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LA - Assignment

Q1 Find out Linear dependence / Independence of vectors for following. If dependent find the relation.

(i) $x_1 = (1,-1,1)$
$x_2 = (2,1,1)$
$x_3 = (3,0,2)$

Sol^n

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_1$; $R_2 \leftarrow R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$R_2 \leftarrow \frac{1}{3} R_2$; $R_3 \leftarrow -1 R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$\therefore x_1 + 2x_2 + 3x_3 = 0$

$x_2 + x_3 = 0$

$x_3 = k$

$x_2 = -k$

$x_1 + 2(-k) + 3(k) = 0$

$x_1 - 2k + 3k = 0$

$x_1 + k = 0$

$x_1 = -k$

$\Rightarrow (x_1, x_2, x_3) = (-k, -k, k)$

$\therefore -x_1 - x_2 + x_3 = 0$

$\therefore \boxed{x_3 = x_1 + x_2}$ Ans

(ii). $x_1 = (3, 2, 7)$

$x_2 = (2, 4, 1)$

$x_3 = (1, -2, 6)$

Sol^n

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 2 & 4 & -2 & 0 \\ 7 & 1 & 6 & 0 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & 4 & -2 & 0 \\ 7 & 1 & 6 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$; $R_3 \leftarrow R_3 - 7R_1$

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$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 8 & -8 & 0 \\ 0 & 15 & -15 & 0 \end{array}\right]$$

$R_2 \leftarrow \frac{1}{8} R_2$; $R_3 \leftarrow \frac{1}{15} R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \quad x_1 - 2x_2 + 3x_3 = 0$

$x_2 - x_3 = 0$

$x_3 = k$

$\therefore \quad x_2 = x_3 = k$

$\therefore \quad x_1 - 2k + 3k = 0$

$x_1 + k = 0$

$x_1 = -k$

$\therefore (x_1, x_2, x_3) = (-k, k, k)$

$-x_1 + x_2 + x_3 = 0$

$\therefore \boxed{x_1 = x_2 + x_3}$ Ans

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(iii) $x_1 = (1,3,4,2)$
$x_2 = (3,-5,2,6)$
$x_3 = (2,-1,3,4)$

Sol^n

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 3 & -5 & -1 & 0 \\ 4 & 2 & 3 & 0 \\ 2 & 6 & 4 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 3R_1$
$R_3 \leftarrow R_3 - 4R_1$
$R_4 \leftarrow R_4 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -14 & -7 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \leftarrow \frac{1}{-7} R_2$; $R_3 \leftarrow \frac{1}{-5} R_3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ $x_1 + 3x_2 + 2x_3 = 0$

$2x_2 + x_3 = 0$

$x_3 = k$

$2x_2 = -k$

$x_2 = -k/2$

$x_1 - \frac{3}{2}k + 2k = 0$

$2x_1 - 3k + 2k = 0$

$2x_1 - k = 0$

$x_1 = k/2$

$\therefore (x_1, x_2, x_3) = \left(\frac{k}{2}, \frac{-k}{2}, k\right)$

$\therefore \frac{k}{2}x_1 - \frac{k}{2}x_2 + kx_3 = 0$

$\boxed{x_1 - x_2 + 2x_3 = 0}$ Ans

Q2 Check whether the sys. of eq^n are consistent. If they are, then find sol^n

(i) $x - 3y - 8z = -10$

$3x + y - 4z = 0$

$2x + 5y + 6z = 13$

Sol^n $AX = B$

$C = [A : B]$

$$\begin{bmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 13 \end{bmatrix}$$

Now, $C = [A \vdots B]$

$$\therefore \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{array}\right]$$

$R_2 \leftarrow R_2 - 3R_1$; $R_3 \leftarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{array}\right]$$

$R_2 \leftarrow \frac{R_2}{10}$; $R_3 \leftarrow \frac{R_3}{11}$

$$\left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$C = \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(C) = 2$

$\rho(A) = 2$ and $n = 3$

$\therefore$ Infinite solⁿˢ

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$\therefore AX = B$

$$\begin{bmatrix} 1 & -3 & -8 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 3 \\ 0 \end{bmatrix}$$

$\therefore \quad x - 3y - 8z = -10 \longleftarrow (1)$

$y + 2z = 3 \longleftarrow (2)$

let $z = k$;

