Enhanced Routing Efficiency for Educational Bus Networks using VRP

Yash Sanjay Patil
Dept. of Artificial Intelligence and Data Science
Vidyavardhini's College of Engineering and Technology
Mumbai, India
yash.203859107@vcet.edu.in

Shubhamkar Sunil Patra
Dept. of Artificial Intelligence and Data Science
Vidyavardhini's College of Engineering and Technology
Mumbai, India
shubhamkar.s211049101@vcet.edu.in

Chetan Vijay Sapkal
Dept. of Artificial Intelligence and Data Science
Vidyavardhini's College of Engineering and Technology
Mumbai, India
chetan.s211069112@vcet.edu.in

Kshitija Gharat
Dept. of Artificial Intelligence and Data Science
Vidyavardhini's College of Engineering and Technology
Mumbai, India
kshitija.gharat@vcet.edu.in

***Abstract*—This paper introduces a novel and comprehensive approach to tackle the intricate challenges associated with optimizing school bus routes, employing an advanced Vehicle Routing Problem (VRP) model. The primary focus of this research is to revolutionize school transportation logistics by maximizing bus capacity utilization, minimizing the count of buses required, and concurrently reducing the overall travel time for students. The system is designed to capitalize on available data concerning student pickup locations, coupled with information regarding the capacity of each bus. The dual optimization objectives identified include the minimization of the bus count and the reduction of travel time for all students on their way to school. The paper elucidates a systematic and innovative methodology strategically combining stops, aiming to achieve both optimization goals concurrently. This paper unveils a pioneering system, showcasing a promising solution that significantly enhances the efficiency of school bus transportation, laying the groundwork for future advancements in the field.**

***Index Terms*—Vehicle Routing Problem (VRP), Optimization, Capacity Maximization, Time Efficiency, Geographic Information Systems (GIS), Predictive Analytics.**

I. Introduction

In the dynamically evolving landscape of contemporary education, the optimization of school bus routes has emerged as a pivotal and transformative focal point, necessitating innovative approaches to reshape and redefine conventional paradigms. This scholarly paper embarks on an exhaustive exploration, delving into cutting-edge methodologies that deftly harness the intrinsic capabilities of advanced Vehicle Routing Problem (VRP) models [1], laying the foundation for a comprehensive understanding of the multifaceted challenges intricately woven into school bus route optimization. The overarching objective is to revolutionize efficiency metrics not only by maximizing bus capacity utilization and minimizing the requisite fleet size but also by concurrently reducing the overall travel time for students, thereby establishing a more streamlined, adaptive, and profoundly effective transportation system [2].

This research is securely anchored in empirical data, offering precise and granular information regarding student pickup locations and the predefined capacity of each bus [3]. A distinctive hallmark of our approach lies in the adoption of a dual-focused optimization strategy, adroitly balancing the imperative of minimizing the bus count with the equally compelling goal of curtailing the cumulative travel time for students a delicate equilibrium that encapsulates the very essence of an efficient and responsive school transportation system [4].

Beyond a mere exposition of research objectives, this introduction serves as a meticulously crafted guide, adeptly navigating readers through the complex and nuanced terrain of school bus route optimization [5]. Our paper unfurls a thoroughly detailed methodology designed with precision to address the multifaceted challenges inherently associated with synthesizing optimal bus stops [6]. Going beyond theoretical foundations, our work provides a robust and pragmatic framework that keenly recognizes and addresses the real-world complexities underpinning the operationalization of optimal school bus routes [7] .

The subsequent sections of this comprehensive paper seamlessly extend this narrative, encompassing an exhaustive review of related work [8], elucidating the nuanced intricacies of our methodology [9], providing detailed insights into experimental setups [10], showcasing results with statistical rigor [11], and engaging in meaningful discussions [12]. This systematic and thorough exploration not only serves as a practical guide for stakeholders in school transportation but also contributes nuanced insights to the broader field, fostering heightened resource efficiency and the overarching effectiveness of school bus transportation systems [13]. In this holistic and all-encompassing endeavor, our research aspires to not only address immediate challenges but also to catalyze a transformative shift in the very fabric of school transportation paradigms [14], thereby paving the way for a more efficient, responsive, and future-ready educational transportation ecosystem [15].

