WAVE OPTICS

→ **Optics** :- This is a breach of physics in which we study about light or phenomenon related to light

Geometrical Optics (Ray optics)
↳ Basis on the particle nature of light

Physical optics (Wave optics)
↳ Basis on the wave nature of light

• **Interference** :- Due to superposition of two coherent light wave of same frequency & nearly equal amplitude the intensity of light is redistributed in space. This phenomenon is known as interference.
The intensity distribution @ any point in space is the resultant of the individual intensities @ that pt.

Types of Interference

• Constructive Interference
→ When two waves are superimposed in the same phase i.e. Phase difference b/w them is integral multiple of 2π.
→ $\delta = 2n\pi$
$n = 0, 1 \ldots$

• Destructive Interference
→ When two waves are superimposed in opposite phase i.e. Phase difference b/w them is odd multiple of π.
→ $\delta = (2n+1)\pi$
$n = 0, 1 \ldots$

$\delta = (2n-1)\pi$
$n = 1, 2 \ldots$

→ Condition for Interference:-

We know that intensity due to superposition of two waves is given by.

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

δ is phase difference b/w waves.

→ Condition for contructive Interference: $(\delta = 2n\pi)$

$$\boxed{\Delta = \frac{\lambda}{2\pi}\cdot\delta}$$

Path diff. Phase diff.

where $n = 0, 1, 2, 3, 4 \ldots$

$\delta = 0, 2\pi, 4\pi, 6\pi \ldots$

$\Delta = 0, \lambda, 2\lambda, 3\lambda \ldots$

$$\boxed{I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}}$$

where $\cos\delta = 1$

$$\Delta = \frac{\lambda}{\cancel{2\pi}}\cdot\cancel{2}n\cancel{\pi}$$

$$\boxed{\Delta = n\lambda}$$

→ Condition for destructive Interference: $(\delta = (2n+1)\pi)$

$$\Delta = \frac{\lambda}{2\cancel{\pi}}\cdot(2n+1)\cancel{\pi}$$

where $n = 0, 1, 2, 3 \ldots$

$\delta = \pi, 3\pi, 5\pi \ldots$

$\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$

$$\boxed{\Delta = (2n+1)\frac{\lambda}{2}}$$

$$\boxed{I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

where $\cos\delta = -1$

• **Coherent Sources:-** Two sources are said to be coherent if they emit light waves of same frequency, nearly same amplitude & const. phase difference.
For the interference process, the coherent source is essential.

→ **Methods to obtain Co-herent source:-**

① By division of wavefront
② By division of Amplitude

1. **By Division of wavefront** → The incident wavefront is divided into two parts. These two parts then travel & superimpose each other & produce interference pattern.
Eg. Young's double-slit, Fresnel's bi-prism.

2. **By Division of Amplitude** → The intensity of the incident light is divided into two parts. These lights beams superimpose afterward & produce interference pattern.
Eg. Newton's Ring Experiment, Michelson's Interferometer.

⇒ **Condition for Interference:-**

→ The light source must be monochromatic i.e, the wavelength or frequency must be same.
→ The light source must be coherent.
→ The amplitude for both waves should must be nearly equal for good contrast b/w fringes.
→ The waves must propogate in same derection

① Reflected System:-

The reflected set of beam R_1 & R_2 come from coherent source so they will interfere

Path diff. (Δ) = path travelled by R_2 − path travelled by R_1

$$\Delta = \mu(BC + BD) - BN$$

In ΔCMB, $\cos r = \frac{CM}{BC} \Rightarrow \boxed{BC = \frac{t}{\cos r}}$

In ΔCMD, $\cos r = \frac{CM}{CD} \Rightarrow \boxed{CD = \frac{t}{\cos r}}$

In ΔBDN, $\sin i = \frac{BN}{BD}$

$BN = \sin i (BD)$

from ΔCMB, $\frac{BM}{CM} = \tan r$

$BM = t \tan r$

from ΔCMD, $\frac{MD}{CM} = \tan r$

$MD = t \tan r$

$BN = \sin i (BM + MD)$

$\boxed{BN = 2t \sin i \tan r}$

$\boxed{BN = 2t\mu \sin^2 r / \cos r}$ (∵ from Snell's law

Path difference (Δ) $= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$

$\boxed{\Delta = 2\mu t \cos r}$

Since, the reflected ray (R_1) is reflected from denser medium (@B) therefore, there occurs an additional path difference of $\frac{\lambda}{2}$ or phase change of π, according to stoke's theorem.

$$\boxed{\text{The effective path diff } (\Delta) = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

→ Condition of Maxima & Minima:-

· For Maxima (Bright bands):→ $2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$

$$\boxed{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}}$$

→ (+) $n = 0, 1, 2 ----$

→ (−) $n = 1, 2, 3 ----$

· For Minima (Dark bands):→ $2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$$\boxed{2\mu t \cos r = n\lambda}$$

→ $n = 0, 1, 2, 3 ----$

Note:- For accessible thin film $\Delta = 2\mu \underset{\approx 0}{\cancel{t \cos r}} \pm \frac{\lambda}{2}$

$$\boxed{\Delta = \pm \frac{\lambda}{2}}$$ ← Minima (Dark)

② Refracted System:-

The path difference b/w transmitted rays T_1 & T_2 is given by.

Path difference (Δ) = Path travelled by T_2 - Path travelled by T_1

$$\Delta = \mu(CD+DE) - CK$$

From ΔCDL & ΔLDE

$$CD = DE = \frac{LD}{\cos r} = \frac{t}{\cos r}$$

From ΔCEK

$$CK = CE \sin i$$
$$= (CL+LE)\sin i$$
$$= (LD\tan r + LD\tan r)\sin i$$
$$= 2t\tan r \sin i$$

$$CK = \frac{2\mu t \sin^2 r}{\cos r} \quad (\because \text{From Snell's law})$$

Hence, $$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$\boxed{\Delta = 2\mu t \cos r}$ ← The effective path difference

The transmitted beam T_2 is reflected at C & D before transmission from rarer medium, therefore, there will be no additional path diff. of $\frac{\lambda}{2}$ or phase change of π, according to Stockes law.

