Queen Mary
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SCHOOL OF BUSINESS AND MANAGEMENT

# GAME THEORY ANALYSIS OF COURNOT MODEL

## BUSM147 – STRATEGIC ANALYSIS

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** Used Both Cournot and Cournot 2 Game Results for Analysis

# Introduction

Oligopoly arises when there is small number of firms in the market, these firms are large in nature and have all or most of the sales in an industry.

In Oligopoly, we have Cournot Model (developed by Augustin Cournot) which is an economic model explaining an Industry structure where firms offer similar product, and they compete on amount of goods produced independently and simultaneously. The amount of good produced by firms will determine price of the product, which will be used to calculate profit of each firm.

There are few assumptions which are taken in Cournot Model, [1]: -

1. Firms involved in Cournot Model produce Homogeneous product and competing firms choose a quantity to produce independently and simultaneously.
2. Demand Curve is Linear.
3. Cost of Production will be considered as zero and firms have same production technology.
4. Every firm decides its output to maximize its own profits and takes the output of competitor as given.

Companies operating in a limited competitive market called oligopoly often compete to take market share from each other. One way to do this is to change the number of items you sell.

The OPEC (Organization of Petroleum exporting Countries) is the perfect example of Cournot oligopoly, as they decide how much quantity of oil will be produced as the amount of oil produce controls the price of the oil.[2] OPEC is basically a group of 13 countries that aims to control supply of Oil.

The game captures the following interactions: -

- **Social interaction**: Quantity impact the price of the product, lower production leads to higher price and higher production leads to lower price.

- **Strategic interaction**: Both firms should consider rival firm's output, to plan for maximizing their profits in next rounds.

- **Strategy**: Firms can form cartel to get maximum profit or can achieve Nash-equilibrium to get optimum results.

# One-Shot and Dynamic Version

## 2. One Shot

In One-Shot, there are two possibilities, either the firms will go into Cournot-Nash Equilibrium, or they will form a Cartel.

### 2.1 Cournot-Nash Equilibrium

Cournot-Nash equilibrium is a concept where firms do not get profits if they deviate from their original Strategy created after considering competitor's choice.

Step 1: Calculate the marginal revenues for both firms.

Let's Start with firm 1. The total revenue of firm 1, R1, is:

$$\mathbf{R1= pQ_1 = (20-(Q_1+Q_2))Q_1 = 20Q_1 - Q_1^2 - Q_2Q_1}$$

To calculate marginal revenue, MR1, we take derivative of R1.

Hence $\mathbf{MR1 = 20- 2Q_1 - Q_2}$

Similarly, total revenue of firm 2, R2 is:

$$\mathbf{R2= pQ_2 = (20-(Q_1+Q_2))Q_2 = 20Q_1-Q_2^2-Q_2Q_1}$$

Similarly marginal revenue of firm 2,

$$\mathbf{MR2 = 20- 2Q_2 - Q_1,}$$

Step 2: Calculate the profit maximizing outputs for both firms.

It is given that Marginal Cost of a Firm is 2 and Profit maximization is possible when Marginal Revenue and Marginal Cost are equal, MR1= MC1.

$$20-2Q_1-Q_2 = 2$$

So, $\mathbf{Q_1= (18-Q_2)/2}$

This is called Reaction Function of Firm 1 (**RF1**)

Equivalently we can compute, $\mathbf{Q_2 = (18-Q_1)/2}$, Reaction Function of Firm2 (**RF2**)

Substituting $Q_2$ in $Q_1$ equation, we get,

$$Q1 = (18 - \frac{18 - Q1}{2})/2$$

$$Q_1 = 6$$

And using $Q_1$=6 we get,

$$Q_2 = 6$$

So $Q_1$=$Q_2$=6 is the best responses for Firm 1 and Firm 2.

So, Market Output = $Q_1$+$Q_2$=12
Price of the product is, P = 20 – ($Q_1$+$Q_2$) = 8
Finally, each Firm's profit is,

Profit = ProductPrice*Output – Marginal Cost *Output
Profit = 8*6-2*6
**Profit = 36**

## 2.2 Cartel Formation

Now let's assume that both Firms decide to form a Cartel, with a goal to decide the individual Outputs to get maximum profits.

