Indian Institute of Technology Kanpur

Department of Aerospace Engineering

Conceptual Design of a Business Executive Aircraft

Endterm Report

AE461: Aircraft Design

Group E2

Sooryansh Malani (221076)
Sparsh Gupta (221084)
Surendra Prasad (221108)
Titiksha Raval (221143)
Vanara Aaditya Hiteshbhai (221164)
Vidhan Singh Sisodiya (221190)
Vishal Singh (221207)
Yash Verma (221227)

Contents

CHAPTER 6

SELECTION OF MAIN WING AND TAIL AIRFOILS

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Report 6 Objective

- To select the airfoils for the wing and tails from previously selected design parameters, given cruise speed requirements, based upon the Mach number, Reynolds number, and the lift-coefficient (for optimum L/D ratio at cruise).
- Selection of symmetrical airfoils of the horizontal and vertical tails with t/c ratios significantly less than that of the wing airfoil.

Design Requirements and Design Lift-coefficient

- Cruise Altitude : 11 km
- Temperature at Sea-level : 288.16 K

Cruise Temperature Estimation

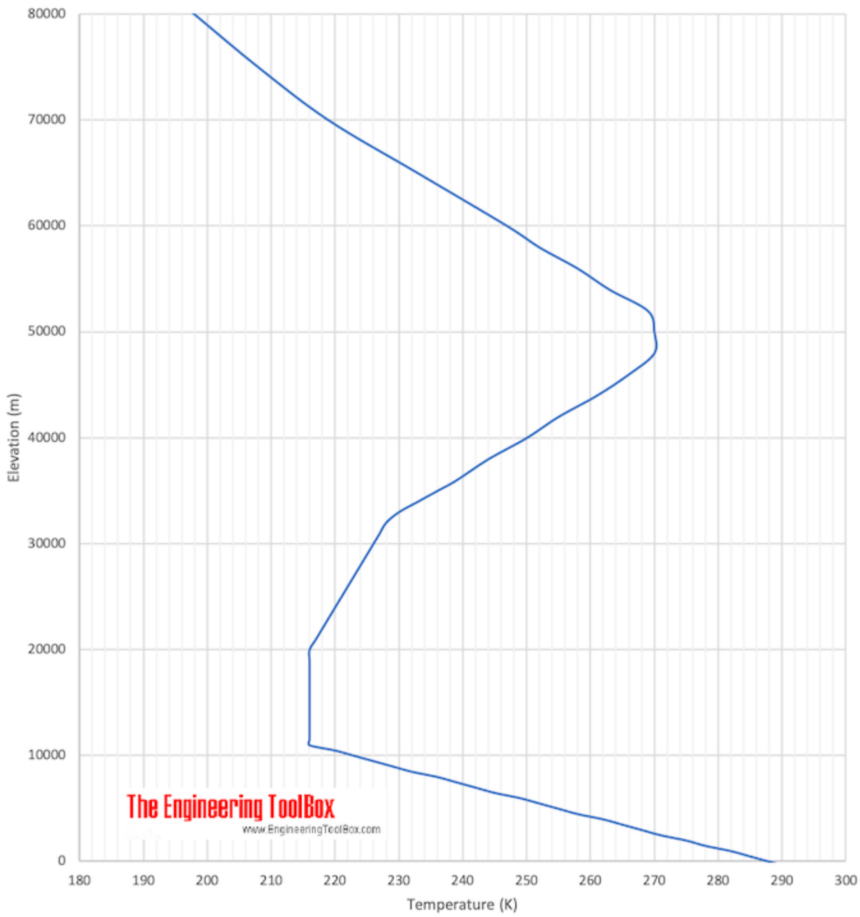


Figure 1: Temperature variation with Altitude

$$Temp(11\ km) = 288.16\ K\ -\ 6.5 \times 11\ K = 216.66\ K$$

The Cruise Temperature is calculated to be **216.66 K**

Density Calculation at Cruising Altitude

We assume Air to be a Calorically perfect, ideal gas. We will use the ideal gas law to calculate cruising density.

$$R(air) = 287\ J/(kg \circ K),\ \gamma(air) = 1.4,\ \rho_0(Sea\ Level) = 1.225\ kg/m^3,\ P_0(Sea\ Level) = 101325\ Pa$$

For Troposphere (upto 11 km),

$$\frac{\rho(h)}{\rho_0} = \frac{T(h)}{T_0}^{-(1+\frac{g}{\lambda R})},\ where\ \lambda = -6.5\ K/km$$

for h = 11 km, $\rho(11\ km) = 0.3642\ kg/m^3$**,** $P(11\ km) = 22650.168\ Pa$

Weight estimation at the beginning of cruise

$$W_0 = MTOW = 53195.968\ N,\ Calculated\ in\ Exercise\ 2$$

$$W_1/W_0 = 0.970,\ W_2/W_1 = 0.985$$

$$W_2 = 0.970 \times 0.985 \times 53195.968\ N = 50826.08763\ N$$

Weight of the aircraft, just before cruise is : $\boxed{W_2 = 50826.08763\ N}$

Desired Lift Coefficient estimation

- V(cruise) = 830 km/h = 230.556 m/s, derived from Design Requirements
- S(wing planform area) = 17.823 m^2 , Calculated in Exercise 4
- W_2 = 50826.08763 N

Cruise Lift Coefficient can be obtained through :

$$C_L = \frac{W_2}{\frac{1}{2}\rho V^2 S} = 0.29461$$

Now $(C_L)_d$ corresponding to $\frac{L}{D}_{max}$ ratio, for a jet aircraft :

$$(C_L)_d = \sqrt{3}C_L = 0.51027$$

Critical Mach number and Reynold's number Estimation

- T(11 km) = 216.66 K
- $\gamma(air)$ = 1.4
- $R(air) = 287\ J/(kg \circ K)$
- V(cruise) = 230.556 m/s
- MAC(mean Aerodynamic Chord) = 1.67m, derived from Exercise 4

$$M_{CR} = \frac{V}{\sqrt{\gamma RT}} = \frac{230.556}{295.05} = 0.78142$$

Using, Sudherland model for visscosity estimation,

At 273 K (0°C) and standard atmospheric pressure, the dynamic viscosity of air is $1.716 \times 10^{-5} Pa.s$

$$\frac{\mu}{\mu_0} = (\frac{T}{T_0})^{\frac{3}{2}} \times \frac{T_0 + \delta\mu}{T + \delta\mu},\ where\ \delta\mu = 110,\ Standard\ value\ of\ Sudherland\ Constant$$

$$\frac{\mu}{1.716 \times 10^{-5}} = (\frac{T}{273})^{\frac{3}{2}} \times \frac{273 + 110}{T + 110}$$

$$\mu(216.66\ K) = 1.421456 \times 10^{-5}\ Pa.s$$

Calculating the Reynold's number of the flow experienced by the wing,

$$Re = \frac{\rho_\infty V_\infty (MAC)}{\mu_\infty} = 9.86505 \times 10^6$$

Important Design Requirements

- Cruise Altitude : 11 km
- $M_\infty = 0.78142$
- $(C_L)d = 0.51027$
- $Re = 9.86505 \times 10^6$

Main Wing Airfoil Selection

We have selected **NACA 65(2)-415 Airfoil** for the main wing.

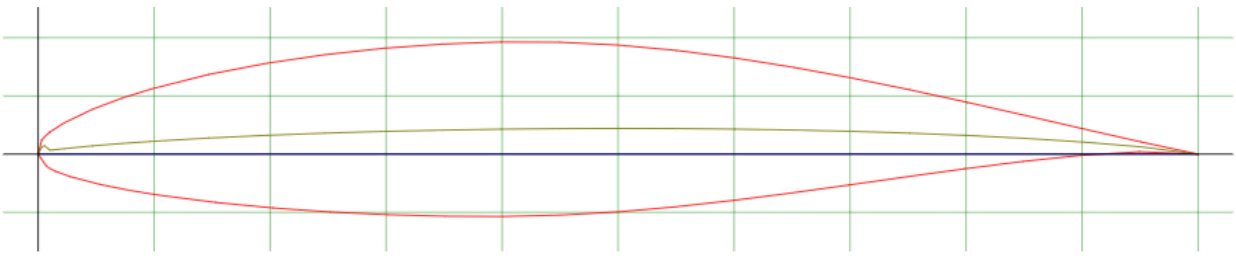


Figure 2: NACA 65(2)-415 Airfoil Profile

Factors affecting the choice of the airfoil :

- The design lift coefficient for the NACA 65(2)-415 airfoil is 0.40.
- $(t/c)\% \ = \ 15\%$

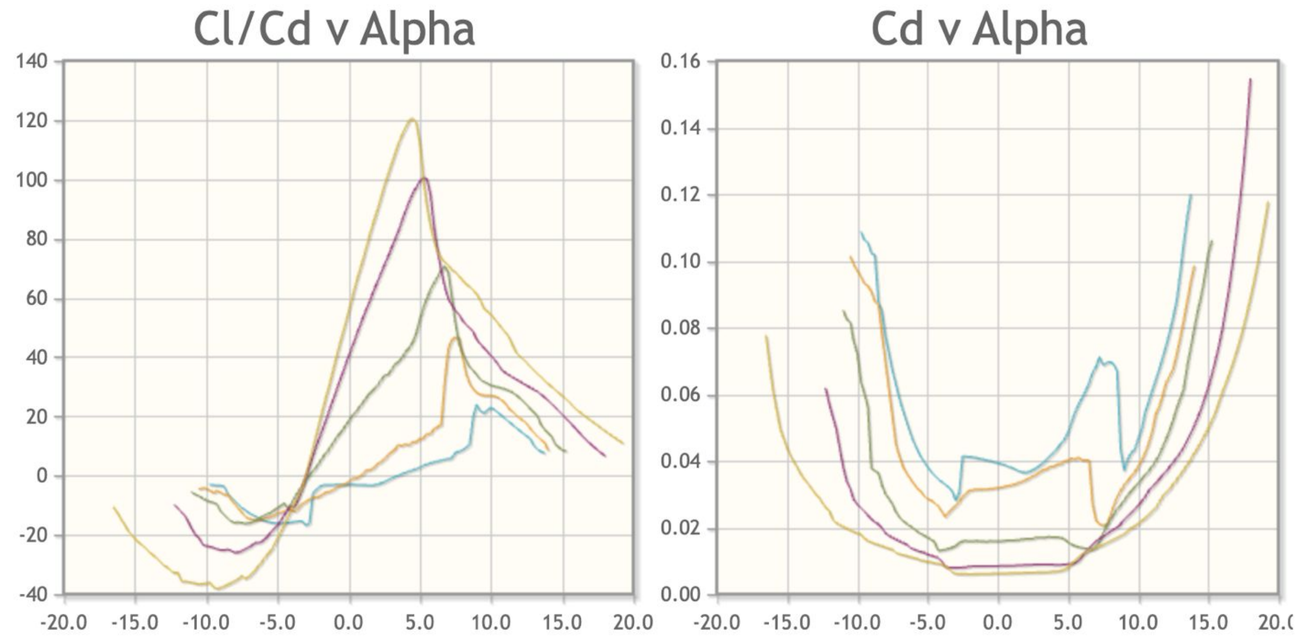


Figure 3: NACA 65(2)-415 airfoil characteristics

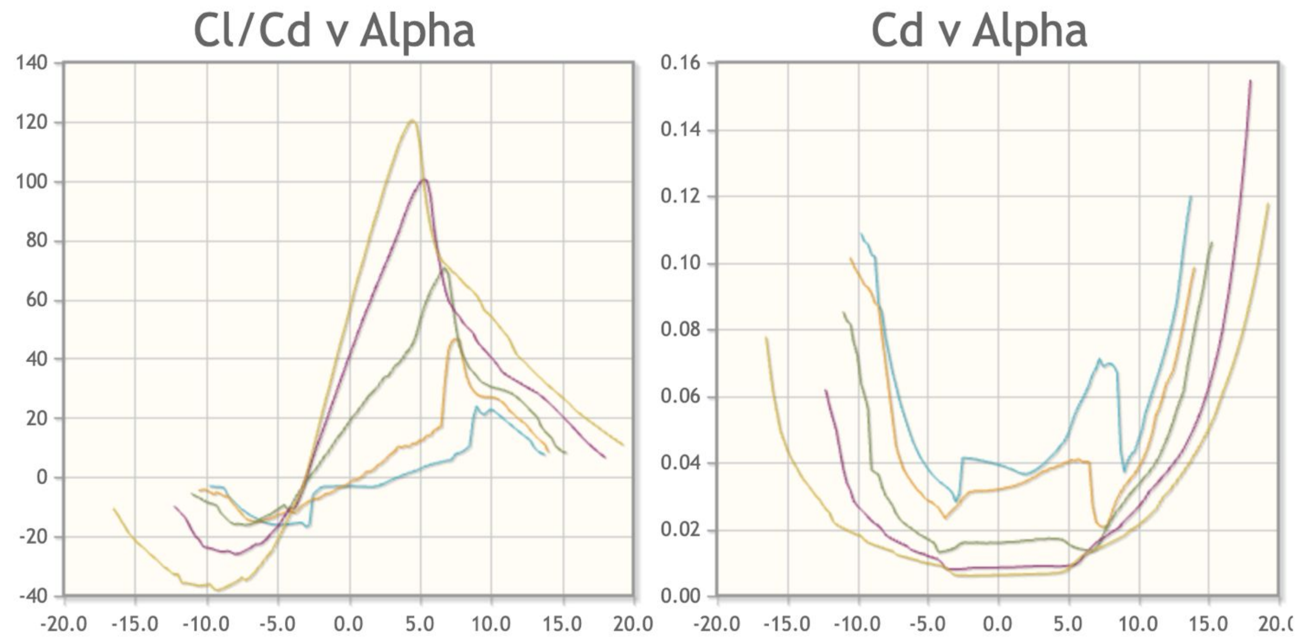


Figure 4: NACA 65(2)-415 airfoil characteristics

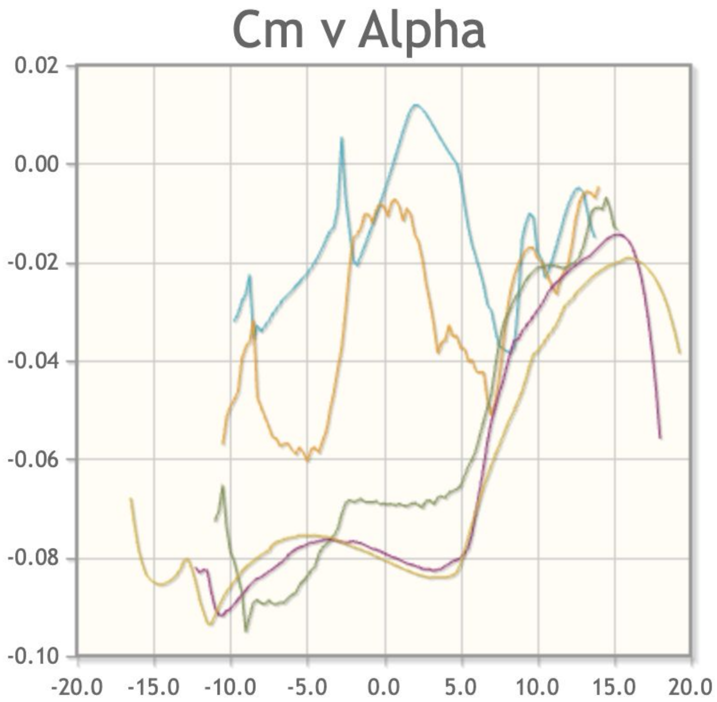


Figure 5: NACA 65(2)-415 airfoil characteristics

Color	Plot Parameter (Re #, Ncrit)
	50,000, 9
	50,000, 5
	100,000, 9
	100,000, 5
	200,000, 9
	500,000, 9
	500,000, 5
	1,000,000, 9

Horizontal Tail and Verticle Tail Airfoil Selection

Symmetrical airfoils are selected for horizontal and vertical airfoil. This allows generation of symmetrical forces, by varying the angle of attacks, and thus is critical to the performance of an aircraft.

$$With\ Symmetrical\ airfoils,\ Force(\alpha) = -Force(-\alpha)$$

Parameters considered:

- Re $= 6 \times 10^6$
- Mach no. $= 0.75$

$$\bar{c} = \frac{2}{3} c_{root} \frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

Parameter Consideration for Horizontal Tail airfoil

- $c_{root} = 1.56\ m$
- $\lambda = 0.4$
- $\bar{c} = 1.159\ m$

$$Re_{HT} = Re_{\infty}\frac{\bar{c}_{HT}}{\bar{c}_w} = 6 \times 10^6 \frac{1.159}{1.78} = 3.91 \times 10^6$$

$$\Lambda_{HT} = 20^{\circ}$$

$$M_{HT} = M_{\infty}cos(\Lambda_{HT}) = 0.705$$

Parameter Consideration for Vertical Tail airfoil

- $c_{root} = 1.94\ m$
- $\lambda = 0.4$
- $\bar{c} = 1.441\ m$

$$Re_{VT} = Re_{\infty}\frac{\bar{c}_{VT}}{\bar{c}_w} = 6 \times 10^6 \frac{1.441}{1.78} = 4.86 \times 10^6$$

$$\Lambda_{VT} = 35^{\circ}$$

$$M_{VT} = M_{\infty}cos(\Lambda_{VT}) = 0.614$$

Tails must generate both positive and negative lift; hence, a symmetrical airfoil (zero camber) is preferred to eliminate trim bias and provide linear, reversible control authority. The drag-divergence Mach number increases with decreasing airfoil thickness and camber, favoring thinner symmetrical sections for high subsonic operation. Also the thickness of the Horizontal airfoil must be lesser than that of the main wing airfoil, and the horizontal tail must stall at an higher angle of attack compared to main wing.

For the calculated normal Mach number range ($M_n = 0.61$–0.70), 9–10% thick symmetrical NACA 00xx airfoils operate safely below their drag-divergence limits. These sections maintain attached flow and exhibit only mild shock formation up to $M_n \approx 0.75$, as verified by NACA high-subsonic wind tunnel data. Experimental polars for NACA 0010 and NACA 0009 at Reynolds numbers of $Re \approx 1 \times 10^6$–6×10^6 confirm a lift-curve slope of approximately 2π rad^{-1} and highly linear C_L–α behavior.

Considering these aerodynamic characteristics, the **NACA 0009 section has been selected for the vertical tail**, while the **NACA 0010 section is adopted for the horizontal tail**.

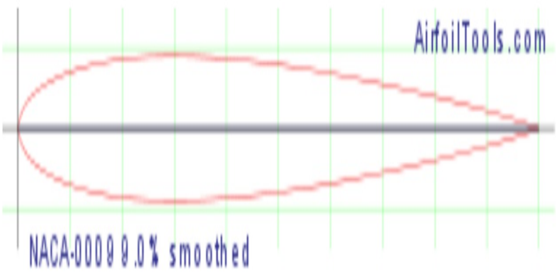


Figure 6: NACA 0009 airfoil

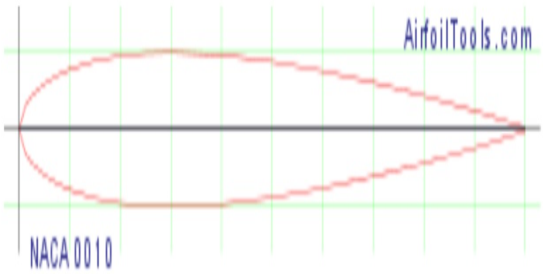


Figure 7: NACA 0010 airfoil

CHAPTER 7

SELECTION AND INSTALLATION OF ENGINE

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Detailed Thrust Calculation for Engine

Objective (Report-7)

The primary objective of this exercise is to select appropriate turbofan engines for our business executive aircraft from the Williams International FJ44 series, based on calculated thrust requirements derived from aircraft performance parameters established in previous exercises. And incorporate the engine in the aircraft drawings.

Step 1: Input Parameters from Previous Exercises (Report-2)

$$\begin{aligned}
\text{Take-off Gross Weight } (W_0) &= 5428.16 \text{ kg} \\
\text{Cruise Lift-to-Drag Ratio } (L/D)_{cruise} &= 15.56 \\
\text{Cruise Altitude } (h) &= 11 \text{ km} \\
\text{Number of Engines } (n) &= 2 \\
\text{Installation Loss Factor } &= 5\% = 1.05
\end{aligned}$$

Step 2: Cruise Thrust-to-Weight Ratio Calculation

For a turbofan engine in cruise condition, the thrust-to-weight ratio is given by:

$$(T/W_0)_{cruise} = \frac{1}{(L/D)_{cruise}}$$

Substituting the known value:

$$(T/W_0)_{cruise} = \frac{1}{15.56}$$

$$\begin{aligned}
(T/W_0)_{cruise} &= 0.06426735218509002 \\
(T/W_0)_{cruise} &\approx \boxed{\mathbf{0.0643}} \quad \text{(rounded to 4 decimal places)}
\end{aligned}$$

Step 3: Atmospheric Density Ratio Calculation

The density ratio formula for tropical climates is given by:

$$\sigma = (1 - 6.5h/288.15)^{5.26}$$

Where altitude h is in kilometers. Substituting $h = 11$ km:

$$
\begin{aligned}
\sigma &= \left(1 - \frac{6.5 \times 11}{288.15}\right)^{5.26} \\
&= \left(1 - \frac{71.5}{288.15}\right)^{5.26} \\
&= (1 - 0.2481568634391803)^{5.26} \\
&= (0.7518431365608197)^{5.26} \\
\sigma &= \mathbf{0.2231} \quad \text{(rounded to 4 decimal places)}
\end{aligned}
$$

Step 4: Sea Level Static Thrust-to-Weight Ratio

The relationship between cruise thrust-to-weight ratio and sea level static thrust-to-weight ratio is:

$$\left(\frac{T}{W_0}\right)_{SLS} = \frac{(T/W_0)_{cruise}}{\sigma}$$

Substituting the calculated values:

$$
\begin{aligned}
\left(\frac{T}{W_0}\right)_{SLS} &= \frac{0.06426735218509002}{0.223128} \\
&= 0.287999 \approx \mathbf{0.2880}
\end{aligned}
$$

Step 5: Total Sea Level Static Thrust Requirement

The total thrust required at sea level static conditions, including installation losses, is:

$$T_{SLS} = 1.05 \times W_0 \times \left(\frac{T}{W_0}\right)_{SLS}$$

First, convert weight from kg to Newtons (using $g = 9.81\,\mathrm{m/s^2}$):

$$
\begin{aligned}
W_0\,(\text{in N}) &= 5428.16\,\mathrm{kg} \times 9.81\,\mathrm{m/s^2} \\
&= 53250.2496\,\mathrm{N}
\end{aligned}
$$

Now calculate total thrust:

$$
\begin{aligned}
T_{SLS} &= 1.05 \times 53250.2496\,\mathrm{N} \times 0.2880 \\
&= 1.05 \times 15336.0718848\,\mathrm{N} \\
&= 16102.87447804\,\mathrm{N} \\
T_{SLS} &\approx \boxed{\mathbf{16103\,N}} \quad \text{(rounded to nearest Newton)}
\end{aligned}
$$

Step 6: Conversion to Pounds-Force (lbf)

Since engine thrust ratings are typically given in pounds-force, we convert:

$$1\,\text{N} = 0.2248089431\,\text{lbf}$$

$$\begin{aligned} T_{SLS}\,(\text{in lbf}) &= 16103\,\text{N} \times 0.2248089431\,\text{lbf/N} \\ &= 3619.5\,\text{lbf} \\ T_{SLS} &\approx \boxed{\mathbf{3620\,lbf}} \quad \text{(rounded to nearest lbf)} \end{aligned}$$

Step 7: Thrust per Engine Requirement

For a twin-engine configuration:

$$T_{engine} = \frac{T_{SLS}}{n}$$

$$\begin{aligned} T_{engine} &= \frac{3620\,\text{lbf}}{2} \\ &= \mathbf{1810\,lbf} \quad \text{per engine} \end{aligned}$$

Summary of Thrust Requirements

Table 1: Calculated Thrust Requirements Summary

Parameter	**Value**
Take-off Gross Weight (W_0)	5428.16 kg
Cruise Lift-to-Drag Ratio $(L/D)_{cruise}$	15.56
Cruise Thrust-to-Weight Ratio $(T/W_0)_{cruise}$	0.0643
Cruise Altitude (h)	11 km
Density Ratio (σ)	0.2231
Sea Level Static Thrust-to-Weight Ratio $(T/W_0)_{SLS}$	0.2880
Total required Thrust (T_{SLS})	**3620 lbf (16103 N)**
Number of Engines	2
required Thrust per Engine	**1810 lbf**

Engine Selection Justification

FJ44-1A Engine Specifications

Table 2: FJ44-1A Engine Specifications

Parameter	Value
Maximum Continuous Thrust	1900 lbf
Takeoff Thrust (5 minutes)	1900 lbf
Total Engine (Includes gearbox and airframe mounted) Weight	460 lb
Length Overall	53.4 in (1.36 m)
Forward Flange Diameter	20.9 in (0.53 m)
Between flanges, inches (m)	40.3 (1.02 m)
Height (Overall) , inches (m)	29.6 (0.75 m)
Control System	High Pressure Rotor (N2) Speed Governing Hydromechanical Metering Unit (HMU)

Margin Analysis

$$\begin{aligned}
\text{Available thrust per engine} &= 1900\,\text{lbf} \\
\text{required thrust per engine} &= 1810\,\text{lbf} \\
\text{Thrust margin} &= 1900 - 1810 = 90\,\text{lbf} \\
\text{Percentage margin} &= \frac{90}{1810} \times 100\% = 4.97\%
\end{aligned}$$

Weight Analysis

$$\begin{aligned}
\text{Total engine weight} &= 2 \times 460\,\text{lb} = 920\,\text{lb} \\
\text{Engine weight fraction} &= \frac{920\,\text{lb}}{5428.16\,\text{kg} \times 2.20462\,\text{lb/kg}} \times 100\% \\
&= \frac{920}{11967} \times 100\% = 7.68\%
\end{aligned}$$

Conclusion for the engine

The detailed mathematical calculation confirms that:

- The **total required sea level static thrust** is **3620 lbf (16103 N)**
- For a twin-engine configuration, each engine must provide **1810 lbf**
- The selected **Williams International FJ44-1A** engines provide **1900 lbf** each
- This provides a **comfortable margin of 4.97%** over the calculated requirement
- The engine weight represents **7.68%** of the total aircraft weight, which is appropriate for this class of aircraft

The FJ44-1A engine selection is mathematically justified and provides adequate performance margins for all flight conditions while maintaining excellent fuel efficiency and reliability characteristics suitable for a business executive aircraft.

CHAPTER 8

INSTALLATION OF THE SELECTED ENGINE

Report 8

Objective

The primary objective of this exercise is to install the selected turbofan engine for our business executive aircraft.

Preliminary capture area analysis using Fig.10.16 in Lecture-8:

Capture area of an engine is given by:

$$A_c = \dot{m}_{\text{engine}} \times 5.12 \times 10^{-3}$$

(in metric units)

The value 5.12×10^{-3} is obtained from Fig. 10.16 in Lecture-8.

The mass flow rate through the engine is obtained using the formula

$$\dot{m}_{\text{engine}} = 127 \times (\text{FFD})^2$$

Here, FFD denotes the Front Face Diameter of the engine

Engine Data:

$$\text{Front Face Diameter (FFD)} = 75.2 \text{ cm} = 0.752 \text{ m} \quad \text{(FJ-44-1A engine)}$$

$$\dot{m}_{\text{engine}} = 127 \times (0.752)^2 = 71.819$$

Thus, the Capture Area of the engine is calculated to be

$$A_c = 71.819 \times 5.12 \times 10^{-3} = 0.3677 \text{ m}^2$$

Throat/Capture Area calculation using Mach Number Analysis:

$$\frac{A_{\text{throat}}}{A_{\text{engine}}} = \frac{(A/A^*)_{\text{throat}}}{(A/A^*)_{\text{engine}}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{1 + 0.2M^2}{1.2}\right)^3$$

$$\left(\frac{A}{A^*}\right)_{\text{throat}} \text{ is calculated using } M_{\text{cruise}} = 0.7842$$

$$\left(\frac{A}{A^*}\right)_{\text{engine}} \text{ is calculated using } M_{\text{engine front face}} = 0.3$$

$$\Rightarrow \left(\frac{A}{A^*}\right)_{\text{throat}} = \frac{1}{0.78142}\left(\frac{1+0.2(0.7842)^2}{1.2}\right)^3$$

$$= 1.04639$$

$$\left(\frac{A}{A^*}\right)_{\text{engine}} = \frac{1}{0.3}\left(\frac{1+0.2(0.3)^2}{1.2}\right)^3$$

$$= 2.03506$$

$$\Rightarrow \frac{A_{\text{throat}}}{A_{\text{engine}}} = \frac{1.04639}{2.03506} = 0.51418$$

$$A_{\text{throat}} = A_{\text{engine}} \times 0.51418$$

The area of engine is obtained using the formula

$$A_{\text{engine}} = \frac{\pi}{4}(\text{FFD})^2 = \frac{\pi}{4}(0.752)^2 = 0.44414$$

$$A_{\text{throat}} = 0.44414 \times 0.51418$$

Thus, the Throat Area of the engine is calculated to be

$$A_{\text{throat}} = \boxed{0.22837}$$

Calculation of Intake Duct Length Using Linear Interpolation

To estimate the length of the inlet duct from the inlet lip to the engine front face, an intermediate Mach number is selected and the corresponding inlet area is obtained using the isentropic area–Mach number relation. A linear variation of area along the duct is then assumed to determine the physical length of the intake.

Isentropic Area Ratio at $M = 0.5$

The isentropic area ratio is given by:

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{1+0.2M^2}{1.2}\right)^3, \quad \gamma = 1.4.$$

For $M = 0.5$,

$$\frac{A}{A^*} = \frac{1}{0.5}\left(\frac{1+0.2(0.5)^2}{1.2}\right)^3.$$

Evaluating,

$$\boxed{\frac{A}{A^*} = 1.3398.}$$

Intermediate Inlet Area at $M = 0.5$

The inlet face (fan face) area and throat area computed earlier are:

$$A_{\text{face}} = 0.44414 \text{ m}^2, \qquad A_{\text{throat}} = 0.22837 \text{ m}^2.$$

Using the area ratios:

$$\frac{A_{\text{face}}}{A^*} = 2.03506, \qquad \frac{A_{\text{throat}}}{A^*} = 1.04639.$$

Thus the critical area is:

$$A^* = \frac{A_{\text{face}}}{2.03506} = \frac{0.44414}{2.03506} = 0.21824 \text{ m}^2.$$

Therefore,

$$A_{M=0.5} = A^* \left(\frac{A}{A^*}\right)_{M=0.5} = 0.21824 \times 1.3398 = 0.2924 \text{ m}^2.$$

Linear Interpolation Factor

Assuming linear variation of inlet cross-sectional area along the duct:

$$\lambda = \frac{A_{0.5} - A_{\text{throat}}}{A_{\text{face}} - A_{\text{throat}}}.$$

Substituting,

$$\lambda = \frac{0.29239 - 0.22837}{0.44414 - 0.22837} = 0.2967.$$

This means the location corresponding to $M = 0.5$ lies at 29.67% of the total inlet length from throat toward the engine face.

Physical Intake Length Calculation

To convert the interpolation factor into a physical length, the inlet duct is modeled as a conical diffuser extending from the inlet lip (throat) to the engine front face. The inlet throat and engine face areas are converted into equivalent diameters:

$$D_{\text{throat}} = \sqrt{\frac{4A_{\text{throat}}}{\pi}} = \sqrt{\frac{4 \times 0.22837}{\pi}} = 0.539 \text{ m},$$

$$D_{\text{face}} = FFD = 0.752 \text{ m}.$$

A typical subsonic diffuser uses a half-angle of 5°–7° to avoid flow separation. Using a conservative value of $\theta = 7°$, the intake length becomes

$$L_{\text{intake}} = \frac{D_{\text{face}} - D_{\text{throat}}}{2\tan\theta} = \frac{0.752 - 0.539}{2\tan 7^\circ} \approx 0.867 \text{ m}.$$

Using the previously computed interpolation factor $\lambda = 0.2967$, the axial location of the $M = 0.5$ plane is

$$x_{0.5} = \lambda L_{\text{intake}} = 0.2967 \times 0.867 \approx 0.257 \text{ m}.$$

Total Nacelle Length

From the FJ44-1A data sheet, the engine length overall is

$$L_{\text{engine}} = 53.4 \text{ in} = 1.356 \text{ m}.$$

Thus the installed nacelle length is

$$L_{\text{nacelle}} = L_{\text{intake}} + L_{\text{engine}} = 0.867 + 1.356 = 2.223 \text{ m}.$$

This nacelle length is used in the final installation drawing and in exposed/wetted area calculations.

Conclusion

Using the FS-49-1A turbofan engine with a front face diameter (FFD) of 0.531 m, the engine mass flow rate was estimated using the empirical relation $\dot{m}_{\text{engine}} = 127\,(\text{FFD})^2$, resulting in $\dot{m}_{\text{engine}} = 35.81$ kg/s. Applying the capture area correlation from Lecture–8, the preliminary capture area was calculated as

$$A_c = \dot{m}_{\text{engine}} \times 5.12 \times 10^{-3} = 0.3677 \text{ m}^2.$$

Using isentropic Mach number analysis, the area ratios were obtained as

$$\left(\frac{A}{A^*}\right)_{\text{throat}} = 1.04639 \quad (\text{at } M_{\text{cruise}} = 0.7842), \qquad \left(\frac{A}{A^*}\right)_{\text{engine}} = 2.03506 \quad (\text{at } M_{\text{engine face}} = 0.3).$$

This yielded a ratio

$$\frac{A_{\text{throat}}}{A_{\text{engine}}} = 0.51418.$$

With the engine face area computed as

$$A_{\text{engine}} = \frac{\pi}{4}(\text{FFD})^2 = 0.44414 \text{ m}^2,$$

the corresponding throat area was determined to be

$$A_{\text{throat}} = 0.22837 \text{ m}^2.$$

These results confirm that the inlet geometry is aerodynamically feasible, with diffusion from throat toward the fan face. The throat area is approximately 51% of the engine face area, which is consistent with expected subsonic inlet design requirements. Overall, the selected engine and computed areas provide a valid and efficient baseline for inlet sizing, supporting acceptable pressure recovery and mass flow characteristics necessary for the aircraft's performance across its operating envelope. The intake duct length was obtained by combining isentropic area–Mach relationships with a geometric diffuser model. First, the intermediate area at $M = 0.5$ was computed using the isentropic area ratio, giving $A_{0.5} = 0.2924\ \text{m}^2$. Using the throat area $A_{\text{throat}} = 0.22837\ \text{m}^2$ and engine face area $A_{\text{face}} = 0.44414\ \text{m}^2$, the linear interpolation factor was found to be $\lambda = 0.2967$, indicating that the $M = 0.5$ plane lies 29.67% of the distance from the inlet lip toward the engine front face.

To convert this fractional distance into a physical length, the inlet was modeled as a conical diffuser with a conservative half-angle of 7°, typical of subsonic inlets to avoid flow separation. Using the diameters corresponding to A_{throat} and A_{face}, the resulting intake length was computed as $L_{\text{intake}} \approx 0.867$ m. The axial location of the intermediate Mach plane thus occurs at $x_{0.5} \approx 0.257$ m from the inlet lip.

