three (translational) degrees of freedom.

- In addition to translational kinetic energy, a diatomic molecule has two rotational
  Kinetic energies

$$E_t + E_r = 1/2mV_x^2 + 1/2mV_y^2 + 1/2mV_z^2 + 1/2I_YW_y^2 + 1/2I_zW_z^2$$



Here the line joining the two atoms has been taken as x-axis about which there is no rotation. So, the degree of freedom of a diatomic molecule is 5, it does not vibrate.

At very high temperature, vibration is also activated due to which two extra degree of freedom emerge from vibrational energy. Hence at very high temperature degree of freedom of diatomic molecule is seven.

***(**Each translational and rotational degree of freedom corresponds to one mole of absorption of energy and has energy $1/2k_BT$).

Internal Energies & specific heats of monoatomic, diatomic& polyatomic gases:

1. If 'f' is degree of freedom then for a gas of polyatomic molecules energy associated with 1 mole of gas,

$$U = \frac{f}{2}RT \quad , \quad C_v = \frac{f}{2}R$$

$$C_p = \left(1 + \frac{f}{2}\right) R, \;\; \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

2. For a monoatomic gas f=3,

$$U = \frac{3}{2}RT \quad , \quad C_v = \frac{3}{2}R$$

$$C_p = \frac{5}{2}\ R, \qquad \gamma = 1.66$$

3. For a diatomic gas with no vibrational mode f=5, so

$$U = \frac{5}{2}RT \quad , \quad C_v = \frac{5}{2}R$$

$$C_p = \frac{7}{2}\ R, \qquad \gamma = 1.4$$

4. For a diatomic gas with vibrational mode f=7, so

$$U = \frac{7}{2}RT \quad , \quad C_v = \frac{7}{2}R$$

$$\mathrm{Cp} = \frac{9}{2}\ \mathrm{R} \;\; , \gamma = 1.28$$

Meanfree path: It is the average distance covered by a molecule between two successive collisions. It is given by,

$$\overline{\lambda} = \frac{1}{\sqrt{2}(n\pi d^2)}$$

Where,n is no. density and 'd' is diameter of the molecule.

Brownian Equation :-The zig-zag motion of gas molecules is Brownian motion which occurs due to random collision of molecules.

Memory Map

Kinetic Theory of gases

1. $V_{rms}=\sqrt{\frac{3p}{\rho}}$ 3. $V_{rms}=\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$

2. $E=\frac{3}{2}\ RT$ 4. $V \propto \sqrt{T}$

Law of Equipartion of Energy

$\frac{1}{2}\ mv_x{}^2=\frac{1}{2}\ mv_y{}^2$

$= \frac{1}{2}\ mv_{2=}{}^2 = \frac{1}{2}\ k_BT$

$P=\frac{1}{3}\ P\overrightarrow{C^2}$

Mean free Path

$\bar{\lambda} = \frac{1}{\sqrt{2}\ n\pi d^2}$

Specific Heats

$r = 1 + \frac{2}{f}$ where $r=\frac{C_p}{C_v}$

and f=degree of freedom

(1 Marks Question)

1. What type of motion is associated with the molecules of a gas?

Ans:- Brownian motion.

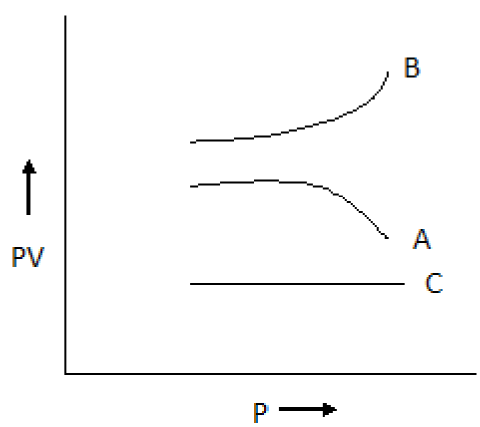
2. On which factors does the average kinetic energy of gas molecules depend?

Ans:- The average K.E. of a gas molecule depends only on the absolute temperature of the gas and is directly proportional to it.

3. Why do the gases at low temperature and high pressure, show large deviations from ideal behaviour?

Ans:- At low temperature and high pressure , the intermolecular attractions become appreciable. So, the volume occupied by the gas molecules cannot be neglected in comparison to the volume of the gas. Hence the real gases show large from ideal gas behaviour.

4. Following fig. shows the variation of the product PV with respect to the pressure (P) of given masses of three gases, A,B,C. The temperature is kept constant. State with proper arguments which of these gases is ideal.



Ans:- Gas 'C' is ideal because PV is constant for it. That is gas 'C' obeys Boyle's law at all pressures.

5. When a gas is heated, its temperature increases. Explain it on the basis of kinetic theory of gases.

Ans:- When a gas is heated, the root mean square velocity of its molecules increases. As $V_{rms} \alpha \sqrt{T}$ so temperature of the gas increases.

6. The ratio of vapour densities of two gases at the same temperature is 8:9. Compare the rms. velocity of their molecules?

Ans :- $\frac{(Vrms)1}{(Vrms)2} = \sqrt{\frac{M2}{M1}} = \sqrt{\frac{\rho 2}{\rho 1}} = \sqrt{\frac{9}{8}} = 3:2\sqrt{2}$

7. Cooking gas containers are kept in a lorry moving with uniform speed. What will be the effect on temperature of the gas molecules?

Ans:- As the lorry is moving with a uniform speed, there will be no change in the translational motion or K.E. of the gas molecules. Hence the temperature of the gas will remain same.

8. What is the mean translational kinetic energy of a perfect gas molecule at temperature T?

Ans:- A perfect gas molecule has only translational K.E.

$$E = 3/2\ k_B T$$

9. Name two factors on which the degrees of freedom of a gas depend?

Ans:- (i) Atomicity of the gas molecule.

(ii) Shape of the molecule.

(iii) Temperature of gas.

10. Define absolute zero, according to kinetic interpretation of temperature?

Ans:- Absolute zero is the temperature at which all molecular motion ceases.

(2 Marks question)

1. Write the relation between the pressure and kinetic energy per unit volume of a gas. Water solidifies into ice at 273 K. What happens to the K.E. of water molecules?

Ans:- P = 2/3 E. The K.E. of water molecules gas partly converted into the binding energy of the ice.

2. The absolute temperature of a gas is increased 4 times its original value. What will be the change in r.m.s. velocity of its molecules?

Ans:- $V_{rms} \alpha \sqrt{T}$

$V'_{rms} \alpha \sqrt{4T}$

$V'_{rms}/ V_{rms} = 2$

$V'_{rms} = 2V_{rms}$

Change in rms velocity of molecules = $V'_{rms} - V_{rms}$

$= V_{rms}$

3.What will be the ratio of the root mean square speeds of the molecules of an ideal gas at 270K and 30K?

Ans :- $V_{rms}/ V'_{rms} = \sqrt{\frac{T}{T'}} = \sqrt{\frac{270}{30}} = 3 : 1$

4.A mixture of Helium and Hydrogen gas is filled in a vessel at 30 degree Celsius. Compare the root mean square velocities of the molecules of these gases at this temperature.

(atomic weight of Hydrogen is 4)

Ans :- $(V_{rms})He/(V_{rms})H_2 = \{(M_{H2})/(M_{He})\}^{1/2} = \sqrt{\frac{2}{4}} = 1 : 2\sqrt{2}$

5.The velocities of three molecules are 3V,4V and 5V.Determine the root mean square velocity.

Ans:- $V_{rms} = \sqrt{\frac{50}{3}}V = 4.08V$

6.Write the equation of state for 16g of O_2**.**

Ans :- No. of moles in 32g of O_2 = 1

No. of moles in 16g of O_2 = 1/9 x 16 = ½

As pv = nRT and n=1/2

So, PV= $\frac{1}{2}$ RT

7.Should the specific heat of monoatomic gas be less than, equal to or greater than that of a diatomic gas at room temperature? Justify your answer.

Ans :- Specific heat of a gas at constant volume is equal to f/2R.

For monoatomic gases f = 3 so C_v = 3/2 R.

For diatomic gases f = 5 so C_v = 5/2 R.

Hence the specific heat for monoatomic gas is less than that for a diatomic gas.

8. A gas in a closed vessel is at the pressure P_o. If the masses of all the molecules be made half and their speeds be made double, then find the resultant pressure?

Ans:- $P_o = \frac{1}{3} \frac{mN}{V} \overline{V^2} \qquad = \frac{1}{3} \frac{mN}{2V} (2V)^2 = 2P_0$

9. A box contains equal number of molecules of hydrogen and oxygen. If there is a fine hole in the box, which gas will leak rapidly? Why?

Ans :- $V_{rms} \propto \frac{1}{\sqrt{M_0}}$

Hence hydrogen gas will leak more rapidly because of its smaller molecular mass.

10. When a gas filled in a closed vessel is heated through 1^0C, its pressure increases by 0.4 %. What is the initial temperature of the gas?

Ans:- P\` = P = 0.4/100. P, T\` = T + 1

By Gay Lussac's law P/T = (P + 0.4/100.P)/T + 1,

$$\frac{P}{T} = \left(P + \frac{.4}{100}\,P\right) \div \ (T+1)$$

$$\frac{(\mathrm{P+.004P})}{\mathrm{T+1}} = \frac{\mathrm{P(1.004)}}{\mathrm{T+1}}$$

T+1= (1.004)T

1=.004T

T=250K

(3 Marks Questions)

1. Show that rms velocity of O_2 is $\sqrt{2}$ times that of SO_2. Atomic wt. of Sulphur is 32 and that of oxygen is 16.

Ans. $V \propto \frac{1}{\sqrt{M}}$. $\frac{V_{O_2}}{V_{SO_2}} = \sqrt{\frac{64}{32}} = \sqrt{2}$

Or $v_{O_2} = \sqrt{2}\ SO_2$.

2. Calculate the temperature at which rms velocity of SO_2 is the same as that of Oxygen at $27℃$.

Ans. For O_2, $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R \times 300}{32}}$

For SO_2, $V_{rms} = \sqrt{\frac{3R\acute{T}}{M_0}} = \sqrt{\frac{3R \times \acute{T}}{64}}$

As $V_0 = V$ $\therefore \sqrt{\frac{3R\acute{T}}{64}} = \sqrt{\frac{3R \times 300}{32}}$

$\acute{T}$ = 600t = 600 – 273 = 327℃.

3. Calculate the total no. of degrees of freedom possessed by the molecules in $1cm^3$ of H_2 gas at NTP

Ans. No. of H_2 Molecules in 22.4 liters or 22400 cm^3 at NTP $= 6.02 \times 10^{23}$.

$\therefore$ No. of H_2 Molecules in 1 cm^3 at NTP $= \frac{6.02 \times 10^{23}}{22400} = 2.6875 \times 10^{19}$.

No. of degrees of freedom associated with each H_2 (a diatomic) molecule = 5

$\therefore$ Total no. of degree of freedom associated with 1cm^3 gas

$$= 2.6875 \times 10^{19} \times 5 = 1.3475 \times 10^{20}.$$

4. Derive Boyle's law on the basis of Kinetic Theory of Gases.

5. Derive Charles's law on the basis of Kinetic Theory of Gases.

6. State Dalton's law of partial pressures. Deduce it from Kinetic Theory of Gases.

7. Using the expression for pressure exerted by a gas, deduce Avogadro's law and Graham's law of diffusion.

8. State the number of degree of freedom possessed by a monoatomic molecule in space. Also give the expression for total energy possessed by it at a given temperature. Hence give the total energy of the atom at 300 K.

9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of helium gas atom at $-20℃$? Atomic mass of argon = 39.9 u and that of helium = 4.0 u.

Ans. Root mean square speed for argon at temperature T

$$V = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

Root mean square speed for helium at temp. $20℃$ is

$$\acute{V} = \sqrt{\frac{3R \times 253}{4}}$$

As V = $\acute{V}$ so we have $\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$

$$= \frac{T}{39.9} = \frac{253}{4} \quad \text{or } T = \frac{253 \times 39.9}{4}$$

$$T = 2523.7 \text{ K}$$

10. From a certain apparatus the diffusion rate of Hydrogen has an average value of 28.7 cm^3 s^{-1}; the diffusion of another gas under the same conditions is measured to have an average rate of 7.2cm^{3}s^{-1}. Identify the gas.

Ans. From Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_1}{M_2}}$$

$$M_2 = \left(\frac{r_1}{r_2}\right)^2 M_1 = \left(\frac{28.7}{7.2}\right)^2 \times 2$$

$$= 31.78 \approx 32$$

Thus the unknown gas is Oxygen.

(Long Questions)

11. Prove that the pressure exerted by a gas is P = $\frac{1}{3}\rho\overline{c^2}$ where ρ is the density and c is the root mean square velocity.

12.What are the basic assumptions of Kinetic Theory of Gases? On their basis derive an expression for the pressure exerted by an ideal gas.

Oscillations and Waves

- **Periodic Motion**: A motion which repeats itself over and over again after a regular interval of time.
- **Oscillatory Motion**: A motion in which a body moves back and forth repeatedly about a fixed point.
- **Periodic function**: A function that repeats its value at regular intervals of its argument is called periodic function. The following sine and cosine functions are periodic with period T.

  $f(t) = \sin\frac{2\pi t}{T}$ and $g(t) = \cos\frac{2\pi t}{T}$

  These are called Harmonic Functions.

  Note :- All Harmonic functions are periodic but all periodic functions are not harmonic.

One of the simplest periodic functions is given by

$f(t) = A\cos\omega t \quad [\omega = 2\pi/T]$

If the argument of this function ωt is increased by an integral multiple of 2π radians, the value of the function remains the same. The function f(t) is then periodic and its period, T is given by

$T = \frac{2\pi}{\omega}$

Thus the function f(t) is periodic with period T

$f(t) = f(t + T)$

Linear combination of sine and cosine functions

$f(t) = A\sin\omega t + B\cos\omega t$

A periodic function with same period T is given as

$A = D\cos\phi$ and $B = D\sin\phi$

$\therefore f(t) = D \sin(\omega t + ø)$

$\therefore D = \sqrt{A^2 + B^2}$ and $ø = \tan^{-1}\frac{x}{a}$

- **Simple Harmonic Motion (SHM):** A particle is said to execute SHM if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from mean position and is always directed towards mean position.

  Restoring Force ∝ Displacement

  $$F \propto x$$

  $$F = -kx$$

  Where 'k' is force constant.
- **Amplitude**: Maximum displacement of oscillating particle from its mean position.

  $x_{Max} = \pm A$
- **Time Period:** Time taken to complete one oscillation.
- **Frequency:** $= \frac{1}{T}$. Unit of frequency is Hertz (Hz).

  $$1 \text{ Hz} = 1\ s^{-1}$$
- **Angular Frequency:** $\omega = \frac{2\pi}{T} = 2\pi\nu$

  $$\text{S.I unit } \omega = \text{rad } s^{-1}$$
- **Phase:**
  1. The Phase of Vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.

     It is denoted by ø.
  2. **Initial phase or epoch**: The phase of particle corresponding to time t = 0.

     It is denoted by ø.
- **Displacement in SHM :**

  $$x = A\cos(\omega t + ø_0)$$

  Where, x = Displacement,

  A = Amplitude

  ωt = Angular Frequency

  $ø_0$ = Initial Phase.

Case 1: When Particle is at mean position x = 0

$$v = -\omega\sqrt{A^2 - 0^2} = -\omega A$$

$$v_{max} = \omega A = \frac{2\pi}{T} A$$

Case 2: When Particle is at extreme position x = $\pm A$

$$v = -\omega\sqrt{A^2 - A^2} = 0$$

<u>Acceleration</u>

Case 3: When particle is at mean position x = 0,

acceleration = $-\omega^2(0)$ = 0.

Case 4: When particle is at extreme position then

$x = A$ acceleration = $-\omega^2 A$

- **Formula Used** :

1. $x = A\cos(\omega t + \phi_0)$

2. $v = \frac{dx}{dt} = -\omega\sqrt{A^2 - x^2}$, $v_{max} = \omega A$.

3. $a = \frac{dv}{dt} = \omega^2 A\cos(\omega t + \phi_0)$

$$= -\omega^2 x$$

$a_{max} = \omega^2 A$

4. *Restoring force* $F = -kx = -m\omega^2 x$

Where k = force constant & $\omega^2 = \frac{k}{m}$

5. *Angular freq.* $\omega = 2\pi\nu = 2\pi/T$

6. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Displacement}}{\textbf{Acceleration}}} = 2\pi\sqrt{\frac{x}{a}}$

7. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Inertia Factor}}{\textbf{Spring Factor}}} = 2\pi\sqrt{\frac{m}{k}}$

8. P.E at displacement 'y' from mean position

$E_P = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \sin^2\omega t$

9. K.E. at displacement 'y' from the mean position

$E_K = \frac{1}{2}k(A^2 - y^2) = \frac{1}{2}m\omega^2(A^2 - y^2)$

$= \frac{1}{2}m\omega^2A^2\cos^2\omega t$

10. Total Energy at any point

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi^2mA^2\nu^2$

11. Spring Factor $K = F/y$

12. Period Of oscillation of a mass 'm' suspended from a massless spring of force constant '*k*'

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For two springs of spring factors k_1 and k_2 connected in parallel effective spring factor

$$k = k_1 + k_2 \qquad \therefore T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

13. For two springs connected in series, effective spring factor 'k' is given as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad Or \quad k = \frac{k_1k_2}{k_1+k_2}$$

$$T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

Note:- When length of a spring is made 'n' times its spring factor becomes $\frac{1}{n}$ times and hence time period increases $\sqrt{n}$ times.

14. When spring is cut into 'n' equal pieces, spring factor of each part becomes '*nk*'.

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

15. Oscillation of simple pendulum

$$T = 2\pi\sqrt{l/g}$$

$$\nu = \frac{1}{2\pi}\sqrt{g/l}$$

16. For a liquid of density ρ contained in a U-tube up to height 'h'

$$T = 2\pi\sqrt{h/g}$$

17. For a body dropped in a tunnel along the diameter of earth

$T = 2\pi\sqrt{R/g}$, where **R** = Radius of earth

18. **Resonance**: If the frequency of driving force is equal to the natural frequency of the oscillator itself, the amplitude of oscillation is very large then such oscillations are called resonant oscillations and phenomenon is called resonance.

Waves

Angular wave number: It is phase change per unit distance.

i.e. $k = \frac{2\pi}{\lambda}$, S.I unit of k is radian per meter.

Relation between velocity, frequency and wavelength is given as :- $V = \upsilon\lambda$

Velocity of Transverse wave:-

(i)In solid molecules having modulus of rigidity 'η' and density 'ρ' is

$$V = \sqrt{\frac{\eta}{\rho}}$$

(ii)In string for mass per unit length 'm' and tension 'T' is $V = \sqrt{\frac{T}{m}}$

Velocity of longitudinal wave:-

(i) in solid $V = \sqrt{\frac{Y}{\rho}}$, Y= young's modulus

(ii) in liquid $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

(iii) in gases $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

According to Newton's formula: When sound travels in gas then changes take place in the medium are isothermal in nature. $V = \sqrt{\frac{P}{\rho}}$

According to Laplace: When sound travels in gas then changes take place in the medium are adiabatic in nature.

$V = \sqrt{\frac{P\gamma}{\rho}}$ 'Where $\gamma = \frac{Cp}{Cv}$

Factors effecting velocity of sound :-

(i) Pressure – No effect

(ii) Density $- V \alpha \frac{1}{\sqrt{\rho}}$ or $\frac{V1}{V2} = \sqrt{\frac{\rho 1}{\rho 2}}$

(iii) Temp-$V \alpha \sqrt{T}$ or $\frac{V1}{V2} = \sqrt{\frac{T1}{T2}}$

Effect of humidity :– sound travels faster in moist air

(iv) Effect of wind –velocity of sound increasing along the direction of wind.

Wave equation:– if wave is travelling along + x-axis

(i) Y=A sin ($\omega t - kx$), Where, $k = \frac{2\pi}{\lambda}$

(ii) Y=A sin $2\pi\ (\frac{t}{T} - \frac{x}{\lambda})$

(iii) Y=A sin $\frac{2\pi}{T}$ (vt-x)

If wave is travelling along –ve x- axis

(iv) Y=A sin ($\omega t + kx$) ,Where ,$k = \frac{2\pi}{\lambda}$

(v) Y=A sin $2\pi\ (\frac{t}{T} + \frac{x}{\lambda})$

(vi) Y=A sin $\frac{2\pi}{T}$ (vt+x)

Phase and phase difference

Phase is the argument of the sine or cosine function representing the wave.

$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Relation between phase difference ($\Delta\phi$) and time interval (Δt) is $\boldsymbol{\Delta\phi = -\frac{2\pi}{T}\Delta t}$

Relation between phase difference (Δp) and path difference (Δx) is $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

Equation of stationary wave:-

(1) Y_1 =a sin $2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (incident wave)

$Y_1 = \pm$ a sin $2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$ (reflected wave)

Stationary wave formed

$Y = Y_1 + Y_2 = \pm$ 2a cos $\frac{2\pi x}{\lambda}$ sin $\frac{2\pi t}{T}$

(2) For (+ve) sign antinodes are at x= $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

And nodes at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

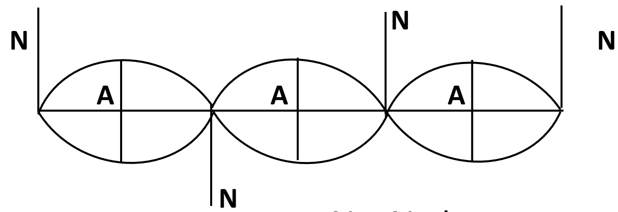
(3) For (-ve) sign antinodes are at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

Nodes at x= $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

(4)Distance between two successive nodes or antinodes are $\frac{\lambda}{2}$ and that between nodes and nearest antinodes is $\frac{\lambda}{4}$.

(5) Nodes-point of zero displacement-

Antinodes- point of maximum displacement-



N = Nodes

A = Antinodes

Mode of vibration of strings:-

a) $\upsilon = \frac{p}{2L}\sqrt{\frac{T}{m}}$ Where ,T= Tension

M= mass per unit length

υ = frequency, V=velocity of second, p = 1, 2, 3, …

b) When stretched string vibrates in P loops $\upsilon_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\upsilon$

c) For string of diameter D and density ρ

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

d) Law of length $\nu \propto \frac{1}{L}, \nu L = constant$

ORGANPIPES

1. In an organ pipe closed at one end only odd harmonics are present

$\upsilon_1 = \frac{V}{4L}$ (fundamental)

$\upsilon_2 = 3\upsilon$ (third harmonic or first overtone)

$\upsilon_3 = 5\upsilon$

$\upsilon_n = (2n-1)\,\upsilon$

2. In an open organ pipe at both ends both odd and even harmonics are present.

$\upsilon'_1 = \frac{V}{2L} = \upsilon'$ (first harmonic)

$\upsilon'_2 = 2\upsilon'$ (second harmonic or first overtone)

$\upsilon'3 = 3\upsilon$ ’

$\upsilon'n = (2n-1)\ \upsilon$ ’

3. Resonance tube: If L_1 and L_2 are the first and second resonance length with a tuning fork of frequency ‘ υ ’then the speed of sound. v = $4v(L_1 + 0.3D)$

Where ,D=internal diameter of resonance tube

$$v = 2v(L_2 - L_2)$$

End correction=0.3D= $\frac{L2 - L1}{2}$

Beats formation

1. Beat frequency = No. of beats per second = Difference in frequency of two sources.

$$b = \nu_1 - \nu_2$$

2. $\nu_2 = \nu_1 \pm b$
3. If the prong of tuning fork is filed, its frequency increases. If the prong of a tuning fork is loaded with a little way, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

Doppler effect in sound

1. If V, Vo, Vs, and Vm are the velocity of sound , observes, source and medium respectively, then the apparent frequency

$$\upsilon 1 = \frac{V + Vm - Vo}{V + Vm - Vs} \times \upsilon$$

2. If the medium is at rest (v_m= o),then

$$\upsilon' = \frac{V - Vo}{V - Vs} \times \upsilon$$

3. All the velocity are taken positive with source to observer ($S \rightarrow O$) direction and negative in the opposite ($O \rightarrow S$) direction

(Questions)

(1 marks questions)

1. Which of the following relationships between the acceleration ‘a’ and the displacement ‘x’ of a particle involve simple harmonic motion?

(a) $a=0.7x$ (b) $a=-200x^2$ (c) $a = -10x$ (d) $a=100x^3$

Ans: - (c) reprent SHM.

2. Can a motion be periodic and not oscillatory?

Ans: - Yes, for example, uniform circular motion is periodic but not oscillatory.

3. Can a motion be periodic and not simple harmonic? If your answer is yes, give an example and if not, explain why?

Ans:- Yes, when a ball is doped from a height on a perfectly elastic surface ,the motion is oscillatory but not simple harmonic as restoring force F=mg=constant and not $F\alpha -x$, which is an essential condition for S.H.M.

4. A girl is swinging in the sitting position. How will the period of the swing change if she stands up?

Ans:-The girl and the swing together constitute a pendulum of time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the girl stands up her centre of gravity is raised. The distance between the point of suspension and the centre of gravity decreases i.e. length ‘l’ decreases .Hence the time period ‘T’ decreases.

5. The maximum velocity of a particle, executing S.H.M with amplitude of 7mm is 4.4 m/s. What is the period of oscillation?

Ans: - $V_{max}=\omega A=\frac{2\pi}{T}A$, $T=\frac{2\pi A}{\text{Vmax}}=\frac{2\times22\times.007}{7\times4.4}=0.01s$

6. Why the longitudinal wave are also called pressure waves?

Ans: - Longitudinal wave travel in a medium as series of alternate compressions and rare fractions i.e. they travel as variations in pressure and hence are called pressure waves.

7. How does the frequency of a tuning fork change, when the temperature is increased?

Ans: -As the temperature is increased, the length of the prong of a tuning fork increased .This increased the wavelength of a stationary waves set up in the tuning fork. As frequency,

$\upsilon = \frac{1}{\lambda}$, So the frequency of tuning fork decreases.

8. An organ pipe emits a fundamental node of a frequency 128Hz. On blowing into it more strongly it produces the first overtone of the frequency 384Hz. What is the type of pipe –Closed or Open?

Ans: - The organ pipe must be closed organ pipe, because the frequency the first overtone is three times the fundamental frequency.

9. All harmonic are overtones but all overtones are not harmonic. How?

Ans: -The overtones with frequencies which are integral multiple of the fundamental frequency are called harmonics. Hence all harmonic are overtones. But overtones which are non-integrals multiples of the fundamental frequency are not harmonics.

10. What is the factor on which pitch of a sound depends?

Ans: - The pitch of a sound depends on its frequency.

(2 Marks questions)

1. At what points is the energy entirely kinetic and potential in S.H.M? What is the total distance travelled by a body executing S.H.M in a time equal to its time period, if its amplitude is A?

Ans. The energy is entirely kinetic at mean position i.e. at y=0. The energy is entirely potential at extreme positions i.e.

$$y = \pm A$$

Total distance travelled in time period T = 2A + 2A = 4A.

2. A simple pendulum consisting of an inextensible length 'l' and mass 'm' is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2. Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards?

Ans. When the lift is stationary, $T = 2\pi\sqrt{\frac{l}{g}}$

When the lift accelerates upwards with an acceleration of 4.5 m/s^2

$T' = 2\pi\sqrt{\frac{l}{g+45}}$

Therefore, the time period decreases when the lift accelerates upwards.

3. Does the function $y = \sin^2\omega t$ represent a periodic or a S.H.M? What is period of motion?

Ans. Displacement $y = \sin^2\omega t$

Velocity $v = \frac{dy}{dt} = 2\sin\omega t \times \cos\omega t \times \omega$

$v = \omega \sin 2\omega t$

Acceleration $a = \frac{dv}{dt} = \omega \times \cos 2\omega t \times 2\omega$

$a = 2\omega^2 \cos 2\omega t.$

As the acceleration is not proportional to displacement y, the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω.

∴ Time Period $T = \frac{2\pi}{Angular\ freq.} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

4. All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?

Ans. All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So these functions cannot be used to represent bounded motion like SHM.

5. A simple Harmonic Motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$. What is its time period?

Ans. $\frac{d^2x}{dt^2} = -\alpha x$ Or a = – αx

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{x}{\alpha x}} = \frac{2\pi}{\sqrt{\alpha}}$$

$$\mathbf{T = \frac{2\pi}{\sqrt{\alpha}}}$$

6. The Length of a simple pendulum executing SHM is increased by 2.1%. What is the percentage increase in the time period of the pendulum of increased length?

Ans. Time Period, $T = 2\pi\sqrt{\frac{l}{g}}$ i.e. $T \propto \sqrt{l}$.

The percentage increase in time period is given by

$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100$ (for small variation)

$= \frac{1}{2} \times 2.1\%$

= 1.05%

7. A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from x = A to x = A/2.

Ans. Displacement from mean position = $A - {}^{A}/_{2} = {}^{A}/_{2}$.

When the motion starts from the positive extreme position, y = A cosωt.

$$\therefore \frac{A}{2} = A \cos\frac{2\pi}{T}t.$$

$\cos\frac{2\pi}{T}t = ½ = \cos\frac{\pi}{3}$

or $\frac{2\pi}{T}t = \frac{\pi}{3}$

$\therefore t = {}^{T}/_{6}$

8. An open organ pipe produces a note of frequency 5/2 Hz at 15^0C, calculate the length of pipe. Velocity of sound at 0^0C is 335 m/s.

Ans. Velocity of sound at 15^0C

$V=V_0+0.61xt = 335+0.61x15 = 344.15$ m/s. (Thermal coefficient of velocity of sound wave is $.61/^0C$)

Fundamental frequency of an organ pipe

$$\upsilon = \frac{V}{4L} \quad , \quad \therefore L = \frac{V}{4\upsilon} = \frac{344.15}{4 \times 512} = 0.336m$$

9. An incident wave is represented by Y(x, t)=20sin(2x-4t).Write the expression for reflected wave

(i) From a rigid boundary

(ii) From an open boundary.

Ans.(i) The wave reflected from a rigid boundary is

$$Y (x, t) = -20\sin (2x+4t)$$

(i)The wave reflected from an open boundary is

$$Y (x, t) = 20\sin (2x+4t)$$

Explain why

(i) in a sound wave a displacement node is a pressure antinode and vice- versa

(ii) The shape of pulse gets- distorted during propagation in a dispersive medium.

Ans. (i) At a displacement node the variations of pressure is maximum. Hence displacement node is the a pressure antinode and vice-versa.

(ii)When a pulse passes through a dispersive medium the wavelength of wave changes.

So, the shape of pulse changes i.e. it gets distorted.

(3 Marks Questions)

1. The speed of longitudinal wave `V` in a given medium of density ρ is given by the formula, use this formula to explain why the speed of sound in air.

   (a) is independent at pressure

   (b) increases with temperature and

   (c) increases with humidity

2. Write any three characteristics of stationary waves.

Ans. (i) in stationary waves, the disturbance does not advance forward. The conditions of crest and trough merely appear and disappear in fixed position to be followed by opposite condition after every half time period. (ii) The distance between two successive nodes or antinodes is equal to half the wavelength. (iii) The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

3. Show that the speed of sound in air increased by .61m/s for every 1^0 C rise of temperature.

Ans. $V \alpha \sqrt{T}$

$$\frac{Vt}{Vo} = \sqrt{\frac{t+273}{0+273}}$$

$V_t = V_0 \left(1 + \frac{t}{273}\right)^{1/2} = V_0 \left(1 + \frac{1}{2} . \frac{t}{273}\right)$

$V_t = V_0 + \frac{V \circ \times t}{546}$

At, 0^0C speed of sound in air is 332 m/s.

$\therefore V_t - V_0 = \frac{332 \times t}{546}$

When t= 1^0C , $V_t - V_0$ = 0.61m/s.

4. Find the ratio of velocity of sound in hydrogen gas $\Upsilon = \frac{7}{5}$ to that in helium gas $\Upsilon = \frac{5}{3}$ at the same temperature. Given that molecular weight of hydrogen and helium are 2 and 4 respectively.

Ans. $V = \sqrt{\frac{\gamma RT}{M}}$

At constant temperature,

$\frac{V_H}{V_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \frac{M_H}{M_{He}}} = \sqrt{\frac{715}{5/3}} . \frac{4}{2}$ = 1.68.

5. The equation of a plane progressive wave is, $y = 10Sin2\pi(t - 0.005x)$ where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, y = $10Sin2\pi(t - 0.005x)$………….. (1)

Standard equation for harmonic wave is, $y = A\ Sin2\pi(\frac{t}{T} - \frac{x}{\lambda})$…………… (2)

Comparing eqn (1) & (2), $A = 10,\ \ \frac{1}{t} = 1,\ \ \frac{1}{\lambda} = 0.005$

(i) Amplitude A = 10cm

(ii) Frequency $\nu = \frac{1}{T} = 1Hz$

(iii) Wavelength $\lambda = \frac{1}{0.005} = 200cm$

(iv) Velocity v = $\nu\,\lambda$ = 1 x 200 = 200cm/s

6. Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. Y = A Sin (2π$\nu t + \phi_o$)

= 0.01 Sin $(1200\pi t + \frac{\pi}{6})$

7. The amplitude of oscillations of two similar pendulums similar in all respect are 2cm & 5cm respectively. Find the ratio of their energies of oscillations.

Ans. $\frac{E_1}{E_2} = (\frac{A_1}{A_2})^2 = \left(\frac{2}{3}\right)^2 = 4:25$

8. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

Ans. A periodic motion repeats after a definite time interval T. So,

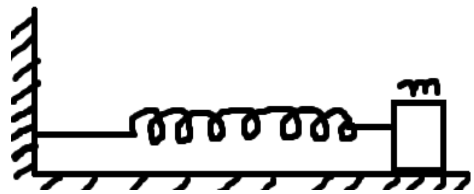
$$y(t) = y(t+T) = y(t+2T)\ etc.$$

9. A spring of force constant 1200N/m is mounted horizontal table. A mass of 3Kg is attached to the free end of the spring, pulled sideways to a distance of 2.0cm and released.

(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?



Ans. Here k = 1200N/m, m = 3Kg, A = 2cm = 2×0.01m

(i) $\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2} \times \frac{1}{3.14}\sqrt{\frac{1200}{3}} = 3.2s^{-1}$

(ii) $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1200}{3}} = 20s^{-1}$

Maximum acceleration = $\omega^2 A = (20)^2 \times 2 \times 10^{-2} = 8m/s^2$

(iii) Maximum speed = $\omega A = 20 \times 2 \times 10^{-2} = 0.40m/s$

10. Which of the following function of time represent, (a) simple harmonic (b) periodic but not SHM and (c) non periodic ?

(i) Sinωt- Cosωt (ii) $Sin^3\omega t$ (iii) $3Cos(\frac{\pi}{2} - 2\omega t)(iv)\exp(-\omega^2 t^2)$

Ans. (i) $x(t) = Sin\omega t - Cos\omega t = \sqrt{2Sin(\omega t - \frac{\pi}{2})}$, so the function is in SHM.

(ii) $x(t) = Sin^3\omega t = \frac{1}{4}(3Sin\omega t - Sin3\omega t)$, represent two separate SHM motion but their combination does not represent SHM.

(iii) $x(t) = 3Cos\left(\frac{\pi}{4} - 2\omega t\right) = 3Cos(2\omega t - \frac{\pi}{4})$, represent SHM.

(iv) $\exp(-\omega^2 t^2) = non\ periodic.$

(5 Marks Questions)

1. (a) A light wave is reflected from a mirror. The incident & reflected wave superimpose to form stationary waves. But no nodes & antinodes are seen, why?

(b) A standing wave is represented by y=2ASinKxCoswt.If one of the component wave is $y_1 = ASin(\omega t - Kx)$, what is the equation of the second component wave?

Ans. (a) As is known, the distance between two successive nodes or two successive antinodes is $\frac{\lambda}{2}$. The wavelength of visible light is of the order of $10^{-7}m$. As such as a small distance cannot be detected by the eye or by a ordinary optical instrument. Therefore, nodes and antinodes are not seen.

(b) As, $2SinACosB = Sin(A+B) + Sin(A-B)$

$$y = 2ASinKxCos\omega t$$

$$= ASin(Kx+\omega t) + ASin(Kx-\omega t)$$

According to superposition principle,

$$y = y_1 + y_2$$

$$and\ y_1 = ASin(\omega t - Kx) = -ASin(Kx - \omega t)$$

$$y_2 = y - y_1 = 2ASinKxCos\omega t + ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) + 2ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) - 2ASin(\omega t - Kx)$$

2. Discuss Newton's formula for velocity of sound in air. What correction was made to it by Laplace and why?

Ans. According to Newton the change in pressure & volume in air is an isothermal process. Therefore he calculated, $v = \sqrt{\frac{p}{\rho}}$ on substituting the require value he found, the velocity of sound was not in close agreement with the observation value. Then Laplace pointed out the error in Newton's formula. According to Laplace the change in pressure and volume is an adiabatic process. So he calculated the value of sound as, $v = \sqrt{\frac{\Upsilon r}{\rho}}$ on putting require value he found velocity of sound as 332m/s very closed to observed theory.

3. (a) What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing wave.

(b) Draw fundamental nodes of vibration of stationary wave in (i) closed pipe, (ii) in an open pipe.

4. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4.

5. Explain Doppler's effect of sound. Derive an expression for the apparent frequency where the source and observer are moving in the same direction with velocity Vs and Vo respectively, with source following the observer.

[Ans. = $\nu^{|} = \frac{v-v_o}{v-v_s} * \nu$]

6. For a travelling harmonic wave, $y = 2Cos(10t - 0.008x + 0.35)$ where x & y are in cm and t in second. What is the phase difference between oscillatory motions at two points separated by a distance of (i) 4cm (ii) 0.5m (iii)$\frac{\lambda}{2}$ $(iv)\frac{3\lambda}{4}$?

Ans.$y = 2Cos(10t - 0.008x + 0.35) \ldots\ldots\ldots.(i)$

We know, $y = ACos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right) \ldots\ldots\ldots.(ii)$

From (i) & (ii), $\frac{2\pi}{\lambda} = 0.008, \lambda = \frac{2\pi}{0,008} cm = \frac{2\pi}{0.80} m$.

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda} * path\ difference = \frac{2\pi}{\lambda} * \Delta x$.

(i) When $\Delta x = 4cm$, $\Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 4 = 3.2rad$.

(ii) When$\Delta x = 0.5m$, $\Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 0.5 = 0.40rad$.

(iii) When $\Delta x = \frac{\lambda}{2}$, $\Delta\phi = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi rad$.

(iv) When$\Delta x = \frac{3\lambda}{4}$, $\Delta\phi = \frac{2\pi}{\lambda} * \frac{3\lambda}{4} = \frac{3\pi}{2} rad$.

7. **(i)** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly exited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).

Ans. (i) For the fundamental mode,

$\lambda = 2\,L = 2 \times 100 = 200$ cm = 2m.

Frequency $\nu = 2.53$ kHz = 2530 Hz

Speed of sound, $v = \nu\lambda = 2530 \times 2 = 5060$ m/s

=5.06 km/s

(ii) Length of pipe L= 20 cm = 0.2 m

Speed of sound v= 340 m/s

Fundamental frequency of closed organ pipe

$$\nu = v/4L = \frac{340}{4\times0.2} = 425 \text{ Hz}$$ sw can be excited

Fundamental frequency of open organ pipe

$$\nu' = \frac{v}{2L} = \frac{340}{2\times0.2} = 850 \text{ Hz}$$

Hence source of frequency 430 Hz will not be in resonance with open organ pipe.

8. A train stands at a platform blowing a whistle of frequency 400 Hz in still air.

(i) What is the frequency of the whistle heard by a man running

(a)Towards the engine 10 m/s.

(b) Away from the engine at 10 m/s?

(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound received by the running man in each case?

Take speed of sound in still air = 340 m/s.

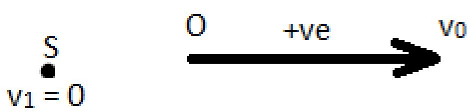
Ans.(i) (a) When the man runs towards the engine

$V_0 = -10$ m/s, $v_1 = 0$

$$v' = \frac{v - v_0}{v - v_s} \times v = \frac{340+10}{340-0} \times 400 = \frac{350}{340} \times 400 = 411.8$$

(b) When the man runs away from the engine

$V_0 = +\,10$ m/s, $v_s = 0$



$$v'' = \frac{v - v_0}{v - v_s} \times v = \frac{340-10}{340-0} \times 400 = \frac{330}{340} \times 400 = 388.2 \text{ Hz}$$

(ii) (a) When the man runs towards the engine , relative velocity of sound

$v' = v + v_0 = 340 + 10 = 350$ m/s

(b) When the man runs away from the engine, relative velocity of sound

$v' = v - v_0 = 340 - 10 = 330$ m/s .

(iii) The wavelength of sound is not affected by the motion of the listener. Its value is

$$\lambda = \frac{v}{v} = 340/400 = 0.85\text{m}$$

9. What is a spring factor? Derive the expression for resultant spring constant when two springs having constants k_1 and k_2 are connected in (i) parallel and (ii) in series.

10. Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation is equal to the average potential energy over the same period. At what distance from the mean position is the kinetic energy in simple harmonic oscillator equal potential energy?

CONTENTS

UNIT - I

UNIT AND MEASUREMENT

- The dimensions of a physical quantity are the powers to which the fundamental (base) quantities are raised to represent that quantity.

- **Dimensionless Physical Quantities :** Angle, solid angle, relative density, specific gravity, strain, Poisson's ratio, Reynold's number, all trigonometric ratios, refractive index, mechanical efficiency, relative permittivity, dielectric constant, relative permeability, electric susceptibility, magnetic susceptibility.

- The **three main uses** of dimensional analysis are :

  (i) **Conversion of one system of units into another for which we use**

  $$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

  where M_1, L_1, T_1 are fundamental units on one system; M_2, L_2, T_2 are fundamental units on the other system; *a, b, c* are the dimensions of the quantity in mass, length and time; n_1 is numerical value of the quantity in one system and n_2 is its numerical value in the other system.

  (ii) Checking the dimensional correctness of a given physical relation.

  (iii) Derivation of formulae.

- **Principle of Homogeneity of Dimensions :** According to this principle, a correct dimensional equation must be homogeneous, *i.e.*, dimensions of all the terms in a physical expression must be same

  LHS = RHS

- **Significant Figures :** In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which is doubtful, are called the significant figures. For counting significant figures rules are as :

1. All the non-zero digits are significant. In 2.738 the number of significant figures is 4.

2. All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. As examples 209 and 3.002 have 3 and 4 significant figures respectively.

3. If the measurement number is less than 1, the zero(s) on the right of decimal point and to the left of the first non-zero digit are non-significant. In 0.00807, first three underlined zeros are non-significant and the number of significant figures is only 3.

4. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 12.3 m = 1230 cm = 12300 mm has only 3 significant figures.

5. The trailing zero(s) in number with a decimal point are significant. Thus, 3.800 kg has 4 significant figures.

6. A choice of change of units does not change the number of significant digits or figures in a measurement.

- In any mathematical operation involving addition, subtraction, decimal places in the result will correspond to lowest number of decimal places in any of the numbers involved.

- In a mathematical operation like multiplication and division, number of significant figures in the product or in the quotient will correspond to the smallest number of significant figures in any of the numbers involved.

- Problems with *accuracy* are due to errors. The *precision* describes the limitation of the measuring instrument.

- Difference between measured value and true value of a quantity represents error of measurement.

  (i) Mean of n measurements

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots a_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} a_i$$

(ii) Mean absolute error

$[|\Delta a_1| + |\Delta a_2| + |\Delta a_3|$

$$\Delta a_{\text{mean}} = \frac{\left[|\Delta a_1| + |\Delta a_2| + |\Delta a_3 + ... + |\Delta a_n|\right]}{n}$$

$$= \frac{\sum_{I=1}^{n} |\Delta a_1|}{n}$$

(iii) Relative error $= \dfrac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$

(iv) Percentage error $= \dfrac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

- The errors are communicated in different mathematical operations as detailed below :

  if $\pm \Delta a$, $\pm \Delta b$ and $\pm \Delta x$ are absolute errors in *a, b* and *x* respectively, then

  (i) for $x = (a + b)$, $\Delta x = \pm [\Delta a + \Delta b]$

  (ii) for $x = (a - b)$, $\Delta x = \pm [\Delta a + \Delta b]$

  (iii) for $x = a \times b$, $\dfrac{\Delta x}{x} = \pm\left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

  (iv) for $x = a/b$, $\dfrac{\Delta x}{x} = \pm\left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

  (v) for $x = \dfrac{a^n b^m}{c^p}$, $\dfrac{\Delta x}{x} = \pm\left[n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b} + p\dfrac{\Delta c}{c}\right]$

VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Name the strongest force in nature. What is its range?
2. Give two discoveries of Physics used in your daily life.

3. What is the relation between light year and par sec.
4. Give the order of magnitude of the following :
   (i) size of atom
   (ii) size of our galaxy.
5. How many kg make 1 unified atomic mass unit?
6. Name same physical quantities that have same dimension.
7. Name the physical quantities that have dimensional formula $[ML^{-1}\ T^{-2}]$
8. Give two examples of dimension less variables.
9. State the number of significant figures in

(i) $0.007 m^2$	(ii) 2.64×10^{24} kg
(iii) 0.2370 g cm^{-3}	(iv) 0.2300m
(v) 86400	(vi) 86400 m

10. Given relative error in the measurement of length is .02, what is the percentage error?
11. If a physical quantity is represented by $X = M^a L^b T^{-c}$ and the percentage errors in the measurements of Mm L and T are α, β and γ. What will be the percentage error in X.
12. A boy recalls the relation for relativistic mass$^{(m)}$ in terms of rest mass (m_0), velocity of particle *v*, but forgets to put the constant *c* (velocity of light). He writes $m = \dfrac{m_0}{\left(1 - v^2\right)^{1/2}}$ correct the equation by putting the missing '*c*'.
13. Name the technique used in locating.
   (a) an under water obstacle
   (b) position of an aeroplane in space.
14. Deduce dimensional formulae of—
   (i) Boltzmann's constant
   (ii) mechanical equivalent of heat.
15. Give examples of dimensional constants and dimensionless constants.

SHORT ANSWER QUESTIONS (2 MARKS)

1. What is a physical standard? What characteristics should it have?

2. Define the term unit. Distinguish between fundamental and derived units.

3. Describe the principle and use of SONAR and RADAR.

4. State the principle of homogeneity. Test the dimensional homogeneity of equations—

   (i) $S = ut + \frac{1}{2}at^2$

   (ii) $S_n = u + \frac{a}{2}(2n - 1)$

5. In van der Wall's gas equation $\left(P + \frac{a}{v^2}\right)(v - b) = RT$. Determine the dimensions of a and b.

6. Using dimensions convert (a) 1 newton into dynes (b) 1 erg into joules.

7. Magnitude of force experienced by an object moving with speed v is given by $F = kv^2$. Find dimensions of k.

8. A book with printing error contains four different formulae for displacement. Choose the correct formula/formulae

   (a) $y = a \sin \frac{2\pi}{T} t$ (b) $y = a \sin vt$

   (c) $y = \frac{a}{T} \sin\left(\frac{t}{a}\right)$ (d) $y = \frac{a}{T}\left(\sin \frac{2\pi}{T} t + \cos \frac{2\pi}{T} t\right)$

9. Give limitations of dimensional analysis.

10. For determination of 'g' using simple pendulum, measurements of length and time period are required. Error in the measurement of which quantity will have larger effect on the value of 'g' thus obtained. What is done to minimise this error?

SHORT ANSWER QUESTIONS (3 MARKS)

1. Give the name of six Indian Scientists and their discoveries.
2. Name the discoveries made by the following scientists :

   (a) Faraday (b) Chadwick

   (c) Hubble (d) Maxwell

   (e) Newton (f) Bohr.

3. Name the scientific principle on which the following technology is based.

   (i) Steam engine (ii) Laser

   (iii) Aeroplane (iv) Rocket propulsion

   (v) Radio and T.V. (vi) Production of Ultra high magnetic field.

4. Describe a method for measuring the molecular size of Oleic acid.
5. What types of phenomena can be used as a time standard. What are the advantages of defining second in terms of period of radiation from cesium –133 atom.
6. Deduce the dimensional formula for the following quantities

   (i) Gravitational constant (ii) Yung's modules

   (iii) Coefficient of viscosity.

7. Define the following units :

(i) Light year (ii) Parsec

(iii) Astronomical unit (Au)

LONG ANSWER QUESTIONS (5 MARKS)

1. Name the four basic forces in nature. Write a brief note of each. Hence compare their strengths and ranges.
2. Distinguish between the terms precision and accuracy of a measurement.

3. Explain

   (i) absolute error (ii) mean absolute error

   (iii) relative error (iv) percentage error

   (v) random error

NUMERICALS

1. Determine the number of light years in one metre.
2. The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.
3. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces 20.15 g and 20.17 g are added to the box.
   (i) What is the total mass of the box?
   (ii) The difference in masses of the pieces to correct significant figures.
4. 5.74 g of a substance occupies 1.2 cm^3. Express its density to correct significant figures.
5. If displacement of a body s = (200 ± 5) *m* and time taken by it t = (20 + 0.2) *s*, then find the percentage error in the calculation of velocity.
6. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy.
7. The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m and 2.48m. Find the average length, absolute error and percentage error. Express the result with error limit.
8. A physical quantity is measured as a = (2.1 ± 0.5) units. Calculate the percentage error in (i) Q^2 (2) 2Q.
9. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be 35.72´´ of arc. Calculate diameter of Jupiter.
10. A lesser light beamed at the moon takes 2.56 and to return after reflection at the moon's surface. What will be the radius of lunar orbit.

11. Convert

 (i) $3m \cdot S^{-2}$ to $km\ h^{-2}$

 (ii) $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$ to $cm^3\ g^{-1}\ S^{-2}$

12. A calorie is a unit of heat or energy and it equals 4.2 J where IJ = 1kg m^2S^{-2}. Suppose we employ a system of units in which unit of mass is α kg, unit of length is β m, unit of time is γ s. What will be magnitude of calorie in terms of this new system.

13. The escape velocity *v* of a body depends on–

 (i) the acceleration due to gravity 'g' of the planet,

 (ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.

14. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

15. One mole of an ideal gas at STP occupies 22.4 L. What is the ratio of molar volume to atomic volume of a mole of hydrogen? Why is the ratio so large. Take radius of hydrogen molecule to be 1°A.

VERY SHORT ANSWER (1 MARK)

3. 1 parsec = 3.26 light year

4. Size of atom = 10^{-10} m (b) size of galaxy = 10^{22} m

 Order = – 10, Order of magnitude = 22.

5. $1u = 1.66 \times 10^{-27}$ kg

6. Work, energy and torque.

7. Stress, pressure, modulus of elasticity.

8. Strain, refractive index.

9. (i) 1, (ii) 3, (iii) 4, (iv) 4, (v) 3, (vi) 5 since it comes from a measurement the last two zeros become significant.

10. 2%

11. % error in measurement of X = $a\alpha + b\beta + c\gamma$.

12. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from 1 but v^2/c^2 can be subtracted from 1.

$$\therefore \quad m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

13. (a) SONAR → Sound Navigation and Ranging.

(b) RADAR → Radio Detection and Ranging.

14. (i) Boltzmann Constant :

$$k = \frac{\text{Heat}}{\text{Temperature}} \Rightarrow [k] = \frac{ML^2T^{-2}}{K} = M^1L^2T^{-2}K^{-1}$$

(ii) $$[J] = \left[\frac{Work}{Heat}\right] = \frac{M^1L^2T^{-2}}{M^1L^2T^{-2}} = \left[m^0L^0T^0\right]$$

15. Dimensional Constants : Gravitational constant, plank's constant.

Dimensionless Constants : π, *e*.

SHORT ANSWER (2 MARKS)

4. (i) Dimension of L.H.S. = [s] = $[M^oL^1T^o]$

Dimension of R.H.S = [ut] + [at^2] = $[M^oL^1T^{-1}] + [M^oL^1T^{-2}.T^2]$

= $M^oL^1T^o$

∴ The equation to dimensionally homogeneous.

(ii) S_n = Distance travelled in n^{th} sec that is $(S_n - S_{nm})$

$$\therefore \quad S_n = u \times l + \frac{a}{2}(2n - 1) \times 1$$

Hence this is dimensionally incorrect.

5. Since dimensionally similar quantities can only be added

$$\therefore \quad [P] = \left[\frac{a}{v^2}\right] \Rightarrow [a] = [Pv^2] = M^1\, L^5\, T^{-2}$$

$[b] = [v] = L^3$

7. $[k] = \frac{[F]}{[v^2]} = \frac{M^1L^1T^{-2}}{[LT^{-1}]^2} = \frac{M^1L^1T^{-2}}{M^0L^2T^{-2}} = [M^1L^{-1}]$

8. The argument of sine and cosine function must be dimension less so (a) is the probable correct formula. Since

(a) $y = a\sin\left(\frac{2\pi}{T}t\right)$ $\quad\because \left[\frac{2\pi t}{T}\right] = [T^o]$ is dimensionless.

(b) y = a sin μt $\because$ [μt]=[L] is dimensional so this equation is incorrect.

(c) $y = \frac{a}{t}\sin\left(\frac{t}{a}\right)$ $\quad\left[\frac{t}{a}\right]$ is dimensional so this is incorrect.

(d) $y = \frac{a}{t}\left(\sin\frac{2\pi}{T}t + \cos\frac{2\pi t}{T}\right)$: Though $\frac{2\pi t}{T}$ is dimensionless $\frac{a}{T}$ does not have dimensions of displacement so this is also incorrect

9. Limitation of dimensional analysis :-

   1. The value of proportionality constant cannot be obtained
   2. Equation containing sine and cosine, exponents, logx etc cannot be analysed.
   3. If fails to derive the exact form of physical relation which depends on more than three fundamental quantities
   4. It does not tell whether a quantity is scaler or vector.

ANSWERS FOR NUMERICALS

(NUMERICAL)

1. 1 ly = 9.46×10^{15} m

$$1\text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16}\text{ly}$$

2. P = 2(l + b)

= 2(10.5 + 5.2) + 2(0.2 + 0.1)

= (31.4 + 0.6) cm

3. (i) Mass of box : 2.3 kg

Mass of gold pieces = 20.15 + 20.17 = 40.32 g = 0.04032 kg.

Total mass = 2.3 + 0.04032 = 2.34032 kg

In correct significant figure mass = 2.3 kg (as least decimal)

(ii) Difference in mass of gold pieces = 0.02 g

In correct significant figure (2 significant fig. minimum decimal) will be 0.02 g.

4. Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{5.74}{1.2}$ = 4.783 g/cm^3

Here least significant figure is 2, so density = 4.8 g/cm^3

5. Percentage error in measurement of displacement $= \frac{5}{200} \times 100$

Percentage error in measurement of time $= \frac{0.2}{20} \times 100$

$\therefore$ Maximum permissible error = 2.5 + 1 = 3.5%

6. K.E. $= \frac{1}{2} mv^2$

$$\therefore \quad \frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} \Rightarrow \frac{\Delta k}{k} \times 100 = \frac{\Delta m}{m} \times 100 + 2\left(\frac{\Delta v}{v}\right) \times 100$$

$\therefore$ Percentage error in K.E. = 3% + 2 × 2% = 7%

7. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} = 2.48 \text{ m}$$

Mean absolute error

$$\frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.013\text{n}$$

$$\text{Percentage error} = \frac{0.01}{2.48} \times 100\% = 0.04 \times 100\%$$

$= 0.40\%$

Correct length = (2.48 ± 0.01)m

Correct length = (2.48 M ± 0.40%)

8. $P = Q^2$

$$\frac{\Delta p}{p} = \frac{2\Delta Q}{Q} \qquad \left(\frac{0.5}{2.1}\right) = \frac{1.0}{2.1} = 0.476$$

$$\frac{\Delta p}{p} \times 100\% = 47.6\% = 48\%$$

$R = 2Q$

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \Rightarrow \frac{0.5}{2.1} = 0.238$$

$$\frac{\Delta R}{R} \times 100\% = 24\%$$

9. Q = 35.72”

1” = 4.85×10^{-6} radian $\Rightarrow = 35.72 \times 4.85 \times 10^{-6}$

$d = DQ = 824.7 \times 10^6 \times 35.72 \times 4.85 \times 10^{-6}$

$= 1.4287 \times 10^5$ km

10. t = 2.56 s

∴ t’ = time taken by laser beam to go to the man $= \frac{t}{2}$

distance between earth and moon = $d = c \times \frac{t}{2}$

$$= 3 \times 10^8 \times \frac{2.56}{2}$$

$= 3.84 \times 10^8$ m

11. (i) $3 \text{ m s}^{-2} = \left(\frac{3}{1000}\text{km}\right)\left(\frac{1}{60 \times 60}\text{hr}\right)^{-2}$

$$= \frac{3 \times (60 \times 60)^2}{1000} = 3.9 \times 10^4 \text{km h}^{-2}$$

(ii) $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

$= 6.67 \times 10^{-11} (\text{kg m s}^{-2})(\text{m}^2 \text{ kg}^{-2})$

$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$$= 6.67 \times 10^{-11} (1000 \text{ g})^{-1} (100 \text{ cm})^3 \left(\text{s}^{-2}\right)$$

$$= 6.67 \times 10^{-11} \alpha \frac{1}{1000} \times 100 \times 100 \times 100$$

$$= 6.67 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$$

12. $n_2 = n_1 \left[\frac{m_1}{m_2}\right]^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$

$$= 4.2 \left(\frac{\text{kg}}{\alpha \text{ kg}}\right)^1 \left(\frac{\text{m}}{\beta \text{ m}}\right)^2 \left(\frac{\text{s}}{\text{r s}}\right)^{-2}$$

$n_2 = 4.2 \ \alpha^{-1} \ \beta^{-2} \ \rho^{+2}$

13. $v \ \alpha \ g^a \ R^b$ Þ $\mu = k \ g^a \ R^b$ K → dimensionless proportionality constant

$[v] = [g]^a [R]^b$

$[M^o L^1 T^{-1}] = [M^0 \ L^1 T^{-2}]^a \ [m^0 \ L^1 \ T^0]^b$

equating powers

$1 = a + b$

$-1 = -2a \Rightarrow a = \frac{1}{2}$

$b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore v = k\sqrt{gR}$

14. n α l^a T^b m^c [l] $= M^o L^1 T^0$

$[T] = M^1 L^1 T^{-2}$ (force)

$[M] = M^1 L^{-1} T^0$

$[M^0 L^0 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$

$b + c = 0$

$a + b - c = 0$

$-2b = -1$ $b = 1/2$

$c = -\frac{1}{2}$ $\qquad a = 1$

$n \quad \alpha \quad \frac{1}{l}\sqrt{\frac{T}{M}}$

15. $1 A^0 = 10^{-10}$m

Atomic volume of 1 mole of hydrogen = Avagadios number × volume of hydrogen molecule

$= 6.023 \times 10^{23} \times \frac{4}{3} \times \pi \times \left(10^{-10}\right)^3$

$= 25.2 \times 10^{-7}$ m^3

Molar volume = 22.4 L = 22.4×10^{-3} m^3

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} = 0.89 \times 10^4 \approx 10^4$$

This ratio is large because actual size of gas molecule is negligible in comparison to the inter molecular separation.

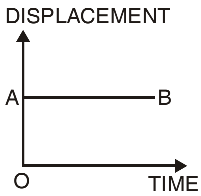
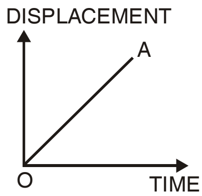
UNIT II

KINEMATICS

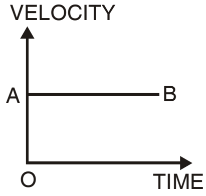
- **One Dimensional Motion.** The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with respect to times.

  (an object moves along any of the three axes X, Y or Z).

- **Two dimensional motion.** The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with respect to time. (the object moves in a plane.)
- **Three dimensional motion.** The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with respect to time. (the object moves in space.)
- **Speed.** The speed of an object is defined as the ratio of distance covered and time taken i.e. speed = distance travelled/(time taken). Speed is a scalar quantity. It can only be zero or positive.
- **Instantaneous velocity.** The velocity of an object at a given instant of time is called its instantaneous velocity. When a body is moving with uniform velocity, its instantaneous velocity = average velocity = uniform velocity.
- **Graphs and Nature**

S.No.	Type of motion	Graph	Features of graph
●	For a stationary body, the time displacement graph is a st. line parallel to time axis.		The slope of st. line represents instantaneous velocity zero slope → zero velocity.
●	When a body, is moving with a constant velocity, then time displacement graph will be a st. line inclined to time axis.		Constant slope. Magnitude of velocity is constant.

- When a body is moving with a constant velocity, the velocity time graph is a st-line parallel to time axis.



The slope of this graph represents the instantaneous acceleration zero acceleration.

- **Relative velocity.** The relative velocity of one object w.r.t another is the velocity with which one object moves w.r.t another object. If $\vec{\nu}_A$ and $\vec{\nu}_B$ are the velocity of two objects A and B, and θ is the angle between them, then relative velocity of object A w.r.t B is given by

$$\vec{\nu}_{AB} = \vec{\nu}_{AB} - \vec{\nu}_B \quad \text{where, } \nu_{AB} = \sqrt{\nu_A^2 + \nu_B^2 - 2\nu_A\nu_B \cos\theta}$$

and
$$\tan\beta = \frac{\nu_B \sin\theta}{\nu_A - \nu_A \cos\theta}$$

Here, b is the angle which $\vec{\nu}_{AB}$ makes with the direction of $\vec{\nu}_A$.

- **Acceleration.** The acceleration of an object is defined as the ratio of change of velocity of the object, and time taken *i.e.,* Acceleration = change in velocity/time taken. Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called ***retardation*** or ***deceleration.***

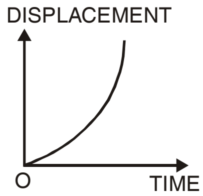
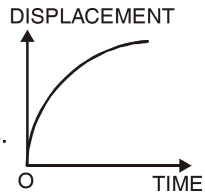
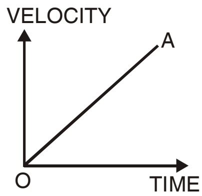
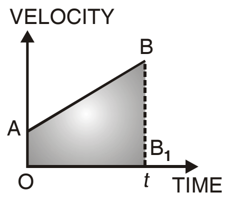
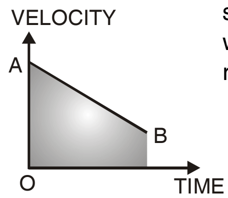
- **Instantaneous acceleration.** The acceleration of an object at a given instant is called its instantaneous acceleration.

Instantaneous acceleration, $a = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

- **Formulae for uniformly accelerated motion along a straight line.**

	For accelerated motion	**For Retarded motion**
1.	$\nu = u + at$	$\nu = u - at$
2.	$s = ut + \frac{1}{2}at^2$	$s = ut - \frac{1}{2}at^2$
3.	$\nu^2 = u^2 + 2\,as$	$\nu^2 = u^2 - 2\,as$
4.	$D_n = u = \frac{a}{2}(2n - 1)$	$D_n = u - \frac{a}{2}(2n - 1)$

- **Graphs and nature**

S.No.	Type of motion	Graph	Features of graph
●	When a body is moving with a constant acceleration the time-displacement graph is a curve with bend upwards.		The slope of time-displacement curve (instantaneous velocity increases with time.
●	When a body is moving with a constant retardation, the time-displacement graph is a curve with bend downwards.		(i) The slope of time-displacement curve (instantaneous velocity) decreases with time.
●	When a body is moving with a constant acceleration and its initial velocity is zero, the velocity-time graph is an oblique st line, passing through origin.		Greater will be the slope of st. line greater will be the instantaneous acceleration.
●	When a body is moving with a constant acceleration and its initial velocity is not zero, the velocity-time graph is an oblique st-line not passing through origin.		The area enclosed by the velocity-time graph with time axis represents the distance travelled by the body.
●	When a body is moving with a constant retardation and its initial velocity is not zero, the velocity-time graph is an oblique st. line not passing through origin.		slope represents acceleration which is negative i.e., retardation.

- **Scalars.** The quantities which have magnitudes and unit only but no direction. For example, mass, length, time, speed, work, temperature etc.

- **Vector.** The quantities which have magnitudes unit as well as direction and obeys vector laws of addition, multiplication etc. For example, displacement, velocity, acceleration, force, momentum etc.

S.No.	Resultant	Properties and results
●	Addition of vectors	(i) Only vectors of same nature can be added. (ii) The addition of two vectors $\vec{A}$ and $\vec{B}$ is a resultant $\vec{R}$, where R = $(A^2 + B^2 + 2AB\cos\theta)^{1/2}$ and $\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$ where θ is the angle between $\vec{A}$ and $\vec{B}$ and β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$ (iii) Vector addition is commutative *i.e.* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (iv) Vector addition is associative *i.e.* $(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$ (v) *R* is maximum if q = 0° and is minimum if q = 180°

- **Substraction of vectors.**

S.No.	Resultant	Properties and results
●	Subtraction of two vectors	(i) Only vectors of same nature can be subtracted. (ii) Subtraction of $\vec{B}$ from $\vec{A}$ = vector addition of $\vec{A}$ and $(-\vec{B})$ *i.e.* $\vec{A}-\vec{B} = \vec{A}+(-\vec{B}) = \vec{R}$ where $R = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$ and $\tan\beta = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)}$ where θ is the angle between $\vec{A}$ and $\vec{B}$: and b is the angle which $\vec{R}$ makes with the direction of $\vec{A}$. (iii) Vector subtraction of two vectors is not commutative *i.e.* $\vec{A}-\vec{B} \neq \vec{B}-\vec{A}$ (iv) Vector subtraction is not associative *i.e.* $\vec{A}-(\vec{B}-\vec{C}) \neq (\vec{A}-\vec{B})-\vec{C}$

- **Rectangular components of a vector in a plane.** If $\vec{A}$ makes an angle θ with x-axis and $\overrightarrow{A_x}$ and $\overrightarrow{A_y}$ be the rectangular components of $\vec{A}$ along x-axis and y-axis respectively, then $\vec{A} = \vec{A} + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$

  Here A_x = A $\cos\theta$ and A_y = A $\sin\theta$

  and $A = \left(A_x^2 + A_y^2\right)^{1/2}$ and $\tan\theta = A_y/A_x$

- *The dot product of two vectors* $\vec{A}$ and $\vec{B}$*, represented by* $\vec{A}.\vec{B}$ *is a scalar, which is equal to the product of the magnitudes of* $\vec{A}$ *and* $\vec{B}$ *and the cosine of the smaller angle between them.*

  If q is the smaller angle between $\vec{A}$ and $\vec{B}$, then $\vec{A}.\vec{B} = AB\cos\theta$

  (i) $\hat{i}.\hat{i} = 1 = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

  (ii) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

  (iii) if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

- The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is represented as $\vec{A} \times \vec{B}$

  If q is the smaller angle between $\vec{A}$ and $\vec{B}$, then $\vec{A} \times \vec{B} = \vec{C} = AB\sin\theta\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

- **Right handed screw rule.** *It states that if a right handed screw placed with its axis perpendicular to the plane containing the two vectors* $\vec{A}$ and $\vec{B}$ *is rotated from the direction of* $\vec{A}$ *to the direction of* $\vec{B}$ *through smaller angle, then the sense of the advancement of the tip of the screw gives the direction of* $\left(\vec{A} \times \vec{B}\right)$

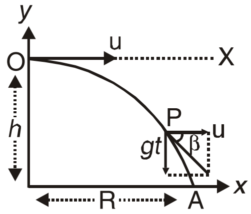
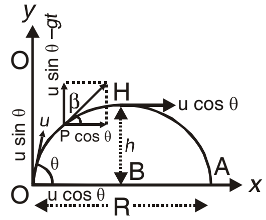
  For unit vectors : (i) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ and

  (ii) $\hat{i}$, $\hat{j}$ and $\hat{k}$: $\hat{i}\times\hat{j} = \hat{k}\,;\hat{j}\times\hat{k} = \hat{i}\,;\hat{k}\times\hat{i} = \hat{j}$

  If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

  $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- **Projectile.** Projectile is the name given to a body which is thrown with some initial velocity with the horizontal direction and then it is allowed to move under the effect of gravity alone.

S.No.	Projectile projected with velocity *u*	Horizontal projection	Projectile with angular projection
●	Path of projectile is a parabola		
●	Velocity of projectile at any instant t	$\nu = \sqrt{u^2 + g^2t^2}$	$\nu = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$
●	Direction of the velocity $\vec{\nu}$ with the horizontal direction	$\beta = \tan^{-1}\left(\frac{gt}{u}\right)$	$\beta = \tan^{-1}\left(\tan\theta - \frac{gt}{u\cos\theta}\right)$
●	Horizontal range	$R = u\sqrt{\frac{2h}{g}}$	$R = \frac{u^2\sin 2\theta}{g}$
●	Time of flight	$t = \sqrt{\frac{2h}{g}}$	$T = \frac{2u\sin\theta}{g}$
●	Maximum height	H = h	$H = \frac{u^2\sin^2\theta}{2g}$

1 MARK QUESTIONS

1. Why can speed of a particle not be negative?

2. Is it possible in straight line motion a particle have zero speed and a non zero velocity?

3. Suggest a situation in which an object is accelerated and have constant speed.

4. Two balls of different masses are thrown vertically upward with same initial velocity. Maximum heights attained by them are h_1 and h_2 respectively what is h_1/h_2?

5. A car moving with velocity of 50 kmh^{-1} on a straight road is ahead of a jeep moving with velocity 75 kmh^{-1}. How would the relative velocity be altered if jeep is ahead of car?

6. Which of the two-linear velocity or the linear acceleration gives the direction of motion of a body?

ANSWERS

1. Because speed is distance travelled per second and distance is never negative.
2. No. it is not possible.
3. Uniform circular motion.
4. Same height $\therefore\ h_1/h_2 = 1$
5. No change
6. Linear velocity

1 MARK

7. Will the displacement of a particle change on changing the position of origin of the coordinate system?
8. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero.?
9. Can a body subjected to a uniform acceleration always move in the straight line?
10. Write an example of zero vector.
11. State the essential condition for the addition of vectors.
12. When is the magnitude of $\left(\vec{A}+\vec{B}\right)$ equal to the magnitude of $\left(\vec{A}-\vec{B}\right)$?
13. What is the maximum number of component into which a vector can be resolved?
14. A body projected horizontally moves with the same horizontal velocity although it moves under gravity. Why?
15. What is the angle between velocity and acceleration at the highest point of a projectile motion?

ANSWERS

7. Will not change.
8. No, (highest point of vertical upward motion under gravity)
9. No example. Projectile motion.
10. The velocity vectors of a stationary object is a zero vectors.
11. They must represent the physical quantities of same nature.
12. When $\vec{A}$ is perpendicular to $\vec{B}$.
13. Infinite.
14. Because horizontal component of gravity is zero along horizontal direction.
15. 90°

1 MARK

16. When does (i) height attained by a projectile maximum? (ii) horizontal range is maximum?
17. What is the angle between velocity vector and acceleration vector in uniform circular motion?
18. A particle is in clockwise uniform circular motion the direction of its acceleration is radially inward. If sense of rotation or particle is anticlockwise then what is the direction of its acceleration?
19. A train is moving on a straight track with acceleration a. A passenger drops a stone. What is the acceleration of stone with respect to passenger?
20. What is the average value of acceleration vector in uniform circular motion over one cycle?
21. Does a vector quantity depends upon frame of reference chosen?
22. What is the angular velocity of the hour hand of a clock?
23. What is the source of centripetal acceleration for earth to go round the sun?

ANSWERS

16. height is maximum at θ = 90

    Range is maximum at θ = 45.

17. 90°

18. Radial inward.

19. $\sqrt{a^2 + g^2}$ where g = Acceleration due to gravity.

20. Null vector.

21. No.

22. $W = \frac{2\pi}{12} = \frac{\pi}{6}\,\text{rad}\,h^{-1}$

23. Gravitation force of sun.

1 MARK

24. What is the unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$?

25. What is the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} \times \vec{B})$?

ANSWERS

24. $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

25. 90°

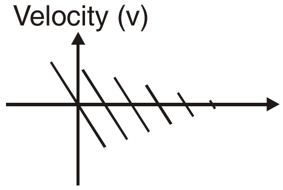
2 MARKS

1. What are positive and negative acceleration in straight line motion?

2. Can a body have zero velocity and still be accelerating? If yes gives any situation.

3. The displacement of a body is proportional to t^3, where t is time elapsed. What is the nature of acceleration- time graph of the body?

5. Suggest a suitable physical situation for the following graph.



6. An object is in uniform motion along a straight line, what will be position time graph for the motion of the object if

   (i) x_0 = positive, v = negative $|\bar{v}|$ is constant

   (ii) both x_0 and v are negative $|\bar{v}|$ is constant

   where x_0 is position at t = 0

ANSWERS

1. If speed of an object increases with time, its acceleration is positive. (Acceleration is in the direction of motion) and if speed of an object decreases with time its acceleration is negative (Acceleration is opposite to the direction of motion).

2. Yes, at the highest point of vertical upward motion under gravity.

3. as $s \propto t^3 \Rightarrow s = kt^3$

   velocity $V = \frac{ds}{dt} = 3kt^2$

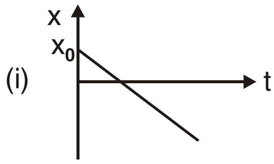
   acceleration $a = \frac{dv}{dt} = 6kt$

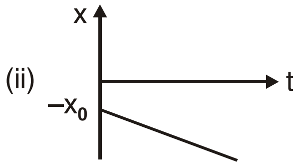
   i.e $a \propto t$

   $\Rightarrow$ motion is uniform, accelerated motion. a – t graph is straight-line.

5. A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit.

6.





7. A vector $\vec{a}$ is turned through a small angle do without a change in its length. What are $|\Delta\vec{a}|$ and Δa

8. What will be the effect on horizontal range of a projectile when its initial velocity is doubled keeping angle of projection same?

9. The greatest height to which a man can throw a stone is h. What will be the greatest distance upto which he can throw the stone?

10. A person sitting in a train moving at constant velocity throws a ball vertically upwards. How will the ball appear to move to an observer.

    (i) Sitting inside the train

    (ii) Standing outside the train

11. A gunman always keep his gun slightly tilted above the line of sight while shooting. Why?

ANSWERS

7. $|\overrightarrow{\Delta a}| = ad\theta \qquad \Delta a = 0$

8. $R = \dfrac{u^2 \sin 2\theta}{g} \Rightarrow R \alpha u^2$

   Range becomes four times.

9. Maximum height ; $H = \dfrac{u^2 \sin^2\theta}{2g}$

   $\Rightarrow H_{max} = \dfrac{u^2}{2g} = h(\text{at } \theta = 90°)$

Max. Range $R_{max} = \frac{u^2}{g} = 2h$

10. (i) Vertical straight line motion

(ii) Parabolic path.

11. Because bullet follow parabolic trajectory under constant downward acceleration.

2 MARKS

12. Is the acceleration of a particle in circular motion not always towards the centre. Explain.

Ans. No, acceleration is towards the centre only in case of uniform circular motion.

3 MARKS

1. Derive the relation

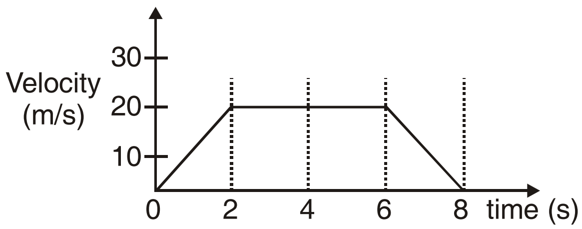
$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

where S_{nth} = distance travelled in n^{th} second

a = Uniform acceleration

u = Initial speed

2. The velocity time graph for a particle is shown in figure. Draw acceleration time graph from it.



3. Draw position-time graphs of two objects, A and B moving along a straight line, when their relative velocity is

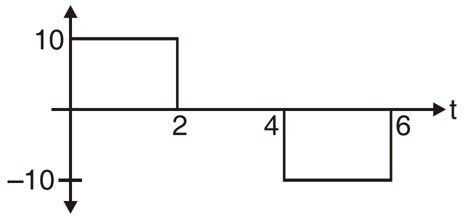
   (i) zero

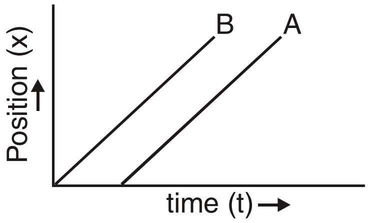
4. For an object projected upward with a velocity V_o, which comes back to the same point after some time, draw

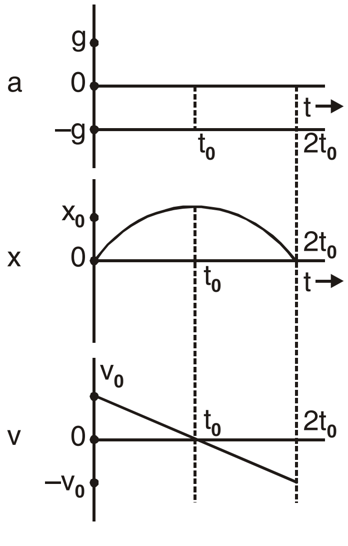
   (i) Acceleration-time graph (ii) Position-time graph

   (iii) Velocity-time graph

ANSWERS

2. 

3. 

4. 

$2t_0 \rightarrow$ total time of the Journey

$x_0 \rightarrow$ highest position.

3 MARKS

5. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

6. Establish the following vector inequalities :

   (i) $\left|\vec{a}+\vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$

   (ii) $\left|\vec{a}+\vec{b}\right| \ge \left|\vec{a}\right| - \left|\vec{b}\right|$

7. A body is projected at an angle θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles θ_1 and θ_2 projections for the same horizontal range. such that $\theta_1 + \theta_2 = 90°$

8. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum range.

9. Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.

10. Derive the relation between linear velocity and angular velocity in a uniform circular motion

5 MARKS

1. Derive the following equations of motion for an object moving with constant acceleration along a straight line using graphical method.

   (i) $v = u + at$

   (ii) $S = ut + \frac{1}{2}at^2$

   (iii) $v^2 = u^2 + 2as$

   Where symbols have usual meanings.

2. A projectile is fired horizontally with a velocity u. Show that its trajectory is a parabola. Also obtain expression for

   (i) time of flight

   (ii) horizontal range

   (iii) velocity at any instant.

3. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

NUMERICALS

1. The V-t graphs of two objects make angle 30° and 60° with the time axis. Find the ratio of their accelerations.

2. When the angle between two vectors of equal magnitudes is $2\pi/3$, prove that the magnitude of the resultant is equal to either.

3. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

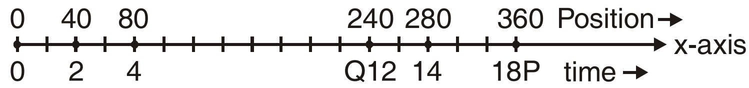
4. What is the angle made by vector $\vec{A} = 2\hat{i} + 2\hat{j}$ with x-axis?

5. What is the vector sum of n coplanar forces, each of magnitude F, if each force makes an angle of $\frac{2\Pi}{n}$ with the preceding force?

7. A car is moving along x-axis. As shown in figure it moves from 0 to P in 18 seconds and return from P to Q in 6 second. What are the average velocity and average speed of the car in going from

   (i) O to P

   (ii) from o to P and back to Q.



ANSWER

1. $\dfrac{a_1}{a_2} = \dfrac{\tan 30}{\tan 60} = \dfrac{1/\sqrt{3}}{\sqrt{3}} = \dfrac{1}{3} \Rightarrow 1 : 3$

2. $R = (P^2 + Q^2 + 2PQ\ \cos\theta)^{1/2}$

$$= \left(P^2 + P^2 + 2P.P\cos\frac{2\Pi}{3}\right)^{1/2}$$

$$= \left[2P^2 + 2P^2\left(\frac{-1}{2}\right)\right]^{1/2} = P$$

3. $\left|\vec{A}\right| = \sqrt{3^2 + 4^2} = 5$

$$\hat{A} = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{3\hat{i} + 4\hat{j}}{5}$$

also $\left|\vec{B}\right| = \sqrt{7^2 + 24^2} = 25$

desired vector $= \left|\vec{B}\right|\hat{A} = 25\dfrac{\left(3\hat{i} + 4\hat{j}\right)}{5}$

$$= 5\left(3\hat{i} + 4\hat{j}\right) = 15\hat{i} + 20\hat{j}$$

4. $\theta = \tan^{-1}\dfrac{2}{2} = 45°$

5. Resultant force is zero.

7. (i) o to p Average velocity = 20 ms^{-1}

average speed = 20 ms^{-1}

(ii) o to P and back to Q

Average velocity = 10 ms^{-1}

Average speed = 20 ms^{-1}

NUMERICALS

8. On a 60 km straight road, a bus travels the first 30 km with a uniform speed of 30 kmh^{-1}. How fast must the bus travel the next 30 km so as to have average speed of 40 kmh^{-1} for the entire trip?

9. The displacement x of a particle varies with time as $x = 4t^2 - 15t + 25$.

   Find the position, velocity and acceleration of the particle at t = 0.

10. A driver take 0.20 second to apply the breaks (reaction time). If he is driving car at a speed of 54 kmh^{-1} and the breaks cause a deceleration of 6.0 ms^{-2}. Find the distance travelled by car after he sees the need to put the breaks.

11. A body covers 12 m in 2nd second and 20 m in 4th second. How much distance will it cover in 4 seconds after the 5th second.

12. A ball thrown vertically upwards with a speed of 19.6 ms^{-1} from the top of a tower returns to the earth in 6s. Find the height of the tower ($g = 9.8\ m/s^2$)

13. Two town A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed do the buses ply of the road?

ANSWER

8. $$V_{avg} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{St\,S}{S\left(\frac{1}{V_1} + \frac{1}{V_2}\right)} = \frac{2\ V_1V_2}{V_1 + V_2}$$

   or $$40 = \frac{2 \times 30 \times v_2}{V_1 + V_2} \Rightarrow V_2 = 60\,kmh^{-1}$$

9. position x = 25 m

   velocity $V = \frac{dx}{dt}\Big/ t = 0 = -15\ ms^{-1}$

   acceleration $a = \frac{dv}{dt} = 8\ ms^{-2}$

10. (distance covered during 0.20 s) +

(distance covered until rest)

= (15 × 0.20) + [18.75] = 21.75 m

11. $S_{2nd} = u + \frac{a}{2}(2 \times 2 - 1) \Rightarrow 4 + \frac{3}{2}a = 12$

$S_{4th} = u + \frac{a}{2}(2 \times 4 - 1) \Rightarrow 4 + \frac{7}{2}a = 20$

$\Rightarrow u = 6ms^{-1}$ and $a = 4ms^{-1}$

According to question = $5g - 5_s$ $\left(s = ut + \frac{1}{2}at^2\right)$

= 136 m

12. using $s = ut + \frac{1}{2}at^2$

$-h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 62$

h = 58.8 m

13. V = 40 kmh^{-1} and T = 9 min

NUMERICALS

14. A motorboat is racing towards north at 25 kmh^{-1} and the water current in that region is 10 kmh^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat.

15. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft position 10 second apart is 30°, what is the speed of the aircraft?

16. A boat is moving with a velocity $\left(3\hat{i} - 4\hat{j}\right)$ with respect to ground. The water in river is flowing with a velocity $\left(-3\hat{i} - 4\hat{j}\right)$ with respect to ground. What is the relative velocity of boat with respect to river?

17. A hiker stands on the edge of a clift 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground and the speed with which it hits the ground ($g = 9.8\ ms^{-2}$)

18. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit the target 5 km away? Assume that the muzzle speed to be fixed and neglect air resistance.

ANSWER

14. $V = 21.8\ kmh^{-1}$

    angle with north $\theta = 23.4°$

15. Speed = $182.2\ ms^{-1}$

16. $\overrightarrow{V_{BW}} = \overrightarrow{V_B} - \overrightarrow{V_W}$

    $= 6\hat{i} + 8\hat{j}$

17. time = 10 seconds

    $$V = \sqrt{V_x^2 + V_y^2} = \sqrt{15^2 + 98^2} = 99.1\ ms^{-1}$$

18. Maximum Range = 3.46 km

    So it is not possible.

NUMERICALS

19. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?

20. A cyclist is riding with a speed of 27 kmh^{-1}. As he approaches a circular turn on the road of radius 30 m, he applies brakes and reduces his speed at the constant rate $0.5\ ms^{-2}$. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

ANSWERS

19. $w = \frac{88}{25}$ rod s^{-1} $\quad w = \frac{2\pi}{T} = \frac{2\pi N}{t}$

$a = 991.2\ cms^{-2}$

20. $a_c = \frac{v^2}{r} = 0.7\ ms^{-2}$

$a_T = 0.5\ ms^{-2}$

$a = \sqrt{a_c^2 + a_T^2} = 0.86 ms^{-2}$

If θ is the angle between the net acceleration and the velocity of the cyclist, then

$\theta = \tan^{-1}\frac{a_c}{a_T} = \tan^{-1}(1.4) = 54°28'$

NUMERICAL

21. If the magnitude of two vectors are 3 and 4 and their scalar product is 6, find angle between them.

22. Find the value of λ so that the vector $\vec{A} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each other.

23. If $\hat{i}$ and $\hat{j}$ are unit vectors along X and y-axis respectively, then what is the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.

ANSWERS

21. $\vec{A}.\vec{B} = AB\cos\theta$

or $6 = (3 \times 4)\cos\theta$

or $\theta = 60°$

22. $\because \vec{A} \perp \vec{B} \Rightarrow \vec{A}.\vec{B} = 0$

$\Rightarrow \lambda = 3$

UNIT – III

NEWTON LAW OF MOTION AND FRICTION

KEY CONCEPTS

- According to the **principle of conservation of linear momentum,** the vector sum of linear momenta of all the bodies in an isolated system is conserved.

  Flight of rockets, jet planes, recoiling of a gun etc. are explained on the basis of this principle.

- Apparent weight of a man in an elevator is given by *W'* = m(g ± a).

  where mg = real weight of the man. Acceleration = (+a), when the lift is accelerating upwards and

  Acceleration = (–a) when the lift is accelerating downwards.

  when lift is *moving* in uniform motion then a = 0, *w'* = mg = real weight.

  in free fall, a = g $\therefore$ *W'* = m(g – g) = 0

  i.e. apparent weight becomes zero.

- When two bodies of masses m_1 and m_2 are tied at the ends of an inextensible string passing over a light frictionless pulley, **acceleration** of the system is given by

$$a = \frac{(m_1 - m_2)\,g}{(m_1 + m_2)} \text{ and}$$

  Tension in the string is $T = \dfrac{2m_1 m_2 g}{(m_1 + m_2)}$

- Impulse is defined as change in momentum.

Impulse $\vec{I} = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1$

where t is the time for which average force acts, $\left(\overrightarrow{p_2} - \overrightarrow{p_1}\right)$ is change in linear momentum of the body.

- Any system is said to be in equilibrium if net force applied on the system is zero. In this case system is either at rest or in uniform motion.
- **Friction** is the opposing force that comes into play when one body is actually moving over the surface of another body or one body is trying to move over the surface of the other.

  Two causes of friction are : roughness of surfaces in contact; (ancient view)

  Force of adhesion between the molecules of the surfaces in contact. (Modern view)
- Type of solid friction :–
  1. Static friction. It comes in to effect when object is at rest but external force is applied.
  2. Dynamic friction- it comes in to effect when object is in motion.
  3. Rolling friction- it comes in to when object is in rolling.
- Limiting friction is the maximum value of static friction. Dynamic/Kinetic friction is somewhat less than the force of limiting friction.
- **Coefficient of friction** $\mu = F/R$

  when F = external force and R = normal reaction
- **Angle of friction** (θ) is the angle which resultant of F and R makes with the direction of R. The relation between μ and θ is $\mu = \tan\theta$
- **Angle of Repose** (a) is the minimum angle of inclination of a plane with the horizontal, such that a body placed on hte plane just begins to slide down. $\mu = \tan\alpha$
- **Centripetal force** is the force required to move a body uniformally in a circle.

- **During motion on level curved road,** the necessary centripetal force is provided by the force of friction between the tyres and the road. The maximum velocity with which a vehicle can go round a level curve without skidding is $v = \sqrt{\mu r g}$.

- To increase speed on turn, **curved roads are usually banked** *i.e.* outer edge of the curved road is raised suitably above the inner edge. If θ is the angle of banking, then

$$\tan\theta = \frac{v^2}{r g}$$

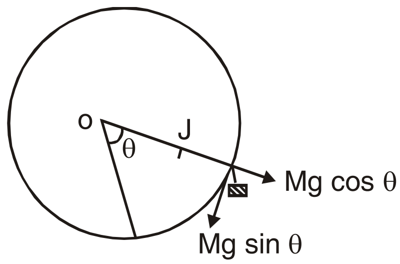
When frictional force is ignored, the optimum speed is $\nu_0 = (rg \tan\theta)^{1/2}$

- While rounding a banked curved road, maximum permissible speed is given by $\nu_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$ when friction is taken in to account.

- When a cyclist takes a turn. He bends a little inwards from his vertical position, while turning. Angle θ of bending from vertical position is given by

$$\tan\theta = \frac{v^2}{r g}$$

- **Motion along a vertical circle is a** non uniform circular motion. Tension in the string at any position is $T = \frac{m\nu^2}{r} + mg\cos\theta$, where θ is the angle of string with vertical line for looping with optimum speed :- (when tension at highest point is zero)



(i) velocity of projection at lowest point *L* is $\nu_L \geq \sqrt{5\,g\,r}$

(ii) velocity at the highest point *H* is $\nu_H \geq \sqrt{gr}$

(iii) Tension at lowest point $T_L \geq 6$ mg

(iv) For oscillation in vertical circle $0 < \nu_L \leq \sqrt{2\,g\,r}$

(v) For leaving the vertical circle somewhere between

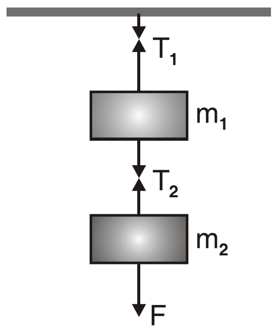
$90° < \theta < 180°, \sqrt{2gr} < \nu_L < \sqrt{5\,g\,r}$

- **Acceleration** of a body down a rough inclined plane $a = g(\sin\theta - \mu \cos\theta)$
- When a person of mass *m* climbs up a rope with acceleration a, the tension in the rope is $T = m(g + a)$

  When the person climbs down the rope with acceleration a, tension in the rope is $T = m(g - a)$
- When a body slides down a smooth inclined plane of inclination θ with the horizontal, its acceleration down the plane is $a = g \sin\theta$
- Suppose two masses m_1 and m_2 are suspended vertically from a rigid support, with the help of strings as shown in Fig. when mass m_2 is pulled down with a force *F*, then

  $$T_2 = F + m_2 g$$

  and $$T_1 = F + (m_1 + m_2)\, g$$

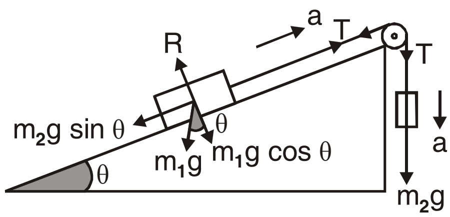


- When the same system of two masses attached to a string passes over a frictionless pulley at the edge of an inclined plane, as shown in Fig. equation wil be...

$m_1\ a = T - m_1\ g\ \sin\theta$

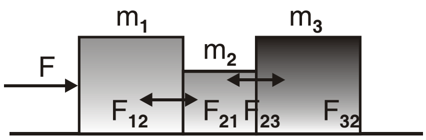
$m_2\ a = m_2\ g - T$

$R = m_1 g \cos\ \theta$



- **Three bodies in contact on a smooth horizontal table**

The action and reaction forces are as shown in Fig.



Common acceleration, $a = \dfrac{F}{m_1 + m_2 + m_3}$

Equation of motion of first body is

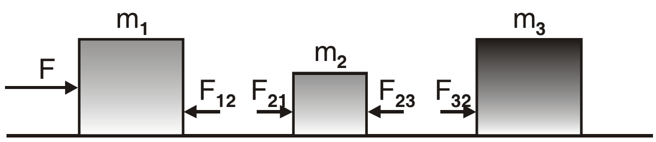
$F - F_{12} = m_1\ a \qquad F_{12} = F_{21} = F_1$

Equation of motion of 2nd body is

$F_{21} - F_{23} = m_2\, a \qquad F_{23} = F_{32} = F_2$

Equation of motion of 3rd body is

$F_{32} = m_3\ a$

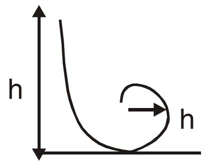


we get

$$F_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

$$F_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$$

- The minimum height through which a motor cyclist has to descend to a vertical loop of radius r is $h = \frac{5}{2} r$
- **Circular Motion**



- **Angular displacement (θ) :** The angular displacement of an object moving around a circular path is defined as the angle subtended by the radius vector at the centre of the circular path in the given time.

- **Angular velocity (ω) :** It is defined as the time rate of change of angular displacement of the object *i.e.* $\omega = d\theta/dt$. Its S.I unit is rad/s.

- **Uniform circular motion :** When a point object is moving on a circular path with a constant speed, then the motion of the object is said to be a uniform circular motion.

- **Centripetal acceleration :** It is defined as the acceleration of as object undergoing uniform circular motion. It always acts along the radius towards the centre of the circular path. The magnitude of centripetal acceleration is, $a = \frac{\nu^2}{r} = r\omega^2 = 4\pi^2 r\nu^2 = \frac{4\pi^2 r}{T^2}$

- The rate of change of angular velocity (ω) is called its angular acceleration (α) *i.e.*

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

- The acceleration which changes the magnitude of the velocity is called tangential acceleration. It is given by $a_T = r\alpha$, where α is the angular acceleration.

  The direction of tangential acceleration is along the tangent to curved path.

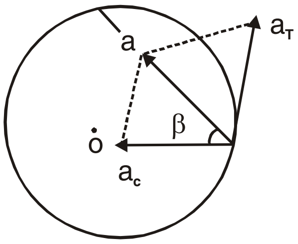
- When a body moves in a circular path with increasing angular velocity, it has two linear accelerations.

  (i) centripetal acceleration, $a_c = v^2/r$; (ii) tangential acceleration, $a_T = r\,\alpha$;

  Resultant acceleration of the body is

$$a = \sqrt{a_c^2 + a_T^2} \qquad \text{and} \qquad \tan\beta = \frac{a_T}{a_c}$$

  where β is the angle between a and a_c



1 MARK QUESTIONS

1. A passenger sitting in a car at rest, pushes the car from within. The car doesn't move. Why?
2. Give the magnitude and directions of the net force acting on a rain drop falling with a constant speed.
3. Why the passengers in a moving car are thrown outwards when it suddenly takes a turn?
4. What is the purpose of using shockers in a car?
5. Why are tyres made of rubber not of steel?
6. Wheels are made circular. Why?
7. A force is required to keep a body in uniform motion in a straight line. Comments.
8. On a rainy day skidding takes place along a curved path. Why?
9. Why does a gun recoils when a bullet is being fired?

10. Why is it difficult to catch a cricket ball than a tennis ball even when both are moving with the same velocity?

11. Calculate the impulse necessary to stop a 1500 kg car moving at a speed of $25ms^{-1}$. (–37500 N-S)

12. Lubricants are used between the two parts of a machine. Why?

13. What provides the centripetal force to a car taking a turn on a level road?

14. What is inertial frame of reference?

15. An athlete runs a certain distance before taking a long jump. Why?

16. Action and reaction forces do not balance each other. Why?

17. The wheels of vehicles are provided with mudguards. Why?

18. China wares are wrapped in straw paper before packing?

20. Why is it difficult to walk on a sand?

21. The outer edge of a curved road is generally raised over the inner edge. Why?

22. Explain why the water doesn't fall even at the top of the circle when the bucket full of water is upside down rotating in a vertical circle?

23. Why does a speedy motor cyclist bends towards the centre of a circular path while taking a turn on it?

24. If the net force acting upon the particle is zero show that its linear momentum remains constant?

2 MARKS QUESTIONS

1. A man getting out of a moving bus runs in the same direction for a certain distance. Comment.

2. The motion of a particle of mass m is described by $h = ut + 1/2gt^2$. Find the force acting on particle. (F = mg)

3. It is difficult to push a box full of clothes than an empty box. Explain.

4. A particle of mass 0.3 kg is subjected to a force of F = -kx with k = 15 Nm^{-1}. What will be its initial acceleration if it is released from a point 20 cm away from the origin? ($a = -10\ ms^{-2}$)

5. A 50 g bullet is fired from a 10 kg gun with a speed of 500 ms^{-1}. What is the speed of the recoil of the gun. (2.5 ms^{-1})

6. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m by applying a force P at the free end of the rope. Find the force exerted by the rope on the block. $F = M\left(\frac{P}{M + m}\right)$

7. Three forces F_1, F_2 and F_3 are acting on the particle of mass m which is stationary. F_2 is perpendicular to F_1 if F_1 is removed, what will be the acceleration of particle? ($a = F_1/m$)

8. A spring balance is attached to the ceiling of a lift. When the lift is at rest spring balance reads 49 N of a body hang on it. If the lift moves :-

   (i) Downward

   (ii) upward, with an acceleration of 5 ms^{-2}

   (iii) with a constant velocity.

   What will be the reading of the balance in each case?
   (24N, 74 N, 49N)

9. It is easier to pull a roller than to push it. Why?

10. A horse cannot pull a cart and run in empty space. Why?

11. A bob of mass 0.1 kg hung from the ceiling of room by a string 2 m long is oscillating. At its mean position the speed of the bob is 1 ms^{-1}. What is the trajectory of the oscillating bob if the string is cut when the bob is :-

    (i) At the mean position (Parabolic)

    (ii) At its extreme position. (vertically downwards)

13. Define force of friction? How does ball bearing reduce friction?

14. Is larger surface area break on a bicycle wheel more effective than smaller surface area brake? Explain?

3 MARKS QUESTIONS

1. A block of mass 500g is at rest on a horizontal table. What steady force is required to give the block a velocity of 200 cms^{-1} in 4 s?

2. A force of 98 N is just required to move a mass of 45 kg on a rough horizontal surface. Find the coefficient of friction and angle of friction? (0.22, 12°24')

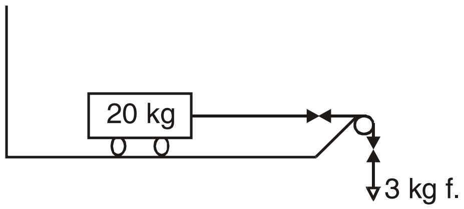
3. Calculate the force required to move a train of 200 quintal up on an incline plane of 1 in 50 with an acceleration of 2 ms^{-2}. The force of friction per quintal is 0.5 N. (40200N0)

4. An aeroplane requires to take off a speed of 80 km h^{-1} on a runway of 100m. Mass of the plane is 10000 kg and coefficient of friction between the plane and the ground is 0.2. If the acceleration of the plane is uniform during take off, Calculate the minimum force required by the engine for the take off. (27.13 N)

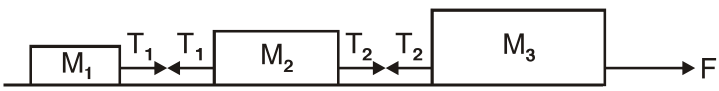
5. A smooth block is released from rest on a 30° incline and travels a distance d. If the time taken to slide on a rough 30° inclined surface is n times large to cover the same distance on a smooth incline. Find the coefficient of friction?

$$\left(1/\sqrt{3}\left(1-1/n^2\right)\right)$$

6. What is the acceleration of the block and trolly system as in fig., if the coefficient of kinetic friction between the trolly and the surface is 0.04? Also calculate tension in the string. Take g = 10 ms^{-2}, mass of string is negligible. (0.957 ms^{-2}, 27.13 N)



7. Three blocks of masses m_1 = 10 kg, m_2 = 20 kg and m_3 = 30 kg are connected by strings on smooth horizontal surface and pulled by a force of 60 N. Find the acceleration of the system and tensions in the string.

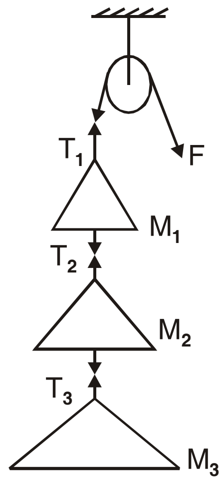


(1ms^{-2} T_1 = 10N, T_2 = 30N)

8. Three masses m_1 = 2 kg, m_2 = 3 kg and m_3 = 5 kg are suspended with a string which is passed over a pulley as shown in fig. Calculate T_1, T_2, T_3 when the system is :-

(i) Stationary

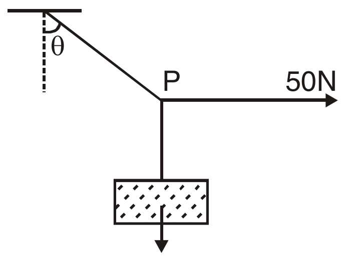
(ii) Moving upward with an acceleration of $2 ms^{-2}$.



(i) (98 N 78.4 N, 49 N)

(ii) (118 N, 94.4N, 59 N)

9. A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N horizontally is applied at the mid – point. P of the rope. Calculate the angle of rope makes with the vertical. Neglect the mass of rope. ($g = 9.8 ms^{-2}$) ($\theta = 40°$)



10. A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2, what is :-

(i) The optimum speed of the race car to avoid wear and tear of the tyres and

(ii) Maximum permissible speed to avoid slipping?
($28.2 ms^{-1}$, $38.2 ms^{-1}$)

11. A bullet of mass 0.01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and surface is 0.25. The bullet remain embedded in the block and combination moves 20 m before coming to rest. Find the speed of the bullet strike the block? (4000 m/s)

12. A glass marble of mass 300 g after falling from a height of 40 m rebounds to a height of 10 m. Find the impulse and the average force between the marble and the floor. The time during which they are in contact is 0.1 s. Take g = 9.8 ms^{-2}. (4.2N ; 42N)

5 MARKS QUESTIONS

13. Define the principle of conservation of linear momentum. Deduce the law of conservation of linear momentum from Newton's third law of motion.

14. Why circular roads are banked? Derive an expression for angle of banking for safe circular turn?

15. Obtain an expression for minimum velocity of projection of a body at the lowest point for Looping a vertical loop.

16. Show that the area under the force-time graph gives the magnitude of the impulse of the given force for the following case when (i) force is constant (ii) variable force.

17. Derive an expression for acceleration of a body down a rough inclined plane? (Sliding only)

UNIT – IV

WORK ENERGY AND POWER

- Work done (W) by a constant force $\vec{F}$ in producing a displacement $\vec{s}$ in a body is

  $W = \vec{F}.\vec{s} = Fs\cos\theta$, where q is smaller angle between $\vec{F}$ and $\vec{s}$.

- If the force is not constant,

  $W = \int \vec{F}.d\,\vec{s}$ = area under the force displacement graph.

- Work done is a *scalar* quantity.

- Work done = *Positive* when θ lies between 0 and $\pi/2$. Work done = *negative* when θ lies between $\frac{\pi}{2}$ and π. Work done = zero, when $\theta = \frac{\pi}{2}$

- SI unit of work is *joule* and the cgs unit of work is *erg.*

S.No.	*Unit*	*Symbol*	*Equivalent in joule*
1.	erg	erg	10^{-7} J
2.	Calorie	Cal.	4.2 J
3.	Kiowatt hour	k Wh	3.6×10^{6} J
4.	electron volt	e V	1.6×10^{-19} J

- **In a conservative force,** work done is independent of the path followed by the body.

- **Work done by a spring force**

  $$W = -\frac{1}{2}k\,x^{2}$$

  where k is spring constant and x is the displacement from normal position of rest.

- Power of a body is defined as the *time rate of doing work* by the body.

  $$P = \frac{dw}{dt} = \frac{\vec{F}.d\vec{s}}{dt} = \vec{F}.\vec{\nu} \qquad \left(\because \vec{\nu} = \frac{d\vec{s}}{dt}\right)$$

  Thus, $P = \vec{F}.\vec{\nu} = F\,\nu\cos\theta$

  Here, θ is angle between $\vec{F}$ and $\vec{\nu}$ of the body.

  The practical unit of power is horse power *(h . p),* where 1 *h.p* = 746 W

- **Energy** of a body is defined as the *capacity of the body* to do the work. Energy is a *scalar* quantity. Energy has the same units and dimensions as those of work.

- "*Principle of conservation of energy",* according to which sum of total energy in this universe remains constant. The amount of energy disappearing in one form is exactly equal to the amount of energy appearing in any other form.

- **Kinetic Energy** of a body is the energy possessed by the body due to its motion.

  K.E. of translation $= \frac{1}{2}m\nu^2$ where *m* is mass and ν is velocity of the body.

- The energy stored in a body or system due to its configuration or shape is called *potential energy.*

  Gravitational P.E. = *mgh*

  where *m* is mass of a body at a height h and acceleration due to gravity is *g*.

  P.E. may be positive or negative when forces involved are repulsive, P.E. is positive

  When forces involved are attractive, P.E. is negative.

- **Mechanical Energy**

  Mechanical energy of a particle or system is defined as the sum of K.E. and P.E. of the system. K.E. is always positive, but The mechanical energy may be zero.

  Positive or negative. Negative mechanical energy represents a *bound state.*

- According to **work energy theorem,** work and energy are equivalent. Work done by a force on a body results in the net increase in K.E. of the body by the same amount.

- The equivalence between mass and energy is expressed in terms of *Einstein relation.*

  $$E = mc^2$$

  Where *m* is the **change in mass** and *E* is the energy and *c* is velocity of light in vaccum.

- **Conservative and Non-conservative Forces**

S.No.	*Conservative forces*		*Non Conservative forces*
1.	Work done by such forces in displacing a particle does not depend upon the path along which particle is displaced.	1.	Work done by such forces in displacing a particle depends upon the path along which particle is displaced.
2.	Work done by such forces in displacing a particle around a closed path is zero.	2.	Work done by such forces in displacing a particle around a closed path is NOT zero.
3.	K.E. of particle remains constant.	3.	K.E. of particle changes.

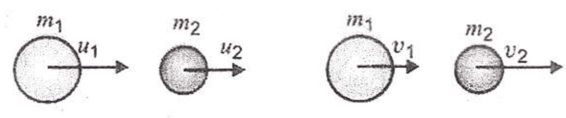
- **Collisions**

  When a body strikes against body or one body influences the other from a distance, **collision is said to be occur.** Collisions are of two types :

  (a) *Perfectly elastic collision* - in which there is *no change in kinetic energy of the system,* i.e., total K.E. before collision = total K.E. after collision.

  (b) *Perfectly inelastic collision* - in which *K.E. it NOT conserved.* the bodies stick together after impact.

- If the initial and final velocities of colliding bodies lie along the same line then it is known as head on collision.



law of conservation of linear momentum. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

principle of conservation of K.E.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1\nu_1^2 + \frac{1}{2}m_2\nu_2^2$$

$$\nu_1 = \frac{(m_1 - m_2)}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2} \text{ and } \nu_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2$$

- *coefficient of restitution or resilience of the two bodies.* $e = -\frac{\nu_1 - \nu_2}{u_1 - u_2}$

  For a perfectly elastic collision, e = 1 and for a *perfectly inelastic collision,* $e = 0$. $(0 \le e \le 1)$

● S.N.	Perfectly elastic collisions	Perfectly Inelastic collisions
1.	Particles do not stick together after collision.	1. Particles stick together after collision.
2.	Rel. vel. of separation after collision = rel. vel. of approach before collision	2. Rel. vel. of separation after collision in zero.
3.	Coeff. of restitution, $e = 1$.	3. Coeff. of restitution, $e = 0$
4.	Linear momentum is conserved.	4. Linear momentum is conserved.
5.	K.E. is conserved.	5. K.E. is *not* conserved.

For Competition

- Work done by a body *does not depend upon the time taken* to complete the work.

- When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$, then

$$W = \frac{-1}{2}k\left(x_2^2 - x_1^2\right)$$

- Some practical units of energy and their relation with SI unit of energy (joule) are :

  (i) 1 calorie = 4.2 J

  (ii) 1 kilowatt hour (kWh) = 3.6×10^6J.

  (iii) 1 electron volt (1 eV) = 1.6×10^{-19} J.

- Suppose a body dropped from a height h_0 above the ground strikes the ground with a velocity ν_0. If body rebound with a velocity ν_1 and go to a height h_1, then coefficient of restitution

$$e = \frac{\nu_1}{\nu_0} = \sqrt{\frac{2g\,h_1}{2g\,h_0}} = \left(\frac{h_1}{h_0}\right)^{1/2}$$

If ν_n is velocity with which the body rebounds after n collisions to a height h_n, then $\frac{\nu_n}{\nu_0} = \left(\frac{h_n}{h_0}\right)^{1/2} = e^n$

- When a machine gun of power P fires n bullets per second, each with K.E. = E, then $P = n\,E$.
- Thus power dissipated by the centripetal force is zero.
- Work done against friction on a horizontal surface is
$W = \mu\, R\, x = \mu\, m\, g\, x$
- Work done against friction while moving up an inclined plane is $W = F\,x = \mu\, R\, x = (\mu\, m\, g \cos\theta)\, x$
- Stopping distance of a vehicle = K.E/stopping force.
- When two vehicles of masses m_1, m_2 moving with velocities ν_1 and ν_2 respectively are stopped by the same force, their stopping distances x_1 and x_2 are $\frac{x_1}{x_2} = \frac{\text{K.E.}_1}{\text{K.E}_2} = \frac{\left(\frac{1}{2}\right)m_1\nu_1^2}{\left(\frac{1}{2}\right)m_2\nu_2^2}$

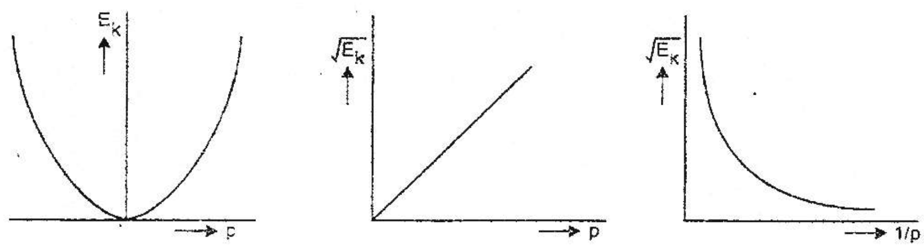
The stopping times t_1 and t_2 of the two vehicles are $\frac{t_1}{t_2} = \frac{p_1}{p_2} = \frac{m_1\nu_1}{m_2\nu_2}$

- When linear momentum of a body increases by a factor n, its kinetic energy is increased by a factor n^2
- When speed of a vehicle becomes n times, its stopping distance becomes n^2 times, provided stopping force applied is **remains same.**

- When the collision is not perfectly elastic, then the expressions for velocities after direct **collision are :**

$$\nu_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} + \frac{(1+e)m_2u_2}{m_1 + m_2}; \nu_2 = \frac{(1+e)m_1u_1}{m_1 + m_2} + \frac{(m_2 - em_1)u_2}{m_1 + m_2}$$

- **Graphs between p and E.**



- **Spring:-**
    - Spring constant $k = F/x$ = restoring force per unit extension/ compression.
    - If a spring of spring constant k is cut into n parts of equal length, spring constant of each part = nk.
    - When a spring of spring constant k is cut into two parts of unequal lengths l_1 and l_2, the spring constants of the two parts are :

    $$k_1 = \frac{(l_1 + l_2)}{l_1} \text{ and } k_2 = \frac{k(l_1 + l_2)}{l_2}$$

    - When two springs of spring constants k_1 and k_2 are joined *in series.*

    the equivalent spring constant k of the combination is given by

    $$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

    - When two springs of spring constants k_1 and k_2 are connected in parallel the equivalent spring constant k is given by $k = k_1 + k_2$

WORK ENERGY AND POWER

VERY SHORT ANSWER QUESTION (MARK 1)

1. Define the conservative and non conservative forces. Give examples of each.
2. A light body and a heavy body have same linear momentum. Which one has greater K.E? (**Ans. :** lighter body has more K.E.)
3. The momentum of the body is doubled what % does its K.E change? (300%)
4. How can we change the momentum of a body without change in its K.E.?
5. Which spring has greater value of spring constant – a hard spring or a delicate spring?
6. Two bodies stick together after collision. What type of collision is in b/w these two bodies?
7. State the two conditions under which a force does no work?
8. How will the momentum of a body changes if its K.E is doubled?
9. K.E of a body is increased by 300 %. Find the % increase in its momentum? (100%)
10. A light and a heavy body have same K.E. which of the two have more momentum and why? (heavier body)
11. Mountain roads rarely go straight up the slop, but wind up gradually. Why?
12. A truck and a car moving with the same K.E on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance?
13. What happens to the P.E of a bubble when it rises in water? (decrease)
14. A body is moving at constant speed over a friction surface. What is the work done by the weight of the body? (W = 0)
15. What type of energy is stored in the spring of a water?
16. Define spring constant of a spring?