Revenue = P*Q, where P is price of the product and Q is the total quantity
= (20-Q)*Q
=20Q- $Q^2$

So, derivative of Revenue will the Marginal Revenue (MR)
**Marginal Revenue = 20-2Q**
And we know that Marginal Cost = 2, which is equal to marginal revenue,

2= 20-2Q
Q=9 (in case of Cartel)

Price = 20-Q = 20-9
Price = 11

And since each firm will produce half of the market quantity, so **$Q_1$ = 4.5, $Q_2$= 4.5**
**Profit of Firm 1**, Profit = ProductPrice*Output – Marginal Cost *Output
**= 11*4.5-2*4.5 = 40.5**
**Profit of Firm 2 = 40.5**

| | Cournot | Cartel |
|---|---|---|
| Output per Firm | 6 | 4.5 |
| Price per Product | 9 | 11 |
| Profit per Firm | 36 | 40.5 |

**Table 1 – Cournot vs Cartel Comparison**

When Firms decide to act according to Nash-equilibrium, they have a profit of 36, and when they form a cartel, they get profit of 40.5; so, we can say firms should continue in accordance with cartel and get maximum profit in each round, but Cartel is very unstable, the cartel agreement does not construct a Nash equilibrium and is vulnerable to deviations. This situation corresponds to **Prisoner's Dilemma** [4], where rational choice of both the firms will be to deviate from Cartel and get maximum profit. But if both firms deviate, then they will end up in Cournot equilibrium. Prisoner's Dilemma is basically a situation which shows that why two Firms might not cooperate, even if it appears to be in best interest of both the firms.

