Debugging Outputs

2.

(c) Your cost function should be a number between 2200-2400

(d) Final tempreture: 3.36K

The Schwefel function

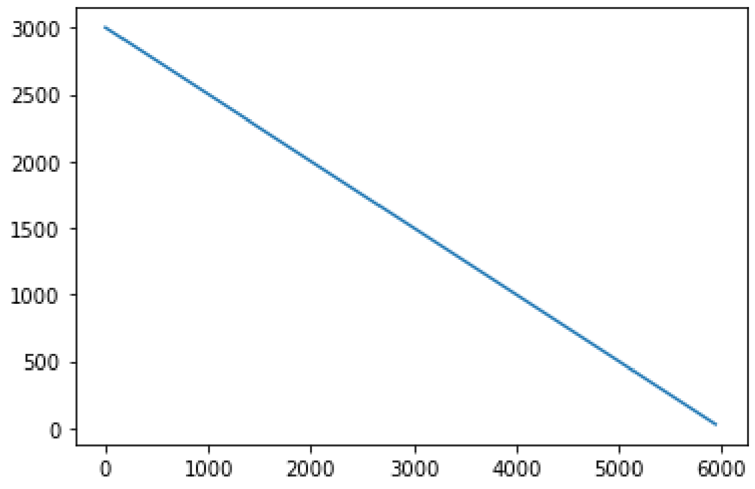
In [ ]:
```
import numpy as np
import pandas as pd
import numba
import matplotlib.pyplot as plt
numba.warnings.filterwarnings("ignore")
def Schwefel(X):
    dim=len(X)
    return 418.9829*dim-np.sum(X*np.sin(np.sqrt(np.abs(X))),axis=0)
```

Visualize a 2d function

In [ ]:
```
def plot_surface(func,x_min=-2,x_max=2,y_min=-2,y_max=2):
    a=np.linspace(x_min,x_max,100)
    b=np.linspace(y_min,y_max,100)
    x,y=np.meshgrid(a,b)
    z=func((x,y))
    fig=plt.figure()
    ax = fig.gca(projection='3d')
    ax.plot_surface(x,y,z)
```

Qestion 1 part a

In [ ]:
```
linear_cooling1 = np.linspace(3000,30,(3000-30)*2)
linear_cooling2 = np.linspace(3000,10,(3000-10)*2)
plt.plot(linear_cooling1)
initial = np.array([400.0,400.0,400.0,400.0,400.0,400.0,400.0,400.0,400.0,400.
```



In [ ]:
```
import numba

def SA(solution,evaluation,delta,boundary,cooling_schedule):
    """ Simulated Annealing for minimization
    solution: np.array. Initial guess of solution
    evaluation: func. Function to evaluate solution
    delta: float. Magnitude of random displacement
    boundary: array of int/float. [lowerbound,upperbound]
    cooling_schedule: np.array. An array of tempretures for simulated annealin
    """
    best_solution=solution.copy()
    lowest_eval=evaluation(best_solution)
    for idx,temp in enumerate(cooling_schedule):
        if idx%500==0:
            print("%d/%d   temp:%f"%(idx,len(cooling_schedule),temp))
        for n in range(len(solution)):
            trial=solution.copy()
            trial[n]+=delta*(2*np.random.random()-1)
            if trial[n]>=boundary[0] and trial[n]<=boundary[1]:
                if np.exp(-(evaluation(trial)-evaluation(solution))/temp)>np.r
                    solution=trial
                    if evaluation(solution)<lowest_eval:
                        best_solution=solution.copy()
                        lowest_eval=evaluation(best_solution)
    return {"solution":best_solution,"evaluation":lowest_eval}


SA(initial,Schwefel,0.5,[-500.0,500.0],linear_cooling2)
```

```
0/5980   temp:3000.000000
500/5980   temp:2749.958187
1000/5980   temp:2499.916374
1500/5980   temp:2249.874561
2000/5980   temp:1999.832748
2500/5980   temp:1749.790935
3000/5980   temp:1499.749122
3500/5980   temp:1249.707309
4000/5980   temp:999.665496
4500/5980   temp:749.623683
5000/5980   temp:499.581870
5500/5980   temp:249.540057
```