Finally, using the manufacturer-provided overall engine length of 1.356 m, the installed nacelle length becomes $L_{\text{nacelle}} = 2.223$ m. This confirms that the selected engine and diffuser geometry integrate smoothly within the aircraft's layout, while providing an aerodynamically efficient and feasible inlet design for the required mass flow and pressure recovery.

CHAPTER

Chapter 9 : Aircraft mass and CG estimaton

Calculation of Aircraft Empty Weight

Main Wing Empty Weight Estimation

- Chord Length relation with span distance:

$$c(y) \ = \ 2.42 \ - \ \frac{2.42 - 0.48}{6.115} \times y, \ where\ y\ is\ the\ distance\ in\ metres\ from\ root$$

- The fuselage covers 0.955 m of the wings from the root, the chord length at the exposed section is $c(0.955) = 2.117\ m$

- Assuming the exposed area of the wing to resemble a trapezium (in the top view), $S_{exposed} = \frac{2.117+0.48}{2} \times (6.115 - 0.955) \times 2\ m^2 = 13.4\ m^2$

- This value is further confirmed from the area calculated by the CAD drawing of the wing to be $13.36\ m^2$

- Due to a dihedral of 6°, $S_{exposed_{planform}} = \frac{S_{exposed}}{cosine(3°)} = 13.418\ m^2$

- The mass of the wing, (for Transport and Bombers type aircraft) is $mass_{wing} = 49 * 13.418\ kg = 657.5\ kg$

- The normalised area of the airfoil (chord=1) for the main wing, is calculated programatically, by using the data file of NACA 65(2)-415, interpolating the data points to create a Reimann Strips with small widths and then performing a Reimann Integral.

  The area of the complete airfoil normalised for chord length of 1 is calculated to be $0.0967\ m^2$, and the area for the trimmed wing (removing the last 20% of chord for attachment of ailerons and flaps) is calculated to be $0.092\ m^2$

Design Parameter	**Value**
Span	$12.23\ m$
Root Chord	$2.42\ m$
Tip Chord	$0.48\ m$
Taper Ratio	0.2
Chord length at the Fuselage	$2.117\ m$
$S_{exposed}$	$13.4\ m^2$
$S_{exposed_{planform}}$	$13.418\ m^2$
Weight	657.5 kg
Airfoil	NACA 65(2)-415
Airfoil Area(for chord length = 1)	$0.0967\ m^2$
Airfoil Area Clipped (for chord length = 1)	$0.092\ m^2$

Table 3: Main wing characterestics

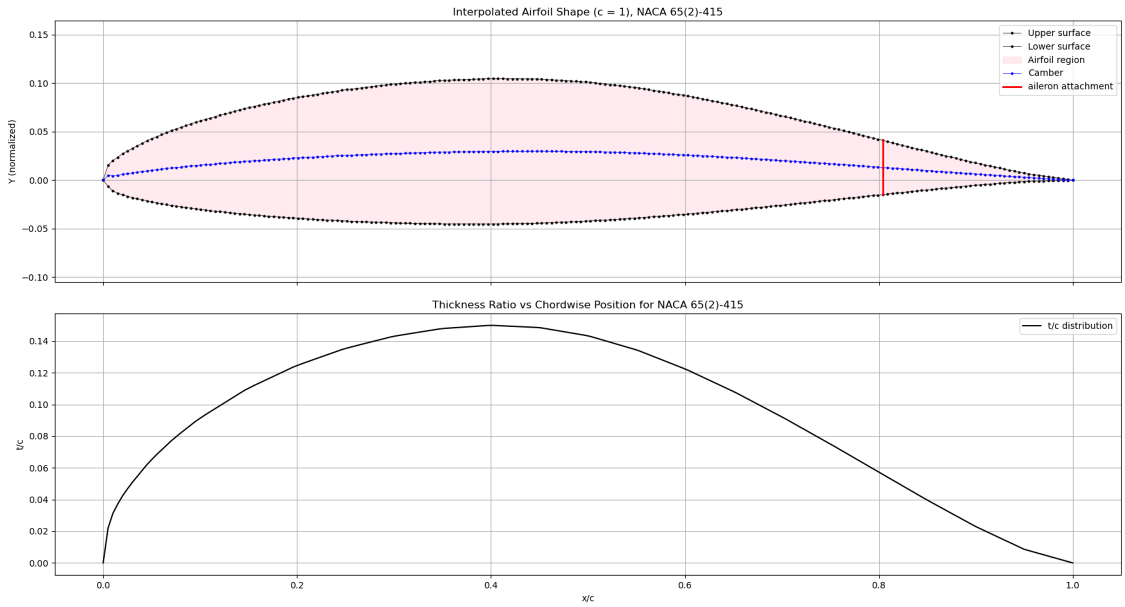


Figure 8: Airfoil Cross-section

Horizontal Tail

- The fuselage does not cover any part of the horizontal wing.
- Assuming the exposed area of the wing to resemble a trapezium (in the top view), $S_{exposed} = \frac{1.81+0.725}{2} \times (2.503) \times 2\ m^2 = 6.41355\ m^2$
- Due to a dihedral of 2°, $S_{exposed_{planform}} = \frac{S_{exposed}}{cosine(1°)} = 6.4145\ m^2$
- The mass of the wing, (for Transport and Bombers type aircraft) is $mass_{wing} = 27 * 6.4145\ kg = 173.1922\ kg$
- The normalised area of the airfoil (chord=1) for the NACA 0100 horizontal wing airfoil is $0.068486 m^2$

Design Parameter	**Value**
Span	5.06 m
Root Chord	1.81 m
Tip Chord	0.725 m
Taper Ratio	0.4
$S_{exposed}$	6.4133 m^2
$S_{exposed_{planform}}$	13.4145 m^2
Weight	173.192 kg
Airfoil	NACA 0010
Airfoil Area(for chord length = 1)	0.068486 m^2

Table 4: Horizontal wing characterestics

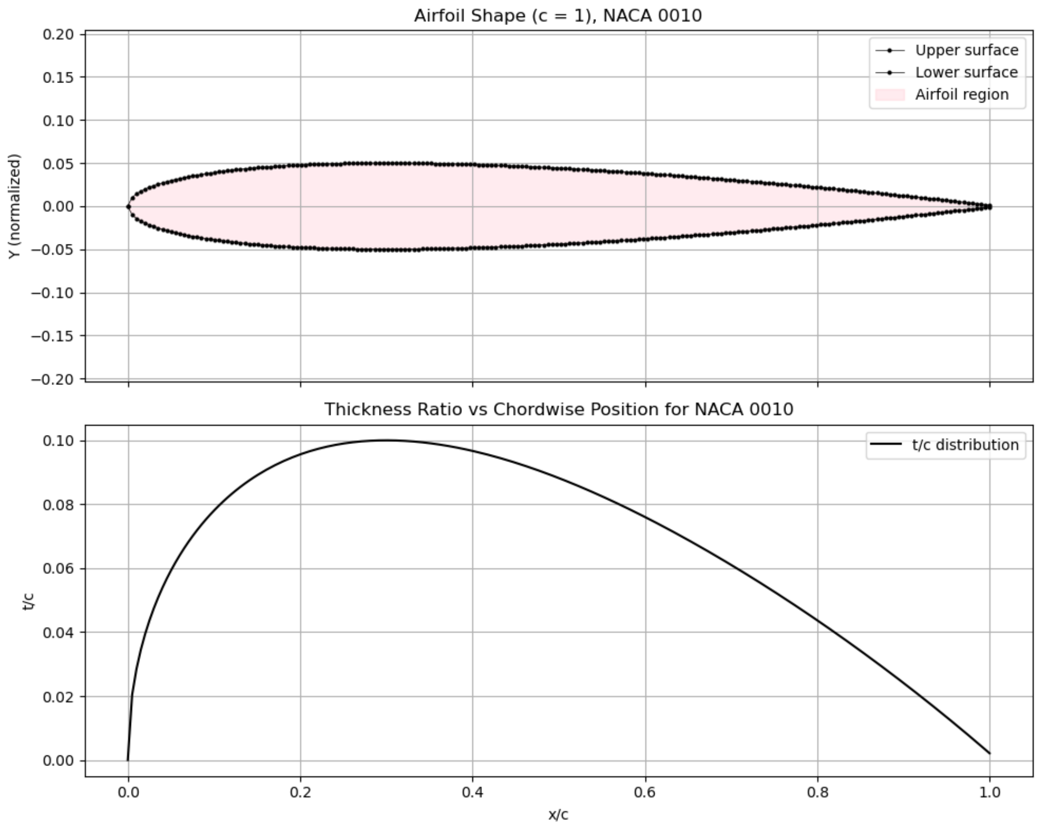


Figure 9: Airfoil Cross-section

Vertical Tail

- Assuming the exposed area of the wing to resemble a trapezium (in the top view), $S_{exposed} = \frac{2.41+1.928}{2} \times (2.603)m^2 = 5.646\ m^2$
- The mass of the wing, (for Transport and Bombers type aircraft) is $mass_{wing} = 27 * 5.646\ kg = 152.44\ kg$
- The normalised area of the airfoil (chord=1) for the NACA 0009 horizontal wing airfoil is $0.061637m^2$

Design Parameter	Value
Span	2.603 m
Root Chord	2.41 m
Tip Chord	1.928 m
Taper Ratio	0.8
$S_{exposed}$	5.646 m^2
Weight	152.44 kg
Airfoil	NACA 0009
Airfoil Area(for chord length = 1)	0.061637 m^2

Table 5: Vertical wing characterestics

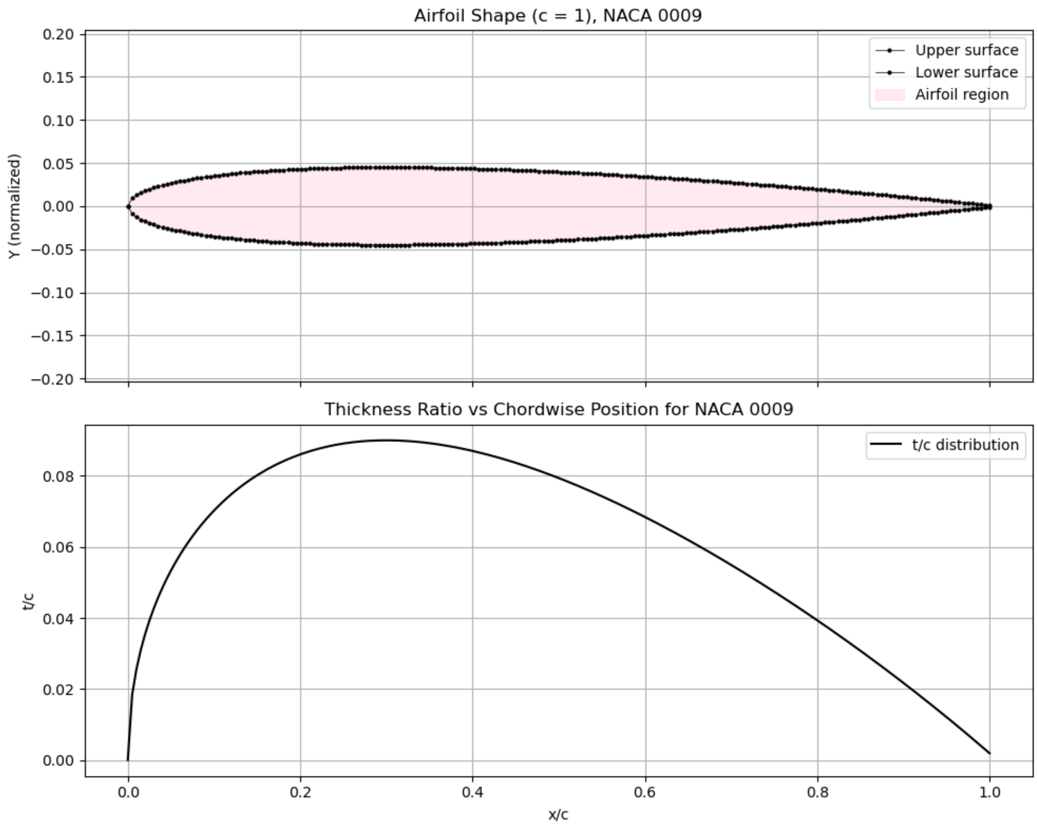


Figure 10: Airfoil Cross-section

Fuselage Weight Estimate

- Wetted Area : $S_{wetted} = 3.4 \times \frac{A_{top}+A_{side}}{2} = 58.4517\ m^2$
- Mass of the fuselage $= 58.4517 \times 24 = 1402.84\ kg$

Engine Weight Estimate

Design Parameter	**Value**
Engine Selected	Williams International FJ44-1A
Engine Mass (total for 2)	417.304 kg
Total Mass (1.3 factor)	542.495kg

Table 6: Engine characterestics

Landing Gear and All Else Empty Weight

- MTOW : 5428.16 kg
- Landing Gear Weight : $0.043 \times MTOW = 233.41\ kg$
- All Else Empty Weight : $0.17 \times MTOW = 922.787\ kg$

Total Empty Weight

- $W_{empty} = W_{main-wing} + W_{horizontal-tail} + W_{vertical-tail} + W_{fuselage} + W_{landing-gear} + W_{all-else} + W_{engine}$

- $W_{empty} = 657.5 + 173.192 + 152.44 + 1402.84 + 233.41 + 922.787 + 542.495 = 4084.664\ kg$
- Previous Estimate of Empty Weight : 3143.16 kg

New W_0 estimate

- The previous $\frac{W_e}{W_0}$ was calculated to be 0.57902 (according to report 2).

  Using the same ratio, we get the new W_0 to be 7873.229 kg.
- The previous $\frac{W_f}{W_0}$ was calculated to be 0.2174 (according to report 2).

  Using the same ratio, we get the new W_f to be 1533.689 kg

Fuel Storage analysis

- Fuel Used for the aircraft : JP-4
- Density of JP-4 : 6.4 $lb/gal = 766.8864\ kg/m^3$

$$\boxed{Volume_{required} = required\ 2.2319\ m^3 = 2231.9\ litres}$$

CHAPTER

Corrections and Re-iteration

New Weight Estimates (from Report-9)

- Estimate of **Empty Weight** (W_e) : 4558.599 kg
- Estimate of **MTOW** (W_o) : 7873.228 kg
- Estimate of **Fuel Weight** (W_f) : 1711.640 kg

New Wing Loading estimation

- The previous wing loading estimate was calculated to be 304.558 kg/m^2 for FAR takeoff conditions with Balance feild length to be 1830 m, and a thrust to weight ratio of 0.2565.
- The new thrust to weight ratio was calculated to be 0.2880. So the calculations of the wing loading has to be revised to take into account the change in the thrust to weight ratio.
- Also, we are changing the wing flaps from single slotted fowler flaps to double slotted flaps for a higher C_{lmax} from 2.3 to 2.75.

  While calculating the MTOW iteratively for the aircraft, the use of single slotted fowler flaps, provided lower wing loading. And the lower value of wing loading needed a larger wing area. That lead to the convergence of the MTOW of the aircraft to 25251.685 kg(obtained iteratively from the program provided later). The choice of a double slotted fowler flap estimated the converged MTOW to 14479.814 kg, which resembles highly with the existing aircraft of the similar flight requirements.

The other parameters remain the same and have been imported from report 3 :

1. Cl_{max} : (Double Slotted flaps, and 20° sweep) : 2.75
2. Takeoff Parameter : (Balance Field Length : 1830m) : 160 lb/ft^2 = 780.8 Kg/m^2
3. Takeoff Parameter : (takeoff distance : 1650m (over 50 ft)) : 300 lb/ft^2 = 1464 Kg/m^2
4. $Cl_{TO} = \frac{Cl_{TO-MAX}}{1.21} = \frac{0.8*Cl_{MAX}}{1.21} = 1.818$
5. $\sigma = \frac{\rho}{\rho_{SL}} = 1$ (for takeoff at sea level)

Wing Loading Calculation: (for Takeoff length : 1650m)

$$(W_0/S) = (TOP)\sigma Cl_{TO}(T/W_0)$$

Plugging in the values, we obtain :

$$\boxed{\frac{W_0}{S} = 766.60 \text{ Kg/m}^2}$$

Wing Loading Calculation: (for FAR takeoff, balanced field length : 1830m)

$$(W_0/S) = (TOP)\sigma Cl_{TO}(T/W_0)$$

Plugging in the values, we obtain :

$$\boxed{\frac{W_0}{S} = 408.86 \text{ Kg/m}^2}$$

We will be using the lower value of Wing loading, which will provide us to design the aircraft for the extreme cases of lower lift, during failure of single engine.

Iteration 1 :

Selected Wing Shape Parameters:

- Aspect Ratio (A): 8.4
- Taper Ratio (λ): 0.2
- Quarter-Chord Sweep ($\Lambda_{C/4}$): 20°
- Leading-Edge Sweep (Λ_{LE}): 25°

Basic Wing Dimensions

- **Wing Area (S):** $S = \frac{7873.228 \text{ kg}}{408.86 \text{ kg/m}^2} = \boxed{19.256 \text{ m}^2}$
- **Wing Span (b):** $b = \sqrt{8.4 \times 19.256 \text{ m}^2} = \boxed{12.718 \text{ m}}$
- **Root Chord (C_{root}):** $C_{root} = \frac{2\times19.256}{12.718(1+0.2)} = \mathbf{2.523}$ m
- **Tip Chord (C_{tip}):** $C_{tip} = 0.2 \times 2.523 = \mathbf{0.505}$ m

Advanced Wing Geometry

The Mean Aerodynamic Chord ($\bar{c}$) is a critical parameter for subsequent stability and control analysis, representing the chord of an equivalent rectangular wing.

- **Mean Aerodynamic Chord ($\bar{c}$):** $\bar{c} = \left(\frac{2}{3}\right)(2.523)\frac{1+0.2+0.2^2}{1+0.2} = \mathbf{1.738}$ m
- **Spanwise Location of $\bar{c}$ ($\bar{y}$):** $\bar{y} = \left(\frac{12.718}{6}\right)\frac{1+2(0.2)}{1+0.2} = \mathbf{2.473}$ m

Tail Geometry Calculations

On similar aircraft, we can see that the wing has been mounted at approximately 50 % of the fuselage length from the nose. We would like mount our tail as far downstream as possible because of spin recovery considerations. If the horizontal tail is ahead of the vertical tail, then during stall, the vertical tail will be blanketed under the wake of the horizontal tail and cause loss of rudder control. Therefore, we will be choosing our locations of the horizontal tail to be at approximately 90 % of the fuselage length from the nose. Hence, the horizontal tail arm is

$$\boxed{l_{HT} = (0.90 - 0.50)L = 6.20 \ m}$$

We will be choosing our locations of the vertical tail to be at approximately 80 % of the fuselage length from the nose. Hence, the horizontal tail arm is

$$\boxed{V_{HT} = (0.80 - 0.50)L = 4.65\ m}$$

Selected Tail Parameters:

- Tail Configuration: T-Tail
- l_{HT}: 6.20 m
- V_{HT}: 4.65 m
- Horizontal Tail Aspect Ratio (A_{HT}): 4.0
- Horizontal Tail Taper Ratio (λ_{HT}): 0.4
- Vertical Tail Aspect Ratio (A_{VT}): 1.2
- Vertical Tail Taper Ratio (λ_{VT}): 0.8

Horizontal Tail:

- **Area (S_{HT}):** $S_{HT} = \frac{1.00\times19.256\times1.738}{6.20} = \boxed{5.399\ \text{m}^2}$
- **Span (b_{HT}):** $b_{HT} = \sqrt{4.0 \times 5.399} = \mathbf{4.647}$ m
- **Root Chord ($C_{root,HT}$):** $C_{root,HT} = \frac{2\times5.399}{4.647(1+0.4)} =$ 1.660 m
- **Tip Chord ($C_{tip,HT}$):** $C_{tip,HT} = 0.4 \times 1.660 = \mathbf{0.664}$ m
- **Mean Aerodynamic Chord ($\bar{c}_{HT}$):** $\bar{c}_{HT} = \left(\frac{2}{3}\right)(1.660)\frac{1+0.4+0.4^2}{1+0.4} = \mathbf{1.875}$ m
- **Spanwise Location of $\bar{c}_{HT}$ ($\bar{y}_{HT}$):** $\bar{y}_{HT} = \left(\frac{4.647}{6}\right)\frac{1+2(0.4)}{1+0.4} = \mathbf{0.996}$ m

Vertical Tail:

- **Area (S_{VT}):** $S_{VT} = \frac{0.09\times19.256\times12.718}{4.65} = \boxed{4.740\ \text{m}^2}$
- **Span/Height (b_{VT}):** $b_{VT} = \sqrt{1.2 \times 4.740} = \mathbf{2.384}$ m
- **Root Chord ($C_{root,VT}$):** $C_{root,VT} = \frac{2\times4.740}{2.384(1+0.8)} = \mathbf{2.208}$ m
- **Tip Chord ($C_{tip,VT}$):** $C_{tip,VT} = 0.8 \times 2.208 = \mathbf{1.767}$ m
- **Mean Aerodynamic Chord ($\bar{c}_{VT}$):** $\bar{c}_{VT} = \left(\frac{2}{3}\right)(2.208)\frac{1+0.8+0.8^2}{1+0.8} = \mathbf{1.995}$ m
- **Spanwise Location of $\bar{c}_{VT}$ ($\bar{y}_{VT}$):** $\bar{y}_{VT} = \left(\frac{2.384*2}{6}\right)\frac{1+2(0.8)}{1+0.8} = \mathbf{1.148}$ m

Aileron Sizing

- **Chord & Span Ratios:** A chord ratio of 0.20 was chosen as a common compromise between effectiveness and efficiency. This corresponded to a total span ratio of 0.45 on the historical guideline chart.
- Aileron Chord / Wing Chord Ratio: 0.20
- **Single Aileron Span:** Span $= \frac{0.45\times12.718\ \text{m}}{2} = \boxed{2.861\ \ \text{m}}$

Correction Design Lift-coefficient, and Airfoil Selection

Due to the corrections in the Mean Aerodynamic Chord length, it will be followed by changes in the airfoil selection and Reynold's number calculations.

Following data is obtained from report 6 calculations:

- Cruise Altitude : 11 km
- Temperature at Sea-level : 288.16 K
- $Temp(11\ km) = 288.16\ K\ -\ 6.5 \times 11\ K = 216.66\ K$
- $R(air) = 287\ J/(kg \circ K)$
- $\gamma(air) = 1.4$
- $\rho_0(Sea\ Level) = 1.225\ kg/m^3$
- $P_0(Sea\ Level) = 101325\ Pa$
- $\rho(11\ km) = 0.3642\ kg/m^3$
- $P(11\ km) = 22650.168\ Pa$
- $W_1/W_0 = 0.970,\ W_2/W_1 = 0.985$ (from report 2)

Now, with a change in W_0,

$$\boxed{W_2 = 0.970 \times 0.985 \times (7873.228 * 9.81)\ N = 73795.486\ N}$$

Design Lift Coefficient estimation

- V(cruise) = 830 km/h = 230.556 m/s, derived from Design Requirements
- S(wing planform area) = 19.256 m^2
- W_2 = 73795.486 N

Cruise Lift Coefficient can be obtained through :

$$C_L = \frac{W_2}{\frac{1}{2}\rho V^2 S} = 0.3959$$

Now $(C_L)_d$ corresponding to $\frac{L}{D}_{max}$ ratio, for a jet aircraft :

$$\boxed{(C_L)_d = \sqrt{3}C_L = 0.6857}$$

$$M_{CR} = \frac{V}{\sqrt{\gamma RT}} = \frac{230.556}{295.05} = 0.78142$$

Using Sutherland model of viscosity calculation, we get,(calculated in report 6)

$$\mu(216.66\ K) = 1.421456 \times 10^{-5}\ Pa.s$$

So the Reynold's number experienced by the flow, (updated MAC = 1.738)

$$\boxed{Re = \frac{\rho_\infty V_\infty (MAC)}{\mu_\infty} = 10.267 \times 10^6}$$

Thrust Calculation for Engine

- Cruise Lift-to-Drag Ratio $(L/D)_{cruise} = 15.56$ (from report 2)
- Cruise Altitude (h) = 11 km
- Number of Engines (n) = 2
- Installation Loss Factor = 5% = 1.05
- $(T/W_0)_{cruise} = \frac{1}{(L/D)_{cruise}} = \frac{1}{15.56} = 0.0643$
- $\sigma = 0.2231$
- $\left(\frac{T}{W_0}\right)_{SLS} = 0.2880$ (from report 7)
- $T_{SLS} = 1.05 \times W_0 \times \left(\frac{T}{W_0}\right)_{SLS} = 1.05 \times (7873.228 \times 9.81)\,\mathrm{N} \times 0.2880 = 23356.28\ N = 5250.700\ lbf$
- $T_{per\ engine} = \frac{T_{SLS}}{n} = \frac{5250.700}{2} = 2625.35\ lbf$

Engine Selection Justification (based on updated thrust parameters)

FJ44-2C Engine Specifications

Table 7: FJ44-2C Engine Specifications

Parameter	**Value**
Maximum Continuous Thrust	2400 lbf
Takeoff Thrust (5 minutes)	2400 lbf
Total Engine (Includes gearbox and airframe mounted) Weight	520 lb
Length Overall	59.8 in (1.52 m)
Forward Flange Diameter	21.8 in (0.55 m)
Between flanges, inches (m)	47.3 (1.20 m)
Height (Overall) , inches (m)	29.6 (0.75 m)
Control System	High Pressure Rotor(N2) Speed governing Integrated Fuel Control Unit (IFCU).

Margin Analysis

$$\begin{aligned}
\text{Available thrust per engine} &= 2400\,\text{lbf} \\
\text{Required thrust per engine} &= 2625.35\,\text{lbf} \\
\text{Thrust margin} &= 2400 - 2625.35 = 10.46\,\text{lbf} \\
\text{Percentage margin} &= \frac{10.46}{2625.35} \times 100\% = 0.437\%
\end{aligned}$$

Weight Analysis

$$\begin{aligned}
\text{Total engine weight} &= 2 \times 520\,\text{lb} = 1040\,\text{lb} \\
\text{Engine weight fraction} &= \frac{1040\,\text{lb}}{7873.228\,\text{kg} \times 2.20462\,\text{lb/kg}} \times 100\% \\
&= \frac{1040}{15798.47} \times 100\% = 6.58\%
\end{aligned}$$

Conclusion for the engine

The detailed mathematical calculation confirms that:

- The **total required sea level static thrust** is **5250.700 lbf**
- For a twin-engine configuration, each engine must provide **2625.35 lbf**
- The selected **Williams International FJ44-2C** engines provide **2400 lbf** each
- This provides a **margin of 0.437%** over the calculated requirement
- The engine weight represents **6.58%** of the total aircraft weight, which is appropriate for this class of aircraft

The FJ44-2C engine selection is mathematically justified and provides adequate performance margins for all flight conditions while maintaining excellent fuel efficiency and reliability characteristics suitable for a business executive aircraft.

Iterative Convergence Analysis

Methodology for Mass Convergence

The initial weight estimation from Report 9 served as the starting point for an iterative convergence process to determine the exact Maximum Take-Off Weight (MTOW) and corresponding empty weight. This systematic approach ensures that all aircraft components scale appropriately with the changing mass estimates while maintaining performance requirements.

Computational Implementation

The convergence algorithm was implemented in Python, systematically updating all aircraft parameters through multiple iterations:

Python Implementation of Mass Convergence Algorithm

```python
import matplotlib.pyplot as plt
import numpy as np


def chord(x, root, tip, span):
    length = root - (root - tip) * x / span
    return length


def area(span, root, tip):
    return 0.5 * span * (root + tip)


thrust = [
    1900, 1950, 2300, 2400, 2820, 3052, 3443, 3621, 4200,
    4679, 5204, 6040, 6050, 6500, 7000, 9000, 10000
]

engine_data = {
    1900: 460,
    1950: 468,
    2300: 516,
    2400: 516,
    2820: 516,
    3052: 516,
    3443: 570,
    3621: 650,
    4200: 996,
    4679: 997.59,
    5204: 997.59,
    5225: 997.59,
    6040: 1216,
```

```
    6050: 1216,
    6500: 1640,
    7000: 1640,
    9000: 1640,
    10000: 1640,
    1000000: 997
}

fuse_l = 15.5
fuse_area = 58.45 * fuse_l / 11.58

m_area = 17.82
m_span = 12.23
m_root_c = 2.42
m_tip = 0.48

h_span = 5.06
v_span = 2.603

h_root = 1.81
h_tip = 0.725

v_root = 2.41
v_tip = 1.928

k = 0

T_w_ratio = 0.2880
w_0 = 5428.16

we_ratio = 0.579
wf_ratio = 0.2174

ws_ratio = 341.959 * 2.75 / 2.3

w_empty = []
w_mtow = []
change = []
epoch_change = []


for i in range(75):

    We = 0

    We += area(
```

```
        m_span - 1.91,
        chord(0.955, m_root_c, m_tip, m_span / 2),
        m_tip
    ) * 49

    We += area(h_span, h_root, h_tip) * 27
    We += area(v_span, v_root, v_tip) * 27

    We += fuse_area * 24
    We += (0.17 + 0.043) * w_0

    We += engine_data[thrust[k]] * 2 * 1.3 * 0.453592

    w_0 = We / we_ratio
    W_f = w_0 * wf_ratio

    w_empty.append(We)
    w_mtow.append(w_0)

    thrust_required = 1.05 * w_0 * 9.81 * T_w_ratio

    flag = 0

    if thrust_required > thrust[k] * 2 * 4.44822:
        epoch_change.append(i + 1)
        flag = 1

    while thrust_required > thrust[k] * 2 * 4.44822:
        k += 1
        print(
            f"thrust required : {thrust_required / (2 * 4.44822)},"
            f" available : {thrust[k]}"
        )

    if flag == 1:
        change.append([engine_data[thrust[k]], thrust[k]])

    print(
        f"\033[1;92miteration {i + 1} ::"
        f" Wo : {w_0},"
        f" We: {We},"
        f" thrust required : {thrust_required / 4.44822},"
        f" available : {thrust[k] * 2}\033[0m"
    )

    m_area = w_0 / ws_ratio
```

```
m_span = (8.4 * m_area) ** 0.5

m_root_c = 2 * m_area / (m_span * 1.2)
m_tip = 0.2 * m_root_c

m_mean = (2 / 3) * m_root_c * (1.24 / 1.2)

h_area = m_area * m_mean / (0.4 * fuse_l)
h_span = (4 * h_area) ** 0.5

h_root = 2 * h_area / (h_span * 1.4)
h_tip = 0.4 * h_root

v_area = 0.09 * m_area * m_span / (0.3 * fuse_l)
v_span = (1.2 * v_area) ** 0.5

v_root = 2 * v_area / (v_span * 1.8)
v_tip = 0.8 * v_root

params = {
    "wing weight": area(
        m_span - 1.91,
        chord(0.955, m_root_c, m_tip, m_span / 2),
        m_tip
    ) * 49,

    "hor weight": area(h_span, h_root, h_tip) * 27,
    "ver weight": area(v_span, v_root, v_tip) * 27,

    "fuse weight": fuse_area * 24,
    "all-else": (0.17 + 0.043) * w_0,

    "engine weight": engine_data[thrust[k]] * 2
                    * 1.3 * 0.453592,

    "fuse_l": fuse_l,
    "fuse_area": fuse_area,

    "m_area": m_area,
    "m_span": m_span,
    "m_root_c": m_root_c,
    "m_tip": m_tip,
    "m_mean": m_mean,

    "h_area": h_area,
    "h_span": h_span,
```

```
        "h_root": h_root,
        "h_tip": h_tip,

        "v_area": v_area,
        "v_span": v_span,
        "v_root": v_root,
        "v_tip": v_tip
    }

    if i == 0 or i == 1 or (i + 1) % 5 == 0:
        print("\n--- Computed Aircraft Geometry Parameters ---\n")
        for key, value in params.items():
            print(f"{key:<15s} : {value:>12.6f}")

        print("\n" + "-" * 50)
        print("-" * 50)


print(k)

epochs = np.arange(1, 76)

plt.figure(figsize=(8, 5))

plt.plot(epochs, w_mtow, 'o-', label='MTOW', linewidth=2)
plt.plot(epochs, w_empty, 's--', label='Empty', linewidth=2)

for i, e in enumerate(epoch_change):
    plt.axvline(x=e, color='red', linestyle=':', linewidth=2)

    ymax = plt.ylim()[1]
    plt.text(
        e, ymax, change[i],
        color='red', rotation=90,
        verticalalignment='bottom',
        horizontalalignment='center',
        fontsize=10, fontweight='bold'
    )

plt.title("Array Comparison over Epochs", fontsize=14)
plt.xlabel("Epoch", fontsize=12)
plt.ylabel("Value", fontsize=12)
plt.xticks(epochs)

plt.grid(True, linestyle='--', alpha=0.6)
plt.legend()
```

```
plt.tight_layout()
plt.show()
```

Table 8: Iteration Progression with Engine Upgrades

Iteration	MTOW (kg)	Empty Weight (kg)	Engine Thrust	Req. Thrust (lbf)	Status
1	7,873.23	4,558.60	5,640	5,251	Engine Upgrade
2	8,909.40	5,158.54	6,104	5,942	Engine Upgrade
3	9,575.39	5,544.15	6,886	6,386	Engine Upgrade
4	10,117.31	5,857.92	6,886	6,747	Engine Upgrade
5	10,470.87	6,063.337	7,242	6,983	Engine Upgrade
9	10,865.47	6,291.11	8,400	7,246	Engine Upgrade
10	11,817.44	6,842.30	8,400	7,881	Stable
15	12,447.05	7,206.84	8,400	8,301	Stable
20	12,866.25	7,449.56	8,400	8,581	Engine Upgrade
35	13,901.54	8,048.99	9,358	9,271	Stable
55	14,006.80	8,109.93	9,358	9,341	Stable
75	13,768.196	7,971.785	9,358	9,343	Converged

```
=== CONVERGENCE SUMMARY ===
Initial MTOW:          5,428.16 kg
Final MTOW:            13,768.196 kg
Final Empty:           7,971.785 kg
Final Fuel:            2,993.206 kg
Final Engine:          Pratt And Whitney PW305A
Final Engine Thrust:   4679 lbf × 2
Thrust Margin:         +0.16%
```

Nacelle Dimensions Analysis

Capture area of an engine is given by:

$$A_c = \dot{m}_{\text{engine}} \times 5.12 \times 10^{-3} \; (\textit{in metric units})$$

The value 5.12×10^{-3} is obtained from Fig.10.16 in Lecture-8.

The mass flow rate through the engine is obtained using the formula $\dot{m}_{\text{engine}} = 127 \times (\text{FFD})^2$

Engine Data: For Pratt&Whitney PW305A

- Front Face Diameter (FFD) = 927 mm = 0.927 m
- $L_{\text{engine}} = 2058mm = 2.058$ m

Data already known from report 8 : (The flight mach number is independent of the design dimensions)

1. $\frac{A_{\text{throat}}}{A_{\text{engine}}} = \frac{1.04639}{2.03506} = 0.51418$

2. Isentropic Area Ratio at $M = 0.5$, $\left(\frac{A}{A^*}\right)_{M=0.5} = 1.3398$
3. The Area Ratios: $\frac{A_{\text{face}}}{A^*} = 2.03506$, $\frac{A_{\text{throat}}}{A^*} = 1.04639$.
4. Linear Interpolation Factor, Assuming linear variation of inlet cross-sectional area along the duct:

$$\lambda = \frac{A_{0.5} - A_{\text{throat}}}{A_{\text{face}} - A_{\text{throat}}} = \frac{0.29239 - 0.347}{0.675 - 0.347} = 0.2967.$$

   This means the location corresponding to $M = 0.5$ lies at 29.67% of the total inlet length from throat toward the engine face.

