Monte Carlo Simulation Lab
Assignment-9

Yash Vanjani
(140123046)
Mathematics and Computing
IIT Guwahati

April 12th, 2016

Q 1 Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} * Z_{i+1}$$

with 5000 generated values for each of the paths where $Z_{i+1} \sim N(0, 1)$. Plot all the sample paths in a single figure. Also estimate E[W(2)] and E[W(5)] from the 10 paths that you have generated.

Code for R

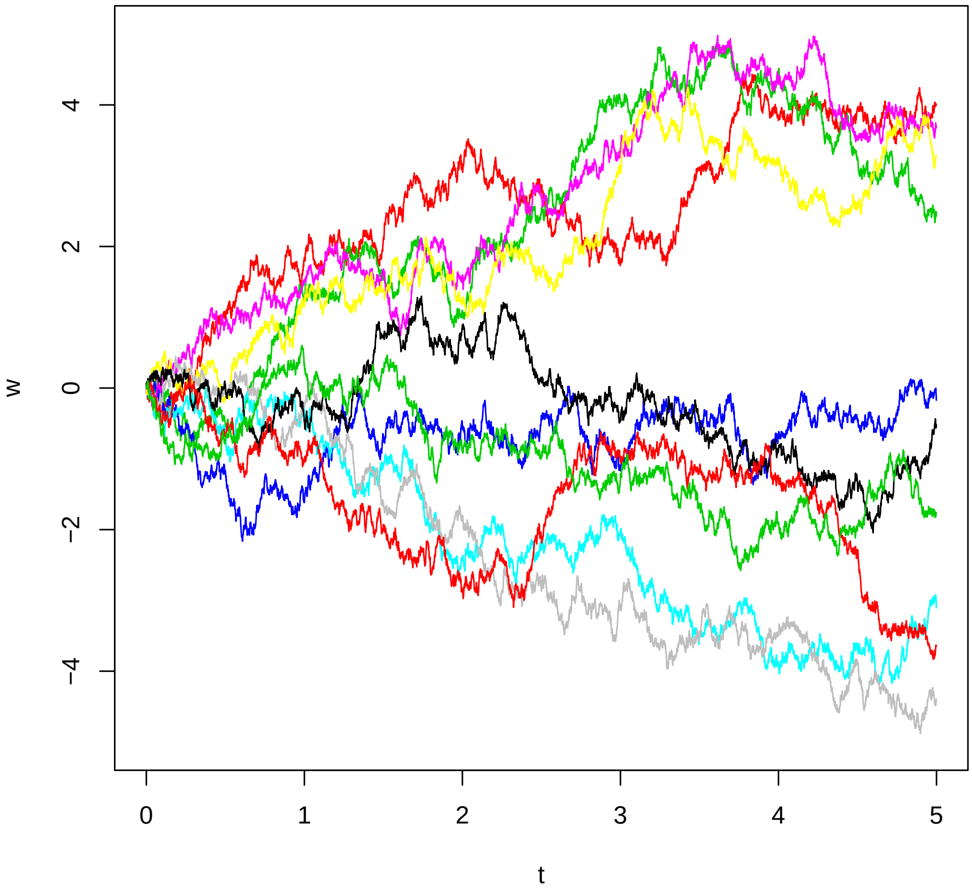
```r
library(stats)
w<-vector("numeric")
t<-vector("numeric")
pal<- palette()
t[1]=0
sec2<-vector("numeric") #for storing BM values at 2nd sec
sec5<-vector("numeric") #for storing BM values at 5th sec
for(i in 1:4999)
{
    t[i+1]=t[i]+0.001
}
for(i in 1:10)
{
    z<-rnorm(5000, mean=0, sd=1)
    w[1]=0
    for (j in 1:4999)
    {
        w[j+1]=w[j]+z[j+1]*sqrt(0.001)
        if(j==2000)
        {
            sec2[i]=w[2000]
        }
        if(j==4999)
        {
            sec5[i]=w[5000]
        }

    }
    if(i==1)
    {
        plot(t,w,type="l", col=pal[i %% 8 +1],ylim=c(-5,5))
    }
    else
    {
        lines(t,w, col=pal[i %% 8 +1],ylim=c(-5,5))
    }
```

```
}
cat("E[W(2)] = ",mean(sec2),"\n")
cat("E[W(5)] = ",mean(sec5),"\n")
```

