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LA (2HS102)

Q1 Find the inverse of following matrix using Gauss-Jordan Method :-

$$\begin{bmatrix} 3 & 4 & 6 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix}$$

Sol^n Now for finding inverse of foll. matrix,

[A : I]

$$\left[\begin{array}{ccc|ccc} 3 & 4 & 6 & 1 & 0 & 0 \\ 6 & 7 & 8 & 0 & 1 & 0 \\ 9 & 10 & 11 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow \frac{R_1}{3}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 4/3 & 2 & 1/3 & 0 & 0 \\ 6 & 7 & 8 & 0 & 1 & 0 \\ 9 & 10 & 11 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - 6R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 4/3 & 2 & 1/3 & 0 & 0 \\ 0 & -1 & -4 & -2 & 1 & 0 \\ 9 & 10 & 11 & 0 & 0 & 1 \end{array}\right]$$

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$R_3 \leftarrow R_3 - 9R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 4/3 & 2 & 1/3 & 0 & 0 \\ 0 & -1 & -4 & -2 & 1 & 0 \\ 0 & -2 & -7 & -3 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow -R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 4/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & 4 & 2 & -1 & 0 \\ 0 & -2 & -7 & -3 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 - \frac{4}{3} R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -10/3 & -7/3 & 4/3 & 0 \\ 0 & 1 & 4 & 2 & -1 & 0 \\ 0 & -2 & -7 & -3 & 0 & 1 \end{array}\right]$$

$R_3 \leftarrow R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -10/3 & -7/3 & 4/3 & 0 \\ 0 & 1 & 4 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 + \frac{10}{3} \times R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -16/3 & 10/3 \\ 0 & 1 & 4 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

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$R_2 \leftarrow R_2 - 4R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -16/3 & 10/3 \\ 0 & 1 & 0 & -2 & 7 & -4 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -16/3 & 10/3 \\ -2 & 7 & -4 \\ 1 & -2 & 1 \end{bmatrix}$$ Soln

$\Rightarrow$ Hence, A^{-1} above is solution by Gauss Jordan method.

Q2 Verify Cayley-Hamilton Theorem for Given matrix :-

$$\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

Soln $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 3 & 7 \\ 4 & 2-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

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$$\Rightarrow (1-\lambda)\begin{vmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} + 3\begin{vmatrix} 3 & 4 \\ 1-\lambda & 1 \end{vmatrix} + 7\begin{vmatrix} 4 & 2-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[(2-\lambda)(1-\lambda) - 6\right] + 3\left[3 - (4-4\lambda)\right] + 7\left[8 - (2-\lambda)\right] = 0$$

$$\Rightarrow (1-\lambda)\left[(2-2\lambda-\lambda+\lambda^2)-6\right] + 3\left[3-4+4\lambda\right] + 7\left[8-2+\lambda\right] = 0$$

$$\Rightarrow (1-\lambda)\left[-3\lambda+\lambda^2-4\right] + \left[-3+12\lambda\right] + \left[42+7\lambda\right] = 0$$

$$\Rightarrow -3\lambda+\lambda^2-4+3\lambda^2-\lambda^3+4\lambda-3+12\lambda+42+7\lambda = 0$$

$$\Rightarrow -\lambda^3+4\lambda^2+20\lambda+35 = 0$$

$\therefore \lambda^3 = 4\lambda^2 + 20\lambda + 35$ ~~characteristic eq of A~~

→ Acc. to Cayley-Hamilton theorem, every matrix should satisfy its own characteristic equation.

$\therefore$ Characteristic equation of matrix A is,

$$-A^3 + 4A^2 + 20A + 35I = 0$$

Now,

$A^2 = A \cdot A$

$$= \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix}$$

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and, $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix} \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 135 & 152 & 232 \\ 140 & 163 & 208 \\ 60 & 76 & 111 \end{bmatrix}$$

Now, RHS = 0

LHS $= -A^3 + 4A^2 + 20A + 35I = 0$

$$- \begin{bmatrix} 135 & 152 & 232 \\ 140 & 163 & 208 \\ 60 & 76 & 111 \end{bmatrix} + 4 \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix} + 20 \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 35 & 0 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} -135+4(20)+20(1)+35 & -152+4(23)+20(3)+0 & -232+4(23)+20(7 \\ -140+4(15)+20(4)+0 & -163+4(22)+20(2)+35 & -208+4(37)+20(3) \\ -60+4(10)+20+0 & -76+36+40+0 & -111+4(14)+20+35 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{RHS}.$$

$\therefore$ LHS = RHS, Hence Proved.

