Assignment-3

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Q 1 Simulate 5000 sample of exponential with mean 5. Draw the histogram and the calculate the mean, maximum and minimum

Code for C++

```
#include <iostream>
#include <cmath>
#include <fstream>

using namespace std;

int main()
{
    ofstream myfile;
    myfile.open("output.txt");
    int a=167, b= 59, x=23, e;
    int m=pow(2,15);
    double E[5000]={0};
    double u, sum=0;
    int freq[35]={0};
    int i;
    for(i=0;i<5000;i++)
    {
        x=(a*x+b)%m;
        u=double(x)/double(m);

        if(u<0.0001)
        {
            i--;
        }
        else
        {
            E[i]=-5*log(u);
            e=int(E[i]);
            freq[e]++;
        }
    }
    for(i=0;i<35;i++)
        myfile<<"freq["<<i<<"]="<<freq[i]<<"\n";

    double max=E[0], min=E[0];
    for(i=0;i<5000;i++)
    {
        if(E[i]>max)
            max=E[i];
        if(E[i]<min)
            min=E[i];
        sum=sum+E[i];
```

```
44      }
45      double mean=double(sum)/5000;
46      myfile<<"\nmean = "<<mean<<"\n";
47      myfile<<"max = "<<max<<"\n";
48      myfile<<"min = "<<min<<"\n";
49
50      myfile.close();
51  }
```

The output of the code is as follows:

```
freq[0]=934
freq[1]=729
freq[2]=587
freq[3]=485
freq[4]=402
freq[5]=340
freq[6]=267
freq[7]=243
freq[8]=181
freq[9]=149
freq[10]=130
freq[11]=102
freq[12]=86
freq[13]=65
freq[14]=57
freq[15]=45
freq[16]=33
freq[17]=30
freq[18]=23
freq[19]=20
freq[20]=13
freq[21]=14
freq[22]=10
freq[23]=10
freq[24]=9
freq[25]=5
freq[26]=6
freq[27]=5
freq[28]=4
freq[29]=4
freq[30]=3
freq[31]=2
freq[32]=0
freq[33]=1
freq[34]=1
```

```
36
37 mean = 5.00691
38 max = 45.0546
39 min = 0.00015259
```

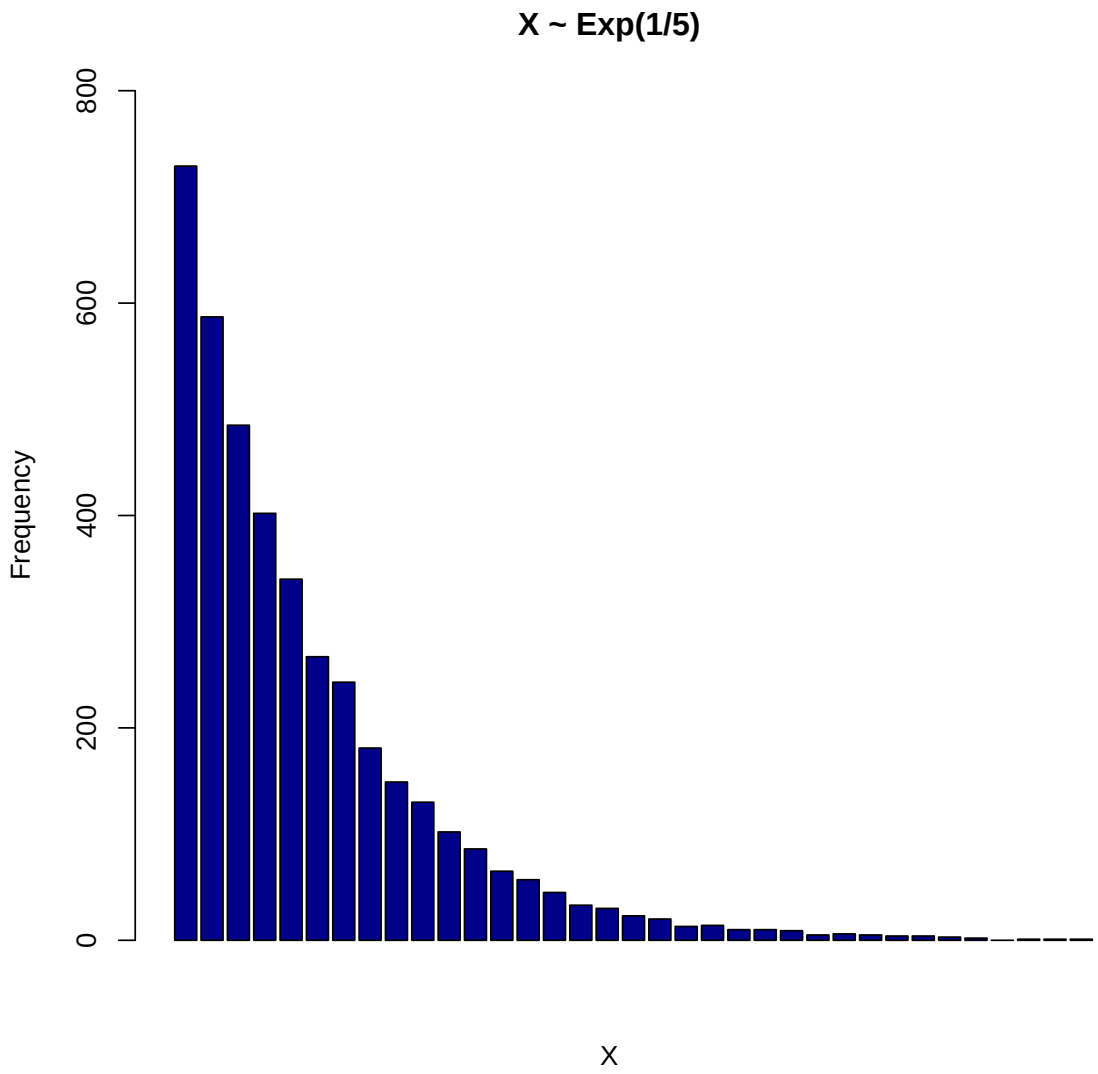
The code in R is shown below:

```
1  m<-2^15
2  a<-167
3  b<-59
4  x<-23
5  E<-array(0,5000)
6  freq<-array(0,35)
7  for(i in 1:5000)
8  {
9      x<-(a*x+b)%%m
10     u<-as.double(x)/m
11     if(u<0.0001)
12     {
13         i<-i-1;
14     }
15     else
16     {
17         E[i]<- -5*log(u)
18         e<- as.integer(E[i])
19         freq[e]=freq[e]+1
20     }
21 }
22 mean=mean(E)
23 min=min(E)
24 max=max(E)
25 cat("Max is:",max,"\n")
26 cat("min is:",min,"\n")
27 cat("mean is:",mean,"\n")
28 barplot(freq,main="X ~ Exp(1/5)", xlab="X",ylab="Frequency",xlim=c(0,40)
       ,ylim=c(0,800),col=c("darkblue"));
```

