

Programming Languages - Assignment Lambda Calculus

1. $(\lambda x.x)(\lambda x.x)$

Answer:

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$
 $= (\lambda x.x)$

2. $(\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$

Answer:

Using alpha conversion (replacing bound 'x' with 'a' in the second term)

$= (\lambda x.x\ x)(\lambda a.\lambda y.a\ a)$

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= (\lambda a.\lambda y.a\ a)(\lambda a.\lambda y.a\ a)$

$= (\lambda y.(\lambda a.\lambda y.a\ a)(\lambda a.\lambda y.a\ a))$

This example is non-terminating as no beta reduction is possible.

3. $((\lambda x.(x\ y))(\lambda z.z))$

Answer:

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= (\lambda z.z)\ y$

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= y$

4. $(\lambda z.z)(\lambda y.y\ y)(\lambda x.x\ a)$

Answer:

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$ to $(\lambda z.z)(\lambda y.y\ y)$

$= (\lambda y.y\ y)(\lambda x.x\ a)$

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= (\lambda x.x\ a)(\lambda x.x\ a)$

Using alpha conversion (replacing bound 'x' with 'b' in the second term)

$= (\lambda x.x\ a)(\lambda b.b\ a)$

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= (\lambda b.b\ a)\ a$

Applying beta reduction: $(\lambda x.e_1)e_2 = e_1[e_2/x]$

$= aa$

5. $(\lambda z.z)(\lambda z.z\ z)(\lambda z.z\ y)$

Answer:

Using alpha conversion (replacing bound 'z' with 'a' in the second term and bound 'z' with 'b' in the third term)

$$= (\lambda z.z) (\lambda a.a) (\lambda b.b) y$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$ to $(\lambda z.z) (\lambda a.a)$

$$= (\lambda a.a) (\lambda b.b) y$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= (\lambda b.b) y (\lambda b.b) y$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= (\lambda b.b) y y$$

$$= yy$$

6. $(\lambda x.\lambda y.x y y) (\lambda a.a) b$

Answer:

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= (\lambda y.(\lambda a.a) y y) b$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= (\lambda a.a) b b$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= bb$$

7. $(\lambda x.x x) (\lambda y.y x) z$

Answer:

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= (\lambda y.y x) (\lambda y.y x) z$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= ((\lambda y.y x) x) z$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= xxz$$

8. $(\lambda x. (\lambda y. (x y)) y) z$

Answer:

Using alpha conversion (replacing bound 'y' with 'a')

$$= (\lambda x. (\lambda a. (x a)) y) z$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$ to $(\lambda a. (x a)) y$

$$= \lambda x. (x y) z$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= zy$$

9. $((\lambda x.x x) (\lambda y.y)) (\lambda y.y)$

Answer:

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$

$$= ((\lambda y.y)(\lambda y.y)) (\lambda y.y)$$

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$
 $= (\lambda y.y) (\lambda y.y)$
 Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$
 $= (\lambda y.y)$

10. $((\lambda x. \lambda y.(x\ y))(\lambda y.y))\ w$

Answer:

Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$
 $= (\lambda y.(\lambda y.y)\ y)\ w$
 Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$
 $= (\lambda y.y)\ w$
 Applying beta reduction: $(\lambda x.e1)e2 = e1[e2/x]$
 $= w$