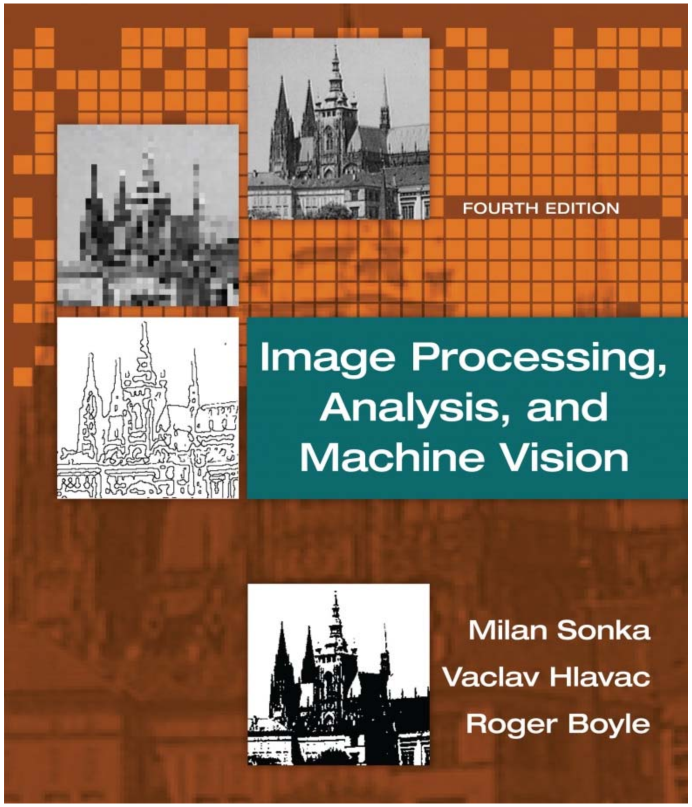
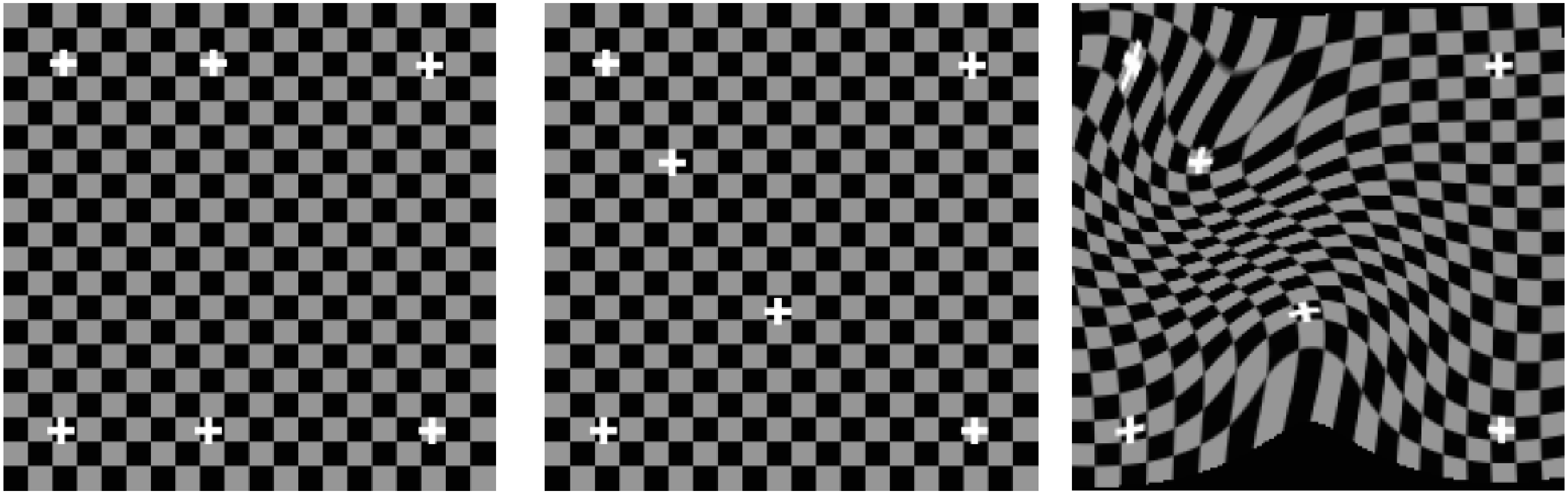


Chapter 5.2 Geometric Transformations II

Sources:
Slides Ross Whitaker (Utah) and G. Gerig

Geometric Transformations

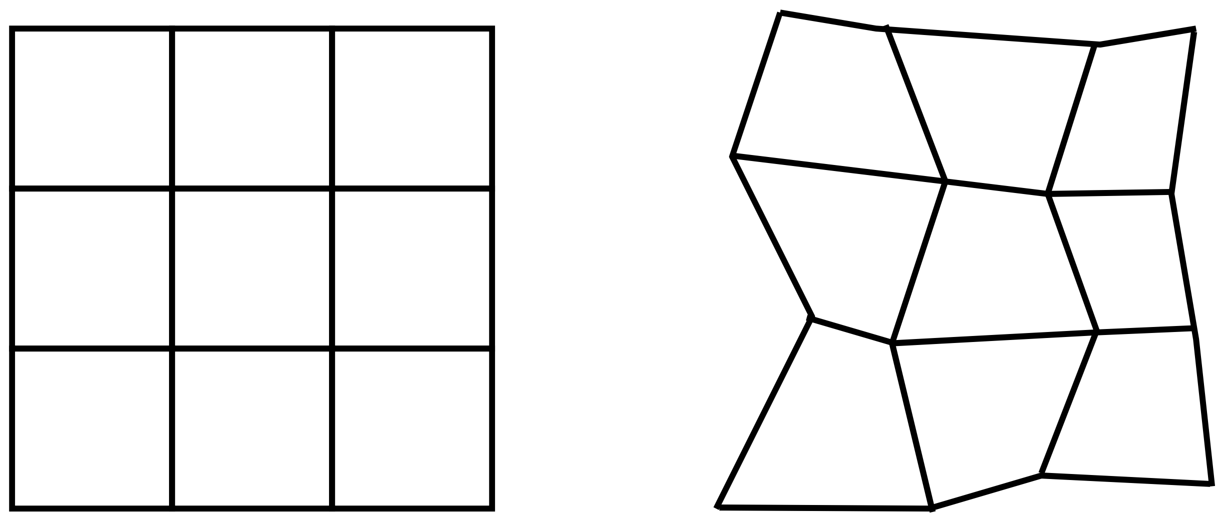
- Greyscale transformations -> operate on range/output
- Geometric transformations -> operate on image domain
  - Coordinate transformations
  - Moving image content from one place to another
- Two parts:
  - Define transformation
  - Resample greyscale image in new coordinates



SPECIFYING "WARPS" VIA SPARSE SET OF LANDMARKS

Specifying Warps – Another Strategy

- Let the # DOFs in the warp equal the # of control points (x1/2)
  - Interpolate with some grid-based interpolation
    - E.g. binlinear, splines



Landmarks Not On Grid

- Landmark positions driven by application
- Interpolate transformation at unorganized correspondences
  - *Scattered data interpolation*
- How do we do scattered data interpolation?
  - Idea: use kernels!
- *Radial basis functions*
  - Radially symmetric functions of distance to landmark

Concept

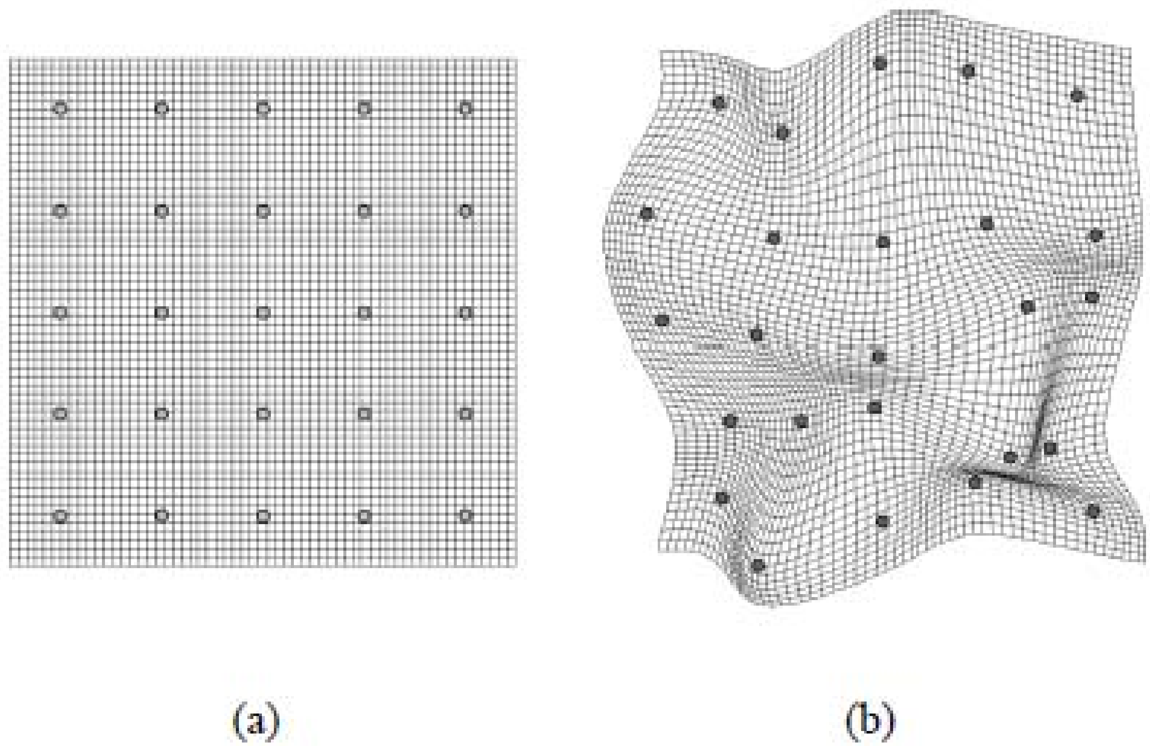


Figure 1. Warping a 2D mesh with RBFs: a) original mesh; b) mesh after warping.

