Engineering Mathematics - 3 ⇒ assignment 4

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Q1] let $U = L\{(1,1,-1,1), (1,-1,1,1), (1,0,0,1)\}$
and $W = L\{(1,2,-1,1), (1,-2,1,1), (1,-2,0,1)\}$

$$\cancel{U = \begin{bmatrix} 1 & 1 & -1 & 1 & x \\ 1 & -1 & 1 & 1 & y \\ 1 & 0 & 0 & 1 & z \end{bmatrix}}$$

$$U = \begin{bmatrix} 1 & 1 & 1 & x \\ 1 & -1 & 0 & y \\ -1 & 1 & 0 & z \\ 1 & 1 & 1 & w \end{bmatrix}$$

$R_2 \to R_2 - R_1$ / $R_3 \to R_3 + R_1$ / $R_4 \to R_4 - R_1$

$$U = \begin{bmatrix} 1 & 1 & 1 & x \\ 0 & -2 & -1 & y-x \\ 0 & 2 & 1 & z+x \\ 0 & 0 & 0 & w-x \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$U = \begin{bmatrix} 1 & 1 & 1 & x \\ 0 & -2 & -1 & y-x \\ 0 & 0 & 0 & z+y \\ 0 & 0 & 0 & w-x \end{bmatrix}$$

ROW ECHELON FORM

System of equations for U:

- $z + y = 0$
- $w - x = 0$

$$\begin{array}{cccc} x & y & z & w \end{array}$$

$(1, 2, -1, 1) \to W_1$

$(1, -2, 1, 1) \to W_2$

$(1, -2, 0, 1) \to W_3$

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checking for intersection :

case 1 : $W_1 \to$ $\overset{x}{1}, \overset{y}{2}, \overset{z}{-1}, \overset{w}{1}$

system of equations for U : $z + y = 0 \to (1)$
$w - x = 0 \to (2)$

$\boxed{w_1 \times\times}$

(1) $-1 + 2 = 0$ (×)
(2) $1 - 1 = 0$ (✓)

case 2 : $W_2 \to$ $\overset{x}{1}, \overset{y}{-2}, \overset{z}{1}, \overset{w}{1}$

system of equations for u : $z + y = 0 \to (1)$
$w - x = 0 \to (2)$

(1) $1 + -2 = 0$ (×)
(2) $1 - 1 = 0$ (✓)

$\boxed{w_2 \times\times}$

case 3 : $W_3 \to$ $\overset{x}{1}, \overset{y}{-2}, \overset{z}{0}, \overset{w}{1}$

system of equations for u : $z + y \to (1)$
$w - x \to (2)$

(1) $0 - 2 = 0$ (×)
(2) $1 - 1 = 0$ (✓)

$\boxed{w_3 \times\times}$

$\because$ w_1, w_2, w_3 doesn't satisfies any system of equations of U.

∘ dimension $U \cap W$: 0 (zero)

basis of $U \cap W$: $(0, 0, 0, 0)$

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Q1].

$$u = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$u = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$u = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$u = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ROW ECHELON

dimension of $U = 2$

basis ~~biasets~~ of $u = \{ (1, 1, -1, 1), (0, -1, 1, 0) \}$

$$w = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$w = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -4 & 2 & 0 \\ 0 & -4 & 1 & 0 \end{bmatrix}$$

$R_3 \longleftrightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

ROW ECHELON :

