

THERMODYNAMICS

Important Points:

1. Zeroth Law of Thermodynamics:

- a) This law gives the concept of temperature.
- b) If the temperatures of two bodies are equal then they are said to be in thermal equilibrium.
- c) When two bodies A and B are in thermal equilibrium with a third body C individually, then the bodies A and B are in thermal equilibrium with each other.

2. Calorie:

The quantity of heat required to rise the temperature of 1gram of water from $14.5^{\circ}C$ to $15.5^{\circ}C$ is called standard Calorie.

3. External Work:

- a) The work done by the system is positive.
- b) The work done on the system is negative.

4. Internal Work:

When the work done by one part of the system on another part of the same system is called Internal Work.

5. Internal Energy:

- a. The sum of potential energy and kinetic energy of individual molecules in the system is called Internal Energy.

Internal energy $U = P.E. + K.E$

\therefore Change in internal energy

$$dU = U_f - U_i$$

Where U_f is final internal energy and

U_i is initial internal energy

b. Internal energy of a system increases with temperature.

6. External work done by an ideal gas during Expansion:

$$dW = PdV$$

7. First law of Thermodynamics:

The quantity of heat energy is supplied to a system is equal to sum of the external work done by the system and increase in its internal energy. This obeys the law of conservation of energy.

$$dQ = dU + dW$$

Where dQ = amount of heat

dU = increase in internal energy

$dw = PdV$ = work done

8. Specific Heat of Gas at Constant Volume:

It is defined as the amount of heat required to rise the temperature of unit mass of gas through at constant volume

$$c_v = \frac{1}{m} \cdot \frac{dQ}{dT}$$

9. Specific Heat of Gas at Constant Pressure:

It is defined as the amount of heat required to rise the temperature of unit mass of a gas through $1^\circ C$ at constant pressure.

$$c_p = \frac{1}{m} \cdot \frac{dQ}{dT}$$

10. Molar Specific Heat at Constant Volume:

It is defined as the amount of heat required to raise the temperature of one mole of gas through 1°C at constant volume.

$$C_v = \frac{1}{n} \cdot \frac{dQ}{dT}$$

11. Molar Specific Heat at Constant Pressure:

It is defined as the amount of heat required to rise the temperature of one mole of gas through 1°C at constant pressure.

$$C_p = \frac{1}{n} \cdot \frac{dQ}{dT}$$

12. Relation between C_p and C_v :

$$C_p - C_v = R \quad ; \quad C_p > C_v \quad \text{and} \quad \frac{C_p}{C_v} = \gamma$$

13. Isothermal Process:

- a) The changes in volume and pressure of a gas taking place at constant temperature is called Isothermal Process.
- b) Isothermal process obeys Boyle's law $PV = \text{constant}$.
- c) This process takes place in conducting vessel.
- d) Internal energy of the system does not change.
- e) This is a slow process.
- f) Molar specific heat is infinity in this process.
- g) Work done in isothermal process is

$$W = 2.302 nRT \log_{10} \left(\frac{v_2}{v_1} \right)$$

14. Adiabatic Process:

- a) The changes in pressure, volume of a gas taking place at constant heat energy is called adiabatic process.
- b) This process is represented by $PV^\gamma = \text{constant}$.
- c) This process takes place in insulated vessel.
- d) Heat content does not change.
- e) Molar specific heat is zero.
- f) It is a quick process.
- g) Work done in adiabatic process is

$$W = \frac{R}{(1-\gamma)}(T_2 - T_1)$$

15. Relation between P, V and T:

- a) Relation between P, V is given by

$$PV^\gamma = \text{constant}$$

- b) Relation between V, T is given by $TV^{\gamma-1} = \text{constant}$
- c) Relation between P, T is given by $P^{1-\gamma}T^\gamma = \text{constant}$

16. Method of Mixtures:

- a) Heat lost by the hot body = heat gained by the cold body
- b) Two substances of masses m_1 and m_2 specific heats c_1 and c_2 are at temperatures θ_1 and θ_2 are mixed then final temperature of mixture is

$$\theta = \frac{m_1c_1\theta_1 + m_2c_2\theta_2}{m_1c_1 + m_2c_2}$$

- c) If $c_1 = c_2$ i.e. substances are made of same material then $\theta = \frac{m_1\theta_1 + m_2\theta_2}{m_1 + m_2}$

17. Joules Law:

The amount of work performed is directly proportional to the amount of heat generated $W \propto Q$

$W = JQ$ Where

J = Joules constant (or) mechanical equivalent of heat.