→ Condition of Maxima & Minima:-

- For maxima (Bright bands) :→ $\boxed{2\mu t \cos r = n\lambda}$
  ↳ $n = 0, 1, 2, 3 \ldots$
- For minima (Dark bands) :→ $\boxed{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}}$
  ↳ (+) $n = 0, 1, 2 \ldots$
  ↳ (−) $n = 1, 2, 3 \ldots$

Note: The reflected & transmitted interference patterns complimentary.

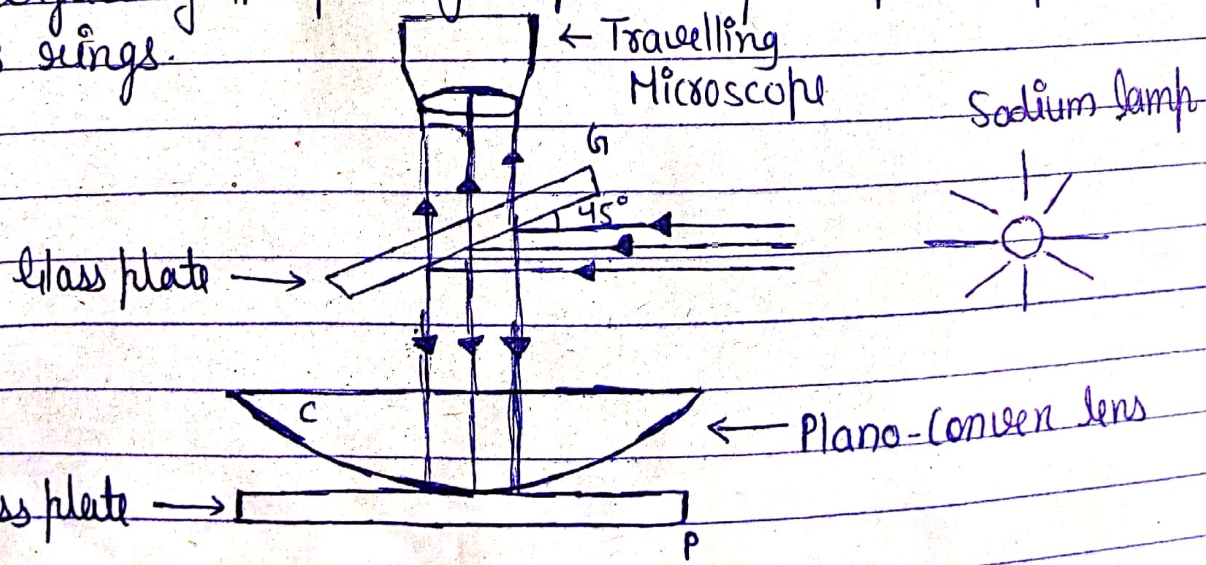
⇒ Newton's Ring Experiment:-

→ What are Newton's Rings?

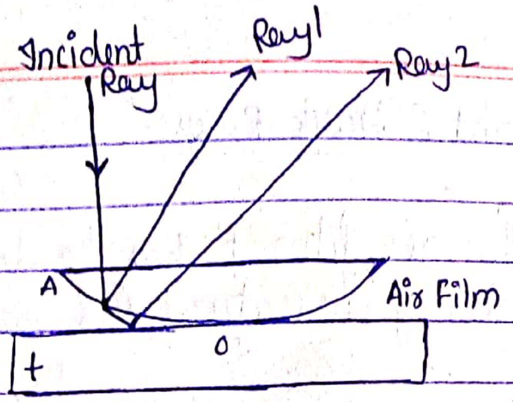
When a plano-convex lens of large radius of curvature is placed with its convex surface in contant with plane glass plate, an air film is enclosed b/w the upper surface of the plate & lower surface of the lens. If a monochromatic light is allowed to fall normally on this film then alternate bright & dark concentric rings with the dark centre is formed. These rings are known as Newton's Rings.

→ Experimental Setup:-

- In the experimental set-up, A plano-convex lens L of large radii of curvature is place in contact with a plane glass plate P.
- Light from a monochromatic source (Sodium lamp) is allowed to fall on a glass plate G inclined at an angle of 45° to the incident beam.
- Light reflected from the lower surface of plano-convex lens & the upper surface of the plane glass plate superimpose & produces Newton's rings.



① Reflected System:-



The path difference b/w R1 & R2 is given by $\Delta = 2\mu t\cos(r) \pm \frac{\lambda}{2}$

For air film $\mu = 1$, For normal incidence i & $r = 0$

Hence, $\boxed{\Delta = 2t \pm \frac{\lambda}{2}}$

• Condition for bright rings:⇒ $\Delta = n\lambda$

$$2t \pm \frac{\lambda}{2} = n\lambda$$

$$\boxed{2t = (2n \pm 1)\frac{\lambda}{2}}$$

↳ (+) $n = 0, 1, 2 \ldots$

↳ (−) $n = 1, 2, 3 \ldots$

• Condition for dark rings:→ $\Delta = (2n \pm 1)\lambda/2$

$$2t \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\boxed{2t = n\lambda}$$

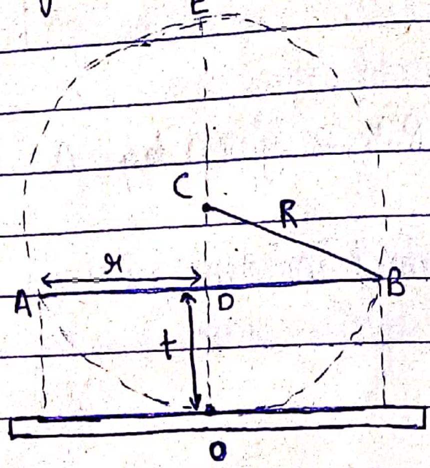
↳ $n = 0, 1, 2, 3 \ldots$

Note:- • At the point of contact $t = 0$, path difference is $\Delta = \frac{\lambda}{2}$, (condition for minima). So, the centre remains dark.