17. What happens when a sphere collides head on elastically with a sphere of same mass initially at rest?

SHORT ANSWER QUESTION (2 MARKS)

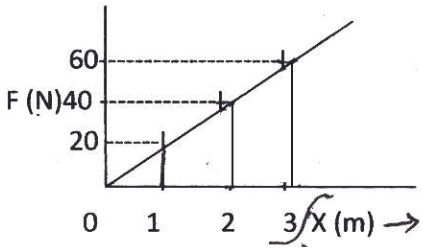
1. A elastic spring is compressed by an amount x. Show that its P.E is 1/2 kx^2 Where k is the spring constant?

2. Derive an expression for it K.E of a body of mass m moving with a velocity v by calculus method.

3. Show that the total mechanical energy of a body falling freely under gravity is conserved.

4. How high must a body be lifted to gain an amount of P.E equal to the K.E it has when moving at speed 20 ms^{-1}. (The value of acceleration due to gravity at a place is 9.8 ms^{-2}). (20.2 m)

5. Calculate the work done in moving the object from x = 2 to x = 3 m from the graph given :-



6. After bullet is fired, gun recoils freely. Compare the K.E. of bullet and the gun. ($KE_b > KE_g$)

7. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10% what is the speed with which the bob arrived at the lowest point.

8. Two springs A and B are identical except that A is harder than B($K_A > K_B$) if these are stretched by the equal force. In which spring will more work be done?

9. Explain the term work? show that work done is equal to the dot product of force and displacement.

10. Find the work done if a particle moves from position $\vec{r_1} = \left(3\hat{i} + 2\hat{j} - 6\hat{k}\right)$ to a position $\vec{r_2} = \left(14\hat{i} + 13\hat{j} - 9\hat{k}\right)$ under the effect of force, $\vec{F} = \left(4\hat{i} + \hat{j} + 3\hat{k}\right)N$.

11. A particle of mass 1 g moving with a velocity $\vec{v_1} = \left(3\hat{i} - 2\hat{j}\right)$ has elastic collision with another particle of mass 2 g moving with a velocity $\vec{v_2} = 4\hat{j} - 6\hat{k}$. Find the velocity of the particle formed. (V = 4.6 ms^{-1})

12. A body of mass m accelerate uniformly from rest to velocity v_1, in time t_1. Derive an expression for the instantaneous power delivered to the body as a function of time (P = mv_1^2t/t_1^2)

13. How much energy is related when 1 mg of U is completely destroyed in an atomic bomb? (9×10^{10})j

14. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is further stretched through 0.1m. Calculate work done. (4000 Nm^{-1}, 60 J)

15. For a particle executing S.H.M, potential energy function is given by V(x) = 1/2 kx^2, K = 0.5 Nm^{-1} is force constant of the oscillator. If the total energy of the particle is 1 J, show that particle turn back when x = ± 2m from its mean position.

16. A pump on the ground floor of a building can pump up water to fill a tank of volume $30m^3$ in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump. The efficiency of the pump is 30%. (43.567 Kw)

17. A force $\vec{F} = 2x\hat{j}N$ acts in a region, where a particle moves clock wise along the sides of a square of length 2m. Find the total amount of work done? (8 J)

18. A mass less pan is placed on an elastic spring. Spring is compressed by 0.01 m when a sand bag of mass 0.1 kg is dropped on it from a height 0.24 m. From what height should the sand bag be dropped to cause a compression of 0.04 m? (3.96 m)

19. State and prove Work Energy Theorem.

20. Show that in an elastic one dimensional collision the relative velocity of approach before collision is equal to the relative velocity of separation after collision.

21. Show that in a head on collision between two balls of equal masses moving along a straight line the balls exchange their velocities.

22. A force acting on a body along Y axis the direction of motion of the body. If this force produces a potential energy $U = A\,x^4$ when $A = 1.2\ Jm^{-4}$. What is the force acting on the body when the body is at $x = 0.8m$. (2.46N)

23. A spring of force constant K is cut into two equal pieces. Calculate force constant of each part.

24. How vibration of a simple pendulum does illustrate the principle of energy conservation?

25. A spring is first stretched by x by applying a force F. Now the extension of the spring is increases to 3x. What will be the new force required to keep the spring in this condition? Calculate the work done in increasing the extension.

LONG ANSWER QUESTIONS (5 MARKS)

26. Show that at any instant of time during the motion total mechanical energy of a freely falling body remains constant. Show graphically the variation of K.E. and P.E. during the motion.

27. Define spring constant, Write the characteristics of the force during the elongation of a spring. Derive the relation for the PE stored when it is elongated by X. Draw the graphs to show the variation of P.E. and force with elongation.

28. How does a perfectly inelastic collision differ from perfectly elastic collision? Two particles of mass m_1 and m_2 having velocities U_1 and U_2 respectively make a head on collision. Derive the relation for their final velocities. Discuss the following special cases.

    (i) $m_1 = m_2$

    (ii) $m_1 >> m_2$ and $U_2 = 0$

    (iii) $m_1 << m_2$ and $u_1 = 0$

29. A body is moving along z-axis of a coordinate system under the effect of a constant force $\vec{F} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$N. Find the work done by the force in moving the body a distance of 2 m along z-axis.

30. In lifting a 10 kg weight to a height of 2m, 230 J energy is spent. Calculate the acceleration with which it was raised?

31. A bullet of mass 0.02 kg is moving with a speed of $10m^{-1}s$. It can penetrate 10 cm of a wooden block, and comes to rest. If the thickness of the target would be 6 cm only find the KE of the bullet when it comes out.

(**Ans :** 0.4 J)

32. A man pulls a lawn roller with a force of 20 kg F. If he applies the force at an angle of 60° with the ground. Calculate the power developed if he takes 1 min in doing so.

33. A ball bounces to 80% of its original height. Calculate the mechanical energy lost in each bounce.

34. A pendulum bob of mass 0.1 kg is suspended by a string of 1 m long. The bob is displaced so that the string becomes horizontal and released. Find its kinetic energy when the string makes an angle of (i) 0°, (ii) 30° with the vertical.

35. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to k/r^2, k is a constant. What is the total energy of the particle. (–k/2r)

36. The K.E. of a particle moving along a circle of radius R depends on the distance covered S as $T = \alpha S^2$ where α is constant. Find the force acting on the particle as a function of S. $\left(F = 2\alpha S\sqrt{1 + s^2/R^2}\right)$

37. Derive the relation $E = \Delta mc^2$.

Solution: As $m = m_o \Big/ \sqrt{1 - v^2/c^2}$ or $m(1 - v^2/c^2)^{1/2} = m_o$ or $m^2(1 - v^2/c^2) = m_o^2$

or $m^2c^2 - m^2v^2 = m_o^2c^2$

Differentiating both sides taking m_o and c as constant

$2m\,dm\,c^2 - 2m\,dm\,v^2 - m^2\,2v\,dv = 0$ or $2m(dm\,c^2 - dm\,v^2 - mv\,dv) = 0$

But $2m \neq 0$ so, $dm\, c^2 - dm\, v^2 - m\, v\, dv = 0$ or $v^2 dm + m\, v\, dv = c^2\, dm$ (1)

Let a force F applied on the body displaces it through a distance dx. The work done in doing so will change the KE of the body. Δ **K.E.=dE =F.dx**

But F = d(mv)/dt = (mdv/dt + vdm/dt) or dw = F.dx

dw = (mdv/dt + vdm/dt)dx or dw = mdv/dt. dx + vdm/dt dx

dw = mdx/dt.dv + vdx/dt dm dw = dE = m v dv + v^2 dm

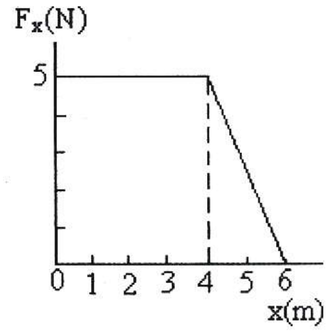
from equ. (i) dE = c^2 dm (2)

let the mass increases from m_o to m as K.E. increases from 0 to E. Integrating eq. 2 for the given limits

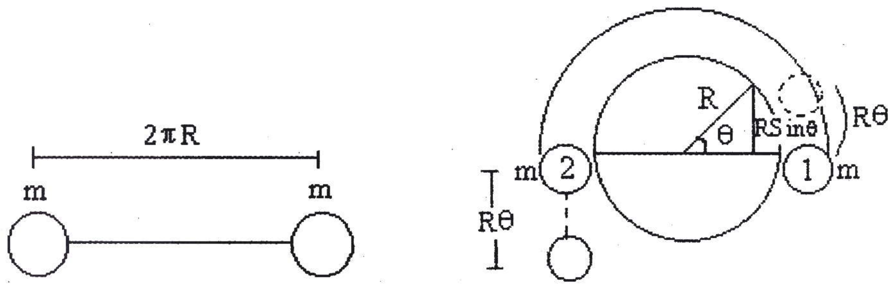
$E - o = c^2(m - m_o)$ As $m_o\, c^2$ rest mass energy = o

therefore ***E = m c²***

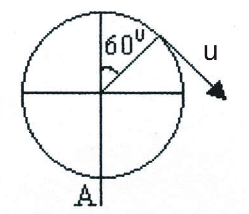
38. Force acting on a particle varies with x as shown in figure, below. Calculate the work done by the force as the particle moves from x = 0 to x = 6m.



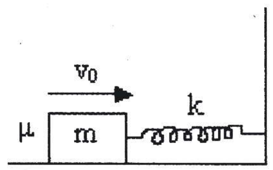
39. The mass system is kept on sphere. Ball 1 is slightly disturbed. What is the velocity of these balls when it is making angle 'θ' with horizontal (friction is absent everywhere).



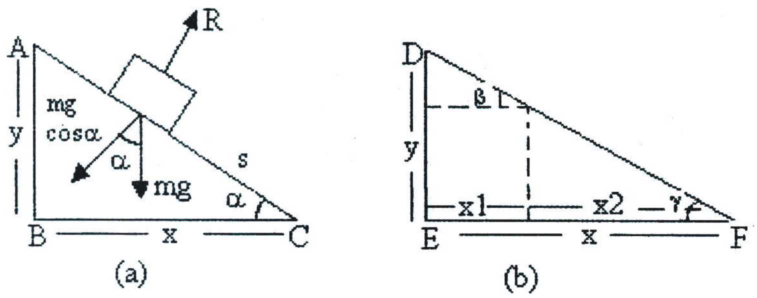
40. What is the minimum value of 'u' for completing circular motion of particle as shown in figure given below?



41. A block is projected horizontally on rough horizontal floor. The coefficient of friction between the block and the floor is μ. The block strikes a light spring of stiffness k with velocity v_0. Find the maximum compression of the spring.



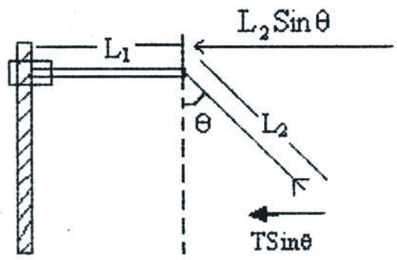
42. In figure a and b, AC, DE and EF are fixed inclined planes BC = EF = x and AB = DE = y. A small block of mass m is released from rest from the point A. It slides down AC and reaches C with a speed V_C. The same block is released from rest from the point D, it slides down DEF and reaches the point F with speed VF. The coefficient of kinetic friction between the block and the surface AC and DEF is a, calculate V_C and V_F.



43. A locomotive of mass m starts moving so that its velocity v is according to the law $v = a\sqrt{s}$, where a is constant and s is distance covered. Find the total work done by all the forces acting the locomotive during the first t seconds after the beginning of motion.

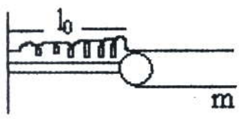
44. The Kinetic energy of a particle moving along a circle of radius R depends on the distance covered S and T = αs^2 where α is constant. Find the force acting on the particle as a function of S.

45. A ball suspended by a string of length 20 cm is fixed to the free end of the pivoted rod of length 40 cm as shown in the figure. The rod is made to rotate in a horizontal plane with constant angular speed. The string makes an angle θ = 30° with the vertical axis. Find the angular speed of the rotation?

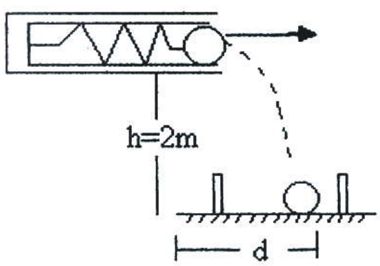


ω = angular speed;   T = Tension in the string.

46. A smooth, light rod AB can rotate about a vertical axis passing through its end A. The rod is fitted with small sleeve of mass m attached to the end A by a weightless spring of length 10, stiffness k. What work must be performed to slowly get this system going and the angular velocity w?



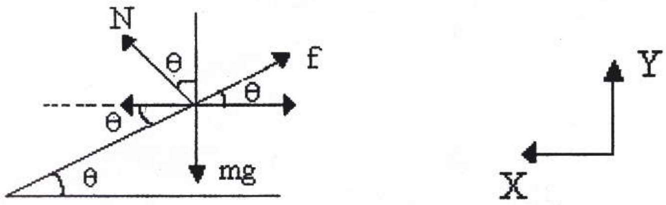
47. A spring gun having spring constant 100 N/m, a small ball of mass 0.1 Kg is placed in its barrel by compressing the spring through 0.05m as shown in figure.



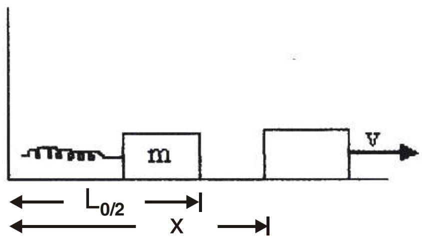
(a) Find the velocity of the ball when spring is released.

(b) Where should a box be placed on ground so that ball falls in it, if the ball leaves the gun horizontally at a height of 2m above the ground.

48. A turn of radius 20m is banked for the vehicle of mass 200kg going at a speed of $10ms^{-1}$. Find the direction and magnitude of frictional force acting on a vehicle if it moves with a speed (a) 5 m/s (b) 15 m/s assume the friction is sufficient to prevent slipping ($g = 10m/s^2$).

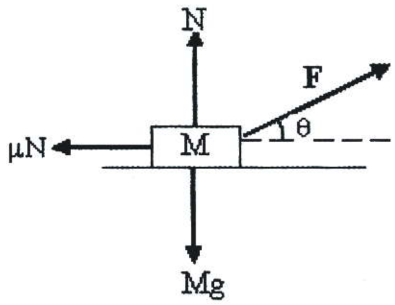


49. A block of mass m is pushed against a spring constant K fixed at one end to a wall. The block can slide on a frictionless table as shown in figure. The natural length of the spring is L_0 and it is compressed to half its natural length when the block is released. Find the velocity of the block as a function of its distance x from the wall.

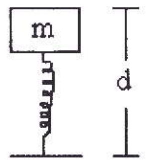


50. A ball falls under gravity from a height 10m, with an initial velocity V_0, it hits the ground, looses 50% of its energy after collision and it rises to the same height. What is the value of V_0?

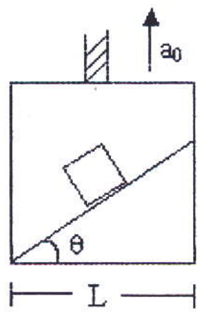
51. A block of mass M is pulled along a horizontal surface by applying a force at an angle θ with horizontal. Co-efficient of friction between block and surface is μ. If the block travels with uniform velocity, Find the work done by this applied force during a displacement d of the block.

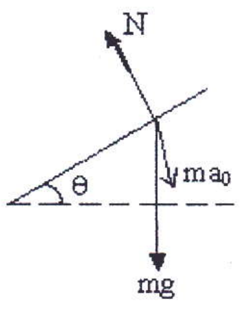


52. A block of mass m released from rest onto an ideal non-deformed spring of spring constant ‘K’ form a negligible height. Neglecting the air resistance, find the compression ‘d’ of the spring.

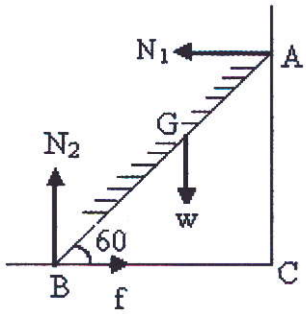


53. A particle slides down a smooth inclined plane of elevation θ, fixed in an elevator going up with an acceleration a_0 as shown in figure. The base of the incline has a length L. Find the time taken by the particle to reach the bottom.

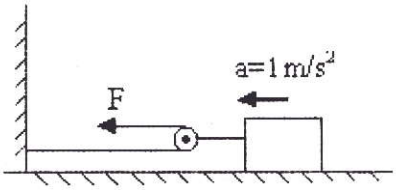




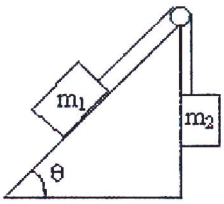
54. 6m long ladder weighting 30Kg rest with its upper end against a smooth wall and lower end on rough ground. What should be the minimum coefficient of friction between the ground and the ladder for it to be inclined at 60° with the horizontal without slipping? Take g = 10m/s^2.

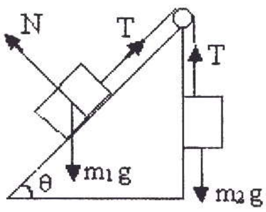


55. A block of mass 200kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in figure. What horizontal force F should be applied to produce in the block an acceleration of 1 m/s^2?



56. Two bodies of masses m_1 and m_2 are connected by a light string going over a smooth light pulley at the end of an incline. The mass m_1 lies on the incline and m_2 hangs vertically. The system is at rest; Find the angle of incline and the force exerted by the incline on the body of mass m_1.





UNIT V

MOTION OF SYSTEMS OF PARTICLES AND RIGID BODY

- **Centre of mass** of a body is a point where the entire mass of the body can be supposed to be concentrated.
- For a system of *n*-particles, the centre of mass is given by

$$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + \ldots\ldots.. + m_n\vec{r_n}}{m_1 + m_2 + m_3 + \ldots\ldots. + m_n} = \frac{\sum_{i=1}^{i=n} m_i \vec{r_1}}{M}$$

where M = $m_1 + m_2 + \ldots m_n$

- **Torque** $(\vec{\tau})$ *The turning effect of a force with respect to some axis, is called moment of force or torque due to the force.* Torque is measured as the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the axis of rotation.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

SI unit of torque is Nm.

- **Angular momentum** $(\vec{L})$**.** *It is the rotational analogue of linear momentum and is measured as the product of the linear momentum and the perpendicular distance of its line action from the axis of rotation.*

If $\vec{P}$ is linear momentum of the particle and $\vec{r}$ its position vector, then angular momentum of the particle, $\vec{L} = \vec{r} \times \vec{p}$

SI unit of angular momentum is kg m^2 s^{-1}.

- **Relation** between torque and angular momentum :

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

- **Law of conservation of angular momentum.** If no external torque acts on a system, then the total angular momentum of the system always remains conserved.

  $\vec{L_1} + \vec{L_2} + \vec{L_3} + ... + \vec{L_n} = \vec{L}_{total} = a$ constant

- **Moment of inertia (*I*).** *The moment of inertia of a rigid body about a given axis is the sum of the products of masses of the various particles* **with** *squares of their respective perpendicular distances from the axis of rotation.*

  $$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_2^2 + ... + m_n r_n^2 = \sum_{i=1}^{i=n} m_i r_i^2$$

  SI unit of moment of inertia is kg m^2.

- **Radius of gyration (*K*).** It is defined as the distance of a point from the axis of rotation at which, if whole mass of the body were concentrated, then

  $$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}} \text{ and } I = MK^2.$$

  SI unit of radius of gyration is m.

- **Theorem of perpendicular axes.** It states that the moment of inertia of a 2-d object about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through the **plane.**

  $I_z = I_x + I_y$

  where *X* and *Y*-axes lie in the plane of the **object** and Z-axis is perpendicular to its plane and passes through the point of intersection of *X* and *Y* axes.

- **Theorem of parallel axes.** It states that the moment of inertia of a rigid body about any axis is equal to moment of inertia of the body about a parallel axis through its centre of mass plus the product of mass of the body and the square of the perpendicular distance between the axes.

  $I = I_c + M h^2$, where I_c is moment of inertia of the body about an axis through its centre of mass and h is the perpendicular distance between the two axes.

- **Moment of inertia of some object :-**

S. No.	Body		Axis of rotation	Moment of Inertia (I)
●	Uniform circular ring of mass *M* and radius *R*	(i)	about an axis passing through centre and perp. to its plane.	MR^2
		(ii)	about a diameter.	$\frac{1}{2}MR^2$
		(iii)	about a tangent in its own plane.	$\frac{3}{2}MR^2$
		(iv)	about a tangent $\perp$ to its plane.	$2\ MR^2$
●	Uniform circular disc of mass *M* and radius *R*.	(i)	about an axis passing through centre and perp. to its plane.	$\frac{1}{2}MR^2$
		(ii)	about a diameter.	$\frac{1}{4}MR^2$
		(iii)	about a tangent in its own plane.	$\frac{5}{4}MR^2$
		(iv)	about a tangent $\perp$ to its plane.	$\frac{3}{2}MR^2$
●	Solid sphere of radius *R* and mass *M*	(i)	about its diameter.	$\frac{2}{5}MR^2$
		(ii)	about a tangential axis.	$\frac{7}{5}MR^2$
●	Spherical shell of radius *R* and mass *M*.	(i)	about is diameter.	$\frac{2}{3}MR^2$
		(ii)	about a tangential axis.	$\frac{5}{3}MR^2$
●	Long thin rod of	(i)	about an axis through	$\frac{ML^2}{12}$

length L.	C.G. and $\perp$ to rod.	
	(ii) about an axis through one end and $\perp$ to rod.	$\frac{ML^2}{3}$

- **Law of conservation of angular momentum.** If no external torque acts on a system, the total angular momentum of the system remains unchanged.

  $I\vec{\omega} =$ constant vector or $I_1\omega_1 = I_2\omega_2$, provided no external torque acts on the system.

- For **translational equilibrium** of a rigid body, $\vec{F} = \sum_i \vec{F}_i = 0$
- For **rotational equilibrium** of a rigid body, $\vec{\tau} = \sum_i \vec{\tau}_i = 0$
- **Analogy between various quantities describing linear motion and rotational motion.**

S.No.	linear motion	S.No.	Rotation motion
●	Distance/displacement (s)	1.	Angle or angular displacement (θ)
●	Linear velocity, $v = \frac{dx}{dt}$	2.	Angular velocity, $\omega = \frac{d\theta}{dt}$
●	Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$	3.	Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
●	Mass *(m)*	4.	Moment of interia (I)
●	Linear momentum, $p = m\,v$	5.	Angular momentum, $L = I\omega$
●	Force, $F = m\,a$	6.	Torque, $\tau = I\alpha$
●	Also, force $F = \frac{dp}{dt}$	7.	Also, torque, $\tau = \frac{dL}{dt}$
●	Translational KE, $K_T = \frac{1}{2}mv^2$	8.	Rotational K.E., $K_R = \frac{1}{2}I\,\omega^2$
●	Work done, $W = F\,s$	9.	Work done, $W = \tau\,\theta$
●	Power, $P = F\,v$	10.	Power, $P = \tau\,\omega$
●	(Principle of conservation of linear momentum)		(Principle of conservation of angular momentum).

- Equations of translatory motion
  (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}a t^2$
  (iii) $v^2 - u^2 = 2as$,

11. Equations of rotational motion
    $\omega_2 = \omega_1 + \alpha t$
    (ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
    (iii) $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

Motion of a body rolling without slipping on an inclined plane acceleration

$$a = \frac{mg\sin\theta}{m + I/r^2}$$

Kinetic energy of a rolling body is

E_K = K.E of translation (K_T) + K.E. of rotation (K_e)

$$E_K = \frac{1}{2}M\nu^2 + \frac{1}{2}I w^2$$

ROTATIONAL MOTION (1 MARK)

1. What is a rigid body?
2. State the principle of moments of rotational equilibrium.
3. Is centre of mass of a body necessarily lie inside the body? Give any example
4. Can the couple acting on a rigid body produce translatory motion?
6. Which component of linear momentum does not contribute to angular momentum?
8. A system is in stable equilibrium. What can we say about its potential energy?
10. Is radius of gyration a constant quantity?
11. Two solid spheres of the same mass are made of metals of different densities. Which of them has a large moment of inertia about the diameter?
12. The moment of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momenta are equal. Which one has a greater kinetic energy?

13. A particle moves on a circular path with decreasing speed. What happens to its angular momentum?

14. What is the value of instantaneous speed of the point of contact during pure rolling?

15. Which physical quantity is conserved when a planet revolves around the sun?

16. What is the value of torque on the planet due to the gravitational force of sun?

17. If no external torque acts on a body, will its angular velocity be constant?

18. Why there are two propellers in a helicopter?

20. A child sits stationary at one end of a long trolley moving uniformly with speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, then what is the effect of the speed of the centre of mass of the (trolley + child) system?

ANSWERS

3. No. example ring

4. No. It can produce only rototory motion.

6. Radial Component

7. No, because centripetal acceleration is not zero.

8. P.E. is minimum.

11. No, it changes with the position of axis of rotation.

12. Sphere of small density will have large moment of inertia.

13. $K = \dfrac{L^2}{2I} \quad \Rightarrow \quad K_B > K_A$

14. as $\vec{L} = \vec{r} \times m\vec{v}$ i : e magnitude $\vec{L}$ decreases but direction remains constant.

15. zero

16. Angular momentum of planet.

17. zero.

18. No. $\omega\alpha\frac{1}{1}$.

19. due to conservation of angular momentum

20. No change in speed of system as no external force is working.

ROTATIONAL MOTION (2 MARKS)

1. Show that in the absence of any external force, the velocity of the centre of mass remains constant.

2. State the factors on which the position of centre of mass of a rigid body depends.

3. What is the turning effect of force called for ? On what factors does it depend?

4. State the factors on which the moment of inertia of a body depends.

5. On what factors does radius of gyration of body depend?

6. Why do we prefer to use a wrench of longer arm?

7. Can a body be in equilibrium while in motion? If yes, give an example.

8. There is a stick half of which is wooden and half is of steel. (i) it is pivoted at the wooden end and a force is applied at the steel end at right angle to its length (ii) it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why?

9. If earth contracts to half its radius what would be the length of the day at equator?

10. An internal force can not change the state of motion of centre of mass of a body. How does the internal force of the brakes bring a vehicle to rest?

11. When does a rigid body said to be in equilibrium? State the necessary condition for a body to be in equilibrium.

12. How will you distinguish between a hard boiled egg and a raw egg by spinning it on a table top?

13. What are binary stars? Discuss their motion in respect of their centre of mass.

14. In which condition a body lying in gravitational field is in stable equilibrium?

15. Give the physical significance of moment of inertia.

ANSWERS

2. (i) Shape of body

   (ii) mass distribution

3. Torque

   Factors (i) Magnitude of force

   (ii) Perpendicular distance of force vector from axis of rotation.

4. (i) Mass of body

   (ii) Size and shape of body

   (iii) Mass distribution w.r.t. axis of rotation

   (iv) position and orientation of rotational axis

5. Mass distribution.

6. to increase torque.

7. Yes, if body has no linear and angular acceleration. Hence a body in uniform straight line motion will be in equilibrium.

8. I (first case) > I(Second case)

   $\because \quad \tau = I\alpha$

   $\Rightarrow \alpha$ (first case) < α (second case)

9. $I_1 = \frac{2}{5}MR^2 \quad \Rightarrow \quad I_2^e = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \quad \Rightarrow \quad I_2 = \frac{I}{4}$

   $L = I_1 w_1 = I_2 w_2$

or $\quad I\left(\frac{2\pi}{T_1}\right) = \frac{1}{4}\left(\frac{2\pi}{T_2}\right)$

or $\quad T_2 = \frac{T_1}{4} = \frac{24}{4} = 6 \text{ hours}$

10. In this case the force which bring the vehicle to rest is friction, and it is an external force.

11. For translation equilibrium

$$\sum \vec{F}_{gt} = 0$$

For rotational equilibrium

$$\sum \vec{\tau}_{ext} = 0$$

12. For same external torque, angular acceleration of raw egg will be small than that of Hard boiled egg

14. When vertical line through centre of gravity passes through the base of the body.

15. It plays the same role in rotatory motion as the mass does in translatory motion.

ROTATIONAL MOTION (3 MARKS)

1. Derive the three equation of rotational motion (i) $\omega = \omega_0 + \alpha t$

   (ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

   (iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

   under constant angular acceleration. Here symbols have usual meaning.

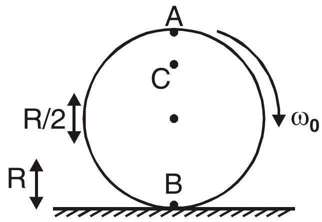
2. Obtain an expression for the work done by a torque. Hence write the expression for power.

3. Prove that the rate of change of angular momentum of a system of particles about a reference point is equal to the net torque acting on the system.

4. Derive a relation between angular momentum, moment of inertia and angular velocity of a rigid body.

5. Show that moment of a couple does not depend on the point about which moment is calculated.

6. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any linear push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the dics shown in figure. Will the disc roll?



7. A uniform circular disc of radius R is rolling on a horizontal surface. Determine the tangential velocity (i) at the upper most point (ii) at the centre of mass and (iii) at the point of contact.

8. Derive an expression for the total work done on a rigid body executing both translational and rotational motions.

9. Prove that the acceleration of a solid cylinder rolling without slipping down an inclined plane is $\frac{2g}{3}\sin\theta$.

10. Show that the angular momentum of a particle is the product of its linear momentum and moment arm. Also show that the angular momentum is produced only by the angular component of linear momentum.

ANSWER

6. For A $V_A = R\omega_0$ in forward direction

   For B = $V_B = R\omega_0$ in backward direction

   For C $V_C = \frac{R}{2}\omega_0$ in forward direction disc will not roll.

NUMERICALS

1. Three masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1m. Locate the centre of mass of the system.

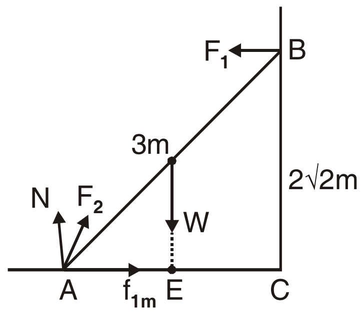
2. Two particles mass 100 g and 300 g at a given time have velocities $10\hat{i}-7\hat{j}-3\hat{k}$ and $7\hat{i}-9\hat{j}+6\hat{k}$ ms^{-1} respectively. Determine velocity of COM.

3. From a uniform disc of radius R, a circular disc of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of original disc. Locate the centre of gravity of the resultant flat body.

4. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (i) What is its angular acceleration (assume the acceleration to be uniform) (ii) How many revolutions does the wheel make during this time?

5. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm, what is the mass of the matre stick?

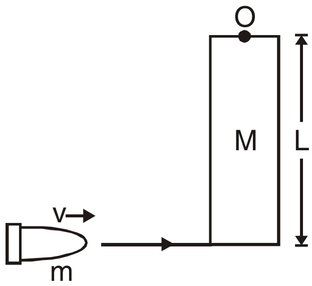
6. A 3m long ladder weighting 20 kg leans on a frictionless wall. Its feet rest on the floor 1 m from the wall as shown in figure. Find the reaction forces of the wall and the floor.



7. Calculate the ratio of radii of gyration of a circular ring and a disc of the same radius with respect to the axis passing through their centres and perpendicular to their planes.

8. An automobile moves on a road with a speed of 54 kmh^{-1}. The radius of its wheels is 0.35 m. What is the average negative torque transmitted by its brakes to a wheel if the vehicle is brought to rest in 15s? The moment of inertia of the wheel about the axis of rotation is 3 kg m^2.

9. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod, as shown in figure. The bullet get embedded in the rod. Find the angular velocity of the system just after the impact.



10. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad s^{-1}. Which of the two will start to roll earlier? The coefficient of kinetic friction is $\mu_k = 0.2$

ANSWERS

1. (x, y) = (0.54 m, 0.36 m)

2. Velocity of COM $= \dfrac{31\hat{i} - 34\hat{j} + 15\hat{k}}{4} ms^{-1}$

3. COM of resulting portion lies at R/6 from the centre of the original disc in a direction opposite to the centre of the cut out portion.

4. $\alpha = 4\pi$ rad s^{-1}

   n = 576

5. m = 66.0 g

6. $F_2 = \sqrt{f^2 + N^2}$

   $= \sqrt{34.6^2 + 196^2} = 199.0\,N$

   If F_2 makes an angle α with the horizontal then

   $\tan\alpha = \dfrac{N}{f} = 5.6568$

   $\alpha = 80°$

7. $\frac{K_{ring}}{K_{disc}} = \frac{R}{R/\sqrt{2}} = \frac{\sqrt{2}}{1}$

8. $\alpha = \frac{w - w_o}{t} = -\frac{1}{0.35} \text{rad s}^{-2}$

$\tau = I\alpha = -8.57 \text{ kgm}^2\text{s}^{-2}$

9. Using conservation of angular momentum

$L_{initial} = L_{final}$

M V L = $I\omega$

or M V L $= \frac{M + 3m}{3} L^2 w$

or $\omega = \frac{3mv}{(M + 3m)L}$

10. The disc begins to roll earlier than the ring.

ROTATIONAL MOTION (5 MARKS)

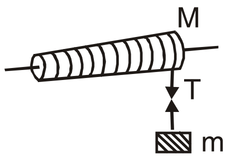
1. Obtain the expression for the linear acceleration of a cylinder rolling down an inclined plane and hence find the condition for the cylinder to roll down without slipping.

2. Prove the result that the velocity V of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

$$v^2 = \frac{2gh}{1 + \frac{k^2}{R^2}}$$

where K = Radius of gysation of body about its symmetry axis, and R is radius of body. The body starts from rest at the top of the plane.

3. A light string is wound round a cylinder and carries a mass tied to it at the free end. When the mass is released, calculate.



(a) the linear acceleration of the decending mass

(b) angular acceleration of the cylinder

(c) Tension in the string.

4. State the theorem of

(i) perpendicular axis (ii) parallel axis.

Find the moment of inertia of a rod of mass M and length L about and axis perpendicular to it through one end. Given the moment of inertia about an axis perpendicular to rod and through COM is $\frac{1}{12}ML^2$

UNIT – VI

GRAVITATION

- **Newton's law of gravitation.** It states that the gravitational force of attraction acting between two bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, *i.e.,* $F = G\frac{m_1 m_2}{r^2}$; where G is the universal gravitational constant.

  The value of $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

- **Gravity :** It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth.

- **Acceleration due to gravity (g).** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is a vector quantity.

  $$g = \frac{GM}{R^2}$$

  where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity.**

  (i) Effect of altitude, $g' = \frac{g R^2}{(R+h)^2}$ and $g' = g\left(1 - \frac{2h}{R}\right)$

  The first relation is valid when h is comparable with R and the second relation is valid when $h << R$. The value of g decreases with increase in h.

  (ii) **Effect of depth,** $g' = g\left(1 - \frac{d}{R}\right)$

  The acceleration due to gravity decreases with increase in depth d and becomes zero at the centre of earth.

- **Gravitational field.** It is the space around a material body in which its gravitational pull can be experienced by other bodies.

  The intensity of gravitational field at a point at a distance *r* from the centre of the body of mass M is given by $I = GM/r^2$ = g (acceleration due to gravity).

- **Gravitational potential.** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point, $V = \dfrac{\text{work done}(\text{W})}{\text{test mass}(\text{m}_0)} = \dfrac{GM}{r}$

- Gravitational potential energy U = gravitational potential × mass of body

  $$= -\frac{GM}{r} \times m$$

  Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, $-\dfrac{\text{dv}}{\text{dr}}$

- **Satellite.** *A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.*

  (i) Orbital speed of a satellite when it is revolving around earth at height *h* is given by

  $$v_0 = R\sqrt{\frac{g}{R+h}}$$

  When the satellite is orbiting close to the surface of earth, *i.e., h < < R,* then

  $$v_0 = R\sqrt{\frac{g}{R}} = \sqrt{g\,R}$$

  (ii) Time period of satellite *(T)*. It is the time taken by the satellite to complete one revolution around the earth.

  $$T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

(iii) Height of satellite above the earth's surface :

$$h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R$$

(iv) Total energy of satellite, E = P.E. + K.E.

$$E = -\frac{GM\,m}{(R+h)} + \frac{1}{2}mv_0^2 = -\frac{GM\,m}{(R+h)} + \frac{1}{2}m\left(\frac{GM}{R+h}\right) = -\frac{GM\,m}{2(R+h)}$$

If the satellite is orbiting close to earth, then $r = R$. Now total energy of satellite.

$$E = -\frac{GM\,m}{2R}$$

(v) **Binding energy of satellite.** $= -E = \dfrac{GM\,m}{2r}$

- **Escape speed.** The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth so that it just crosses the gravitational field of earth. Escape velocity ν_e is given by,

$$\nu_e = \sqrt{\frac{2GM}{R}} = \sqrt{2\,g\,R}$$

For earth, the value of escape speed is 11.2 kms^{-1}.

VERY SHORT ANSWER TYPE QUESTION (1 MARK)

Q 1. The mass of moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the earth?

Q 2. Why does one feel giddy while moving on a merry round?

Q 3. Name two factors which determine whether a planet would have atmosphere or not.

Q 4. The force of gravity due to earth on a body is proportional to its mass, then why does a heavy body not fall faster than a lighter body?

Q 5. The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside is zero, so can a body be shielded from gravitational influence?

Q 6. The gravitational force between two bodies in 1 N if the distance between them is doubled, what will be the force between them?

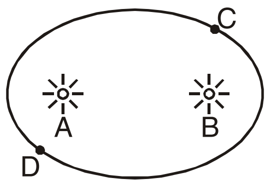
Q 7. A body of mass 5 kg is taken to the centre of the earth. What will be its (i) mass (ii) weight there.

Q 8. Why is gravitational potential energy negative?

Q 9. A satellite revolves close to the surface of a planet. How is its orbital velocity related with escape velocity of that planet.

Q 10. Does the escape velocity of a body from the earth depend on (i) mass of the body (ii) direction of projection

Q 11. Identify the position of sun in the following diagram if the linear speed of the planet is greater at C than at D.



Q 12. A satellite does not require any fuel to orbit the earth. Why?

Q 13. A satellite of small mass burns during its desent and not during ascent. Why?

Q 14. Is it possible to place an artificial satellite in an orbit so that it is always visible over New Delhi?

Q 15. If the density of a planet is doubled without any change in its radius, how does 'g' change on the planet.

Q 16. Mark the direction of gravitational intensity at (i) centre of a hemispherical shell of uniform mass density (ii) any arbitrary point on the upper surface of hemisphere.

Q 17. Why an astronaut in an orbiting space craft is not in zero gravity although weight less?

Q 18. Write one important use of (i) geostationary satellite (ii) polar satellite.

Q 19. A binary star system consists of two stars A and B which have time periods T_A and T_B, radius R_A and R_B and masses m_A and m_B which of the three quantities are same for the stars. Justify.

Q 20. The time period of the satellite of the earth is 5 hr. If the separation between earth and satellite is increased to 4 times the previous value, then what will be the new time period of satellite.

Q 21. The distance of Pluto from the sun is 40 times the distance of earth if the masses of earth and Pluto he equal, what will be ratio of gravitational forces of sun on these planets.

Q 22. If suddenly the gravitational force of attraction between earth and satellite become zero, what would happen to the satellite?

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q 1. If the radius of the earth were to decrease by 1%, keeping its mass same, how will the acceleration due to gravity change?

Q 2. If 'g' be the acceleration due to gravity on earth's surface. Calculate the gain in potential energy of an object of mass m raised from the surface of earth to a height equal to the radius or earth in term of 'g'.

Q 3. A satellite is moving round the earth with velocity v_0 what should be the minimum percentage increase in its velocity so that the satellite escapes.

Q 4. Two planets of radii r_1 and r_2 are made from the same material. Calculate the ratio of the acceleration due to gravity on the surface of the planets.

Q 5. If earth has a mass 9 times and radius 4 times than that of a planet 'P'. Calculate the escape velocity at the planet 'P' if its value on earth is 11.2 kms^{-1}

Q 6. A black hole is a body from whose surface nothing can escape. What is the condition for a uniform spherical body of mass M to be a black hole? What should be the radius of such a black hole if its mass is nine times the mass of earth?

Q 7. At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of earth.

Q 8. At what depth is the value of 'g' same as at a height of 40 km from the surface of earth.

Q 9. The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate mass of the sun if $G = 6.67 \times 10^{-11}$ N m^2/kg^{-2}?

Q 10. Draw graphs showing the variation of acceleration due to gravity with (i) height above earth is surface (ii) depth below the earth's surface.

Q 11. Which planet of the solar system has the greatest gravitational field strength? What is the gravitational field strength of a planet where the weight of a 60 kg astronaut is 300 N.

Q 12. Two satellites are at different heights from the surface of earth which would have greater velocity. Compare the speeds of two satellites of masses m and 4m and radii 2R and R respectively.

Q 13. What is (i) inertial mass, (ii) gravitational mass. Are the two different?

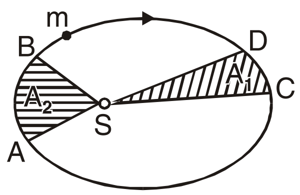
Q 14. Why the space rockets are generally launched west to East?

Q 15. Explain why a tennis ball bounces higher on hills than in plane?

Q 16. The gravitational force on the earth due to the sun is greater than moon. However tidal effect due to the moon's pull is greater than the tidal effect due to sun. Why?

Q 17. The mass of moon is $\frac{M}{81}$ (where M is mass of earth). Find the distance of the point where the gravitational field due to earth and moon cancel each other. Given distance of moon from earth is 60 R, where R is radius of earth.

Q 18. The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from D to C and t_2 is time to move from A to B, what is the relation between t_1 and t_2?



Q 19. Calculate the energy required to move a body of mass m from an orbit of radius 2R to 3R.

Q 20. A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one quarter that of the earth and whose radius is one third of the earth.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Q 1. Define gravitational potential at a point in the gravitational field. Obtain a relation for it. What is the position at which it is (i) maximum (ii) minimum.

Q 2. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side. Also obtain the potential at the centre of the square.

Q 3. Three mass points each of mass m are placed at the vertices of an equilateral triangle of side l. What is the gravitational field and potential at the centroid of the triangle due to the three masses.

Q 4. Briefly explain the principle of launching an artificial satellite. Explain the use of multistage rockets in launching a satellite.

Q 5. In a two stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the 2^{nd} stage gives it the necessary critical speed to put it in a circular orbit. Which stage requires more expenditure of fuel? Given mass of earth = 6.0×10^{24} kg, radius of earth = 6400 km

Q 6. The escape velocity of a projectile on earth's surface is 11.2 kms^{-1} A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Q 7. A satellite orbits the earth at a height 'R' from the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence?

Q 9. Deduce the law of gravitation from Kepler's laws of planets ary motion.

Q 10. Mention at least three conditions under which weight of a person can become zero.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q 1. What is acceleration due to gravity?

Obtain relations to show how the value of 'g' changes with (i) attitude (ii) depth

Q 2. Define escape velocity obtain an expression for escape velocity of a body from the surface of earth? Does the escape velocity depend on (i) location from where it is projected (ii) the height of the location from where the body is launched.

Q 3. State Kepler's three laws of planetary motion. Prove the second and third law.

Q 4. Derive expression for the orbital velocity of a satellite and its time period.

What is a geostatinary satellite. Obtain the expression for the height of the geostationary satellite.

Q 5. State universal law of gravitation. Explain briefly how Newton discovered the universal law of gravitation.

Q 6. Define the term gravitational potential energy. Is it a scalar or vector? Derive an expression for the gravitational potential energy at a point in the gravitational field of earth.

NUMERICALS

Q 1. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30}kg. The mean distance of Jupiter from the Sun is 7.8×10^{11}m. Calculate gravitational force which sun exerts on Jupiter, and the speed of Jupiter.

Q 2. A mass 'M' is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum.

Q 3. If the radius of earth shrinks by 2%, mass remaining constant. How would the value of acceleration due to gravity change ?

Q 4. A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch at a place 1600 k/m above the earth's surface? Radius of earth 6400 km.

Q 5. Imagine a tunnel dug along a diameter of the earth. Show that a particle dropped from one end of the tunnel executes simple harmonic motion. What is the time period of this motion?

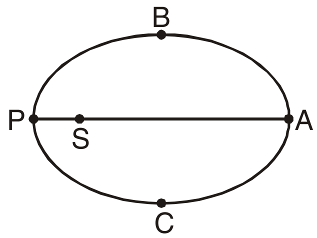
Q 6. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1}. Calculate gravitational potential at that point.

Q 7. A geostationary satellite orbits the earth at a height of nearly 36000 km. What is the potential due to earth's gravity at the site of this satellite (take the potential energy at ∞ to be zero). Mass of earth is 6×10^{24} kg, radius of earth is 6400 km.

Q 8. How much faster than the present speed should the earth rotate so that bodies lying on the equator may fly off into space.

Q 9. The distance of Neptune and Saturn from the sun is nearly 10^{13}m and 10^{12}m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.

Q 10. Let the speed of the planet at perihelion P in fig be ν_p and Sun planet distance SP be r_p Relate (r_p, ν_p) to the corresponding quantities at the aphelion (r_A, ν_A). Will the planet take equal times to traverse BAC and CPB?



ANSWER FOR VERY SHORT QUESTIONS (1 MARK)

1. Both forces will be equal in magnitude as gravitational force is a mutual force between the two bodies.

2. When moving in a merry go round, our weight appears to decrease when we move down and increases when we move up, this change in weight makes us feel giddy.

3. (i) Value of acceleration due to gravity (ii) surface temperature of planet.

4. $\because F = \frac{G\,Mm}{R^2}$ $F \propto m$ but $g = \frac{Gm}{R^2}$ and does not depend on 'm' hence they bodies fall with same 'g'.

5. No, the gravitational force is independent of intervening medium.

6. F = 1 $F' = \frac{F}{4}$

7. Mass does not change.

8. Because it arises due to attractive force of gravitation.

9. $\nu_e = \sqrt{2}\ \nu_o \quad \because\ \nu_e = \sqrt{\frac{2\,GM}{R}} \quad \nu_o = \sqrt{\frac{GM}{R}}$ when $r = R$

11. No, $\nu_e = \sqrt{\frac{2\,GM}{R}}$

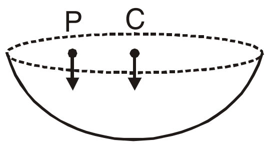
12. Sun should be at B as speed of planet is greater when it is closer to sun.

13. The gravitational force between satellite and earth provides the necessary centripetal force for the satellite to orbit the earth.

14. The speed of satellite during descent is much larger than during ascent, and so heat produced is large.

15. No, A satellite will be always visible only if it revolves in the equatorial plane, but New Delhi does not lie in the region of equitorial plane.

16. 'g' gets doubled as g α ρ (density)

17. 

In both cases it will be downward

18. The astronaut is in the gravitational field of the earth and experiences gravity. However, the gravity is used in providing necessary centripetal force, so is in a state of free fall towards the earth.

19. Geostationary satellite are used for tele communication and polar satellite for remote rensing.

20. Angular velocity of binary stars are same is $w_A = w_B$,

$$\frac{2\pi}{T_A} = \frac{2\pi}{T_B} \quad \Rightarrow \quad T_A = T_B$$

21. $\frac{T_2^2}{T_1^2} = \left(\frac{R_2}{R_1}\right)^3 \Rightarrow T_2^2 = 64 \times 25 \Rightarrow T_2 = 40\,\text{hr}$

22. $\frac{F_{es}}{F_{ps}} = \left(\frac{r_p}{r_e}\right)^2 = \left(\frac{40\,r_e}{r_e}\right)^2 = 1600:1$

23. The satellite will move tangentially to the original orbit with a velocity with which it was revolving.

SHORT ANSWER (2 MARKS)

1. $g = \frac{GM}{R^2}$ If R decreases by 1% it becomes $\frac{99}{100}R$

$\therefore g' = \frac{GM}{(.99\,R)^2} = 1.02\frac{GM}{R^2} = (1+0.02)\frac{GM}{R^2}$

$\therefore$ g' increases by 0.02 $\frac{GM}{R^2}$, therefore increases by 2%.

2. Gain in PE $= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R} = \frac{g\,R^2\,m}{2\,R} = \frac{1}{2}mg\ R$

3. The maximum orbital velocity of a satellite orbiting near its surface is $\nu_o = \sqrt{gR} = \frac{\nu_e}{\sqrt{2}}$

For the satellite to escape gravitational pull the velocity must become v_e

But $\nu_e = \sqrt{2}\nu_o = 1.414\ \nu_o = (1+0.414)\nu_o$

This means that it has to increases 0.414 in 1 or 41.4%

$\therefore$ The minimum increase required, as the velocity of satellite is maximum when it is near the earth.

4. $g = \frac{GM}{r^2} = \frac{G\rho\frac{4}{3}\pi r^3}{r^2} = \frac{4\pi}{3}G\rho r$

$g \propto r$

$\therefore \frac{g_1}{g_2} = \frac{r_1}{r_2}$

5. $\nu_e = \sqrt{\frac{2GM}{R_e}}$ $\nu_p = \sqrt{\frac{2\,GM_p}{R_p}}$ $M_p = \frac{M}{9}, R_p = \frac{R_e}{4}$

$\therefore\ \nu_p = \sqrt{2\,G\frac{M}{9}\times\frac{4}{R_e}} \quad = \quad \frac{2}{3}\sqrt{\frac{2GM}{R_e}} \quad = \quad \frac{2}{3}\times 11.2 = \frac{22.4}{3}$

= 7.47 km/sec

6. According to Einstein's theory of relatively, the maximum speed of an object cannot exceed speed of light. Hence for a body to be a black hole

$$\nu_e = \sqrt{\frac{2GM}{R}} \;\le\; C$$

If M = 9 M_e = 9 × 6 × 10^{24} kg then

$$R = \frac{2GM}{c^2} = \frac{2\times 6.67\times 10^{-11}\times 9\times 6\times 10^{24}}{\left(3\times 10^{8}\right)^2}$$

= 8 × 10^{-2} m

= 8 cm

7. g' = 64% of g $= \frac{64}{100}g$

$$g' = g\,\frac{R^2}{(R+h)^2} = \frac{64}{100}g$$

$$\therefore\ \frac{R}{R+h} = \frac{8}{10}$$

$$h = \frac{R}{4} = 1600\,\text{km}$$

8. $g_d = g_h$

$$g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$$

d = 2h = 2 × 40 = 80 km

9. $R = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

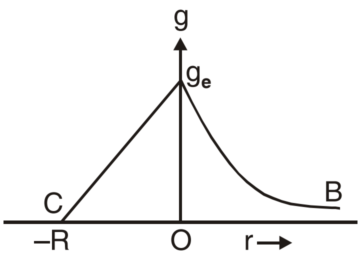
$T = 365$ days $= 365 \times 24 \times 3600$ s

Centripetal force = gravitational force

$$\frac{m\nu^2}{R} = \frac{GMm}{R^2} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{GMm}{R^2}$$

$$M_s = \frac{4\pi^2 R^3}{G\,T^2} = \frac{4 \times 9.87 \times \left(1.5 \times 10^{11}\right)^3}{6.64 \times 10^{-11} \times \left(365 \times 24 \times 3600\right)^2}$$

$M_s = 2.01 \times 10^{30}$ kg

10. 

$g \,\alpha\, \frac{1}{r^2}$ for $r > 0$ above surface of earth

$g \,\alpha\, (R - d)$ for $r < 0$ below surface of earth

g is max for r = 0 on surface.

11. Jupiter has maximum gravitational field strength gravitational field strength

$$= \frac{F}{m} = \frac{300}{60}$$

$= 5$ N kg^{-1}

12. $\nu_0 = \sqrt{\frac{GM}{(R+h)}}$

$\therefore$ Velocity of satellite closer to the earth's surface will be greater

$$\nu_{01} = \sqrt{\frac{GM}{2R}} \qquad \nu_{02} = \sqrt{\frac{GM}{R}}$$

$\therefore \quad \frac{\nu_{01}}{\nu_{02}} = \left(\frac{1}{\sqrt{2}}\right)$, where M is mass of the planet,

v_0 is independent of mass of the satellite.

13. Inertial mass is the measure of inertia of the body $= m_i = \frac{F}{a}$

Gravitational mass of a body determine the gravitational pull between earth and the body.

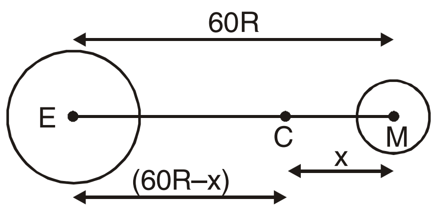
$$m_g = \frac{FR^2}{GM}$$

Both inertial mass and gravitational mass are not different but are equivalent.

14. Since the earth revolves from west to east, so when the rocket is launched from west to east the relative velocity of the rocket increases which helps it to rise without much consumption of fuel.

15. The value of 'g' on hills is less than at the plane, so the weight of tennis ball on the hills is lesser force than at planes that is why the earth attract the ball on hills with lesser force than at planes. Hence the ball bounces higher.

16. The tidal effect depends inversely on the cube of the distance, while gravitational force depends on the square of the distance.

17. 

