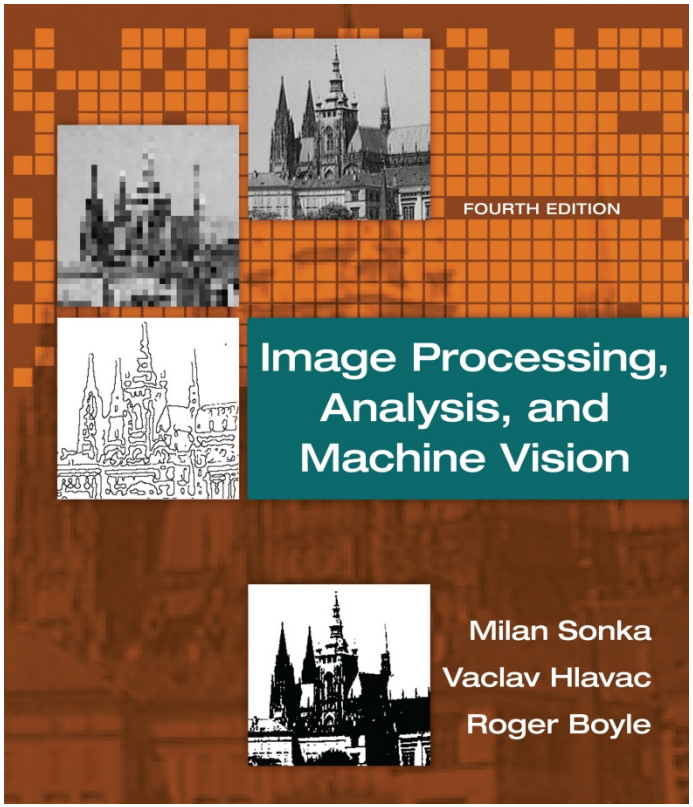


Chapter 2
The image, its representations and properties

Part I

Sources:

- Sonka Textbook
- Gonzalez/Woods DIP textbook

Overview

- Introduction of concepts and mathematical tools used in image analysis
- Image representation concepts
- Image digitization
- Digital image properties
- Color images
- Camera overview

Introduction

- We will explore concepts widely used in image analysis
- This chapter will focus on the essential basics
- We will concentrate on the intuitive meaning of the basic concepts of image analysis

2.1 Image representations, a few concepts

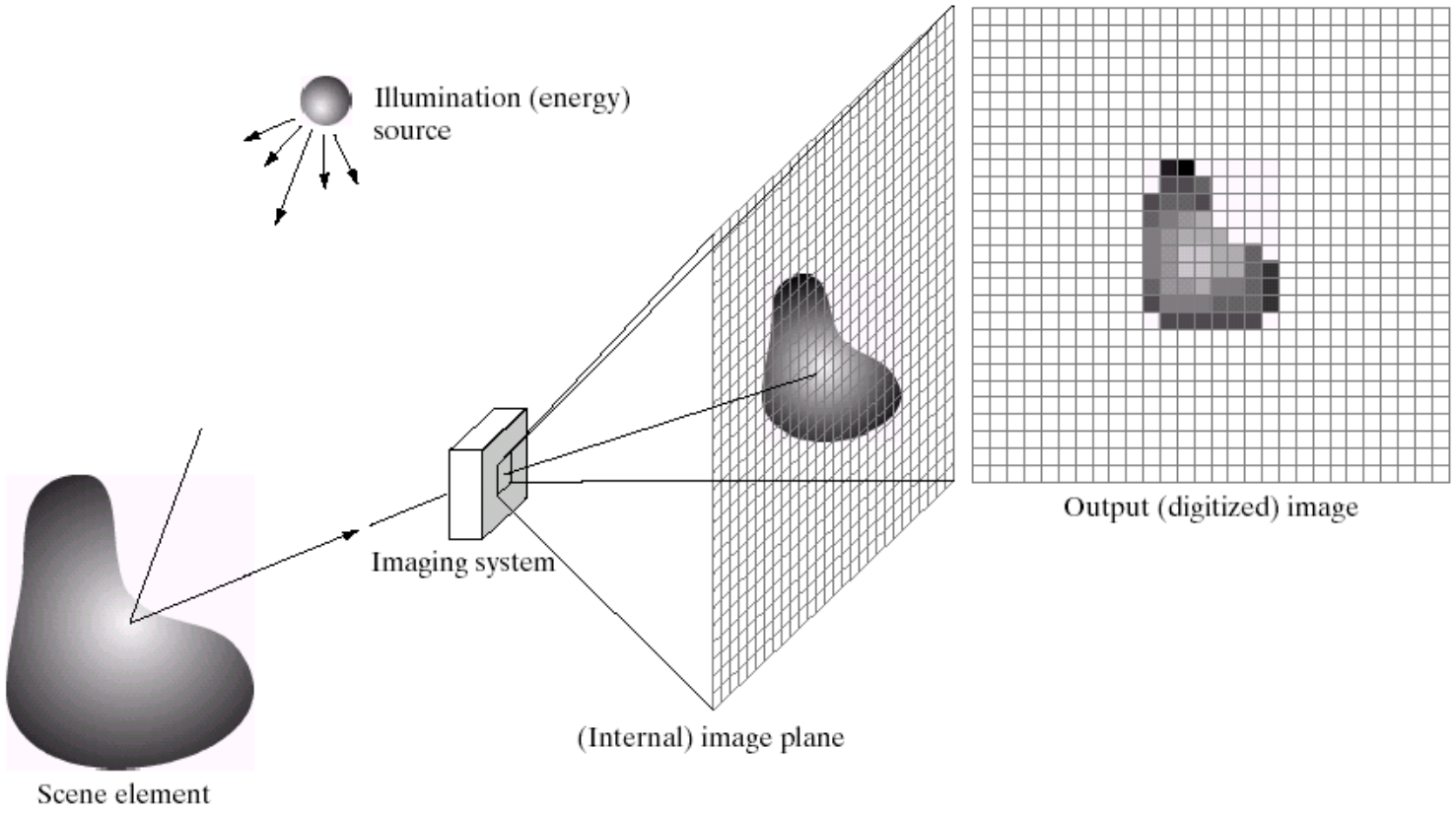
- Mathematical models used to describe images and other signals
- Signal
  - A function depending on some variable with physical meaning
  - May be on 1D (time dependent), 2D (image dependent on two coordinates in plane), or 3D (describing volumetric object in space)

- Functions Used
  - Continuous
    - continuous domain and range
  - Discrete
    - discrete domain set
  - Digital
    - discrete domain and range
- Image
  - Usual intuitive definition
    - Image on human retina
    - Image captured by video camera
  - Modeled by continuous (image) function of two variables
    - F(x,y) where (x,y) is co-ordinated in plane
    - F(x,y,t) where t is time
  - May be acquired in any way

The Continuous Image Function

- Image function values
  - Correspond to brightness at image points
  - Can express other physical quantities
    - Temperature *pressure distribution *distance from observer
- Brightness
  - Integrates different optical quantities
  - Allows us to avoid complicated process of image formation

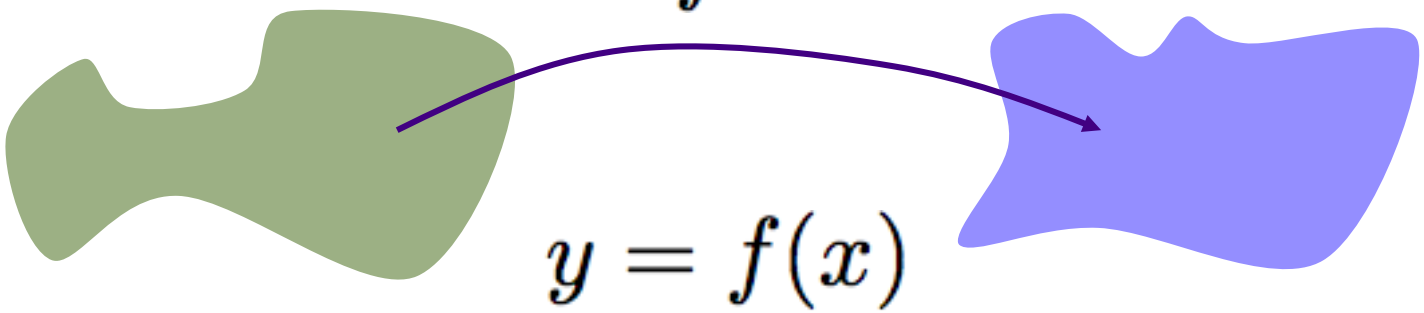
Di



a
b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image As A Mapping (Function)



