* Cauchy's Root Test:-

In a series $\sum u_n$, if $\lim_{n\to\infty} (u_n)^{1/n} = \lambda$; then the series converges for $\lambda < 1$ & series diverges for $\lambda > 1$.

Obs:. For $\lambda = 1$; Root test fails.

Ex. 1 Test the convergence of the series:

$$\sum \frac{n^3}{3^n}$$

Solution:- $u_n = \frac{n^3}{3^n} \Rightarrow (u_n)^{1/n} = \frac{n^{3/n}}{3}$

$$\lim_{n\to\infty} (u_n)^{1/n} = \lim_{n\to\infty} \left(\frac{n^{3/n}}{3}\right)$$

$= \frac{1}{3} < 1$; Hence the series is convergent.

(2) $\sum (\log n)^{-2n}$

↳ Convergent.

(3) $\sum \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

↳ Convergent.

$$u_n = \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

$$= \frac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^{n^{3/2}}}$$

$$(u_n)^{1/n} = \frac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^{n^{(3/2-1)}}}$$

$$= \frac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^{\sqrt{n}}}$$

If $\sqrt{n} = y$, then

$$= \frac{1}{\left(1+\frac{1}{y}\right)^{y}}$$

$$\lim_{y\to\infty} = \frac{1}{(1+1/y)^y} = \frac{1}{e} < 1$$

(4)

$$\lim_{y\to\infty} = \frac{1}{(1+1/y)^y} = \frac{1}{e} < 1$$

Series is convergent.

(4) $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots + \infty \dots$ $(x>0)$

Solution:-

$$u_n = \left(\frac{n}{n+1}\right)^{n-1} \cdot x^{n-1} \qquad u_n = \left(\frac{n+1}{n+2}\right)^n \cdot x^n$$

$x < 1$; Convergent

$x > 1$; Divergent

[Zero Test] $\Rightarrow$ $x=1 \Rightarrow$ Divergent.

if $x=1$; $u_n = \left(\frac{n+1}{n+2}\right)^n$

$$\lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)^n \qquad \lim_{n\to\infty}\left(\frac{1+1/n}{1+2/n}\right)^n$$

$(1+1/n)^n \Rightarrow e$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n+1}\right)^n}$$

$$= \lim_{n\to\infty} \frac{\left(1+\frac{1}{n+1}\right)}{\left(1+\frac{1}{n+1}\right)^{n+1}}$$

$= \frac{1}{e} \neq 0$; The series is divergent.

Ex.3 Test for convergence the series $\sum_{n=1}^{\infty} \left[\frac{(n+1)x}{n^{n+1}}\right]^n$

$\Rightarrow$ Convergent for $x \geq 0$ & $x < 1$ } convergent

$x \geq 1$ divergent.

---

* Alternating series:-

" A series in which the terms are alternatively positive & negative is an alternating series:

For example.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-----$$

$$1-2+3-4+5-6---$$

$|u_n|, |u_{n+1}|$

* <u>Leibnitz</u> <u>Test</u>:-

An alternating series $\sum (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + ----$ converges if following conditions are satisfied.

(1) The u_n's are all positive (as it is multiplied with $\underline{(-1)}^{n+1}$)

(2) The positive u_n's are non increasing ;
i.e., $u_n \geq u_{n+1}$; for all $n > \underline{m}$.

(3) $\lim_{n\to\infty} u_n = 0$

<u>Proof</u>:-

- Consider that the given series is
$u_1 - u_2 + u_3 - u_4 - - -$ & $u_1 > u_2 > u_3 > u_4 - - -$
also $\lim_{n\to\infty} u_n = 0$.

- Now consider the sum of $2n$ terms;

$$S_{2n} = u_1 - u_2 + u_3 - u_4 + u_5 - u_6 + --- + u_{2n-1} - u_{2n}$$

$u_1 > u_2 > u_3 > u_4 ...$

$$= \underbrace{(u_1 - u_2)}_{+ve} + \underbrace{(u_3 - u_4)}_{+ve} + \underbrace{(u_5 - u_6)}_{+ve} + -- + \underbrace{(u_{2n-1} - u_{2n})}_{+ve} \quad -- (1)$$

$$= u_1 - \underbrace{(u_2 - u_3)}_{+ve} - \underbrace{(u_4 - u_5)}_{+ve} - \underbrace{(u_6 - u_7)}_{+ve} --- - \underbrace{(u_{2n-2} - u_{2n-1})}_{+ve} - u_{2n} \quad --- (2)$$

→ Eq.(1) All brackets are positive, so it can be considered that as n increases, S_{2n} also increases.

