Monte Carlo Simulation Lab
Assignment-10

Yash Vanjani
140123046
Mathematics and Computing
IIT Guwahati

April 18 , 2016

Q 1 a financial asset. The process $S(t)$ is a GBM with drift parameter μ, volatility parameter σ, and initial value $S(0)$ if
$S(t) = S(0)exp([\mu - \frac{\sigma^2}{2}]t + \sigma W(t))$.
where $W(t)$ is a standard BM. As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < ...t_n$ as
$S(t_{i+1} = S(t_i)exp([\mu - \frac{\sigma^2}{2}](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}Z_{i+1})$
where $Z_1, Z_2, ..., Z_n$ are independent $N(0, 1)$ variates. In the interval $[0, 5]$, taking both positive and negative values for μ and for at least two different values of σ^2 , simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$. Calculate expectation and variance of $S(5)$ and match it with the theoretical values.

Solution

Code for R

```
library(stats)
S<-vector("numeric")
t<-vector("numeric")
pal<- palette()
t[1]=0
sec5<-vector("numeric") #for storing BM values at 5th sec
mu<-(1.2)#mu=mean
sigma<-0.6 #sigma=standard deviation

for(i in 1:4999)
{
    t[i+1]=t[i]+0.001
}
for(i in 1:10)
{
    z<-rnorm(5000, mean=0, sd=1)
    S[1]=1
    for (j in 1:4999)
    {
        S[j+1]=S[j]*exp(((mu-(sigma*sigma/2))*(0.001))+(sigma*z[j+1]*sqrt
            (0.001)))
        if(j==4999)
        {
            sec5[i]=S[5000]
        }

    }
```

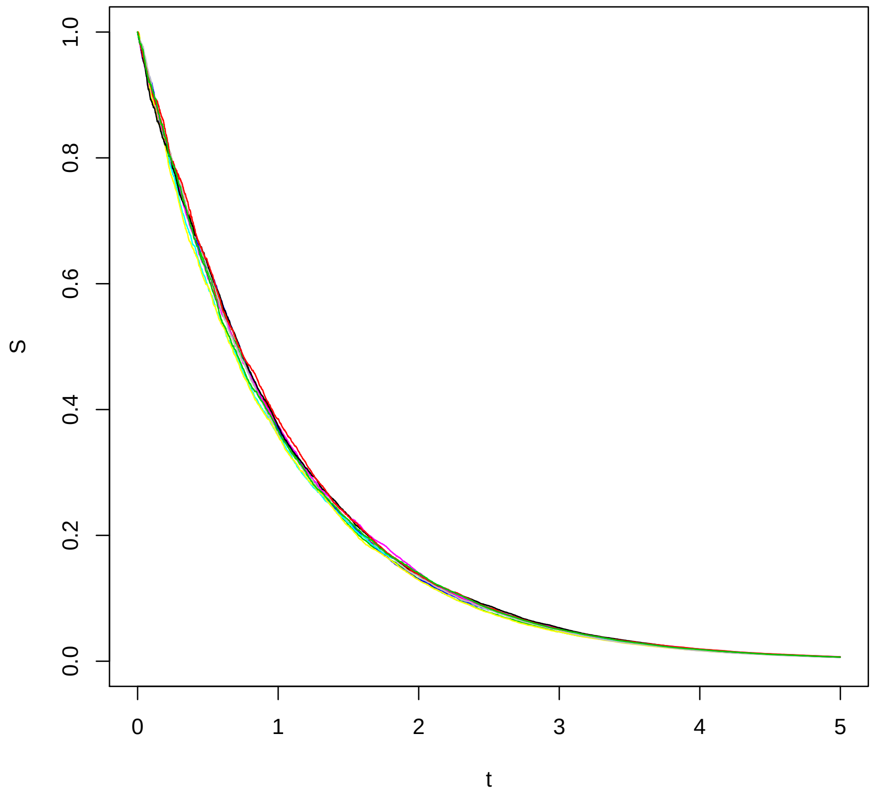
```
    if(i==1)
    {
        plot(t,S,type="l", col=pal[i %% 8 +1],ylim=c(0,600))
    }
    else
    {
        lines(t,S, col=pal[i %% 8 +1],ylim=c(0,600))
    }
}
cat("\nFor S(t=0) = ",S[1],", mu = ",mu,", var = ",(sigma*sigma),"\n")
cat("experemntal E[S(5)] = ",mean(sec5),"\n")
cat("experemntal Var[S(5)] = ",var(sec5),"\n")
mu_theoritical<-S[1]*exp(mu*5)
var_theoritical<-S[1]*S[1]*exp(2*mu*5)*(exp(sigma*sigma*5)-1)
cat("theoritical E[S(5)] = ",mu_theoritical,"\n")
cat("theoritical Var[S(5)] = ",var_theoritical,"\n\n")
```

The above code is run for both positive and negative values of μ and three different values of σ^2.