Out[ ]:
```
{'solution': array([409.92200032, 422.89364451, 412.03130478, 400.89365856,
        382.27468065, 418.95898455, 433.01440276, 405.33740188,
        410.30543057, 418.43413549]),
 'evaluation': 311.23619487956466}
```

Results of cooling to 30k:

{'solution': array([401.95961952, 401.73384909, 399.77923077, 402.58062509, 398.20568883, 393.69259645, 400.08385456, 406.5751065 , 401.87068183, 402.32257805]), 'evaluation': 505.67854948603235}

{'solution': array([398.19367804, 402.71537622, 402.29245314, 402.09846683, 398.84873886, 401.88885117, 402.03392677, 398.02615728, 403.84803147, 398.99766267]), 'evaluation': 498.6607704304565} {'solution': array([396.24683702, 400.94075755, 399.68220039, 402.45384345, 403.36741374, 400.48218127, 401.13933858, 406.12415339, 398.05183186, 402.20182853]), 'evaluation': 493.31579366268033}

Results of cooling to 10k:

{'solution': array([404.39670429, 408.51932237, 401.29860695, 403.90097084, 405.86960325, 406.9867223 , 406.0797239 , 421.91470371, 402.7028205 , 386.90141679]), 'evaluation': 394.0468481978796}

{'solution': array([408.2951065 , 404.35795534, 399.38883255, 396.48701909, 398.33884911, 393.40359247, 404.03689135, 413.38134821, 401.78260699, 399.87631444]), 'evaluation': 479.54843193471834}

{'solution': array([397.0983453 , 414.14925898, 407.01958924, 401.77358943, 404.63403286, 410.46927417, 406.78679262, 395.03358842, 400.42851645, 419.5898875]), 'evaluation': 349.32836491771513}

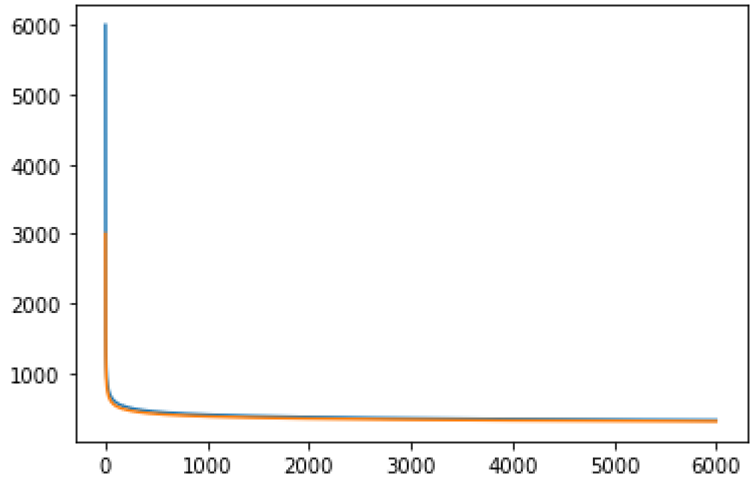
On average, better solutions are obtained from cooling to a lower temperature

Question 1 Part b

In [ ]:
```
logarithmic_cooling1 = np.array([])
for i in range(6001):
    logarithmic_cooling1 = np.append(logarithmic_cooling1,(6000/(1+ 6000*np.lo
plt.plot(logarithmic_cooling1)

logarithmic_cooling2 = np.array([])
for i in range(6001):
    logarithmic_cooling2 = np.append(logarithmic_cooling2,(3000/(1+ 3000*np.lo
plt.plot(logarithmic_cooling2)
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x7f7a3acc8100>]
```



