Monte Carlo Simulation: Assignment 1

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Question 1

$$x_{i+1} = (ax_i + b)\ mod\ m$$

$$u_{i+1} = x_{i+1}/m$$

Generate the sequence of numbers x_i for $a = 6, b = 0$,
$m = 11$ and x_0 ranging from 0 to 10. Also, generate the sequence of numbers x_i for $a = 3, b = 0, m = 11$, and $x_0$ranging from 0 to 10. Observe the sequence of numbers generated and observe the repetition of values. Tabulate these for each group of values. How many distinct values are appearing before repetitions? Which in your view are the best choices and why?

Solution

C++ Code:

```
1  #include <iostream>
2  #include <fstream>
3
4
5  using namespace std;
6
7  int main()
8  {
9      ofstream myfile;
10     myfile.open("output.txt", ios::app);
11     int i,j;
12     int a,b,m;
13     float x;
```

```
14      float u;
15      a=6;
16      b=0;
17      m=11;
18      cout<<"For a=6, b=0, m=11:\n";
19      myfile<<"For a=6, b=0, m=11:\n";
20
21      for (i=0;i<11;i++)
22      {
23          cout<<"\nRandom x[i] for x[0]="<<i<<"\n";
24          myfile<<"\nRandom x[i] for x[0]="<<i<<"\n";
25          x=i;
26          for(j=0;j<15;j++)
27          {
28              cout<<x<<" ";
29              myfile<<x<<" ";
30              x=((a*int(x))+b)%m;
31          }
32      }
33      for(i=0;i<11;i++)
34      {
35          cout<<"\n";
36          cout<<"Random u[i] for x[0]="<<i<<"\n";
37          myfile<<"\n";
38          myfile<<"Random u[i] for x[0]="<<i<<"\n";
39          x=i;
40          u=float(x/m);
41          for(j=0;j<15;j++)
42          {
43              cout<<u<<" ";
44              myfile<<u<<" ";
45              x=((a*int(x))+b)%m;
46              u=float(x/m);
47          }
48          cout<<"\n";
49      }
50      a=3;
51      b=0;
52      m=11;
53      cout<<"\n\nFor a=3, b=0, m=11:\n";
54      myfile<<"\n\nFor a=3, b=0, m=11:\n";
55      for (i=0;i<11;i++)
56      {
57          cout<<"\nRandom x[i] for x[0]="<<i<<"\n";
58          myfile<<"\nRandom x[i] for x[0]="<<i<<"\n";
59          x=i;
60          for(j=0;j<15;j++)
61          {
62              cout<<x<<" ";
63              myfile<<x<<" ";
64              x=((a*int(x))+b)%m;
65          }
66      }
67      for(i=0;i<11;i++)
68      {
69          cout<<"\n";
70          cout<<"Random u[i] for x[0]="<<i<<"\n";
71          myfile<<"\n";
72          myfile<<"Random u[i] for x[0]="<<i<<"\n";
```

```
        x=i;
        u=float(x/m);
        for(j=0;j<15;j++)
        {
            cout<<u<<" ";
            myfile<<u<<" ";
            x=((a*int(x))+b)%m;
            u=float(x/m);
        }
        cout<<"\n";
        myfile<<"\n";
    }
    myfile.close();
}
```