$J = 4.2 \text{ J/cal}$

Triple Point:

The pressure and temperature at which the three states (liquid, solid and gas) of a substance exist is known as triple point. Pressure and temperatures of triple point of water are 610.13 Pa and 273.16K

18. Second law of thermo dynamics:

a) Claudius Statement:

It is impossible for a self acting machine unaided by any external agency to transfer heat from a body at lower temperature to a body at higher temperature (Or)

Heat cannot by itself flow from a colder body to hotter body.

b) Kelvin Plank Statement:

It is impossible to construct a heat engine operating in a cycle to convert the heat energy completely into work without any change of working system.

19. Heat engine is a device which converts heat energy into mechanical energy.

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

20. Refrigerator (heat pump) is reverse of heat engine.

$$\text{Co-efficient of performance } K = \frac{Q_3}{W} = \frac{Q_3}{Q_1 - Q_3} = \frac{1}{\left(\frac{T_1}{T_2} - 1\right)}$$

Very Short Answer Questions

1. Define thermal equilibrium. How does it lead to Zeroth Law of Thermodynamics?

A. Thermal Equilibrium:

If two bodies are at the same temperature then they are said to be in thermal equilibrium. When bodies are in thermal equilibrium, no exchange of heat takes place.

Explanation:

Consider two systems A and B separated by an insulating wall. These systems are in turn contact with a third system C by a conducting wall. The states of A and B change until they come to thermal equilibrium with C. Now the insulating wall between A and B is replaced by a conducting wall and C is insulated from A and B. Then no change is observed in the states of A and B. This led to Zeroth Law of Thermodynamics.

2. Define calorie. What is the relation between calorie and mechanical equivalent of heat?

A. Calorie:

The amount of heat energy required to rise the temperature of 1gm of water through 1°C is called calorie.

Relation between calorie and mechanical equivalent of heat:

$$J = 4.2 \text{ Joule/Cal}$$

Where J is called mechanical equivalent of heat. (or) Joule's constant.

3. What thermodynamic variables can be defined by a) Zeroth Law b) First Law?

A. a) Zeroth Law:

The thermodynamic variables that can be defined by Zeroth Law is temperature.

b) First law:

The thermodynamic variables that can be defined by first law is internal energy, pressure and volume.

4. What is specific heat capacity of the substance? On what factors does it depend?

A. Specific Heat Capacity:

Specific heat capacity is defined as the quantity of heat required to raise the temperature of unit mass of the body through 1°C . $C = \frac{1}{m} \frac{dQ}{dT}$

Specific heat capacity depends on the nature of the substance and its temperature.

5. Define molar specific heat capacity.

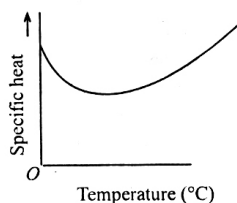
A. Molar specific heat capacity is defined as the quantity of heat required to raise the temperature of unit mole of the body through 1°C . $C = \frac{1}{\mu} \frac{dQ}{dT}$

6. For a solid, what is the total energy of an oscillator?

A: Total energy = potential energy + kinetic energy

7. Indicate the graph showing the variation of specific heat of water with temperature. What does it signify?

A:



The specific heat of water varies slightly with temperature, so defining calorie as amount of heat required to raise the temperature of 1gm of water by 1°C is not exactly correct. So for a precise definition of calorie it is necessary to specify unit temperature interval. One calorie is defined to be amount of heat required to raise the temperature of 1gm of water from 14.5°C to 15.5°C .

8. Define state variables and equation of state?

A. State Variables:

A thermodynamic system is completely described by certain macroscopic variables like pressure, volume, temperature and mass. These are called state variables.

Equation of State:

A relation among the any of the three thermodynamic variables of the system is called its equation of state.