In [ ]:
```
def SA(solution,evaluation,delta,boundary,cooling_schedule):
    """ Simulated Annealing for minimization
    solution: np.array. Initial guess of solution
    evaluation: func. Function to evaluate solution
    delta: float. Magnitude of random displacement
    boundary: array of int/float. [lowerbound,upperbound]
    cooling_schedule: np.array. An array of tempretures for simulated annealin
    """
    best_solution=solution.copy()
    lowest_eval=evaluation(best_solution)
    for idx,temp in enumerate(cooling_schedule):
        if idx%500==0:
            print("%d/%d   temp:%f"%(idx,len(cooling_schedule),temp))
        for n in range(len(solution)):
            trial=solution.copy()
            trial[n]+=delta*(2*np.random.random()-1)
            if trial[n]>=boundary[0] and trial[n]<=boundary[1]:
                if np.exp(-(evaluation(trial)-evaluation(solution))/temp)>np.r
                    solution=trial
                    if evaluation(solution)<lowest_eval:
                        best_solution=solution.copy()
                        lowest_eval=evaluation(best_solution)
    return {"solution":best_solution,"evaluation":lowest_eval}


SA(initial,Schwefel,0.5,[-500.0,500.0],logarithmic_cooling2)
```

```
0/6001   temp:3000.000000
500/6001   temp:415.707877
1000/6001   temp:379.326466
1500/6001   temp:360.842054
2000/6001   temp:348.780336
2500/6001   temp:339.964687
3000/6001   temp:333.085339
3500/6001   temp:327.482176
4000/6001   temp:322.778491
4500/6001   temp:318.740173
5000/6001   temp:315.212364
5500/6001   temp:312.087616
6000/6001   temp:309.288509
```

Out[ ]:
```
{'solution': array([404.11095893, 405.57347469, 405.30179787, 393.69514849,
        405.26888806, 405.88541661, 414.58332046, 415.12101214,
        407.57774999, 385.68359892]),
 'evaluation': 419.93072065387514}
```

Results with initial temperature of 6000k:

{'solution': array([410.65174856, 420.33424829, 408.16342106, 401.01755326, 410.41874023, 402.57349059, 394.80321195, 412.00031086, 410.30582654, 396.97093843]), 'evaluation': 314.9030902553077}

{'solution': array([411.09118894, 408.84039108, 406.73419803, 411.74630376, 426.98342836, 394.49921394, 414.99659845, 411.67969076, 406.67299525, 410.72714753]), 'evaluation': 209.09523720553761}

{'solution': array([420.97139186, 420.74145283, 402.71537012, 385.74316384, 417.33904163, 399.84789883, 396.19735922, 412.76173187, 412.8566653 , 399.20549608]), 'evaluation': 390.86217888393}

Results with intial temperature of 3000k:

{'solution': array([390.00344074, 421.15700647, 433.5379708 , 423.67024766, 411.19886594, 420.54256908, 420.85926799, 429.27645829, 405.86011888, 416.62559737]), 'evaluation': 185.8894200003233}

{'solution': array([403.97662781, 408.95918128, 397.16082029, 411.08906659, 406.05990034, 394.39803512, 405.1748975 , 412.61899037, 410.43166142, 403.43692085]), 'evaluation': 338.6942017456463}

{'solution': array([408.06924782, 428.46455521, 400.68681417, 383.74369953, 396.03608294, 415.37470597, 409.79720893, 415.18983855, 419.55647374, 409.37347281]), 'evaluation': 354.48627510371716}

On average, both logarithmic cooling schedules lead to better solutions than linear cooling

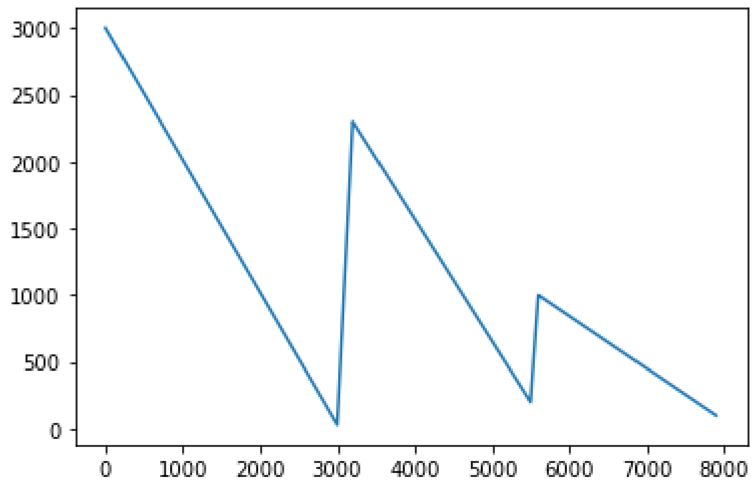
Question 1 Part c

In [ ]:

```
linear_cooling1 = np.linspace(3000,30,3000)
heating=np.append(linear_cooling1,np.linspace(linear_cooling1[-1],2300,200))
heating=np.append(heating,np.linspace(heating[-1],200,2300))
heating=np.append(heating,np.linspace(heating[-1],1000,100))
schedule=np.append(heating,np.linspace(heating[-1],100,2300))
plt.figure()
plt.plot(schedule)
```