Output:

```
For a=6, b=0, m=11:

Random x[i] for x[0]=0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Random x[i] for x[0]=1
1 6 3 7 9 10 5 8 4 2 1 6 3 7 9
Random x[i] for x[0]=2
2 1 6 3 7 9 10 5 8 4 2 1 6 3 7
Random x[i] for x[0]=3
3 7 9 10 5 8 4 2 1 6 3 7 9 10 5
Random x[i] for x[0]=4
4 2 1 6 3 7 9 10 5 8 4 2 1 6 3
Random x[i] for x[0]=5
5 8 4 2 1 6 3 7 9 10 5 8 4 2 1
Random x[i] for x[0]=6
6 3 7 9 10 5 8 4 2 1 6 3 7 9 10
Random x[i] for x[0]=7
7 9 10 5 8 4 2 1 6 3 7 9 10 5 8
Random x[i] for x[0]=8
8 4 2 1 6 3 7 9 10 5 8 4 2 1 6
Random x[i] for x[0]=9
9 10 5 8 4 2 1 6 3 7 9 10 5 8 4
Random x[i] for x[0]=10
10 5 8 4 2 1 6 3 7 9 10 5 8 4 2
Random u[i] for x[0]=0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Random u[i] for x[0]=1
0.0909091 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636
    0.181818 0.0909091 0.545455 0.272727 0.636364 0.818182

Random u[i] for x[0]=2
0.181818 0.0909091 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273
    0.363636 0.181818 0.0909091 0.545455 0.272727 0.636364

Random u[i] for x[0]=3
0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.0909091
    0.545455 0.272727 0.636364 0.818182 0.909091 0.454545

Random u[i] for x[0]=4
0.363636 0.181818 0.0909091 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545
    0.727273 0.363636 0.181818 0.0909091 0.545455 0.272727
```

```
39
40 Random u[i] for x[0]=5
41 0.454545 0.727273 0.363636 0.181818 0.0909091 0.545455 0.272727 0.636364 0.818182
       0.909091 0.454545 0.727273 0.363636 0.181818 0.0909091
42
43 Random u[i] for x[0]=6
44 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818
       0.0909091 0.545455 0.272727 0.636364 0.818182 0.909091
45
46 Random u[i] for x[0]=7
47 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.0909091 0.545455
       0.272727 0.636364 0.818182 0.909091 0.454545 0.727273
48
49 Random u[i] for x[0]=8
50 0.727273 0.363636 0.181818 0.0909091 0.545455 0.272727 0.636364 0.818182 0.909091
       0.454545 0.727273 0.363636 0.181818 0.0909091 0.545455
51
52 Random u[i] for x[0]=9
53 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.0909091 0.545455 0.272727
       0.636364 0.818182 0.909091 0.454545 0.727273 0.363636
54
55 Random u[i] for x[0]=10
56 0.909091 0.454545 0.727273 0.363636 0.181818 0.0909091 0.545455 0.272727 0.636364
       0.818182 0.909091 0.454545 0.727273 0.363636 0.181818
57
58
59 For a=3, b=0, m=11:
60
61 Random x[i] for x[0]=0
62 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
63 Random x[i] for x[0]=1
64 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4
65 Random x[i] for x[0]=2
66 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8
67 Random x[i] for x[0]=3
68 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1
69 Random x[i] for x[0]=4
70 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5
71 Random x[i] for x[0]=5
72 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9
73 Random x[i] for x[0]=6
74 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2
75 Random x[i] for x[0]=7
76 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6
77 Random x[i] for x[0]=8
78 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10
79 Random x[i] for x[0]=9
80 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3
81 Random x[i] for x[0]=10
82 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7
83 Random u[i] for x[0]=0
84 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
85
86 Random u[i] for x[0]=1
87 0.0909091 0.272727 0.818182 0.454545 0.363636 0.0909091 0.272727 0.818182 0.454545
       0.363636 0.0909091 0.272727 0.818182 0.454545 0.363636
88
89 Random u[i] for x[0]=2
90 0.181818 0.545455 0.636364 0.909091 0.727273 0.181818 0.545455 0.636364 0.909091
```

```
    0.727273 0.181818 0.545455 0.636364 0.909091 0.727273
91
92 Random u[i] for x[0]=3
93 0.272727 0.818182 0.454545 0.363636 0.0909091 0.272727 0.818182 0.454545 0.363636
    0.0909091 0.272727 0.818182 0.454545 0.363636 0.0909091
94
95 Random u[i] for x[0]=4
96 0.363636 0.0909091 0.272727 0.818182 0.454545 0.363636 0.0909091 0.272727 0.818182
    0.454545 0.363636 0.0909091 0.272727 0.818182 0.454545
97
98 Random u[i] for x[0]=5
99 0.454545 0.363636 0.0909091 0.272727 0.818182 0.454545 0.363636 0.0909091 0.272727
    0.818182 0.454545 0.363636 0.0909091 0.272727 0.818182
100
101 Random u[i] for x[0]=6
102 0.545455 0.636364 0.909091 0.727273 0.181818 0.545455 0.636364 0.909091 0.727273
    0.181818 0.545455 0.636364 0.909091 0.727273 0.181818
103
104 Random u[i] for x[0]=7
105 0.636364 0.909091 0.727273 0.181818 0.545455 0.636364 0.909091 0.727273 0.181818
    0.545455 0.636364 0.909091 0.727273 0.181818 0.545455
106
107 Random u[i] for x[0]=8
108 0.727273 0.181818 0.545455 0.636364 0.909091 0.727273 0.181818 0.545455 0.636364
    0.909091 0.727273 0.181818 0.545455 0.636364 0.909091
109
110 Random u[i] for x[0]=9
111 0.818182 0.454545 0.363636 0.0909091 0.272727 0.818182 0.454545 0.363636 0.0909091
    0.272727 0.818182 0.454545 0.363636 0.0909091 0.272727
112
113 Random u[i] for x[0]=10
114 0.909091 0.727273 0.181818 0.545455 0.636364 0.909091 0.727273 0.181818 0.545455
    0.636364 0.909091 0.727273 0.181818 0.545455 0.636364
```

