

1)

Post A: ["cats", "are", "cute", "and", "funny"]

Post B: ["dogs", "are", "funny", "animals"]

Post C: ["cats", "and", "dogs", "rarely", "get",
"along"]

Word vector:

$V = \begin{bmatrix} \text{"along"}, \text{"and"}, \text{"animals"}, \text{"are"}, \text{"cats"}, \\ \text{"dogs"}, \text{"funny"}, \text{"get"}, \text{"rarely"} \end{bmatrix}$

Vec A: [0, 1, 0, 1, 1, 1, 0, 1, 0, 0]

Vec B: [0, 0, 1, 1, 0, 0, 1, 1, 0, 0]

Vec C: [1, 1, 0, 0, 1, 0, 1, 0, 1, 1]

Matrix $X = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$$P(\text{cat}) = p$$

$$P(\text{not cat}) = 1 - p \quad \begin{matrix} 10 \text{ words} \\ (1-p)^{10} \end{matrix}$$

$$P(\text{not cat}) = (1-p)^L$$

$$P(\text{atleast one cat}) = 1 - P(\text{not cat}) \\ = 1 - (1-p)^L$$

$$(5(\text{Post A}) + 4(\text{Post B}) + 6(\text{Post C}))$$

Total words in corpus = 15
occurrences of "cat": 2

$$p = \frac{2}{15}$$

→ for Post A ($L=5$)

$$P(\text{Post A is Cat-type}) = 1 - \left(1 - \frac{2}{15}\right)^5$$

$$= 1 - \left(\frac{13}{15}\right)^5$$

$$P(\text{Post A is Cat-type}) \approx 1 - 0.489 \\ \approx 0.511$$

Total Posts: 3 (A, B, C)

Cat-Type Post: {A, C} (count = 2)

Posts containing "cute": {A} (count = 1)

$$p\left(\frac{\text{cute}}{\text{cat-type}}\right) = \frac{\text{Number of cat-type posts with "cute"}}{\text{Total number of cat-type posts}}$$

$$p\left(\frac{\text{cute}}{\text{cat-type}}\right) = \frac{1(\text{Post A})}{2(\text{Post A \& Post D})} = 0.5$$

→ 50%

$$p\left(\frac{\text{cat-type}}{\text{cute}}\right) = \frac{p(\text{cute/cat-type}) \times P(\text{cat-type})}{P(\text{cute})}$$

$$= \frac{0.5 \times 2/3}{1/3} = 1$$

→ 100%

$$U(L) = -\frac{1}{20}L^2 + 3L$$

first derivative $U'(L)$

$$U'(L) = -\frac{1}{10}L + 3$$

$$L = 30 \quad \text{at } U'(L) = 0$$

$$U''(L) = -\frac{1}{10} \quad \therefore L = 30 \text{ is max}$$

$$G(L, p) = P(L, p) \cdot U(L)$$

$$= [1 - (1-p)^L] \cdot \left[-\frac{1}{20}L^2 + 3L \right]$$