

Question 3

a)

Since this is a continuous valued histogram we will obtain a uniform distribution after equalization. Say the pdf of uniform distribution in the region $[0, a]$ is p_1 and in the region $(a, 1]$ it is p_2 , then:

$$\begin{aligned}\int_0^a p_1 dx &= \alpha \\ \implies p_1 &= \frac{\alpha}{a} \\ \int_a^1 p_2 dx &= 1 - \alpha \\ \implies p_2 &= \frac{1 - \alpha}{1 - a}\end{aligned}$$

Now for mean intensity or expected intensity ($E(I)$) we need to calculate $\int_0^1 I(x)p(x)dx$ where $x \in [0, 1]$ and $I(x) = x$:

$$\begin{aligned}\int_0^1 I(x)p(x)dx &= \int_0^1 xp(x)dx \\ \int_0^1 xp(x)dx &= \int_0^a x \frac{\alpha}{a} dx + \int_a^1 x \frac{1 - \alpha}{1 - a} dx \\ \implies E(I) &= \frac{\alpha}{a} * \frac{x^2}{2} \Big|_0^a + \frac{1 - \alpha}{1 - a} * \frac{x^2}{2} \Big|_a^1 \\ \implies E(I) &= \frac{a\alpha}{2} + \frac{(1 - \alpha)(1 + a)}{2} \\ \implies E(I) &= \frac{a\alpha + 1 - \alpha + a - a\alpha}{2} \\ \implies E(I) &= \frac{1 - \alpha + a}{2}\end{aligned}$$

Therefore the mean intensity is $\frac{1 - \alpha + a}{2}$

b)

Since a is the median intensity therefore equal amount of mass must be distributed around $a \implies \alpha = \frac{1}{2}$. Putting this in the above formula we get the value of mean intensity as:

$$E(I) = \frac{\frac{1}{2} + a}{2}$$

c)

The scenario in which histogram equalization in above manner will do a better job is where we have a image containing contrasting colors or a central bright object and black background. Example a person with fair complexion wearing a black t-shirt. In this case with normal histogram equalization we will get a grey face for the person (due to low intensity values from the t-shirt) which is bad. But in the above case black and white parts will be separated and equalized independently which will not impose the low intensity values on the person's face. This is basically because in the above equalization low and high intensities are treated differently so it does not mix different intensities.

d)

To generate the two images run `hist.py` (without any arguments) keeping `img.jpg` (original image) in the same folder

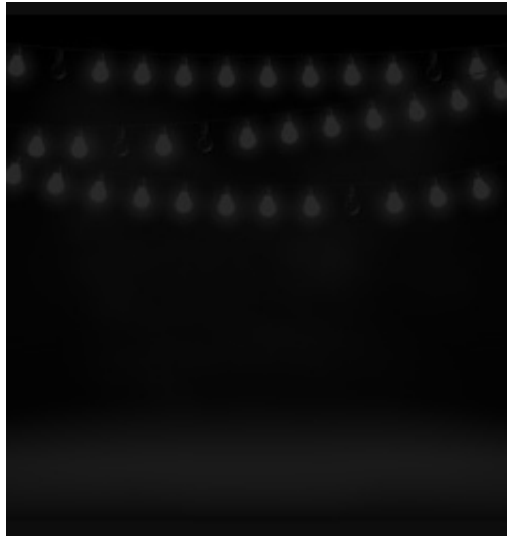


Figure 1: Original Image

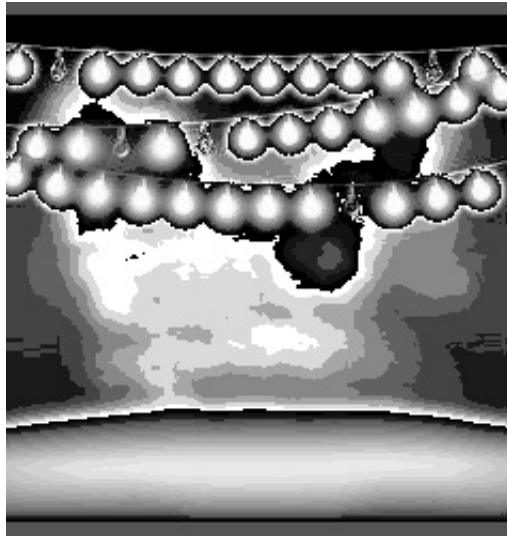


Figure 2: Image with normal Histogram Equalization



Figure 3: Image with median based Histogram Equalization