Efficient Frontier Analysis

Date: 16th Sep 2024

Problem Statement:

Industry_Portfolios.xlsx contains monthly nominal (net) returns (expressed as percentages) for ten industry portfolios, over the ten-year period from Jan 2004 through Dec 2013. Use these returns to estimate the vector of mean returns and the covariance matrix of returns for the ten industry portfolios:

1. Create a table showing the mean return and standard deviation of return for the ten industry portfolios.

2. Plot the minimum-variance frontier (without the riskless asset) generated by the ten industry portfolios:
- This graph must have expected (monthly) return on the vertical axis vs standard deviation of (monthly) return on the horizontal axis.
- This graph must cover the range from 0% to 2% on the vertical axis, in increments of 0.1% (or less).

3. Briefly explain (in words, without mathematical equations or formulas) the economic significance and relevance of the minimum-variance frontier to an investor.

4. Now suppose that the (net) risk-free rate is 0.13% per month:
- Plot the efficient frontier (with the riskless asset) on the same graph as the minimum-variance frontier generated by the ten industry portfolios.
- The two frontiers will intersect at single point: the tangency portfolio

5. Briefly explain the economic significance and relevance of the efficient frontier to an investor.

6. Calculate the Sharpe ratio for the tangency portfolio, and also the tangency portfolio weights for the ten industry portfolios.

7. Briefly explain the economic significance and relevance of the tangency portfolio to an investor.

Solution:

1. The entire analysis was done on Python. The mean, standard deviation and covariance matrix were calculated and the former 2 were built into a table:

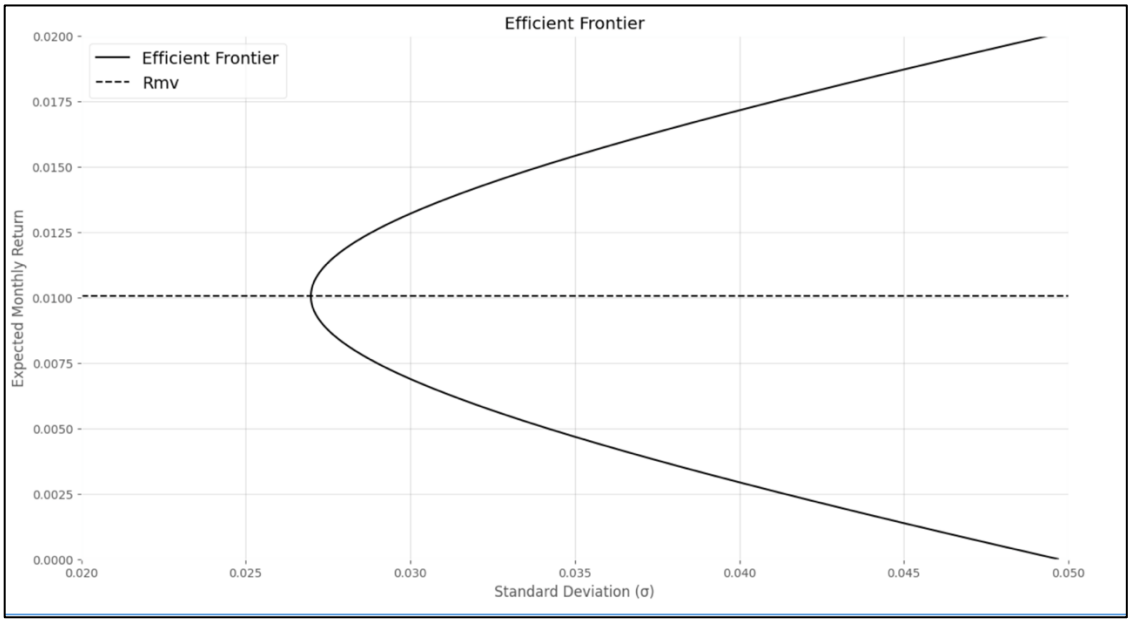
	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
Mean Return	0.009028	0.007333	0.010128	0.012312	0.007662	0.008814	0.009163	0.007838	0.009072	0.004891
Standard Deviation	0.033457	0.083619	0.053103	0.060815	0.053812	0.044483	0.040938	0.037872	0.037018	0.055825

As we can see:
- The mean returns are the highest for the “Energy” sector, and the lowest for the “Other” sector.
- The standard deviation is the highest for the “Durables” sector, and the lowest for the “Non-Durables” sector.