For $a = 6, b = 0, m = 11$:

Sequence of numbers generated for seed = 0 : 0, 0, 0, ...
Sequence of numbers generated for seed > 0 : 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, ...

Seeds	0	1	2	3	4	5	6	7	8	9	10
Period	1	10	10	10	10	10	10	10	10	10	10

For $a = 3, b = 0, m = 11$:

Sequence of numbers generated for seed = 0: 0, 0, 0, ...
Sequence of numbers generated for seed > 0: 1, 3, 9, 5, 4, ...

Seeds	0	1	2	3	4	5	6	7	8	9	10
Period	1	5	5	5	5	5	5	5	5	5	5

Observations:

Number of distinct values appearing before repetition for $a = 6(\text{seed} > 0) = 10$

Number of distinct values appearing before repetition for $a = 3(\text{seed} > 0) = 5$

Best Choice: $a = 6$ and m $\neq 0$, because the random numbers generated have more period(=10) than generated by a=3 (period = 5).

Question 2

Generate a sequence u_i with $m = 244944$, $a = 1597$ (take x_0 as per your choice). Try to group the values in the ranges $0-0.05$, $0.05-0.10$, $0.10-0.15$, ... and see their frequencies (i.e. the number of values falling in a group). For at least 5 different values of the number of values generated, tabulate the frequencies in each case, draw bar diagrams of these data and put in your observations.