Gravitational field at C due to earth

= Gravitational field at C due to earth moon

$$\frac{GM}{(60R-x)^2} = \frac{GM/81}{x^2}$$

$$81x^2 = (60R - x)^2$$

$$9x = 60\ R - x$$

$$x = 6\ R$$

18. According to Kepler's IInd law areal velocity for the planet is constant

$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2} \quad A_1 = 2A_2$$

$$\therefore \quad \frac{2A_2}{t_1} = \frac{A_2}{t_2}$$

$$t_1 = 2t_2$$

19. Gravitational P.E of mass m in orbit of radius R = $U = -\frac{GMm}{R}$

$$\therefore \quad U_i = -\frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

$$\Delta U = U_f - U_i = GMm\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{GMm}{6R}$$

20. $g = \frac{4}{3}\pi\, G\, R\, \rho$

$$g' = \frac{4}{3}\pi\, G\, R'\, \rho'$$

The gain in P.E at the highest point will be same in both cases. Hence

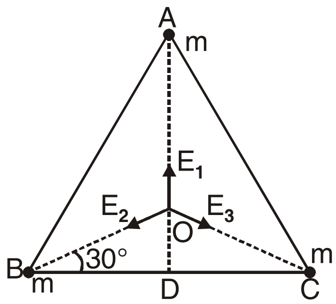
mg'h' = mgh

$$h' = \frac{mgh}{mg'} = \frac{m \times \frac{4}{3}\pi G R \rho\, h}{m \frac{4}{3}\pi\, G R'\, \rho'}$$

$$= \frac{R\rho h}{R'\rho'} = \frac{3R' \times 4\rho' \times 1.5}{R' \times \rho'}$$

= 18 m

ANSWER FOR 3 MARKS QUESTIONS

3. 

$$E_1 = \frac{GM}{(OA)^2}$$

$$E_2 = \frac{GM}{(OB)^2}$$

$$E_3 = \frac{GM}{(OC)^2}$$

From Δ ODB Cos 30° $= \frac{BD}{OB} = \frac{\ell/2}{OB}$

$$OB = \frac{\ell/2}{\cos 30^\circ} = \frac{\ell}{2}\frac{2}{\sqrt{3}} = \ell/\sqrt{3}$$

Gravitational field at O due to m at A, B and C is say $\overrightarrow{E_1}, \overrightarrow{E_2}$ & $\overrightarrow{E_3}$

$$E = \sqrt{E_2^2 + E_3^2 + 2E_2E_3 \text{Cos}120^\circ}$$

$$= \sqrt{\frac{(GM3)^2}{l^2} + \left(\frac{3Gm}{l}\right)^2 + 2\left(\frac{3GM}{l}\right)\left(\frac{3GM}{l}\right)\left(-\frac{1}{2}\right)}$$

$$= \frac{3\,G\,M}{l} \quad \text{along} \quad OD$$

$\bar{E}$ is equal and opposite to $\overrightarrow{E_1}$

$\therefore$ net gravitational field = zero

As gravitational potential is scalar

$V = V_1 + V_2 + V_3$

$$= -\frac{GM}{OA} - \frac{GM}{OB} - \frac{GM}{OC}$$

$$V = -\frac{3GM}{l/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{l}$$

5. Work done on satellite in first stage = W_1 = PE at 150 km – PE at the surface

$$W_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

Work done on satellite in 2nd stage = W_2 = energy required to give orbital velocity ν_o

$$= \frac{1}{2}m\nu_0^2 = \frac{1}{2}\left(\frac{GMm}{R+h}\right)$$

$$\frac{W_1}{W_2} = \frac{2h}{R} = \frac{2 \times 150}{6400} = \frac{3}{64} < 1$$

$\therefore$ $W_2 > W_1$ so second stage requires more energy

6. v_e = 11.2 kms^{-1}, velocity of projection = ν = $3\nu_e$ Let m be the mass of projectile and ν_o the velocity after it escapes gravitational pull.

By law of conservation of energy

$$\frac{1}{2}m\nu_0^2 = \frac{1}{2}m\nu^2 - \frac{1}{2}m\nu_e^2$$

$$\nu_0 = \sqrt{\nu^2 - \nu_e^2} = \sqrt{9\nu_e^2 - \nu_e^2} = \sqrt{8\nu_e^2}$$

$$= 22.4\sqrt{2}$$

$$= 31.68 \text{ km s}^{-1}$$

7. The energy required to pull the satellite from earth influence should be equal to the total energy with which it is revolving around the earth.

The K.E. of satellite $= \frac{1}{2}m\nu^2 = \frac{1}{2}m\frac{GM}{R+h} \qquad \therefore \nu = \sqrt{\frac{GM}{R+h}}$

The P.E of satellite $= -\frac{GMm}{R+h}$

$$\therefore \text{ T.E.} = \frac{1}{2}\frac{mGM}{(R+h)} - \frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$$

$\therefore$ Energy required will be $\left(+\frac{1}{2}\frac{GMm}{(R+h)}\right)$

9. Suppose a planet of mass m, moves around the sun in a circular orbit of radius 'r' with velocity ν.

Then centripetal force $F = \frac{m\nu^2}{r}$

But $= \nu = \frac{2\pi r}{T}$

$$F = m\frac{4\pi^2 r^2}{rT^2} = \frac{mr4\pi^2}{T^2}$$

According to Kepler's IIIrd law

$T^2 \; \alpha \; r^3$

$T^2 = k \, r^3$

$$\therefore \; F = \frac{mr \, 4\pi^2}{k \, r^3} = \frac{4\pi^2 m}{kr^2} = \frac{4\pi^2}{k}\frac{m}{r^2}$$

The force between planet and sun must be mutual, so must be proportional to mass of sun.

$$\frac{4\pi^2}{k} \, \alpha \, m \quad \Rightarrow \quad \frac{4\pi^2}{k} = GM$$

$\therefore \; F = \dfrac{GMm}{r^2}$ This is Newton's law of gravitation

10. (i) When the person is at centre of earth.

 (ii) When the person is at the null points in space (at these points the gravitational forces due to different masses cancel each other)

 (iii) when a person is standing in a freely falling lift.

 (iv) When a person is inside a space craft which is orbiting around the earth.

ANSWER FOR NUMERICALS

1. $F = \dfrac{GMm}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{\left(7.8 \times 10^{11}\right)^2}$$

$F = 4.1 \times 10^{23}$ N

$$\therefore \; F = \frac{m\nu^2}{r} \quad \Rightarrow \quad \nu = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2} \times \frac{r}{m}}$$

$$\nu = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.9 \times 10^{30}}{7.8 \times 10^{11}}}$$

$\nu = 1.3 \times 10^4$ m s^{-1}

2. Let m_1 = m then m_2 = M – m

Force between them when they are separated by distance 'r'

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(Mm - m^2\right)$$

For F to be maximum, differentiate F w.r.t m and equate to zero

$$\frac{dF}{dm} = \frac{G}{r^2}(M-2m) = 0$$

M = 2m; $m = \frac{M}{2}$

$$\therefore m_1 = m_2 = \frac{M}{2}$$

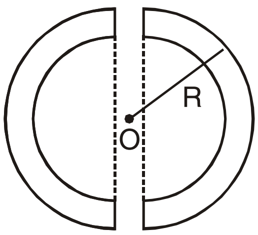
3. increases by 4%

4. In equilibrium mg = kx, $g = \frac{GM}{R^2}$

at height h mg' = kx', $g' = \frac{GM}{(R+h)^2}$

$$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2}$$

$$\frac{x'}{x} = \frac{(6400)^2}{(6400+1600)^2} = \frac{16}{25} \qquad \therefore\ x' = \frac{16}{25} \times 1\,\text{cm} = 0.64\,\text{cm}$$

5. 

The acceleration due to gravity at a depth below the earth's surface is given by

$$g_d = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$

$$= \frac{g}{R}y$$ where y is distance from centre of earth

$g_d \; \alpha \; y$

As acceleration is proportional to displacement and is directed towards mean position, the motion would be S.H.M

$$T = \text{Time period} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{\frac{y}{g_d}}$$

$$= 2\pi\sqrt{\frac{R}{a}}$$

6. Gravitational intensity = $E = \frac{GM}{R^2}$

   Gravitational potential $V = -\frac{GM}{R}$

   $\therefore \; \frac{V}{E} = -R$

   or, V = –E × R

   or V = –4.8 × 10,000 × 10^3 = -4.8 × 10^7 J kg^{-1}

7. U = Potential at height $h = -\frac{GM}{R+h}$

$$U = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 36 \times 10^6} = -9.44 \times 10^6 \text{ J/kp}$$

8. The speed of earth = $\nu = \omega R = \frac{2\pi}{T} \times R$

at present

$$\nu = \frac{2\pi \times 6400 \times 10^3}{24 \times 3600}$$

The gravitational force should be equal to the centripetal force so that centrifugal force, given by $\frac{mv'^2}{R}$

$$F = \frac{GMm}{R^2} = \frac{m\,\nu'^2}{R}$$

$$\nu' = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}}$$

$\therefore \; \frac{\nu'}{\nu} = 17$. $\therefore$ The velocity should become 17 times the present velocity

9. By kepler's III[rd] law

$$\left(\frac{T_n}{T_s}\right)^2 = \left(\frac{R_n}{R_s}\right)^3$$

$$\frac{T_n}{T_5} = \left(\frac{R_n}{R_s}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$= 10\sqrt{10} = 10 \times 3.16 = 31.6$$

$\therefore \; T_n : T_s = 36.6 : 1$

10. The magnitude of angular momentum at P is $L_p = m_p\ r_p\ \nu_p$

Similarly magnitude of angular momentum at A is $L_A = m_A r_A \nu_A$

From conservation of angular momentum

$m_p r_p \nu_p = m_A \nu_A r_A$

$$\frac{\nu_P}{\nu_A} = \frac{r_A}{r_P}$$

$\because\ r_A > r_P,\ \therefore \nu_P > \nu_A$

area bound by SB & SC(SBAC > SBPC)

$\because$ By 2nd law equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to traverse CPB

UNIT – VII

PROPERTIES OF BULK MATTER

KEY CONCEPTS

- **Elasticity :** It is the property of the body by virtue of which the body regains its original configuration (length, volume or shape) when the deforming forces are removed.
- **Stress :** The internal restoring force acting per unit area of a deformed body is called stress, *i.e.,*

  Stress = restoring force/area.
- **Strain :** It is defined as the ratio of change in configuration to the original configuration of the body *i.e.,* $\text{Strain} = \dfrac{\text{change in configuration}}{\text{original configuration}}$

  Strain can be of three types : (i) Longitudinal strain (ii) Volumetric strain (iii) Shearing strain.
- **Hooke's law :** It states that the stress is directly proportional to strain within the elastic limit.
- **Modulus of Elasticity** or **Coefficient of elasticity** of a body is defined as the ratio of the stress to the corresponding strain produced, within the elastic limit.

  Modulus of elasticity is of three types:

  (i) *Young's Modulus of elasticity (Y).* It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit, *i.e.,*

  $$Y = \frac{\text{normal stress}}{\text{longitudinal strain}} = \frac{F / a}{\Delta l / l} = \frac{F}{a} \times \frac{l}{\Delta t}$$

  (ii) *Bulk Modulus of elasticity (K).* It is defined as the ratio of normal stress to the volumetric strain within the elastic limit, *i.e.,*

$$B = \frac{\text{normal stress}}{\text{longitudinal strain}} = \frac{F/a}{\Delta V/V} = -\frac{F}{a} \times \frac{V}{\Delta V} = -p\frac{V}{\Delta V}$$

(iii) *Modulus of Rigidity* (η). It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit, *i.e.,*

$$\eta = \frac{\text{tangential stress}}{\text{shearing strain}} = \frac{F/a}{\theta} = \frac{F}{A\,\theta}$$

- Compressibility $= \dfrac{1}{\text{Bulk modulus}} = -\dfrac{\Delta V}{pV}$

- **Poisson's ratio** (σ) is defined as the ratio of lateral strain to the longitudinal strain, *i.e.,*

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\Delta R/R}{\Delta l/l} = \frac{-\Delta R.\,l}{R.\,\Delta l}$$

Theoretical value of σ lies between –1 and $\frac{1}{+2}$. The practical value of σ lies between 0 and +1/2. If there is no change in the volume of wire on loading, then its Poisson's ratio is 0.5.

- **Elastic potential energy stored per unit volume of a strained body**

$$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{(\text{stress})^2}{2\text{Y}} = \frac{\text{Y} \times (\text{strain})^2}{2}$$

where Y is the Young's modulus of elasticity of a solid body.

- Workdone in a stretched wire, $W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$$= \frac{1}{2}\frac{F}{A} \times \frac{\Delta L}{L} \times AL = \frac{1}{2}F \times \Delta L = \frac{1}{2}\ \text{load} \times \text{extension}$$

- Total pressure at a point inside the liquid of density ρ at depth h is $P = h\,\rho\,g + P_0$ where P_0 is the atmospheric pressure.

- **Pascal's law :** This law states for the principle of transmission of pressure in liquids or gases. Pascal's law states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

- **Viscosity :** Viscosity is the property of a fluid by virtue of which an internal frictional force comes into play when the fluid is in motion and opposes the relative motion of its different layers.

  Viscous drag F acting between two layers of liquid each of area A, moving with velocity gradient dv/dx is given by $F = \eta \, A \, dv/dx$

  where h is the coefficient of viscosity of liquid.

  SI unit of η is poiseuille of N s m^{-2} or Pascal-second

- **Stoke's law :** It states that the backward dragging force F acting on a small spherical body of radius r, moving through a medium of viscosity η, with velocity v is given by $F = 6 \pi \eta r v$.

- **Terminal velocity :** It is the maximum constant velocity acquired by the body while falling freely in a viscous medium. Terminal velocity v of a spherical body of radius r, density ρ while falling freely in a viscous medium of viscosity η, density σ is given by $\nu = \dfrac{2r^2(\rho - \sigma)g}{9\eta}$

- **Stream line flow of a liquid** is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.

- **Turbulent flow :** It is that flow of liquid in which the motion of the particles of liquid becomes disorderly or irregular.

- **Critical velocity :** It is that velocity of liquid flow, upto which the flow of liquid is a streamlined and above which its flow becomes turbulent. Critical velocity of a liquid (v_c) flowing through a tube is given by

  $$\nu_c = \frac{N \eta}{\rho D}$$

  where ρ is the density of liquid flowing through a tube of diameter D and η is the coefficient of viscosity of liquid. N = Reynold number

- **Equation of continuity,** a v = constant **where a = area of cross section v = velocity of flow of liquid.**

- **Bernoulli's Theorem :** It states that for the stream line flow of an ideal liquid, the total energy (the sum of pressure energy, the potential energy and kinetic energy) per unit volume remains constant at every cross-section throughout the tube, *i.e.,*

$$P+\rho g h+\frac{1}{2}\rho \nu^2 = \text{a constant}, \qquad \text{or} \qquad \frac{P}{\rho g}+h+\frac{1}{2}\frac{\nu^2}{g} = \text{another constant}$$

- **Torricelli's Theorem :** It states that the velocity of efflux, *i.e.,* the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid. Quantitatively velocity of efflux, $\nu=\sqrt{2gh}$, where h is the depth of orifice below the free surface of liquid.

- **Surface tension :** It is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum surface area and as such it behaves as a stretched membrane. Surface tension,

$$S=\frac{F}{l}$$

where F is the force acting on the imaginary line of length l, drawn tanentially to the liquid surface at rest. SI unit of surface tension is Nm^{-1}

- **Surface energy** = surface tension × area of the liquid surface

  (i) Excess of pressure inside a liquid drop, $P = 2\ S/R$

  (ii) Excess of pressure inside a soap bubble, $P = 4\ S/R$

  where S is the surface tension and R is the radius of the drop or bubble.

- **Angle of contact :** The angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

- **Capillarity :** The phenomenon of rise or fall of liquid in a capillary tube is called capillarity. The height (h) through which a liquid will rise in a capillary tube of radius r which wets the sides of the tube will be given by

$$h=\frac{2S\cos\theta}{r\rho g}$$

where S is the surface tension of liquid, θ is the angle of contact, ρ is the density of liquid and g is the acceleration due to gravity.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Define term (i) stress (ii) strain
2. Differentiate the properties plasticity and elasticity of material
3. Draw stress – strain curve for elastomers (elastic tissue of Aorta)
4. How are we able to break a wire by repeated bending?
5. What is the value of bulk modulus for an incompressible liquid?
6. Define Poisson's ratio? Does it have any unit?
7. What is elastic fatigue?
8. Why is it easier to swim in sea water than in the river water?
9. Railway tracks are laid on large sized wooden sleepers. Why?
10. The dams of water reservoir are made thick near the bottom. Why?
11. Why is it difficult to stop bleeding from a cut in human body at high attitudes?
12. The blood pressure in human is greater at the feet than at the brain. Why?
13. Define coefficient of viscosity and write its SI unit.
14. Why machine parts get jammed in winter?
15. Why are rain drops spherical?
16. Why do paints and lubricants have low surface tension?
17. What will be the effect of increasing temperature on (i) angle of contact (ii) surface tension.
18. For solids with elastic modulus of rigidity, the shearing force is proportional to shear strain. On what factor does it depend in case of fluids?
19. How does rise in temperature effect (i) viscosity of gases (ii) viscosity of liquids.

20. Explain why detergents should have small angle of contact?
21. Write the dimensions of coefficient of viscosity and surface tension.
22. Obtain a relation between SI unit and cgs unit of coefficient of viscosity.
23. Explain, how the use of parachute helps a person jumping from an aeroplane.
24. Why two ships moving in parallel directions close to each other get attracted?
25. Why the molecules of a liquid lying near the free surface possess extra energy?
26. Why is it easier to wash clothes in hot water soap solution?
27. Why does mercury not wet glass?
28. Why ends of a glass tube become rounded on heating?
29. What makes rain coats water proof?
30. What happens when a capillary tube of insufficient length is dipped in a liquid?
31. Does it matter if one uses gauge pressure instead of absolute pressure in applying Bernoulli's equation?
32. State Wein's displacement law for black body radiation.
33. State Stefan Boltzmann law.
34. Name two physical changes that occur on heating a body.
35. Distinguish between heat and temperature.
36. Which thermometer is more sensitive a mercury or gas thermometer?
37. Metal disc has a hole in it. What happens to the size of the hole when disc is heated?
38. Name a substance that contracts on heating.
39. A gas is free to expand what will be its specific heat?

40. What is Deby's temperature?

41. What is the absorptive power of a perfectly black body?

42. At what temperature does a body stop radiating?

43. If Kelvin temperature of an ideal black body is doubled, what will be the effect on energy radiated by it?

44. In which method of heat transfer does gravity not play any part?

45. Give a plot of Fahrenheit temperature versus celsius temperature

46. Why birds are often seen to swell their feather in winter?

47. A brass disc fits snugly in a hole in a steel plate. Should we heat or cool the system to loosen the disc from the hole.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

1. State Hooke's law. Deduce expression for young's modulus of material of a wire of length 'l', radius of crossection 'r' loaded with a body of mass M producing an extension Δl in it.

2. A wire of length l area of crossection A and young's modulus Y is stretched by an amount x. What is the work done?

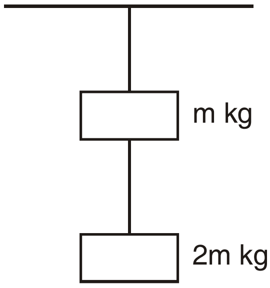
3. Prove that the elastic potential energy per unit volume is equal to $\frac{1}{2} \times \text{stress} \times \text{strain}$.

4. Define the term bulk modulus. Give its SI unit. Give its SI unit. Give the relation between bulk modulus and compressibility.

5. Define shear modulus. With the help of a diagram explain how shear modulus can be calculated.

6. Which is more elastic steel or rubber. Explain.

7. Two wires P and Q of same diameter are loaded as shown in the figure. The length of wire P is L m and its young's modulus is Y N/m^2 while length of wire a is twice that of P and its material has young's modulus half that of P. Compute the ratio of their elongation.



8. What is Reynold's number? Write its significance. On what factors does it depend.

9. Define surface tension and surface energy. Obtain a relation between them.

10. State and prove Torricelli's theorem for velocity of efflux.

11. Using dimensional method obtain, Stoke's law expression for viscous force $F = 6\pi\ \eta\ a\ v$

12. The fig (a) & (b) refer to the steady flow of a non viscous liquid which of the two figures is incorrect? Why?

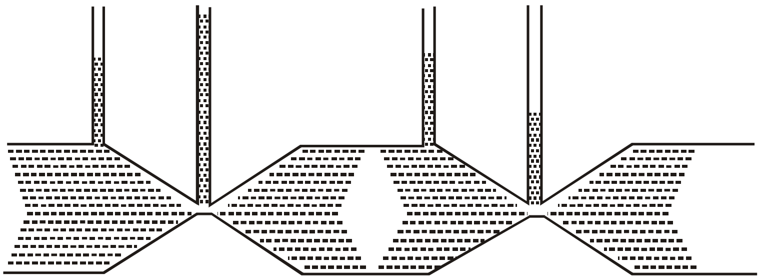
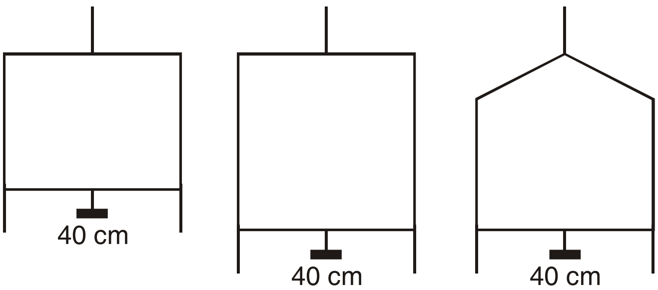


Fig. (a) Fig. (b)

13. The fig below shows a thin liquid supporting a small weight 4.5×10^{-2}N. What is the weight supported by a film of same liquid at the same temperature in fig (b) & (c) Explain your answer.



(a) (b) (c)

14. Two soap bubbles of different diameter are in contact with a certain portion common to both the bubbles. What will be the shape of the common boundary as seen from inside the smaller bubble? Support your answer with a neat diagram and justify your answer.

15. During blood transfusion the needle is inserted in a vein where gauge pressure is p_g and atmospheric pressure is p. At what height must the blood container be placed so that blood may just enter the vein. Given density of blood is ρ.

16. Why we cannot remove a filter paper from a funnel by blowing air into narrow end.

17. On a hot day, a car is left in sunlight with all windows closed. Explain why it is considerably warmer than outside, after some time?

18. Name the suitable thermometers to measure the following temperatures

    (a) –100°C (b) 80° C

    (c) 780°C (d) 2000°C

19. If a drop of water falls on a very hot iron, it does not evaporate for a long time. Why?

20. The earth without its atmosphere would be inhospitably cold. Why?

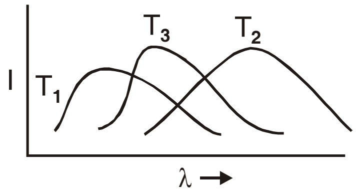
21. The coolant used in chemical or in a nuclear plant should have high specific heat. Why?

22. A sphere, a cube and a disc made of same material and of equal masses heated to same temperature of 200°C. These bodies are then kept at same lower temperature in the surrounding, which of these will cool (i) fastest, (ii) slowest, explain.

23. (a) Why pendulum clocks generally go faster in winter and slow in summer.

    (b) Why the brake drums of a car are heated when it moves down a hill at constant speed.

24. The plots of intensity versus wavelength for three blackbodies at temperature T_1, T_2 and T_3 respectively are shown.



Arrange the temperature in decreasing order. Justify your answer.

25. The triple point of water is a standard fixed point in modern thermometry. Why? Why melting point of ice or boiling point of water not used as standard fixed points.

SHORT ANSWER TYPE QUESTION (3 MARKS)

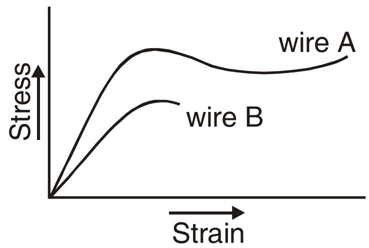
1. How is the knowledge of elasticity useful in selecting metal ropes used in cranes for lifting heavy loads.

2. If the torque required to produce unit twist in a solid shaft of radius r, length l and made of material of modulus of rigidity η is given by

$$\tau = \frac{\pi \eta r^4}{2l}$$

Explain why hollow shafts are preferred to solid shafts for transmitting torque?

3. Stress strain curve for two wires of material A and B are as shown in Fig.



(a) which material in more ductile?

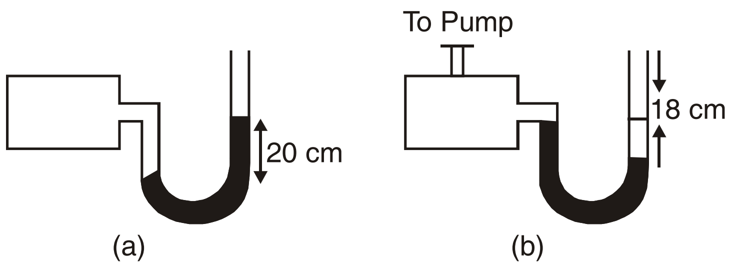
(b) which material has greater value of young modulus?

(c) which of the two is stronger material?

(d) which material is more brittle?

4. State Pascal's law for fluids with the help of a neat labelled diagram explain the principle and working of hydraulic brakes.

5. A manometer reads the pressure of a gas in an enclosure as shown in the fig (a) when some of the gas is removed by a pump, the manometer reads as in fig (b) The liquid used in manometer is mercury and the atmospheric pressure is 76 cm of mercury. (i) Give absolute and gauge pressure of the gas in the enclosure for cases (a) and (b).



6. How would the levels change in (b) if 13.6 cm of H_2O (immensible with mercury) are poured into the right limb of the manometer in the above numerical.

7. Define Capillarity and angle of contact. Derive an expression for the ascent of a liquid in a capillary tube.

8. The terminal velocity of a tiny droplet is v. N number of such identical droplets combine together forming a bigger drop. Find the terminal velocity of the bigger drop.

9. Two spherical soap bubble coalesce. If v be the change in volume of the contained air, A is the change in total surface area then show that 3PV + 4AT = 0 where T is the surface tension and P is atmospheric pressure.

10. Give the principle of working of venturimeter. Obtain an expression for volume of liquid flowing through the tube per second.

11. A big size balloon of mass M is held stationary in air with the help of a small block of mass M/2 tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations.

12. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on

the base of the vessel the same? Why do the vessels filled to same height give different reading on weighing scale.

13. A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of liquid as σ.

14. Define the coefficients of linear expansion. Deduce relation between it and coefficient of superficial expansion and volume expansion.

15. Describe the different types of thermometers commonly used. Used the relation between temperature on different scales. Give four reasons for using mercury in a thermometer.

16. Two rods of different metals of coefficient of linear expansion α_1 and α_2 and initial length l_1 and l_2 respectively are heated to the same temperature. Find relation in α_1, α_2, l_1 and l_2 such that difference between their lengths remain constant.

17. Explain the principle of platinum resistance thermometer.

18. Draw a graph to show the anomalous behaviour of water. Explain its importance for sustaining life under water.

19. A steel rail of length 5m and area of cross section 40cm^2 is prevented from expanding while the temperature rises by 10°C. Given coefficient of linear expansion of steel is 1.2×10^{-5} k^{-1}. Explain why space needs to be given for thermal expansion.

20. Define (i) Specific heat capacity (ii) Heat capacity (iii) Molar specific heat capacity at constant pressure and at constant volume and write their units.

21. Plot a graph of temperature versus time showing the change in the state of ice on heating and hence explain the process (with reference to latent heat)

22. What is the effect of pressure on melting point of a substance? What is regelation. Give a practical application of it.

23. What is the effect of pressure on the boiling point of a liquid. Describe a simple experiment to demonstrate the boiling of H_2O at a temperature much lower than 100°C. Give a practical application of this phenomenon.

24. State and explains the three modes of transfer of heat. Explains how the loss of heat due to these three modes is minimised in a thermos flask.

25. Define coefficient of thermal conductivity. Two metal slabs of same area of cross-section, thickness d_1 and d_2 having thermal conductivities K_1 and K_2 respectively are kept in contact. Deduce expression for equivalent thermal conductivity.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Draw and discuss stress versus strain graph, explaining clearly the terms elastic limit, permanent set, proportionality limit, elastic hysteresis, tensible strength.

2. Show that there is always an excess pressure on the concave side of the meniscus of a liquid. Obtain an expression for the excess pressure inside (i) a liquid drop (ii) liquid bubble (iii) air bubble inside a liquid.

3. State and prove Bernoullis theorem. Give its limitation. Name any two application of the principle.

4. Define terminal velocity. Obtain an expression for terminal velocity of a sphere falling through a viscous liquid. Use the formula to explain the observed rise of air bubbles in a liquid.

5. State Newton's law of cooling. Deduce the relations :

$$\log_e(T - T_0) = -kt + c$$

and $$T - T_0 = C\, e^{-kt}$$

where the symbols have their usual meanings. Represent Newton's law of cooling graphically by using each of the above equation.

6. On what factors does the rate of heat conduction in a metallic rod in the steady state depend. Write the necessary expression and hence define the coefficient of thermal conductivity. Write its unit and dimensions.

7. Distinguish between conduction, convection and radiation.

8. What is meant by a block body. Explain how a black body may be achieved in practice. State and explain Stefan's law?

9. Explain the construction and working of a constant volume gas thermometer. What are its main draw backs?

10. Discuss energy distribution of a block body radiation spectrum and explain Wein's displacement law of radiation and Stefan's law of heat radiation.

NUMERICALS

1. An aluminium wire 1m in length and radius 1mm is loaded with a mass of 40 kg hanging vertically. Young's modulus of Al is 7.0×10^{-10} N/m^2 Calculate (a) tensile stress (b) change in length (c) tensile strain and (d) the force constant of such a wire.

2. The average depth of ocean is 2500 m. Calculate the fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the bottom of ocean, given that the bulk modulus of water is 2.3×10^9 N/m^2.

3. A force of 5×10^3 N is applied tangentially to the upper face of a cubical block of steel of side 30 cm. Find the displacement of the upper face relative to the lower one, and the angle of shear. The shear modulus of steel is 8.3×10^{10} pa.

4. How much should the pressure on one litre of water be changed to compress it by 0.10%.

5. Calculate the pressure at a depth of 10 m in an Ocean. The density of sea water is 1030 kg/m^3. The atmospheric pressure is 1.01×10^5 pa.

6. In a hydraulic lift air exerts a force F on a small piston of radius 5cm. The pressure is transmitted to the second piston of radius 15 cm. If a car of mass 1350 kg is to be lifted, calculate force F that is to be applied.

7. How much pressure will a man of weight 80 kg f exert on the ground when (i) he is lying and (2) he is standing on his feet. Given area of the body of the man is 0.6 m^2 and that of his feet is 80 cm^2.

8. The manual of a car instructs the owner to inflate the tyres to a pressure of 200 k pa. (a) What is the recommended gauge pressure? (b) What is the recommended absolute pressure (c) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read?

9. Calculate excess pressure in an air bubble of radius 6mm. Surface tension of liquid is 0.58 N/m.

10. Terminal velocity of a copper ball of radius 2 mm through a tank of oil at 20°C is 6.0 cm/s. Compare coefficient of viscosity of oil. Given $\rho_{cu} = 8.9 \times 10^3$ kg/m^3, $\rho_{oil} = 1.5 \times 10^3$ kg/m^3

11. Calculate the velocity with which a liquid emerges from a small hole in the side of a tank of large cross-sectional area if the hole is 0.2m below the surface liquid ($g = 10\ ms^{-2}$).

12. A soap bubble of radius 1 cm expands into a bubble of radius 2cm. Calculate the increase in surface energy if the surface tension for soap is 25 dyne/cm.

13. A glass plate of 0.20 m^2 in area is pulled with a velocity of 0.1 m/s over a larger glass plate that is at rest. What force is necessary to pull the upper plate if the space between them is 0.003m and is filled with oil of $\eta = 0.01\ Ns/m^2$

14. The area of cross-section of a water pipe entering the basement of a house is $4 \times 10^{-4}\ m^2$. The pressure of water at this point is $3 \times 10^5\ N/m^2$, and speed of water is 2 m/s. The pipe tapers to an area of cross section of $2 \times 10^{-4}\ m^2$, when it reaches the second floor 8 m above the basement. Calculate the speed and pressure of water flow at the second floor.

15. A metal black of area 0.10 m^2 is connected to a 0.010 kg mass via a string that passes over an ideal pulley. A liquid with a film thickness of 0.30 mm is placed between the block and the table when released the block moves with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.

16. Water rises to a height of 9 cm in a certain capillary tube. If in the same tube, level of Hg is depressed by 3 cm, compare the surface tension of water and mercury. Specific gravity of Hg is 13.6 and the angle of contact for Hg is 135°

17. Two stars radiate maximum energy at wavelength, 3.6×10^{-7} m respectively. What is the ratio of their temperatures?

18. Find the temperature of 149°F on kelvin scale.

19. A metal piece of 50 g specific heat 0.6 cal/g°C initially at 120°C is dropped in 1.6 kg of water at 25°C. Find the final temperature or mixture.

20. A iron ring of diameter 5.231 m is to be fixed on a wooden rim of diameter 5.243 m both initially at 27°C. To what temperature should the iron ring be heated so as to fit the rim (Coefficient of linear expansion of iron is $1.2 \times 10^5\ k^{-1}$?

21. 100g of ice at 0°C is mixed with 100 g of water at 80°C. The resulting temperature is 6°C. Calculate heat of furion of ice.

22. Calculate heat required to covert 3kg of water at 0°C to steam at 100°C Given specific heat capacity of H_2O = 4186J kg^{-1} k^{-1} and latent heat of stem = 2.256×10^6 J/kg

23. Given length of steel rod 15 cm; of copper 10 cm. Their thermal conductivities are 50.2 j – s m^{-1} k^{-1} and 385 j – s – m^{-1} k^{-1} respectively. Area of crossection of steel is double of area of Copper rod?

24, A body at temperature 94°C cools to 86°C in 2 min. What time will it take to cool from 82°C to 78°C. The temperature of surrounding is 20°C.

25. A body re-emits all the radication it receives. Find surface temperature of the body. Energy received per unit area per unit time is 2.835 watt/m^2 and $\sigma = 5.67 \times 10^{-8}$ W m^{-2} k^{-4}.

26. At what temperature the resistance of thermometer will be 12% more of its resistance at 0°C (given temperature coefficient of resistance is 2.5×10^{-3} $°C^{-1}$?

VERY SHORT ANSWER (1 MARK)

4. Repeated bending of wire decreases elastric strength and therefore it can be broken easily.

5. $K = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\text{o}} = \infty$ Infinity

6. Poisson's ratio is the ratio of lateral strain to the longitudinal strain. It has no units.

7. It is the loss in strength of a material caused due to repeated alternating strains to which the material is subjected

8. The density of sea water is more than the density of river water, hence sea water gives more up thrust for the same volume of water displaced.

9. This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably and in turn prevents yielding of the ground under the weight of the train.

10. Pressure exerted by liquid column = $h \rho g$ so as 'h' increases ρ increases so to with stand high pressure dams are made thick near the bottom.

11. The atmospheric pressure is low at high altitudes. Due to greater pressure difference in blood pressure and the atmospheric pressure, it is difficult to stop bleeding from a cut in the body.

12. The height of blood clumn is quite large at feet than at the brain, hence blood pressure at feet is greater.

14. In winter i.e. at low temperature the viscosity of lubricants increases.

15. Due to surface thension the drops try to occupy minimum surface area, and for a given volume sphere has minimum surface area.

16. Low surface tension makes paints and lubricants to spread more effectively.

17. Angle of contact increases with increase of temperature while surface tension generally decreases with increase of temperature

18. Rate of Shear Strain.

19. Viscosity of gases increases while viscosity of liquid decreases.

20. Detergents should have small angle of contact so that they have low surface tension and greater ability to wet a surface. Further as $h = \frac{2T\ \cos\theta}{r\rho g}$ i.e. θ is small Cos θ will be large so *h* i.e. penetration will be high.

21. $[\eta] = M^{1}L^{-1}T^{-1}$

$[T] = M^{1}T^{-2}L^{0}$

22. c.g.s unit of η = poise

S.I Unit of η = poiseuille or deca poise

1 poise = 1 g cm^{-1} s^{-2} = 10^{-1} kg m^{-1} s^{-1}

= 0.1 poiseuille

23. Viscous force on the parachute is large as $F = 6\pi\ \eta\ r\ v$, $F\ \alpha\ r$, so its terminal velocity becomes small so the person hits the ground with this small velocity and does not get injured.

24. According to Bernoulli's theorem for horizontal flow $P + \frac{1}{2}\rho\upsilon^2 = \text{constant}$. As speed of water between the ships is more than outside them pressure between them gets reduced & pressure outside is more so the excess pressure pushes the ships close to each other therefore they get attracted.

25. The molecules in a liquid surface have a net downward force (cohesion) on them, so work done in bringing them from within the body of liquid to the surface increases surface energy.

26. Hot water soap solution has small surface tension therefore can remove the dirt from clothes by wetting them effectively.

27. Mercury does not wet glass because of larger cohesive force between Hg-Hg molecules than the adhesive forces between mercury-glass molecules.

28. When glass is heated, it melts. The surface of this liquid tends to have a minimum area. For a given volume, the surface area is minimum for a sphere. This is why the ends of a glass tube become rounded on heating.

29. The angle of contact between water and the material of the rain coat is obtuse. So the rain water does not wet the rain coat.

30. When a capillary tube of insufficient length is dipped in a liquid, the radius of curvature of the mensicus increase so that hr = constant. That is pressure on concave side becomes equal to pressure exerted by liquid column so liquid does not overflow.

31. No. Unless the atmospheric pressures at the two points where Bernoulli's equation is applied, are significantly different.

34. Volume and electrical resistance.

36. Gas thermometer is more sensitive as coefficient of expansion of Gas is more than mercury.

37. Expansion is always outward, therefore the hole size increased on heating.

38. Ice

39. Infinity

40. The temperature above which molar heat capacity of a solid substance becomes constant.

41. One.

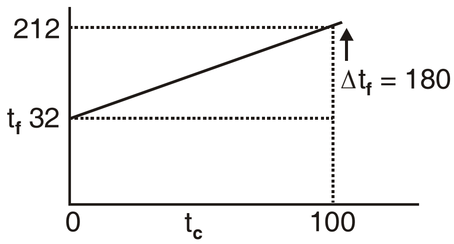
42. At oK.

43. $E \alpha T^4 \therefore \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2T_1}{T_1}\right)^4 = 16$

$\therefore E_2 = 16\ E_1$

44. In conduction and radiation

45.



46. When birds swell their feathers, they trap air in the feather. Air being a poor conductor prevents loss of heat and keeps the bird warm.

47. The temp. coefficient of linear expansion for brass is greater than that for teel. On cooling the disc shrinks to a greater extent than the hole, and hence brass disc gets lossened.

SHORT ANSWER TYPE (2 MARKS)

2. Restoring force in extension x = F = $\frac{AYx}{L}$

Work done in stretching it by dx = dw = F. dx

Work done in stretching it from zero to x = W = $\int dw = \int_0^x F\ dx$

$$W = \int_0^x \frac{AYx}{L} dx = \frac{1}{2} \frac{AYx^2}{L}$$

3. Energy Density $= \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}\frac{AY}{L}x^2}{AL}$

$$= \frac{1}{2}\left(\frac{AYx}{AL}\right) \times \frac{x}{L}$$

$$= \frac{1}{2}\frac{F}{A} \times \frac{x}{L}$$

$$= \frac{1}{2} \text{ Stress} \times \text{Strain}$$

6. $Y_s = \frac{F}{A}\frac{l}{\Delta l_s}$

$$Y_r = \frac{F}{A}\frac{l}{\Delta l_r}$$

For same force applied to wires made of steel & rubber of same length and same area of cross section

$$\Delta l_s < \Delta l_r$$

$$\frac{Y_s}{Y_r} = \frac{\Delta l_r}{\Delta l_s} > 1$$

$\therefore \quad Y_s > Y_r$

7. $\Delta l_p = \frac{3mg}{A} \times \frac{L}{Y}$ $\qquad$ $\Delta l_Q = \frac{2mg}{A} \cdot \frac{2L}{Y/2} = \frac{8mg}{A}\frac{L}{Y}$

$\therefore \frac{\Delta l_P}{\Delta l_Q} = \frac{3}{8}$

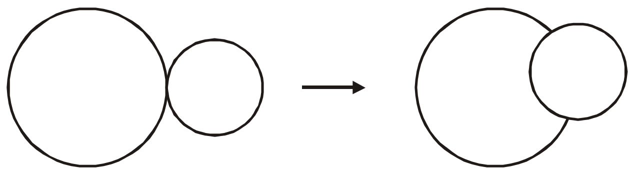
8. Definition of Reynold number N_R is the ratio of inertial force to viscous force.

12. Fig. (a) is correct.

At the constriction, the area of cross section is small so liquid velocity is large, consequently pressure must be small so height of liquid must be less.

13. The weight supported by (b) & (c) are same as that in (a) and is equal to 4.5×10^{-2} N.

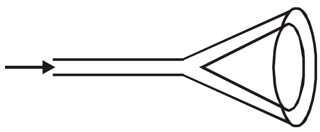
The weight supported = $2\,\sigma\, l$, where σ is surface tension and l is the length which is same in all the three cases, hence weight supported is same.

14. 

When seen from inside the smaller bubble the common surface will appear concave as (1) the pressure (excess) $= \frac{2T}{R}$ will be greater for concave surface & as R is small for the smaller bubble, the pressure will be greater.

15. $p_g = \rho g$

$$h = \frac{P_g}{\rho g}$$

16. 

When air is blown into the narrow end its velocity in the region between filter paper and glass increases. This decreases the pressure. The filter paper gets more firmly held with the wall of the tunnel.

17. Glass transmits 50% of heat radiation coming from a hot source like sun but does not allow the radiation from moderately hot bodies to pass through it.

18. (*a*) Gas thermometer;

(*b*) Mercury thermometer;

(*c*) Platinum resistance thermometer;

(*d*) Radiation pyrometer.

19. A vapour film is formed between water drop and the hot iron. Vapour being a poor conductor of heat makes the water droplet to evaporate slowly.

20. Due to green house effect, the presence of atmosphere prevents heat radiations received by earth to go back. In the absence of atmosphere radiation will go back at night making the temperature very low and inhospitable.

21. So, that it absorbs more heat with comparatively small change in temperature and extracts large amount of heat.

22. Rate of energy emission is directly proportional to area of surface for a given mass of material. Surface area of sphere is least and that of disc is largest. Therefore cooling of (i) disc is fastest and (ii) sphere is slowest.

23. (*a*) Time period of pendulum = $T = 2\pi\sqrt{\frac{l}{g}}$ or $T \times \sqrt{l}$

In winter *l* becomes shorter so its time period reduces so it goes faster. In summer *l* increases resulting in increase in time period so the clock goes slower.

(*b*) When the car moves down hill, the decrease in gravitational potential energy is converted into work against force of friction between brake shoe and drum which appears as heat.

24. $\lambda_m^1 < \lambda_m^3 < \lambda_m^2$

$\therefore$ from Wein displacement law

$T_1 > T_3 > T_2$

25. The melting point of ice as well as the boiling point of water changes with change in pressure. The presence of impurities also changes the melting and boiling points. However the triple point of water has a unique

temperature and is independent of external factors. It is that temperatures at which water, ice & water vapour co-exist that is 273.16K and pressure 0.46 cm of Hg.

ANSWERS FOR SHORT QUESTIONS (3 MARKS)

1. The ultimate stress should not exceed elastic limit of steel (30×10^7 N/m^2)

$$U = \frac{F}{A} = \frac{M9}{\pi r^2} = \frac{10^5 \times 9.8}{\pi r^2} = 30 \times 10^7$$

$\therefore$ r = 3.2 cm

So to lift a load of 10^4kg, crane is designed to withstand 10^5kg. To impart flexibility the rope is made of large number of thin wires braided.

2. Torque required to produce unit twist in hollow shaft of internal radius r_1 and external radius r_2 is

$$\tau' = \frac{\pi\eta\left(r_2^4 - r_1^4\right)}{2l}$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^4 - r_1^4}{r^4} = \frac{\left(r_2^2 + r_1^2\right)\left(r_2^2 - r_1^2\right)}{r^4}$$

If the shafts are made of material of equal volume.

$$\pi r^2 l = \pi\left(r_2^2 - r_1^2\right) l \text{ or } r_2^2 - r_1^2 = r^2$$

$\therefore$ Since $r_2^2 + r_1^2 > r^2 \quad \tau^1 > \tau$

3. (*a*) Wire with larger plastic region is more ductile material A

(*b*) Young's modulus is $\frac{\text{Stress}}{\text{Strain}}$

$\therefore Y_A > Y_B$

(*c*) For given strain, larger stress is required for A than that for B.

$\therefore$ A is stronger than B.

(*d*) Material with smaller plastic region is more brittle, therefore B is more brittle than A.

5. (*i*) In case (*a*) Pressure head, h = + 20 cm of Hg

Absolute pressure = P + h = 76 + 20 = 96 cm of Hg.

Gauge Pressure = h = 20 cm of Hg.

In case (b) Pressure Head h = –18 cm of Hg

Absolute Pressure = 76 – 18 = 58 cm of Hg

Gauge Pressure = h = – 18 cm of Hg

6. as $h, \rho, g = h_2 \rho_2 g$

$h_1 \times 13.6 \times g = 13.6 \times 1 \times g$

$h_1 = 1$ cm

Therefore as 13.6 cm of H_2O is poured in right limb it will displace Hg level by 1 cm in the left limb, so that difference of levels in the two limbs will become 19 cm.

8. $$\upsilon = \frac{2}{9}\left[\frac{9(\sigma - \rho) r^2}{\eta}\right]$$

$$\upsilon \times r^2$$

If N drops coalesce, then

Volume of one big drop = volume of N droplets

$$\frac{4}{3}\pi R^3 = N\left(\frac{4}{3}\pi r^3\right)$$

$R = N^{1/3}\, r$ ....(2)

$\therefore$ Terminal velocity of bigger drop $= \left(\frac{R}{r}\right)^2 \times \upsilon$ from eq.(1)

$= N^{2/3}$ v from eq. (2)

9. Let P_1 & P_2 be the pressures inside the two bubbles then

$$P_1 - P = \frac{4T}{r_1} \Rightarrow P_1 = P + \frac{4T}{r_1}$$

$$P_2 - P = \frac{4T}{r_2} \Rightarrow P_2 = P + \frac{4T}{r_2}$$

When bubbles coalesce

$P_1V_1 + P_2 V_2 = PV$ (1)

$\therefore$ The pressure inside the new bubble $p = P + \frac{4T}{r}$

substituting for P_1 & V_1 in eq.(1)

$$\left(P + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(P + \frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3 = \left(P + \frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

or $\frac{4}{3}\pi P\left(r_1^3 + r_2^3 - r^3\right) + \frac{16\pi T}{3}\left[r_1^2 + r_2^2 - r^2\right] = 0$

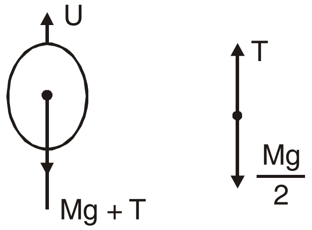
Given change in volume.

$V = \frac{4}{3}\pi\, r_1^3 + \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r^3$ (3)

Change in Area

$A = 4\pi\, r_1^2 + 4\pi r_2^2 - 4\pi r^2$ (4)

Using eq. (3) & (4) in (2) we get $PV + \frac{4T}{3}A = 0 = 3\ PV + 4TA = 0$

11. 

When the balloon is held stationary in air, the forces acting on it get balance

Up thrust = Wt. of Balloon + Tension in string

$$U = Mg + T$$

For the small block of mass $\frac{M}{2}$ floating stationary in air

$$T = \frac{M}{2}g$$

$$\therefore \qquad U = Mg + \frac{M}{2}g = \frac{3}{2}Mg$$

When the string is cut T = 0, the small block begins to fall freely, the balloon rises up with an acceleration 'a' such that

$$U - Mg = Ma$$

$$\frac{3}{2}Mg - Mg = Ma$$

$$a = \frac{g}{2} \text{ in the upward direction.}$$

12. (*i*) As the two vessels have liquid to same height and the vessels have same base area, the force exerted = pressure × base area will be same as pressure = h ρ g.

(*ii*) Since the volume of water in vessel 1 is greater than in vessel (2), the weight of water = volume × density × h, so weight of first vessel will be greater than the water in second vessel.

13. Radius of larger drop = $\frac{D}{2}$

radius of each small drop = r

$$\therefore \quad 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^2 \Rightarrow r = \frac{D}{6}$$

Initial surface area of large drop $4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$

Final surface area of 27 small drop = $27 \times 4\pi r^2 = 27 \times 4\pi \frac{D^2}{36} = 3\pi D^2$

$\therefore$ Change in energy = Increase in area × σ

$$= 2\pi D^2 \sigma$$

16. $l_1^1 = l_1\left[1 + \alpha_1(t_2 - t_1)\right]$

$l_2^1 = l_2\left[1 + \alpha_2(t_2 - t_1)\right]$

Given that the difference in their length remain constant

$$\therefore \quad l_2^1 - l_1^1 = l_2 - l_1$$

$$l_2[1 + \alpha_2 (t_1 - t_1)] - l_1[1 + \alpha_1 (t_2 - t_1)] = l_2 - l_1$$

$$\therefore \quad l_2 \alpha_2 = l_1 \alpha_1$$

19. The Compressive strain $\frac{\Delta l}{l} = \alpha T = 1.2 \times 10^{-5} \times 10 = 1.2 \times 10^{-4}$

Thermal stress $= \frac{\Delta F}{A} = Y\frac{\Delta L}{L} = 2 \times 10^{11} \times 1.2 \times 10^{-4}$

$$= 2.4 \times 10^7 Nm^{-2}$$

which corresponds to an external force

$$\Delta F = 2.4 \times 10^{7} \times 40 \times 10^{-4} = 10^{5} \text{ N.}$$

A force of this magnitude can easily bend the rails, hence it is important to leave space for thermal expansion.

25. Definition of coefficient of thermal conductivity.

In steady state the heat passing in unit time through the rod remain same that is

$$\frac{Q}{t} = \frac{K_1 A (T_1 - T)}{d_1} = \frac{K_2 A (T - T_2)}{d_2} = \frac{KA (T_1 - T_2)}{d_1 + d_2}$$

where k is the coefficient of thermal conductivity

Also $T_1 - T_2 = (T_1 - T) + (T - T_2)$

$$\therefore \quad \frac{d_1 + d_2}{kA} = \frac{d_1}{K_1 A} + \frac{d_2}{K_2 A}$$

$$\therefore \quad \frac{d_1 + d_2}{K} = \frac{d_1}{K_1} + \frac{d_2}{K_2} = \frac{K_2 d_1 + K_1 d_2}{K_1 K_2}$$

$$K = \frac{K_1 K_2 (d_1 + d_2)}{K_2 d_1 + K_1 d_2}$$

ANSWER FOR NUMERICALS

1. (*a*) $\text{Stress} = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{40 \times 10}{\pi \times \left(1 \times 10^{-3}\right)^2} = 1.27 \times 10^{8} \text{N / m}^2$

(*b*) $\Delta L = \frac{FL}{AY} = \frac{40 \times 10 \times 1}{\pi \times \left(1 \times 10^{-3}\right)^2 \times 7 \times 10^{10}} = 1.8 \times 10^{-3} \text{m}$

(c) Strain $= \dfrac{\Delta L}{L} = \dfrac{1.8 \times 10^{-3}}{1} = 1.8 \times 10^{-3}$

(d) $F = Kx = K\Delta L$ K = Force constant

$$K = \frac{F}{\Delta L} = \frac{40 \times 10}{1.8 \times 10^{-3}} = 2.2 \times 10^{5} N / m$$

2. Pressure exerted at the bottom layer by water column of height h is

$$P = h\rho g = 2500 \times 1000 \times 10$$

$$= 2.5 \times 10^7 \text{ N m}^{-2}$$

$$= \text{Stress}$$

Bulk modulus $K = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{P}{\Delta V / V}$

$$\therefore \quad \frac{\Delta V}{V} = \frac{P}{K} = \frac{2.5 \times 10^{7}}{2.3 \times 10^{9}} = 1.08 \times 10^{-2}$$

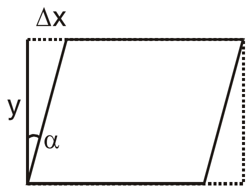
$$= 1.08\%$$

3. Area A of the upper face = $(0.30)^2$ m^2

The displacement Δx of the upper face relative to the lower one is given by

$$\Delta x = \frac{yF}{\eta A} \quad \because \eta = \frac{F / A}{\Delta x / y}$$

$$= \frac{0.30 \times 5 \times 10^{3}}{8.3 \times 10^{10} \times (0.30)^{2}} = 2 \times 10^{-7} / n$$



$\therefore$ Angle of shear $\propto$ is given by $\tan \propto = \dfrac{\Delta x}{y}$

$$\propto = \tan^{-1}\left(\frac{\Delta x}{y}\right)$$

$$= \tan^{-1}\left(\frac{2 \times 10^{-7}}{0.30}\right) = \tan^{-1}\left(0.67 \times 10^{-6}\right)$$

4. $V = 1$ litre $= 10^{-3} m^3$

$$\frac{\Delta V}{V} = 0.10\% = \frac{0.10}{100} = 0.001$$

$$K = \frac{P}{\Delta V / V} \Rightarrow P = \frac{K\Delta V}{V} = 2.2 \times 10^{9} \times 0.001$$

$P = 2.2 \times 10^6$ N m^{-2}

5. Pressure at a depth of 10m = $h\rho g$

$= 10 \times 1030 \times 10 = 1.03 \times 10^5$ N/m^2

ATM. pressure $= 1.01 \times 10^5$ pa.

Total pressure at a depth of 10 m $= 1.03 \times 10^5 + 1.01 \times 10^5$

$= 2.04 \times 10^5$ pa.

6. $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

$$F_1 = F_2 \frac{A_1}{A_2} = F_2 \left(\frac{\pi r_1^2}{\pi r_2^2}\right)$$

$$F_1 = 1350 \times 9.8 \left(\frac{5 \times 10^{-2}}{15 \times 10^{-2}}\right)^2$$

$= 1470$ N.

7. (i) When man is lying $P = \frac{F}{A} = \frac{80 \times 9.8}{0.6} = 1.307 \times 10^3 N / m^{-2}$

(ii) When man is standing then A = 2 × 80 cm^2 = 160 × 10^{-4} m^2

$\therefore \qquad P = \frac{80 \times 9.8}{160 \times 10^{-4}} = 4.9 \times 10^4 N\ m^{-2}$

8. (*a*) Pressure Instructed by manual = P_g = 200K Pa

(*b*) Absolute Pressure = 101 kP + 200 k Pa = 301 k Pa

(*c*) At mountain peak Pa' is 10% less

Pa' = 90 k Pa

If we assume absolute pressure in tyre does not change during driving then $P_g = P - P'_a$ = 301 – 90 = 211 k Pa

So the tyre will read 211 k Pa, pressure.

