Advanced Computational Mathematics for Data Science (ACM-DS)

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Intended Audience

This program is designed for graduate students with a solid background in computer science and practical industry experience in machine learning. The curriculum aims to substantially augment the theoretical understanding of these students so they can engage in advanced research and make meaningful contributions to both academia and industry.

Part 1: Foundational Mathematics

1. Core Foundations
   - Propositional Logic, Set Theory (ZFC), Predicate Logic
   - Relations, Functions (Isomorphisms, Equivalence, Cardinality, Ordering)
   - Algebraic Structures (Magmas, Semigroups, Monoids, Groups)
   - Numbers (Natural numbers, Integers, Rationals, Rings, Fields)
   - Number Theory (Factorization, Primes, Modular Arithmetic)
   - Discrete Math (Combinatorics, Basic Probability, Graph Theory)
   - Algebra, Polynomials (Abelian Groups, Finite Fields)
2. Geometry
   - Trigonometry
   - Conic Sections
   - Explicit
   - Implicit
   - Parametric
   - Polyhedra
   - Quadric Surfaces

3. Group Theory
   - Symmetry
   - Cayley's Theorem
   - Group Actions
   - Subgroups
   - Group Homomorphisms
   - Lagrange's Theorem
   - Orbit-Stabilizer Theorem
   - Normal Subgroups
   - Quotient Groups
   - Isomorphism Theorems
   - Sylow Theorems

4. Linear Algebra
   - Vector Spaces
   - Matrices
   - Determinants
   - Eigenvalues
   - Eigenvectors
   - Singular Value Decomposition
   - Matrix Factorizations
   - Linear Transformations
   - Linear Systems
   - Multilinear Tensor Algebra

5. Real Analysis
   - Metric Spaces
   - Topology
   - Sequences
   - Series
   - Limits
   - Continuity
   - Differentiability
   - Integrability
   - Multivariable Calculus
   - Nonstandard Analysis

Part 2: Advanced Topics

1. Asymptotics, Dynamical Systems, PDEs
2. Probability Theory (Measure Theory)
3. Statistics
   - Learning Theory
   - Information Theory
   - Machine Learning
   - Hypothesis Testing
4. Bayesian Methods, Stochastic Processes
   - Markov Chains
   - Martingales
   - Brownian Motion
   - Stochastic Calculus
   - Diffusion Processes
5. Matrix Calculus, Numerical Methods
   - Precision
   - Conditioning
   - Root Finding
   - Optimization
   - Interpolation
   - Quadrature
   - Differential Equations
   - Matrix Decompositions
   - Iterative Methods
   - Monte Carlo Methods (Sketching)
6. Signal Processing
   - Fourier Analysis
   - Wavelets
   - Filter Banks
   - Sampling Theory
   - Shannon-Nyquist Theorem
7. Decision making under uncertainty: Bandits, Reinforcement Learning
   - Markov Decision Processes
   - Policy Gradients
   - Q-Learning

- Actor-Critic
- Temporal Difference Learning
- Model-Based RL
- Inverse RL
- Multi-Agent RL

8. Nonlinear Dynamics, Perturbation Theory
   - Chaos
   - Bifurcations
   - Catastrophe Theory

9. Functional Analysis (Nonparametric Methods)

10. Optimization Theory (Convex Optimization, Non-Convex Optimization)

11. High Dimensional Statistics
    - High Dimensional Geometry
    - Concentration
    - Metric Entropy
    - RMT
    - Functional Calculus
    - Chaining

12. Differential Geometry
    - Manifolds
    - Tangent Bundles
    - Lie Groups, Lie Algebras
    - Grassmann Algebras
    - Exterior Algebras
    - Differential Forms
    - De Rham Cohomology
    - Connections
    - Curvature
    - Riemannian Geometry
    - Symplectic Geometry
    - Geodesics

13. Category Theory, Type Theory, Homotopy Type Theory

14. Advanced Algorithms
    - Hashing
    - Integer Programming
    - Coding Theory

- Randomized Algorithms
- Approximation Algorithms
- Online Algorithms
- Quantum Algorithms

15. Computational Complexity Theory (Circuit Complexity, P, NP, PSPACE, L, NL, etc.)

Additional Skills

- **Computer Architecture**: Superscaler CPUs, GPUs, FPGAs, ASICs, SoCs
- **Systems**: Operating Systems, Networking, Security
- **Deep Learning Systems**: PyTorch, JAX, CUDA, OpenCL, Metal
- **Tools**:
  - Text Editors, IDEs: vim/emacs, VSCode, CoPilot
  - Version Control: Git, GitHub, GitLab
  - Command Line Skills: Bash scripts, ssh, tmux, zsh
  - Linux Basics: File system, permissions, processes, systemd, cron, package management, graphics, audio, video, printing
  - Environment Management: conda/mamba, pip, virtualenv
  - Containerization: Docker, Singularity
  - Cloud Services: AWS, GCP, Azure
  - CI/CD: Automated testing and deployment
  - Monitoring and Logging: Grafana, Prometheus
  - Data Storage: SQL, NoSQL, data lakes
  - Data Processing: Spark, Hadoop, Hive, Pig, Kafka, Storm, Flink, Beam
  - Data Visualization: matplotlib, seaborn, plotly, bokeh, d3.js
  - Web Development: HTML, CSS, JavaScript, React, Node.js, Express.js
  - Mobile Development: Android, iOS, React Native, Flutter
  - Desktop Development: Electron
  - Game Development: Unity, Unreal Engine
  - Graphics/Vision: OpenGL, WebGL, Vulkan, DirectX, Blender, Maya, SolidWorks, OpenCV
  - DevOps: Kubernetes, Terraform, Ansible, Chef, Puppet