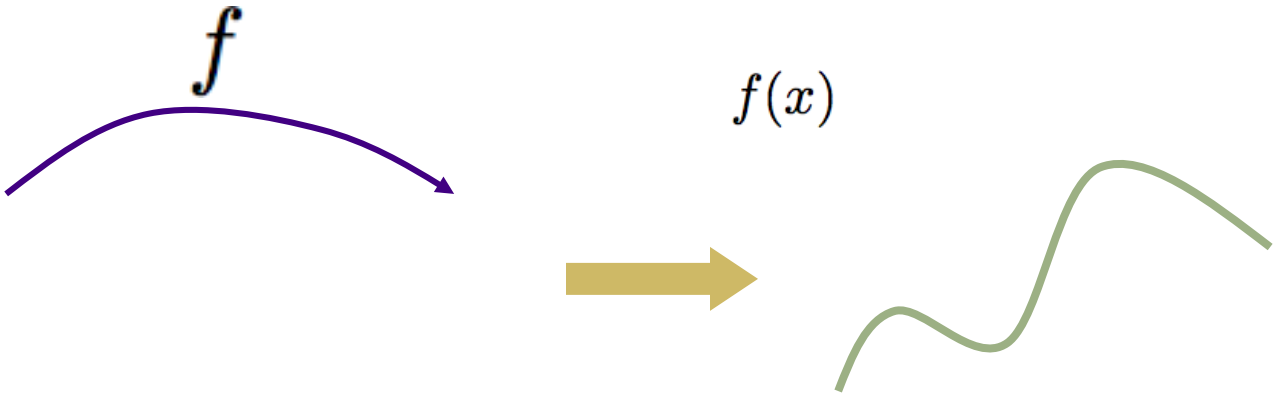
$$f : \mathcal{D} \mapsto \mathcal{R}$$

$$\mathcal{D} \subset \Re^n \text{ and } \mathcal{R} \subset \Re^m$$

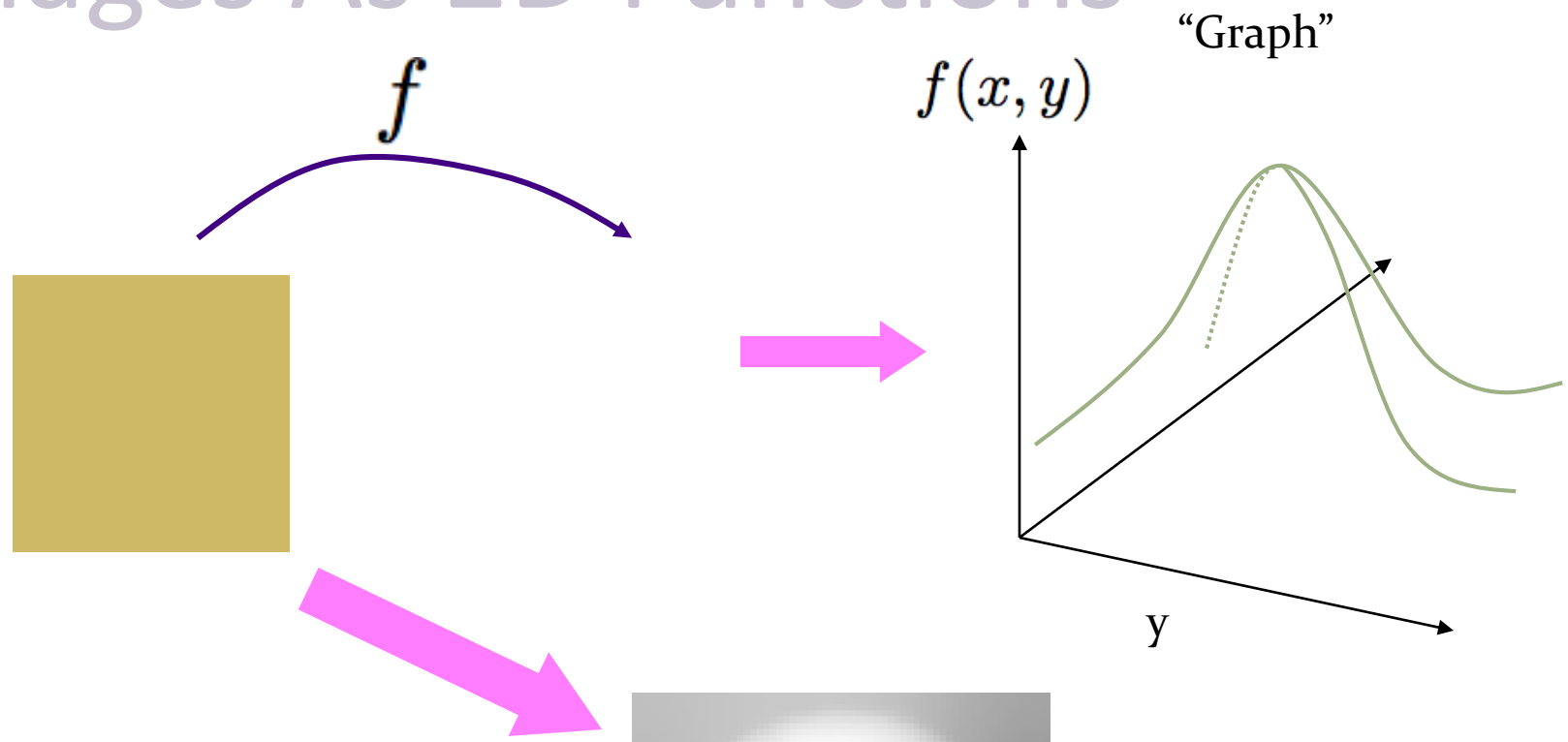
Image As A Mapping: Issues

- Dimensionality of domain (n = ?)
- Dimensionality of range (m = ?)
- Typically use shorthand of R^n or R^m
- Discrete or continuous
  - Discrete reasoning/math
  - Continuous math (calculus) –> discrete approximation
  - Issues for both domain and range

Examples of "Images" as Functions



Images As 2D Functions



Project brightness on a screen with values at coordinates (x,y) proportional to value of f(x,y)

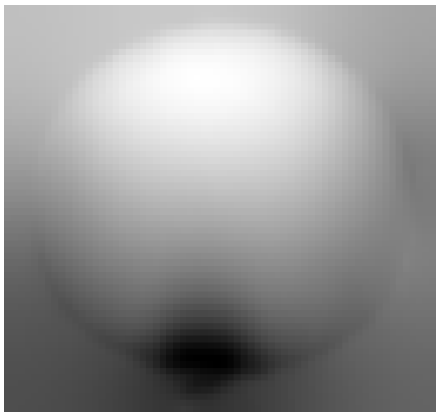
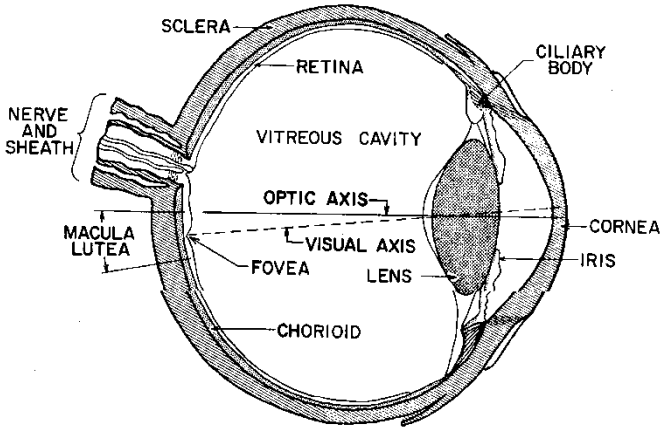


Image Formation

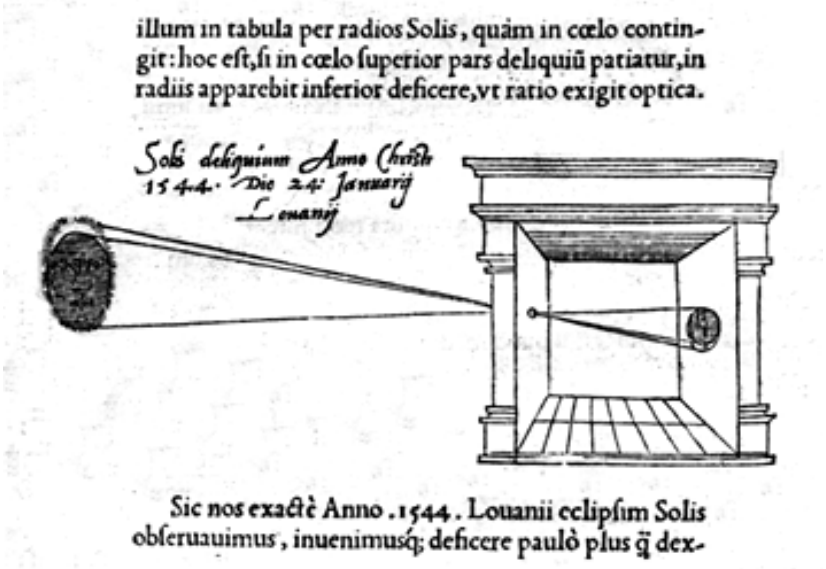


Animal eye:

a looonnng time ago

Photographic camera:

Niepce, 1816.



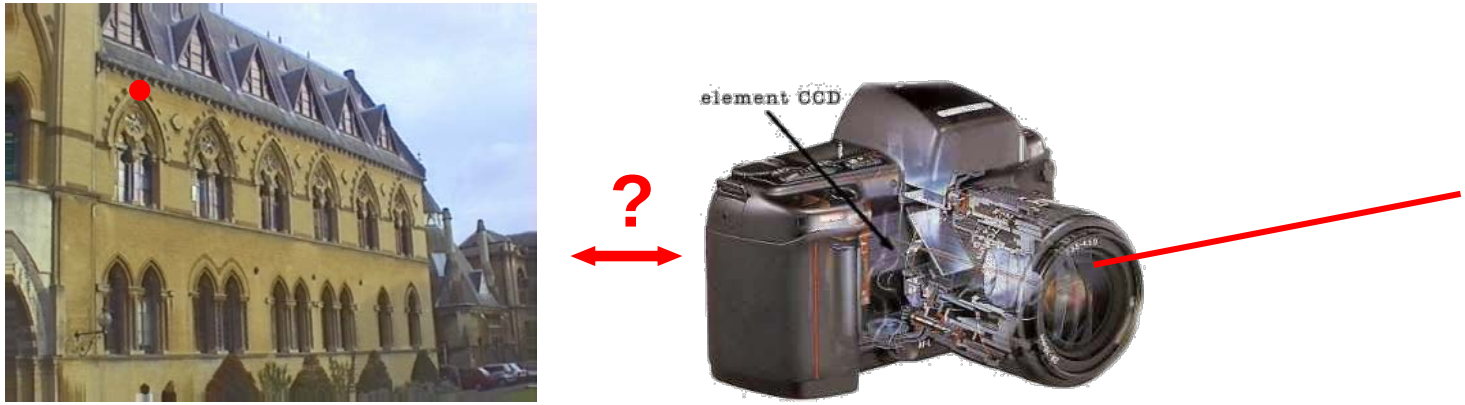
illum in tabula per radios Solis, quàm in cœlo contingit: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