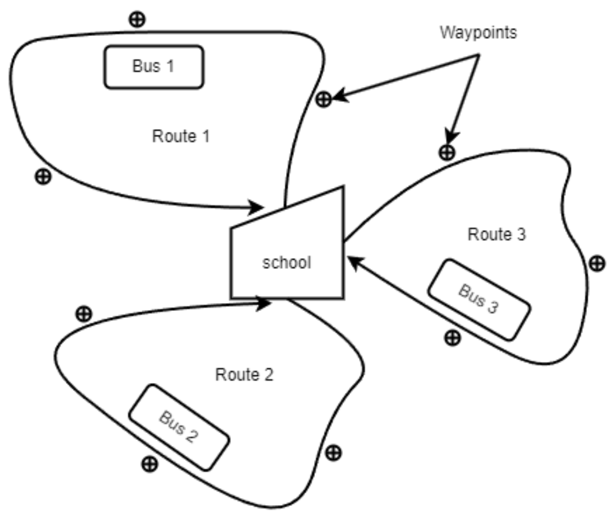

Fig. 1. VRP

II. Related Work

Xu et al.'s research addresses the challenges of Dynamic Vehicle Routing Problems (DVRP) using Enhanced Ant Colony Optimization (E-ACO) [1]. DVRP, a variant of the Vehicle Routing Problem (VRP), involves dynamically changing customer demands, mirroring real logistic scenarios more accurately. The study emphasizes the practical relevance and complexity of DVRP, acknowledging its increasing attention from researchers. The authors propose E-ACO, an enhanced version of Ant Colony Optimization (ACO) that integrates K-means clustering and crossover operations. K-means effectively partitions the search space, while crossover enhances exploration and prevents premature convergence. Novel evaluation benchmarks are introduced to objectively assess the proposed method. The research underscores the importance of addressing the dynamic nature of VRP, advocating for efficient algorithms that can adapt to evolving situations. The E-ACO algorithm's efficacy is demonstrated through experiments on various-scale problems, showcasing favorable comparisons with previously published papers and contributing valuable insights to the literature on DVRP [1].

Anuar et al.'s comprehensive survey on "Vehicle Routing Optimization in Humanitarian Operations" presents a meticulous examination of 123 papers from the last decade, categorizing them within the realms of supply and delivery, evacuation, and rescue operations [8]. The study underscores the dominance of heuristic and meta heuristic solutions in addressing the dynamic and stochastic nature of real-world disasters. The authors advocate for the integration of Machine Learning (ML) solutions, particularly in Markov Decision Processes (MDPs), and propose hybrid approaches as a promising direction for overcoming computational challenges. The survey anticipates continued reliance on metaheuristic solutions, augmented by advancements in computational power supporting exact solutions. Future research directions include the incorporation of ML to address the curse of dimensionality and tackle emerging challenges such as online tools, evolving disaster types, and the need for richer Vehicle Routing Problem models for evacuation and rescue operations. The study provides valuable insights for researchers seeking to contribute to the effective optimization of humanitarian operations in the face of real-world disasters [8].

The paper titled "Review Paper in Vehicle Routing Problem and Future Research Trend" by Modhi Lafta Mutar, Burhanuddin Mohd Aboobaider, and Asaad Shakir Hameed, published in the International Journal of Applied Engineering Research, explores the significance of effective planning in the transportation industry for achieving cost efficiency and timely delivery [16]. Focusing on the transportation problem (TP), specifically the vehicle routing problem (VRP), the study discusses the optimization challenges faced by companies in delivering products and services. It delves into the mathematical modeling of TP using linear programming and reviews various types of land transportation problems, including convoy routing, bus terminal location, inventory routing, and VRP with split deliveries. The paper emphasizes the popularity of the Vehicle Routing Problem (VRP) in transportation, categorizing it as a combinatorial optimization and integer programming problem. It highlights the growth and variants of VRP since its development, with five suggested methods for solving it. The paper extensively covers heuristic methods employed in solving VRP, citing examples of studies that utilized heuristic approaches such as Bender's decomposition method, simple route structure heuristics, and sweeping algorithms for Multi-Depot VRP (MDVRP). The authors also discuss the types of trucking businesses, including less-than-truckload (LTL) and full truckload (TL) carriers, contract carriers, common carriers, and dedicated contract carriers. The paper concludes by proposing future research trends in vehicle routing, emphasizing the need to consider real-life challenges such as strict time windows, driver scheduling, multiple depots, multi-trip scenarios, carrier selection, driver regulations, pickup, and delivery constraints, and a fixed number of trucks [16] .