Q3 $x_1 = (1, 2, 4)$
$x_2 = (2, -1, 3)$
$x_3 = (0, 1, 2)$
$x_4 = (-3, 7, 2)$

Sol^n Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ be ~~vectors~~ four scalars.
consider,

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0$$

$$\lambda_1(1,2,4) + \lambda_2(2,-1,3) + \lambda_3(0,1,2) + \lambda_4(-3,7,2) = 0$$

$$\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$$
$$2\lambda_1 - \lambda_2 + \lambda_3 + 7\lambda_4 = 0$$
$$4\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 = 0$$

$$\underset{A}{\begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & -1 & 1 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

$C = [A : B]$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 2 & -1 & 1 & 7 & 0 \\ 4 & 3 & 2 & 2 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$; $R_3 \leftarrow R_3 - 4R_1$

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$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 1 & 13 & 0 \\ 0 & -5 & 2 & 14 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 1 & 13 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

Here, $\rho = 3$

$\therefore$ It is linearly dependent

$\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$ —— (1)

$-5\lambda_2 + \lambda_3 + 13\lambda_4 = 0$ —— (2)

$\lambda_3 + \lambda_4 = 0$ —— (3)

Acc. to (3), $\lambda_3 + \lambda_4 = 0$

$\lambda_3 = -\lambda_4$

let, $\boxed{\lambda_4 = k}$

$\therefore \boxed{\lambda_3 = -k}$

acc. to (2), $-5\lambda_2 + \lambda_3 + 13\lambda_4 = 0$

$-5\lambda_2 - k + 13k = 0$

$-5\lambda_2 + 12k = 0$

$$\boxed{\lambda_2 = \frac{12k}{5}}$$

Acc. to (1), $\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$

$\lambda_1 + \frac{24}{5}k - 3k = 0$

$5\lambda_1 + 24k - 15k = 0$

$5\lambda_1 + 9k = 0$

$$\boxed{\lambda_1 = -\frac{9k}{5}}$$

Now, $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0$

$-\frac{9k}{5} x_1 + \frac{12}{5} k x_2 - k x_3 + k x_4 = 0$

$k\left[-\frac{9}{5} x_1 + \frac{12}{5} x_2 - x_3 + x_4\right] = 0$

$-9x_1 + 12x_2 - 5x_3 + 5x_4 = 0$

$12x_2 + 5x_4 = 9x_1 + 5x_3$ Ans

Q4 Find Interval of convergence for series :-

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^3 \cdot 7^n}$$

Sol^n Applying D'Alemberts ratio test,

$$|u_n| = \frac{(x-3)^n}{n^3 \cdot 7^n}$$

$$|u_{n+1}) = \frac{(x-1)^{n+1}}{(n+1)^3 \cdot 7^{n+1}}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left|\frac{\overset{(x-3)}{\cancel{(x-3)^{n+1}}} \times n^3 \cancel{7^n}}{(n+1)^3 \cdot \underset{7}{\cancel{7^{n+1}}} \times \cancel{(x-3)^n}}\right|$$

$$= \left|\frac{x-3}{7}\right| \lim_{n\to\infty} \left|\frac{\cancel{n^3}}{\cancel{n^3}\left(1+\frac{1}{n^3}\right)}\right|$$

$$= \left|\frac{x-3}{7}\right|$$

Now, $\frac{|x-3|}{7} < 1$

$|x-3| < 7$

$-7 < x-3 < 7$

$-4 < x < 10$

let $x = -4$

$$u_n = \frac{(-4-3)^n}{n^3 \cdot 7^n} = \frac{(-7)^n}{n^3 \cdot 7^n} = \frac{(-1)^n}{n^3}$$

