Assignment-4

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Q 1 Generate 1000 standard normal variates using standard Double-exponential distri- bution by acceptance-rejection method. Calculate the necessary constant c, where$f(x)/g(x)c$, $f(x)$ and $g(x)$ are the pdfs of standard normal and standard Double- exponential distribution respectively. Calculate the theoretical and simulated accep- tance probability. How do you justify your generated random numbers are correct ? Provide as many verification as you can.

Code for R

```
f<-function(x)
{
        return((exp(-(x^2)/2))/sqrt(2*pi))
}

g<-function(x)
{
        return ((exp(-abs(x))/2))
}

g_inv<-function(x)
{
        if(x<0.5)
                {return(log(2*x))}
        else
                {return(((log(2*(1-x)))*(-1)))}
}
c<-sqrt(2*exp(1)/pi)
u<-runif(1000)
v<-runif(1000)
count<-0
sum<-0

x<-array(0,1400)
for(i in 1:1400)
{
        x[i]<-g_inv(u[i])
}

fdist<-vector('numeric')
for (i in 1:1000)
{
        if((c*g(x[i])*v[i])<=(f(x[i])))
                {
                        count<-count+1
                        sum<-sum+x[i]
                        fdist<-c(fdist , x[i])
                }
```

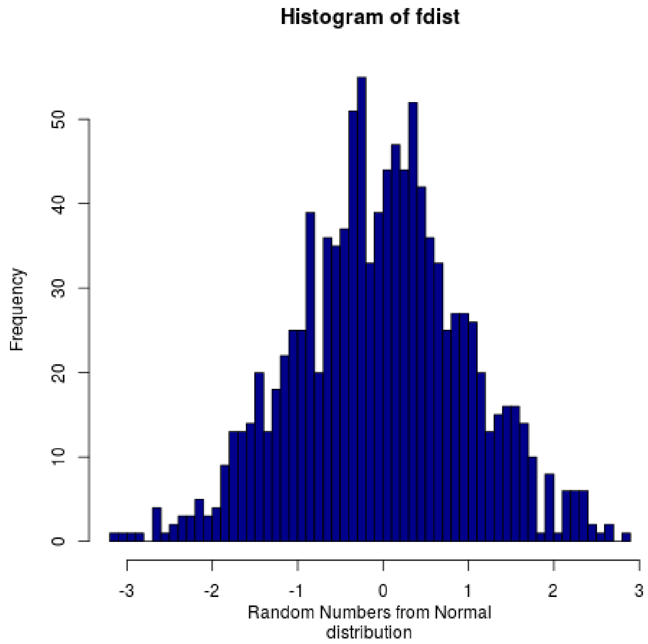
```
}

mean=as.double(sum/count)
sqsum<-0
for (i in 1:1000)
{
        if((c*g(x[i])*v[i])<=(f(x[i])))
                {
                        sqsum<-sqsum+((mean-x[i])^2)
                }
}
var=as.double(sqsum/count)
cat("Mean : ",mean,"\n");
cat("Standard Deviation : ",sqrt(var),"\n");
cat("Observed Acceptance probability : ",(count/1400),"\n");
cat("Theoritical Acceptance probability : ",(1/c),"\n");
png("plot_1_1000.png")
hist(fdist, col="dark blue", breaks=50, xlab="Random Numbers from Normal
distribution");
```

The output of the code is as follows:

```
Mean :  -0.09066503
Standard Deviation :  1.041097
Observed Acceptance probability :  0.7564286
Theoritical Acceptance probability :  0.7601735
```

The corresponding graph obtained is shown below:



(a)

Observations

- The theoretically calculated acceptance probability is $1/c$ which is 0.7601.
- The accpetance probability calculated from the code is 0.761.
- As these two values are close it shows that the random numbers generated follow standard normal variation.

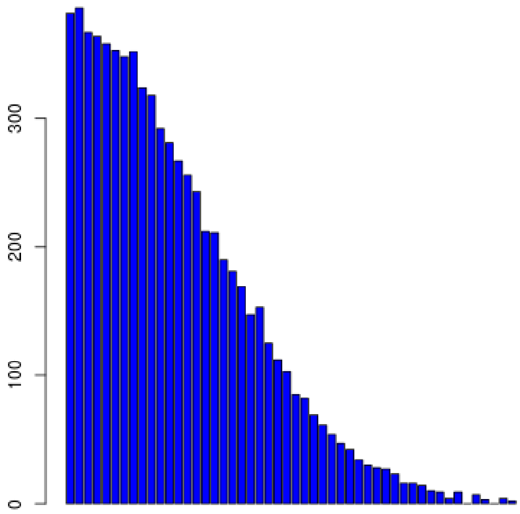
Q 2 Do the same exercise for generating random numbers from half-standard normal dis- tribution using exponential distribution with mean 1 by acceptance-rejection method.