9. Why a heat engine with 100% efficiency can never be realized in practice?

A. For 100% efficiency $T_1 = \infty$ (or) $T_2 = 0$ K. As absolute zero and infinite temperature cannot be realized in practice, the efficiency of a heat engine cannot have 100%. The efficiency of a heat engine is always less than unity.

10. In summer, when the valve of a bicycle tube is opened, the escaping air appears cold.

A. When the valve of the cycle tube is opened, the gas expands suddenly which is adiabatic expansion. Hence temperature decreases producing cooling effect.

11. Why does the brake drum of an automobile get heated up while moving down at constant speed?

A: Brakes are to be applied during the driving of an automobile for constant speed. As a result of friction between wheel and brake drum, the temperature of wheel and brake drum of an automobile increase.

12. Can a room be cooled by leaving the door of an electric refrigerator open?

A. A refrigerator is a heat engine that works in backward direction i.e., it extracts heat from a low temperature reservoir and transfers it to a high temperature reservoir. Thus it exhausts more heat into the room than it extracts from it. Hence the room gets heated.

- 13. Which of the two will increase the pressure more, an adiabatic or an isothermal process, in reducing the volume to 50%?**

A: For isothermal process $P_1V_1 = P_2V_2$ since $V_2 = \frac{V_1}{2}$

$$P_{2iso} = 2P_1$$

For adiabatic process $P_1V_1^\gamma = P_2V_2^\gamma$

$$P_{2adia} = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1(2)^\gamma$$

γ is the ratio of specific heats. It is always greater than one so $P_{2adi} > P_{2iso}$

- 14. A thermos flask containing a liquid is shaken vigorously. What happens to its temperature?**

A: Its temperature increases because liquid moves against the viscous forces. Work done against viscous forces is converted into heat.

- 15. A sound wave is sent into a gas pipe. Does the internal energy change?**

A: As the propagation of sound wave in a gas is an adiabatic process then its temperature and hence its internal energy will increase.

- 16. How much will be the internal energy change in i) isothermal process ii) adiabatic process?**

A. i) Isothermal process:

As temperature is constant, internal energy does not change.

ii) Adiabatic process:

During adiabatic expansion internal energy decreases and adiabatic compression internal energy increases.

17. The coolant in a chemical or a nuclear plant should have high specific heat. Why?

A. $S = \frac{1}{m} \frac{d\theta}{dt} \Rightarrow s \propto \frac{1}{dt}$

The coolant in a chemical or a nuclear plant should absorb large amount of heat for a small rise in temperature. Hence the specific heat of the coolant must be high.

18. Explain the following processes. i) Isochoric Process, ii) Isobaric Process.

A. **i) Isochoric Process:**

A thermodynamic process in which volume remains constant is known as isochoric process.

ii) Isobaric Process:

A thermodynamic process in which pressure remains constant is known as isobaric process.

Short Answer Questions

1. State and explain the first law of Thermodynamics?

A. Statement:

The amount of heat supplied to a system is equal to the algebraic sum of the change in internal energy of the system and the amount of external work done. This obeys the law of conservation of energy.

Explanation:

If dQ is the amount of heat supplied to the system, dU is the increase in internal energy and dW is the external work, then $dQ = dU + dW$

Limitations:

- 1) It does not explain the direction of heat flow.
- 2) It does not tell us about the efficiency of conversion heat into work.

2. Define two principal specific heats of a gas. Which is greater and why?

A. 1. Principal Specific Heat At Constant Pressure (C_p):

It is defined as the amount of heat required to increase temperature of one gram of a gas through 1°C at constant pressure.

2. Principal Specific Heat At Constant Volume (C_v):

It is defined as the amount of heat required to increase the temperature of one gram of gas through 1°C at constant volume.

$C_p > C_v$ of a gas:

The heat supplied to a gas at constant volume is used only to increase the temperature (internal energy) of the gas. But heat supplied to a gas at constant pressure it is used to

- 1) Increase the temperature of the gas and

2) Do external work against constant pressure.

Hence more heat is required.

Hence for a given mass of a gas for the same rise in temperature heat supplied to the gas at constant pressure is greater than that supplied at constant volume i.e. $C_P > C_V$.