2. Next, alpha, delta and zeta were calculated to be the following:

	Value
α	13.794324
δ	1373.875974
ζ	0.196409

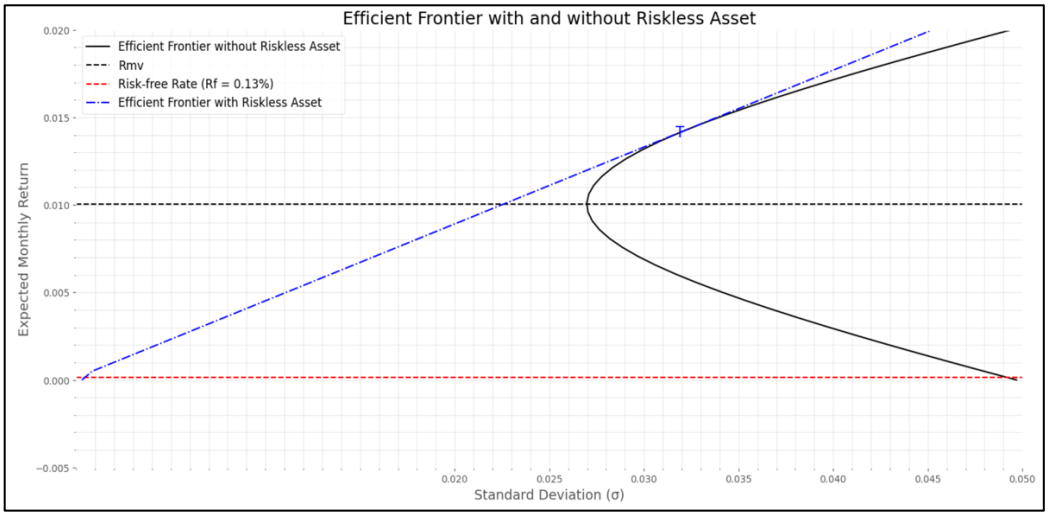
The ‘σ2’ values while varying Rp at ‘0.1’ intervals, beginning from ‘0.0’ till we reach ‘2.0’. The resulting Efficient frontier graph was plotted as below:



3. The minimum-variance frontier helps pick out the most efficient portfolios, i.e., highest upside for the lowest possible risk possible. The portfolios below the mean-variance (Rmv) are less desired as they represent higher risk with lower returns and the opposite for the portfolios above the mean-variance (Rmv). The investor can, simply put, apply the minimum-variance frontier to optimize their portfolios and gain the maximum returns with the lowest amount of risk possible.

4. The Efficient Frontier with the Riskless Asset was calculated and plotted as shown below:

   Since Rf < Rmv, the efficient frontier with the riskless asset is only tangential at the upper half of the efficient frontier without the riskless asset parabola.



5. The efficient frontier portfolio, graphically, is essentially a tangential line that takes into account a riskless asset, in our case, a bond with a yield of 0.13%. The concept here is to add a bond with a lower yield rate compared to the Rmv to hedge the risk the investor takes while picking a portfolio on the minimum-variance frontier. It helps investors asses a portfolio that fits their expected utility.

6. The CAL or the Sharpe Ratio is calculated with the following formula:

$$\underline{\text{Sharpe Ratio (CAL)}} = (\zeta - 2\,\alpha\,R_f + \delta\,R_f^{\,2})^{1/2}$$

   The output was calculated to be:

	Sharpe Ratio
0	0.439142

7. The tangency portfolio represents the highest return for each unit of risk. The tangency portfolio is the point on the Capital Allocation Line with the highest Sharpe ratio. The tangency portfolio allows each investor to fulfil their expected utility by varying their portfolios with a mix of the tangency portfolio and the risk-free asset (Rf). It helps both conservative and aggressive investors balance their portfolios and make the most of their investments. Aggressive investors can borrow at the risk-free rate and invest more in the tangency portfolio while conservative investors can invest more in risk free assets and lend more.