Solution

C++ Code:

```
#include <iostream>
#include <fstream>

using namespace std;

int main()
{
    ofstream myfile;
    myfile.open("output.txt", ios::app);

    int i,j,k,l;
    int a[2],b,m;
    float x[500000];
    int n[5];
    char filename[6];
    n[0]=1000;
    n[1]=500;
    n[2]=5000;
    n[3]=50000;
    n[4]=500000;
    float u[250000];
    int f[20];
    a[0]=1597;
    a[1]=51749;
    b=0;
    m=244944;
    char freq[20][100]={"0.00-0.05","0.05-0.10","0.10-0.15","0.15-0.20","0.20-0.25","
        0.25-0.30","0.30-0.35","0.35-0.40","0.40-0.45","0.45-0.50","0.50-0.55","
        0.55-0.60","0.60-0.65","0.65-0.70","0.70-0.75","0.75-0.80","0.80-0.85","
        0.85-0.90","0.90-0.95","0.95-1.00"};
    {
        myfile<<"For a="<<a[0]<<", b="<<b<<", m="<<m<<":\n";
        for (k=0;k<5;k++)
        {


            x[0]=12345;
            myfile<<"\nFor n="<<n[k];
            u[0]=float(x[0]/m);
```

```
37            for(j=0;j<n[k]-1;j++)
38            {
39                x[j+1]=((a[0]*int(x[j]))+b)%m;
40                u[j+1]=float(x[j+1]/m);
41            }
42            myfile<<"\n";
43
44            for(i=0;i<20;i++)
45            {
46                f[i]=0;
47            }
48
49            for(i=0;i<n[k];i++)
50            {
51                if(u[i]>=0 && u[i]<0.05)
52                    f[0]++;
53                if(u[i]>=0.05 && u[i]<0.10)
54                    f[1]++;
55                if(u[i]>=0.10 && u[i]<0.15)
56                    f[2]++;
57                if(u[i]>=0.15 && u[i]<0.20)
58                    f[3]++;
59                if(u[i]>=0.20 && u[i]<0.25)
60                    f[4]++;
61                if(u[i]>=0.25 && u[i]<0.30)
62                    f[5]++;
63                if(u[i]>=0.30 && u[i]<0.35)
64                    f[6]++;
65                if(u[i]>=0.35 && u[i]<0.40)
66                    f[7]++;
67                if(u[i]>=0.40 && u[i]<0.45)
68                    f[8]++;
69                if(u[i]>=0.45 && u[i]<0.50)
70                    f[9]++;
71                if(u[i]>=0.50 && u[i]<0.55)
72                    f[10]++;
73                if(u[i]>=0.55 && u[i]<0.60)
74                    f[11]++;
75                if(u[i]>=0.60 && u[i]<0.65)
76                    f[12]++;
77                if(u[i]>=0.65 && u[i]<0.70)
78                    f[13]++;
79                if(u[i]>=0.70 && u[i]<0.75)
80                    f[14]++;
81                if(u[i]>=0.75 && u[i]<0.80)
82                    f[15]++;
83                if(u[i]>=0.80 && u[i]<0.85)
84                    f[16]++;
85                if(u[i]>=0.85 && u[i]<0.90)
86                    f[17]++;
87                if(u[i]>=0.90 && u[i]<0.95)
88                    f[18]++;
89                if(u[i]>=0.95 && u[i]<=1)
90                    f[19]++;
91            }
92
93            for(i=0;i<20;i++)
94            {
95                myfile <<i<<"   "<<freq[i]<<"   "<<f[i]<<"\n";
```

```
            }
            myfile<<"\n";
        }
    }
    myfile.close();
}
```

Output:

```
For a=1597, b=0, m=244944:

For n=1000
0   0.00-0.05    48
1   0.05-0.10    53
2   0.10-0.15    49
3   0.15-0.20    50
4   0.20-0.25    50
5   0.25-0.30    48
6   0.30-0.35    50
7   0.35-0.40    48
8   0.40-0.45    50
9   0.45-0.50    53
10  0.50-0.55    48
11  0.55-0.60    50
12  0.60-0.65    49
13  0.65-0.70    51
14  0.70-0.75    51
15  0.75-0.80    51
16  0.80-0.85    49
17  0.85-0.90    49
18  0.90-0.95    51
19  0.95-1.00    52


For n=500
0   0.00-0.05    26
1   0.05-0.10    25
2   0.10-0.15    26
3   0.15-0.20    28
4   0.20-0.25    18
5   0.25-0.30    24
6   0.30-0.35    29
7   0.35-0.40    22
8   0.40-0.45    24
9   0.45-0.50    25
10  0.50-0.55    22
11  0.55-0.60    28
12  0.60-0.65    24
13  0.65-0.70    21
14  0.70-0.75    32
15  0.75-0.80    27
16  0.80-0.85    20
17  0.85-0.90    26
18  0.90-0.95    26
19  0.95-1.00    27


For n=5000
0   0.00-0.05    248
```