3. Derive the relation between the two heat capacities of a gas on the basis of first law of Thermodynamics.

A. Relation between C_P and C_V :

Consider one mole of an ideal gas in a cylinder fitted with an air tight movable piston of area of cross section 'A'. Let the temperature of the gas is increased through 1°C at constant pressure. Let 'dL' is the displacement in the piston.

Work done by the gas on the piston, $dW = F.dL$

$$\text{Or } dW = P.A.dL \quad (\because F = PA)$$

$$\therefore dW = P.dV \dots\dots\dots (1)$$

$$\text{Also, } C_p = \frac{1}{n} \frac{dQ}{dT} \Rightarrow C_p = dQ \dots\dots\dots (2)$$

From the first law of thermodynamics $dQ = dU + dW$,

$$\text{Or } C_p = dU + PdV \dots\dots\dots (3)$$

Also, from $PV = RT$

$$\text{Or } PdV = RdT$$

$$\therefore PdV = R \dots\dots\dots (4)$$

From equations (3) and (4)

$$C_p = dU + R \dots\dots\dots (5)$$

$$\text{At constant volume, } C_v = \frac{1}{n} \frac{dQ}{dT} \Rightarrow C_v = dQ \dots\dots\dots (6)$$

Again from first law of thermodynamics, $dQ = dU$ (7) [$\because dW = 0$]

From equations (6) and (7) $dU = C_v$

Or $C_p = C_v + R$

$$\therefore C_p - C_v = R$$

4. Obtain an expression for the work done by an ideal gas during isothermal change.

A. Work done in Isothermal Process:

Let a gas expands isothermally through small volume dv .

Work done by the gas $dW = PdV$

Work done by the gas in changing its volume from v_1 to v_2 is given by,

$$W = \int_{v_1}^{v_2} P dV = \int_{v_1}^{v_2} \frac{RT}{V} dv = RT \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\Rightarrow W = RT [\log v]_{v_1}^{v_2}$$

$$\Rightarrow W = RT [\log v_2 - \log v_1]$$

$$\Rightarrow W = RT \log_e \frac{v_2}{v_1}$$

$$W = 2.303RT \log_{10} \left(\frac{v_2}{v_1} \right)$$

But, $p_1 v_1 = p_2 v_2$

$$\therefore W = 2.303RT \log_{10} \left(\frac{p_1}{p_2} \right)$$

5. Obtain an expression for the work done by an ideal gas during adiabatic change and explain?

A. Work done in Adiabatic Process:

Let a certain mass of an ideal gas undergoes an adiabatic process. Let dv be the change in volume at constant pressure P . then the work done for change in volume from v_1 to v_2 is given by

$$W = \int_{v_1}^{v_2} P dv$$

$$\text{But } PV^\gamma = \text{constant or } P = \frac{K}{V^\gamma}$$

$$W = \int_{v_1}^{v_2} \frac{K}{V^\gamma} dv = \frac{K}{(1-\gamma)} V^{-\gamma+1} = \frac{K}{1-\gamma} (V^{1-\gamma})_{v_1}^{v_2}$$

$$W = \frac{K}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma})$$

$$\text{But } P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$\therefore W = \frac{1}{1-\gamma} (P_2 V_2^\gamma V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma})$$

$$\text{Or } W = \frac{1}{1-\gamma} (P_2 V_2 - P_1 V_1)$$

$$\text{For } n \text{ moles of a gas, } W = \frac{nR}{\gamma-1} [T_1 - T_2]$$

6. Compare isothermal Process and adiabatic Process.

Isothermal Process	Adiabatic Process
1. The pressure and volume of gas change at constant temperature.	1. The pressure, volume and temperature of a gas change.
2. It is a slow process.	2. It is a quick process.