9. Excess pressure in soap bubble = $p = \frac{4T}{r} = \left(\frac{4 \times 0.58}{6 \times 10^{-3}}\right)$

= 387 N m^{-2}

10. $v_t = \frac{2}{9}\left[\frac{g(\sigma - \rho)r^2}{\eta}\right]$

$$\eta = \frac{2}{9}\left[\frac{9.8\left(8.9 \times 10^3 - 1.5 \times 10^3\right)\left(2 \times 10^{-3}\right)^2}{6 \times 10^{-2}}\right]$$

= 1.08 kg m^{-1} s^{-1}

11. From Torricelli theorem, velocity of efflux $\upsilon = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 0.2}$

= 2m/s

12. Surface energy per unit area is equal to surface tension.

E = increase in surface area × ST

$$= 4\pi\left(2^2 - 1^2\right) \times 2.5$$

$$= 4\pi \times 3 \times 25$$

$$= 1.02 \times 10^3 \text{ erg}$$

13. $F = \eta A \dfrac{dv}{dy}$

$$= 0.01 \times 0.20 \times \frac{0.1}{0.003} = 66.7 \times 10^{-3} N$$

14. Since $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{2 \times 4 \times 10^{-4}}{2 \times 10^{-4}} = 4 m / s$$

Using Bernoulli's Theorem

$$P_2 = P_1 + \frac{1}{2}\rho\left(\upsilon_1^2 - \upsilon_2^2\right) + \rho g\left(h_1 - h_2\right)$$

$\therefore$ $v_2 > v_1$

$h_2 > h_1$

$$= 3 \times 10^5 \frac{1}{2}(1000)\left[(4)^2 - (2)^2\right] - 1000 \times 9.8 \times 8$$

$$= 2.16 \times 10^5 \text{ N/m}^2$$

15. Shear Stress $= \dfrac{F}{A} = \dfrac{9.8 \times 10^{-2}}{0.10} = F^1$

Strain Rate $= \dfrac{\upsilon}{l} = \dfrac{0.085}{0.30 \times 10^{-3}}$

$$\eta = \frac{\text{Stress}}{\text{Strain}} = \frac{9.8 \times 10^{-2} \times 0.30 \times 10^{-3}}{0.085 \times 0.10}$$

$$= 3.45 \times 10^{-3} \text{ Pa s}$$

16. $$\frac{\sigma_w}{\sigma_m} = \frac{h_1 \rho_1 \text{Cos } \theta_2}{h_2 \rho_2 \text{ Cos } \theta_1} = \frac{10 \times 1 \times \text{Cos } 135°}{-3.42 \times 13.6 \times \text{Cos } o°}$$

$$= \frac{10 \times (-0.7071)}{-3.42 \times 13.6}$$

$$= 0.152$$

17. By Wien's Displacement Law

$$\lambda_m T = \lambda_m^1 T^1$$

$$\therefore \frac{T}{T'} = \frac{\lambda m'}{\lambda m} = \frac{4.8 \times 10^{-7}}{3.6 \times 10^{-7}}$$

$$= 4:3$$

18. $$\frac{F - 32}{180} = \frac{T - 273}{100}$$

$$\frac{149 - 32}{180} = \frac{T - 273}{100} \Rightarrow \frac{117}{9} = T - 273$$

$$T = 286k$$

19. $$m_1 c_1 (\theta_1 - \theta) = m_2 c_2 (\theta - \theta_2)$$

$\because$ $C_2 = 1$ cal/gm°c

$$\therefore \quad 50 \times 0.6 \times (120 - \theta) = 1.6 \times 10^3 \times 1 \times (\theta - 25)$$

$$\theta = 26.8°C$$

20. $d_2 = d_1[1 + \propto \Delta t]$

5.243 = 5.231[1 + 1.2 ×10^{-5} (T –300)]

$$\left[\frac{5243}{5231} - 1\right] = 1.2 \times 10^{-5}(T - 300)$$

T = 191 + 300 = 491 k = 218°C

21. $\underset{\text{at } 0°C}{\text{ice}} \rightarrow \underset{\text{at } 0°C}{\text{water}} \rightarrow \underset{\text{at } 6°C}{\text{water}}$

m_1c_1 (80 –6) = m_2 L + m_2c_2(6 –0)

100 × 1 × 74 = 100 L + 100 × 1 × 6

L = (1 × 74) –6

= 68 cal/g.

22. Heat required to convert H_2O at 0° to H_2O at 100° = m_1c_1 t

= 3 × 4186 × 100

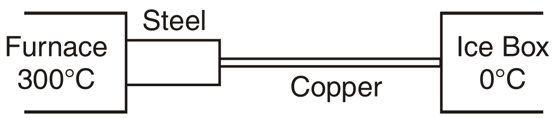
= 1255800 J

Heat required to convert H_2O at 100°C to steam at 100°C is = mL

= 3 × 2.256 × 10^6

= 6768000J

Total heat = 8023800 J

23. 

$$\frac{Q}{t} = \frac{k_1A_1(T_1 - T)}{d_1} = \frac{k_2A_2(T - T_2)}{d_2}$$

$A_1 = 2A_2$

$$\therefore \frac{50.2 \times 2\ A_2 (573 - T)}{15 \times 10^{-2}} = \frac{385 \times A_2 (T - 273)}{10 \times 10^{-2}}$$

T = 317.43 k = 44.43°C

24. $$\frac{\theta_1 - \theta_2}{t} \propto \left(\frac{\theta_1 + \theta_2}{2}\right) - \theta_0$$

$$\frac{94 - 86}{2} = k\left[\frac{94 + 86}{2} - 20\right] \Rightarrow 4 = k \times 70 \qquad ...1$$

$$\frac{82 - 78}{t'} = k\left[\frac{82 + 78}{2} - 20\right] \Rightarrow \frac{4}{t'} = k60 \qquad ...2$$

dividing eq. (1) by eq (2)

$$4 / \frac{4}{t'} = \frac{k \times 70}{k \times 60} \qquad t' = \frac{7}{6} = 1.16 \text{ min}$$

25. $$E = \sigma T^4$$

$$T = \left(\frac{E}{\sigma}\right)^{1/4} = \left(\frac{2.835}{5.670 \times 10^{-8}}\right)^{1/4} = 84.92k$$

= 85 k

26. $$R_t = R_0 (1 + \alpha \Delta t)$$

R_t = R_0+ 12% of R_0 = 1.12 R_0

$1.12R_0 = R_0\ (1 + 2.5 \times 10^{-3}\ \Delta t)$

$$\Delta t = \frac{0.12}{2.5 \times 10^{-3}} = 48°C$$

UNIT VIII

THERMODYNAMICS

KEY CONCEPTS

- The ratio of work done (W) to the amount of heat produced (Q) is always a constant, represented by J.

  i.e., $$\frac{W}{Q} = J$$

  where J is called **Joule's mechanical equivalent of heat.** The value of J = 4.186 joule/calorie.

- If temperature of a body of mass m rises by ΔT, then $Q = mc\,\Delta\,T$ where c is *specific heat* of the material of the body,

  When the state of body of mass m changes at its melting point/boiling point, then $Q = m\,L$, where L is *latent heat* of the body.

- All solids expand on heating. The coefficient of linear expansion (α), coefficient of superficial expansion (β) and coefficient of cubical expansion (γ) are related as

  $$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}.$$

- In case of liquids, $\gamma_r = \gamma_a + \gamma_c$ where γ_r is coefficient of real expansion of a liquid, γ_a is coefficient of apparent expansion of the liquid and γ_r is coefficient of cubical expansion of the vessel.

- **Principle of calorimetry :** When two substances at different temperatures are mixed together, they exchange heat. If we assume that no heat is lost to the surroundings, then according to principle of calorimetry.

  $$\underset{\text{by on subs tan ce}}{\text{Heat one}} = \underset{\text{by another substance}}{\text{Heat gained}}$$

- **Specific heat of gases :** Specific heat of a gas is the amount of heat required to raise the temperature of one gram of gas through 1°C.

  Principal specific heat of a gas :

  (*i*) Specific heat at constant volume (c_v)

  (*ii*) Specific heat at constant pressure (c_p)

  $C_v \rightarrow$ molar heat capacity at constant volume

  $C_p \rightarrow$ molar heat capacity at constant pressure

  $$C_p - C_V = R/J$$

- Coefficient of thermal conductivity (K) of a solid conductor is calculated from the relation

  $$\frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{\Delta t}\right)$$

  where A is area of hot face, Δx is distance between the hot and cold faces, ΔQ is the small amount of heat conducted in a small time (Δt), ΔT is difference in temperatures of hot and cold faces.

  Here ($\Delta T/\Delta x$) *temperature gradient, i.e.,* rate of *fall* of temperature with distance in the direction of flow of heat.

  All liquids and gases are heated by convection. Heat comes to us from the sun by radiation.

- **Newton's Law of Cooling :** It states that the rate of loss of heat of a liquid is directly proportional to difference in temperatures of the liquid and the surroundings provided the temperature difference is small (= 30°C).

  $$-\frac{dQ}{dt} \propto (T - T_0) \qquad \text{or} \qquad \frac{dQ}{dt} = -K(T - T_0)$$

  As, $\dfrac{dQ}{dt} = \dfrac{ms\,dT}{dt} = -K(T - T_0)$ or $\dfrac{dT}{dt} \propto (T - T_0)$.

- When the temperature difference between body and surroundings is large, then Stefan's law for cooling of body is obeyed. According to it,

  $$E = \sigma\ (T^4\ T_0^4)$$

where E is the amount of thermal energy emitted per second per unit area of a black body. T is the temperature of black body and T_0 is the temperature of surroundings, σ is the Stefan's constant.

Wien's Displacement Law : The wavelength λ_{max} at which the maximum amount of energy is radiated decreases with the increase of temperature and is such that

$$\lambda_{max}T = a \text{ constant}$$

where T is the temperature of black body in Kelvin.

- **Thermodynamical system :** An assembly of extremely large number of gas molecules is called a thermodynamical system. The pressure P, volume V, temperature T and heat cotent Q are called *Thermodynamical parameters.*

- **Zeroth Law of Thermodynamics :** (Concept of temperature) According to this law, when thermodynamic systems A and B are separately in thermal equilibrium with a third thermodynamic system C, then the systems A and B are in thermal equilibrium with each other also.

- **Internal Energy of a Gas** is the sum of kinetic energy and the potential energy of the molecules of the gas.

  K.E./molecule $= \frac{1}{2}mc^2 = \frac{3}{2}k\,T$ where k is Boltzmann's constant.

  internal energy of an ideal gas is wholly kinetic.

- **First Law of Thermodynamics** (principle of conservation of energy)

  According to this law $dQ = dU + dW$

  where dQ is the small amount of heat energy exchange with a system, dU is small change in internal energy of the system and dW is the small external work done by or on the system.

- *Sign conventions* used in thermodynamics.

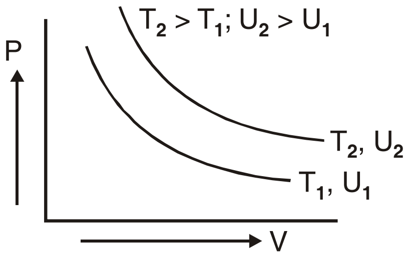
  (a) Heat *absorbed* by the system = *positive* and heat rejected by the system = *negative*.

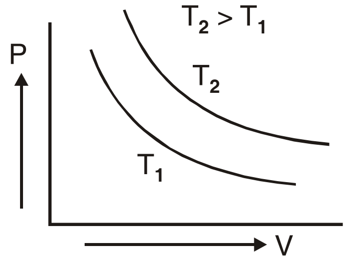
  (b) When temperature of the system rises, its internal energy increases ΔU = *positive*.

When temperature of the system falls, its internal energy decreases, ΔU = *negative*.

(c) When a gas expands, work is done by the system. It is taken as positive. When a gas is compressed, work is done *on* the system. It is taken as negative.

Isothermal changes	Adiabatic changes
● Temperature (T) remains constant, *i.e.,* $\Delta T = 0$	1. Heat content and entropy are constant, *i.e.,* Q = const ; S = constant, $\Delta Q = 0$; $\Delta S = 0$
● Changes are slow.	2. Changes are fast.
● System is thermally conducting.	3. System is thermally insulated.
● Internal energy, U = constant $\therefore \Delta U = 0$	4. Internal energy changes, *i.e.,* $\Delta U \neq 0$
● Specific heat, $c = \infty$	5. Specific heat, $c = 0$
● Equation of isothermal changes, PV = constant.	6. Eqn. of adiabatic changes (*i*) PV^{γ} = constant (ii) $TV^{\gamma-1}$ = constant (*iii*) $P^{1-\gamma}\, T^{\gamma}$ = constant





● Slope of isothermal curve, $\dfrac{dP}{dV} = -\left(\dfrac{P}{V}\right)$	7. Slope of adiabatic curve, $\dfrac{dP}{dV} = -\gamma\left(\dfrac{P}{V}\right)$
● Coeff, of isothermal elasticity, $K_i = P$	8. Coeff. of adiabatic elasticity, $K_a = \gamma P$
● Work done in isothermal expansion	9. Work done in adiabatic expansion
$W = 2.303\ nRT \log_{10} (V_2/V_1)$	$W = \dfrac{nR(T_2 - T_1)}{1-\gamma}$
n → number of mole	
$W = 2.303\ P_1 V_1 \log_{10} (V_2/V_1)$	$\dfrac{P_2V_2 - P_1V_1}{1-\gamma}$
$W = 2.303\ nRT \log_{10} (P_1/P_2)$	$W = C_v (T_1 - T_2)$

● **Second Law of Thermodynamics :** It is impossible for self acting machine, unaided by an external agency to convey heat from the body at lower temperature to another at higher temperature. This statement of the law was made by **Clausius.**

According to **Kelvin,** it is impossible to derive a continuous supply of work by cooling a body to a temperature lower than that of the coldest of its surroundings.

Heat Engines : A heat engine is a device which converts heat energy into mechanical energy. Efficiency of a heat engine is the ratio of work done (W) by the engine per cycle to the energy absorbed from the source (Q_1) per cycle.

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$ where Q_2 = heat rejected to the sink

- **Carnot Engine :** is an ideal heat engine which is based on Carnot's reversible cycle. Its working consists of four steps. (Isothermal expansion, Adiabatic expansion isothermal compression and adiabatic compression).

The efficiency of Carnot engine is given by $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

where Q_1 is heat energy absorbed from the source maintained at high temperature $T_1 K$ and Q_2 is amount of heat energy rejected to the sink at low temperature $T_2 K$.

A Refrigerator absorbs heat Q2 from a sink (substance to be cooled) at lower temperature TK. Electric energy W has to be supplied for this purpose

$Q_1 = Q_2 + W$

Coefficient of performance (b) *of a refrigerator is the ratio of the heat absorbed per cycle from the sink (Q2) to the electric energy supplied (W) for this purpose per cycle, i.e.,*

$$\beta = \frac{Q_2}{W}, \text{ i.e., } \beta = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2} = \frac{1-\eta}{\eta}$$

QUESTIONS

1. Why spark is produced when two substances are struck hard against each other?

2. What is the specific heat of a gas in an isothermal process.

3. On what factors, does the efficiency of Carnot engine depend?

4. What are two essential features of Carnot's ideal heat engine.

5. Plot a graph between internal energy *U* and Temperature (T) of an ideal gas.

6. Refrigerator transfers heat from cold body to a hot body. Does this violate the second law of thermodynamics.

7. What is heat pump?

8. Give two example of heat pump?

9. What is heat engine?

10. Why a gas is cooled when expanded?

11. Can the temperature of an isolated system change?

12. Which one a solid, a liquid or a gas of the same mass and at the same temperature has the greatest internal energy.

13. Under what ideal condition the efficiency of a Carnot engine be 100%.

14. Which thermodynamic variable is defined by the first law of thermo-dynamics?

15. Give an example where heat be added to a system without increasing its temperature.

16. What is the efficiency of carnot engine working between ice point and steam point?

17. Two blocks of the same metal having masses 5g and 10g collide against a target with the same velocity. If the total energy used in heating the balls which will attain higher temperature?

18. What is the specific heat of a gas in an adiabatic process.

SHORT ANSWER (1 MARK)

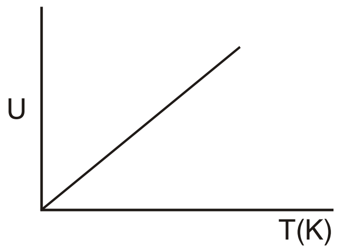
1. Work is converted into heat.

2. Infinite

3. $\eta = 1 - T_2/T_1$

4. (i) Source and sink have infinite heat capacities.

   (ii) Each process of the engine's cycle is fully reversible

5. 

6. No, External work is done

7. A heat pump is a device which uses mechanical work to remove heat.

8. Refrigerator, Air Conditioner.

9. Heat engine is a device which convert heat energy into mechanical energy.

10. Decrease in internal energy.

11. Yes in an adiabatic process

12. Gas has greatest internal energy and solid has least internal energy.

13. If the temperature of sink is OK.

14. Internal energy.

15. Melting.

16. $\eta = 1 - T_2/T_1 = 1 - 273/373 = 26.8\%$

17. Both the balls will undergo the same rise in temperature.

18. Zero.

SHORT ANSWER TYPE QUESTION (2 MARKS)

1. A thermos bottle containing tea is vigorously shaken. What will be the effect on the temperature of tea.

2. Write two limitation of the first law of thermodynamics.

3. Write the expressions for C_v and C_p of a gas in terms of gas constant R and γ where

$$\gamma = C_p/C_v$$

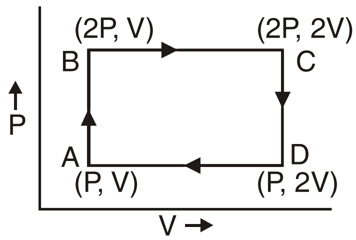
4. No real engine can have an efficiency greater that of a carnot engine working between the same tow temperatures. Why?

5. Why water at the base of a waterfall is slightly warmer than at the top?

6. When ice melts, the change in internal energy is greater than the heat supplied. Why?

7. Explain why two isothermal curves never intersect.

8. An ideal monatomic gas is taken round the cycle ABCDA as shown. Calculate the work done during the cycle.



9. Can a room be cooled by opening the door of refrigerator in a closed room?

10. Explain what is meant by isothermal and adiabatic operations.

11. Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$ Explain?

SHORT ANSWERS (2 MARKS)

1. Temperature of tea will rise.

2. (i) It does not give the direction of flow of heat.

   (ii) It does not explain why heat cannot be spontaneously converted into work.

3. $\gamma = C_p/C_v$

$C_p - C_v = R$

$C_p = \gamma C_v$

$$(\gamma - 1)C_v = R; C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

4. In carnot engine.

   (i) There is absolutely no friction between the wall of cylinder and piston.

   (ii) Working substance is an ideal gas

   In real engine these condition cannot be fulfilled.

5. Potential energy converted in to kinetic energy, some part of kinetic energy is converted in to heat.

6. dq = du + dw

   du = dq − pdv

   When ice melt change in volume is negative.

7. PV

8. PV

9. No, It a voilets seconds law.

10. Adiabatic a Process – Pressure, volume and temperature of the system changes but there is no exchange of heat.

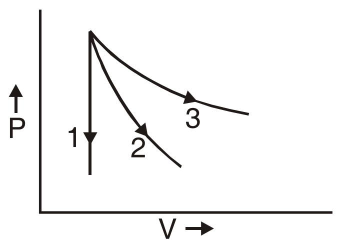
    Isothermal Process – Pressure, volume changes temperature remain constant.

11. Heat flows from higher temperature to lower temperature until the temperature become equal only where the thermal capacities of two bodies are equal.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. Obtain an expression for work done in an isothermal process.

2. Identify and name the Thermodynamic processes 1,2,3 as shown in figure.



3. Two samples of gas initially at the same temperature and pressure are compressed from volume V to V/2 one sample is compressed isothermally and the other adiabatically in which case the pressure will be higher? Explain?

4. Explain briefly the principle of a heat pump. What is meant by coefficient of performance?

5. When you blow on the back of your hand with your mouth wide open your breath feels warm about if you partially close your mouth form an "O" and then blow on your hand breath fells cool. Why?

6. Is it a violation of the second law of thermodynamics to convert

   (a) Work completely in to heat

   (b) Heat completely in to work

   Why or why not?

7. State first law of thermodynamics on its basis establish the relation between two molar specific heat for a gas.

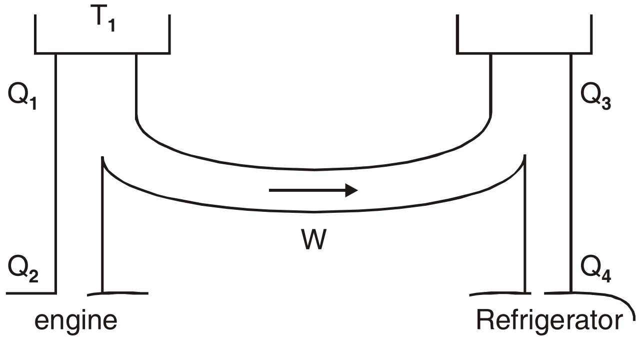
8. Explain briefly the working principle of a refrigerator and obtain an expression for its coefficient of performance.

9. State zeroth law of thermodynamics. How does it lead to the concept of temperature?

10. What is a cyclic process? Show that the net work done during a cyclic process is numerically equal to the area of the loop representing the cycle.

11. State second law of thermodynamics.

12. An ideal engine works between temperatures T_1 and T_2 . It drives an ideal refrigerator that works between temperatures T_3 and T_4. Find the ratio Q3/Q1 in terms of T_1, T_2, T_3 and T_4



LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Describe briefly carnot engine and obtain an expression for its efficiency.

2. Define adiabatic process. Derive an expression for work done during adiabatic process.

3. Why a gas has two principle specific heat capacities? What is the significance of $C_p - C_v$ and C_p/C_v where symbols have usual meaning.

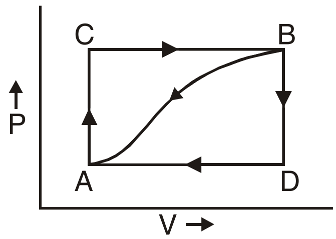
NUMERICALS

1. When a system is taken from state A to state B along the path ACB, 80 k cal of heat flows into the system and 30 kcal of work is done.

   (a) How much heat flows into the system along path ADB if the work done is 10 k cal?

   (b) When the system is returned from B to A along the curved path the work done is 20 k cal. Does the system absorb or librate heat.

   (c) If $U_A = 0$ and $U_D = 40$ k cal, find the heat absorbed in the process AD



2. $\frac{1}{2}$ mole of helium is contained in a container at S.T.P. How much heat energy is needed to double the pressure of the gas, keeping the volume constant? Heat capacity of gas is 3 J g^{-1} K^{-1}.

3. The volume of steam produced by 1g of water at 100°c is 1650 cm^3. Calculate the change in internal energy during the change of state given

   $J = 4.2 \times 10^7$ erg cal^{-1} g = 98 J cm/s^2

   latent heat of steam = 540 cal/g

4. What is the coefficient of performance (β) of a carnot refrigerator working between 30°C and 0°C?

5. Calculate the fall in temperature when a gas initially at 72°C is expanded suddenly to eight times its original volume. (γ = 5/3)

6. A steam engine intake steam at 200°C and after doing work exhausts it directly in air at 100°C calculate the percentage of heat used for doing work. Assume the engine to be an ideal engine?

7. A perfect carnot engine utilizes an ideal gas the source temperature is 500K and sink temperature is 375K. If the engine takes 600k cal per cycle from the source, calculate

   (i) The efficiency of engine

   (ii) Work done per cycle

   (iii) Heat rejected to sink per cycle.

8. Two carnot engines A and B are operated in series. The first one A receives heat at 900 K and reject to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 400 K calculate the temperature T when

   (i) The efficiencies of the two engines are equal

   (ii) The work output of the two engines are equal

9. Ten mole of hydrogen at NTP is compressed adiabatically so that its temperature become 400°C How much work is done on the gas? what is the increase in the internal energy of the gas

$$R = 8.4 \text{ J mol}^{-1}\text{K}^{-1} \ \gamma = 1.4$$

10. The temperature T_1 and T_2 of the two heat reservoirs in an ideal carnot engine be 1500°C and 500°C respectively. which of these increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine.

ANSWERS

1. (a) $dw_{ADB} = +10$ k cal

Internal energy is path independent

$du_{ADB} = du_{ACB} = 50$ k cal

$dQ_{ADB} = 50 + 10 = 60$ k cal.

(b) $dw_{BA} = -20$ k cal

$du_{BA} = -du_{ADB}$

$dQ_{BA} = du_{BA} + dW_{BA}$

$= -50 - 20 = -70$ k cal

(c) $U_A = 0 \ U_D = 40$ k cal

$du_{AD} = 40$ k cal

$dw_{ADB} = 10$ k cal

$dw_{DB} = 0$ since $dV = 0$

$dQ_{AD} = 40 + 10 = 50$ k cal

2. $n = \frac{1}{2}$, $C_v = 3$J/gK. M = 4

$C_v = MC_v = 12$ J/mole k M $\rightarrow$ Molecular mass

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = 2$$

$\Delta T = 2T_1 - T_1 = 273$ k

$\Delta Q = n\ c_v\ \Delta T = 1638$ J

3. Mass of water = 1 g = 10^{-3} kg

$$\text{volume of water} = \frac{\text{Mass}}{\text{Density}} = \frac{10^{-3}}{10^{3}} = 10^{-6} m^{3}$$

$= 1\ cm^3$

Change in volume = 1650 – 1 = 1649 cm^3

$dQ = m\ L = 540$ cal $= 540 \times 4.2 \times 10^7$ erg

$P = 1$ atm $= 76 \times 13.6 \times 981$

$du = dQ - pdv = 22.68 \times 10^9 - 1.67 \times 10^9$

$= 21.01 \times 10^9$ erg.

4. $$\beta = \frac{T_2}{T_1 - T_2} = \frac{273}{303 - 273} = 9.1$$

5. $T_1 V_1^{v-1} = T_2 V_2^{v-1}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{v-1} = 345\left(\frac{x}{8x}\right)^{2/3}$$

$$= 345 \times \frac{1}{4} = 86.25 \text{ k}$$

6. $T_1 = 200°C = 473$ K $T_2 = 100°C = 373$ K

$$\eta = \frac{w}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{473 - 373}{473}$$

$$= \frac{100}{473} = 0.21$$

= 21%

7. $T_1 = 500$ K $T_2 = 375$ K

Q_1 = Heat absorbed = 600 k cal

$$\eta = 1 - T_2/T_1 = \frac{125}{500} = 0.25$$

$$= 25\%$$

(b) $\eta = \dfrac{w}{Q_1}$

$$W = \eta Q_1 = 0.25 \times 60 \text{ k cal} = 150 \text{ k cal}$$

$$= 450 \text{ k cal}$$

(c) $w = Q_1 - Q_2 \; Q_2 = Q_1 - W = 600 - 150$

$$= 450 \text{ k cal}$$

8. $W_A = W_B$

$$\frac{W}{Q_1} = \left(1 - \frac{T_2}{T_1}\right)$$

$$W = Q_1(1 - T_2/T_1)$$

$$Q_2\left(1 - T_3/T_2\right) = Q_1\left(1 - T_2/T_1\right)$$

$$\left(1 - T/900\right) \quad Q_1 = \left(1 - \frac{400}{T}\right) Q_2$$

$$\left(1 - T/900\right) \quad Q_1 = \left(1 - \frac{400}{T}\right) T/900$$

$$1 - T/900 = \frac{T}{900} - \frac{400}{900}$$

$$\frac{2T}{900} = 13/9$$

$$T = 650 \text{ K}$$

$$\eta_A = \eta_B$$

$$1 - T/900 = \frac{1 - 400}{T}$$

$$T^2 = 900 \times 400$$

= 600 k

$T_1 = 273$ k $T_2 = 673$ k

mass of gas = 10 mole

$$w_{adi} = \frac{10\ R}{(\gamma - 1)}(T_1 - T_2)$$

$$= \frac{10 \times 8.4}{(1.4 - 1)}(273 - 673)$$

$= -8.4 \times 10^4$ J work being done on the gas

$du = -dw = 8.4 \times 10^4$ J

10. $\eta = 1 - T_2/T_1$

(i) T_1 is increased from 1500°C to 1600°C

$T_1 = 1873$ k

T_2 remain constant $T_2 = 773$ k

$$\eta_1 = \frac{1873 - 773}{1873} = 58.73\%$$

(ii) T_1 remain constant 1500°C

$T_1 = 1500 + 273 = 1773$ k

T_2 is decreased by 100 i.e. 400°C

$T_2 = 400 + 273 = 673$ k

$$\eta_2 = \frac{1773 - 673}{1773} = \frac{1100}{1773} = 62.04\%$$

$\eta_2 > \eta_1$

UNIT IX

BEHAVIOUR OF PERFECT GAS AND KINETIC THEORY

POINTS TO REMEMBER

- **Pressure exerted by a gas :** It is due to continuous collision of gas molecules against the walls of the container and is given by the relation

  $P = \frac{Mc^2}{3V} = \frac{1}{\rho}\rho c^2$ where c is the rms velocity of gas molecules.

- **Average K.E. per molecule** of a gas $\frac{1}{2}mC^2 = \frac{3}{2}k_B T$. It is independent of the mass of the gas but depends upon the temperature of the gas.

- **Absolute zero :** It is that temperature at which the root mean square velocity of the gas molecules reduces to zero.

- **Different types of speed of gas molecules**

  (i) Most probable speed $\quad c_{mp} = \sqrt{\frac{2k_B T}{m}}$

  $k_B \rightarrow$ Boltzmann's Constant

  (ii) **Mean speed** or average speed

  $$c_{av} = \frac{c_1 + c_2 + ...c_n}{n} = \sqrt{\frac{8k_B T}{m\pi}}$$

  (iii) The number of degrees of freedom = total number of independent co-ordinates required to describe completely, the position and configuration of the system.

  For monoatomic gases, $f = 3$

For diatomic gases, $f = 5$

For linear triatomic gas molecules, $f = 7$

For non-linear triatomic gas molecules, $f = 6$

- According to the **law of equipartition of energy,** for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom. The average energy associated with each molecule per degree of freedom $= \frac{1}{2} k_B T$, where k_B is Boltzmann constant and T is temperature of the system.

- **Mean free path** of gas molecules is the average distance travelled by a molecule between two successive collisions. It is represented by λ.

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

where d = diameter of molecule and n = number of molecules per unit volume of the gas.

also $\lambda = \dfrac{k_B T}{\sqrt{2}\pi d^2 p}$

where k_B is Boltzmann constant; p is pressure and T is temperature of the gas.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write two condition when real gases obey the ideal gas equation ($PV = nRT$). n $\rightarrow$ number of mole.
2. If the number of molecule in a container is doubled. What will be the effect on the *rms* speed of the molecules?
3. Draw the graph between P and 1/V (reciprocal of volume) for a prefect gas at constant temperature.
4. Name the factors on which the degree of freedom of gas depends.
5. What is the volume of a gas at absolute zero of temperature?
6. How much volume does one mole of a gas occupy at NTP?

7. What is an ideal gas?
8. The absolute temperature of a gas is increased 3 times what is the effect on the root mean square velocity of the molecules?
9. What is the Kinetic Energy per unit volume of a gas whose pressure is P?
10. A container has equal number of molecules of hydrogen and carbon dioxide. If a fine hole is made in the container, then which of the two gases shall leak out rapidly?
11. What is the mean translational Kinetic energy of a perfect gas molecule at temperature T?
12. Why it is not possible to increase the temperature of a gas while keeping its volume and pressure constant.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

1. When an automobile travels for a long distance the air pressure in the tyres increases. Why?
2. A gas storage tank has a small leak. The pressure in the tank drop more quickly if the gas is hydrogen than if it is oxygen. Why?
3. Why the land has a higher temperature than the ocean during the day but a lower temperature at night.
4. Helium is a mixture of two isotopes having atomic masses 3g/mol and 4g/mol. In a sample of helium gas, which atoms move faster on average?
5. State Avogadro's law. Deduce it on the basis of Kinetic theory of gases.
6. Although the velocity of air molecules is nearly 0.5 km/s yet the smell of scent spreads at a much slower rate why.
7. The root mean square (rms) speed of oxygen molecule at certain temperature 'T' is 'V'. If temperature is doubled and oxygen gas dissociates into atomic oxygen what is the speed of atomic oxygen?
8. Two vessels of the same volume are filled with the same gas at the same temperature. If the pressure of the gas in these vessels be in the ratio 1 : 2 then state

   (i) The ratio of the rms speeds of the molecules.

(ii) The ratio of the number of molecules.

9. Why gases at high pressure and low temperature show large deviation from ideal gas behaviour.

10. A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Why the pressure of the gas decreases when the piston is pulled out.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

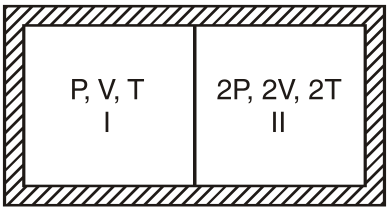
1. On what parameters does the λ (mean free path) depends.

2. Equal masses of oxygen and helium gases are supplied equal amount of heat. Which gas will undergo a greater temperature rise and why?

3. Why evaporation causes cooling?

4. Two thermally insulated vessels 1 and 2 are filled, with air at temperatures (T_1, T_2), volume (V_1, V_2) at pressure (P_1, P_2) respectively. If the value joining the two vessels is opened what is temperature of the vessel at equilibrium.

5. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartment. What is the ratio of the number of molecules in compartments I and II?



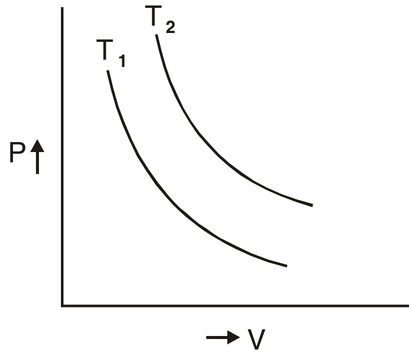
6. Prove that for a perfect gas having n degree of freedom

$$\frac{C_p}{C_v} = 1 + \frac{2}{n}$$

where C_p and C_v have their usual meaning.

7. The ratio of specific heat capacity at constant pressure to the specific heat capacity at constant volume of a diatomic gas decreases with increase in temperature. Explain.

8. Isothermal curves for a given mass of gas are shown at two different temperatures T_1 and T_2 state whether $T_1 > T_2$ or $T_2 > T_1$, justify your answer.



9. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic) the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not in which case is V_{rms} the largest?

10. State Grahm's law of diffusion. How do you obtain this from Kinetic Theory of gases.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Prove that the pressure exerted by a gas is given by

$$P = \frac{1}{3}\rho c^2$$

where p is density and c is root mean square velocity.

2. What are the fundamental postulates of the Kinetic theory of gases?

3. Given that $P = \frac{1}{3}\rho c^2$ where P is the pressure, ρ is the density and c is the rms. Velocity of gas molecules. Deduce Boyle's law and Charles law of gases from it.

4. What do you understand by mean speed, root mean square speed and most probable speed of a gas. The velocities of ten particles in m/s are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9 calculate.

   (i) Average speed (ii) r.m.s. speed

5. What is law of equipartition of energy? Find the value of $\gamma = C_p/C_v$ for diatomic and monatomic gas. Where symbol have usual meaning.

NUMERICALS

1. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface which is at a temperature of 35°C?

2. An electric bulb of volume 250 cm^3 was realed off during manufacture at a pressure of 10^{-3} mm of Hg at 27°C. Find the number of air molecules in the bulb–

   ($R = 8.31$ J $mole^{-1}$ $K^{-1,}$ NA = 6.02×10^{-23} $mole^{-1}$)

   (density of mercury $\rho = 13.6 \times 10^3$ kg m^{-3})

3. An ideal gas has a specific heat at constant pressure ($C_p = 5\ R/2$). The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 k and a pressure of 1.6×10^6 Nm^{-2}. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas

   ($R = 8.3$ J K^{-1} mol^{-1})

4. An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atmosphere and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drop to 17°C. Estimate the mass of oxygen taken out of the cylinder

   ($R = 8.31$ J $mol^{-1\ k-1}$)

   (molecular mass of $O_2 = 32$)

5. At what temperature the rms speed of oxygen atom equal to r.m.s. speed of heliums gas atom at –10°C

   Atomic mass of helium = 4

   Atomic mass of oxygen = 32

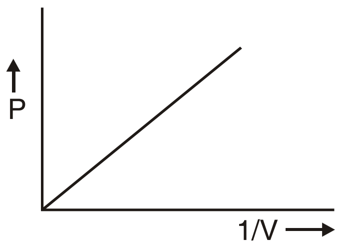
6. The density of Carbon dioxide gas at 0°C and at a pressure of 1.0×10^5 newton/m^2 is 1.98 kg/m^3. Find the root mean square velocity of its molecules at 0°C and 30°C. Pressure is kept constant.

7. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27°C. How much heat has to be transferred to the gas to double the rms speed of its molecules.

ANSWERS (1 MARK)

1. (i) Low pressure (ii) High temperature

2. No effect

3. 

4. Atomicity and temperature

5. 0

6. 22.4 litre

7. Gas in which intermolecular forces are absent

8. increases $\sqrt{3}$ times

9. 3P/2

10. Hydrogen (rms speed is greater)

11. $\frac{3}{2}RT$

12. $P = \frac{1}{3}\frac{M}{V}K\ T \qquad T\ \alpha\left(PV\right)$

    P and V are constant then T is also constant.

ANSWERS (2 MARKS)

1. Work is done against friction. This work done is converted into heat.

2. Rate of diffusion of a gas is inversely proportional to the square root of the density. So hydrogen leaked out more rapidly.

3. Specific Heat of water is more than land (earth). Therefore for given heat change in temp. of land is more than ocean (water).

6. The air molecules travel along a zigzag path due to frequent collision as a result their displacement perunit time is very small.

7. $C = \sqrt{\frac{3RT}{M}} = v$ $\qquad C' = \sqrt{\frac{3R(2T)}{M/2}} = 2\sqrt{\frac{3RT}{M}}$

$C^1 = 2V$

8. $P = \frac{1}{3}\frac{mnc^2}{V}$ $\qquad P \alpha nc^2$; $c a \sqrt{T}$

as the temperature is same rms speeds are same.

Pan $\quad$ i.e. $\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{1}{2}$

9. When temp is low and pressure is high the intermolecular forces become appreciable thus the volume occupied by the molecular is not negligibly small as composed to volume of gas.

10. When piston is pulled out the volume of the gas increases, Now losses number of molecules colliding against the wall of container per unit area decreases. Hence pressure decreases.

ANSWERS (3 MARKS)

1. (i) diameter of molecule (iii) $\gamma \propto \frac{1}{P}$

2. (ii) $\lambda \propto T$ (iv) $\lambda \propto \frac{1}{\int}$ (v) $\lambda \propto \frac{1}{n}$ $(iv)\ \lambda \propto m$

3. During evaporation fast moving molecules escape a liquid surface so the average kinetic energy of the molecules left behind is decreased thus the temperature of the liquid is lowered.

4. number of mole = Constant

$\mu_1 + \mu_2 = \mu$

$$\frac{P_1V_1}{R\,T_1} + \frac{P_2V_2}{RT_2} = \frac{P(V_1 + V_2)}{RT}$$

from Boyles law $P(V_1 + V_2) = P_1V_1 + P_2V_2$

5. $n = \dfrac{pV}{kT}$ $\qquad h' = \dfrac{2p\ 2v}{kT}$

$n / n' = \dfrac{1}{4}$

8. $T = \dfrac{PV}{\mu R}$ T α P V (man is constant)(μ is constant)

since PV is greater for the curve at T_2 than for the curve T_1 therefore $T_2 > T_1$

Three vessels at the same pressure and temperature have same volume and contain equal number of molecules

$$Vrms = \sqrt{\frac{3\,RT}{m}} \qquad V_{rms} \propto \frac{1}{\sqrt{m}}$$

rms speed will not same, neon has smallest mans therefore rms speed will be largest for neon.

ANSWERS NUMERICALS

1. $v_1 = 10^{-6}m^3$

Pressure on bubble P_1 = water pressure + Atmospheric pressure

$= \rho gh + Patm$

$= 4.93 \times 10^5$ Pa

$T_1 = 285$ k. $T_2 = 308$ k

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{4.93 \times 10_5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} m^3$$

2. $V = 250$ cc $= 250 \times 10^{-6}$ m^3

   $P = 10^{-6}$ m $\rho = 13.6 \times 10^3$ kg/m^3

   $T = 300$ K

   $$\mu = \frac{PV}{RT} = \frac{h\rho gV}{RT} = 1.3 \times 10^{-8} \text{mole}$$

   number of molecule $= \mu N_A = 8 \times 10^{15}$

3. $PV = nRT \Rightarrow \quad n = \dfrac{PV}{RT}$

   $$n \rightarrow \text{number of mole of gas} = \frac{1.6 \times 10^6 \times .0083}{8.314 \times 300}$$

   $$\approx 4$$

   Heat has been supplied at constant volume

   $\therefore \quad \Delta Q_V = nC_v \, \Delta T$ When $C_v = C_P - R$

   $$= \frac{5}{2}R - R = \frac{3}{2}R$$

   $$2.49 \times 10^6 = 4 \times \frac{3}{2} \times 8.3\,(T'-300) \quad \Delta T = T' - 300$$

   $\Rightarrow T' = 800$ K $T' \rightarrow$ final temperature

   If P' be the final pressure then

   $P'V = nRT'$

   $$P' = \frac{nRT'}{V} = \frac{4 \times 8.3 \times 800}{.0083}$$

   $$= 4 \times 8 \times 10^5 = 3.2 \times 10^6 \text{ N/m}^2$$

4. V_1 = 30 litre = 30 × 10^3 cm^3 = 3 × 10^{-2} m^3

P_1 = 15 × 1.013 × 10^5 N/m^2

T_1 = 300 K

$$\mu_1 = \frac{P_1V_1}{RT_1} = 18.3$$

$$\mu_2 = \frac{P_2V_2}{RT_2}$$

P_2 = 11 × 1.013 × 10^5 N/m^2

V_2 = 3 × 10^{-2} m^3

T_2 = 290 k

μ_2 = 13.9

= 18.3 – 13.9 = 4.4

Mans of gas taken out of cylinder = 4.4 × 32 g

= 140.8 g

= 0.140 kg.

5. $$\text{Vrms} = \left[\frac{3\,PV}{M}\right]^{1/2} = \left[\frac{3\,RT}{M}\right]^{1/2}$$

Let r.m.s speed of oxygen is $(\text{Vrms})_1$ and of helium is $(\text{Vrms})_2$ is equal at temperature T_1 and T_2 respectively.

$$\frac{(\text{Vrms})_1}{(\text{Vrms})_2} = \sqrt{\frac{M_2T_1}{M_1T_2}}$$

$$\left[\frac{4T_1}{32 \times 263}\right]^{1/2} = 1$$

$$T_1 = \frac{32 \times 263}{4} = 2104 \text{ k}$$

6. $$P = \frac{1}{3}Pv^2$$

$$Vrms = \sqrt{\left(v^2\right)} = \sqrt{\frac{3P}{\rho}}$$

$$Vrms = \sqrt{\frac{3 \times 1 \times 10^5}{1.98}} = 389\, m/sec$$

Vrms $\propto \sqrt{T}$

$$\frac{(Vrms)_{20}}{(Vrms)_0} = \sqrt{\frac{273 + 30}{273}} = \sqrt{\frac{303}{273}} = 1.053$$

$(Vrms)_{30}$ = 389 × 1.053 = 410 m/s

7. Number of mole in 0.014 kg of Nitrogen.

$$n = \frac{0.014 \times 10^3}{28} = \frac{1}{2}\text{mole}$$

$$C_V = \frac{5}{2}R = \frac{5}{2} \times 2 = 5 \quad \text{cal / mole k}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \qquad T_2 = 4T_1$$

$\Delta T = T_2 - T_1 = 4T_1 - T_1 = 3T_1$

$= 3 \times 300 = 900$ K

$$\Delta Q = n\, c_v\, \Delta T = \frac{1}{2} \times 5 \times 900 = 2250 \text{ cal}$$

UNIT X

OSCILLATIONS AND WAVES

- **Periodic Motion :** It is that motion which is identically repeated after a fixed interval of time. The fixed interval of time after which the motion is repeated is called period of motion.

  For example, the revolution of earth around the sun

- **Oscillatory Motion or Vibratory Motion :** It is that motion in which a body moves to and for or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.

- **Simple Harmonic Motion :** It is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean (*i.e.,* equilibrium) position and the magnitude of force acting on the particle at any instant is directly proportional to the displacement of the particle from the mean (*i.e.,* equilibrium) position at that instant *i.e.* $F = -k\,y$.

  where k is known as force constant. Here, –ve sign shows that the restoring force (F) is always directed towards the mean position.

- **Displacement in S.H.M :** The displacement of a particle executing S.H.M at an instant is defined as the distance of the particle from the mean position at that instant. It can be given by the relation

  $$y = a \sin \omega t \text{ or } y = a \cos \omega t$$

  The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M.

- **Velocity in S.H.M :** It is defined as the time rate of change of the displacement of the particle at the given instant. Velocity in S.H.M. is given by

$$V = \frac{dy}{dt} = \frac{d}{dt}\left(a \sin \omega t\right) = a\omega \cos \omega t = a\omega\sqrt{1 - \sin^2 \omega t} = a\omega\sqrt{1 - y^2/a^2}$$

$= \omega\sqrt{a^2 - y^2}$

- **Acceleration in S.H.M :** It is defined as the time rate of change of the velocity of the particle at the given instant, *i.e.,*

$$a = \frac{dv}{dt}$$

$$\frac{d}{dt} a\omega \cos \omega t = -\omega^2 a \sin \omega t = -\omega^2 y$$

- **Time period in S.H.M** is given by

$$T = 2\Pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}, \qquad \text{or} \qquad T = 2\Pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

- **Frequency of vibration in S.H.M.,**

$$\gamma = \frac{1}{T} = \frac{1}{2\Pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}, \qquad \text{or} \qquad v = \frac{1}{2\Pi}\sqrt{\frac{\text{spring factor}}{\text{inertia factor}}}$$

- **Total energy in S.H.M**

$$= \text{P.E.} + \text{K.E.} = \frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\omega^2\left(a^2 - y^2\right) = \frac{1}{2}m\omega^2 a^2 = \text{a constant.}$$

- **Expression for time period**

  (i) in case of simple pendulum $T = 2\pi\sqrt{\ell/g}$

  (iv) Oscillations of a loaded spring $T = 2\pi\sqrt{m/k}$

  where, m is the mass of body attached at the free end of spring and K is the force constant of spring.

- **Spring constant (K) of a spring :** It is defined as the force per unit extension or compression of the spring.

  (i) The spring constant of the combination of two springs in series is

$$K = \frac{k_1 k_2}{k_1 + k_2}$$

(ii) The spring constant of the combination of two springs in parallel's

$$K = k_1 + k_2$$

- **Undamped oscillations :** When a simple harmonic system oscillates with a constant amplitude (which does not change) with time, its oscillations are called undamped oscillations.

- **Damped oscillations :** When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called damped oscillations.

- **Free, forced and resonant oscillations**

  (a) **Free oscillations :** When a system oscillates with its own natural frequency without the help of an external periodic force, its oscillations are called free oscillations.

  (b) **Forced oscillations :** When a system oscillates with the help of an external periodic force of frequency, other than its own natural frequency, its oscillations are called forced oscillations.

  (c) **Resonant oscillations :** When a body oscillates with its own natural frequency, with the help of an external periodic force whose frequency is the same as that of the natural frequency of the oscillating body, then the oscillations of the body are called resonant oscillations.

- **A Wave Motion** is a form of disturbance which travels through a medium on account of repeated periodic vibrations of the particles of the medium about their mean position, the motion being handed on from one particle to the adjoining particle.

  A material medium is a must for propagation of waves. It should possess the properties of inertia, and elasticity. The two types of wave motion are :

  (i) **Transverse wave** motion that travels in the form of crests and troughs.

  (ii) **Longitudinal wave** motion that travels in the form of compressions and rarefactions.

- **Speed of longitudinal waves** in a long solid rod is $\nu = \sqrt{y/\rho}$

  where, *Y* is Young's modulus of the material of solid rod and P is density of the material. The speed of longitudinal waves in a *liquid* is given by $\nu = \sqrt{k/\rho}$, where *K* is *bulk modulus of elasticity* of the liquid.

- The expression for speed of longitudinal waves in a gas, as suggested by **Newton and modified late by Laplace is**

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{where} \quad \gamma = C_p / C_v$$

  P is pressure exerted by the gas.

- **The speed of transverse waves** over a string is given by $\nu = \sqrt{T/m}$

  where, *T* is *tension* in the string and *m* is mass of unit length of the string.

- **Equation of plane progressive waves** travelling with a velocity ν along positive direction of X-axis is

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

  where, λ is wavelength of the wave, a is amplitude of particle, and x is the distance from the origin.

- **Superposition principle** enables us to find the resultant of any number of waves meeting at a point. If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...\vec{y}_n$ are displacements at a point due to *n* waves, the resultant displacement $\vec{y}$ at that point is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + ...\vec{y}_n$

- **On a string, transverse stationary waves** are formed due to superimposition of direct and the reflected transverse waves.

- The wavelength of n^{th} mode of vibration of a stretched string is

$$\lambda_n = \frac{2L}{n} \text{ and its frequency, } v_n = n\, v_1$$

  This note is called n^{th} harmonic or (n – 1)th overtone.

- **Nodes are the points,** where amplitude of vibration is zero, In the n^{th} mode of vibration, there are (n + 1) nodes located at distances (from one end)

$$x = 0, \frac{L}{n}, \frac{2L}{n} .....L$$

Antinodes are the points, where amplitude of vibration is *maximum*. In the n^{th} mode of vibration, there are n antinodes, located at distances (from one end)

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots\ldots\ldots \frac{(2n-1)}{2n}L$$

- *In* **an organ pipe closed at one end,** longitudinal stationary waves are formed.

  frequency of n^{th} mode of vibration, $v_n = (2n - 1)v_1$

- **In an organ pipe open at both ends,** antinodes are formed at the two ends, separated by a node in the middle in the first normal mode of vibration and so on. The fundamental frequency in this case is twice the fundamental frequency in a closed organ pipe of same length.

  In an open organ pipe, all harmonics are present, whereas in a closed organ pipe, even harmonics are missing.

- **Beats :** Beats is the phenomenon of regular variation in the intensity of sound with time when two sources of nearly equal frequencies are sounded together.

- If v_1 and v_2 are the frequencies of two sources producing beats, then time interval between two successive maxima $= \frac{1}{v_1 - v_2}$

  time interval between two successive minima $= \frac{1}{v_1 - v_2}$

  Beat frequency = $v_1 - v_2$

$$F_1 = F_2 \frac{A_1}{A_2} = F_2 \left(\frac{\pi r_1^2}{\pi r_2^2}\right)$$

- **Doppler's Effect :** Whenever there is a relative motion between a source of sound and listener, the apparent frequency of sound heard is different from the actual frequency of sound emitted by the source.

  If γ is actual frequency of sound emitted and v', the apparent frequency, then $v' = \frac{(v + v_L)}{(v - v_s)} v$

  where, v is velocity of sound in air v_s is velocity of source (*S*) and v_L is velocity of listener (*L*), both moving along *SL.* Note that velocity along *SL* is taken positive and velocity along *LS* is taken negative.

ONE MARK QUESTIONS

1. How is the time period effected, if the amptitude of a simple pendulum is increased?

2. Define force constant of a spring.

3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?

4. How is the frequency of oscillation related with the frequency of change in the of K.E and P.E of the body in S.H.M.?

5. What is the frequency of total energy of a particle in S.H.M.?

6. How is the length of seconds pendulum related with acceleration due gravity of any planet?

7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.

8. How is the time period of the pendulum effected when pendulum is taken to hills or in mines?

9. A transverse wave travels along x-axis. The particles of the medium must move in which direction?

10. Define angular frequency. Give its S.I. unit.

11. Sound waves from a point source are propagating in all directions. What will be the ratio of amplitudes at distances of x meter and y meter from the source?

12. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

13. What is the time period for the function $f(t) = \sin \omega t + \cos \omega t$ may represent the simple harmonic motion?

14. When is the swinging of simple pendulum considered approximately SHM?

15. Can the motion of an artificial satellite around the earth be taken as SHM?

16. What is the phase relationship between displacement, velocity and accelection in SHM?

17. What forces keep the simple pendulum in motion?

18. How will the time period of a simple pendulum change when its length is doubled?

19. What is a harmonic wave function?

20. If the motion of revolving particle is periodic in nature, give the nature of motion or projection of the revolving particle along the diameter.

21. In a forced oscillation of a particle, the amplitude is maximum for a frequency w_1 of the force, while the energy is maximum for a frequency w_2 of the force. What is the relation between w_1 and w_2?

22. Which property of the medium are responsible for propagation of waves through it?

23. What is the nature of the thermal change in air, when a sound wave propagates through it?

24. Why does sound travel faster in iron than in water or air?

25. When will the motion of a simple pendulum be simple harmonic?

26. A simple harmonic motion of acceleration 'a' and displacement 'x' is represented by $a + 4\Pi^2 x = 0$. What is the time period of S.H.M?

27. What is the main difference between forced oscillations and resonance?

28. Define amplitude of S.H.M.

29. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

30. Why the pitch of an organ pipe on a hot summer day is higher?

31. Under what conditions does a sudden phase reversal of waves on reflection takes place?

32. The speed of sound does not depend upon its frequency. Give an example in support of this statement.

33. If an explosion takes place at the bottom of lake or sea, will the shock waves in water be longitudinal or transverse?

34. Frequency is the most fundamental property of wave, why?

35. How do wave velocity and particle velocity differ from each other?

36. If any liquid of density higher than the density of water is used in a resonance tube, how will the frequency change?

37. Under what condition, the Doppler effect will not be observed, if the source of sound moves towards the listener?

38. What physical change occurs when a source of sound moves and the listener is stationary?

39. What physical change occurs when a source of sound is stationary and the listener moves?

40. If two sound waves of frequencies 480 H_z and 536 H_z superpose, will they produce beats? Would you hear the beats?

41. Define non dispessive medium.

2 MARKS QUESTIONS

1. Which of the following condition is not sufficient for simple harmonic motion and why?

   (i) acceleration and displacement

   (ii) restoring force and displacement

2. The formula for time period T for a loaded spring, $T = 2\Pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

   Does-the time period depend on length of the spring?

3. Water in a U-tube executes S.H.M. Will the time period for mercury filled up to the same height in the tube be lesser of greater than that in case of water?

4. There are two springs, one delicate and another hard or stout one. For which spring, the frequency of the oscillator will be more?

5. Time period of a particle in S.H.M depends on the force constant K and mass m of the particle $\left(T = \frac{1}{2\Pi}\sqrt{\frac{m}{k}}\right)$. A simple pendulum for small angular

displacement executes S.H.M approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

6. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is falling freely?

7. Why can the transverse waves not be produced in air?

8. The velocity of sound in a tube containing air at 27°C and pressure of 76 cm of Hg is 330 ms^{-1}. What will be its velocity, when pressure is increased to 152 cm of mercury and temperature is kept constant?

9. Even after the breakup of one prong of tunning fork it produces a round of same frequency, then what is the use of having a tunning fork with two prongs?

10. Why is the sonometer box hollow and provided with holes?

11. The displacement of a particle in S.H.M may be given by

$$y = a \sin(\omega t + \phi)$$

show that if the time t is increased by $2\Pi/\omega$, the value of y remains the same.

12. What do you mean by the independent behaviour of waves?

13. Define wave number and angular wave number and give their S.I. units.

14. Why does the sound travel faster in humid air?

15. Use the formula $v = \sqrt{\frac{\gamma p}{\rho}}$ to explain, why the speed of sound in air

    (a) is independent of pressure

    (b) increase with temperature

16. Differentiate between closed pipe and open pipe at both ends of same length for frequency of fundamental note and harmonics.

17. Bats can ascertain distances, directions, nature and size of the obstacle without any eyes, explain how?

18. In a sound wave, a displacement node is a pressure antinode and vice-versa. Explain, why.

19. How does the frequency of a tuning fork change, when the temperature is increased?

20. Explain, why can we not hear an echo in a small room?

21. What do you mean by reverberation? What is reverbevation time?

THREE MARKS QUESTIONS

1. Show that for a particle in linear simple harmonic motion, the acceleration is directly proportional to its displacement of the given instant.

2. Show that for a particle in linear simple harmonic motion, the average kinetic energy over a period of oscillation, equals the average potential energy over the same period.

3. Deduce an expression for the velocity of a particle executing S.H.M when is the particle velocity (i) Maximum (ii) minimum?

4. Draw (a) displacement time graph of a particle executing SHM with phase angle ϕ equal to zero (b) velocity time graph and (c) acceleration time graph of the particle.

5. Show that a linear combination of sine and cosine function like $x(t) = a \sin\omega t + b \cos\omega t$ represents a simple harmonic Also, determine its amplitude and phase constant.

6. Show that in a S.H.M the phase difference between displacement and velocity is $\Pi/2$, and between displacement and acceleration is Π.

7. Derive an expression for the time period of the horizontal oscillahons of a massless loaded spring.

8. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period.

9. Distinguish with an illustration among free, forced and resonant oscillations.

10. In reference to a wave motion, define the terms

    (i) amplitude (ii) time period

    (iii) frequency (iv) angular frequency

    (v) wave length and wave number.

