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5/7/21

LA (2HS102)

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Q3 Find the eigen values and eigen vectors for following matrix :-

$$\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$

Sol^n given matrix, $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

The characteristic matrix $\Rightarrow |A - \lambda I| = 0$

$$\begin{vmatrix} -3-\lambda & -7 & -5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$-3-\lambda \begin{vmatrix} 4-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} + (-7) \begin{vmatrix} 3 & 2 \\ 2-\lambda & 1 \end{vmatrix} + (-5) \begin{vmatrix} 2 & 4-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$(-3-\lambda)\left[(4-\lambda)(2-\lambda)-(6)\right] + (-7)\left[3-(4-2\lambda)\right] + (-5)\left[4-(4-\lambda)\right] = 0$$

$$(-3-\lambda)\left[8-4\lambda-2\lambda+\lambda^2-6\right] + (-7)\left[3-4+2\lambda\right] + (-5)\left[4-4+\lambda\right] = 0$$

$$(3-\lambda)\left[2-8\lambda+\lambda^2\right] + (-7)\left[-1+2\lambda\right] + (-5)\left[\lambda\right] = 0$$

$$6-24\lambda+3\lambda^2-2\lambda+8\lambda^2-\lambda^3+(7-14\lambda)+(-5\lambda) = 0$$

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$\cancel{6 - 24\lambda + 3\lambda^2 - 2\lambda + 8\lambda^2 - \lambda^3 + 7 - 14\lambda - 5\lambda = 0}$

$\cancel{53 - 45\lambda + 11\lambda^2 - \lambda^3 = 0}$

$(-3-\lambda)(\lambda^2 - 6\lambda + 2) + (-7)(-1+2\lambda) + (-5)(\lambda) = 0$

$-3\lambda^2 + 18\lambda - 6 - \lambda^3 + 6\lambda^2 - 2\lambda + 7 - 14\lambda - 5\lambda = 0$

$3\lambda^2 - 3\lambda - 6 - \lambda^3 + 7 = 0$

$3\lambda^2 - 3\lambda + 1 - \lambda^3 = 0$

$-(\lambda^3 - 3\lambda^2 + 3\lambda - 1) = 0$

using $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$,

we get,

$-(\lambda-1)^3 = 0$

$(\lambda-1)^3 = 0$

$\therefore \lambda_1 = 1 \quad \lambda_2 = 1 \quad \lambda_3 = 1$

$\therefore$ when $\lambda = 1$,

$|A - \lambda I|$

$$= \begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

Now, let $[A - \lambda I]X = 0$

$$\therefore \begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

let $C = [A : B]$

$$\left[\begin{array}{ccc|c} -4 & -7 & -5 & 0 \\ 2 & 3 & 3 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 3 & 3 & 0 \\ -4 & -7 & -5 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$; $R_3 \leftarrow R_3 + 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Now, $AX = B$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$x_1 + 2x_2 + x_3 = 0$$
$$-x_2 + x_3 = 0$$

let $x_3 = k$

$$x_2 = x_3$$

$\therefore \quad x_2 = x_3 = k$

$$x_1 + 2k + k = 0$$
$$x_1 = -3k$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3k \\ k \\ k \end{bmatrix}$$ Ans

Q2 find the interval of convergence for the series :-

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 . 3^n}$$

Sol^n Here, applying D'Alembert's ratio test,

$$u_n = \frac{(x-1)^n}{n^3 . 3^n} \qquad u_{n+1} = \frac{(x-1)^{n+1}}{(n+1)^3 \, 3^{n+1}}$$

$$\lim_{n \to \infty} \frac{u_{n+1}}{u_n}$$

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$$\lim_{x\to\infty} \frac{\frac{(x-1)^{n+1}}{(n+1)^3 \times 3^{n+1}}}{\frac{(x-1)^n}{n^3 \times 3^n}}$$

$$\lim_{n\to\infty} \frac{(x-1)^{n+1} \times n^3 \times \cancel{3^n}}{(n+1)^3 \times 3^{n+1} \times \cancel{(x-1)^n}}$$

$$\lim_{n\to\infty} \frac{(x-1) n^3}{3 (n+1)^3}$$

$$\lim_{n\to\infty} \frac{\cancel{n^3} (x-1)}{3\cancel{n^3} (1+1/n)^3}$$

$$\lim_{n\to\infty} \frac{(x-1)}{3 (1+1/n)^3}$$

on applying $\lim_{n\to\infty}$, we got

$$\lim_{n\to\infty} \frac{(x-1)}{3 (1+1/n)^3} = \left|\frac{x-1}{3}\right|$$

Now, for series to be convergence

$$\therefore \left|\frac{x-1}{3}\right| < 1$$

$$\therefore \frac{|x-1|}{3} < 1$$

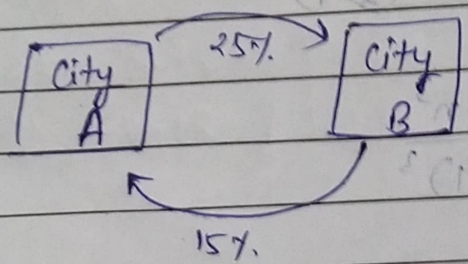
$\therefore$ $|x-1| < 3$

$\therefore$ $-2 < x < 4$

$\Rightarrow$ Hence, the interval of convergence of series is when $-2 < x < 4$.

Q1

Sol^n



initial population of city A = 1000 $= A_0$
Initial population of city B = 1000 $= B_0$

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

Now, 25% population leaves city A
$\therefore$ 100% − 25% = 75% remaining in city A.

and 15% migrates from city B :

15% population leaves city B
$\therefore$ (100 − 15)% = 85% remaining in city B.

$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$ $A_1 = 0.75A_0 + 0.15B_0$

$B_1 = 0.25A_0 + 0.85B_0$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0.75 & 0.15 \\ 0.25 & 0.85 \end{bmatrix} \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 & 0.15 \\ 0.25 & 0.85 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 \times 1000 + 0.15 \times 1000 \\ 0.25 \times 1000 + 0.85 \times 1000 \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 900 \\ 1100 \end{bmatrix}$$

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0.75 & 0.15 \\ 0.25 & 0.85 \end{bmatrix}}_{C} \begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

$$|C - \lambda I| = \begin{vmatrix} 0.75 - \lambda & 0.15 \\ 0.25 & 0.85 - \lambda \end{vmatrix} = 0$$

$$(0.75 - \lambda)(0.85 - \lambda) - (0.15)(0.25) = 0$$

$$0.6375 - 1.6\lambda + \lambda^2 - 0.0375 = 0$$

$$\lambda^2 - 1.6\lambda + 0.6 = 0$$

$$\lambda^2 - \frac{8}{5}\lambda + \frac{3}{5} = 0$$

$$5\lambda^2 - 8\lambda + 3 = 0$$

$$5\lambda^2 - 3\lambda - 5\lambda + 3 = 0$$

$$(5\lambda - 3)(\lambda - 1) = 0$$

$$\lambda = 3/5 \quad \text{or} \quad \lambda = 1$$

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~~at λ=1,~~

at $\lambda = 1$,

$[C - \lambda I]\,x = 0$

$$\begin{bmatrix} -0.25 & 0.15 \\ 0.25 & -0.15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$