

Assignment - 3

(~ Eigen Values & Vectors)

$$1.) A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$A - \lambda I \Rightarrow \begin{bmatrix} -2 - \lambda & 2 & -3 \\ 2 & 1 - \lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

$$-(2+\lambda) \left((1-\lambda)(-\lambda) - 12 \right) \\ - 2(-2\lambda + 6) - 3(-4 + (1-\lambda)) \Rightarrow 0$$

$$-(\lambda+2) \left[-\lambda + \lambda^2 - 12 \right] + 4\lambda + 12 \\ + 12 - 3 + 3\lambda \Rightarrow 0$$

$$\cancel{\lambda^2 - \cancel{\lambda^3}} + 12\lambda + 2\lambda - 2\lambda^2 + 24 + 4\lambda + 12 \\ + 12 - 3 + 3\lambda = 0$$

$$\cancel{-\lambda^3 - \cancel{\lambda^2}} + 15\lambda + 3 \Rightarrow 0 \\ \cancel{\lambda^3 + \cancel{\lambda^2}} - 15\lambda + 3 \Rightarrow 0$$

$$- \lambda^3 - \lambda^2 + 21\lambda + 45 \Rightarrow 0 \\ \lambda = -3, 5$$

for $\lambda = -3$

$A \Rightarrow$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$x + 2y - 3z = 0$$

$$2x + 4y - 6z = 0$$

$$-x - 2y + 3z = 0$$

2.)

$$A \rightarrow \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$A - \lambda I \Rightarrow \begin{pmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{pmatrix}$$

$$\Rightarrow (4-\lambda)((1-\lambda)^2) + 1(+2(1-\lambda)) = 0$$

$$\Rightarrow (4-\lambda)(1-\lambda)^2 + 2(1-\lambda) = 0$$

$$(1-\lambda)[(4-\lambda)(1-\lambda) + 2] = 0$$

$$(1-\lambda)[6-5\lambda+\lambda^2] = 0$$

$$(\lambda-1)(\lambda-2)(\lambda-3) = 0$$

Eigen Values, $\lambda = 1, 2, 3$

for $\lambda = 1$

$$\begin{pmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$-2x = 0, \quad |x=0, z=0 \\ 3x+z=0, \quad |y=k$$

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Eigen vector = $K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$$\begin{aligned} -2x - y &= 0 & -2x - z &= 0 \\ 2x &= -y & z &\Rightarrow -2x \end{aligned}$$

$$x = k, y = -2k, z = -2k$$

Eigen vector $\Rightarrow K \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ Ans

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -3 \end{bmatrix}$$

$$\begin{aligned} -2x - 2z &= 0 \\ 2x &\Rightarrow -z \end{aligned}$$

$$\begin{aligned} -2x - 2y &= 0 \\ 2x &\Rightarrow -y \end{aligned}$$

$$x = k, y = -k, z = -1k$$

Eigen vectors \rightarrow K $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$$3.) \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I \Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda) \cdot (-\lambda)(3-\lambda) = 0$$

$$(\lambda-5)(\lambda)(\lambda-3) = 0$$

Eigen vectors : $\lambda = 5, 0, 3$

for $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \quad \begin{array}{l} -5y=0 \\ \underline{y=0} \end{array}$$

$$-x-2z=0$$

$$x = -2k, y=0, z=k \quad \underline{x=-2z}$$

Eigen vectors \rightarrow K $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ Ans

for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \rightarrow \begin{array}{l} x=0 \\ -x+3z=0 \\ \cancel{x=3z} \\ z=0 \end{array}$$

$$x=0, y=k, z=0$$

Eigen vectors $\Rightarrow K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 3 \end{bmatrix} \rightarrow \begin{array}{l} y=0 \\ x=0 \\ z=k \end{array}$$

Eigen vectors $\Rightarrow K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$4.) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$$A - \lambda I \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

$$\Rightarrow -1((3-\lambda)(-2-\lambda)) = 0$$

$$\Rightarrow (3-\lambda)(\lambda+2) = 0$$

Eigen Values $\lambda = 3, -2$

for $\lambda = 3$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \quad \begin{array}{l} x=0 \\ y=0 \\ z=k \end{array}$$

Eigen vectors $\Rightarrow k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda = -2$

$$x=0, y=-4k$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 8 & 5 & 4 \\ 8 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} 5y+4z=0 \\ y=-\frac{4}{5}z \end{array}$$

Eigen Vectors $\in K$

$$\begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$