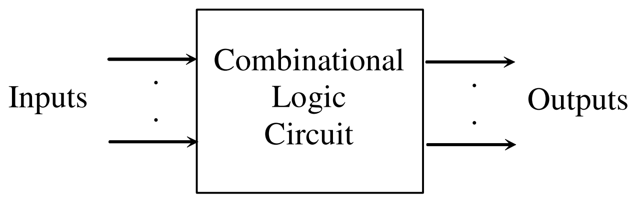
Ch. 2: Combinational Systems

2.1: Design Process for Combinational Systems



To design a logic circuit:
- Describe input/output relationships
- Translate these relationships into a digital logic circuit

2.2: Switching Algebra

An algebraic system to analyze and design *combinational logic circuits*.
Consists of *binary variables* and a set of *logical operations*:
- Binary variables: A, B, C, w, x, y, z, etc (as in algebra), taking values 0 or 1
- Three basic logical operations: AND, OR, NOT

AND $F = x \cdot y$ (F is equal to x AND y) x, y are inputs; F is output
Definition: output is 1 if and only if **both** inputs are 1.

Symbol Truth Table Timing Diagram

x	y	
0	0	
0	1	
1	0	
1	1	

OR $F = x + y$ (F is equal to x OR y) x, y are inputs; F is output
Definition: output is 1 if and only if either input is 1.

Symbol Truth Table Timing Diagram

x	y	
0	0	
0	1	
1	0	
1	1	

NOT $F = x'$ (F is equal to x prime) x is input; F is output
Definition: output is opposite (inverse) of input. (This takes one input only)

Symbol Truth Table Timing Diagram

x	
0	
1	

The **AND** and **OR** operators can be extended to more than two inputs:
- Definition of **AND**: output is 1 if and only if **<u>all</u>** inputs are 1.
- Definition of **OR** : output is 1 if and only if **<u>any</u>** input is 1.

3-input **AND**
$F = x \cdot y \cdot z = x\, y\, z$
(*dot* is often omitted)

4-input **OR**
$G = A + B + C + D$

<u>Precedence of Operations (p. 46)</u>
Order or operations - First: **NOT**; Second: **AND**; Last: **OR**
Parentheses are used to indicate or modify order of logical operations.

$a \cdot (b')$ ← Evaluate b' first, then **AND** the result with a

$(x \cdot y)'$ ← Evaluate $x \cdot y$ first, then take the **NOT** of the result

$(a \cdot b) + c$ ← Evaluate $a \cdot b$ first, then **OR** the result with c

Not the same!

$a \cdot (b + c)$ ← Evaluate $b + c$ first, then **AND** the result with a

$(X \cdot Y')' + Z$ ← Evaluate Y' first, then $(X \cdot Y')$, then $(X \cdot Y')'$, finally the **OR**

Properties (Postulates or Theorems) of Switching Algebra

Summarized in textbook's inside front cover.