Out[ ]:

```
[<matplotlib.lines.Line2D at 0x7f7a3accfbe0>]
```



In [ ]:

```
def SA(solution,evaluation,delta,boundary,cooling_schedule):
    """ Simulated Annealing for minimization
    solution: np.array. Initial guess of solution
    evaluation: func. Function to evaluate solution
    delta: float. Magnitude of random displacement
    boundary: array of int/float. [lowerbound,upperbound]
    cooling_schedule: np.array. An array of tempretures for simulated annealin
    """
    best_solution=solution.copy()
    lowest_eval=evaluation(best_solution)
    for idx,temp in enumerate(cooling_schedule):
        if idx%500==0:
            print("%d/%d   temp:%f"%(idx,len(cooling_schedule),temp))
        for n in range(len(solution)):
            trial=solution.copy()
            trial[n]+=delta*(2*np.random.random()-1)
            if trial[n]>=boundary[0] and trial[n]<=boundary[1]:
                if np.exp(-(evaluation(trial)-evaluation(solution))/temp)>np.r
                    solution=trial
                    if evaluation(solution)<lowest_eval:
                        best_solution=solution.copy()
                        lowest_eval=evaluation(best_solution)
    return {"solution":best_solution,"evaluation":lowest_eval}


SA(initial,Schwefel,0.5,[-500.0,500.0],heating)
```

```
0/5600   temp:3000.000000
500/5600   temp:2504.834945
1000/5600   temp:2009.669890
1500/5600   temp:1514.504835
2000/5600   temp:1019.339780
2500/5600   temp:524.174725
3000/5600   temp:30.000000
3500/5600   temp:2025.967812
4000/5600   temp:1569.247499
4500/5600   temp:1112.527186
5000/5600   temp:655.806873
5500/5600   temp:200.000000
```

Out[ ]:

```
{'solution': array([421.71141513, 418.36476996, 380.60976251, 415.66529137,
        417.95774491, 401.57406357, 409.00766093, 422.72338768,
        420.97440962, 388.2584955 ]),
 'evaluation': 381.8520533628648}
```

Results of heating and cooling schedule:

{'solution': array([410.26658686, 399.30241289, 404.27520702, 412.39865061, 399.96141002, 408.58010571, 389.16993348, 413.53807862, 405.91184769, 386.68723041]), 'evaluation': 480.66399341508395}

{'solution': array([399.0088453 , 408.13542331, 403.00331785, 391.27241485, 404.3833026 , 397.61922611, 406.28683651, 399.35870841, 403.35910734, 401.22351143]), 'evaluation': 494.43808579542656}

{'solution': array([402.73716751, 408.80072617, 400.08274249, 400.14761476, 395.83010505, 405.28408989, 392.72808628, 407.60438496, 397.01760472, 403.67275408]), 'evaluation': 497.0578001261583}

This cooling and heating schedule seems to be doing worse than both linear and logarithmic cooling schedules for this particular funciton and set of hyperparameters.

Question 2 Part a

In [ ]:

```
import pandas as pd


df=pd.read_csv('~/Downloads/wines.csv')
df
```

Out[ ]:

	Alcohol %	Malic Acid	Ash	Alkalinity	Mg	Phenols	Flavanoids	Phenols.1	Proantho-cyanins	Color intensity
0	14.23	1.71	2.43	15.6	127	2.80	3.06	0.28	2.29	5.64
1	13.24	2.59	2.87	21.0	118	2.80	2.69	0.39	1.82	4.32
2	14.83	1.64	2.17	14.0	97	2.80	2.98	0.29	1.98	5.20
3	14.12	1.48	2.32	16.8	95	2.20	2.43	0.26	1.57	5.00
4	13.75	1.73	2.41	16.0	89	2.60	2.76	0.29	1.81	5.60
...	...	...	...	...	...	...	...	...	...	...
173	13.40	4.60	2.86	25.0	112	1.98	0.96	0.27	1.11	8.50
174	13.27	4.28	2.26	20.0	120	1.59	0.69	0.43	1.35	10.20
175	13.17	2.59	2.37	20.0	120	1.65	0.68	0.53	1.46	9.30
176	14.13	4.10	2.74	24.5	96	2.05	0.76	0.56	1.35	9.20
177	12.25	1.73	2.12	19.0	80	1.65	2.03	0.37	1.63	3.40