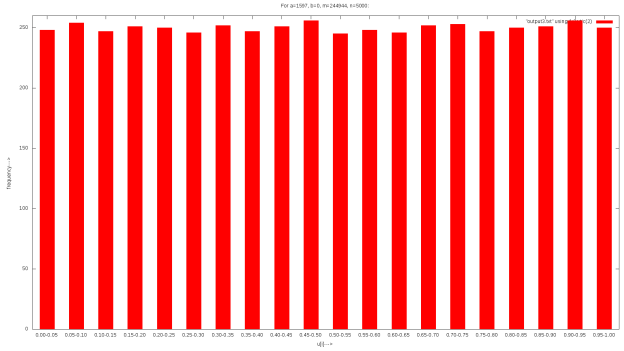
```
1   0.05-0.10   254
2   0.10-0.15   247
3   0.15-0.20   251
4   0.20-0.25   250
5   0.25-0.30   246
6   0.30-0.35   252
7   0.35-0.40   247
8   0.40-0.45   251
9   0.45-0.50   256
10  0.50-0.55   245
11  0.55-0.60   248
12  0.60-0.65   246
13  0.65-0.70   252
14  0.70-0.75   253
15  0.75-0.80   247
16  0.80-0.85   250
17  0.85-0.90   251
18  0.90-0.95   256
19  0.95-1.00   250


For n=50000
0   0.00-0.05   2469
1   0.05-0.10   2521
2   0.10-0.15   2471
3   0.15-0.20   2525
4   0.20-0.25   2515
5   0.25-0.30   2467
6   0.30-0.35   2525
7   0.35-0.40   2468
8   0.40-0.45   2520
9   0.45-0.50   2518
10  0.50-0.55   2468
11  0.55-0.60   2522
12  0.60-0.65   2468
13  0.65-0.70   2517
14  0.70-0.75   2530
15  0.75-0.80   2470
16  0.80-0.85   2516
17  0.85-0.90   2471
18  0.90-0.95   2523
19  0.95-1.00   2516


For n=500000
0   0.00-0.05   24690
1   0.05-0.10   25205
2   0.10-0.15   24693
3   0.15-0.20   25208
4   0.20-0.25   25201
5   0.25-0.30   24691
6   0.30-0.35   25212
7   0.35-0.40   24692
8   0.40-0.45   25204
9   0.45-0.50   25205
10  0.50-0.55   24691
11  0.55-0.60   25204
12  0.60-0.65   24690
13  0.65-0.70   25201
```

14	0.70 − 0.75	25216
15	0.75 − 0.80	24692
16	0.80 − 0.85	25200
17	0.85 − 0.90	24693
18	0.90 − 0.95	25210
19	0.95 − 1.00	25202

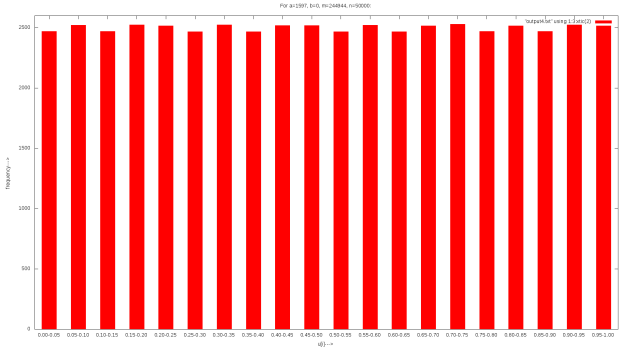
Plots:

(a) $n = 500$

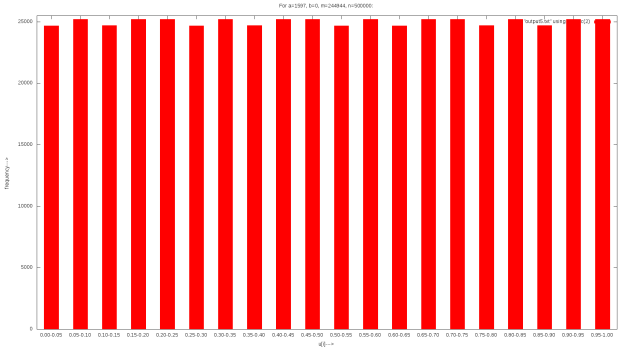
(b) $n = 1000$



(c) $n = 5000$



(d) $n = 50000$



(e) $n = 500000$

Figure 1: Histograms for (a) n = 500, (b) n = 1000 and (c) n = 5000 (d) n = 50000 (e) n = 500000

Observations:

- As number of random numbers generated increases, the graph becomes more uniform.
- Odd seed (= 12345) is taken to get more uniformly generated random numbers.
- Overflow of $x[i]$ is observed for $a = 51749$.

Question 3

Generate a sequence u_i with $a = 1229, b = 1, m = 2048$. Plot a two-dimensional graphthe points (u_{i-1}, u_i), i.e. the points $(u_1, u_2), (u_2, u_3), (u_3, u_4), ...$ What are your observations?