• From path difference itself it is clear that the fringes will depend on 't' & 'λ'. The air is symmetrical about the pt. of contact, Therefore, the fringes are circular.

→ Diameters of Bright & Dark Rings:-

The thickness of the air film 't' has to determined. This can be done in terms of radius of curvature R & radius of the rings formed, r.



From the geometrical property of the circle, AD × DB = OD × DE

$$r \times r = OD\,(2R - OD)$$

$$r^2 = 2Rt - t^2$$

$$r^2 = 2Rt \qquad (t^2 <<< R)$$

$$\boxed{t = \frac{r^2}{2R}}$$

· For bright rings :→ $2t = (2n \pm 1)\,\lambda/2$

$$r^2 = (2n+1)\frac{\lambda R}{2} \qquad (+\ \text{sign})$$

If r_n is radius of n^{th} bright ring, then

$$r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}$$

$$D_n = \sqrt{(2n+1)\,2\lambda R}$$

$$\boxed{D_n \propto \sqrt{2n+1}}$$

[Thus, the diameter for bright ∝ root of odd natural number

- For dark ring:→ $2t = n\lambda$

$$r^2 = n\lambda R$$

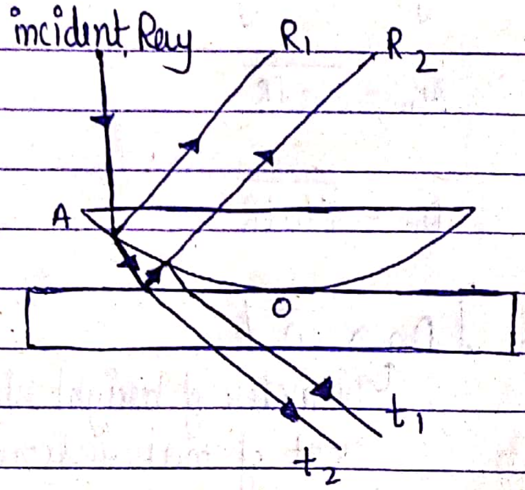
$$r_n = \sqrt{n\lambda R}$$

$$D_n = \sqrt{4n\lambda R}$$

$$\boxed{D_n \propto \sqrt{n}}$$

Thus, The diameter for dark rings is proportional to root of natural number.

② Refracted System:-



The path diff. b/w T_1 & T_2 $(\Delta) = 2\mu t \cos r$

For air film $\mu = 1$, $i = r = 0$

Hence, $\boxed{\Delta = 2t}$

Note:- At centre $t=0$, $\Delta = 0$, which is condition for constructive interference. Thus the centre will be bright for transmitted system

- Condition for Bright ring:→ $\Delta = n\lambda$

$$\boxed{2t = n\lambda} \leftarrow n = 0, 1, 2, 3 \ldots$$

- Condition for dark ring:→ $\Delta = (2n \pm 1)\lambda/2$

$$\boxed{2t = (2n \pm 1)\frac{\lambda}{2}}$$

← (+) $n = 0, 1, 2 \ldots$

← (−) $n = 1, 2, 3 \ldots$

→ **Diameter Bright & Dark Rings:-**

$$t = \frac{r^2}{2R}$$

· For Bright rights:- $2t = n\lambda$

$$r^2 = n\lambda R$$

If r_n is the radius of n^{th} bright ring, then

$$r_n = \sqrt{n\lambda R}$$

$$D_n = \sqrt{4n\lambda R}$$

$$\boxed{D_n \propto \sqrt{n}}$$

Diameter of bright rings are proportional to root of natural number.

· For Dark rings:- $2t = (2n \pm 1)\frac{\lambda}{2}$

$$r^2 = (2n+1)\frac{\lambda R}{2}$$

If r_n is the radius of n^{th} dark ring,

$$r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}$$

$$D_n = \sqrt{2(2n+1)\lambda R}$$

$$\boxed{D_n \propto \sqrt{(2n+1)}}$$

Diameter is dark ring are proportional to root of odd natural number.

→ Application of Newton's Ring Experiment:-

⇒ Determination of wavelength of sodium light using Newton's Rings Experiment:→

· When the rings are obtained in Newton's Rings experiment, the diameter of these rings can be measure with the help of travelling microscope.
· Let the diameter of the n^{th} dark ring, then.

$D_n^2 = 4n\lambda R$

& the diameter of the $(n+p)^{th}$ dark ring, will be

$D_{n+p}^2 = 4(n+p)\lambda R$ {P is integer}

· From the above Eq^n

$$D_{n+p}^2 - D_n^2 = 4\lambda R[(n+p)-n]$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

· Procedure to measure the diameters of rings:

After focusing the crosswire on the eye piece the microscope is moved extreme left of rings system & the crosswire is adjusted tangentially on any bright or dark ring at its outer edge & reading of micrometer is noted. On reaching the central spot crosswire further moved to the outside & take the reading of micrometer at alternate bright & dark ring.

· Procedure to measure the radius of curvature (R) of plano convex lens:

The radius of curvature of lens is measured with help of spherometer.

$$\boxed{R = \frac{l^2}{6h} + \frac{h}{2}}$$

l ← distance b/w legs of spherometer
h ← convexity of lens.

⇒ Michelson Interferometer :-

→ Instruments which are based upon the principle of interference are called interferometer. Michelson defines one such interferometer that is called Michelson Interferometer.