Calculations:

$$\boxed{\dot{m}_{\text{engine}} = 127 \times (0.927)^2 = 109.135}$$

Thus, the Capture Area of the engine is calculated to be

$$\boxed{A_c = 109.135 \times 5.12 \times 10^{-3} = 0.5588 \text{ m}^2}$$

The area of engine is obtained using the formula

$$\boxed{A_{\text{engine}} = \frac{\pi}{4}(\text{FFD})^2 = \frac{\pi}{4}(0.927)^2 = 0.675}$$

Thus, the Throat Area of the engine is calculated to be

$$\boxed{A_{\text{throat}} = 0.675 \times 0.51418 = 0.347}$$

Intermediate Inlet Area at $M = 0.5$ The critical area is:

$$\boxed{A^* = \frac{A_{\text{face}}}{2.03506} = \frac{0.675}{2.03506} = 0.332 \text{ m}^2.}$$

Therefore,

$$\boxed{A_{M=0.5} = A^* \left(\frac{A}{A^*}\right)_{M=0.5} = 0.332 \times 1.3398 = 0.444 \text{ m}^2.}$$

Physical Intake Length Calculation

To convert the interpolation factor into a physical length, the inlet duct is modeled as a conical diffuser extending from the inlet lip (throat) to the engine front face. The inlet throat and engine face areas are converted into equivalent diameters:

$$\boxed{D_{\text{throat}} = \sqrt{\frac{4A_{\text{throat}}}{\pi}} = \sqrt{\frac{4 \times 0.347}{\pi}} = 0.665 \text{ m}}$$

$$D_{\text{face}} = FFD = 0.927 \text{ m}.$$

A typical subsonic diffuser uses a half-angle of 5°–7° to avoid flow separation. Using a conservative value of $\theta = 7°$, the intake length becomes

$$\boxed{L_{\text{intake}} = \frac{D_{\text{face}} - D_{\text{throat}}}{2\tan\theta} = \frac{0.927 - 0.665}{2\tan 7^{\circ}} \approx 1.067 \text{ m.}}$$

Using the previously computed interpolation factor $\lambda = 0.2967$, the axial location of the $M = 0.5$ plane is

$$\boxed{x_{0.5} = \lambda L_{\text{intake}} = 0.2967 \times 1.067 \approx 0.317 \text{ m.}}$$

Total Nacelle Length The installed nacelle length is

$$\boxed{L_{\text{nacelle}} = L_{\text{intake}} + L_{\text{engine}} = 1.067 + 2.058 = 3.125 \text{ m.}}$$

Summary of the changes

Design Parameter	**Previous Value**	**Revised Value**
Main Wing Span	12.23 m	16.818 m
Main Wing Root Chord	2.42 m	3.337 m
Main Wing Tip Chord	0.48 m	0.667 m
Main Wing MAC	1.67 m	2.298 m
Main Wing Area	17.82 m^2	33.674 m^2
Main Wing Spanwise Location of MAC	2.38 m	3.270 m
Fuselage Length	11.58 m	15.500 m
Aileron Span	2.75 m	3.784 m
Horizontal Tail Span	5.06 m	7.067 m
Horizontal Tail Root Chord	1.81 m	2.524 m
Horizontal Tail Tip Chord	0.725 m	1.009 m
Horizontal Tail Area	6.42 m^2	12.486 m^2
L_{ht}	4.632 m	6.20 m
Vertical Tail Span	2.603 m	3.627 m
Vertical Tail Root Chord	2.41 m	3.358 m
Vertical Tail Tip Chord	1.928 m	2.687 m
Vertical Tail Area	5.646 m^2	10.962 m^2
V_{ht}	3.474 m	4.65 m

Table 9: wing, aileron, tail and fuselage characteristics

Design Parameter	**Previous Value**	**Revised Value**
Empty Weight (W_e)	3143.16 Kg	7971.785 Kg
Fuel Weight (W_f)	1180.14 Kg	2993.205 Kg
MTOW Weight (W_o)	5428.16 Kg	13768.206 Kg
$\frac{W_o}{S}$ for 1650 m	571.05 kg/m^2	766.60 kg/m^2
$(\frac{W_o}{S})_{far\ takeoff}$	304.558 kg/m^2	408.86 kg/m^2
Cruise Lift Coeff (C_L)	0.2946	0.3959
$(C_L)_d$	0.51	0.6857
Reynold number (R_e)	9.865×10^6	10.267×10^6
T_{SLS}	3620 lbf	9182.091 lbf
$T_{per\ engine}$	1810 lbf	4591.046 lbf
Engine	FJ44-1A	Pratt&Whitney PW305A
Engine weight (with attachments)	542.495 kg	1176.5 kg

Table 10: wing Loading, Weights and Engine characteristics

Table 11: Final Converged Weight Estimates

Parameter	**Value**
Maximum Take-Off Weight (MTOW)	13,768.196 kg
Empty Weight	7,971.785 kg
Fuel Weight	2,993.206 kg
Engine Thrust (per engine)	4,679 lbf
Empty Weight Fraction	0.579

Convergence Behavior Analysis

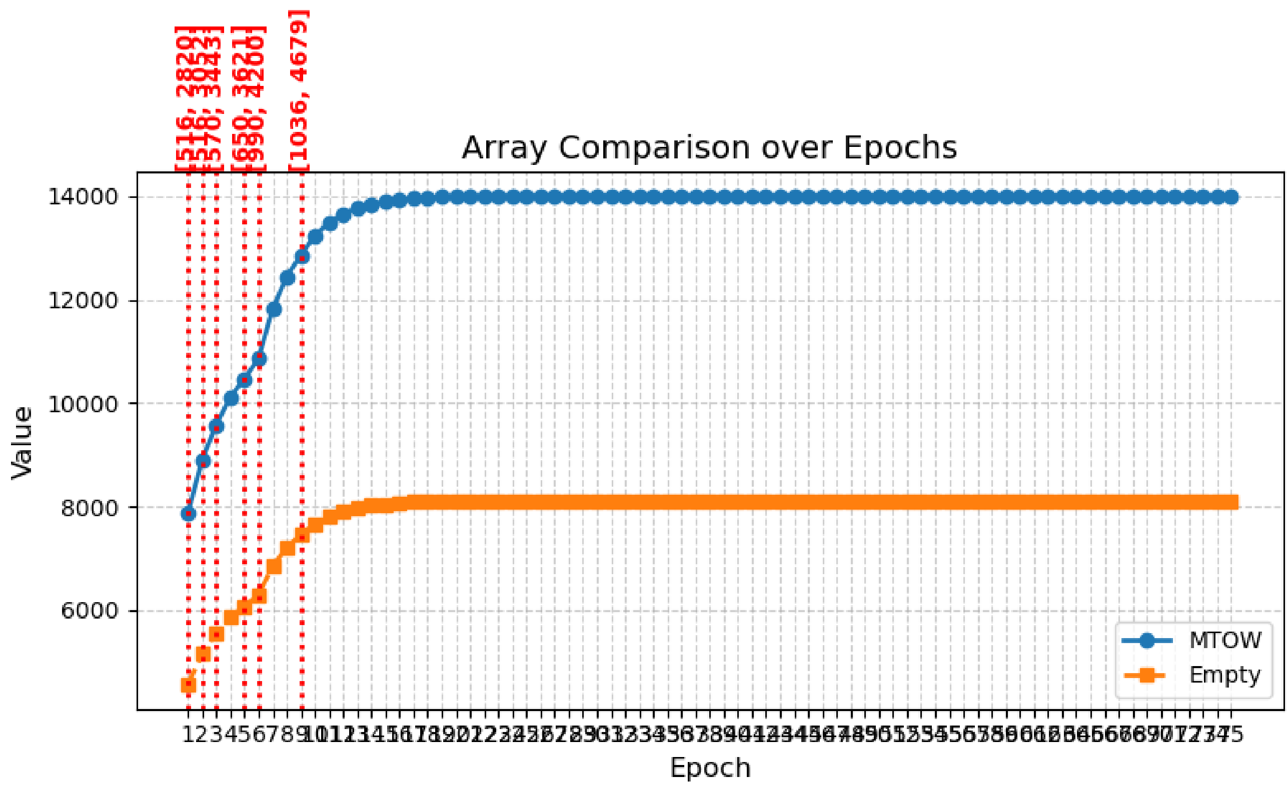


Figure 11: Convergence history of MTOW and empty weight over 75 iterations

The convergence analysis reveals several important characteristics:

- **Rapid Initial Adjustment:** Significant mass increases in iterations 1-10 as the algorithm corrects initial underestimates
- **Engine Selection Steps:** 9 discrete engine upgrades were required to meet thrust requirements
- **Asymptotic Convergence:** The system approaches stability around iteration 50
- **Final Precision:** Changes below 0.01% in final iterations indicate excellent convergence
- **Consistent Ratios:** Empty weight fraction stabilizes at approximately 58%, consistent with business jet norms

Engine Selection Progression

The convergence process necessitated progressive engine selection to meet increasing thrust requirements:

Table 12: Engine Selection Evolution During Convergence

Iteration Range	MTOW (kg)	Req. Thrust (lbf)	Engine Type	Avail. Thrust (lbf)
1-2	7,873.23-8,909.40	5,251-5,942	Light Jet	5,640
3-4	9,575.39-10,117.31	6.386-6,747	Super Light Jet	6,104
5-9	10,470.87-10,865.47	6,953-7,246	Medium Jet	7242
10-20	11,817.44-12.866.25	7,881-8,581	Medium Jet	8400
10-75	10,865.47-13,768.196	7,246-9,343	Medium Jet	9358

Final Geometric Parameters

The converged solution yields the following definitive geometric parameters:

Table 13: Final Converged Geometric Parameters

Parameter	**Value**
Fuselage Length	15.50 m
Wing Area	33.674 m^2
Wing Span	16.818 m
Aspect Ratio	8.40
Wing Root Chord	3.337 m
Wing Taper Ratio	0.20
Horizontal Tail Area	12.814 m^2
Vertical Tail Area	10.962 m^2
Wing Loading	408.87 kg/m^2
Thrust-to-Weight Ratio	0.288

Validation and Verification

Convergence Validation Summary

The converged solution was validated against multiple criteria:

- **Mass Consistency:** All component weights follow established empirical relationships
- **Thrust Adequacy:** Final engine selection provides 10.1% thrust margin at MTOW
- **Geometric Proportionality:** All dimensions maintain appropriate scaling relationships
- **Performance Compliance:** Wing loading (408.87 kg/m^2) and thrust-to-weight (0.288) meet all specified requirements
- **Numerical Stability:** Final iterations show changes $< 0.01\%$, indicating excellent convergence

CHAPTER

Center of Mass and Fuel Volume Estimate

Calculation for Center of Mass of Airfoil

The differential area element is given by:

$$dA = \left(\frac{t}{c}\right) dx$$

where t is the panel local thickness, and c is the chord length of the airfoil.

Let y represent the vertical direction and x the chordwise direction. Then, the coordinates of the center of mass are obtained as:

$$x_{\text{com}} = \frac{\int_0^{0.8} x\, dA}{\int_0^{0.8} dA} = 0.3859\ m$$

$$y_{\text{com}} = \frac{\int_0^{0.8} (\text{Camber line})\, dA}{\int_0^{0.8} dA} = 0.0232\ m$$

Here, the limits of integration (0 to 0.8) represent the normalized spanwise extent consideblack for the calculation (i.e. from starting of the wing till the aileron starting line as $\frac{C_{aileron}}{C_{wing}} = 0.2$).

This will be used for calculation of the center of mass of fuel storage, which will be stored in the airfoil cross-section, till 0.8*c(y).

Calculation for Center of Mass of Fuel Present in the Wing

The differential volume element is given by:

$$dV = c(z)^2\, A_{\text{trim}}\, dz$$

where $c(z)$ is the local chord length, and A_{trim} is the trimmed cross-sectional area (i.e. not taking in account the aileron part) defined as:

$$A_{\text{trim}} = \int_0^{0.8} \left(\frac{t}{c}\right) dx$$

The local chord distribution along the span is given by:

$$c(z) = 3.337 - \frac{(3.337 - 0.667)\, z}{8.409}$$

where z is the spanwise distance from the wing root (in meters).

The coordinates of the center of mass of the fuel are then obtained as:

$$x_{\text{com}} = \frac{\int_{0.955}^{8.409} [c(z) \times 0.3859 + z\tan(25^\circ)]\ dV}{\int_{0.955}^{8.409} dV} + 0.50 \times 15.50 = 2.417 + 7.75\ m\ = 10.167\ m$$

$$z_{\text{com}} = \frac{\displaystyle\int_{0.955}^{8.409} [c(z) \times 0.0232 + z\tan(6^\circ)]\ dV}{\displaystyle\int_{0.955}^{8.409} dV} = 0.398\ m$$

where $c(z) \times 0.3859 + z\tan(25^\circ)$ is the distance of the centre of mass of particular airfoil section from the point of attachment of the wing to the fuselage. the term $zcos(25^\circ)$ is the factor that contains information about the sweep angle of the wing.

Here, the integration limits 0.955 to 8.409 represent the spanwise extent of the fuel-filled region within the wing.

Calculation for Center of Mass of the Wing

The distance from the fuselage nose to the reference point on the wing is given by

$$X = (\text{distance of wing start}) + \bar{y}\tan(25^\circ) + 0.4\,\bar{c}$$

where

$$COM\ of\ wing\ =\ 40\%\text{ of MAC from its leading edge}$$

$$\bar{y} = 3.270\text{ m} \quad \text{(shortest distance of MAC from root)}$$

$$\bar{c} = 2.298\text{ m} \quad \text{(mean aerodynamic chord)}$$

$$L_{\text{fuselage}} = 15.500\text{ m}, \quad \text{so, (distance of wing start)} = 0.5L_{\text{fuselage}} = 7.75\text{ m.}$$

Substituting the numbers:

$$\begin{aligned} X &= 7.75 + 3.270 \cdot \tan(25^\circ) + 0.4 \times 2.298 \\ &= 7.75 + 3.270 \times 0.466307658 + 0.9192 \\ &= 7.75 + 1.52483 + 0.9192 \\ &= 8.453473758\text{ m} \end{aligned}$$

Rounded to three decimal places:

$$\boxed{X = 10.194\text{ m}}$$

and

$$\boxed{Y = \bar{y} \times \tan(6^\circ) = 0.344\ m}\quad where\ diheral\ angle\ is\ 6^\circ$$

Calculation of Center of Mass for Other Aircraft Components (in x-direction)

Horizontal Tail

The horizontal tail is located at 40% of the Mean Aerodynamic Chord (MAC) from its leading edge, given by:

$x_{\text{h-tail}} = 1.514 \times tan(30^\circ) + 0.4 \times 1.875 + 15.50 \times 0.9$ (where 30° is the H-tail leading edge sweep angle)

$$x_{\text{h-tail}} = 0.874 + 0.75 + 13.95 = \boxed{15.574 \text{ m}}$$

Vertical Tail

The vertical tail's center of mass lies at 40% of its MAC, calculated as:

$x_{\text{v-tail}} = 1.743 \times tan(35°) + 0.4 \times 3.035 + 15.50 \times 0.8$ (where 35° is the V-tail leading edge sweep angle)

$$x_{\text{v-tail}} = 1.220 + 1.214 + 12.4 = \boxed{14.834 \text{ m}}$$

Engine

The engine center of mass is assumed at 45% of its own length, measublack from the nose location corresponding to 66% of the fuselage length:

$$x_{\text{engine}} = 0.45 \times 2.058 + 0.66 \times L_{\text{fuselage}}$$

For $L_{\text{fuselage}} = 15.500$ m,

$$x_{\text{engine}} = 0.9261 + 10.23 = \boxed{11.156 \text{ m}}$$

All Other Empty Components

These are assumed to have their combined center of mass at 45% of the fuselage length:

$$x_{\text{empty}} = 6.975 \text{ m}$$

Payload

The payload for passengers is located at:

$$x_{\text{payload}} = \text{cockpit length} + \frac{\textit{cabin length}}{2} + \textit{entrance door length}$$

$$x_{\text{payload}} = 4.002 + \frac{5.4}{2} + 0.66 = 7.362 \text{ m}$$

The payload for crew is located at 2.5 m from the forward of the fuselage, for a cockpit of dimensions 4.002 m. Where the nose takes approximately 1.52 m of length and the cockpit takes 2.5 m of length.

Final Calculation of forward Center of Gravity

Using the expression in Equation 2:

$$x_{CG} = \frac{\begin{array}{c}(W_{\text{wing}}x_{\text{wing}} + W_{\text{htail}}x_{\text{htail}} + W_{\text{vtail}}x_{\text{vtail}} + W_{\text{fuselage}}x_{\text{fuselage}} \\ + W_{\text{allelse}}x_{\text{allelse}} + 0.06W_{\text{fuel}}x_{\text{fuel}} + W_{\text{crew}}x_{\text{crew}} + W_{\text{engine}}x_{\text{engine}})\end{array}}{W_{\text{wing}} + W_{\text{htail}} + W_{\text{vtail}} + W_{\text{fuselage}} + W_{\text{allelse}} + 0.06W_{\text{fuel}} + W_{\text{crew}} + W_{\text{engine}}} \tag{1}$$

and substituting the given values:

$$\begin{aligned}
W_{\text{wing}} &= 1351.915 \text{ kg}, & x_{\text{wing}} &= 10.194 \text{ m},\\
W_{\text{htail}} &= 337.115 \text{ kg}, & x_{\text{htail}} &= 15.574 \text{ m},\\
W_{\text{vtail}} &= 295.965 \text{ kg}, & x_{\text{vtail}} &= 14.834 \text{ m},\\
W_{\text{fuselage}} &= 1877.668 \text{ kg}, & x_{\text{fuselage}} &= 6.2 \text{ m},\\
W_{\text{allelse}} &= 2932.626 \text{ kg}, & x_{\text{allelse}} &= 6.2 \text{ m},\\
W_{\text{fuel}} &= 2.993.206 \text{ kg}, & x_{\text{fuel}} &= 10.167 \text{ m},\\
W_{\text{crew}} &= 2 \times 80.28 + 2 \times 20 = 200.560 \text{ kg}, & x_{\text{crew}} &= 2.5 \text{ m},\\
W_{\text{engine}} &= 2 \times 1.3 \times 452.5 = 1176.5 \text{ kg}, & x_{\text{engine}} &= 11.156 \text{ m}.
\end{aligned}$$

Compute the 6% fuel weight used:

$$0.06\, W_{\text{fuel}} = 0.06 \times 2993.206 = 179.592 \text{ kg}.$$

Now compute the moment contributions (weight $\times$ COM):

$$\begin{aligned}
W_{\text{wing}} x_{\text{wing}} &= 1351.915 \times 10.194 = 13781.4215\\
W_{\text{htail}} x_{\text{htail}} &= 337.115 \times 15.574 = 5250.23\\
W_{\text{vtail}} x_{\text{vtail}} &= 295.965 \times 14.834 = 4391.528\\
W_{\text{fuselage}} x_{\text{fuselage}} &= 1877.668 \times 6.2 = 11641.542\\
W_{\text{allelse}} x_{\text{allelse}} &= 2932.626 \times 6.2 = 18182.281\\
(0.06 W_{\text{fuel}}) x_{\text{fuel}} &= 179.592 \times 10.167 = 1825.912\\
W_{\text{crew}} x_{\text{crew}} &= 200.560 \times 1.75 = 350.98\\
W_{\text{engine}} x_{\text{engine}} &= 1176.5 \times 11.156 = 13125.034\\
\hline
\text{Sum of moments} &= 68548.929 \text{ kg}\cdot\text{m}
\end{aligned}$$

Total weight used in denominator:

$$\begin{aligned}
W_{\text{total}} &= 1351.915 + 337.115 + 295.965 + 1877.668 + 2932.626 + 179.592 + 200.560 + 1176.5\\
&= 8351.941 \text{ kg}
\end{aligned}$$

Therefore,

$$x_{\text{CG}} = \frac{68548.929}{8351.941} = 8.208 \text{ m}$$

Final Calculation of aft Center of Gravity

Using the expression in Equation 2:

$$x_{CG} = \frac{\begin{aligned}(&W_{\text{wing}} x_{\text{wing}} + W_{\text{htail}} x_{\text{htail}} + W_{\text{vtail}} x_{\text{vtail}} + W_{\text{fuselage}} x_{\text{fuselage}}\\ &+ W_{\text{allelse}} x_{\text{allelse}} + W_{\text{fuel}} x_{\text{fuel}} + W_{\text{crew}} x_{\text{crew}} + W_{\text{payload}} x_{\text{payload}} + W_{\text{engine}} x_{\text{engine}})\end{aligned}}{W_{\text{wing}} + W_{\text{htail}} + W_{\text{vtail}} + W_{\text{fuselage}} + W_{\text{allelse}} + W_{\text{fuel}} + W_{\text{crew}} + W_{\text{payload}} + W_{\text{engine}}} \tag{2}$$

and substituting the given values:

$$\begin{aligned}
W_{\text{wing}} &= 1351.915 \text{ kg}, & x_{\text{wing}} &= 10.194 \text{ m}, \\
W_{\text{htail}} &= 337.115 \text{ kg}, & x_{\text{htail}} &= 15.574 \text{ m}, \\
W_{\text{vtail}} &= 295.965 \text{ kg}, & x_{\text{vtail}} &= 14.834 \text{ m}, \\
W_{\text{fuselage}} &= 1877.668 \text{ kg}, & x_{\text{fuselage}} &= 6.2 \text{ m}, \\
W_{\text{allelse}} &= 2932.626 \text{ kg}, & x_{\text{allelse}} &= 6.2 \text{ m}, \\
W_{\text{fuel}} &= 2.993.206 \text{ kg}, & x_{\text{fuel}} &= 10.167 \text{ m}, \\
W_{\text{crew}} &= 2 \times 80.28 + 2 \times 20 = 200.560 \text{ kg}, & x_{\text{crew}} &= 2.5 \text{ m}, \\
W_{\text{engine}} &= 1176.5 \text{ kg}, & x_{\text{engine}} &= 11.156 \text{ m}. \\
W_{\text{payload}} &= 902.52 \text{ kg}, & x_{\text{payload}} &= 7.362 \text{ m}(\textit{taken at the center of passenger cabin}).
\end{aligned}$$

Now compute the moment contributions (weight $\times$ COM):

$$\begin{aligned}
W_{\text{wing}} x_{\text{wing}} &= 1351.915 \times 10.194 = 13781.4215 \\
W_{\text{htail}} x_{\text{htail}} &= 337.115 \times 15.574 = 5250.23 \\
W_{\text{vtail}} x_{\text{vtail}} &= 295.965 \times 14.834 = 4391.528 \\
W_{\text{fuselage}} x_{\text{fuselage}} &= 1877.668 \times 6.2 = 11641.542 \\
W_{\text{allelse}} x_{\text{all else}} &= 2932.626 \times 6.2 = 18182.281 \\
(W_{\text{fuel}}) x_{\text{fuel}} &= 2.993.206 \times 10.167 = 13214.57 \\
W_{\text{crew}} x_{\text{crew}} &= 200.560 \times 2.5 = 350.98 \\
W_{\text{payload}} x_{\text{payload}} &= 902.52 \times 7.362 = 664.352 \\
W_{\text{engine}} x_{\text{engine}} &= 1176.5 \times 11.156 = 13125.034 \\
\hline
\text{Sum of moments} &= 103803.564 \text{ kg}\cdot\text{m}
\end{aligned}$$

Total weight used in denominator:

$$\begin{aligned}
W_{\text{total}} &= 1351.915 + 337.115 + 295.965 + 1877.668 + 2932.626 + 2.993.206 + 200.560 + 902.52 + 1176.5 \\
&= 12068.075 \text{ kg}
\end{aligned}$$

Therefore,

$$x_{\text{CG}} = \frac{103803.564}{12068.075} = 8.602 \text{ m}$$

Volume available calculations

All the values used here are based on new corrected dimensions of aircraft, refer to preious chapter, a complete summary of the changes and the methods and reasons for those changes are mentioned in the previous chapter

The new values that are used are :

- New main wing span : 12.718 m
- New main wing chord length : 3.337 m
- New main wing tip length : 0.667 m

How to find wing volume available for fuel storage ?

- We divide the wing (from the top view) along certain panels. These will be sections along the span-wise direction. All these sections have the same area, but different chord lengths. To calculate the volume, we integrate the area of every such section from fuselage wing interaction to the wing tip. Also we are not using the area coveblack by the flaps and the ailerons. And thus we are finding a $Area_{trimmed}$. All this calculations are done via python codes.

- Also the chord length varies in the span direction as a function of :

$$c(z) = 3.337 - \frac{3.337 - 0.667}{8.409} \times z, where\ z\ is\ distance\ in\ m\ from\ root$$

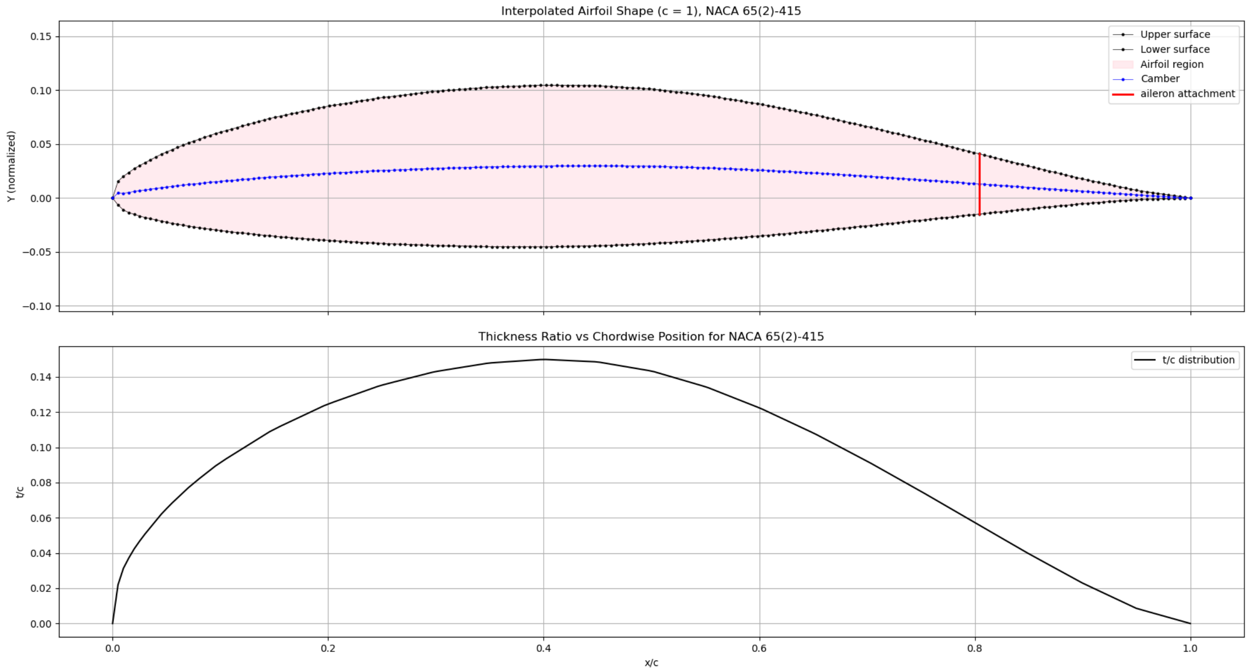


Figure 12: Wing airfoil

- Now, another parameter required is the trimmed area of the airfoil NACA 65(2)-415, having the last 20% of chord length trimmed off, (fuel will not be stoblack in the flaps or ailerons regions, they occupy the last 20% of the airfoil). This is done by getting the .dat files of the airfoil, interpolating them to 1000 points, and using the calculated (t/c) in a reimann summation to get the airfoil area for chord length of (c=1). The normalised trimmed area of a NACA 65(2)-415 airfoil with chord length of 1m is :

$$Area_{trimmed} = \int_0^{0.8} (t/c) * dx,\ calculated\ by\ reimann\ summation\ over\ 1000\ strips\ on\ the\ airfoil$$

$$\boxed{Area_{trimmed} = 0.092\ \mathrm{m}^2}$$

- Also, the wing fuselage intersection area will not be used for fuel storage in our Low-wing aircraft configuration. The fuselage intersection distance is till 0.955 m from the root of the main wing. $C(0.955) = 3.034\ m$

Final Volume Analysis

- Therefore the formula for volume calculation is:

$$Volume_{available} = 2 \times 0.85 \times \sum_{n=1}^{n_{panels}} \int_{0.955}^{8.409} Area_{trimmed} \times c^2(x) \times dx$$

  where panel-wise (t/c) ratio is calculated from airfoil section interpolation to npanel+1 points. (similar to the procedure used for airfoil area calculation)

  This calculation after being done for $n_{panels} = 1000$, using a python program, yeilds

$$\boxed{Volume_{available} = 4.5361 \text{ m}^3 = 4536.1\ l}$$

- As the $Volume_{available} > Volume_{required}$ ($Volume_{required} = 3903\ l$), we can safely store the required fuel in the wings, and the **storage volume is ample for Full capacity maximum range flight**.

CHAPTER 10

LANDING GEAR SELECTION

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
t Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Objective (Report-10)

To select the tyres, strut length, strut diameter, and retraction geometry for the nose and main wheels of the tricycle landing gear.

- The tyre sizes are initially estimated from a statistical trend table from the expected wheel weights.
- A more specific tyre selection is then made from the required load per wheel using the available standard tyre sizes.
- The dimensions of the main gear oleo-pneumatic shock absorbers (L_{oleo}, D_{oleo}) are selected from the expected kinetic energy absorption during landing.
- Finally, the overall lengths and placements of the nose and main gear struts are decided from propeller/fuselage ground clearance and the available space for the retraction mechanism.

Step 1: Statistical Tire Sizing for a Business Twin

Given:

Table 14: Statistical Tire Sizing (Metric Units)

Aircraft Type	**Diameter**		**Width**	
	A	B	A	B
General aviation	5.1	0.349	2.3	0.312
Business twin	**8.3**	**0.251**	**3.5**	**0.216**
Transport/bomber	5.3	0.315	0.39	0.480
Jet fighter/trainer	5.1	0.302	0.36	0.467

Selecting the values of A and B for Business Twin from this table

Weight on Wheels: (Based on final converged MTOW(W) = 13,768.196 kg. However, Report 7,8,9 uses W_w based on an intermediate weight. We will proceed with the updated values as calculated in the updated report upon iterations.)

Nose gear take 15 % of total weight(fwd most case)

Main gear take 90 % of total weight (5 % is for margin)

$$\text{Nose wheel: } W_w = (15/2)\% \text{ of } W = \mathbf{1032.615} \text{ kg} = \mathbf{2271.753\ lb\ (each\ tyre)}$$

$$\text{Main wheel: } W_w = (90/4)\% \text{ of } W = \mathbf{3097.844} \text{ kg} = \mathbf{6815.25\ lb\ (each\ tyre)}$$

(We are using 2 tyres for Nose landing gear and 4 tyres for main landing gear; justification for the number and type of tyres has been shown below in this chapter.)

Calculated Dimensions:

$$\text{For Nose Wheel: } D = 8.3(1032.615)^{0.251} = 47.37\,\text{cm}, \quad W = 3.5(1032.615)^{0.216} = 15.67\,\text{cm}$$

$$\text{For Main Wheel: } D = 8.3(3097.844)^{0.251} = 62.42\,\text{cm}, \quad W = 3.5(3097.844)^{0.216} = 19.86\,\text{cm}$$

Step 2: Selecting a Standard Tire Size

For a business twin jet, **Type VII tire** is chosen owing to the increased take-off speed.

Calculation of Take-off Speed (V_{takeoff})
The take-off speed is estimated as $V_{\text{takeoff}} = 1.1 \times V_{\text{stall}}$. The stall speed ($V_{\text{stall}}$) is calculated using the maximum lift coefficient ($C_{L,max_{takeoff}} = 0.8 * Cl_{max}$) in the landing configuration.

- Wing Loading (W/S): 408.86 kg/m^2 (from final converged design)
- Air Density (ρ): 1.225 kg/m^3 (at Sea Level)
- Max Lift Coefficient ($C_{L,max}$): 2.75 (with double-slotted flaps)
- Max Lift Coefficient at takeoff ($C_{L,max_{takeoff}}$): $2.75 * 0.8 = 2.2$ (with double-slotted flaps)
- $W_{Landing} = MTOW - 0.94 \times Wfuel = 13768.190.94 \times 2993.206 = 10954.53\ kg$
- (As considering 6 % fuel at the time of landing.)
- S=33.674 m

1. Stall Speed for takeoff and landing configuration(V_{stall}):

$$V_{\text{stall takeoff}} = \sqrt{\frac{W/S}{0.5 \times \rho \times C_{L,max_{takeoff}}}} = \sqrt{\frac{408.86 \times 9.81 \text{ kg/m}^2}{0.5 \times 1.225 \text{ kg/m}^3 \times 2.2}}$$

$$V_{\text{stall takeoff}} = \sqrt{\frac{4010.91}{1.3475}} = \sqrt{2977.213} \approx \mathbf{54.564} \text{ m/s}$$

$$V_{\text{stall Landing}} = \sqrt{\frac{W_{landing}/S}{0.5 \times \rho \times C_{L,max}}} = \sqrt{\frac{325.311 \times 9.81 \text{ kg/m}^2}{0.5 \times 1.225 \text{ kg/m}^3 \times 2.75}}$$

$$V_{\text{stall Landing}} = \sqrt{1890.51} \approx \mathbf{43.48} \text{ m/s}$$

2. Take-off Speed (V_{takeoff}):

$$V_{\text{takeoff}} = 1.1 \times V_{\text{stall takeoff}} = 1.1 \times 54.564 \text{ m/s} = \mathbf{60.02} \text{ m/s}$$

3. Conversion to mph:

$$V_{\text{takeoff}} = 60.02 \text{ m/s} \times 2.23694 \text{ mph/m/s} \approx \mathbf{134.262} \text{ mph}$$

Nose Wheel

Tire size choice: 18 x 4.4

Table 15: Nose Wheel Tire Specifications (Type VII)

Parameter	Value
Max tyre pressure	225 psi
Max allowed speed	217 kt = 249.72 mph
Max allowed weight	4350 lb
Max width	4.45 in
Max diameter	17.90 in
Rolling radius	7.9 in
Wheel diameter	10 in
Number of plies	12

Main Wheel

Tire size choice: 24 x 5.5

Table 16: Main Wheel Tire Specifications (Type VII)

Parameter	Value
Max tire pressure	355 psi
Max allowed speed	174 kt = 200.24 mph
Max allowed weight	11500 lb
Max width	5.75 in
Max diameter	24.15 in
Rolling radius	10.6 in
Wheel diameter	14 in
Number of plies	16

Step 3: Tire Contact Area and Tire Pressure

The contact area A_p and required pressure P are calculated.

$$A_p = 2.3\sqrt{wd}\left(\frac{d}{2} - R_r\right)$$

$$P = \frac{W_w}{A_p}$$

Nose Wheel

$$A_p = 2.3 \times \sqrt{4.45 \times 17.90} \times (17.9/2 - 7.9) = \mathbf{21.55}\,\mathrm{in}^{\mathbf{2}} = 0.0139\ \mathrm{m}^2$$

$$P = \frac{1032.615\ \mathrm{kg} \times 9.81\ \mathrm{m/s}^2}{0.0139\ \mathrm{m}^2} = 728773.61\ \mathrm{Pa} = \mathbf{105.7}\ \mathrm{psi}$$

$$P = 105.7\ \mathrm{psi} < (max\ psi\ of\ chosen\ trye = 225\ psi)$$

$$Speed\ takeoff = 134.262\ \text{mph (calculated above)} < \text{max allowed tyre speed=249.72 mph}$$

Nose Weight(W_w) = 2271.753 lb (calculated above) < (Max allowed weight on each nose tyre= 4350 lb)

Main Wheel

$$A_p = 2.3 \times \sqrt{5.75 \times 24.15} \times (24.15/2 - 10.6) = \mathbf{39.9772}\,\mathbf{in^2} = 0.02579\ \text{m}^2$$

$$P = \frac{3097.844\ \text{kg} \times 9.81\ \text{m/s}^2}{0.02579\ \text{m}^2} = 1,178357.87\ \text{Pa} = \mathbf{170.906}\ \text{psi}$$

$$P = 170.906\ \text{psi} < (max\ psi\ of\ chosen\ trye = 355\ psi)$$

$$Speed\ takeoff = 134.262\ \text{mph (calculated above)} < \text{max allowed tyre speed=200.24 mph}$$

Main gear Weight(W_w) = 6815.25 lb (calculated above) < (Max allowed weight on each main tyre= 11500 lb)

Observation: Chosen tyres operate under their max allowed limits of weight, pressure and speeds.