Solis deliquium Anno Christi 1544. Die 24. Januarij Louanij

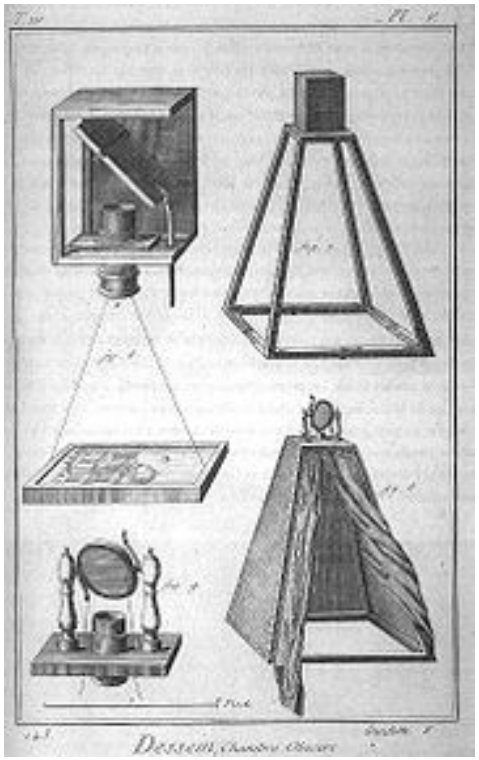
Sic nos exactè Anno .1544. Louanii eclipſim Solis obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

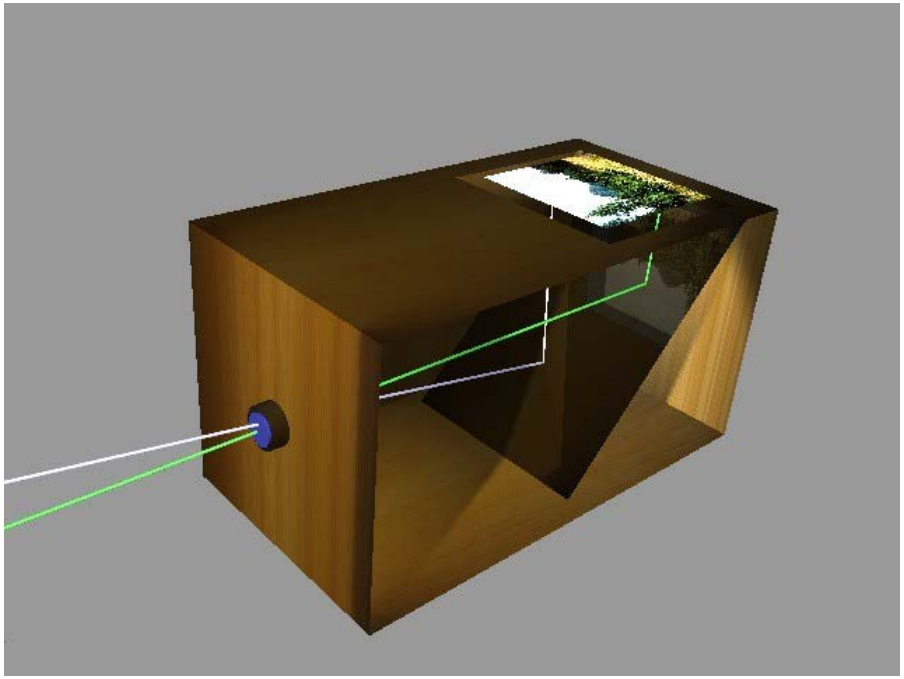
Camera model

Relation between pixels and rays in space



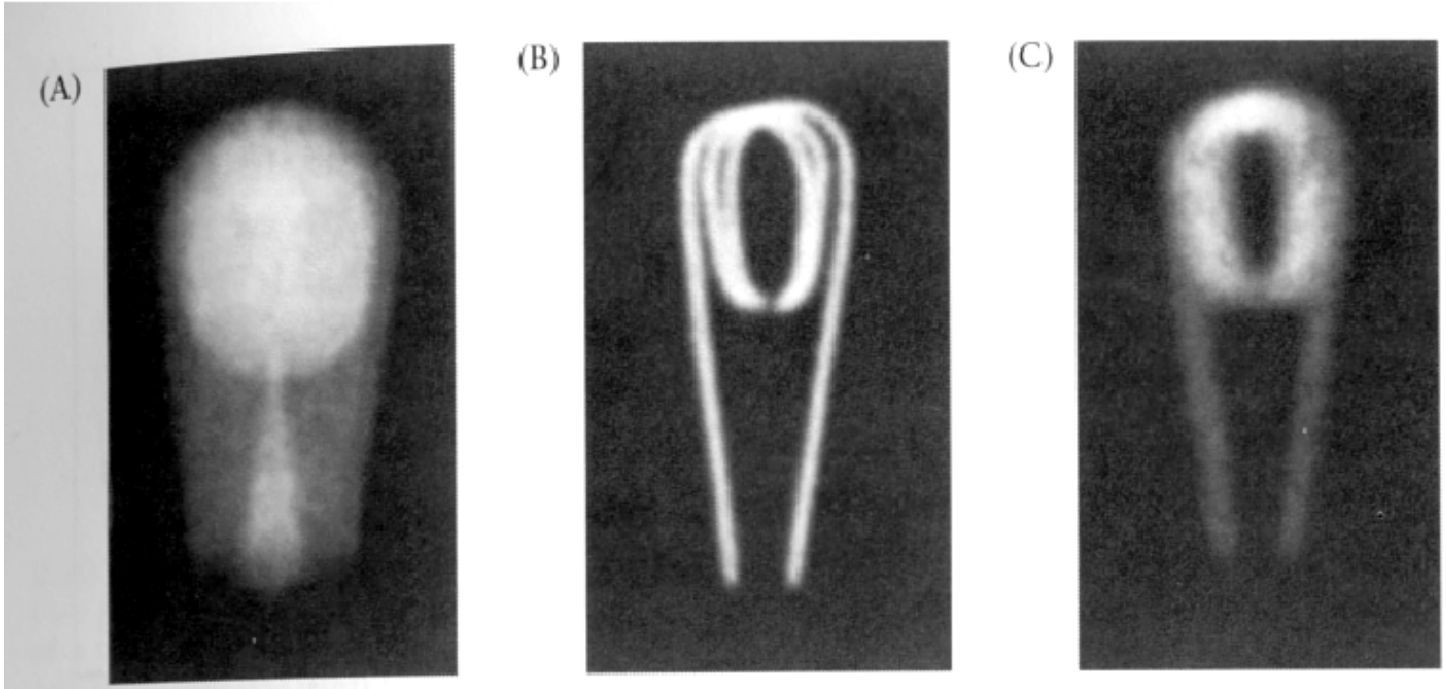
Camera obscura + lens





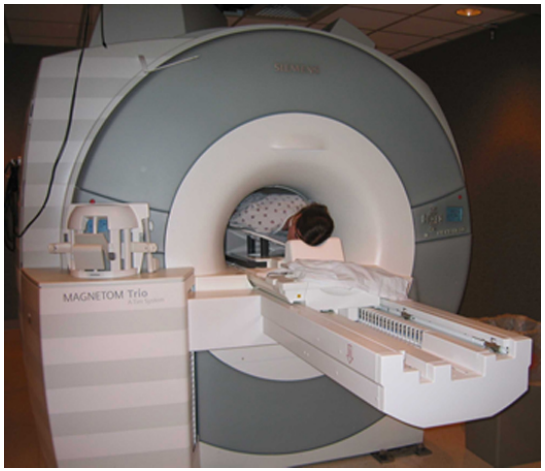
image

Limits for pinhole cameras



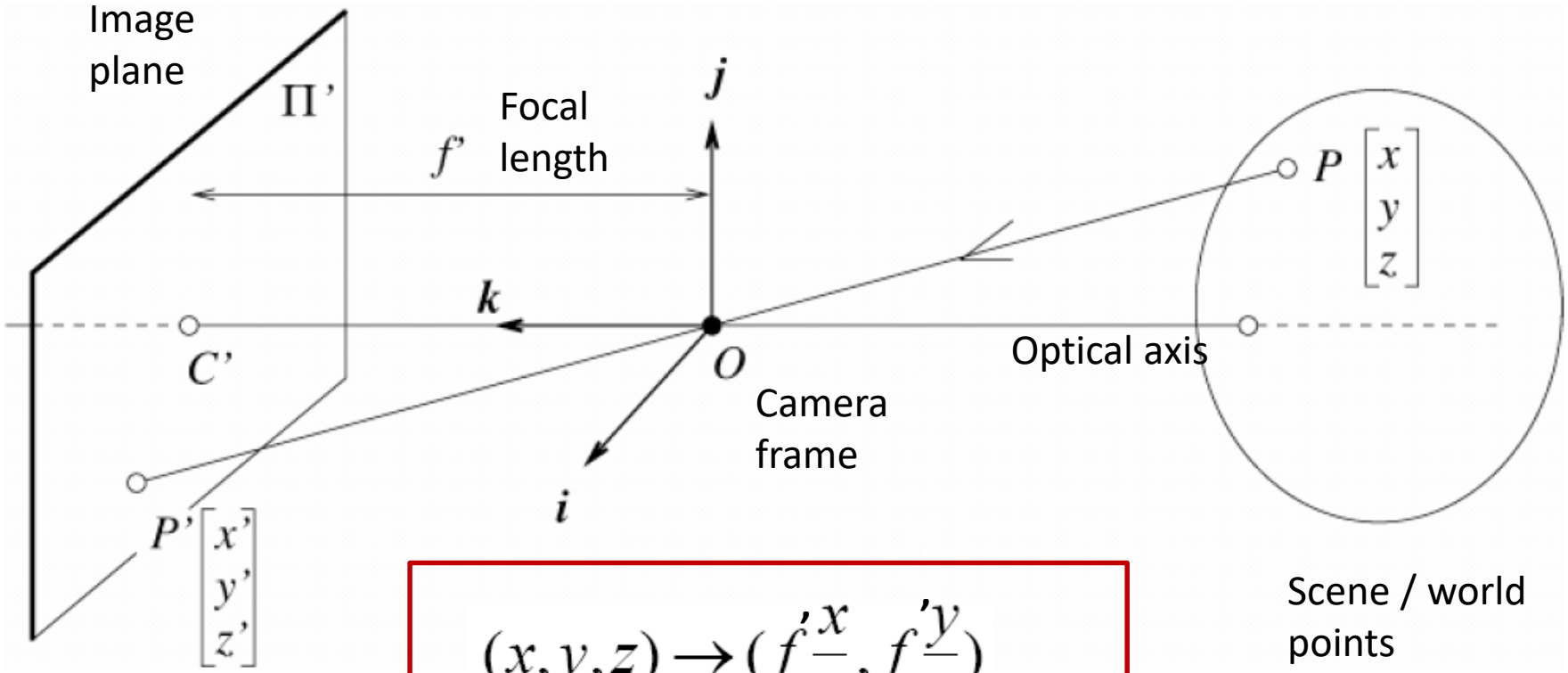
2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Image Sensors



