Problem 3

Given $s \in S$ & $s^{\perp} \in S^{\perp}$, $\langle s, s^{\perp} \rangle = 0$. (by definition).

So let's inspect the trace of an arbitrary s (as in problem).

$$tr(AB^{*}) = \sum_{j} (AB^{*})_{jj} = \sum_{j} \sum_{k} A_{jk} B^{*}_{kj}$$

By definition of matrix mult.

In textbook.

$$= \cancel{\sum_{j} \sum_{i} A_{ji} B^{*}_{ij}} = \sum_{j} \sum_{i} A_{ji} \overline{B}_{ji}$$

We see that $tr(AB^{*})$ is really shorthand for $A_{ij} \cdot \overline{B}_{ij}$ by componentwise multiplication (ie. standard dot product!).

As $\alpha \cdot 0 = 0 \ \forall \alpha \in F$, it follows that:

$$S^{\perp} = \begin{bmatrix} 0 & x & b & c & \\ y & 0 & a & & \cdots \\ z & q & 0 & & \\ & \cdots & & 0 & \\ & & & & \cdots \end{bmatrix}$$

etc; ie All matrices with a diagonal of zero.

As $Dim(M_{n \times n}(F)) = n^2$ and $dim(S) = n$,

$Dim(S^{\perp}) = n^2 - n$. This can be shown by counting the variables in our description of $S^{\perp}$.