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AEM (2HS306)

(BDA)

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Q5

x	0	0.1	0.2	0.3	0.4
$f(x)$	1	1.1052	1.2214	1.3499	1.4918

find $f(0.38)$.

Solⁿ

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		0.1052			
0.1	1.1052		0.011		
		0.1162		0.0013	
0.2	1.2214		0.0123		-0.0002
		0.1285		0.0011	
0.3	1.3499		0.0134		
		0.1419			
0.4	1.4918				

$\Rightarrow$ $h = x_1 - x_0 = 0.1 - 0 = 0.1$

$\Rightarrow$ $p = \dfrac{x - x_0}{h} = \dfrac{0.38 - 0}{0.1} = 3.8$

$\therefore$ Now, according to Newton's forward difference interpolation formula,

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

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$$\therefore y(0.38) = 1 + (3.8 \times 0.1052) + \frac{(3.8)(3.8-1)(0.011)}{2}$$

$$+ \frac{(3.8)(3.8-1)(3.8-2)(0.0013)}{6}$$

$$+ \frac{(3.8)(3.8-1)(3.8-2)(3.8-3)(-0.0002)}{24}$$

$$\therefore y(0.38) = 1 + 0.3998 + 0.0585 + 0.0041 - 0.000012$$

$$\therefore \boxed{y(0.38) = 1.4623} \quad \text{Ans}$$

Q4 System of equations are,

$x - 2y + 10z = 30.6$

$2x + 5y - z = 10.5$

$3x + y + z = 9.3$

using Gauss seidel method

Sol^n Rearranging the following given equations as the co-efficient matrix of given system is not diagonally dominant.

Hence, the system of equation are

$3x + y + z = 9.3$

$2x + 5y - z = 10.5$

$x - 2y + 10z = 30.6$

from above equations, we get,

$$x = \frac{1}{3}(9.3 - y - z)$$

$$y = \frac{1}{5}(10.5 - 2x + z)$$

$$z = \frac{1}{10}(30.6 - x + 2y)$$

<u>Iteration table</u>:-

$Iter^n$	x_i	y_i	z_i
0	0	0	0
1	3.1	0.86	2.92
2	1.84	1.95	3.27
3	1.36	2.21	3.37
4	1.24	2.28	3.39
5	1.21	2.29	3.4
6	1.2	2.3	3.4
7	1.2	2.3	3.4

$\therefore$ The solution of following system of equation is $(x, y, z) = (1.2, 2.3, 3.4)$

$\Rightarrow$ $x = 1.2$
$y = 2.3$ Ans
$z = 3.4$

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Q3

x	5	6	9	11
$f(x)$	12	13	14	16

Find $f(10)$ using Lagrange Interpolation.

Sol^n

$$f(x) = \frac{(x-6)(x-9)(x-11)(12)}{(5-6)(5-9)(5-11)}$$

$$+ \frac{(x-5)(x-9)(x-11)(13)}{(6-5)(6-9)(6-11)}$$

$$+ \frac{(x-5)(x-6)(x-11)(14)}{(9-5)(9-6)(9-11)}$$

$$+ \frac{(x-5)(x-6)(x-9)(16)}{(11-5)(11-6)(11-9)}$$

Placing value of $x=10$, we get,

$$f(10) = \frac{(10-6)(10-9)(10-11)(12)}{(5-6)(5-9)(5-11)}$$

$$+ \frac{(10-5)(10-9)(10-11)(13)}{(6-5)(6-9)(6-11)}$$

$$+ \frac{(10-5)(10-6)(10-11)(14)}{(9-5)(9-6)(9-11)}$$

$$+ \frac{(10-5)(10-6)(10-9)(16)}{(11-5)(11-6)(11-9)}$$

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$$\therefore f(10) = \frac{(\cancel{4})(1)(\cancel{-1})(\cancel{12})}{(+1)(\cancel{-4})(+6)} + \frac{(\cancel{8})(1)(-1)(13)}{(1)(-3)(-5)}$$

$$+ \frac{(5)(\cancel{4})(+1)(\cancel{14})}{(\cancel{4})(3)(\cancel{+2})} + \frac{(\cancel{8})(\cancel{4})(1)(16)}{(\cancel{6})(\cancel{8})(2)}$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3}$$

$$\boxed{f(10) = 14.6667}$$ Ans

Q1 given, $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-t}\sin t$, where

