Monte Carlo Simulation Lab Assignment-8

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Q 1 Use the following Monte Carlo estimator to approximate the expected value

$$I = E(exp(\sqrt{U}))$$

where $U \backsim u[0,1]$: $I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i$, where $Y_i = exp(\sqrt{U_i}$ with $U_i \backsim u[0,1]$. Take all values of M to be $10^2, 10^3, 10^4$ and 10^5. Determine the 95% confidence interval for I_M for all the four values of M that you have taken

Solution: R code:

```
m<-vector("numeric")
m[1]=10^2
m[2]=10^3
m[3]=10^4
m[4]=10^5
for(i in 1:4)
{
    u<-runif(m[i])
    Y<-vector("numeric")
    for(j in 1:m[i])
    {
        Y[j]=exp(sqrt(u[j]))
    }
    I=sum(Y)/m[i]
    var=var(Y)
    min=I-(1.95*sqrt(var)/sqrt(m[i]))
    max=I+(1.95*sqrt(var)/sqrt(m[i]))
    cat("\nI_",m[i]," = ",I,"\n")
    cat("var_",m[i]," = ",var,"\n")
    cat("min_",m[i]," = ",min,"\n")
    cat("max_",m[i]," = ",max,"\n")
}
```

Observations :-

The result is given as the following table with columns M, mean of random variable, variance of random variable and upper and lower endpoint of 95% confidence interval.

M	Mean	Variance	Lower endpoiont	Upper endpoint
10^2	2.013587	0.2133722	1.923512	2.103662
10^3	2.004278	0.1839995	1.977826	2.030729
10^4	2.005147	0.1936897	1.996565	2.013729
10^5	1.999698	0.1940106	1.996982	2.002414

Results :-

1. The theoretical mean is 2.
2. The empirical mean tends to 2 as number of iterations increase.
3. 95% confidence interval for 10^5 is $(1.996982, 2.002414)$.

Q 2 Repeat the above exercise using antithetic variates via t he following estimator and calculate the percentage of variance reduction:
$I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i$
where $Y_i = \frac{exp(\sqrt{U_i})+exp(\sqrt{1-U_i}}{2}$
with $U_i \backsim u[0,1]$

Solution: R code:

```
m<-vector("numeric")
m[1]=10^2
m[2]=10^3
m[3]=10^4
m[4]=10^5
for(i in 1:4)
{
    u<-runif(m[i])
    Y<-vector("numeric")
    for(j in 1:m[i])
    {
        Y[j]=exp(sqrt(u[j]))
    }
    I_y=sum(Y)/m[i]
    var_y=var(Y)
    Z<-vector("numeric")
    for(j in 1:m[i])
    {
        Z[j]=(exp(sqrt(u[j]))+exp(sqrt(1-u[j])))/2
    }
    I_z=sum(Z)/m[i]
    var_z=var(Z)
    min=I_z-(1.95*sqrt(var_z)/sqrt(m[i]))
    max=I_z+(1.95*sqrt(var_z)/sqrt(m[i]))
    cat("\nI_",m[i]," = ",I_z,"\n")
    cat("var_",m[i]," = ",var_z,"\n")
    cat("min_",m[i]," = ",min,"\n")
    cat("max_",m[i]," = ",max,"\n")
    delta=(var_y-var_z)/var_y*100
    cat("Percentage of variance reduction for ",m[i]," random numbers = ",delta,"\n")
}
```

Observations :-

The result is given as the following table with columns M, mean of random variable, variance of random variable and upper and lower endpoint of 95% confidence interval and variance reduction percentage.

M	Mean	Variance	Lower endpoiont	Upper endpoint	Variance Reduction (%age)
10^2	1.99814	0.001078075	1.991738	2.004543	99.48891
10^3	1.998315	0.001081175	1.996287	2.000342	99.47121
10^4	1.999851	0.001102568	1.999204	2.000499	99.4355
10^5	2.00008	0.001068032	1.999878	2.000281	99.45017

Results :-

1. The theoretical mean is 2.
2. The empirical mean tends to 2 as number of iterations increase.
3. 95% confidence interval for 10^5 is $(1.999878, 2.000281)$.
4. The variance reduction for 10^5 iterations is 99.45017% which is desirable.

Q 3 Use $\sqrt{U}$ to construct control variate estimate and repeat the above exercise. Calculate the percentage of variance reduction.

Solution: R code: Using $\sqrt{U}$ as control variate estimate

```
m<-vector("numeric")
m[1]=10^2
m[2]=10^3
m[3]=10^4
m[4]=10^5
for(i in 1:4)
{
    u<-runif(m[i])
    Y<-vector("numeric")
    for(j in 1:m[i])
    {
        Y[j]=exp(sqrt(u[j]))
    }
    I_y=sum(Y)/m[i]
    var_y=var(Y)

    v<-runif(m[i])
    Z<-vector("numeric")
    for(j in 1:m[i])
    {
        Z[j]=sqrt(u[j])
    }
    I_z=sum(Z)/m[i]
    var_z=var(Z)
    c<-(-1)*cov(Y,Z)/var(Z)

    W<-vector("numeric")
    for(j in 1:m[i])
    {
        W[j]=Y[j]+(c*(Z[j]-I_z))
    }
    I_w=sum(W)/m[i]
    var_w=var(W)

    min=I_z-(1.95*sqrt(var(W))/sqrt(m[i]))
    max=I_z+(1.95*sqrt(var(W))/sqrt(m[i]))

    cat("\nI_",m[i]," = ",I_w,"\n")
    cat("var_",m[i]," = ",var_w,"\n")
    cat("min_",m[i]," = ",min,"\n")
    cat("max_",m[i]," = ",max,"\n")
    delta=(var_y-var_w)/var_y*100
```

```
    cat("Percentage of variance reduction for ",m[i]," random numbers = "
        ,delta,"\n")
}
```

Observations :-

Using $\sqrt{U}$ as control variate estimate

The result is given as the following table with columns M,mean of random variable, variance of random variable and upper and lower endpoint of 95% confidence interval and variance reduction percentage

M	Mean	Variance	Lower endpoiont	Upper endpoint	Variance Reduction (%age)
10^2	1.877849	0.002499348	0.5927332	0.6122307	98.70001
10^3	1.97944	0.002635716	0.6537804	0.660112	98.59785
10^4	2.004715	0.002702267	0.6682152	0.6702425	98.60314
10^5	2.003103	0.002697464	0.6679026	0.6685431	98.61584

Results :-

1. The theoretical mean is 2.
2. The empirical mean tends to 2 as number of iterations increase.
3. 95% confidence interval for 10^5 using $\sqrt{U}$ is $(0.6679026, 0.6685431)$.
5. The variance reduction is significantly less than antithetic method.