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AEM [2HS306]

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(1) $y' + 2y = \sin 3x$
$y(0) = 1$
$h = 0.2$
$y(0.2) = ?$ } given

Sol^n $y' = \sin 3x - 2y$
$y_0 = 1 \quad x_0 = 0 \quad h = 0.2$

For Fourth - Order Runge - kutta Method,

$k_1 = h f(x_0, y_0)$
$= (0.2) f(0, 1)$
$= (0.2)(-2)$
$= -0.4$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{(-0.4)}{2}\right)$

$= (0.2) f(0.1, 0.8)$

$= (0.2)(-1.30447)$

$= -0.26089$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.2) f\left(0.1, 1 + \frac{(-0.26089)}{2}\right)$

$= (0.2) f(0.1, 0.86955)$

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$= (0.2)(-1.44357)$

$= -0.288714$

$k_4 = h f(x_0+h, y_0+k_3)$

$= h(0.2) f(0.2, 1+(-0.288714))$

$= (0.2) f(0.2, 0.71129)$

$= (0.2)(-0.85793)$

$= -0.171586$

$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$= \frac{(-0.4) + 2(-0.26089) + 2(-0.288714) + (-0.171586)}{6}$

$= -0.278646$

$y(0.2) = y_0 + k$

$= 1 + (-0.27846)$

$= 0.72154$

Ans $\boxed{y(0.2) = 0.72154}$ is the answer using 4th order Runge-Kutta Method

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Q3 Solve fol^{n} by Gauss seidel - method :-

$5x - y + 2z = 10$

$2x + 4y = 12$

$x + y + 5z = -1$

Rewriting eq^{ns};

$$x = \frac{1}{5}[10 + y - 2z]$$

$$y = \frac{1}{4}[12 - 2x]$$

$$z = \frac{1}{5}[-1 - x - y]$$

Iteration Table :-

Itr^{n}	x_i	y_i	z_i
0	0	0	0
1	2	2	-1
2	2.8	1.6	-1.08
3	2.752	1.624	-1.0752
4	2.75488	1.62256	-1.07549
5	2.75471	1.62265	-1.07547
6	≈ 2.754	≈ 1.6226	≈ -1.0754

∴ By Gauss-Seidel method roots are

$x = 2.754$ $y = 1.6226$ $z = -1.0754$

Q2

x	0.5	0.6	0.7	0.8
y	1.27626	1.185465	1.225169	1.337435

Soln

	x	y	Δ	Δ²	Δ³
x_0	0.5	$y_0 = 1.127626$	Δy_0		
			0.057839	$\Delta^2 y_0$	
x_1	0.6	$y_1 = 1.185465$	Δy_1	−0.018135	$\Delta^3 y_0$
			0.039704	$\Delta^2 y_1$	0.090697
x_2	0.7	$y_2 = 1.225169$	Δy_2	0.072562	
			0.112266		
x_3	0.8	$y_3 = 1.337435$			

$$\Rightarrow p = \frac{x - x_0}{h} = \frac{0.56 - 0.5}{0.1} = 0.6$$

By Newton Forward Interpolation formula,

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)\Delta^2 y_0}{2!} + \frac{p(p-1)(p-2)\Delta^3 y_0}{3!}$$

$$= 1.127626 + (0.6)(0.057839) + \frac{(0.6)(-0.4)(-0.018135)}{2}$$

$$+ \frac{(0.6)(-0.4)(-1.4)(0.090697)}{6}$$

$$y_{(0.56)} = 1.169584632$$

$$\boxed{f(0.56) = 1.169584632}$$