Name : Yash Manharbhai Patel

Student ID : A20451170

Course : CS512 - Computer Vision

Semester : Fall20

CS-512 - Assignment-4

Ans-1) a) Projection equation $p = MP$,

Problem of :

forward projection -

=> one is radial lens distortion,
It gives large shrink away from the center
once parameters are determined, the image can be corrected

$$p^{(i)} = \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} K^{*}[R^{*}|T^{*}]\, P^{(w)}$$

$$\lambda = 1 + k_1 d + k_2 d^2$$

d = distance from center
k_1 = linear distortion coefficient
k_2 = quadratic distortion coefficient

=> second is weak perspective camera,
It is correct when depth variation in the scene is small compared with distance from camera.

$$e = |M_{\infty}P - MP| = \frac{\overbrace{\Delta}^{\text{depth variation}}}{\underbrace{d_0}_{\text{distance from camera}}}\underbrace{(MP - P_0)}_{\text{distance from center}}$$

calibration -

camera uses 3x4 projection Matrix, $M = K^*[R^* | T^*]$, which includes internal and external parameters.

Here we need to find 12 unknown parameters of M. It requirs minimum 6 points Every point give us two equation. So, finaly we will get 12 equations

Parameters $\rightarrow K^*, R^*, T^*$

$\rightarrow R, T, f, K_u, K_v, u_0, v_0, S,$

reconstruction -

one is correspodence is not possible.

- object points not visible in both views (occlesion)
- ambignous points (multiple candidates)
- uniform regions (inside points cannot be distiguished)

forward projection problem is the easiest.
reconstruction problem is the most difficult

b) In camera calibration we need to find values of projection matrix M which is 3x4.

we need to find 12 values of M.

we required points $\{p_i\}_{i=1}^{m} \longleftrightarrow \{\underline{P_i}\}_{i=1}^{m}$

Here, p_i is 2DH image coordinates and $\underline{P_i}$ is 3DH word coordinates.

to find 12 unknowns, we need 12 equations. but each point gives us 2 equations. so, we need atleast 6 points

using these 12 equation:
- find projection matrix M
- find parameters from M.

$M \rightarrow K^*, R^*, T^*$

$\rightarrow R, T, f, K_u, K_v, u_0, v_0, \theta$

c) In non-coplaner calibration,

Given: $\{p_i\}_{i=1}^{M} \longleftrightarrow \{\underline{P}_i\}_{i=1}^{M}$

2DH image points — 3DH world points

steps: - find projection matrix M

Here, projection matrix M is 3×4. we need to find 12 values.

$$M = \begin{bmatrix} — m_1^T — \\ — m_2^T — \\ — m_3^T — \end{bmatrix}$$

- using m_1, m_2, m_3 find parameters

$M \rightarrow K^*, R^*, T^* \rightarrow f, K_u, K_v, u_0, v_0, s, R, T.$

M_1, M_2, M_3 have 12 unknowns, it requires 12 equations, so we need minimum 6 points. each point gives 2 equations.

d) $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, $P_i^w = \underset{3D}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \underset{3DH}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}}$

$$P_i^i = M P_i^w = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \underset{2DH}{\begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix}}$$

2D image coordinates $= \left(\frac{18}{7}, \frac{14}{7}\right)$

e) $\underset{3D}{(1,2,3)} \longleftrightarrow \underset{2D}{(100, 200)}$

$\underset{3DH}{(1,2,3,1)} \longleftrightarrow \underset{2DH}{(100, 200, 1)}$

Ans : $\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & -200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}$

f) M is a 3×4 matrix which has 12 unknown values

to find 12 unknown values, we need 12 equations.

every point gives us 2 equations, so, we need at least 6 points.

