Wave Optics

Introduction to Wave Optics

Wave optics, also known as physical optics, is the study of light as a wave phenomenon. This branch of optics explains various optical phenomena such as interference, diffraction, and polarization that cannot be explained by ray optics alone. The wave nature of light was firmly established through numerous experiments demonstrating these characteristic wave behaviors.

Light exhibits wave properties when it interacts with obstacles or apertures of sizes comparable to its wavelength. The wavelength of visible light ranges from approximately 400 nm (violet) to 700 nm (red). Understanding wave optics is fundamental to comprehending how light behaves in various optical systems and natural phenomena.

Interference of Light

Concept of Interference

Interference is a phenomenon that occurs when two or more coherent light waves superpose to form a resultant wave of greater, lower, or the same amplitude. This superposition can be either constructive or destructive, depending on the phase relationship between the interfering waves.

Constructive Interference: When two waves meet in phase (crest meets crest, trough meets trough), their amplitudes add together, resulting in a brighter region. The resultant amplitude is the sum of individual amplitudes, and the intensity is maximum.

Destructive Interference: When two waves meet out of phase (crest meets trough), they cancel each other out, resulting in darkness. The resultant amplitude is the difference of individual amplitudes, and the intensity is minimum or zero.

Conditions for Interference

For sustained and observable interference patterns, certain conditions must be satisfied:

1. **Coherence:** The light sources must be coherent, meaning they must maintain a constant phase relationship. This is typically achieved using a single source split into two parts.
2. **Same Frequency:** The interfering waves must have the same frequency or wavelength.
3. **Same Polarization:** For maximum effect, the waves should have the same polarization state.
4. **Small Path Difference:** The path difference between interfering waves should be of the order of the wavelength of light.

Mathematical Description

The path difference between two interfering waves determines whether interference is constructive or destructive:

For Constructive Interference (Bright Fringe):

$$\Delta = n\lambda$$

where $n = 0, 1, 2, 3, \ldots$

For Destructive Interference (Dark Fringe):

$$\Delta = (2n + 1)\frac{\lambda}{2}$$

where $n = 0, 1, 2, 3, \ldots$

Here, Δ is the path difference, λ is the wavelength of light, and n is the order of interference.

Newton's Rings

Introduction

Newton's rings are a classic demonstration of interference in a thin film of varying thickness. The phenomenon produces a series of concentric, alternating bright and dark rings when monochromatic light is reflected between a convex lens surface and a flat glass plate.

This phenomenon was first observed by Sir Isaac Newton in 1666, though it was initially described by Robert Hooke in 1665. Newton's detailed study of these rings contributed significantly to understanding the wave nature of light, despite Newton himself favoring a corpuscular theory.

Experimental Setup

The Newton's rings apparatus consists of:

1. **Plano-convex lens:** A lens with one flat surface and one convex surface, placed with its convex side down on a flat glass plate. The lens typically has a large radius of curvature.
2. **Flat glass plate:** An optically flat glass surface on which the lens rests.
3. **Monochromatic light source:** Usually a sodium lamp emitting light of wavelength approximately 589 nm.
4. **Glass plate at 45°:** A semi-transparent glass plate inclined at 45° to direct light downward onto the lens-plate combination and reflect the interference pattern toward the observer.
5. **Traveling microscope:** Used to measure the diameters of the rings with precision.

Formation of Newton's Rings

When monochromatic light is incident on the setup, part of it reflects from the lower surface of the lens, and part transmits through the air gap and reflects from the upper surface of the glass plate. These two reflected rays interfere with each other.

The thickness of the air film between the lens and plate is nearly zero at the point of contact and gradually increases outward. This varying thickness creates different path differences at different radial distances from the center, resulting in alternating bright and dark circular fringes.

Important Note on the Central Spot: The center of Newton's rings pattern appears dark when viewed in reflected light. This occurs because at the point of contact, the air film thickness is essentially zero. However, the ray reflecting from the glass-air boundary (the top surface of the plate) undergoes a phase change of π radians (or path difference of $\lambda/2$), while the ray reflecting from the lens does not. This π phase change causes destructive interference at the center, producing a dark spot.