178 rows × 15 columns

In [ ]:

```
def normalise(df):
    df_n = df.drop(columns = ["Start assignment","ranking"])
    df_n = (df_n - df_n.mean())/df_n.std()
    df_n = df_n.join(df["Start assignment"])
    df_n = df_n.join(df["ranking"])
    return df_n
df_n = normalise(df)
df_n
```

Out[ ]:

	Alcohol %	Malic Acid	Ash	Alkalinity	Mg	Phenols	Flavanoids	Phenols.1
0	1.514341	-0.560668	0.231400	-1.166303	1.908522	0.806722	1.031908	-0.657708
1	0.294868	0.227053	1.835226	0.450674	1.278379	0.806722	0.661485	0.226158
2	2.253415	-0.623328	-0.716315	-1.645408	-0.191954	0.806722	0.951817	-0.577356
3	1.378844	-0.766550	-0.169557	-0.806975	-0.331985	-0.151973	0.401188	-0.818411
4	0.923081	-0.542765	0.158499	-1.046527	-0.752080	0.487157	0.731565	-0.577356
...	...	...	...	...	...	...	...	...
173	0.491955	2.026281	1.798775	1.648436	0.858284	-0.503494	-1.070491	-0.738059
174	0.331822	1.739837	-0.388260	0.151234	1.418411	-1.126646	-1.340800	0.547563
175	0.208643	0.227053	0.012696	0.151234	1.418411	-1.030776	-1.350811	1.351077
176	1.391162	1.578712	1.361368	1.498716	-0.261969	-0.391646	-1.270720	1.592131
177	-0.924604	-0.542765	-0.898568	-0.148206	-1.382223	-1.030776	0.000731	0.065455

178 rows × 15 columns

Question 2 Part b

In [ ]:

```
df_n1 = df_n.loc[df["Start assignment"] == 1]
df_n2 = df_n.loc[df["Start assignment"] == 2]
df_n3 = df_n.loc[df["Start assignment"] == 3]
centroid1 = df_n1.drop(columns = ["Start assignment","ranking"]).mean()
centroid2 = df_n1.drop(columns = ["Start assignment","ranking"]).mean()
centroid3 = df_n1.drop(columns = ["Start assignment","ranking"]).mean()
print(centroid1, centroid2, centroid3)
```

```
Alcohol %            -0.026247
Malic Acid           -0.022814
Ash                   0.039092
Alkalinity           -0.011393
Mg                    0.001194
Phenols               0.046102
Flavanoids           -0.014459
Phenols.1            -0.092477
Proantho-cyanins      0.015299
Color intensity      -0.122335
Hue                   0.071956
OD280 315            -0.021072
Proline              -0.001085
dtype: float64 Alcohol %            -0.026247
Malic Acid           -0.022814
Ash                   0.039092
Alkalinity           -0.011393
Mg                    0.001194
Phenols               0.046102
Flavanoids           -0.014459
Phenols.1            -0.092477
Proantho-cyanins      0.015299
Color intensity      -0.122335
Hue                   0.071956
OD280 315            -0.021072
Proline              -0.001085
dtype: float64 Alcohol %            -0.026247
Malic Acid           -0.022814
Ash                   0.039092
Alkalinity           -0.011393
Mg                    0.001194
Phenols               0.046102
Flavanoids           -0.014459
Phenols.1            -0.092477
Proantho-cyanins      0.015299
Color intensity      -0.122335
Hue                   0.071956
OD280 315            -0.021072
Proline              -0.001085
dtype: float64
```