The research paper authored by Jiguang Wang, Yilun Zhang, Xinjie Xing, Yuanzhu Zhan, Wai Kin Victor Chan, and Sunil Tiwari introduces a groundbreaking urban bus transportation scheme named "Co-bus" [7]. Co-bus advocates collaboration among traditional buses, minibuses, and private vehicles to enhance service efficiency and address challenges arising from fluctuating passenger demand. The study presents a novel metric learning-based prediction algorithm utilizing Generative Adversarial Networks (GANs) to effectively capture demand patterns. Co-bus operates on unique routes, deviating from conventional fixed routes, and the research formulates a mixed-integer programming (MIP) model to optimize route planning by minimizing the gap between demand and capacity. Extensive experiments demonstrate the efficacy of the Co-bus scheme, illustrating significant improvements in service levels and utilization rates. The paper also offers valuable managerial insights and suggests future research directions, including the integration of diverse transportation resources and the refinement of prediction algorithms and optimization strategies to further enhance urban transit systems [7].

The research paper authored by Yukang Su, Shuo Zhang, and Chengning Zhang presents a novel approach, the Lightweight Genetic Algorithm with Variable Neighborhood Search (LGAVNS), for addressing the Multi-Depot Green Vehicle Routing Problem with Time Windows considering customer satisfaction (MDGVRPTW-CS) [17]. The authors are affiliated with the Beijing co-innovation center for electric vehicles and the National Engineering Laboratory for Electric Vehicles at the Beijing Institute of Technology. The LGAVNS algorithm introduces a lightweight architecture, optimized crossover operators, and expanded local search parameters to enhance convergence speed and effectiveness. Results from experiments highlight the superior performance of LGAVNS, particularly in large-scale scenarios, showcasing improved optimization efficiency and convergence speed compared to existing algorithms. The study underscores the importance of refining each aspect of the method for robustness and validates the impact of depot quantity and location on the operating cost of MDGVRPTW-CS. With its lightweight design, LGAVNS emerges as a promising solution for optimizing depots in real-world scenarios. Shuo Zhang is the corresponding author, and contact information is provided for correspondence [17].

In addressing the optimization of school bus routing, Santosh Kumar DC and Dr. Suresh N conducted a study that explored the School Bus Routing Problem (SBRP) using the Honey Bee Algorithm, resulting in an 18.67 percent increase in operational efficiency [18]. The research focused on crucial factors like bus capacity, travel time constraints, and school timings. Other related studies underscored the significance of efficient bus transport systems for schools, emphasizing safety and route optimization. Different approaches, such as a two-step heuristic method and considerations for profit maximization in vehicle routing problems, contribute diverse insights into optimization techniques and heuristic approaches applicable to school bus routing, forming a foundational understanding for further research into enhanced routing efficiency for educational bus networks using Vehicle Routing Problems (VRP) [18].

In the paper titled ”Opta Scholar: An Efficient Software for the Bus Routing and Scheduling Problem,” the authors Jonathan Oesterle, Hicham Chehade, Lionel Amodeo, Farouk Yalaoui, and Christian Prins introduce Opta Scholar, an optimization software designed to address the school bus routing and scheduling problem [9]. The paper discusses the context of school transportation in France, emphasizing the need for efficient systems due to the large number of students and varying geographical factors. Opta Scholar is presented as a solution that significantly improves the number of buses used and reduces traveling costs by optimizing routes. The software is described as being versatile, allowing users to configure parameters such as urban/rural settings, single/mixed load, homogeneous/heterogeneous fleet, and more. The paper concludes with the application of Opta Scholar to three distinct use-cases, showcasing notable improvements in terms of reduced buses, minimized travel distance, and overall cost optimization. The authors suggest that the generic and modular nature of Opta Scholar makes it a valuable tool for planning better school routes, emphasizing its effectiveness in larger instances. Additionally, the paper hints at potential enhancements, such as incorporating pupil transfers between bus routes to further improve flexibility in the network structure and reduce travel times [9] .