Concept

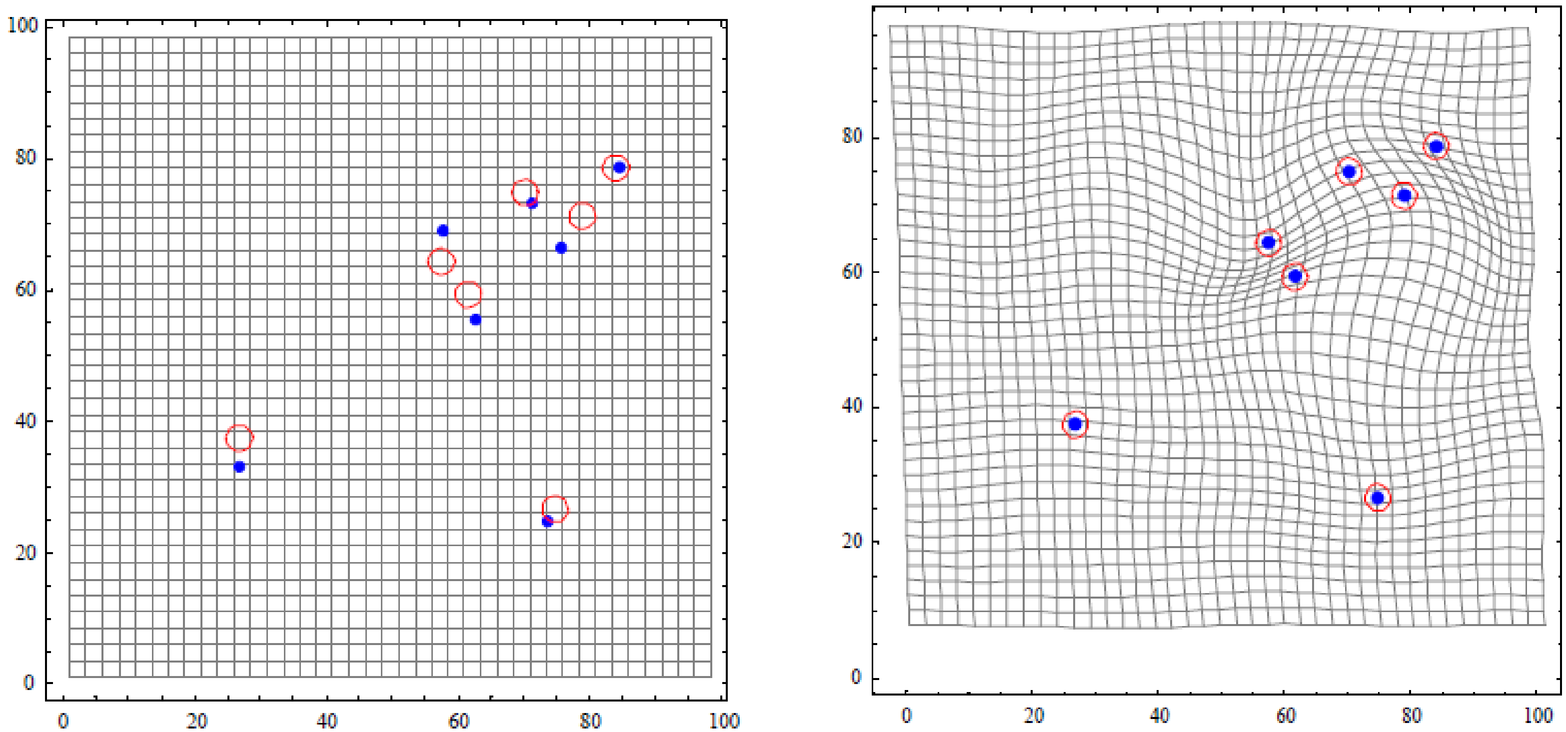


Fig. 5 Radial basis interpolation of a regular grid, based on the random motion of 7 landmarks.

Warping a Neuro-Anatomy Atlas on 3D MRI Data with Radial Basis Functions H.E. Bennink, J.M. Korbeeck, B.J. Janssen, B.M. ter Haar Romeny

Concept

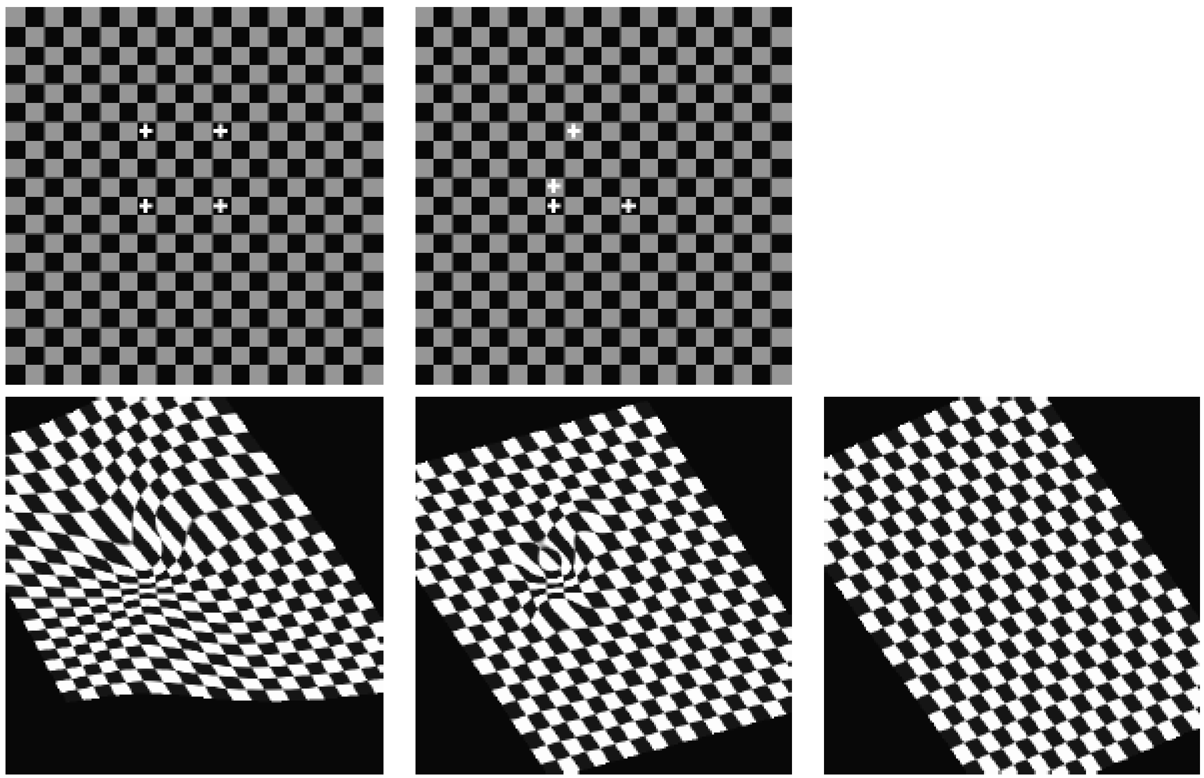
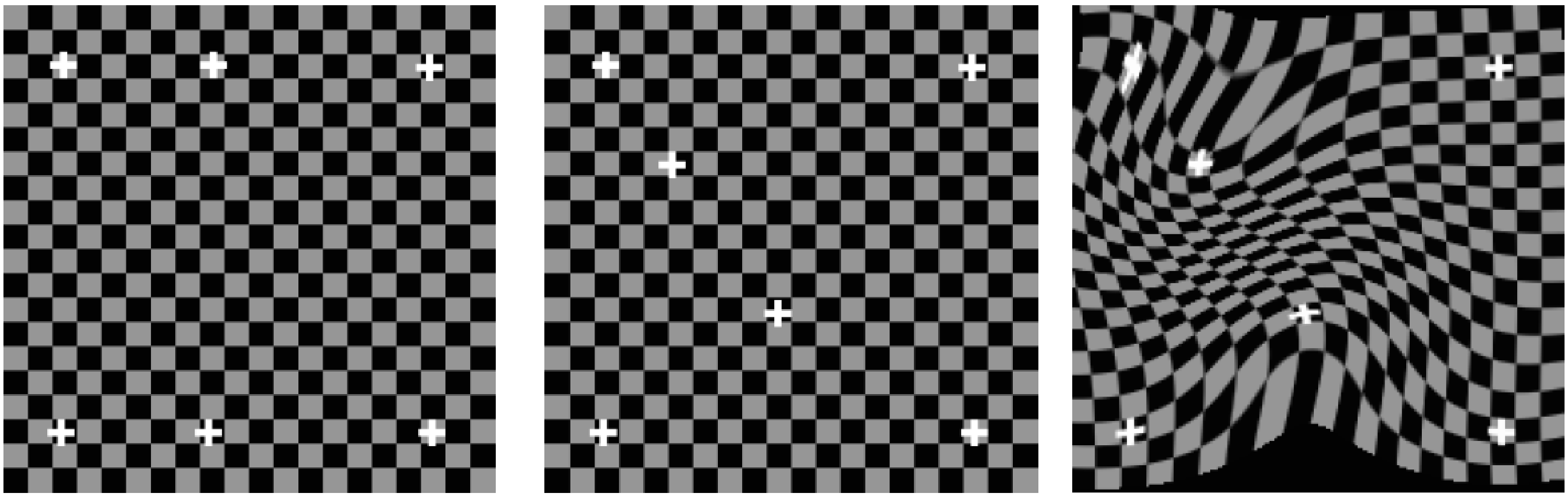


Figure 8: Generalizing affine mappings in different ways. **Top**: Position of source and target anchor points. **Bottom** (left to right): thin-plate warp, Gaussian warp, affine least-square warp ($\lambda = \infty$). In all cases the mapping can be well approximated by an affine mapping far away from the anchors. In the thin-plate case this affine map is different at different regions, unlike the Gaussian case in which the same affine component appearing in the definition of the mapping dominates the transformation in all areas away from the anchors.