The output of the R is shown below:

```
1 Max is: 45.05457
2 Min is: 0.0001525902
3 Mean is: 5.006914
```

The histogram is shown below:



(a) $X \sim Exp(1/5)$

Q 2 Simulate 5000 sample of Gamma with parameter $n = 5$ and $\lambda = 5$. Draw the histogram and the calculate the mean, maximum and minimum.

Code for C++

```
#include <iostream>
#include <cmath>
#include <fstream>
#include <cstdio>


using namespace std;

int main()
{
    ofstream myfile;
    int m[5]={(int)pow(2,17),(int)pow(2,19),(int)pow(2,23),(int)pow(2,27)
        ,(int)pow(2,29)};
    int a[5]={167,93,7,123,135};
    int b[5]={371,33,294,4,357};
    int x[5]={55,29,435,99,123};
    double u[5];
    double E[5000][5]={log(0.5)};
    double G[5000],sum=0;
    int freq[50]={0},e;
    for(int j=0;j<5;j++)
    {
        for(int i=0;i<5000;i++)
        {
            x[j]=(x[j]*a[j]+b[j])%m[j];
            u[j]=(double)x[j]/m[j];
            if(u[j]<0.0001)
                i--;
            else
                E[i][j]=log(u[j]);
        }
    }
    double max=G[0];
    double min=G[0];
    for(int i=0;i<5000;i++)
    {
        G[i]=0;
        for(int j=0;j<5;j++)
            G[i]+=E[i][j];
        G[i]=-0.2*G[i];
        e=(int)(G[i]*10);
        sum+=G[i];
        max=(G[i]>max)?G[i]:max;
```

```
43          min=(G[i]<min)?G[i]:min;
44          freq[e]++;
45      }
46      myfile.open("output1.txt");
47      FILE* fp=fopen("output.txt","w");
48      double M=sum/5000;
49      myfile<<"Mean of the distribution = "<<M<<endl;
50      myfile<<"Min = "<<min<<endl<<"Max = "<<max<<endl;
51      for(int i=0;i<30;i++)
52          fprintf(fp,"%d\n",freq[i]);
53      fclose(fp);
54      myfile.close();
55  }
```

The output of the code is as follows:

```
1 Mean of the distribution = 0.994363
2 Min = 0
3 Max = 5.13151
```

The code in R is shown below:

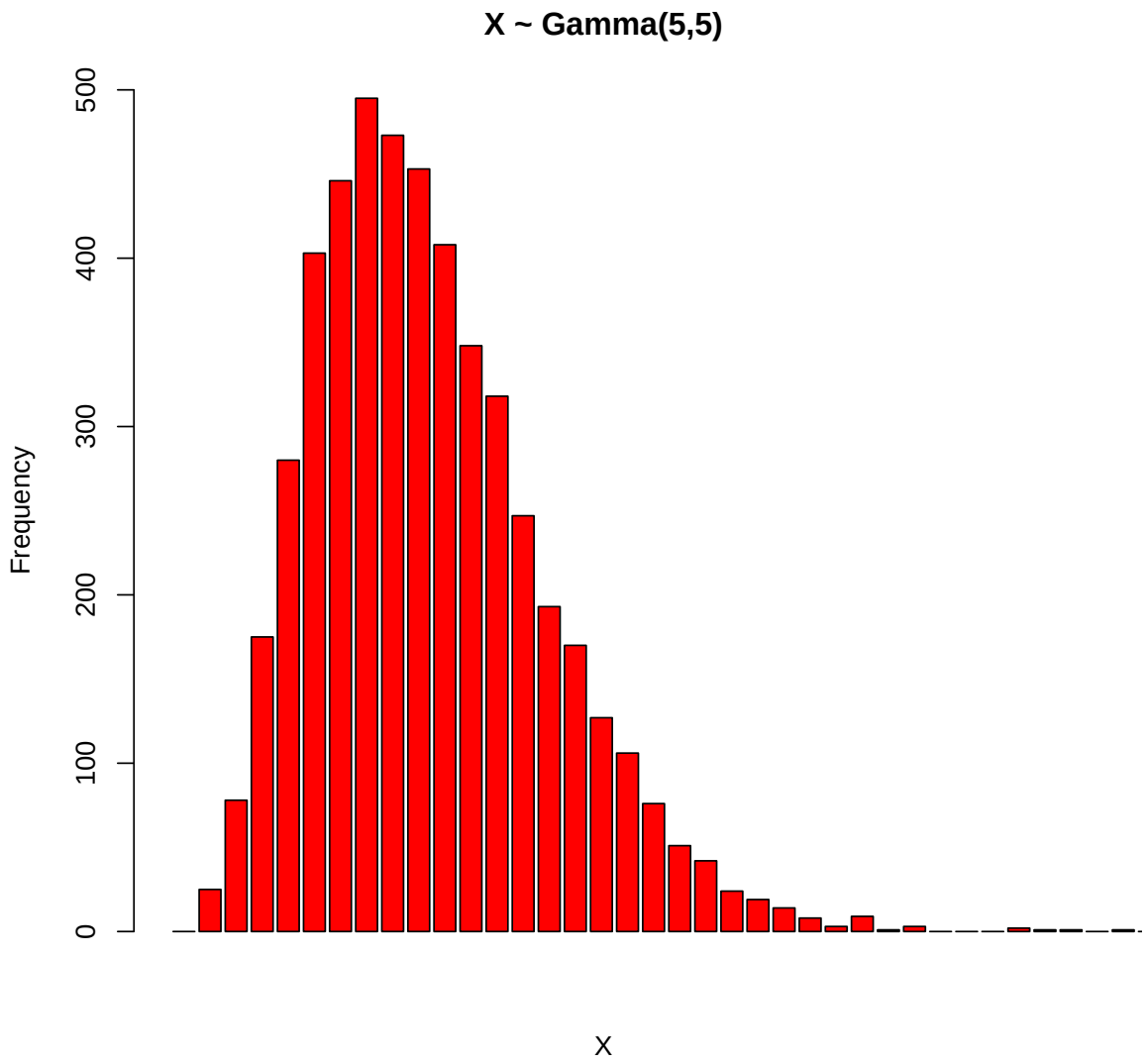
```
1  m<-c(2^17,2^19,2^23,2^27,2^29);
2  a<-c(167,93,7,123,135);
3  b<-c(371,33,294,4,357);
4  x<-c(55,29,435,99,123);
5  E<-matrix(log(0.5),nrow=5000,ncol=5);
6  freq<-array(0,50);
7  u<-array(0,5);
8  for(j in 1:5)
9  {
10     for(i in 1:5000)
11     {
12         x[j]<-(a[j]*x[j]+b[j])%%m[j];
13         u[j]<-as.double(x[j])/m[j];
14         if(u[j]<0.0001)
15         {
16             i<-i-1;
17         }
18         else
19         {
20             E[i,j]<- log(u[j]);
21         }
22     }
23 }
```

```
G<-array(0,5000);
for(i in 1:5000)
{
    G[i]<-sum(E[i,]);
    G[i]<- -0.2*G[i];
    e<-as.integer(G[i]*10);
    freq[e+1]<-freq[e+1]+1;
}
M<-mean(G);
max<-max(G);
min<-min(G);
cat("Max is:",max,"\n")
cat("min is:",min,"\n")
cat("mean is:",M,"\n")
barplot(freq,main="X ~ Gamma(5,5)", xlab="X",ylab="Frequency",xlim=c
    (0,40),ylim=c(0,500),col="red");
```

