Assignment 2b

EE698V – Machine Learning for Signal Processing

- Take a printout of this pdf and write your answers here. Or else, write your answers on A4 sheets at the same locations as in this pdf.
- You can attach extra sheets at the end. But these first three sheets should contain the final answer.
- Submissions should be hand-written and handed over to your TAs before the deadline.

Q1. DSP:

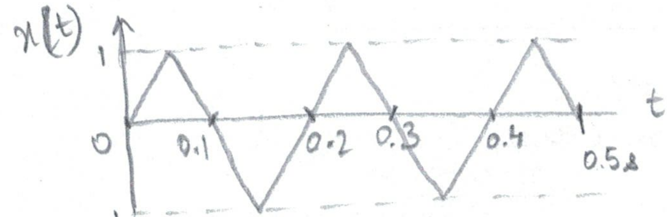
a. Write your last name (surname): ARORA

b. Plot a triangular wave signal x (amplitude spanning between -1 to +1) of 0.5s duration with frequency ~~Fs~~ (in Hz), where ~~Fs~~=length of your last name (surname). Mark the axes limits and wherever the signal is 0. **(2 Marks)**

$F_0 = 5\text{ Hz}$

In 0.5 s, we will see

$\#\text{cycles} = 0.5 \times 5 = 2.5$



or

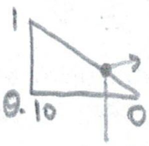
c. Sample x at a frequency 7Hz and quantize it to 4 levels between -1 to +1 to obtain x_1. Plot the resulting time series x_1. **(4 Marks)**

$T_s = \frac{1}{7}\text{ s} = 0.143\text{ s}$

Choose the 4 levels as $\{-1, -\frac{1}{3}, +\frac{1}{3}, 1\}$

$x[0] = -1$

$x[1] = x(t = \frac{1}{7})$



$\frac{1}{0.05} = \frac{x}{0.15 - 0.143} = \frac{x}{0.007}$

$\Rightarrow x = 0.7/5$

So, $x[1] = \frac{0.7}{5} < \frac{2}{3}$

$\therefore x[1] = \frac{1}{3}$

$x[2] = x(t = \frac{2}{7})$

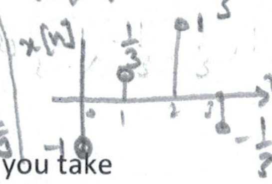
$\frac{1}{0.05} = \frac{x}{0.036}$

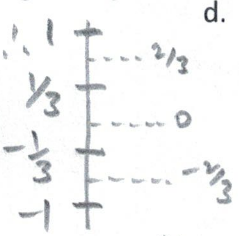
$x = \frac{3.6}{5} > \frac{2}{3}$

$\therefore x[2] = 1$

$x[3] = x(t = 3/7)$

$x[3] = -\frac{1}{3}$





(boundaries)

d. Sample x at a frequency Fs >> Nyquist frequency to obtain a time series x_2. If you take an N-point DFT of x_2, where N>Fs, you will get the most prominent peak at k in the magnitude of the DFT? Express k in terms of N and Fs. **(4 Marks)**

$x(t) \approx \sin(2\pi f_0 t)$

$x[n] = \sin\left(2\pi f_0 \frac{n}{F_s}\right) = Re\left\{ e^{2\pi f_0 \frac{n}{F_s}} \right\}$

$X[k] \propto \sum_n \left\{ e^{2\pi f_0 \frac{n}{F_s}} \times e^{-2\pi k \frac{n}{N}} \right\}$

Peak occurs when $2\pi f_0 \frac{n}{F_s} = 2\pi k \frac{n}{N}$

$\Rightarrow k = f_0 \frac{N}{F_s} = \frac{5N}{F_s}$ in my case

Q2. Evaluation Metrics

A classifier is designed to classify rose R, lotus L and jasmine J. There are 8 test samples, on which the classifier predicts the probability of each class as follows:

P(R)	P(L)	P(J)	True label
0.37	0.63	0.	R
0.56	0.27	0.17	R
0.2	0.37	0.43	L
0.33	0.26	0.42	L
0.18	0.79	0.02	L
0.41	0.25	0.34	J
0.12	0.17	0.7	J
0.49	0.16	0.35	R

a. If the hard label is decided as $y = arg\max_{l} P(l)$,

a. the accuracy of the classifier will be: **(1 Marks)** $\frac{4}{8} = 50\%$

b. the confusion matrix will look like: **(2 Marks)**

$$CM = \begin{array}{c|ccc} \text{true} \backslash \text{est.} & R & L & J \\ \hline R & \text{II} & \text{I} & \\ L & & \text{I} & \text{II} \\ J & \text{I} & & \text{I} \end{array} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