Perspective projection equations

- 3d world mapped to 2d projection in image plane



$$(x,y,z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Scene point $\rightarrow$ Image coordinates

Forsyth and Ponce

- Image on retina or camera sensor
  - Intrinsically two-dimensional (2D)
    - Commonly result of projection of 3D scene
      - Perspective projection
  - Intensity image
    - Image bearing information about brightness point
  - Mathematical Model
    - Pin-hole camera
      - Models perspective projection of 3D scene

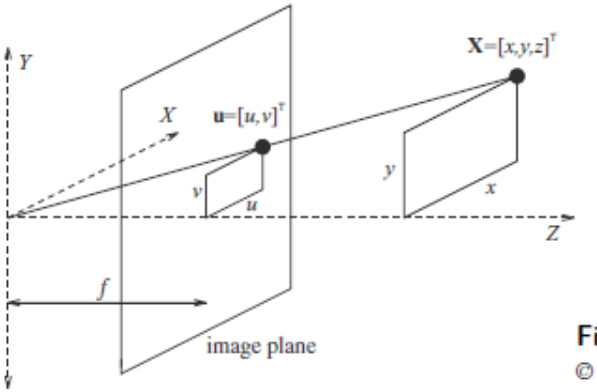


Figure 2.1: Perspective projection geometry.
© Cengage Learning 2015.

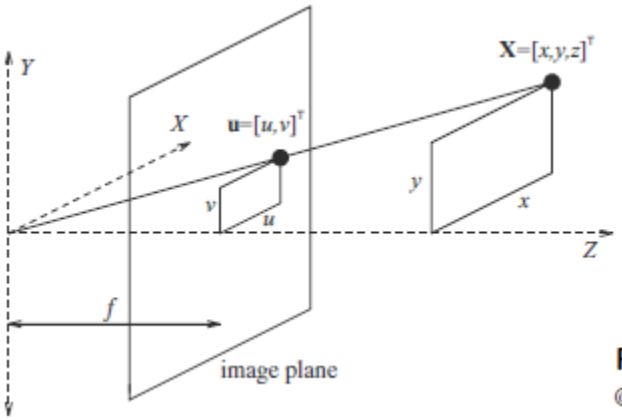


Figure 2.1: Perspective projection geometry.

- Image plane has been reflected with respect to XY plane
- Quantities x, y, z are co-ordinates of point X in 3D scene
- f is distance from pinhole to image plane (commonly called focal length)
- Projected point **u** has co-ordinates (u,v) in 2D image plane

- Non-linear perspective projection
  - Often approximated by linear parallel (or orthographic) projection
    - $f \rightarrow \infty$
  - Orthographic projection
    - Limiting case of perspective projection
    - $z \rightarrow \infty$
- 3D objects mapped into camera plane using perspective projection
  - A lot of information disappears
  - Such a transform is not one-to-one

- Static images
  - Time is constant
  - Monochromatic static image – continuous image function f(x,y)
- Computerized image processing
  - Uses digital image functions
  - Represented by matrices
    - Co-ordinates are natural numbers
  - Domain of image is region R in the plane

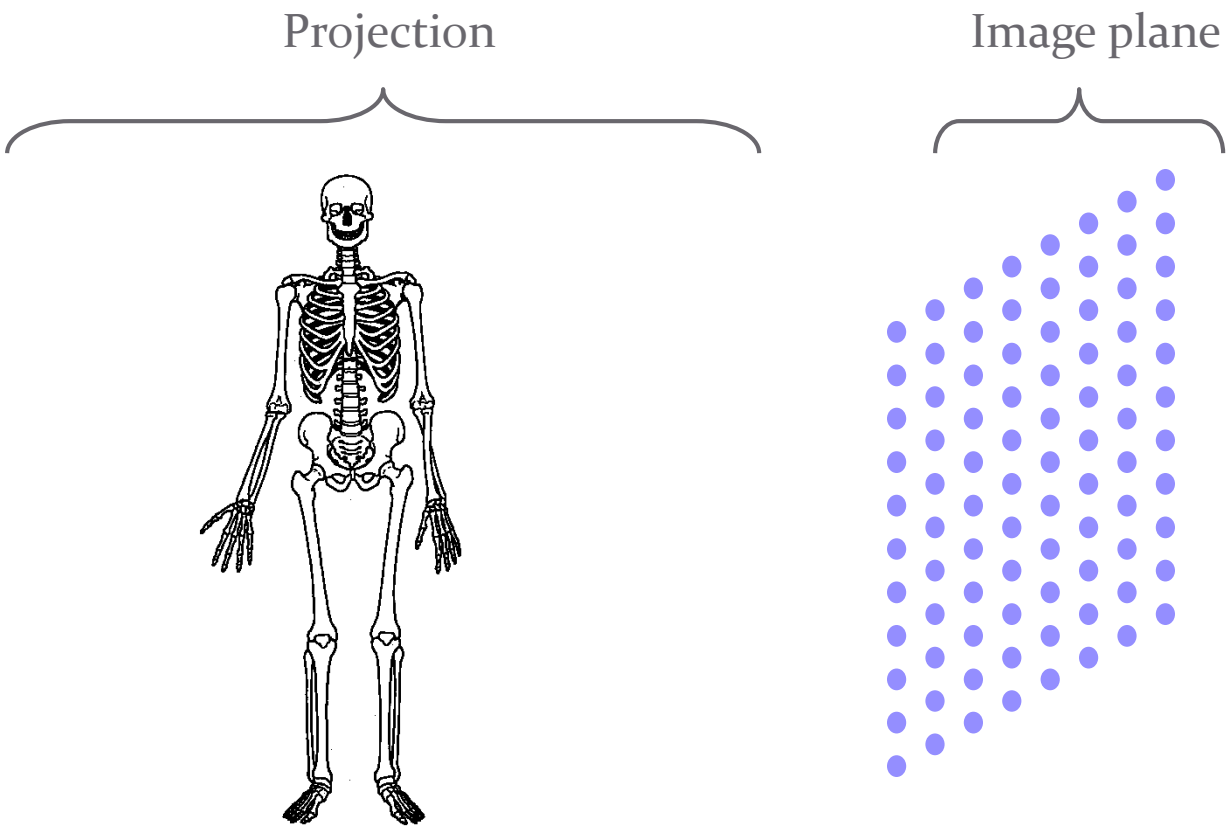
$$R = \{(x, y), 1 \leq x \leq x_m, 1 \leq y \leq y_n\}$$

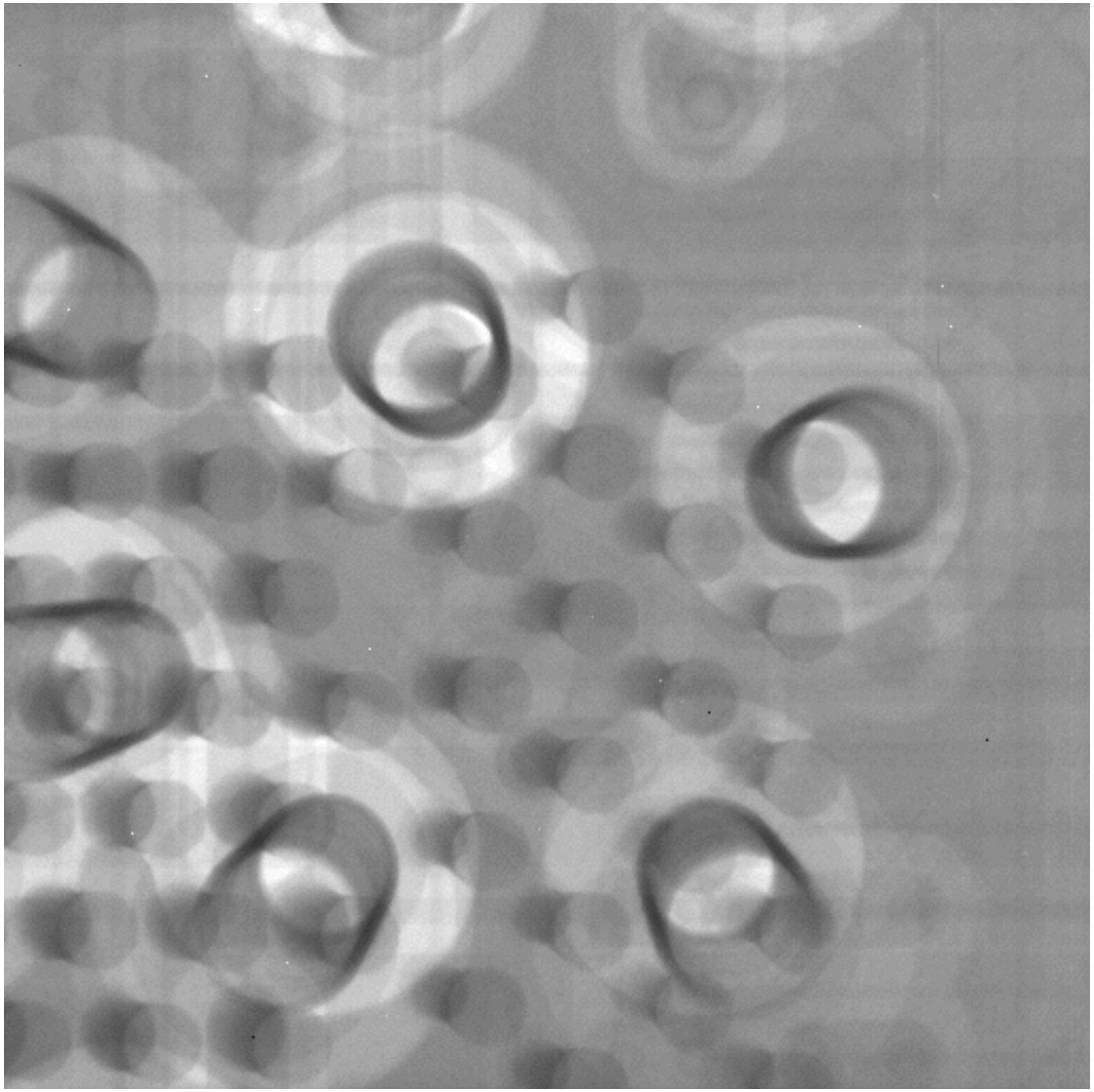
    - Where x_m, y_n represent maximal co-ordinates
    - Limited domain – infinite summation/integration limits may be used

