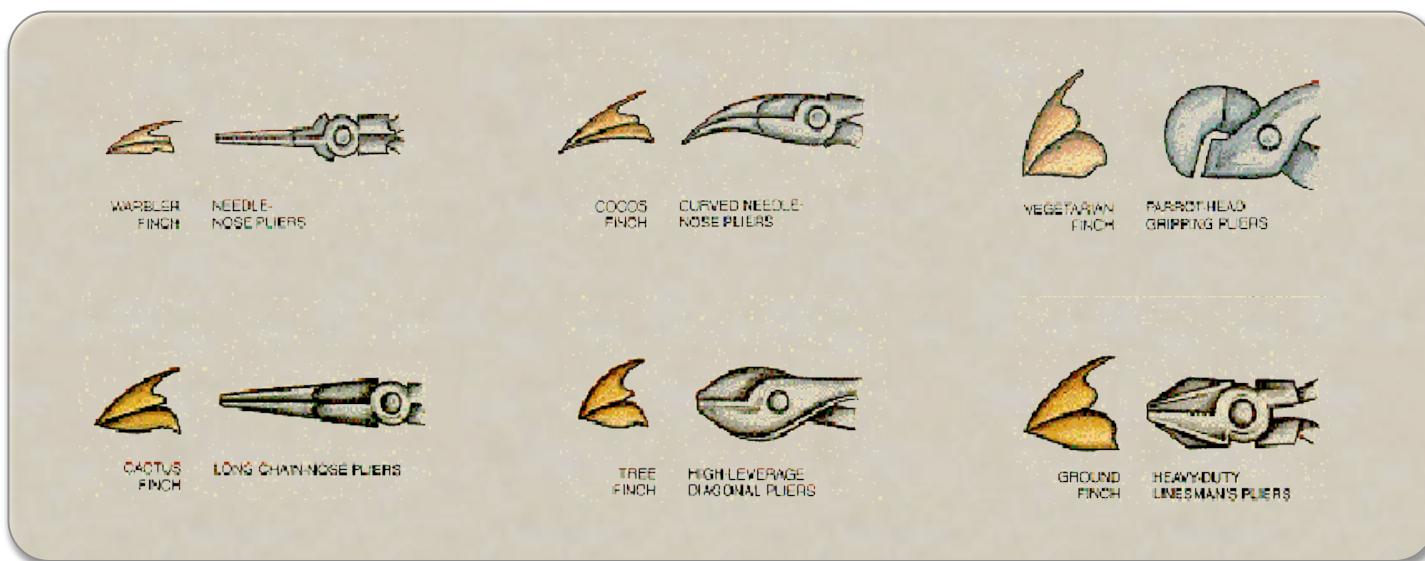


codon substitution models and the analysis of natural selection pressure

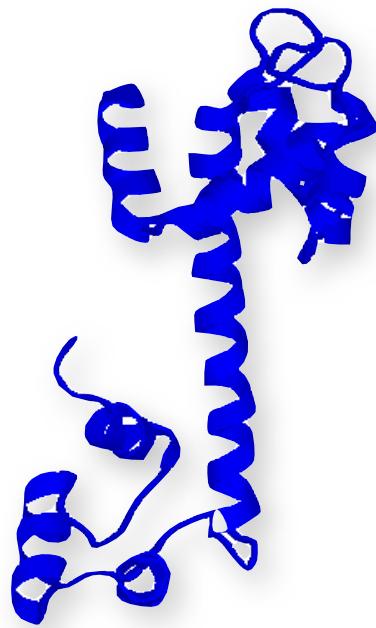
Joseph P. Bielawski
Department of Biology
Department of Mathematics & Statistics
Dalhousie University



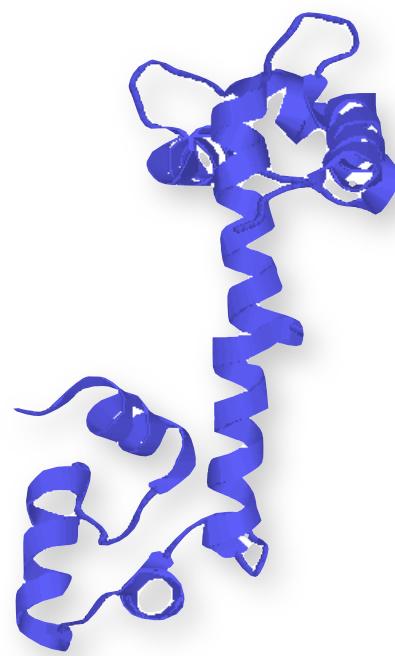
morphological adaptation



protein structure



Troponin C: fast skeletal



Troponin C: cardiac and
slow skeletal

gene sequences

human

COW

rabbit

rat

opossum

```

GTG CTG TCT CCT GCC GAC AAG ACC AAC GTC AAG GCC GCC TGG GGC AAG GTT GGC GCG CAC
... ... ... G.C ... ... T... .T ... ... ... ... ... ... ... ... ... ... ... ... ... .GC A...
... ... ... ... C ... T ... ... ... A... ... A.T ... ... AA ... A.C ... AGC ...
... ... C ... G.A AT ... A ... A... ... AA. TG. ... G ... A... ... T .GC ... T
... ... C ... G GA ... T ... ... T C ... G ... A ... AT ... T ... G ... A ... GC ...

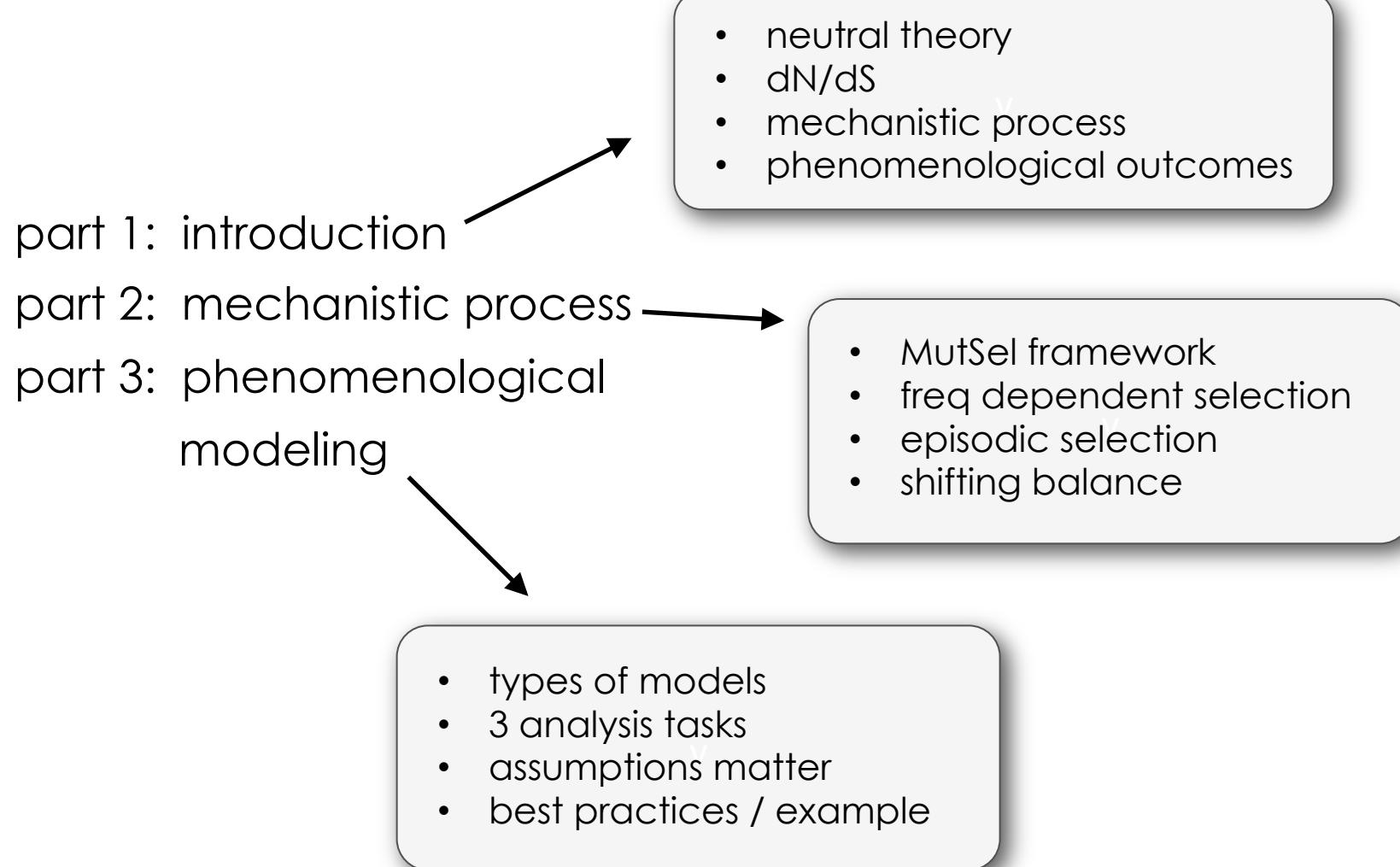
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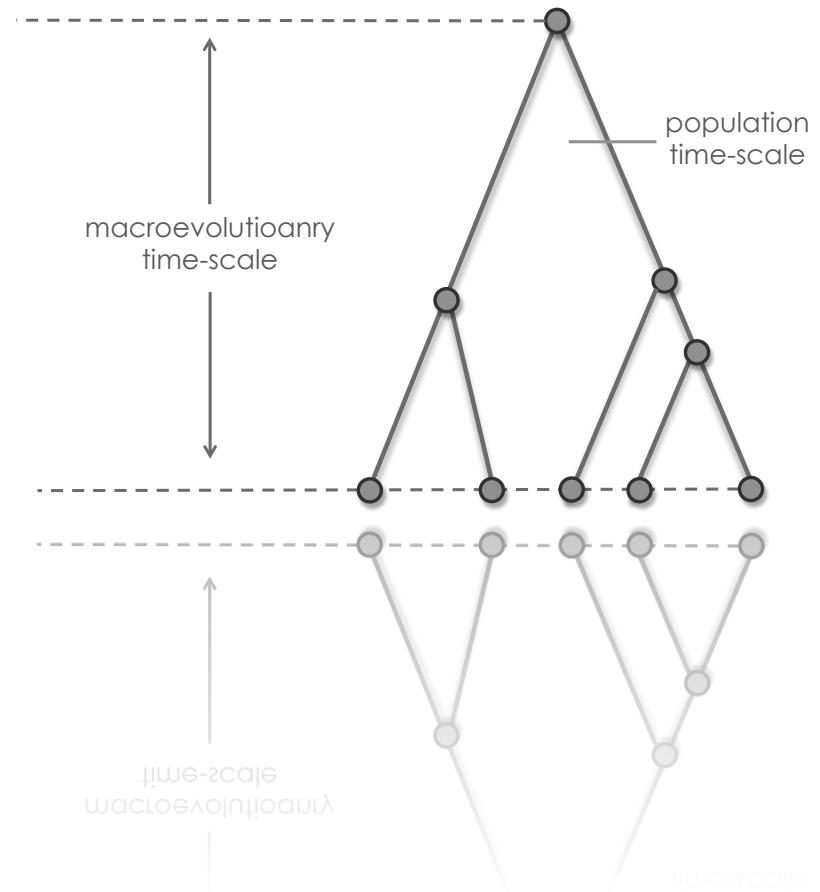
GCT GGC GAG TAT GGT GCG GAG GCC CTG GAG AGG ATG TTC CTG TCC TTC CCC ACC ACC AAG
... .A .CT ... .C .A ... .T ... . . . . . . . . . . . . . . . . . . . . . . . . .
.G. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
.G. . .T . .A . . .C .A. . . . . .A C.. . . . . . . . . . . . . . . . . . . . . .
.C . .T .CC . .C .CA . .T . .A . .T . .T .CC . .A .CC . . .C . . . . . . . . .T . . .A

```

The goals and the plan



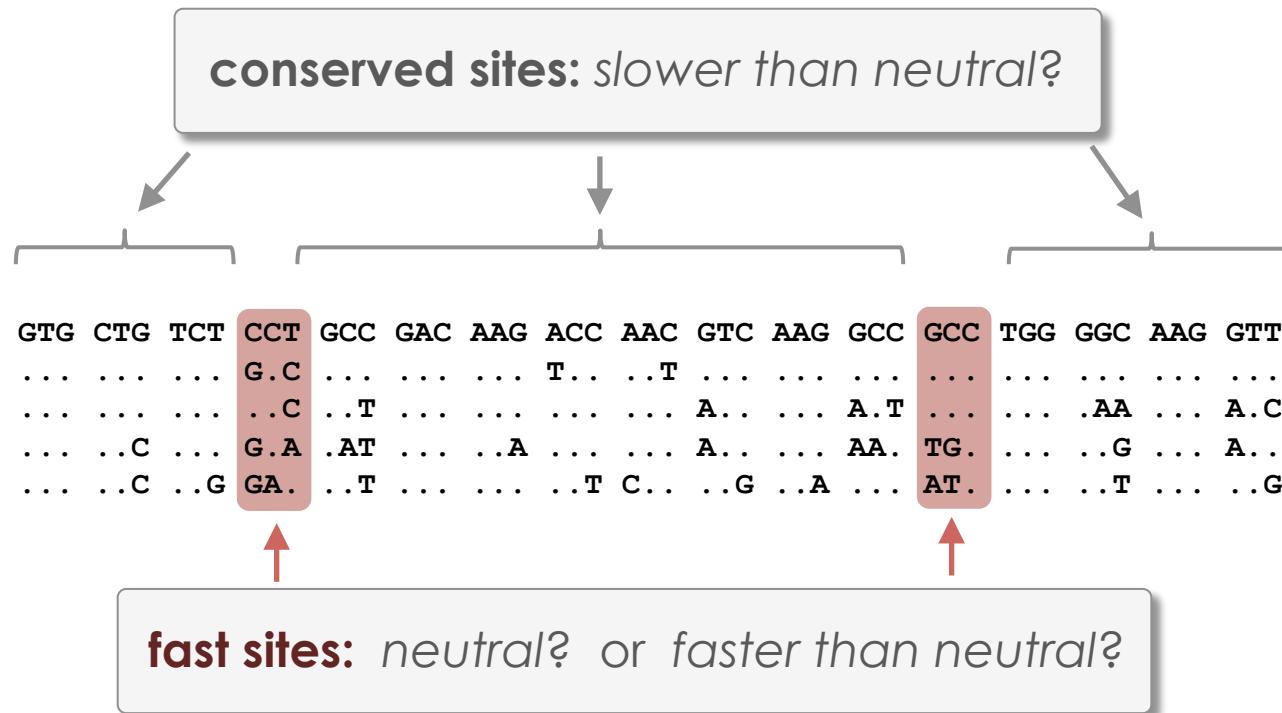
part 1: introduction



evolutionary rate depends on intensity of selection

selectively constrained = slower than neutral (drift alone)

adaptive divergence = faster than neutral (drift alone)



What is the neutral expectation?

neutral theory of molecular evolution (Kimura 1968)

the **number of new mutations** arising in a diploid population

$$2N\mu$$

the **fixation probability** of a new mutant by drift

$$1/2N$$

The **substitution (fixation) rate**, k

$$k = 2N\mu \times 1/2N$$

the elegant simplicity of **neutral theory**: $k = \mu$

genetic code determines impact of a mutation

	U	C	A	G	
U	UUU Phe UUC Phe UUU Leu UUG Leu	UCU Ser UCC Ser UCA Ser UCG Ser	UAU Tyr UAC Tyr UAA Stop UAG Stop	UGU Cys UGC Cys UGA Stop UGG Trp	U C A G
C	CUU Leu CUC Leu CUA Leu CUG Leu	CCU Pro CCC Pro CCA Pro CCG Pro	CAU His CAC His CAA Gln CAG Gln	CGU Arg CGC Arg CGA Arg CGG Arg	U C A G
A	AUU Ile AUC Ile AUA Ile AUG Met	ACU Thr ACC Thr ACA Thr ACG Thr	AAU Asn AAC Asn AAA Lys AAG Lys	AGU Ser AGC Ser AGA Arg AGG Arg	U C A G
G	GUU Val GUC Val GUA Val GUG Val	GCU Ala GCC Ala GCA Ala GCG Ala	GAU Asp GAC Asp GAA Glu GAG Glu	GGU Gly GGC Gly GGA Gly GGG Gly	U C A G

<http://www.langara.bc.ca/biology/mario/Assets/Geneticode.jpg>

The genetic code determines how random changes to the gene brought about by the process of mutation will impact the function of the encoded protein.