RBF Formulation

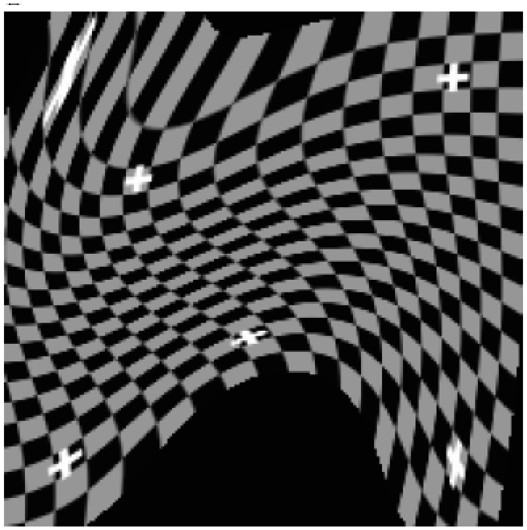
- Please see Gerig handouts for formulation

RBF Warp – Example

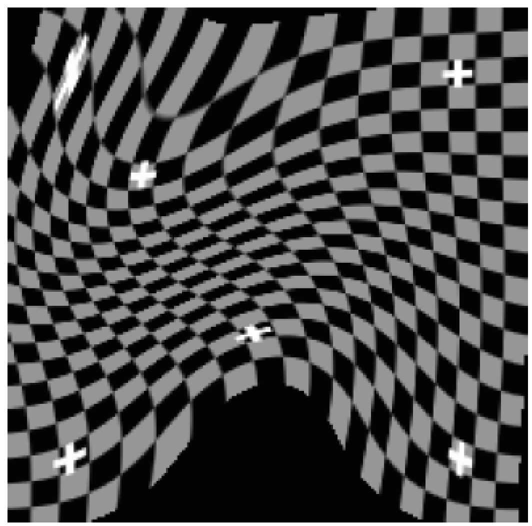


What Kernel Should We Use

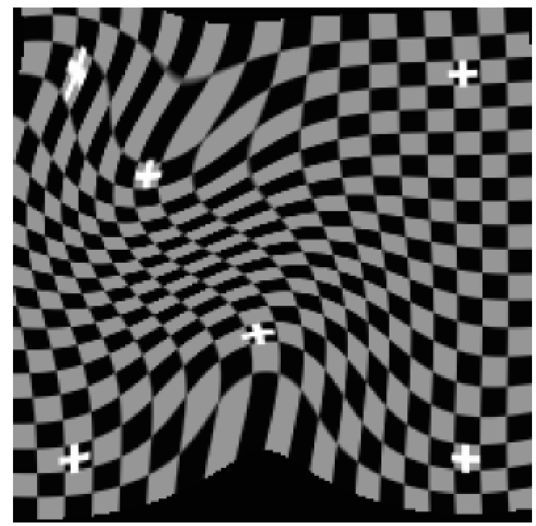
- Gaussian
  - Variance is free parameter – controls smoothness of warp



$\sigma = 2.5$



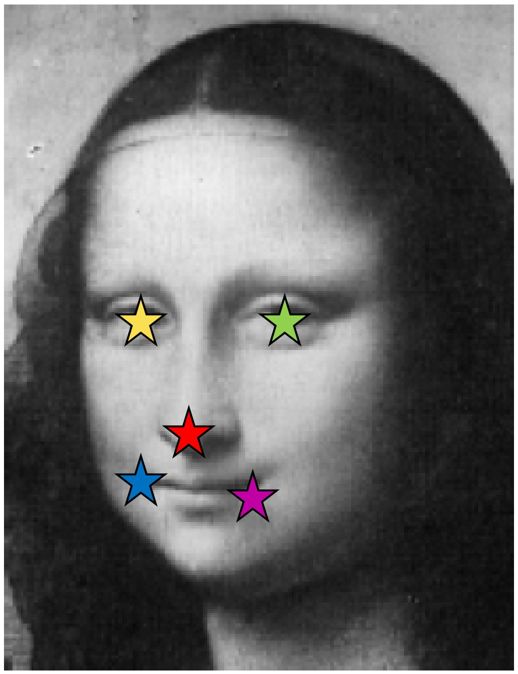
$\sigma = 2.0$



$\sigma = 1.5$

From: Arad et al. 1994

RBFs – Aligning Faces



Mona Lisa – Target

Venus – Source

Venus – Warped

Symmetry?

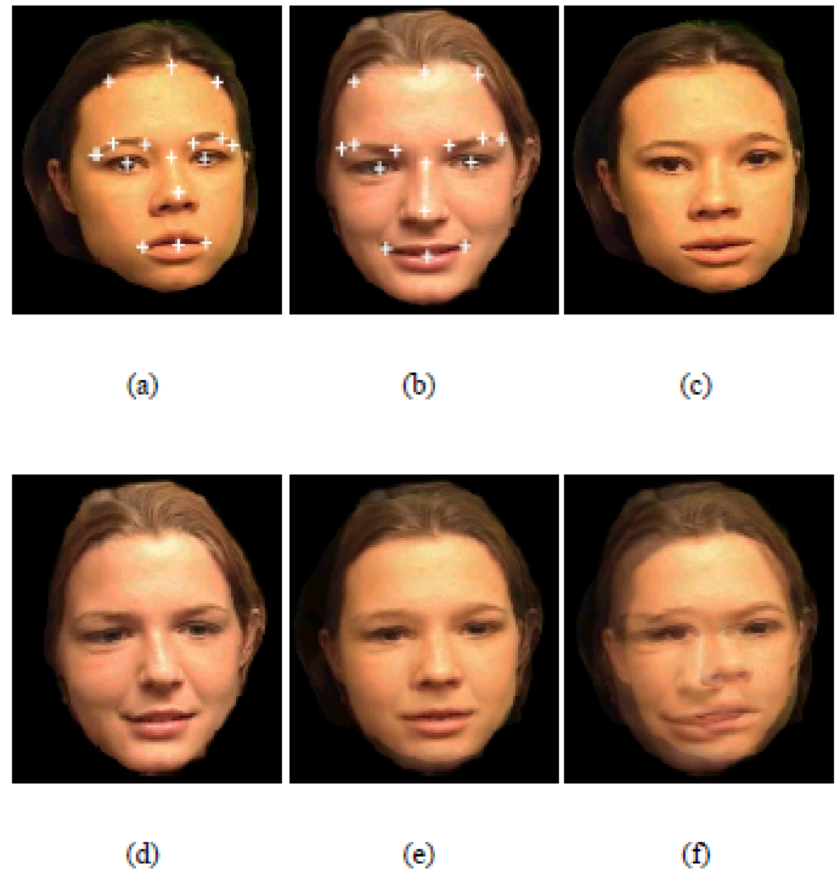


Figure 2. Image metamorphasis with RBFs: a) source image I_0; b) destination image I_1; c) forward warping I_0 with $d_{0\rightarrow 1}$; d) backward warping I_1 with $d_{1\rightarrow 0}$; e) result of morphing between I_0 and I_1; f) cross-dissolved image.

Image-based Talking Heads using Radial Basis Functions James D. Edge and Steve Maddock

Symmetry?

What can we say about symmetry: A->B and B->A ?

Application

Figure 4. Synthesized viseme transitions. Central column contains transitional frames between the source and destination visemes.

- Modeling of lip motion in speech with few landmarks.
- Synthesis via motion of landmarks.

