Problem 2

$$f(x)_w = \frac{\langle x, \sin(x) \rangle}{\|\sin(x)\|^2} \sin(x) + \frac{\langle x, \cos(x) \rangle}{\|\cos(x)\|^2} \cos(x) + \frac{\langle x, 1 \rangle}{\|1\|^2} 1$$

$$= \frac{\int_0^{2\pi} t \sin(t)\, dt}{\|\sin(x)\|^2} \sin(x) + \frac{\int_0^{2\pi} t \cos(t)}{\|\cos(t)\|^2} \cos(t) + \frac{\int_0^{2\pi} t \cdot 1 \, dt}{\|1\|^2} \cdot 1$$

Now let's evaluate some subexpressions to avoid confusion.

- $\int_0^{2\pi} t \sin(t)\, dt = \ldots \text{ calculus } = \ldots -t\cos(t) + \sin(t) \Big|_0^{2\pi} = -2\pi$
- $\int_0^{2\pi} t \cos(t)\, dt = t\sin(t) + \cos(t) \Big|_0^{2\pi} = (0+1) - (0+1) = 0$
- $\|1\|^2 = \int_0^{2\pi} 1 \cdot dt = 2\pi$
- $\|\sin(x)\|^2 = \int_0^{2\pi} \sin^2(t)\, dt = \pi$
- $\|\cos(x)\|^2 = \int_0^{2\pi} \cos^2(t)\, dt = \|\sin(x)\|^2 = \pi$
- $\int_0^{2\pi} t \cdot 1 \, dt = \frac{1}{2} t^2 \Big|_0^{2\pi} = \frac{1}{2}(4\pi^2) = 2\pi^2$

$$\therefore \; f(x)_w = \frac{-2\pi}{\pi} \sin(x) + \frac{0}{\text{who cares}} \cos(t) + \frac{2\pi^2}{2\pi} \cdot 1$$

$$= \boxed{-2\sin(x) + \cancel{2}\pi}$$