# Homework 4

Due: Friday May 1, 2020, 23:59:00 hours

Answer all questions. As before, please show details of your work, not merely the answer. For all the problems involving Jacobians below, $J = [linear\ velocities, angular\ velocities]^T$ representation must be assumed. That is,

$$J = \begin{bmatrix} \cdots & z_i \times a_{ie} & \cdots \\ \cdots & z_i & \cdots \end{bmatrix}$$

1. [25 points]For the three-link planar manipulator below in Figure 1a, compute the linear velocity v and the angular velocity ω of the **center of link 2 (shown by the location O_c)** and derive the manipulator Jacobian matrix. Show all steps of your calculation.
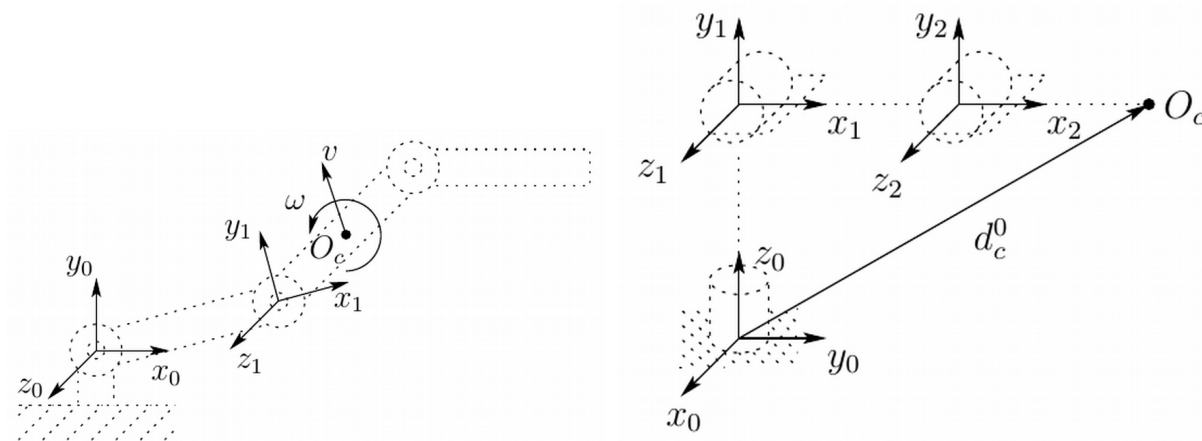
(a) Three-link Planar Arm.   (b) The Elbow Manipulator.

Figure 1: Robot schematics for problem 1 and 2.

2. [20+5=25 points]Consider the three-link elbow manipulator shown in Figure 1b with frame assignments shown. The manipulator Jacobian J will be a 3×6 matrix (3 columns and 6 rows). Let J_{11} be the first 3×3 block of J; that is, J_{11} is the comprised of the three columns and the first three rows of J.

Show that:

$$J_{11} = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} & -a_2 s_2 c_1 - a_3 s_{23} c_1 & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_2 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{bmatrix}$$

Explain the effect of $\theta_3 = 0$ on the solution of the Jacobian.

3. [20 points]Two frames $o_0 x_0 y_0 z_0$ and $o_1 x_1 y_1 z_1$ are related by the homogenous transformation matrix

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_1(t) = [3, 1, 0]^T$ relative to frame $o_1 x_1 y_1 z_1$. What is the velocity of the particle in frame $o_0 x_0 y_0 z_0$? Assume displacement units are in meters and velocities in meters/second.

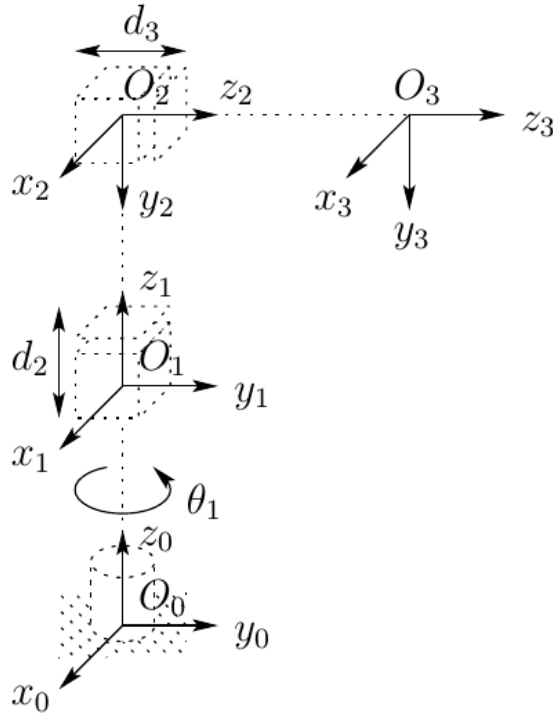4. [30 points] Find the 6×3 Jacobian for the three links of the cylindrical manipulator of Figure 2. Show that there are no singular configurations for this arm.

Figure 2: A 3-link cylindrical manipulator.