3. The system is in thermal contact with the surroundings.	3. The system is in thermally insulated from the surroundings.
4. The internal energy of the system does not change i.e, $du = 0$	4. Heat content does not change i.e. $dQ = 0$
5. It obeys the Boyle's law i.e., $PV=K$.	5. The adiabatic process is represented by $PV^\gamma = \text{constant}$.
6. Work done $W=2.303RT\log_{10} \left(\frac{V_2}{V_1} \right)$	6. Work done $W = \frac{nR}{\gamma-1} [T_1 - T_2]$

7. Explain the following processes. i) Cyclic process with example.ii) Non-cyclic process with example.

A. Cyclic process:

The process in which the system after passing through various stages (pressure, volume and temperature changes) returns to its initial condition is called a cyclic process.

i) In cyclic process, $dU = 0$. Hence $dQ = dW$

ii) P – V graph for a cyclic process is a closed curve.

In cyclic process total heat absorbed by the system equals the work done by the system.

Ex. A gas in a cylinder at pressure P_1 , volume V_1 and temperature T_1 are taken to different stages and again taken back to the initial conditions of pressure P_1 , volume V_1 and temperature T_1 . This process is called a cyclic process.

ii) Non-cyclic process:

The process in which the system does not return to its initial conditions is called a non-cyclic process.

Ex. Consider a gas in a cylinder provided with an air tight movable piston. Let its pressure P_1 and volume V_1 are changed to pressure P_2 and volume V_2 . The work done in this process is the area under the P-V curve with volume axis. This is a non-cyclic process.

8. Explain Quasi-Static process?

A. Quasi-Static Process:

An infinitesimally slow process in which at each and every intermediate stage, the system remains in thermal and thermodynamic equilibrium with the surroundings throughout the entire process is called quasi - static process.

Explanation:

1) Consider a gas enclosed in a cylinder by piston. If the piston is suddenly pushed down, the gas inside will undergo sudden compression. During the compression, the gas inside the cylinder passes through several states that are not in equilibrium. This is because the pressure and temperature of the system will rapidly change. But after certain time, the gas will attain the thermodynamic equilibrium with surroundings.

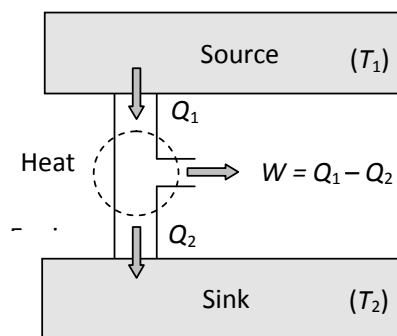
2) Instead of pushing the piston in a sudden manner. Let us lower it down in infinitely slow manner such that at every stage of the system, the pressure difference, and temperature difference between system and surroundings should be infinitesimally small. It is called quasi-static process.

9. Explain qualitatively the working of a heat engine?

A. Heat Engine:

It is a device which converts heat energy into mechanical work.

Working:



There are three parts in a heat engine.

- 1) Source (hot body)
- 2) Working substance
- 3) Sink (cold body)

1) Source:

It is at a very high temperature T_1 K and Q_1 heat is extracted from this body (Hot reservoir)

2) Working Substance:

In steam engine working substance is steam and in diesel engine working substance is mixture of fuel vapour and air.

3) Sink:

It is at a very low temperature T_2 K and Q_2 heat is rejected by the working substance to this (Cold reservoir).

$$\text{Work done } W = Q_1 - Q_2$$

The efficiency of heat engine is defined as ratio of work done by the engine to the amount of heat absorbed by the engine.

$$\therefore \eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Long Answer Questions

1. Explain reversible and Irreversible Processes.

Describe the working of a Carnot engine. Obtain the expression for efficiency.

A. Reversible Process:

A process that can be retraced back in opposite direction in such a way that the system passes through the same states as in the direct process and finally the system and surroundings return to their original states is called reversible process.

Example:

- 1) Peltier effect and Seebeck effect.
- 2) Fusion of ice and vaporization of water.

Irreversible processes:

A process that cannot be retraced back in opposite direction is called irreversible process. All the spontaneous natural processes are irreversible.

Examples:

- 1) Diffusion of gases.
- 2) Work done against friction.

Carnot's Heat Engine:

Carnot ideal heat engine consists of four components.

1) Cylinder:

The walls are perfectly non-conductive and base is perfectly conducting, fitted with a non-conducting frictionless movable piston.

2) Source:

It is at a high temperature T_1 K and has infinite thermal capacity.

3) Sink:

It is at a lower temperature T_2 and has infinite thermal capacity.