Application: Image Morphing

- Combine shape and intensity with time parameter t
  - Just blending with amounts t produces "fade"

$$I(t) = (1-t)I_1 + tI_2$$

  - Use control points with interpolation in t

$$\bar{c}(t) = (1-t)\bar{c}_1 + t\bar{c}_2$$

  - Use c_1, c(t) landmarks to define T_1, and c_2,c(t) landmarks to define T_2

Image Morphing

- Create from blend of two warped images

$$I_t(\bar{x}) = (1-t)I_1\left(T_1(\bar{x})\right) + tI_2\left(T_2(\bar{x})\right)$$

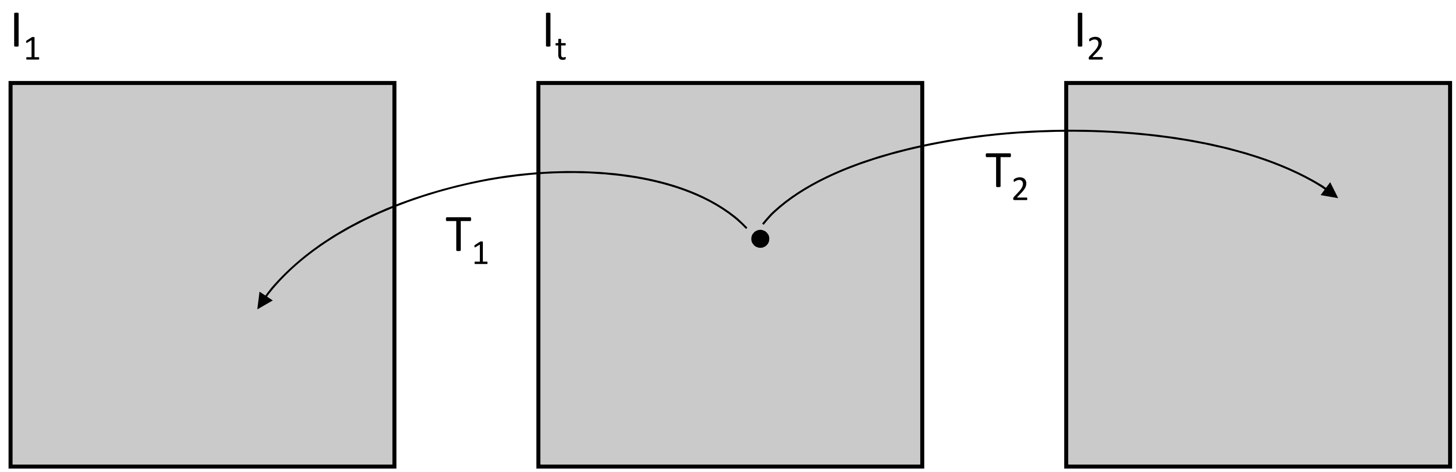


Image Morphing

Application: Image Templates/Atlases

- Build image templates that capture statistics of class of images
  - Accounts for shape and intensity
  - Mean and variability
- Purpose
  - Establish common coordinate system (for comparisons)
  - Understand how a particular case compares to the general population

Templates – Formulation

- N landmarks over M different subjects/samples

Correspondences

Images

$$I^j(\bar{x}) \qquad \bar{c}_i^j \qquad \begin{pmatrix} \bar{c}_1^1 & \dots & \bar{c}_N^1 \\ \vdots & & \vdots \\ \bar{c}_1^M & \dots & \bar{c}_N^M \end{pmatrix}$$

Mean of correspondences (template)