Kimura (1968)

d_S : number of synonymous substitutions per synonymous site (K_S)

d_N : number of nonsynonymous substitutions per nonsynonymous site (K_A)

ω : the ratio d_N/d_S ; it measures selection at the protein level

an index of selection pressure

rate ratio	mode	example
$dN/dS < 1$	purifying (negative) selection	histones
$dN/dS = 1$	Neutral Evolution	pseudogenes
$dN/dS > 1$	Diversifying (positive) selection	MHC, Lysin

an index of selection pressure

Why use d_N and d_S ? (Why not use raw counts?)

example of counts:

300 codon gene from a pair of species
5 synonymous differences
5 nonsynonymous differences

$$5/5 = 1$$

why don't we conclude that rates are equal (i.e.,
neutral evolution)?

the genetic code & mutational opportunities

Relative proportion of different types of mutations in hypothetical protein coding sequence.					
Type	Expected number of changes (proportion)				
	All 3 Positions	1 st positions	2 nd positions	3 rd positions	
Total mutations	549 (100)	183 (100)	183 (100)	183 (100)	
Synonymous	134 (25)	8 (4)	0 (0)	126 (69)	
Nonsynonymous	392 (71)	166 (91)	176 (96)	57 (27)	
nonsense	23 (4)	9 (5)	7 (4)	7 (4)	

Modified from Li and Graur (1991). Note that we assume a hypothetical model where all codons are used equally and that all types of point mutations are equally likely.

Why do we use d_N and d_S ?

same example, but using d_N and d_S :

Synonymous sites = 25.5%

$$S = 300 \times 3 \times 25.5\% = 229.5$$

Nonsynonymous sites = 74.5%

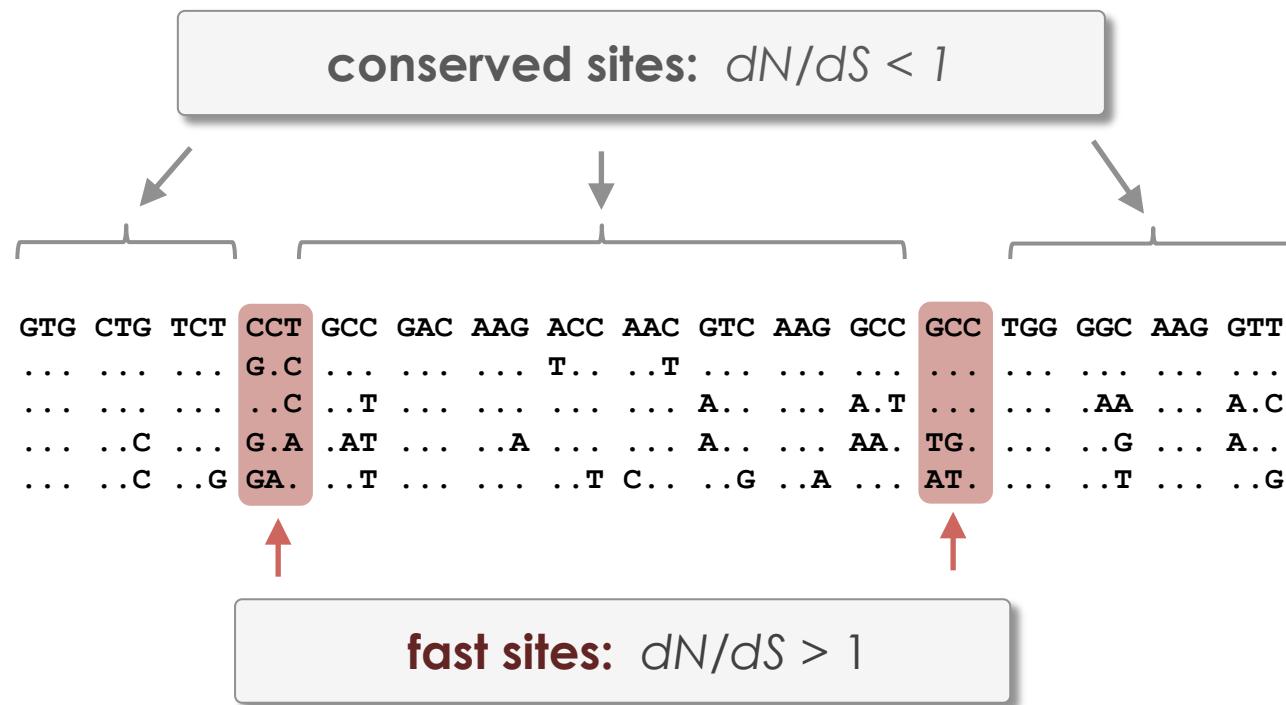
$$N = 300 \times 3 \times 74.5\% = 670.5$$

So, $d_S = 5/229.5 = 0.0218$

$$d_N = 5/670.5 = 0.0075$$

$$d_N/d_S (\omega) = 0.34, \text{ **purifying selection !!!**}$$

an index of selection pressure acting on the protein



conclusion: dN differs from dS due to the effect of selection on the protein.

mutational opportunity vs. physical site

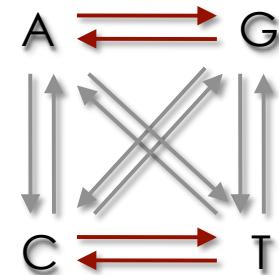
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Note that by framing the counting of sites in this way we are using a “mutational opportunity” definition of the sites. Thus, a synonymous or non-synonymous site is not considered a physical entity!

Note that we assume a hypothetical model where all codons are used equally and that all types of point mutations are equally likely.

real data have biases (*Drosophila GstD1* gene)

transitions vs. transversions:



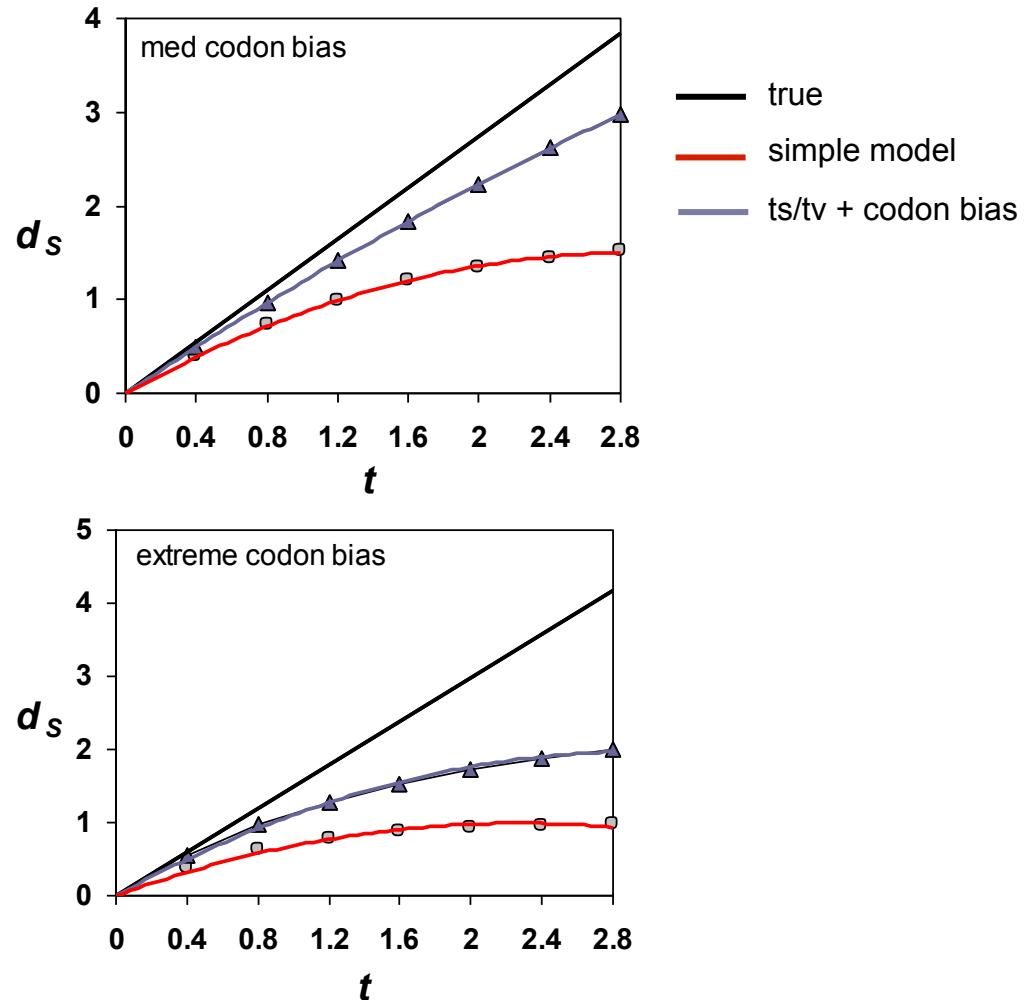
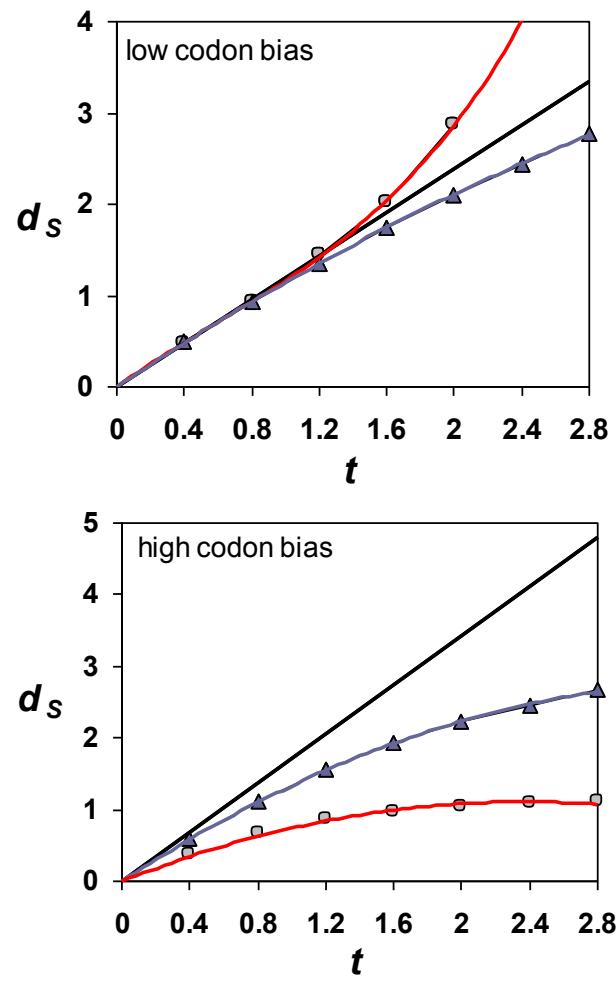
$$ts/tv = 2.71$$

preferred vs. un-preferred codons:

partial codon usage table for the *GstD* gene of *Drosophila*

Phe F TTT	0		Ser S TCT	0		Tyr Y TAT	1		Cys C TGT	0
TTC	27		TCC	15		TAC	22		TGC	6
Leu L TTA	0		TCA	0		*** * TAA	0		*** * TGA	0
TTG	1		TCG	1		TAG	0		Trp W TGG	8
<hr/>										
Leu L CTT	2		Pro P CCT	1		His H CAT	0		Arg R CGT	1
CTC	2		CCC	15		CAC	4		CGC	7
CTA	0		CCA	3		Gln Q CAA	0		CGA	0
CTG	29		CCG	1		CAG	14		CGG	0

uncorrected evolutionary bias leads to estimation bias



data from: Dunn, Bielawski, and Yang (2001) Genetics, 157: 295-305

recap

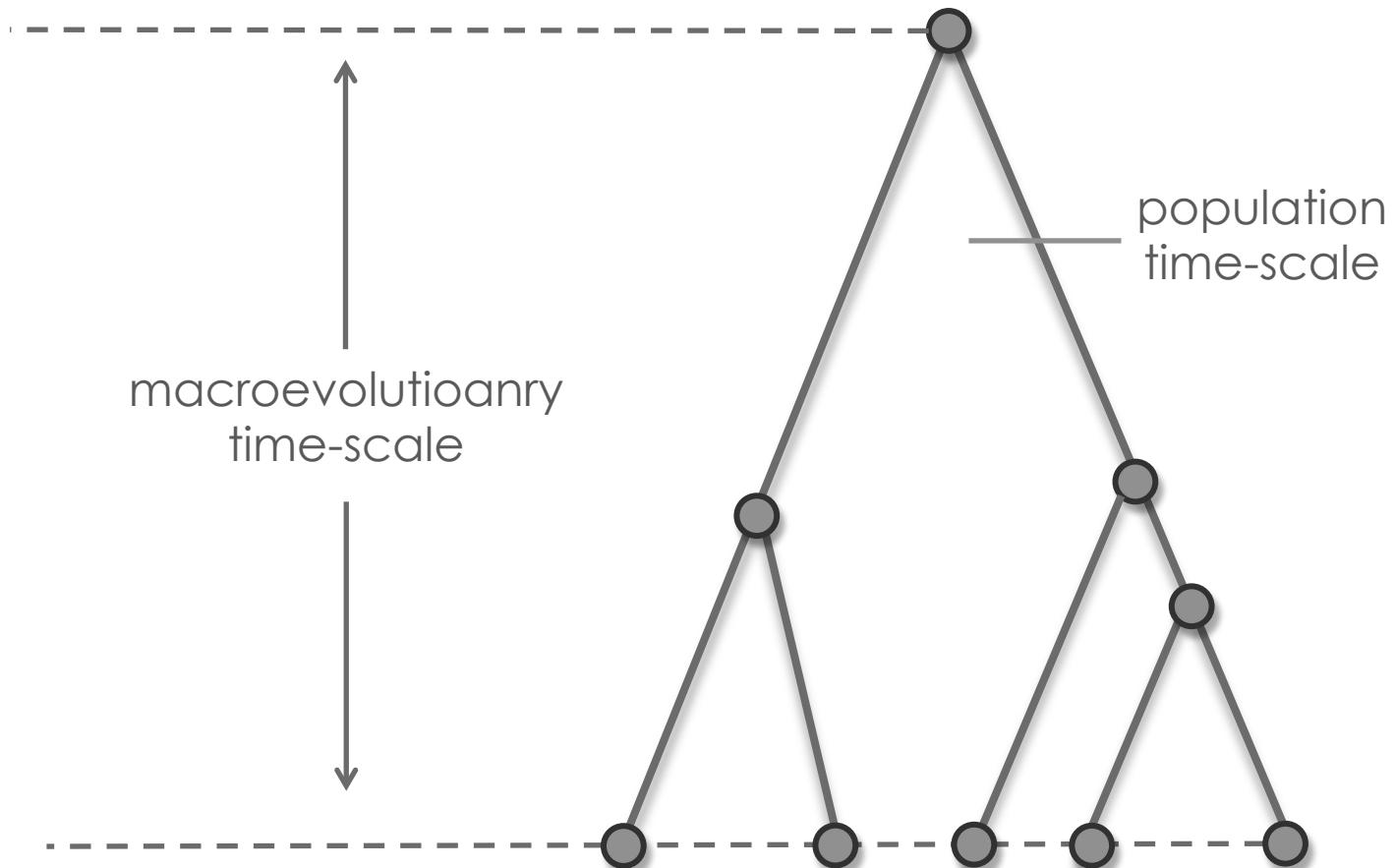
dS and dN must be corrected for BOTH the structure of genetic code and the underlying mutational process of the DNA