4) Insulating Stand:

It is a perfectly non-conducting stand on which the cylinder containing the working substance can be put and it can be made perfectly insulated from the surroundings.

The working substance (perfect gas) in the cylinder is subjected to cycle of four operations and it is known as Carnot cycle.

1. Isothermal Expansion:

The cylinder containing ideal gas is allowed to expand slowly at this constant

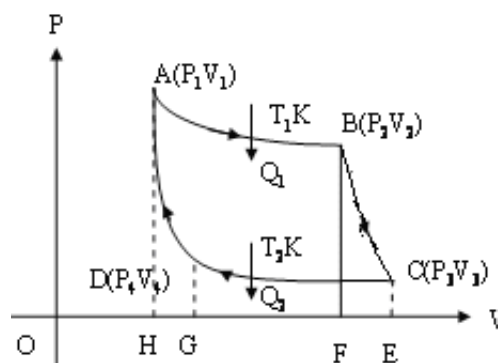
temperature T_1 . $W_1 = Q_1 = RT_1 \log_e \left(\frac{v_2}{v_1} \right) = \text{area ABFH}$

2. Adiabatic Expansion:

Now the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2 .

$$W_2 = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{area BCEF}$$

3. Isothermal Compression:



The gas is compressed at constant temperature T_2 .

Work done = Heat released by the system

$$Q_2 = W_3 = -RT_2 \log_e \frac{V_4}{V_3} = \text{area } CDGE$$

$$(\text{Or}) Q_2 = +RT_2 \log_e \frac{V_3}{V_4}$$

4. Adiabatic Compression:

Finally the compression is continued so that gas returns to its initial stage.

$$W_4 = + \frac{R(T_2 - T_1)}{\gamma - 1} = \frac{-R(T_1 - T_2)}{\gamma - 1} = \text{area } ADGH$$

Net work done by the engine in complete cycle is given by,

$$W = W_1 + W_2 - W_3 - W_4$$

$$\text{But } W_2 = W_4.$$

$$W = W_1 - W_3$$

Efficiency:

The efficiency of engine is defined as the ratio of work done to the heat supplied.

$$\eta = \frac{\text{work done}}{\text{heat input}} = \frac{W}{Q_1}$$

Net work done during the complete cycle

$$W = W_1 + W_2 + (-W_3) + (-W_4) = W_1 - W_3 = \text{Area } ABCD$$

$$[\text{As } W_2 = W_4]$$

$$\eta = \frac{W}{Q_1} = \frac{W_1 - W_3}{W_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{W_3}{W_1} = 1 - \frac{Q_2}{Q_1}$$

$$\text{Or } \eta = 1 - \frac{RT_2 \log_e (V_3 / V_4)}{RT_1 \log_e (V_2 / V_1)}$$

Since points *B* and *C* lie on same adiabatic curve

$$T_1 V_1^{r-1} = T_2 V_3^{r-1} \quad \text{Or} \quad \frac{T_1}{T_2} = \left(\frac{V_3}{V_1} \right)^{r-1} \quad \dots\dots(i)$$

Also point D and A lie on the same adiabatic curve.

$$T_1 V_1^{r-1} = T_2 V_4^{r-1} \quad \text{Or} \quad \frac{T_1}{T_2} = \left(\frac{V_4}{V_1} \right)^{r-1} \quad \dots\dots(ii)$$

$$\text{From (i) and (ii), } \frac{V_3}{V_2} = \frac{V_4}{V_1} \quad \text{or} \quad \frac{V_3}{V_4} = \frac{V_2}{V_1} \quad \text{or} \quad \log_e \left(\frac{V_3}{V_4} \right) = \log_e \left(\frac{V_2}{V_1} \right)$$

$$\text{Hence the efficiency of Carnot engine } \eta = 1 - \frac{T_2}{T_1}$$

2. State second law of thermodynamics. How is heat engine different from refrigerator?

A. Classius Statement:

It is impossible for a self acting machine unaided by an external agency to transfer heat from a body at lower temperature to another at a higher temperature.

Kelvin Statement:

It is impossible to construct a heat engine operating in a cycle to convert the heat energy completely into work without any change of working system.

