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Q. $x+y+2z=6$
$x+2y+3z=10$
$x+2y+\lambda z=\mu$

, Investigate for what values of λ & μ, eq^ns have
(i) No sol^n
(ii) Infinite sol^n
(iii) Unique sol^n.

Sol^n Matrix for given sets of eq^n are,

$$\underset{A}{\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}}$$

For (i) No solution,

$\lambda = 3$; $\lambda - 3 = 0$

$\mu \neq 10$

$\Rightarrow \rho(A) = 2$ $\therefore \rho(A) \neq \rho(A:B)$

$\Rightarrow \rho(A:B) = 3$ $\therefore$ No solution

~~For (ii) Infinite sol^n~~
~~if $\lambda = 3$, $\Rightarrow \rho(A) = 3$~~
~~$\mu \in R$ $\Rightarrow \rho(A:B) = 3 = n$~~

For Infinite sol^ns,
$\lambda = 3$; $\rho(A) = 2$
$\mu = 10$; $\rho(A:B) = 2$

$\therefore$ Infinite sol^ns.

For (iii) Unique sol^n.

if $\lambda \neq 3 \Rightarrow \rho(A) = 3$
$\mu \in R \Rightarrow \rho(A:B) = 3 = n$

$\therefore$ Unique solution.

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Q2

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix} = A$$

Ans [P] [A] [Q]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 \longleftrightarrow R_1$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 5R_1$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3/4$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1/4 & 0 & -5/4 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 + R_2$; $R_3 \leftarrow R_3 - 2R_2$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & -2 & -\frac{5}{4} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_3 \leftarrow \frac{R_3}{-3}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & -\frac{2}{3} & -\frac{5}{12} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_3 \leftarrow C_3 - 4C_1 \; ; \; C_4 \leftarrow C_4 - C_1$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & -\frac{2}{3} & -\frac{5}{12} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_3 \leftarrow C_3 - 2C_2 \; ; \; C_4 \leftarrow C_4 - C_2$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & -\frac{2}{3} & -\frac{5}{12} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_4 \leftarrow C_4 + \frac{1}{3} C_3$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & -\frac{2}{3} & -\frac{5}{12} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 & -\frac{7}{3} \\ 0 & 1 & -2 & -\frac{5}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & -\frac{2}{3} & -\frac{5}{12} & 0 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 & -4 & -\frac{7}{3} \\ 0 & 1 & -2 & -\frac{5}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} I_3 & 0 \end{bmatrix}$$

Q3 $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. Find Inverse.

Soln

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 4R_1 \; ; \; R_3 \leftarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & -5 & -15 & 0 & 1 & -4 \\ 0 & -1 & -7 & 1 & 0 & -2 \end{array}\right]$$

$R_2 \leftarrow -\frac{1}{5} R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & -1 & -7 & 1 & 0 & -2 \end{array}\right]$$

$R_1 \leftarrow R_1 - 2R_2 \; ; \; R_3 \leftarrow R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & -4 & 1 & -1/5 & -6/5 \end{array}\right]$$

$R_3 \leftarrow \frac{R_3}{-4}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & 1 & -\frac{1}{4} & -1/20 & -6/20 \end{array}\right]$$

$R_1 \leftarrow R_1 + 2R_3 \quad ; \quad R_2 \leftarrow R_2 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & \frac{3}{10} & -6/5 \\ 0 & 1 & 0 & 3/4 & -1/20 & \frac{17}{10} \\ 0 & 0 & 1 & -1/4 & -1/20 & -6/20 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1/2 & 3/10 & -6/5 \\ 3/4 & -1/20 & 17/10 \\ -1/4 & -1/20 & -6/20 \end{bmatrix}$$

Q4 Verify that the following matrix is orthogonal or not:

$$\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/4 \end{bmatrix}$$

Solⁿ $A = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/4 \end{bmatrix}$ $A' = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/4 \end{bmatrix}$

For orthogonality,

$$AA' = I$$

$$\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LHS

$$\cancel{\left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)} \quad \cancel{\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right)}$$

$$a_{11} = \left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right) = \frac{1}{9}+\frac{4}{9}+\frac{4}{9} = 1$$

$$a_{12} = \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right) = \frac{2}{9}+\frac{2}{9}-\frac{4}{9} = 0$$

$$a_{13} = \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right)+\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right) = \frac{2}{9}-\frac{4}{9}+\frac{2}{9} = 0$$

$$a_{21} = \left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)+\left(-\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right) = \frac{2}{9}+\frac{2}{9}-\frac{4}{9} = 0$$

$$a_{22} = \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)+\left(-\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right) = \frac{4}{9}+\frac{1}{9}+\frac{4}{9} = 1$$

$$a_{23} = \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)+\left(-\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right) = \frac{4}{9}-\frac{2}{9}-\frac{2}{9} = 0$$

$$a_{31} = \left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)+\left(-\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right) = \frac{2}{9}-\frac{4}{9}+\frac{2}{9} = 0$$

$$a_{32} = \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(-\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)+\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right) = \frac{4}{9}-\frac{2}{9}-\frac{2}{9} = 0$$

$$a_{33} = \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)+\left(-\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right)+\left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) = \frac{4}{9}+\frac{4}{9}+\frac{1}{9} = 1$$

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$\therefore$ LHS $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ RHS

→ Hence, Matrix A is orthogonal matrix.

Q5
$5x + 3y + 7z = 4$
$3x + 26y + 2z = 9$
$7x + 2y + 10z = 5$

Sol^n

Matrix form for given eq^n is

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

A X = B

Let, $|A| = \begin{vmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{vmatrix}$

$= 5((26)(10) - (2)(2)) + 3((2)(7) - (3)(10)) + 7[(3)(2) - (7)(26)]$

$= 5(260 - 4) + 3(14 - 30) + 7(6 - 182)$
$= 5(256) + 3(-16) + 7(-176)$
$= 1280 - 48 - 1232$
$= 1280 - 1280$
$= 0$
$|A| = 0$

As, $|A| = 0$, the system of equation has infinite ~~no~~ solutions.

——— x ——— x ———