"jai shree Ganeshay Namah"

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Number Line. (1)

$-\infty$ --- -4 -3 -2 -1 0 1 2 3 4 --- ∞

Types of Numbers.

① Natural no. (N)

1, 2, 3, 4, 5, 6, 7, 8, 9

② Whole Numbers (W):

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

③ Integers (I or Z)

$-\infty$, ---- , -2, -1, 0, 1, 2, ---, ∞

Numbers without fractional components.

④ Real Numberys: (R) :

$-\infty$ ---- -2.0, -1.5, -1.0, -0.5, 0, 0.5, 1.0, 1.5, 2.0 --- ∞

Numbers with fractional value i.e, decimal value

R^+ : 0 and positive real numbers

R^- : Negative real numbers

Basic operations of Maths :-

① Addition

② Substraction

③ Multiplication

④ Division

$\bigcirc \rightarrow$ Larger Magnitude $\quad$ $\circ \rightarrow$ Smaller Magnitude

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1 Rules of Addition

a) $(+) + (+) = (+)$ $\quad$ Eg $\rightarrow$ $2+2=4$

b) $(-) + (-) = (-)$ $\quad$ Eg $\rightarrow$ $-2+(-2)=-4$

c) $(+) + (-) = (+)$ $\quad$ Eg $\rightarrow$ $4+(-3)=1$

d) $(+) + (-) = (-)$ $\quad$ Eg $\rightarrow$ $4+(-6)=-2$

2 Substraction :-

a) $(+) - (+) = (-)$ $\quad$ Eg $\rightarrow$ $2-4=-2$

b) $(+) - (+) = (+)$ $\quad$ Eg $\rightarrow$ $6-3=3$

c) $(+) - (-) = (+)$ $\quad$ Eg $\rightarrow$ $3-(-3)=3+3=6$

3 Multiplication :-

Examples

a) $(+) \times (+) = (+)$ $\quad$ $2\times 3 = 6$

b) $(+) \times (-) = (-)$ $\quad$ $2\times(-3) = -6$

c) $(-) \times (-) = (+)$ $\quad$ $(-2)\times(-3) = 6$

d) $(-) \times (+) = (-)$ $\quad$ $(-2)\times 3 = -6$

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(4) Division :-

Examples

a) $\frac{(+)}{(+)} = (+)$ $\frac{6}{3} = 2$

b) $\frac{(-)}{(+)} = (-)$ $\frac{-6}{3} = -2$

c) $\frac{(+)}{(-)} = (-)$ $\frac{6}{3} = -2$

d) $\frac{(-)}{(-)} = (+)$ $\frac{-6}{-3} = 2$

BODMAS Rule for Simplification of Expression

Bracket	$(x+y)$	$(2+3)$
POWER	(x^y)	2^3
Division	(x/y)	$2/3$
Multiplication	$(x \times y)$	2×3
Addition	$(x+y)$	$2+3$
Substraction	$(x-y)$	$2-3$

Ex-1 Solve $= 3\times(5+2)$

$= 3\times 7$

$= 21$ A

(2) Solve $3^2 + (5-7)\times 3 + 2$

$= 3^2 + (-2)\times 3 + 2$

$= 9 - 6 + 2$

$= 5$ A

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(3) $25-[20-\{10-(7-\overline{5-3})\}]$

$= 25-[20-\{10-(7-2)\}]$

$= 25-[20-\{10-5\}]$

$= 25-[20-5]$

$= 25-15 = 10$ Ans

(4) $[72-12\div 3]+(18-6)\div 4$

$= [72-4]+(12)\div 4$

$= 68+3 = 71$ Ans

(5) Find value of x

$42\div 2+x\times 3-22=8$

$\Rightarrow 21+3x-22=8$

$\Rightarrow 3x = 8+22-21$

$\Rightarrow 3x = 30-21$

$\Rightarrow 3x = 9$

$\Rightarrow x = \frac{9}{3}$

$\Rightarrow x = 3$ Ans

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functions
Relations

function:-
It is a relation b/w two or more variables / constant.
for eg → (a) $Y = 3x^2+1$, (b) $Y = 7x$, (c) $3y = \frac{x^2+1}{5}$

Constant :- A value or number which never changes in an expression it remains fixed.
for eg → (a) $Y = 3x+5$ here 5 is a constant (b) $Y = 5$ here Y is constant with value 5.

Variable :- A quantity that changes within the context of mathematical problem.
Ex: A father is 30 years older than his son.
Let age of son be x and age of father be Y.
$\therefore Y = x + 30$
here x and y both are variable.