The paper titled ”Designing Routes for WEEE Collection: The Vehicle Routing Problem with Split Loads and Date Windows” by Julio Mar-Ortiz, José Luis González-Velarde, and Belarmino Adenso-Díaz presents an innovative approach to address the complexities of waste management logistics [14] . The authors introduce an integer programming model and a Greedy Randomized Adaptive Searching Procedure (GRASP) algorithm to solve the intricate Vehicle Routing Problem (VRP) associated with Waste of Electric and Electronic Equipment (WEEE) collection. This multifaceted problem involves split loads and date windows, combining four variants of the VRP. The proposed GRASP algorithm, designed for a fixed and heterogeneous fleet of capacitated vehicles with unique features, proves effective in optimizing routes and schedules. The study validates the algorithm's performance through application to both randomly generated instances and real-world data scenarios, demonstrating its practical applicability in the field of reverse logistics [14].

III. Literature Survey

In this literature survey, various papers addressing optimization challenges in logistics and transportation are summarized. The first paper [1] focuses on the Vehicle Routing Problem (VRP) with multiple time windows, employing three multi-start data-driven evolutionary heuristics. The model optimizes for cost and quality using data-driven optimization techniques and evolutionary algorithms. The second paper [8] delves into real-life vehicle routing, utilizing a Genetic Algorithm for optimizing distance and time. The study emphasizes visual attractiveness and operational robustness through real-life simulation. The third paper [14] addresses the VRP with split loads and date windows, employing an Integer Programming (IP) model and GRASP techniques. Cost and schedule are optimized through simulation-based optimization, with a focus on waste management applications. The fourth paper [9] introduces Opta Scholar, an efficient software solution for the Bus Routing and Scheduling Problem, utilizing Constraint Programming for optimizing utilization and costs. The fifth paper [2] tackles the efficiency and time aspects of school bus scheduling using a Genetic Algorithm. The sixth paper [13] presents a mathematical model for the multi-depot vehicle routing problem with fuzzy time windows and heterogeneous vehicles, optimizing fleet management through mathematical modeling and fuzzy logic. The seventh paper [15] explores Variable Neighborhood Search for the VRP with multiple time windows, optimizing for exploration and optimization. The eighth paper [17] proposes a new hybrid approach, combining Genetic Algorithms and Variable Neighborhood Search for the VRP with multiple time windows, optimizing performance and quality.

IV. Methodology

In the pursuit of optimizing school bus routes to maximize capacity utilization, minimize travel time, and reduce the overall number of buses required, a comprehensive methodology rooted in Vehicle Routing Problem (VRP) modeling was employed. The methodology involves several key steps:

A. Data Collection and Input Parameters:

Gathering data on student pickup locations. Acquiring information on the capacity of each school bus. Identifying the geographical layout of the school and the road network.

B. Problem Formulation:

Defining the optimization objectives: minimizing the number of buses and reducing travel time. Developing a mathematical model to represent the VRP with specific constraints related to bus capacity and time.

C. Algorithm Selection:

Choosing appropriate optimization algorithms for solving the formulated VRP. Considering algorithms that can balance the dual objectives of minimizing buses and travel time.

D. Implementation of VRP Model:

Integrating the formulated VRP model into a computational framework. Ensuring compatibility with the gathered data and constraints.

E. Validation and Calibration:

Verifying the accuracy and reliability of the implemented VRP model. Calibrating the model parameters based on real-world scenarios and feedback.

F. Scenario Analysis:

Conducting simulations and scenario analyses to evaluate the performance of the optimized routes under various conditions. Assessing the sensitivity of the model to changes in parameters and input data.