Code for R

```
f<-function(x)
{
    return (exp(-x*x/2)*(2/pi)^(1/2));
}
g<-function(x)
{
    return (exp(-x));
}
h<-function(x)
{
    return (-log(1-x))
}
c<-1.4;
m<-2^13;
a<-113;
b<-83;
x<-4;
y<-91;
count<-0
freq<-array(0,50);
for(i in 1:10000)
{
    x<-(a*x+b)%%m;
    u<-as.double(x)/m;
    y<-(a*y+b+80)%%m;
    v<-as.double(y)/m;
    Y<-h(v);
    if(c*g(Y)*u<=f(Y))
        {
            freq[Y*15+1]<-freq[Y*15+1]+1;
            count=count+1;
        }
}
png("q2.png")
print(1/c)
print(count/10000)
barplot(freq, col="blue");
```

```
Observed Acceptance probability :  0.7142857
Theoritical Acceptance probability :  0.7175
```

The corresponding graph obtained is shown below:



(b)

Observations

- The theoretically calculated acceptance probability is $1/c$ which is 0.7175.
- The accpetance probability calculated from the code is 0.7142857.
- As these two values are close it shows that the random numbers generated follow standard normal variation.

Q 3 Consider the following discrete distribution.

j	1	2	3	4	5
p_j	0.05	0.25	0.45	0.15	0.10

- Generate 10 random numbers from the above probability mass function using usual procedure (inverse transform) of generating random number from discrete distribution defined on finite number of points. Calculate mean and variance of the generated numbers.
- Generate 10 random numbers from the same probability mass function by acceptance rejection principle. Calculate mean and variance of the generated numbers.

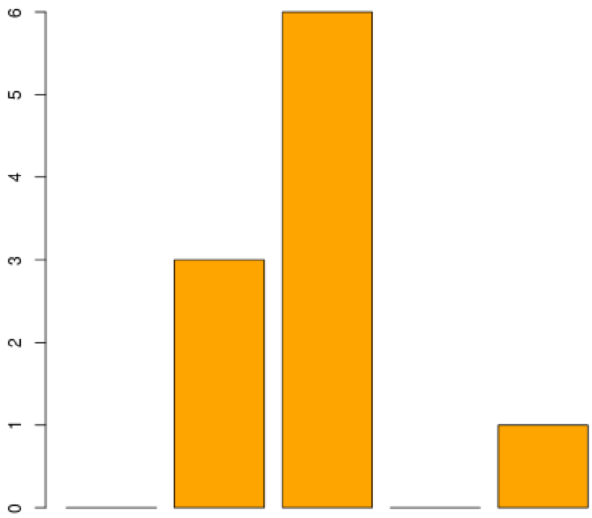
Part a:
Code for R

```
m<-2^13;
a<-113;
b<-91;
x<-70;
p<-c(0.05,0.25,0.45,0.15,0.10);
c<-cumsum(p);
sum<-0;
sqsum<-0;
freq<-array(0,5);
for(i in 1:10)
{
    x<-(a*x+b)%%m;
    u<-as.double(x)/m;
    for(j in 1:5)
        if(u<c[j])
        {
            freq[j]<-freq[j]+1;
            sum<-sum+j;
            break;
        }
}
mean<-as.double(sum)/10;
for(i in 1:5)
    sqsum<-sqsum+freq[i]*(mean-i)^2;
var<-as.double(sqsum)/10;
cat("Mean : ",mean,"\n");
cat("Variance : ",var,"\n");
png("q3_a.png");
barplot(freq,col=c("orange"));
```

The output of the code is as follows:

```
Mean :   2.9
Variance :   0.69
```

The corresponding graph obtained is shown below:



(c)

Part b:

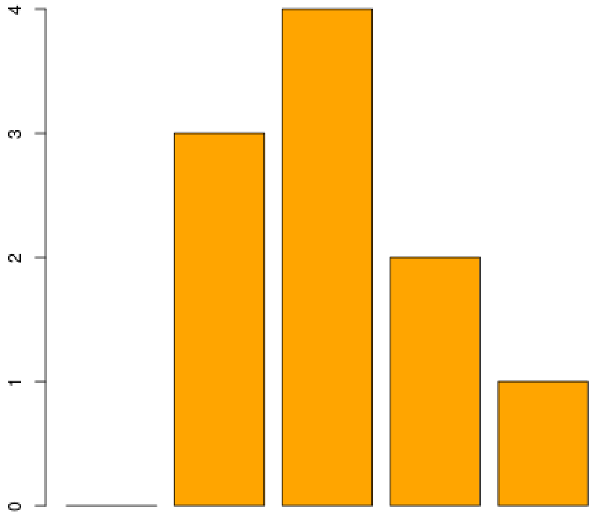
Code for R

```
Acceptance_Rejection<-function()
{
    pmf<-c(0.05,0.25,0.45,0.15,0.10);
    f=vector(length=10);
    i=1;
    while(i<=10)
    {
        u1=runif(1);
        u2=runif(1);
        y=floor(1+(5*u2));
        if (u1<=(pmf[y]/0.45))
        {
            f[i]=y;
            print(f[i]);
            i=i+1;
        }
    }
    var_rand=0;
    for (i in 1:10)
    {
        var_rand=var_rand+((f[i]-mean(f))^2);
    }
    var_rand=var_rand/10;
    cat("\nMean : ",mean(f), "\nVariance : ", var_rand);
}
Acceptance_Rejection();
```

The output of the code is as follows:

```
Mean :   3.1
Variance :   0.89
```

The corresponding graph obtained is shown below:



(d)