DOMS Page No. Date / /

(a)

Practise Assignment - 1

(i) $2x-3y+7z = 5$
$3x+y-3z = 13$
$2x+19y-47z = 32$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \quad \begin{array}{l} (1)\ R_2 - \frac{3}{2}R_1 \\ (2)\ R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 22 & -54 & 27 \end{bmatrix} \quad (3)\ R_3 - 4R_2$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\rho(A) = 2$
$\rho(A:B) = 3$

$\rho(A) \neq \rho(A:B) \longrightarrow$ Inconsistent
$\downarrow$
No solⁿ

Date / /

(ii) $2x - y + 3z \Rightarrow 8$
$-x + 2y + z \Rightarrow 4$
$3x + y - 4z \Rightarrow 0$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_2 \longleftrightarrow R_1$

$$\begin{bmatrix} -1 & 2 & 1 & 4 \\ 2 & -1 & 3 & 8 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_2 + 2R_1$
$R_3 + 3R_1$

$$\begin{bmatrix} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 7 & -1 & 12 \end{bmatrix}$$

$R_3 - \frac{7}{3} R_2$

$$\begin{bmatrix} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -38/3 & -68/3 \end{bmatrix}$$

DOMS | Page No.
Date / /

$\rho(A) \Rightarrow 3$

$\rho(A:B) \Rightarrow 3$

$n \Rightarrow 3$

$\rho(A) = \rho(A:B) = n = 3$

consistent, unique sol^n

$-\frac{38}{\cancel{8}} z \Rightarrow -\frac{68}{\cancel{8}}$

$z \Rightarrow \frac{\cancel{68}\,34}{\cancel{38}\,19}$ $\quad z \Rightarrow \frac{34}{19}$

$3y + 5z \Rightarrow 16$

$3y + 5\left(\frac{34}{19}\right) \Rightarrow 16$

$3y \Rightarrow 16 - \frac{170}{19}$

$y \Rightarrow \frac{134}{57}$

$-x + 2y + z \Rightarrow 4$

$-x + \frac{268}{57} + \frac{102}{57} \Rightarrow \frac{228}{57}$

$x \Rightarrow \frac{268}{57} + \frac{102}{57} - \frac{228}{57}$

$x \Rightarrow \frac{142}{57}$

$x \Rightarrow \frac{142}{57}$, $y \Rightarrow \frac{134}{57}$, $z \Rightarrow \frac{34}{19}$ Ans

(iii) $4x - y \Rightarrow 12$

$-x + 5y - 2z \Rightarrow 0$

$-2x + 4z \Rightarrow -8$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{bmatrix}$$

① $R_2 + \frac{1}{4} R_1$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ -1 & -1/4 & 4 & -5 \end{bmatrix}$$

~~$\rho(A) \Rightarrow 3$~~

~~$\rho(A:B) \Rightarrow 3$~~

② $R_3 + \frac{1}{4} R_1$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & -1/2 & 4 & -2 \end{bmatrix}$$

③ $R_3 + \frac{2}{19} R_2$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & 0 & 72/19 & -32/19 \end{bmatrix}$$

$$\left.\begin{array}{l} \rho(A) \Rightarrow 3 \\ \rho(A:B) \Rightarrow 3 \\ n \Rightarrow 3 \end{array}\right\} \rightarrow \text{Consistent \& Unique.}$$

$$\left.\begin{array}{ll} \frac{72}{19} z \Rightarrow -\frac{32}{19}, & z \Rightarrow -\frac{4}{9} \\ \frac{19}{4} y - 2z \Rightarrow 3, & y \Rightarrow \frac{4}{9} \\ 4x - y \Rightarrow 12, & x \Rightarrow \frac{28}{9} \end{array}\right\} \text{Ans}$$

(b)

$x + y + z \Rightarrow 6$
$x + 2y + 3z \Rightarrow 10$
$x + 2y + \lambda z \Rightarrow \mu$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$$

$$R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

(i) no sol^n

$$\rho(A) \neq \rho(A:B)$$

$$\boxed{\mu \neq 10}\ ,\ \boxed{\lambda = 3}$$

(ii) unique sol^n

$$\rho(A) = \rho(A:B) = n$$

$$\boxed{\lambda \neq 3}\ ,$$ ~~μ can be~~ $$\boxed{\mu \in R}$$

(iii) infinite no of sol^n

$$\rho(A) = \rho(A:B) \neq n$$

$$\boxed{\lambda = 3}\ ,\ \boxed{\mu = 10}$$

(c) $x + y + z \Rightarrow 1$

$x + 2y + 4z \Rightarrow \lambda$

$x + 4y + 10z \Rightarrow \lambda^2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & (\lambda-1)(\lambda-2) \end{bmatrix}$$

1) can't be unique, $n = 3$, $\rho(A) \Rightarrow 2$

2) for infinitely $\lambda \Rightarrow 1$ or $\lambda \Rightarrow 2$

~~let $z \to k$~~

Date / /

Case-1, $\lambda = 1$

$z \Rightarrow k$

$y + 3k \Rightarrow 0$

$\boxed{y \Rightarrow -3k}$

$x + y + z = 1$

$x - 3k + k \Rightarrow 1$

$\boxed{x \Rightarrow 2k+1}$

$\boxed{x \Rightarrow 2k+1, \quad y \Rightarrow -3k, \quad z \Rightarrow k}$ Ans

Case-2 , $\lambda \Rightarrow 2$, $z \Rightarrow k$

$y + 3k \Rightarrow 1$

$\boxed{y \Rightarrow 1-3k}$

$x + y + z \Rightarrow 1$

$x + 1 - 3k + k \Rightarrow 1$

$\boxed{x \Rightarrow 2k}$

$\boxed{x \Rightarrow 2k, \quad y \Rightarrow 1-3k, \quad z \Rightarrow k}$ Ans

DOMS Page No.
Date / /

(d) $x + 3y - 2z \Rightarrow 0$
$2x - y + 4z \Rightarrow 0$
$x - 11y + 14z \Rightarrow 0$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -5 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

$R_3 \to R_3 - \frac{14}{5} R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -5 & 8 & 0 \\ 0 & 0 & -32/5 & 0 \end{bmatrix}$$

$\rho(A) \Rightarrow \rho(A:B) \Rightarrow n \Rightarrow 3$
$\downarrow$
Consistent, Unique solⁿ

$\boxed{z \Rightarrow 0}$, $\boxed{y \Rightarrow 0}$, $\boxed{x \Rightarrow 0}$

Ans

Date / /

e) $3x + y - \lambda z \Rightarrow 0$

$4x - 2y - 3z \Rightarrow 0$

$2\lambda x + 4y + \lambda z \Rightarrow 0$

$$\begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 4 & -2 & -3 & 0 \\ 2\lambda & 4 & \lambda & 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{4}{3} R_1$

$R_3 \to R_3 - \frac{2\lambda R_1}{3}$

$$\begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 0 & -10/3 & \frac{4\lambda}{3} - 3 & 0 \\ 0 & \frac{12-2\lambda}{3} & \frac{2\lambda^2 + \lambda}{3} & 0 \end{bmatrix}$$

from here we get to know that rank should not be equal to 3.

$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} \Rightarrow 0$$

$3(-2\lambda + 12) - 1(4\lambda + 6\lambda)$
$- \lambda(16 + 4\lambda) \Rightarrow 0$

$-6\lambda + 36 - 10\lambda - 16\lambda - 4\lambda^2 \Rightarrow 0$

$$4\lambda^2 + 32\lambda - 36 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda+9) - 1(\lambda+9) = 0$$

$$\lambda = 1, -9$$