



Q1. Find the first and last occurrence of an element in a sorted array.

Approach: Finding First Occurrence.

→ Applying binary search, if at any instance mid is such that $arr[mid] == key$, then:

- ① Either this is the first occurrence itself.
- ② Or there are more 'key' towards the left of mid .

So we'll temporarily store mid and then search in the left half by making $high = mid - 1$.

If $arr[mid] < key$, then search in right half by making $low = mid + 1$.

If $arr[mid] > key$, then search in left half by making $high = mid - 1$.

Example: $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$ $key = 4$.

① $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$
 \uparrow \uparrow \uparrow
 l mid h

$arr[mid] < key \Rightarrow$ Definitely in the right half.

$low = mid + 1$ // $low = 4$

② $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$
 \uparrow \uparrow \uparrow
 l mid h

$arr[mid] == key \Rightarrow$ Either mid or before mid .

store mid in a variable ans . // $ans = 5$.

Then make $high = mid - 1$. // $high = 4$

③ $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$
 \downarrow
 mid

③ $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

mid
↓
l h

$arr[mid] == key \Rightarrow$ Either mid or before mid.

store mid in a variable ans. // $ans = 4$

Then make $high = mid - 1$. // $high = 3$

④ $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

h l

$low > high \Rightarrow$ break out. Last value of $ans = 4$

Note: low, mid, high and ans are storing the indices and not values.

Finding Last Occurance.

→ Applying binary search, if at any instance mid is such that $arr[mid] == key$, then:

① Either this is the last occurrence itself.

② Or there are more 'key' towards the right of mid.

So we'll temporarily store mid and then search in the right half by making $low = mid + 1$.

If $arr[mid] < key$, then search in right half by making $low = mid + 1$.

If $arr[mid] > key$, then search in left half by making $high = mid - 1$.

Example: $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$ $key = 4$.

① $arr[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

l mid h

$\text{arr}[\text{mid}] < \text{key} \Rightarrow$ Definitely in the right half.

$low = mid + 1$. // $low = 4$

② arr[] = {1, 1, 2, 3, 4, 4, 4, 4}

arr[mid] == key \Rightarrow Either mid or after mid.
store mid in a variable ans. // ans = 5.
Then make low = mid + 1. // low = 6

③ arr[] = {1, 1, 2, 3, 4, 4, 4, 4}

arr[mid] == key \Rightarrow Either mid or after mid.
store mid in a variable ans. // ans = 6
Then make low = mid + 1. // low = 7

④ arr[] = {1, 1, 2, 3, 4, 4, 4}

arr[mid] == key \Rightarrow Either the last occurrence is at index mid or to the right of mid.
Store mid in a variable ans. // ans = 7
Then make low = mid + 1.

⑤ arr[] = {1, 1, 2, 3, 4, 4, 4, 4}

low > high \Rightarrow break out. Last value of `ans = 7`

Code :

```
int firstOcc(int arr[], int n, int key) {
    int s = 0, e = n-1;
    int mid = s + (e-s)/2;
    int ans = -1;
    while(s <= e) {
        if(arr[mid] == key){
            ans = mid;
            e = mid - 1;
        }
        else if(key > arr[mid]) { //Right me jao
            s = mid + 1;
        }
        else if(key < arr[mid]) { //Left me jao
            e = mid - 1;
        }
        mid = s + (e-s)/2;
    }
    return ans;
}
```

```
int lastOcc(int arr[], int n, int key) {
    int s = 0, e = n-1;
    int mid = s + (e-s)/2;
    int ans = -1;
    while(s <= e) {
        if(arr[mid] == key){
            ans = mid;
            s = mid + 1;
        }
        else if(key > arr[mid]) { //Right me jao
            s = mid + 1;
        }
        else if(key < arr[mid]) { //Left me jao
            e = mid - 1;
        }
        mid = s + (e-s)/2;
    }
    return ans;
}
```

Follow-up question:

Q. Count the number of occurrences of an element in a sorted array.

→ (Last Occurance Index) - (First Occurance Index) + 1

Q. Peak in mountain array.

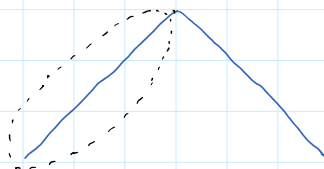
Eg: arr[]: {0, 10, 5, 2}
ans = 10.



Approach: If for 'mid' index,

1. arr[mid] < arr[mid+1] :

This element lies in the increasing slope / ascend. Search in the right half. ⇒ low = mid + 1

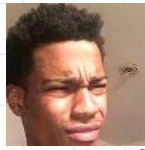


2. In all other cases.

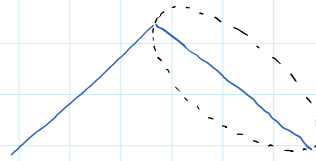
This element lies in the decreasing slope / descend. or is the peak element itself. Since mid can be the peak element, search in the left half including mid.

element, search in the left half including mid.

$\Rightarrow \text{high} = \text{mid}.$



Confused?



Example: $\text{arr}[] : \{1, 10, 5, 2, 0\}$

① $\text{low} = 0, \text{high} = 4, \text{mid} = 2$

$\text{arr}[] : \{1, 10, 5, 2, 0\}$
 $\underset{l}{1} \quad \underset{\text{mid}}{10} \quad \underset{h}{5}$

$\text{arr}[\text{mid}]$ is not smaller than $\text{arr}[\text{mid}+1]$. $\Rightarrow \text{high} = \text{mid}$

② $\text{low} = 0, \text{high} = 2, \text{mid} = 1$

$\text{arr}[] : \{1, 10, 5, 2, 0\}$
 $\underset{l}{1} \quad \underset{\text{mid}}{10} \quad \underset{h}{5}$

$\text{arr}[\text{mid}]$ is not smaller than $\text{arr}[\text{mid}+1]$. $\Rightarrow \text{high} = \text{mid}$

③ $\text{low} = 0, \text{high} = 1, \text{mid} = 0$

$\text{arr}[] : \{1, 10, 5, 2, 0\}$
 $\underset{l}{1} \quad \underset{h}{10}$

$\text{arr}[\text{mid}]$ is smaller than $\text{arr}[\text{mid}+1]$ $\Rightarrow \text{low} = \text{mid} + 1.$

④ $\text{low} = 1, \text{high} = 1, \text{mid} = 1$

$\text{arr}[] : \{1, 10, 5, 2, 0\}$
 $\underset{l}{1} \quad \underset{\text{mid}}{10} \quad \underset{h}{5}$

Do this till $\text{low} < \text{high}$

Code:

```
int peakIndexInMountainArray(vector<int>& arr) {  
    int s = 0;  
    int e = arr.size() - 1;  
  
    int mid = s + (e-s)/2;  
  
    while(s<e) {  
        if(arr[mid] < arr[mid+1]){  
            s = mid + 1;  
        }  
        else  
        {  
            e = mid;  
        }  
        mid = s + (e-s)/2;  
    }  
    return s;  
}
```

Homework: Find pivot in a rotated sorted array.

Discussed in the next Lecture. 😊