Wing Design
Group 10

Shashank S AE11B044
Yash Murthy AE11B049
Ashok Kumar AE11B050
Rohit Grover AE11B043
Ravi Prakash AE11B048

Chapter 1

Wing design

This report summarizes the wing design of our airplane

1.1 Wing load distribution

Schrenk method is one of the best and simple method for approximating the aerodynamic wing loading.
According to this method, the average of the actual chord and the chord of the semi-ellipse wing with the same span b and area S of the wing will be taken as the chord length at any point of the wing.

$$c(y) = \frac{c_{actual}(y) + c_{cell}(y)}{2} \tag{1.1}$$

and,

$$c_{cell} = \frac{4S}{\pi b}\sqrt{1 - \left(\frac{y}{b})\right)^2}$$

where, y is the coordinate measured from wing tip, c(y) is chord length as a function of y, c is the chord length of the wing (0.14m), b is half wing span (0.58m) and S is the area of the wing ($0.086m^2$)

Since our wing is rectangular, the chord length is same throughout the wing. Hence, the eliptical approximation gives us

$$c(y) = \frac{0.14 + c_{cell}}{2} \tag{1.2}$$

The lift per unit span of the wing is given by

$$q = \frac{1}{2}\rho V^2 C_l c(y) \tag{1.3}$$

Since we are designing for maximum loads, we consider loads during takeoff

- $\rho = 1.23$ kg/m^3
- $V = 10.78$ m/s
- $C_l = 2.43$

Therefore,

$$q = 8.6 \times 10^{-5}(146 + \sqrt{586.5^2 - y^2}) \tag{1.4}$$

Using $\int q dy = L(y)$

$$L(y) = 8.6 \times 10^{-5} \left(146 + 0.315 \left[\frac{y}{2}\sqrt{586.5^2 - y^2} + \frac{586.5^2}{2} sin^{-1}\frac{y}{586.5}\right]\right) \tag{1.5}$$

Where, y in mm

The follwing plots outline the shear force and bending moment diagrams. Fig.1.1 gives the lift distribution on the wing, Fig.1.2 gives the bending moment diagram and Fig.1.3 gives the shear force diagram