P1a.	$a + b = b + a$	**P1b.**	$a \cdot b = b \cdot a$	**Commutative**
P2a.	$a + (b + c) = (a + b) + c$	**P2b.**	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	**Associative**
P3a. **P3aa.**	$a + \mathbf{0} = a$ $\mathbf{0} + a = a$	**P3b.** **P3bb.**	$a \cdot \mathbf{1} = a$ $\mathbf{1} \cdot a = a$	**Identity**
P4a. **P4aa.**	$a + \mathbf{1} = \mathbf{1}$ $\mathbf{1} + a = \mathbf{1}$	**P4b.** **P4bb.**	$a \cdot \mathbf{0} = \mathbf{0}$ $\mathbf{0} \cdot a = \mathbf{0}$	**Null**
P5a. **P5aa.**	$a + a' = \mathbf{1}$ $a' + a = \mathbf{1}$	**P5b.** **P5bb.**	$a \cdot a' = \mathbf{0}$ $a' \cdot a = \mathbf{0}$	**Complement**
P6a.	$a + a = a$	**P6b.**	$a \cdot a = a$	**Idempotency**
P7.	$(a')' = a$			**Involution**
P8a.	$a \cdot (b + c) = a \cdot b + a \cdot c$	**P8b.**	$a + b \cdot c = (a + b) \cdot (a + c)$	**Distributive**
P9a. **P9aa.**	$a \cdot b + a \cdot b' = a$ $a' \cdot b' + a' \cdot b + a \cdot b + a \cdot b' = \mathbf{1}$	**P9b.** **P9bb.**	$(a + b) \cdot (a + b') = a$ $(a' + b') \cdot (a' + b) \cdot (a + b) \cdot (a + b') = \mathbf{0}$	**Adjacency**
P10a.	$a + a' \cdot b = a + b$	**P10b.**	$a \cdot (a' + b) = a \cdot b$	**Simplification**
P11a. **P11aa.**	$(a + b)' = a' \cdot b'$ $(a + b + c \ldots)' = a' \cdot b' \cdot c' \ldots$	**P11b.** **P11bb.**	$(a \cdot b)' = a' + b'$ $(a \cdot b \cdot c \ldots)' = a' + b' + c' \ldots$	**DeMorgan**
P12a.	$a + a \cdot b = a$	**P12b.**	$a \cdot (a + b) = a$	**Absorption**
P13a.	$a \cdot t_1 + a' \cdot t_2 + t_1 \cdot t_2 = a \cdot t_1 + a' \cdot t_2$	**P13b.**	$(a + t_1) \cdot (a' + t_2) \cdot (t_1 + t_2)$ $= (a + t_1) \cdot (a' + t_2)$	**Consensus**
P14a.	$a \cdot b + a' \cdot c = (a + c) \cdot (a' + b)$			

P1 thru P8, and P11 are basic properties. The rest can be derived from them.

Some points worth noting:

P6a means that $xyz + xyz$ can be replaced with xyz. (Eliminate duplicates)
It also means that xyz can be replaced with $xyz + xyz$. (Duplicate a term)

P8a is the distributive property over **OR** function, similar in regular algebra.

P8b means that the distributive property also applies over **AND**. (There is no analogous property in regular algebra)

Proving Basic Properties of Switching Algebra

One way: use a ***truth table***. A truth table shows all possible combinations of the input variables, where each input can take on value of 0 or 1. Show if the left-hand side (LHS) is equal to the right-hand side (RHS) for every combination.

Ex. Prove $a \cdot (b + c) = a{\cdot}b + a{\cdot}c$ [**P8a**] (See book p. 48 for **P8b** proof)

a	b	c	$b + c$	$a \cdot (b + c)$	$a{\cdot}b$	$a{\cdot}c$	$a{\cdot}b + a{\cdot}c$

In a truth table: n inputs $\rightarrow 2^n$ rows

Another Example: Determine whether $(a+b)(a'+b') \stackrel{?}{=} a'b + a\,b'$

a	b	$(a+b)$	$(a'+b')$	LHS	$a'b$	$a\,b$	RHS

Determine whether $(A + B)' \stackrel{?}{=} A' + B'$ [Incorrect application of P11]

A	B	$A+B$	$(A+B)'$	A'	B'	$(A' + B')$
0	0					
0	1					
1	0					
1	1					

Prove Property P11aa $(X + Y + Z)' \stackrel{?}{=} X' \cdot Y' \cdot Z'$ [DeMorgan's Property]

X	Y	Z	$X+Y+Z$	$(X+Y+Z)'$	X'	Y'	Z'	$(X' \cdot Y' \cdot Z')$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Is the equality $((A' + B)' \cdot C')' + A) = B$ true?

A	B	C		
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Determine whether $a(a + b)(a\, b')' = \mathbf{1}$ is true

a	b		
0	0		
0	1		
1	0		
1	1		

Is the equality $A{\cdot}B + A{\cdot}B' + C{\cdot}D + C'{\cdot}D = A + D$ true?

A B C D		

Is the equality $(a + b)\,(a' + c)\,(b + c) = (a + b)\,(a' + c)$ true?

a	b	c		
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Truth table method is cumbersome when more than 4 variables are involved.

As alternative, we will learn how to algebraically manipulate the Boolean expressions using the *Properties of Boolean Algebra*. (Next class)