

1)

Post A: ["cats", "are", "cute", "and", "funny"]

Post B: ["dogs", "are", "funny", "animals"]

Post C: ["cats", "and", "dogs", "rarely", "get", "along"]

Word vector:

$v = ["\text{along}", "\text{and}", "\text{animals}", "\text{are}", "\text{cats}", "\text{dogs}", "\text{funny}", "\text{get}", "\text{rarely}"]$

Vec A: [0, 1, 0, 1, 1, 1, 0, 1, 0, 0]

Vec B: [0, 0, 1, 1, 0, 0, 1, 1, 0, 0]

Vec C: [1, 1, 0, 0, 1, 0, 1, 0, 1, 1]

$$\text{Matrix } x = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$P(\text{cat}) = P$$

$$P(\text{not cat}) = 1 - P$$

10 words
 $(1 - P)^{10}$

$$P(\text{not cat}) = (1 - P)^L$$

$$\begin{aligned} P(\text{at least one cat}) &= 1 - P(\text{not cat}) \\ &= 1 - (1 - P)^L \end{aligned}$$

$$(5(\text{Post A}) + 4(\text{Post B}) + 6(\text{Post C}))$$

Total words in corpus = 15

occurrences of "cat": 2

$$P = \frac{2}{15}$$

→ for Post A ($L = 5$)

$$P(\text{Post A is cat-type}) = 1 - \left(1 - \frac{2}{15}\right)^5$$

$$= 1 - \left(\frac{13}{15}\right)^5$$

$$\begin{aligned} P(\text{Post A is cat-type}) &\approx 1 - 0.489 \\ &\approx 0.511 \end{aligned}$$

Total Posts: 3 (A, B, C)

Cat-Type Post: {A, C} (count = 2)

Posts containing "cute": {A} (count = 1)

$P\left(\frac{\text{cute}}{\text{cat-type}}\right) = \frac{\text{Number of cat-type posts with "cute"}}{\text{Total number of cat-type posts}}$

$$P\left(\frac{\text{cute}}{\text{cat-type}}\right) = \frac{1(\text{Post A})}{2(\text{Post A \& Post B})} = 0.5$$

$\rightarrow 50\%$

$$P\left(\frac{\text{cat-type}}{\text{cute}}\right) = \frac{P(\text{cute}/\text{cat-type}) \times P(\text{cat-type})}{P(\text{cute})}$$
$$= \frac{0.5 \times 2/3}{1/3} = 1$$

$\rightarrow 100\%$

$$U(L) = -\frac{1}{20}L^2 + 3L$$

First derivative $U'(L)$

$$U'(L) = -\frac{1}{10}L + 3$$

$$L = 30 \quad \text{at } U'(L) = 0$$

$$U''(L) = -\frac{1}{10} \quad \therefore L=30 \text{ is Max}$$

$$G(L, p) = p(L, p) \cdot U(L)$$

$$= [1 - (1-p)^L] \cdot \left[-\frac{1}{20}L^2 + 3L \right]$$