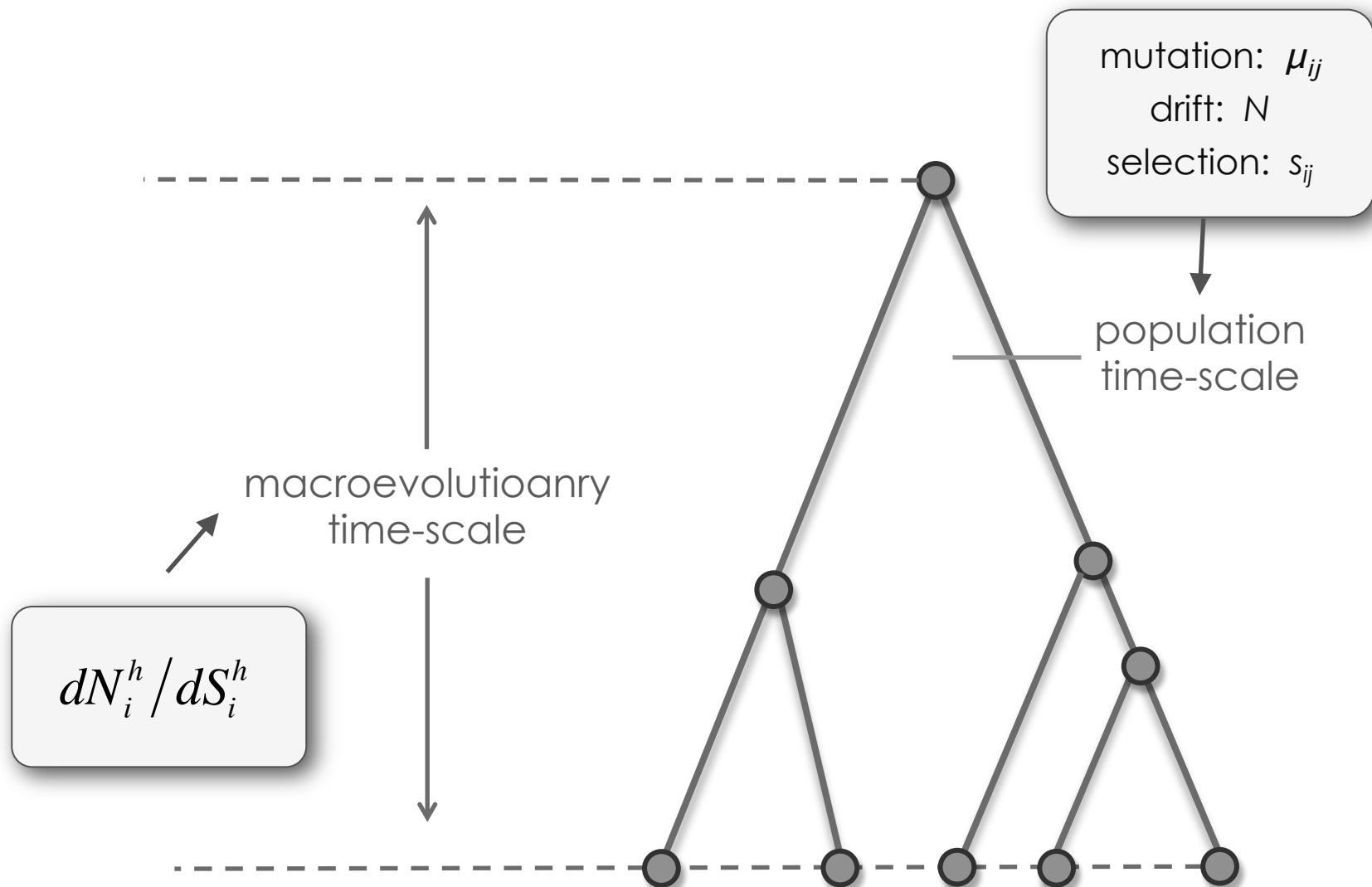
but, this can differ among lineages and genes!

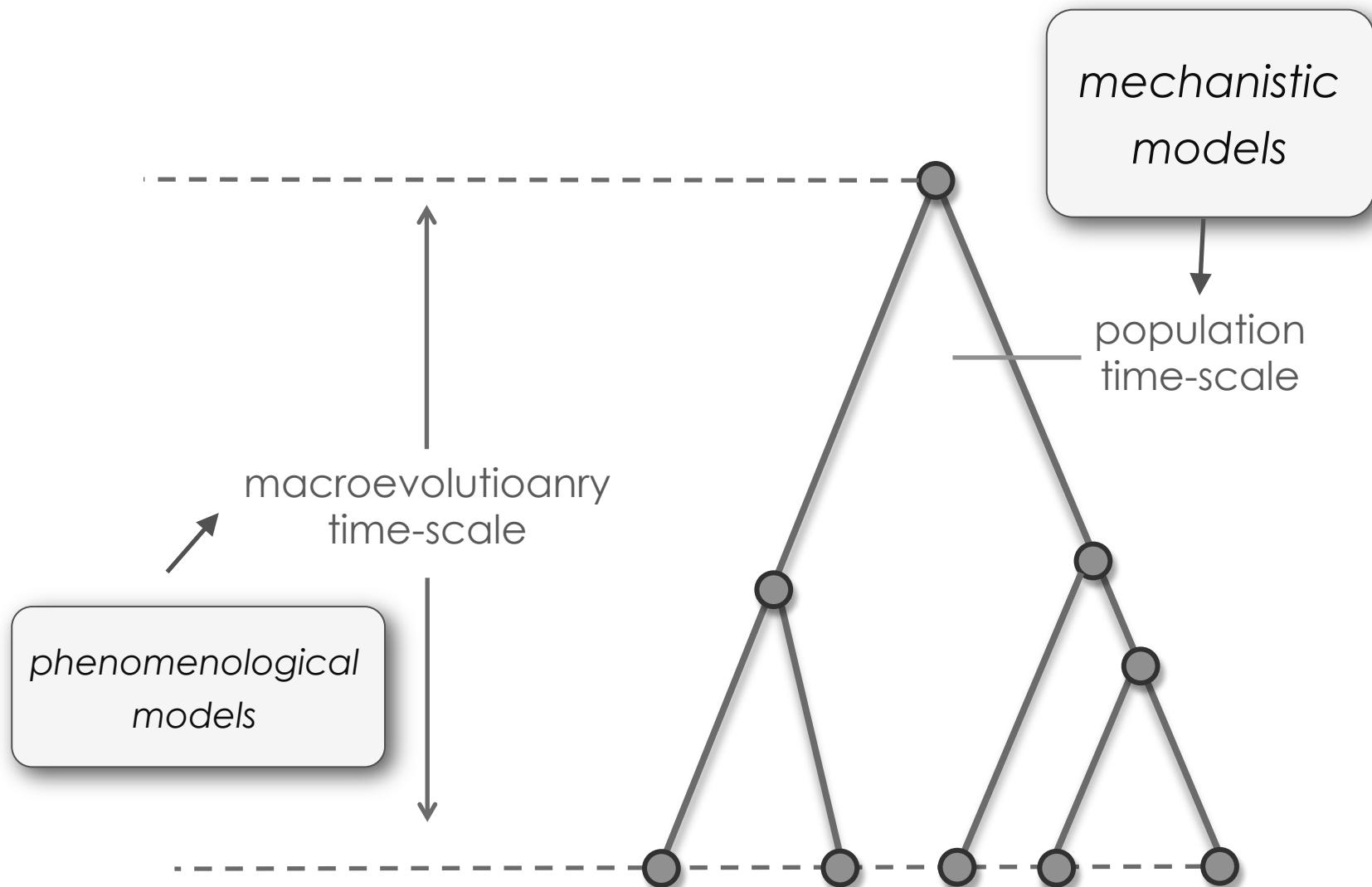
correcting dS and dN for underlying mutational process of the DNA makes them **sensitive to assumptions about the process of evolution!**

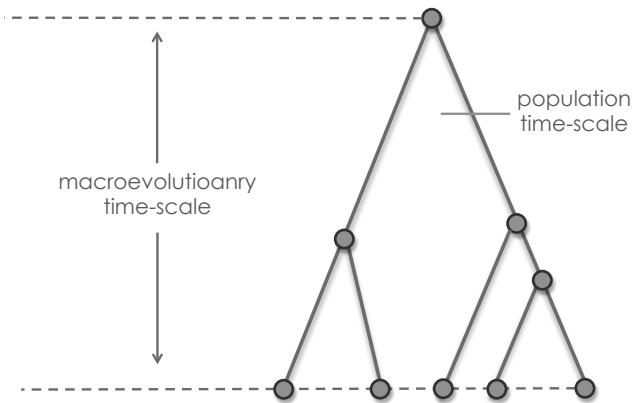
but, the process of evolution occurs at the population genetic level (micro-evolution)

reconciling evolutionary time scales









- Wright-Fisher population
- drift: \mathbf{N}
- mutation: $\boldsymbol{\mu}$
- selection: s_{ij}
- s_{ij} vary among sites AND amino acids
- expected dN^h/dS^h

“MUTSEL MODELS”

$$\Pr = \begin{cases} \mu_{ij} N \times \frac{1}{N} = \mu_{IJ} & \text{if neutral} \\ \mu_{ij} N \times \frac{2s_{ij}}{1 - e^{-2Ns_{ij}}} & \text{if selected} \end{cases}$$

$$s_{ij} = \Delta f_{ij}$$

Halpern and Bruno (1998)

fixation probability with selection

population genetics at a single codon site (h)

fitness coefficients

$$f^h = \langle f_1, \dots, f_{61} \rangle$$

selection coefficients

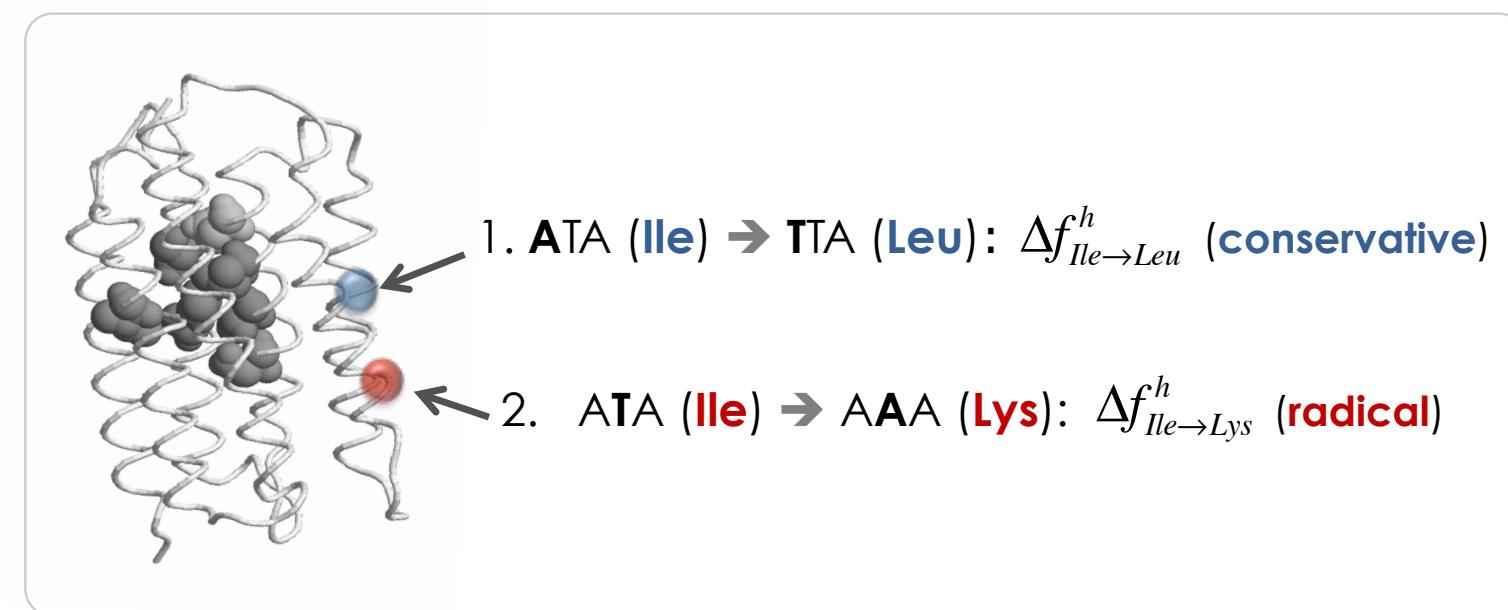
$$s_{ij}^h = f_j^h - f_i^h$$

fixation probability (Kimura, 1962)