→ Eq(2) S_{2n} is always less than u_1 as all the brackets (u_2-u_3) -- etc are positive & they are subtracted from u_1.

→ Here the S_{2n} is upper bounded by u_1.

→ Hence S_{2n} must reach to a finite limit. $(<u_1)$

$\lim_{n\to\infty} u_n = 0$

$\lim_{n\to\infty} u_{2n+1} = 0$

Now,

→ $\lim_{n\to\infty} \underbrace{S_{2n+1}} = \underbrace{\lim_{n\to\infty}(S_{2n} + u_{2n+1})}$

$= \underbrace{\lim_{n\to\infty} S_{2n}} + 0$

$=$ same finite limit

$S_{2n} = S_{2n+1}$ ⇒ They are reaching to same finite limit; regardless of n is even or odd.

obs:

$\lim_{n\to\infty} u_n \neq 0$; $\lim_{n\to\infty} S_{2n} \neq \lim_{n\to\infty} S_{2n+1}$; Hence the series becomes oscillatory.

* Understanding the Leibnitz theorem with an example:-

(1) Suppose we have a series $\sum (-1)^{n+1} \frac{1}{n}$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

So, here $u_1 > u_2 > u_3 > u_4 \cdots$ is satisfied.

$\lim_{n\to\infty} u_n = 0$ is also satisfied.

→ If we consider S_{2n} (Let n be 5 for understanding purpose); then

$$S_{2n} = S_{10} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10}$$

$$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{7}-\frac{1}{8}\right)$$

$$+\left(\frac{1}{9}-\frac{1}{10}\right)$$

$0.5 + \longrightarrow$ $$= \frac{1}{2}+\frac{1}{12}+\frac{1}{30}+\frac{1}{56}+\frac{1}{90} \quad --\ (a)$$

In other form :

$$= 1-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{4}-\frac{1}{5}\right)-\left(\frac{1}{6}-\frac{1}{7}\right)$$

$$-\left(\frac{1}{8}-\frac{1}{9}\right)-\frac{1}{10}$$

$$= 1-\frac{1}{6}-\frac{1}{20}-\frac{1}{42}-\frac{1}{72}-\frac{1}{10} \quad --\ (b)$$

$\boxed{< 1}$ so result in $< u_1$

$S_{2n} < u_1$

→ Also, lets see the series progression :

$S_1 = 1$;

$S_2 = 1 - 1/2 = 1/2 = 0.5$

$S_3 = 1 - \frac{1}{2} + \frac{1}{3} = 0.833333--$

$S_4 = \frac{14}{24} = \frac{7}{12} = 0.5833333$

$S_5 = \frac{94}{120} = \frac{47}{60} = 0.7833333$

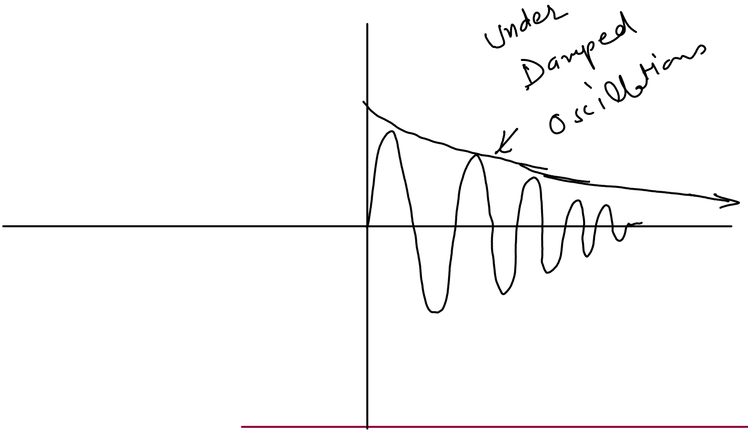
$S_6 = \frac{444}{720} = \quad = 0.6166667$