11. What do you understand by phase of a wave? How does the phase change with time and position.

LONG ANSWER QUESTIONS

1. Derive expressions for the kinetic and potential energies of a simple harmonic oscillator. Hence show that the total energy is conserved in S.H.M. in which positions of the oscillator, is the energy wholly kinetic or wholly potential?
2. Find the total energy of the particle executing S.H.M. and show graphically the variation of potential energy and kinetic energy with time in S.H.M. What is the frequency of these energies with respect to the frequency of the particle executing S.H.M.?
3. Discuss the Newton's formula for velocity of sound in air. What correction was applied to it by Laplace and why?
4. What are standing waves? Desire and expression for the standing waves. Also define the terms node and antinode and obtain their positions.
5. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4,
6. Give the differences between progressive and stationary waves.
7. Give a qualitative discussion of the modes of vibrations of a stretched string fixed at both the ends.
8. Give a qualitative discussion of the different modes of vibration of an open organ pipe.
9. Describe the various modes of vibrations of a closed organ pipe.
10. What are beats? How are they produced? Briefly discuss one application for this phenomenon.
11. State Doppler's effect in sound obtain an expression for apparent frequency when source and listener move away from each other

NUMERICALS

1. The time period of a body executing S.H.M is 1s. After how much time will

its displacement be $\frac{1}{\sqrt{2}}$ of its amplitude.

2. A point describes SHM in a line 6 cm long. Its velocity, when passing through the centre of line is 18 cm s^{-1}. Find the time period.

3. Find the period of vibrating particle (SHM), which has accelesation of 45 cm s^{-2}, when displacement from mean position is 5 cm

4. A 40 gm mass produces on extension of 4 cm in a vertical spring. A mass of 200 gm is suspended at its bottom and left pulling down. Calculate the frequency of its vibration.

5. The acceleration due to gravity on the surface of the moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the moon, if its time period on the earth is 3.5 s? [g = 9.8 ms^{-2}]

6. Calculate the energy possessed by stone of mass 200 g executing S.H.M of amplitude 1 cm and time period 4s.

7. A particle executes S.H.M of amplitude 25 cm and time period 3s. What is the minimum time required for the particle to move between two points 12.5 cm on eitherside of the mean position?

8. The vertical motion of a huge piston in a machine is approximately S.H.M with a frequency of 0.5 s^{-1}. A block of 10kg is placed on the piston. What is the maximum amplitude of the piston's S.H.M. for the block and piston to remain together?

9. At what temperature will the speed of sound be double its value at 273°K?

10. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body?

11. A steel wire 80 cm long has a mess 8 mg. If the wire is under tension of 400 N, what is the speed of transverse waves in the wire?

12. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation, Estimate the values of (a) the spring constant and (b) the damping constant 'b' for the spring and shock absorber system of one wheel assuming that each wheel supports 750 kg.

13. A string of mass 2.5 kg is under a tension of 200N. The length of the stretched string is 20m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

14. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in day air at 20°C (is equal to 343 ms^{-1})?

15. The equation of a plane progressive wave is given by the equation $y = 10 \sin 2\pi (t - 0.005x)$ where y and x are in cm and t in seconds. Calculate the amplitude, frequency, wave length and velocity of the wave.

16. Find the frequency of note emitted (fundamental note) by a string 1m long and stretched by a load of 20 kg, if this string weighs 4.9 g. Given, $g = 980 \text{ cms}^{-2}$

17. A pipe 20 cm long is closed at one end, which harmonic mode of the pipe is resonantly excited by a 430 H_z source? Will this same source can be in resonance with the pipe, if both ends are open? Speed of sound = 340 ms^{-1}

18. One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tunning folk of frequency 256 H_z. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive x direction. The amplitude of wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

19. The transverse displacement of a string (clamped at its two ends) is given by

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos(120\pi t)$$

where x,y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2}kg. Answer the following.

(a) Does the function represent a travelling or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength frequency and speed of propagation of each wave?

(c) Determine the tension in the string.

20. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency 45 H_z. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-}. What is (a) the speed of transverse wave on the string and (b) the tension in the string?

21. A steel sod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod as given to be 2.53 kH_z. What is the speed of sound in steel?

ANSWERS OF ONE MARK QUESTIONS

1. No effect on time period when amptitude of pendulum is increased or decreased.

2. The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.

3. Not at the mid point, between mean and extreme position. it will be at

$$x = a/\sqrt{2}.$$

4. P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of S.H.M

5. The frequency of total energy of particle is S.H.M is zero because it remains constant.

6. Length of the seconds pendulum proportional to (acceleration due to gravity)

7. Increased

8. As $T \propto \frac{1}{\sqrt{g}}$, T will increase.

9. In the y–z plane or in plane perpendicular to x-axis.

10. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.

    $\omega = 2\pi$ v, S.I. unit is rad s^{-1}

11. Intensity = amplitude^2 $\propto \frac{1}{(\text{distance})^2}$

$\therefore$ required ratio = y/x

12. No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

13. $T = 2\pi/\omega$

14. Swinging through small angles.

15. No, it is a circular and periodic motion but not SHM.

16. In SHM, –The velocity leads the displacement by a phase $\Pi/2$ radians and acceleration leads the velocity by a phase $\Pi/2$ radians.

17. The component of weight (mg $\sin\theta$)

18. $\sqrt{2}$ times, as $T \propto \sqrt{\ell}$

19. A harmonic wave function is a periodic function whose functional form is sine or cosine.

20. S.H.M

21. Both amplitude and energy of the particle can be maximum only in the case of resonance, for resonance to occur $\omega_1 = \omega_2$.

22. Properties of elasticity and inertia.

23. When the sound wave travel through air adiabatic changes take place in the medium.

24. Sound travel faster in iron or solids because iron or solid is highly elastic as compared to water (liquids) or air (gases).

25. When the displacement of bob from the mean position is so small that $\sin\theta \approx \theta$.

26. $a = -4\pi^2 x = -\omega^2 x \Rightarrow \omega = 2\pi$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1\,s$$

27. The frequency of external periodic force is different from the natural frequency of the oscillator in case of forced oscillation but in resonance two frequencies are equal.

28. The maximum displacement of oscillating particle on either side of its mean position is called its amplitude.

29. A periodic motion repeats after a definite time interval T. So, $y(t) = y(t + T) = y(t + 2T)$ etc.

30. On a hot day, the velocity of sound will be more since (frequency proportional to velocity) the frequency of sound increases and hence its pitch increases.

31. On reflection from a denser medium, a wave suffers a sudden phase reversal.

32. If sounds are produced by different musical instruments simultaneously, then all these sounds are heard at the same time.

33. Explosion at the bottom of lake or sea create enormous increase in pressure of medium (water). A shock wave is thus a longitudinal wave travelling at a speed which is greater than that of ordinary wave.

34. When a wave passes through different media, velocity and wavelength change but frequency does not change.

35. Wave velocity is constant for a given medium and is given by $v = \nu\lambda$ But particle velocity changes harmonically with time and it is maximum at mean position and zero at extreme position.

36. The frequency of vibration depends on the length of the air column and not on reflecting media, hence frequency does not change.

37. Doppler effect will not be observed, if the source of sound moves towards the listener with a velocity greater than the velocity of sound. Same is also true if listener moves with velocity greater than the velocity of sound towards the source of sound.

38. Wave length of sound changes.

39. The number of sound waves received by the listener changes.

40. Yes, the sound waves will produce 56 beats every second. But due to persistence of hearing, we would not be able to hear these beats.

41. A medium in which speed of wave motion is independent of frequency of wave is called non-dispersive medium. For sound, air is non dispersive medium.

ANSWERS OF TWO MARKS QUESTIONS

1. Condition (i) is not sufficient, because direction of acceleration is not mentioned. In SHM, the acceleration is always in a direction opposite to that of the diplacement.

2. Although length of the spring does not appear in the expression for the time period, yet the time period depends on the length of the spring. It is because, force constant of the spring depends on the length of the spring.

3. The time period of the liquid in a U-tube executing S.H.M. does not depend upon density of the liquid, therefore time period will be same, when the mercury is filled up to the same height in place of water in the U-tube.

4. We have, $v = \frac{1}{2\Pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

   So, when a hard spring is loaded with a mass m. The extension l will be lesser w.r.t. delicate one. So frequency of the oscillation of the hard spring will be more and if time period is asked it will be lesser.

5. Restoring force in case of simple pendulum is given by

$$F = \frac{mg}{l}y \Rightarrow K = mg / \ell$$

   So force constant itself proportional to m as the value of k is substituted in the formula, m is cancelled out.

6. The pendulum is in a state of weight less ness i.e. g = 0. The frequency of pendulum

$$v = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = 0$$

7. For air, modulus of rigidity is zero or it does not possess property of cohession. Therefore transverse waves can not be produced.

8. At a given temperature, the velocity of sound is independent of pressure, so velocity of sound in tube will remain $330 ms^{-1}$.

9. Two prongs of a tunning fork set each other in resonant vitorations and help to maintain the vibrations for a longer time.

10. When the stem of the a tunning fork gently pressed against the top of sonometer box, the air enclosed in box also vibrates and increases the intensity of sound. The holes bring the inside air incontact with the outside air and check the effect of elastic fatigue.

11. The displacement at any time t is

$y = a \sin(wt + \phi)$

$\therefore$ displacement at any time $(t + 2\Pi/w)$ will be

$$y = a \sin [w(t + 2\Pi/w) + \phi] = [\sin\{wt + \phi) + 2\Pi\}]$$

$$\Rightarrow \quad y = a \sin(wt + \phi) \; [\because \sin(2\Pi + \phi) = \sin\phi]$$

Hence, the displacement at time t and $(t + 2\Pi/w)$ are same.

12. When a number of waves travel through the same region at the same time, each wave travels independently as if all other waves were absent. This characteristic of wave is known as independent behaviour of waves. For example we can distinguish different sounds in a full orchestra.

13. Wave number is the number of waves present in a unit distance of medium. $\left(\bar{\nu} = 1/\lambda\right)$. S.I. unit is m^{-1}.

Angular wave number or propagation constant is $2\Pi/\lambda$. It represents phase change per unit path difference and denoted by $k = 2\Pi/\lambda$. S.I. unit of k is rad m^{-1}.

14. Because the density of water vapour is less than that of the dry air hence density of air decreases with the increase of water vapours or humidity and velocity of sound inversely proportional to square root of density.

15. Given, $v = \sqrt{\dfrac{\gamma P}{\rho}}$

(a) Let V be the volume of 1 mole of air, then

$$\rho = \frac{M}{V} \quad \text{or} \quad V = \frac{M}{\rho}$$

for 1 mole of air PV = RT

$$\therefore \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M}$$

$$\Rightarrow \quad v = \sqrt{\frac{\gamma\, RT}{M}} \quad \text{(i)}$$

So at constant temperature v

is constant as γ, R and M are constant

(b) From equation (i) we know that $v \propto \sqrt{T}$, so with the increase in temperature velocity of sound increases.

16. (i) In a pipe open at both ends, the frequency of fundamental note produced is twice as that produced by a closed pipe of same length.

(ii) An open pipe produces all the harmonics, while in a closed pipe, the even harnonics are absent.

17. Bats emit ultrasonic waves of very small wavelength (high frequencies) and so high speed. The reflected waves from an obstacle in their path give them idea about the distance, direction, nature and size of the obstacle.

18. At the point, where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, pressure difference is maximum i.e. at the same point it is a pressure antinode. On the other hand, at the mid point of compression or a rarefaction, the displacement variation is maximum i.e. such a point is pressure node, as pressure variation is minimum at such point.

19. As the temperature increases, the length of the prong of the tunning fork increases. This increases the wavelength of the stationary waves set up in the tuning fork. As frequency, $\nu \propto \sqrt{\frac{1}{\lambda}}$, so frequency of the tunning for**k** decreases.

20. For an echo of a simple sound to be heard, the minimum distance between the speaker and the walls should be 17m, so in any room having length less than 17 m, our ears can not distinguish between sound received directly and sound received after reflection.

21. The phenomenon of persistence or prolongation of sound after the source has stopped emitting sound is called reverberation. The time for which the sound persists until it becomes inaudible is called the reverberation time.

SOLUTION / HINTS OF NUMERICALS

1. Soln : $y = r \sin wt = r \sin \frac{2\Pi}{T} t$

   Here $y = \frac{1}{3} r$ and $T = 1s$

   $\therefore \frac{1}{\sqrt{2}} r = r \sin \frac{2\Pi}{T} t \Rightarrow 2\Pi t = \Pi/4$

   $\Rightarrow \quad t = \frac{1}{8} s$

2. Soln : Here amplitude r = 6/2 = 3cm

   When $\quad y = 0, \nu = 18 \text{ cms}^{-1}$

   Now $\quad \nu = w\sqrt{r^2 - y^2} \Rightarrow 18 = w\sqrt{3^2 - 0}$

   or $\quad 3w = 18 \Rightarrow w = 6 \text{ rad s}^{-1}$

   We know $T = \frac{2\Pi}{w} = \frac{2\Pi}{6} = 1.047 \text{ s}$

3. Soln : Here y = 5 cm and acceleration $a = 45 \text{ cms}^{-2}$

   We know $\quad a = w^2 y$

   $\therefore \quad 45 = w^2 \times 5$ or $w = 3 \text{ rad s}^{-1}$

   And $\quad T = \frac{2\pi}{w} = \frac{2\pi}{3} = 2.095 \text{ s}$

4. Soln :

   Here mg' = 40 g = 40 × 980 dyne ; l = 4 cm.

   say k is the force constant of spring, then

   mg = kl or $k = mg/\ell$

$$k = \frac{40 \times 980}{4} = 9800 \text{ dyne cm}^{-1}$$

when the spring is loaded with mass m = 200 g

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9800}{200}}$$

$= 1.113 \text{ s}^{-1}$

5. Soln : Here on earth, T = 3.5 s ; g = 9.8 ms^{-2}

for simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$3.5 = 2\pi\sqrt{\frac{\ell}{9.8}} \qquad \text{(i)}$$

on moon, g´ = 1.7 ms^{-2} and if T´ is time period

then $T' = 2\pi\sqrt{\frac{\ell}{1.7}}$ (ii)

Dividing eq(ii) by eq.(i), we get

$$\frac{T'}{3.5} = \sqrt{\frac{9.8}{1.7}} \quad \text{or} \quad T' = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4 \text{ s}$$

6. Solu. : $E = 2\Pi^2 m r^2 \nu^2 = \frac{2\Pi^2 mr^2}{T^2}$

m = 0.2 kg, r = 0.01 m, T = 4s

$$\therefore \quad E = \frac{2\Pi^2 \times 0.2 \times (0.1)^2}{4^2} = 0.00246 \text{ J}$$

7. Soln :

Given, r = 25 cm; T = 3s ; y = 12.5 cm

The displacement $y = r \sin\frac{2\Pi}{T} t$

$$12.5 = 25 \sin \frac{2\pi}{3} t \quad \text{or} \quad \frac{2\pi}{3} t = \frac{\pi}{6} \quad \text{or} \quad t = 0.25 \text{ s.}$$

The minimum time taken by the particle 2t = 0.5 s

8. Soln : Given, $\nu = 0.5 \text{ s}^{-1}$ g = 9.8 ms^{-1}

$$a = w^2y = (2\Pi\nu)^2y = 4\,\Pi^2\nu^2y$$

a_{max} at the extreme position i.e. r = y

$a_{max} = 4\Pi^2\nu^2r$ and a_{max} = g to remain in contact.

$$\text{or } r = \frac{g}{4\Pi^2\nu^2} = \frac{9.8}{4\Pi^2 \times (0.5)^2} = 0.993 \text{ m}$$

9. Say ν_1 in the velocity of sound at T_1 = 273°k and $\nu_2 = 2\nu_1$ at temperature T_2

$$\text{Now } \frac{v_2}{v_2} = \sqrt{\frac{T_2}{T_1}} \quad \therefore \frac{2\nu_1}{\nu_1} = \sqrt{\frac{T_2}{273}}$$

or T_2 = 4 × 273 = 1092°K

10. Here m = 50 kg, l = 0.2 m

$$\text{we know mg = kl or } k = \frac{mg}{\ell} = \frac{50 \times 9.8}{0.2} = 2450 \text{ Nm}^{-1}$$

T = 0.60 s and M is the mass of the body, then using

$$T = 2\Pi\sqrt{\frac{M}{k}} \Rightarrow M = \frac{2450 \times (0.60)^2}{4\,\Pi^2} = 22.34 \text{ kg}$$

Weight of body Mg = 22.34 × 9.8 = 218.93 N.

11. Soln : Speed of wave (transverse) in stretched string

$$v = \sqrt{\frac{T}{m}} \text{ T is Tension, m is mass per unit length}$$

$$\nu = \sqrt{\frac{400}{\frac{8 \times 10^{-3}}{0.8}}} = 200 \text{ ms}^{-1}$$

12. Soln :

(a) Here m = 3000 kg, x = 0.15m

If k is the spring constant of each spring, then spring constant for four spring connected in parallel will be 4k.

$\therefore$ 4k x = mg or $k = \frac{mg}{x}$

$$k = \frac{3000 \times 10}{4 \times 0.15} = 5 \times 10^4 \text{Nm}^{-1}$$

(b) As $A' = A\ e^{-bt/2m}$ $\therefore \frac{A}{2} = A\,e^{-bt/2m}$ or $2 = e^{bt/2m}$

or $\log_e 2 = \frac{bt}{2m} \log_e e = \frac{bt}{2m}$ or $b = \frac{2m \log_e 2}{t}$

But $t = 2\Pi\sqrt{\frac{m}{4k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{3000}{4 \times 5 \times 10^4}} = \frac{40}{70}\sqrt{\frac{3}{2}}s$

Hence $b = \dfrac{2 \times 750 \times 0.693}{\frac{40}{70}\sqrt{\frac{3}{2}}} = 1350.4 \text{ kg s}^{-1}$

13. Soln : Given T = 200N, length of sting l = 20m

total mass of the string = 2.5 kg

$\therefore$ mass per unit length of the string

$$m = \frac{2.5}{20} = 0.125 \text{ kg m}^{-1}$$

Now $\nu = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{0.125}} = 40 \text{ ms}^{-1}$

Hence time taken by the transverse wave to reach other end

$$t = \frac{\ell}{v} = \frac{20}{40} = 0.5 \text{ s}$$

14. Soln : Given speed of sound in air, $\nu = 343 \text{ ms}^{-1}$

length of wire, l = 12.0 m, total mass of wire M = 2.10 kg

$$m = \frac{M}{l} = \frac{2.10}{12.0} = 0.175 \text{ kgm}^{-1}$$

Now

$$\nu = \sqrt{\frac{T}{m}} \Rightarrow T = \nu^2 m = (343)^2 \times 0.175 = 20,588.6N = 2.06 \times 10^4 N$$

15. Solu, here $y = 10 \sin 2\Pi (t - 0.005x)$

$$y = 10 \sin \frac{2\pi}{200}(200\,t - x) \qquad \text{(i)}$$

The equation of a travelling wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \text{(ii)}$$

Comparing the equation (i) and (ii), we have

$$\alpha = 10 \text{ cm}, \lambda = 200 \text{ cm and } \nu = 200 \text{ ms}^{-1}$$

NOW $\quad \nu = \frac{\nu}{\lambda} = \frac{200}{200} = 1 \text{ H}_z$

16. Solu, L = 100 cm T = 20 kg = 20 × 1000 × 980 dyne

$$m = \frac{4.9}{100} = 0.049 \text{ g cm}^{-1}$$

Now the frequency of fundamental note produced,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\nu = \frac{1}{2 \times 100}\sqrt{\frac{20 \times 1000 \times 980}{0.049}} = 100\text{H}_z$$

17. Solu : The frequency of n^{th} mode of vibration of a pipe closed at one end is given by

$$\nu_n = \frac{(2n-1)\nu}{4L}$$

river $\quad \nu = 340 \text{ ms}^{-1}$, L = 20 cm = 0.2 m ; $\nu_n = 430 \text{ H}_z$

$$\therefore \quad 430 = \frac{(2n-1)\times 340}{4 \times 0.2} \Rightarrow n = 1$$

Therefore, first mode of vibration of the pipe is excited, for open pipe since n must be an integer, the same source can not be in resonance with the pipe with both ends open

18. Solu : The wave is travelling along x-axis and its equation is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) = a \sin\left(\frac{2\pi}{\lambda} vt - \frac{2\pi}{\lambda} x\right)$$

$$y = a \sin (2\pi\upsilon t - kx) = a \sin(\omega t - kx) \qquad \text{(i)}$$

To determine a, ω and k :

$a = 5.0 \text{ cm} = 0.05 \text{ m}, \upsilon = 256 \text{ H}_z$

$w = 2\pi\nu = 2\pi \times 256 = 1.61 \times 10^3 \text{ s}^{-1}$

$m = 8.0 \times 10^{-3} \text{ kg m}^{-1}, T = 90 \times 9.8 \text{ N}$

$$\nu = \sqrt{\frac{T}{m}} = \sqrt{\frac{90 \times 9.8}{8.0 \times 10^{-3}}} = 332 \text{ ms}^{-1}$$

$$\therefore \quad \lambda = \frac{v}{\upsilon} = \frac{332}{256} = 1.297 \text{ m}$$

and $\quad k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.297} = 4.84 \text{ m}^{-1}$

substituting for a, w and k in equ (i) we have

$y = 0.05 \sin (1.61 \times 10^3 \, t - 4.84 \, x)$

19. $$y(x,t) = 0.06 \quad \sin\frac{2\pi}{3}x \quad \cos 120\,\pi t \qquad \text{(i)}$$

(a) The displacement which involves harmonic functions of x and t separately represents a stationary wave and the displacement, which is harmonic function of the form (vt ± x), represents a travelling wave. Hence, the equation given above represents a stationary wave.

(b) When a wave pulse $y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$ travelling along x-axis is superimposed by the reflected pulse.

$y_2 = -a \sin \frac{2\Pi}{\lambda}(vt + x)$ from the other end, a stationery wave is formed and is given by

$$y = y_1 + y_2 = -2a \sin \frac{2\pi}{\lambda} \times \cos \frac{2\pi}{\lambda} vt \quad \text{(ii)}$$

comparing the eqs (i) and (ii) we have

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \quad \text{or} \quad \lambda = 3\,\text{m}$$

and $\frac{2\pi}{\lambda} v = 120\pi$ or $v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

Now frequency $\gamma = \frac{v}{\lambda} = \frac{180}{3} = 60 \text{ H}_z$

(c) Velocity of transverse wave in a string is given by

$$v = \sqrt{\frac{T}{m}}$$

Here $m = \frac{3 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kgm}^{-1}$

Also $v = 180 \text{ ms}^{-1}$

$\therefore$ $T = v^2 m = (180)^2 \times 2 \times 10^{-2} = 648\text{N}$

20. Solution : frequency of fundamental mode, $\nu = 45\text{H}_z$

Mass of wire $M = 3.5 \times 10^{-2}$ kg; mass per unit length, $m = 4.0 \times 10^{-2} \text{kgm}^{-1}$

$$\therefore \text{Length of wire} = L = \frac{M}{m} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875\,\text{m}$$

(a) for fundamental mode $L = \frac{\lambda}{2}$ or $\lambda = 2L = 0.875 \times 2 = 1.75\,\text{m}$

$\therefore$ velocity $v = \nu\lambda = 45 \times 1.75 = 78.75 \text{ ms}^{-1}$

(b) The velocity of transverse wave

$$v = \sqrt{\frac{T}{m}} \Rightarrow T = v^2 m = (78.75)^2 \times 4.0 \times 10^{-2} = 248.6\text{N}$$

21. Solution :

Given : $\upsilon = 2.53 \text{ kH}_z = 2.53 \times 10^3 \text{H}_z$

(L) Length of steel rod = 100 cm = 1m.

when the steel rod clamped at its middle executes longitudinal vibrations of its fundamental frequency, then

$$L = \frac{\lambda}{2} \quad \text{or} \quad \lambda = 2L = 2 \times 1 = 2\text{m}$$

The speed of sound in steel

$$\upsilon = \upsilon\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3 \text{ ms}^{-1}$$

UNSOLVED QUESTION PAPER : PHYSICS

CLASS - XI

QUESTIONS (1 MARK)

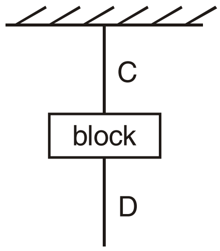
1. Name the fundamental forces of nature.
2. What is moment of Inertia of a solid sphere about its diameter.
3. At what points the velocity and acceleration are zero for a particle executing simple harmonic motion
4. What is the efficiency of a Carnot engine operating between boiling point and freezing point of water?
5. What are the S.I. and C.G.S. unit of Heat? How are they related?
6. What provides restoring forces in the following cases
   (i) A spring compressed and then left free to vibrate.
   (ii) Pendulum disturbed from its mean position.
7. What is the value of bulk modulus for an incompressible liquid?
8. Why the aeroplanes and cars are given a streamline shape?

QUESTIONS (2 MARKS)

1. The radius of the earth is 6.37×10^6m and its mass is 5.975×10^{24} kg. Find the earth's average density to appropriate significant figure.
2. If the position vectors of P and Q be respectively

   $\left(\hat{i} + 3\hat{J} - 7\hat{k}\right)$ and $\left(5\hat{i} - 2\hat{J} + 4\hat{k}\right)$ find $\overrightarrow{PQ}$

3. A block is supported by a cord c from a rigid support and another cord D is attached to the bottom of the block. If you give a sudden jerk to D, it will break, but if you pull D steadily, C will break. Why?



4. If earth Contracts to half its radius, what would be the duration of the day.

5. What is the height at which the value of g is the same as at a depth of R/2? (R is radius of earth)

6. Prove that the elastic potential energy density of a stretched wire is equal to half the product of stress and strain.

7. A particle is executing simple harmonic motion according to equation.

   $$x = 5 \sin \pi t$$

   where x is in cm. How long the particle take to move from the position of equilibrium to the position of maximum displacement?

8. At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth

   man of one molecule of oxygen = 5.34×10^{-26}kg

   Boltzman constant k = 1.38×10^{-3} Joule/k

   escape velocity from the earth = 11.0 km/s

9. Explain why

   (a) Two bodies at different temperature T_1 and T_2, if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$

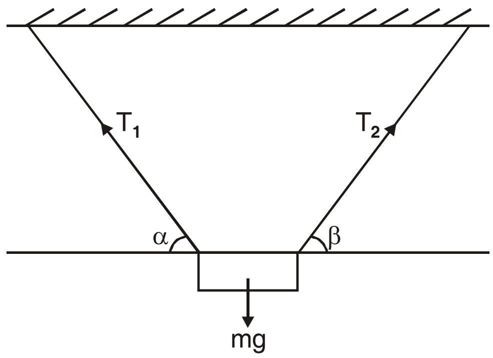
   (b) Air pressure in a car tyre increases during driving.

10. Why no real engine can have an efficiency greater than that of a cannot engine working between the same two temperature?

QUESTIONS (3 MARKS)

1. A car travels first half of a length S with velocity v_1. The second half is covered with velocity v_2 and v_3 for equal time intervals. Find the average velocity of the motion

2. A body of mass m is suspended by two strings making angles α and β with the horizontal as shown. Calculate the tensions in the two string



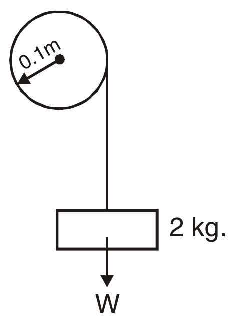
3. Define elastic collision. Prove that bodies of identical mass exchange their velocities after head on collision.

4. A body attached to a string of length l describe a vertical circle. Derive the expression for the velocity of the body and tension in the string at any point.

5. A small sphere of mans 0.1 kg and radius 2.5 cm rolls without sliding with a uniform velocity of 0.1 m/s along a straight line on a smooth horizontal table. Calculate the total energy of the sphere.

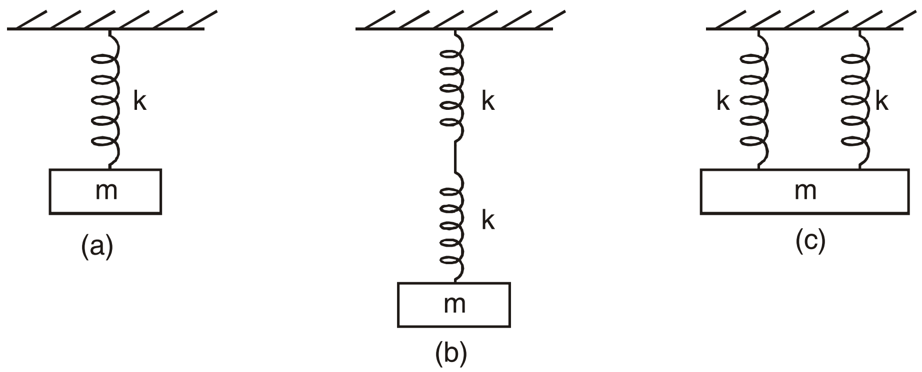
OR

The moment of inertia of a solid flywheel about its axis is 0.1 kg – m^2. A tangential force of 2 kg wt. is applied round the circumference of the flywheel with the help of a string and mass arrangement as shown. If the radius of the wheel is 0.1 m. Find the acceleration of the mass.



6. State and derive Keplar's law of periods for circular orbits.

7. Show that the time periods for vertical harmonic oscillations of the three system shown in fig (a), (b), and (c) in the ratio of $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$



spring constant of each spring is k.

8. Two perfect gases at absolute temperature T_1 and T_2 are mixed. These is no loss of energy. Find the temperature of the mixture if masses of molecules are m_1 and m_2 and the number of molecules in the gases are μ_1 and μ_2 respectively.

9. Plot the corresponding difference circles for each of the following simple harmonic motion. Indicate the initial (t = 0) position of the particle, the radius of the circle and the angular speed of the rotating particle.

   (i) $x = -2 \sin(3t + \pi/3)$ (ii) $x = \cos(\pi/6 - t)$

   (The sense of rotation is taken to be anti clockwise)

QUESTIONS (5 MARKS)

1. Derive the three basic kinematic equations by calculus method.

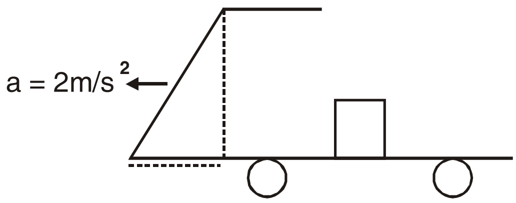
   OR

   A body is projected with some initial velocity making an angle θ with the horizontal. Show that its path is parabola, find the maximum height attained and its horizontal range.

2. Why are circular road banked? Deduce an expression for safe velocity in a banked road. (with friction)

OR

The rear side of a truck is open and a box of 40 kg mass is placed 5 m away form the open end as shown in fig. The coefficient of friction between the box and the surface below it is 0.15 on a straight road the truck starts from rest and accelerates with 2 m/s^2. At what distance from the starting point does the box fall off the truck?



3. (i) What is the phenomenon of capillary? Derive an expression for the rise of liquid in a capillary tube.

   (ii) What happen if the length of the capillary tube is smaller than the height to which the liquid rises. Explain?

OR

Discuss stress vs strain graph explaining clearly the term elastic limit, permanent set, elastic hysteresis and tensile strength.

PHYSICS

CLASS - XI

General Instructions

(a) Questions from question no. 1-4 carry 1 marks each, 5-12 carry 2 marks each, 13-27 carry 3 marks each and 28-30 carry 5 marks each.

(b) There is no overall choice but one choice is given in 2 marks question, two choice in 3 marks question and all three choices in five marks question

(c) You may use the following physical constant where ever necessary:

Speed of light C = 3×10^8 ms^{-1}

Gravitational constant G = 6.6×10^{-11} NM^2 Kg^{-2}

Gas constant R = 8.314 J Mol^{-1} k^{-1}

Mass of electron = 9.110×10^{-31} Kg

Mechanical equivalent of heat = 4.185 J Cal^{-1}

Standard atmospheric pressure = 1.013×10^5 Pa

Absolute zero 0K = -273.15^0C

Acceleration due to gravity = 9.8 Ms^{-2}

Use of calculator is not permitted. However you may use log table, if required. Draw neat labelled diagram wherever necessary to explain your answer.

1. A light body and heavy body have equal momentum, which one have greater kinetic energy?

2. What does speedometer of a car indicates?

3. Write down the dimensions of viscosity coefficient

4. Why do we use ball-bearings?

5. How errors are combined in following mathematical operations of physical quantities?

   (i) Subtraction (ii) Product

6. Draw the Velocity - Time graph for following cases when (i) Object is moving in positive direction with acceleration (ii) An object is under free fall.

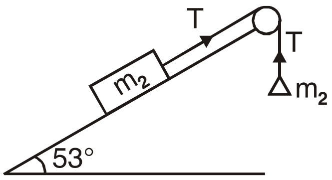
7. Derive the necessary relation for safest velocity of an automobile on a banked road of radius r and friction coefficient μ.

8. If variation of position with time t is given by $x = a + bt + ct^2$. Write the dimensions of a, b & c.

9. The forces whose magnitude is in the ratio of 3:5 give a resultant of 35 N. If the angle b/w them is 60°. Find the magnitude of each force.

10. What is an impulse? A ball coming towards a batsman with a certain velocity U. He deflects the ball by an angle Q and its velocity increases to V. Draws A vector diagram to show initial momentum, find momentum and impulse.

11. In the given system of masses m_1 = 5 kg, and coefficient of friction for each constant is 0.2. Calculate the mass m_2, if m_1 is sliding down with an acceleration of 2 ms^{-2}. What will be the tension in the string?



12. The radius and length of a solid cylinder is measured as R = (10.0 +/- 02) cm, l = (20.0 + /- 0.5) cm. Calculate the volume and surface area of the cylinder and error in them.

13. A bomb is exploded into three fragments of mass 1:2:3. The fragment having lighter masses move with a speed of 40 m/s in mutually perpendicular to each other. Calculate the velocity of the third fragment.

14. If $\vec{A}\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{B} = \left(3\hat{i} + 4\hat{j}\right)$. Determine the vector having same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

15. A force acting on an object is given by $\vec{F} = \left(3\hat{i} + 4\hat{j} - 6\hat{k}\right)$

N and the displacement made by it is given by $\overline{X} = \left(6\hat{i} - 2\hat{j} - \hat{k}\right)$. Calculate the work done and power if work is done in 2 s.

16. Define and prove conservation of linear momentum.

17. If the momentum of an object is increased by 50%, Calculate the percentage changes in its K.E.

OR

Two particles having mass ratio of 4:5 have same K.E. Calculate the ration of their linear momentum.

18. The velocity- Time relation of a particle is given by V = $(3t^2 - 2t - 1)$m/s. Calculate using calculus method, the position and acceleration of the particle when the velocity of the particle is zero. Given the initial position of the object is 5m.

19. Express 10J of energy in a new system of units in which 100g, 10 cm, 30 sec are the fundamental units. Determine which one of them is bigger unit of energy.

20. The escape velocity (v) of a body depends upon the mass (m) of body, gravitational acceleration (g) and radius (R) of the planet. Derive the relation for escape velocity dimensionally.

21. State and Prove Work- Energy Theorem. OR Define uniform velocity of an object moving along a straight line. What will be shape of velocity time and position-time graphs of such a motion?

22. If a composite physical quantity in terms of moment of inertia I, force F, velocity V, work W and length L is define as, Q = $(IF\ V^2/WL^3)$. Find the **dimension** of Q and identify it.

23. Explain why a man who fall from a height on a cemented floor receive more injury then when he fall from the same height on the heap of sand.

24. Is it possible to have collision in which all the kinetic energy is **lost**? If so cite an example.

OR

Prove that mechanical energy remains conserved during motion when a body of mass *m* is dropped from a height *h.*

25. Two masses 8 kg and 12 kg are connected at the two ends of an inextensible string that passes over a frictionless pulley. Find the acceleration of the masses and tension in the string when masses are released.

26. A body of mass 1 Kg initially at rest is moved by a horizontal force of 0.5 N on a smooth friction less table. Calculate the work done by the force in 10 S and show that it is equal to the change in kinetic energy of the body

27. Two bodies of masses m_1 *and* m_2 ($m_1 \neq m_2$) moving with initial velocities u_1 and u_2 ($u_1 > u_2$), along a straight line in the same direction, suffer perfect head on collision. Find their velocities after collision.

28. State Parallelogram law of vector addition. Find the magnitude and direction of the resultant of two vectors A and B interms of their magnitudes and angle between them.

OR

28 (i) Explain why it is easier to pull a roller than to push it.

(ii) State Newton's laws of motion with at least one example of each. Show that Newton's second law is the real law.

29. What do you understand by friction? Explain static friction, limiting friction and kinetic friction. Which of them self adjusting in nature? Draw a graph to show the variation of frictional force with applied force.

OR

(i) Derive the equation $S = ut + 1/2\ at^2$ using graphical method.

(ii) Show that the velocity of particle in a circular is always tangential to the circle.

30. A projectile is fired in air making an angle θ with horizontal. Show that

(i) Its path is parabolic in nature.

(ii) $Tan\ \theta = 4H/R$ where H is maximum height attained and R is the range of projectile.

EXAMINATION PAPER : 2010

Time : 3 hours **Maximum Marks : 70**

General Instructions :

(i) All questions are compulsory.

(ii) There are 30 questions in total. Questions 1 to 8 carry one mark each, questions 9 to 18 carry two marks each, questions 19 to 27 carry three marks each and question 28 to 30 carry five marks each.

(iii) There is no overall choice. However, an internal choice has been provided in the one question of two marks; one question of three marks and all three questions of five marks each. You have to attempt only one of the choices in such questions.

(iv) Use of calculators in not permitted.

(v) Please write down the serial number of question before attempting it.

(vi) You may use the following values of physical constant wherever necessary.

Boltzmann's constant K = 1.38×10^{-23} JK^{-1}

Avogadro's number $N_A = 6.022 \times 10^{23}$/mol

Radius of Earth R = 6400 km.

1. Express one micron in metre. 1

2. What does the slope of velocity-time graph represent? 1

3. Are the magnitude and direction of $\left(\vec{A} - \vec{B}\right)$ same as that of $\left(\vec{B} - \vec{A}\right)$? 1

4. What is the principle of working of a rocket? 1

5. Why do we slip on a rainy day? 1

6. What is the source of the kinetic energy of the falling rain drop? 1

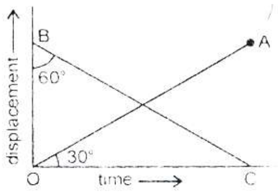
7. Two bodies move in two concentric circular paths of radii r_1 and r_2 with same time period. What is the ratio of their angular velocities? 1

8. State second law of thermodynamics. 1

9. If $x = at + bt^2$, where x is in metre and t in hour, what will be the unit of 'a' and 'b'? 2

10. The displacement-time graph of two bodies P and Q are represented by OA and BC respectively. What is the ratio of velocities of P and Q?

$\angle OBC = 60^\circ$ and $\angle AOC = 30^\circ$



OR

A car moving with a speed of 50 kmh^{-1} can be stopped by brakes after at least 6 m. What will be the minimum stopping distance, if the same car is moving at speed of 100 kmh^{-1} ? 2

11. A particle of mass m is moving in an horizontal circle of radius 'r', under a centripetal force equal to (k/r^2), where k is a constant. What is its potential energy? 2

12. Two solid spheres of the same are made of metals of different densities, which of them has larger moment of inertia about its diameter? Why? 2

13. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, what will happen to the satellite? 2

14. When air is blown in between two balls suspended close to each other, they are attracted towards each other. Give reason. 2

15. What is an isothermal process?

    Also give essential conditions for an isothermal process to take place. 2

16. Calculate the fall in temperature of helium initially at 15°C, when it is suddenly expanded to 8 times its volume. Given $\gamma = 5/3$. 2

17. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second vessel contains chlorine(diatomic) and third contain polyatomic gas. Do the vessel contain equal number of molecules? Is the root-mean square speed of molecules same in three cases? 2

18. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart, Take the direction from A to B as positive direction and give the signs of velocity and acceleration on the particle when it is

(i) at the end B

(ii) at 3 cm away from A going towards B 2

19. Draw displacement-time, velocity-time and acceleration-time graphs for a particle executing simple harmonic motion.

OR

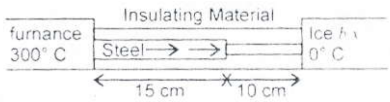
A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound (b) transmitted sound? Speed of sound in air v_a = 340 ms^{-1}, in water v_w = 1486 ms^{-1} 3

20. (i) Define Absolute Zero.

(ii) Deduce the dimensional formula for R, using ideal gas equation PV = nRT

(iii) Find degree of freedom of a monoatomic gas. 1+1+1=3

21. What is the temperature of the steel-copper junction in the steady state of the system shown in figure. The area of cross-section of steel rod is twice that of the copper rod, K_{steel} = 50.2 Js^{-1} $m^{-1}K^{-1}$, K_{cu} = 385 Js^{-1} $m^{-1}K^{-1}$



3

22. Define the term gravitational potential. Give its S.I. unit. Also derive expression for the gravitational potential energy at a point in the gravitational field of the earth. 3

23. Write S.I. unit of torque and angular momentum. Also deduce the relation between angular momentum and torque. 1/2+1/2+2=3

24. Show that the total mechanical energy of a freely falling body remains constant through out the fall. 3

25. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration

of the masses, and the tension in the string when the masses are released. [g = 9.8 ms^{-2}] 3

26. The position of a particle is given by

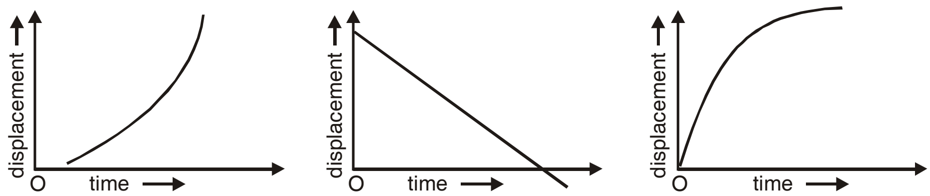
$$\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 4\hat{k} \text{ m}$$

where t is in seconds, $\vec{r}$ is in metres and the coefficients have the proper units.

(a) Find the velocity v and acceleration a. 3

(b) What is the magnitude of velocity of the particle at t = 2 s? 3

27. Discuss the nature of the motion from the given displacement-time graph. 3



(i) (ii) (iii)

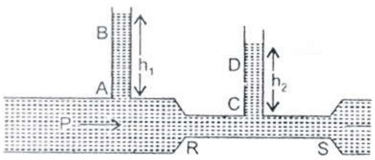
28. What is the need for Banking of road? Obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at an angle θ. The coefficient of friction between the wheel and the road is μ.

OR

What do you understand by friction? Discuss about static friction, limiting friction, kinetic friction, rolling friction. Show how the force of friction f varies with the applied force *F*. 5

29. (i) State Pascal's law of transmission of fluid pressure. Explain how is Pascal's law applied in a hydraulic lift. (with suitable diagram)

(ii) As shown in figure water flows from *P* to *Q*. Explain why height h_1 of column *AB* of water is greater that height h_2 of column *CD* of water.



OR

(i) Explain the terms specific heat and heat capacity.

(ii) State Newton's law of cooling. Derive mathematical expression for it. 5

30. Show that total mechanical energy *E* of a particle executing simple harmonic motion is constant and equal to $1/2KA^2$, where $K = m\omega^2$, m = mass and ω is angular velocity and A is the maximum amplitude. Also draw the graph of potential energy, kinetic energy and total energy of the particle.

OR

Obtain an expression for a standing wave formed and obtain the position of nodes and antinodes. 5

EXAMINATION PAPER : 2011

Time : 3 hours **Maximum Marks : 70**

General Instructions :

(i) All questions are compulsory.

(ii) There are 30 questions in total. Questions 1 to 8 carry one mark each, questions 9 to 18 carry two marks each, questions 19 to 27 carry three marks each and question 28 to 30 carry five marks each.

(iii) There is no overall choice. However, an internal choice has been provided in the one question of two marks; one question of three marks and all three questions of five marks each. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted.

(v) Please write down the serial number of question before attempting it.

(vi) You may use the following values of physical constants wherever necessary.

Boltzmann's constant K = 1.38×10^{-23} JK^{-1}

Avogadro's number $N_A = 6.022 \times 10^{23}$/mol

Radius of Earth R_e = 6400 km.

= 1.013×10^5 Pa

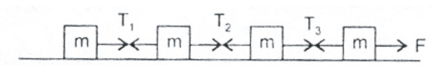
1 Atmospheric Pressure g = 9.8 m/s^2

R = 8.3 J mol^{-1} K^{-1}

1. Write the dimensional formula of torque. 1

2. Draw velocity-time graph for an object, starting from rest. Acceleration is constant and remains positive. 1

3. Arrange increasing order the tension T_1, T_2 and T_3 in the figure.



4. Why there is lack of atmosphere on the surface of moon? 1

5. The triple point of carbon dioxide is 216.55 K. Express this temperature on Fahrenheit scale. 1

6. In an open organ pipe, third harmonic is 450 Hz. What is the frequency of fifth harmonic?

7. Which type of substances are called elastomers? Give one example 1

8. A simple harmonic motion is described by a = –16 x where a is acceleration, x $\rightarrow$ displacement in m. What is the time period? 1

9. Percentage error in the measurement of height and radius of cylinder are x and y respectively. Find percentage error in the measurement of volume. Which of the two measurements height or radius need more attention? 2

OR

The length and breadth of a rectangle are measured as $(a \pm \Delta a)$and$(b \pm \Delta b)$ respectively. Find (i) relative error. (ii) absolute error in the measurement of area. 2

10. An object moving on a straight line covers first half of the distance at speed ν and second half of the distance at speed 2v. Find (i) average speed, (ii) mean speed.

11. An object moving on a circular path in horizontal plane. Radius of the paths is *r* and constant speed is v. Deduce expression for centripetal acceleration. 2

12. Find the height from the surface of earth at which weight of a body of mass *m* will be reduced by 36% of its weight on the surface. (R_e = 6400 km) 2

13. Define gravitational potential. Give its S.I. unit.

14. An engine has been designed to work between source and sink at temperature 177°C and 27°C respectively. If energy input is 3600 J. What is the work done by the engine? 2

15. Explain :

    (i) Why does the air pressure in a car tyre during driving increase?

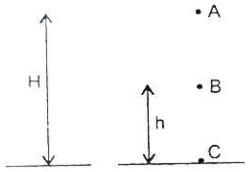
    (ii) Why coolant used in a chemical plant should have high specific heat? 2

16. Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dynes cm^{-1}. 2

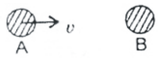
17. Show that Newton's second law of motion is the real law of motion. 2

18. A block initially at rest breaks into two parts of masses in the ratio 2 : 3. The velocity of smaller part is $\left(8\hat{i} + 6\hat{j}\right)$ m/s. Find the velocity of bigger part. 2

19. A body of mass *m* is released in vacuum from the position A at a height H above the ground. Prove that sum of kinetic and potential energies at A, B and C remains constant. 3



20. Give two points of difference between elastic and inelastic collisions. Two balls A and B with A in motion initially and B at rest. Find their velocities after collision (perfectly elastic). Each ball is of mass "m". 3



21. A liquid is in streamlined flow through a tube of non-uniform cross-section. Prove that sum of its kinetic energy, pressure energy and potential energy per unit volume remains constant. 3

22. Give reason :

    (i) fog particles appear suspended in atmosphere.

    (ii) two boats being moved parallel to each other attract.

    (iii) bridges are declared unsafe after long use. 3

23. State Kepler's law of Planetary motion. Name the physical quantities which remain constant during the planetary motion. 3

24. What is the law of equipartition of energy? Determine the value of γ for diatomic gas N_2 at moderate temperature. 3

25. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Drive an expression for its time period. Does it depend on the mass of the bob? 3

OR

A SHM is described by $y = r \sin \omega t$. What is :

(i) the value of displacement y at which speed of the body executing SHM is half of the maximum speed?

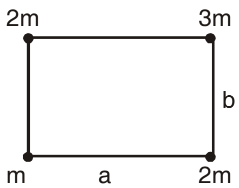
(ii) the time at which kinetic and potential energies are equally shared? 3

26. A solid sphere of mass m and radius r is impure rolling on a horizontal surface. What fraction of total energy of rotation?

(a) kinetic energy of rotation?

(b) kinetic energy of translation?

27. Four bodies have been arranged at the corners of a rectangle shown in figure. Find the centre of mass of the system. 3



28. A body is projected with velocity m at angle θ_0 upward from horizontal. Prove that the trajectory is parabolic. Deduce expression for 5

(i) horizontal range, (ii) maximum height attained.

OR

A body is projected horizontally from the top of a building of height h. Velocity of projection is u. Find : 5

(i) the time it will take to reach the ground.

(ii) horizontal distance from foot of building where it will strike the ground.

(iii) velocity with which the body reach the ground.

29. Drive an expression for maximum speed a vehicle should have, to take a turn on a banked road. Hence deduce expression for angle of banking at which there is minimum wear and tear to the tyres of the vehicle. 5

OR

Define angle of friction. The inclination θ of a rough plane is increased gradually. The body on the plane just comes into motion when inclination θ becomes 30°. Find coefficient of friction the inclination is further increased to 45°. Find acceleration of the body along the plane (g = 10 m/s^2) 5

30. A progressive wave is given by y(x,t) = 8 cos(300t – 0.15x). Where x in metre y in cm and t in second. What is the

    (i) direction of propagation

    (ii) wavelength

    (iii) frequency

    (iv) wave speed

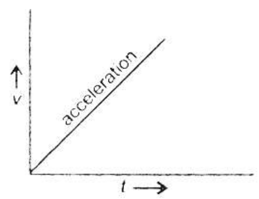
    (v) phase difference between two points 0.2 in apart?

OR

Give any three differences between progressive waves and stationary waves. A stationary wave is y = 12 sin300t cos 2x. What is the distance between two nearest nodes. 5

ANSWERS

1. [ML^{2}T^2]

2. 

3. $T_1 < T_2 < T_3$

4. Since, the value of acceleration due to gravity 'g' is less on moon, escape velocity on surface of the moon is small and so the molecules of gases escape from the surface of the moon.

5. K = °C + 273

$\Rightarrow$ 216.55 – 273 = °C

$\Rightarrow$ °C = –56.55

$\because \dfrac{9}{5}C + 32 = °F$

$\Rightarrow$ $°F = -69.8$

OR

Use $\dfrac{°F - 32}{180} = \dfrac{°C}{100} = \dfrac{216.55 - 273.15}{100}$

$$\frac{°F - 32}{180} = -0.566$$

$$°F - 32 = -0.566 \times 180$$

$$F = -101.88 + 32 = 69.8$$

6. $\because$ $v_3 = 3v_1$

$v_3 = 450$ Hz

$\therefore$ $450 = 3v_1$

$\Rightarrow$ $v_1 = 150$ Hz

Fifth harmonic, $v_5 = 5v_1$

$= 5 \times 150$

$v_5 = 750$ Hz

7. Those materials for which stress-strain variation is not a straight line within elastic limit *e.g.* Rubber.

8. For S.H.M., $a = -\omega^2 x$

Comparing with $a = -16\,x$

$\because$ $\omega = \dfrac{2\pi}{T} = \sqrt{16} = 4$

$\therefore \quad T = \frac{\pi}{2}$ second

9. Height of cylinder = x

   Radius of cylinder = y

   Volume of cylinder $V = \pi y^2 x$

   Percentage error in measurement of volume

$$\frac{\Delta V}{V} \times 100 = \pm\left(2\frac{\Delta y}{y} + \frac{\Delta x}{x}\right) \times 100$$

   Hence, radius needs more attention because any error in its measurement is multiplied two times.

OR

(i) Relative error in area

$$\frac{\Delta A}{A} = \left[\frac{\Delta a}{a} + \frac{\Delta b}{b}\right]$$

as $\quad A = ab$

(ii) Absolute error in area

$$\Delta A = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)A = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)ab$$

$$\Delta A = [(\Delta a)b + (\Delta b)a]$$

10. Let total distance be x. Distance of first half $= \frac{x}{2}$

   Speed = v

   Time taken $t_1 = \frac{\frac{x}{2}}{v} = \frac{x}{2v}$

   Distance of second half $= \frac{x}{2}$

Speed = 2v

$$\text{Time taken } t_2 = \frac{\frac{x}{2}}{2v} = \frac{x}{4v}$$

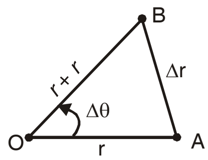
(i) $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{x}{\frac{x}{2v} + \frac{x}{4v}} = \frac{4v}{3}$$

(ii) $$\text{Mean speed} = \frac{v + 2v}{2} = \frac{3v}{2}$$

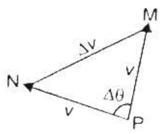
11. Consider a body in a circular path of radius *r*, with a speed V. The velocity direction is tangential at any point in the path.

The position vectors at A and B are represented by two sides of an isosceles triangle first.



(i) The change in position vector is indicated by AB = AB. The velocity at A and B are along the tangenis at these points and the change in velocity will complete an isosceles triangle of velocities.

(ii) $\overline{MN} = \overline{\Delta v}$ Since the triangles are similar,



$$\frac{\Delta V}{\Delta r} = \frac{V}{r} \Rightarrow \Delta V = \frac{V}{r}, \Delta r$$

$$\underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta V}{\Delta r} = \underset{\Delta t \to 0}{\text{Lt}} \frac{V}{r} \frac{\Delta r}{\Delta t}$$

$$\therefore \qquad \frac{dv}{dt} = \frac{v}{r}.v \Rightarrow a = \frac{v^2}{r}$$

12. h = ?, R_e = 6400 km

$$g' = g\left(1 - \frac{2h}{R}\right) = g - \frac{2hg}{R}$$

$$\Rightarrow \qquad g - g' = \frac{2gh}{R}$$

Percentage decrease in weight $= \frac{mg - mg'}{mg} \times 100 = \frac{g - g'}{g} \times 100$

$$\frac{g - g'}{g} \times 100 = \frac{2gh}{gR} \times 100 = \frac{2h}{R} \times 100$$

$$36 = \frac{2 \times h}{6400} \times 100$$

$$\Rightarrow \qquad h = 1.152 \text{ km}$$

13. Gravitational potential at a point in gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Its SI unit is J-kg^{-1}

14. Q_1 = 3600 J

T_1 = 177 °C = 177 + 273 = 450 K

T_2 = 27 °C = 27 + 273 = 300 K

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$Q_2 = Q_1 \times \frac{T_2}{T_1} = 3600 \times \frac{300}{450} = 2400 \text{ J}$$

15. (i) Because work done against friction is converted into heat. Due to which the gas in tyre gets heated and hence pressure of gas increases as $P \propto T$ at constant volume.

(ii) Because heat absorbed by a substance is directly proportional to specific heat of substance.

16. σ = 30 dynes/cm, r_1 = 2 cm, r_2 = 3 cm

Since, bubble has two surfaces, initial surface area of bubble = $2 \times 4\pi r_1^2$

$= 2 \times 4\pi \times (2)^2$

$= 32\pi$ cm^2

Final surface area of bubble = $2 \times 4\pi r_2^2 = 2 \times 4\pi \times (3)^2 = 72\pi$ cm^2

Increase in surface area = $72\pi - 32\pi = 40\pi$ cm^2

Work done = s × Increase in surface area = $30 \times 40\pi$ = 3768 ergs

17. **I law from II law :** According to II law, force experienced is the product of mass and acceleration. When there is no force, the mass does not accelerate and retains the same status. So the view of I law, *i.e.,* when there is no force the body maintains the status of motion is confirmed.

III law and II law : Consider two masses m_1 and m_2 exerting force on each other (internal forces). If their change in momentum are *dp,* and dp_2, then,

$dp_1 + dp_2 = 0$. Since no external force acts on the system of two masses.

$$\therefore \quad \frac{dp_1}{dt} = -\frac{dp_2}{dt}$$

i.e., $f_1 = -f_2$

i.e., force experienced by m_1 due to m_2 and by m_1 are equal and opposite, confirming action and reaction.