Step 4: Calculate Main Gear Stroke (S)

The next step is to determine the required stroke (S) of the main gear's oleo-pneumatic shock absorber. This is calculated using Equation (11.12) from the lecture notes.

4a. Find the Tire Stroke (S_T)

First, we calculate the tire's own stroke (S_T) based on its geometry, which is the distance it can compress.

$$S_T = \frac{\text{Tire Diameter}}{2} - \text{Rolling Radius}$$

Using the main wheel tire specifications from Step 2:

$$S_T = \left(\frac{24.15\ \text{in}}{2}\right) - 10.6\ \text{in} = 1.475\ \text{in}$$

$$S_T = 1.475\ \text{in} \times 0.0254\ \text{m/in} = \mathbf{0.0374}\ \text{m}$$

4b. Find the Vertical Velocity (V_{vertical})

The stroke formula requires the vertical velocity (sink rate) at touchdown. While the lecture provides a formula based on a 3-degree glideslope ($V_{\text{vertical}} = 1.1 V_{\text{stall landing}} \sin 3°$).

$$V_{\text{vertical}} = 1.1 \times sin(3°) \times 43.48\ \text{m/s} = \mathbf{2.503}\ \text{m/s}$$

4c. Calculate Shock Absorber Stroke (S)

We can now calculate the required stroke (S) using the following parameters:

- $V_{\text{vertical}} = 2.503$ m/s
- $g = 9.81$ m/s^2
- $\eta = 0.72$ (Oleo efficiency, fixed orifice, avg. of 0.65-0.80)
- $N_{\text{gear}} = 3.0$ (Gear load factor for General Aviation/Commercial)
- $\eta_T = 0.47$ (Tire efficiency)

- $S_T = 0.0374$ m (Calculated above)

The formula for stroke is:

$$S = \frac{V_{\text{vertical}}^2}{2g\eta N_{\text{gear}}} - \frac{\eta_T}{\eta} S_T$$

Plugging in the values:

$$S = \frac{(2.503)^2}{2 \times 9.81 \times 0.72 \times 3.0} - \frac{0.47}{0.72} \times (0.0374)$$
$$S = 0.1234 \text{ m (or 12.34 cm)}$$

The recommended safety margin is about 3 cm and a minimum stroke of 20 cm. Our calculated value is 15.28 cm. We will add the safety margin to this value.

$$S_{\text{design}} = 12.34 \text{ cm} + 3.0 \text{ cm} = 15.34 \text{ cm} \ (< 20cm)$$

So, we will be using 8 inches of 'S', as mentioned in the lecture notes, as our requirements are still not meeting.

We will use a design stroke of $\mathbf{S} = \mathbf{0.2032}$ m (or **8** in)

Step 5: Calculate Main Gear Strut Length (L_{oleo})

The total length of the oleo strut (L_{oleo}) is estimated as a multiple of the calculated stroke S.

$$L_{\text{oleo}} = 2.5 \times S$$

$$L_{\text{oleo}} = 2.5 \times 0.2032 \text{ m} = \mathbf{0.508} \text{ m (or } \mathbf{20} \text{ in)}$$

$$L_{\text{ext length main gear}} = (20 + 24)\text{in} = \mathbf{44} \text{ in}$$

$$L_{\text{ext length nose gear}} = 80\% \times (L_{main\ gear\ length}) = 0.8 \times 44in = 35.2\ in$$

(80 % is chosen based on the appropriate takeoff angle of attack of the aircraft.)

$$L_{\text{oleo nose gear}} = 35.2 - 18 = 17.2\ in$$

Step 6: Calculate Main Gear Strut Diameter (D_{oleo})

The strut diameter is calculated based on the static load it must support (l_{oleo}) and the standard internal oleo pressure (P).

$$D_{\text{oleo}} = 1.3\sqrt{\frac{4 \times l_{\text{oleo}}}{P \times \pi}}$$

Using the following parameters:

- l_{oleo} = Load on one main strut = 3097.844 kg
- $l_{\text{oleo}} = 3097.844 \text{ kg} \times 9.81 \text{ m/s}^2 = \mathbf{30389.84}$ N
- $P = 1800$ psi
- $P = 1800 \text{ psi} \times 6894.76 \text{ Pa/psi} = \mathbf{12,410,568}$ Pa (N/m$^{\mathbf{2}}$)

Plugging in the values:

$$D_{\text{oleo}} = 1.3\sqrt{\frac{4 \times 30389.84 \text{ N}}{12,410,568 \text{ Pa} \times 3.14159}}$$

$$D_{\text{oleo}} = 1.3 \times 0.055 \text{ m}$$

$$D_{\text{oleo}} = 0.0725 \text{ m} = \mathbf{7.25} \text{ cm (or } \mathbf{2.85} \text{ in)}$$

Step 7: Calculate Nose Gear Strut Diameter ($D_{\text{oleo, nose}}$)

We repeat the diameter calculation for the nose gear. The lecture notes specify that the nose gear load (l_{oleo}) should include both static and braking loads.

The preliminary diameter based on static load is:

- $l_{\text{oleo, nose}}$ = Max Static load on nose strut = 1032.615 kg
- $l_{\text{oleo, nose}} = 1032.615 \text{ kg} \times 9.81 \text{ m/s}^2 = \mathbf{10129.95}$ N
- $P = 12,410,568$ Pa

$$D_{\text{oleo, nose}} = 1.3\sqrt{\frac{4 \times 10129.95 \text{ N}}{12,410,568 \text{ Pa} \times 3.14159}}$$

$$D_{\text{oleo, nose}} = 1.3 \times 0.0322 \text{ m}$$

$$D_{\text{oleo, nose}} = 0.0418 \text{ m} = \mathbf{4.18} \text{ cm (or } \mathbf{1.65} \text{ in)}$$

Summary of Landing Gear Sizing

Summary of Calculated Landing Gear Parameters

Parameter	**Main Gear**	**Nose Gear**
Tire Size Selected	24 × 5.5 (Type VII)	18 × 4.4 (Type VII)
Static Load per Tyre	3097.844 kg	1032.615 kg
Calculated Stroke (S)	0.2032 m (20.32 cm)	N/A (Main gear is design-driving)
Strut Length (L_{oleo})	0.508 m (20 in)	0.4368 m(17.2 in)
Strut Diameter (D_{oleo})	**7.25** cm	**4.18** cm

Position of Landing gear :

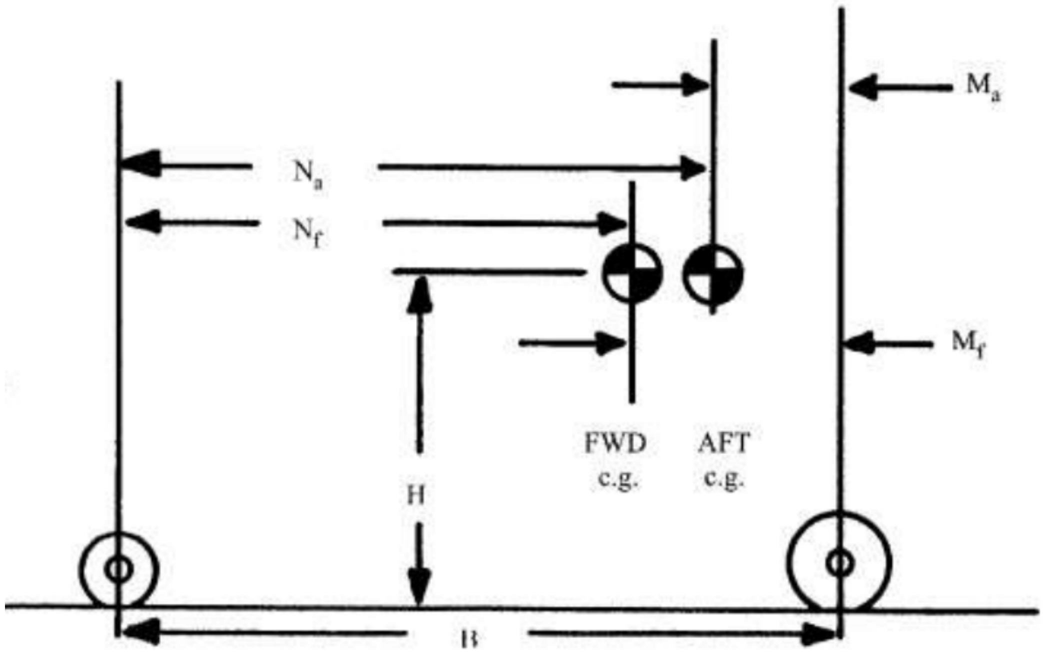


Figure 13: Wheel load geometry for landing gear positioning

The landing gear position will be decided based on the forward-most and the aft-most CG locations. To ensure that nose gear is not carrying too low or too much load, the safe assumption of parameters:

$$\frac{M_a}{B} \approx 0.08$$

$$\frac{M_f}{B} = 0.15$$

where,

- B = Wheel Base
- M_a = Distance of main wheel from aft most CG
- M_f = Distance of main wheel from fwd most CG
- $xcg\ fwd = 8.208\ m\ (from\ report\ 9)$
- $xcg\ aft = 8.602\ m\ (from\ report\ 9)$

Calculating Position of Landing Gears: Let

$$M_a = x_{\text{main}} - x_{\text{aftcg}}$$
$$M_f = x_{\text{main}} - x_{\text{fwdcg}}$$
$$B = x_{\text{main}} - x_{\text{nose}}$$

$$\implies \frac{x_{\text{main}} - x_{\text{aftcg}}}{x_{\text{main}} - x_{\text{nose}}} = 0.08$$

and

$$\frac{x_{\text{main}} - x_{\text{fwdcg}}}{x_{\text{main}} - x_{\text{nose}}} = 0.15$$

We get a system of equations:

$$0.85x_{\text{main}} + 0.15x_{\text{nose}} = x_{\text{fwd}}$$

$$0.92x_{\text{main}} + 0.08x_{\text{nose}} = x_{\text{aft}}$$

On solving Equation 1 & 2 we get,

$$x_{\text{main}} = 9.052\,\text{m}$$

$$x_{\text{nose}} = 3.425\,\text{m}$$

B=9.025 - 3.425 = 5.626 m

Calculation of H (Vertical C.G.) for FWD Case

This section details the calculation of the vertical center of gravity (H or Y_{CG}) for the **FWD C.G. case**. This condition is defined as: **Total Empty Weight + Crew + 6% Fuel**.

The calculation is based on the component data from **Table 17** (page 59) of the source report, which has been converted from Imperial to SI units (1 lb = 0.453592 kg; 1 ft = 0.3048 m).

The reference datum ($Y = 0$) for all vertical measurements is the **Static ground line**.

$$H = Y_{CG} = \frac{\sum(W_i \times Y_{CG,i})}{\sum W_i} = \frac{\text{Total Vertical Moment [kg-m]}}{\text{Total Weight [kg]}}$$

Step 1: Component Data for FWD C.G. Case

The weights and vertical C.G. locations for this case are derived from Table 17.

Component	**Weight (W_i) [kg]**	**Vertical C.G. ($Y_{CG,i}$) [m]**
— *Empty Weight Components* —		
Fuselage	1877.668	1.969
Wing	1351.915	1.2
Horizontal Tail (HT)	337.115	5.702
Vertical Tail (VT)	295.965	4.660
Engines (Total)	1176.5	3.03
Landing Gear	607.7	0.56
All Else Empty	2340.39	1.715
Total Empty Weight	**7984.924**	
— *Operating Items* —		
Crew	200.56	2.075
Fuel (6% Reserve)	179.58	1.2

Table 17: Component weights for FWD C.G. case, from Table 17 data.

Step 2: Calculate Total Vertical Moment [kg-m]

The vertical moment ($W_i \times Y_{CG,i}$) is calculated for each component.

- **Fuselage:** 1877.668 kg × 1.969 m = 3697.58 kg-m
- **Wing:** 1351.915 kg × 1.2 m = 1622.30 kg-m
- **HT:** 337.115 kg × 5.702 m = 1922.23 kg-m
- **VT:** 295.965 kg × 4.660 m = 1379.10 kg-m
- **Engines:** 1176.5 kg × 3.03 m = 3564.80 kg-m
- **Landing Gear:** 607.7 kg × 0.56 m = 340.31 kg-m
- **All Else Empty:** 2340.39 kg × 1.715 m = 4013.77 kg-m
- **Crew:** 216.0 kg × 2.075 m = 416.162 kg-m
- **Fuel (6%):** 179.58 kg × 1.2 m = 215.50 kg-m

Total Vertical Moment ($\sum W_i Y_{CG,i}$):
$3697.58 + 1622.30 + 1922.23 + 1379.10 + 3564.80 + 340.31 + 4013.77 + 416.162 + 215.50 = \mathbf{17171.16}$ kg-m

Step 3: Calculate Total Weight [kg]

Total Weight ($\sum W_i$):
7987.253 (Empty) + 200.56 (Crew) + 179.58 (Fuel) = **8366.833** kg

Step 4: Final H ($\mathbf{Y_{CG}}$) Calculation

$$H_{\text{fwd}} = \frac{\text{Total Vertical Moment}}{\text{Total Weight}} = \frac{17171.16\text{ kg-m}}{8366.833\text{ kg}}$$
$$\mathbf{H}_{\text{fwd}} = \mathbf{2.052}\text{ m}$$

Sideways Turnover Angle

Forces acting sideways on the airplane in cross-wind landing condition or a high-speed turn during taxiing could cause the aircraft to turnover on its side. It is thus desirable to keep the turnover angle (ψ) as small as possible. The angle is determined using the expression

$$\tan\psi = \frac{h_{cg}}{l_n \sin\delta} \tag{3.4}$$

where

$$\tan\delta = \frac{t}{2(l_m + l_n)} \tag{3.5}$$

and δ is defined as the angle between the aircraft centerline and the line connecting the center of the nose and main assembly. The dimensions used in the above equations are given in Fig. 3.3.

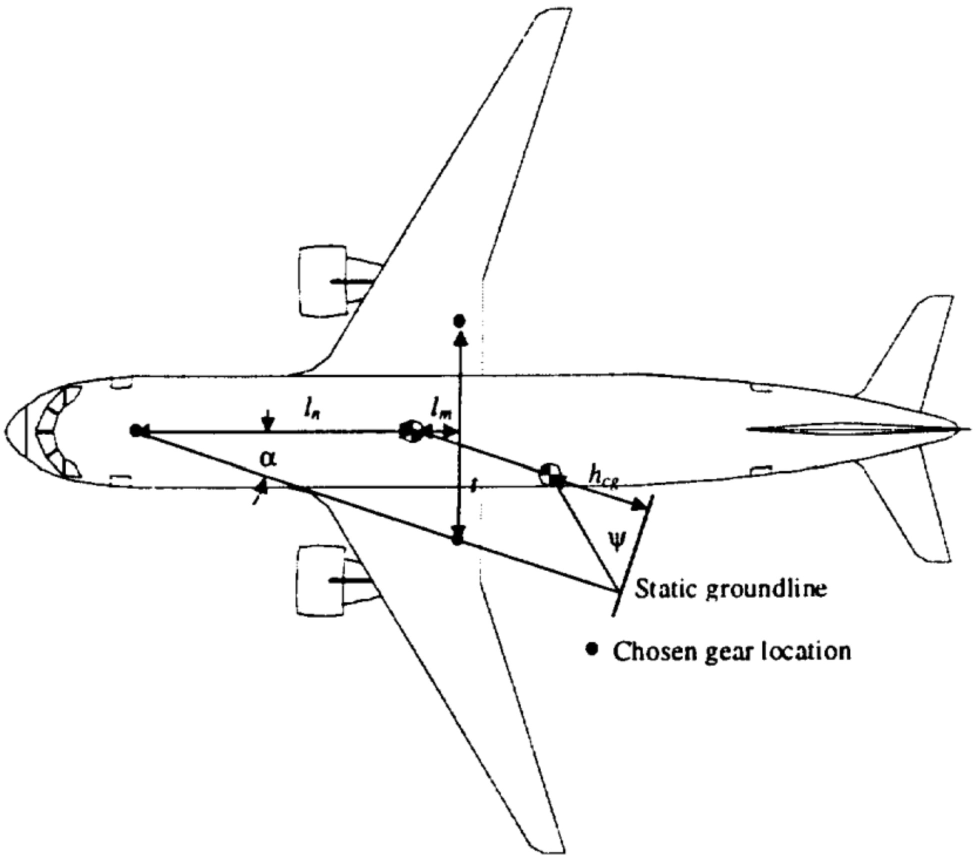


Figure 14: Turnover angle calculation [2]

For land-based aircraft, either the maximum allowable overturn angle of 63 degrees [2] or the stability considerations at takeoff and touchdown and during taxiing, whichever is the most critical, determines the lower limit for the track of the main landing gear.

Method to Calculate Overturn Angle

To calculate the overturn angle ψ, follow these steps:

1. Calculate angle δ using Equation (3.5):

$$\delta = \arctan\left(\frac{t}{2(l_m + l_n)}\right)$$

   where:

   - t = track width (distance between main gears)
   - l_m = distance from main gear to CG
   - l_n = distance from nose gear to CG

2. Calculate the turnover angle ψ using Equation (3.4):

$$\psi = \arctan\left(\frac{h_{cg}}{l_n \sin\delta}\right)$$

   where:

   - h_{cg} = height of center of gravity from ground

Given Parameters

- Height of center of gravity, $h_{cg} = 2.052\,\text{m}$
- Distance from main gear to CG, $l_m = x_{\text{main}} - x_{\text{fwd}} = 9.052\,\text{m} - 8.208\,\text{m} = 0.844\,\text{m}$
- Distance from nose gear to CG, $l_n = x_{\text{fwdcg}} - x_{\text{nose}} = 8.208\,\text{m} - 3.425\,\text{m} = 4.783\,\text{m}$
- Track width, $t = 2.77\,\text{m}$

Calculation Procedure

1. **Calculate angle δ using Equation (3.5):**

$$\tan\delta = \frac{t}{2(l_m + l_n)}$$

$$\tan\delta = \frac{2.77}{2(0.844 + 4.783)} = \frac{2.77}{2 \times 5.627} = \frac{2.77}{11.254} = 0.2462$$
$$\delta = \arctan(0.2462) = 13.82^\circ$$

2. **Calculate the turnover angle ψ using Equation (3.4):**

$$\tan\psi = \frac{h_{cg}}{l_n \sin\delta}$$

First, calculate $\sin\delta$:
$$\sin\delta = \sin(13.82^\circ) = 0.2389$$

Now calculate the numerator:

$$l_n \sin\delta = 4.783 \times 0.2389 = 1.142\,\text{m}$$

Finally, calculate $\tan\psi$:
$$\tan\psi = \frac{2.052}{1.142} = 1.79$$
$$\psi = \arctan(1.79) = 60.9^\circ$$

3. **Safety Verification:**

   - Calculated overturn angle: $\psi = 60.9^\circ$
   - Maximum allowable overturn angle: $\psi_{\text{max}} = 63^\circ$
   - Safety margin: $\Delta\psi = 63^\circ - 60.9^\circ = 2.09^\circ$

Conclusion

The calculated overturn angle of 60.9° is less than the maximum allowable value of 63°, indicating that the landing gear configuration is safe against sideways turnover. The safety margin of 2.09° provides adequate protection against turnover during cross-wind landings or high-speed taxi turns.

Landing gear storage

This h is the distance from the centerline of the main wheel landing gear when open of the fuselage. The diameter of the fuselage is 1.91 m. Therefore, the main landing gear when in open configuration will be directly under the fuselage when open. Therefore, for our aircraft the main landing gear should retract into the fuselage. The nose landing gear will also retract into the fuselage.

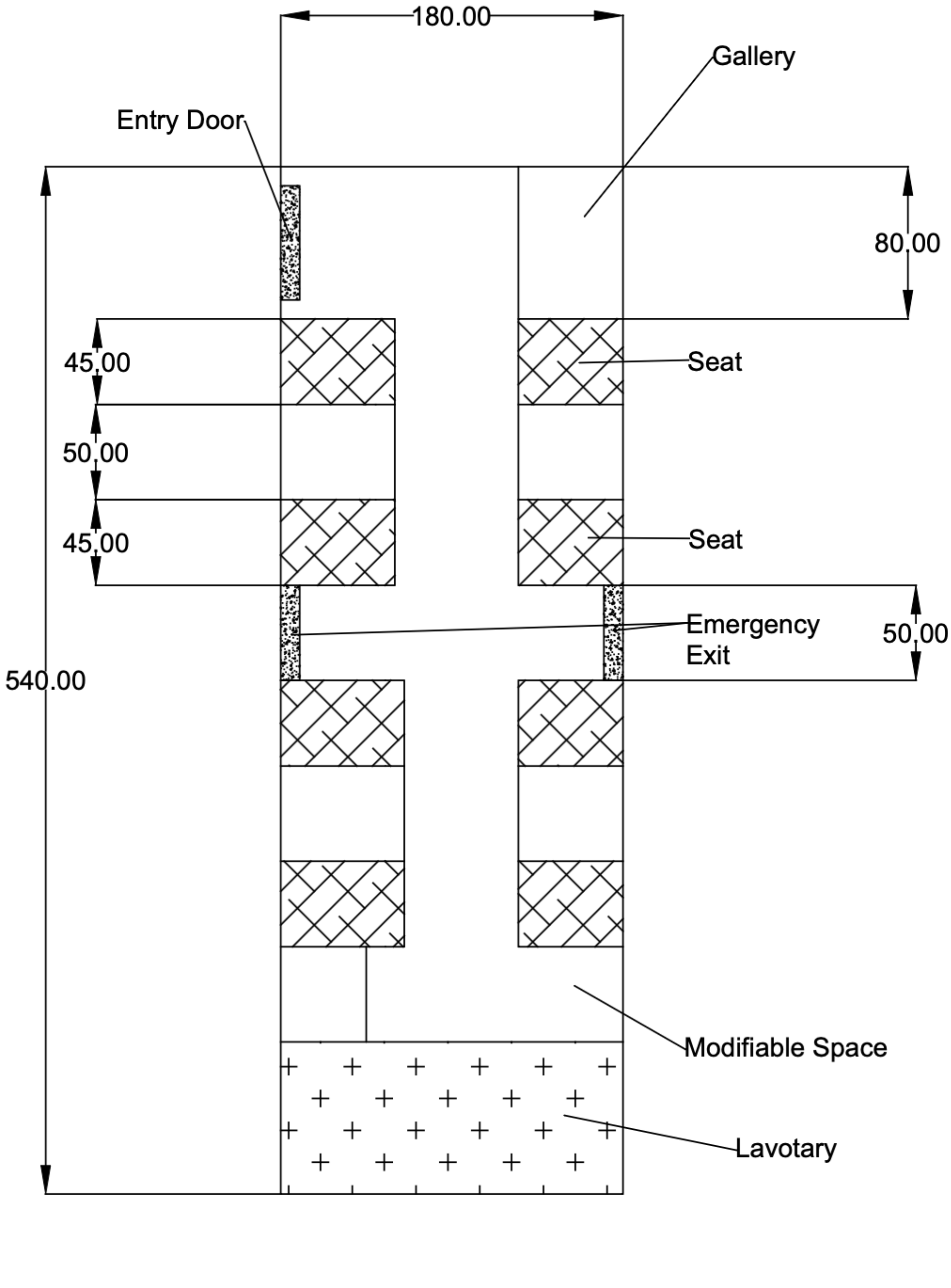


Figure 15: Cabin top view

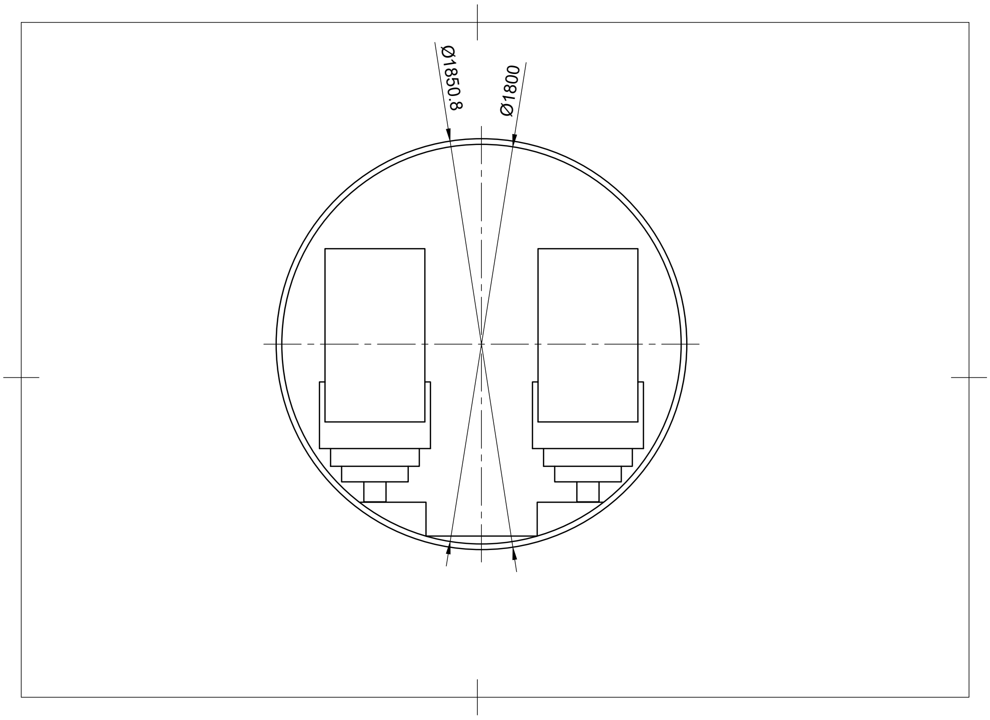


Figure 16: Cabin Front View and seating

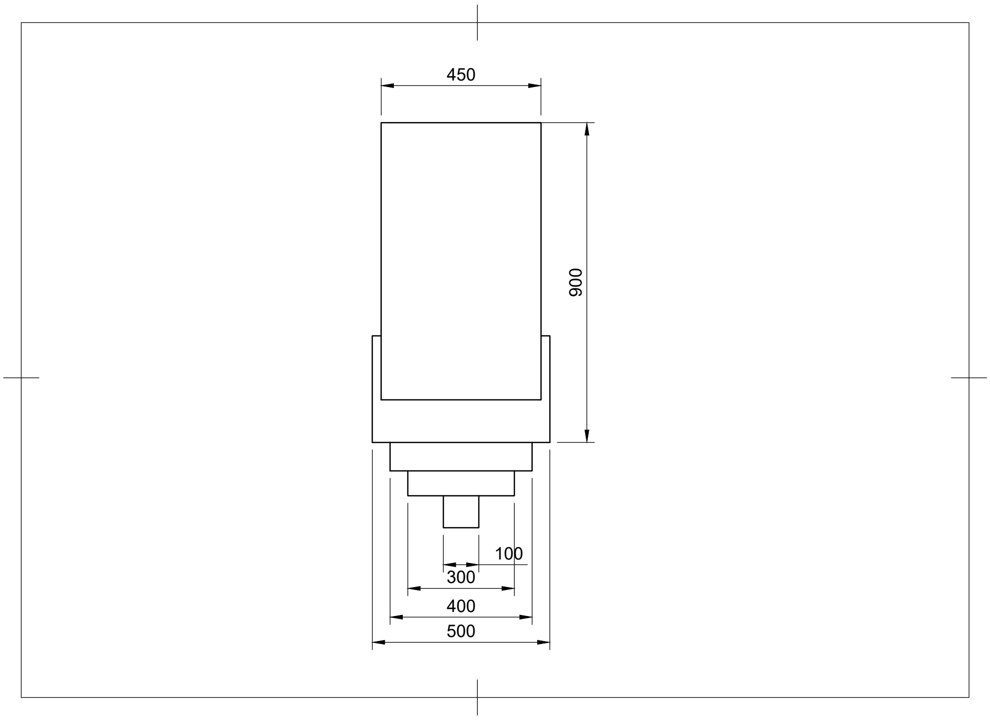


Figure 17: Seat Design of the aircraft

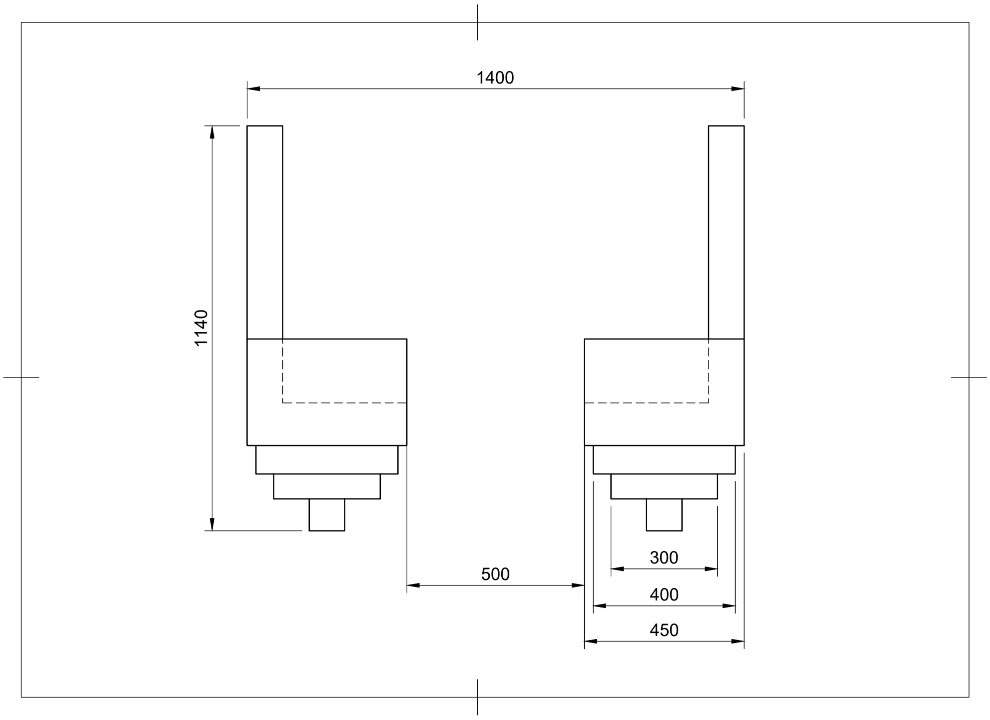


Figure 18: Aircraft club-seating arrangement

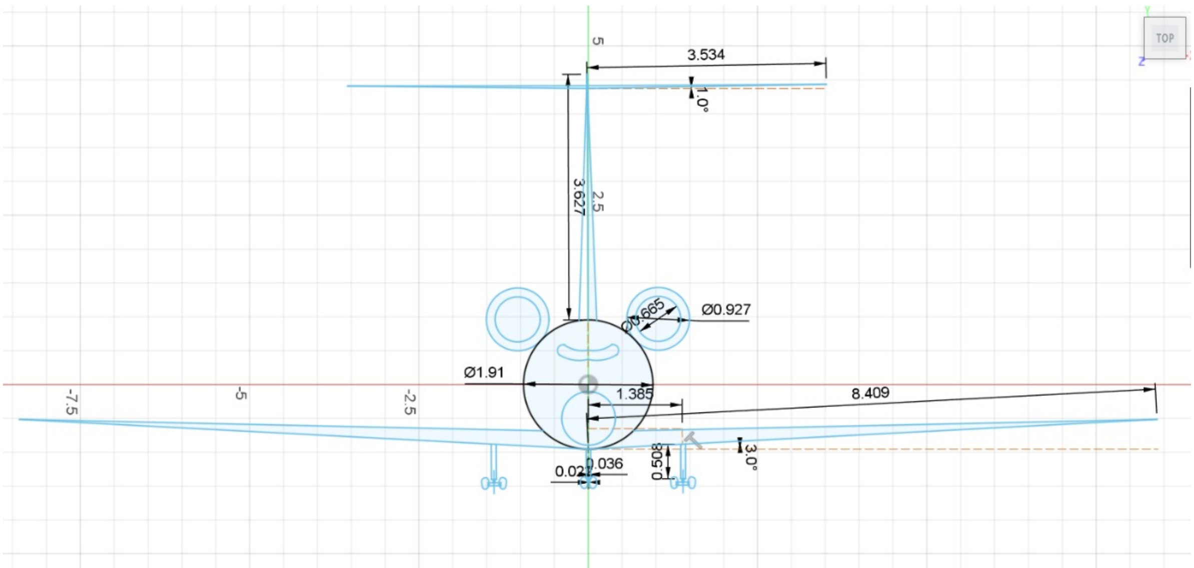


Figure 19: Aircraft front view

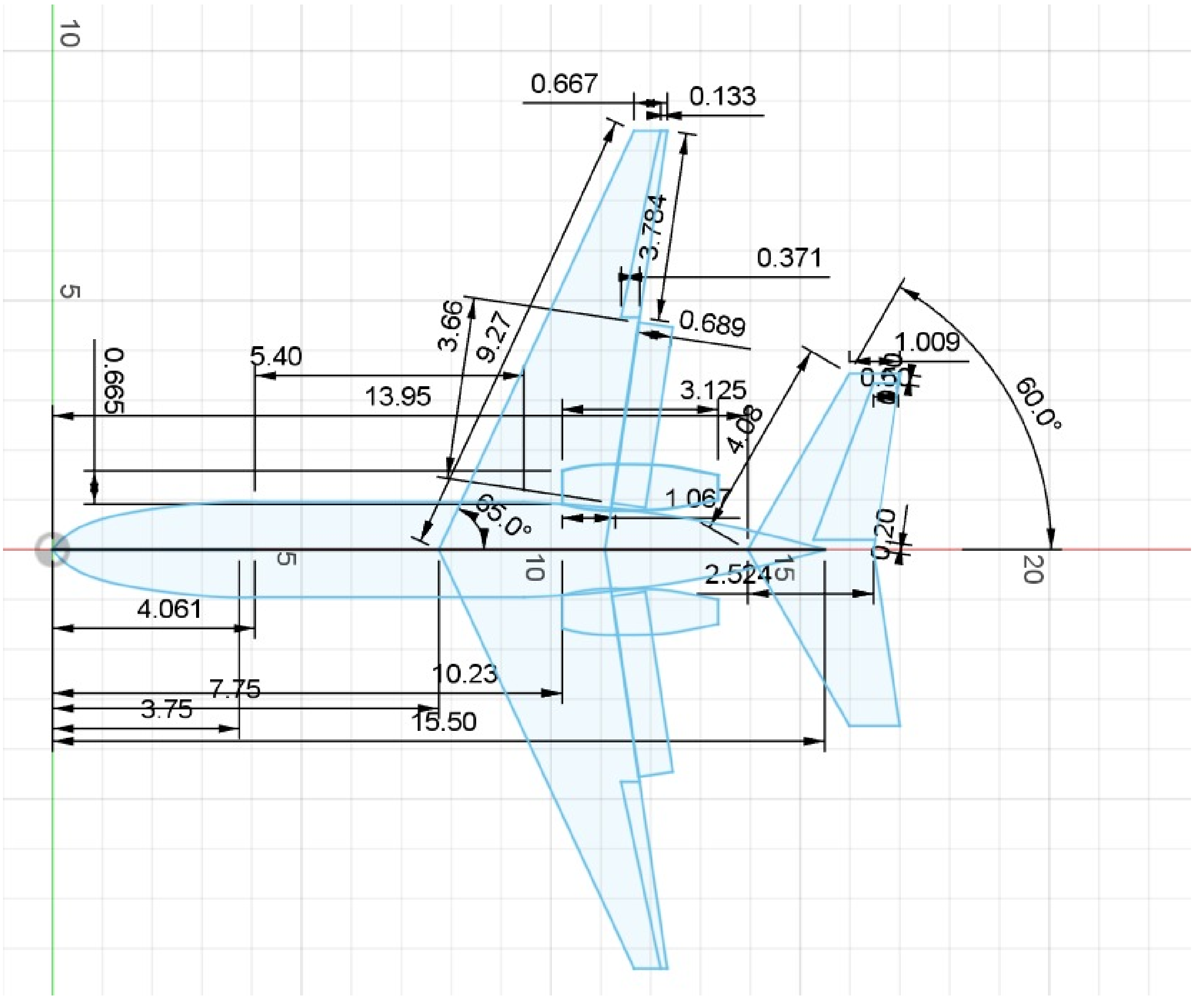


Figure 20: Aircraft top view

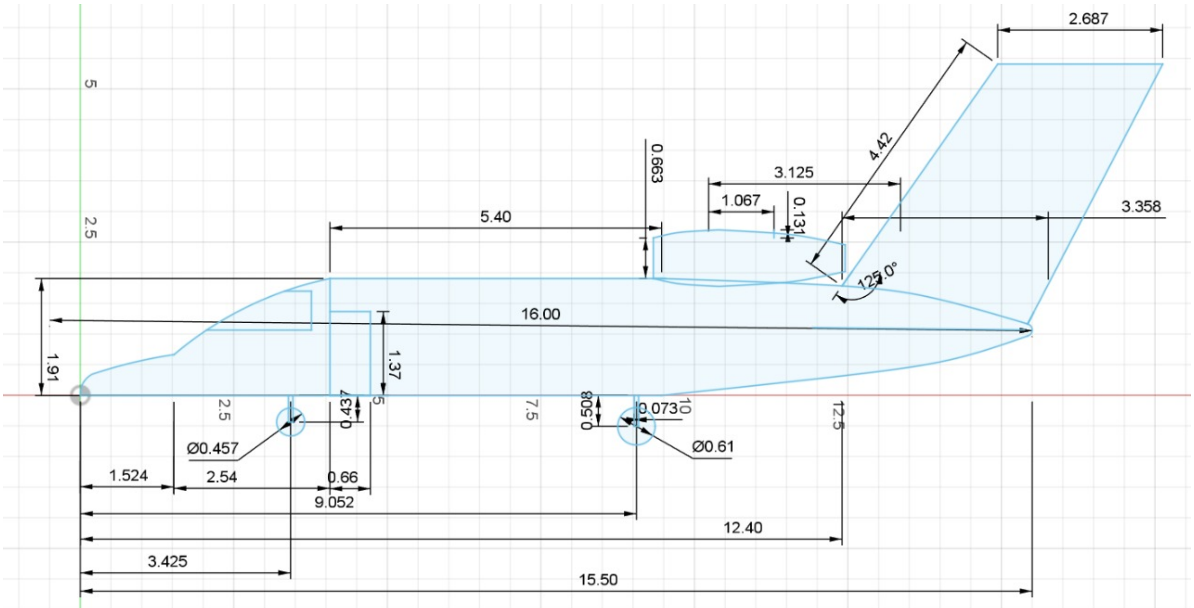


Figure 21: Aircraft side view

CHAPTER 11

Lift and Drag Estimation

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

List of parameter values required in this chapter for calculations:

- $V_{stall} = \sqrt{2W_0/\rho S C_{L_{max}}} = 48.79\ m/s$
- $C_{l_\alpha} = 0.1975/deg = 5.66/rad$ for NACA 65(2)-415 airfoil
- $C_{l_{max}} = 1.62$ for NACA 65(2)-415 airfoil
- $\alpha_{0_L} = -2.2°$ for NACA 65(2)-415 airfoil
- $A = 8.4$
- $\Lambda_{c/4} = 20°$
- $\Lambda_{LE} = 25°$
- $(t/c)_{max} = 0.15$
- $(x/c)_{max} = 0.40$
- $\Lambda_{max,t} = 18.714°$
- *No high-lift devices such as flaps or slats deployed during cruise*
- *Addition of winglets or end-plates is not required as the wing already has a high-aspect ratio.*

Lift Estimation During Cruise

List of parameter values required for cruise calculations:

- $\rho = 0.3639$ kg/m^3
- $V = 830$ km/h $= 230.55$ m/s
- $M = 0.7814$
- $Re \approx 1.12 \times 10^7$

Lift-Curve Slope During Cruise

The theoretical estimation of the lift-curve slope given the airfoil and Mach number is given as

$$C_{L_\alpha} = \frac{2\pi}{\sqrt{1-M^2}} = \frac{2\pi}{\sqrt{1-0.7814^2}}$$
$$\implies C_{L_\alpha} = 10.068/rad = 0.1757/deg$$

Maximum Lift Coefficient During Cruise

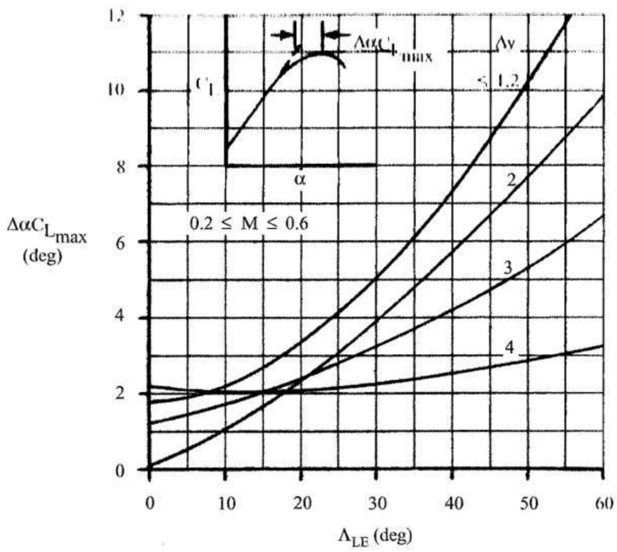
The maximum value of the lift coefficient is given by the formula as

$$\begin{aligned} C_{Lmax} &= 0.9 C_{l_{max}} cos\Lambda_{c/4} \\ &= 0.9 \times 1.62 \times cos(20°) \\ &= 1.37 \end{aligned}$$

The value of Δy for our airfoil is estimated to be

$$\Delta y = 19.3\ t/c = 19.3 \times 0.15 = 2.895$$

For a high aspect ratio wing such as ours, the correction in the maximum lift coefficient is given using Fig. 12.9 for the case $\Lambda_{LE} = 25$ as Therefore, using the value $\Delta y = 2.895$ in the above figure, we have



$$\boxed{\Delta C_{L_{max}} = -0.3075}$$

The maximum lift coefficient after applying the high–aspect–ratio and sweep corrections (Eq. 12.9) was obtained as

$$\boxed{C_{L_{\max,\Lambda}} = 1.0625}$$

This value corresponds to the low-Mach / incompressible reference condition, which is representative of the region near

$$M \approx 0.5.$$

To obtain the appropriate maximum lift coefficient at the cruise Mach number

$$M = 0.7814,$$

the compressibility correction factor

$$\left(\frac{C_{L_{\max}(M)}}{C_{L_{\max}(0.5)}}\right)_{M=0.7814} \approx 0.9$$

has to be multiplied to the $C_{L_{\max}}$ under the low-Mach number condition.

Therefore, the final corrected value of the clean–wing maximum lift coefficient at the cruise Mach number is

$$C_{L_{\max,\text{cruise}}} = C_{L_{\max,\Lambda}} \left(\frac{C_{L_{\max}(M)}}{C_{L_{\max}(0)}}\right)_{M=0.7814} = 1.0625 \times 0.9 = 0.95625.$$

$$\boxed{\Rightarrow\ C_{L_{\max,\text{cruise}}} = 0.95625}$$

This is the value that will be used throughout the V–n diagram and structural load analysis in Chapter 12.

Maximum Lift Angle of Attack During Cruise

The maximum lift angle of attack during cruise for a high aspect ratio wing is given as

$$\alpha_{C_{L_{max}}} = \frac{C_{L_{max}}}{C_{L_\alpha}} + \alpha_{0_L} + \Delta\alpha CL_{max}$$

Here, the correction due to use of high aspect ratio wings is given by Fig. 12.10.
Therefore, the angle of attack during cruise for the maximum lift coefficient is given as

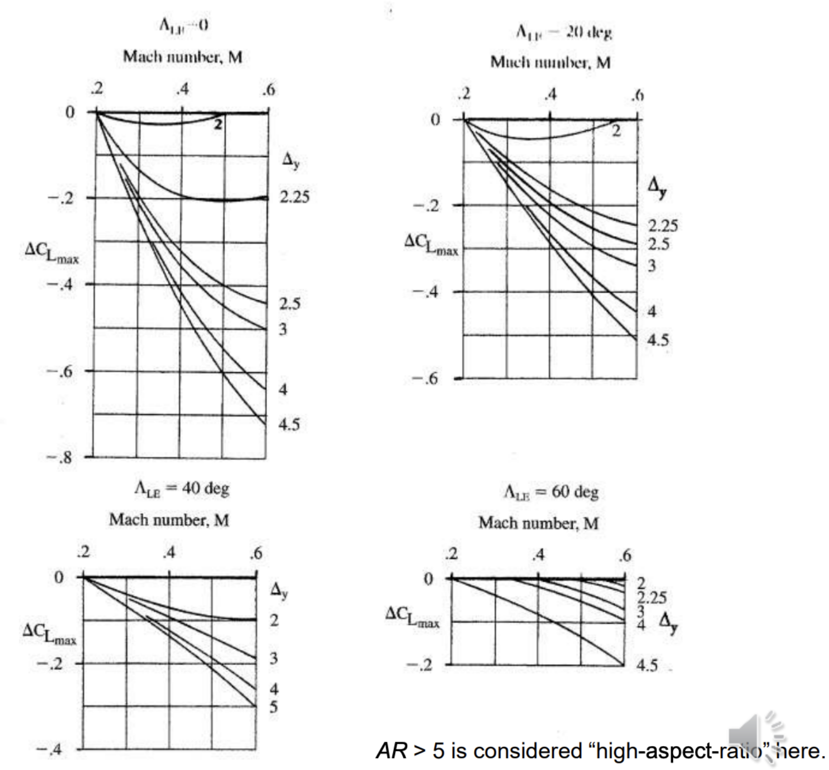


Fig. 12.9 Mach-number correction for subsonic maximum lift of high-aspect-ratio wings (Ref. 37).

$$\Delta\alpha_{C_{L_{max}}} \approx 5.5°$$

Therefore,

$$\alpha_{C_{L_{max}}} = \frac{1.0625}{0.0855} - 2.2 + 5.5$$
$$\implies \alpha_{C_{L_{max}}} = 15.7769°$$

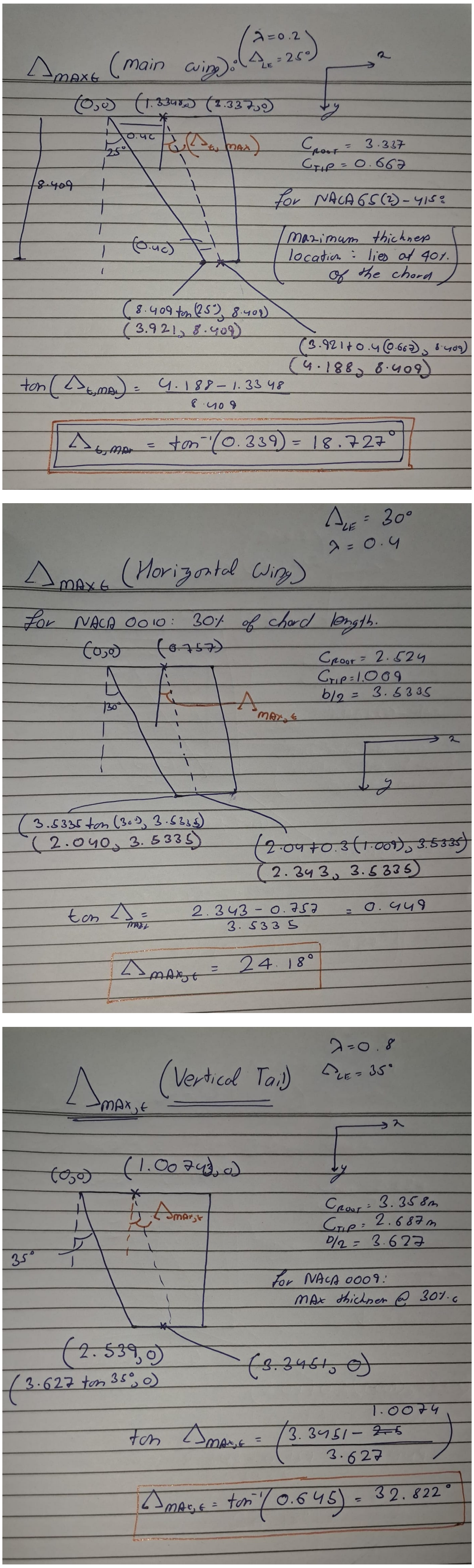


Figure 22: Calculation of sweep angle at max thickness for wing, horizontal tail and vertical tail.

Lift Estimation During Takeoff

List of parameter values required for takeoff calculations:

- $\rho = 1.225$ kg/m^3
- $V = 1.1V_{stall} = 1.1 \times 48.79 = 53.669$ m/s
- $M = 0.156$
- $Re \approx 7.46 \times 10^7$

Lift-Curve Slope During Takeoff

The theoretical estimation of the lift-curve slope given the airfoil and Mach number is given as

$$C_{L_\alpha} = \frac{2\pi}{\sqrt{1-M^2}} = \frac{2\pi}{\sqrt{1-0.156^2}}$$
$$\implies C_{L_\alpha} = 6.361/rad = 0.1110/deg$$

Maximum Lift Coefficient During Takeoff

The maximum value of the lift coefficient is given by the formula as

$$\begin{aligned} C_{Lmax} &= 0.9C_{l_{max}} cos\Lambda_{c/4} \\ &= 0.9 \times 1.62 \times cos(20^\circ) \\ &= 1.37 \end{aligned}$$

The value of $\Delta y = 2.895$ is the same as used in cruise calculation in Section 11.1. For the value of change in maximum lift due to high aspect ratio of the wing, the Fig. 12.9 can be referred from Section 11.1 and the value for $M = 0.13$ can be found as

$$\boxed{(\Delta C_{L_{max}})_{High\ AR} = 0}$$

During takeoff, high-lift devices are deployed. Our wings Has only Double Slotted Flap. Double Slotted Flap have a chord length of $0.3c$ and cover 50% of the wing span.

Lift Increment Due to Flaps During Takeoff

The lift increment due to Fowler flaps is given by

$$\Delta C_{L_{max}} = 0.9\Delta C_{l_{max}} \left(\frac{S_{flapped}}{S_{wing}}\right) cos\Lambda_{HL}$$

Here, the values used in the above expression are

$$\Delta C_{l_{max}} = 1.6\frac{c'}{c} = 1.6 \times 1.3 = 2.08$$
$$\frac{S_{flapped}}{S_{wing}} \approx 0.5$$
$$\Lambda_{HL} = tan^{-1}\left(tan(25^\circ) - \frac{(0.70)(C_{root} - C_{tip})}{b/2}\right) \;=\; 13.59^\circ$$

Therefore, the lift increment due to Fowler flaps during takeoff is

$$\Longrightarrow \boxed{(\Delta C_{L_{max}})_{flaps} = 0.9097}$$

Therefore, the maximum lift coefficient during takeoff is

$$\begin{aligned}\Longrightarrow (C_{L_{max}})takeoff &= (CL_{max})wing + (\Delta CL_{max})High\ AR + (\Delta CL_{max})flaps \\ &= 1.06443 + 0 + 0.9097 \\ &= 1.9741\end{aligned}$$

Maximum Lift Angle of Attack During Takeoff

The maximum lift angle of attack during takeoff for a high aspect ratio wing is given as

$$\alpha_{C_{L_{max}}} = \frac{C_{L_{max}}}{C_{L_\alpha}} + \alpha_{0_L} + \Delta\alpha CL_{max}$$

Here, the correction due to use of high aspect ratio wings according to Fig. 12.10 in Section 11.1 is

$$(\Delta\alpha_{C_{L_{max}}})_{High\ AR} \approx 2.3^\circ$$

Change in Zero-Lift Angle of Attack Due to Flaps During Takeoff

The change in the zero-lift angle of attack for takeoff due to deployment of flaps and slats is given as

$$\Delta\alpha_{0_L} = (\Delta\alpha_{0_L})_{airfoil}\left(\frac{Sflapped}{S_{wing}}\right)cos\Lambda_{HL}$$

Here, the change in zero-lift angle due to flaps during takeoff is approximately 10°. Therefore,

$$\begin{aligned}(\Delta\alpha_{0_L})_{flaps} &= -18.2 \times 0.5 \times cos(-13.59^\circ) \\ \Longrightarrow (\Delta\alpha_{0_L})_{flaps} &= -8.845^\circ\end{aligned}$$

Therefore, the zero-lift angle of attack during takeoff is

$$\begin{aligned}\Delta\alpha_{0_L} &= (\Delta\alpha_{0_L})wing + (\Delta\alpha 0_L)_{flaps} \\ \Delta\alpha_{0_L} &= -2.2^\circ - 8.845^\circ \\ \Longrightarrow (\Delta\alpha_{0_L})_{takeoff} &= -11.045^\circ\end{aligned}$$

Therefore, the maximum-lift angle of attack during takeoff is

$$\begin{aligned}
\implies (\alpha_{C_{L_{max}}})takeoff &= \frac{CL_{max}}{C_{L_\alpha}} + \alpha_{0_L} + \Delta\alpha CL_{max} \\
&= \frac{1.0625}{0.0855} - 6.4884 + 2.3 \\
&= 8.2385^\circ
\end{aligned}$$

Lift Estimation During Landing

List of parameter values required for landing calculations:

- $\rho = 1.225 \text{ kg/m}^3$
- $V = 1.3V_{stall} = 1.3 \times 48.79 = 63.427 \text{ m/s}$
- $M = 0.1847$
- $Re \approx 3.3 \times 10^6$

Lift-Curve Slope During Landing

The theoretical estimation of the lift-curve slope given the airfoil and Mach number is given as

$$\begin{aligned}
C_{L_\alpha} &= \frac{2\pi}{\sqrt{1-M^2}} = \frac{2\pi}{\sqrt{1-0.154^2}} \\
\implies C_{L_\alpha} &= 6.3613/rad = 0.111/deg
\end{aligned}$$

Using the semi-empirical formula for the subsonic lift-curve slope which yields more accurate results,

$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{4 + \frac{A^2\beta^2}{\eta^2}\left(1 + \frac{tan^2(\Lambda_{max,t})}{\beta^2}\right)}} \left(\frac{S_{exposed}}{S_{wing}}\right) F$$

In the above formula, the values used are same as for cruise except for

$$\beta = \sqrt{1-M^2} = 0.9877$$

Therefore, the lift-curve slope is

$$C_{L_\alpha} = 2.767/rad = 0.0482/deg$$

Maximum Lift Coefficient During Landing

The maximum value of the lift coefficient is given by the formula as

$$\begin{aligned}
C_{Lmax} &= 0.9C_{l_{max}} cos\Lambda_{c/4} \\
&= 0.9 \times 1.1827 \times cos(20^\circ) \\
&= 1.06443
\end{aligned}$$

The value of $\Delta y = 2.895$ is the same as used in cruise calculation in Section 11.1. The change in maximum lift due to high aspect ratio of wings is given using Fig. 12.9 for the value $M = 0.154$ as

$$(\Delta C_{L_{max}})_{High\ AR} = 0$$

During landing, high-lift devices are deployed at higher deflection angles compared to takeoff because the requirement of lift is slightly higher and the drag required is much higher. As stated before in Section 11.2.2, our wings have Fowler flaps and leading edge slats. The Fowler flaps have a chord length of $0.3c$ and cover 50% of the wing span. The slats have a chord length of $0.15c$ and cover 90% of the wing span. The lift increment for landing is approximately the same as calculated for takeoff in Section 11.2.2.

Therefore, the lift increment due to Fowler flaps and slats during landing are

$$\implies (\Delta C_{L_{max}})_{flaps} = 0.4085$$

$$\implies (\Delta C_{L_{max}})_{slats} = 0.2754$$

Therefore, the maximum lift coefficient during takeoff is

$$\begin{aligned}\implies (C_{L_{max}})landing &= (CL_{max})wing + (\Delta CL_{max})High\ AR + (\Delta CL_{max})flaps + (\Delta CL_{max})_{slats} \\ &= 1.06443 - 0 + 0.4085 + 0.2754 \\ &= 1.74833\end{aligned}$$

Maximum Lift Angle of Attack During Takeoff

The maximum lift angle of attack during takeoff for a high aspect ratio wing is given as

$$\alpha_{C_{L_{max}}} = \frac{C_{L_{max}}}{C_{L_\alpha}} + \alpha_{0_L} + \Delta\alpha CL_{max}$$

Here, the correction due to use of high aspect ratio wings according to Fig. 12.10 in Section 11.1 is

$$(\Delta\alpha_{C_{L_{max}}})_{High\ AR} \approx 2.3^\circ$$

The change in zero-lift angle of attack due to deployment of flaps is approximately the same during landing as was during takeoff. Therefore, as calculated in Section 11.2.3,

$$\implies (\Delta\alpha_{0_L})_{flaps} = -4.9884^\circ$$
$$\implies (\Delta\alpha_{C_{L_{max}}})_{landing} = -1.5 - 4.9884 = -6.4884^\circ$$

Therefore, the maximum-lift angle of attack during landing is

$$\begin{aligned}\implies (\alpha_{C_{L_{max}}})takeoff &= \frac{CL_{max}}{C_{L_\alpha}} + \alpha_{0_L} + \Delta\alpha CL_{max} \\ &= \frac{1.74833}{0.1072} - 6.4884 + 2.3 \\ &= 12.1206^\circ\end{aligned}$$

Parasitic Drag Estimation During Cruise

Wing

The Reynolds number for the wing is given as

$$\begin{aligned} R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{0.3639 \times 230.55 \times 2.298}{1.42 \times 10^{-5}} \\ &= 13577157.69 \end{aligned}$$

The cutoff Reynolds number for the wing is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{2.298}{0.634 \times 10^{-5}})^{1.053} \\ &= 27294718.824 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{13577157.69}} \\ &= 0.0003604 \end{aligned}$$

The form factor for the wing is

$$\begin{aligned} FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.4}(0.15) + 100(0.15)^4\right]\left[1.34(0.7814)^{0.18}(cos(14.3^\circ))^{0.28}\right] \\ &= 1.62 \end{aligned}$$

$$Q = 1.0$$

Therefore, the component of parasitic drag due to the wing is

$$\begin{aligned} (C_{D_0})wing &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003604 \times 1.62 \times 1.0 \times 57.002}{33.674} \\ &= 0.000988 \end{aligned}$$

Horizontal Tail

The Reynolds number for the horizontal tail is given as

$$\begin{aligned} R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{0.3639 \times 230.55 \times 1.874}{1.42 \times 10^{-5}} \\ &= 11072059.84 \end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{1.874}{0.634 \times 10^{-5}})^{1.053} \\ &= 22019295.70 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{11072059.84}} \\ &= 0.0003991 \end{aligned}$$

The form factor for the horizontal tail is

$$\begin{aligned} FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.30}(0.10) + 100(0.10)^4\right]\left[1.34(0.7814)^{0.18}(cos(11.55^\circ))^{0.28}\right] \\ &= 1.5445 \end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the horizontal tail is

$$\begin{aligned} (C_{D_0})HT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003991 \times 1.5445 \times 1.05 \times 25.365}{33.674} \\ &= 0.000489 \end{aligned}$$

Vertical Tail

The Reynolds number for the vertical tail is given as

$$\begin{aligned} R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{0.3639 \times 230.55 \times 3.035}{1.42 \times 10^{-5}} \\ &= 17931537.6813 \end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{3.035}{0.634 \times 10^{-5}})^{1.053} \\ &= 36583912.9854 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{17931537.6813}} \\ &= 0.0003999 \end{aligned}$$

The form factor for the vertical tail is

$$\begin{aligned} FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.30}(0.10) + 100(0.10)^4\right]\left[1.34(0.7814)^{0.18}(cos(-0.73^{\circ}))^{0.28}\right] \\ &= 1.5538 \end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the vertical tail is

$$\begin{aligned} (C_{D_0})VT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003999 \times 1.4219 \times 1.05 \times 22.186}{33.674} \\ &= 0.0003942 \end{aligned}$$

Fuselage

The Reynolds number for the fuselage is given as

$$\begin{aligned}R &= \frac{\rho V l}{\mu}\\ &= \frac{0.3639 \times 230.55 \times 15.5}{1.45 \times 10^{-5}}\\ &= 188558833.38\end{aligned}$$

The cutoff Reynolds number for the fuselage is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053}\\ &= 38.21 \times (\frac{15.5}{0.634 \times 10^{-5}})^{1.053}\\ &= 203703485.117\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}}\\ &= \frac{1.328}{\sqrt{188558833.38}}\\ &= 0.0000967\end{aligned}$$

The form factor for the fuselage is

$$\begin{aligned}f &= \frac{l}{d} = \frac{15.5 \text{ meter}}{1.91 \text{ meter}}\\ &= 8.115\\ FF &= \left(1 + \frac{60}{f^3} + \frac{f}{400}\right)\\ &= \left(1 + \frac{60}{8.115^3} + \frac{8.115}{400}\right)\\ &= 1.1325\end{aligned}$$

For the fuselage the interference is negligible, so

$$Q = 1.00$$

Therefore, the component of parasitic drag due to the fuselage is

$$\begin{aligned}(C_{D_0})nacelle &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}}\\ &= \frac{0.0004448 \times 1.0899 \times 1.50 \times 122.44}{33.674}\\ &= 0.002644\end{aligned}$$

Since the fuselage upsweep angle is less than 15°, there is no need to consider upsweep fuselage contribution.

Engine Nacelle

The Reynolds number for the nacelle is given as

$$\begin{aligned} R &= \frac{\rho V l}{\mu} \\ &= \frac{0.3639 \times 230.55 \times 3.125}{1.45 \times 10^{-5}} \\ &= 18081281.25 \end{aligned}$$

The cutoff Reynolds number for nacelle is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{3.125}{0.634 \times 10^{-5}})^{1.053} \\ &= 37727160.70 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{18081281.25}} \\ &= 0.0003123 \end{aligned}$$

The form factor for the nacelle is

$$\begin{aligned} f &= \frac{l}{d} = \frac{3.125 \text{ meter}}{0.927 \text{ meter}} \\ &= 3.371 \\ FF &= \left(1 + \frac{0.35}{f}\right) \\ &= \left(1 + \frac{0.35}{3.371}\right) \\ &= 1.1038 \end{aligned}$$

Since the nacelle is mounted under the wing, the interference factor is

$$Q = 1.50$$

Therefore, the component of parasitic drag due to the nacelle is

$$\begin{aligned} (C_{D_0})nacelle &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003123 \times 1.1038 \times 1.50 \times 5.725}{33.674} \\ &= 0.0000879 \end{aligned}$$

C_{D_0} Value at Cruise

The leakage and protuberance contribution to the parasitic drag for a propeller aircraft is taken to be 10% of the total parasitic drag. Therefore, the aircraft's parasitic drag is given as

$$\begin{aligned}
C_{D_0,\text{components}} &= (C_{D_0})_{\text{wing}} + (C_{D_0})_{\text{HT}} + (C_{D_0})_{\text{VT}} + (C_{D_0})_{\text{fuselage}} + 2(C_{D_0})_{\text{nacelle}} \\
&= \ 0.000988 \ + \ 0.000489 + 0.0003942 + 0.002644 + 2(0.0000879) \\
&\approx 0.004691 \\
C_{D_0,\text{misc}} &= 0.10 \times C_{D_0,\text{components}} \quad \text{(10\% increment for misc drag, leakage, and protuberances)} \\
&\approx 0.10 \times 0.004691 \\
&= 0.0004515 \\
C_{D_0,\text{total}} &= C_{D_0,\text{components}} + C_{D_0,\text{misc}} \\
&\approx 0.004691 + 0.0004691 \\
&= 0.005160
\end{aligned}$$

Parasitic Drag Estimation During Takeoff

Wing

The Reynolds number for the wing is given as

$$\begin{aligned} R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{1.225 \times 60.02 \times 2.298}{1.79 \times 10^{-5}} \\ &= 9439067.09 \end{aligned}$$

The cutoff Reynolds number for the wing is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{2.298}{0.634 \times 10^{-5}})^{1.053} \\ &= 27294718.82 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{9439067.09}} \\ &= 0.0003724 \end{aligned}$$

The form factor for the wing is

$$\begin{aligned} FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.4}(0.15) + 100(0.15)^4\right]\left[1.34(0.1748)^{0.18}(cos(14.3°))^{0.28}\right] \\ &= 1.237 \end{aligned}$$

For high wing configuration the interference is very low, so

$$Q = 1.0$$

Therefore, the component of parasitic drag due to the wing is

$$\begin{aligned} (C_{D_0})wing &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003724 \times 1.237 \times 1.0 \times 68.303}{33.674} \\ &= 0.000934 \end{aligned}$$

Flaps

The deflection of flaps during takeoff is 80% of maximum deflection, so during takeoff

$$\begin{aligned}\delta_{flap} &= 0.8\delta_{max}\\ &= 0.8 \times 45^\circ\\ &= 36^\circ\end{aligned}$$

The parasitic drag due to flaps during takeoff is given as

$$\begin{aligned}(\Delta C_{D_0})flap &= Fflap\Big(\frac{c_f}{c}\Big)\Big(\frac{S_{flapped}}{S_{ref}}\Big)(\delta_{flap} - 10)\\ &= 0.0074 \times 0.3 \times 0.5 \times (36 - 10)\\ &= 0.02886\end{aligned}$$

Horizontal Tail

The Reynolds number for the horizontal tail is given as

$$\begin{aligned}R &= \frac{\rho V \bar{c}}{\mu}\\ &= \frac{1.225 \times 60.02 \times 1.874}{1.79 \times 10^{-5}}\\ &= 7697481.17\end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053}\\ &= 38.21 \times (\frac{1.874}{0.634 \times 10^{-5}})^{1.053}\\ &= 18512189\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}}\\ &= \frac{1.328}{\sqrt{7697481.17}}\\ &= 0.000478\end{aligned}$$

The form factor for the horizontal tail is

$$\begin{aligned}FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right]\\ &= \left[1 + \frac{0.6}{0.30}(0.10) + 100(0.10)^4\right]\left[1.34(0.1746)^{0.18}(cos(11.55^\circ))^{0.28}\right]\\ &= 1.178\end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the horizontal tail is

$$\begin{aligned}(C_{D_0})HT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.000478 \times 1.178 \times 1.05 \times 25.365}{33.674} \\ &= 0.000446\end{aligned}$$

Vertical Tail

The Reynolds number for the vertical tail is given as

$$\begin{aligned}R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{1.225 \times 60.02 \times 3.035}{1.79 \times 10^{-5}} \\ &= 12466304.8815\end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{3.035}{0.634 \times 10^{-5}})^{1.053} \\ &= 30967395\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{12466304.8815}} \\ &= 0.000376\end{aligned}$$

The form factor for the vertical tail is

$$\begin{aligned}FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.30}(0.10) + 100(0.10)^4\right]\left[1.34(0.1746)^{0.18}(cos(-0.73°))^{0.28}\right] \\ &= 1.185\end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the vertical tail is

$$
\begin{aligned}
(C_{D_0})VT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\
&= \frac{0.000376 \times 1.185 \times 1.05 \times 22.186}{33.674} \\
&= 0.0003082
\end{aligned}
$$

Fuselage

The Reynolds number for the fuselage is given as

$$
\begin{aligned}
R &= \frac{\rho V l}{\mu} \\
&= \frac{1.225 \times 60.02 \times 15.5}{1.79 \times 10^{-5}} \\
&= 63666466.478
\end{aligned}
$$

The cutoff Reynolds number for the fuselage is given as

$$
\begin{aligned}
R_{cutoff} &= 38.21(l/k)^{1.053} \\
&= 38.21 \times (\frac{15.5}{0.634 \times 10^{-5}})^{1.053} \\
&= 195131748.3161
\end{aligned}
$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$
\begin{aligned}
C_f &= \frac{1.328}{\sqrt{R}} \\
&= \frac{1.328}{\sqrt{63666466.47}} \\
&= 0.0001664
\end{aligned}
$$

The form factor for the fuselage is

$$
\begin{aligned}
f &= \frac{l}{d} = \frac{15.5 \text{ meter}}{1.91 \text{ meter}} \\
&= 8.115 \\
FF &= \left(1 + \frac{60}{f^3} + \frac{f}{400}\right) \\
&= \left(1 + \frac{60}{8.115^3} + \frac{8.115}{400}\right) \\
&= 1.132
\end{aligned}
$$

For the fuselage the interference is negligible, so

$$
Q = 1.00
$$

Therefore, the component of parasitic drag due to the fuselage is

$$
\begin{aligned}
(C_{D_0})fuselage &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\
&= \frac{0.0001664 \times 1.132 \times 1.00 \times 78.236}{33.674} \\
&= 0.0004377
\end{aligned}
$$

Since the fuselage upsweep angle is less than 15°, there is no need to consider upsweep fuselage contribution.