$\therefore$ from eq^n (2),

$y - 2k = 3$

$y = 3 + 2k$

from eq^n (1),

$x - (3+2k) - 8(k) = -10$

$x - 3 - 2k - 8k = -10$

$x - 3 - 10k = -10$

$x = -10 + 3 + 10k$

$\boxed{x = 10k - 7}$

$\therefore (x, y, z) = (10k - 7, 3+2k, k)$

Ans

~~Now,~~

~~as $\rho(A) \neq \rho(A|B)$~~

~~$\therefore$ The system of eq^n is inconsistent.~~

(ii) $4x - 2y + 6z = 8$

$x + y - 3z = -1$

$15x - 3y + 9z = 21$

Sol^n

$$\underset{A}{\begin{bmatrix} 4 & -2 & 6 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 8 \\ -1 \\ 21 \end{bmatrix}}$$

$C = [A : B]$

$$\therefore \quad C = \left[\begin{array}{ccc|c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$R_1 \leftarrow \frac{1}{4} R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & 2 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1 \quad ; \quad R_3 \leftarrow R_3 - 15R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & \frac{3}{2} & -\frac{9}{2} & -3 \\ 0 & -\frac{21}{2} & -\frac{27}{2} & -9 \end{array}\right]$$

$R_3 \leftarrow \frac{1}{-3} R_3$; $R_2 \leftarrow \frac{1}{3} R_2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & \frac{1}{2} & -\frac{3}{2} & -1 \\ 0 & \frac{7}{2} & \frac{9}{2} & 3 \end{array}\right]$$

$R_2 \leftarrow R_2 \times 2$; $R_3 \leftarrow R_3 \times 2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & 1 & -3 & -2 \\ 0 & 7 & 9 & 6 \end{array}\right]$$

$R_3 \leftarrow R_3 - 7R_2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 30 & 20 \end{array}\right]$$

$R_3 \leftarrow \frac{R_3}{10}$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right]$$

$\rho(C) = \rho(A) = 3$ and $n = 3$

$\therefore$ Given system is consistent.

$\therefore$ It has unique sol^n.

$AX = B$

$$\begin{bmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$x - \frac{y}{2} + \frac{3z}{2} = 2$$

or $\quad 2x - y + 3z = 4 \quad \text{--- (1)}$

$\quad y - 3z = -2 \quad \text{--- (2)}$

$\quad 3z = 2 \quad \text{--- (3)}$

$\therefore$ from eq^n (3), $\boxed{z = \frac{2}{3}}$

$$y - \cancel{3}\left(\frac{2}{\cancel{3}}\right) = -2$$

$$y - 2 = -2$$

$$\boxed{y = 0}$$

$$2x - y + 3z = 4$$

$$2x - 0 + \cancel{3}\left(\frac{2}{\cancel{3}}\right) = 4$$

$$2x + 2 = 4$$

$$2x = 4 - 2$$

$$2x = 2$$

$$\boxed{x = 1}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2/3 \end{bmatrix} \text{ Ans}$$

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Q4 Find eigen values and eigen vectors for following matrices:-

(1) $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

Sol^n $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{vmatrix} = 0$$

$$3-\lambda \begin{vmatrix} -3-\lambda & -4 \\ 5 & 7-\lambda \end{vmatrix} \cancel{+ 10 \begin{vmatrix} -2 & (-3-\lambda) \\ 3 & 5 \end{vmatrix}} + 5 \begin{vmatrix} -2 & -3-\lambda \\ 3 & 5 \end{vmatrix}$$

$$+ 10 \begin{vmatrix} -4 & -2 \\ 7-\lambda & 3 \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)\left[(-3-\lambda)(7-\lambda) + 20\right) + 5(-10 - (-9-3\lambda))$

$+ 10((-12) + \cancel{2}(14-2\lambda))$

$\Rightarrow (3-\lambda)(-21 + 3\lambda - 7\lambda + \lambda^2 + 20) + 5(-10+9+3\lambda)$

$+ 10(-12+14-2\lambda)$

$\Rightarrow (3-\lambda)(\lambda^2 - 4\lambda - 1) + 5(3\lambda - 1) + 10(-2\lambda + 2)$

$\Rightarrow 3\lambda^2 - 12\lambda - 3 - \lambda^3 + 4\lambda^2 + \lambda + 15\lambda - 5 - 20\lambda + 20$

$\Rightarrow -\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$

$-\lambda^3 + 2\lambda^2 + 5\lambda^2 - 10\lambda - 6\lambda + 12 = 0$

$-\lambda^2(\lambda-2)+5\lambda(\lambda-2)-6(\lambda-2)=0$

$-(\lambda-2)(\lambda^2-5\lambda+6)=0$

$-(\lambda-2)(\lambda-2)(\lambda-3)=0$

$(\lambda-2)^2(\lambda-3)=0$

$\boxed{\lambda_1=2}$ or $\boxed{\lambda_2=3}$

$\therefore$ when $\lambda=2$,

$[A-\lambda I]X=0$

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$|A|=0$ and $B=0$

$\therefore$ Infinite solns.