- Range of images
  - Also limited
  - Monochromatic images
    - Lowest value corresponds to black
    - Highest value corresponds to white
    - Gray-levels
      - Brightness values bounded by lowest and highest value
- Quality of digital image grows in proportion to spatial, spectral, radiometric, and time resolutions
  - Spatial resolution – given by proximity of image sample in image plane

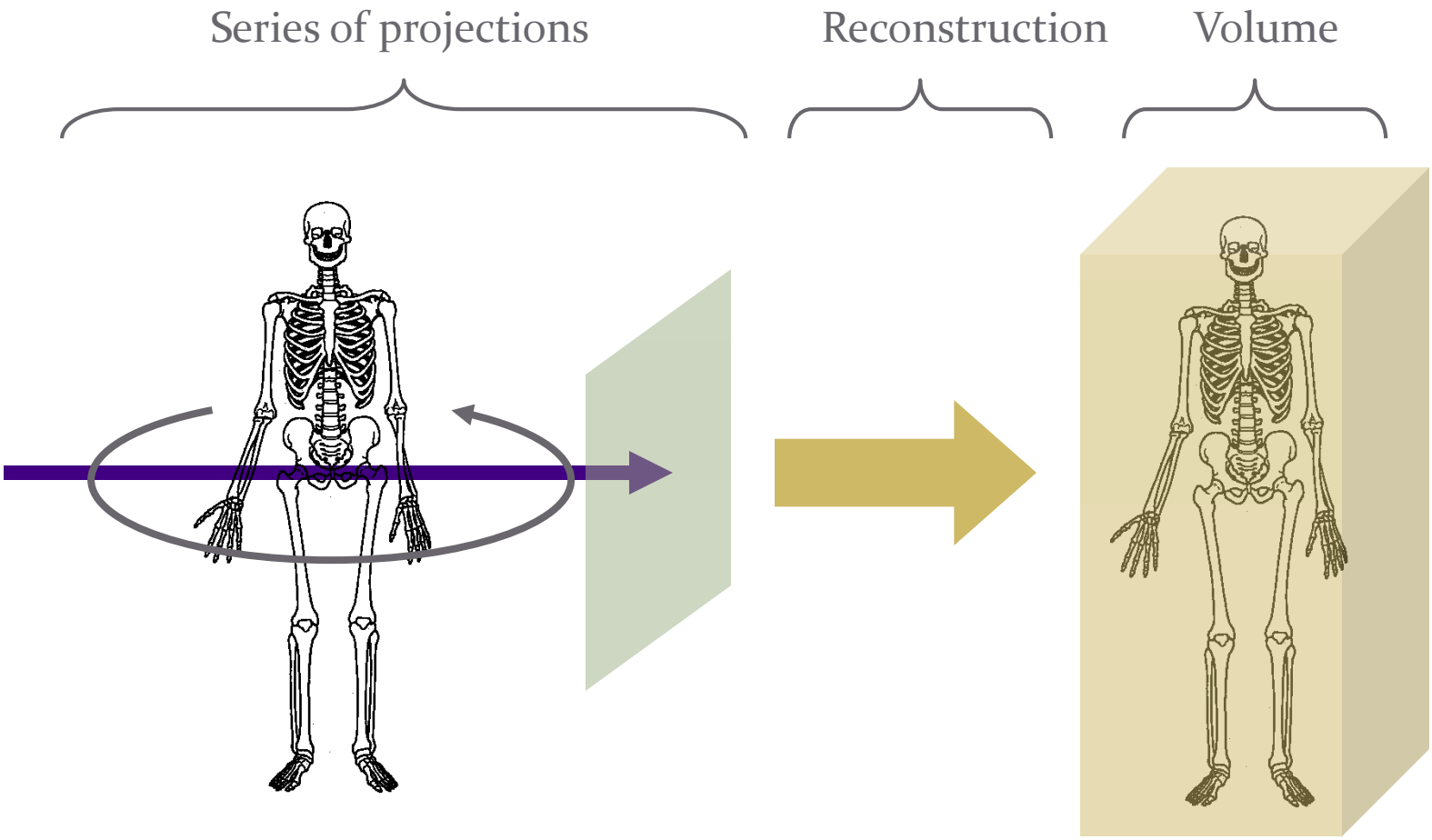
- Spectral resolution – given by bandwidth of light frequencies captured by sensor
- Radiometric resolution – corresponds to number of distinguishable gray-levels
- Time resolution – given by interval between time sample at which images are captured
  - Important in dynamic image analysis

X-Rays

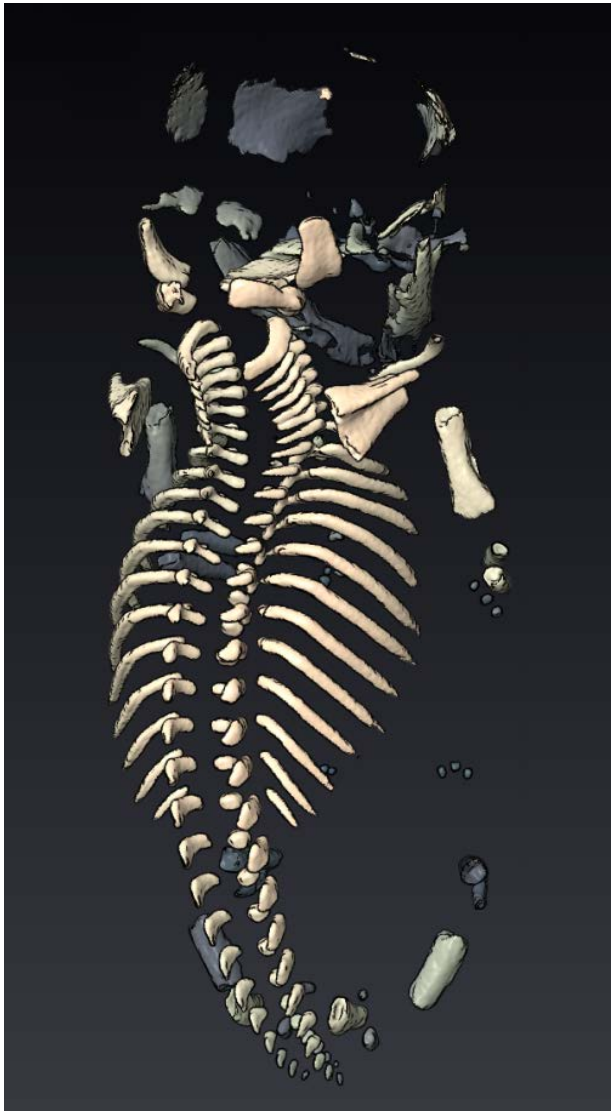




Computed Tomography



CT (CAT)

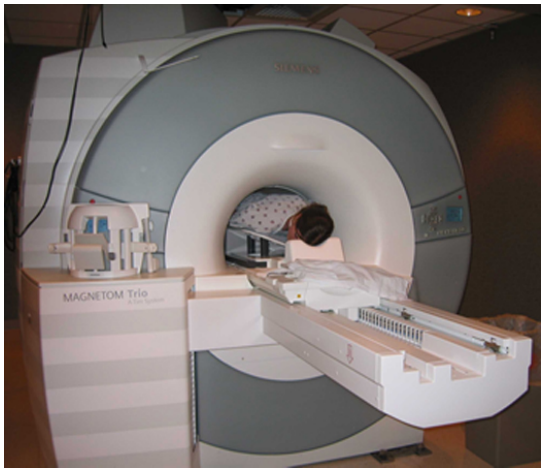


2.2 Image Digitization

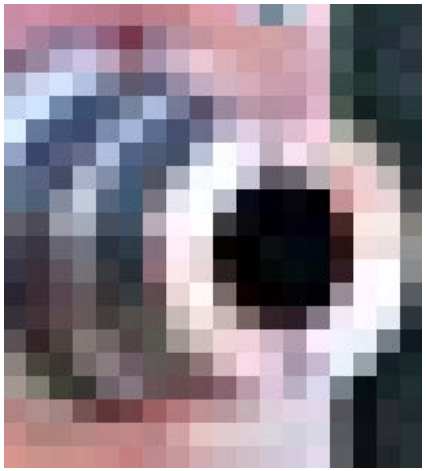
- Image to be processed by computer must be represented using appropriate discrete data structure
  - Ex. A matrix
- Image captured by sensor is expressed as continuous function f(x,y)
- Image digitization – function f(x,y) is sampled into matrix *M* rows and *N* columns.
- Image quantization – assigns integer value to each continuous sample