Mathematical Analysis

Consider a point at distance r from the center where the air film has thickness t. For a lens of radius of curvature R, the thickness can be approximated as:

$$t \approx \frac{r^2}{2R}$$

For Dark Rings (Destructive Interference):

The condition for dark fringes, accounting for the phase change at reflection:

$$2t = n\lambda$$

Substituting the expression for t:

$$2 \times \frac{r_n^2}{2R} = n\lambda$$

Therefore, the radius of the nth dark ring is:

$$r_n = \sqrt{n\lambda R}$$

The diameter of the nth dark ring:

$$D_n = 2r_n = 2\sqrt{n\lambda R}$$

For Bright Rings (Constructive Interference):

$$r_n = \sqrt{(2n-1)\frac{\lambda R}{2}}$$

where $n = 1, 2, 3, \ldots$ is the order of the ring.

Applications

1. **Wavelength Measurement:** By measuring the diameters of Newton's rings, the wavelength of monochromatic light can be determined using the formula:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

2. **Radius of Curvature:** The radius of curvature of a lens can be determined if the wavelength is known.
3. **Testing Optical Surfaces:** Newton's rings are used to test the flatness and quality of optical surfaces. Any irregularity in the surface produces distortions in the ring pattern.
4. **Refractive Index Measurement:** By introducing a liquid between the lens and plate, its refractive index can be determined from the change in ring diameters.

Example Problem

Problem: In a Newton's rings experiment using sodium light ($\lambda = 589$ nm) with a plano-convex lens of radius of curvature 100 cm, calculate the diameter of the 5th dark ring.

Solution:
Given: $n = 5$, $\lambda = 589 \times 10^{-9}$ m, $R = 1$ m

Using the formula for dark rings:

$$D_n = 2\sqrt{n\lambda R}$$

$$D_5 = 2\sqrt{5 \times 589 \times 10^{-9} \times 1}$$

$$D_5 = 2\sqrt{2945 \times 10^{-9}}$$

$$D_5 = 2 \times 5.427 \times 10^{-5}$$

$$D_5 = 1.085 \times 10^{-4} \text{ m} = 0.1085 \text{ mm}$$

Michelson Interferometer

Introduction

The Michelson interferometer is one of the most important and versatile optical instruments ever invented. Developed by Albert A. Michelson in 1881, this precision instrument produces interference fringes by splitting a beam of light into two paths and then recombining them.

The interferometer played a crucial role in the famous Michelson-Morley experiment (1887), which aimed to detect the luminiferous aether. The negative result of this experiment was pivotal in the development of Einstein's theory of special relativity. Michelson was awarded the Nobel Prize in Physics in 1907 for his optical precision instruments and spectroscopic investigations.

Construction and Working Principle

Principle: The Michelson interferometer works on the principle of interference of light waves created by division of amplitude. A beam of light is split into two beams that travel different paths and are then recombined to produce an interference pattern.

Components:

1. **Light Source (S):** A monochromatic or extended light source (e.g., sodium lamp, laser).
2. **Beam Splitter (P):** A partially silvered glass plate inclined at 45° to the incident beam. It reflects approximately 50% of the incident light and transmits the remaining 50%.
3. **Compensator Plate (P_1):** A transparent glass plate identical in thickness to the beam splitter, placed parallel to it. This ensures both beams travel through the same thickness of glass, compensating for dispersion effects.
4. **Mirror M_1:** A movable plane mirror perpendicular to one beam path. It can be moved precisely using a micrometer screw.
5. **Mirror M_2:** A fixed plane mirror perpendicular to the other beam path. Fine adjustment screws allow precise alignment.
6. **Telescope/Detector (T):** Used to observe the interference pattern.

Working:

1. Light from source S is collimated by lens L and falls on the beam splitter P at 45°.
2. At P, the beam splits into two:
    - **Reflected beam:** Travels toward mirror M_1, reflects back, and passes through P toward the observer.
    - **Transmitted beam:** Passes through P, travels toward mirror M_2, reflects back, and is partially reflected by P toward the observer.
3. These two beams, having traveled different optical paths, superpose and create an interference pattern visible through telescope T.
4. When mirrors M_1 and M_2 are exactly perpendicular and equidistant from P, the path difference is zero, and uniform illumination is observed.
5. When M_1 is moved, the path difference changes, causing fringes to shift. Moving M_1 by $\lambda/2$ causes one complete fringe shift.

Types of Fringes

Circular Fringes: When mirrors M_1 and M_2 are exactly perpendicular to each other, circular fringes are observed. These fringes are concentric circles centered on the normal to the mirrors.

Localized Fringes: When mirrors M_1 and M_2 are slightly tilted with respect to each other, straight-line fringes are observed. These fringes are localized near the mirror surfaces and form a wedge-shaped air film pattern.