(b) $Y = 7x^2 + x + 5$
here Y is a variable and x is also a variable

Types of Variables

(a) **Independent variables :** Variables whose value does not depend on other variable and can be choosen in context of mathematical problem.
for eg → $Y = x+5$, where $x \in N$ i.e, x is a natural no. $x \in \{1,2,3,4,5,6,7,8,9\}$

•→ Multiply

(6)

(b) Dependent Variable :- Variables whose value depend on another variable in a function is called dependent variable.

for eg → $Y = x + 2$ where, $x \in N$

Here Y is dependent variable whose value depend on variable x.

Ex :- A father is 30 years older than his son

$Y = x + 30$ where

Independent variable → x → age of son

dependent variable → Y → age of father

Some Examples of functions

Eg 1 $Y = 2x + 1$

Independent →	X	-3	-2	-1	0	1	2	3
dependent →	Y	-5	-3	-1	1	3	5	7

Eg-2 $Y = x^2 - 1$

X	-3	-2	-1	0	1	2	3
Y	8	3	0	-1	0	3	8

Ex-3 $Y = 2x^2 - x + 1$

x	-3	-2	-1	0	1	2	3
Y	22	11	4	1	2	7	16

for $x = 0$ $\quad y = 2x^2 - x + 1$
$= 2 \cdot 0^2 - 0 + 1 = 1$

$x = 1$ $\quad y = 2x^2 - x + 1$
$= 2 \cdot 1^2 - 1 + 1$
$= 2$

$x = 2$ $\quad y = 2x^2 - x + 1$
$= 2 \cdot 2^2 - 2 + 1$
$= 2 \cdot 4 - 2 + 1$
$= 8 - 2 + 1$
$= 7$

$x = 3$ $\quad y = 2x^2 - x + 1$
$= 2 \cdot 3^2 - 3 + 1$
$= 2 \cdot 9 - 3 + 1$
$= 18 \cdot 3 + 1$
$= 16$

$x = -1$ $\quad y = 2(x^2) - x + 1$
$= 2(-1)^2 - (-1) + 1$
$= 2 \cdot 1 + 1 + 1$
$= 4$

$x = -2$ $\quad y = 2x^2 - x + 1$
$= 2(-2)^2 - (-2) + 1$
$= 2 \times 4 + 2 + 1$
$= 11$

$x = -3$ $\quad y = 2x^2 - x + 1$
$= 2(-3)^2 - (-3) + 1$
$= 2 \times 9 + 3 + 1$
$= 22$

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Domain of a function :- Domain of a function is the set of permissible values of independent variable.

Range of a function :- Range of a function is the set of values of dependent variable obtain after putting every possible value of independent variable from the domain into the function.

Powers of two (2)

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$
$2^{11} = 2048$
$2^{12} = 4096$
$2^{13} = 8192$
$2^{14} = 16384$
$2^{15} = 32768$

H.C.F and L.C.M

→ HCF of two or more no. is the greatest no. which divides all the numbers.

Example:- HCF of 36 & 96

2	36
2	16
2	8
2	4
2	2
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

$96 = 2\times2\times2\times2\times2\times3$

$36 = 2\times2\times3\times3$

$\text{HCF} = 2\times2\times3$

$= 12.$

Q① find HCF & LCM of 48, 96, 128

2	48
2	24
2	12
2	6
3	3
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$48 = 2\times2\times2\times2\times3$

$96 = 2\times2\times2\times2\times2\times3$

$128 = 2\times2\times2\times2\times2\times2\times2$

$\text{HCF} = 2\times2\times2\times2 = 16$

$\text{LCM} = 2\times2\times2\times2\times2\times3\times2\times2 = 384$

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Q. Find HCF & LCM of 512, 768, 1024

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$HCF = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 = 3072$

Q. Find HCF & LCM of $3^3 \times 2^4$, $9^2 \times 16$, 18×2^2

$3^3 \times 2^4$, $3^4 \times 2^4$, $3^2 \times 2^3$

$HCF = 3^2 \times 2^3 = 72$

$LCM = 3^2 \times 2^3 \times 3 \times 3 \times 2 = 1296$

Basic Maths formulae

① $(a+b)^2 = a^2 + 2ab + b^2$

$LHS = (a+b)^2 = (a+b)(a+b)$

$= a(a+b) + b(a+b) = a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2 = RHS$

