

Proportion

Tuesday, April 6, 2021 9:27 AM

→ two ratios are equal.

$$\text{i.e. } \frac{3}{12} = \frac{1}{4} \text{ is a proportion}$$

How to express?

$$3:12 = 1:4 \text{ or } 3:12::1:4 //$$

⇒ Types of proportion.

i) Third proportion:

- if $a:b::b:c$, then c is called 3rd proportional to a and b .

$$a:b::b:c \text{ or } a:b = b:c \text{ or } \frac{a}{b} = \frac{b}{c}$$

$$\therefore c = \frac{b^2}{a} //$$

Ex find the third proportion of 0.36 & 0.48.

$$c = \frac{b^2}{a} \text{ where } a = 0.36 \text{ \& } b = 0.48$$

$$= \frac{0.48 \times 0.48}{0.36} = 0.64 //$$

ii) Fourth proportion:

if $a:b::c:d$ then d is called 4th proportional
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $a \quad b \quad c \quad d$ to a, b & c .

$$a:b::c:d \text{ or } a:b=c:d \text{ or } \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow a \times d = b \times c \Rightarrow d = \frac{bc}{a} //$$

ex find the fourth proportion of 3, 6 and 12.

$$d = \frac{bc}{a} \text{ where } a=3, b=6 \text{ \& } c=12$$

$$= \frac{6 \times 12}{3} = 24 //$$

iii) Mean proportion:

if a and b are given then \sqrt{ab} is mean proportion.

if mean proportion betⁿ a and b is A

$$a:A::A:b \Rightarrow A^2 = ab \Rightarrow A = \sqrt{ab} //$$

ex find the mean proportion betⁿ 9 and 16.

$$\Rightarrow \sqrt{ab}$$

$$\Rightarrow \sqrt{9 \times 16} = \sqrt{144} = 12 //$$

ix) Continued Proportion:-

Three quantities a, b, c of some kind and if said to be continued it

$$a:b = b:c$$

$$b^2 = ac$$

(middle no.)² = first no. + last number.

Ex If 3, x , 27 are in continued proportion,
 $\begin{matrix} a & b & c \end{matrix}$
 then find x .

$$b^2 = ac$$

$$= 3 \times 27$$

$$b^2 = 81$$

$$b = \sqrt{81} = 9 //$$

\Rightarrow Relation among the quantities more than four

$$\text{Given } a:b = x:y$$

$$b:c = m:n$$

then these three quantities are related as.

$$\begin{array}{ccc} a:b & = & x:y \\ \downarrow & \nearrow & \downarrow \\ b:c & = & m:n \end{array}$$

$$a:b:c = xm:my:yn$$

$$a:c = xm:yn$$

=> some useful results on proportion:

if four quantities a, b, c, d are said to be proportion it and only it.

$$\overbrace{a:b}^{\downarrow \quad \downarrow} = \overbrace{c:d}^{\downarrow \quad \downarrow}$$

=> extremes

\downarrow => middle or mean.

$$ad = bc \quad (\because \text{product of extremes} = \text{product of means})$$

=> Componendo and Dividendo:

if $a:b$ is equal to $c:d$

$$\frac{a}{b} = \frac{c}{d}$$

$$a+b$$

$$c+d$$

b d

i) Componendo Rule - $\frac{a+b}{b} = \frac{c+d}{d}$

ii) Dividendo Rule - $\frac{a-b}{b} = \frac{c-d}{d}$

iii) Componendo & Dividendo - $\frac{a+b}{a-b} = \frac{c+d}{c-d} //$