# Explaining the Solution Using Number Theory

The problem involves finding the specific position and time where a group of robots on a grid forms a cool pattern. The grid has dimensions `length` $\times$ `height`, and the robots move in a fixed pattern. When a robot reaches the boundary of the grid, it teleports to the opposite side and continues moving. At a certain integer time $t$, the robots align to form an interesting pattern.

Instead of generating thousands of images of the grid to search for this pattern, we can leverage insights from number theory to predict when and where the alignment occurs. Here's how it works:

## Robot Movement and Alignment Patterns

1. Each robot's position at any time $t$ can be described modulo the grid's dimensions:

- The *horizontal position* of the robot is governed by the grid's `length`, looping back to 0 when the end is reached.
- The *vertical position* of the robot is governed by the grid's `height`, looping back similarly.

2. At some integer time $t$, the robots align in a recognizable pattern. To determine the time $t$ when this happens, instead of analyzing thousands of grid images, we reduce the problem to observing the positions of robots along the *horizontal axis* and *vertical axis*.

3. These alignments can be encoded as two separate congruences:

- The robots align horizontally at position $a$ (mod `length`).
- The robots align vertically at position $b$ (mod `height`).

## Solving Using Linear Congruences

The problem reduces to solving the following system of linear congruences:

$$x \equiv a \pmod{\texttt{length}}$$

$$x \equiv b \pmod{\texttt{height}}$$

Using the Chinese Remainder Theorem (CRT), we can determine the smallest positive integer $x$ that satisfies both congruences. This value of $x$ represents the earliest time when the robots align at the specified positions on both the horizontal and vertical axes.

## Why This Works

By reducing the problem to modular arithmetic: - We take advantage of the periodicity of the robot movements. The horizontal and vertical positions repeat cyclically with periods `length` and `height`, respectively. - Instead of examining every possible image, we exploit the structure of the problem to focus only on critical alignment points, dramatically reducing computation.

Thus, the value of $x$ obtained from the congruences gives us the precise time when the pattern occurs, without needing to generate and inspect thousands of images.