Output:

$E[W(2)] = -0.0492709$
$E[W(5)] = -0.02063341$

The corresponding plot of "W v/s t" is shown below:



(a) "W v/s t" plot

Q 2 Repeat the above exercise with the following Brownian motion (BM(, 2)) discretization $X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} * Z_{i+1}$. Take $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$

Code for R

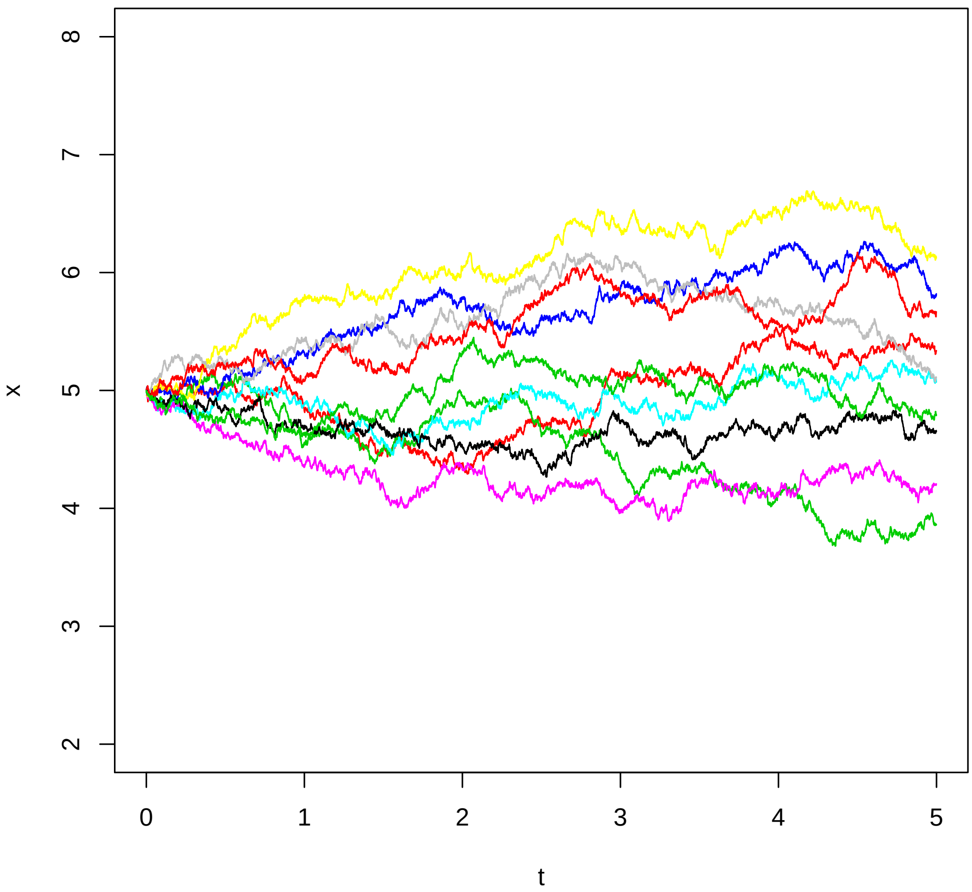
```r
library(stats)
x<-vector("numeric")
t<-vector("numeric")
pal<- palette()
t[1]=0
sec2<-vector("numeric") #for storing BM values at 2nd sec
sec5<-vector("numeric") #for storing BM values at 5th sec
for(i in 1:4999)
{
    t[i+1]=t[i]+0.001
}

mu<-0.06
sigma<-0.3

for(i in 1:10)
{
    z<-rnorm(5000, mean=0, sd=1)
    x[1]=5
    for (j in 1:4999)
    {
        x[j+1]=x[j]+(z[j+1]*sqrt(0.001)*sigma)+(mu*0.001)

        if(j==2000)
        {
            sec2[i]=x[2000]
        }
        if(j==4999)
        {
            sec5[i]=x[5000]
        }

    }
    if(i==1)
    {
        plot(t,x,type="l", col=pal[i %% 8 +1],ylim=c(2,8))
    }
    else
    {
        lines(t,x, col=pal[i %% 8 +1],ylim=c(2,8))
    }
}
```

```
cat("E[W(2)] = ",mean(sec2),"\n")
cat("E[W(5)] = ",mean(sec5),"\n")
```