$$\left.\begin{aligned} m_1^T \underline{P_i} - x_i m_3^T \underline{P_i} &= 0 \\ m_2^T \underline{P_i} - y_i m_3^T \underline{P_i} &= 0 \end{aligned}\right\} \quad x_i = \frac{x_i}{w_i} = \frac{m_1^T \underline{P_i}}{m_3^T \underline{P_i}} \qquad y_i = \frac{y_i}{w_i} = \frac{m_2^T \underline{P_i}}{m_3^T \underline{P_i}}$$

for single point:

$$\underbrace{\begin{bmatrix} \underline{P_i}^T & 0 & -x_i \underline{P_i}^T \\ 0 & \underline{P_i}^T & -y_i \underline{P_i}^T \end{bmatrix}}_{2\times12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times1}$$

for m points:

$$\underbrace{\begin{bmatrix} \underline{P_1}^T & 0 & -x_1 \underline{P_1}^T \\ 0 & \underline{P_1}^T & -y_1 \underline{P_1}^T \\ \vdots & \vdots & \vdots \\ \underline{P_m}^T & 0 & -x_m \underline{P_m}^T \\ 0 & \underline{P_m}^T & -y_m \underline{P_m}^T \end{bmatrix}}_{2m\times12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{2m\times1}$$

$$AX=0$$

To solve $Ax=0$ use SVD

$A = UDV^T$

Solution = column of V belonging to zero singular value

$$\hat{x} = \begin{bmatrix} \hat{M}_1 \\ \hat{M}_2 \\ \hat{M}_3 \end{bmatrix} \Rightarrow \hat{M} = \begin{bmatrix} — \hat{M}_1^T — \\ — \hat{M}_2^T — \\ — \hat{M}_3^T — \end{bmatrix}$$

The solution is not unique. if $A\hat{x} = 0$ we also have $A(\rho\hat{x}) = 0 \Rightarrow \rho\hat{x}$ is a solution

2) finding parameters from M,

$$M = \rho\hat{M} = K^*[R^* | T^*]$$

$$[K^*R^* | K^*T^*] = \rho\hat{M}$$

$$\rho\hat{M} = \underbrace{\rho}_{\text{unknown}}\underbrace{\left[\begin{array}{c|c} — a_1^T — & \\ — a_2^T — & b \\ — a_3^T — & \end{array}\right]}_{\text{Known}} = \underbrace{\left[\begin{array}{c|c} \alpha_u r_1^T + s r_2^T + u_0 r_3^T & \\ \alpha_v r_2^T + v_0 r_3^T & K^*T^* \\ r_3^T & \end{array}\right]}_{\text{unknown.}}$$

$\rightarrow \alpha_u r_1^T + s r_2^T + u_0 r_3^T = \rho a_1^T$

$\rightarrow \alpha_v r_2^T + v_0 r_3^T = \rho a_2^T$

$\rightarrow r_3^T = \rho a_3^T$

$\rightarrow K^*T^* = \rho b$

Known

a_1, a_2, a_3, b

unknown

$r_1, r_2, r_3, T^*, (K^*)$

$s, \alpha_u, \alpha_v, u_0, v_0, \rho$

$$|\rho| = 1/|a_3|$$
$$u_0 = |\rho|^2 \, a_1 \cdot a_3$$
$$v_0 = |\rho|^2 \, a_2 \cdot a_3$$
$$\alpha_v = (|\rho|^2 a_2 \cdot a_2 - v_0^2)^{1/2}$$
$$s = |\rho|^4/\alpha_v \, (a_1 \times a_3) \cdot (a_2 \times a_3)$$
$$\alpha_u = (|\rho|^2 a_1 \cdot a_1 - s^2 - u_0^2)^{1/2}$$

$$K^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\epsilon = \mathrm{sgn}(b_3)$$

$$T^* = \epsilon |\rho| (K^*)^{-1} b$$
$$r_3 = \epsilon |\rho| a_3$$
$$r_1 = |\rho|^2/\alpha_v \, a_2 \times a_3$$
$$r_2 = r_3 \times r_1$$
$$R^* = [r_1^T \; r_2^T \; r_3^T]^T$$

h) Given $\{p_i\}_{i=1}^{m} \longleftrightarrow \{\underline{P_i}\}_{i=1}^{m}$ and

image point   world point

estimated $M = \begin{bmatrix} — m_1^T — \\ — m_2^T — \\ — m_3^T — \end{bmatrix}$

assess the quality of fit using:

$$E = \frac{1}{m} \sum_i \left(\left\| x_i - \frac{m_1^T \underline{P_i}}{m_3^T \underline{P_i}} \right\|^2 + \left\| y_i - \frac{m_2^T \underline{P_i}}{m_3^T \underline{P_i}} \right\|^2 \right)$$

distance between known and predicted positions.