$$\hat{c}_i = \frac{1}{M}\sum_{j=1}^{M} \bar{c}_i^j$$

Transformations from mean to subjects

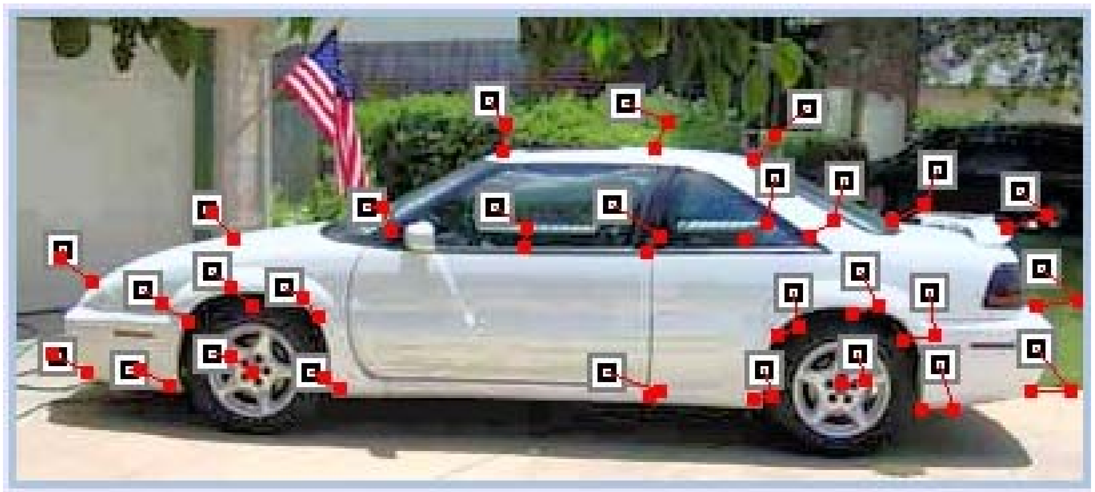
$$\bar{c}_i^j = T^j(\hat{c}_i)$$

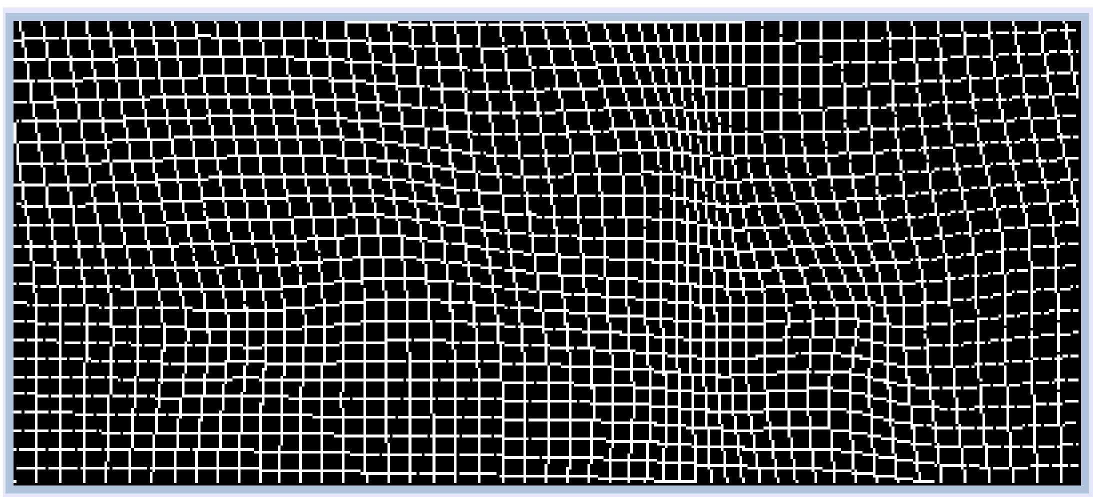
Templated image

$$\hat{I}(\bar{x}) = \frac{1}{M}\sum_j I^j\left(T^j(\bar{x})\right)$$

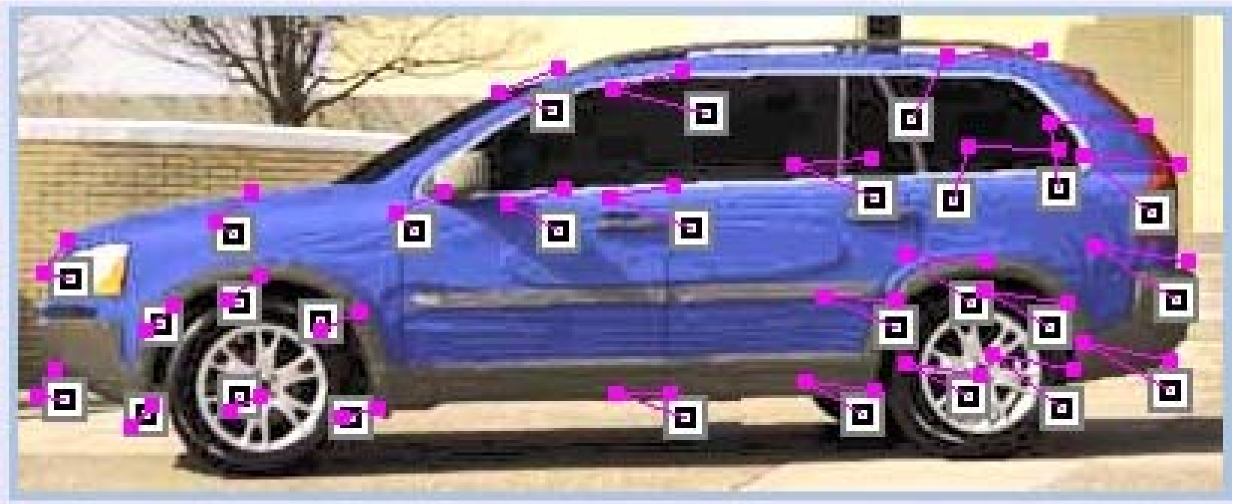
Cars

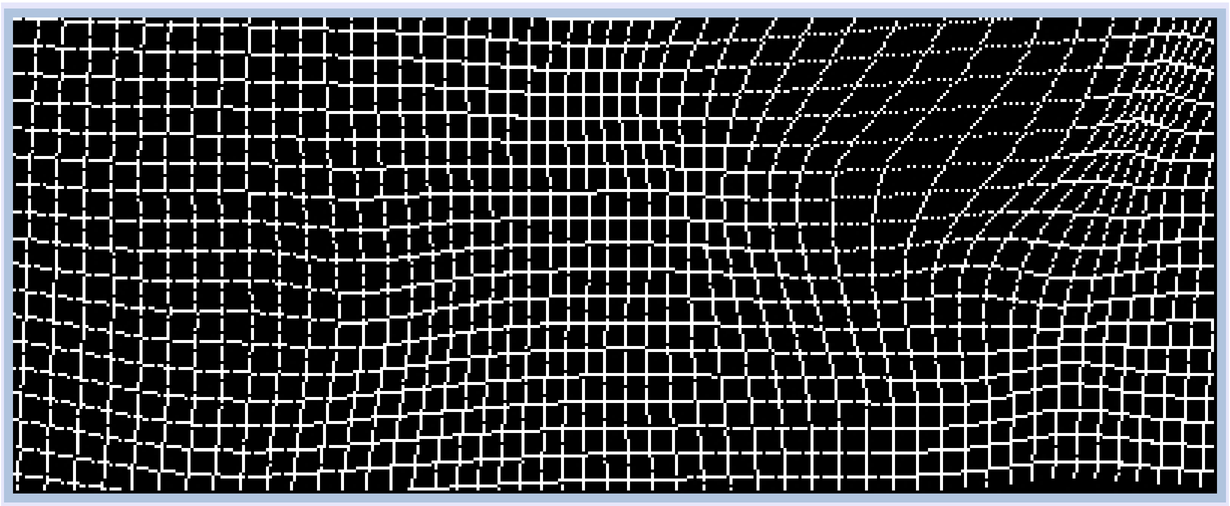
Car Landmarks and Warp



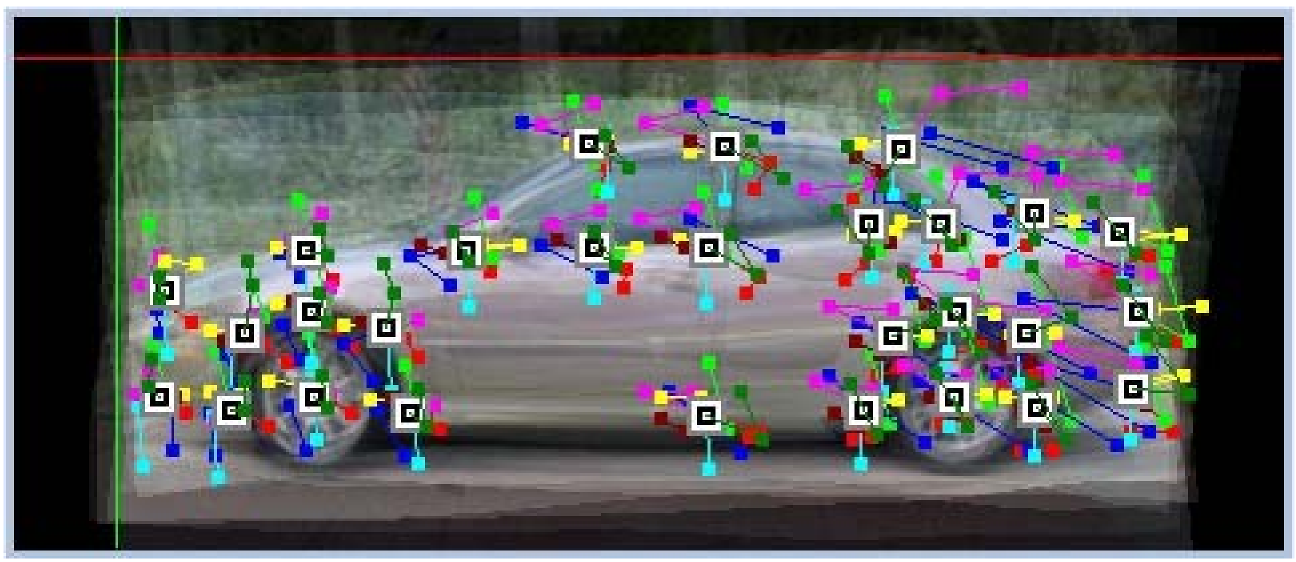


Car Landmarks and Warp

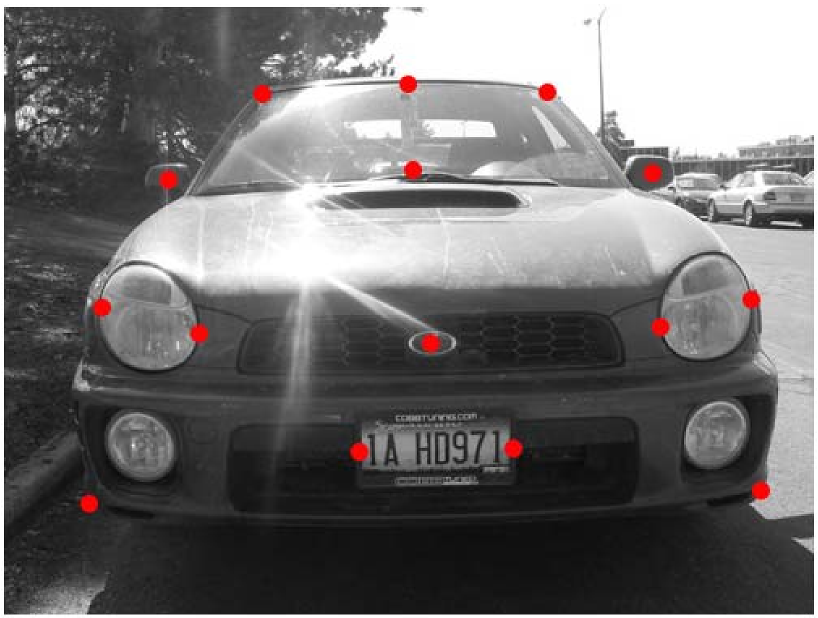




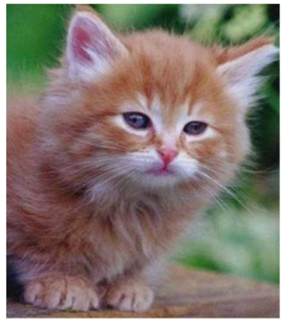