1) $\mu = -1$ **and** $\sigma^2 = 0.0009$

```
experemntal E[S(5)] =  0.006568828
experemntal Var[S(5)] =  1.110003e-07
theoritical E[S(5)] =  0.006737947
theoritical Var[S(5)] =  2.0476e-07
```

The corresponding plot obtained is shown below:

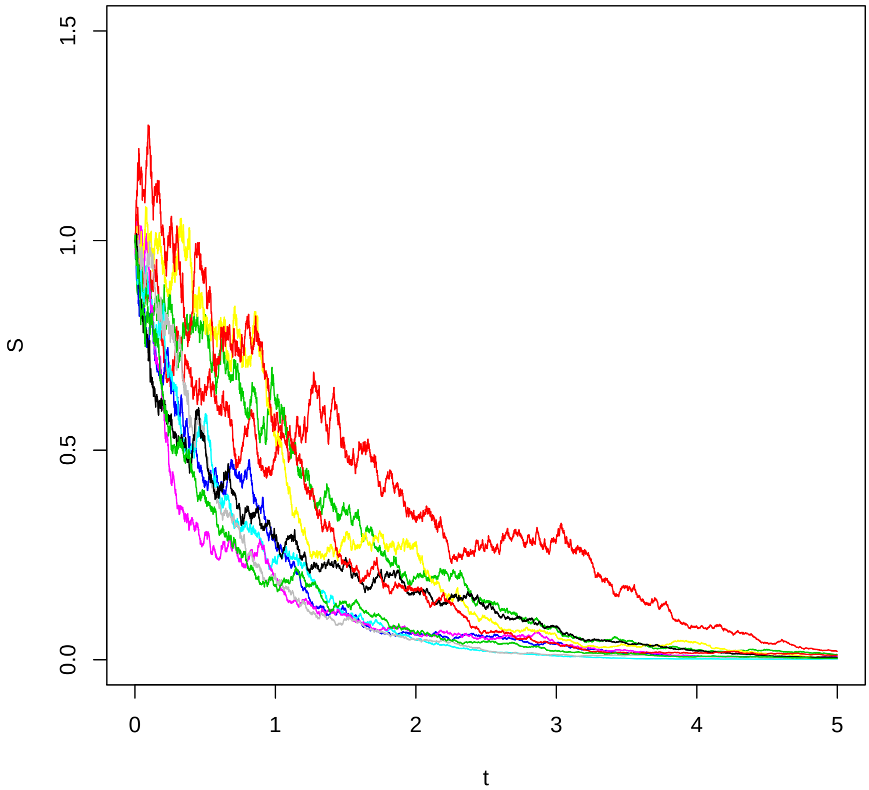


(a) For $\mu = -1, \sigma^2 = 0.0009$

2) $\mu = -1$ **and** $\sigma^2 = 0.16$

```
experemntal E[S(5)] =  0.007180603
experemntal Var[S(5)] =  3.251476e-05
theoritical E[S(5)] =  0.006737947
theoritical Var[S(5)] =  5.563947e-05
```

The corresponding plot obtained is shown below:

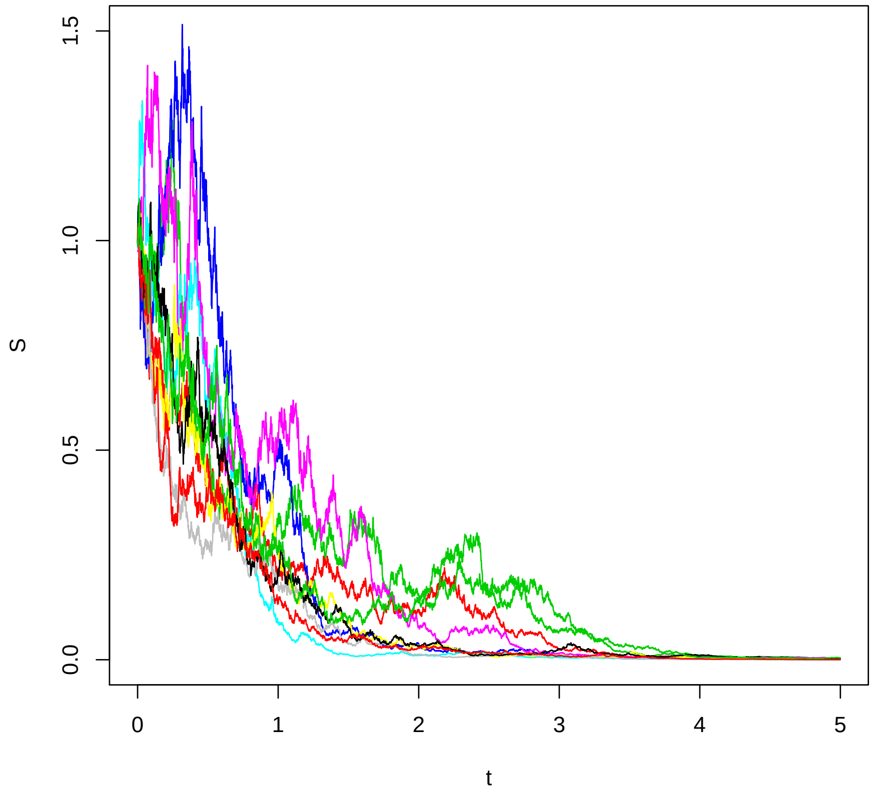


(b) For $\mu = -1, \sigma^2 = 0.16$

3) $\mu = -1$ **and** $\sigma^2 = 0.64$

```
experemntal E[S(5)] =  0.001537008
experemntal Var[S(5)] =  3.101096e-06
theoritical E[S(5)] =  0.006737947
theoritical Var[S(5)] =  0.001068375
```

The corresponding plot obtained is shown below:

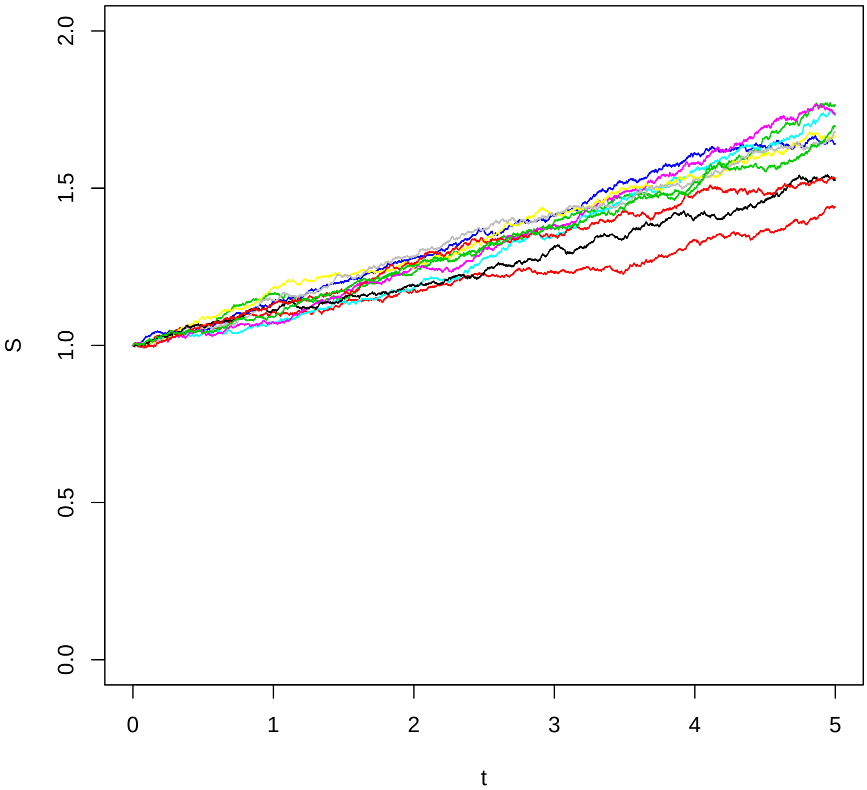


(c) For $\mu = -1, \sigma^2 = 0.64$

4) $\mu = 0.1$ **and** $\sigma^2 = 0.0009$

```
experemntal E[S(5)] =  1.640698
experemntal Var[S(5)] =  0.01144312
theoritical E[S(5)] =  1.648721
theoritical Var[S(5)] =  0.01225983
```

The corresponding plot obtained is shown below:

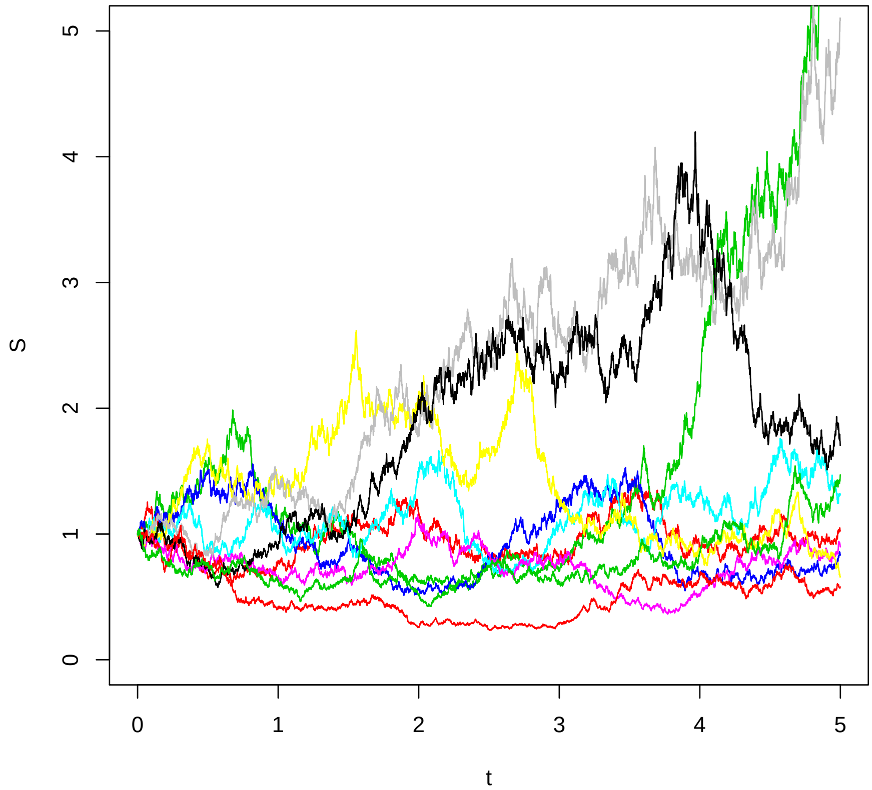


(d) For $\mu = 0.1, \sigma^2 = 0.0009$

5) $\mu = 0.1$ **and** $\sigma^2 = 0.16$

```
experemntal E[S(5)] =  1.965153
experemntal Var[S(5)] =  3.815481
theoritical E[S(5)] =  1.648721
theoritical Var[S(5)] =  3.331366
```

The corresponding plot obtained is shown below:

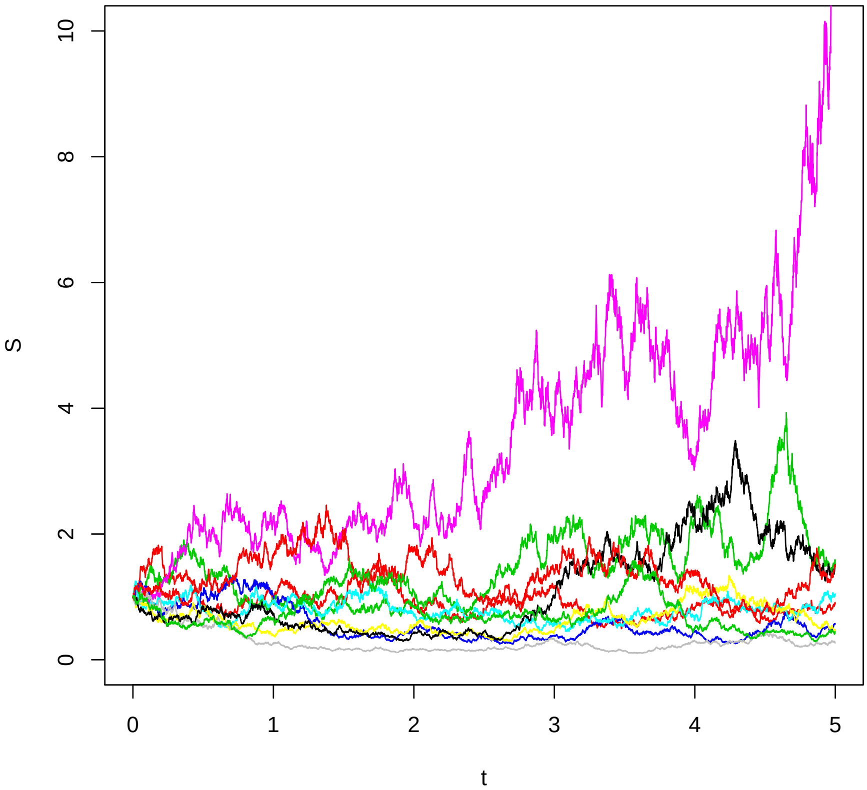


(e) For $\mu = 0.1, \sigma^2 = 0.16$

6) $\mu = 0.1$ **and** $\sigma^2 = 0.36$

```
experemntal E[S(5)] =  2.060723
experemntal Var[S(5)] =  13.35654
theoritical E[S(5)] =  1.648721
theoritical Var[S(5)] =  13.72636
```

The corresponding plot obtained is shown below:

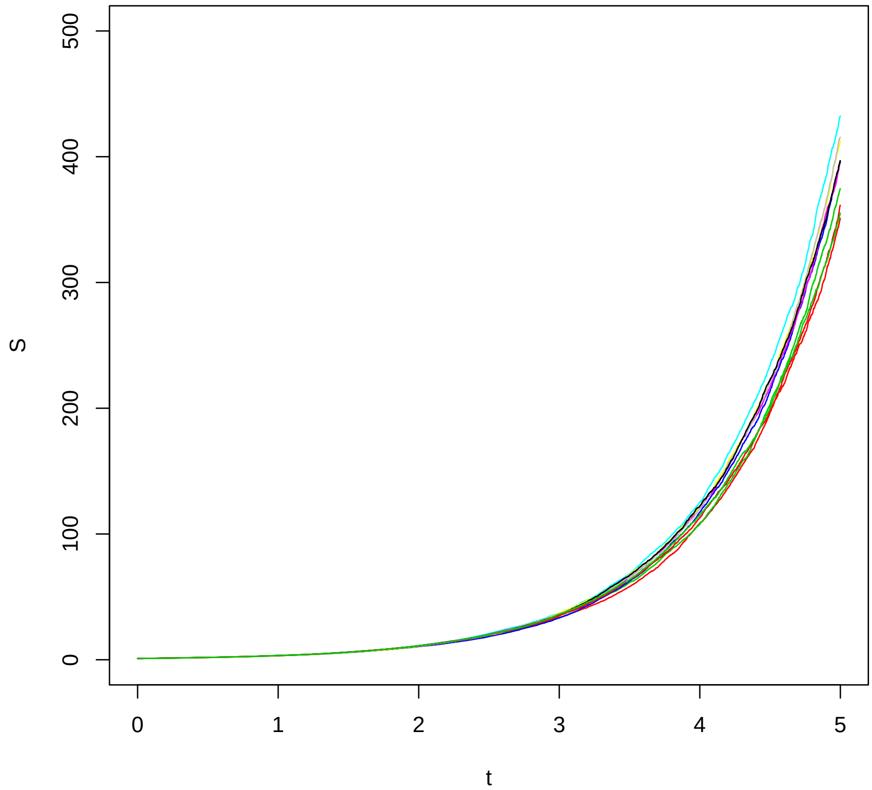


(f) For $\mu = 0.1, \sigma^2 = 0.36$

7) $\mu = 1.2$ **and** $\sigma^2 = 0.0009$

```
experemntal E[S(5)] =  389.121
experemntal Var[S(5)] =  759.7764
theoritical E[S(5)] =  403.4288
theoritical Var[S(5)] =  734.0469
```

The corresponding plot obtained is shown below:

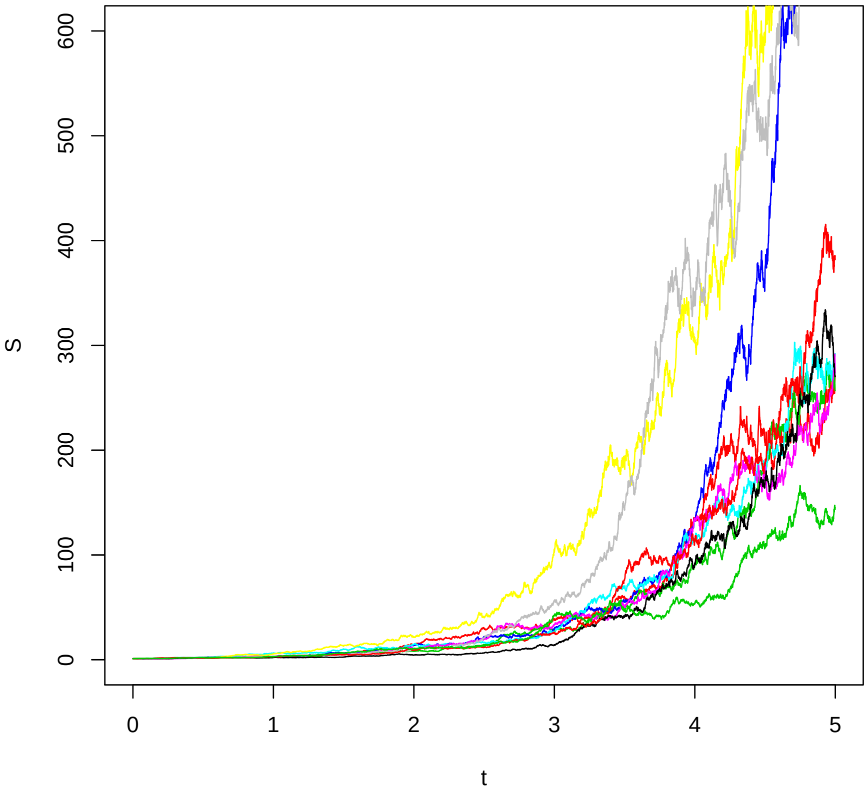


(g) For $\mu = 1.2, \sigma^2 = 0.0009$

8) $\mu = 1.2$ **and** $\sigma^2 = 0.16$

```
experemntal E[S(5)] =  481.3793
experemntal Var[S(5)] =  143927.7
theoritical E[S(5)] =  403.4288
theoritical Var[S(5)] =  199462.7
```

The corresponding plot obtained is shown below:

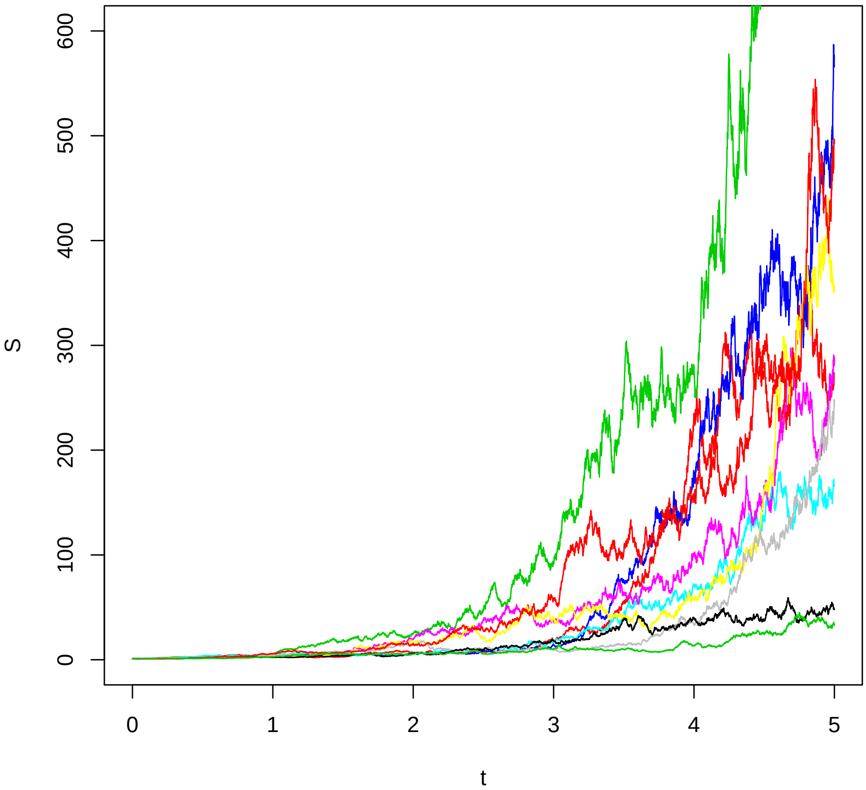


(h) For $\mu = 1.2, \sigma^2 = 0.16$

9) $\mu = 1.2$ **and** $\sigma^2 = 0.36$

```
experemntal E[S(5)] =  343.1025
experemntal Var[S(5)] =  77828.83
theoritical E[S(5)] =  403.4288
theoritical Var[S(5)] =  821854.3
```

The corresponding plot obtained is shown below:



(i) For $\mu = 1.2, \sigma^2 = 0.36$