e.g. $(7.3, 12.1, 15.3) \longleftrightarrow (5,3)$ known $\longrightarrow \Sigma \longrightarrow$ error

$\longrightarrow (6,2)$ predicted using M.

i) To recover M we need to solve,

$$\begin{bmatrix} \underline{P}_1^T & 0 & -x_1 \underline{P}_1^T \\ \underline{0} & \underline{P}_1^T & -y_1 \underline{P}_1^T \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$$

- when P_i are all on the same plane π, we have $\pi^T \underline{P}_i = 0 \quad \forall \underline{P}_i$

$\pi = (\underbrace{a, b, c}_{\text{normal}}, \underbrace{-d}_{\text{distance}})$, $\underline{P}_i = (x, y, z, 1)$

- there are many possible solutions

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \alpha \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ \pi \end{bmatrix}$$

e.g. $\alpha = 1, \beta = 0, \gamma = 0$

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Pi \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \end{bmatrix}$$

any combination of α, β, γ could be a solution.

Additional degenerate configurations are points on a quadratic surface.

Here, infinite number of solutions are possible. So we cannot get unique solution of projection matrix M. but we can get unique solution in non-coplanar one.

i) difference between the homography H (2D projective map) and the projection matrix M.

$$\underset{2DH}{p_i} = \overset{3\times4}{M}\underset{3DH}{\underline{P_i}} = S\hat{M}\,\underline{P_i} \Rightarrow \hat{M} = \begin{bmatrix} \text{---}\ \hat{M}_1^T\ \text{---} \\ \text{---}\ \hat{M}_2^T\ \text{---} \\ \text{---}\ \hat{M}_3^T\ \text{---} \end{bmatrix}$$

$$\underset{2DH}{p_i} = \overset{3\times3}{H}\underset{2DH}{\underline{p_i}} \Rightarrow \underset{2DH}{p_f} = \underset{3\times4}{K^*[R^* \mid T^*]}\,\underset{3DH}{\underline{P_i}}$$

$$= K^*[r_1 \; r_2 \; r_3 \; T^*] \begin{bmatrix} x_i \\ y_i \\ 0 \\ 1 \end{bmatrix}$$

$$= K^*[x_i r_1 + y_i r_2 + T^*]$$

$$= \underbrace{K^* [r_1 \quad r_2 \quad T^*]}_{3\times3} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

2D projection map $\qquad$ 2DH

To get a 2D projective map we assume: Z coordinate is 0.

$\{P_i\}_z = 0 \iff P_i^o = (x_i, y_i, 0)$

$$\underbrace{\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}}_{2DH} = \underbrace{H}_{3\times3} \underbrace{P_i^*}_{2DH} = \underbrace{\begin{bmatrix} — h_1^T — \\ — h_2^T — \\ — h_3^T — \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} x_i^o \\ y_i^o \\ 1 \end{bmatrix}}_{2DH}$$

Projection matrix M is 3X4, and 2D projection map is 3X3

Given $\underbrace{\{P_i\}_{i=1}^m}_{2DH} \longleftrightarrow \underbrace{\{P_i\}_{i=1}^m}_{3DH}$ $\quad P_i = \underbrace{\begin{bmatrix} x_i \\ y_i \\ 0 \\ 1 \end{bmatrix}}_{3DH}$