- Continuous range of image function is split into K intervals
- Image Function Sampling – poses two questions
  - Sampling period should be determined
    - This is distance between two neighboring sampling points in image
  - Geometric arrangement of sampling points should be set

Image Sensors



Digital Image



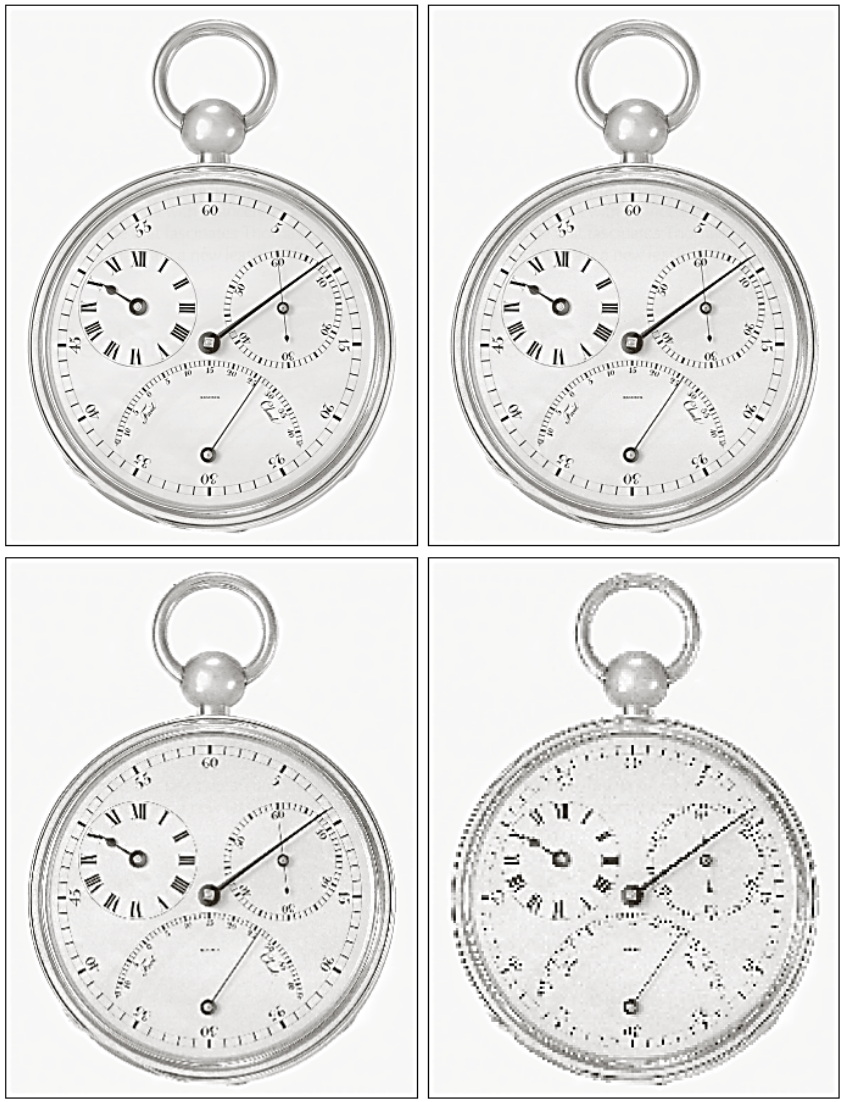
Digital Image

2.2.1 Sampling

Relationship exists between density of digital sampling and the detail that the image will contain

Format	Sampling Grid
Television image	512 x 512
PAL	768 x 576
NTSC	640 x 480
HDTV	1920 x 1080
Highest resolution avail.	10000 x 7096

Sampling Effect of spatial resolution



a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

- Continuous image is digitized at sampling points
  - Sampling points are ordered in the plane
  - Their geometric relation is called the **grid**
- Digital image is then a data structure (usually a matrix)
  - Grids used in practice are usually square (a) or hexagonal (b)
- Grid vs. Raster
  - Raster is grid on which neighborhood relation between points is defined

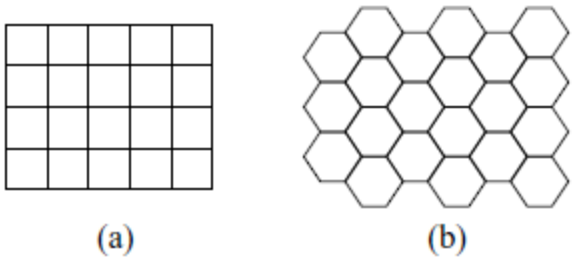


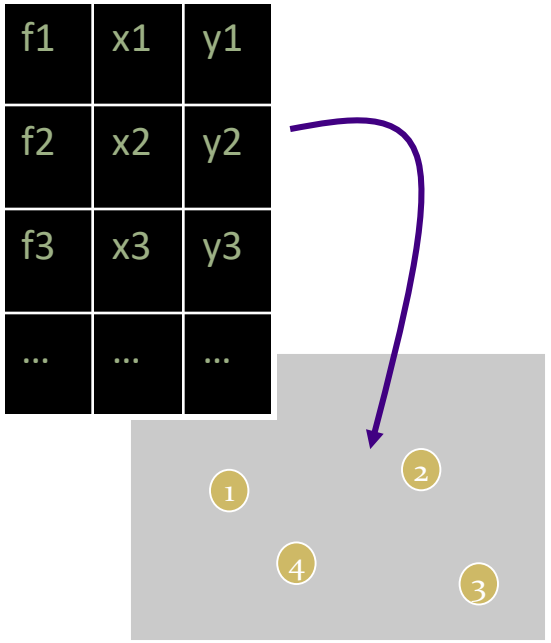
Figure 2.2: (a) Square grid. (b) Hexagonal grid.
© Cengage Learning 2015.

- One infinitely small sampling point in grid corresponds to one picture element called a **pixel** or **image element** in digital image
  - Pixel is a unit which is not further divisible from image analysis point of view
  - Called a **voxel** in 3-D image
  - Pixel often referred to as a “point”
- Set of pixels together cover entire image
  - Pixel captured by real digitization device has finite size

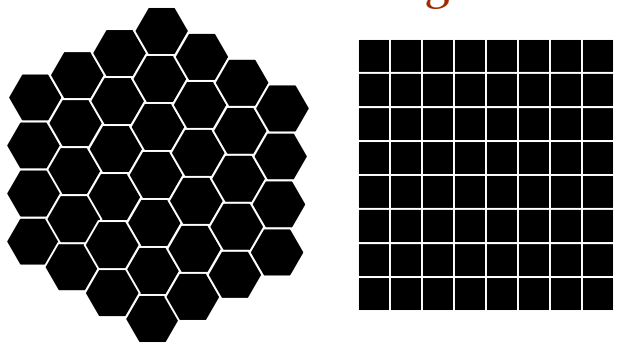
Image As Grid of Values

- Two views
  - Domain is a discrete set of samples
  - Samples are points from an underlying continuous function
- How is the grid organized?
  - Unstructured
    - Points specified by position and value
  - Structured grids
    - Position inferred from structure/index
    - 1D, 2D, 3D, ….
    - Sizes w, w x h, w x h x d

Unstructured grid



Structured grids



2.2.2 Quantization

- $f_x(j\Delta x, k\Delta y)$ - value of sampled image
  - Expressed as digital value in image processing
- Quantization – transition between continuous values of image function and its digital equivalent
- Images quantize with insufficient brightness levels
  - Occurrence of false contours
  - Can be reduced when quantization into intervals of unequal length is used