Car Mean

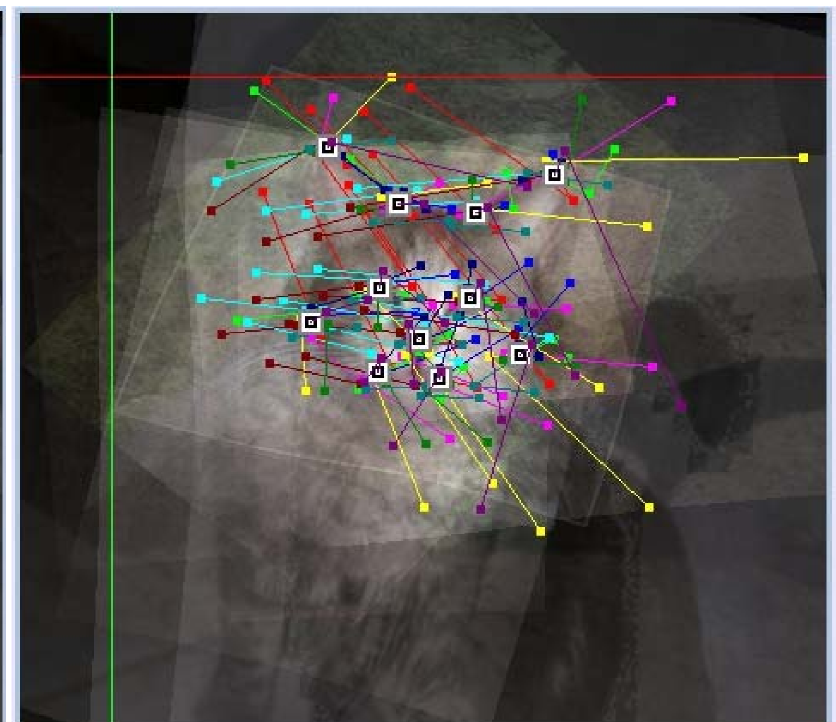


Cars

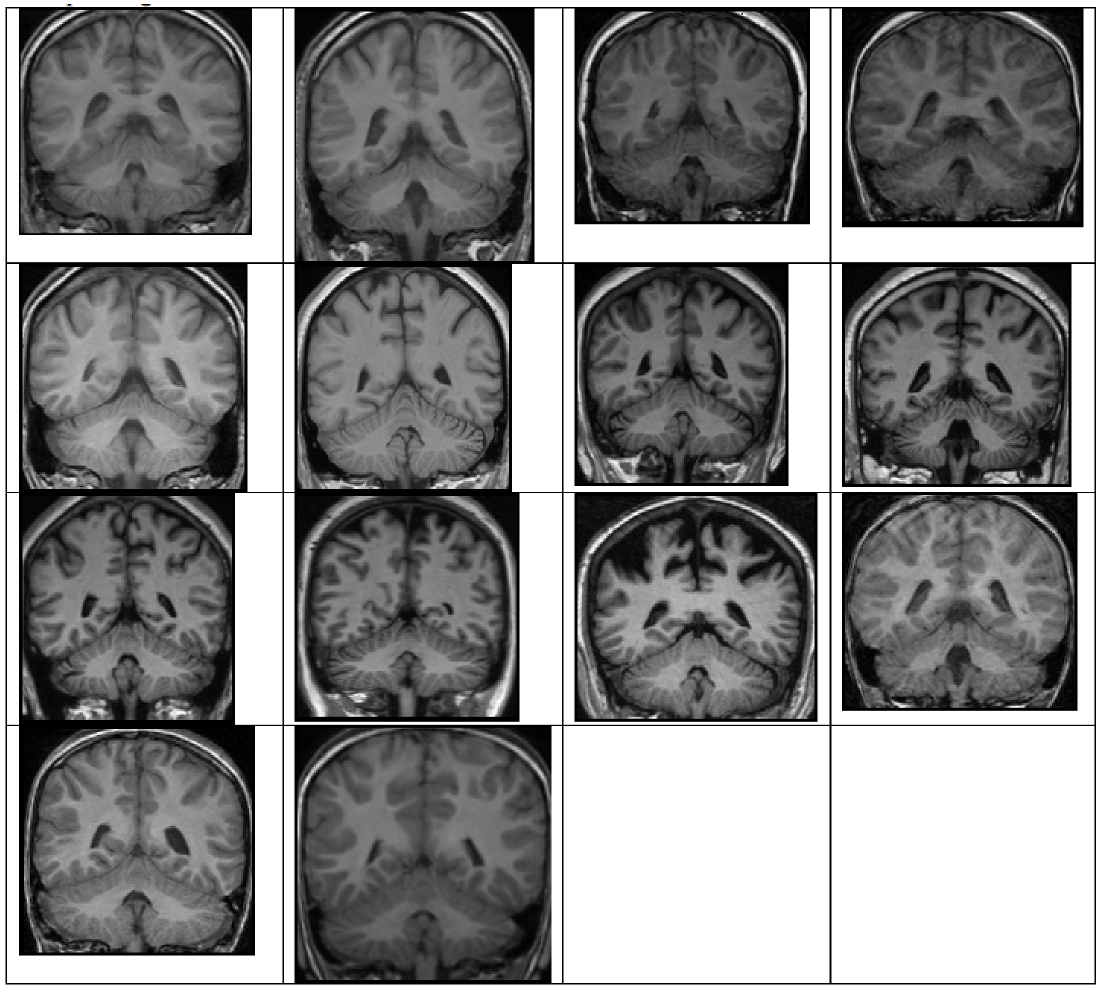


Cats

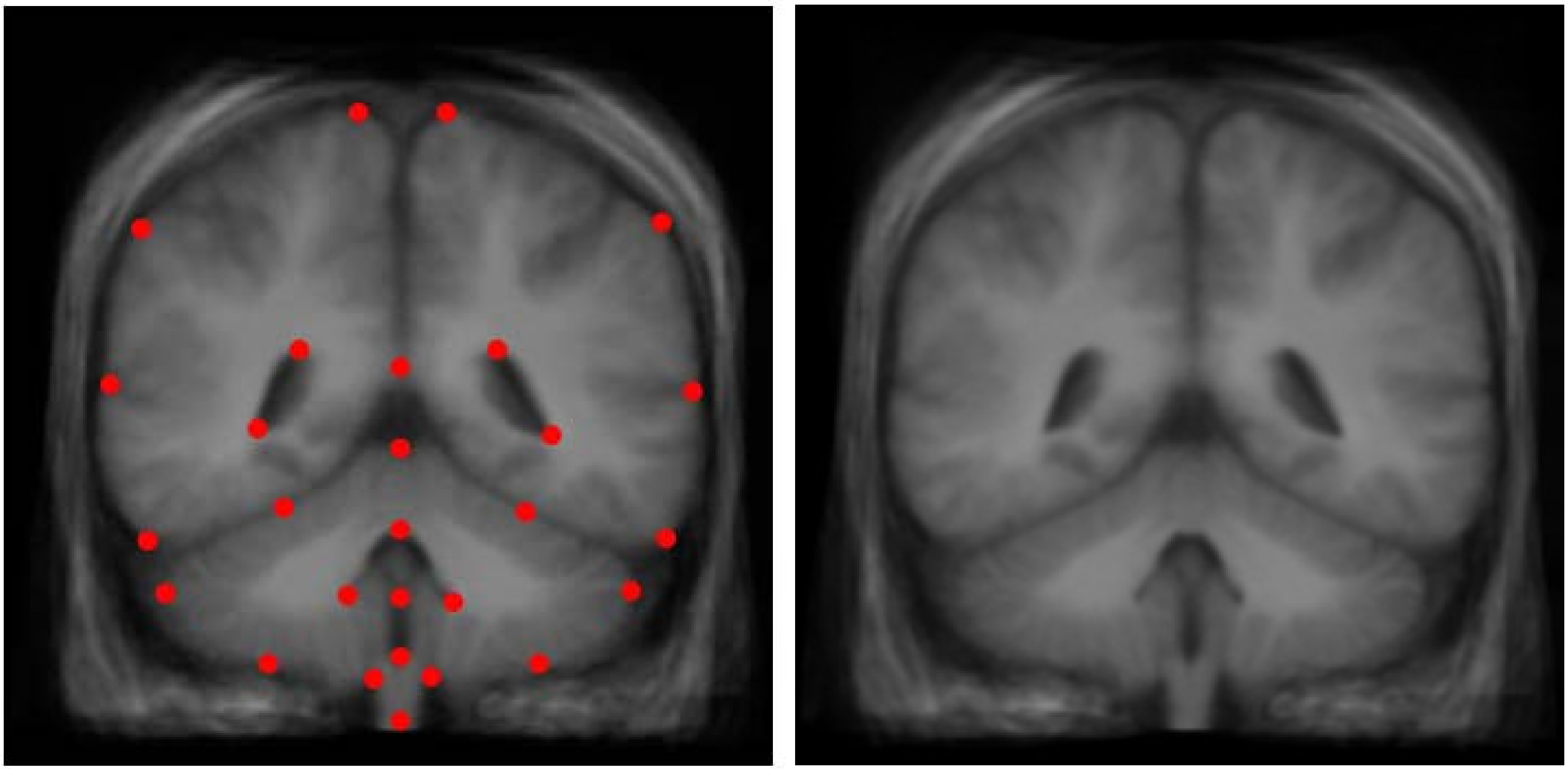




Brains



Brain Template

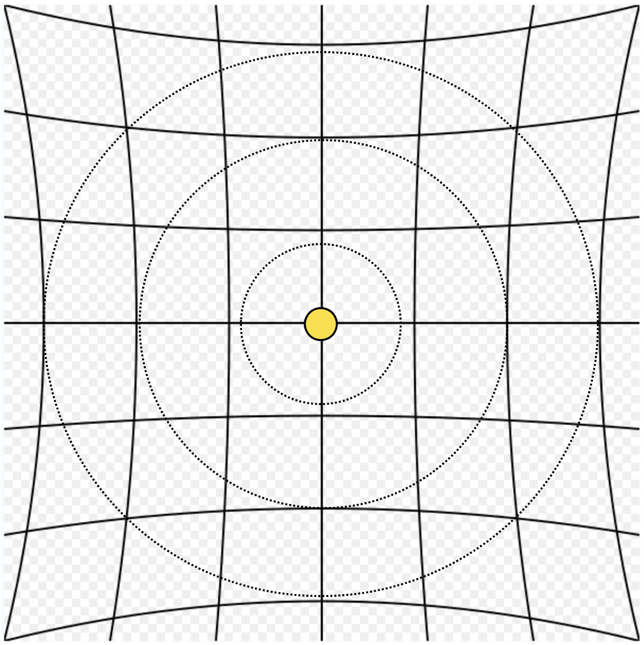


APPLICATIONS

Warping Application: Lens Distortion

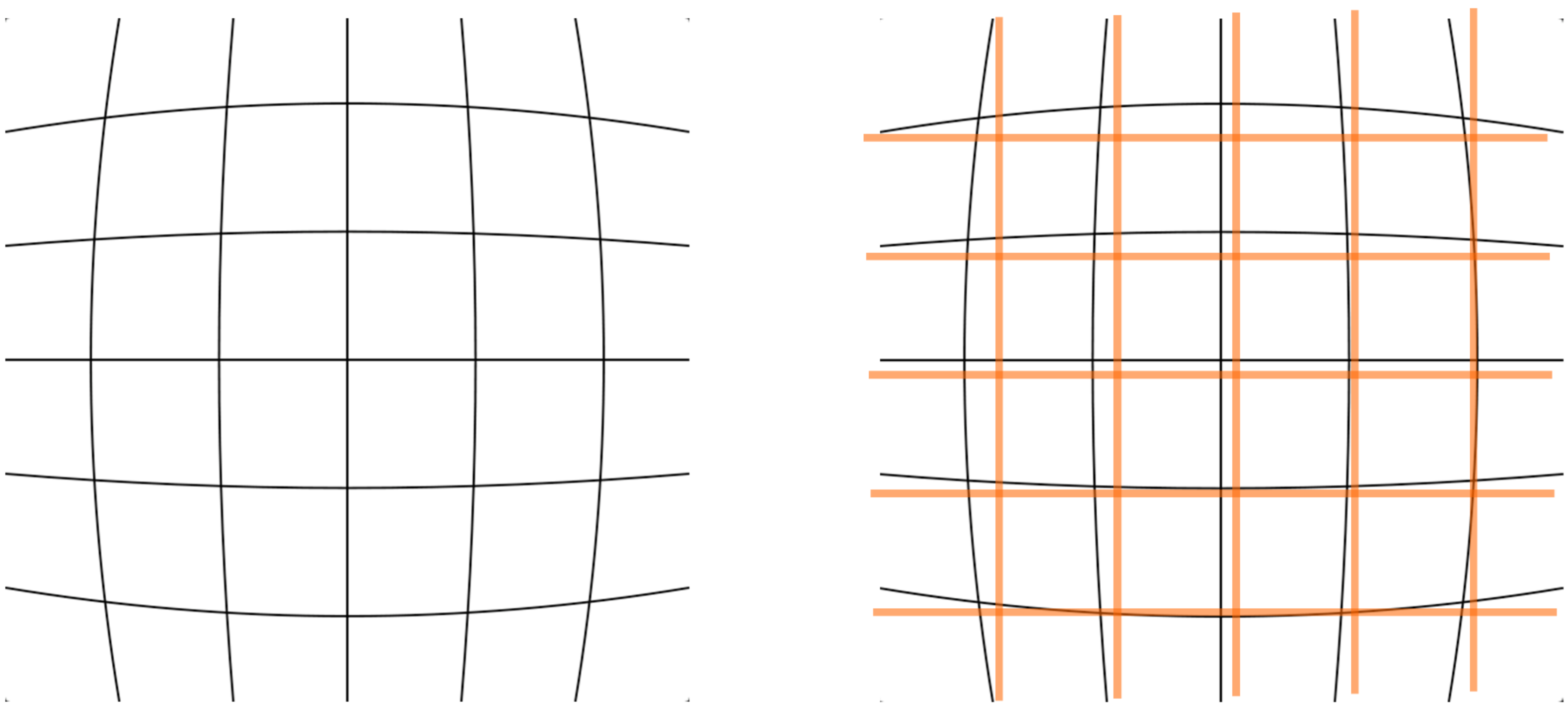
- Radial transformation – lenses are generally circularly symmetric
  - Optical center is known
  - Model of transformation:

$$\bar{x}' = \bar{x}\left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \ldots\right)$$