$$\Pr(s_{ij}^h) = \frac{2s_{ij}^h}{1 - e^{-2Ns_{ij}^h}}$$

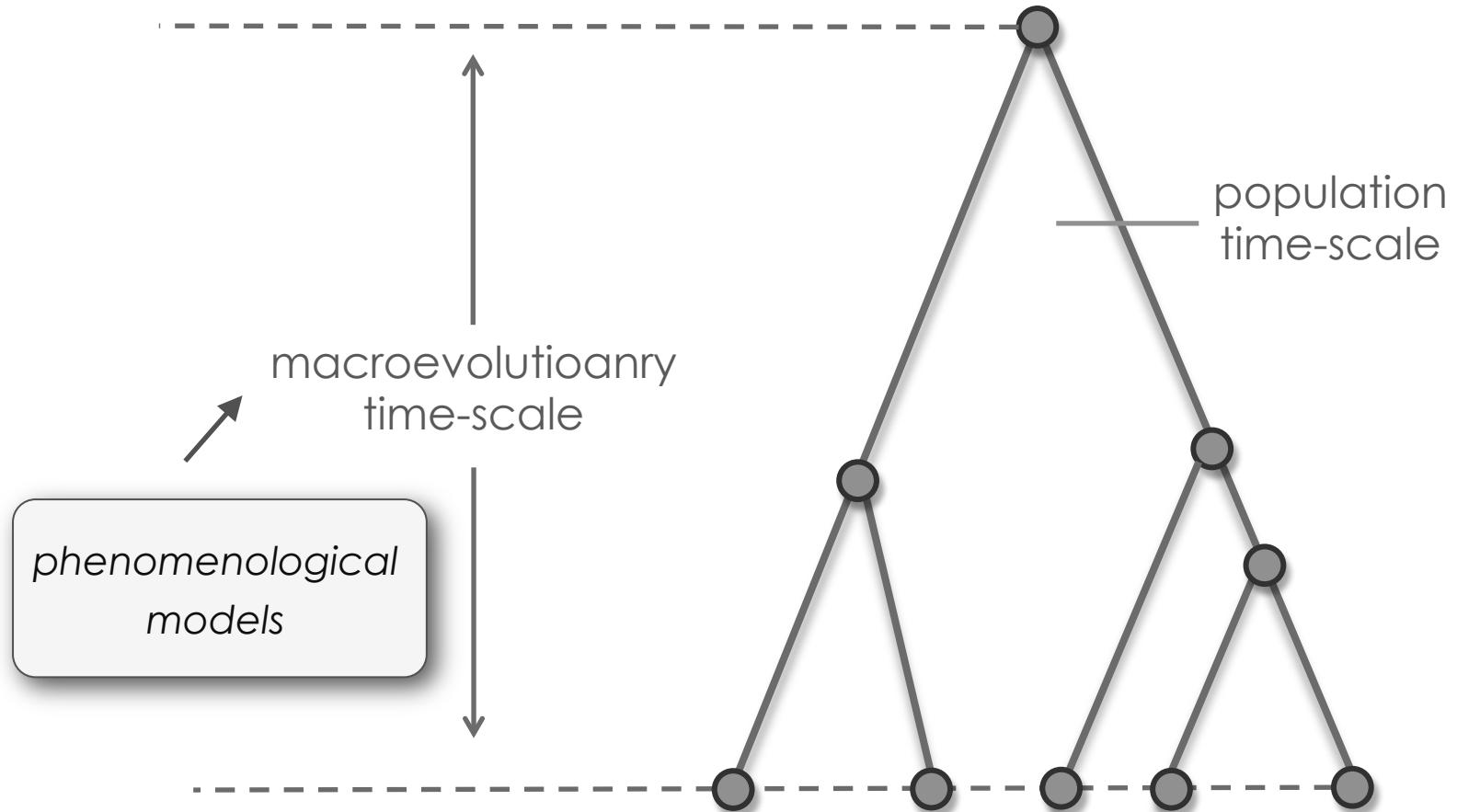
fixation probability with selection

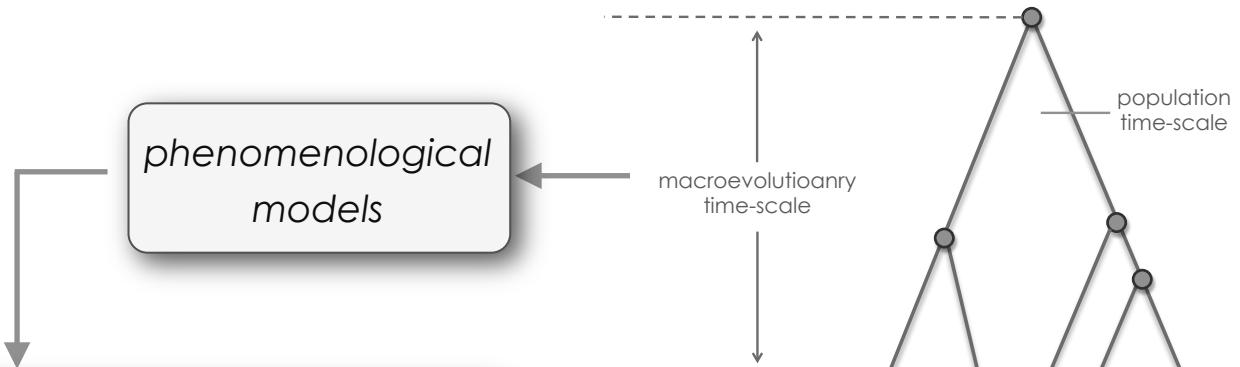
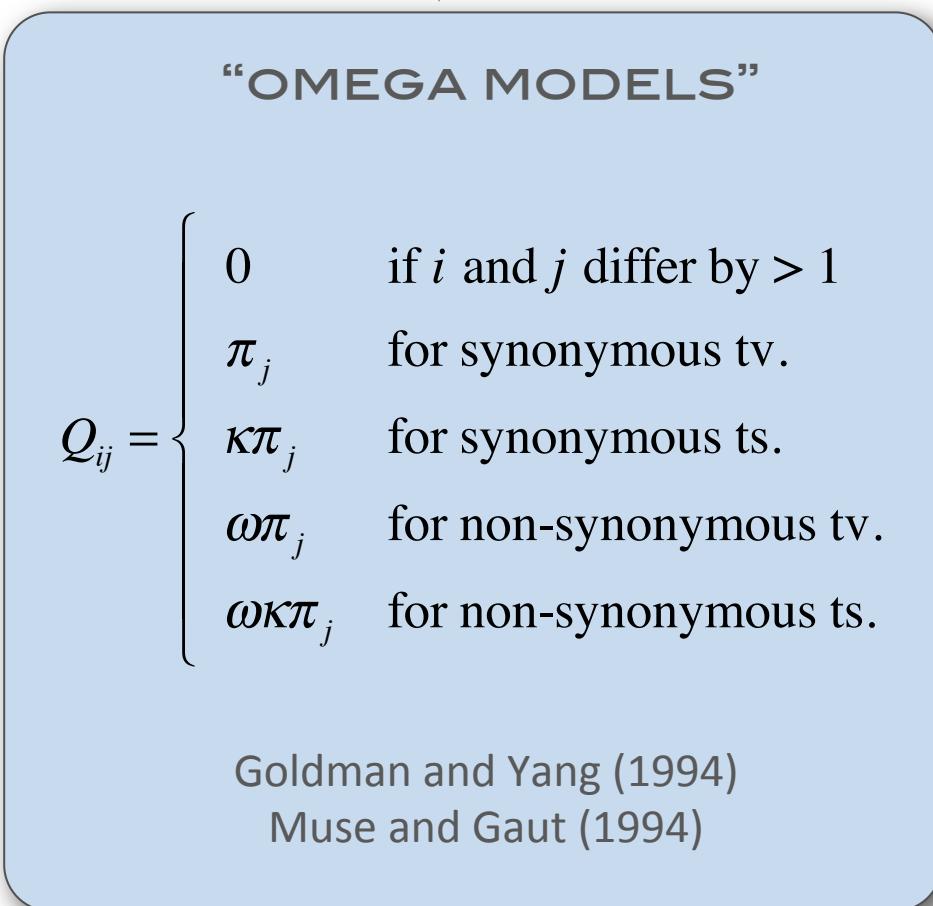
MutSel: selection favours amino acids with higher fitness (if N is large enough)



realism: fitness expected to differ among sites and amino acids according to protein function

the cost of realism: too complex to fit such a model to real data





- phenomenological parameters
- $\text{ts/tv ratio: } \kappa$
- codon frequencies: π_j
- $\omega = dN/dS$
- parameter estimation via ML
- stationary process

the instantaneous rate matrix, Q , is very big: 61×61

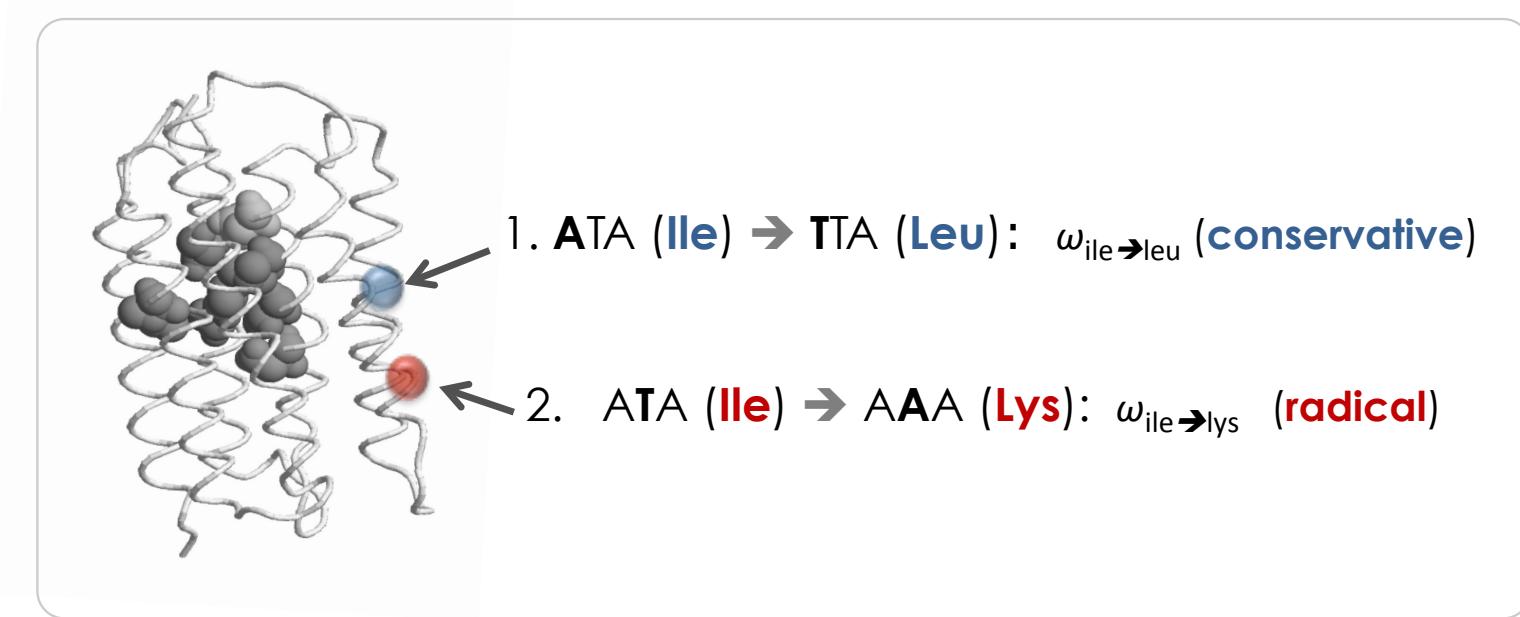
phenomenological codon models: just a few parameters are needed to cover the 3721 transitions between codons!

From codon below:	to codon below:							
	TTT (Phe)	TTC (Phe)	TTA (Leu)	TTG (Leu)	CTT (Leu)	CTC (Leu)	...>	GGG (Gly)
TTT (Phe)	---	$\kappa\pi_{TTC}$	$\omega\pi_{TTA}$	$\omega\pi_{TTG}$	$\omega\kappa\pi_{TTT}$	0	...>	0
TTC (Phe)	$\kappa\pi_{TTT}$	---	$\omega\pi_{TTA}$	$\omega\pi_{TTG}$	0	$\omega\kappa\pi_{CTC}$...>	0
TTA (Leu)	$\omega\pi_{TTT}$	$\omega\pi_{TTC}$	---		0	0	...>	0
TTG (Leu)	$\omega\pi_{TTT}$	$\omega\pi_{TTC}$	$\kappa\pi_{TTA}$	---	0	0	...>	0
CTT (Leu)	$\omega\kappa\pi_{TTT}$	0	0	0	---	$\kappa\pi_{CTC}$...>	0
CTC (Leu)	0	$\omega\kappa\pi_{TTC}$	0	0	$\kappa\pi_{TTT}$	---	...>	0
↓	↓	↓	↓	↓	↓	↓	↓	↓
GGG (Gly)	0	0	0	0	0	0	0	---

* This is equivalent to the codon model of Goldman and Yang (1994). Parameter ω is the ratio d_N/d_S , κ is the transition/transversion rate ratio, and π_i is the equilibrium frequency of the target codon (i).

substitution probability with selection

intentional simplification: all amino acid substitutions have the same ω !



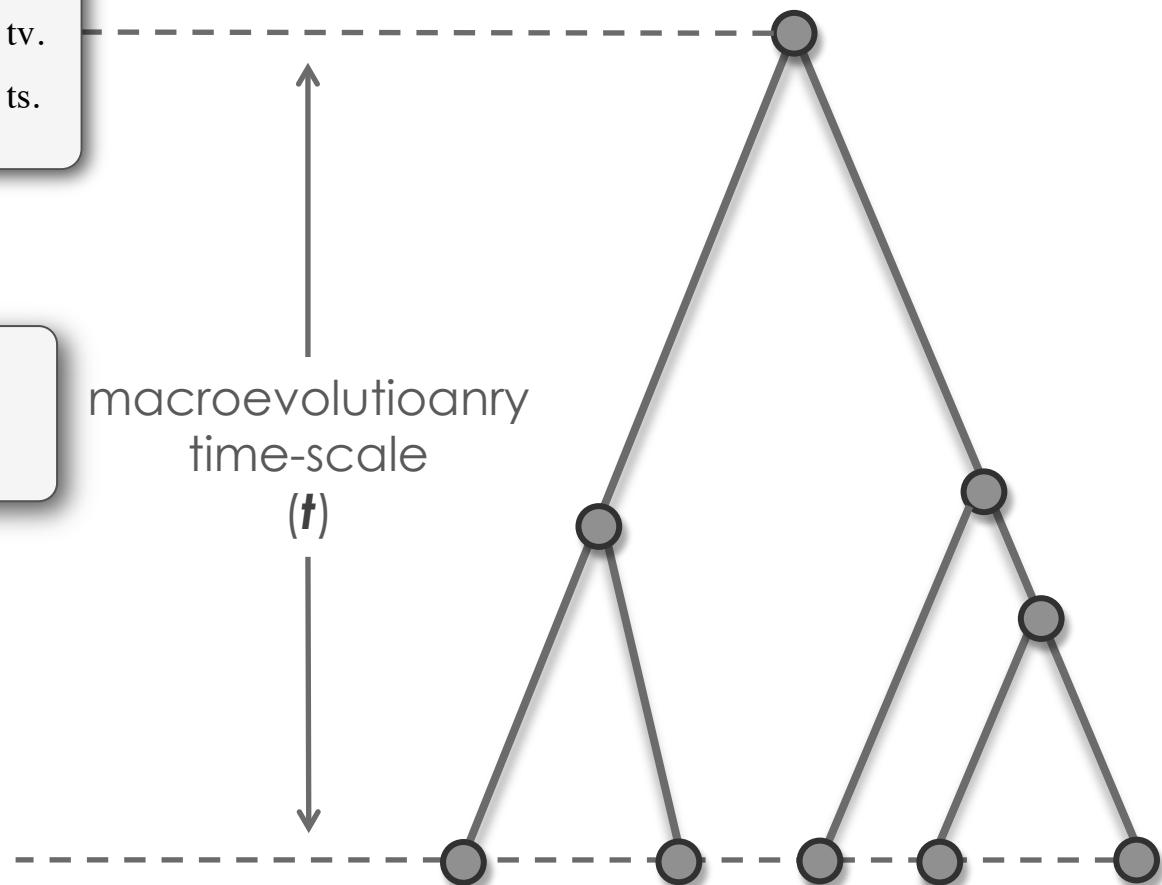
contradiction? selection should favour amino acids with higher fitness.