② $(a-b)^2 = a^2 - 2ab + b^2$

$L.H.S = (a-b)^2 = (a-b)(a-b)$

$= a(a-b) - b(a-b) = a^2 - ab - ba + b^2$

$= a^2 - 2ab + b^2 = R.H.S$

③ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$L.H.S = (a+b+c)^2 = (a+b+c)(a+b+c)$

$= a(a+b+c) + b(a+b+c) + c(a+b+c)$

$= a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$

④ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$L.H.S = (a+b)^3 = (a+b)(a+b)^2$

$= (a+b)(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$= a^3 + b^3 + 3ab(a+b)$

⑤ $(a-b)^3 = (a-b)(a^2 - 2ab + b^2)$

$= a^3 - b^3 - 3ab(a-b)$

$L.H.S = (a-b)^3 = (a-b)(a-b)^2$

$= (a-b)(a^2 - 2ab + b^2)$

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$= a(a^2 - 2ab + ab^2) - b(a^2 - 2ab + b^2)$

$= a^3 - 2a^2b + ab^2 - ba^2 + 2ab^2 - b^3$

$= a^3 - b^3 - 3a^2b + 3ab^2$

$= a^3 - b^3 - 3ab(a-b) \quad = RHS.$

(6) $a^2 - b^2 = (a+b)(a-b)$

RHS $= (a+b)(a-b)$

$= a^2 - ab + ba - b^2$

$= a^2 - b^2 = LHS$

(7) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

RHS $= (a-b)(a^2 + ab + b^2)$

$= a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3$

$= a^3 - b^3 \quad = RHS$

(8) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

RHS $= (a+b)(a^2 - ab + b^2)$

$= a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3$

$= a^3 + b^3 = RHS.$

(9) $a^n = a \times a \times a \times \cdots \times a$ (n times)

Eg → $a^4 = a \times a \times a \times a$

Eg → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(10) $a^m \times a^n = a^{m+n}$

Ex: $a^2 \times a^3 = a^{2+3} = a^5$

Ex: $2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$

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(11) (a) $\frac{a^m}{a^n} = a^{m-n}$ if $\underline{m > n}$

Ex: $\frac{a^7}{a^4} = a^{7-4} = a^3$

Ex:- $\frac{2^7}{2^5} = 2^{7-5} = 2^2 = 4$

Ex = $\frac{256}{64} = 4 = \frac{2^8}{2^6} = 2^{8-6} = 2^2 = 4$

(b) $\frac{a^m}{a^n} = \frac{a^m}{a^m} = 1$ if $\underline{m = n}$

Ex: $\frac{2^5}{2^5} = \frac{32}{32} = 1$

(c) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if $\underline{m < n}$

Ex: $\frac{a^4}{a^7} = a^{4-7} = a^{-3} = \frac{1}{a^3}$

$\frac{a^4}{a^7} = \frac{1}{a^{7-4}} = \frac{1}{a^3}$

Ex:- $\frac{2^4}{2^7} = \frac{1}{2^{7-4}} = \frac{1}{2^3}$

or

$\frac{2^4}{2^7} = 2^{4-7} = 2^{-3} = \frac{1}{2^3}$

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(12) $(ab)^n = a^n \times b^n$

$(2\times3)^4 = 2^4 \times 3^4 = 16 \times 81$

(13) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$Ex = \left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3}$

(14) $a^0 = 1$ where $a \in R,\ a \neq 0$

(15) $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$

$Ex = 2^{-3} = \frac{1}{2^3}$, $2^3 = \frac{1}{2^{-3}}$

(16) $(a-b-c)^2 = a^2+b^2+c^2-2ab-2ac+2bc$

$= (a-b-c)(a-b-c)$

$= a(a-b-c) - b(a-b-c) - c(a-b-c)$

$= a^2 - ab - ac - ba + b^2 + bc - ca + cb + c^2$

$= a^2 + b^2 + c^2 - 2ab - 2ac + 2bc.$

(17) $(a^m)^n = a^{mn}$

$Ex \rightarrow (2^3)^3 = 2^{3\times3} = 2^9 = 512$

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Equations

\# $2x - 4 = 0$

$$2x = 4$$

$$x = \frac{4}{2} = 2$$

\# Types of Equations :-

1. Linear equation. ~~in two variables~~
2. Quadratic equation.
3. Cubic Equation.
4. Polynomial Equation.

Linear equation in one variable

Ex 1) $5x - 9 = -3x + 19$

$$5x + 3x = 19 + 9$$

$$8x = 28$$

$$x = \frac{28}{8} = \frac{7}{2} \text{ Ans}$$

Ex 2) $4(x-3) + 12 = 15 - 5(x+6)$

$$4x - 12 + 12 = 15 - 5x - 30$$

$$4x = 15 - 5x - 30$$

$$4x = -5x - 15$$

$$4x + 5x = -15$$

$$9x = \cancel{30} - 15$$

$$x = \frac{-15}{9} = \frac{-5}{3}$$

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Ex-3 $4x - 11 - x = 2 + 2x - 20$