Solution

C++ Code:

```
#include <iostream>
#include <fstream>

using namespace std;

int main()
{
    ofstream myfile;
    myfile.open("output.txt", ios::app);

    int i,j;
    int a,b,m;
    float x[1000];
    float u[1000];
    a=1229;
    b=1;
    m=2048;
    x[0]=157;
    for(i=0;i<1000;i++)
    {
        x[i+1]=(int((a*x[i])+b))%m;
        u[i+1]=x[i+1]/m;
    }
    u[0]=x[0]/m;
    for(i=0;i<1000;++i)
        myfile<<u[i]<<"        "<<u[i+1]<<"\n";
    myfile.close();
}
```

Output:

```
0.0766602       0.21582
0.21582     0.243652
0.243652    0.449219
0.449219    0.090332
0.090332    0.0185547
0.0185547       0.804199
0.804199    0.361328
0.361328    0.0727539
0.0727539       0.415039
0.415039    0.0834961
0.0834961       0.617188
0.617188    0.523926
0.523926    0.905273
```

```
0.905273    0.581543
0.581543    0.716797
0.716797    0.943848
0.943848    0.989258
0.989258    0.79834
0.79834     0.160156
0.160156    0.83252
...             ...
...             ...
...             ...
```

Graph:

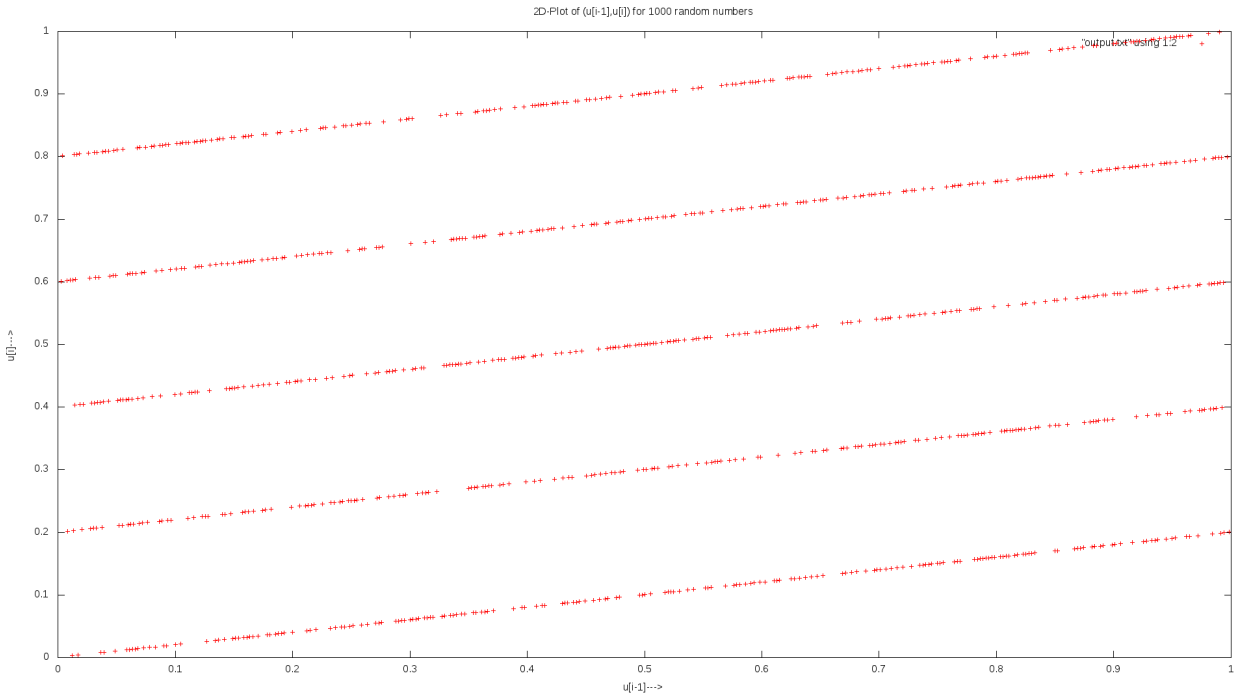


Figure 2: 2D-plot of (u_{i-1}, u_i) for 1000 random numbers

Observations:

- Plotted observations are observed to lie on a line.
- As the number of random numbers generated is increased, the lines get darker.
- Distance between two lines is very high which implies the random numbers generated are not that uniform for given values of a and b. This distance can be decreased by taking better values of a and b.