probability of substitution between codons over time, $P(t)$

$$Q_{ij} = \begin{cases} 0 & \text{if } i \text{ and } j \text{ differ by } > 1 \\ \pi_j & \text{for synonymous tv.} \\ \kappa\pi_j & \text{for synonymous ts.} \\ \omega\pi_j & \text{for non-synonymous tv.} \\ \omega\kappa\pi_j & \text{for non-synonymous ts.} \end{cases}$$

$$P(t) = \{p_{ij}(t)\} = e^{Qt}$$

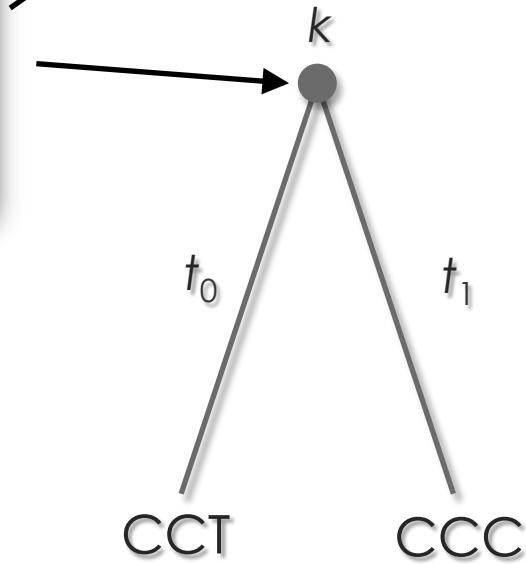
recall that **Paul Lewis** introduced **Q matrices** and how to obtain **transition probabilities**



likelihood of the data at a site

$$L_h(CCC, CCT) = \sum_k \pi_k p_{kCCC}(t_0) p_{kCCT}(t_1)$$

the likelihood is a **sum over all possible ancestral codon states** that could have been observed at node k



recall that **Paul Lewis** described how to compute the likelihood of the data at a site for a DNA model. The only difference here is that the states are codons rather than nucleotides.

note: analysis is typically done by using an unrooted tree

likelihood of the data at all sites

The likelihood of observing the entire sequence alignment is the product of the probabilities at each site.

Paul Lewis covered this with the “**AND**” rule in his likelihood lecture.

$$L = L_1 \times L_2 \times L_3 \times \dots \times L_N = \prod_{h=1}^N L_h$$

See **Paul Lewis's** lecture slides for more about likelihoods vs. log-likelihoods.

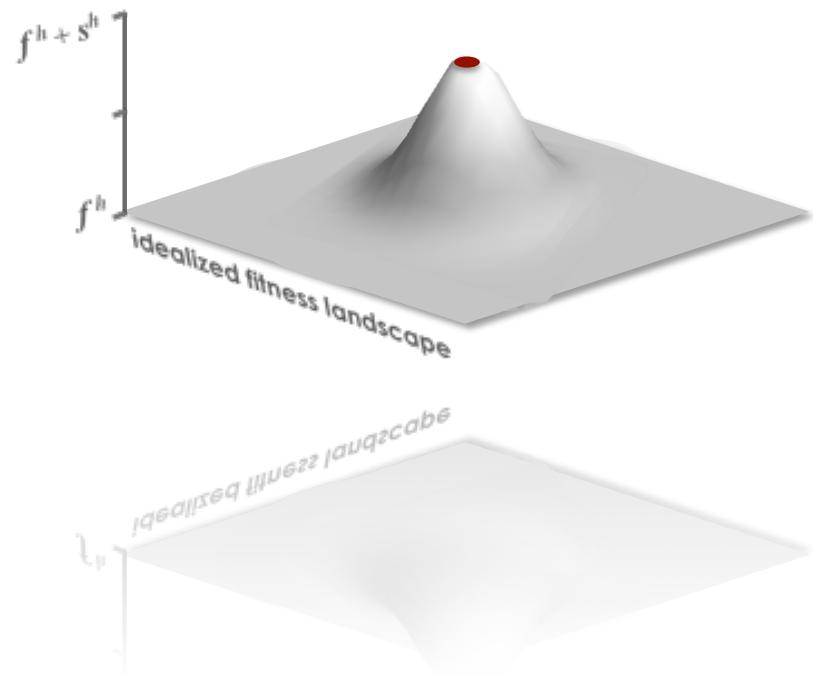
The log likelihood is a sum over all sites.

$$\ell = \ln\{L\} = \ln\{L_1\} + \ln\{L_2\} + \ln\{L_3\} + \dots + \ln\{L_N\} = \sum_{h=1}^N \ln\{L_h\}$$

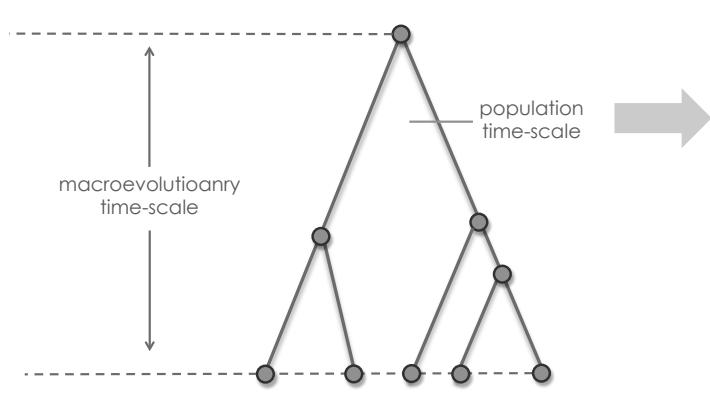
summary

- dN/dS is a measure of selection pressure that can be connected to a mechanistic process of population genetic evolution (MutSel models)
- dN/dS can be estimated from multi-sequence alignments as a parameter (ω) in a phenomenological model of sequence evolution
- estimates of dN/dS for real data must be corrected for the underlying process of evolution for those data
- estimates of dN/dS can be sensitive to assumptions about the underlying process of evolution
- phenomenological estimates of dN/dS are highly simplistic summaries of a much more complex evolutionary process

part 2: mechanistic processes of codon evolution



site-specific MutSel rate matrix



MUTSEL RATE MATRIX

$$A_{ij}^h = \begin{cases} \mu_{ij} & \text{if } s_{ij}^h = 0 \\ \mu_{ij}N \times \frac{2s_{ij}^h}{1 - e^{-2Ns_{ij}^h}} & \text{otherwise} \end{cases}$$

- MutSel time-scale is infinitesimal compared to substitution scale
- MutSel probabilities approximate the instantaneous site-specific rate matrix, A
- μ_{ij} = nucleotide GTR process (before the effect of selection)

site-specific MutSel rate matrix

two explicit ways to reconcile **population genetics**
and **macroevolution**:

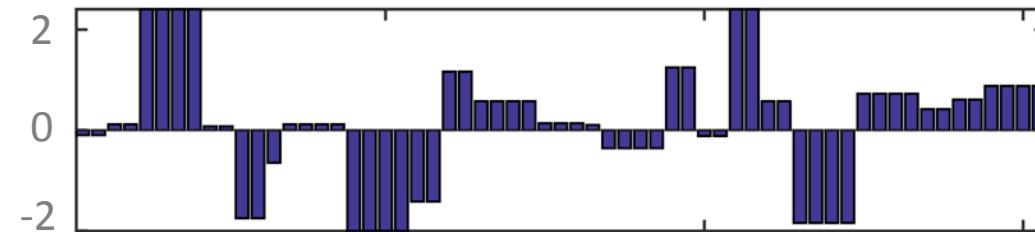
1. map fitness to equilibrium frequencies

2. macroevolution index of selection intensity

fitness coefficients map to stationary codon frequencies

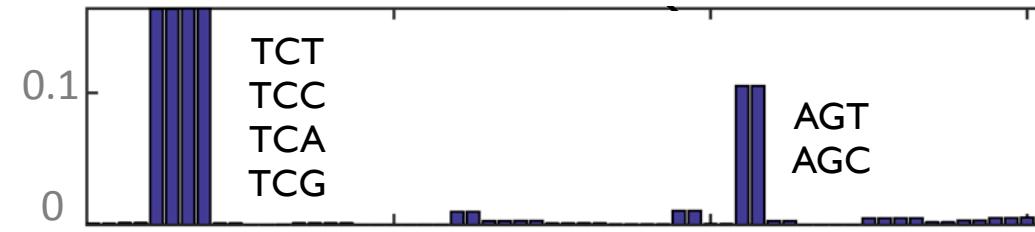
fitness
coefficients

$$f^h = \langle f_1, \dots, f_{61} \rangle$$



codon
frequencies

$$\pi^h = \langle \pi_1, \dots, \pi_{61} \rangle$$



from fitness coefficients to dN/dS

MUTSEL RATE MATRIX

$$dN^h / dS^h = \frac{E[\text{evolution w/ selection}]}{E[\text{drift away from equilibrium set by selection}]}$$

$$dN^h / dS^h = \frac{\sum_{i \neq j} \pi_i^h A_{ij}^h I_N}{\sum_{i \neq j} \pi_i^h \mu_{ij} I_N}$$

- $dN/dS = \omega$ when matrix A^h is replaced by matrix Q of model M0
- dN/dS is an analog of ω under MutSel

positive selection: 3 evolutionary scenarios

1

frequency dependent
selection

2

episodic adaptation

3

shifting balance

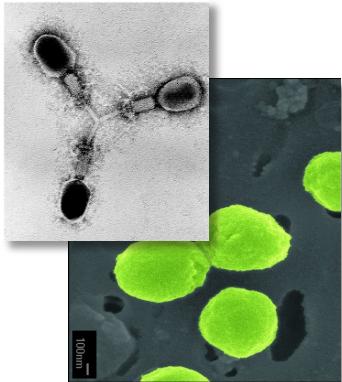
dynamic
fitness
landscape

static
fitness
landscape

scenario 1: frequency dependent selection

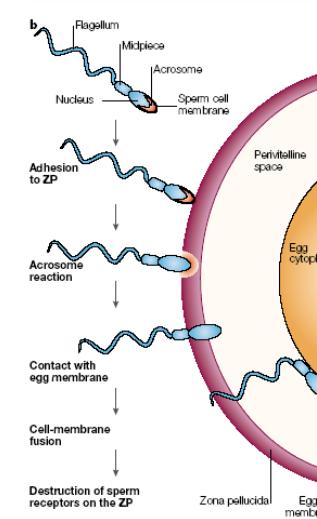
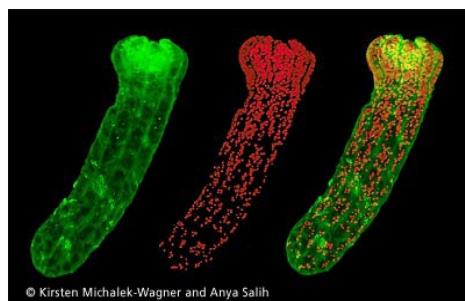
1. antagonistic evolutionary interaction

host-pathogen



sexual-conflict

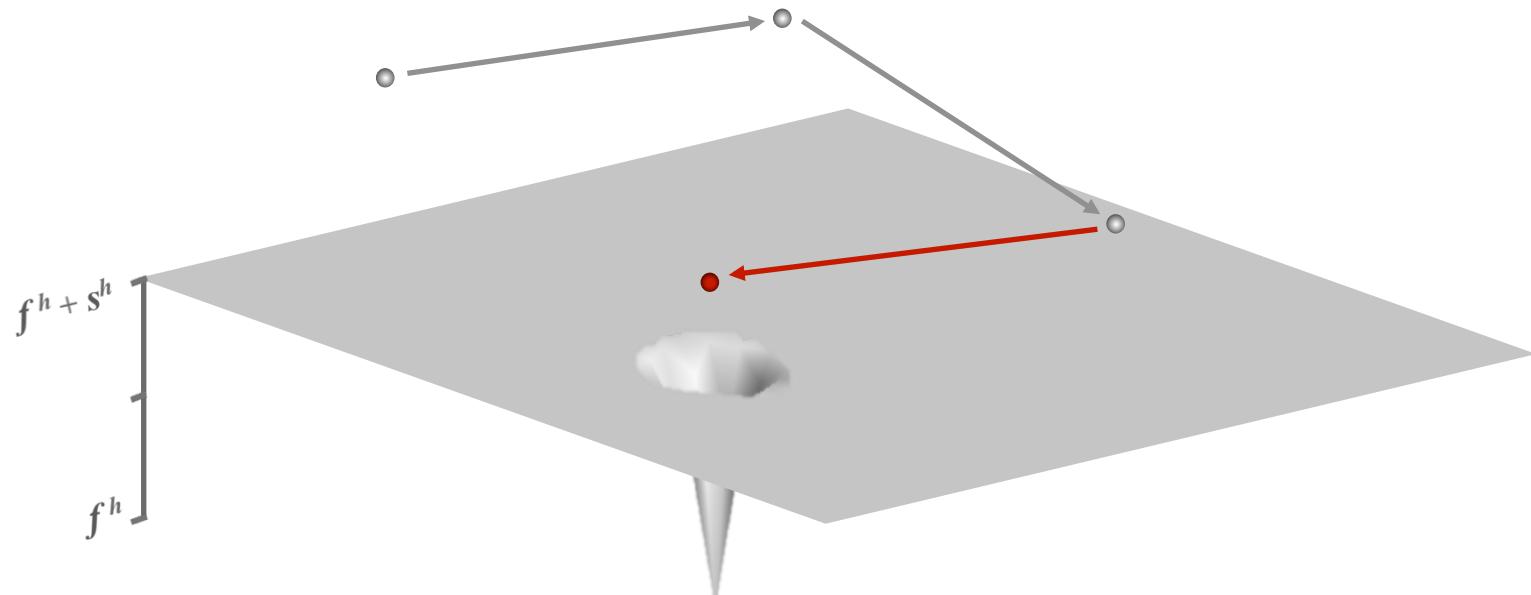
molecular-interactions



© Kirsten Michalek-Wagner and Anya Salih

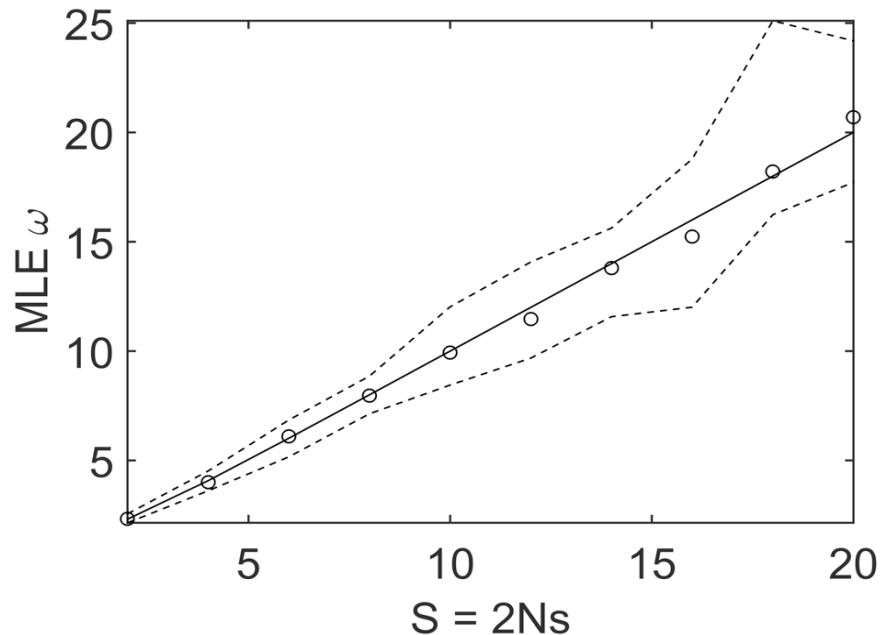
frequency-dependent selection: MutSelM0

1. amino acid at a site has f^h ; all others have $f^h + s$
2. fitness values swap when a substitution occurs