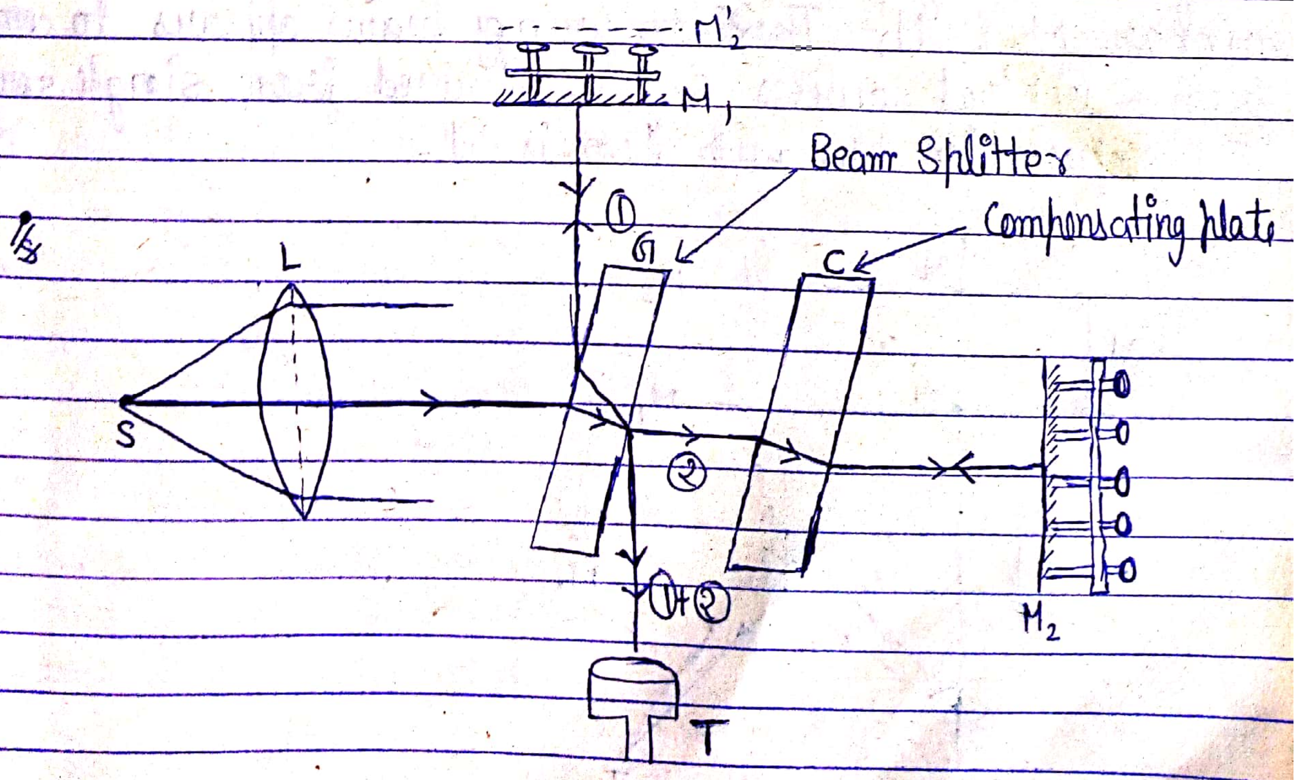
· Michelson Interferometer is used to determine :-

→ Wavelength of light

→ Thickness of thin film.

→ Construction & working :→

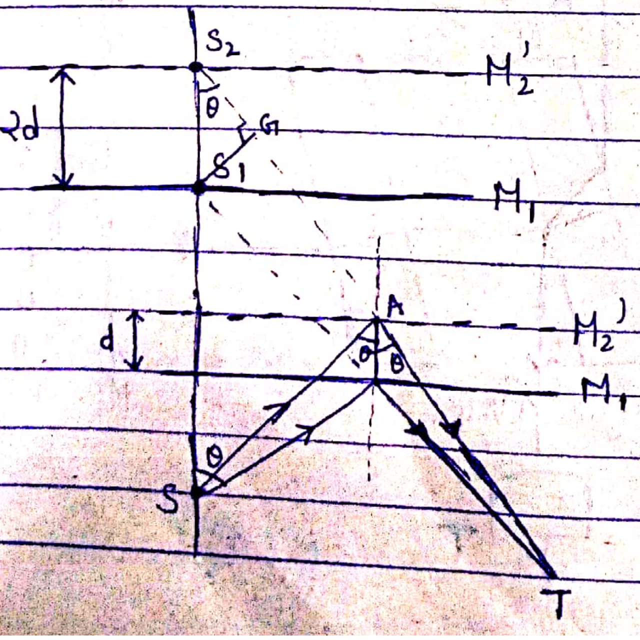
It consist of a source (S) of monochromatic light placed at the focus of convex lens L. Lens L converts light coming from source (S) with a parallel beam of light which is then allowed to fall on glass plate G (Partially silvered at its back surface) which is inclined at an angle of 45°. Half of light energy falling on plate G is reflected towards mirror M_1 in the form of beam ① & other half is transmitted towards mirror M_2 as beam ②



The beam ① after reflection from highly polished plane mirror M_2 gets transmitted through plate G towards ~~telescope~~ telescope T.
The beam ② after reflection from highly polished plane mirror M_1 gets reflected from plate G towards telescope T. These 2 rays travelling towards telescope T interfere with each other & fringes are observed by the eye looking into mirror M_1 through telescope T.
Another glass plate C of same material as that of G parallel to G is introduced in the path of beam ② b/w mirror M_2 & plate G. It ensures that beam ① & ② suffers equal no. of reflection & refraction before travelling towards telescope T. plate C is known as compensating plate.

→ Types of Fringes :-

① Circular Fringes :- When mirror M_1 is seen directly through telescope T, in addition to its virtual image M_2'. (This forms) a system equivalent to air film enclosed b/w two mirror M_1 & M_2'. The 2 interfering beams appears to come from 2 virtual sources S_1 & S_2 derived from single source S & seperation b/w with them is 2d.