Define $P_i^* = \underbrace{\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}}_{2DH}$

$$\underbrace{\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}}_{2DH} = \underbrace{K^* [r_1 \quad r_2 \quad T^*]}_{3\times3} \underbrace{P_i^*}_{2DH}$$

$$x_i = \frac{u_i}{w_i} = \frac{h_1^T \underline{P}_i^*}{h_3^T \underline{P}_i^*} \Rightarrow h_1^T \underline{P}_i^* - x_i h_3^T \underline{P}_i^* = 0$$

$$y_i = \frac{v_i}{w_i} = \frac{h_2^T \underline{P}_i^*}{h_3^T \underline{P}_i^*} \Rightarrow h_2^T \underline{P}_i^* - y_i h_3^T \underline{P}_i^* = 0$$

for projection matrix (M) : 12 unknowns
require 12 eq^n to solve
require 6 points minimum

for 2D-projection map (H) : 9 unknowns
(8 independent)
require 8 eq^n to solve
require 4 points minimum

each point gives two eq^{n}s.

$$H = K^* [r_1 \;\; r_2 \;\; T^*] = \alpha \hat{H}$$

$$\hat{H} = [\hat{h}_1 \;\; \hat{h}_2 \;\; \hat{h}_3]$$ columns of H

$$\hat{M} = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix}$$ rows of M

Ans-25 a) $P_{im} = (1, 2)$
$P_w = (3, 4, 5)$ $\Bigg\} \Rightarrow$ $P_i = (1, 2, 1)^T$
$\underline{P}_i = (3, 4, 5, 1)^T$ $\Bigg] \Rightarrow$ $x_i = 1$
$y_i = 2$

$$\text{Ans: } \begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 & 0 & 0 & -3 & -4 & -5 & -1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 5 & 1 & -6 & -8 & -10 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \underline{P}_i^T & 0 & -x_i \underline{P}_i^T \\ 0 & \underline{P}_i^T & -y_i \underline{P}_i^T \end{bmatrix}$$

b) $$\hat{M} = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{array}\right]$$ $u_0 = ?$
$v_0 = ?$

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \qquad a_3 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$|\rho| = 1/|a_3| = 1/\sqrt{50} = 1/5\sqrt{2}$$

$$u_0 = |\rho|^2 a_1 \cdot a_3 = \frac{1}{50} \cdot 26 = \frac{13}{25} = 0.52$$

$$v_0 = |\rho|^2 a_2 \cdot a_3 = \frac{1}{50} \cdot 38 = \frac{19}{25} = 0.76$$

c) $P_{im} = (1, 2)$
$P_w = (3, 4, 5)$, $$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

$m = 1$ $\qquad m_1^T = (1\ 2\ 3\ 4)$

$x_i = 1$ $\qquad m_2^T = (2\ 3\ 4\ 5)$

$y_i = 2$ $\qquad m_3^T = (3\ 4\ 5\ 6)$

$P_i = (3, 4, 5, 1)^T$

$$E = \frac{1}{m}\sum_i \left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)$$

$$= \frac{1}{1}\left[\left\| 1 - \frac{30}{56} \right\|^2 + \left\| 2 - \frac{43}{56} \right\|^2 \right]$$

$$= \| 0.464 \|^2 + \| -1.232 \|^2$$

$$= 0.215 + 1.518$$

$$= \underline{\underline{1.733}}$$

d) $I + a = R^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$T^* = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, $\quad R = ?$, $\quad T = ?$

$R^* = R^T \implies R = (R^*)^T$

$T^* = -R^T T \implies T = -(R^*)^T T^*$

$$R = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = -\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = -\begin{bmatrix} 6 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} -6 \\ -2 \\ -3 \\ -1 \end{bmatrix}}_{3DH} = \underbrace{\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}}_{3D}$$

e) $P_{im} = (1, 2)$

$P_w = (3, 4, 0) \Rightarrow P_w^* = (3, 4)$

$x_i = 1 \quad , \quad y_i = 2 \quad , \quad P_i^{*T} = (3, 4, 1)$

Ans:
$$\begin{bmatrix} P_i^{*T} & 0 & -x_i \, P_i^{*T} \\ 0 & P_i^{*T} & -y_i \, P_i^{*T} \end{bmatrix}_{2\times 9}$$

$$: \begin{bmatrix} 3 & 4 & 1 & 0 & 0 & 0 & -3 & -4 & -1 \\ 0 & 0 & 0 & 3 & 4 & 1 & -6 & -8 & -2 \end{bmatrix}_{2\times 9}$$

Ans-3) a) In sparse approach we detect feature points and then match. These feature points are not everywhere. So, it is sparse it is at some locations only.

