Monte Carlo Simulation Lab
Assignment-7

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Q 1 Consider the multivariate normal, $X = (X_1, X_2) \sim N(\mu, \sum)$ where $\mu = (5, 8)$ and $\sum = (1, 2a, 2a, 4)$. For the cases $a = -0.25, 0, 0.25$, generate 1000 values of X and calculate sample means, sample variances and sample correlations. Make empirical contour plots based on above generated samples.

Code for R

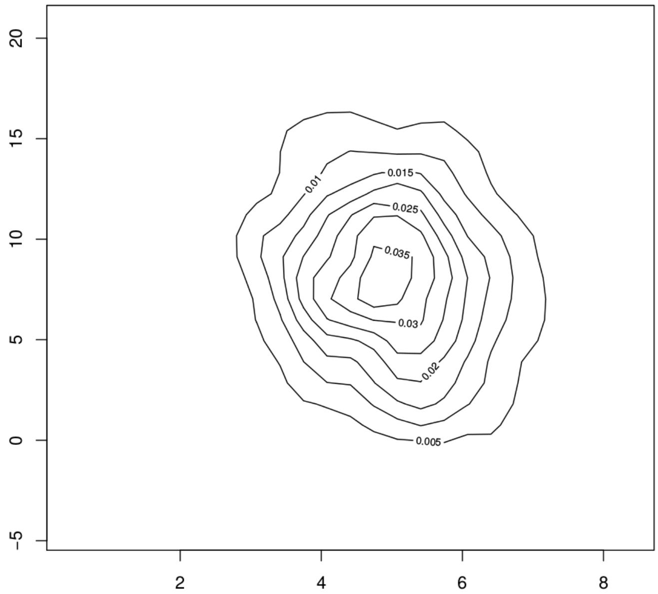
```
library(MASS)
a<-vector("numeric")
a[1]=-0.25
a[2]=0
a[3]=0.25
u<-vector("numeric")
u[1]=5
u[2]=8
E11=1
E12=2*a
E21=2*a
E22=4
Z1<-rnorm(1000)
Z2<-rnorm(1000)
X1<-vector("numeric")
X2<-vector("numeric")
for(j in 1:3)
{   for (i in 1:1000)
    {
        X1[i]=u[1]+Z1[i]
        X2[i]=u[2]+(2*a[j]*Z1[i])+(4*Z2[i])
    }
    g<-kde2d(X1,X2)
    contour(g)
    dev.copy(png,paste(j,".png"))
    dev.off()
    cat("For a = ",a[j],"\n")
    cat("The mean of X1 is ",mean(X1),"\n")
    cat("The mean of X2 is ",mean(X2),"\n")
    cat("The variance of X1 is ",var(X1),"\n")
    cat("The variance of X2 is ",var(X2),"\n")
    cat("The covariance is ",cov(X1,X2),"\n")
    cat("The correlation is ",cor(X1,X2),"\n")

}
```

Output:

For $a = -0.25$:

```
The mean of X1 is  4.967631
The mean of X2 is  8.030552
The variance of X1 is  1.02416
The variance of X2 is  16.0848
The covariance is  -0.6624957
The correlation is  -0.1632268
```

The corresponding contour plot obtained is shown below:

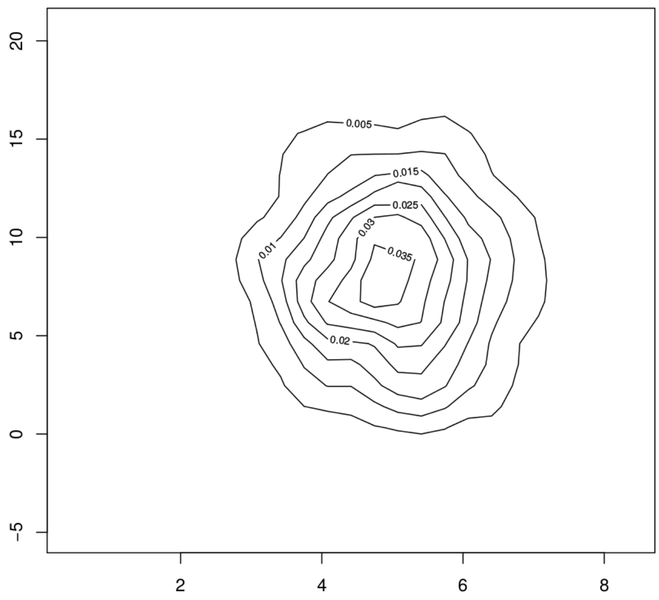


(a) Contour plot for $a = -0.25$

For $a = 0$:

```
The mean of X1 is  4.967631
The mean of X2 is  8.014367
The variance of X1 is  1.02416
The variance of X2 is  15.67834
The covariance is  −0.1504157
The correlation is  −0.03753697
```

The corresponding contour plot obtained is shown below:

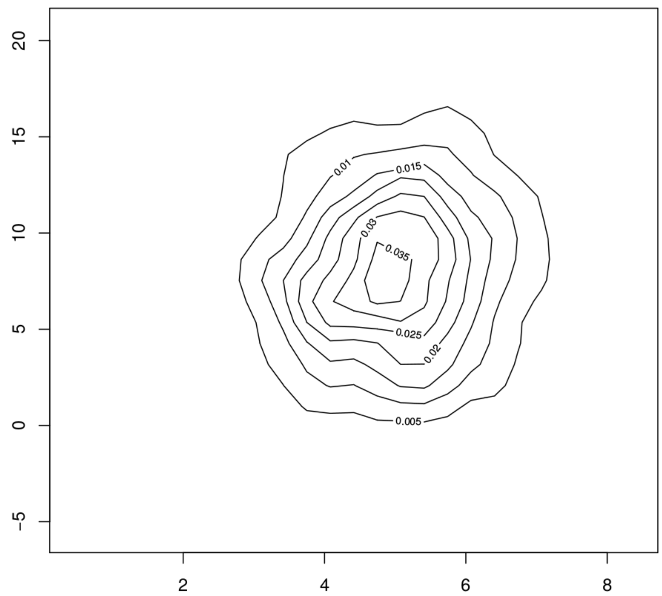


(b) Contour plot for $a = 0$

For $a = 0.25$:

```
The mean of X1 is  4.967631
The mean of X2 is  7.998183
The variance of X1 is  1.02416
The variance of X2 is  15.78397
The covariance is  0.3616643
The correlation is  0.08995259
```

The corresponding contour plot obtained is shown below:



(c) Contour plot for $a = 0.25$

Q 2 Also, plot the actual and empirical marginal cdfs of X_1 and X_2 .

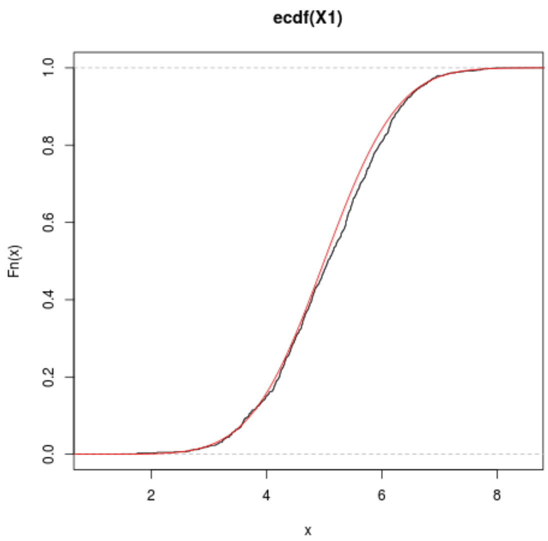
Code for R

```
library(MASS)
a<-vector("numeric")
a[1]=-0.25
a[2]=0
a[3]=0.25
u<-vector("numeric")
u[1]=5
u[2]=8
E11=1
E12=2*a
E21=2*a
E22=4
Z1<-rnorm(1000)
Z2<-rnorm(1000)
X1<-vector("numeric")
X2<-vector("numeric")

for (i in 1:1000)
{
    X1[i]=u[1]+Z1[i]
    X2[i]=u[2]+(2*a[1]*Z1[i])+(4*Z2[i])
}

png("X1_a=-0.25.png")
plot(ecdf(X1))
d1<-seq(0,10,length=1000)
hx<-pnorm(d1, mean=5, sd=1)
lines(d1,hx,col="red")
dev.off()

png("X2._a=-0.25png")
plot(ecdf(X2))
d2<-seq(-5,21,length=1000)
hx<-pnorm(d2, mean=8, sd=2)
lines(d2,hx,col="red")
dev.off()

for (i in 1:1000)
{
    X1[i]=u[1]+Z1[i]
    X2[i]=u[2]+(2*a[2]*Z1[i])+(4*Z2[i])
}

png("X1_a=0.png")
```