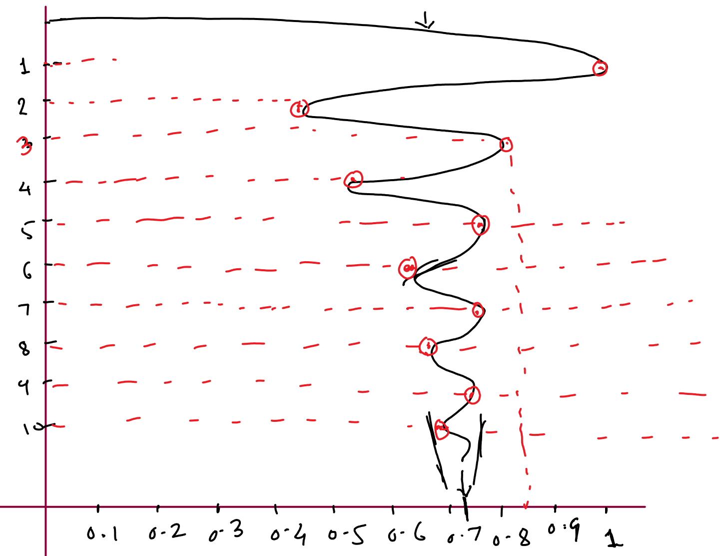
$S_7 = \frac{269}{420} \quad = 0.7595238---$

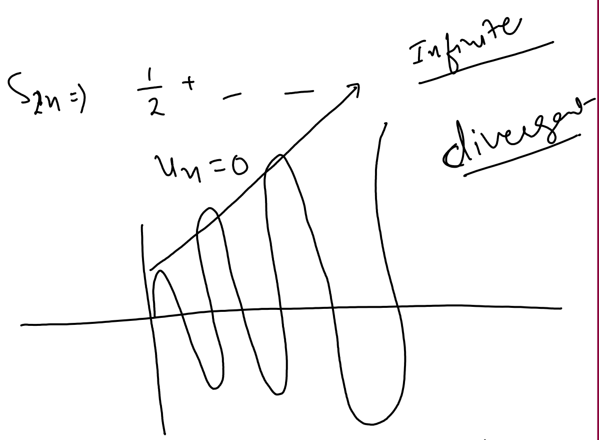
$S_8 = \quad = 0.6345238$

$S_9 = \quad = 0.7456349---$

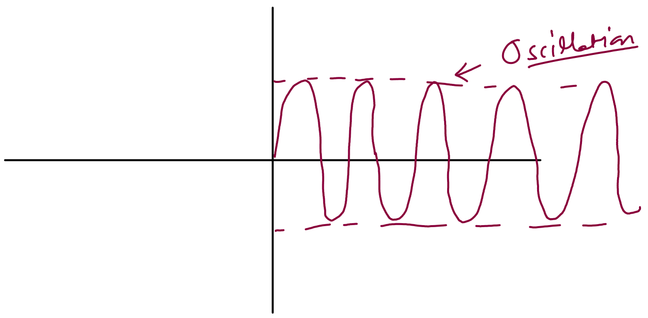
$S_{10} = \quad = 0.6456349--$







$(-1)^n$ / $(-1)^{n+1}$ / $(-1)^{n-1}$



$\Rightarrow \lim_{n\to\infty} u_n \neq 0$

$(-1)^n$

* Test the convergence of the alternating series :-

(1) $\quad 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \cdots$ where : $(0 < x < 1)$

Solution :-

$$|u_n| = n \cdot x^{n-1} \;;\; u_{n+1} = (n+1) \cdot x^n$$

$\therefore$ $(u_n > u_{n+1})$ - (1) ?

$$u_{n+1} - u_n = (n+1)(x^n) - (n \cdot x^{n-1})$$

$$= x^{n-1} \underbrace{(x(n+1) - n)}_{-ve} \;;\; (0 < x < 1)$$

m

$n = 10000$;

$x = 0.8$

10000×0.8

$= 8000$

Satisfying first condition. i.e. $\boxed{u_n > u_{n+1}}$

Now for second condition;

$(0 < x < 1)$

$x = \frac{1}{y} \Rightarrow y > 1$

$\lim_{n\to\infty} \frac{n}{y^{n-1}} = \frac{\infty}{\infty}$

L' Hospi -

$\lim_{n\to\infty} \frac{1}{(n-1)\,y^{n-2}} \Rightarrow 0$

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} x^{n-1} \cdot n$

$= \frac{1}{x} \cdot \lim_{n\to\infty} \boxed{x^n \cdot n} \Rightarrow \boxed{0}$

$\frac{x^n}{1/n}$

Hence satisfying second condition.

So, according Leibnitz Test, the series is convergent.