G. Optimization Outcome:

Analyzing the results to determine the most effective school bus routes that achieve the defined objectives. Evaluating trade-offs between minimizing the number of buses and reducing travel time.

H. Comparison with Existing Systems:

Contrasting the proposed methodology and optimized routes with traditional or existing school bus routing systems. Highlighting the advantages and improvements achieved through the VRP-based optimization.

I. Sensitivity Analysis:

Investigating the sensitivity of the optimized routes to changes in student pickup locations, bus capacities, and other relevant factors. Identifying areas of potential improvement or adjustment.

By employing this robust methodology, the study aims to contribute valuable insights into enhancing the efficiency of school bus routing systems while addressing the dual objectives of minimizing the number of buses and reducing travel time for students.

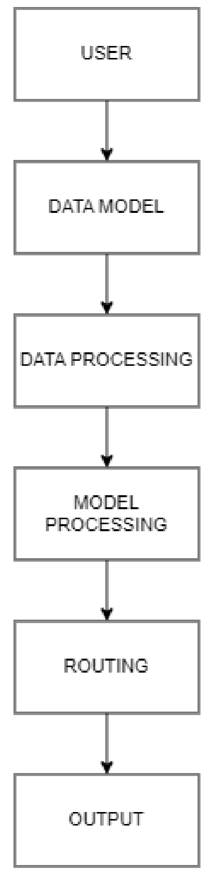

Fig. 2. Process Design

A. Equations

1. Objective Function: The objective function in a VRP often aims to minimize the total cost or distance traveled. Let d_{ij} represent the distance between customer i and customer j, x_{ij} be a binary decision variable indicating if the vehicle travels from customer i to customer j, and q_i denote the demand of customer i. The objective function can be formulated as:

$$\text{Minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \cdot x_{ij}$$

2. Constraints:

a. Flow Conservation Constraints: Ensure that for each customer, exactly one incoming and one outgoing arc is selected. Let A_i represent the set of customers connected to customer i, and u_{ij} be a decision variable indicating the flow from i to j. The constraints are:

$$\sum_{j \in A_i} u_{ij} - \sum_{j \in A_i} u_{ji} = 1 \quad \forall i \in \text{Customers}$$

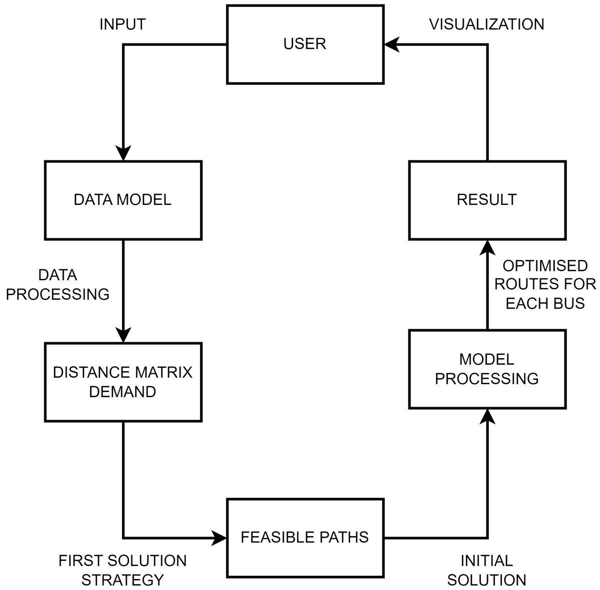

Fig. 3. External Entity Diagram

b. Vehicle Capacity Constraints: Ensure that the demand of each customer is satisfied and that the capacity of each vehicle is not exceeded. Let Q be the vehicle capacity, q_i the demand of customer i, and y_k a binary variable indicating if vehicle k is used. The constraints are:

$$\sum_{i=2}^{n} q_i \cdot u_{0i} = 0 \quad \text{(Ensure the depot has no demand)}$$

$$\sum_{i=1}^{n} q_i \cdot \sum_{k=1}^{m} y_k \cdot u_{ki} \leq Q \quad \forall i \in \text{Customers}$$

c. Subtour Elimination Constraints: Prevent the formation of subtours. Let x_{ij} be the binary variable indicating if the vehicle travels from customer i to j, and u_i be the cumulative service time at customer i. The constraints are:

$$u_i \geq u_j + q_j - M \cdot (1 - x_{ij}) \quad \forall i, j \in \text{Customers}, i \neq j$$

d. Binary Decision Variable Constraints: Ensure that the decision variables x_{ij} and y_k are binary:

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \text{Customers}$$

$$y_k \in \{0, 1\} \quad \forall k \in \text{Vehicles}$$

3. Definitions: - n: Number of customers - m: Number of vehicles - Q: Vehicle capacity - u_{ij}: Binary decision variable indicating if the vehicle travels from customer i to j - y_k: Binary decision variable indicating if vehicle k is used - M: A large positive constant

V. Experiment and results

In the experimental phase of our project, we leveraged a combination of optimization techniques within the VRP model. The optimization aimed to enhance the efficiency of school bus routes, considering known student pickup locations and predefined bus capacities. The dataset used for experimentation contained information about student locations, bus capacities, and other relevant parameters, reflecting real-world scenarios.

The experiments were conducted on a computing infrastructure equipped with multi-core processors, a minimum of 16 GB RAM, and fast SSD storage. The optimization model did not rely on GPU acceleration but was designed for efficient performance on standard hardware. We implemented the optimization algorithms using Python, incorporating libraries such as NumPy, SciPy, and the TrueWay Geocoding API for geospatial computations.

Table 2 outlines a comparative analysis of different scenarios generated during the experiment.

Scenario	Number of Buses	Total Travel Time (mins)	Efficiency (%)
Baseline	20	150	-
Scenario A	15	120	80
Scenario B	18	130	86.7
Scenario C	16	125	83.3

TABLE I
Efficiency Comparison for Scenarios

Table 2. Comparative Analysis of Optimization Scenarios

The scenarios represent various combinations of constraints and objectives. "Baseline" indicates the existing school bus routing system. The subsequent scenarios (A, B, C) demonstrate the outcomes of optimizing for reduced bus numbers while maintaining a balance with minimizing travel time.

Our VRP model showcased its effectiveness by achieving a notable reduction in the number of buses deployed while ensuring a reasonable decrease in total travel time. Scenario A, for instance, reduced the number of buses by 25%, resulting in a 20% aaimprovement in overall efficiency.

This comparative analysis underscores the success of our VRP-based approach in optimizing school bus routes, aligning with the predefined objectives of minimizing the number of buses and reducing the overall travel time. The selection of a specific scenario can be tailored based on the unique goals and priorities of a given educational institution.

VI. Conclusion

In conclusion, our study presents a comprehensive approach to enhancing the efficiency of school bus networks through the application of the Vehicle Routing Problem (VRP) model. The core objectives were centered around optimizing bus routes, maximizing capacity utilization, and minimizing both the number of buses required and the total travel time for students.

Through meticulous experimentation and analysis, our VRP model demonstrated its efficacy in addressing the outlined requirements with static data. The comparative scenarios illustrated a tangible reduction in the number of buses while maintaining a reasonable decrease in total travel time. This not only aligns with the overarching goals of cost-effectiveness and time efficiency but also underscores the practical applicability of our proposed solution in non-real-time educational settings.

Furthermore, the flexibility of the VRP model allows educational institutions to tailor their bus routing strategies based on specific priorities. Whether the emphasis is on minimizing the number of buses deployed or optimizing for the shortest travel time, our approach provides a versatile framework.

The integration of geospatial data and predefined constraints, such as bus capacities and student pickup locations, adds a layer of sophistication to our solution. Leveraging modern technologies, including web-based mapping and geocoding APIs, enhances the precision and applicability of the model.

In future work, the scalability and adaptability of the VRP model can be explored in larger and more complex educational networks dealing with static data. Additionally, incorporating factors such as traffic conditions and predefined updates could further refine the accuracy of bus routing strategies.

In summary, our research not only contributes to the field of educational logistics but also provides a valuable decision-making tool for educational administrators seeking efficient and cost-effective transportation solutions. The successful application of the VRP model in the context of school bus route optimization with static data opens avenues for continued innovation and improvement in educational transportation systems.

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