Correspondences

- Take picture of known grid – crossings



- Measure set of landmark pairs → Estimate transformation, correct images

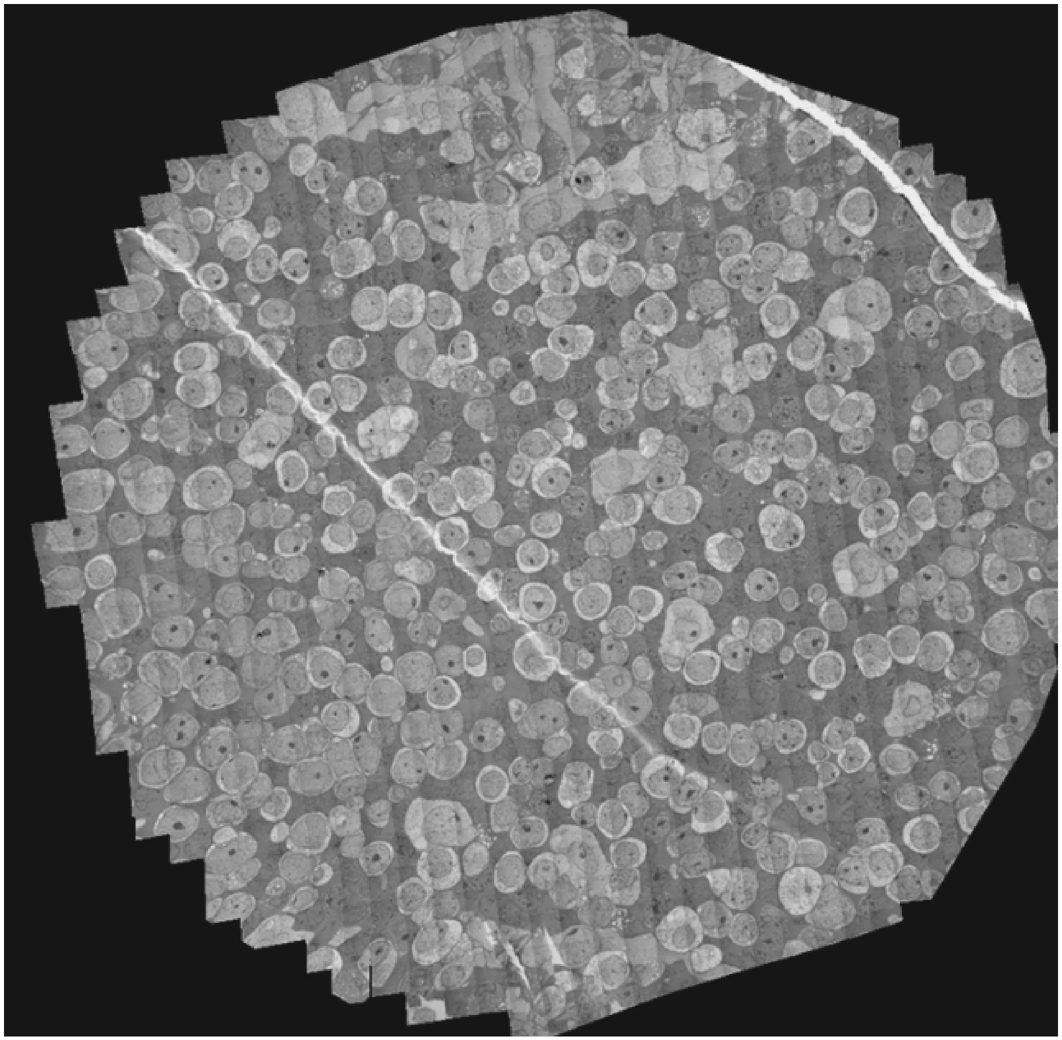
Image Mosaicing

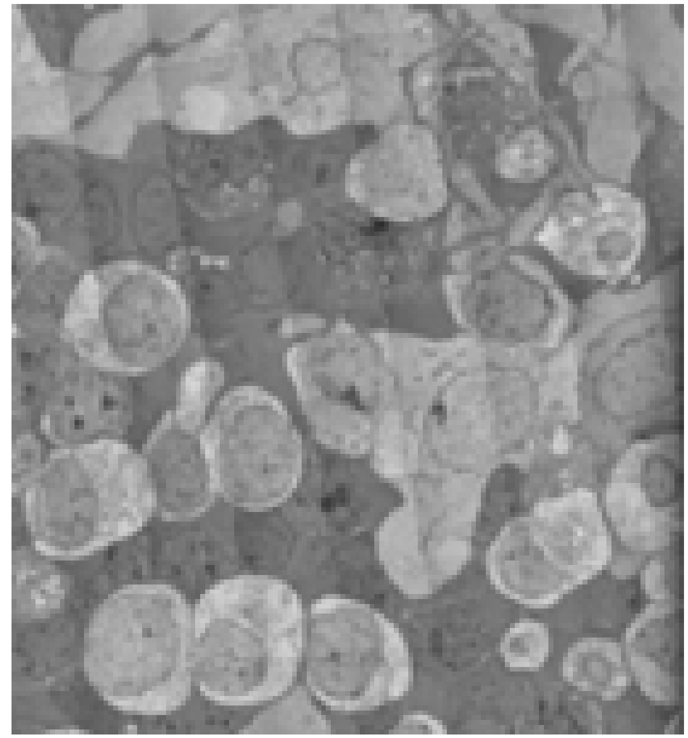
- Piecing together images to create a larger mosaic
- Doing it the old fashioned way
  - Paper pictures and tape
  - Things don't line up
  - Translation is not enough
- Need some kind of warp
- Constraints
  - Warping/matching two regions of two different images only works when…

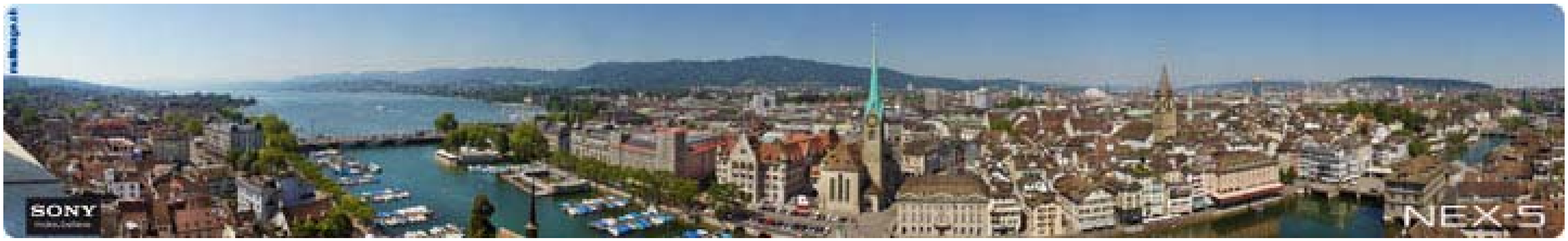
Applications

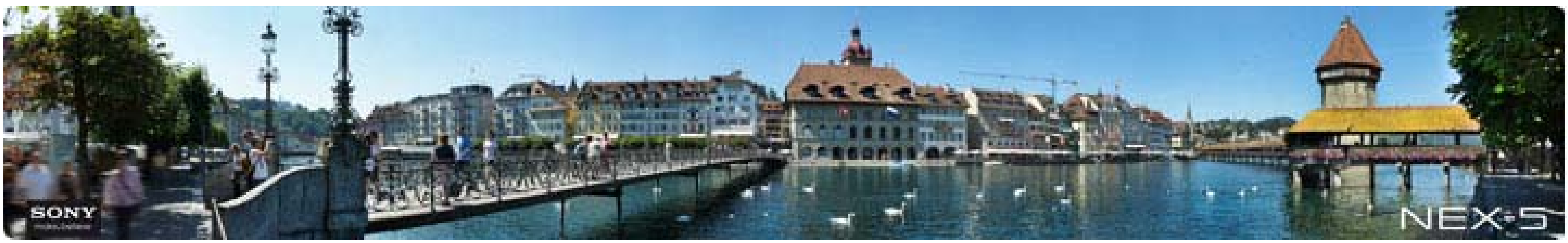
Saint-Guénolé Church of Batz-sur-Mer Equirectangular 360º by Vincent Montibus

Microscopy (Morane Eye Inst, UofU, T. Tasdizen et al.)





