

Hello,

I was told to expose you students to some discrete mathematics that might be relevant in relation to Olympiad-level competitions. Although I have been (majorly!) unsuccessful in granting you a look at everything important, I believe that I have at least been successful in giving you a look, taste and feel of this side of the mathematics.

Here I am giving you some general resources that might assist you in nurturing your growth in Mathematics.

For olympiads

Syllabus (The italicized topics were covered in classes):

- **Algebra:** basic set theory, principle of Mathematical Induction, inequalities (AMGM and Cauchy-Schwarz), theory of equations (remainder theorem, *relation between roots and coefficients, symmetric expressions in roots*, applications of the Fundamental theorem of algebra and its applications), functional equations
- **Geometry** similarity, congruence, concurrence, collinearity, parallelism and orthogonality, tangency, concyclicity, theorems of Apollonius, Ceva, Menelaus and Ptolemy, special points of a triangle such as circumcentre, in-centre, ex-centres, ortho-centre and centroid
- **Combinatorics** Basic counting numbers such as factorial, number of permutations and combinations, cardinality of a power set, problems based on induction and bijection techniques, existence problems, pigeonhole principle, *double counting*, graph theory
- **Number theory** *divisibility, gcd and lcm, primes, fundamental theorem of arithmetic (canonical factorisation), congruences, Fermat's little theorem*, Wilson's theorem, integer and fractional parts of a real number, *Pythagorean triplets, polynomials with integer coefficients*.

Some lists of resources:

- Large number of references!
http://artofproblemsolving.com/wiki/index.php?title=Mathematics_competitions_resources
- Indian references
<http://olympiads.hbcse.tifr.res.in/subjects/mathematics/references>
- This is a set of notes made by Vipul Naik (IMO silver twice). It contains more than books
http://www.cmi.ac.in/~vipul/olymp_resources/preparationinformation/preparingforolympiads.pdf

Some books: (Can be found on <http://gen.lib.rus.ec/>)

- Mathematical Olympiad Treasures (Titu Andreescu, Bogdan Enescu)
- Problem-Solving Strategies (Arthur Engel) — Contains good puzzles
- The IMO Compendium — free book containing past questions with solution. This book is highly recommended for olympiads!

General resources

Here are some interesting math related resources. Even if you're not interested in olympiads and simply interested in math, this may be of interest to you.

Some topics that you can explore:

- Impossible geometric constructions, squaring a circle, trisecting an angle, constructing a cube of a given length — Why are they not possible?
- The story of Evariste Galois — What did he do (and what is the Abel-Ruffini theorem)? How did Galois die? You can read more about some incidents of his life.
- Group theory — What is it? How does it relate to Rubik's cube? What is Polya's enumeration theory? Try this link :<http://dogschool.tripod.com/>
- Catalan numbers — What are they? How does the formula come? What are the various things it represents?
- Fractals — Mandelbrot's set, Dragon curve, Sierpinski's triangle, Koch snowflake. What is the dimension of a fractal? How can it be between 1d and 2d?
- Pell's equation — What are the solutions of Pell's equation? How do you prove that these are all?
- Flexagons — What are they? Can you make your own? How do they work? How are they related to Catalan numbers?
- Cauchy-Schwartz inequality — Proofs, Geometric interpretations?
- Fibonacci series — What is it? What is the rabbit-population interpretation? What is the connection with golden ratio? How can a computer calculate nth number in the series quickly? Formulae for nth number in the series? What are the generalizations?
- Carpets theorem — How to prove? What are the applications?
- q-factorials, q-Binomial numbers, q-Binomial theorem, q-Catalan numbers, other q-analogues — Can you find a q-analogue of a number yourself?

- Barycentric trigonometry — Some applications?
- Mersenne primes — What was Euler's proof? How to prove infinitude of primes with Mersenne primes?
- Who was Paul Erdos? Read some stories about his life.
- Bezout's identity, Extended euclid's algorithm
- Properties of orthocentre, nine-point circle
- Pole-Polar concept, Apollonius circle, Connections with physics
- Gaussian primes
- Peano's axioms, natural numbers, generalizations beyond Complex numbers (Quaternions, octonions)
- Knot theory

Some books

- Men of mathematics (E T Bell)
- One, two, three ... infinity (George Gamow)
- Riot at the Calc Exam (Collin Adams) — The only math humour book I have ever seen

You can also look at some youtube channels like 'vihart' and 'numberphile'.

This website is a repository of links from 4chan.org/sci: <https://sites.google.com/site/scienceandmathguide/subjects/mathematics>

4chan.org/sci is an interesting board and the repository has many good non-math links.

<http://spikedmath.com/> is a math-comic. Sometimes <http://xkcd.com/> has some good math humour.

Some career advice/blog links:

<https://terrytao.wordpress.com/career-advice/>

http://www.maa.org/external_archive/devlin/devangle.html

A poem: <http://www.aleph.se/Trans/Cultural/Art/tensor.html>

Some writings of Prof. B Sury: <http://www.isibang.ac.in/~sury/blah.html>

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Best of luck!