Linear Regression Model

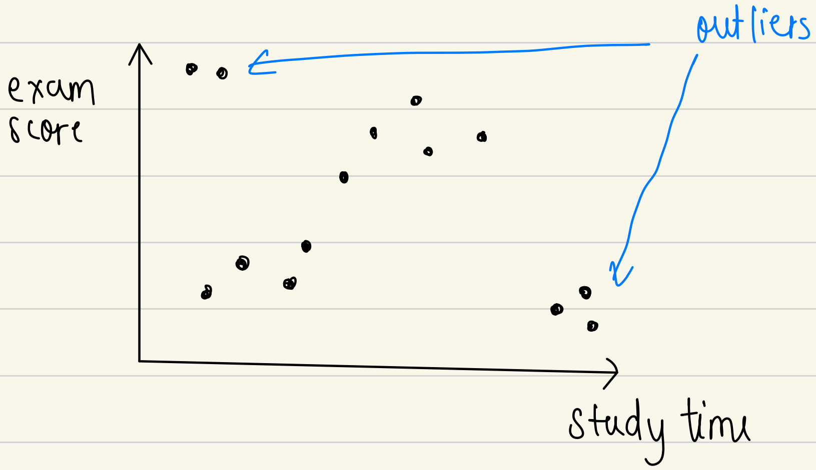
- Understanding Linear Regression

Linear Regression is used to predict the value of one variable based on the other variable

- → The value we want to predict is called dependant variable
- → The value we know and are using to predict other variable is called independent variable

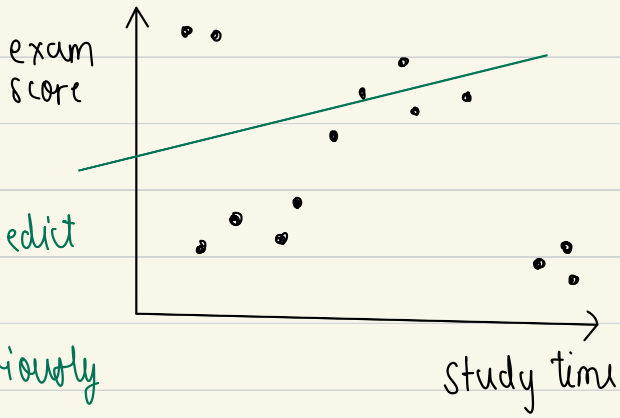
- Example

Suppose we have student data with their study time and exam score



Linear regression tries to find a line which fits this points the best

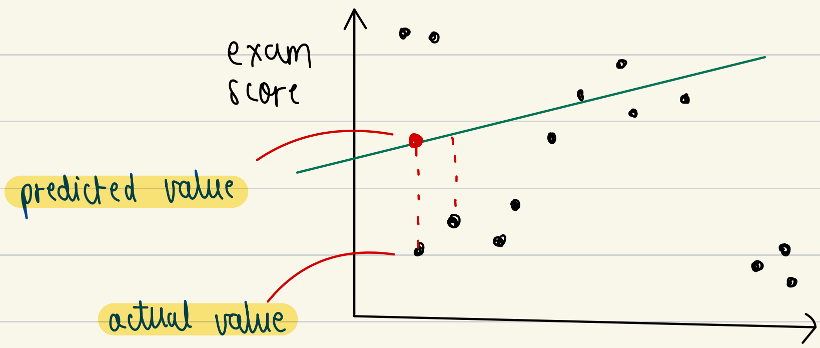
we basically have to minimize the error



suppose we predict this line which is obviously not the best

so now how do we calculate the error for this line?

we calculate error for each point



so basically we want to minimize this gap (error) we have to define a error function first, so that we can minimize it

↳ with Gradient Descent

we have

$$\boxed{y = mx + b}$$

which is our predicted line

Error function is defined as follows

$$\boxed{E = \frac{1}{n} \sum_{i=0}^{n} \left(y_i - \hat{y}_i \right)^2}$$

$\hat{y}_i$ = predicted point we have

y_i = point we want

$$E = \frac{1}{n} \sum_{i=0}^{n} \left(y_i - (m \cdot x_i + b) \right)^2$$

this is called the mean squared error function

- Error Function - Mean Squared Error

$$E = \frac{1}{n} \sum_{i=0}^{n} \left(y_i - (m \cdot x_i + b)\right)^2$$

so for each point $\left(\sum_{i=0}^{n}\right)$ we will take the difference of
actual y value (y_i) and predicted y value ($m \cdot x_i + b$)
and we square that difference
then we
take all these errors and divide by total number of points ($1/n$)

- Finding lowest possible E

now the only thing we can influence is m & b in

$$\boxed{y = \underline{m}x + \underline{b}}$$

we want to find m and b so that we can minimize E

$$\boxed{\underline{E} = \frac{1}{n} \sum_{i=0}^{n} (y_i - (m \cdot x_i + b))^2}$$

how can we do that?

taking partial derivative with respect to m and b because that gives us the direction of the steepest ascend wrt m and b

- Gradient Descent

Taking partial derivative of error function wrt to m & b

$$\frac{\partial E}{\partial m} = \frac{1}{n} \sum_{i=0}^{n} 2\left(y_i - (m x_i + b)\right) \cdot (-x_i)$$

$$= \frac{-2}{n} \sum_{i=0}^{n} (y_i - m x_i + b)(x_i)$$

similarly wrt b

$$\frac{\partial E}{\partial b} = -\frac{2}{n} \sum_{i=0}^{n} \left(y_i - (m \cdot x_i + b)\right)$$

So now to improve m and b
what we need to do for every iteration is :

$$\boxed{\begin{aligned} m &= m - L \cdot \frac{\partial E}{\partial m} \\ b &= b - L \cdot \frac{\partial E}{\partial b} \end{aligned}}$$

where L = learning rate

↳ how large are the steps we take towards improvement