Ex.2 Examine the convergence of the series:

$$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \frac{1}{9.10} - \cdots$$

Solution:-

↳ Convergent.

Ex.3

$$\frac{6}{3} - \frac{8}{5} + \frac{10}{7} - \frac{12}{9} + \frac{14}{11} - \cdots$$

$\boxed{1 - 1 + 1 - 1}$

↳ $u_n = \frac{2n+4}{2n+1}$; $u_{n+1} = \frac{2(n+1)+4}{2(n+1)+1}$

(1) $u_n > u_{n+1}$ → Satisfied.

(2) $\lim_{n\to\infty} u_n = 1 \neq 0$; Hence it is oscillatory.

Ex.4

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \cdots$$

$(0 < x < 1)$

↳ Convergent.

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{x^n}{1+x^n}$$

$$= \lim_{n\to\infty} \frac{1}{1 + 1/x^n}$$

$(0<x<1)$

$n \to \infty \Rightarrow x^n \to 0$

$\frac{1}{x^n} \to \infty$

$= 0$

Ex.5

$$\sum^{\infty} (-1)^{n-1} x^n$$

Ex. 5

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n(n-1)} \; ; \; (0<x<1)$$

$$\Downarrow$$

$$\frac{u_n}{u_{n+1}} = \frac{n+1}{(n-1).x} > 1$$

$\boxed{n>1}$

↳ Convergent ; $n >$ ___

Ex. 6

$$\sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{5n+1} \Rightarrow \text{oscillatory.}$$

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* Series of Positive & Negative Terms :-

↳ (1) Absolutely convergent series

(2) Conditionally convergent series.

(1) Absolute convergent series :-

$|u_1| + |u_2| + |u_3| + \cdots + \infty$ is convergent

then $u_1 + u_2 + u_3 + \cdots \infty$ is also convergent.

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} \Rightarrow$$ as per p series it is convergent.

$|u_1| + |u_2| + |u_3| + \cdots$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \Rightarrow \underbrace{1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots}$$

↳ Convergent.

(2) Conditionally convergent :-

$u_1 + u_2 + u_3$ is convergent but $|u_1| + |u_2| + |u_3| + \cdots \infty$ is not convergent then that series $\sum u_n$ can be said as conditionally convergent.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n} \Rightarrow u_n \Rightarrow 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \rightarrow \text{convergent}$$

$|u_n| \Rightarrow \quad 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ divergent

* Examine the following series for absolute or conditional convergence

(1) $$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots$$

↳ Conditionally convergent.

Solution:

$$u_n = \frac{(1+2+3\cdots+n)}{(n+1)^3}$$

$$= \frac{n(n+1)}{2(n+1)^3}$$

$$u_n = \frac{n}{2(n+1)^2}$$

Cond.2 $$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{2n(1+1/n)^2} = 0$$

Con.1 $$\frac{u_n}{u_{n+1}} = \frac{n}{\cancel{2}(n+1)^2} \cdot \frac{\cancel{2}(n+2)^2}{(n+1)}$$

$$= \frac{n(n+2)^2}{(n+1)^3}$$

$$= \frac{n^3+4n^2+4n}{n^3+3n^2+3n+1} > 1 \Rightarrow u_n > u_{n+1}$$

$$u_{n+1} - u_n = \frac{n+1}{2(n+2)^2} - \frac{n}{2(n+1)^2}$$

$$= \frac{(n+1)^3 - n(n+2)^2}{2(n+1)^2(n+2)^2}$$

$$= \frac{\cancel{n^3}+3n^2+3n+1 - \cancel{n^3}-4n^2-4n}{2(n+1)^2(n+2)^2}$$

$$= \frac{\cdot}{2(n+1)^2(n+2)^2}$$

$$= \frac{-n^2-n+1}{2(n+1)^2(n+2)^2} <0 \quad n=1$$

hence $u_n > u_{n+1}$

As per the Leibnitz Test, the series is convergent.

If all terms are taken $|u_n|$ => (positive)

$$u_n = \frac{n}{2(n+1)^2}$$

Zero
Con

Comparision Test :- $\Sigma v_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{2\cancel{n}(1+1/n)^2} \cdot \frac{\cancel{n}}{1}$$

$= \frac{1}{2} \neq 0$ Hence,

Both will converge or diverge.

Now $\Sigma v_n = \Sigma \frac{1}{n}$ is divergent as per p-series, hence $\Sigma|u_n|$ is divergent.

