Problem 4

We've shown $W_{odd} + W_{even} = V$. (previous HW) Now, we'll show $\forall\, o \in W_{odd}$ & $e \in W_{even}$, $\langle o, e\rangle = 0$.

$$\langle o, e\rangle = \int_{-1}^{1} o(t)\, e(t)\, dt = \int_{-1}^{0} o(t) e(t)\, dt + \int_{0}^{1} o(t) e(t)\, dt$$

$$= \int_{0}^{1} o(-t)\, e(-t)\, dt + \int_{0}^{1} o(t)\, e(t)\, dt$$

$$= -\int_{0}^{1} o(t)\, e(t)\, dt + \int_{0}^{1} o(t)\, e(t)\, dt$$

$$= 0.$$

We change variables: $t_{new} = -t_{old}$.

This lets us pull out a "-", which lets us change boundary order.

As $\langle o, e\rangle = 0$ $\forall\, o \in Odd, e \in Even$, $W_{odd} \perp W_{even}$.

As shown before, $W_{odd} + W_{even} = V$.

Thus $W_{odd}^{\perp} = W_{even}$.

Problem 5

$$\langle v_w, x\rangle = \left\langle \sum_{w \in B} \frac{\langle v, w\rangle}{\|w\|^2} w, x \right\rangle = \sum_{w\in B} \left(\frac{\langle v, w\rangle}{\|w\|^2}\right) \langle w, x\rangle$$

for some basis B of W; scalar; inner prod

$$= \sum_{w\in B} \frac{\langle w, x\rangle}{\|w\|^2} \langle v, w\rangle = \left\langle v, \sum_{w\in B} \frac{\overline{\langle w, x\rangle}}{\|w\|^2} w \right\rangle = \left\langle v, \sum_{w\in B} \frac{\langle x, w\rangle}{\|w\|^2} w \right\rangle$$

$\|w\| \in \mathbb{R}$, so $\overline{\|w\|} = \|w\|$

$$= \langle v, x_w\rangle.$$