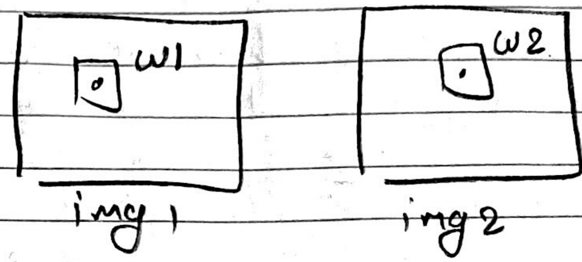
In dense approach we match every point of one image with every point in another image. There are lot of match everywhere. So, it is dense. we use correlation or sum of square distances. it produce more points

advantages:
sparse → It handle large disparities
→ more robust
→ if big change in scene then it can handle it
dense → it limited to very small change of the view

disadvantages:
sparse → ambiguous points (multiple candidates)
dense → object points not visible in both views (occlusion)
→ uniform regions (inside points cannot be distinguished)

b) - Insted of features points compare all patches
- Insted of distance b/w feature vectors measure correlation or SSD
- Apply regularization to reduce errors and find correspodence in difficult areas (e.g uniform)

match all patches of img1 with all patches of img2



Correlation : $\varphi(w_1, w_2) = \sum_i w_1(x_i, y_i) \cdot w_2(x_i, y_i)$

High means good similarity and vise a versa

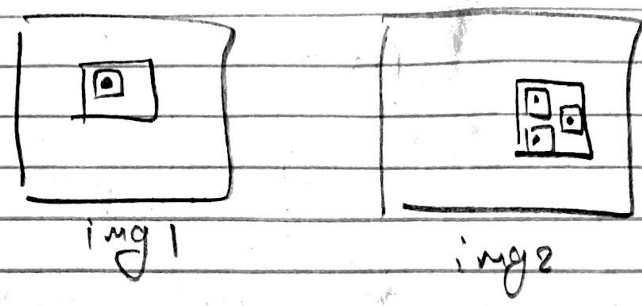
SSD : $\varphi(w_1, w_2) = -\sum_i \left(w_1(x_i, y_i) - w_2(x_i, y_i)\right)^2$

we take euclidean distance. large distance bad similarity and vise a versa
High value = good correspondence

ZNCC : $\varphi(w_1, w_2) = \sum_i \frac{(w_1(x_i, y_i) - \mu_{w_1})(w_2(x_i, y_i) - \mu_{w_2})}{\sigma_{w_1} \sigma_{w_2}}$

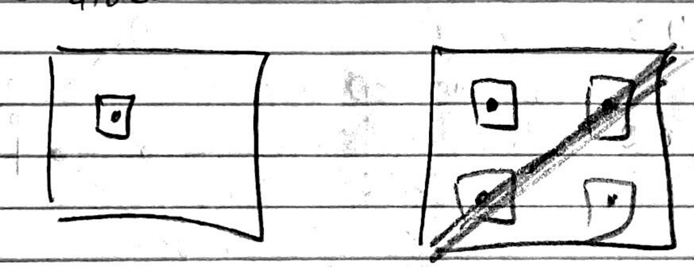
ZNSSD : $\varphi(w_1, w_2) = -\sum_i \left(\frac{(w_1(x_i, y_i) - \mu_{w_1})}{\sigma_{w_1}} - \frac{(w_2(x_i, y_i) - \mu_{w_2})}{\sigma_{w_2}}\right)$

→ The risk in allowing the search space to be the entire image is difficult areas like uniform regions



because of uniform regions many solutions are possible.
So, apply regularization to reduce error

→ To reduce search space to a line, define constraints for reducing # of candidates



during search for corresponding points, only search for points along the line, it will reduce # of candidates