Question 2 Part c

In [ ]:

```
def cost(rankings):
    df_n['trial'] = rankings.tolist()
    df_n1 = df_n.loc[df_n["trial"] == 1]
    df_n2 = df_n.loc[df_n["trial"] == 2]
    df_n3 = df_n.loc[df_n["trial"] == 3]
    centroid1 = df_n1.drop(columns = ["Start assignment","ranking", "trial"]).
    centroid2 = df_n1.drop(columns = ["Start assignment","ranking", "trial"]).
    centroid3 = df_n1.drop(columns = ["Start assignment","ranking", "trial"]).
    d1 = np.square((df_n1 - centroid1).drop(columns = ["Start assignment", "ra
    d2 = np.square((df_n2 - centroid2).drop(columns = ["Start assignment", "ra
    d3 = np.square((df_n3 - centroid3).drop(columns = ["Start assignment", "ra
    return d1.sum().sum() + d2.sum().sum() + d3.sum().sum()
J = cost(df['Start assignment'].to_numpy())
J
```

Out[ ]:
```
2307.1548828673654
```

Question 2 Part d

In [ ]:
```
@numba.jit
def simulated_annealing(feats,ranks,centers,start_temp,alpha,steps=10000):
    """ Simulated Annealing for clustering
    feats: pd.DataFrame. Normalized chemical descriptors
    ranks: np.array shape(178,). Initial assignment.
    centers: np.array shape (3,13). Fixed centers
    start_temp: float. Initial tempreture
    alpha: float. Hyperparameter for geometric cooling
    steps: int.
    """
    best_rank=ranks.copy()
    # Use your cost function to evaluate the current best rank
    new_eval = lowest_eval = cost(ranks)
    for step in (range(steps)):
        # update tempture according to geometric cooling schedule
        temp = (alpha**step)*start_temp
        if step == steps-1:
            print('Final temperature:', temp)
        if step%500==0:
            print(step,temp,lowest_eval)
        for n in range(len(ranks)):
            trial=ranks.copy()
            rand_choice=np.random.randint(3)+1
            trial[n]=rand_choice
            trialcost = cost(trial)
            # How much does the cost function of trial and cost function of ra
            diff = new_eval - trialcost
            # Metropolis acceptance criterion
            if np.exp(-diff/temp)>=np.random.random():
                ranks=trial
                # update evaluation
                new_eval = trialcost
                if new_eval<lowest_eval:
                    #update best rank and lowest_eval
                    best_rank = trial.copy()
                    lowest_eval = new_eval
    return best_rank, lowest_eval
solution, lowest_eval = simulated_annealing(df_n, df['Start assignment'].to_nu
```

```
0 500.0 2307.1548828673654
500 303.18947243059233 2301.946531099956
1000 183.84771238548186 2301.946531099956
1500 111.48138185145105 2301.946531099956
2000 67.59996269874972 2301.946531099956
2500 40.9911940539233 2301.946531099956
3000 24.85619699901808 2301.946531099956
3500 15.07227450952633 2301.946531099956
4000 9.139509913744707 2301.946531099956
4500 5.542006378044853 2301.946531099956
Final temperature: 3.3639198998326267
```

In [ ]:
```
solutiony, lowest_evaly = simulated_annealing(df_n, df['Start assignment'].to_
solutionz, lowest_evalz = simulated_annealing(df_n, df['Start assignment'].to_
0 500.0 2307.1548828673654
500 303.18947243059233 2303.032508028119
1000 183.84771238548186 2302.3912000539167
1500 111.48138185145105 2302.3912000539167
2000 67.59996269874972 2302.3912000539167
2500 40.9911940539233 2302.2591724210874
3000 24.85619699901808 2302.2591724210874
3500 15.07227450952633 2302.2591724210874
4000 9.139509913744707 2302.2591724210874
4500 5.542006378044853 2302.2591724210874
Final temperature: 3.3639198998326267
0 500.0 2307.1548828673654
500 303.18947243059233 2302.473446221185
1000 183.84771238548186 2302.227787973779
1500 111.48138185145105 2302.227787973779
2000 67.59996269874972 2302.227787973779
2500 40.9911940539233 2302.227787973779
3000 24.85619699901808 2302.227787973779
3500 15.07227450952633 2302.227787973779
4000 9.139509913744707 2302.227787973779
4500 5.542006378044853 2302.227787973779
Final temperature: 3.3639198998326267
```

