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2HS306

AEM (BDA)

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Q3 Using Newton divided Interpolation Formula, find value of $f(3)$ from following data :-

x	-1	2	4	5
$f(x)$	-5	13	255	625

Sol^n

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	-1	-5			
			$18/3 = 6$		
x_1	2	13		$\frac{121-6}{4-(-1)} = 23$	$\frac{83-23}{5-(-1)} = 10$
			$242/2 = 121$		
x_2	4	255		$\frac{370-121}{5-2} = 83$	
			370		
x_3	5	625			

Acc. to NDIF,

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)$$

$$f(x) = -5 + (x-(-1))(6) + (x-(-1))(x-2)(23) + (x-(-1))(x-2)(x-4)(10)$$

Now, acc. to ques, $x = 3 = f(3)$

$$f(3) = -5 + (3-(-1))(6) + (3-(-1))(3-2)(23) + (3-(-1))(3-2)(3-4)(10)$$

$$= -5 + (4)(6) + (4)(1)(23) + (4)(1)(-1)(10)$$

$$= -5 + 24 + 92 - 40 = 71$$

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$\boxed{f(3) = 71}$ Ans

Q2 Solve $y'''' - k^4 y = 0$ where $k \neq 0$ & $y(0) = y'(0) = y''(0) = 0$ Using Laplace Transform.

Sol^n Eq^n is $y'''' - k^4 y = 0$

Taking Laplace Transform on both sides and considering k^4 as some constant as $k \neq 0$, we get

$$\left[s^4 \bar{y} - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0)\right] - k^4 \bar{y} = 0$$

Using the given conditions,

$$\left[s^4 \bar{y} - y'''(0)\right] - k^4 \bar{y} = 0$$

$$\cancel{s^4 \bar{y} - y'''(0) - k^4 \bar{y} = 0} \quad \left\{ \cancel{y'''(0) = 0 \text{ as } y''(0) = 0, y'(0) = 0} \right.$$

$$\cancel{s^4 \bar{y} - k^4 \bar{y} = 0}$$

$$\cancel{(s^4 - k^4)\bar{y} = 0}$$

$$s^4 \bar{y} - y'''(0) - k^4 \bar{y} = 0$$

$$(s^4 - k^4)\bar{y} = y'''(0)$$

$$\bar{y} = \frac{y'''(0)}{s^4 - k^4}$$

$$y = L^{-1}\left(\frac{y'''(0)}{(s^2)^2 - (k^2)^2}\right)$$

$$y = y'''(0)\, L^{-1}\left(\frac{1}{(s^2)^2 - (k^2)^2}\right)$$

$$y = y'''(0)\, \frac{1}{k^2} \sinh k^2 t \qquad \left\{ \because L^{-1}\left(\frac{1}{s^2 - a^2}\right) = \frac{1}{a}\sinh at \right.$$

$$\boxed{y = \frac{y'''(0)}{k^2} \sinh(k^2 t)}$$ Ans

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Q1 Express $f(x) = x(2\pi - x)$ as a Fourier series in $0 < x < 2\pi$ where $f(x+2\pi) = f(x)$

Solⁿ $f(x) = x(2\pi - x)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Now,

$$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\,dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} (2\pi x - x^2)\,dx$$

$$= \frac{1}{\pi} \left[\frac{2\pi x^2}{2} - \frac{x^3}{3} \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[\pi(2\pi)^2 - \frac{(2\pi)^3}{3} \right]$$

$$= \frac{1}{\pi} \left(4\pi^3 - \frac{8\pi^3}{3} \right)$$

$$= \left(\frac{12\pi^3 - 8\pi^3}{3} \right) \frac{1}{\pi}$$

$$= \left(\frac{4\pi^3}{3} \right) \frac{1}{\pi} = \boxed{\frac{4\pi^2}{3} = a_0}$$

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$$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{0}^{2\pi} (2\pi x - x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{(2\pi x - x^2)(\sin nx)}{n} - \frac{(2\pi - 2x)(-\cos nx)}{n^2} + \frac{(-2)(-\sin nx)}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(2\pi x - x^2)(\sin nx)}{n} + \frac{(2\pi - 2x)(\cos nx)}{n^2} + \frac{(2)(\sin nx)}{n^3}\right]_0^{2\pi}$$

Placing limits,

$$= \frac{1}{\pi}\left[\frac{((4\pi^2 - 4\pi^2)(\sin 2n\pi))}{n} + \frac{(2\pi - 4\pi)(\cos 2n\pi)}{n^2} + \cancel{\frac{(2)(\sin 2n\pi)}{n^3}} - \cancel{\frac{(0-0)(\sin(n(0)))}{n}} + \frac{(2\pi)(1)}{n^2} + \cancel{\frac{(2)(0)}{n^3}}\right]$$

$$= \frac{1}{\pi}\left[\left(0 + \frac{-2\pi}{n^2} + 0\right) - \left(0 + \frac{2\pi}{n^2} + 0\right)\right]$$

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$$= \boxed{\frac{1}{\pi}\left[\frac{-4\pi}{n^2}\right] = a_n}$$

$$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cdot \sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} (2\pi x - x^2) \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\frac{(2\pi x - x^2)(-\cos nx)}{n} - \frac{(2\pi - 2x)(-\sin nx)}{n^2} + \frac{(-2)(\cos nx)}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(0)(-1)}{n} + \frac{(2\pi - 4\pi)(0)}{n^2} + \frac{(-2)(1)}{n^3} - \left((0) + (0) - \frac{2}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[0 + 0 - \frac{2}{n^3} - 0 + \frac{2}{n^3}\right] = \frac{1}{\pi}(0) = 0$$

$$\cancel{b_n = \frac{2}{\pi n^3}} \qquad \boxed{b_n = 0}$$

$\therefore$ Fourier series of $x(2\pi - x)$

$$= \boxed{\frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{-4}{n^2} \cos nx}$$

Ans