Endsem - Lab Examination
MA 226 : Monte Carlo Simulation

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Note:

- Please login to MA226 account with password : exam226
- After login, you have to create a folder on the desktop with your roll number (a complete 8 digit number). You have to keep all the programs inside this folder. Anything kept outside will not be collected by our automated script.
- At the end of examination, you can atmost logout the PC or keep as it is. Please DO NOT shutdown your PC.
- There will be no internet connection in the lab during the exam hours.
- Please do not violate the Honor Code during the examination.
- The exam time is 120 minutes starting from 2-00 pm.
- Total Marks 40.

Use R or C/C++ for all questions [5 marks]

1. Describe and write down an algorithm to simulate a bivariate normal random variable (X, Y). Given four standard normal variate 0.9597, -1.3404, 1.2238, 0.2551, generate

samples for the bivariate normal random variable (X, Y), where $X \sim N(1, 2)$, $Y \sim N(2, 4)$ and the correlation co-efficient between X and Y is 0.05.

Use R or C/C++ for all questions [5 marks]

2. Calculate the sample mean based on the sample of size 200 generated from mixture of two exponential distributions with mean 2 and 3. Please take proportion of mixture as 0.4.

   Use R or C/C++ for all questions [15 marks]

3. Use Naive Monte carlo, Antithetic estimate, Control Variate estimate with $Y = 2e^{-2U}$ to estimate the integral

$$\int_0^2 e^{-x^2} dx$$

   Tabulate the estimates, its variances and variance reductions for $N = 100, 1000, 10000$.

   Hint :

$$\int_0^2 e^{-x^2} dx = 2 \int_0^1 e^{-(2u)^2} du.$$

   Use R or C/C++ for all questions [15 marks]

4. Write a separate code to generate random number from standard normal distribution by box-muller method. In the interval $[0, 5]$, taking $\mu = 0.2$, $\sigma = 0.1$ and $S(0) = 100$, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path). Also, by generating a large number of sample paths, calculate expectation and variance of $S(5)$ and match it with the theoretical values.