To find path difference, Draw $S_1G \perp S_2A$.
In right angled ΔS_1S_2G

$$\frac{S_1G}{S_2G} = \cos\theta$$

$$\boxed{S_1G = 2d\cos\theta}$$

Due to reflection of beam ② from glass plate 'G' since it reflected through denser medium hence there will be an additional path difference of $\frac{\lambda}{2}$ is added according to stokes law.

$$\boxed{\text{The effective path difference } (\Delta) = 2d\cos\theta \pm \frac{\lambda}{2}}$$

→ For bright fringe: $\Delta x = n\lambda$

$$2d\cos\theta - \frac{\lambda}{2} = n\lambda$$

$$\boxed{2d\cos\theta = \left(n+\frac{1}{2}\right)\lambda}$$

↑ Maxima

→ For dark fringe: $\Delta x = (2l+1)\frac{\lambda}{2}$

$$2d\cos\theta - \frac{\lambda}{2} = l\lambda + \frac{\lambda}{2}$$

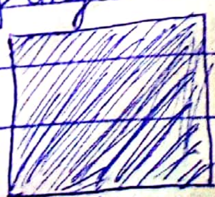
$$2d\cos\theta = (l+1)\lambda$$

$$\boxed{2d\cos\theta = n\lambda}$$

↑ Minima where $n = l+1$

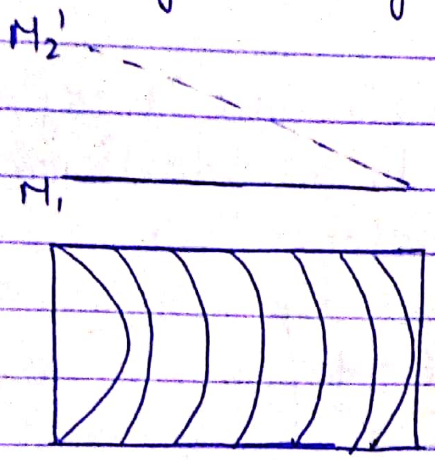
When mirror M_1 & M_2 are exactly $\perp r$ to eachother then M_1 & M_2' are ||el to each other. In this case we get circular fringes which is also known as Hardinger's fringes.

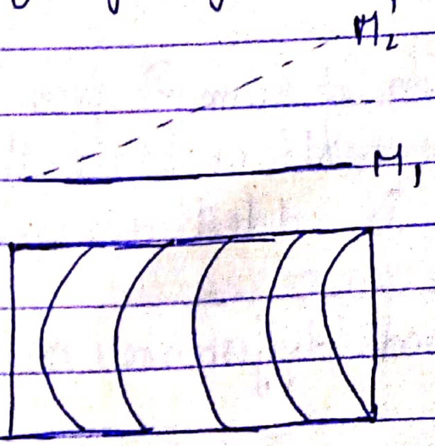
$d \neq 0$

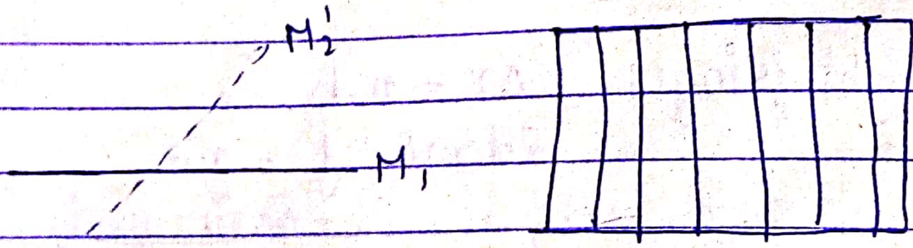


$d = 0$

(2) **Localised Fringes**:- When M_1 & M_2 are not exactly $\perp^r$ to each other therefore M_1 & M_2' are not $\parallel$el to each other. In this case the thickness of air film is varying b/w M_1 & M_2' (Wedge Shaped Film)
In such a film we get localized fringes.







(3) **Fringes in White light**:- In case of white light source the centre fringes is found to be dark & all around it there 8 to 10 coloured fringes & beyond that white light is seen due to overlapping of different colour.

• Diffraction:-

When light falls on obstacles or small aperatures whose size is comparable with the wavelength of light, the light bends rounds the corners of the obstacles & enters in geometrical shadow. This bending of light is called diffraction.