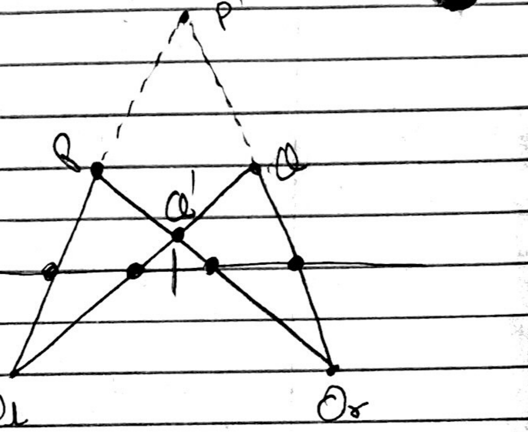
c) $P_l = (100, 200) = (X_l, y_l)$
$P_r = (103, 200) = (X_r, y_r)$
$f = 10$
$T = 100$
$Z = ?$

depth, $Z = f \cdot \frac{T}{d} = \frac{10 \times 100}{3} = 1000/3$
$= \underline{\underline{333.33}}$

$d = X_r - X_l = 103 - 100 = 3$

d) In stereo matching, there is an ambiguity in correspondence between points matching

Correct : P, Q
incorrect : P', Q'



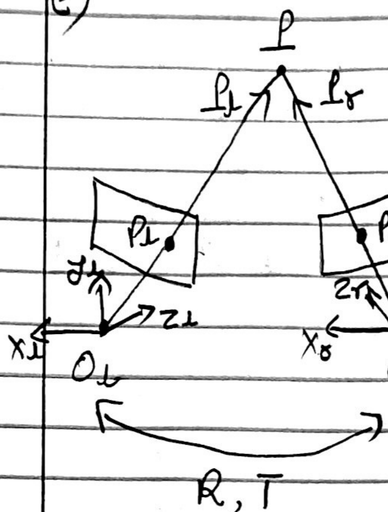
Sceane has two 3D Points
project both points on both image

P, Q correct correspondence
P', Q' incorrect correspondence

because of the triangulation we get Points P', Q' on intersection.

in incorrect matching, P' = Q, Q' = P
here P', Q' are far from P, Q.
In simple correspondance mistake we get big outliers.

e)



P = 3D point in world coordinate
P_l = 3D point in left coordinate
P_r = 3D point in right coordinate
p_l = image point (2D) in left coordinates
p_r = image point (2D) in right coordinates

calibration ⇒ $\begin{cases} R_l, T_l \\ R_r, T_r \end{cases}$ external rotation / translation of left / right camera w.r.t world.

$$M_{l \leftarrow r} = \underbrace{M_{l \leftarrow w}}_{R_l^T T_l^{-1}} \underbrace{M_{w \leftarrow r}}_{(R_r^T T_r^{-1})^{-1}} = TR$$

$$= R_l^T T_l^T T_r R_r = TR$$

$$= \begin{bmatrix} R_l^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -T_l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & T_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_r & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_l^T & -R_l^T T_l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_r & T_r \\ 0 & 1 \end{bmatrix}$$

rotation translation

$$= \begin{bmatrix} R_l^T R_r & R_l^T (T_r - T_l) \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow R = R_l^T R_r, \quad T = R_l^T (T_r - T_l)$$

Ans-4

a) $f = 10$ mm
$T = 20$ mm
$d = 30$ mm
$Z = ?$

$$Z = f \cdot \frac{T}{d} = \frac{10 \cdot 20}{30} = \frac{20}{3} = 6.667$$

b) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$$A \times B = \begin{bmatrix} x & y & z \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

$$A \times B = [A]_\times B = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

$$[A]_\times = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

c) $F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ $P_L = (1, 2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$P_r = (2, 3) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$P_r^T F P_l = ?$

$$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 17 & 23 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \underline{\underline{68}}$$

d) $p' = (x', y') = (1, 2)$
$p = (x, y) = (2, 3)$

Ans $= \begin{bmatrix} xx' & xy' & x & yx' & yy' & y & x' & y' & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 & 2 & 3 & 6 & 3 & 1 & 2 & 1 \end{bmatrix}$