# Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

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## 1 Mason's Gain Formula

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### *2.1 Introduction*

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## 3 Second order System

### *3.1 Damping*

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3.1. A second-order real system has the following properties:
a) the damping ratio $\zeta = 0.5$ and undamped natural frequency $\omega_n = 10 rad/s$
b) the steady state value of the output, to a unit step input, is 1.02.
The transfer function of the system is
(A) $\frac{1.02}{s^2+5s+100}$ (B) $\frac{102}{s^2+10s+100}$
(C) $\frac{100}{s^2+10s+100}$ (D) $\frac{102}{s^2+5s+100}$
**Solution:** Characteristic equation of second order system is as follows

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{3.1.1}$$

Given

$$\zeta = 0.5 \tag{3.1.2}$$

$$\omega_n = 10 rad/s \tag{3.1.3}$$

The damping coefficient is between 0 and 1. Thus, the system is Underdamped.
and the Characteristic equation becomes

$$s^2 + 10s + 100 = 0 \tag{3.1.4}$$

Denominator of the Transfer Function is characteristic equation. Considering this, we can eliminate A and D options.

We know that output of the system in s domain is

$$ \tag{3.1.5} $$

$$C(s) = T(s)R(s) \tag{3.1.6}$$

$$R(s) = \frac{1}{s} \tag{3.1.7}$$

as it is unit step input.

Steady state output is given by

$$C(\infty) = \lim_{s\to 0} sC(s) \tag{3.1.8}$$

Given, steady state output is 1.02 and is the same for only option B

Therefore, transfer function of the system is

$$\frac{102}{s^2+10s+100} \tag{3.1.9}$$

The step response of the transfer function

$$C(t) = \mathscr{L}^{-1}\left\{\frac{102}{s(s^2+10s+100)}\right\} \tag{3.1.10}$$

Dividing into partial fractions

$$C(t) = \mathscr{L}^{-1}\left\{\frac{51}{50s}\right\} - \mathscr{L}^{-1}\left\{\frac{-51s-510}{50(s^2+10s+100)}\right\} \tag{3.1.11}$$

$$C(t) = \left(\frac{-51}{50}\right)\mathscr{L}^{-1}\left\{\frac{s+5}{(s+5)^2+75}\right\} + \left(\frac{-51}{10}\right)\mathscr{L}^{-1}\left\{\frac{1}{(s+5)^2+75}\right\} + \mathscr{L}^{-1}\left\{\frac{51}{50s}\right\} \tag{3.1.12}$$

Applying Inverse Laplace Transform, we get

$$\mathscr{L}^{-1}\left\{\frac{s+5}{(s+5)^2+75}\right\} = \exp^{-5t} cos(5\sqrt{3}t) \tag{3.1.13}$$

$$\mathscr{L}^{-1}\left\{\frac{1}{(s+5)^2+75}\right\} = \frac{1}{5\sqrt{3}}\exp^{-5t} cos(5\sqrt{3}t) \tag{3.1.14}$$

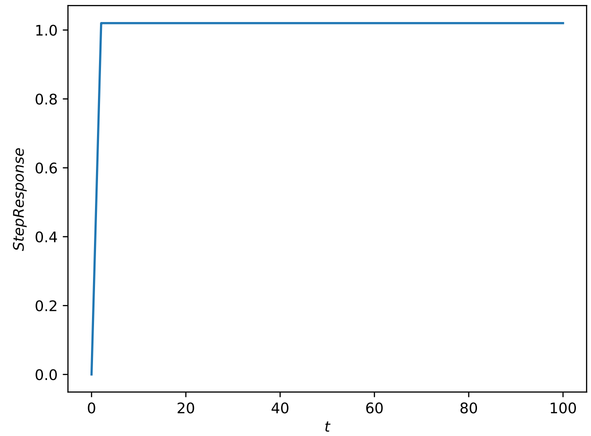


Fig. 3.1

Therefore,

$$C(t) = \left(\frac{-51}{50}\right)\exp^{-5t} cos(5\sqrt{3}t) - \left(\frac{17\sqrt{3}}{50}\right)\exp^{-5t} sin(5\sqrt{3}t) + \frac{51}{50}u(t) \tag{3.1.15}$$

The following code is used to get the step response plot

```
stepresponseplot.py
```

## 4 Routh Hurwitz Criterion

### 4.1 Routh Array

### 4.2 Marginal Stability

### 4.3 Stability

## 5 State-Space Model

### 5.1 Controllability and Observabiity

### 5.2 Second Order System

## 6 Nyquist Plot

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