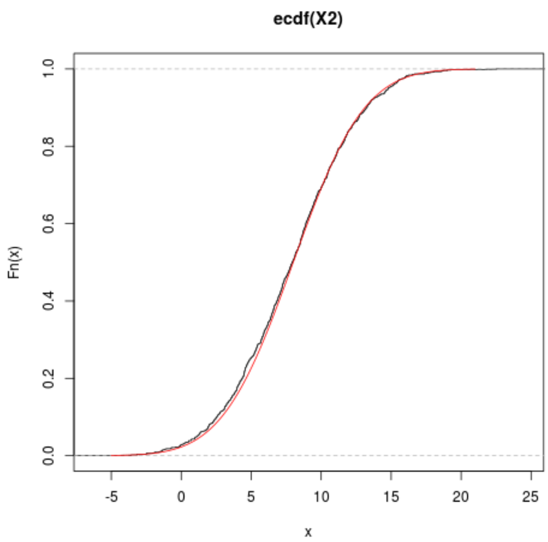
```
plot(ecdf(X1))
d1<-seq(0,10,length=1000)
hx<-pnorm(d1, mean=5, sd=1)
lines(d1,hx,col="red")
dev.off()

png("X2_a=0.png")
plot(ecdf(X2))
d2<-seq(-5,21,length=1000)
hx<-pnorm(d2, mean=8, sd=2)
lines(d2,hx,col="red")
dev.off()

for (i in 1:1000)
{
    X1[i]=u[1]+Z1[i]
    X2[i]=u[2]+(2*a[3]*Z1[i])+(4*Z2[i])
}

png("X1_a=0.25.png")
plot(ecdf(X1))
d1<-seq(0,10,length=1000)
hx<-pnorm(d1, mean=5, sd=1)
lines(d1,hx,col="red")
dev.off()

png("X2_a=0.25.png")
plot(ecdf(X2))
d2<-seq(-5,21,length=1000)
hx<-pnorm(d2, mean=8, sd=2)
lines(d2,hx,col="red")
dev.off()
}
```

Plots of actual and emperical marginal cdfs:

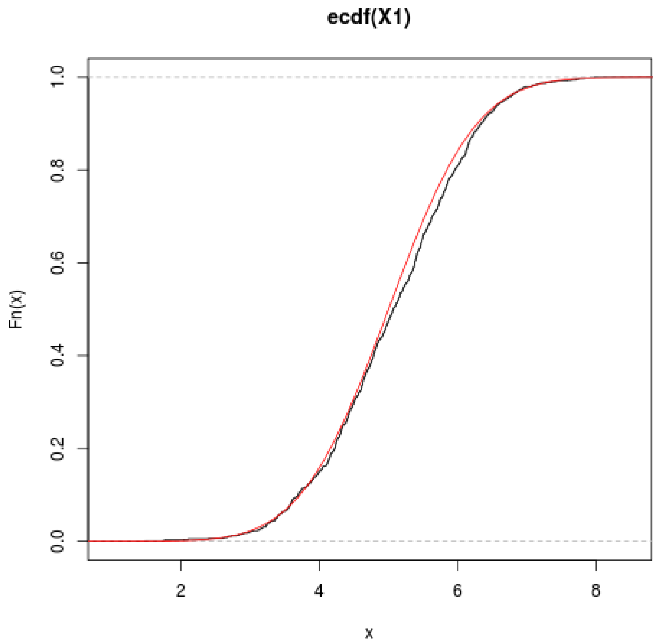
Black-Empirical
Red-Actual



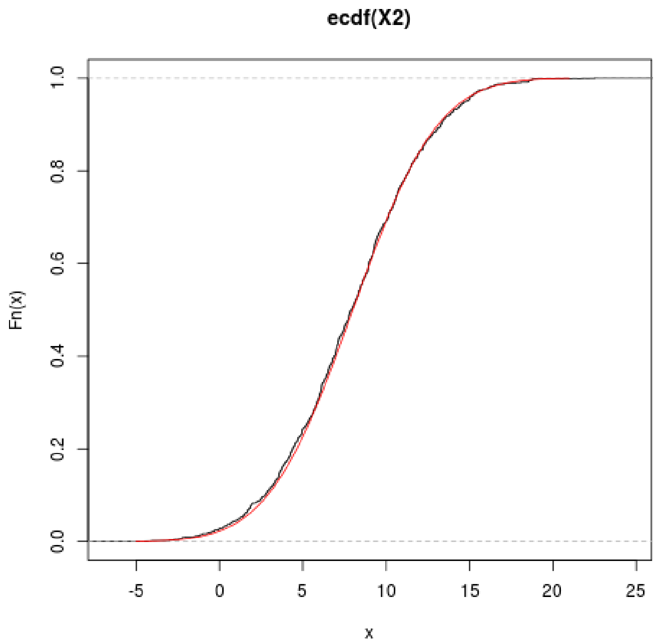
(d) cdf of X1 for $a = -0.25$



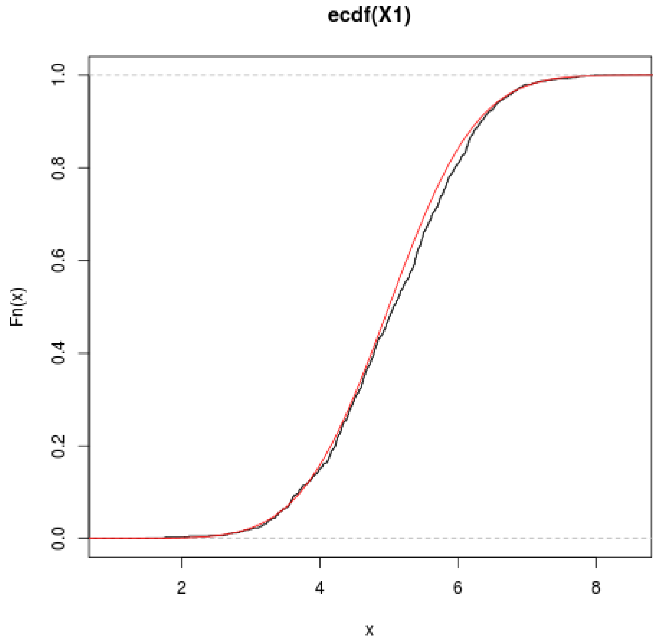
(e) cdf of X2 for $a = -0.25$



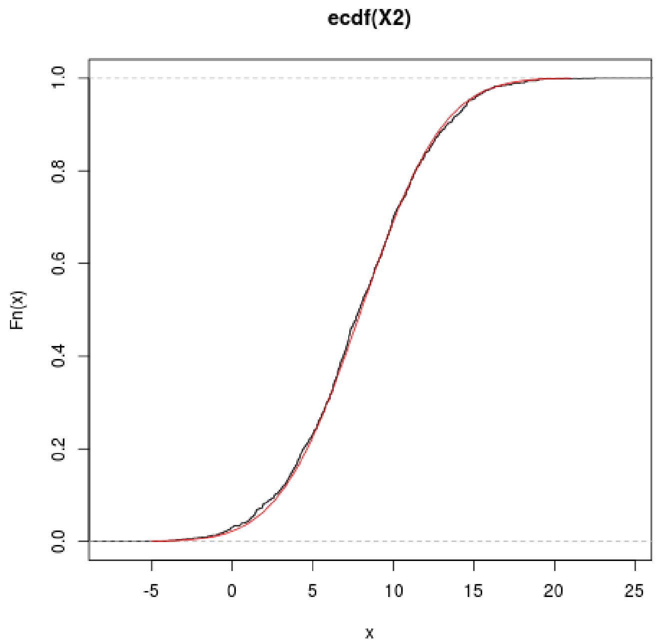
(f) cdf of X1 for $a = 0$



(g) cdf of X2 for $a = 0$



(h) cdf of X1 for $a = 0.25$



(i) cdf of X2 for $a = 0.25$

Q 3 Let us recall generating a bivariate normal with the help of conditional distributions. Suppose that $X_1 \sim N(\mu_1, {\sigma_1}^2), X_2 \sim N(\mu_2, \sigma_2^2)$ and the conditional distribution of X_2 given $X_1 = x$ is $N(\mu_2 + \rho\sigma_2/\sigma_1(x - \mu_1), \sigma_2^2(1 - \rho^2))$ where $|\rho| < 1$ is the correlation coefficient between X_1 and X_2 . The vector (X_1, X_2) is said to have a bivariate normal distribution. Simulate the vector for a particular set of parameter values, using this idea of conditional distributions. Estimate the sample quantities (mean, etc.) and compare with actual values. Take same $\mu_1, \mu_2, \rho_1, \rho_2$ and ρ.

Code for R

```
library(MASS)
a<-vector("numeric")
a[1]=-0.25
a[2]=0
a[3]=0.25
u<-vector("numeric")
u[1]=5
u[2]=8
E11=1
E12=2*a
E21=2*a
E22=4
Z1<-rnorm(1000)
Z2<-rnorm(1000)
X1<-vector("numeric")
X2<-vector("numeric")
for(j in 1:3)
{   for (i in 1:1000)
    {
        X2[i]=u[2]+(sqrt(E22)*Z2[i])
        X1[i]=(u[1]+((X2[i]-8)*a[j]/sqrt(E22)))+(Z1[i]*sqrt(1-(a[j]*a[j])))
    }