Hence here Σu_n is convergent & $\Sigma|u_n|$ is divergent, hence the series is conditionally convergent.

Ex. 2

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)}$$

↳ Conditionally convergent.

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* Power Series :-

↳ Mclaurian Series }
Taylor Series

→ Mclaurian Series
→ Taylor Series

$a_0 + a_1x^1 + a_2x^2 + a_3x^3 - \cdots -$

⇒ GP series :

$a + ax + ax^2 + ax^3 + \cdots$

$a(1 + x + x^2 + x^3 + \cdots)$

→ $|x| < 1$ convergent

Region of Convergence

$-1 < x < 1$

$|x| < 1$

$1 + x + x^2 + x^3 - \cdots = \dfrac{1}{1 - x}$

→ Common Ratio

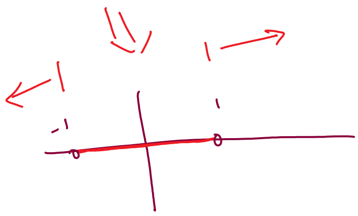
$CR = \dfrac{\text{term}_{n+1}}{\text{term}_n} \Rightarrow$

$\Rightarrow \dfrac{1}{1-x} \overset{|x|<1}{\Rightarrow} 1 + x + x^2 + x^3 \cdots - \Rightarrow \sum_{n=1}^{\infty} x^{n-1}$

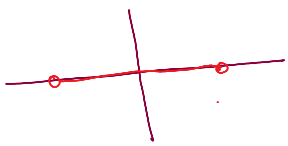
$\dfrac{1}{1+x} \overset{|x|<1}{\Rightarrow} \dfrac{1}{1-(-x)} \Rightarrow 1 + (-x) + (-x)^2 + (-x)^3 + \cdots$

$= 1 + (-1)\cdot x + (-1)^2 \cdot x^2 + (-1)^3 \cdot x^3 - \cdots$

$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot x^{n-1}$



$-1 < x \leq 1$



Ex. 1) $x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$

Leibnitz

→ Ratio Test / Root Test ⇒ $|x| < a$

$-a < x < a$

$\left|\dfrac{u_{n+1}}{u_n}\right| = |x|$

Applying Ratio test $\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \to \infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right|$

$= \lim_{n \to \infty} \left| \frac{nx}{n+1} \right|$

$= |x| < 1$

$\Rightarrow -1 < x < 1$

$\Rightarrow$ Let $x = 1$; Then the series will be

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

As per the Leibnitz test; it is convergent

$\Rightarrow$ Let $\underline{x = -1}$; then

$- \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \right]$

$- \left(\frac{1}{n} \right) \Rightarrow$ divergent as per the p-series.

$\underline{-1 < x \leq 1}$

Eg. 2

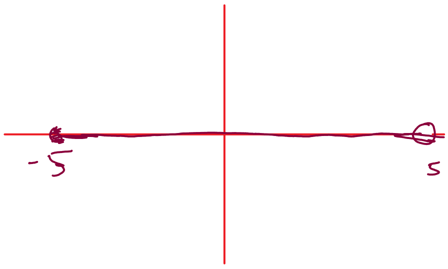
$\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 5^n}$; $\Rightarrow$ $|x| < 5 \Rightarrow$

$\underline{-5 < x < 5}$

Final interval of convergence is $-7 \leq x < 7$
$-5 \leq x < 5$
$\underline{[-5, 5)}$

$\rightarrow$ Let $x = 5 \Rightarrow \sum \frac{\cancel{(5)^n}}{n \cdot \cancel{5^n}}$ $\sum \frac{1}{n} \Rightarrow$ as per p series it is divergent.

$x = -5 \Rightarrow \sum \frac{(-5)^n}{n \cdot 5^n} \Rightarrow \sum \frac{(-1)^n \cancel{5^n}}{n \cancel{5^n}}$



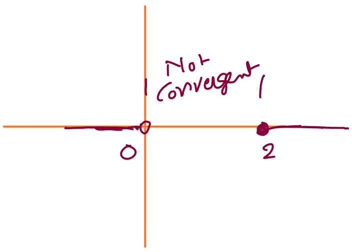
as per Leibnitz test, the series convergent.