Example

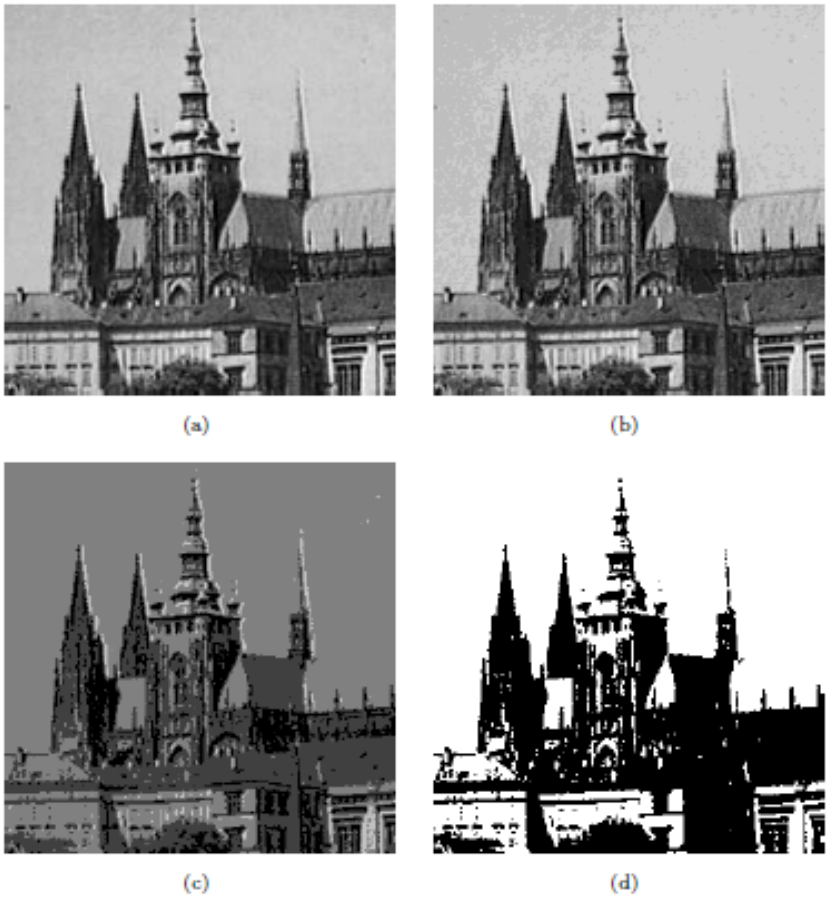
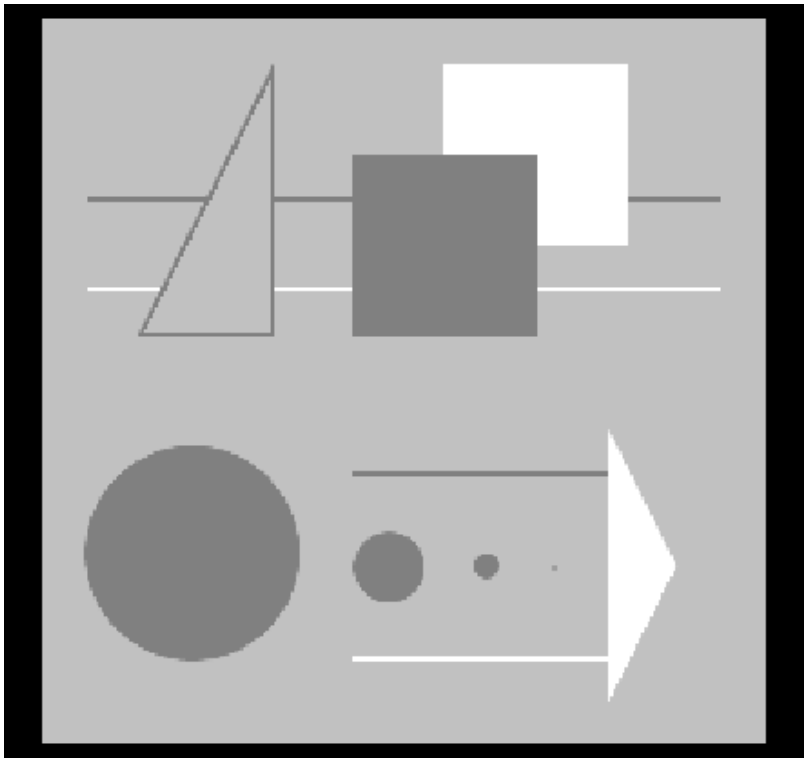
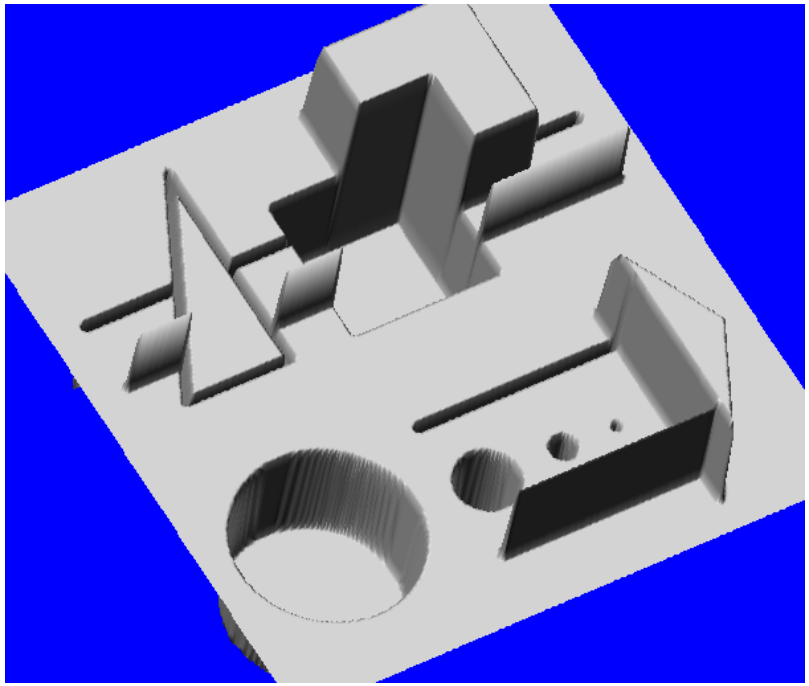


Figure 2.3: Brightness levels. (a) 64. (b) 16. (c) 4. (d) 2. © *Cengage Learning 2015.*

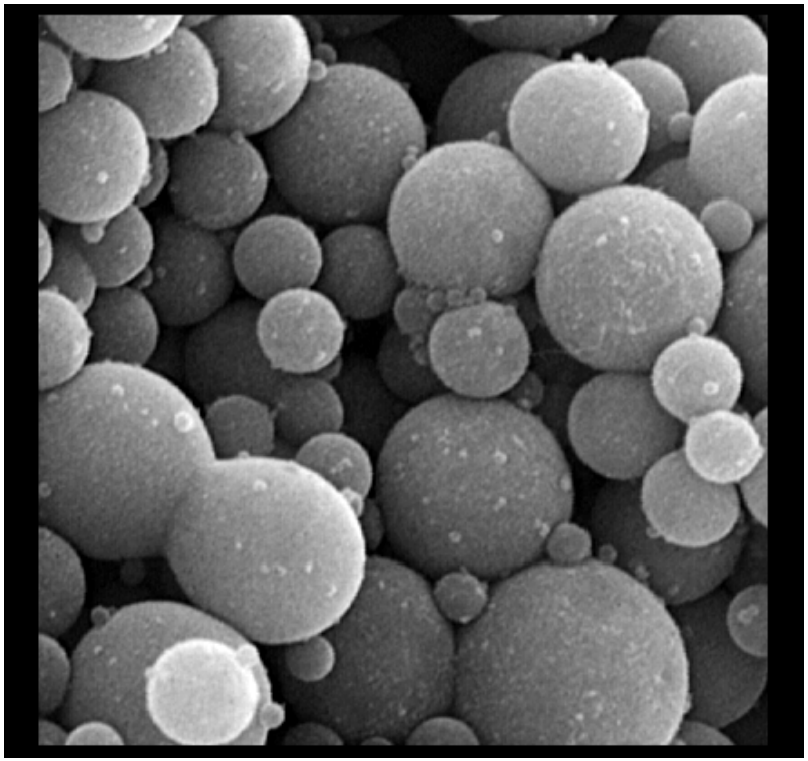
Figure 2.3 a-d demonstrate effect of reducing number of brightness level in image

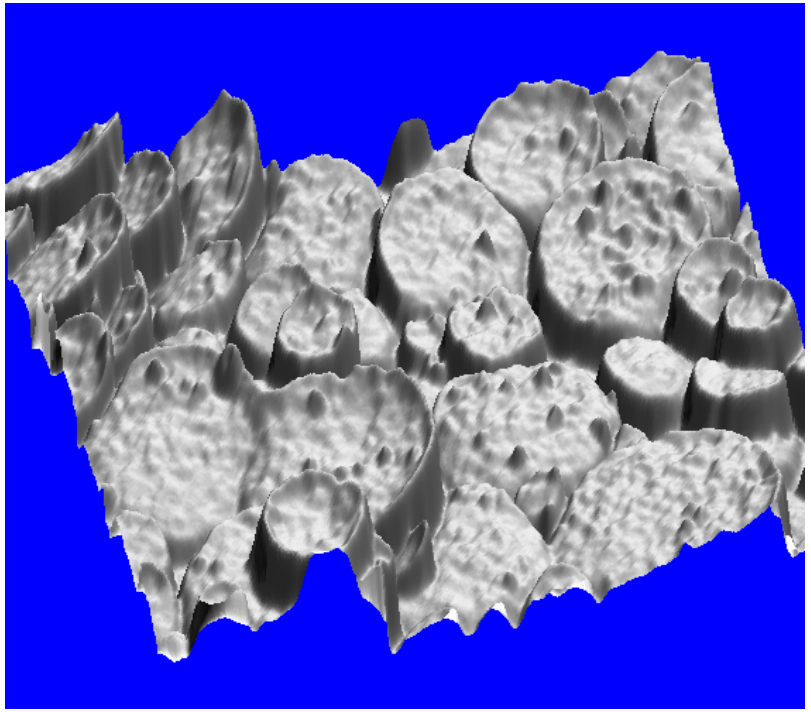
Digital Images





Digital Images





Arithmetic Operations on Images

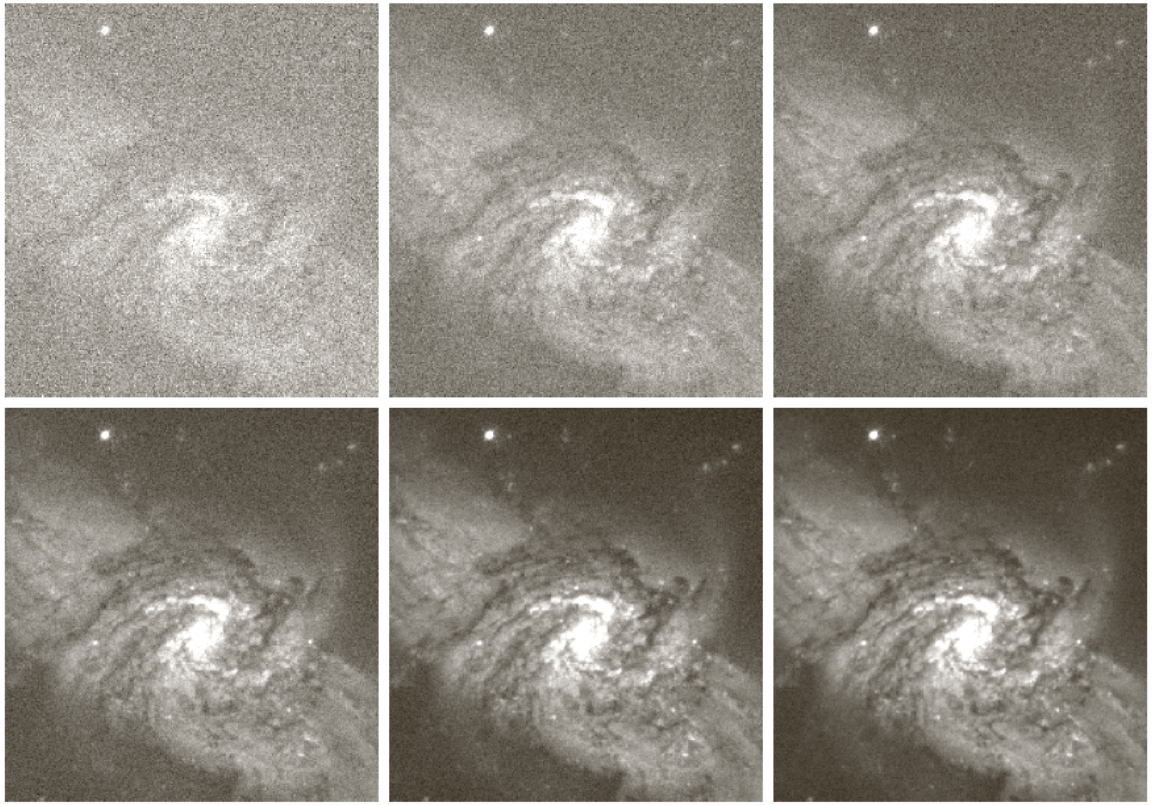
- Arithmetic operations on pixel values
  - Multiple images with the same domain
  - Image become arguments
    - Implied that the operation is applied pointwise across the domain
  - Addition, subtraction, multiply, divide, boolean