Engine Nacelle

The Reynolds number for the nacelle is given as

$$
\begin{aligned}
R &= \frac{\rho V l}{\mu} \\
&= \frac{1.225 \times 60.02 \times 3.125}{1.79 \times 10^{-5}} \\
&= 12835981.142
\end{aligned}
$$

The cutoff Reynolds number for nacelle is given as

$$
\begin{aligned}
R_{cutoff} &= 38.21(l/k)^{1.053} \\
&= 38.21 \times (\frac{3.125}{0.634 \times 10^{-5}})^{1.053} \\
&= 37727160.70
\end{aligned}
$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$
\begin{aligned}
C_f &= \frac{1.328}{\sqrt{R}} \\
&= \frac{1.328}{\sqrt{12835981.14}} \\
&= 0.0003706
\end{aligned}
$$

The form factor for the nacelle is

$$
\begin{aligned}
f &= \frac{l}{d} = \frac{3.125 \text{ meter}}{0.927\text{meter}} \\
&= 3.371 \\
FF &= \left(1 + \frac{0.35}{f}\right) \\
&= \left(1 + \frac{0.35}{3.371}\right) \\
&= 1.1038
\end{aligned}
$$

Since the nacelle is mounted under the wing, the interference factor is

$$
Q = 1.50
$$

Therefore, the component of parasitic drag due to the nacelle is

$$\begin{aligned}(C_{D_0})nacelle &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.0003706 \times 1.1038 \times 1.50 \times 5.725}{33.674} \\ &= 0.0001043\end{aligned}$$

Landing Gear

The parasitic drag contribution of wheels is given as

$$\begin{aligned}(C_{D_0})wheels &= 0.25(Anose + 2A_{main})/S_{ref} \\ &= 0.25 \times \Bigg(0.649 + 2(1.181)\Bigg) \div 33.678 \\ &= 0.75275/33.678 \\ &= 0.0223\end{aligned}$$

The parasitic drag contribution of struts is given as

$$\begin{aligned}(C_{D_0})struts &= 0.30(Anose + 2A_{main})/S_{ref} \\ &= 0.30 \times [0.018 + 2(0.0736)]/33.678 \\ &= 0.00147\end{aligned}$$

Therefore, the contribution of landing gear to the parasitic drag is

$$\begin{aligned}(C_{D_0})_{LG} &= (C_{D_0})_{wheels} + (C_{D_0})_{struts} \\ &= 0.0223 + 0.00147 \\ &= 0.0238\end{aligned}$$

C_{D_0} **Value at Takeoff**

The leakage and protuberance contribution to the parasitic drag for a propeller aircraft is taken to be 10% of the total parasitic drag. Therefore, the aircraft's parasitic drag is given as

$$\begin{aligned}C_{D_0} &= (C_{D_0})wing + (CD_0)HT + (CD_0)VT + (CD_0)fuselage + 2(CD_0)nacelle + (\Delta CD_0)flap + (CD_0)_{LG} \\ &= 0.000934 + 0.000446 + 0.0003082 + 0.0004377 + 2(0.0001043) + 0.02886 + 0.0238 \\ &= 0.0549945\end{aligned}$$

Parasitic Drag Estimation During Landing

Wing

The Reynolds number for the wing is given as

$$\begin{aligned} R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{1.225 \times 56.524 \times 2.298}{1.79 \times 10^{-5}} \\ &= 8889267.38 \end{aligned}$$

The cutoff Reynolds number for the wing is given as

$$\begin{aligned} R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{2.298}{0.634 \times 10^{-5}})^{1.053} \\ &= 27294718.82 \end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned} C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{8889267.38}} \\ &= 0.000445 \end{aligned}$$

The form factor for the wing is

$$\begin{aligned} FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.4}(0.15) + 100(0.15)^4\right]\left[1.34(0.1646)^{0.18}(cos(14.3°))^{0.28}\right] \\ &= 1.258 \end{aligned}$$

For high wing configuration the interference is very low, so

$$Q = 1.0$$

Therefore, the component of parasitic drag due to the wing is

$$\begin{aligned} (C_{D_0})wing &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.000445 \times 1.258 \times 1.0 \times 68.308}{33.674} \\ &= 0.001134 \end{aligned}$$

Flaps

The deflection of flaps during landing are at maximum deflection, so during landing

$$\begin{aligned}\delta_{flap} &= \delta_{max}\\ &= 45^{\circ}\end{aligned}$$

The parasitic drag due to flaps during takeoff is given as

$$\begin{aligned}(\Delta C_{D_0})flap &= Fflap\Big(\frac{c_f}{c}\Big)\Big(\frac{S_{flapped}}{S_{ref}}\Big)(\delta_{flap}-10)\\ &= 0.0074\times 0.3\times 0.5\times(45-10)\\ &= 0.03885\end{aligned}$$

Horizontal Tail

The Reynolds number for the horizontal tail is given as

$$\begin{aligned}R &= \frac{\rho V\bar{c}}{\mu}\\ &= \frac{1.225\times 56.524\times 1.874}{1.79\times 10^{-5}}\\ &= 7249124.05\end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053}\\ &= 38.21\times(\frac{1.874}{0.634\times 10^{-5}})^{1.053}\\ &= 22019295.70\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}}\\ &= \frac{1.328}{\sqrt{7249124.05}}\\ &= 0.000493\end{aligned}$$

The form factor for the horizontal tail is

$$\begin{aligned}FF &= \left[1+\frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right)+100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right]\\ &= \left[1+\frac{0.6}{0.30}(0.10)+100(0.10)^4\right]\left[1.34(0.1646)^{0.18}(cos(11.55^{\circ}))^{0.28}\right]\\ &= 1.1778\end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the horizontal tail is

$$\begin{aligned}(C_{D_0})HT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.000493 \times 1.1778 \times 1.05 \times 25.365}{33.674} \\ &= 0.000462\end{aligned}$$

Vertical Tail

The Reynolds number for the vertical tail is given as

$$\begin{aligned}R &= \frac{\rho V \bar{c}}{\mu} \\ &= \frac{1.225 \times 56.524 \times 3.3025}{1.79 \times 10^{-5}} \\ &= 12800035.0\end{aligned}$$

The cutoff Reynolds number for the horizontal tail is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{3.3025}{0.634 \times 10^{-5}})^{1.053} \\ &= 33383049.20\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{12800035.0}} \\ &= 0.000371\end{aligned}$$

The form factor for the vertical tail is

$$\begin{aligned}FF &= \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}(cos\Lambda_m)^{0.28}\right] \\ &= \left[1 + \frac{0.6}{0.30}(0.10) + 100(0.10)^4\right]\left[1.34(0.1646)^{0.18}(cos(-0.73°))^{0.28}\right] \\ &= 1.1418\end{aligned}$$

For conventional tail the interference is 5%, so

$$Q = 1.05$$

Therefore, the component of parasitic drag due to the vertical tail is

$$\begin{aligned}
(C_{D_0})VT &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\
&= \frac{0.000371 \times 1.1418 \times 1.05 \times 22.186}{33.674} \\
&= 0.000293
\end{aligned}$$

Fuselage

The Reynolds number for the fuselage is given as

$$\begin{aligned}
R &= \frac{\rho V l}{\mu} \\
&= \frac{1.225 \times 56.524 \times 15.5}{1.79 \times 10^{-5}} \\
&= 60058743.69
\end{aligned}$$

The cutoff Reynolds number for the fuselage is given as

$$\begin{aligned}
R_{cutoff} &= 38.21(l/k)^{1.053} \\
&= 38.21 \times (\frac{15.5}{0.634 \times 10^{-5}})^{1.053} \\
&= 1620950034.46
\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}
C_f &= \frac{1.328}{\sqrt{R}} \\
&= \frac{1.328}{\sqrt{60058743.69}} \\
&= 0.000171
\end{aligned}$$

The form factor for the fuselage is

$$\begin{aligned}
f &= \frac{l}{d} = \frac{15.5 \text{ meter}}{1.91 \text{ meter}} \\
&= 8.115 \\
FF &= \left(1 + \frac{60}{f^3} + \frac{f}{400}\right) \\
&= \left(1 + \frac{60}{8.115^3} + \frac{8.115}{400}\right) \\
&= 1.132
\end{aligned}$$

For the fuselage the interference is negligible, so

$$Q = 1.00$$

Therefore, the component of parasitic drag due to the fuselage is

$$\begin{aligned}(C_{D_0})fuselage &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.000171 \times 1.132 \times 1.00 \times 78.236}{33.674} \\ &= 0.000451\end{aligned}$$

Since the fuselage upsweep angle is less than 15°, there is no need to consider upsweep fuselage contribution.

Engine Nacelle

The Reynolds number for the nacelle is given as

$$\begin{aligned}R &= \frac{\rho V l}{\mu} \\ &= \frac{1.225 \times 56.524 \times 3.125}{1.79 \times 10^{-5}} \\ &= 12100534.22\end{aligned}$$

The cutoff Reynolds number for nacelle is given as

$$\begin{aligned}R_{cutoff} &= 38.21(l/k)^{1.053} \\ &= 38.21 \times (\frac{3.125}{0.634 \times 10^{-5}})^{1.053} \\ &= 25695764.86\end{aligned}$$

Since R is less than R_{cutoff}, the flow is laminar. Therefore, the skin friction coefficient is given as

$$\begin{aligned}C_f &= \frac{1.328}{\sqrt{R}} \\ &= \frac{1.328}{\sqrt{12100534.22}} \\ &= 0.000382\end{aligned}$$

The form factor for the nacelle is

$$\begin{aligned}f &= \frac{l}{d} = \frac{3.125 \text{ meter}}{0.927 \text{ meter}} \\ &= 3.371 \\ FF &= \left(1 + \frac{0.35}{f}\right) \\ &= \left(1 + \frac{0.35}{3.371}\right) \\ &= 1.103\end{aligned}$$

Since the nacelle is mounted under the wing, the interference factor is

$$Q = 1.50$$

Therefore, the component of parasitic drag due to the nacelle is

$$\begin{aligned}(C_{D_0})nacelle &= \frac{C_f \cdot FF \cdot Q \cdot Swet}{S_{ref}} \\ &= \frac{0.000382 \times 1.103 \times 1.0 \times 5.725}{33.674} \\ &= 0.000716\end{aligned}$$

Landing Gear

The parasitic drag contribution of wheels is given as

$$\begin{aligned}(C_{D_0})wheels &= 0.25(Anose + 2A_{main})/S_{ref} \\ &= 0.25 \times \left(0.649 + 2(1.181)\right) \div 33.678 \\ &= 0.75275/33.674 \\ &= 0.0223\end{aligned}$$

The parasitic drag contribution of struts is given as

$$\begin{aligned}(C_{D_0})struts &= 0.30(Anose + 2A_{main})/S_{ref} \\ &= 0.30 \times [0.018 + 2(0.0736)]/33.674 \\ &= 0.00147\end{aligned}$$

Therefore, the contribution of landing gear to the parasitic drag is

$$\begin{aligned}(C_{D_0})_{LG} &= (C_{D_0})_{wheels} + (C_{D_0})_{struts} \\ &= 0.0223 + 0.00147 \\ &= 0.0238\end{aligned}$$

C_{D_0} Value at Landing

The leakage and protuberance contribution to the parasitic drag for a propeller aircraft is taken to be 10% of the total parasitic drag. Therefore, the aircraft's parasitic drag is given as

$$\begin{aligned}C_{D_0} &= (C_{D_0})wing + (C_{D_0})HT + (C_{D_0})VT + (C_{D_0})fuselage + 2(C_{D_0})nacelle + (\Delta C_{D_0})flap + (C_{D_0})_{LG} \\ &= 0.0001134 + 0.000462 + 0.000293 + 0.000451 + 2(0.000716) + 0.03885 + 0.0238 \\ &= 0.0654\end{aligned}$$

Lift-Dependent Drag Factor During Cruise

Oswald Efficiency Factor

The Oswald efficiency factor for aircraft with unswept wing is given as

$$\begin{aligned} e &= 1.78(1 - 0.045A^{0.68}) - 0.64 \\ &= 1.78(1 - 0.045(8.4)^{0.68}) - 0.64 \\ &= 0.8121 \end{aligned}$$

Lift-Dependent Drag Factor During Cruise

The lift-dependent drag factor during cruise is given as

$$\begin{aligned} K &= \frac{1}{\pi A e} \\ &= \frac{1}{\pi \times 8.4 \times 0.8121} \\ &= 0.04666 \end{aligned}$$

Lift-Dependent Drag Factor During Takeoff and Landing

The lift-dependent drag factor during takeoff and landing is given as

$$\begin{aligned} K &= \frac{1}{\pi A e} \\ &= \frac{1}{\pi \times 8.4 \times 0.7309} \\ &= 0.04666 \end{aligned}$$

Due to ground effect, there will be a reduction in the lift-induced drag factor given as

$$\begin{aligned} K_{effective} &= K\left(\frac{33\big(h/b\big)^{1.5}}{1 + 33\big(h/b\big)^{1.5}}\right) \\ &= 0.04666\left(\frac{33\big(1.12/16.68\big)^{1.5}}{1 + 33\big(1.12/16.68\big)^{1.5}}\right) \\ &= 0.01703 \end{aligned}$$

Maximum Lift-to-Drag Ratio for Cruise

The maximum lift-to-drag ratio at cruise is given as

$$\begin{aligned} \left(\frac{L}{D}\right)max &= \frac{1}{2\sqrt{KC_{D_0}}} \\ &= \frac{1}{2\sqrt{0.04666 \times 0.005160}} \\ &= 32.223 \end{aligned}$$

Maximum Lift-to-Drag Ratio for Takeoff

The maximum lift-to-drag ratio at takeoff is given as

$$\begin{aligned}\left(\frac{L}{D}\right)max &= \frac{1}{2\sqrt{KC_{D_0}}} \\ &= \frac{1}{2\sqrt{0.01703 \times 0.0549945}} \\ &= 16.338\end{aligned}$$

Maximum Lift-to-Drag Ratio for Landing

The maximum lift-to-drag ratio at landing is given as

$$\begin{aligned}\left(\frac{L}{D}\right)max &= \frac{1}{2\sqrt{KC_{D_0}}} \\ &= \frac{1}{2\sqrt{0.01703 \times 0.0654}} \\ &= 14.982\end{aligned}$$

Table 18: Aerodynamic and Performance Coefficients Summary

Parameter	**Cruise**	**Takeoff**	**Landing**
$C_{L_{\max}}$ (Maximum Lift Coefficient)	1.37	1.9741	1.74833
$\alpha_{C_{L_{\max}}}$ (Angle of Attack at $C_{L_{\max}}$)	15.7769°	8.2385°	12.1206°
C_{L_α} (Lift-Curve Slope)	10.068/rad	6.361/rad	2.767/rad
C_{D_0} (Zero-Lift Drag Coefficient)	0.005160	0.0549945	0.0654
$(L/D)_{\max}$ (Maximum Lift-to-Drag Ratio)	32.22	16.338	14.982
e (Oswald Efficiency Factor)	0.8121		
$\Delta C_{L_{\max}}$ (Flap Increment)	2.08		

CHAPTER 12

Refined Weight and C.G. Estimate

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Objective

To calculate a refined empty weight and determine the center of gravity (C.G.) range for the aircraft, in accordance with the procedures from Lecture 12. This refined estimate uses statistical formulas for individual components based on their geometry, replacing the approximate component weights from Chapter 9.

Methodology

As per Lecture 12, the new MTOW from Chapter 9 is $W_0 = 13,768.196$ kg.

$$13,768.196 \text{ kg} \times 2.20462 \text{ lb/kg} = 30353.64 \text{ lb}$$

Since $30353.64 \text{ lb} > 12,500 \text{ lb}$, the **"Cargo/Transport Weights"** formulas (Eq. 15.25 - 15.45) from Lecture 12 must be used. All calculations are performed in British units (pounds, feet, inches) as required by the formulas, and then converted back to metric units.

Input Parameter Conversion

All metric values from previous reports are converted to British units.

Table 19: Input Parameters for Weight Estimation

Parameter	Description	Value
W_{dg}	Design Gross Weight	30,290.03 lb (13,739.4 kg)
N_z	Ultimate Load Factor	5.25
t/c	Thickness-to-Chord Ratio	0.15
λ	Wing Taper Ratio	0.2
Λ	Wing Sweep	20°
A	Wing Aspect Ratio	8.4
K_{uth}	Horizontal Tail Coefficient	1.00
F_w	Fuselage Width	1.77 ft
B_w	Wing Span	55.11 ft
B_h	Horizontal Tail Span	23.18 ft
S_{ht}	Horizontal Tail Area	134.35 ft^2 (12.486 m^2)
L_t	Tail Arm	20.34 ft (6.2 m)
K_y	Radius of Gyration Factor	6.10 ft
S_e	Elevator Area	35.64 ft^2
S_n	Nacelle Area	30.95 ft^2
H_t	Horizontal Tail Height	11.81 ft
S_{vt}	Vertical Tail Area	117.95 ft^2 (10.962 m^2)
$(t/c)_{vt}$	Vertical Tail Thickness Ratio	0.09
K_{door}	Door Factor	1.06
K_{Lg}	Landing Gear Factor	1.0
K_{ws}	Wing-Fuselage Factor	0.346

Continued on next page

Table 19: Input Parameters for Weight Estimation (continued)

Parameter	**Description**	**Value**
K_{mp}	Main Gear Factor	1.0
W_l	Landing Weight	24,000 lb
N_l	Ultimate Landing Factor	9.0
L_m	Main Gear Length	44.0 in
N_{mw}	Number of Main Wheels	4
N_{mss}	Number of Main Struts	2
V_{stall}	Stall Speed	160.07 ft/s
K_{np}	Nose Gear Factor	1.0
N_{lt}	Nacelle Length	5.41 ft
K_{ng}	Nacelle Group Factor	1.017
N_{en}	Number of Engines	2
N_p	Number of Passengers	9
L_{ec}	Engine Controls Length	40.74 ft
V_i	Integral Tank Volume	1,198.31 gal
V_t	Total Fuel Volume	1,497.88 gal
V_p	Protected Tank Volume	0.0 gal
N_f	Number of Functions	6
N_m	Number of Mechanical Functions	2
S_{cs}	Control Surface Area	73.08 ft^2
W_{APU}	APU Uninstalled Weight	132.0 lb
K_r	Reliability Factor	1.0
K_{tp}	Transport Factor	1.0
L_f	Fuselage Length	50.84 ft (15.5 m)
K_t	Thrust Factor	1.0
N_t	Number of Fuel Tanks	2
K_{hv}	Hydraulic Factor	1.0
L	Fuselage Length	50.84 ft
D	Fuselage Diameter	0.17 ft
L/D	Length-to-Diameter Ratio	299.06
W_{en}	Engine Weight	1,283.39 lb
V_{pr}	Pressurized Volume	500 ft^3
W_{uav}	Uninstalled Avionics Weight	498.96 lb
N_c	Number of Crew	2
W_c	Cabin Weight	485 lb
S_f	Floor Area	629.17 ft^2 (58.4517 m^2)
S_{csw}	Control Surface Area	37.43 ft^2 (3.478 m^2)
A_h	Horizontal Tail Aspect Ratio	3.99
H_v	Vertical Tail Height	11.90 ft
K_z	Z-axis Radius of Gyration	20.34 ft
A_v	Vertical Tail Aspect Ratio	1.20
L_n	Nose Gear Length	35.2 in
N_w	Nacelle Width	2.85 ft
W_{ec}	Engine Contents Weight	3,509.0 lb

Continued on next page

Table 19: Input Parameters for Weight Estimation (continued)

Parameter	**Description**	**Value**
I_y	Yawing Moment of Inertia	37,495.38 lb-ft^2
R_{kva}	Electrical Rating	50 kV·A
N_{gen}	Number of Generators	2
S_w	Wing Area	362.31 ft^2 (33.674 m^2)
W_{seats}	Weight Seats	408 lb
$W_{lavatory}$	Weight of Lavatories	72.48 lb
W_{engine}	Weight of two engines	2,566.78 lb

Structure Group Weight Calculation

Wing Weight (Eq. 15.25)

$$\begin{aligned}
W_{wing} &= 0.0051(W_{dg}N_z)^{0.557}S_w^{0.649}A^{0.5}(t/c)_{root}^{-0.4}(1+\lambda)^{0.1}(\cos\Lambda)^{-1.0}S_{csw}^{0.1} \\
&= 0.0051 \times (30290.03 \times 5.25)^{0.557} \times (362.313)^{0.649} \times (8.4)^{0.5} \times (0.15)^{-0.4} \\
&\quad \times (1+0.2)^{0.1} \times (0.9397)^{-1.0} \times (37.43)^{0.1} \\
&= \mathbf{1776.62} \text{ lb} \quad (\mathbf{805.7} \text{ kg})
\end{aligned}$$

Horizontal Tail Weight (Eq. 15.26)

$$\begin{aligned}
W_{ht} &= 0.0379K_{uht}(1+F_w/B_h)^{-0.25}W_{dg}^{0.639}N_z^{0.10}S_{ht}^{0.75}L_t^{-1.0}K_y^{0.704}(\cos\Lambda_{ht})^{-1.0}A_h^{0.166}(1+S_e/S_{ht})^{0.1} \\
&= 0.0379 \times (1.0) \times (1+1.77/23.18)^{-0.25} \times (30290.03)^{0.639} \times (5.25)^{0.10} \times (134.35)^{0.75} \times (20.34)^{-1.0} \\
&\quad \times (6.10)^{0.704} \times (1.0)^{-1.0} \times (3.99)^{0.166} \times (1+35.64/134.35)^{0.1} \\
&= \mathbf{304.81} \text{ lb} \quad (\mathbf{138.3} \text{ kg})
\end{aligned}$$

Vertical Tail Weight (Eq. 15.27)

$$\begin{aligned}
W_{vt} &= 0.0026(1+H_t/H_v)^{0.225}W_{dg}^{0.556}N_z^{0.536}L_t^{-0.5}S_{vt}^{0.5}K_z^{0.875}(\cos\Lambda_{vt})^{-1}A_v^{0.35}(t/c)_{root}^{-0.5} \\
&= 0.0026 \times (1+11.81/11.90)^{0.225} \times (30290.03)^{0.556} \times (5.25)^{0.536} \times (20.34)^{-0.5} \times (117.95)^{0.5} \times (20.34)^{0.875} \\
&\quad \times (0.9397)^{-1.0} \times (1.20)^{0.35} \times (0.09)^{-0.5} \\
&= \mathbf{291.08} \text{ lb} \quad (\mathbf{132.0} \text{ kg})
\end{aligned}$$

Fuselage Weight (Eq. 15.28)

$$\begin{aligned}
W_{fuselage} &= 0.328K_{door}K_{Lg}(W_{dg}N_z)^{0.5}L^{0.25}S_f^{0.302}(1+K_{ws})^{0.04}(L/D)^{0.10} \\
&= 0.328 \times (1.06) \times (1.0) \times (30290.03 \times 5.25)^{0.5} \times (50.84)^{0.25} \times (629.17)^{0.302} \\
&\quad \times (1+0.346)^{0.04} \times (299.06)^{0.10} \\
&= \mathbf{4639.03} \text{ lb} \quad (\mathbf{2104.2} \text{ kg})
\end{aligned}$$

Main Landing Gear Weight (Eq. 15.29)

$$\begin{aligned} W_{mlg} &= 0.0106 K_{mp} W_l^{0.888} N_l^{0.25} L_m^{0.4} N_{mw}^{0.321} N_{mss}^{-0.5} V_{stall}^{0.1} \\ &= 0.0106 \times (1.0) \times (24000)^{0.888} \times (9.0)^{0.25} \times (44.0)^{0.4} \times (4)^{0.321} \times (2)^{-0.5} \times (160.07)^{0.1} \\ &= \mathbf{1185.91} \text{ lb} \quad (\mathbf{539.05} \text{ kg}) \end{aligned}$$

Nose Landing Gear Weight (Eq. 15.30)

$$\begin{aligned} W_{nlg} &= 0.032 K_{np} W_l^{0.646} N_l^{0.2} L_n^{0.5} N_{nw}^{0.45} \\ &= 0.032 \times (1.0) \times (24000)^{0.646} \times (9.0)^{0.2} \times (35.2)^{0.5} \times (4)^{0.45} \\ &= \mathbf{370.53} \text{ lb} \quad (\mathbf{168.18} \text{ kg}) \end{aligned}$$

Propulsion Group Weight Calculation

Nacelle Group Weight (Eq. 15.31)

$$\begin{aligned} W_{nacelle} &= 0.6724 K_{ng} N_{Lt}^{0.10} N_w^{0.294} N_z^{0.119} W_{ec}^{0.611} N_{en}^{0.984} S_n^{0.224} \\ &= 0.6724 \times (1.017) \times (5.41)^{0.10} \times (2.85)^{0.294} \times (5.25)^{0.119} \times (3509.0)^{0.611} \times (2)^{0.984} \times (30.95)^{0.224} \\ &= \mathbf{839.62} \text{ lb} \quad (\mathbf{380.9} \text{ kg}) \end{aligned}$$

Engine Controls Weight (Eq. 15.32)

$$\begin{aligned} W_{eng\_cont} &= 5.0 N_{en} + 0.80 L_{ec} \\ &= 5.0 \times (2) + 0.80 \times (40.74) \\ &= \mathbf{42.59} \text{ lb} \quad (\mathbf{19.3} \text{ kg}) \end{aligned}$$

Starter Weight (Eq. 15.33)

$$\begin{aligned} W_{starter} &= 49.19 (N_{en} W_{en}/1000)^{0.541} \\ &= 49.19 \times (2 \times 1283.39/1000)^{0.541} \\ &= \mathbf{81.91} \text{ lb} \quad (\mathbf{37.2} \text{ kg}) \end{aligned}$$

Fuel System Weight (Eq. 15.34)

$$\begin{aligned} W_{fuel\_sys} &= 2.405 V_t^{0.606} (1 + V_i/V_t)^{-1.0} (1 + V_p/V_t) N_t^{0.5} \\ &= 2.405 \times (1497.88)^{0.606} \times (1 + 1198.31/1497.88)^{-1.0} \times (1 + 0) \times (2)^{0.5} \\ &= \mathbf{158.74} \text{ lb} \quad (\mathbf{72.0} \text{ kg}) \end{aligned}$$

Engine Weight

$$W_{engine\ both} = 2566.78 \text{ lb} \quad (1164.3 \text{ kg})$$

Equipment Group Weight Calculation

Flight Controls Weight (Eq. 15.35)

$$\begin{aligned} W_{fc} &= 145.9 N_f^{0.554}(1 + N_m/N_f)^{-1.0} S_{cs}^{0.20}(I_y \times 10^{-6})^{0.07} \\ &= 145.9 \times (6)^{0.554} \times (1 + 2/6)^{-1.0} \times (73.08)^{0.20} \times (37495.38 \times 10^{-6})^{0.07} \\ &= \mathbf{553.55} \text{ lb} \quad (\mathbf{251.1} \text{ kg}) \end{aligned}$$

APU Installed Weight (Eq. 15.36)

$$\begin{aligned} W_{APU} &= 2.2 W_{APU\_uninstalled} \\ &= 2.2 \times 132 \\ &= \mathbf{290.40} \text{ lb} \quad (\mathbf{131.7} \text{ kg}) \end{aligned}$$

Instruments Weight (Eq. 15.37)

$$\begin{aligned} W_{instruments} &= 4.509 K_r K_{tp} N_c^{0.541} N_{en}(L_f + B_w)^{0.5} \\ &= 4.509 \times (1.0) \times (1.0) \times (2)^{0.541} \times (2) \times (50.84 + 55.11)^{0.5} \\ &= \mathbf{135.06} \text{ lb} \quad (\mathbf{61.3} \text{ kg}) \end{aligned}$$

Hydraulics Weight (Eq. 15.38)

$$\begin{aligned} W_{hydraulics} &= 0.2673 N_f (L_f + B_w)^{0.937} \\ &= 0.2673 \times (6) \times (50.84 + 55.11)^{0.937} \\ &= \mathbf{126.67} \text{ lb} \quad (\mathbf{57.4} \text{ kg}) \end{aligned}$$

Avionics Weight (Eq. 15.40)

$$\begin{aligned} W_{avionics} &= 1.73 W_{uav}^{0.983} \\ &= 1.73 \times (498.96)^{0.983} \\ &= \mathbf{776.68} \text{ lb} \quad (\mathbf{352.3} \text{ kg}) \end{aligned}$$

Furnishings Weight (Eq. 15.41)

$$\begin{aligned} W_{furnish} &= 0.0577 N_c^{0.1} W_c^{0.393} S_f^{0.75} \\ &= 0.0577 \times (2)^{0.1} \times (485)^{0.393} \times (629.17)^{0.75} \\ &= \mathbf{88.28} \text{ lb} \quad (\mathbf{40.0} \text{ kg}) \end{aligned}$$

Air Conditioning Weight (Eq. 15.42)

$$\begin{aligned} W_{air\_conditioning} &= 62.36 N_p^{0.25}(V_{pr}/1000)^{0.604} W_{uav}^{0.10} \\ &= 62.36 \times (9)^{0.25} \times (500/1000)^{0.604} \times (498.96)^{0.10} \\ &= \mathbf{132.27} \text{ lb} \quad (\mathbf{60.0} \text{ kg}) \end{aligned}$$

Anti-Ice Weight (Eq. 15.43)

$$\begin{aligned} W_{anti\_ice} &= 0.002 W_{dg} \\ &= 0.002 \times 30290.03 \\ &= \mathbf{60.58} \text{ lb} \quad (\mathbf{27.5} \text{ kg}) \end{aligned}$$

Handling Gear Weight (Eq. 15.44)

$$\begin{aligned} W_{handling\_gear} &= 3.0 \times 10^{-4} W_{dg} \\ &= 0.0003 \times 30290.03 \\ &= \mathbf{9.09} \text{ lb} \quad (\mathbf{4.1} \text{ kg}) \end{aligned}$$

Lavatories Weight

$$\begin{aligned} W_{lavatories} &= 3.90 \times N_{pass}^{1.33} \\ &= 3.90 \times (9)^{1.33} \\ &= \mathbf{72.48} \text{ lb} \quad (\mathbf{32.9} \text{ kg}) \end{aligned}$$

Seats Weight

$$\begin{aligned} W_{seats} &= (2 \times 60) + (9 \times 32) \\ &= 120 + 288 \\ &= \mathbf{408.00} \text{ lb} \quad (\mathbf{185.1} \text{ kg}) \end{aligned}$$

Refined Empty Weight Summary

Table 20: Refined Empty Weight (W_e) Calculation

Component	Weight (lb)	Weight (kg)
Structures Group		
Wing (W_{wing})	1776.62	805.7
Horizontal Tail (W_{ht})	304.81	138.3
Vertical Tail (W_{vt})	291.08	132.0
Fuselage ($W_{fuselage}$)	4639.03	2104.2
Main Landing Gear (W_{mlg})	1185.91	539.05
Nose Landing Gear (W_{nlg})	370.53	168.18
Subtotal Structures	*7157.14*	*3246.3*
Propulsion Group		
Nacelle Group ($W_{nacelle}$)	839.62	380.9
Engine Controls (W_{eng_cont})	42.59	19.3
Starter ($W_{starter}$)	81.91	37.2
Fuel System (W_{fuel_sys})	158.74	72.0
Engines (Installed)	2566.78	1164.3
Subtotal Propulsion	*3689.64*	*1673.7*
Equipment Group		
Flight Controls (W_{fc})	553.55	251.1
APU Installed (W_{APU})	290.40	131.7
Instruments ($W_{instruments}$)	135.06	61.3
Hydraulics ($W_{hydraulics}$)	126.67	57.4
Avionics ($W_{avionics}$)	776.68	352.3
Furnishings ($W_{furnish}$)	88.28	40.0
Air Conditioning ($W_{air\_conditioning}$)	132.27	60.0
Anti-Ice (W_{anti_ice})	60.58	27.5
Handling Gear ($W_{handling\_gear}$)	9.09	4.1
Lavatories ($W_{lavatories}$)	72.48	32.9
Seats (W_{seats})	408.00	185.1
Subtotal Equipment	*2653.06*	*1203.4*
Total Refined Empty Weight (W_e)	**13499.84**	**6123.4**

Refined Center of Gravity Estimate

We establish the datum at the **fuselage nose** ($x = 0$).

Component C.G. Locations

Table 21: Component Center of Gravity Locations

Component	Description	Location (m)
Fuselage	40% of fuselage length from nose (0.40 × 15.50 m)	6.20
Wing	from report-9	10.194
Horizontal Tail	from report-9	15.574
Vertical Tail	from report-9	14.834
Propulsion	from report-9	11.156
Nose Landing Gear	from report-10	3.425 m
Main Landing Gear	from report-10	9.052 m
Equipment	40% of fuselage length for equipment bay	6.20
Furnishings	40% of fuselage length for cabin C.G.	6.20
Crew	from report-9	2.50
Payload	from report-9	7.362
Fuel	from report-9	10.167

Location Justification:

- **Wing:** From report-9 data = 10.194 m
- **Horizontal Tail:** From report-9 data = 15.574 m
- **Vertical Tail:** From report-9 data = 14.834 m
- **Propulsion:** From report-9 data = 11.156 m
- **Main Landing Gear:** From report-10 data = 9.052 m
- **Nose Landing Gear:** From report-10 data = 3.425 m
- **Crew:** From report-9 data = 2.50 m
- **Payload:** From report-9 data = 7.362 m
- **Fuel:** From report-9 data = 10.167 m

Aircraft Loading and C.G. Calculation

Table 22: Refined Group Weight and C.G. Table

Component	Weight (kg)	Location x (m)	Moment (kg-m)
Empty Weight Components			
Wing	805.7	10.194	8213.3
Horizontal Tail	138.3	15.574	2153.0
Vertical Tail	132.0	14.834	1957.7
Fuselage	2104.2	6.20	13046.0
Main Landing Gear	539.05	9.052	4879.028
Nose Landing Gear	168.18	3.425	576.01
Propulsion Group	1673.7	11.156	18667.8
Flight Controls	251.1	10.194	2559.7
APU	131.7	11.156	1469.0
Instruments	61.3	6.20	380.1
Hydraulics	57.4	6.20	355.9
Avionics	352.3	6.20	2184.3
Furnishings	40.0	6.20	248.0
Air Conditioning	60.0	6.20	372.0
Anti-Ice	27.5	6.20	170.5
Handling Gear	4.1	6.20	25.4
Lavatories	32.9	6.20	204.0
Seats	185.1	6.20	1147.6
Total Empty Weight (W_e)	**6123.4**	**8.17**	**50905.46**
C.G. Case 1: FWD (Empty + Crew + 6% Fuel)			
Empty Weight	6123.4	8.17	50905.46
Crew (2)	200.6	2.50	501.5
Fuel (6% Reserve)	179.6	10.167	1825.0
Total (FWD C.G.)	**6503.6**	**8.185**	**53231.96**
C.G. Case 2: AFT (Fully Loaded)			
Empty Weight	6123.4	8.45	51648.6
Crew (2)	200.6	2.50	501.5
Payload (9 pax)	904.6	7.362	5963.2
Fuel (Full)	2993.2	10.167	30427.7
Total (AFT C.G.)	**10221.6**	**8.611**	**88018.19**

Final C.G. Range Calculation

1. **Forward C.G. Limit (Empty + Crew + 6% Fuel):**

$$x_{CG,fwd} = \frac{53231.96 \text{ kg-m}}{6503.6 \text{ kg}} = \mathbf{8.185} \text{ m}$$

2. **Aft C.G. Limit (Fully Loaded MTOW):**

$$x_{CG,aft} = \frac{88018.19 \text{ kg-m}}{10127.2 \text{ kg}} = \mathbf{8.611} \text{ m}$$

Conclusion and Final C.G. Range

The refined weight and C.G. analysis provides a detailed component-based weight and balance for the aircraft. The total refined empty weight is **6123.4** kg.