MutSelM0: (1) and (2) above imply Markov chain properties with the same rate matrix Q as **codon model M0**

frequency-dependent selection: MutSelM0



generating process:

MutSelM0

expectation = dN^h/dS^h

symbol = —

fitted model:

model M0

inference = MLE ω

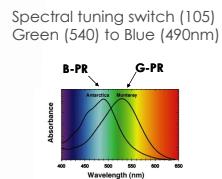
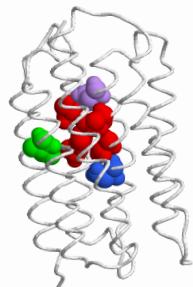
symbol = ○

conclusion: phenomenological codon models
assume frequency-dependent selection

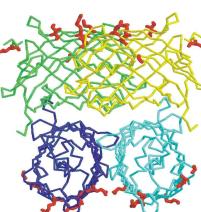
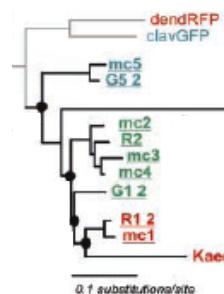
scenario 2: adaptive peak shift

2. episodic Darwinian adaptation

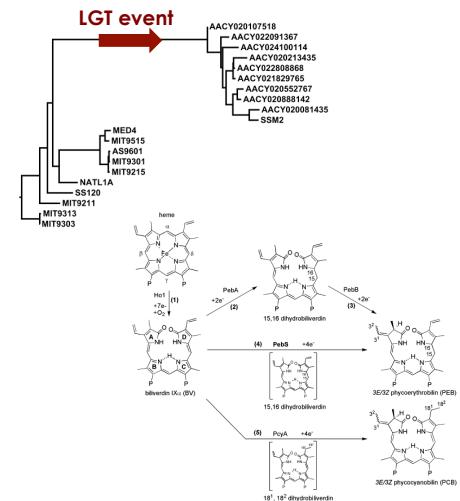
exploitation of a new niche



gene duplication

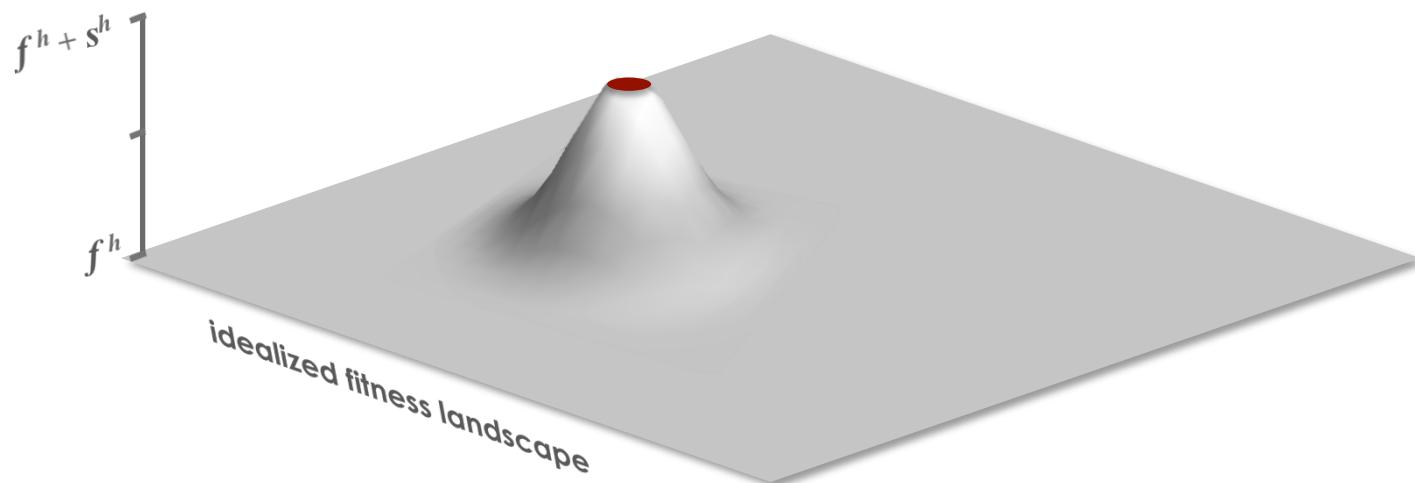


lateral gene transfer (LGT)



adaptive peak shift: evolution of novel function

optimal function in a stable environment



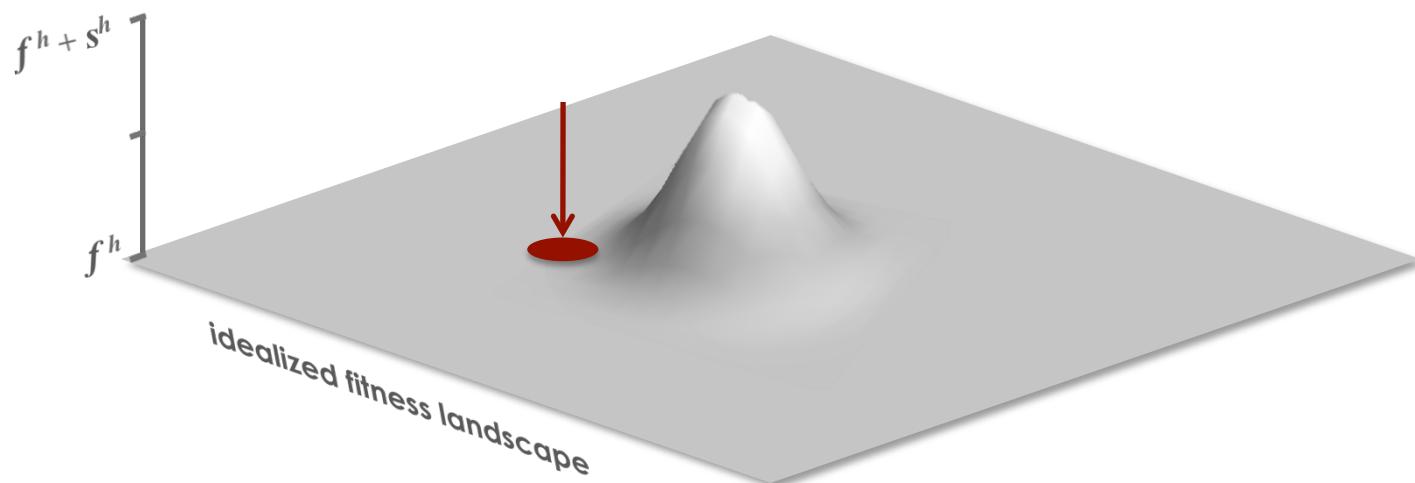
population: at fitness peak

fitness peak: stationary

FFTNS: keeps population at peak

adaptive peak shift: evolution of novel function

sub-optimal function in a novel environment



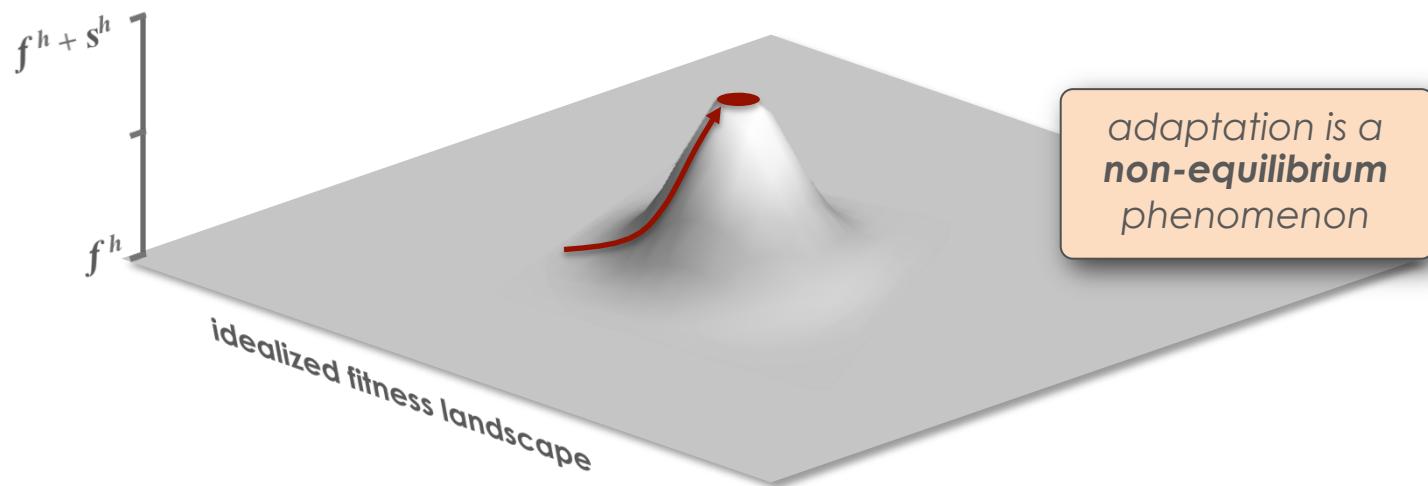
population: lower fitness

fitness peak: moving

FFTNS: increase population mean fitness
(non-stationary process)

adaptive peak shift: evolution of novel function

episodic adaptive evolution of a novel function



population: returns to peak

fitness peak: stabilized

FFTNS: increases population mean

fitness until at peak

adaptive peak shift: MutSelES model

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Research



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<http://dx.doi.org/10.1098/rsbl.2014.1031>

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Molecular evolution

How to calculate the non-synonymous to synonymous rate ratio of protein-coding genes under the Fisher–Wright mutation–selection framework

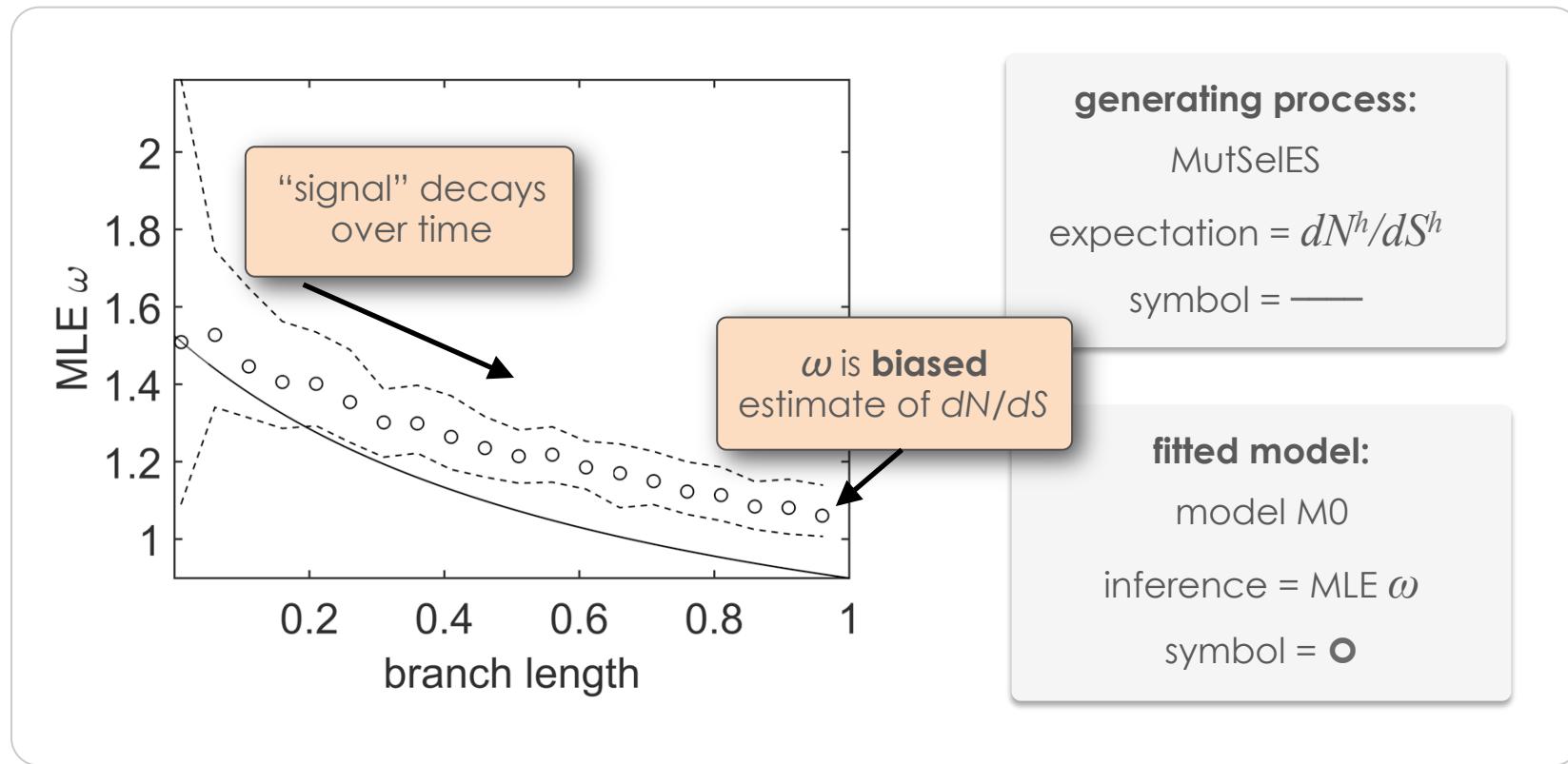
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First principles of population genetics are used to obtain formulae relating the non-synonymous to synonymous substitution rate ratio to the selection coefficients acting at codon sites in protein-coding genes. Two theoretical cases are discussed and two examples from real data (a chloroplast gene and a virus polymerase) are given. The formulae give much insight into the dynamics of non-synonymous substitutions and may inform the development of methods to detect adaptive evolution.