$3x - 2x = -18 + 11$

$x = -7$ A

Ex-4 $7(3x - 11) + 4x = 8(x-4) + 2(x-1)$

$21x - 77 + 4x = 8x - 32 + 2x - 2$

$25x - 77 = 10x - 34$

$25x - 10x = -34 + 77$

$15x = 43$

$x = \frac{43}{15}$

* Linear equation in two Variables

(i) $10x - 3y = 5$ — (i)

$2x + 4y = 7$ — (ii)

(ii) $\times 5$

$\cancel{10x} + 20y = 35$

$-\cancel{10x} + 3y = -5$

$23y = 30$

$y = \frac{30}{23}$

Put $y = \frac{30}{23}$ in (i)

$10x - 3 \times \frac{30}{23} = 5$

$230x - 90 = 115$

$230x = 115 + 90$

$230x = 205$

$x = \frac{205}{230} = \frac{41}{46}$ A

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(2) $8x + 7y = 38$ — (I)

$3x - 5y = -1$ — (II)

By Elimination method.

(I) × 3 $\quad \cancel{24x} + 21y = 114$

(II) × 8 $\quad -\cancel{24x} + 40y = +8$

$$61y = 122$$

$$y = \frac{122}{61} = 2$$ Put in (II)

$$3x - 5(2) = -1$$

$$3x - 10 = -1$$

$$3x = -1 + 10$$

$$3x = 9$$

$$x = \frac{9}{3} = 3$$ A

II method → Substituting Method

from eqn (II)

$$3x - 5y = -1$$

$$3x = 5y - 1$$

$$x = \frac{1}{3}(5y - 1)$$ Put in (I)

$$8 \times \frac{1}{3}(5y - 1) + 7y = 38$$

$$8(5y - 1) + 21y = 114$$

$$40y - 8 + 21y = 114$$

$$61y = 122$$

$$y = \frac{122}{61} = 2$$ ~~Put in~~

$$x = \frac{1}{3}(5(2) - 1)$$

$$x = \frac{1}{3}(9) = 3$$ A

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Q1) $4x - 3y = 8$ — (i)

$x - 2y = -3$ — (ii)

(ii) $\times 4$

$$\cancel{4x} - 8y = -12$$
$$-4x + 3y = -8$$

$$-5y = -20$$
$$\boxed{y = 4}$$ Put in (ii)

$$x - 2(4) = -3$$
$$x - 8 = -3$$
$$\boxed{x = 5}$$

(2) $8x + 5y = 9$ — (i)

$3x + 2y = 4$ — (ii)

(i) $\times 2$ $\quad 16x + \cancel{10y} = 18$

(ii) $\times 5$ $\quad 15x + \cancel{10y} = 20$

$- \quad - \quad -$

$$\boxed{x = -2}$$ Put in (ii)

$$3(-2) + 2y = 4$$
$$-6 + 2y = 4$$
$$2y = 10$$
$$\boxed{y = 5}$$

(3) $x + y = 5$ — (i)

$3x - 4y = 1$ — (ii)

(i) $\times 3$

$$\cancel{3x} + 3y = 15$$
$$-\cancel{3x} + 4y = -1$$

$$7y = 14$$
$$\boxed{y = 2}$$ Put in (i)

$$x + 2 = 5$$
$$\boxed{x = 3}$$

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② Quadratic Equations

$$ax^2 + bx + c = 0$$

eg → $5x^2 + 15x + 3 = 0$

$a = 5$, $b = 15$, $c = 3$

~~These equations solve by 2 two methods~~

→ quadratic eqn are equations of second degree.
→ quadratic eqn can be of one or two variables
→ quadratic eqn in one variable eg → $3x^2 + 5x + 2 = 0$

→ Standard form of quadratic equation is

$$\boxed{ax^2 + bx + c = 0}$$ where $a \neq 0$

→ quadratic eq. in two variables eg → $2x^2 - 5xy - 3y^2 = 0$

→ Standard form is

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

→ we will focus on quad. eqn in one variable
i.e. $ax^2 + bx + c = 0$

→ Methods of solving ~~quad~~ quad. eqn.

