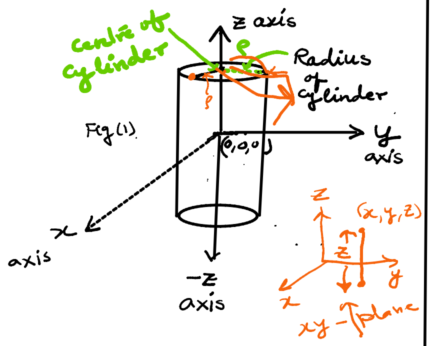
# Vector Analysis (Cylindrical coordinate System)



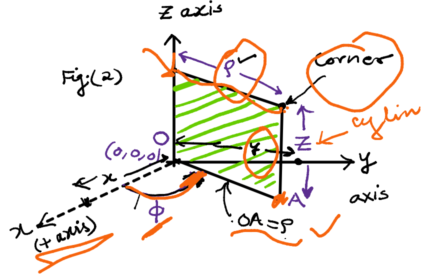

(a) Point to be located is represented by

$(\rho, \phi, z)$ or $(r, \phi, z)$

where $\rho$ = Radius of cylinder which is centred, kept along z-axis

$\phi$ = angle between $+x$ axis and line OA anticlockwise w.r.t $+x$ axis

$z = z$ (Cartesian Coordinate system)
(cylindrical Coordinate system)




$\rho, \phi, z = f(x, y, z)$
$(x, y, z) = f(\rho, \phi, z)$

(b) In cylindrical coordinate system 3 sets of unit vectors along radius $\rho$, angle $\phi$ & z $\rightarrow \pm \bar{a}_\rho, \pm \bar{a}_\phi, \pm \bar{a}_z$

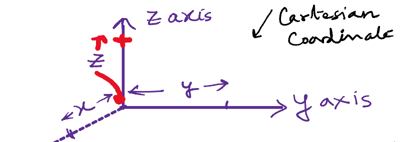
(c) (i) Transformation from cylindrical to Cartesian coordinate

Cylin is given $(\rho, \phi, z)$ known

$x = \rho \cdot \cos\phi$ — (1)

$y = \rho \cdot \sin\phi$ — (2)

$z = z$ cylindrical — (3)
(Cartesian)




$x^2 = \rho^2 \cos^2\phi$ — (A)
$y^2 = \rho^2 \sin^2\phi$ — (B)

$x^2 + y^2 = \rho^2(\cos^2\phi + \sin^2\phi)$

$\rho^2 = x^2 + y^2$; $\rho = \sqrt{x^2+y^2}$

ems A & B

(ii) Transformation from Cartesian to cylindrical — (1) (2)

$(x, y, z)$ known

$\boxed{\rho = \sqrt{x^2+y^2}}$ $\left| \frac{y}{x} = \frac{\rho \sin\phi}{\rho \cos\phi} \right.$ ; $\tan\phi = \frac{y}{x}$

$\phi = \tan^{-1}(y/x)$ ; $z_{cy} = z_{car}$

(d) Position Vector in Cylindrical Coord.

$\boxed{\bar{R}_A = \rho \bar{a}_\rho + z \bar{a}_z}$

Point A $(\rho, \phi, z)$

Origin $\xrightarrow{\bar{R}_A}$ A $(\rho, \phi, z)$

# Spherical Coordinate System

(a) In spherical Coordinate system any point is represented by $(r, \theta, \phi)$.

where (i) $r$ = radius of the sphere centre of which is centered on the origin (0,0,0)

(ii) $\theta$ = angle between $+z$ axis and line joining origin O(0,0,0) and point in question $(r, \theta, \phi)$ (line OA).

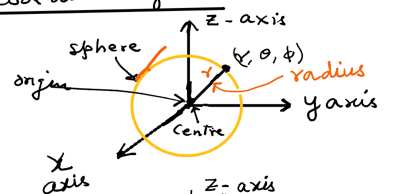
$\boxed{\theta \text{ varies from } 0^\circ \text{ to } 180^\circ}$

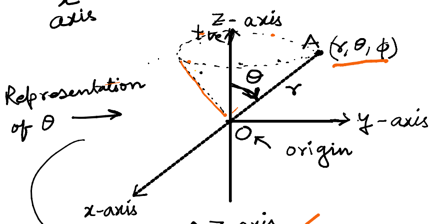
(iii) $\phi$ = Angle between +ve x-axis and line OB

