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LAMBDA CALCULUS

(1) $(\lambda x.x)(\lambda x.x)$

$\Rightarrow (\lambda x.x)(\lambda z.z)$ (α-substitution rename x with z)

$\Rightarrow [(\lambda z.z)/x]\, x$ (β-reduction replace λx with $\lambda z.z$)

$\Rightarrow (\lambda z.z) = I$

(2) $(\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$

$\Rightarrow (\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$ (α-substitution z with z)

$\Rightarrow (\lambda x.x\ x)(\lambda z.\lambda y.z\ z)$ (β-reduction replace x with $\lambda z.\lambda y.z\ z$)

$\Rightarrow (\lambda z.\lambda y.z\ z)(\lambda z.\lambda y.z\ z)$ (α-substitution z with x)

$\Rightarrow (\lambda z.\lambda y.z\ z)(\lambda x.\lambda y.x\ x)$ (β-reduction replace z with $\lambda x.\lambda y.x$)

$\Rightarrow \lambda y.(\lambda x.\lambda y.x\ x)(\lambda x.\lambda y.x\ x)$

$\Rightarrow (\lambda x.\lambda y.x\ x)(\lambda x.\lambda y.x\ x)$ (Same as step 3)

∴ It is recursive.

(3) $((\lambda x.(x\ y))(\lambda z.z))$

$\Rightarrow \left[\frac{(\lambda z.z)y}{x}\right] x$ (β-reduction replace x with $\lambda z.z$)

$\Rightarrow (\lambda z.z)\, y$

$\Rightarrow [y/z]\, z$ (β-reduction replace z with y)

$\Rightarrow y$

(4) $(\lambda z.z)(\lambda y.yy)(\lambda x.x\ a)$ (β-reduction replace z with $\lambda y.yy$)

$\Rightarrow [(\lambda y.y\ y)]_z.(\lambda x.x\ a)$ (β-reduction)

$\Rightarrow (\lambda y.y\ y).(\lambda x.x\ a)$

$\Rightarrow (\lambda x.x.a)(\lambda x.x.a)$ (β-reduction replace y with $\lambda x.x\ a$)

$\Rightarrow (\lambda x.xa)\ a$ (β-reduction replace x with a)

$\Rightarrow a\ a$

(5) $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$

$\Rightarrow (\lambda z.z)(\lambda z.z\ z)(\lambda z.z\ y)$ (β-reduction replace z with $(\lambda z.zz)$)

$\Rightarrow (\lambda z.z\ z)(\lambda z.z\ y)$ (β-reduction replace z with $\lambda z.zy$)

$\Rightarrow (\lambda z.zy)(\lambda z.z\ y)$ (β-reduction replace z with $\lambda z.zy$)

$\Rightarrow (\lambda z.zy)\ y$ (β-reduction replace z with y)

$\Rightarrow yy$

(6) $(\lambda x.\lambda y.x\ y\ y)(\lambda a.a)b$

$\Rightarrow (\lambda x.\lambda y.x\ yy)(\lambda a.a)\ b$ (β-reduction replace x with $\lambda a.a$)

$\Rightarrow (\lambda y(\lambda a.a)\ yy)\ b$ (β-reduction replace y with b)

$\Rightarrow (\lambda a.a)bb$ (β-reduction replace a with b)

$\Rightarrow b.b$

(7) $(\lambda x.x\ x)\ (\lambda y.y\ x)\ z$

$\Rightarrow (\lambda x.x\ x)(\lambda y.y\ x)\ z$ (β-reduction replace x with $\lambda y.y\ x$)

$\Rightarrow (\lambda y.y\ x)(\lambda y.y\ x)\ z$ (β-reduction replace y with $\lambda y.y\ x$)

$\Rightarrow ((\lambda y.y\ x)\ x)\ z$ (β-reduction replace y with z)

$\Rightarrow x\ x\ z$

(8) $\lambda x.(\lambda y.(x\ y))\ y)\ z$

~~$\Rightarrow \lambda x.(\lambda y.(x\ y))\ y)\ z$~~ (~~β-reduction replace~~

$\Rightarrow \lambda x.(\lambda y.(x\ y))\ y)\ z$ (α-conversion y to a)

$\Rightarrow \lambda x.(\lambda a.(x\ a))\ y)\ z$ (β-reduction replace x with z)

$\Rightarrow \lambda a.(z\ a)\ y$ (β-reduction replace a with y)

$\Rightarrow z\ y$

(9) $((\lambda x.x\ x)(\lambda y.y))(\lambda y.y)$

$\Rightarrow ((\lambda x.x\ x)(\lambda y.y))(\lambda y.y)$ (β-reduction replace x with $\lambda y.y$)

$\Rightarrow ((\lambda y.y)(\lambda y.y))(\lambda y.y)$ (β-reduction replace y with $\lambda y.y$)

$\Rightarrow (\lambda y.y)(\lambda y.y)$ (β-reduction replace y with $\lambda y.y$)

$\Rightarrow (\lambda y.y)$

$\Rightarrow I$ (Identity element)

(10) $(((\lambda x.\ \lambda y.\ (x\ y))(\lambda y.\ y))\ w)$

$\Rightarrow (((\lambda x.\lambda y.(x\ y))(\lambda y.y))\ w)$ (α-conversion rename y to a)

$\Rightarrow (((\lambda x.\ \lambda a.\ (x\ a))\ (\lambda y.\ y))\ w)$ (β-reduction replace x with $\lambda y.y$)

$\Rightarrow ((\lambda a.((\lambda y.y)\ a)))\ w)$ (β-reduction replace a with w)

$\Rightarrow (\lambda y.y)\ w$ (β-reduction replace y with w)

$\Rightarrow$ 10