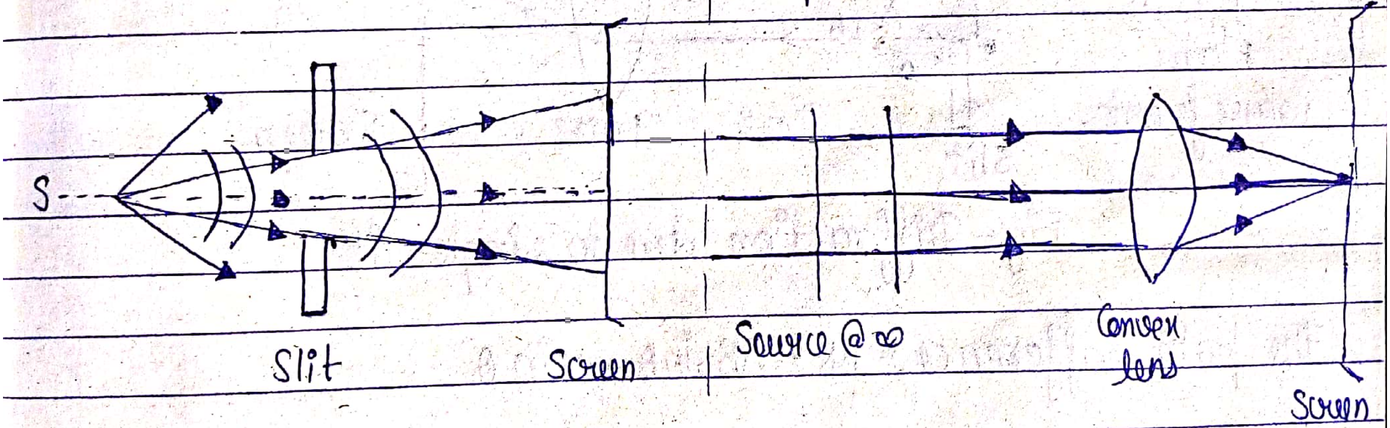
Types of Diffraction

Fresnel diffraction

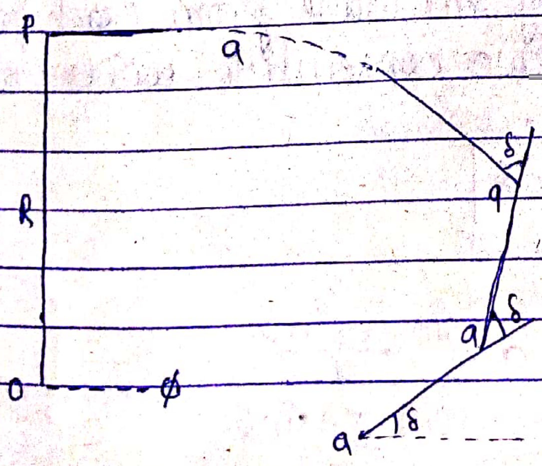
→ In this class of diffraction source & the screen are placed @ finite distance from the aperature

Fraunhofer diffraction

→ In this class of diffraction source & the screen are placed @ infinite distance from the aperature



→ **Resultant of n simple harmonic motion:-**



$$\boxed{R = \frac{a\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}}$$

→ a = amplitude

→ n = no. of light ray

→ δ = angle b/w each

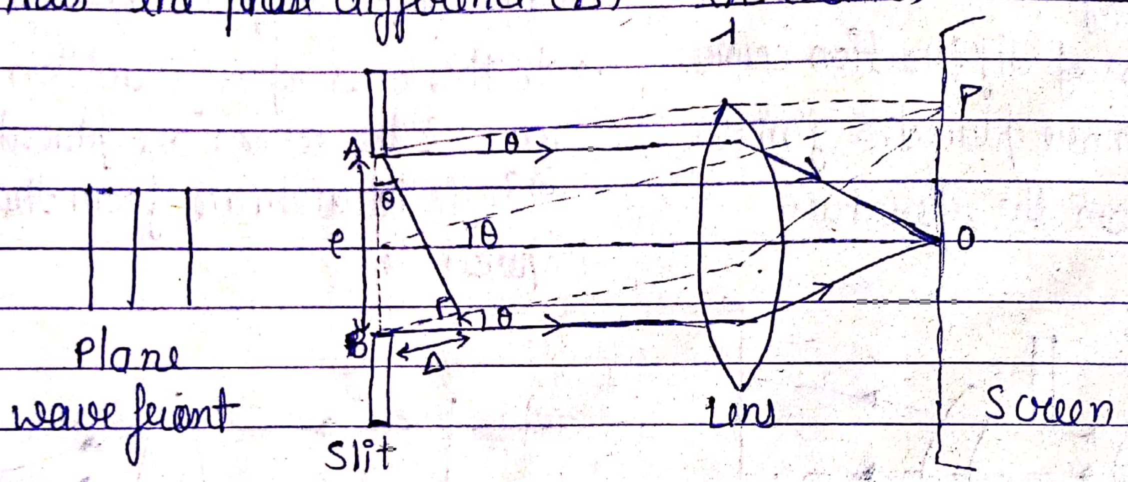
Scanned with CamScanner

⇒ Fraunhofer Diffraction Due to single slit:-

A light beam is incident normally from S on narrow slit 'AB' of width 'e'. According to Huygens wave theory, every point of the plane wave front in the plane of slit is a source of secondary wavelets, which spreads out to the right in all direction. The secondary wavelets travelling @ an angle θ with the normal are focused @ pt. P.

The path difference b/w secondary wavelets from A & B in direction θ is

Thus the phase difference $(\Delta) = \frac{2\pi}{\lambda}(e\sin\theta)$



Fig← Diffraction due to single slit.

The path difference = BK = AB$\sin\theta$ = $e\sin\theta$

Let us consider that the width of the slit is divided into n equal parts & the amplitude of the wave from each part is 'a'. The phase difference any two consecutive waves from these parts would be

$$\boxed{\frac{1}{n}\frac{2\pi}{\lambda}(e\sin\theta) = \delta}$$

$$R = a\frac{\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} = a\frac{\sin\frac{\pi e\sin\theta}{\lambda}}{\sin\frac{\pi e\sin\theta}{n\lambda}}$$

let $\frac{\pi e \sin\theta}{\lambda} = \alpha$

$R = a \frac{\sin\alpha}{\sin\frac{\alpha}{n}}$ {R = Resultant Amplitude}

for large n, $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$.

$R = a \frac{\sin\alpha}{\frac{\alpha}{n}} = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$ $(A = na)$

$$\boxed{\text{Intensity } (I) = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2}$$

→ Conditions for Maxima & Minima:

• Condition for central maximum:-

For central point on a screen → $\theta = 0$, $\alpha = 0$

But $\lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$

Thus $\boxed{I = I_0 = A^2}$

• Condition for minima:-

∵ minimum when, $\sin\alpha = 0$ & $\alpha \neq 0$

if $\sin\alpha = 0$

$\alpha = \pm m\alpha$

↳ $m = 1, 2, 3 ----$

$\alpha = \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$

$$\boxed{e\sin\theta = \pm m\lambda}$$

• **Condition for secondary Maxima:-**

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left(I_0 \frac{\sin^2\alpha}{\alpha^2}\right) = 0$$

$$I_0\left[\sin^2\alpha\left(\frac{-2}{\alpha^3}\right) + \frac{1}{\alpha^2}(2\sin\alpha\cos\alpha)\right] = 0$$

$$I_0 \frac{2\sin\alpha}{\alpha^3}\left[-\sin\alpha + \alpha\cos\alpha\right] = 0$$

Hence, $\sin\alpha = 0$ or $\sin\alpha = \alpha\cos\alpha$

pos^n of minima Intensity

The position of secondary maxima is given by.

