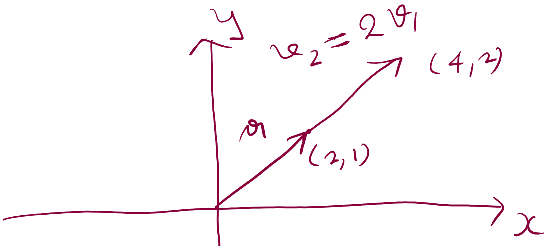
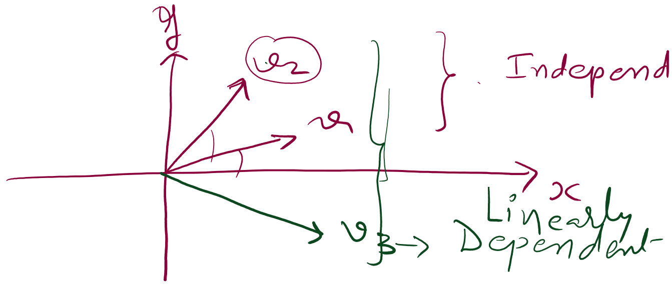
Class A Vectors and Dependency

Monday, May 10, 2021   11:24 AM

* Vectors :-



$v_1 \Rightarrow 2x + y = 0$
$v_2 \Rightarrow 4x + 2y = 0$ } Dependent



$$\underbrace{v_3 = a_1 v_1 + a_2 v_2}_{\text{Linear equation}}$$

$A X = 0$ } Homogeneous system

$n =$ no. of unknowns

$r =$ rank

$[A] \Rightarrow r = n \Rightarrow$ Unique solution

↳ Trivial Solution (0,0,0)

↳ Independent vectors

$[A] \Rightarrow r < n \Rightarrow$ Infinite solutions

↳ Non Trivial Solutions

↳ Dependent vectors

m equations & n unknowns

$\underset{1}{A} \underset{n \times 1}{X} = 0$

$$A_{m\times n}\, x_{n\times 1} = 0$$

→ Dependent vectors

$$a_1x_1 + a_2x_2 + a_3x_3 = 0 \quad \text{--(1)}$$

$$a_4x_1 + a_5x_2 + a_6x_3 = 0 \quad \text{--(2)}$$

$m = 2$; $n = 3$

$$\underbrace{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}}_{[2\times 3]} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{(3\times 1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$[A] \Rightarrow r < n$ = Dependent vectors

↳ relationships

↳ some variables are free & non zero values

Result of midsem 1

	stud 1	stud 2	- - -	- - stud 94
subject 1	$\square$			
subject 2	$\square$			
⋮	$\square$			
Subject 5				
Total [sub 1 + sub 2 + .. sub 5]				

$$\begin{bmatrix} 0 & 0 & 0 & 0 & - & - & 0 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} \; x_{94\times 1}$$

independent

$$\boxed{x_6} = (1)x_1 + (1)x_2 + (1)x_3 \ldots x_4 + x_5$$

$\lambda_1 = 1$; $\lambda_2 = 1$; $\lambda_3 = 1$ . . .

$$\lambda_6 x_6 = [\lambda_1 x_1 + \lambda_2 x_2 + \ldots + \lambda_5 x_5]$$

→ Row 6 → Total →

$A \Rightarrow (6\times 94)$

rank A = 5 < 6 ⇒ Total = sub 1 + sub 2 + sub 3 + sub 4 + sub 5

sub 1 = Total − sub 2 − sub 3 . . . −

$- sub\,4 - sub\,5$

$sub\,5 = \_\_\_\_\_\_$

$$\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \Leftarrow$$

$\hookrightarrow \Delta = 0$

$A\cancel{x} \exists B$

$$x = \frac{B}{A} = BA^{-1}$$

Linear Dependence :- The vectors $x_1, x_2 \ldots x_n$ are said to be <u>linearly dependent</u> if there exists the numbers $\lambda_1, \lambda_2 \ldots \lambda_r$ (<u>not all zero</u>) such that

$$\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_r x_r = 0$$

If $\underline{\lambda_1 \neq 0}$;

$$\lambda_1 x_1 = -\left[\lambda_2 x_2 + \lambda_3 x_3 + \cdots + \lambda_r x_r\right]$$

$$\underline{x_1} = -\frac{1}{\lambda_1}\left[\lambda_2 x_2 + \lambda_3 x_3 + \cdots \lambda_r x_r\right]$$

$$= \mu_2 x_2 + \mu_3 x_3 + \cdots + \mu_r x_r$$

Linearly dependent

↳ Linear combinations of $x_2, x_3 \ldots x_r$.