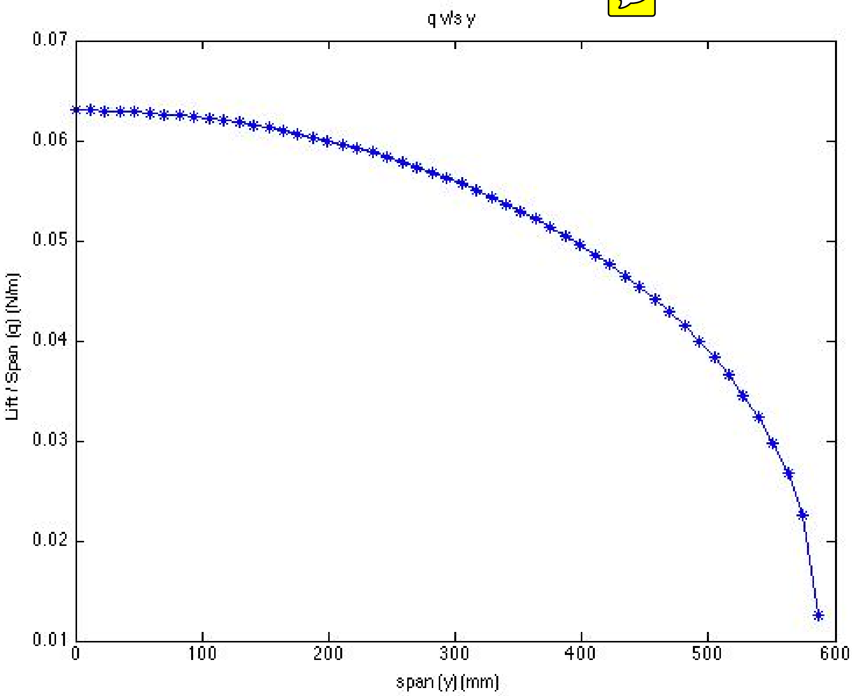

Figure 1.1: Lift Distribution per unit span of wing

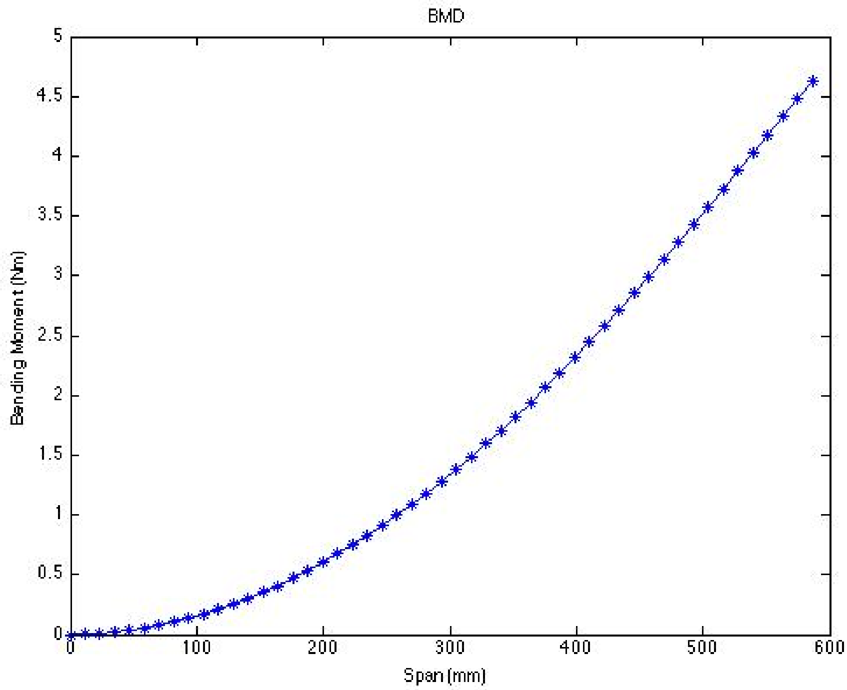

Figure 1.2: Bending Moment diagram due to lift

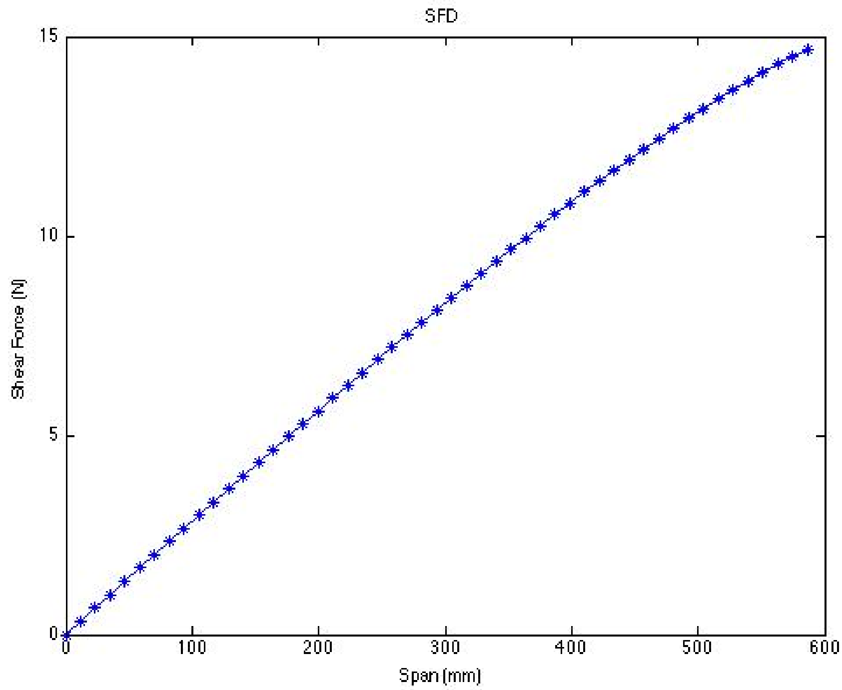

Figure 1.3: Shear force Diagram due to lift

1.2 Spar design

The maximum yield strength of spar material aluminium is $\sigma_{yield} = 19$ MPa. Taking a box section as the spar section, we get moment of inertia as the section as

$$I_{yy} = 2\left(\frac{bd^3}{12} + \frac{btd^2}{4}\right) \tag{1.6}$$

where, b is the breadth and d is the depth of the rectangular box spar. Fig.1.4 shows the spar cross-section.