    cat("For a = ",a[j],"\n")
    cat("The mean of X1 is ",mean(X1),"\n")
    cat("The mean of X2 is ",mean(X2),"\n")
    cat("The variance of X1 is ",var(X1),"\n")
    cat("The variance of X2 is ",var(X2),"\n")
    cat("The covariance is ",cov(X1,X2),"\n")
    cat("The correlation is ",cor(X1,X2),"\n")
}
```

The output of the code is as follows for $a = -0.25$:

```
The mean of X1 is 4.982625
The mean of X2 is 8.037845
The variance of X1 is 1.020662
The variance of X2 is 4.014131
The covariance is -0.5149089
The correlation is -0.2543862
```

The output of the code is as follows for $a = 0$:

```
The mean of X1 is 4.986941
The mean of X2 is 8.037845
The variance of X1 is 1.018299
The variance of X2 is 4.014131
The covariance is -0.01357344
The correlation is -0.006713615
```

The output of the code is as follows for $a = 0.25$:

```
The mean of X1 is 4.992087
The mean of X2 is 8.037845
The variance of X1 is 1.014091
The variance of X2 is 4.014131
The covariance is 0.488624
The correlation is 0.2421813
```

Observations:

1. Mean of X1 is very close to 5 as required for all values of a.
2. Mean of X2 is very close to 8 as required for all values of a.
3. Variance of X1 is very close to 1 as required for all values of a.
4. Variance of X2 is very close to 4 as required for all values of a.
5. Correlation is very close to the values required for all values of a.
6. Covariance is almost equal to 0 for $a = 0$ as expected.