CAPE LAB
ASSIGNMENT 2

Team B

2023

Yash Jangade Aman Kumar Diya Sarkar
Akshat Soni Jaya Anand

Q2] A}

Double Pipe Heat Exchanger

Given: {D. Q. Kern, Ex 6.3, Pg no. 121}

Double Pipe Lube oil – Crude oil Heat Exchanger

Hot Stream – 26^o API Lube oil

Cold Stream – 34^o API Crude oil

Mass flow rate of Hot Stream, m_h – 6900 lb/hr

Mass flow rate of Cold Strem, m_c – 72500 lb/hr

$T_{h, I}$ – 450^o F

$T_{c, I}$ – 300^o F

f – 0.003

Allowable Pressure Drop – 10 psi

$C_{p, h}$ - 0.62 Btu/(lb ^{o}F)

$C_{p, c}$ - 0.585 Btu/(lb ^{o}F)

U - 28.2 Btu/(hr ft^2 ^{o}F)

A - 173 ft^2

To find:

The outlet temperature of hot stream ($T_{h,o}$) and cold stream ($T_{c,o}$).

Solution:

Using the conservation of energy principle,

$$m_h * C_{p,\ h} * \left(T_{h,\ i} - T_{h,\ o}\right) = m_c * C_{p,\ c} * (T_{c,\ o} - T_{c,\ i})$$

Rearranging the equation,

$$T_{h,\ o} = T_{h,\ i} - \frac{m_c * C_{p,\ c} * \left(T_{c,\ o} - T_{c,\ i}\right)}{m_h * C_{p,\ h}}$$

Substituting the value, we get

$$T_{h,\ o} = 450 - \frac{72500 * 0.585 * \left(T_{c,\ o} - 300\right)}{6900 * 0.62}$$

Now, using LMTD formula,

$$lmtd = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

a) <u>Co – Current Flow</u>

$$\Delta T_1 = T_{h,\ i} - T_{c,\ i}$$
$$\Delta T_2 = T_{h,\ o} - T_{c,\ o}$$

Therefore,

$$lmtd = \frac{150 - (T_{h,\ o} - T_{c,\ o})}{\ln\left(\frac{150}{T_{h,\ o} - T_{c,\ o}}\right)}$$

Now, we can use the relation,

$$U * A * lmtd = m_h * C_{p,\ h} * \left(T_{h,\ i} - T_{h,\ o}\right)$$

Now, let's assume a function f which we can iterate over to get the value of $T_{c,o}$

$$f = U * A * lmtd - m_h * C_{p,\ h} * \left(T_{h,\ i} - T_{h,\ o}\right)$$

Substituting values and rearranging the equation,

$$f = (U * A * \frac{C_{p,\ c} * m_c * \left(T_{c,\ o} - T_{c,\ i}\right)}{\ln\left(\frac{T_{h,\ i} - T_{c,\ i}}{T_{h,\ i} - T_{c,\ o} - \frac{C_{p,\ c} * m_c * \left(T_{c,\ o} - T_{c,\ i}\right)}{m_h * C_{p,\ h}}}\right.} - m_h * C_{p,\ h} * \left(T_{h,\ i} - T_{h,\ o}\right)$$

Substituting values in the equation,

$$f = -42412.5T_{c,\ o} + (4878.6 * \frac{10.914095371669T_{c,\ o} - 3274.2286115007}{\log\left(\frac{150}{3424.2286115007 - 10.914095371669}\right)} + 12727$$

Differentiating with respect to $T_{c,\ o}$, we get,

$$\frac{df}{dT_{c,o}} = -42412.5 + \frac{53245.5056802244}{\log\left(\frac{150}{3424.2286115007 - 10.914095371669T_{c,\ o}}\right)} - \frac{0.681159332966027.(22.828190743338 - 0.0727606358111267T_{c,\ o})(10.914095371669T_{c,\ o} - 3274.2286115007)}{\left(1 - 0.00318731504520833T_{c,\ o}\right)^2 \log\left(\frac{150}{3424.2286115007 - 10.914095371669T_{c,\ o}}\right)^2}$$

Using the iterative method of Newton Raphson,

$$T_{c,o,\ next} = T_{c,o,\ prev} - \frac{f}{f'}$$

Hence, we get (for tolerance = 10^{-3})

```
T_co: 309.8273 degF
Time taken: 0.1018369197845459 second
```

itr	Vm_prev	Vm_next	fn(Vm_prev)	fn(Vm_next)	error
1	301	310.501	662420	-58237.3	9.5013
2	310.501	309.841	-58237.3	-1175.09	0.660099
3	309.841	309.827	-1175.09	-4.6556	0.0138016
4	309.827	309.827	-4.65557	3.7524	9.91821e-05

$\therefore$ $T_{c,o}$ = 309.8273 °F

$\therefore$

$$\therefore T_{h,\ o} = 450 - \frac{72500 * 0.585 * (309.8273 - 300)}{6900 * \ 0.62}$$