4. The non-synonymous rate during adaptive evolution

adaptive peak shift: MutSelES



conclusion : episodic models “work” because $w>1$ is a consequence of a system moving towards a new fitness peak.

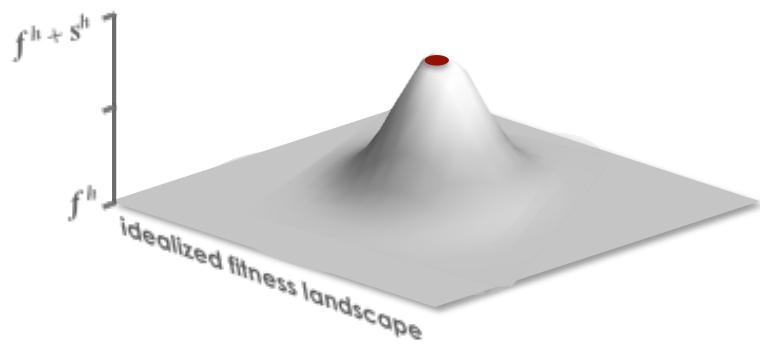
conclusion : episodic models “work” because they are sensitive to non-stationary behavior

Scenario 3: non-adaptive evolution

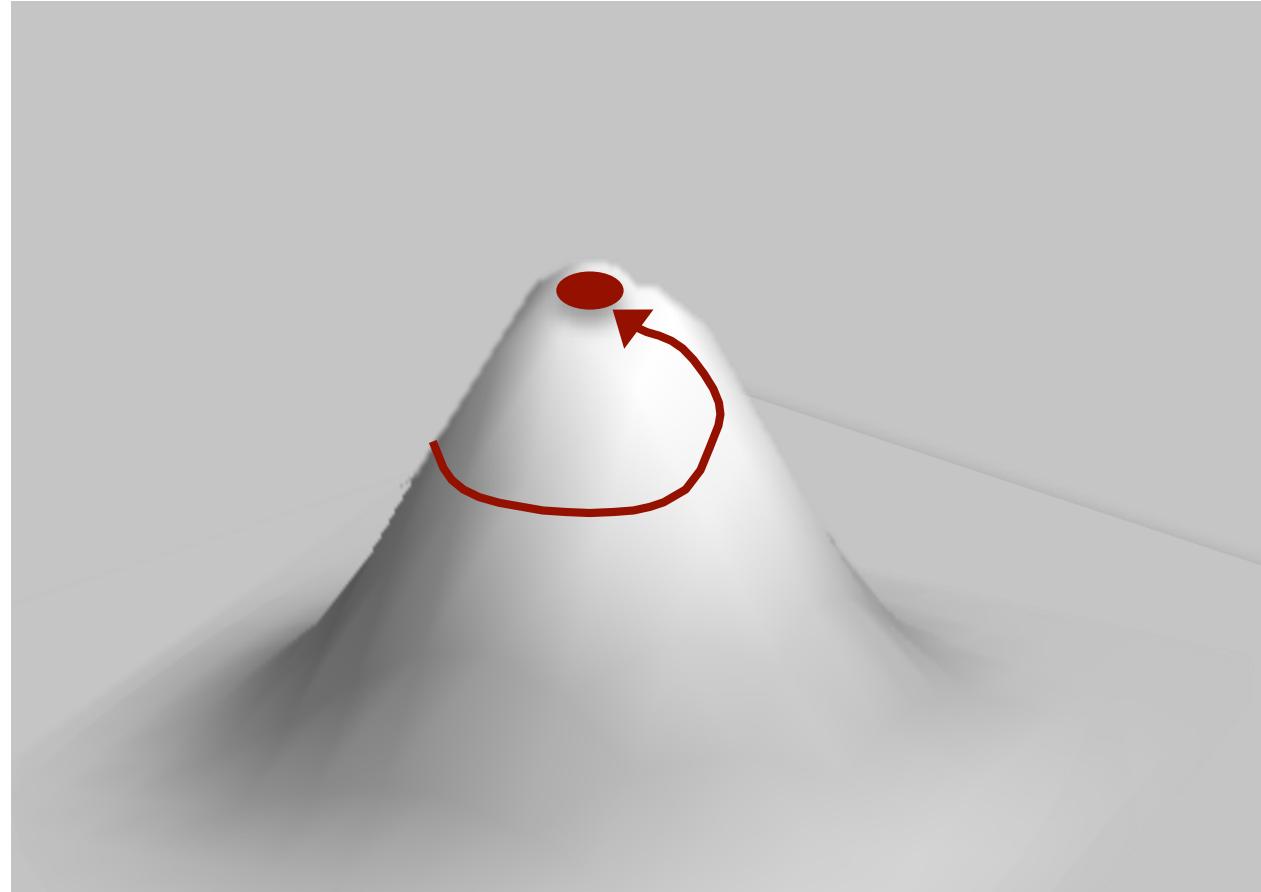
**3. fitness
coefficients are
constant
(fixed-peak)**

Spielman and Wilke (2015)

- dN/dS must be ≤ 1 when fitness coefficients are fixed.
- positive selection is not possible on a stationary fitness peak

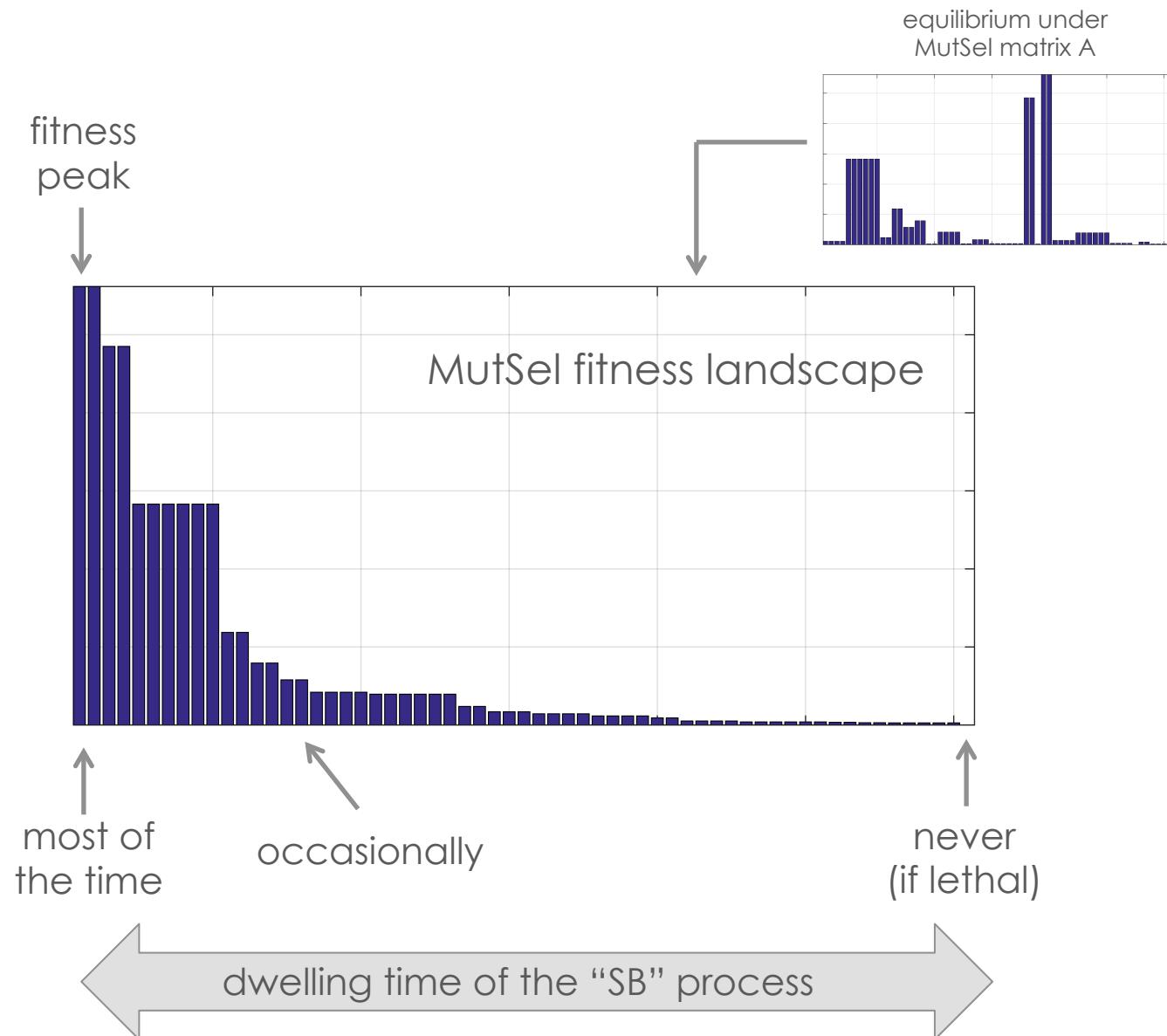


shifting balance: movement around peak

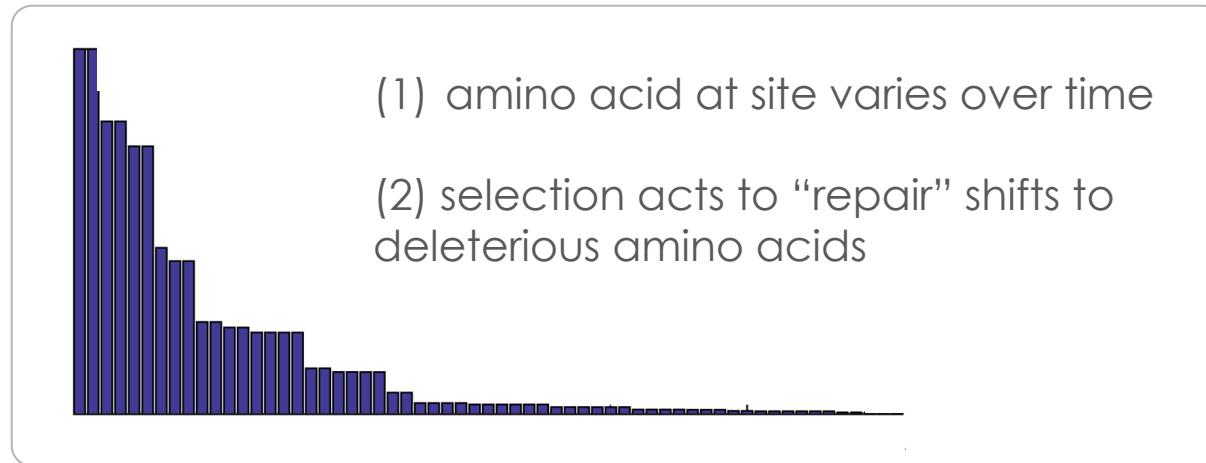


mutation and **drift** can move a pop. off a fitness peak

shifting balance: the MutSel landscape



shifting balance: positive selection on a MutSel landscape



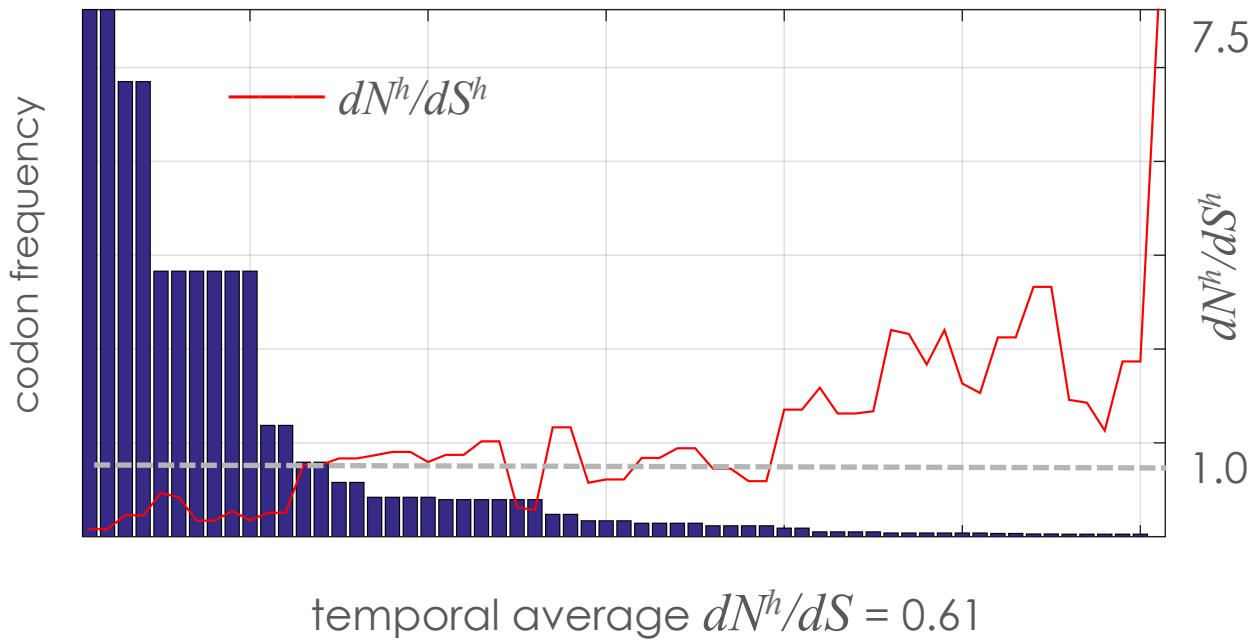
EXPECTED PROPORTION OF
MUTATIONS FIXED BY SELECTION

$$p_+^h = \frac{\sum_{(i,j)} \pi_i^h (A_{ij}^h - \mu_i) I_+}{\sum_{i \neq j} \pi_i^h A_{ij}^h}$$

conclusion: $p_+ > 0$ as long as number of viable amino acids > 1 at a site

shifting balance: the MutSel landscape

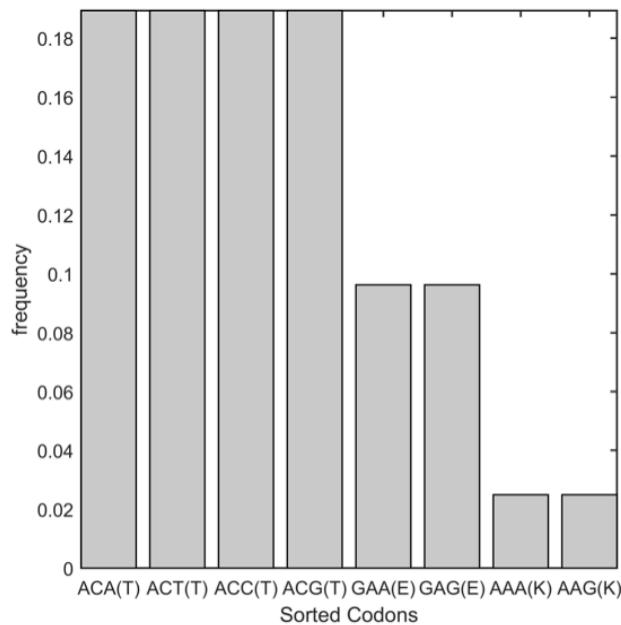
dN^h/dS^h depends on the current amino acid