Mathematical Analysis

For Circular Fringes:

If the distance between M_1 and its image M_2' is d, the path difference for a ray at angle θ is:

$$\Delta = 2d\cos\theta$$

Condition for bright fringe:

$$2d\cos\theta = n\lambda$$

Condition for dark fringe:

$$2d\cos\theta = (2n+1)\frac{\lambda}{2}$$

where n is the order of interference.

Applications

1. **Wavelength Measurement:** The wavelength of monochromatic light can be determined by counting the number of fringes (N) that shift when mirror M_1 is moved by a distance Δd:

$$\lambda = \frac{2\Delta d}{N}$$

2. **Refractive Index Measurement:** By placing a gas chamber in one arm and gradually filling it with gas, the change in optical path determines the refractive index:

$$\mu = 1 + \frac{N\lambda}{2L}$$

where L is the length of the gas chamber and N is the number of fringes shifted.

3. **Thickness Measurement:** The thickness of thin transparent sheets can be measured precisely.
4. **Standardization of Length:** Michelson used his interferometer to measure the standard meter in terms of wavelengths of light.
5. **Fourier Transform Spectroscopy:** Modern versions use the Michelson interferometer for spectroscopic analysis.
6. **Gravitational Wave Detection:** The LIGO (Laser Interferometer Gravitational-Wave Observatory) uses a modified Michelson interferometer with arms several kilometers long to detect gravitational waves.

Example Problem

Problem: In a Michelson interferometer experiment with sodium light ($\lambda = 589$ nm), when one mirror is moved through 0.233 mm, 790 fringes cross the field of view. Verify the wavelength.

Solution:
Given: $\Delta d = 0.233$ mm = 0.233×10^{-3} m, $N = 790$

Using the formula:

$$\lambda = \frac{2\Delta d}{N} = \frac{2 \times 0.233 \times 10^{-3}}{790}$$

$$\lambda = \frac{0.466 \times 10^{-3}}{790} = 5.898 \times 10^{-7} \text{ m}$$

$$\lambda = 589.8 \text{ nm}$$

This closely matches the sodium D-line wavelength of 589 nm.

Diffraction of Light

Introduction

Diffraction is the bending of light waves around obstacles or through apertures, resulting in the spreading of light into regions that would otherwise be in shadow according to geometrical optics. This phenomenon is a clear demonstration of the wave nature of light.

Diffraction becomes significant when the size of the obstacle or aperture is comparable to the wavelength of light. For visible light (wavelength ~400-700 nm), diffraction effects are observed with very small apertures or obstacles.

Types of Diffraction

Diffraction phenomena are classified into two main types based on the distance between the source, obstacle, and observation screen:

Fresnel Diffraction (Near-Field Diffraction)

Characteristics:

1. **Source and screen at finite distances:** Either the light source or the observation screen (or both) is at a finite distance from the diffracting aperture or obstacle.
2. **Spherical or cylindrical wavefronts:** The incident and diffracted wavefronts are spherical or cylindrical, not plane.
3. **Complex patterns:** The diffraction pattern is more complex, with intensity varying across the pattern.
4. **No lenses required:** Fringes can be observed without using any additional lenses.
5. **Fresnel zones:** The wavefront can be divided into Fresnel zones to analyze the pattern.

The Fresnel diffraction pattern changes with the distance from the diffracting object. As the observation point moves farther from the aperture, the pattern gradually transitions from Fresnel to Fraunhofer diffraction.

Applications: Fresnel diffraction is important in understanding light propagation in optical systems with finite distances, such as near-field optics and some optical instrument designs.

Fraunhofer Diffraction (Far-Field Diffraction)

Characteristics:

1. **Source and screen at infinite distances:** Both the light source and observation screen are effectively at infinite distances from the diffracting aperture. This is achieved practically by using lenses.
2. **Plane wavefronts:** Both incident and diffracted wavefronts are plane waves.
3. **Simpler patterns:** The diffraction pattern is simpler and well-defined, consisting of a central maximum surrounded by secondary maxima and minima.
4. **Lenses used:** A converging lens is used to produce parallel incident light, and another lens focuses the diffracted light onto the observation screen.
5. **Pattern at focal plane:** The Fraunhofer diffraction pattern is observed at the focal plane of the focusing lens.

The Fraunhofer condition is satisfied when:

where d is the distance from aperture to screen, a is the aperture size, and λ is the wavelength.

Applications: Fraunhofer diffraction is used extensively in optical spectroscopy, X-ray crystallography, and analysis of optical systems.