Now, when $\lambda=3$,

$[A-\lambda I]X=0$

$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$|A|=0$ and $B=0$

$\therefore$ Infinite solns.

(ii) $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

$R_2 \leftarrow R_1 - R_3$

Sol^n $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 1 & 0 & -1 \end{bmatrix}$

$R_2 \leftarrow R_1 - R_2$

$$\begin{bmatrix} 11 & -4 & -7 \\ 4 & -2 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$R_1 \leftarrow R_1 - 2R_2$

$$\begin{bmatrix} 3 & 0 & -3 \\ 4 & -2 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$R_1 \leftarrow \frac{1}{3} R_1$

$$\begin{bmatrix} 1 & 0 & -1 \\ 4 & -2 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & -1 \\ 4 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} = A$$

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 0 & -1 \\ 4 & -2-\lambda & -2 \\ 0 & 0 & -\lambda \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & -\lambda \\ 4 & -2-\lambda & -2 \\ 1-\lambda & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -\lambda \begin{vmatrix} 4 & -2-\lambda \\ 1-\lambda & 0 \end{vmatrix} = 0$$

$$\Rightarrow (-\lambda)\left(0 - (-2-\lambda)(1-\lambda)\right) = 0$$

$$(-\lambda)(2+\lambda)(1-\lambda) = 0$$

$\therefore \lambda_1 = 0 \quad \lambda_2 = -2 \quad \lambda_3 = 1$

let $\lambda = 0$,

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 4 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - x_3 = 0$

$4x - 2x_2 - 2x_3 = 0$

$\therefore \boxed{x_3 = k}$

$\therefore \boxed{x_1 = k}$

$\therefore 4k - 2x_2 - 2k = 0$

$-2k = 2x_2$

$\therefore \boxed{x_2 = -k}$

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let $\lambda_2 = 1$,

$$\cancel{\begin{bmatrix} 0 & 0 & -1 \\ 4 & -3 & -2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 4 & -3 & -2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_1 = -1$

$x_3 = -1$

$4x_1 - 3x_2 - 2x_3 = 0$

$-4 - 3x_2 - 2 = 0$

$-6 - 3x_2 = 0$

$-3x_2 = 6$

$x_2 = -2$

$\therefore (x_1, x_2, x_3) = (-1, -2, -1)$

Now,

let $\lambda_3 = -2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 - x_3 = 0$ $\qquad x_2 = 0$

$4x_1 - 2x_3 = 0$ $\qquad x_3 = 2$

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$4 - x_1 - 2 = 0$

$x_1 = -2$

$\therefore (x_1, x_2, x_3) = (-2, 0, 2)$ Ans

(iii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Sol^n let, $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & -1 & 3 \\ -2 & 3 & -1 \\ 6 & -2 & 2 \end{bmatrix}$$

$R_2 \leftarrow R_2 + R_1$; $R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 2 \\ 0 & 1 & -7 \end{bmatrix}$$

$R_2 \leftarrow \frac{R_2}{2}$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & -7 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -8 \end{bmatrix}$$

$R_3 \leftarrow \frac{1}{-8} R_3$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A$$

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Now, $|A - \lambda I| = 0$

$$\therefore \begin{bmatrix} 2-\lambda & -1 & 3 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$\therefore 1-\lambda \begin{vmatrix} 2-\lambda & -1 \\ 0 & 1-\lambda \end{vmatrix}$$

$$(1-\lambda)\left[(2-\lambda)(1-\lambda) - (0)(-1)\right]$$

$$(1-\lambda)(2-\lambda)(1-\lambda) = 0$$

$$(1-\lambda)^2 (2-\lambda) = 0$$

$\therefore \underline{\lambda = 1}$ or $\underline{\lambda = 2}$

Now, let $\lambda_1 = 1$

$$[A - \lambda I]X = 0$$

$$\therefore \begin{bmatrix} 1 & -1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 - x_2 + 3x_3 = 0$$

$$x_3 = 0$$

$$\therefore x_1 - x_2 + 0 = 0$$

$$x_1 = x_2 = k$$

$$\therefore (x_1, x_2, x_3) = (k, k, 0)$$

Now, let $\lambda = 2$,

$$\therefore \begin{bmatrix} 0 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore -x_2 + 3x_3 = 0$

$\therefore -x_2 + x_3 = 0$

$\therefore -x_3 = 0$

$\therefore x_2 = x_3 = 0$.

$x_1 = k$

$\therefore (x_1, x_2, x_3) = (k, 0, 0)$

—x———x————x—