"Assignments"

Q. Find set of vectors are LD or LID.

1.) [1 0 0], [1 1 0], [1 1 1]

here no. of unknowns = 3
$n = 3$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) \Rightarrow 3$
$n = 3$
$\rho(A) = n$

So, It's Linearly Independent

Date / /

2.) $[7\ -3\ 11\ -6]$, $[-56\ 24\ -88\ 48]$

$n = \cancel{4}\,2$

$$A \Rightarrow \begin{bmatrix} 7 & -3 & 11 & -6 \\ -56 & 24 & -88 & 48 \end{bmatrix}$$

here max rank of A can be 2. $\rho(A)_{max} \Rightarrow 2$

$\rho(A)_{max} = \cancel{\phi}\ 2 \neq n$

So, It's Linearly Dependent.

3.) $[-1\ 5\ 0]$, $[16\ 8\ -3]$, $[-64\ 56\ 9]$

$n = 3$

$$A \Rightarrow \begin{bmatrix} -1 & 5 & 0 \\ 16 & 8 & -3 \\ -64 & 56 & 9 \end{bmatrix} \begin{array}{l} R_2 \to R_2 + 16R_1 \\ \\ R_3 \to R_3 - 64R_1 \end{array}$$

$$\Rightarrow \begin{bmatrix} -1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & -264 & 9 \end{bmatrix} \quad R_3 \to R_3 + 3R_2$$

$$\Rightarrow \begin{bmatrix} -1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

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$\rho(A) = 2$

$n = 3$

$\rho(A) \neq n$

So, It's Linearly Dependent

4.) $[1 \ -1 \ \ 1], [1 \ 1 \ -1] \ [-1 \ 1 \ 1] \ [0 \ 1 \ 0]$

$n = 4$

$$A \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$A \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \quad R_4 \to R_4 - \tfrac{1}{2} R_2$$

$$A \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad R_4 - R_4 - \tfrac{1}{2} R_3$$

$$\begin{bmatrix} 3 & 5 & -6 & +2 \\ -2 & 0 & 1 & 0 \\ 4 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{4}{3} R_1$$

$$R_2 \to R_2 + \frac{2}{3} R_1$$

$$A \to \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10/3 & -3 & 4/3 \\ 0 & -17/3 & 9 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A \Rightarrow \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & -17 & 27 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \to R_3 + \frac{17}{10} R_2$$

$$A \to \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & 0 & 117/10 & 78/10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) \Rightarrow \cancel{2}3$
$n = 4$

So, it's Linearly Dependent.

7.) $[3\ 4\ 7], [2\ 0\ 3], [8, 2, 3], [5, 5, 6]$

$n \Rightarrow 4$

$$A \Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & \cancel{3}5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$$

$\rho(A)_{max} \Rightarrow 3$
$n = 4$

So, It's Linearly Dependent

8.) $[6\ 0\ 1\ 3\ 4\ 2]\ [0\ -1\ 2\ 7\ 0\ 5]$
$[12\ 3\ 0\ -19\ 8\ -11]$
$n \Rightarrow 3$

$$A \Rightarrow \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 1 & 2 & 0 \\ 3 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}_{6\times 3}$$

$\cancel{R_2 \to R_2}$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 6 & 0 & 12 \\ 3 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_3 \to R_3 - 6R_1,\ R_4 \to R_4 - 3R_1,\ R_5 \to R_5 - 4R_1,\ R_6 - 2R_1$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & -12 & 12 \\ 0 & 1 & -19 \\ 0 & -8 & 8 \\ 0 & 1 & -11 \end{bmatrix}$$

$R_3 \to R_3 + 12R_2$

$R_4 \to R_4 + R_2$

$R_5 \to R_5 - 8R_2$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & -24 \\ 0 & 0 & -16 \\ 0 & 0 & -16 \\ 0 & 0 & -8 \end{bmatrix}$$

$R_3 \to \frac{1}{24} R_3$ $\quad \cancel{R_3 \to R_3 + 8}$ $\quad R_4 \to \frac{1}{16} R_4$, $R_5 \to \frac{1}{16} R_5$

$R_6 \to \frac{1}{8} R_6$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_4$

$$R_2 \quad A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_5$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_5 \to R_5 - R_6$

$$A \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$R_6 \longleftrightarrow R_3$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) \Rightarrow 3$

$n \Rightarrow 3$

It's linearly Independent

—x———x———x—