$[-5, 5)$

Ex.2 $\dfrac{1}{1-x} + \dfrac{1}{2(1-x)^2} + \dfrac{1}{3(1-x)^3} + \cdots + \infty$

$\underline{u_{n+1}} < \underline{u_n}$

$\left|\dfrac{1}{1-x}\right| < 1$



$|1-x| < 1$

$\therefore -1 < (1-x) < 1$

$\therefore -1 > x-1 > 1$

$\therefore 0 > x > 2 \Rightarrow \Rightarrow x < 0$

$\cancel{x} > 2$

$x < 0 \;\&\; x \geq 2$ $\qquad (-1)^n \Rightarrow$

Ex. 3 Find the interval of convergence for the series:

$\dfrac{x}{\sqrt{3}} - \dfrac{x^2}{\sqrt{5}} + \dfrac{x^3}{\sqrt{7}} - \dfrac{x^4}{\sqrt{9}} + \cdots$

$\Rightarrow \underline{|x| < 1} \Rightarrow \underline{-1 < x < 1}$

Solution:-

$|u_n| = \left|\dfrac{x^n}{\sqrt{2n+1}}\right|$; $|u_{n+1}| = \left|\dfrac{x^{n+1}}{\sqrt{2(n+1)+1}}\right|$

$= \left|\dfrac{x^{n+1}}{\sqrt{2n+3}}\right|$

Now applying ratio test:

$\underbrace{\lim_{n\to\infty}\left|\dfrac{u_{n+1}}{u_n}\right|}_{<1} = \lim_{n\to\infty}\left|\dfrac{x^{n+1}}{\sqrt{2n+3}} \cdot \dfrac{\sqrt{2n+1}}{x^n}\right|$

$\therefore \;\; \left| x \;\; \sqrt{n} \;\; \sqrt{2+1/n} \right|$

< 1

$$= \lim_{n \to \infty} \left| \frac{x \cdot \cancel{\sqrt{n}} \sqrt{2 + 1/n}}{\cancel{\sqrt{n}} \sqrt{2 + 3/n}} \right|$$

$$= |x| < 1 \Rightarrow \underbrace{-1 < x < 1}_{\text{Interval of convergen.}}$$

$\Rightarrow$ Now checking end points :-

Let $x = 1$;

$$\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{9}} + \cdots$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{2n+1}}$$

Applying Leibnitz test:

Cond. 1 $\quad u_{n+1} - u_n < 0$

$$u_n = \frac{1}{\sqrt{2n+1}} \quad \& \quad u_{n+1} = \frac{1}{\sqrt{2n+3}}$$

$$u_{n+1} - u_n = \frac{1}{\sqrt{2n+3}} - \frac{1}{\sqrt{2n+1}} < 0$$

Cond 1 is satisfied.

Cond. 2 $\quad \lim_{n \to \infty} u_n = \frac{1}{\sqrt{2n+1}} = 0$

Hence at $x = 1$; the series is convergent.

Now at $x = -1$;

$$\frac{-1}{\sqrt{3}} - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{9}} \cdots$$

$$= -\left[\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} + \cdots \right]$$

1 $\quad$ -1 $\quad$ using

$= \quad u_n \Rightarrow \dfrac{1}{\sqrt{2n+1}} \Rightarrow$ using Comparison test,

let $\sum v_n = \sum \dfrac{1}{\sqrt{n}}$

$v_n = \dfrac{1}{\sqrt{n}}$

Applying comparison test:

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\sqrt{n}\left(\sqrt{2+1/n}\right)} \cdot \frac{\sqrt{n}}{1}$$

$$= \frac{1}{\sqrt{2}} \neq 0$$

Hence either both converge or both diverge.

But as $\sum v_n = \sum \dfrac{1}{\sqrt{n}}$ is divergent as per p-series,

the series is divergent at $x = -1$.

Hence the find interval of convergence is

$$\underline{-1 < x \leq 1}$$

Ex. 4 Find the interval of convergence for the series:

$$\sum_{n=1}^{\infty} (x+5)^n$$

$$\underline{-6 < x < -4}$$

Ex. 5 Find out the interval of convergence for following series:

$$\sum_{n=1}^{\infty} \frac{n \cdot x^n}{n+2}$$

$\underline{x < 1} \Rightarrow 0 < x < 1$

; $\underline{|x| < 1}$

$\Downarrow$

$\underline{-1 < x < 1}$

$\rightarrow$

Ex. 6 $\quad \sum^{\infty} \quad x^n$

~~Ex. 6~~

$\rightarrow$

$$\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\cdot 3^n} \quad ; -3 \leq x \leq 3$$