$\therefore$ $T_{h,o}$ = 352.5712 °F

b) Counter-Current Flow:

$$\Delta T_1 = T_{h,\ i} - \ T_{c,\ o}$$
$$\Delta T_2 = T_{h,\ o} - \ T_{c,\ i}$$

Therefore,

$$lmtd = \frac{750 - (T_{h,\ o} + \ T_{c,\ o})}{\ln\left(\frac{450 - \ T_{c,\ o}}{T_{h,\ o} - \ 300}\right)}$$

Now, we can use the relation,

$$U * A * lmtd = \ m_h * C_{p,\ h} * \left(T_{h,\ i} - \ T_{h,\ o}\right)$$

Now, lets assume a function f which we can iterate over to get the value of $T_{c,o}$

$$f = \ U * A * lmtd - \ m_h * C_{p,\ h} * \left(T_{h,\ i} - \ T_{h,\ o}\right)$$

Substituting values and rearranging the equation,

$$f = (U * A * \frac{C_{p,\ c} * m_c * \left(T_{c,\ o} - \ T_{c,\ i}\right)}{\ln\left(\frac{T_{h,\ i} - \ T_{c,\ i}}{T_{h,\ i} - T_{c,\ o} - \frac{C_{p,\ c} * m_c * \left(T_{c,\ o} - \ T_{c,\ i}\right)}{m_h * C_{p,\ h}}}\right.} - \ m_h * C_{p,\ h} * \left(T_{h,\ i} - \ T_{h,\ o}\right)$$

Substituting values in the equation,

$$f = \ -42412.5T_{c,\ o} + (4878.6 * \frac{8.914095371669T_{c,\ o} - 2674.2286115007}{\log\left(\frac{450 - T_{c,\ o}}{3124.2286115007 - 9.914095371669T_{c,\ o}}\right)} + 12723750.0$$

Differentiating with respect to $T_{c,o}$, we get,

$$\frac{df}{dT_{c,o}} = -42412.5 + \frac{43488.3056}{\log\left(\frac{450-T_{c,\ o}}{3124.2286-9.9140T_{c,\ o}}\right)} - \frac{4878.6*\left(3124.2286-9.91409T_{c,\ o}\right)\left(8.91409T_{c,\ o}-2674.2286\right)*\left(-\frac{1}{3124.2286-9.9140T_{c,\ o}}+\frac{1.0157*10^{-6}*(450-T_{c,\ o})}{(1-0.0031T_{c,\ o})^2}\right)}{(450-T_{c,\ o})*\log\left(\frac{450-T_{c,\ o}}{3124.2286-9.9140T_{c,\ o}}\right)^2}$$

Using the iterative method of Newton Raphson,

$$T_{c,o,\ next} = T_{c,o,\ prev} - \frac{f}{f'}$$

Hence, we get (for tolerance = 10^{-3}):

```
T_co: 310.0676 degF
Time taken: 0.12883567810058594 second
```

itr	Vm_prev	Vm_next	fn(Vm_prev)	fn(Vm_next)	error
1	301	310.534	662531	-37276.6	9.5341
2	310.534	310.071	-37276.6	-301.768	0.4627
3	310.071	310.068	-301.768	-0.516403	0.00379944
4	310.068	310.068	-0.516396	-0.516403	0

∴ $T_{c,o}$: 310.0676 °F

$$\therefore T_{h,\ o} = 450 - \frac{72500 * 0.585 * (310.0676 - 300)}{6900 * \ 0.62}$$

∴ $T_{h,o}$ = 350.1889 °F

Final Result:

Flow	T_hi (degF)	T_ci (fegF)	T_co (degF)	T_ho (degF)
co-current	450	300	309.827	352.571
counter-current	450	300	310.068	350.189

Resistance to Absurd Values:

1. We have applied check system to the input values so as to avoid any wrong input from the user's side.
   We have applied check on the $T_{h,i}$ and $T_{c,i}$ so that $T_{h,i}$ is always greater than $T_{c,i}$.
2. Second check we have applied for the value under the logarithm. The number whose logarithm is to be taken should be positive so as to avoid any error.
3. Any of the following values should not be zero: m_h, m_c, f, Allowable Pressure Drop, $C_{p,h}$, $C_{p,c}$, U, A

Comments:

1. We have found out the outlet temperature values for both hot stream and cold stream for both cases: co-current flow and counter current flow. We have found that counter current has shown better result by cooling the hot stream to a lower value than the co-current flow for same setup.
2. That implies maximum amount heat transfer can be attained by counter-current flow.
3. The reason for such result is the fact that counter-current flow maintains a slowly declining temperature difference or gradient across the length which result in effective heat transfer.
4. We have used Newton Raphson method for iterating the $T_{c,o}$ to get final value. This is because Newton Raphson method has the highest efficiency in terms of the number of iterations for a given tolerance value. This observation can be attributed to the fact that the order of convergence is two for Newton Raphson.

Q2] B}

Given a tank with a certain diameter full of water. A hole is made at its bottom to which a pipe of a certain diameter and length is attached. Given the initial and final height of water in the tank, plot a graph between the discharge time versus length of the pipe.