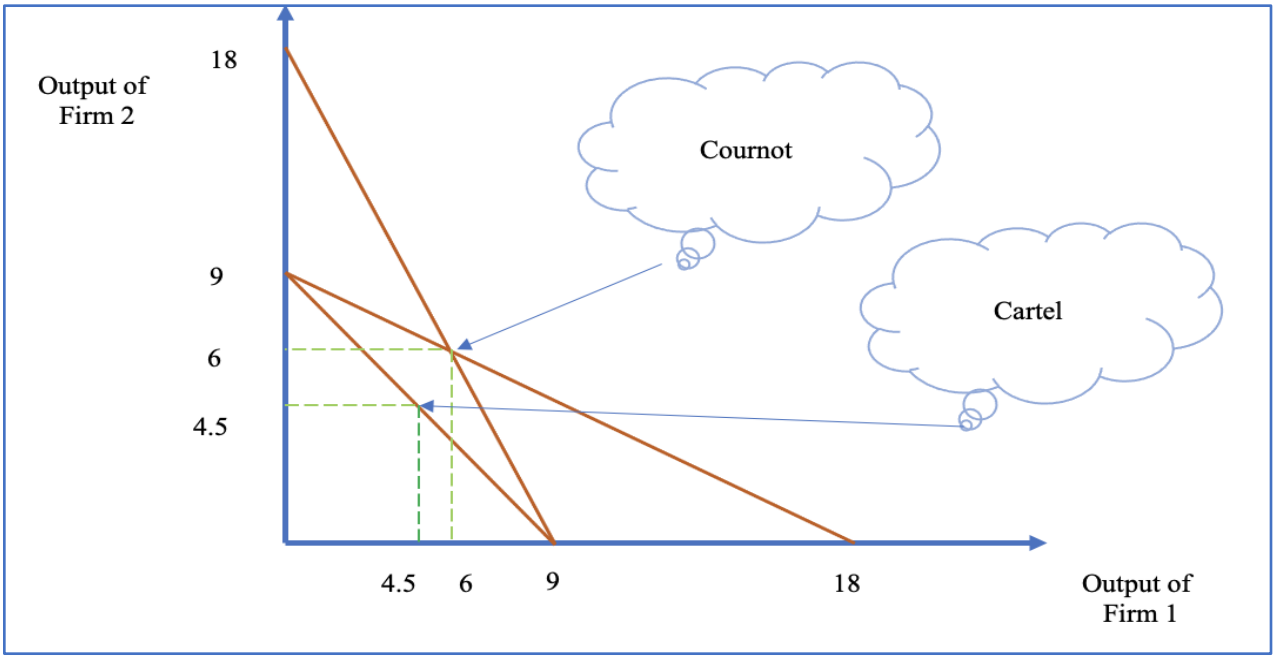


**Figure 1 - Reaction Curve**

For better understanding, let's say that Firm 1 deviates from cartel agreement and produce quantity as per equilibrium, which was 6.

$$P = 20 - (Q1 + Q2)$$
$$P = 20 - (6 + 4.5)$$
$$P = 9.5$$

So, profit of each firm will be, Profit = Price*Quantity – Marginal Cost*Quantity

$$\text{Firm 1} = 9.5*6 - 2*6 = 45$$
$$\text{Firm 2} = 9.5*4.5 - 2*4.5 = 33.75$$

Firm 2 profit reduces as Firm 1 has not follow Cartel agreement, so in One-Shot, it's quite clear that Firms will be in Dilemma and might not follow Cartel agreement.

| | | Firm 2 | |
|---|---|---|---|
| | Output Level | Low (Cartel) | High (RF) |
| Firm 1 | Low (Cartel) | 40.5/40.5 | 33.75/45 |
| | High (RF) | 45/33.75 | 36/36 |

**Table 2 – Profit vs Output Level of Firms**

## 3. Dynamic Version

In Dynamic Version, there can be two situations, one where there are fixed number of rounds and second where its Infinite rounds.

Fixed number of Rounds: -

In this scenario, both the firms know that they must play fixed number of rounds and in each round, firm plays simultaneously, but they also know about actions performed by opponents in last round. It's quite natural that after playing few rounds, firms can decide that they will form Cartel to maximize their profits. But this Cartel has high probability of breaking in last 2 rounds where each firm can try to maximize the profit by breaking the cartel and going for output which maximizes their individual profit.

Infinite number of rounds: -

When we have infinite number of rounds, there is high probability that both firms will decide to form a Cartel and stick to it forever so that both get maximum profit in each round.
Let's say Firm 1 decide to deviate from agreement of Cartel in a round, their profit for that round will be 45, (4.5 more than cartel), it can be called short term gain. After this Firm 2 will go for Reaction function and Firm 1 will also follow same strategy.

So, by this strategy Firm 1 profits will be 36, which is a drop of 4.5 and this will their profit in each upcoming round.

Eventually, the long run losses are by far heavier than the short-term gain.

# Data and Outcome

Now if we look at all 8 Games, we can clearly see that there was no Cartel formation, and looks like in each Game, focus was on winning the individual rounds. Players could have stick to producing total quantity of 9 to gain maximum profit as we have limited number of rounds. One Player could have opted to produce 5 quantity and next player to produce 4 quantities, and later in next round they could have interchanged the quantities respectively, so that Overall Profit remains the same.

If we look at Game 3, played between Player 4 and 3, we see that after initial 2 rounds, they have acquired Nash equilibrium where both players produce 6 quantities to gain profit of 36. But in last 3 rounds maybe due to some technical errors Player 4 starts producing 5 quantities, which reduces its profit in market and as an impact of that Player 3 profits increase by 6, as he is still stuck with quantities as per equilibrium.

So, the best strategy when we have limited, and lesser number of rounds is to form Cartel as soon as possible and get maximum profit.

In Game 4, between Player 2 and 1, Both Players has followed dominant strategy and has tried to maximize his profit by producing more quantities, but eventually it has led to 0 or negative profits.