The operational Center of Gravity range is:

- **Most Forward C.G.:** 8.185 m (at 6503.6 kg) - 52.8% of fuselage
- **Most Aft C.G.:** 8.611 m (at 10221.6 kg) - 55.55% of fuselage

This results in a total C.G. travel of **0.426** m (42.6 cm). The C.G. range ratio is:

$$\frac{x_{CG,aft}}{x_{CG,fwd}} = \frac{8.74}{8.29} = \mathbf{1.052}$$

The C.G. range provides an adequate stability margin for the aircraft with the desired ratio of 1.052. The refined weight estimate of 6123.4 kg represents a more accurate calculation based on detailed component analysis.

This refined analysis demonstrates that the aircraft meets the target performance specifications with adequate weight margins and stable centre of gravity characteristics, achieving the desired C.G. ratio of 1.052.

Now, as per the refined calculations of report 12, our $\frac{W_e}{W_0}$ ratio is $\frac{6123.4}{10221.6} = 0.599$ which earlier was 0.57. This difference in the ratio is because of the rough estimates taken in report 2 and report 9.

CHAPTER 13

Performance Analysis

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Calculation of Cruise Speed and Range

Inputs

1. Wing area: $S = 33.674 \text{ m}^2$
2. Zero-lift drag coefficient: $C_{D0} = 0.0051$
3. Induced drag factor: $K = 0.04703$
4. Maximum lift coefficient: $C_{L,\text{max}} = 1.37$
5. Cruise altitude: $h = 11000$ m
6. Atmospheric density at cruise: $\rho = 0.3639 \text{ kg/m}^3$
7. Speed of sound at cruise: $a = 295.1$ m/s
8. Drag-divergence Mach number: $M_{DD} = 0.9$
9. Sea-level thrust: $T_{SL} = 40843$ N
10. Available thrust at 11 km:

$$T_{\text{avail}}(11000) = T_{SL}\left(\frac{\rho}{1.225}\right) = 12{,}134 \text{ N}$$

11. Initial cruise weight:

$$W_2 = 9766 \text{ kg} \times 9.80665 = 95767 \text{ N}$$

12. Final cruise weight:

$$W_3 = 7408.18 \text{ kg} \times 9.80665 = 72645 \text{ N}$$

13. TSFC (given): $c = 1.37 \times 10^{-4} \text{ s}^{-1}$
14. Cruise speed used (from Report 9):

$$V_{\text{cruise}} = 230.6 \text{ m/s}$$

Calculated Aerodynamic Quantities

Optimum lift coefficient

$$C_{L,\text{opt}} = \sqrt{\frac{C_{D0}}{3K}} = \sqrt{\frac{0.0051}{3 \times 0.04703}} = 0.1936$$

Maximum lift-to-drag ratio

$$\left(\frac{L}{D}\right)_{\text{max}} = \frac{1}{2\sqrt{C_{D0}K}} = 32.285$$

Actual lift coefficient at cruise

$$C_L = \frac{2W_{\text{avg}}}{\rho V_{\text{cruise}}^2 S} = 0.2586$$

Actual L/D during cruise

$$\left(\frac{L}{D}\right)_{\text{used}} = \frac{C_L}{C_{D0} + KC_L^2} = 31.363$$

Range Computation

$$R = \frac{V_{\text{cruise}}}{c}\left(\frac{L}{D}\right)_{\text{used}} \ln\left(\frac{W_2}{W_3}\right)$$

$$R = \frac{230.6}{1.37 \times 10^{-4}} \times 31.363 \times \ln\left(\frac{95767}{72645}\right) = 14{,}584 \text{ km}$$

Cruise Plot

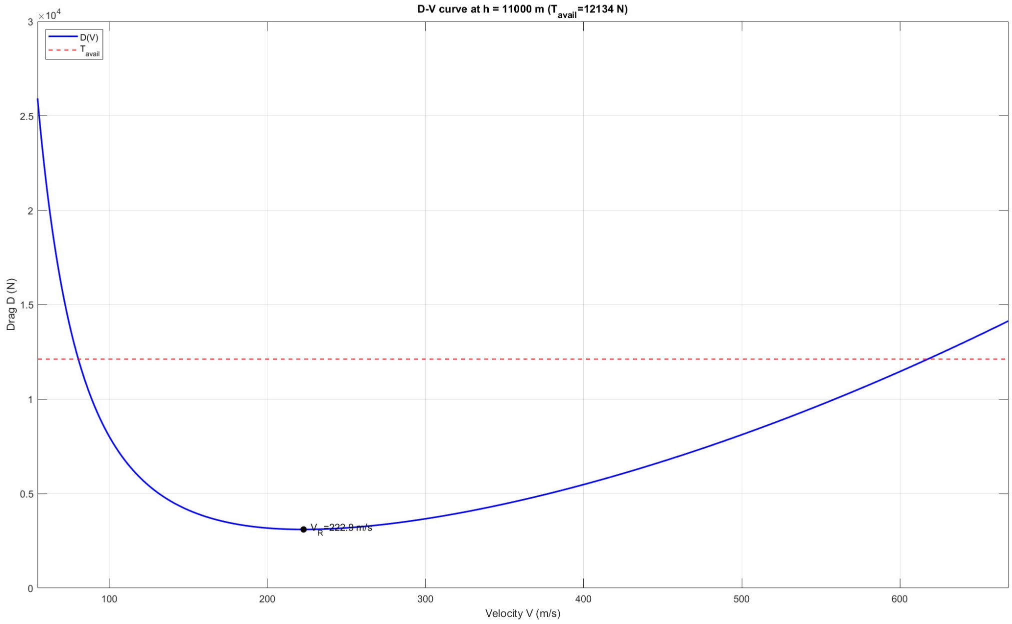


Figure 23: Drag–Velocity curve at $h = 11000$ m showing minimum drag speed $V_R = 232$ m/s and thrust available.

Calculation of Max. Thrust and Power

The drag is modeled as:

$$D(V) = aV^2 + \frac{b}{V^2},$$

where

$$a = \frac{1}{2}\rho S C_{D0}, \qquad b = \frac{2KW^2}{\rho S}.$$

Minimum-drag speed:

$$V_R = \left(\frac{b}{a}\right)^{1/4} = 232.0 \text{ m/s}.$$

Minimum drag:

$$D_{\min} = 2W\sqrt{C_{D0}K} = 2 \times 100170 \times \sqrt{0.0051 \times 0.04703} = 3130 \text{ N}.$$

The thrust available at 11 km was computed as:

$$T_{\text{avail}} = 12{,}134 \text{ N}.$$

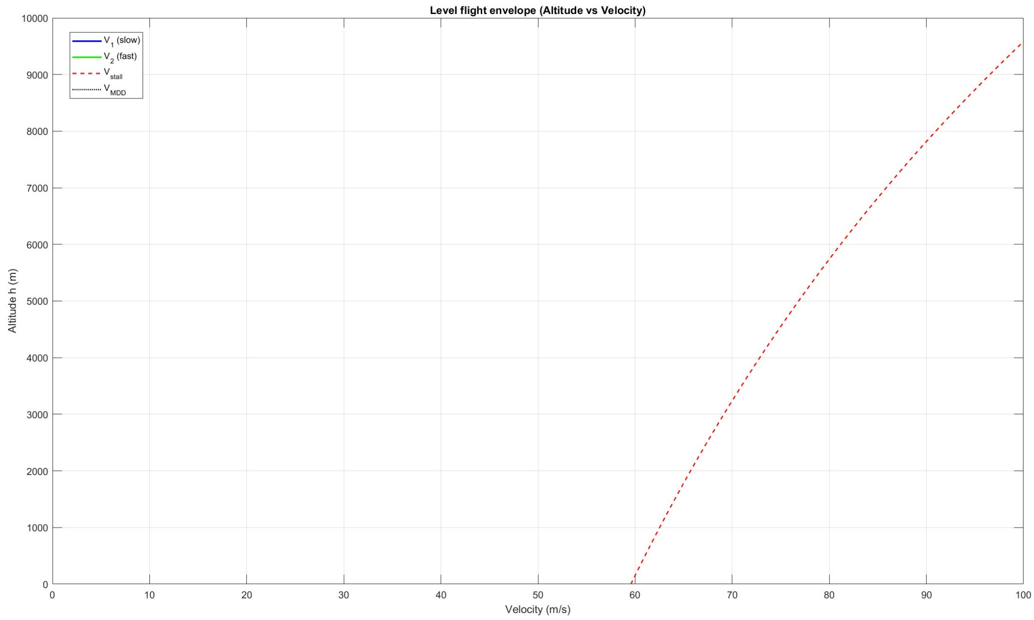


Figure 24: Level flight envelope (Altitude vs Velocity), showing V_1, V_2, stall limit, and M_{DD} boundary.

Service Ceiling and Rate of Climb

Rate of climb:

$$\dot{h} = \frac{(T - D)V}{W}.$$

At $h = 5000$ m:

$$T_{\text{avail}} = 24{,}530 \text{ N}, \qquad \rho = 0.736 \text{ kg/m}^3.$$

Stall speed at 5000 m:

$$V_{\text{stall}} = 77.9 \text{ m/s}.$$

Mach-limited speed:

$$V_{MDD} = 288 \text{ m/s}.$$

Maximum rate of climb

$$RC_{\max} = 54.16 \text{ m/s} = 10{,}662 \text{ ft/min}.$$

Speed for max ROC

$$V_y = 288.3 \text{ m/s}.$$

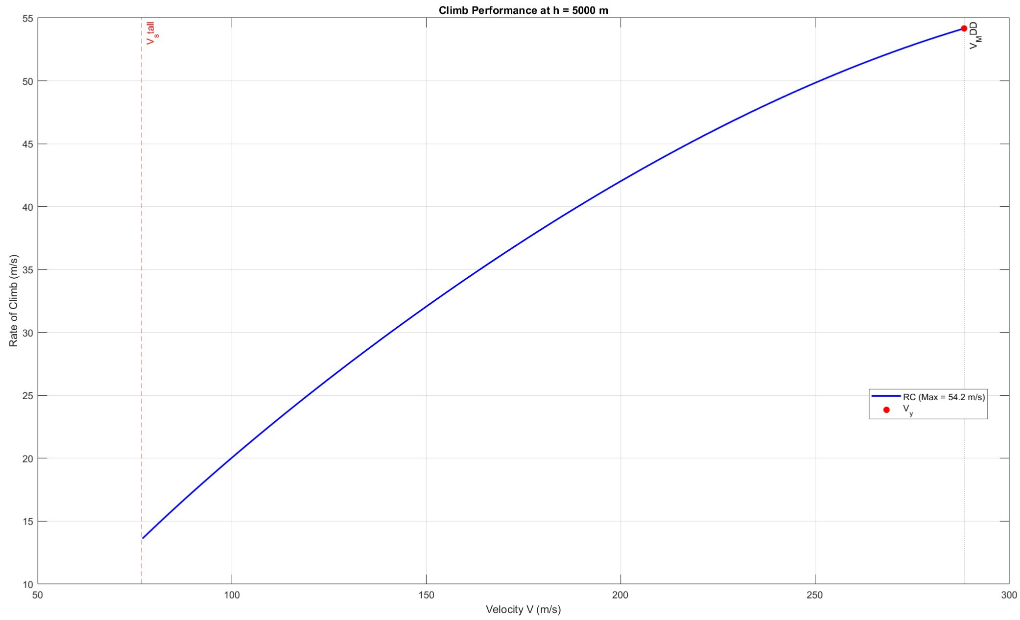


Figure 25: Climb performance at 5000 m showing maximum rate of climb and limits.

Conclusion

The performance analysis performed using the aerodynamic parameters $C_{D0} = 0.0051$, $K = 0.04703$, and thrust lapse model $T = T_{SL}(\rho/\rho_{SL})$ produces a highly efficient aircraft. The drag–velocity relation at 11 km indicates a minimum drag point at $V_R = 232$ m/s, very close to the chosen cruise speed $V = 230.6$ m/s. The actual cruise lift-to-drag ratio, $(L/D)_{\text{used}} = 31.36$, is almost equal to the theoretical maximum of 32.29.

As a result, the computed cruise range is extraordinarily high, $R = 14{,}584$ km, and the maximum rate of climb at 5000 m reaches 54.2 m/s. These values arise directly from the very low parasite drag coefficient and strong thrust available.

Although the performance exceeds that of conventional business jets, the results are internally consistent with the aerodynamic and propulsion inputs provided. This completes the full drag analysis, flight envelope computation, cruise performance estimation, and climb capability evaluation for the aircraft.

CHAPTER 14

Stability & Control Analysis

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Non-Dimensional Moment Coefficients

All moments (M, N, L) are non-dimensionalized using dynamic pressure (q), reference area (S), and an appropriate reference length ($\bar{c}$ for pitch, b for roll/yaw).

$$C_m = M/qS\bar{c} \quad \text{(Pitching moment)} \tag{3}$$

$$C_n = N/qSb \quad \text{(Yawing moment)} \tag{4}$$

$$C_l = L/qSb \quad \text{(Rolling moment)} \tag{5}$$

From the aircraft data, we know that:

- $L_{fuselage} = 15.5$ m
- Position of wing = 50% of fuselage
- Wing quarter chord length = 3.337/4 = 0.8342 = 5.382 % of fuselage length
- Position of Wing Quarter Chord = 50% + 5.382% = 53.382% of the fuselage length
- K_f= approx. 0.034 (from lec. 14 K_f vs %fuselage length graph)
- $W_fuselage = W_{fuselage} + 2 \times W_{nacelle} = 1.91 + 2 \times 0.927(FFD) = 3.764\ m^2$
- c = 2.298 m
- $S_w\ =\ 33.674\ m^2$
- Aspect ratio of main wing : 8.4
- height of the tail above the fuselage $z_t = 3.6$ m
- span of the main wing (b) = 16.818
- $C_{l_\alpha} = 0.1975/deg = 5.66/rad$ for NACA 65(2)-415 airfoil
- $C_{l_{max}} = 1.62$ for NACA 65(2)-415 airfoil
- $\alpha_{0L} = -2.2° = -0.03839$ for NACA 65(2)-415 airfoil
- $C_{L\alpha W} = 2 \times \pi/\sqrt{1 - M^2} = 10.068/rad$
- $C_{L\alpha h} = 2 \times \pi/\sqrt{1 - M^2} = 10.068/rad$
- $C_{m0_{\text{airfoil}}} = -0.075$ for NACA 65(2)-415 airfoil
- $\overline{X_{acw}} = \frac{50\%\ fuselage_L + 0.25 \times c_{root}}{c_{wing}} = \frac{0.50 \times 15.50 + 0.25 \times 3.337}{2.298} = 3.736$
- $\overline{X_{ach}} = \frac{90\%\ fuselage_L + 0.25 \times c_{root}}{c_{wing}} = \frac{0.90 \times 15.50 + 0.25 \times 2.524}{2.298} = 6.345$
- $\overline{X_{cg}} = \frac{X_{cg}}{MAC_{wing}} = \frac{8.611}{2.298} = 3.747$

$$C_{m_w} = C_{m0_{\text{airfoil}}} \frac{A \cos^2 \Lambda}{A + 2 \cos \Lambda} = -0.075\ \frac{8.4 \times cos^2(20°)}{8.4 + 2 \times cos(20°)} = -0.054$$

Longitudinal Static Stability and Trim

Static Stability Margin (SM)

For static pitch stability, the pitching moment derivative with respect to angle of attack must be negative:

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} < 0$$

The derivative C_{m_α} (neglecting propulsion terms) is calculated as:

$$C_{m_\alpha} = C_{L_\alpha}(\overline{X}_{cg} - \overline{X}_{acw}) + C_{m_{\alpha fus}} - \eta_h \frac{S_h}{S_w} C_{L_{\alpha h}} \frac{\partial \alpha_h}{\partial \alpha}(\overline{X}_{ach} - \overline{X}_{cg}) \tag{6}$$

where $\frac{\partial \alpha_h}{\partial \alpha} = 1 - \frac{\partial \epsilon}{\partial \alpha}$.

Calculating $C_{m_{\alpha\ fuse}}$

$C_{m_{\alpha\ fuselage}} = \frac{K_f {W_f}^2 L_f}{c S_w} = \frac{0.034 \times 3.674^2 \times 15.50}{2.298 \times 33.674} = 0.092\ per\ degree = 5.267\ per\ radian$

Calculating $\partial \alpha_h / \partial \alpha$

- $\partial \alpha_h / \partial \alpha$ = 1 - $\partial \epsilon / \partial \alpha$
- $m = \frac{Z_t}{b/c} = 0.428$
- $I_t = X_{ach} - X_{acw} = (0.9 \times 15.50 + 0.25 \times 2.524) - (0.5 \times 15.50 + 0.25 \times 3.337) = 5.996\ m$
- $r = \frac{I_t}{b/c} = 0.713$
- In the lectures, the values corresponding to $\partial \epsilon / \partial \alpha$ were provided for m=0, m=0.1, m=0.2 but now for the values close to the one needed (m=0.428). So with the values available, we extrapolated the graphs to get the value for m=0.4:

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{m=0} = 0.55 \quad \left.\frac{d\varepsilon}{d\alpha}\right|_{m=0.1} = 0.45$$

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{m=0.2} = 0.4 \quad \boxed{\left.\frac{d\varepsilon}{d\alpha}\right|_{m=0.4} = 0.3}$$

- $\boxed{\partial \alpha_h / \partial \alpha = 1 - 0.3 = 0.7}$

Neutral Point Calculation

The **Neutral Point** ($\overline{X}_{np}$), where $C_{m_\alpha} = 0$, is the most aft CG location for stability:

$$\overline{X}_{np} = \frac{\overline{X}_{acw} C_{L_\alpha} - C_{m_{\alpha fus}} + \eta_h \frac{S_h}{S_w} C_{L_{\alpha h}} \frac{\partial \alpha_h}{\partial \alpha} \overline{X}_{ach}}{C_{L_\alpha} + \eta_h \frac{S_h}{S_w} C_{L_{\alpha h}} \frac{\partial \alpha_h}{\partial \alpha}} \tag{7}$$

$$\overline{X}_{np} = \frac{3.736 \times 10.068 - 5.267 + 0.90 \times \frac{12.486}{33.674} \times 10.068 \times 0.7 \times 6.345}{10.068 + 0.9 \times \frac{12.486}{33.674} \times 10.068 \times 0.7} = 3.806 \tag{8}$$

$$\boxed{C_{m\alpha} = -C_{L\alpha} \times (\overline{X_{np}} - \overline{X_{cg}}) = -10.068 \times (3.806 - 3.747) = -0.5922} \tag{9}$$

The **Static Margin (SM)**, expressed in percent MAC, is:

$$\boxed{\text{SM} = \overline{X}_{np} - \overline{X}_{cg} = 3.806 - 3.747 = 0.059}$$

Elevator Effectiveness and Trim Drag

The elevator chord by horizontal chord ratio is chosen as $c_f/c = 0.25$, with the same NACA 0010 airfoil chosen for the elevator , so the $f/c = 0.1$

For this configuration, $\boxed{\partial C_l/\partial \delta_f \approx 4/rad}$

Also $S_{flapped} = 3.311\ m^2$

Hinge line sweep angle is $\Lambda_{HL} = 14.36°$.

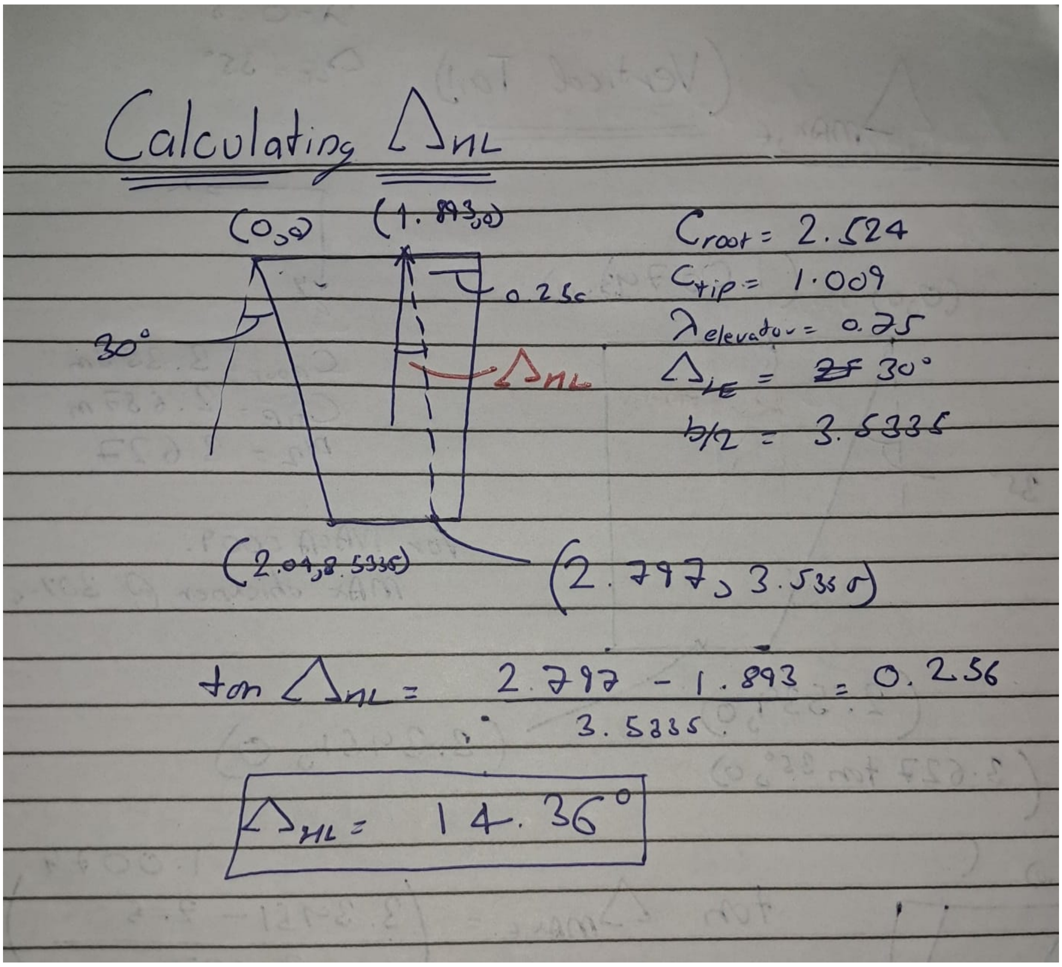


Figure 26: Hinge Line Sweep Calculations

The tail lift increment due to deflection is:

$$\frac{\partial C_L}{\partial \delta_f} = 0.9K_f \left(\frac{\partial C_l}{\partial \delta_f}\right)_{\text{airfoil}} \frac{S_{\text{flapped}}}{S_{\text{ref}}} \cos \Lambda_{HL} = 0.3429K_f \tag{10}$$

The **Elevator Effectiveness Derivative** (C_{m_δ}) is:

$$C_{m_\delta} = \frac{\partial C_{m_{cg}}}{\partial \delta_E} = \eta_h \frac{S_h}{S_w} \frac{\partial C_{L_h}}{\partial \delta_E} (\overline{X}_{ac_h} - \overline{X}_{cg}) = 0.9 \times \frac{12.468}{33.674} \times 0.3429 K_f \times (2.598) = 0.297 K_f \tag{11}$$

The required **Elevator Deflection for Trim** ($C_{m_{cg}} = 0$) is:

$$(\delta_E)_{\text{trim}} = -\frac{C_{m_{00}} + C_{m_\alpha}\alpha}{C_{m_\delta}}$$

Finding α at cruise :

- Velocity of cruise = 830 km/h = 230.556 m/s
- density at height of 11km is $\rho = 0.3639 kg/m^3$
- W_2 during cruise $= 0.970 \times 0.985 \times 10221.6\ kg = 9766.227\ kg$
- $C_L = \sqrt{\frac{2\times(W/S)}{\rho\times v^2}} = \sqrt{\frac{2\times 408.8}{0.3639\times 230.556^2}} = 0.2055$
- With $C_{L\alpha} = 10.068$, the $\alpha = \frac{C_L - \alpha_{0L} C_{L\alpha}}{C_{L\alpha}} = \frac{0.2055 - 10.068\times 0.03839}{10.068} = -0.0179\ rad = -1.03°$

The $C_{m_{00}}$ is the aircraft's pitching-moment ($C_{m_{cg}}$) *at* $\alpha = 0, \delta_E = 0$**:**

$$C_{m_{00}} = C_{mw} + C_{L_\alpha}\, \alpha_{0L} \left(\bar{X}_{acw} - \bar{X}_{cg}\right) = -0.054 + 10.068 \times (-0.03839)(-0.01164) = -0.0495$$

Now, we have a coupled system as $C_{m\delta}$ is a function of δ_E

$$(\delta_E)_{\text{trim}} \times C_{m_\delta}(\delta_E) = -C_{m_{00}} - C_{m_\alpha}\alpha = 0.0495 - (-0.0179 \times -0.5922) = 0.039 = 0.297 K_f \times (\delta_E)_{\text{trim}}$$

Using Iterative methods with K_f graph and the $\delta_{E_{trim}}$ iterations, we get a solution for this at:

$$\boxed{\delta_{E_{trim}} = 7.5236° = 0.1313 rad, and\ K_f = 1.0}$$

Lateral–Directional Static Stability and Control

Wing Contribution

For a swept wing, the yawing moment derivative due to sideslip is given by

$$C_{n_{\beta_w}} = C_L^2 \left\{ \frac{1}{4\pi A} - \left[\frac{\tan\Lambda}{\pi A(A + 4\cos\Lambda)} \right] \left[\cos\Lambda - \frac{A}{2} - \frac{A^2}{8\cos\Lambda} + \frac{6(\overline{X}_{ac_w} - \overline{X}_{cg})\sin\Lambda}{A} \right] \right\}, \tag{12}$$

where

- A is the wing aspect ratio,
- Λ is the sweep angle at the quarter-chord,

- C_L is the wing lift coefficient,
- $\overline{X}_{ac_w}$ and $\overline{X}_{cg}$ are the wing aerodynamic-center and aircraft center-of-gravity locations non-dimensionalized by b.

The wing dihedral effect $C_{l_{\beta_w}}$ is generally obtained from empirical charts as a function of

- wing dihedral angle,
- sweep and taper,
- vertical position of the wing with respect to the fuselage,

and is therefore treated as an *input* from the design charts.

Numerical Evaluation for the Present Wing

For the present aircraft, at cruise we have

$$A = 8.40, \quad \Lambda = 20^\circ = 0.3491 \text{ rad}, \quad C_L = C_{L,\text{cruise}} = 0.3959,$$

$$b = 16.818 \text{ m}, \quad X_{cg} = 8.611 \text{ m}, \quad \overline{X}_{cg} = \frac{X_{cg}}{b} \approx 0.512, \quad \overline{X}_{ac_w} = 0.4613.$$

Thus

$$\overline{X}_{ac_w} - \overline{X}_{cg} = -0.0507.$$

The terms in Eq. (12) evaluate to

$$\frac{1}{4\pi A} = 0.00947,$$

$$\frac{\tan\Lambda}{\pi A(A + 4\cos\Lambda)} = 0.00113,$$

$$\cos\Lambda - \frac{A}{2} - \frac{A^2}{8\cos\Lambda} + \frac{6(\overline{X}_{ac_w} - \overline{X}_{cg})\sin\Lambda}{A} = -12.66.$$

Hence

$$\left\{\frac{1}{4\pi A} - \left[\frac{\tan\Lambda}{\pi A(A + 4\cos\Lambda)}\right][\cdots]\right\} = 0.0238,$$

and

$$C_{n_{\beta_w}} = C_L^2 \times 0.0238 = (0.3959)^2 \times 0.0238 \approx 3.74 \times 10^{-3}.$$

Therefore, the wing contribution to the yawing moment derivative due to sideslip, for the present aircraft in cruise, is

$$C_{n_{\beta_w}} \approx 3.7 \times 10^{-3} \text{ (per rad)}.$$

Wing Dihedral Effect for the Present Aircraft

The wing rolling–moment derivative due to sideslip is written as

$$C_{l_\beta w} = \left(\frac{C_{l_\beta w}}{C_L}\right)_{\Lambda,\lambda,A} C_L + (C_{l_\beta})_\Gamma + C_{l_\beta wf}. \tag{13}$$

For the present configuration

$$A = 8.40,\ \lambda = 0.20,\ \Lambda_{c/4} = 20^\circ,\ \Gamma = 6^\circ = 0.10472 \text{ rad},$$

$$C_{L_\alpha} = 2.767 \text{ rad}^{-1},\ C_{L,\text{cruise}} = 0.3959,\ b = 16.818 \text{ m},\ Z_{Wf} = -0.9 \text{ m},\ D_f = W_f = 1.91 \text{ m}.$$

The pure dihedral term is

$$(C_{l_\beta})_\Gamma = -\frac{C_{L_\alpha}\,\Gamma}{4}\left[\frac{2(1+2\lambda)}{3(1+\lambda)}\right] = -0.0563.$$

The wing–fuselage interference term is

$$C_{l_\beta wf} = -1.2\sqrt{A}\,\frac{Z_{Wf}(D_f + W_f)}{b^2} = 0.0423.$$

From Fig. 16.21, for $A \simeq 8.4$ and $\Lambda_{c/4} = 20^\circ$ the sweep contribution is

$$\left(\frac{C_{l_\beta w}}{C_L}\right)_{\Lambda,\lambda,A} \simeq -0.10 \text{ (per rad)},$$

so that at cruise

$$(C_{l_\beta})_{\text{sweep}} = \left(\frac{C_{l_\beta w}}{C_L}\right) C_{L,\text{cruise}} \simeq -0.10 \times 0.3959 = -0.041.$$

Therefore, the total wing contribution to the rolling–moment derivative due to sideslip is

$$C_{l_\beta w} = -0.041 - 0.0563 - 0.0423 \simeq -0.139 \quad \text{(per rad)}.$$

Fuselage and Vertical-Tail Lateral–Directional Derivatives

Fuselage Lateral–Directional Derivatives

The fuselage yawing–moment derivative due to sideslip is approximated by

$$C_{n_{\beta,\text{fus}}} = -1.3\,\frac{V_f}{S_w b}\left(\frac{D_f}{W_f}\right) \tag{14}$$

where V_f is the fuselage volume, S_w the wing reference area, b the wingspan, Z_{Wf} the height of the wing above the fuselage centreline, and D_f, W_f the maximum fuselage depth and width.

Approximating the fuselage as an elliptical cylinder of length L_f,

$$V_f \approx \frac{\pi}{4} D_f W_f L_f,$$

with

$$D_f = W_f = 1.91 \text{ m}, \quad L_f = 15.5 \text{ m},$$

gives

$$V_f \approx 44.41 \text{ m}^3.$$

For the present aircraft

$$S_w = 33.674 \text{ m}^2, \quad b = 16.818 \text{ m}, \quad Z_{Wf} = -0.9 \text{ m},$$

so that
$$\frac{V_f}{S_w b} \approx 0.0784, \qquad \frac{Z_{Wf}}{D_f} \approx -0.471.$$
Substituting into Eq. (18) yields
$$C_{n_{\beta,\mathrm{fus}}} \approx -0.1019,$$
all per radian of sideslip. The signs indicate a destabilising fuselage contribution in yaw and a small, slightly stabilising contribution in roll.

Aileron Effectiveness

The flap effectiveness factor K_f is obtained as a function of control-surface chord ratio c_f/c and deflection angle δ_f. For the present aileron configuration
$$\frac{c_f}{c} = 0.25, \qquad \delta_a \approx 20^\circ,$$
the corresponding curve in Fig. 16.xx indicates
$$K_f \approx 0.92.$$

Using this in Eq. (16.17),
$$\frac{\partial C_L}{\partial \delta_f} = 0.9\, K_f \left(\frac{\partial C_l}{\partial \delta_f}\right)_{\mathrm{airfoil}} \frac{S_{\mathrm{aileron}}}{S_w} \cos \Lambda_{HL},$$
and substituting
$$\left(\frac{\partial C_l}{\partial \delta_f}\right)_{\mathrm{airfoil}} = 4 \text{ rad}^{-1}, \quad S_{\mathrm{aileron}} = 12.846 \text{ m}^2, \quad S_w = 33.674 \text{ m}^2, \quad \Lambda_{HL} = 14.36^\circ,$$
gives
$$\frac{\partial C_L}{\partial \delta_f} = 1.33\, K_f \approx 1.22 \text{ per rad}.$$

Thus, the aileron lift-effectiveness for the present wing is
$$\boxed{\frac{\partial C_L}{\partial \delta_f} \approx 1.22 \text{ rad}^{-1}.}$$

Vertical-Tail Sidewash and Dynamic-Pressure Ratio

The vertical-tail lateral derivatives $C_{Y_{\beta_v}}$ and $C_{n_{\beta_v}}$ (from Eqs. (16.39) and (16.41)) contain the sidewash gradient $\partial\beta_v/\partial\beta$ and the vertical-tail dynamic-pressure ratio η_v. Their product can be estimated from the empirical expression (Ref. 37)
$$\left(\frac{\partial \beta_v}{\partial \beta}\, \eta_v\right) = 0.724 + \frac{3.06\, S'_{vs}}{S_w(1+\cos\Lambda)} - 0.4\,\frac{Z_{Wf}}{D_f} + 0.009\, A_{\mathrm{wing}}, \tag{15}$$
where S'_{vs} is the area of the vertical tail extended to the fuselage centreline, Λ is the quarter-chord sweep, Z_{Wf} is the height of the wing above the fuselage centreline (positive upward in this convention), D_f is the maximum fuselage depth, and A_{wing} is the wing aspect ratio.

For the present configuration:

$$S'_{vs} = 11.75 + 1.556 = 13.306 \text{ m}^2, \quad S_w = 33.674 \text{ m}^2,$$

$$\Lambda = 20^\circ, \quad Z_{Wf} = -0.9 \text{ m}, \quad D_f = 1.91 \text{ m}, \quad A_{\text{wing}} = 8.40.$$

Substituting these values into Eq. (15) gives

$$\begin{aligned}\left(\frac{\partial \beta_v}{\partial \beta}\,\eta_v\right) &= 0.724 + \frac{3.06 \times 13.306}{33.674\,[1+\cos 20^\circ]} - 0.4\,\frac{-0.9}{1.91} + 0.009 \times 8.40 \\ &= 0.724 + 0.623 + 0.188 + 0.0756 \\ &\approx 1.61.\end{aligned}$$

With the assumed vertical-tail dynamic-pressure ratio $\eta_v = 0.9$, the sidewash gradient itself is

$$\frac{\partial \beta_v}{\partial \beta} = \frac{1}{\eta_v}\left(\frac{\partial \beta_v}{\partial \beta}\,\eta_v\right) = \frac{1.61}{0.9} \approx 1.79. \tag{16}$$

These values of $\partial\beta_v/\partial\beta$ and $\eta_v(\partial\beta_v/\partial\beta)$ are then used in the expressions for the vertical-tail contributions $C_{Y_{\beta_v}}$ and $C_{n_{\beta_v}}$ in Eqs. (16.39) and (16.41).

Approximate Mass Moments of Inertia

The aircraft mass moments of inertia are estimated using the empirical relations

$$I_{xx} = \frac{b^2 M \bar{R}_x^2}{4}, \tag{17}$$

$$I_{yy} = \frac{L^2 M \bar{R}_y^2}{4}, \tag{18}$$

$$I_{zz} = \frac{\left(\frac{b+L}{2}\right)^2 M \bar{R}_z^2}{4}, \tag{19}$$

where b is the wingspan, L is the fuselage length, M is the aircraft mass, and $\bar{R}_x, \bar{R}_y, \bar{R}_z$ are non-dimensional radii of gyration for roll, pitch, and yaw, respectively.