18. Let mass of the block = m

After breaking, $m_1 = \frac{2}{5}m$ and $m_2 = \frac{3}{5}m$

Initial momentum $P_i = 0$

Final momentum $P_f = m_1\vec{\nu}_1 + m_2\vec{\nu}_2$

According to law of conservation of momentum

$P_f = P_i \Rightarrow m_1\vec{\nu}_1 + m_2\vec{\nu}_2 = 0$

$\vec{\nu}_1$ = Velocity of smaller part, $\vec{\nu}_2$ = Velocity of bigger part

$$\Rightarrow \frac{2}{5}m\left(8\hat{i} + 6\hat{j}\right) + \frac{3}{5}m\left(\vec{\nu}_2\right) = 0$$

$$\Rightarrow \frac{3}{5}m\vec{\nu}_2 = -\frac{1}{5}m\left(16\hat{i} + 12\hat{j}\right)$$

$$\vec{\nu}_2 = -\left(\frac{16}{3}\hat{i} + 4\hat{j}\right)$$

19. Let a body of mass *m* be dropped from a point A at a height *H.*

P.E. at A = *mgh*

K.E. = 0

Total energy at A = *mgh*

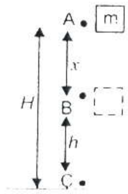
As it reaches B, it would have lost some P.E. and gained K.E.

Velocity on reaching B = $\sqrt{2gx}$

P.E. at B = mg(h – x)

$$\text{K.E.} = \frac{1}{2}m\nu_B^2 = \frac{1}{2}m2gx = mgx$$

Total energy at B = mg(h – x) + mgx = mgh



On reaching the ground C the mass must have gained a velocity $\sqrt{2}$ gh and the P E must be zero

PE at C = 0

$$\text{K.E. at C} = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$$

Total energy at C = mgh

Thus it is proved that the energy at any point in its path is mgh.

20. **Elastic collisions :** (i) K.E. is conserved, (ii) Forces involved must be conservative.

Inelastic collisions : (i) K.E. is not conserved., (ii) Some or all forces involved may be non-conservative.

Mass of ball A and B = m

Initial velocity of ball A = $u_1 = \nu$

Initial velocity of ball B = $u_2 = 0$

Final velocity ball A = ν_1 and of ball B = ν_2

According to conservation of momentum

$m\nu + m_2(0) = m\nu_1 + m\nu_2$

$m\nu = m\nu_1 + m\nu_2$........(i)

Conservation of K.E.

$$\frac{1}{2}m\nu^2 = \frac{1}{2}m\nu_1^2 + \frac{1}{2}m\nu_2^2 \text{ ....(ii)}$$

$$m\nu^2 = m\nu_1^2 + m\nu_2^2 \text{ From (ii)}$$

$$m\left(\nu^2 - \nu_1^2\right) = m\nu_2^2 \text{ (iii)}$$

From (i)

$m(\nu - \nu_1) = m\nu_2$ (iv)

Dividing respecting sides of (iii) by (iv)

$$\frac{m(\nu + \nu_1)(\nu - \nu_1)}{m(\nu - \nu_1)} = \frac{m\nu_2^2}{m\nu_2}$$

$$\Rightarrow \qquad \nu_2 = \nu + \nu_1$$

Substituting in (i)

$$m\nu = m\nu_1 + m(\nu + \nu_1)$$

$\Rightarrow$ $\quad \nu_1 = 0$

Similarly solving,

$$\nu_2 = \nu$$

i.e., Ball A comes to rest and ball B starts moving with velocity ν i.e, they exchange their velocities on collision.

21. Consider an incompressible non-viscous liquid entering the cross-section A_1 at A with a velocity ν_1 and coming out at a height h_2 at B with velocity ν_2.

The P.E. and K.E. increases since h_2 and ν_2 are more than h_1 and ν_1 respectively. This is done by the pressure doing work on the liquid. If P_1 and P_2 are the pressure at A and B, for a small displacement at A and B,

The work done on the liquid A = (P_1A_1)

$$\Delta x_1 = P_1A_1\nu_1\Delta t$$

The work done by the liquid at B = $-(P_2A_2)$

$$\Delta x_2 = P_2A_2\nu_2\Delta t$$

(Considering a small time Δt so that area may be same)

Net work done by pressure = $(P_1 - P_2)\ A\nu\ \Delta t$ since $A_1\nu_1 = A_2\nu_2$

From conservation of energy,

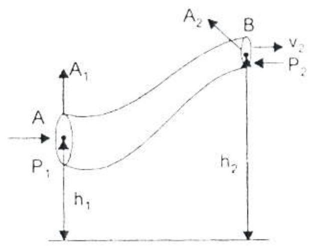
$$(P_1 - P_2)A\nu\ \Delta t = \text{change in (K.E. + P.E.)}$$

$$(P_1 - P_2)A\nu\,\Delta t = A\nu\rho\Delta t g(h_2 - h_1) + \frac{1}{2}A\nu\Delta t\rho\left(\nu_2^2 - \nu_1^2\right)$$

$\therefore$ $$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{\rho}{2}\left(\nu_2^2 - \nu_1^2\right)$$

(i.e.) $$P_1 + \rho g h_1 + \frac{\rho}{2}\nu_1^2 = P_2 + \rho h g_2 + \frac{\rho}{2}\nu_2^2$$

$$\frac{P}{\rho g} + h + \frac{\nu^2}{2g} = \text{constant}$$



22. (i) Fog particles are formed due to condensation of water vapour as they rise up. Due to condensation, they become heavy and appear suspended.

(ii) When two boats move in parallel directions close to each other the stream of water between boat is set into vigorous motion. As a result pressure exerted by water in between the boats becomes less than pressure of water beyond the boats. Due to this difference in pressure the boats attract each other.

(iii) A bridge undergoes alternating stress and strain for a large number of times during its use. When bridge is used for long time, it loses its elastic strength. Therefore, the amount of strain in the bridge for a given stress will become large and ultimately, the bridge will collapse. So, they are declared unsafe after long use.

23. (i) All the planets move around in elliptical orbits with the sun at its focus.

(ii) The line joining the sun and the planet sweeps out equal areas in equal intervals of time.

(iii) The square of the time period of revolution of the planet is directly proportional to the cube of the semi-major axis of the elliptical orbit $T^2 \propto a^3$.

Areal velocity and angular momentum remain constant.

24. According to law of equipartition of energy for any dynamical system in thermal equilibrium the total energy is distributed equally amongst all the degrees of freedom and the energy associated with **each molecule per**

degree of freedom is $\frac{1}{2}k_BT$, where k_B is Boltzmann constant and T is temperature of the system.

Diatomic gas N_2 has 5 degrees of freedom. Using law of equipartition of energy total internal energy of one mole of gas is

$$u = 5 \times \left(\frac{1}{2}k_BT\right) \times N_A = \frac{5}{2}RT$$

$$C_v = \left(\frac{du}{dt}\right)$$

$$C_v = \frac{d}{dt}\left(\frac{1}{2}RT\right) = \frac{5}{2}R$$

$$C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{9}R$$

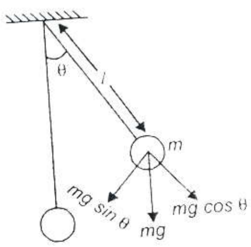
$$\gamma = \frac{C_p}{C_u} = \frac{(7/2)R}{(5/2)R} = 1.4$$

25. Restoring force is provided by the portion mg sin θ of gravitational force, Since, it acts perpendicular to length l, the restoring torque = –mg sin θ.l

Also, $\tau = I\alpha = ml_2\alpha$

$\therefore \quad ml^2\alpha = -mg \sin\theta.l$

$$\alpha = -\frac{g\sin\theta}{l}$$



For small angles of oscillation, $\sin\theta = \theta$

$$\alpha = -\frac{g}{l}.\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}.\theta \quad \text{i.e.} \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

giving $\omega = \sqrt{\frac{g}{l}}$ and $T = 2\pi\sqrt{\frac{l}{g}}$

Time period doesn't depend on mass of bob.

OR

(i) Let the particle be at R when its velocity $V = \frac{V_{max}}{2}$ and its displacement from the mean position O be y.

As $\quad \nu = \omega\sqrt{A^2 - y^2}$

So $\quad y = \sqrt{A^2 - \nu^2/\omega^2}$

Given $\nu = A\ \omega/2$,

then $\quad y = \sqrt{A^2 - \frac{A^2\omega^2}{4\omega^2}} = \frac{\sqrt{3}}{2}A$

(ii) $\text{P.E.} = U_t = \frac{1}{2}m\omega^2[A^2 \sin^2 \omega t]$

$\text{K.E.} = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$

u = K.E.

$\sin^2\omega t = \cos^2\omega t \Rightarrow \sin \omega t = \cos \omega t$

$$\therefore \quad \omega t = \frac{\pi}{4} \quad \Rightarrow \quad \frac{2\pi}{T}t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{T}{8}$$

26. Mass of sphere = m, Radius of sphere = r

Moment of Inertia $I = \frac{2}{5}mr^2$

Total energy = $K_R + K_T$

$$K_{tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}m\nu^2$$

$$= \frac{1}{2}\cdot\frac{2}{5}mr^2\left(\frac{\nu^2}{r^2}\right) + \frac{1}{2}m\nu^2 \qquad (\nu = r\omega)$$

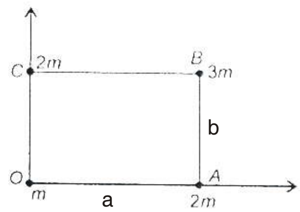
$$K_{tot} = \frac{1}{2}\left(\frac{7}{5}\right)m\nu^2$$

Fraction of K.E. of rotation $= \frac{K_R}{K_{tot}} = \frac{\frac{1}{2}\left(\frac{2}{5}\right)m\nu^2}{\frac{1}{2}\left(\frac{7}{5}\right)m\nu^2} = \frac{2}{7}$

$\therefore$ Fraction of K.E. of translation $= \frac{K_T}{K_{tot}} = \frac{5}{7}$

27. Let $m_1 = m, m_2 = 2\ m,\ m_4 = 2m$

Let mass m_1 be at origin



$\therefore$ For $m_1; x_1 = 0,\ y_1 = 0$

For m_2; $x_2 = a\hat{i},\ y_2 = 0$

For m_3; $x_3 = a\hat{i},\ y_3 = b\hat{j}$

For m_4; $x_4 = 0$, $y_4 = b\hat{j}$

Coordinates of COM of the system are

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{m \times 0 + 2m \times a\hat{i} + 3m \times a\hat{i} + 2m \times 0}{m + 2m + 3m + 2m}$$

$$x = \frac{5ma\hat{i}}{8m} = \frac{5a\hat{i}}{8}$$

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{m \times 0 + 2m \times 0 + 3m \times b\hat{j} + 2m \times b\hat{j}}{m + 2m + 3m + 2m}$$

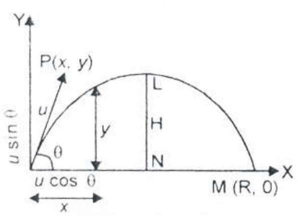
$$= \frac{5mb\hat{j}}{8m} = \frac{5}{8}b\hat{j}$$

$\therefore$ Centre of mass of system is $= \frac{5}{8}\left(a\hat{j} + b\hat{j}\right)$

Students may note that choice of the mass at the origin, may lead to varying result.

28. A body thrown up in space and allowed to proceed with effect of gravity alone is called projectile.

Suppose a body is projected with velocity *u* at an angle θ with the horizontal, P(x, y) is any point on its trajectory at time *t*. Horizontal component of velocity is unaffected by gravity, but the vertical component ($u \sin\theta$) changes due to gravity



$x = (u \cos\theta)t$

$$y = (u \sin \theta)\, t - \frac{1}{2} g t^2$$

$$= u \sin q \times \frac{x}{u \cos \theta} - \frac{1}{2} g \left(\frac{x}{u cis\theta}\right)^2$$

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} \quad ....(i)$$

It represents the equation of a parabola, hence the path followed by a projectile is a parabola.

(i) When the body returns to the same horizontal level y = 0

$$\therefore \qquad 0 = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} \quad \text{[From (i)]}$$

$$\text{or} \qquad x \tan \theta = \frac{g x^2}{2 u^2 \cos^2 \theta}$$

But coordinates of M are (r, 0). Putting x = R,

$$\text{we have} \qquad R = \frac{u^2 \sin 2\theta}{g}$$

(ii) The greatest vertical distance attained by the projectile above the horizontal plane from the point of projection is called maximum height.

Maximum height, LN = H

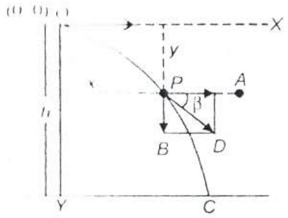
At maximum, height $\nu = 0$

$$\therefore \qquad \nu^2 - \nu_y^2 = -2gH,$$

where $u_y = u \sin\theta$

or $(u \sin \theta)^2 = 2gH$

or $$H = \frac{u^2 \sin^2 \theta}{2g}$$



(i) Time taken to reach the ground (Time of flight)

Let it be *T*

h = vertical height of point of projection *O* from *C*

Taking motion of object along *OY* direction

$y_0 = 0$, $y = h$, $u_y = 0$, $a_y = g$, $t = T$

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

Putting values of y_0, y and, u_y, ay, we have

$$h = \frac{1}{2} g T^2$$

$$\Rightarrow T = \sqrt{\frac{2h}{g}}$$

(ii) Horizontal distance from foot of building where it will strike the ground (Horizontal range R). Taking motion of object along *OX*-direction; we have $x_0 = 0$, $x = R$, $u_x = u$, $a_x = 0$

$$t = T = \sqrt{\frac{2h}{g}}$$

As $$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

Putting the values we have

$$R = u\sqrt{\frac{2h}{g}}$$

(iii) Velocity with which body reach the ground. At any instant to the object posses two perpendicular velocities.

Horizontal velocity $\nu_x = u$ represented by PA

Vertical velocity ν_x represented by PB

$\nu_x = u_x + a_x t$

$\nu_y = 0$, $a_y = g$, we have

$\nu_y = gt$

Resultant velocity ν^1at ν_s^1 and ν_s^1 is given by

$$\nu = \sqrt{\nu_x^2 + \nu_y^2}$$

$$\nu = \sqrt{\nu^2 + g^2 t^2}$$

Let $\vec{\nu}$ makes an angle β with horizontal direction then

$$\tan\beta = \frac{\nu_y}{\nu_x} = \frac{gt}{u} \quad \text{or} \quad \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

29. From the forces acting on the vehicle in a banked curve (θ).

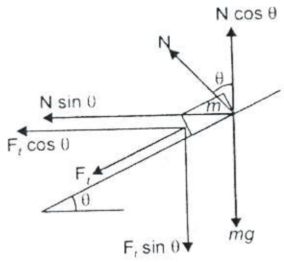
$N\cos\theta - F_f\sin\theta = mg$

$N\sin\theta + F_f\cos\theta = m\nu^2/r$. $F_f = \mu N$

Dividing the equation, we have,

$$\frac{\nu^2}{rg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta + \mu N\sin\theta}$$

$$\nu^2 = rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]$$ [dividing each term of right side by N $\cos\theta$]



$$\nu = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

If $\mu = 0$ i.e., banked road is perfectly smooth. Then from above

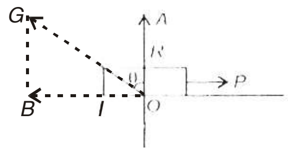
$\nu_0 = (rg \tan \theta)^{1/2}$

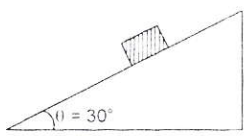
$\nu_0 = rg \tan\theta$

or $\tan\theta = \dfrac{\nu_0^2}{rg}$

OR

Angle which the resultant of force of limiting friction *F* and normal reaction *R* makes with direction of normal reaction *R.*



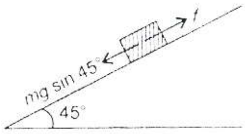


$\theta = \tan^{-1}(\mu) = 30°$

$\Rightarrow \quad \mu = \tan 30° = \dfrac{1}{\sqrt{3}}$

Max friction $f_{max} = \mu mg \cos 45°$

$$= \frac{1}{\sqrt{3}} mg \frac{1}{\sqrt{2}}$$



Net force

F = (mg sin 45° – μ mg cos 45°)

$$ma = \left(\frac{mg}{\sqrt{2}} - \frac{1}{\sqrt{3}}\frac{mg}{\sqrt{2}}\right)$$

$$\therefore \qquad a = \frac{g}{\sqrt{2}}\left[1 - \frac{1}{\sqrt{3}}\right]$$

$$= \frac{10}{\sqrt{2}}\left[\frac{\sqrt{3}-1}{\sqrt{3}}\right] = 10\left[\frac{1.73-1}{\sqrt{6}}\right]$$

$$a = \frac{7.3}{\sqrt{6}} = 2.99\,\mathrm{m/s}^2$$

30. y(x,t) = 8 cos(300t – 0.15x)

On comparing with y = $a \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

(i) Direction of propogation is + x-axis.

(ii) Wavelength $\frac{2\pi}{\lambda} = 0.15$

$$\lambda = \frac{2\pi}{0.15} = 41.87\,\mathrm{m}$$

(iii) $\frac{2\pi}{T} = 300$

$2\pi\upsilon = 300$

$$\gamma = \frac{300}{2\pi} = 47.7\ H_z$$

(iv) $v = \lambda v = \frac{2\pi}{0.15} \times \frac{300}{2\pi} = 3000\,\mathrm{m/s}$

(v) $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$

$$= \frac{2\pi}{\lambda} \times 0.2 = \frac{2\pi \times 0.2 \times 0.15}{2\pi} = 0.03 \text{ radian}$$

OR

Progressive Wave	*Stationary Wave*
(i) All particles have same phase and amplitude.	(i) Amplitude varies with position.
(ii) Speed of motion is same.	(ii) Speed varies with position.
(iii) Energy is transported.	(iii) Energy is not transported.
(iv) Same change in pressure and density is with every point.	(iv) Pressure and density varies with point.

$y = 12 \sin 300t \cos 2x$

Comparing with equation of stationary wave

$y = 2A \sin \omega t \cos Kx$

$K = 2$

Distance between two consecutive nodes $= \dfrac{\lambda}{2}$

Where λ is wavelength

$$K = \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \frac{\pi}{(\lambda/2)} = 2$$

$$\therefore \quad \frac{\lambda}{2} = \frac{\pi}{2}$$

So, the distance between two nearest nodes is $\dfrac{\pi}{2}$.

STUDY MATERIAL FOR CLASS XI SUBJECT: PHYSICS

CHAPTER -I

PHYSICAL WORLD

MAIN POINTS

- Physics deals with the study of the basic laws of nature and their manifestation in different phenomena. The basic laws of physics are universal and apply in widely different contexts and conditions.
- The scope of physics is wide, covering a tremendous range of magnitude of physical quantities.
- Physics and technology are related to each other. Sometimes technology gives raise to new physics at other times physics generates new technology. Both have direct impact on society.
- There are four fundamental forces in nature that govern the diverse phenomena of the macroscopic and the microscopic world. These are the 'gravitational force ', the electromagnetic force', 'the strong nuclear force', and the weak nuclear force'
- The physical quantities that remain unchanged in a process are called conserved quantities. Some of the general conservation laws in nature include the law of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc.
- Conservation laws have a deep connection with symmetries of nature .symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.
- Gravitational force is the force of mutual attraction between any two objects by virtue of their masses. It is always attractive
- Electromagnetic Force is the force between charged particles .It acts over large distances and does not need any intervening medium. Enormously strong compared to gravity. It can be attractive or repulsive.
- Strong nuclear force is the force that binds the nucleons together.It is the strongest of all the fundamental forces. It is charge independent. And very short range.
- Weak nuclear force appears only in certain nuclear processes such as β-decay. Weak nuclear force is not as weak as the gravitational force.
- In a chemical reaction if the total binding energy of the reacting molecules is less than that of the product molecules the difference appears as heat and the reaction is exothermic
- In a chemical reaction if the total binding energy of the reacting molecules is more than that of the product molecules the difference amount of energy is absorbed and the reaction is endothermic.

- In a nuclear process mass gets converted into energy. This is the energy which gets released in a nuclear power generation and nuclear explosions.

UNITS AND MEASUREMENT

CONCEPTS INVOLVED

- The International system of units
- Measurement of length
- Measurement of mass
- Measurement of Time
- Accuracy, Precision of instruments and errors in measurement
- Significant figures
- Dimensions of physical quantities
- Dimensional formulae and dimensional equations
- Dimensional analysis and its applications

Main points

- Physics is a quantitative science, based on measurement of physical quantities.Certain physical quantities have been chosen as fundamental or base quantities. The fundamental quantities that are chosen are Length, Mass, Time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.
- Each base quantity is defined in terms of a certain basic arbitrarily chosenbut properly standardised reference standard called unit (such as metre,kilogram,second,ampere,kelvin,mole,and candela.The units for the fundamental base quantities are called fundamental or base units and two supplementary units in relation to quantities plane angle and solid angle radian, Ste radian..
- Other physical quantities derived from the base quantities can be expressed as a combination of the base units and are called derived units.A complete set of units both fundamental and derived units are called a system of units.
- The International System of units based on seven base unitsis at present internationally acceptedunit system and is widely used throughout the world
- The SI units are used in all physical measurements, for both the base quantitiesand the derived quantities obtained from them. Certain derived units are expressed by means of SI units of special names such as joule, newton, watt etc.
- In computing any physical quantity the units for derived quantities involved in the relationships are treated as though they were algebraic quantities till the desired units are obtained

- In SI system that is System Internationale d' Units there are 7 base units' andtwo supplementary units.
- Direct and indirect methods can be used for the measurement of physical quantities. In measured quantities while expressing theresult, the accuracy and precision of measuring instrumentsalong with errors in measurement should be taken into account.
- In measured and computed quantitiesproper significant figures only should be retained.

  <u>Rules for determining the number of significant figures,carrying out arithmetic operations with them and rounding off the uncertain digits must be followed.</u>
- The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities .Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among physical quantities etc. A dimensionally consistent equation need not be actually an exact equation, but a dimensionally wrong or inconsistent equation must be wrong.
- The uncertainty in the measurement of a physical quantity is called an error.
- The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.
- Precision tells us to what limit the quantity is measured.
- The errors in measurement can be classified as

  (i) Systematic errors and (ii) Random errors
- SYSTEMATIC ERRORS: These are the errors that tend to be either positive or negative.Sources of systematic errors are

  (i) Instrumental errors

  (ii) Imperfection in experimental technique or procedure

  (iii) Personal errors
- RANDOM ERRORS :Those errors which occur irregularly .These errors arise due to unpredictable fluctuations in experimental conditions
- Least count error is the error associated with the resolution of the instrument.
- The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.

  Ex: $\Delta a = | a - a_{mean} |$
- The relative error or the percentage error is the ratio of the mean absolute error to the mean value of the quantity measured. When the relative error is expressed in per cent it is called the percentage error.

  Ex: (i) **Relative error = $\Delta a_{mean}/a_{mean}$** (ii) **% error = ($\Delta a_{mean}/a_{mean}$) X 100**

<u>COMBINATION OF ERRORS</u>

- ✓ **<u>ERROR OF A SUM OR A DIFFERENCE</u>**

  When two quantities are added or subtracted, the absolute error in the final result is thesums of the absolute errors in the individual quantities.

  IF Z=A+ B then the max possible error in Z, $\Delta Z = \Delta A + \Delta B$

  IF Z=A- B then the max possible error in Z, $\Delta Z = \Delta A + \Delta B$

- ✓ **ERROR OF A PRODUCT OR A QUOTIENT**

  When two quantities are multiplied or divided the relative error is the sum of the relative errors in the multipliers

  Suppose **Z= A*B or Z=A/B then the max relative error in 'Z' = ΔZ/Z= (ΔA/A) + (ΔB/B**)

- ✓ **ERROR IN CASE OF A QUANTITY RAISED TO A POWER**

  The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

  Suppose **$Z = A^k$ then ΔZ/Z = K (ΔA/A)**

SIGNIFICANT FIGURES

The reliable digits plus the first uncertain digit in a measurement arecalled Significant figures.

RULES FOR FINDING THE SIGNIFICANT FIGURES IN A MEASUREMENT

- ✓ All the non-zero digits are significant
- ✓ All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all
- ✓ If the number is less than 1, the zero(s) on the right side of decimal point but to the left of the first non-zero digit are not significant.

  Ex: In 0.000 35 the underlined zeros are not significant.

- ✓ The terminal or trailing zeros in a number without a decimal point are not significant

  Ex: 1795 m = 179500cm =1795000mm has four significant figures.

- ✓ The trailing zeros in a number with a decimal point are significant

  For ex: The numbers 75.00 or 0.06700 have four significant figures each.

RULES FOR ARITHMETIC OPERATIONS WITH SIGNIFICANT FIGURES.

- ✓ In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

  Suppose F= 0.04 Kg X 0.452 m/sec^2 =0.0108 kg-m/sec^2

The final result is F = 0.01Kg-m/Sec ^{2}

- ✓ In addition or subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

  For ex: A= 334.5 kg; B= 23.45Kg then A + B =334.5 kg + 23.43 kg = 357.93 kg

  The result with significant figures is 357.9 kg

ROUNDING OFF:

While rounding off measurements the following rules are applied

- ✓ **Rule I:** If the digit to be dropped is smaller than 5,then the preceding digit should be left unchanged.
  For ex: 9.32 is rounded off to 9.3
- ✓ **Rule II:** If the digit to be dropped is greater than 5, then the preceding digit should be raised by 1
  For ex: 8.27 is rounded off to 8.3
- ✓ **Rule III:** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1
  For ex: 9.351 on being rounded off to first decimal, becomes 9.4
- ✓ **Rule IV**: If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even, is raised by 1 if it is odd.
  For ex: 5.45, on being rounded off,become 5.4
  5.450 on being rounded off, becomes 5.4

  (ii) 7.35 , on being rounded off, becomes 7.4

DIMENSIONS, DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION

- Dimensions of a derived unit are the powers to which the fundamental units of mass, length and time etc. must be raised to represent that unit.
  For ex: Density = Mass / Volume= $M/L^3 = M^1L^{-3}$

- Dimensional formula is an expression which shows how and which of the fundamental units are required to represent the unit of a physical quantity.
  For Ex: $M^1 L^1 T^{-2}$ is the dimensional formula of Force.

CATEGORIES of PHYSICAL QUANTITIES

- Dimensional Cost ants: These are the quantities which possess dimensions and have a fixed value.
  Ex: Gravitational Constant
- Dimensional Variables These are the quantities which possess dimensions and do not have a fixed value
  For ex: velocity, acceleration etc.
- Dimensionless Constants: these are the quantities which do not possess dimensions and have a fixed value.
  For ex: πetc.
- Dimensionless Variables: These are the quantities which are dimensionless and do not have a fixed value.
  For ex: Strain, Specific Gravity etc.

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

- A given physical relation is dimensionally correct if the dimensions of the variousterms on either side of the relation are the same.

USES OF DIMENSIONAL EQUATIONS

- **TO CONVERT PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER**

  - ✓ Consider a physical quantity whose dimensions are $M^aL^bT^c$ Let n_1 be its numerical value in a system of fundamental units M_1, L_1, T_1. Then the magnitude of the physical quantity in this system is n_1 $[M_1^a\ L_1^b\ T_1^{c]}$.
    Let n_2 be the numerical value in another system of fundamental units M_2, L_2 and T_2. The magnitude of the quantity in this system is n_2 $[M_2^a\ L_2^b\ T_2^{c]}$.
    Since the value of the quantity is the same in all systems

    $n_2\ [M_2^a\ L_2^b\ T_2^{c]}. = n_1\ [M_1^a\ L_1^b\ T_1^{c]}.$
    $n_2 = n_1\ [M_1^a\ L_1^b\ T_1^{c]}.\ /[M_2^a\ L_2^b\ T_2^{c]}.$

- **TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION**
  Ex: v = u + a t, Here v represents the velocity of the body after t secs, a, is the acceleration and u the initial velocity of the body.
  Dimensional formula of u is $M^0\ L^1\ T^{-1}$
  Dimensional formula of V is $M^0\ L^1\ T^{-1}$
  Dimensional formula of at is $\{M^oL^1\ T^{-2}\}\{T^1\}= \{M^0L^1\ T^{-1\}}$
  The dimensions of every term in the given physical relation is same, hence according to principle of homogeneity the given physical relation is dimensionally correct.
- **TO ESTABLISH RELATION BETWEEN DDIFFERENT PHYSICAL QUANTITIES**

  To find an expression for the time period of a simple pendulum given that the time period(t) may depend upon (i) massof the bob (ii) length of the pendulum (iii) acceleration due to gravity,(iv) angle of swing Θ

  Let (i) $t \propto m^a$ (ii) $t\propto l^b$(iii) $t\propto g^c$(iv) $t\propto \Theta^d$

  Or $t= K\ m^a l^b g^c\ \Theta^d$ Where K is a Dimensionlessconstant of proportionality.

  Writing down the dimensions on either side of the equation we get
  $[T] = [M^a][L^b]\ [L\ T^{-2}]^c = [M^aL^{b+C}\ T^{-2c}]$
  Comparing the dimensions on either side

  a=0; b+c=0; -2c=1 i.e., c= -1/2 , b= +1/2 ,a = 0

  $t= K\ l^{½}\ \ g^{-1/2}$ or $t =K\sqrt{l/g}$

The value of K as found by experiment comes out to be 2π

And **hence** $t = 2\pi\sqrt{l/g}$

- **LIMITATIONS OF DIMENSIONAL ANALYSIS**
  - It supplies no information about dimensionless constants. They have to be determined either by experiment or by mathematical investigation.
  - This method applicable only in the case of power functions. It fails in case of exponential and trigonometric relations.
  - It fails to derive a relation which contains two or more than two quantities of like nature.
  - It can only check whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not
  - It cannot identify all the factors on which the given physical quantity depends upon.

ANSWER THE FOLLOWING EACH QUESTION CARRIES 1 MARK.

1. Define physical quantities

2.Distinguish between fundamental and derived quantities

3. Define one metre

4.Define one kilogram

5.Define one second

6.Define the SI unit of the following physical quantities

(i) Temperature (ii) Luminous intensity

7.Define one radian

8. Define one Ste radian

9.Give the relation between light year and metre

10. Write two advantages in choosing the wavelength of light radiation as a standard

of length

11.What is the difference between 5.0 and 5.000?

12.Write the uses of the dimensional analysis

13.Write the dimensional equation for force

14.Write the dimensional representation for torque

15. Give the relationship between calorie and joule

16.Write two advantages in choosing the wavelength of light radiation as a standard

of length.

17.What is the difference between 4.0 and 4.0000?

18. Write the uses of Dimensional Analysis.

19. Define the term significant figures.

Answer the following. Each question carries 2 marks.

20. Write four limitations of dimensional analysis

21. If $(P + a/V^2)(V-b)= RT$, Where the difference symbols have their usual meaning then what are the dimensions of (a/V^2) and b.

22.Write the dimensions of the following

(i) Electric intensity (ii) Electric Potential (iii) E.M.F. of a cell (iv)Electrical resistance

23 Write the dimensions of the following

(i)Specific Resistance (II) Magnetic flux (III) Electric flux (IV) Magnetic Induction

24Write the dimensions of the following

(i) Conductance (ii)Electric Permittivity (iii)Magnetic Permeability (iv)Coefficient of Self Inductance

23.Solve the following to correct significant figures

(i) 5.1 + 13.235 (ii)7.54 + 18.1295 (iii)14.632 – 5.52345 (iv)3.021 X 11

24.Define mean scalar second

25.Define second in terms of Cs-133 vibrations

Answer the following. Each question carries 3 marks.

26. Answer the following

(a) You aregiven a thread and a meter scale. How will you estimate the diameter of this thread?

(b)A screw gaugehas a pitch of 1.00mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gaugearbitrarily by increasing the number of divisions on the circular scale?

27. Explain briefly how you will estimate the size of the molecule of oleic acid.

28.Explain how will you estimate the distance of a planet or star by using parallax method.

29.Find the area of a circle of radius 3.458 cm up to correct significant figures.

30. If the % errorin the measurement of the radius R of a sphere is 0.2%, then calculate the % error in its volume.

31. A laser light beamed towards the moon takes 2.56 sec to return to the earth after the reflection on themoon‘s surface. What is the distance of the moon from theearth?

3. Motion in a straight line

IMPORTANT POINTS

- Study of motion of objects along a straight line is known as rectilinear motion.

- If a body does not change its position with time it is said to be at rest. If it changes its position with time it is said to be in motion. The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight line, position to the right of the origin is taken as positive and to the left as negative.

- Path length is defined as the total length of the path traversed by an object.

- Displacement is the change in position : $\Delta x = x_2 - x_1$, Path length is greater than or equal to the magnitude of the displacement between the two positions

- An object is said to be in uniform motion in a straight line if its displacement is equal in equal intervals of time. Otherwise the motion is said to be non-uniform.

- Average velocity is the ratio of the displacement and time interval in which the displacement occurs.

  $\bar{V} = \Delta x / \Delta t$

  On an x-t graph, the average velocity over a time interval is the slope of the line connecting the initial and final positions corresponding to that interval.
- Average Speed is ratio of the total path length traversed and the corresponding time interval. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time.

- Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$V_{inst} = \lim_{\Delta t \to 0} \overline{V} = \lim_{\Delta t \to 0} \Delta x/\Delta t = dx/dt$$

The velocity at a particular instant is equal to the slope of the tangent drawn on position –time graph at that instant.

- Average acceleration is the change velocity divided by the time interval during which he change occurs.

$$\overline{a} = \Delta V/\Delta t$$

- Instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt goes to zero.

$$a = \lim_{\Delta t \to 0} (\overline{a}) = \lim_{\Delta t \to o} (\Delta V/\Delta t) = dv/dt$$

The acceleration of an object at a particular time is the slope of the velocity- time e curve at that instant of time.For uniform motion, acceleration is zero and x-t graph is a straight line inclined to the time axis. And v-t graph is a straight line parallel to the time axis.For motion with uniformacceleration, x-t graph is a parabola, while the v-t graph is a straight line inclined to the time axis.

- The area under the velocity- time curve between times t_1 and t_2 *is* equal to the displacement of the object during that interval of time.
- For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity u, final velocity v and acceleration are related by a setoff simple equations called kinematic equations of motion.

(i) $V = u + at$

(ii) $X = ut + \frac{1}{2} a t^2$

(iii) $V^2 - u^2 = 2 a x$

Solve the following. Each question carries 1 mark.

1. Distinguish between scalar and vector quantities.Give an example in support of each.

2. Define speed. How is it different from velocity?

3.Define uniform velocity and variable velocity?

4.Plot the velocity- time graph for a uniform motion. What does the areaunder the graphs indicate?

5.Write any two equations of motion of a body moving with uniform acceleration.

6. Plot a velocity- time graph for a body moving with uniform acceleration.

7.Plot position – time graph for a body having uniformly retarded motion

8. What does the speedometer of car indicate?

9. Two cars are running at velocities of 60 km /hr and 45 km/hr respectively. Calculate the relative velocity of car A, if (i) they are both travelling eastwards; and (ii) car A is travelling eastwards and car B is travelling westwards.

10. A body goes from A to B with a velocity of 40 m/sec, and comes back from B to A with a velocity of 60 m/sec.What is the average velocity of the body during the whole journey.

Answer the following questions each question carries 2 marks

11. A player throws a ball upwards with an initial speed of 39.2 m/sec.

(a) What is the direction of acceleration during the upward motion?

(b) Find the velocity and acceleration of the ball at the highest point.

(c) Find the height through which the ball rises, and the time after which it returns to the player's hands.

12. From the top of a tower 100m height, balls is dropped, and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/sec. find when and where the two balls meet. Take g = 9.8 m/sec^2

13.The distance travelled by a body is found to be directly proportional to the square of time. Is the body moving with uniform velocity or with uniform acceleration?

14.The displācement (x) of a particle moving in one dimension, under the action of a constant force related to time t by the relation $t = \sqrt{x} + 3$ where x is in meters, and t is in seconds. Find the displacement of the particle when its velocity is zero.

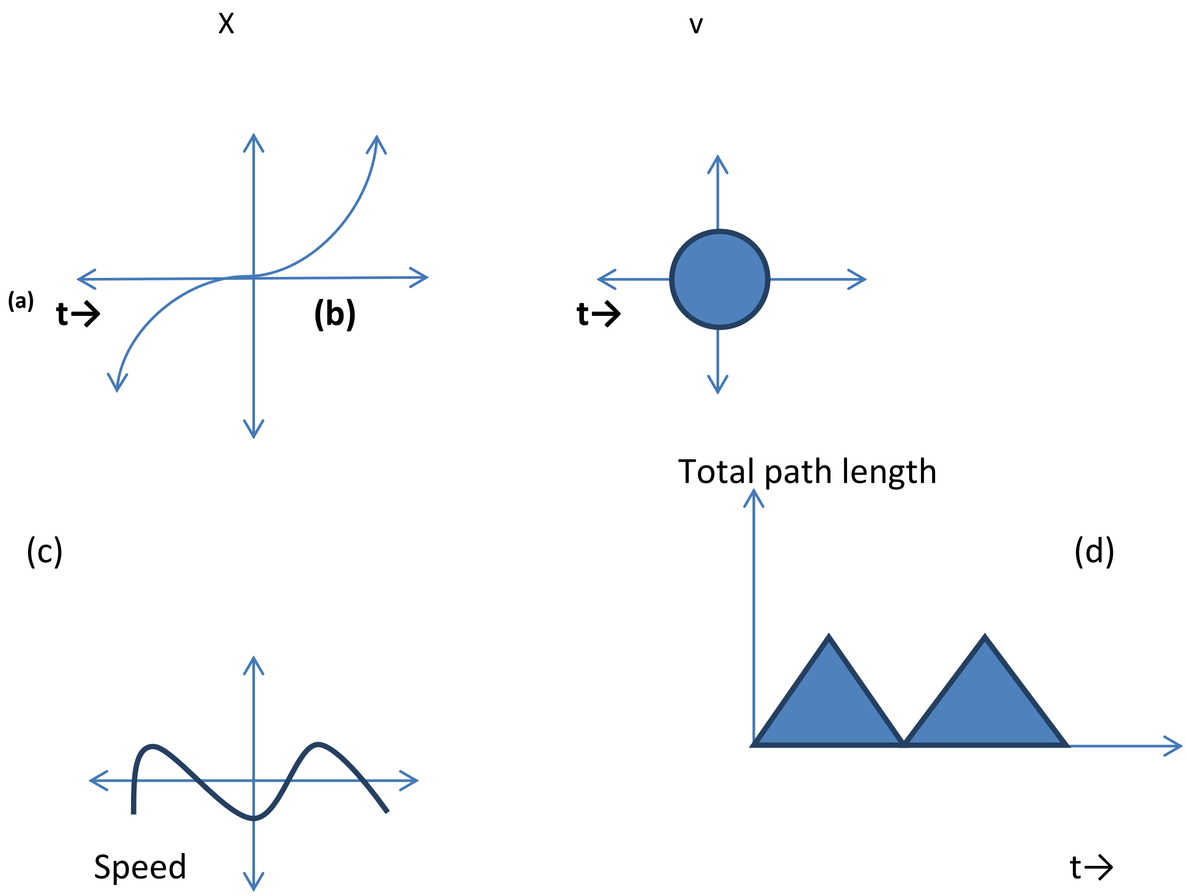
15. Can a body have zero velocity, and finite acceleration?

16. Can a body have constant speed, but a varying velocity?

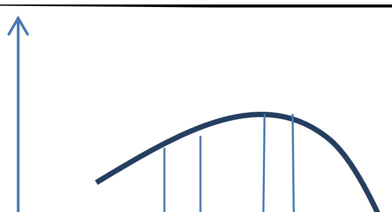
17.Can a body have constant velocity, but a varying speed?

18. Why is itthat a parachute descends slowly whereas a stone dropped from the same height fallsrapidly?

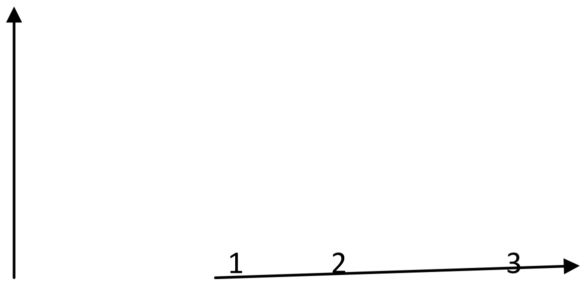
19.Look at the graphs (a) to (d) in fig carefully and state with reasons which of these cannot possibly represent one dimensional motion of a particle.



20. Fig gives the x-t plot of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval



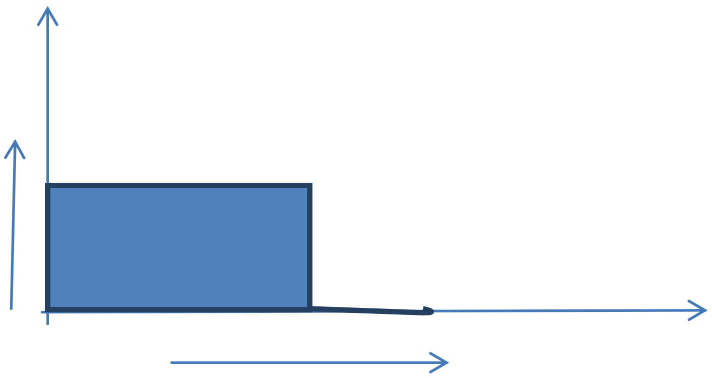
x



t→

21.The acceleration –time graph for a body is shown below. Plot the corresponding velocity-time graph

a



t

21. Suggest a suitable physical situation for each of the following graphs

x

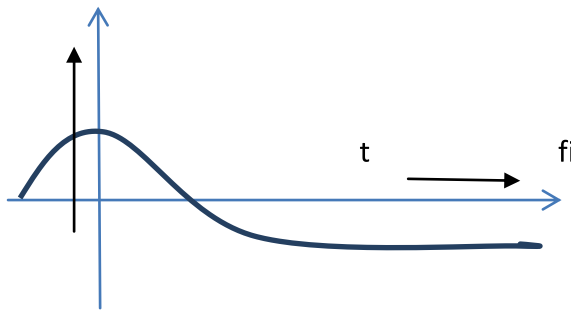
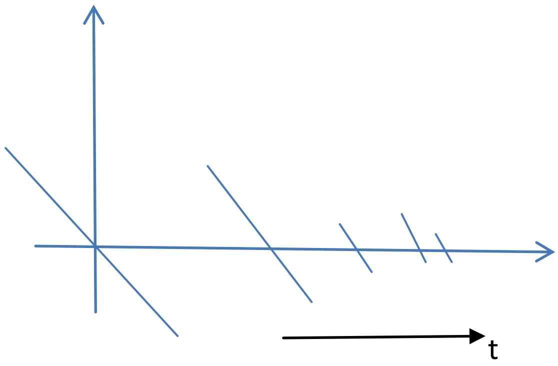


fig (i)

v

fig (ii)



22. Give the equations of motion of a body falling under gravity. Also give the graphs showing the variation of (i) acceleration of a body with time

(ii) Velocity of a body with time

(iii) Distance with time in case of a freely falling body.

23. Discuss the motion of an object under free-fall.

24. Derive a relation between the position and time for a particle moving with uniform acceleration.

25. Derive a relation for the distance covered in the nth secondby a uniformly accelerated body.

26. Show that when a body has uniformly accelerated motion, the distance covered it in a certain interval is equal to the area under the velocity-time graph for that time interval.

27. A body is moving with uniform acceleration its velocity after 5 seconds is 25 m/sec and after 8 seconds is 34 m/sec. Calculate the distance it will cover in the 10^{th} second.

28. The speed of a train increases at a constant rate α from zero, to v, and then remains constant for an interval, and finally decreases to zero at a constant rate β. If L be the total distance described, prove that the total time taken is

(L/v) + (v/2) (1/α + 1/β)

29. A body moving with a uniform acceleration describes 12 m in the third second of its motion and 20m in the fifth second. Find the velocity after 10 seconds.

<u>Description of Motion in Two and Three Dimensions</u>

<u>Main Points:</u>

1.Scalar quantities with magnitudes only. Examples aredistance, speed mass and temperature.

2.Vector quantities with magnitude and direction both. Examples aredisplacement, velocity and acceleration. They obey special rules of vector algebra.

3.A vector '**A**' multiplied by a real number**'λ' is** also a vector, whose magnitude is**'λ'**times the magnitude of the vector**' A '**and whose direction is same or opposite depending upon whether**'λ'** is positive or negative.

4.Two vectors **A** and **B** may be added graphically using head to tail method or parallelogram method.

5. Vector addition is **commutative:**

A + B = B + A

It also obeys the **associative law:**

(A + B) + C = A + (B +C)

6.A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has the properties:

A + 0 =A

λ0 =0

$\bar{0}\ .\bar{A}=\bar{0}$

7.The subtraction of vector **B** from **A** is defined as the sum of **A** and **–B**:

A – B = A + (-B)

8.A vector **A** can be resolved into components along two given vectors a and b lying in the same plane:

A= λ a + μb whereλ and μ are real numbers.

9.A unit vector associated with a vector A has magnitude one and is along the vector A :

$n^\wedge = A/|A|$ The unit vectors i^,j^,k^ are vectors of unit magnitude and point in the direction of x, y, and z- axes respectively in a right handed coordinate system.

10. A vector 'A' can be expressed as

$A = A_x i^\wedge + A_y j^\wedge$ where A_x , A_y are components along x- and y- axes . If vector A makes an angle Θ with the x- axis, then $A_x = A \cos\Theta$, $A_y = A \sin\Theta$ and $A = |A| = \sqrt{A_x^2 + A_y^2}$,

$\tan\Theta = A_y/A_x$.

11. Vectors can be conveniently added using analytical method. If sum of two vectors' A' and 'B' , that lie in x-y plane is 'R', then:

$R = R_x i^\wedge + R_y j^\wedge$, where $R_x = A_x + B_x$ and $R_y = A_y + B_y$

12. The position vector of an object in x-y plane is given by $r = x i^\wedge + y j^\wedge$ and the displacement from position r to position r' is given by

$\Delta r = r' - r$

$$= (x'-x)i^\wedge + (y'-y)j^\wedge$$

$$= \Delta x\, i^\wedge + \Delta y\, j^\wedge$$

13. If an object undergoes a displacement Δr in time Δt, its average velocity is given by

$\bar{V} = \Delta r/\Delta t$. The velocity of an object at time t is the limiting value of the average velocity as Δt tends to zero..

$V = \lim_{\Delta t \to 0} \Delta r/\Delta t = dr/dt$ .It can be written in unit vector notation as

$V = v_x i^\wedge + v_y j^\wedge + v_z k^\wedge$ where $V_x = dx/dt$, $V_y = dy/dt$,$V_z = dz/dt$

When position of an object is plotted on a coordinate system v is always tangent to the curve representing the path of the object.

14. If the velocity of an object changes from v to v' in time Δt, then its average acceleration is given by $\bar{a} = (v-v')/\Delta t = \Delta v/\Delta t$

The acceleration a at any time t is the limiting value of $\bar{a}$ as $\Delta t \to 0$

$a = \lim_{\Delta t \to 0} \Delta v/\Delta t = dv/dt$

In component form, we have $a = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

Where $a_x = dv_x/dt$, $a_y = dv_y/dt$, $a_z = dv_z/dt$

15. If an object is moving in a plane with constant acceleration $a = |a| = \sqrt{a_x^2 + a_y^2}$

And its position vector at time t = 0 is r_0, then at any other time t, it will be at a point given by

$r = r_0 + V_0 t + \frac{1}{2} a t^2$

and its velocity is given by :

$V = V_0 t + at$ where V_0 is the velocity at time t = 0

In component form

$X = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$

$\mathbf{Y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2}$

$\mathbf{V_x = V_{0x} + a_x t}$

$\mathbf{V_y = V_{0y} + a_y t}$

Motion in a plane can be treated as superposition of two separate simultaneous one dimensional motions along two perpendicular directions.

16. An object that is in flight after being projected is called a projectile.If an object is projected with an initial velocity $\mathbf{V_0}$ making an angle Θ_0, with x=axis and if assume its initial position to coincide with the origin of the coordinate system, then the position and velocity of the projectile at time t are given by

$\mathbf{X = (V_0 \cos\Theta_0)t}$

$\mathbf{Y = (V_0 \sin \Theta_0)t - \frac{1}{2} g t^2}$

$\mathbf{V_x = V_{0x} = V_0 \cos\Theta_0}$

$\mathbf{V_y = V_0 \sin\Theta_0 - gt}$

The path of a projectile is parabolic and is given by

$\mathbf{Y = (\tan \Theta_0)\, x - gx^2/2V_0\cos \Theta_0)^2}$

The maximum height that a projectile attains is

$H_{m=}(V_0 \sin\Theta_0)^2/2g$

The time taken to reach this height is

$t_m = V_0 \text{Sin } \Theta_0/g$

The horizontal distance travelled by a projectile from its initial position to the position it passes y = 0 during its fall is called the RANGE, R of the projectile. It is:

$R = V_0^2 \text{Sin } 2\Theta_0/g$

Projectile motion

Motion along vertical direction

Motion along horizontal direction

force of gravity acting on the body

Equations of motion of a body moving with constant accelaration

No force in the horizontal direction and hence horizontal velocity remains constant.

17. When an object follows a circular path at constant speed. The motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2/R$. The direction of a_c is always towards the centre of the circle.

The angular speed is the rate of change of angular distance. It is related velocity v by

$V = \omega R$. The acceleration is $ac = \omega^2 R$.

- If T is the time period of revolution of the object in circular motion and ν is the frequency then we have $\omega = 2\pi\nu V = 2\pi\nu R$ $\qquad a = 4\pi^2\nu^2 R$
- Centripetal force is the name given to the force that provides inward radial acceleration to a body in circular motion.We should always look for some material

force like tension, gravitational force, electrical force, frictionetc. as the centripetal force.

18. The path length traversed by an object between two points is not the same as the magnitude of displacement always. The displacement depends only on the end points; whereas the path length depends on the actual path. The two quantities are equal only if the object does not change its direction during the course of its motion. In all other cases, the path length is greater than the magnitude of displacement.

19. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time. The two are equal only if the path length is equal to the magnitude of the displacement.

Answer the following

1.When are two vectors equal?

2.What are co-initial and collinear vectors?

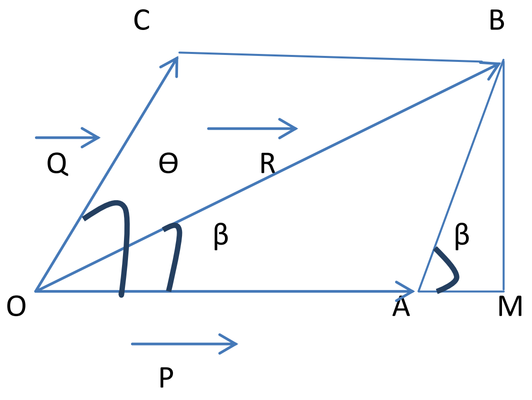
3.When are two vectors equal?

4.State the triangle law of vector addition

5.State the parallelogram lawof vector addition.

6.Using the parallelogram law of vectors, find the magnitude and direction of the resultant R in the following fig

Discuss cases for (i) $\Theta = 0^\circ$ (ii) $\Theta = 90^\circ$ (iii) $\Theta = 180^\circ$



7. State three important properties of vector addition.

8.What do you mean by the null vector or zero vector.

9.Write properties of zero vector.

10. Define scalar product of two vectors. Give one example.

11. Define vector product of two vectors? Give one example.

12.Express the area of a parallelogram in terms of cross product of two vectors.

13.Define the following terms associated with projectile motion.

(i) Projectile (ii) trajectory (iii) time of flight (IV) maximum height (v) Horizontal Range

14. Give two examples of projectile motion.

15.Write two applications of projectile motion

16.At what angle with the vertical should a projectile be fired so that its range is maximum?

17. A projectile is fired with 'u' at an angle of projection Θ. Write

(i) The equation of trajectory

(ii) Expression for the maximum height reached

(iii) Expression for the time of flight

(iv)Expression for the time taken to reach the maximum height.

(v)expression for the horizontal range.

(VI) the other angle of projection for the same horizontal range.

18.Define average angular velocity. Write its units and dimensions.

19.Write the relation between angularvelocity (ω), timeperiod (T) and frequency (v).

20.Write relation between angular acceleration and linear acceleration.

21.Find the min velocity for which the horizontal range is 39.2m

22. Prove that the Max horizontal range is 4 times the max height attained by a projectile.

23. Calculate the magnitude of the vector $\vec{r}$ = (3i^ + 4 j^ + 7 k^)

24. Prove that the vectors (i ^ + 2j^ + 3 k^)and (2i^ – j^) are perpendicular to each other.

25.A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circleusing a 1m length of the string?

26. A bullet is fired at an angle of 30˚ with the horizontalhits the ground 3 km away. By adjusting its angle of projection, can one hope to hita target 5 km away? Assume the muzzle speed to be fixed, and neglect air resistance. Take g = 10 m/sec$^{2.}$

27.Show that the trajectory of a projectile is a parabola.

28.From the top of a building 19.6 m high, a ball is projected horizontally, after how long does it strikes the ground? If the line joining the point of projection to the point, where it hits the ground makes an angle of 45˚ with the horizontal, what is the initial velocity of the ball?

29. A projectile is fired at an angle ϴ with the horizontal. Obtain the expressions for (i) the maximum height attained (ii) the time of flight (iii) the horizontal range.

30. What do you mean by centripetal force ? Derive an expression for the centripetal force.

31. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 seconds. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?

32. A cricket ball is thrown at a speed of 28 m/sec in a direction 30˚ above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to thesame level and (c) the distance from the thrower to the point where the ball returns to the same level.

33. A particle starts rom origin at t=0 with a velocity of 5.0 i^ m/sec and moves in x-y plane under the action of a force which produces a constant acceleration of (3.0i^ + 2.0 j^) m/sec^2. What is the y-coordinate of the particle at the instant its x – coordinate is 84 m? (b)What is the speed of the particle at this time?

Newton’s Laws of Motion

(10 Marks)

Main points

1.**Newton’s first law of motion** states that everybody continues to be in its state of rest or of uniform motion unless it is acted by an external force or it can also be stated as ‘‘ If external force on a body is zero, its acceleration is zero.”

2.Momentum (p) of a body is the product of mass (m) and velocity (v):

P=mv

3. Newton’s Second law of motion : It states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which force acts.

Thus **F= k dp/dt= k ma**

The SI unit of force is: 1N= 1kgms^{-2}

- The second law is consistent with the First law (F=0 implies a=0)
- It is a vector equation
- It is applicable to a particle, and also to a body or a system of particles, provided F is the total external force on the system and a is the acceleration of the system as a whole.
- F at a point at a certain instant determines acceleration at the same point at that instant.Acceleration at an instant does not depend on the history of motion’
- Force is not always in the direction of motion .Depending on the situation F may be along V, opposite to v, normal to v, or may make some other angle with v. In every case it is parallel to acceleration.
- If v=0 at an instant, i.e., if a body is momentarily at rest, it does not mean that force or acceleration are necessarily zero at that instant. For ex: When a ball thrown upward reaches its maximum height, but the force continues to be its weight ‘mg‘ and the acceleration is ‘g’ the acceleration due to gravity.

4. Impulse is the product of force and time which equals change in momentum. The notion of impulse is useful when a large force acts for a short time to produce a

measurable change in momentum. Since the time of action of the force is very short,one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.

(i) Impulse = F X Δt = m (v- u)

(ii) Unit of measurement of Impulse is Ns

5.**Newton's third law of motion:**

"**To every action, there is an equal and opposite reaction."**

The law can be stated:

Forces in nature always occur between pairs of bodies. Force on a body A by a body B is equal and opposite to the force on the body B by A.

- Action and reaction forces are simultaneous forces.
- There is no cause-effect relation between action and reaction.
- Any of the two mutual forces can be called action and the other reaction.
- Action and reaction can act on different bodies and so they cannot be cancelled out.
- The internal action and reaction forces between parts of a body do however sum to zero.
- The equation mg = R for a body on a table is true only if the body is in equilibrium. The two forces mg and R can be different. The equality of mg and R has no connection with the third law of motion.
- $\vec{F}_{AB} = -\vec{F}_{BA}$

6. **Law of conservation of momentum:**

"The total momentum of an isolated system of particles is conserved. "This law follows from the second law of motion.