To validate how good is your solution

In [ ]:

```
def validate(solution,df):
    """Prints out how many wines are corretly assigned to its cultivar
    solution: np.array shape(178,). Your solution.
    df: pd.DataFrame. Read-in of the wines.csv dataset
    """
    # correct classification
    ranking = df['ranking'].values
    cluster_1 = list(df[df['ranking']==1].index)
    cluster_2 = list(df[df['ranking']==2].index)
    cluster_3 = list(df[df['ranking']==3].index)
    clusters =[cluster_1,cluster_2,cluster_3]

    for i in range(3):
        scores=[]
        for j in range(1,4):
            sol_j= [idx for idx,k in enumerate(solution) if k==j]
            scores.append(len(np.intersect1d(sol_j, clusters[i])))

        print(f'Class {i+1} - cultivar {np.argmax(scores)+1}: {np.max(scores)}
of {len(clusters[np.argmax(scores)])} are classified correctly')

validate(solution, df)
validate(solutiony, df)
validate(solutionz, df)
```

```
Class 1 - cultivar 2: 22 out of 71 are classified correctly
Class 2 - cultivar 3: 25 out of 48 are classified correctly
Class 3 - cultivar 2: 19 out of 71 are classified correctly
Class 1 - cultivar 1: 21 out of 59 are classified correctly
Class 2 - cultivar 1: 27 out of 59 are classified correctly
Class 3 - cultivar 3: 22 out of 48 are classified correctly
Class 1 - cultivar 2: 22 out of 71 are classified correctly
Class 2 - cultivar 1: 28 out of 59 are classified correctly
Class 3 - cultivar 1: 18 out of 59 are classified correctly
```

The algorithm did not seem to do better than randomly assigning each wine to a cluster

Qestion 2 Part e

In [ ]:

```
def assign_ranks(feats,centers):
    """
    feats: pd.DataFrame. Normalized chemical descriptors
    centers: np.array shape (3,13). Position of centers
    """
    result=np.zeros(len(feats))
    for i in range(len(feats)):
        dist=np.zeros(3)
        for j in [0,1,2]:
            dist[j]=np.sum((feats[i] -centers[j,:])**2)
        result[i]=np.argmin(dist)+1
    return result
```

