Assignment 3

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Q1

1)

Maximum Likelihood estimate $\hat{\mu}_{ML} = \frac{\sum x_i}{n}$

$\hat{\mu}_{MAP1}$ (with gaussian as the prior)

$$= \frac{\bar{x}\,\sigma_0^2 + \mu_0\,\sigma^2/N}{\sigma_0^2 + \sigma^2/N}$$

$\bar{x} = \frac{\sum x_i}{N}$ $\quad \sigma_0 = 1 \quad \sigma = 4 \quad \mu_0 = 10.5$

$\hat{\mu}_{MAP2}$:-

Prior:- $P(\mu) = 1/2$ if $\mu \in (9.5, 11.5)$
$= 0$ otherwise.

Posterior $\propto$ Likelihood $\times$ Prior

$\propto \prod_i e^{-\frac{(x_i-\mu)^2}{2\sigma_0^2}} \cdot 1/2$ if $\mu \in (9.5, 11.5)$

$= 0$ otherwise.

$-\log(\text{Posterior}) = -2\sum \mu x_i + \sum x_i^2 + n\mu^2 + k$.

$$\frac{\partial(-\log(\text{Posterior}))}{\partial \mu} = -2\sum x_i + 2n\mu.$$

for negative log posterior (nlp) to be minm.

$\mu = \frac{\sum x_i}{n}$ if $\frac{\sum x_i}{n} \in [9.5, 11.5]$

$= 9.5$ if $\frac{\sum x_i}{n} < 9.5$ because $\frac{d(nlp)}{d\mu} > 0$

$= 11.5$ if $\frac{\sum x_i}{n} > 11.5$ because $\frac{d(nlp)}{d\mu} < 0$

(b)
As can be seen from the graph, as N increases, the error for all three estimates decreases, along with their spread around the error value.
However, for smaller values of N, the maximum a posteriori estimator with Gaussian is a better estimator as it has the least error. Hence it is preferred
Both the graphs have been plotted seperately, since plotting them on the same graph decreased visibility.