7. Friction

- Frictional force opposes relative motion between two surfaces in contact.
- It is the component of the contact force along the common tangent to the surface in contact.

- **Static frictional force' f_s'**oppose impending relative motion: **kinetic frictional force ' f_k '**opposes actual relative motion.
  - They are independent of the area of contact.
  - They satisfy the following approximate laws:

    $f_s \leq (f_s)_{max} = \mu_s R$

    $f_k = \mu_k R$

    whereμ_s(coefficient of static friction) and μ_{k}(Coefficient of kinetic friction) are constants characteristic of the pair of surfaces in contact.
  - **$\mu_k < \mu_s$**
  - Static frictional force is a self-adjusting forceup to its limit $\mu_s N$ ($f_s \leq \mu_s N$)

Answer the following questions. Each question carries 1 mark.

1. State the Galileo's law ofinertia?
2. Define force? Give the SI unit of force.
3. Give the Dimensional formula of force.
4. State Newton' first law of motion.
5. Define linear Momentum of a body. Give the SI unit of it.
6. Give the Dimensional formula of linearmomentum. Is it a scalar or a Vector quantity?
7. State the law of Conservation of linear momentum.
8. Express Newton's second law of motion in the vector form
9. What is meant by Impulse? Give the SI unit of it.
10. Give the Dimensional formula of Impulse.
11. Is weight a force or a mass of a body? Name the unit in which force is measured.
12. Name some basic forces in nature.
13. Constant force acting on a body of mass 3 kg changes its speed from 2 m/sec to 3.5 m/sec.If the direction of motion of the body remains u changed, what is the magnitude and direction of motion of the force?
14. A charged comb is able to attract bits of paper against the huge gravitational pull of the earth. What does it prove?
15. What is centripetal Force? Give the expression for finding the centripetal force.
16. Define force of friction .What is the cause friction?
17. Define coefficient of kinetic friction
18. Define angle of friction
19. Define angle of Repose
20. Compare $\mu_k$withμ_s. Is it reasonable to expect the value of coefficient of friction to exceed unity?
21. State Newton's third law of motion.

Answer the following. Each question carries 2 marks

1.What do you mean by inertial frame of reference? Why is it so important? Give an example of inertial frame of reference.

2.A force acts for 20 sec on a body of mass 10 kg after which the force ceasesand the body describes 50 m in the next 10 secs. Find the magnitude of the force.

3.A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Find the magnitude and direction of the acceleration.

4.A constant retarding force of 50 N is applied to a body of mass 10 kg moving initially with a speed of 15m/sec. How long the body does takes to stop.

5.What is linear momentum of a body how is it related to impulse?

6.Write two applications of the concept of Impulse.

7. What do you mean by free-body diagram?

8. Why does a cricket player lower his hands while catching a ball?

9. How does the spring balance weigh the weight of a body?

10. Two masses $m_1$and m_2 are connected at the ends of a light inextensible string that passes over frictionless pulley. Find the acceleration, tension in the string and thrust on the pulley when the masses are released.

Answer the following. Each question carries 3 marks.

1. Show that Newton's second law is the real law of motion
2. State and prove the law of conservation of linear momentum
3. A force 10 N acts on a body for 3 X 10 $^{-6}$sec calculate the impulse. If mass of the body is 5 g . Calculate the change in velocity.
4. A man weighs 60 kg. He stands on a weighing machine in a lift, which is moving
   (a) Upwards with a uniform speed of 6 m/s
   (b) Downward with a uniform acceleration of 6m/s^2.
   (c) Upwards with a uniform acceleration of 6 m/s^2

   What would be the reading of the scale in each case? What would be the reading if the lift mechanism fails, and fall freely?

5. A shell of mass 20 g is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s. What is the recoil speed of the gun? What is the recoil speed of the gun?

6. Given the magnitude and direction of the net force acting on
   (a) A drop of rain falling down with a constant speed
   (b) A cork of mass 10 g floating on water
   (c) A kite skilfully held stationery in the sky
   (d) A car moving with a constant velocity of 30 km/hron a rough road.
   (e) A high-speed electron in space far from all gravitating objects and free of electric and magnetic fields.

7.A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction

and magnitudeof the net force on the pebble

(a) During its upward motion
(b) During its downward motion
(c) At the highest point where it is momentarily at rest.Do your answers alter if the pebble were thrown at an angle of 45˚ with the horizontal direction?

8. What are the laws of limiting friction?

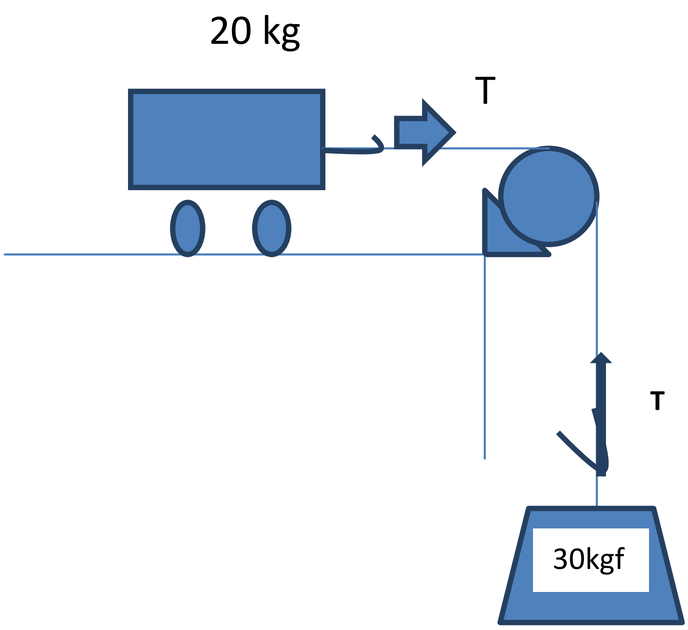
9. State and prove the law of conservation of linear momentum

10. Discuss the variation of frictional force with the applied force on a body.

Also show the variation by plotting a graph between them.

Answer the following questions. Each question carries 5 marks.

1. Show that Newton's second law is the real law of motion.
2.
3. (a)State Newton's second law of motion.
   (b) What is the acceleration of the block and trolley system sown in fig. if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? Take g = 10 m/s^2. Neglect the mass of the string.



4. (a) State Newton’s laws of motion

   (b) A machine gun has a mass of 20 kg. It fires 35 g bullets at the rate of 400 bullets per second with a speed of 400m/s.What force must be applied to the gun to keep it in position?

4. (a) State the principle of conservation of momentum.

   (b) A hammer of mass 1 kg moving with a speed of 6 m/sstrikes a wall and comes to

rest in 0.1 s, Calculate

(i) The impulse of force
(ii) The retardation of the hammer, and
(iii) The retarding force that stops the hammer.

5. What is meant by banking or roads? Discuss the motion of a car on a banked road.

Main points

1. Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of force.
2. $W = FS \cos\Theta$
3. Unit of measurement Work done is Joule or Nm
4. Work done is a scalar quantity.
5. If a graph is constructed of the components $F\cos\Theta$ of a variable force, then the work done by the force can be determined by measuring the area between the curve and the displacement axis.
6. Energy of a body is defined as the capacity of the body to do work.
7. Energy is a scalar quantity.
8. The Dimensional formula of Energy is same as Work and is given by ML^2T^{-2}
9. The SI unit of Energy is same as that of Work. i.e., Joule.
10. The work-Energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body.

$$K_f - K_i = W_{net}$$

11. Momentum of a body is related to Kinetic Energy by

$$P = \sqrt{2m E_k}$$

12. A force is said to be conservative if work done by or against the force in

Movinga body depends only on the initial and final positions of the body

not on the nature of the path followed between the initial and final positions.

For ex: Gravitational Force,Elastic Force, Electrostatic Force etc.

13. A force is said to be non-conservative if work done by or against the force

in moving a body depends upon the path between the initial and final positions.

14. For a conservative force in one dimension, we may define potential energy

Function U(x) such that

$$F(x) = -dU(x)/dx$$

$$\text{Or} \quad U_i - U_f = \int_{x_i}^{x_f} F(x)\, dx$$

15. The principle of conservation of mechanical energy states that the total mechanical energy of a body remains constant if the only forces that act on the body are conservative.

16. The gravitational potential energy of a particle of mass m at a height x above the earth's surface is U(x)=mgx

Where the variation of g with height is ignored.

17. Theelastic potential energy of a spring of force constant k and extension x is

$U(x) = ½ k x^2$

18. Power is defined as the time rate of doing work.

19. Average Power is given by P_{av} =W/t

20 Power is a scalar quantity, P= F.v = FvcosΘ, P = dW/dt

21. SI unit of Power is Watt

22. The Dimensional Formula of Power is $M^1 L^2 T^{-3}$

23. The Scalar or dot product of two vectors A and B is written as A.B and is a scalar quantity given by : A.B = AB cosΘ where Θ is the angle between A and B . It can be positive, negative or zero depending upon the value of the scalar product of the two vectors can be interpreted as the product of magnitude of one vector and the component of the other vector in the direction of the first vector.

For unit vector I^.i^=j^.j^=k^.k^= 1 and i^.j^= j^.k^=k^.i^=0

24.For two bodies, the sum of the mutual forces exerted between them is zero from Newton's third law, $F_{12} + F_{21}$ =0

But the sum of the work done by the two forces need not always cancel. i.e.,

$$W12 + W21 \neq 0$$

However, it may sometimes be true.

25.Elastic collision is a collision in which both momentum and kinetic energy are conserved

26. In inelastic collision momentum is conserved but kinetic energy is not conserved.

27.In a collision between two bodies the Coefficient of restitution can be defined as the ratio between the velocity of separation to the velocity of approach i.e.

e = velocity of separation /velocity of approach

If e = 1 then the collision is perfectly elastic, if e=0, then the collision is perfectly inelastic.

Answer the following questions.Each question carries one mark.

1.Define work. How can you measure the work done by a force?

2. Write the Dimensional formula of work. Give its SI unit of measurement

3. Give examples of zero work done

4. A body is compelled to over along the x-direction by a force given by

F= (2i^-2j^+k^) N

What is the work done in moving the body?

(i)a distance of 2m along x-axis

(ii) a distance of 2m along y-axis.

5. What do you mean by a conservative force? Is frictional force a conservative force.

6. Define power. Is it a scalar or vector?

7. Define one electron volt

8. State work – energy theorem

9. Compare 1kwh with electron volt

10. Two bodies of mass 1 kg and 4 kg have equal linear momentum. What is the ratio of their kinetic energies?

11. Distinguish between elastic and inelastic collision.

12. A light and a heavy body have the same momentum. Which one will have the greater kinetic energy?

13. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the law of conservation of momentum?

14. Define coefficient of restitution.

15. What is the amount of work done, when a body of mass m moves with a uniform speed v in a circle?

Answer the following questions. Each question carries 2 marks.

1. Plot a graph showing the variation of Force (F) with displacement(x), if its force equation is given by F = -kxName at least four commonly used units of energy.

2. Differentiate between a conservative force and a non-conservative force.

3. What is Einstein's energy-mass equivalence relation?

4.A man whose mass is 75kg walks up 10 steps each 20 cm high, in 5 sec. Find the power he develops Take g= 10m/s2 .

5. A block of mass 2 kg moving at a speed of 10 m/s accelerates at 3 $m/s^2$for 5 sec. Compute the final kinetic energy.

6. The sign of work done by a force on a body is important to understand.State carefully if he following quantities are positive or negative.

(a)Work done by a man in lifting a bucket out of a well

by means of a rope tied to the bucket.

(b) Work done by gravitational force in the above case.

(c) Work done by friction on a body sliding down an inclined plane

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

7. What happens to the potential energy of a body when the work done by the conservative force is positive?

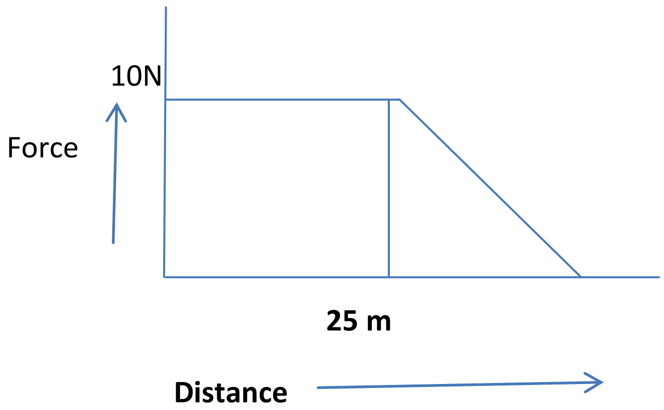
Answer the following. Each question carries 3 marks.

1. State and prove work-energy theorem
2. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to $-k/r^{2,}$ where k is a constant. What is the total energy of the particle?
3. State and prove the law of conservation of energy.
4. Show that Potential energy y = mgh
5. Show that kinetic energy = ½ mv^2
6. Prove the law of conservation of energy at every point of the motion of a body under freefall.
7. Derive an expression to find the potential energy of a spring.

8. Plot graphs between K.E, P.E, and total energy of an elastic spring with displacement (x).
9. What is the power output of the sun if 4 x 10^{9}kg of matter per second is converted into energy in the sun?
10. A body of mass 5kg initially at rest is moved by a horizontal force of 20 N on a frictionless table. Calculate the work done by the force in 10 seconds and prove that this equals the change in kinetic energy.
11. An engine pumps out 40 kg of water per second. If water comes out with a velocity of 3 m/sec, then what is the power of the engine?

Answer the following questions. Each question carries 5 marks.

1. Discuss elastic collision in one dimensional motion.
2. Discuss the elastic collision in two dimension
3. Derive the expression for the work done by (i) Constant force and (ii) a variable force.
4. (a)What are conservative and non-conservative forces? Give examples.
   (b)A body of mass 5 kg is acted upon by a variableforce. The force varies with the distance covered by the body as shown in fig. What is the speed of the body when the body has covered 25 m. Assume that the body starts from rest?

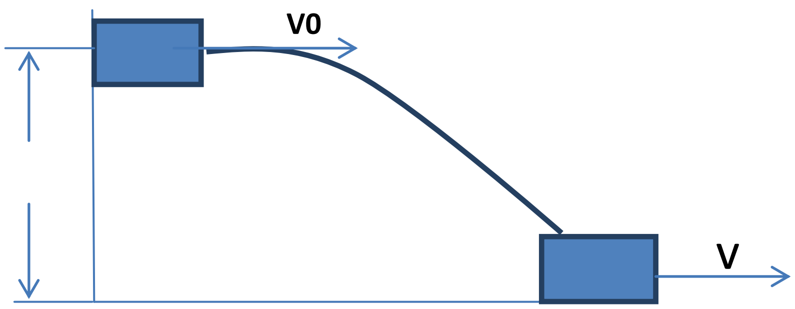


5. A 10 kg ball and a 20 kg ball approach each other with velocities 20m/s and 10 m/s respectively. What are their velocities after collision if the collision is perfectly elastic. Also show that kinetic energy before collision is equal to kinetic energy after collision.

6. Starting with an initial speed v_0,a mass m slides down a curved frictionless track

Arriving at the bottom with a speed v. From what height did it start?

h



7. A ball dropped from a height of 8m hits the ground and bounces back

to a height of 6m only. Calculate the frictional loss in kinetic energy.

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

MAIN POINTS

1. A rigid body is one for which the distance between different particles of the body do not change, even though there is a force acting on it.
2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure rotation or a combination of translation and rotation.
3. In pure translation every particle of the body moves with the same velocity at any instant of time.
4. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity at any instant of time.

5. Angular velocity is a vector. Its magnitude is $\omega = d\Theta/dt$ and it is directed along the axis of rotation. For rotation about a fixed axis, this vector ω has a direction.
6. The vector or a cross product of two vectors A and B is a vector written as A X B. The magnitude of this vector is $AB \sin \Theta$ and its direction is given by right handed screw or the right hand rule.
7. The linear velocity of a particle of rigid body rotating about a fixed axis is given by $v = \omega \times r$ wherre r is the position vector of the particle with respect to an origin along the fixed axis. The relation applies even to more general rotation of a rigid body with one point fixed. In that case r is the position vector of the particle with respect to the fixed point taken as the origin.
8. The centre of mass of the system particles is defined as the point whose position vector *Is* $\boldsymbol{R} = (\sum m_i r_i)/\boldsymbol{M}$
9. Velocity of the centre of mass of a system of particles is given by $\mathbf{V} = \mathbf{p}/\mathbf{M}$, **where P is the linear momentum of the system.** The centre of mass moves as if all the mass of the system is concentrated at this point and all the external forces act at it .If the total external force is zero, then the total linear momentum of the system is constant.
10. The angular momentum of a system of n particles about the origin is

$$\mathbf{L} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{p}_i)$$

11. The Torque or moment of force on a system of n particles about the origin is given by

$$T = \sum_{i=1}^{n} (r_i X F_i)$$

The force F_i acting on the i^{th} particle includes the external as well as the internal forces.

Assuming Newton’s third law and that forces between any two particles act along the

line joining the particles, we can show that $T_{int} = 0$ and $dL/dt = T_{ext}$.

12. A rigid body is in mechanical equilibrium if
(i)It is I n translational equilibrium, i.e.’ the total external force on it is zero:
$\sum F_i = 0$and
(ii) It is in rotational equilibrium, i.e., the total external torque on it is zero:

$$\sum \tau_{i=} \sum r_i \times F_i = 0$$

13. The centre of gravity of an extended body is that point where the total gravitational torque on the body is zero.

14. The moment of inertia of a rigid body is defined by the formula $\mathbf{I = \sum m_i r_i^2}$
 Where ri Is the perpendicular distance of the i^{th} point of the body from the axis. The kinetic energy of rotation is $K = ½ I \omega^2$

15. The theorem of parallel axes :$I' = I_G + M a^2$ where $I_{G\ moment}$ of inertia bout the axis passing through centre of gravity.

Allows us to determine the moment of inertia of a rigid body about an axis as the sum of the moment of inertia of the rigid body about a parallel axis passing through its centre of mass and the product of its mass and the square of the perpendicular distance between the two parallel axes.

16. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.

17. For a rigid body rotating about a fixed axis of rotation $L_z = I_z\omega$ whereIz is the moment of inertia about z-axis. In general , the angular momentum about the axis of rotation , L is along the axis of rotation. I n that case $|L| = L_z = I\omega$. The angular acceleration of a rigid body rotating about a fixed axis is given by $I\alpha = T$. If the external torque acting on the body T = 0 , the component of angular momentum about the fixed axis of such a rotating body is constant,

18. For rolling motion without slipping $v_{cm} = r\omega$, where vcmis the velocity of translation.' r ' is he radius and m is the mass of the body. The kinetic energy of such rolling motion of the body is the sum of kinetic energies of translation and rotation.

$$K = ½ m v_{cm}^2 + ½ I \omega^2$$

19. To determine the motion of the centre of mass of a system, we need to know external forces acting on the body.
20. The time rate of change of angular momentum is the Torque acting on the body,
21. The total torque on a system is independent of the origin, if the total external force is zero.
22. The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one part of the body to the other part of the body.

23. **Principle of conservation of angular momentum:**
 It states that if there is no external torque acting on the system the total angular momentum of the system remains constant.

 i.e. If $\tau_{ext} = 0$, $dL/dt = 0$, Hence $L =$ constant.

24.**Kepler'slaws:**

(i) All planets revolve around the sun in elliptical orbits with sun at one of its foci.

(ii) The line joining the sun to the planet sweeps out equal areas in equal intervals of time

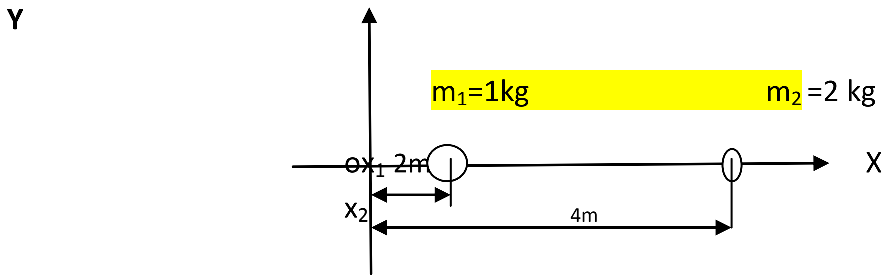
That is the areal velocity of a planet remains constant.

(iii)The square of the timeperiod of revolution of the planet is proportional to the cube of the semi major axis.

$$T^2 \propto R^3$$

Answer the following questions.Each question carries one mark

1. Define the term 'Centre of mass of a system of particles.
2. What will be the centre of mass of the pair of particles described belowin fig on the x-axis?



3. If two masses are equal where does their centre of mass lie?
4. Define a rigid body?
5. Is centre of mass same as the centre of gravity of a body? How can a rigid body be balanced?
6. Write an expression for the the velocity of the centre of mass of particles.
7. Does the total momentum of a system of particles depend upon the velocity of the centre of mass?
8. Write the expression for the acceleration of the centre of mass of particles. A projectile fired into the air suddenly explodes into several fragments. What can you say about motion of the fragments after collision?
9. Briefly explain about the centre of mass of the earth- moon system.
10. Define one radian.
11. Convert one radian into degrees.
12. Define angular velocity? What is its SI unit?

13. Define angular acceleration. What are its SI units?
14. Write the dimensional formula of angular acceleration.
15. Write the dimensional formula of angular velocity.
16. Considering rotational motion about some fixed axis, write equations corresponding to
    (i) $x(t) = x(0) + v(0)\,t + \frac{1}{2} a t^2$
    (ii) $v^2(t) = v^2(0) + 2a\,[x(t) - x(0)$
    (iii) $v(t) = v(0) + at$
    (iv) $v(t) = \dfrac{[\ v(t) + v(0)\ ]}{2}$
17. Define angular momentum. Write the SI unit of angular momentum.
18. Name the dimensional constant whose dimensions are same as that of angular momentum.
19. Does the magnitude and direction of angular momentum $\bar{L}$ depend on the choice of the origin?
20. Express torque in terms of the rate of change of linear momentum.
21. Define moment of inertia of a body.
22. Is moment of inertia scalar or vector physical quantity? Write the SI unit of moment of inertia
23. Why is the most of the mass of a fly wheel placed on the rim?
24. Why are the spokes fitted in a cycle wheel?
25. The cap of pen can be opened with help of two fingers than with one finger. Explain why?
26. State the Work-Energy theorem for rotational motion.
27. State the law of conservation of anguar momentum.
28. For an isolated system plot a graph between moment of inertia (I) and angular velocity (ω)
29. What is the law of rotation?
30. State the theorem of parallel axes
31. State the theorem of perpendicular axes.

Answer the following questions.Each question carries 2 marks.

1. Write an expression for the moment of inertia of a ring of mass M and radius R,
    (i) About an axis passing through the centre , and perpendicular to its plane
    (ii) About a diameter
    (iii) About a tangent to its plane
    (iv) About a tangent perpendicular to the plane of the ring.
2. Write an expression for the moment of inertia of a circular disc of mass M and radius R,

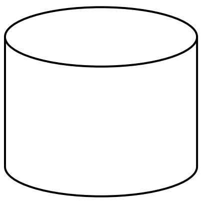
(i) About an axis passing through the centre , and perpendicular to its plane
(ii) About a diameter
(iii) About a tangent to its plane
(iv) About a tangent perpendicular to the plane of the disc.

3.Calculate the angular momentum of the earth rotating about its own axis.

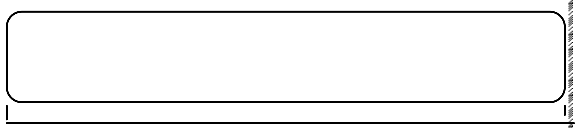
Mass of the earth = 5.98 X 10^{27} kg, radius of the earth = 6.37 X 10^{6} m.

5. A thin metal hoop of radius 0.25 m and mass 2 kg starts from rest, and rolls downan inclined plane. Its linear velocity on reaching the foot of the plane is $4ms^{-1}$. What is the rotational kinetic energy when it reaches the foot of the inclined plane?
6. Three mass points m_1, m2, m3 are located at the vertices of an equilateral triangle of length 'a'. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?
7. If the angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved?
8. A sphere of radius 10 cm weighs 1 kg. Calculate the moment of inertia
   (i) About the diameter
   (ii) about the tangent
9. A wheel rotates with a constant angular acceleration of 3.6 rad/s2. If the angular velocity of the wheel is 4.0 rad/s at t= 0. What angle does the wheel rotate in 1 s? What will be its angular velocity at t= 1s?
10. Mark the centre of mass of the following figures.

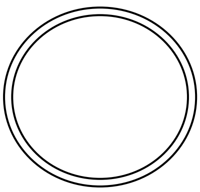
(i) Right circular cylinder



(ii) Cylindrical rod

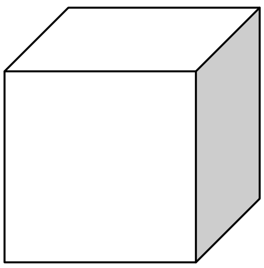


(iii)



Circular ring

(iv)



Symmetrical cube.

11. To maintain a rotor at a uniform angular speed of 1oo s^{-1} an engine needs

to transmit a torque of 200 Nm. What is the power of the engine required?

12. Two cars are going around two concentric circular paths at the same angular speed. Does the inner or the outer car have the larger speed?

Answer the following questions. Each question carries 3 marks.

1. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 A° ($1A^{\circ} = 10^{-10}$m). Find the approximate location of the centre of mass of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom, and nearly all the mass of an atom is concentrated in its nucleus.
2. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
3. A Long playing record revolves with a speed of 33⅓ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0.15, which of the two coins will revolve with the record.
4. State and prove the law of conservation of angular momentum.
5. Derive an expression for Torque acting on a body.
6. Explain the motion of centre of mass of a body with examples.
7. Find the torque of a force 7i^ + 3j^ -5k^ about the origin. The force acts on a particle whose position vector is I^ – j^+ K^?
8. What constant torque should be applied to a disc of mass 16 kg and diameter 0.5m; so that it acquires an angular velocity of 4π rad/s in 8 s? The disc is initially at rest, and rotates about an axis through the centre of the disc in a plane perpendicular to the disc.
9. A uniform ring of radius 0.5 m has a mass of 10 kg. A uniform circular disc of same radius has a mass of 10 kg. Which body will have the greater? moment of inertia?Justify your answer.
10. Obtain a relation between torque applied to a body and angular acceleration produced. Hence define moment of inertia.
11. If the earth were to suddenly contract to half of its present size without change in its mass, what will be the duration of the new day.
12. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground? with maximum velocity?

Gravitation

MAIN POINTS

(06 Marks)

1. Newton's' law of universal gravitation states that the gravitational force of attraction between any two particles of masses m1 and ma2 separated by a distance r has a magnitude

$$F = G \frac{m_1 m_2}{r^2}$$ where G is the Universal gravitational constant, which has the value 6.672 X 10^{-11} N $m^2 kg^{-2}$.

2. In considering motion of an object under the gravitational influence of another object the following quantities are conserved,
(i) Angular momentum
(ii) Total mechanical energy.

3. If we have to find the resultant gravitational force acting onthe particle m due to a number of masses M_1, M_2,….M_n eachgiven by the law of gravitation, From the principle of superposition each force acts independently anduninfluenced by the other bodies. The resultant force $F_n$is then found by vector addition

$$F_n = F_1 + F_2 + F_3 + \ldots\ldots\ldots F_n = \sum_{i=1}^{n} F_n$$

4. Kepler's laws of planetary motion state that
   (i) All planets move in elliptical orbits with the Sun at one of the focal points
   (ii) The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.
   (iii) The square of the orbital period of a planet is proportional to the cube of the semi major axis of the elliptical orbit of the planet.

   The period T and radius R of the circular orbit of a planet about the Sun are related by

$T^2 = (4\pi^2/ GMs)$ R3where Ms is the mass of the Sun. Most planets have clearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis ' a'.

5. Angular momentum conservation leads to Kepler's second law. It holds for any central force.
6. According to Kepler's third law $T^2= K R^3$ .The constant K is same for all planets in circular orbits. This applies to satellites orbiting the earth.
7. An astronaut experiences weightlessness in a space satellite. This is because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in 'free fall' towards the Earth.
8. The acceleration due to gravity

(a)At a height ' h 'above the Earth's surface

$g (h) = GM_E/(R_E +h)^2$

$= (GME/R_E^2) [1-(2h/R_{E)}]$ for $h<<R_E$

$g (h) = g (0) [1-(2h/R_{E)}]$ where $g (0) = GME/R_E^2$

(b) At a depth 'd' below the Earth's surface is

$g (d) = GME/R_E^2 [1- (d/ R_E)] = g (0) [1- (d/ R_E)]$

9. The gravitational force is a conservative force, and therefore s potential energy function can be defined. The gravitational potential energy associated with two particles separated by a distance r is given by

$$V = - (Gm_1m_2/r) + \text{constant}$$

The constant can be given any value. The simplest choice is to take the value of it to be zero. Withthis V becomes

$$V = -Gm_1m_2/r \quad \text{where V is taken to be zero at } r\to\infty.$$

10. The total potential energy for a system of particles is the sum of the energies for all pairs of particles, with each pair represented by a term of the form given byabove equation .This follows from the principle of superposition.

If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = ½ mv^2 – \left(\frac{G M m}{r}\right)$$

That is the total mechanical energy is the sum of kinetic and potential energies.

The total energy is a constant of motion.

11. If m moves in a circular orbit of radius a about M , where M >>> m , the total energy of the system is

$$E = -\frac{GMm}{2a}$$

The total energy is negative for any bound system, that is , one in which the orbit is closed, such as an elliptical orbit. The kinetic and potential energies are

$$K = \frac{GMm}{2a}$$

$$V = -\frac{GMm}{a}$$

12. The escape speed from the surface of the earth is

$$V_e = \sqrt{2GM_E/R_E} = \sqrt{2g\,R_E}$$

and has a value of 11.2 kms^{-1}.

13. If a particle is outside a uniform spherical shell or solid sphere with a spherically symmetric internal mass distribution, the sphere attracts the particle as though the mass of the sphere or shell were concentrated at the centre of the sphere.

14. If a particle is inside a uniform spherical shell, the gravitational force on the particle is zero. If a particle is inside a homogeneous solid sphere, the force on the particle acts toward the centre of the sphere. This force is exerted by the spherical mass interior to the particle.

15. A geostationary satellite moves in a circular orbit in the equatorial plane at a approximate distance of 4.22 X 10^4km from the Earth's surface.

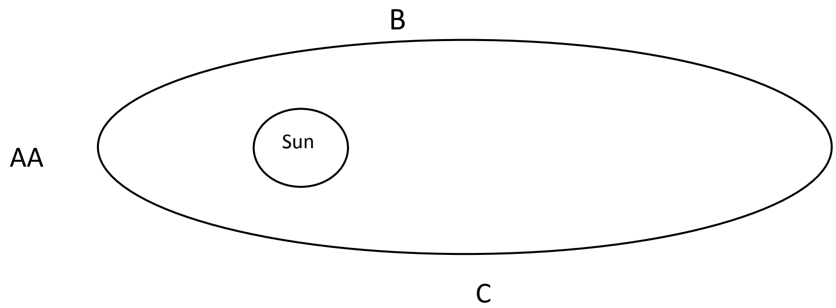
Answer the following questions. Each question carries 1 mark

1. State Newton's law of Gravitation
2. Write the SI unit of Gravitational constant. Also give its dimensional formula
3. What is the effect of medium on the value of G?
4. Define gravitational potential.
5. Name the physical quantity whose dimensional formula is same as that of the gravitational potential.
6. Define gravitational potential energy.
7. Define Orbital velocity.
8. Define escape velocity. Give its value for the earth.
9. What is the value of gravitational potential energy at infinity?
10. What do you mean by the earth's satellite?
11. Write an expression for the escape velocity of a body from the surface of the earth. Also give the factors on which it depends,
12. What does a low value of escape velocity indicates?
13. What is the period of the moon?
14. What is geocentric theory? Who propounded it?
15. What is geostationary satellite? Is it same as synchronous satellite?
16. What is heliocentrictheory? Whopropounded it?
17. StateKepler's laws of planetary motion.
18. Name the force that provides the necessary centripetal force for a planet to move around the sun in a nearly circular orbit.
19. Does speed increase, decrease, or remain constant when a planet comes closer to the sun?
20. Where does a body weigh more near the poles or the equator? Why?
21. The value of G on the earth is 6.7 X 10^{-11}Nm^{-2} kg^{-2}.What is its value on the moon?

Answer the following questions. Each question carries 2 marks.

1. Give the differences between weight and mass
2. You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
3. An astronaut inside a small space-ship orbiting around the Earth has a large size. Can he hope to detect gravity?
4. The mass and diameter of a planet have twice the value of the corresponding parameters of the earth. What is the acceleration due to gravity on the surface of the planet?

5. The planet earth is revolving in an elliptical orbit around sun as shown in the fig. At what point on the orbit will the kinetic energy be (i) minimum (ii) maximum?



6. Does a rocket really need the escape velocity of 11.2km/s initially to escape from the earth? Plot a graph showing the variation of gravitational force (F) with square of the distance, Plot a graph showing the variation of acceleration due to gravity with height or depth.
7. Two planets A and B have their radii in a ratio ‘r’. The ratio of the acceleration due to gravity on the planets is’ x’. What is the ratio of the escape velocity from the two planets?
8. Write formula for the variation of g with (i) height above the surface of the earth (ii) depth below the surface of the earth (iii) rotation of the earth.
9. Give the differences between inertial mass and gravitational mass.
10. Does the escape velocity of a body from the earth depend on :
    (i) The mass of the body,
    (ii) The location from where it is projected,
    (iii) The direction of projection,
    (iv) The height of the location from where the body is launched.

<u>Answer the following questions. Each question carries 3 marks.</u>

1. Derive an expression for finding the escape velocity of a body from the surface of the earth.
2. Deduce an expression for the velocity required by a body so that it orbits around the earth.
3. At what height above the surface of the earth will the acceleration due to gravity become 1% of its value at the earth’s surface .Take the radius of the earth , R =6400km.
4. The mass and diameter of a planet are twice those of the earth. What will be the period of oscillation of a pendulum on this planet, If it is a second’s pendulum on the earth?
5. Two masses 100kg and 10000kg are at a distance 1m from each other. At which point on the line joining them, the intensity of the gravitational field will be zero.
6. Assuming the earth to be a sphere of uniform density, what is the value of g in a mine 100 km below the earth’s surface? Given R= 6380 km and g = 9.8ms^{-2}.

7. A body of mass m is raised to a height h above the earth's surface. Show that the loss in weight due to variation in g is approximately 2mgh/R.
8. Calculate the orbital velocity for a satellite revolving near the earth's surface. Radius of the earth's surface is 6.4 X 10^{6}m and g = 10 ms^{-2}.
9. An artificial satellite circles around the earth at a distance of 3400km. Calculate the mass of the sun. Given 1 year = 365 days and G = 6.7 x $10^{-11}$$Nm^2$ $kg^{-2.}$
10. Show that the gravitational potential at a point of distance r from the mass M is given by V = - (GM/r).
11. A satellite orbits the earth at a height of 500 km from its surface. Calculate the kinetic energy, potential energy and total energy of the satellite.
    Given: Mass of the satellite = 300kg
    Mass of the earth = 6 x 10^{24} kg
    Radius of the earth = 6.4 X 10^{6}m
    G = 6.67 x 10^{-11} Nm^2 kg^{-2}

Class XI and XII students of Central Board of Secondary Education (CBSE) will be required to answer value based questions in their final examination from the academic session 2012-13.

The Central Board of Secondary Education (CBSE), whose educational process is inclusive of co-scholastic areas of life skills, attitude and values, sports and games as well as co-curricular activities, is aiming to strengthen its education system in the area of value education. For the same, the board will be introducing value-based questions in the papers of final examinations in all major subjects for classes XI and XII from the academic session 2012-13 .

The questions will be for 5 marks in a paper of 100 marks and 3-4 marks in a question paper of 70-90 marks. This will be effective from the forthcoming final examinations — classes XI and XII — in 2013.

“Whatever you are doing, put your whole mind on it. If you are shooting, your mind should be only on the target. Then you will never miss. If you are learning your lessons, think only of the lesson. In India boys and girls are taught to do this."

~ **Swami Vivekananda quotes**

1. An old woman crossing the road was holding a money purse. She was not able to walk .A pick pocket snatches away her purse. A school student of class X having seen this incident tries to help that old lady. He informs the police Inspector who stands nearby. The Inspector collects the money purse from the pickpocket and hand it over to the old lady.

(a)What values do you find in the school student?

(b)Also the police inspector in a jeep is chasing the pickpocket on a straight road. The jeep is going at its maximum speed ‘v’. The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance ‘**d**’ away. and the motorcycle starts with a constant acceleration ‘a’. Show that the pickpocket will be caught if $v \geq \sqrt{2ad}$.

Ans: (a) The student is sympathetic towards others, helping, and applies his presence of mind in solving the problems, knows how to use public services.

(b) $s = \frac{1}{2}at^2$, the distance covered by the this time interval $= s + d = vt$

$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$ The pickpocketter will be caught if t is real ad positive

This will be possible if $v^2 \geq 2ad$ or, $v \geq 2ad$.

2. Sita a student of class XII was suffering from malaria. The area is full of mosquitoes. She was not having mosquito net. Her friend Geeta has an extra net. She gave it to Sita. Also she took Gita to a Doctor, got her medicines. After a week Sita became normal

(a) Comment upon the qualities of Sita.

(b) The mosquito net over a 7 m X 4mbed is 3m high. The net has a hole at one corner of the bed through which a mosquito enters the net. It flies and sits at the diagonally opposite upper corner of the net(i) Find the magnitude of the displacement of the mosquito (ii)Taking the hole as the origin, the length of the bed as the X-axis, its width as the Y-axis and vertically up as the Z-axis, with the components of the displacement vector.

Ans: (a) Sita has a caring attitude, and concern for others.

(b)(i)$\sqrt{74}$m i.e.,$\{ (\sqrt{7^2+4^2+3^2}) \}$ (ii)The components of the vector are 7m,4m, and 3m

3. Krishna went for sight seeing to a nearby river along with his physics teacher. He noticed that the wind was blowing from the side and the sailboat still continued to move forward. He was surprised. He asked his physics teacher the explanation of this situation. The teacher having noticed his interest explained the concept through a small example. The physics of sailing is very interesting in that sailboats do not need the wind to push from behind in order to move. The wind can blow from the side and the sailboat can still move forward.

The answer lies in the well-known principle of aerodynamic lift. Imagine you are a passenger in a car as it's moving along, and you place your right hand out the window. If you tilt your hand in the clockwise sense your hand will be pushed backwards and up. This is due to the force of the air which has a sideways component and upwards component (therefore your hand is pushed backwards and up).

(a) What values could you find in Krishna?

(b) Also explain what Magnus effect is.

Ans: (a) Krishna is very interested in learning the subject; also he is interested in knowing how science helps in understanding the day to day experiences, observant, has courage to ask questions.

(b) Refer NCERT TEXT BOOK.

4. Having found his mother suffering from fever *Venkat took* her to the doctor for treatment. While checking the status, the doctor used a thermometer to know the temperature of the body. He kept the thermometer in the mouth of the patient and noted the reading as 102° F. Doctor gave the necessary medicines. After coming home, Venkat asked his mother, who is a science teacher , why mercury is used in a thermometer when there are so many liquids. Then his mother explained the reason.

(a) Comment upon the values of the mother.

(b)A newly designed thermometer has its lower fixed point and upper fixed point marked at $5^{\circ}$and 95° respectively. Compute the temperature on this scale corresponding to $50^{\circ}C$

Ans: (a) Mother has interest in educating her son and explained that Mercury has got the following properties for being used in thermometers

(i) The expansion of Mercury is fairly regular and uniform.

(ii) It is opaque and shining, hence can be easily seen through the glass tube.

(iii) Mercury is a good conductor of heat ad has low thermal capacity,

(iv) Mercury does not wet the sides of the glass tube in which it is filled.

(b) Let Θ be the temperature on the scale corresponding to 50◦C, then

(Θ -5)/(95-5) =(C- 0)/(100-0) =C/100 or Θ = 50◦

Thus, the required temperature on the scale of the designed thermometer is 50◦.

5. Having seen a big stone falling from the top of a tower Ravi pulled his friend Kiran away. The stone hit Ravi slightly and he got hurt. But he was saved from a major accident.
   (a)What made Ravi act in such a way.

   (b)From the top of a tower 100 m in height, a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/s.Find when and where the two balls meet. Take g = 9.8 m/sec2.

   Ans: (a) More observation, presence of mind & Concern (ii) In the first case h = $\frac{1}{2}gt^2$

   (b) For the second case 100-h = 25 t - $\frac{1}{2}gt^2$ where h is the height at which the two stones meet.

   i.e., 100-h = 25t- h; 25t = 100; or t= 100/25 = 4 sec. ; h=78.4m

6. A monkey is sitting on a tree. Rahim seeing the Monkey brought some fruits

   and gave them to the Monkey, and ran into the house immediately. On hearing the

   sound produced when Rahim was running the monkey was scared and climbed the

   nearby tree.

   (a) What values of Rahim inspired you?

   (b)A monkey of mass 40 Kg climbs on a rope which can stand a maximum tension of 600 N .

   In which of the following cases will the rope will break. The monkey

   (I) Climbs up with an acceleration of 6 m/s^2
   (II) Climbs down with an acceleration of 4 m/s^2
   (III) Climbs up with a uniform speed of 5 m/s
   (IV) Falls down the rope nearly under gravity?
   (Take g= 10 m/s^2) (Ignore the mass of the rope)

   (a) Ans: (a1) Rahim loves animals and feeds them, don't frighten animals with

   (b) (I) The tension developed in the string when the monkey climbs up with an acceleration of 6 m/s2 is given by $T = m(g + a) = 40 (10 + 6) = 640$ N

(II) The tension developed when the monkey climbs down with an acceleration of 4m/s2

Is given by $T = m (g-a) = 40(10-4) = 40 \times 6 = 240$ N

(III)When the monkey climbs with a uniform speed of 5m/s acceleration is zero and

the tension in the string is $T = mg\ 40 \times 10 = 400$ N

(IV)As the monkey falls down the rope nearly under gravity, the tension in the

string is given by, $T = m(g-a) = m(g-g)\ 0$

Since the string can withstand a maximum tension of 600 N, hence the rope will

break only in the first case (I)

7. Radha found the wheel getting detached from her uncle's car . She took it to workshop and got it repaired. She informed her uncle, who is a mechanical engineer, about this matter.

(a) What according to you the values displayed by Radha?

(b)A thin wheel can stay up-right on its rim for a considerable length of time when rolled with a considerable velocity, while it falls from its upright position at the slightest disturbance, when stationery. Explain.

Ans: (a) Radha takes care of things and has concern for others. Practical in finding the solutions to problems.

(b) When the wheel is rolling, the angular momentum is conserved. However, due to frictional force, it continues to decrease. Thus, the wheel can stay upright on its rim only for a certain interval of time. In the stationary position, the wheel falls due to unstable equilibrium.

8. Suresh noticed a big Granite Rock in his locality. He thought that if they worked upon it they could earn money. He took permission from the Government, completed all the formalities .He broke the Rock using a bomb. The rock was made into slices .They established a Granite industry. Many of the people in the surroundings started to earn and live comfortably.

(a) What values of Suresh impress you?

(b)A bomb is thrown in a horizontal direction with a velocity of 50 m/s. It explodes into two parts of masses 6 Kg and 3 Kg. the heavier fragment continues to move in the horizontal direction with a velocity of 80 m/s .Calculate the velocity of the lighter fragment.

Ans: (a) Suresh knows how to utilize the natural resources, has got concern for others. Also he knows how to complete all legal formalities before taking up any work.

(b) According to law of conservation of momentum

Total momentum of fragments = Momentum of the Bob

$m_1v_1 + m_2v_2 = MV$

$6X80 + 3X\ v_2 = 9\ X\ 50;\ v_2 = -10$ m/s

9. Rakesh with the intention to win in the interschool sports practiced high jump every day for about a month. He participated and won I position in the interschool sports.

(a) Comment upon the values Rakesh possesses.

(b) Why does an athlete run some steps before taking a jump?

Ans: (a) Rakesh has determination, he Plans and executes his plan accordingly.

(b) An Athlete runs some steps before taking a jump to gain some initial momentum, which helps him to jump more?

10. A sports teacher was training the children for march-past. On their way they come across a bridge .Then the physical education teacher stopped the children from marching on the bridge.

(a) Comment upon the values of sports teacher.

b) Also explain what is meant by Resonance.

Ans: (a) The sports teacher is responsible, cares not only for public property but also children.

(b) When the frequency of marching coincides with the natural frequency of oscillation of the bridge then the bridge oscillates with maximum amplitude to such an extent that the bridge may even collapse. This condition is called “Resonance’’.

11. Suraj went to Big Bazaar to purchase certain goods .There he has noticed an old lady struggling with her shopping. Immediately he showed her the lift and explained to her how it carries the load from one floor to the next. Even then the Old lady was not convinced.. Then suraj took her in the lift and showed her how to operate it.. That old lady was very happy.

(a)What values does Suraj possess?

(b)An elevator can carry a maximum load of 1800 kg is moving up with a constant speed of 2 m/s , The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

Ans: (a) Suraj is sympathetic and also has the attitude of helping others. He has patience

(b) The downward force on the elevator is F= mg + Ff = (1800 X 10) + 4000 = 22000N

The motor must supply enough power to balance this force.

Hence P = F.V = 22000 X 2 = 44000W = 59 hp

12. Jagat and Ram are working in the same company.Jagat has noticed that Ram is suffering from Cancer. Ram is not aware of this.When Jagat asks him to go for a checkup, Ram refuses .He gets convinced how even when he realizes it is very important to get checkup done once a year.

(a) What according to you, are the values displayed by Jagat in helping Ram

(b) A hospital uses an ultrasonic scanner to locate tumors in a tissue. What is the wavelength of sound in the tissue in which the speed of sound in the tissue in which the speed of sound is 1.7 km/s? The operating frequency of the scanner is 4.2 MHz

(Ans: (a,) his concern for his friend, also he has the knowledge of medical facilities available

(b) $\lambda = \mathbf{V}/\nu$; = $1700/4.2 \times 10^6$m; that is $\lambda = 4.05 \times 10^{-4}$ m)

13. Preeti a student of class XI was reading the newspaper, The Headlines in the News paper

were about the earth quake that had taken place in Assam on the previous day. She was very

depressed seeing the loss to life and property.. She approached her physics teacher

and got the information about how earth quake occurs.

(a) What can you say about the inquisitiveness of Preeti?

(b)Earth quake generates sound waves inside the earth. Unlike a gas, the earth can experience both transverse(S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4 km/s, and that of P wave is 8km/s. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the fires S wave. Assuming the waves travel in straight line, how far away does the earthquake occur?

(a)She has concern for society and is sympathetic towards others

b)($Vs = d/t_s$; $v_p = d/t_p$; $v_s t_s = v_p t_p$;$4t_s = 8t_p$;$t_s = 2t_p$; $t_s-t_p = 4min = 240sec$;$t_p=240s$;$t_s=480s$;$d = 1920km$)

14. A group of students went to a place on excursion. While boating on sea water, the students identified a submerged Torpedo shaped structure. The boys debated among themselves on what they saw. A student by name Sharath considering it as a threat informed the police. The police took necessary steps to protect the country from the enemy submarine. Sharath was rewarded.

(a) What can you say about the qualities exhibited by Sharath?

(b) A SONAR system fixed in a submarine operates at a frequency 40 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/hr. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450m/s.

Ans: (a) Navigator is a responsible citizen, he is duty minded, having presence of mind

(b)Apparent frequency received by an enemy submarine, $v' = \{(v + v0)/v\}\ v=$

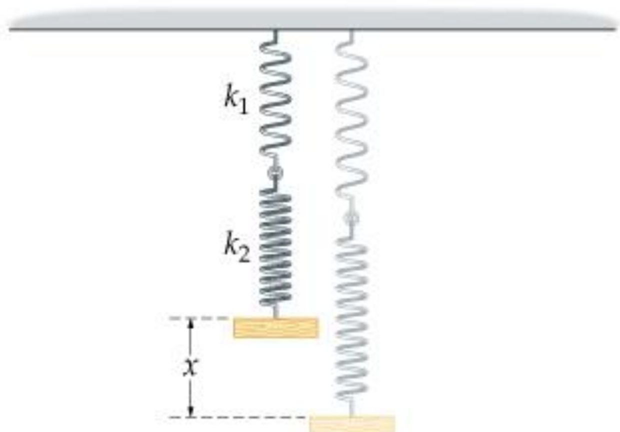
$= \{(1450 + 100)/1450\} \times 40 \times 10^3 Hz = 4,276 \times 10^4 Hz$. This frequency is reflected by the enemy submarine (source) and is observed by SONAR (now observer)

In this case Apparent frequency $v'' = \{v/ (v-vs.)\}\ X\ v = [1450/(1450-100)] X\ 4.276\ X\ 10^4 Hz = 45.9\ kHz$.

15.) The Physics Teacher of class XI has assigned the work of finding the resultant spring constant when two springs of spring constant s k_1, k_2 are joined in series. Two students Sabita and Shirin. Sabita made a theoretical study as well as verified experimentally. Whereas Shirin could not complete the work. When the teacher enquired the next day Sabita could give the answer. Whereas Shirin could not.

(a) Comment upon the qualities of Sabita.

(b) Two springs are joined in series and connected to a mass m as shown in fig. If spring constants are k_1 and k_2, calculate the period of oscillation of mass m.



Ans: (a) Sabita is Sincere and hard working and having scientific temper

(b)[k = k1k2/(k1+k2)]; $T = 2\pi\sqrt{m(k_1+k_2)/k_1k_2}$

16. Adarsh a student of class XI has found the factors on which the time period of oscillation of a pendulum depends and arrived at the expression T = (constant) X $(l/g)^{½}$. He wants to know how the length of the pendulum gets affected on the surface of the moon for the same pendulum and arrived at the conclusion that it is l/6. (a) What values does Adarsh possesses?.

(b)The length of a seconds' pendulum on the surface of the earth is 1m What will be the length on the surface of the moon?

(a) Adarsh is hardworking, thinks logically,having scientific temper , able to find solutions with patience.
(b) Since' **l** 'is proportional to' **g '** the length of the pendulum on the surface of the moon will be 1/6m

17.(a)Ravi has to attend to an interview . He was not well. He took the help of his friend Raghavan. On the way office Ravi felt giddy, He vomited on his dress. Raghavan washed his shirt. He made Ravi to drink enough amount of water. In spite of doing ,a foul smell was coming from the shirt. Then Raghavan purchased a scent bottle from the nearby cosmetics shop and applied on Ravi. Ravi attended the interview, Performed well .Finally he was selected.

(a)What values do you find in Raghavan?

(b)The velocity of air molecules is nearly 500m/s. But the smell of scent spreads very slowly, Why?

Ans: (a) He has presence of mind, serves others in need.

(b) This is because the air molecules can travel only along a zigzag path due to frequent collisions. Consequently, the displacement per unit time is considerably small.

18(a). Ratan noticed that his grandfather to be suffering from fever. He took him to the doctor The doctor gave him some pills .When the pills were used he sweated much, after some time became normal.Rahim enquired the Doctor about how his grandfather became normal .

(a)According to you what values are possessed by Ratan?

(b) A child running a temperature of 101°Fis given an Antipyria which causes an increase in the rte of evaporation of the body. If the fever is brought down to 98°F in 20 mts, what is the amount of heat lost by the body? . The mass of the child is 30 kg.

Ans: (a) Ratan is responsible and he has concern for others, inquisitiveness in gaining the knowledge

(b) Heat lost by the body = mst = 30kg X 1000cal/kg/°C X 1.67 °C = 50100cal [(where loss in tem = (101-98) °C =3 x 5/9 °F = 1.67°C.]

19. Vineet saw his uncle planting seeds in the land. His uncle does not know methods of growing plants. Then he decided to make his uncle aware of this. He explained the importance of ploughing the land before planting the seeds. Uncle is convinced with his ideas. He planted accordingly. The plants grown successfully.

(a) What can you say about Vineet?

(b) What is the utility of ploughing a field? Does it help the soil to retain moisture?

Ans: (a) Vineet has good knowledge of agriculture. He is very much interested in putting his ideas into practice, uses his knowledge to convince his uncle.

(b) When the field is ploughed, the capillaries are broken. So water cannot rise to the surface and the soil is able to retain its moisture.

20. Padma's little sister was crying. Then she took a piece of camphor and put it in water. By seeing the camphor piece dancing on the surface of water, the little one stopped crying.

(a) What can you say about the qualities of Padma?

(b)Why do small pieces of camphor dance on the surface of water?

Ans (a) Padma is responsible, helps her mother in looking after her younger sister.

(b) When camphor is dissolved in water, the surface tension of water is reduced. Since camphor has irregular shape therefore it may dissolve more at one end than at the other end. This produces an unbalanced force due to which it moves .When it reaches a different region, the same process is repeated.

21. A physics teacher explained about conservation of Angular momentum in the class .After the completion of her explanation she want to test how far how far the students are able to understand the topic. In the process she selected two students by name Babita and Ram. Both could explain the topic with examples..

(a) What qualities of them impress you?

(b) A physics teacher sits on a stool that is free to rotate nearly without friction about a vertical axis .Her outstretched hands each hold a large mass so that the rotational inertia is 12kgm^2.By pulling her arms close to her body she is able to reduce her rotational inertia to 6 kg m^2. If her student starts spinning at 0.5 rad/s, what is her speed after she draws her arms in?

Ans: (a) Both were doing group study, discussing together they have given answers.

(b) In the absence of external torque, her angular momentum stays constant so that $I\omega = I'\omega'$

i.e.' = $I\omega/I'$ =(12 x 0.5)/6 =1 rad/s; When her rotational inertia halves, her angular velocity doubles.

22. Suresh was struggling to understand the Kepler's second law of planetary motion. Then his friend Raman who came to him explained how the planet moves around the sun obeying Kepler's law of planetary motion.

(a) Comment upon the values of Raman.

(b) State and the Keplers 'laws of planetary motion.

(Ans: (a) Raman shares his knowledge with his friends and wants to improve his knowledge in the subject, has concern towards his friends.

(b) Refer NCERT Text book)

23. (a) Savita was surprised to see oil spreading on to the surface of water and asked her mother to explain why oil spreads on to the surface of water. Her mother explained her daughter the reason behind it. By going through the explanation she thought of learning more about the other scientific phenomenon also. What qualities do you can find in Savita?

(b) Oil spreads over the surface of water whereas water does not spread over the surface of oil. Why?

(Ans: (a) she has inquisitiveness; she wants know the scientific reason behind the phenomena.

(b)The surface tension of the water is more than that of oil, therefore when oil is poured over water, the greater value of surface tension of water, pulls the oil in all directions. On the other hand, when water is poured over oil, it does not spread over it because surface tension of oil is less than that of water.

24.) Ram and his friend Ramesh while going to the school on a motorcycle noticed that a bidge had collapsed. Immediately they went to their physics teacher and enquired about the reasons for falling of the bridge. After knowing the reasons that very interesting they have decided to pursue their career as civil engineers and vowed to construct 100 % quality dams and bridges.

(a) Comment upon the values possessed by them.

(b) Name the property that helps in constructing bridges. Also define the property.

(Ans :(a) Sympathy, determination, and concern for society, honesty and integrity (b) Elasticity and for definition refer NCERT Text Book)

25. (a) A small hair piece has fallen into the eye of Suresh. It caused itching sensation in the eye of Suresh seeing that Hari, who is a friend of Suresh, took him to the eye specialist. The Doctor removed it. Suresh expressed his gratitude to Hari.Comment upon the values of Hari.

(b) A student measure the thickness of a human hair by looking at it through a microscope of magnification 100.He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5mm. What is the estimate on the thickness of hair?

(Ans: (a) Hari has presence of mind, a caring attitude towards his friend, & Concern towards others.

(b) the estimated thickness of hair = 3.5/100 = 0.035mm)