In [ ]:

```
@numba.jit
def simulated_annealing_2(feats,ranks,centers,delta,start_temp,alpha,steps=100
    """ Simulated Annealing for clustering
    feats: pd.DataFrame. Normalized chemical descriptors
    ranks: np.array shape(178,). Initial assignment.
    centers: np.array shape (3,13). Fixed centers
    start_temp: float. Initial tempreture
    alpha: float. Hyperparameter for geometric cooling
    steps: int.
    """
    ranks=np.array(ranks,dtype=int)
    best_center=centers.copy()
    best_rank=ranks.copy()
    # Use your cost function to evaluate the current best rank
    lowest_eval = cost(ranks)
    for step in (range(steps)):
        # update temptуre according to geometric cooling schedule
        temp = (alpha**step)*start_temp
        if step%500==0:
            print(step,temp,lowest_eval)
        for n in range(len(ranks)):
            trial_center=centers.copy()
            for i in range(len(trial_center)):
                trial_center[i,:]+=delta*(2*np.random.random(trial_center.shap
            trial_ranks = assign_ranks(feats.drop(columns = ["Start assignment
            new_eval = cost(trial_ranks)
            diff = lowest_eval - new_eval
            # Fill in Metropolis acceptance criterion
            if np.exp(-diff/temp)>=np.random.random():
                ranks=trial_ranks
                centers=trial_center
                # update evaluation
                if new_eval<lowest_eval:
                    best_center=centers.copy()
                    best_rank=ranks.copy()
                    lowest_eval=new_eval
    return best_rank, best_center, lowest_eval
solution2x, centers2x,lowest_eval2x = simulated_annealing_2(df_n, df['Start as
solution2y, centers2y,lowest_eval2y = simulated_annealing_2(df_n, df['Start as
solution2z, centers2z,lowest_eval2z = simulated_annealing_2(df_n, df['Start as

validate(solution2x, df)
validate(solution2y, df)
validate(solution2z, df)
```

```
0 500.0 2307.1548828673654
500 303.18947243059233 2307.1548828673654
1000 183.84771238548186 2303.948207163066
1500 111.48138185145105 2303.948207163066
2000 67.59996269874972 2302.7334376998274
2500 40.9911940539233 2301.0
3000 24.85619699901808 2301.0
3500 15.07227450952633 2301.0
4000 9.139509913744707 2301.0
4500 5.542006378044853 2301.0
0 500.0 2307.1548828673654
500 303.18947243059233 2307.1548828673654
1000 183.84771238548186 2307.1548828673654
1500 111.48138185145105 2307.1548828673654
2000 67.59996269874972 2307.1548828673654
2500 40.9911940539233 2307.1548828673654
3000 24.85619699901808 2307.1548828673654
3500 15.07227450952633 2307.1548828673654
4000 9.139509913744707 2301.0
4500 5.542006378044853 2301.0
0 500.0 2307.1548828673654
500 303.18947243059233 2307.1548828673654
1000 183.84771238548186 2307.1548828673654
1500 111.48138185145105 2307.1548828673654
2000 67.59996269874972 2307.1548828673654
2500 40.9911940539233 2307.1548828673654
3000 24.85619699901808 2307.1548828673654
3500 15.07227450952633 2301.1317660958575
4000 9.139509913744707 2301.0
4500 5.542006378044853 2301.0
Class 1 - cultivar 1: 59 out of 59 are classified correctly
Class 2 - cultivar 1: 71 out of 59 are classified correctly
Class 3 - cultivar 1: 48 out of 59 are classified correctly
Class 1 - cultivar 1: 59 out of 59 are classified correctly
Class 2 - cultivar 1: 71 out of 59 are classified correctly
Class 3 - cultivar 1: 48 out of 59 are classified correctly
Class 1 - cultivar 1: 59 out of 59 are classified correctly
Class 2 - cultivar 1: 71 out of 59 are classified correctly
Class 3 - cultivar 1: 48 out of 59 are classified correctly
```

Results over the three trials

0 500.0 2307.1548828673654 500 303.18947243059233 2307.1548828673654 1000 183.84771238548186 2303.948207163066 1500 111.48138185145105 2303.948207163066 2000 67.59996269874972 2302.7334376998274 2500 40.9911940539233 2301.0 3000 24.85619699901808 2301.0 3500 15.07227450952633 2301.0 4000 9.139509913744707 2301.0 4500 5.542006378044853 2301.0 0 500.0 2307.1548828673654 500 303.18947243059233 2307.1548828673654 1000 183.84771238548186 2307.1548828673654 1500 111.48138185145105 2307.1548828673654 2000 67.59996269874972 2307.1548828673654 2500 40.9911940539233 2307.1548828673654

```
3000 24.85619699901808 2307.1548828673654 3500 15.07227450952633
2307.1548828673654 4000 9.139509913744707 2301.0 4500 5.542006378044853 2301.0
0 500.0 2307.1548828673654 500 303.18947243059233 2307.1548828673654 1000
183.84771238548186 2307.1548828673654 1500 111.48138185145105
2307.1548828673654 2000 67.59996269874972 2307.1548828673654 2500
40.9911940539233 2307.1548828673654 3000 24.85619699901808
2307.1548828673654 3500 15.07227450952633 2301.1317660958575 4000
9.139509913744707 2301.0 4500 5.542006378044853 2301.0 Class 1 - cultivar 1: 59 out of
59 are classified correctly Class 2 - cultivar 1: 71 out of 59 are classified correctly Class 3 -
cultivar 1: 48 out of 59 are classified correctly Class 1 - cultivar 1: 59 out of 59 are classified
correctly Class 2 - cultivar 1: 71 out of 59 are classified correctly Class 3 - cultivar 1: 48 out of
59 are classified correctly Class 1 - cultivar 1: 59 out of 59 are classified correctly Class 2 -
cultivar 1: 71 out of 59 are classified correctly Class 3 - cultivar 1: 48 out of 59 are classified
correctly
```

This algorithm did significantly better than the previous one and yielded about an 88% accuracy