$\alpha = \tan\alpha$

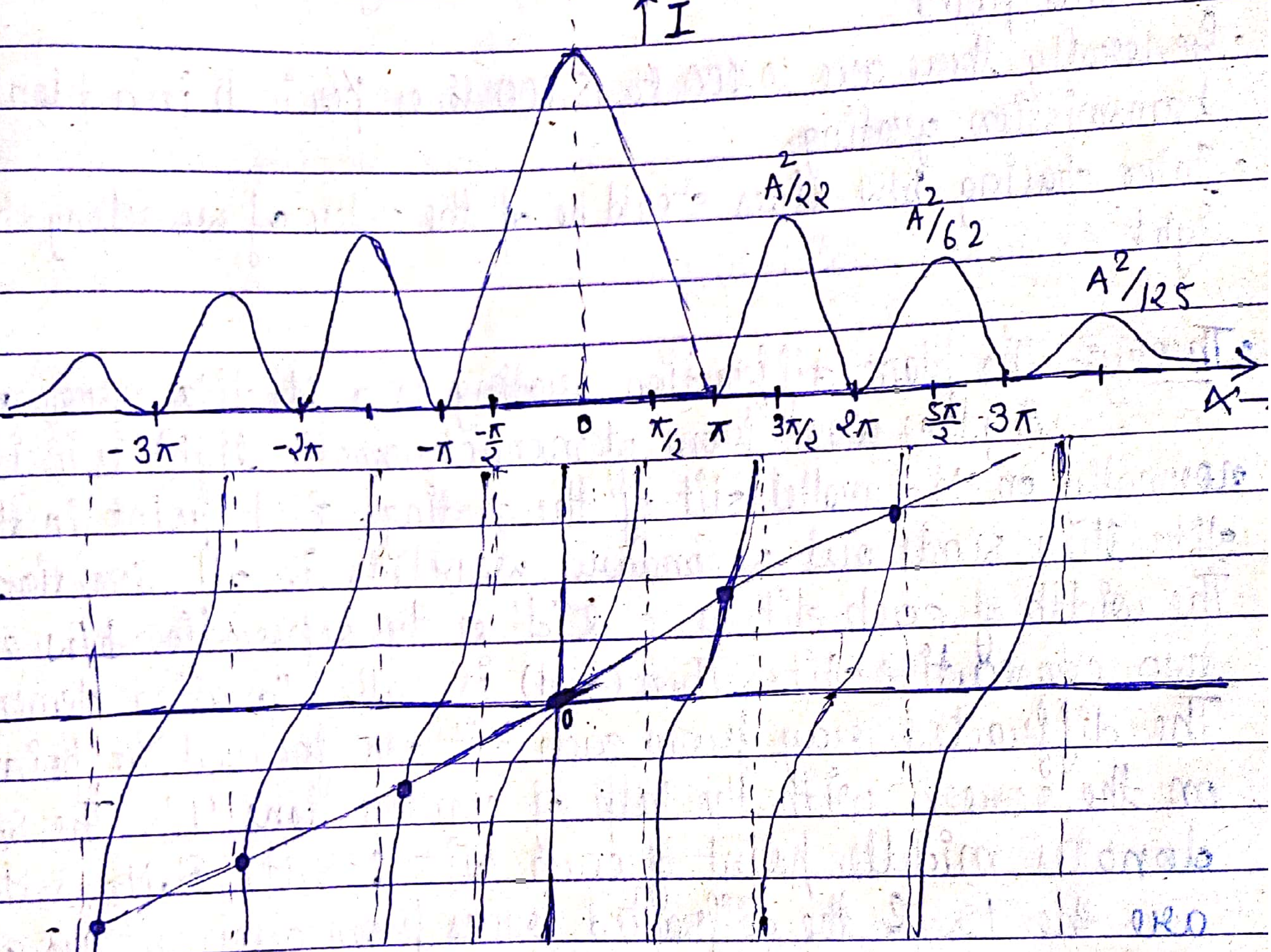
$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$ ---------

→ **Intensity of secondary Maxima:-**

• First secondary Maximum → $I_1 = I_0 \frac{\sin^2\left(\frac{3\pi}{2}\right)}{\left(\frac{3\pi}{2}\right)^2} = \frac{4I_0}{9\pi^2}$

• Second secondary Maximum → $I_2 = I_0 \frac{\sin^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2} = \frac{4I_0}{25\pi^2}$

The relative intensities of the principal, the first, the second maxima are, $1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \text{------}$