Solution:

Let the initial and final height of the liquid in the tank be 5m and 0.5m respectively.

The length of the pipe 'L' is varied from 1m to 10m.

As stated above

$H_i = 5m$
$H_f = 0.5m$

And let us consider

$g = 10 m/s^2$
$D = 0.5$ m
$d = 0.01$ m
$\rho = 1000 \; Kg/m^3$
$\mu = 0.0010016$ Pa.s

Applying Bernoulli's equation between the exit point from the pipe and the point at the top of the tank we get velocity of fluid at exit,

$$v = \sqrt{2g(H+L)}$$

However, the pipe exerts frictional resistance to the flow of the fluid and pressure drop is caused because of it.

Friction factor 'f' is described by an implicit equation as,

$$f = \frac{1}{\left(4 * \log\left(\rho * v * d * \frac{\sqrt{f}}{\mu}\right) - 0.4\right) * 2}$$

The ideal pressure drop,

$$P_{ideal} = \rho * g(H+L)$$

The pressure drop in pipe flow, **'ΔP'** is calculated using Darcy-Weisbach equation as follows,

$$\Delta P = \frac{f * L * v^2 * \rho}{2 * d}$$

So,

$$P_{actual} = P_{ideal} - \Delta P$$

$$P_{actual} = \rho * g * (H + L) - \frac{f * L * v^2 * \rho}{2 * d}$$

$$V_{actual} = \sqrt{2 * \frac{P_{actual}}{\rho}}$$

Applying equation of continuity, we get

$$A_0 * \left(-\frac{dH}{dt}\right) = A_1 * V_{actual}$$

$$-\frac{dH}{dt} = K * V_{actual} \ \left(where\ K = \frac{A_1}{A_0} = \left(\frac{d}{D}\right)^2\right.$$

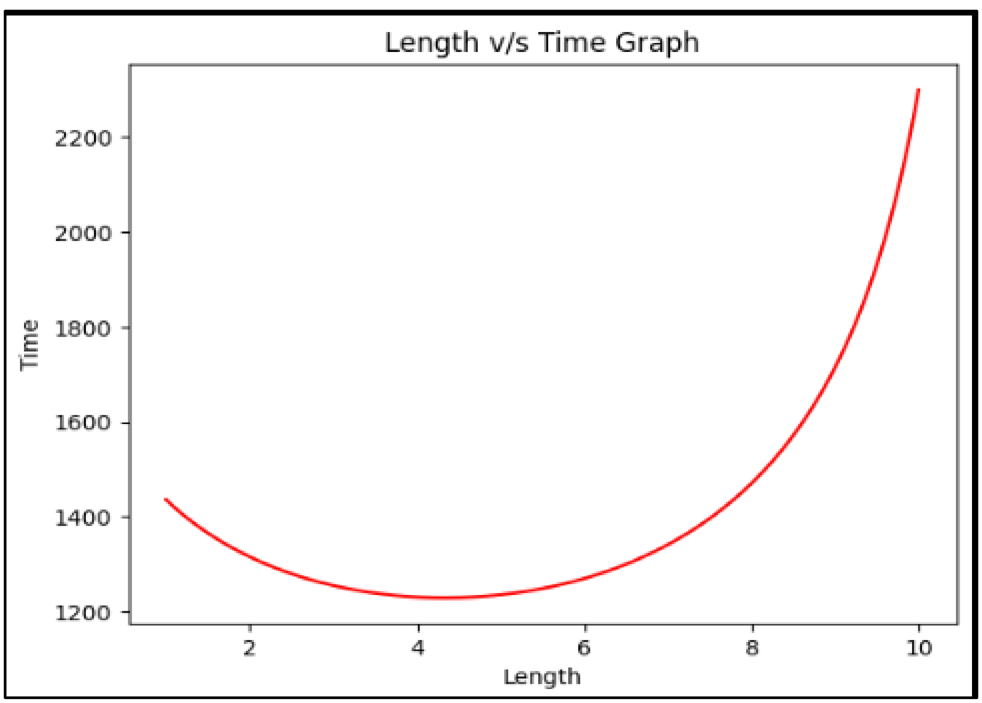
Time 't' is given by integrating dH/(K*Vactual)
i.e,

$$t = \int -\frac{dH}{\left(K * \sqrt{2 * \frac{P_{actual}}{\rho}}\right)}$$

between 5m to 0.5m.

After assuming a particular L, f was found using Newton-Raphson method followed by the evaluation of time.

The final plot of length of pipe vs time can be shown as the output of the code as shown below:



Effect of tolerance value in convergence criteria:

It can be clearly observed from the graphs and data, that for each method, with decreasing tolerance value, the number of iterations increases, i.e., the smaller values of tolerance lead to an increase in the number of steps to meet the convergence criteria.

Link to Code:

Question 2 (a): https://github.com/aman74git/CAPE-LAB/blob/main/assg2/2a.ipynb

Question 2 (b): https://github.com/yashjangade1/CAPE-LAB/blob/main/Assgn%202/Cape_Assgn_2.ipynb