Comparison Table: Fresnel vs. Fraunhofer Diffraction

Feature	Fresnel Diffraction	Fraunhofer Diffraction
Distance	Source/screen at finite distances	Source/screen at infinite distances (using lenses)
Wavefronts	Spherical or cylindrical	Plane waves
Pattern complexity	Complex, varies with distance	Simpler, well-defined
Lenses required	No	Yes (for practical setup)
Mathematical analysis	More complex	Relatively simpler (Fourier transform)
Observation	Close to aperture	Far from aperture (focal plane)
Fresnel number	Greater than 1	Much less than 1

Single-Slit Fraunhofer Diffraction

When plane waves pass through a narrow single slit of width a, Fraunhofer diffraction produces a characteristic pattern with a bright central maximum flanked by alternating dark and bright fringes of decreasing intensity.

Condition for minima (dark fringes):

$$a \sin\theta = n\lambda$$

where $n = \pm 1, \pm 2, \pm 3, \ldots$ (n ≠ 0)

Position of minima on screen:

$$y_n = \frac{n\lambda D}{a}$$

Width of central maximum:

$$w = \frac{2\lambda D}{a}$$

where D is the distance from slit to screen, θ is the diffraction angle, and a is the slit width.

The intensity distribution follows:

$$I(\theta) = I_0\left(\frac{\sin\beta}{\beta}\right)^2$$

where $\beta = \frac{\pi a \sin\theta}{\lambda}$

Diffraction Grating

Introduction

A diffraction grating is an optical component with a periodic structure that splits and diffracts light into several beams traveling in different directions. It is one of the most powerful tools in spectroscopy for analyzing the wavelength composition of light.

Construction: A diffraction grating consists of a large number of equally spaced parallel slits or grooves ruled on a transparent material (transmission grating) or a reflective surface (reflection grating). A typical grating may have 300 to 10,000 lines per centimeter.

Grating Equation

When light of wavelength λ is incident normally on a grating with grating spacing d (distance between adjacent slits), the condition for constructive interference (bright fringes) at angle θ is:

$$d\sin\theta = n\lambda$$

where $n = 0, \pm1, \pm2, \pm3, \ldots$ is the order of diffraction.

Key Points:

1. **n = 0:** Zero order (central maximum) - all wavelengths overlap.
2. **n = ±1:** First order diffraction - wavelengths are spatially separated.
3. **n = ±2, ±3, ...:** Higher orders show greater separation of wavelengths.

Applications of Diffraction Gratings

1. **Spectroscopy:** Gratings disperse white light into its constituent wavelengths, allowing precise wavelength measurement and spectral analysis.
2. **Wavelength Selection:** Used in monochromators to select specific wavelengths from broadband sources.

3. **Laser Tuning:** Used in tunable lasers to select specific wavelengths.
4. **Optical Communications:** Used in wavelength division multiplexing (WDM) systems.

Determination of Wavelength Using Grating

A diffraction grating can be used to accurately determine the wavelength of monochromatic light.

Experimental Method:

1. Set up the grating perpendicular to the incident light beam.
2. Observe the diffraction pattern through a telescope.
3. Measure the angle θ_n for the nth order bright fringe.
4. Use the grating equation to calculate wavelength:

$$\lambda = \frac{d \sin \theta_n}{n}$$

Example Problem:

Problem: A diffraction grating has 5000 lines per cm. When light is incident normally, the second-order maximum is observed at an angle of 30°. Calculate the wavelength of light.

Solution:
Given: Number of lines = 5000 per cm
Therefore, $d = \frac{1}{5000}$ cm = 2×10^{-6} m
$\theta = 30^\circ$, $n = 2$

Using grating equation:

$$\lambda = \frac{d \sin \theta}{n} = \frac{2 \times 10^{-6} \times \sin 30^\circ}{2}$$

$$\lambda = \frac{2 \times 10^{-6} \times 0.5}{2} = 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$$

This wavelength corresponds to green light.

Conclusion

Wave optics provides a comprehensive framework for understanding light behavior through interference, diffraction, and related phenomena. Newton's rings demonstrate interference in thin films, the Michelson interferometer offers precise measurements through path difference analysis, and diffraction reveals the wave nature of light through bending around obstacles. The diffraction grating extends these principles to practical spectroscopic applications, enabling wavelength determination and spectral analysis. These concepts form the foundation for modern optics, photonics, and numerous technological applications ranging from optical instruments to telecommunications systems.