Intensity Distribution @ sigle slit

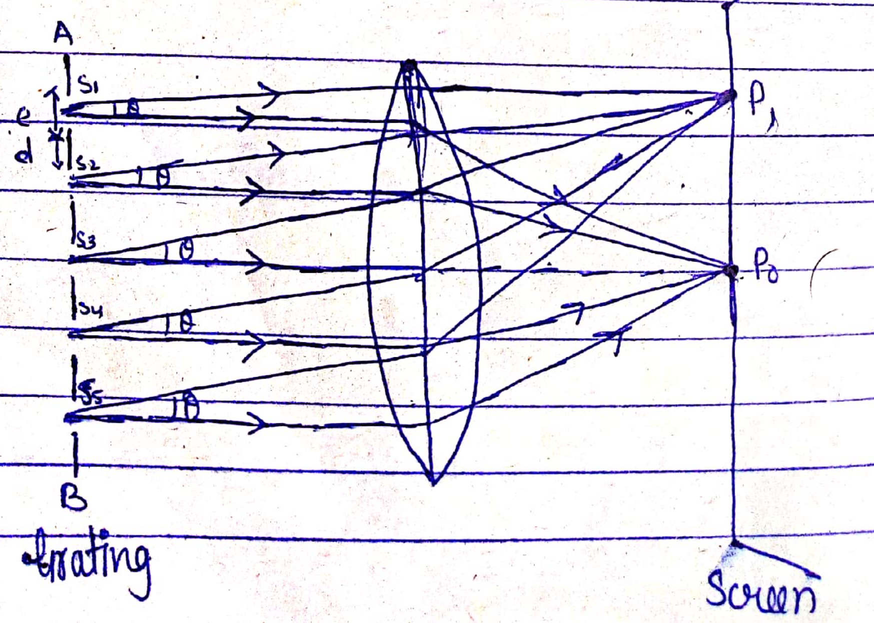
⇒ Diffraction Grating:-

Grating is made ruling a large number of line, equidistant & parallel lines on a plane transparent glass plate with a fine diamond point.

- Generally there are 10,000 to 15,000 lines per inch in a plane transmission grating.
- Inter spacing b/w lines should be of the order of wavelength of light

Theory:- The plane diffraction grating is a N-slits arrangement. A plane wave-front of monochromatic light is incident normally on N-parallel slit of the grating. Each point in the slits then sends out secondary wavelets in all directions. The width of each slit is 'e' & 'd' is the separation b/w any two consecutive slits. Then (e+d) is called "grating element". The diffracted ray from each slit are focused at point P on the screen with the help of convex lens 'L'. The S_1, S_2, S_3 donates middle point of each slit & S_1M_1, S_2M_2, S_3M_3... are the ⊥s & the diffracted waves from each slit has amplitude as;

$$R = \frac{A \sin\alpha}{\alpha}$$



Grating element: The distance b/w centers of any two consecutive slits is called grating element $(e+d)$

The path difference b/w waves from S_1 & S_2 is given by:

$$S_2M_1 = (e+d)\sin\theta$$

Similarly the path difference b/w S_1 & S_2 are also $(e+d)\sin\theta$

Hence the corresponding phase difference $= \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$

Hence the problem of determining the intensity in a dirⁿ θ is reduces to finding the resultant amplitude of N vibration each of amplitude. $\frac{A\sin\alpha}{\alpha}$

And having a common difference 2β.
By the method of vector addition of amplitudes,

$$R = \frac{a\sin\left(\frac{n\delta}{2}\right)}{\frac{\delta}{2}}$$

Here, $a = \frac{A\sin\alpha}{\alpha}$

$n = N$ & $\delta = 2\beta$

Thus, the resultant amplitude @ P_1 will be the resultant amplitude of N-waves as:

$$R' = R\cdot\frac{\sin\left(\frac{2N\beta}{2}\right)}{\sin\left(\frac{2\beta}{2}\right)}$$

$$R' = R\cdot\frac{\sin(N\beta)}{\sin\beta} = A\cdot\frac{\sin\alpha}{\alpha}\cdot\frac{\sin(N\beta)}{\sin\beta}$$

Therefore resultant intensity @ P, is,

$$I = (R')^2 = \underbrace{\frac{A^2 \sin^2\alpha}{\alpha^2}}_{\text{Intensity pattern due to single slit}} \cdot \underbrace{\frac{\sin^2(N\beta)}{\sin^2\beta}}_{\text{Intensity Distribution due to interference from N slits}}$$

⇒ Intensity distribution in Grating:-

- Condition for principal maxima:-

$$I = (R')^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2(N\beta)}{\sin^2\beta}$$

The intensity will be maximum when,

$\sin\beta = 0$ $\qquad \beta = \pm n\pi$

$\hookrightarrow n = 0, 1, 2, \ldots$

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} = \lim_{\beta\to n\pi} \frac{N\cdot\cos\beta}{\cos\beta} = N$$

Hence, $I \propto N^2$

Thus the condition for principal maxima becomes:-

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \beta = \pm n\pi$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda}$$

Hence, for $n=0$, we get the direction of 0 order principal maximum & values of $n = 1, 2, 3 \ldots$ gives the direction of first, second, third ———— order of principal maxima

· **Condition for Minima:-**

The intensity will be minimum when $\sin N\beta = 0$

$$N\beta = \pm m\pi$$

$$N \cdot \frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N((e+d)\sin\theta) = \pm m\lambda$$

↳ $m \neq 0, N, 2N, 3N \ldots\ldots nN$

↳ $\sin\theta = 0$

↳ pos^n of principal maxima

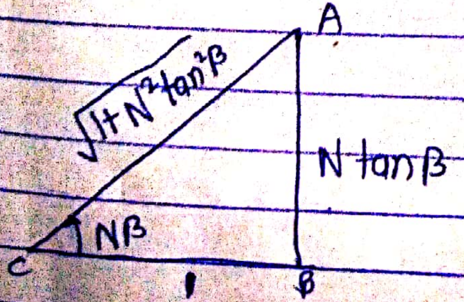
· **Condition for secondary Maxima:-**

From above conditions, there are $(N-1)$ minima b/w two successive principal maxima. Thus, there are $(N-2)$ other maxima with alternate minima b/w two successive principal maxima. These $(N-2)$ maxima are called secondary maxima.

$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2} \cdot \frac{2\sin N\beta}{\sin\beta}\left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

$$N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0$$

$$\tan N\beta = N\cdot\tan\beta$$



$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

$$I' = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{N^2 \tan^2\beta}{(1+N^2\tan^2\beta)\sin^2\beta}$$

$$I' = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{N^2}{(1+N^2\tan^2\beta)\cos^2\beta}$$

$$I' = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot N^2 \cdot \frac{1}{1+(N^2-1)\sin^2\beta}$$

$$\frac{\text{Intensity of secondary Maxima}}{\text{Intensity of principal Maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

- when N increases the intensity of secondary maxima will decreases.