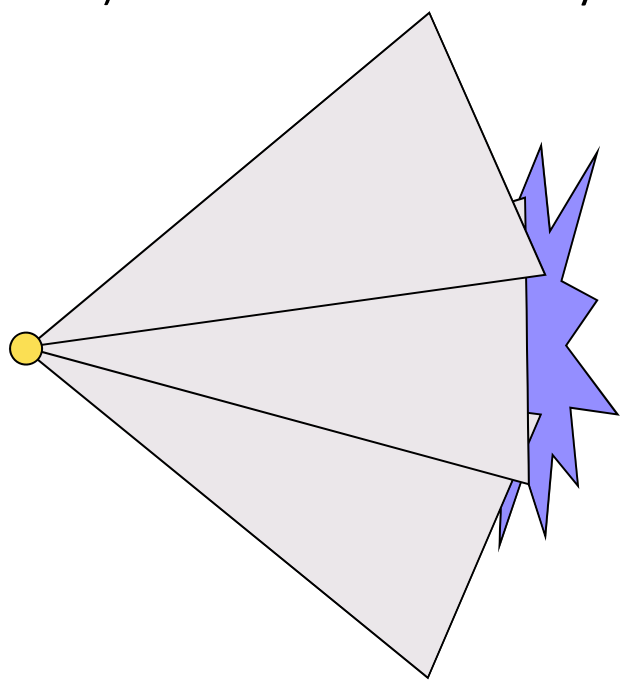

SONY
NEX-5


SONY
NEX-5

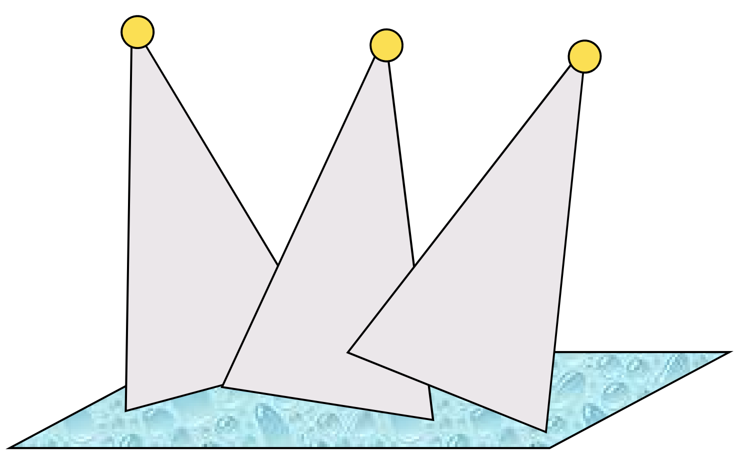
Special Cases

- Nothing new in the scene is uncovered in one view vs another
  - No ray from the camera gets behind another

1) Pure rotations–arbitrary scene

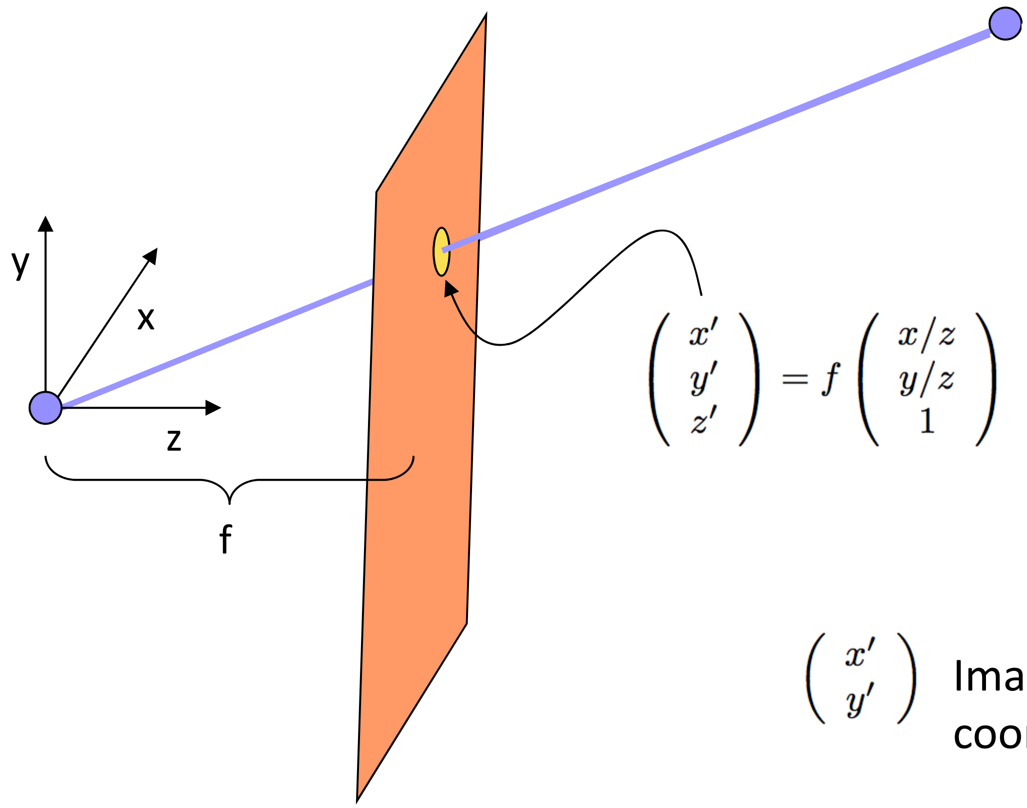


2) Arbitrary views of planar surfaces



3D Perspective and Projection

- Camera model



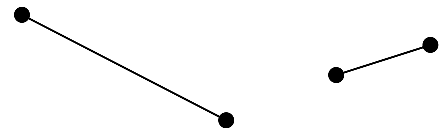
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \\ 1 \end{pmatrix}$$

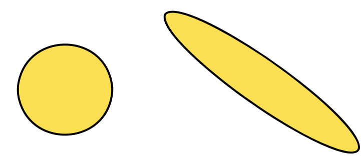
$\begin{pmatrix} x' \\ y' \end{pmatrix}$ Image coordinates

Perspective Projection Properties

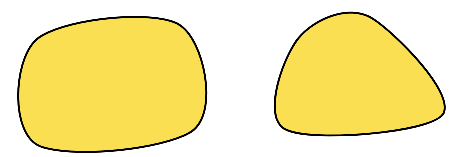
- Lines to lines (linear)



- Conic sections to conic sections



- Convex shapes to convex shapes



- Foreshortening

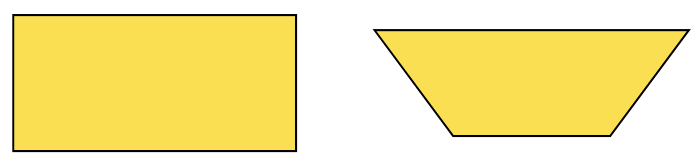


Image Homologies

- Images taken under cases 1,2 are perspectively equivalent to within a linear transformation
  - Projective relationships – equivalence is

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} d \\ e \\ f \end{pmatrix} \iff \begin{pmatrix} a/c \\ b/c \\ 1 \end{pmatrix} = \begin{pmatrix} d/f \\ e/f \\ 1 \end{pmatrix}$$

Transforming Images To Make Mosaics

Linear transformation with matrix P

$$\bar{x}^* = P\bar{x} \qquad P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & 1 \end{pmatrix} \qquad \begin{aligned} x^* &= p_{11}x + p_{12}y + p_{13} \\ y^* &= p_{21}x + p_{22}y + p_{23} \\ z^* &= p_{31}x + p_{32}y + 1 \end{aligned}$$

Perspective equivalence

$$\begin{aligned} x' &= \tfrac{p_{11}x+p_{12}y+p_{13}}{p_{31}x+p_{32}y+1} \\ y' &= \tfrac{p_{21}x+p_{22}y+p_{23}}{p_{31}x+p_{32}y+1} \end{aligned}$$

Multiply by denominator and reorganize terms

$$\begin{aligned} p_{31}xx' + p_{32}yx' - p_{11}x - p_{12}y - p_{13} &= -x' \\ p_{31}xy' + p_{32}yy' - p_{21}x - p_{22}y - p_{23} &= -y' \end{aligned}$$

Linear system, solve for P

$$\begin{pmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & x_1x'_1 & y_1x'_1 \\ -x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2x'_2 & y_2x'_2 \\ & & & \vdots & & & & \\ -x_N & -y_N & -1 & 0 & 0 & 0 & x_Nx'_N & y_Nx'_2 \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1y'_1 & y_1y'_1 \\ 0 & 0 & 0 & -x_2 & -y_2 & -1 & x_2y'_2 & y_2y'_2 \\ & & & \vdots & & & & \\ 0 & 0 & 0 & -x_N & -y_N & -1 & x_Ny'_N & y_Ny'_N \end{pmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{21} \\ p_{23} \\ p_{23} \\ p_{31} \\ p_{32} \end{pmatrix} = \begin{pmatrix} -x'_1 \\ -x'_2 \\ \vdots \\ -x'_N \\ -y'_1 \\ -y'_2 \\ \vdots \\ -y'_N \end{pmatrix}$$

Image Mosaicing

4 Correspondences

5 Correspondences

6 Correspondences

Mosaicing Issues

- Need a canvas (adjust coordinates/origin)
- Blending at edges of images (avoid sharp transitions)
- Adjusting brightnesses
- Cascading transformations