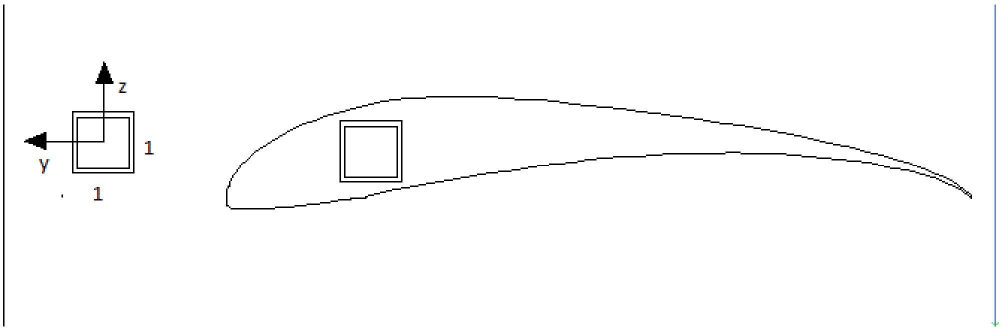

Figure 1.4: Spar cross section

We are using a spar section of 1cm by 1cm with a thickness of 1mm (least dimensions available on the market). Now, we need to check if this spar design is sufficient to carry all types of stresses.

1.2.1 Spar under bending stress

The maximum bending moment, M_{max} is 4.62 Nm,which is got from the bending moment diagram. Now,

$$\sigma = \frac{M_z}{I_{yy}} \tag{1.7}$$

Using our spar cross section, we get $I_{yy} = 2.16 \times 10^{-9}$ m^4, $\sigma = 10.7$ MPa .This value of bending stress is lesser than the yield stress with a factor of safety of almost 2. Hence, our spar is safe w.r.t bending stresses.

1.2.2 Spar under shear stresses

Now that we know our spar cross-section, we can plot the SFD and BMD including the weight of the aircraft. Using the thumb rule of 1 rib per chord of wing, we choose 8 ribs for our wing. Using balsa wood as the material, we get our ribs for the wing to weigh 1.8g. Similarly, we get our spar to be 108g. We neglect the weight of the ribs for simplicity.
The revised SFD and BMD are shown below