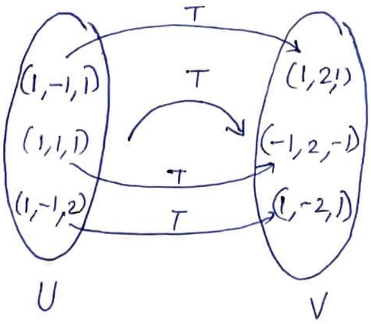
$$w = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

dim. of $W = 3$

basis of $W = \{ (1, 2, -1, 0),$
$(0, -4, 2, 0),$
$(0, 0, -1, 0) \}$

Q2.] Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation such that

- $T(1, -1, 1) = (1, 2, 1)$
- $T(1, 1, 1) = (-1, 2, -1)$
- $T(1, -1, 2) = (1, -2, 1)$



formula for T is:

$$\boxed{T(x,y,z) = (-y,\ 2x-4y-4z,\ -y)}$$

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Q3. given spanning vectors: $(1,2,1,1)$ and $(1,-2,1,1)$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 \\ 0 & -4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \text{ROW ECHELON FORM}$$

$$x + y = 0 \longrightarrow x = 0$$
$$-4y = 0 \longrightarrow y = 0$$

homogeneous system:

1. $x + y = 0$
2. $-4y = 0$

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Q4. rank and nullity theorem says that:

$\dim V = 3$

$\dim. V = \text{rank } T + \text{nullity } T$

① dim. of null (T) = nullity of T

② dim. of range (T) = rank of T

null = kernel

range = image

given linear transformation $\Rightarrow T: \mathbb{R}^3 \to \mathbb{R}^3$ given as:

$$T(x,y,z) = (x+y-z,\ 2x+y-2z,\ x-y-z)$$

To find basis of null space:

Let $v = (x,y,z) \in \mathbb{R}^3$ such that:

$$\text{Nullity of } T = \{v \in V \mid T(v) = 0\}$$

$$\therefore = \{(x,y,z) \mid (x+y-z,\ 2x+y-2z,\ x-y-z) = 0\}$$

$$\Rightarrow \{(x,y,z) \mid (x+y=-z,\ 2x+y=2z,\ x=z+y)\}$$

$x+y=-z$
$2x+y=2z$
$x=z+y$
$\Rightarrow$ $z+y+y=-z$
$2y=-2z$
$y=-z$

$2x+-z=2z$
$2x=3z$
$x=\frac{3}{2}z$

$\frac{3}{2}z = z+-z$
~~$\frac{3}{2}z=0$~~
~~$z=0$~~

$$\Rightarrow \{(x,y,z) \mid (2x=3z,\ y=-z,\ z=-y)\}$$

$$\Rightarrow \{(x,y,z) \mid (x=\tfrac{3}{2}z,\ y=-z,\ z=-y)\}$$

$$\Rightarrow \{(\tfrac{3}{2}z, -z, z)\} \Rightarrow \{z(\tfrac{3}{2}, -1, 1)\}$$

$\therefore \{(\frac{3}{2}, -1, 1)\}$ is the basis for null space.

$$\Rightarrow \text{Nullity}(T) = 1$$

So, if nullity of T is 1, we need rank(T) = 2 to prove theorem.

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Q4 continued]

To find basis of range space :

The standard basis of $\mathbb{R}^3$: $\{(e_1), (e_2), (e_3)\}$

$\Rightarrow$ Standard basis : $\{(1,0,0), (0,1,0), (0,0,1)\}$ generates $\mathbb{R}^3$

$\Rightarrow$ $\{T(1,0,0), T(0,1,0), T(0,0,1)\}$ generates range T

$$T = (x+y-z,\ 2x+y-2z,\ x-y-z)$$

$\Rightarrow$ $\{(1,2,1), (1,1,-1), (-1,-2,-1)\}$ generates range T

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ -1 & -2 & -1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$ Row echelon form

Thus, $\{(1,2,1), (0,-1,-2)\}$ forms a basis of range space of T.