The output of the R is shown below:

```
Max is: 3.654517
Min is: 0.1060267
Mean is: 0.9912688
```

The histogram is shown below:



(b) $X \sim Gamma(5, 5)$

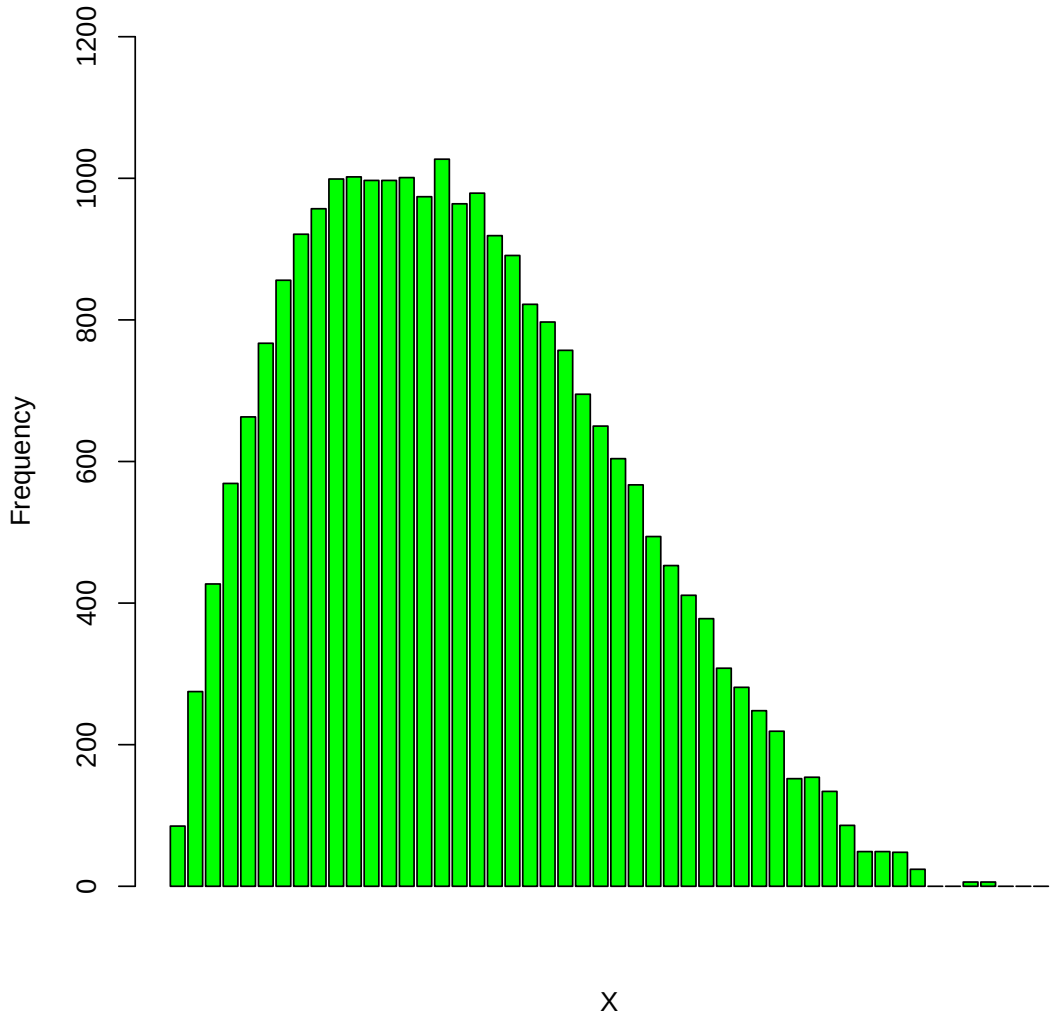
Q 3 Use the rejection method to generate from

$$f(x) = 20x(1-x)^3, 0 < x < 1$$

The code in R is shown below:

```
f<-function(x)
{
    return (20*x*(1-x)^3);
}
m<-2^15;
a<-167;
b<-59;
x<-23;
y<-2471;
cg<-2;
freq<-array(0,50);
for(i in 1:50000)
{
    x<-(a*x+b)%%m;
    u<-as.double(x)/m;
    y<-(a*y+b+7)%%m;
    v<-as.double(y)/m;
    if(cg*u<=f(v))
        freq[v*50+1]<-freq[v*50+1]+1;
}
barplot(freq,ylim=c(0,1200),col="green");
```

The histogram formed is as follows:



(c) $f(x) = 20x(1-x)^3, 0 < x < 1$