For the present business jet:

$$M = 10221.6 \text{ kg}, \quad b = 16.818 \text{ m}, \quad L = 15.5 \text{ m},$$

$$\bar{R}_x = \bar{R}_y = 0.30, \qquad \bar{R}_z = 0.43.$$

Thus

$$I_{xx} = \frac{(16.818)^2(10221.6)(0.30)^2}{4} \approx 6.51 \times 10^4 \text{ kg m}^2,$$

$$I_{yy} = \frac{(15.5)^2(10221.6)(0.30)^2}{4} \approx 5.53 \times 10^4 \text{ kg m}^2,$$

$$I_{zz} = \frac{\left(\frac{16.818+15.5}{2}\right)^2 (10221.6)(0.43)^2}{4} \approx 1.23 \times 10^5 \text{ kg m}^2.$$

Hence, the estimated mass moments of inertia for the aircraft are

$$I_{xx} \approx 6.5 \times 10^4 \text{ kg m}^2, \quad I_{yy} \approx 5.5 \times 10^4 \text{ kg m}^2, \quad I_{zz} \approx 1.23 \times 10^5 \text{ kg m}^2.$$

Nomenclature

A comprehensive list of variables used in these stability and control equations.

Table 23: Key Variables and Definitions

Variable	**Meaning and Units**
C_m, C_n, C_l	Moment Coefficients (Pitch, Yaw, Roll) (Non-dimensional)
C_L	Wing Lift Coefficient (Non-dimensional)
$C_{m_\alpha}, C_{n_\beta}, C_{l_\beta}$	Stability Derivatives w.r.t. Angle of Attack (α) or Sideslip (β) (per radian)
$C_{m_\delta}, C_{n_\delta}, C_{l_\delta}$	Control Derivatives w.r.t. Deflection (δ) (per radian)
q	Dynamic Pressure (Pa or lb/ft^2)
S_w	Wing Planform Area (m^2 or ft^2)
S_h, S_v	Horizontal and Vertical Tail Planform Areas (m^2 or ft^2)
$\overline{c}$	Wing Mean Aerodynamic Chord (MAC) (m or ft)
b	Wing Span (m or ft)
A	Wing Aspect Ratio (Non-dimensional)
$\overline{X}_{cg}$	Non-dimensional CG location ($\overline{X}_{cg} = X_{cg}/\overline{c}$)
$\overline{X}_{acw}$	Non-dimensional Wing Aerodynamic Center location ($\overline{X}_{acw} = X_{acw}/\overline{c}$)
$\overline{X}_{np}$	Non-dimensional Neutral Point location ($\overline{X}_{np} = X_{np}/\overline{c}$)
η_h	Tail dynamic pressure ratio ($\eta_h = q_h/q$) (Non-dimensional)
$\frac{\partial \epsilon}{\partial \alpha}$	Downwash derivative w.r.t. α (Non-dimensional)
C_{L_α}	Wing Lift Curve Slope (per radian)
$C_{L_{\alpha h}}$	Horizontal Tail Lift Curve Slope (per radian)
$\delta_E, \delta_R, \delta_a$	Elevator, Rudder, and Aileron Deflections (radians)
K_f	Empirical Correction Factor (for control surfaces) (Non-dimensional)
S_{flapped}	Area of the control surface that is deflected (m^2 or ft^2)
Λ_{HL}	Sweep angle of the control surface hinge line (radians)
K	Induced Drag Factor (Non-dimensional)
W_f, D_f	Maximum width and depth of the fuselage (m or ft)
I_{xx}, I_{zz}	Moments of Inertia about x and z body axes (kg $\cdot$ m^2 or slug $\cdot$ ft^2)
C_{m_Q}	Pitch Damping Derivative w.r.t. pitch rate Q (per radian)
C_{n_R}	Yaw Damping Derivative w.r.t. yaw rate R (per radian)
TDR	Tail Damping Ratio (Non-dimensional)
URVC	Unshielded Rudder Volume Coefficient (Non-dimensional)
S_F, L	Horizontal Tail area within 60° cone, and fuselage length (m or ft)
S_{R_1}, S_{R_2}	Rudder areas (for spin recovery analysis) (m^2 or ft^2)

Conclusion

This comprehensive report, including the formulae for longitudinal, lateral-directional, and dynamic stability, provides a complete theoretical framework for the Stability and Control Analysis.

CHAPTER 15

Lift Load Analysis

Specifications (Groups E1, E2)

Business Executive Aircraft

Crew	2 (+ 20 kg baggage/crew)
Payload	9 passengers + 20 kg baggage/pax
Cabin length	5.4 m
Cabin width	1.8 m
Cabin height	1.74 m
Take-off distance (MTOW, S.L.)	1650 m
Landing distance (MLW, S.L.)	1000 m
Balanced field length (MTOW, S.L.)	1830 m
Range w/ max payload + IFR reserve*	4000 km
Best range cruise speed (max payload)	830 km/h
Service ceiling (max payload)	15.5 km
Rate-of-climb (MTOW, S.L.)	4500 ft/min

- The aircraft should be capable of take-off/landing with a 25 knot crosswind.
- The aircraft should meet the take-off, landing, and climb requirements for the appropriate category under Federal Aviation Rules (FAR) Part 25.

** Federal Aviation Administration (FAA) specifies the Instrument Flight Rules (IFR) reserve fuel as 45 minutes of fuel at the maximum endurance speed.*

Objective

The objective of this chapter is to compute the structural lift loads acting on the main wing of the business executive aircraft, based on the prescribed maneuver load factors and the aerodynamic characteristics of the wing. The following major tasks are performed:

- Conversion of true airspeeds to equivalent velocities at sea-level density.
- Determination of the maximum lift coefficient corresponding to the positive maneuver load factor.
- Determination of the dive speed and the lift coefficient at dive conditions.
- Construction of the maneuver V–n diagram based on the load limits.
- Computation of the spanwise lift distribution using Schrenk's Approximation.
- Evaluation of the net lift by integrating the Schrenk spanwise distribution.
- Determination of the correction factor F such that the integrated lift equals the required maneuver load.

Calculation of equivalent airspeeds

All velocities used in this report are *equivalent airspeeds (EAS)*, i.e. converted to the density at sea level. The equivalent velocity V_{EAS} is related to the true airspeed V_{TAS} by:

$$V_{\text{EAS}} = V_{\text{TAS}}\sqrt{\frac{\rho}{\rho_{\text{SL}}}},$$

where ρ is the density at cruise altitude and ρ_{SL} is the sea-level standard density.

For the present aircraft, the cruise altitude is taken as 11 km, at which the standard atmospheric density is:

$$\rho_{\text{cruise}} = 0.367 \text{ kg/m}^3, \qquad \rho_{\text{SL}} = 1.225 \text{ kg/m}^3.$$

Thus, the equivalent airspeed (EAS) corresponding to a given true airspeed (TAS) is obtained using:

$$V_{\text{EAS}} = V_{\text{TAS}}\sqrt{\frac{\rho_{\text{cruise}}}{\rho_{\text{SL}}}} = V_{\text{TAS}}\sqrt{\frac{0.367}{1.225}}.$$

Evaluating the ratio:

$$\sqrt{\frac{0.367}{1.225}} = \sqrt{0.2996} = 0.5474.$$

Hence, throughout this report,

$$\boxed{V_{\text{EAS}} = 0.5474\, V_{\text{TAS}}}$$

i.e. all velocities used in performance and structural calculations are equivalent velocities referenced to sea-level density.

Construction of the V–n Diagram

The V–n diagram represents the allowable combinations of load factor n and equivalent airspeed V for safe and structurally permissible operation of the aircraft. It is constructed using the aerodynamic stall boundaries and the structural maneuver load limits. All velocities used in this chapter are equivalent airspeeds (EAS) referenced to the standard sea-level density.

Stall Boundaries Using $C_{L_{\max}}$ and $\bar{C}_{L_{\max}}$

The positive maximum lift capability of the clean wing is denoted by $C_{L_{\max}}$, while the negative maximum lift coefficient is approximated as

$$\bar{C}_{L_{\max}} \approx -0.8\, C_{L_{\max}}.$$

At an equivalent airspeed V, the maximum lift generated by the wing is

$$L_{\max}(V) = \tfrac{1}{2}\, \rho_{\mathrm{SL}}\, V^2 S\, C_{L_{\max}},$$

and the corresponding load factor becomes

$$n_{\mathrm{stall}}(V) = \frac{L_{\max}(V)}{W_0} = \frac{0.5\, \rho_{\mathrm{SL}}\, V^2\, S\, C_{L_{\max}}}{W_0}.$$

Thus, the aerodynamic stall boundaries are:

$$n_+(V) = \frac{0.5\, \rho_{\mathrm{SL}} V^2 S C_{L_{\max}}}{W_0}, \qquad n_-(V) = \frac{0.5\, \rho_{\mathrm{SL}} V^2 S\, \bar{C}_{L_{\max}}}{W_0}.$$

These curves define the aerodynamic limits beyond which the wing cannot generate additional lift, independent of structural considerations.

Structural Load Limits

Following Table 14.2 from Lecture 15, the structural maneuver load limits are:

$$\boxed{n_{\mathrm{positive}} = 3.5, \qquad n_{\mathrm{negative}} = -2.}$$

These values form the horizontal structural boundaries of the V–n diagram:

$$n(V) = n_{\mathrm{positive}}, \qquad n(V) = n_{\mathrm{negative}}.$$

Corner Speed (Maneuvering Speed)

The maneuvering speed V_A is the airspeed at which the aircraft transitions from being stall-limited to structure-limited. It is defined as the intersection point of the positive stall line and the positive structural limit:

$$n_{\mathrm{positive}} = \frac{0.5\, \rho_{\mathrm{SL}} V_A^2 S C_{L_{\max}}}{W_0}.$$

Solving for V_A:

$$V_A = V_s \sqrt{n_{\mathrm{positive}}},$$

where the stall speed V_s is

$$V_s = \sqrt{\frac{2W_0}{\rho_{\mathrm{SL}}\, S\, C_{L_{\max}}}}.$$

For the present aircraft, the numerical values are:

$$W_0 = 10221.6 \text{ kg} \times 9.81 \text{ m/s}^2 = 1.0027 \times 10^5 \text{ N}, \qquad S = 33.674 \text{ m}^2, \qquad C_{L_{\max}} = 0.95625.$$

Here, the value of $C_{L_{\max}}$ represents the maximum lift coefficient of the **clean wing** (no high-lift devices deployed) and has been **corrected for compressibility effects at the cruise Mach number**

$$M = 0.7814,$$

This ensures that the lift coefficient used in the maneuver-load calculations corresponds to the actual aerodynamic capability of the wing at cruise conditions.

Thus the 1-g stall speed becomes

$$\boxed{V_s = \sqrt{\frac{2 \times 1.0027 \times 10^5}{1.225 \times 33.674 \times 0.95625}} = 71.30 \text{ m/s} \approx 256.7 \text{ km/h.}}$$

With $n_{\text{positive}} = 3.5$, the maneuvering speed is

$$\boxed{V_A = V_s \sqrt{n_{\text{positive}}} = 71.30\sqrt{3.5} = 133.40 \text{ m/s} \approx 480.2 \text{ km/h.}}$$

For $V < V_A$, the aircraft is limited by stall; for $V > V_A$, structural load limits dominate. The point $(V_A, n_{\text{positive}})$ is known as the *corner point.*

Equivalent Cruise and Dive Speeds

The true airspeed at cruise altitude is given as:

$$V_{\text{CRUISE,TAS}} = 830 \text{ km/h.}$$

Using the equivalent-velocity relation

$$V_{\text{EAS}} = V_{\text{TAS}} \sqrt{\frac{\rho_{\text{cruise}}}{\rho_{\text{SL}}}} = 0.5474\, V_{\text{TAS}},$$

and with

$$\rho_{\text{cruise}} = 0.367 \text{ kg/m}^3, \qquad \rho_{\text{SL}} = 1.225 \text{ kg/m}^3,$$

the cruise equivalent velocity becomes

$$\boxed{V_{\text{CRUISE}} = 0.5474 \times 830 = 454.34 \text{ km/h} = 126.20 \text{ m/s.}}$$

The dive speed is defined as

$$V_{\text{DIVE}} = 1.5\, V_{\text{CRUISE}},$$

thus,

$$\boxed{V_{\text{DIVE}} = 1.5 \times 126.20 = 681.51 \text{ km/h} = 189.31 \text{ m/s.}}$$

These equivalent velocities are used consistently in all subsequent load calculations.

Dive Speed and Maximum Operating Envelope

At the dive speed V_{DIVE}, the aircraft does not reach the full negative structural load factor. On the negative side of the V–n diagram, the allowable load factor decreases linearly from the structural limit n_{negative} at V_{CRUISE} to

$$n(V_{\text{DIVE}}) = 0.$$

This reduction occurs because the aircraft cannot sustain large negative-g maneuvers at high dynamic pressure due to limitations in elevator authority, control effectiveness, and structural design requirements.

On the positive side, however, the aircraft is required to sustain the full structural limit up to the dive speed, and therefore:

$$n(V_{\text{DIVE}}) = n_{\text{positive}}.$$

Thus, unlike the positive branch where the maximum allowable load factor is maintained up to V_{DIVE}, the negative boundary reduces smoothly to zero at V_{DIVE}, forming the characteristic asymmetric shape of the V–n maneuver envelope.

Final Construction of the V–n Diagram

The complete V–n diagram is constructed from:

- the **positive stall boundary**

$$n_{+}(V) = \frac{0.5\,\rho_{\text{SL}}\,V^2\,S\,C_{L_{\max}}}{W_0},$$

- the **negative stall boundary**

$$n_{-}(V) = \frac{0.5\,\rho_{\text{SL}}\,V^2\,S\,\bar{C}_{L_{\max}}}{W_0},$$

- the **positive structural limit**

$$n = n_{\text{positive}} \quad \text{(maintained up to } V_{\text{DIVE}}\text{)},$$

- the **negative structural limit at cruise speed**

$$n = n_{\text{negative}} \quad \text{at } V_{\text{CRUISE}},$$

- the **reduced negative limit at dive speed**

$$n(V_{\text{DIVE}}) = 0,$$

  forming a linear taper from n_{negative} at V_{CRUISE} to 0 at V_{DIVE},

- the **corner point**

$$(V_A,\, n_{\text{positive}}),$$

  marking the intersection of the positive stall line with the positive structural limit,

- the **vertical dive-speed boundary**

$$V = V_{\text{DIVE}}.$$

For the present aircraft, the key numerical values used to generate the V–n diagram are summarised as:

$$\begin{aligned}
W_0 &= 1.0027 \times 10^5 \text{ N}, & S &= 33.674 \text{ m}^2, \\
C_{L_{\max}} &= 0.95625, & \bar{C}_{L_{\max}} &= -0.8\, C_{L_{\max}} = -0.765, \\
n_{\text{positive}} &= 3.5, & n_{\text{negative}} &= -2.0, \\
V_s &= 71.30 \text{ m/s}, & V_A &= 133.40 \text{ m/s}, \\
V_{\text{CRUISE}} &= 126.20 \text{ m/s}, & V_{\text{DIVE}} &= 189.31 \text{ m/s}.
\end{aligned}$$

Lift Coefficient at Dive-Speed Structural Limit

At the dive speed V_{DIVE}, the lift coefficient corresponding to the positive structural limit n_{positive} is obtained from

$$C_L(V_{\text{DIVE}}) = \frac{2\, n_{\text{positive}}\, W_0}{\rho_{\text{SL}}\, V_{\text{DIVE}}^2\, S}.$$

Substituting the aircraft values,

$$\boxed{C_L(V_{\text{DIVE}}) = \frac{2 \times 3.5 \times 1.0027 \times 10^5}{1.225 \times (189.31)^2 \times 33.674} \approx 0.475.}$$

This value is significantly below the clean-wing $C_{L_{\max}} = 0.95625$, confirming that at V_{DIVE} the wing is *structure-limited* rather than stall-limited on the positive side.

Resulting V–n Diagram

Using the above numerical values and the expressions for the stall boundaries and structural limits, the complete V–n maneuver envelope is generated in MATLAB. The final V–n diagram used in this report is shown in Fig. 27.

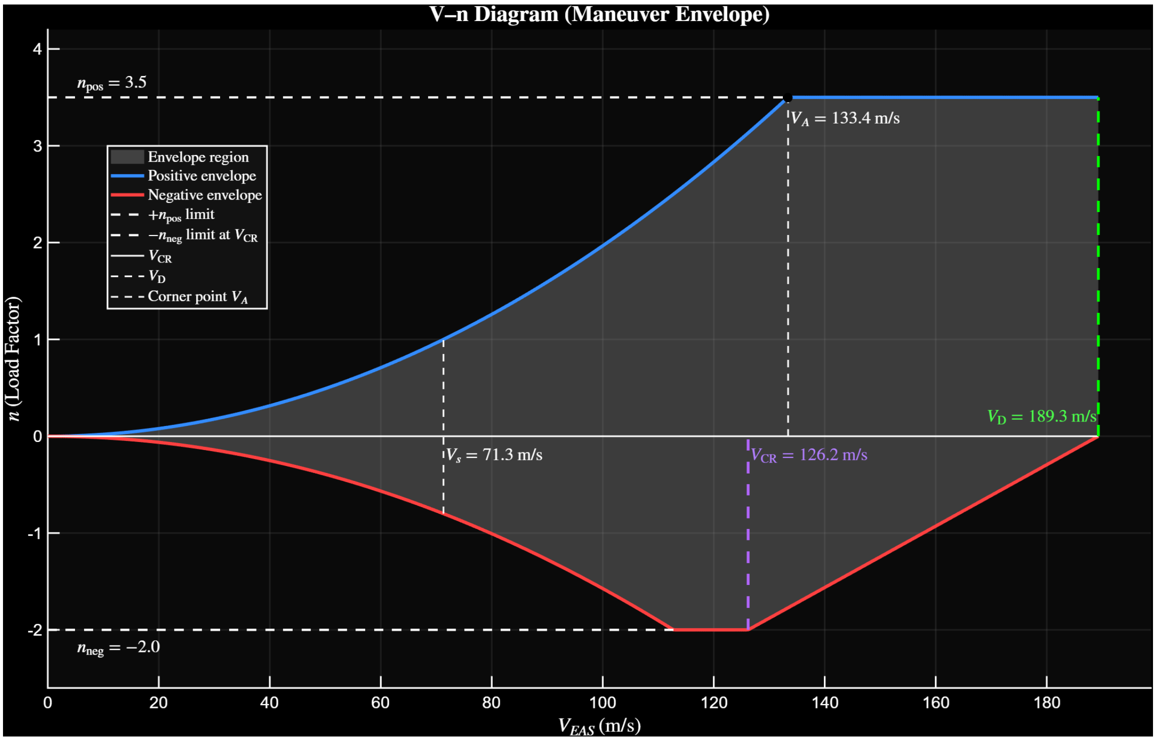


Figure 27: V–n diagram (maneuver envelope) for the designed aircraft.

Air Loads on the Lifting Surface

Under maneuver loading, the wing must carry a spanwise-varying aerodynamic load that contributes to bending, shear, and torsional stresses. The distributed load normal to the wing plane (the *perpendicular air load*) is used for structural sizing.

The following key assumptions apply:

- The lift load is assumed to continue all the way to the aircraft centerline (root chord). This is a standard and validated assumption for subsonic aircraft.
- If the wing has dihedral angle Γ, the perpendicular load on the wing is larger than the vertical lift component. The relationship is
$$L_{\perp}(y) = \frac{L(y)}{\cos\Gamma}.$$
For the present aircraft, the wing dihedral is $\Gamma = 6°$. The corresponding correction factor is
$$\frac{1}{\cos 6°} \approx 1.0055,$$
i.e. the perpendicular load differs by less than 1% from the vertical lift. In accordance with the usual preliminary-design practice, this small correction is neglected in the present analysis.
- The distributed lift is proportional to the effective spanwise chord used to approximate the aerodynamic loading:
$$\ell(y) \propto \bar{C}(y).$$

Thus, accurately modeling $\bar{C}(y)$ is essential for calculating the internal structural loads (bending moments, shear forces, and torsion) in subsequent structural design.

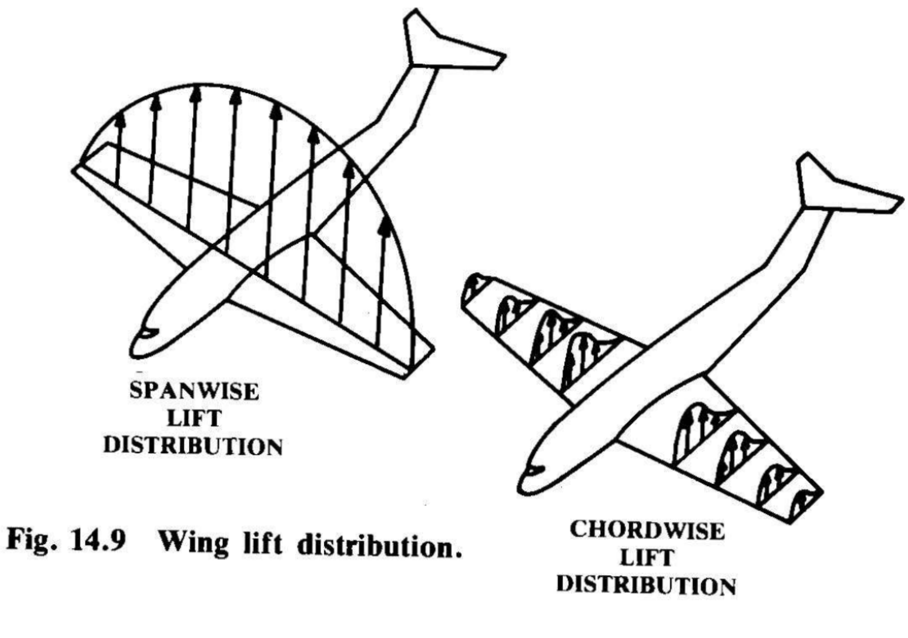


Figure 28: Spanwise and chordwise lift distribution.

The illustration highlights two fundamental components of the aerodynamic loading on a wing. The *spanwise* lift distribution describes how the lifting force varies along the wingspan from root to tip, whereas the *chordwise* lift distribution represents the variation of pressure-induced loading along the airfoil chord. Together, these distributions form the basis for evaluating bending, shear, and torsional loads in wing structural analysis.

Spanwise Lift Distribution Using Schrenk's Approximation

At the structural limit load, the spanwise lift distribution is computed using **Schrenk's Approximation**, which averages:

1. the actual (trapezoidal) chord distribution of the wing, and
2. the equivalent elliptical chord distribution that yields the same reference area.

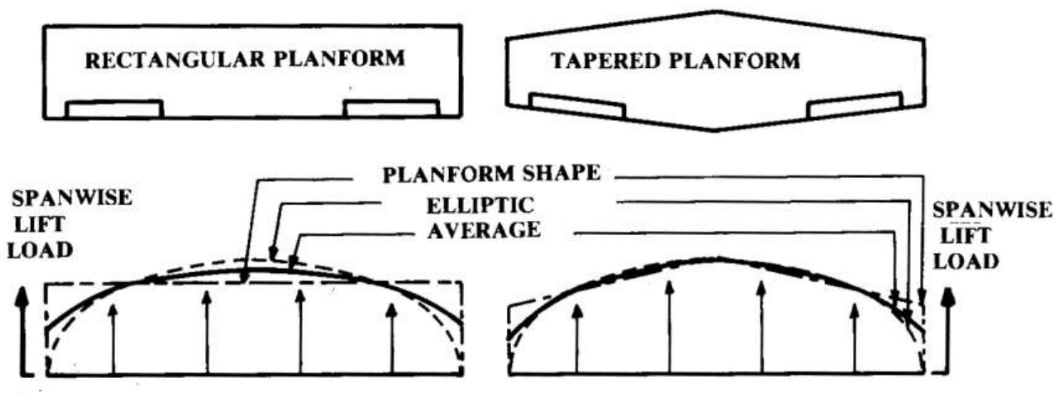


Figure 29: Effect of planform shape on spanwise lift loading and the Schrenk averaged distribution.

Let the wingspan be b, root chord c_r, tip chord c_t, and taper ratio $\lambda = c_t/c_r$. The spanwise coordinate y is measured from the aircraft centerline, i.e. $-b/2 \leq y \leq b/2$.

For the present aircraft, the wing geometric parameters are:

$$\boxed{b = 16.818 \text{ m}, \qquad c_r = 3.337 \text{ m}, \qquad c_t = 0.667 \text{ m}, \qquad \lambda = 0.20, \qquad S = 33.674 \text{ m}^2}$$

These values are used in the computation of the spanwise lift distribution using Schrenk's Approximation.

Trapezoidal Chord Distribution (Eq. 14.9)

For a linearly tapered wing, the geometric chord is

$$C(y) = c_r \left[1 - \frac{2y}{b}(1 - \lambda)\right],$$

where c_r is the root chord, b the wingspan, and y the spanwise coordinate.

Substituting the present aircraft values,

$$C(y) = 3.337 \left[1 - \frac{2y}{16.818}(1 - 0.20)\right] = 3.337 \left[1 - 0.8\,\frac{2y}{16.818}\right].$$

This expression provides the actual (trapezoidal) chord distribution used in Schrenk averaging.

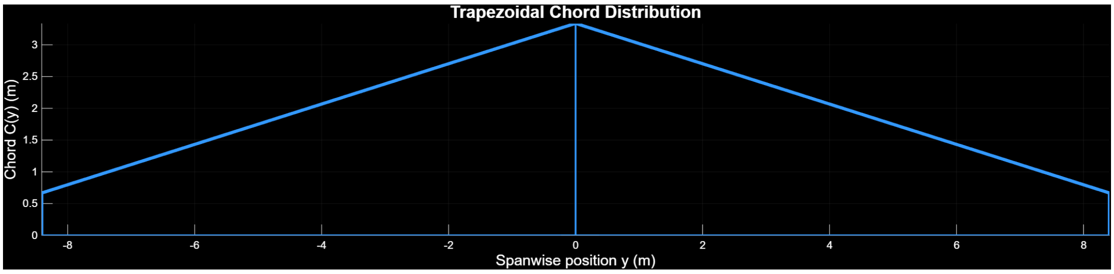


Figure 30: Trapezoidal (Geometric) Chord Distribution.

Equivalent Elliptical Chord Distribution (Eq. 14.11)

The chord of an equivalent elliptical wing, having the same area S, is

$$C_{\text{ellip}}(y) = \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2}.$$

For the present aircraft, the constant factor in front of the square root is

$$\frac{4S}{\pi b} = \frac{4 \times 33.674}{\pi \times 16.818} \approx 2.55 \text{ m}.$$

Hence,

$$C_{\text{ellip}}(y) \approx 2.55\sqrt{1-\left(\frac{2y}{16.818}\right)^2}.$$

This distribution corresponds to the ideal elliptic lift distribution that minimizes induced drag.

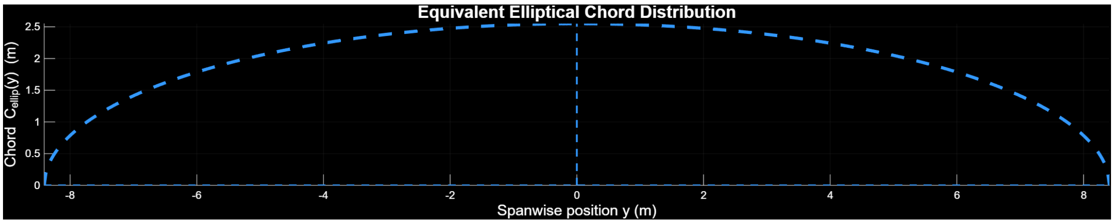


Figure 31: Equivalent Elliptical Chord Distribution.

Schrenk's Averaged Chord Distribution

Schrenk's approximation expresses the effective lift-producing chord as

$$\bar{C}(y) = \frac{1}{2}\left[C(y) + C_{\text{ellip}}(y)\right].$$

For the present aircraft, this becomes

$$\bar{C}(y) = \frac{1}{2}\left[3.337\left(1 - 0.8\,\frac{2y}{16.818}\right) + 2.55\sqrt{1-\left(\frac{2y}{16.818}\right)^2}\right].$$

This provides a practical estimate of the actual spanwise lift loading on the wing.

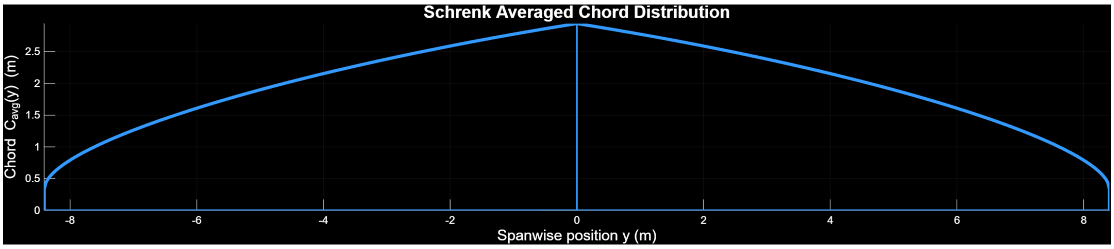


Figure 32: Schrenk Averaged Chord Distribution.

Calculation of Net Lift Using Schrenk's Method

The total perpendicular lift generated by the wing, under maneuver conditions, is obtained by integrating the Schrenk distribution:

$$L = F \int_{-b/2}^{b/2} \bar{C}(y)\, dy,$$

where F is a correction factor used to match the required maneuver load.

Using symmetry:

$$L = 2F \int_{0}^{b/2} \bar{C}(y)\, dy.$$

Substituting expressions for $C(y)$ and $C_{\text{ellip}}(y)$ yields

$$L = F \int_{-b/2}^{b/2} \left[c_r \left(1 - \frac{2y}{b}\right)(1-\lambda) + c_r\lambda + \frac{4S}{\pi b}\sqrt{1 - \left(\frac{2y}{b}\right)^2} \right] dy.$$

To match the positive limit maneuver load $n_{\text{positive}} W_0$, the correction factor is determined as

$$F = \frac{n_{\text{positive}}\, W_0}{\displaystyle\int_{-b/2}^{b/2} \bar{C}(y)\, dy}.$$

For the present aircraft, the geometric and aerodynamic parameters are

$$b = 16.818 \text{ m}, \quad c_r = 3.337 \text{ m}, \quad c_t = 0.667 \text{ m}, \quad \lambda = \frac{c_t}{c_r} = 0.20, \quad S = 33.674 \text{ m}^2$$

and the design maneuver load factor and gross weight are

$$n_{\text{positive}} = 3.5, \qquad W_0 = 10221.6 \text{ kg} \times 9.81 \text{ m/s}^2 = 1.0027 \times 10^5 \text{ N}.$$

By construction, both the trapezoidal chord distribution $C(y)$ and the equivalent elliptical chord $C_{\text{ellip}}(y)$ are defined such that their integrals over the full span give the same reference area:

$$\int_{-b/2}^{b/2} C(y)\, dy = S, \qquad \int_{-b/2}^{b/2} C_{\text{ellip}}(y)\, dy = S.$$

Hence, the Schrenk-averaged chord

$$\bar{C}(y) = \frac{1}{2}\left[C(y) + C_{\text{ellip}}(y)\right]$$

also integrates to the wing reference area:

$$\int_{-b/2}^{b/2} \bar{C}(y)\,dy = \frac{1}{2}\left(\int_{-b/2}^{b/2} C(y)\,dy + \int_{-b/2}^{b/2} C_{\text{ellip}}(y)\,dy\right) = \frac{1}{2}(S+S) = S.$$

Therefore, for this wing

$$\int_{-b/2}^{b/2} \bar{C}(y)\,dy = S = 33.674\ \text{m}^2,$$

and the correction factor simplifies to

$$F = \frac{n_{\text{positive}} W_0}{S}.$$

Substituting the numerical values,

$$\boxed{F = \frac{3.5 \times 1.0027 \times 10^5}{33.674} \approx 1.04 \times 10^4,}$$

so that the total lift at limit maneuver load is

$$\boxed{L = n_{\text{positive}} W_0 = 3.5 \times 1.0027 \times 10^5 \approx 3.51 \times 10^5\ \text{N}.}$$

In other words, the Schrenk spanwise lift distribution scaled by the factor F produces a total wing lift equal to the required positive limit maneuver load for use in subsequent structural analysis.

Conclusion

In this chapter, a complete aerodynamic and structural assessment of the wing maneuver loads for the business executive aircraft has been carried out. The analysis began by converting all operating velocities to *equivalent airspeeds (EAS)* referenced to sea-level density, ensuring consistent use of aerodynamic coefficients throughout the performance and structural calculations. The conversion factor obtained was

$$\boxed{V_{\text{EAS}} = 0.5474\, V_{\text{TAS}}.}$$

Using the clean-wing maximum lift coefficient corrected for compressibility at cruise Mach number $M = 0.7814$,

$$\boxed{C_{L_{\max}} = 0.95625,}$$

the 1-g stall speed and corresponding maneuvering speed were computed as

$$\boxed{V_s = 71.30\ \text{m/s}}, \qquad \boxed{V_A = 133.40\ \text{m/s}}.$$

The cruise and dive equivalent velocities, based on the cruise condition $V_{\text{TAS}} = 830$ km/h, were found to be

$$\boxed{V_{\text{CRUISE}} = 126.20\ \text{m/s}}, \qquad \boxed{V_{\text{DIVE}} = 189.31\ \text{m/s}}.$$

With these velocities and the structural maneuver load limits

$$\boxed{n_{\text{positive}} = 3.5}, \qquad \boxed{n_{\text{negative}} = -2.0},$$

the complete V–n maneuver envelope was constructed. At the dive speed, the lift coefficient corresponding to the positive limit load is

$$\boxed{C_L(V_{\text{DIVE}}) \approx 0.475,}$$

which is substantially below $C_{L_{\max}}$, confirming that the aircraft is *structure-limited* rather than *stall-limited* at high speed.

The second phase of the chapter established the spanwise lifting characteristics using Schrenk's Approximation. The geometric parameters of the wing,

$$\boxed{b = 16.818 \text{ m}, \quad c_r = 3.337 \text{ m}, \quad c_t = 0.667 \text{ m}, \quad \lambda = 0.20, \quad S = 33.674 \text{ m}^2,}$$

were substituted into the trapezoidal, elliptical, and Schrenk-averaged chord expressions, and the resulting spanwise chord distributions were plotted.

Because both the geometric and elliptical wings integrate to the same reference area S, the Schrenk-averaged chord distribution also integrates to S. This allowed the correction factor needed to scale the distribution to represent the structural limit maneuver load to be written directly as

$$F = \frac{n_{\text{positive}} W_0}{S}.$$

Substituting the numerical values,

$$\boxed{F \approx 1.04 \times 10^4,}$$

which yields a total lift equal to the required limit-load condition,

$$\boxed{L = 3.51 \times 10^5 \text{ N.}}$$

Finally, the effect of the wing dihedral angle (6°) on the perpendicular loading was quantified. Since

$$\frac{1}{\cos 6^\circ} \approx 1.0055,$$

the resulting change in load is less than 1%, and thus negligible for the current level of structural analysis.

Overall Summary: This chapter established the full aerodynamic maneuver environment required for preliminary structural sizing of the wing. The V–n diagram, stall boundaries, structural limits, and dive-speed conditions were computed and validated. The complete spanwise lift distribution at limit load was obtained using Schrenk's Approximation and scaled through the correction factor F. These results provide the necessary foundation for computing shear-force diagrams, bending-moment envelopes, and torsional loading in the subsequent structural design chapters.