Applying Leibnitz Test,

$$u_n = \frac{1}{n^3} \; ; \; u_{n+1} = \frac{1}{(n+1)^3}$$

Condition 1 :-

$$u_{n+1} - u_n = \frac{1}{(n+1)^3} - \frac{1}{n^3}$$

$$= \frac{n^3 - (n+1)^3}{n^3 (n+1)^3} = \frac{\cancel{n^3} - \cancel{n^3} - 1 - 3n^2 - 3n}{n^3 (n+1)^3}$$

$$= \frac{-1 - 3n^2 - 3n}{n^3 (n+1)^3} < 0$$

$\therefore$ Condition 1 is satisfied $u_n > u_{n+1}$

Condition 2 :-

$$\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{1}{n^3} = 0$$

$\therefore$ Condition 2 is also satisfied as $\lim_{n \to \infty} u_n = 0$

⇒ As both conditions are satisfied, $\therefore \sum \frac{(x-3)^n}{n^3 \cdot 7^n}$

is convergent at $x = -4$

Now, let $x = 10$

$$\sum u_n = \frac{\cancel{7^n}}{n^3 \cdot \cancel{7^n}} = \frac{1}{n^3}$$

$$\sum V_n = \frac{1}{n^3}$$

$$\lim_{n\to\infty} \frac{u_n}{V_n} = \lim_{n\to\infty} \frac{\cancel{n^3}}{\cancel{n^3}} = 1 \neq 0$$

Hence, either both converge or diverge

But $\sum V_n = \frac{1}{n^3}$ so by comparison with p-series

we get $p = 3 > 1$

$\therefore \sum V_n$ is convergent & therefore $\sum U_n$ is also convergent by p-series test.

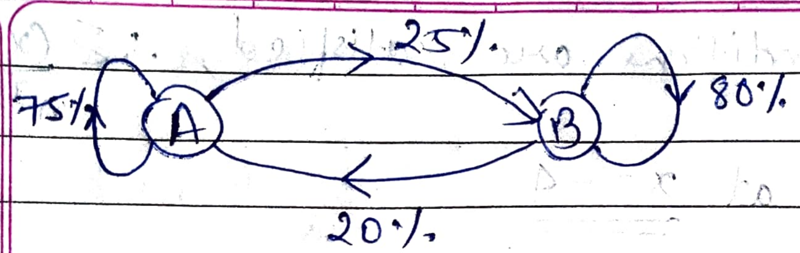
$\therefore$ The intervals are

$[-4, 10]$

or

$-4 \leq x \leq 10$ Ans

Q5



Soln

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1500 \end{bmatrix}$$

$$A_1 = 0.75 A_0 + 0.20 B_0$$
$$B_1 = 0.25 A_0 + 0.80 B_0$$

Applying concept of eigen values & eigen vectors

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0.75 & 0.20 \\ 0.25 & 0.80 \end{bmatrix}}_{C} \begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$[C - \lambda I] = \begin{bmatrix} 0.75-\lambda & 0.20 \\ 0.25 & 0.80+\lambda \end{bmatrix}$$

$$|C - \lambda I| = 0$$

$$(0.75-\lambda)(0.80-\lambda) - (0.20)(0.25) = 0$$

$$(0.6 - 1.55\lambda + \lambda^2) - 0.05 = 0$$
$$\lambda^2 - 1.55\lambda + 0.55 = 0$$
$$\lambda^2 - \lambda - 0.55\lambda + 0.55 = 0$$

$\lambda(\lambda-1) - 0.55(\lambda-1) = 0$

$(\lambda-1)(\lambda-0.55) = 0$

$\lambda = 1$ or $\lambda = 0.55$

let $\lambda = 1$,

$$\begin{bmatrix} 0.75-1 & 0.20 \\ 0.25 & 0.8-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.25 & 0.2 \\ 0.25 & -0.2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-0.25x_1 + 0.2x_2 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.25 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

eigen vector $x_1 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

let $\lambda = 0.55$,

$$\begin{bmatrix} 0.75-0.55 & 0.20 \\ 0.25 & 0.8-0.55 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 0.2 \\ 0.25 & 0.25 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$0.20x_1 = 0.20x_2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

eigen vector $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = c_1 (\lambda_1)^{n+1} x_1 + c_2 (\lambda_2)^{n+1} x_2$$

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = c_1 (\lambda_1)^0 x_1 + c_2 (\lambda_2)^0 x_2$$

$$\begin{matrix} 1500 \\ 1500 \end{matrix} = c_1 \begin{bmatrix} 4 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$1500 = 4c_1 - \cancel{c_2}$

$1500 = 5c_1 + \cancel{c_2}$

$3000 = 9c_1$

$c_1 = \frac{3000}{9} = \frac{1000}{3}$

$c_2 = 1500 - 5\left(\frac{1000}{3}\right)$

$= 1500 - \frac{5000}{3}$

$= \frac{4500 - 5000}{3}$