If $\lambda_1 x_1 + \lambda_2 x_2 \cdots + \lambda_r x_r = 0$ ($\lambda_1 \to k_1$)

only if $\lambda_1 = \lambda_2 = \cdots \lambda_r = 0$, then they are linearly independent.

<u>Ex. 1</u> Given vectors $x_1(1,2,3)$; $x_2(5,0,2)$ &

$x_3(7,4,8)$, check whether the vectors are Linearly Dependent or Independent.

Solution:- Let's assume, that they are Linearly Dependent.

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = [0]$$

$$\lambda_1(1,2,3) + \lambda_2(5,0,2) + \lambda_3(7,4,8) = [0,0,0]$$

$$\lambda_1 + 5\lambda_2 + 7\lambda_3 = 0 \quad \text{--- (1)}$$

$$2\lambda_1 + 0\lambda_2 + 4\lambda_3 = 0 \quad \text{--- (2)}$$

$$3\lambda_1 + 2\lambda_2 + 8\lambda_3 = 0 \quad \text{--- (3)}$$

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & 0 & 4 \\ 3 & 2 & 8 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = 0$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 2 & 0 & 4 & 0 \\ 3 & 2 & 8 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$; $R_3 \leftarrow R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 0 & -10 & -10 & 0 \\ 0 & -13 & -13 & 0 \end{array}\right]$$

$R_3 \leftarrow \frac{1}{-13} R_3$; $R_2 \leftarrow \frac{1}{-10} R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$- \quad \rightarrow \quad 1 \quad 0$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let ; $\lambda_3 = k$; $k \neq 0$

$\lambda_2 + \lambda_3 = 0$

$\therefore \lambda_2 = -\lambda_3 = -k$

$\lambda_1 + 5\lambda_2 + 7\lambda_3 = 0$

$\lambda_1 = -5\lambda_2 - 7\lambda_3$

$= -5(-k) - 7k$

$= 5k - 7k$

$= -2k$

$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$-2k x_1 - k x_2 + k x_3 = 0$

$\boxed{x_3 = 2x_1 + x_2}$ --- (a)

Ex. 2 $x_1 = (1, 2, 3)$; $x_2 = (3, -2, 1)$ & $x_3 = (1, -6, -5)$

Ans. - Linearly Dependent ; $x_2 = 2x_1 + x_3$

$2x_1 = x_2 - x_3$

E. 3 $x_1(2, 1, 1)$; $x_2(2, 0, 1)$; $x_3(4, 2, 1)$

Q.3 $x_1(2,1,1)$; $x_2(2,0,1)$; $x_3(4,2,1)$

$r = 3 \Rightarrow$ L.I. LI

LD

$r \Rightarrow$ rank

n-dimensional space; if you have $n-r$ variables free; then the system is definitely Linearly Dependent.

n-variable $\Rightarrow$ 3-D

$r = 2$; $n = 3 \Rightarrow n - r \Rightarrow 3 - 2 = 1$ Variable is free.

free $\rightarrow \lambda_3 = k \neq 0$

$n = 4$; $r = 2 \Rightarrow n - r = 4 - 2 = 2$ variable are free

---

Q.4 $x_1 = (1, 2, 4)$; $x_2 = (2, -1, 3)$;

$x_3 = (0, 1, 2)$ & $x_4 = (-3, 7, 2)$.

Solution:

$r = 3$; $\lambda_1, \lambda_2, \lambda_3$ & λ_4

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 0 & 12 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 \rightarrow$

$\lambda_3 + \lambda_4 = 0$; $\lambda_4 = k$ — (1)

$\lambda_3 = -k$ — (2)

$-5\lambda_2 + 12\lambda_4 = 0$

$$\therefore \lambda_2 = \frac{12}{5}k \quad --(3)$$

$$\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$$

$$\therefore \lambda_1 = 3\lambda_4 - 2\lambda_2$$

$$= 3k - \frac{2 \times 12}{5}k$$

$$= \frac{15k - 24k}{5} = -\frac{9}{5}k \quad -(4)$$

$k \neq 0$

$$-\frac{9}{5}kx_1 + \frac{12}{5}kx_2 - kx_3 + kx_4 = 0$$

hence,

$$\frac{12}{5}x_2 + x_4 = \frac{9}{5}x_1 + x_3 \quad --$$