c. For lotus, the precision, recall and F1 measure will be: **(3 Marks)**

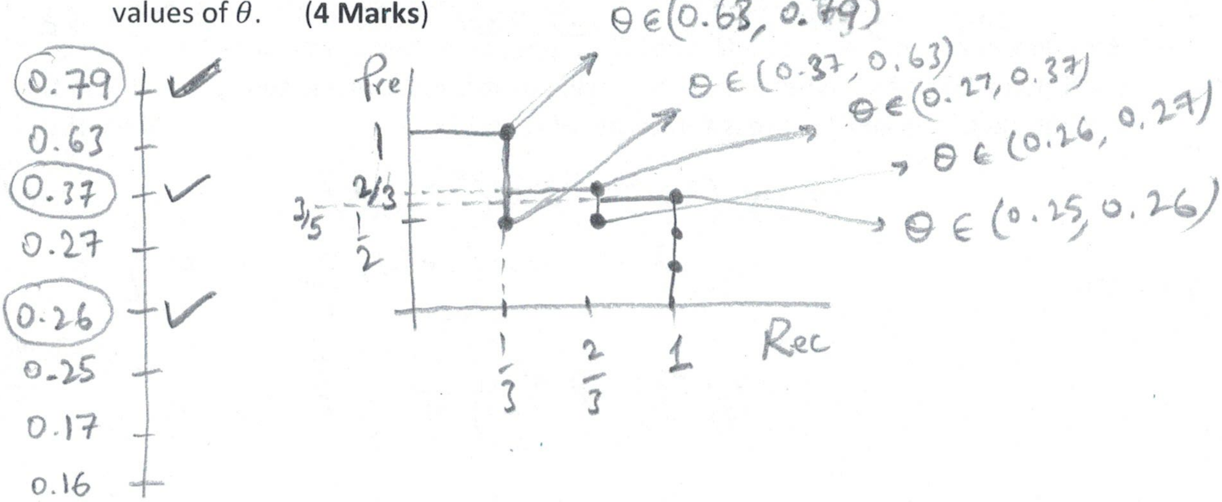
gt \ est	not L	L
not L	4	1
L	2	1

$Pre = \frac{1}{2}$

$Rec = \frac{1}{3}$

$$F_1 = \frac{2 \cdot \frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}} = \frac{2}{5}$$

b. If I want to use this classifier to detect lotuses, I can decide the label using a threshold $\theta > 0$ as $y = L, if\ P(L) > \theta$. Draw the precision recall curve (PRC) by varying the values of θ. **(4 Marks)**



Q3. Image Filtering

An image is given as a matrix F. It is filtered with a filter matrix H.

a. Write the output G, which is a correlation of F and H, with zero padding. **(4 Marks)**
Hint: See reference: http://www.robots.ox.ac.uk/~az/lectures/ia/lect1.pdf

$$F = \begin{array}{|c|c|c|c|c|}\hline 0.6 & 0.2 & 1.2 & 0.4 & 0.3 \\ \hline 0.9 & 1.7 & 0.3 & 1.1 & 0.1 \\ \hline 0.6 & 0.8 & 1.1 & 0.2 & 0.5 \\ \hline 0.4 & 0.1 & 0.4 & 0 & 0.2 \\ \hline \end{array} \qquad H = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

-0.1	-0.1	-0.2	-0.2	-0.2	-0.1	0
-0.2	0.8	-0.1	1.9	0.4	0.4	0
-0.2	1.3	2.6	-0.2	1.6	-0.1	-0.1
-0.2	0.7	0.9	1.6	0	0.8	-0.1
-0.1	0.6	-0.2	0.5	-0.3	0.3	-0.1
-0.	-0.1	-0.1	-0.1	-0.1	0	0

b. This operation is called as (image) Sharpening. **(1 Mark)**

Q4. DFT

a. Prove that for the DFT of a real time series x, just half the spectrum (i.e., k=0 to Nfft/2) is sufficient to reconstruct the entire spectrum (i.e., for any k). **(4 Marks)**
Hint: use the definition of X[k].

b. Does this hold for a complex time series too? **(1 Mark)**

a) $X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$ --- (1)

Consider, $X^*[k] = \sum_{n=0} x^*[n]\, e^{j2\pi\frac{k}{N}n}$ $\quad$ ($x^*[n] = x[n]$ for real $x[n]$) --- assumption (a)

$= X[-k]$

We know $|X^*| = |X|$ and $\measuredangle X^* = -\measuredangle X$, for all $X \in \mathbb{Z}$ (complex no.)

$\therefore |X^*[k]| = |X[k]|$

$\Rightarrow |X[-k]| = |X[k]|$

$\Rightarrow |X[k]|$ is an EVEN function

b) Does NOT hold for complex $x[n]$ $\because$ of assumption (a)

Also, $X[k+N] = X[k]$ (from (1))

$\Rightarrow |X[k]|$ is periodic with period N

$\therefore |X[k]| = |X[k \% N]|$ and $|X[k]| = |X[N-k]|$

can be used to find $|X[k]|$ for any k, given $|X[k]|$, k = 0, ...