Hence, Rank (T) = 2.

as Rank (T) = 2 and Nullity (T) = 1

$$\dim(V) = \text{Rank}(T) + \text{Nullity}(T)$$
$$3 = 2 + 1$$

$\therefore$ Rank - nullity theorem is verified

Q5] Need to find matrix representation for $T(x,y,z) = (x+y-z,\ 2x+y-2z,\ x-y-z)$ w.r.t basis:

$\{(1,-1,1), (1,1,1), (1,-1,2)\}$

matrix A of T on $\mathbb{R}^3$ relative to basis $B = (v_1, v_2, v_3)$ is:

$$A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

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Q5 contd.]

from the matrix:

$\cancel{T(x,y)}$ $T(v_1) = a v_1 + b v_2 + c v_3$

$T(v_2) = d v_1 + e v_2 + f v_3$

$T(v_3) = g v_1 + h v_2 + i v_3$

- we have:

$T(x,y,z) = (x+y-z,\ 2x+y-2z,\ x-y-z)$

$B = \{\underset{v_1}{(1,-1,1)},\ \underset{v_2}{(1,1,1)},\ \underset{v_3}{(1,-1,2)}\}$

- using this:

$T(v_1) = T\begin{bmatrix}1\\-1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\\1\end{bmatrix}$

$T(v_2) = T\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$

$T(v_3) = T\begin{bmatrix}1\\-1\\2\end{bmatrix} = \begin{bmatrix}-2\\-3\\0\end{bmatrix}$

- a, b, c:

$\begin{bmatrix}-1\\-1\\1\end{bmatrix} = a\begin{bmatrix}1\\-1\\1\end{bmatrix} + b\begin{bmatrix}1\\1\\1\end{bmatrix} + c\begin{bmatrix}1\\-1\\2\end{bmatrix}$

$a+b+c = -1$
$-a+b-c = -1$
$a+b+2c = 1$

$c = 0$, $a = 0$, $b = -1$

$a+b = -1$ $\quad a+a-1 = -1$
$b-a = -1 \Rightarrow b = a-1$ $\quad 2a = 0$ $\quad a = 0$

$a = 0,\ b = -1,\ c = 0$

- d, e, f calculations:

$\begin{bmatrix}1\\1\\-1\end{bmatrix} = d\begin{bmatrix}1\\-1\\1\end{bmatrix} + e\begin{bmatrix}1\\1\\1\end{bmatrix} + f\begin{bmatrix}1\\-1\\2\end{bmatrix}$

$d+e+f = 1$
$-d+e-f = 1$
$d+e+2f = -1$

$e = 1+d+f$, $\quad e = -2f-1-d$

$d + 1 + d + f + f = 1$
$2d + 2f = 0$
$d+f = 0$

$e = 1 + \underset{d+f}{0}$
$e = 1$

$-2f-1-d = 1 + \underset{d+f}{0}$
$d - 2f = 2$
$d = 2+2f$
$d = 2(1+f)$
$d = 2(1+f)$

$2+2f+f = 0$
$2+3f = 0$

$f = -2/3$ $\quad d = 2/3$ $\quad e = 1$

$f = -0.66$ $\quad d = 0.66$

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Q5 continued :

- g, h, i calculations :

$$\begin{bmatrix} -2 \\ -3 \\ 0 \end{bmatrix} = g \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + h \begin{bmatrix} 1 \\ +1 \\ 1 \end{bmatrix} + i \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\left.\begin{aligned} g + h + i &= -2 \\ -g + h - i &= -3 \\ g + h + 2i &= 0 \end{aligned}\right\} \text{equations}$$

$g + h = -2i$

$-2i + i = -2$

$-i = -2$

$\boxed{i = 2}$

$g + h = -4$

$-g + h = -3 + 2$

$-g + h = -1$

$\left.\begin{aligned} g + h &= -4 \\ g + 1 &= h \end{aligned}\right\}$ $g + g + 1 = -4$

$2g = -5$

$g = -2.5$

$h = -1.5$

Substituting in the matrix :

$$A = \begin{bmatrix} 0 & 0.66 & -2.5 \\ -1 & 1 & -1.5 \\ 0 & -0.66 & 2 \end{bmatrix}$$

A is the matrix representation required.