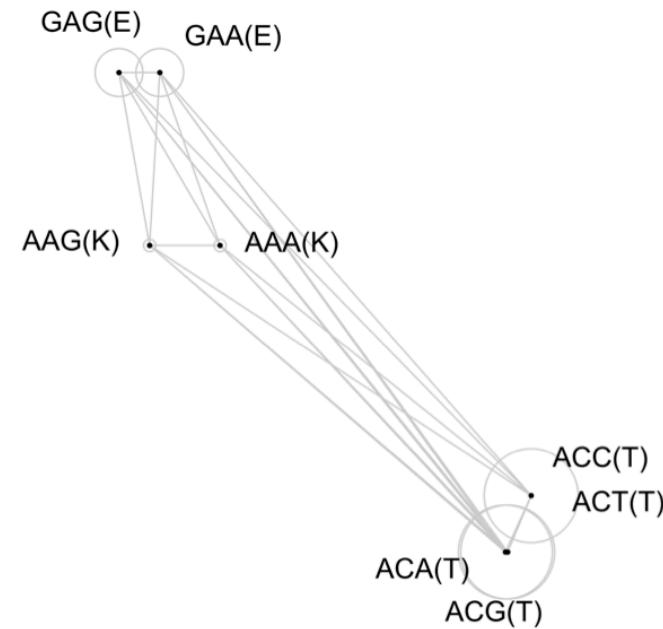
conclusion: positive selection operates on a stationary fitness peak in the same way as when there is an adaptive peak shift

landscapes have unique structures

MutSel landscape



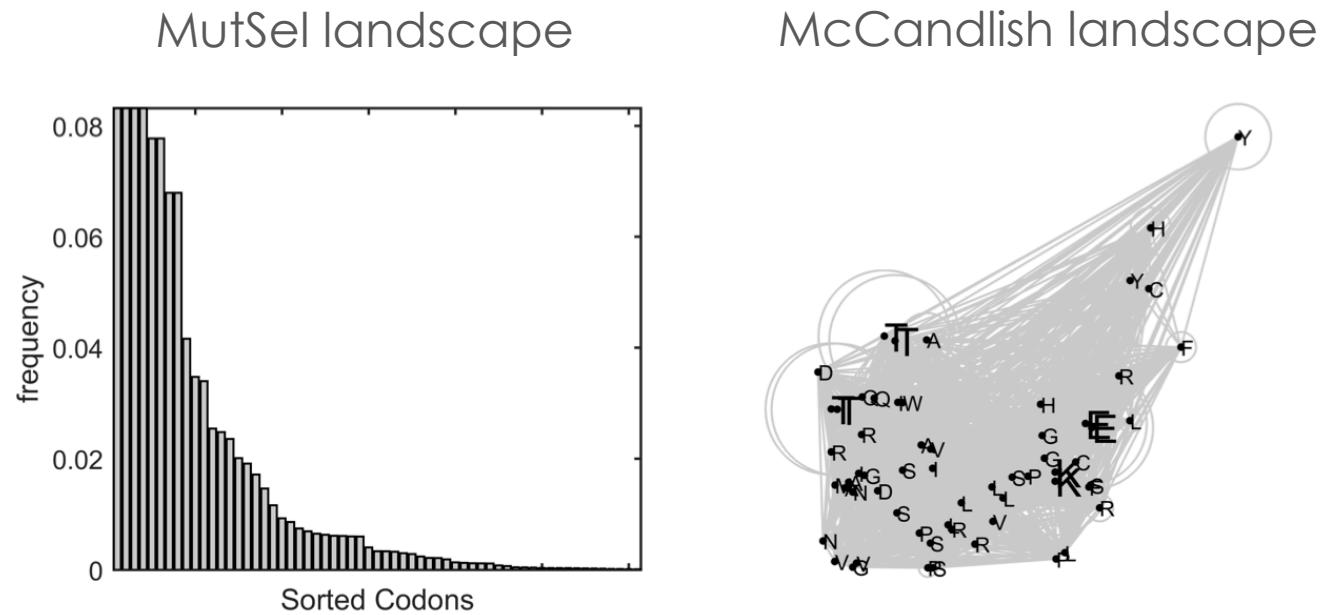
McCandlish landscape



conclusion: A population can get to a sub-optimal codon (E) by drift and reside there for some time (b/c moving between T and E requires changes ≥ 2 codons).

landscape structure depends on N

same site... 10x decrease in N (f^h have not changed!)

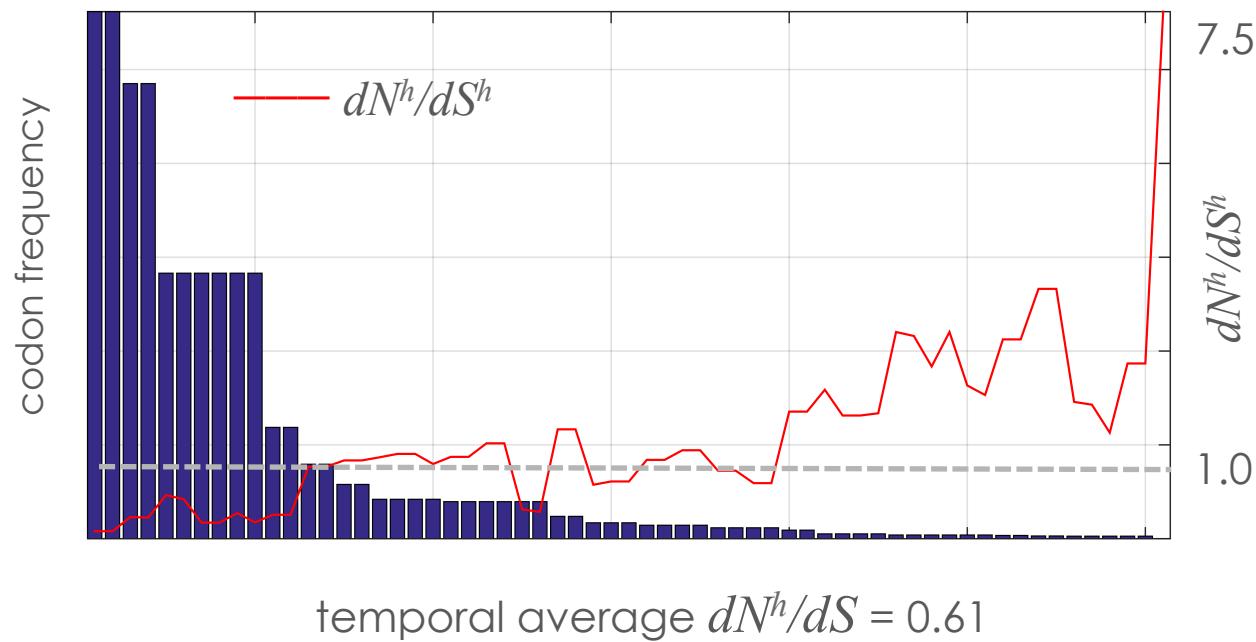


conclusion: decreasing N changes:

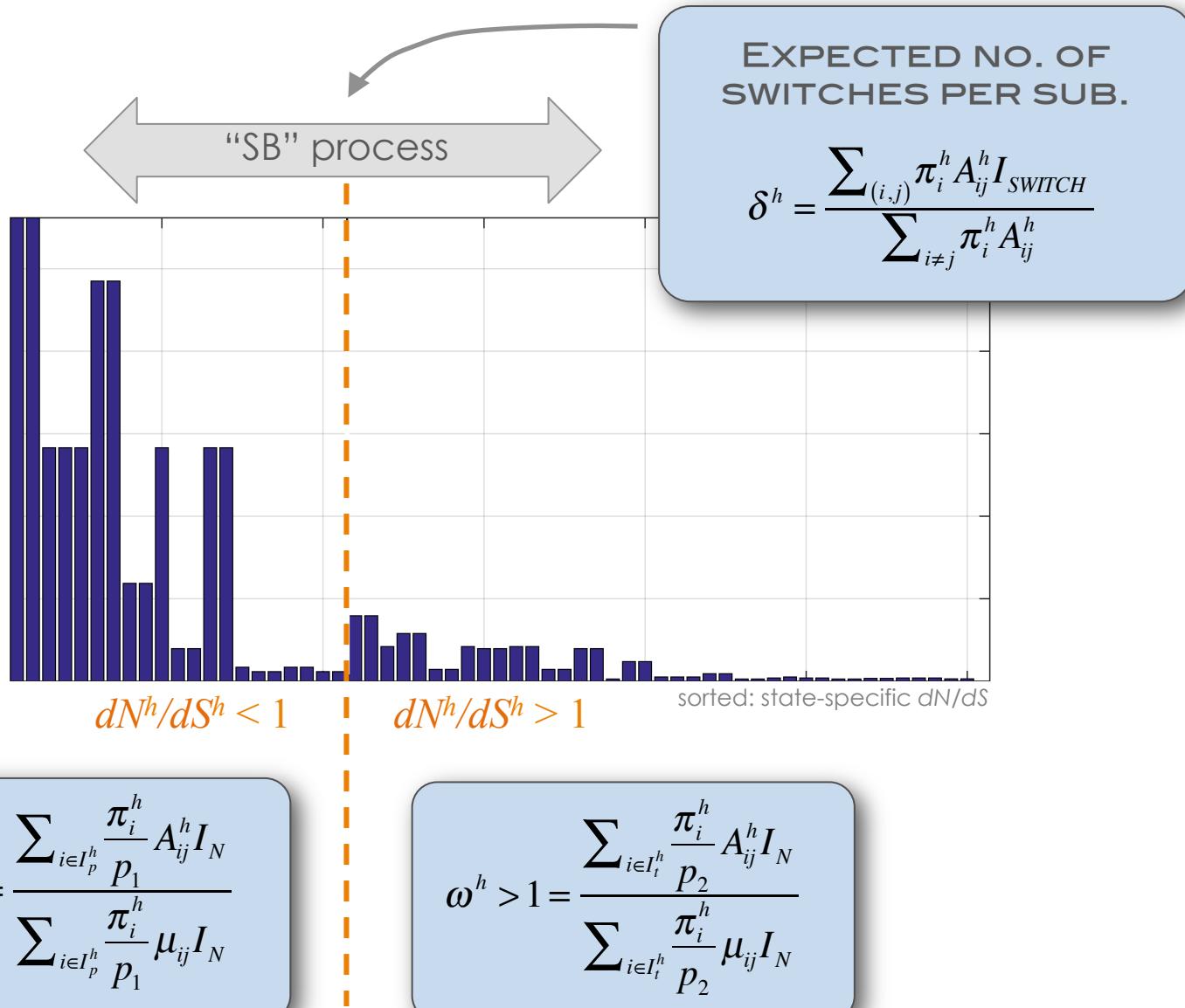
- i. the “space” for shifting balance
- ii. mean dN/dS
- iii. equilibrium frequencies

shifting balance: the MutSel landscape

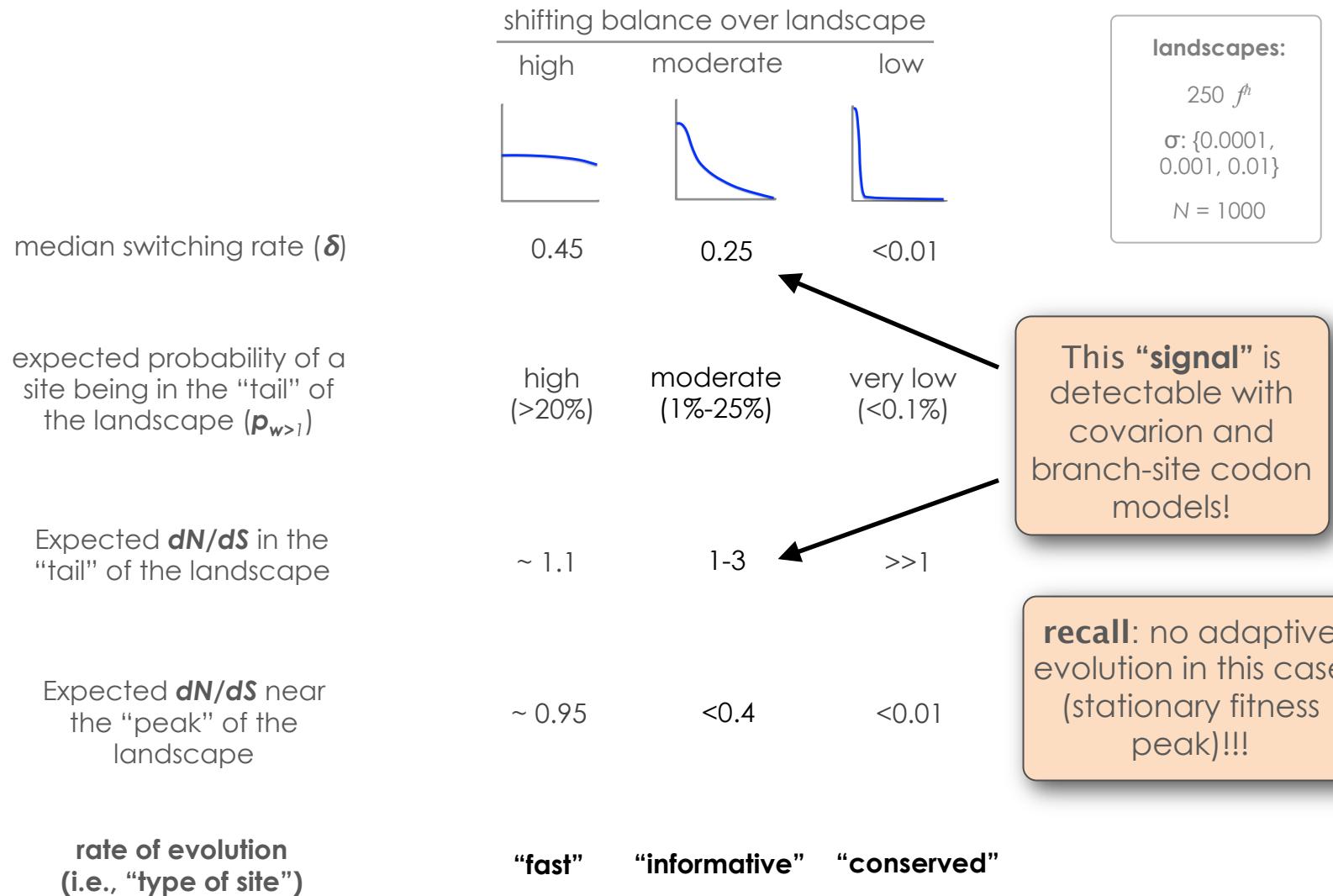
dN^h/dS^h depends on the current amino acid



shifting balance: a mechanistic model



shifting balance: a mechanistic model



summary

- standard codon models (single ω) assume frequency dependent selection, which yields a persistent $dN/dS > 1$
- episodic adaptive evolution leads to transient $dN/dS > 1$
- phenomenological codon models assume a stationary evolutionary process; adaptive evolution is non-stationary
- estimates of ω for episodic adaptive evolution are upwardly biased because adaptive evolution is non-stationary
- protein evolution on a static fitness landscape has temporal dynamics that include positive selection
- MutSel landscapes can be complex and a site can reside at a sub-optimal state for extended periods of time
- rate variation among sites reflects the interplay between mutation, drift, and selection (i.e., shifting balance dynamics)