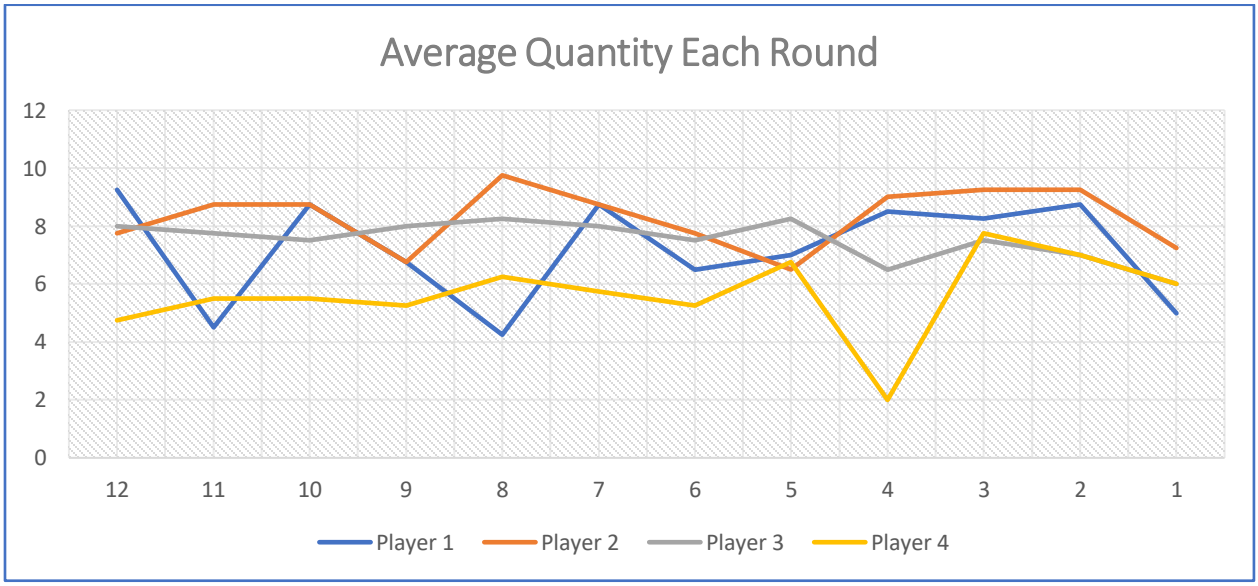


**Figure 2 – Average quantity produced by each player in each Round**

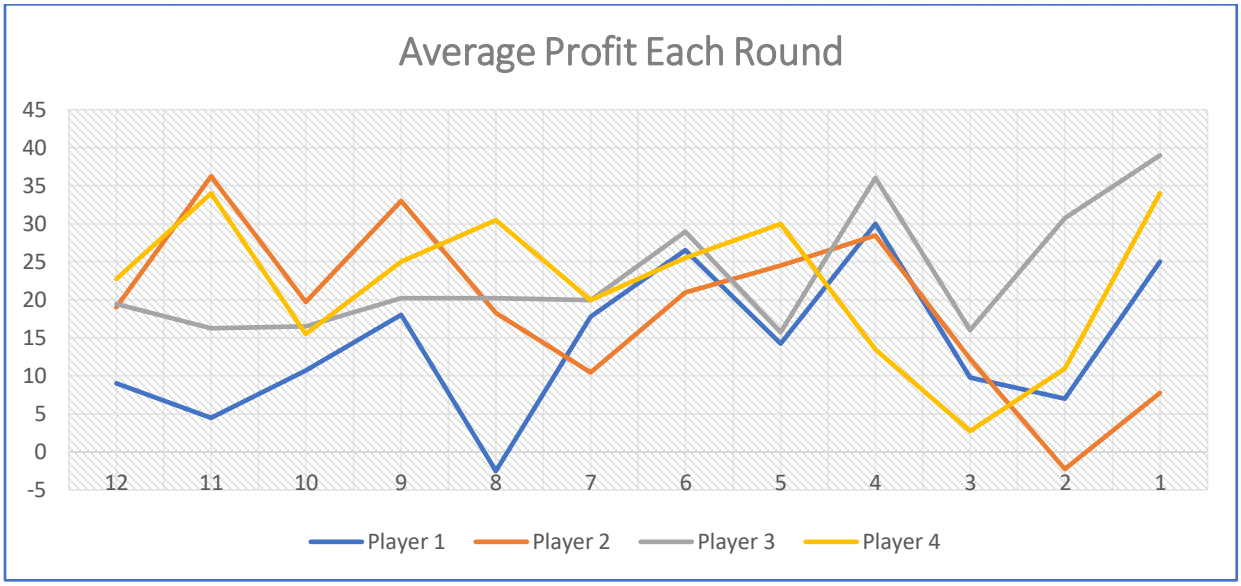


**Figure 3 – Average Profit achieved by each player in each Round**

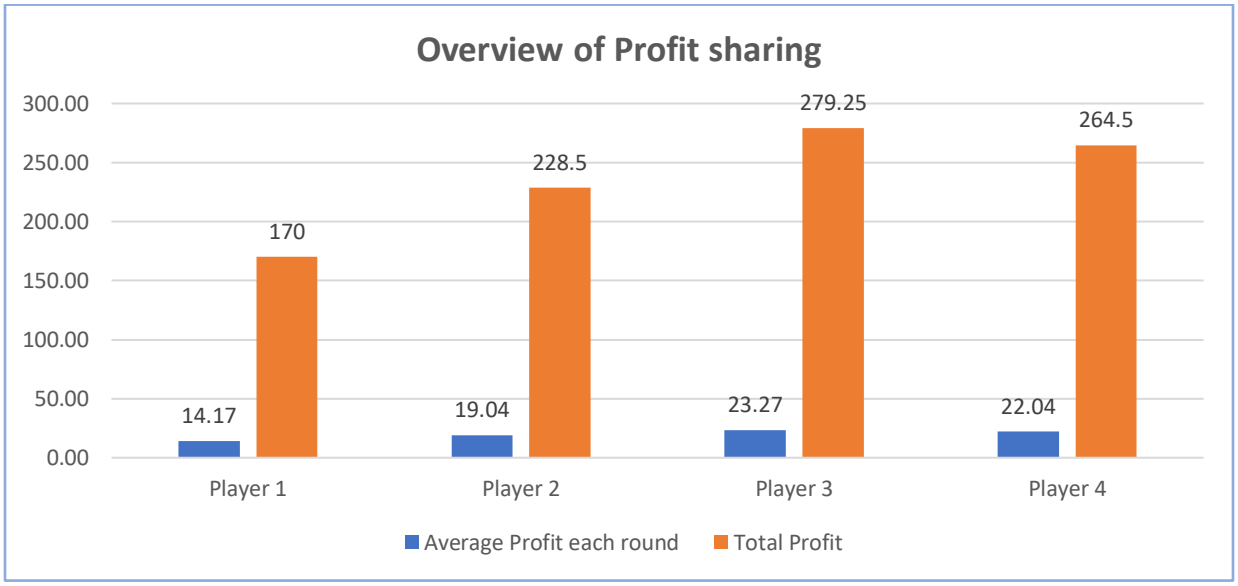


**Figure 4 – Total profit of each player and Average Profit**

In Figure 2 and 4, we can see that Player 3 has tried to play the game in best way and there is very less change in his production quantities and has achieved highest profits in long run, and Player 2 has been greedy and produced way more quantities than equilibrium, and his overall profits are much lower than Player 3.

# Conclusion

To conclude, analysis of Cournot Model shows that cooperating each other should be an obvious choice between the players. So, neither Firm should feel the need to dominate the other and both should be equally altruistic in their choices and should behave optimally after considering the length of the game.

Analysis shows that this model offers a biased explanation of actual world as this model is constrained by some rigid assumptions which limits its application, such as maintaining homogeneity in product by each firm is difficult, as firms always try to change some aspect in product to attract more customers.

Cournot Model will only act best in industry where output and capacity of firms are difficult to adjust, such as airplanes manufacturers, Boeing and Airbus.

When there are only two firms competing in market, they will not work in vacuum and will be responsive to each other strategies.

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## References:

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[4] Picardo, E., 2019. The Prisoner's Dilemma in business and the economy. *Retrieved May*, *25*, p.2019.