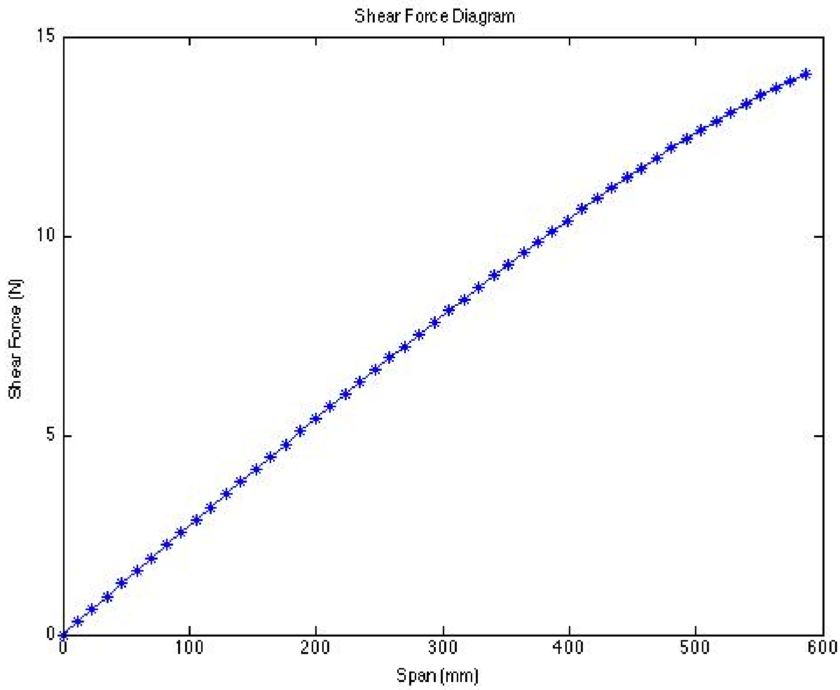

Figure 1.5: Shear force Diagram

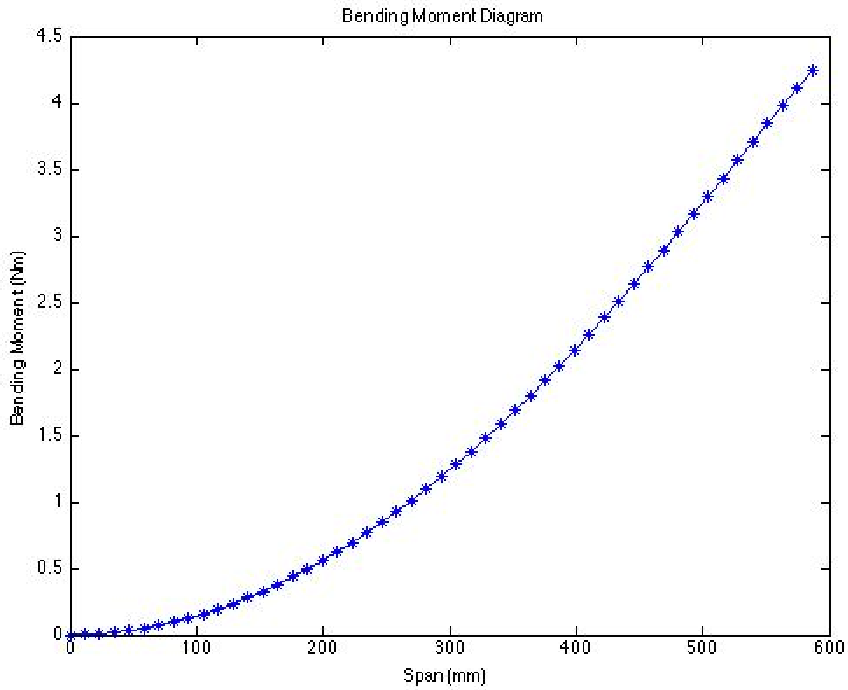

Figure 1.6: Bending Moment diagram

Shear stresses are caused by wing pitching moment (twist) and vertical forces. We can rule out twist due to lift forces as our spar runs through the aerodynamic center of the wing. The torque due to pitching moment is

$$T = \frac{1}{2}\rho v^2 S C_m c \tag{1.8}$$

The shear flow through the section is given by

$$q = -\frac{V_z t}{I_{yy}} \int y ds + \frac{T}{2A} \tag{1.9}$$

where, $V_z = 14.06$ N, $t = 1$ mm, $T = 0.48$ Nm, $A = 1$ cm^2

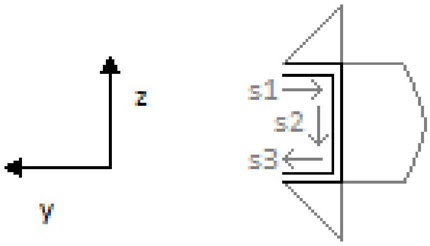

Figure 1.7: Shear flow around spar section

A standard shear flow analysis gives us

$$q_1 = -32.4s_1$$

$$q_2 = -6.48(5s_2 - 0.5s_2^2) - 162$$

where, s is in mm and as directed in 1.7. The maximum shear stress is calculated from the shear flow using

$$\tau_{max} = q_{max}/t \tag{1.10}$$

Looking at the shear flow, we get $q_{max} = 243$ N/m (at $s_2 = 5$ mm) and due to torque $q = 2398$ N/m, Hence, $q_{total} = 2641$ N/m and therfore $\tau_{max} = 2.641$ MPa

1.2.3 Spar under bucking stresses - web

To check if the spar web buckles, we use the following web buckling formula

$$\sigma_{max} = KE\left(\frac{t}{b}\right)^2 \tag{1.11}$$

Substituting, K = 3.67 and E = 70 GPa for Aluminium, we get $\sigma_{max} = 2.6$ GPa

Our maximum stresses are far lesser than this. Hence, no buckling problems will be encountered for our spar.