$y(0) = 0$ & $y'(0) = 1$

Soln $y'' + 2y' + 5y = e^{-t}\sin t$

Taking Laplace of equation:-

$L(y'') + 2L(y') + 5L(y) = L(e^{-t}\sin t)$

By formula,

$L(f^n(t)) = s^n L(f(t)) - s^{n-1}(f(0)) - s^{n-1}f'(0) - \ldots$

$\therefore \quad s^2 L(y) - s y(0) - y'(0) + 2s L(y) - 2y(0) + 5 L(y)$

$$= \frac{1}{(s+1)^2+1}$$

$\therefore \quad s^2 L(y) - 1 + 2s L(y) + 5 L(y) = \frac{1}{(s+1)^2+1}$

$\therefore \quad (s^2+2s+5)(L(y)) = \frac{1}{s^2+2s+2} + 1$

$\therefore \quad (s^2+2s+5)(L(y)) = \frac{1+s^2+2s+2}{s^2+2s+2}$

$$L(y) = \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$$

Now using partial fraction,

$$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{As+B}{s^2+2s+2} + \frac{Cs+D}{s^2+2s+5}$$

$\Rightarrow \quad s^2+2s+3 = (As+B)(s^2+2s+5) + (Cs+D)(s^2+2s+2)$

$\Rightarrow \quad s^2+2s+3 = s^3A + s^3C + 2s^2A + s^2B + 2s^2C + s^2D + 5sA + 2sB + 2sC + 2sD + 5B + 2D$

$= s^3(A+C) + s^2(2A+B+2C+D) + s(5A+2B+2C+2D) + 5B+2D$

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Now, consider,

$A+C=0$, $\therefore A=-C \Rightarrow C=0$

$\therefore 2A+B+2C+D=1 \Rightarrow B+D=1$

$\Rightarrow 5A+2B+2C+2D=2$ ~~B+2D = ... A = 0~~

$5A-2A + 2(B+D)=2$

$3A+2(1)=2$

$3A=2-2$

$A=0$.

$\Rightarrow 5B+2D=3 \Rightarrow 5(1-D)+2D=3$

$5-5D+2D=3$

$-3D=-2$

$D=\frac{2}{3}$

$\therefore B=1-D$

$=1-\frac{2}{3}$

$\therefore B=\frac{1}{3}$

$\therefore A=0,\ B=\frac{1}{3},\ C=0,\ D=\frac{2}{3}$

$\therefore$ Equation $= \dfrac{1/3}{(s^2+2s+2)} + \dfrac{2/3}{(s^2+2s+5)}$

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$$L(y) = \left(\frac{1/3}{(s+1)^2+1} + \frac{2/3}{(s+1)^2+4} \right)$$

$$y = \left[L^{-1}\left(\frac{1/3}{(s+1)^2+1} \right) + L^{-1}\left(\frac{2/3}{(s+1)^2+2^2} \right) \right]$$

$$y = \frac{1}{3}\left[L^{-1}\left(\frac{1}{(s+1)^2+1} \right) + L^{-1}\left(\frac{2}{(s+1)^2+2^2} \right) \right]$$

$$y = \frac{1}{3}\left[\sin t\, e^{-t} + \sin 2t\, e^{-t} \right]$$

$$\boxed{y = \frac{e^{-t}}{3}\left[\sin t + \sin 2t \right]}$$ Ans

Q2 Fourier series, $f(x) = x + |x|$, $-\pi < x < \pi$
$f(x+2\pi) = f(x)$

Soln In fourier series,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin nx \right]$$

where, $a_0 = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx$

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$$= \frac{1}{\pi}\int_{-\pi}^{0} 0 \cdot dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left(\frac{2x^2}{2}\right)_0^{\pi}$$

$$= \frac{1}{\pi}(x^2)_0^{\pi} = \frac{1}{\pi}(\pi^2 - 0) = \pi$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} 2x \cos nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x \cos nx \, dx$$

$$= \frac{2}{\pi}\left[\frac{x \sin nx}{n} - \int \frac{\sin nx}{n} \cdot dx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(0 + \frac{(-1)^n}{n^2}\right) - \left(0 + \frac{1}{n^2}\right)\right]$$

$$\boxed{a_n = \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)}$$

Now, $b_n = \frac{1}{\pi}\int_0^{\pi} 2x \sin nx \, dx$

$$= \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi}\left[-\frac{x \cos nx}{n} - \int \frac{-\cos nx}{n} \, dx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left(-\frac{x \cos nx}{n} - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n} + 0\right]$$

$$= \frac{2}{\cancel{\pi}}\left(\frac{\cancel{\pi}(-1)^{n+1}}{n}\right)$$

$$b_n = \frac{2(-1)^{n+2}}{n}$$

$\therefore$ $f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\left[\frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right]\cos nx + \left(\frac{2(-1)^{n+1}}{n}\right)\sin nx\right]$

Ans