The second law always states that the heat flows from a body at higher temperature to a body at lower temperature. Also it is impossible to construct any heat engine with 100% efficiency.

Refrigerator:

Any device capable of transferring heat from a cold body to a relatively hotter body is called Refrigerator. An ideal heat engine operating in the reverse processes is called refrigerator.

The working substance absorbs heat energy from the sink at lower temperature, a net amount of work is done on it by an external agent and a large amount of heat is refused to the source. The working substance is called refrigerant. In a house hold refrigerator, work is done by an electric motor and the working substance (refrigerant) is Freon.

If Q_2 is the heat energy absorbed from the sink at T_2 K and Q_1 is the heat given to the source at T_1 K. Then

$$W = Q_1 - Q_2 \quad (\text{or}) \quad W = Q_2 \left(\frac{Q_1}{Q_2} - 1 \right)$$

Since $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$, $W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$

The ratio of heat extracted from the sink and to the work required to be done on the refrigerator

is called the coefficient of performance $K = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\left(\frac{T_1}{T_2} - 1 \right)}$

Heat engine and Refrigerator:

An ideal heat engine operating in the reverse processes is called refrigerator. In a heat engine, the efficiency can never exceed 100%. But in the case of a refrigerator, the coefficient of performance may be much higher than 100%.

Problems

14. If a mono-atomic ideal gas of volume 1 litre at N.T.P is compressed adiabatically to half of its volume, find the work done on the gas. Also find the work done if the compression is isothermal. ($\gamma = 5/3$)

Sol: i) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-2}$

But, $T_1 = 273\text{K}$; $V_2 = \frac{V_1}{2}$; $\gamma = \frac{5}{3}$

$$\therefore 273 V_1^{\frac{5}{3}-1} = T_2 \left(\frac{V_1}{2}\right)^{\frac{5}{3}-1}$$

$$T_2 = (2)^{\frac{2}{3}} 273 = 431.6\text{K}$$

Number of moles, $n = \frac{1\text{litre}}{22.4\text{litre}} = \frac{1}{22.4}$

Work done = $\frac{nR}{\gamma-1}(T_1 - T_2)$

$$= \frac{nR}{\gamma-1}(T_1 - T_2) = \frac{8.314}{22.4 \left(\frac{5}{3}-1\right)} [273 - 431.6] = \frac{8.34 \times 3}{22.4 \times 2} (-158.6) = -89\text{J}$$

ii) Work done during isothermal compression is $W = 2.3026 nRT \log_{10} \left(\frac{V_2}{V_1}\right)$

$n = \text{number of moles} = \frac{1}{22.4}$; $T = 273\text{K}$; $R = 8.314\text{J mol}^{-1} \text{K}^{-1}$

$$\frac{V_2}{V_1} = \frac{1}{2} = 0.5$$

$$W = \frac{2.3026 \times 8.314 \times 273 \log_{10}(0.5)}{22.4}$$

or $W = -79\text{J}$

2. Five moles of hydrogen when heated through 20K expand by an amount of $8.3 \times 10^{-3} \text{ m}^3$ under a constant pressure of 10^5 N / m^2 . If $C_v = 20 \text{ J / mole K}$, find C_p .

A: $C_p - C_v = R$

Or $nC_p \Delta T - nC_v \Delta T = nR \Delta T$

Or $n \Delta T (C_p - C_v) = P \Delta V$ ($\because nR \Delta T = P \Delta V$)

Or $5 \times 20 (C_p - 20) = 10^5 \times 8.3 \times 10^{-3}$

$\therefore C_p = 28.3 \text{ J / mole K}$

2. How much steam at 100°C is to be passed into water of mass 100gm at 20°C to raise its temperature by 5°C ? (Latent heat of steam is 540cal/gm and specific heat of water is 1cal / gm $^\circ\text{C}$)

A: heat lost by steam = heat gained by water

$$m_s L_s + m_s s (100 - t) = m_w s (t - 20)$$

$$m_s \times 540 + m_s \times 1 (100 - 25) = 100 \times 1 (25 - 20)$$

$$m_s = \frac{500}{615} = 0.813 \text{ gm}$$