$$h = f + g \Rightarrow h(i,j) = f(i,j) + g(i,j)$$

$$\forall (i,j) \in \mathcal{D}$$

Arithmetic operations: f + g

Averaging (adding) multiple images can reduce noise



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Arithmetic operations: f - g

Digital Subtractive Angiography (DSA)

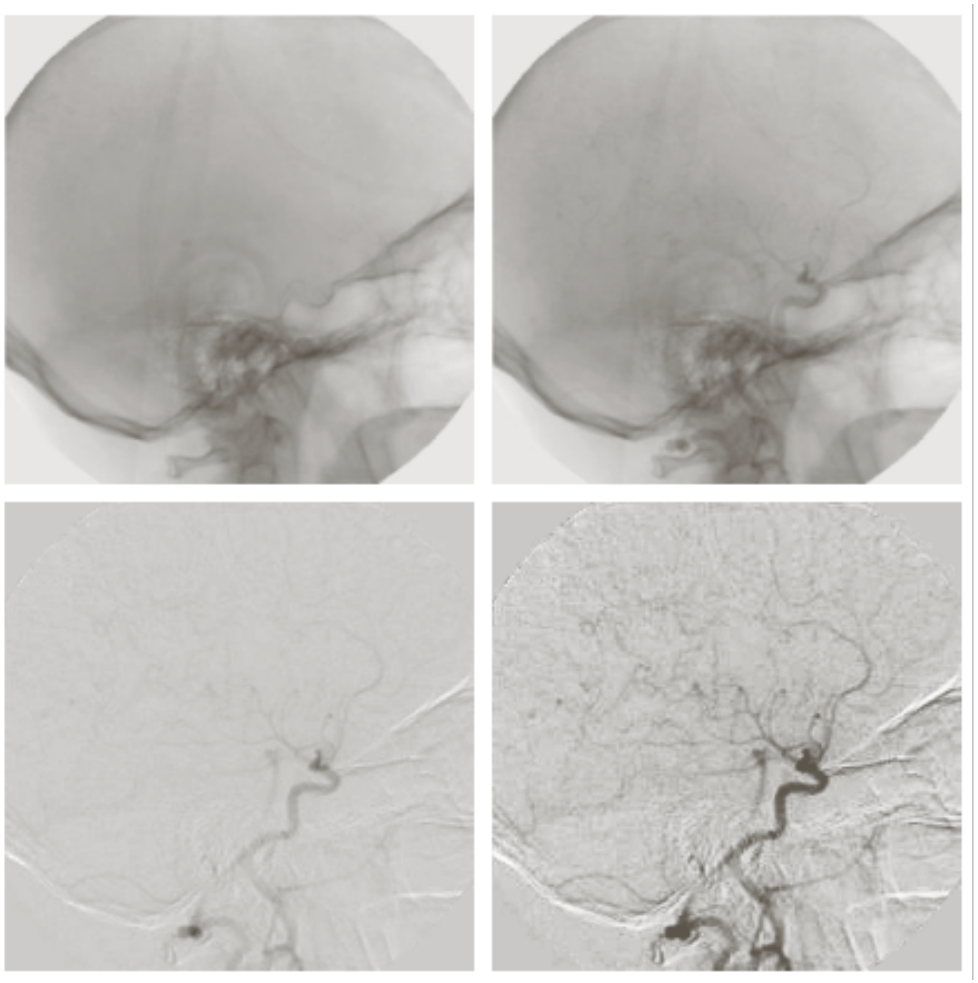
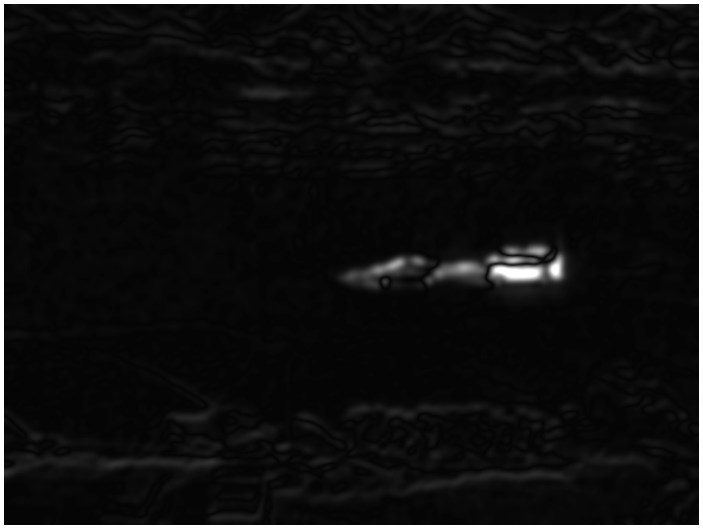


Image Subtraction: Motion Detection

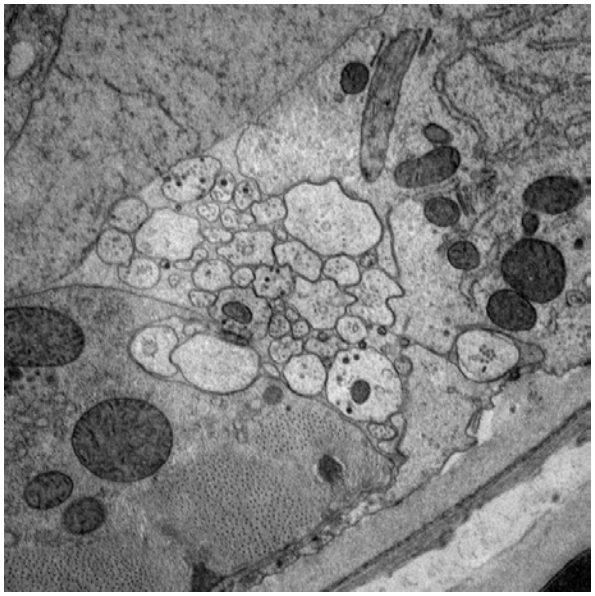


Arithmetic operations: f x g

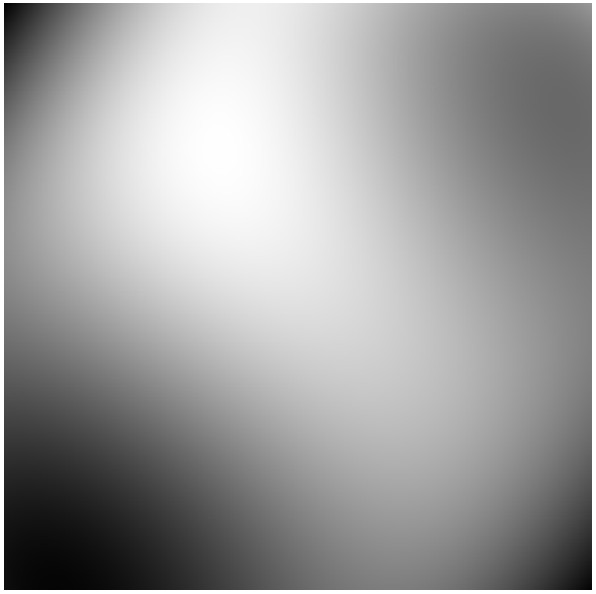
a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

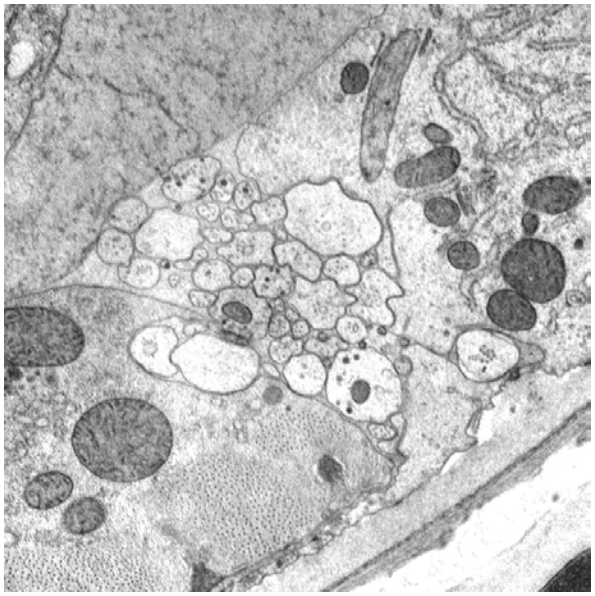
Arithmetic operations: f / g



Captured image



Illumination



Corrected image