① Factorization ② Quadratic formula.

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(1) Factorization Method.

$$ax^2 + bx + c = 0$$

(1) Make factors of $a \times c$ i.e ac
Let a_1, a_2 be the factors of ac
if c is +ve then $a_1 + a_2 = b$

if c is -ve then $a_1 - a_2 = b$

(2) Put $a_1 + a_2$ or $a_1 - a_2$ in place of b and solve for x.

Ex-1 $3x^2 + 5x - 2 = 0$

$a = 3$, $b = 2$, $c = -2$ (-ve)

$\therefore$ factors of 3×2 whose difference is 5 are 6 and 1

Now $5x = 6x - x$

$\therefore$ $3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - 1(x+2) = 0$

$(3x-1)(x+2) = 0$

Now $3x - 1 = 0$ or $x + 2 = 0$

$\Rightarrow 3x = 1$ $\quad x = -2$

$x = \frac{1}{3}$ A

(2) Direct formula or "Quadratic formula"

(Roots of Q E)

The solution of quadratic ~~formula~~ equation $ax^2 + bx + c = 0$ is

$$\Rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

(Roots of Q.E)

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Ex-1 $3x^2 + 7x + 4 = 0$

$a = 3, \ b = 7, \ c = 4$

put in formula

$\Rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-7 \pm \sqrt{7^2 - 4 \times 3 \times 4}}{2 \times 3}$

$\Rightarrow \quad x = \dfrac{-7 \pm \sqrt{49 - 48}}{6}$

$\Rightarrow \quad x = \dfrac{-7 \pm \sqrt{1}}{6}$

$\Rightarrow \quad x = \dfrac{-7 \pm 1}{6}$

$\Rightarrow \quad x = \dfrac{-7+1}{6}$ or $x = \dfrac{-7-1}{6}$

$\Rightarrow \quad x = \dfrac{-6}{6}$ or $x = \dfrac{-8}{6}$

$\Rightarrow \quad x = -1$ or $x = \dfrac{-4}{3}$ A

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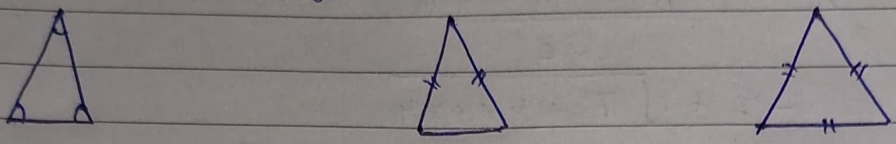
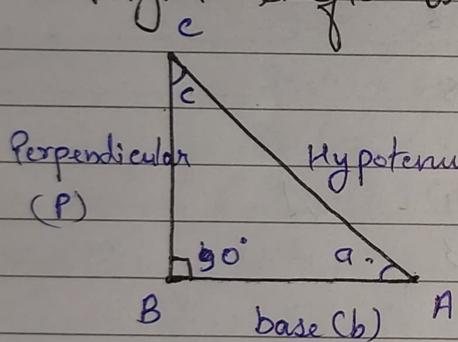
TRIGNOMETRY.

3 types of Triangles.



Scalant Triangle	Isosceles Triangle	Equilateral Triangle.
{all sides are different }	{ Two sides are equal }	{ All sides are equal }

Right Angle Triangle



$\angle B = \angle \theta$

☆ Pythagoras Theorem

$$(P)^2 + (B)^2 = (H)^2$$

Formula of

① $\sin\theta = \frac{P}{H}$

② $\cos\theta = \frac{B}{H}$

③ $\tan\theta = \frac{P}{B}$

④ $\cot\theta = \frac{1}{\tan\theta} = \frac{B}{P}$

⑤ $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{H}{P}$

⑥ $\sec\theta = \frac{1}{\cos\theta} = \frac{H}{B}$

☆ Proof of identity $\sin^2\theta + \cos^2\theta = 1$ by Pythagoras theorem

<u>LHS</u> $\sin^2\theta + \cos^2\theta$

$\Rightarrow \left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2$

$\Rightarrow \frac{P^2}{H^2} + \frac{B^2}{H^2}$

$\Rightarrow \frac{P^2 + B^2}{H^2}$

$\Rightarrow \frac{P^2 + B^2}{H^2}$ {by Pythagoras theorem

P

$\Rightarrow \frac{H^2}{H^2}$

$\Rightarrow 1 = RHS$