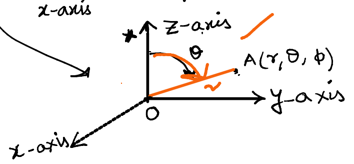
$\boxed{\phi \text{ varies from } 0 \text{ to } 360^\circ}$

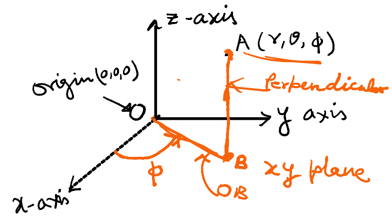
$\boxed{\text{Since } r \text{ is radius. Varies from } 0 \text{ to } \infty}$

(b) There are 3 sets of unit vectors (i) $\pm \bar{a}_r$ along radius
(ii) $\pm \bar{a}_\theta$ along variation of $\theta$



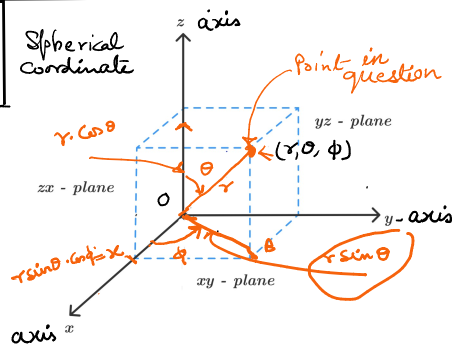

(i) Draw a perpendicular from pt. A on the plane xy. (Point B)
(ii) Join point O and point B

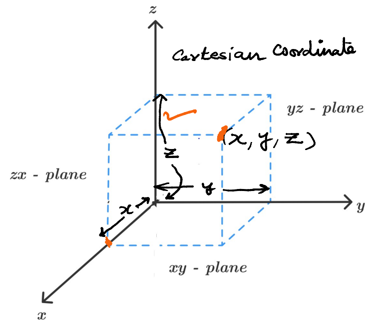
Vectors (i) $\pm \bar{a}_r$ along radius
(ii) $\pm \bar{a}_\theta$ along variation of $\theta$
(iii) $\pm \bar{a}_\phi$ along variation of $\phi$

(i) Draw a perpendicular from pt. A on the plane xy. (Point B)
(ii) Join point O and point B. Line OB is obtained.

ⓒ (i) Transformation from spherical to Cartesian coordinate system :- $(r, \theta, \phi)$

$x = r \sin\theta \cdot \cos\phi$ —— ①
$y = r \sin\theta \, \sin\phi$ —— ②
$z = r \cos\theta$ —— ③




(ii) Square equations ① & ②

$x^2 = r^2 \sin^2\theta \cdot \cos^2\phi$ —— ④
$y^2 = r^2 \sin^2\theta \cdot \sin^2\phi$ —— ⑤

Add ④ & ⑤

$$x^2 + y^2 = r^2 \sin^2\theta \cdot \cos^2\phi + r^2 \sin^2\theta \sin^2\phi$$
$$= r^2 \sin^2\theta \underbrace{[\cos^2\phi + \sin^2\phi]}_{1}$$

$x^2 + y^2 = r^2 \sin^2\theta$ —— ⑥

Square eqn. ③

$z^2 = r^2 \cos^2\theta$ —— ⑦

Add ⑥ & ⑦

$$x^2 + y^2 + z^2 = r^2 \sin^2\theta + r^2 \cos^2\theta$$
$$= r^2 \underbrace{[\cos^2\theta + \sin^2\theta]}_{1}$$

$r^2 = x^2 + y^2 + z^2$ —— ⑧

$\boxed{r = \sqrt{x^2 + y^2 + z^2}}$ —— ⑨

Equation ② is divided by ①  $\dfrac{y}{x} = \dfrac{\cancel{r \sin\theta} \sin\phi}{\cancel{r \sin\theta} \cos\phi}$ —— ⑩

From eqn ⑩  $\tan\phi = \dfrac{y}{x}$ —— ⑪

$\boxed{\phi = \tan^{-1}\left(\dfrac{y}{x}\right)}$ —— ⑫

From eqn ③  $\boxed{\cos\theta = \dfrac{z}{r} \; ; \text{ where } r = \sqrt{x^2 + y^2 + z^2} \quad \theta = \cos^{-1}(z/r)}$ —— ⑬

ⓓ Position Vector in Spherical Coordinate

Point A is represented by $(r, \theta, \phi)$

$\boxed{\bar{R}_A = r \, \bar{a}_r}$

$(0,0,0) \xrightarrow{\bar{R}_A} A(r, \theta, \phi)$
origin