Output:

E[W(2)] = 5.075252
E[W(5)] = 5.301343

The corresponding plot of "X v/s t" is shown below:



(b) "X v/s t" plot

Q 3 The Euler approximated recursion with time dependent μ and σ is given by:

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i} * Z_{i+1}$$

Repeat the above exercise by taking

$$Y(0) = 5, \mu(t) = 0.0325 - 0.05t, \sigma(t) = 0.012 + 0.0138t + 0.00125t^2.$$

Code for R

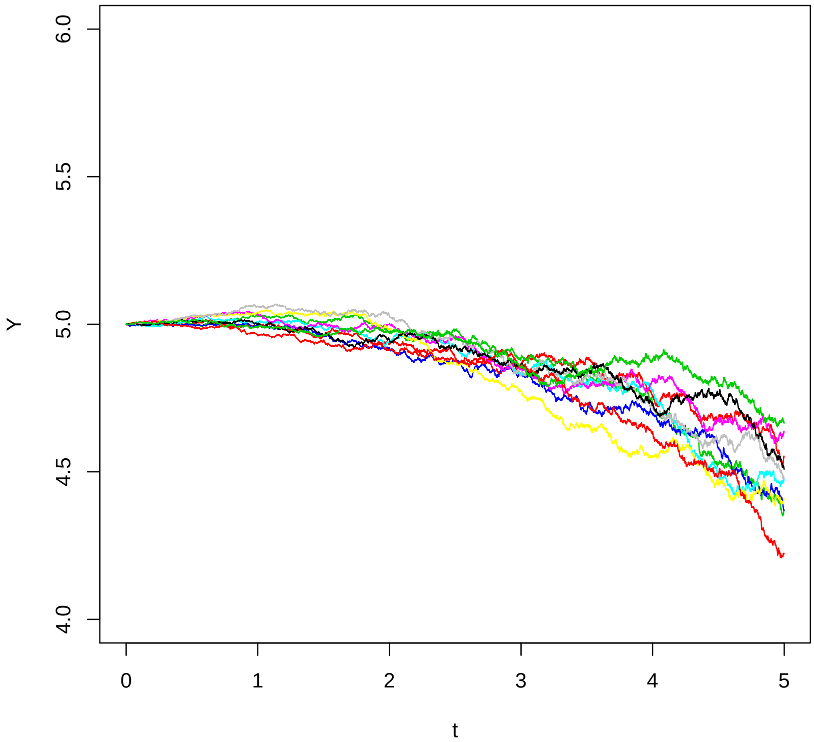
```
library(stats)
Y<-vector("numeric")
t<-vector("numeric")
pal<- palette()
t[1]=0
sec2<-vector("numeric") #for storing BM values at 2nd sec
sec5<-vector("numeric") #for storing BM values at 5th sec
mu<-vector("numeric")
sigma<-vector("numeric")
for(i in 1:4999)
{
    t[i+1]=t[i]+0.001
}
for(i in 1:5000)
{
    mu[i]=0.0325-(0.05*(t[i]))
    sigma[i]=0.012+(0.0138*t[i])+(0.00125*t[i]*t[i])
}

for(i in 1:10)
{
    z<-rnorm(5000, mean=0, sd=1)
    Y[1]=5
    for (j in 1:4999)
    {
        Y[j+1]=Y[j]+(z[j+1]*sqrt(0.001)*sigma[j])+(mu[j]*0.001)

        if(j==2000)
        {
            sec2[i]=Y[2000]
        }
        if(j==4999)
        {
            sec5[i]=Y[5000]
        }

    }
    if(i==1)
```

```
    {
        plot(t,Y,type="l", col=pal[i %% 8 +1],ylim=c(4,6))
    }
    else
    {
        lines(t,Y, col=pal[i %% 8 +1],ylim=c(4,6))
    }
}
cat("E[W(2)] = ",mean(sec2),"\n")
cat("E[W(5)] = ",mean(sec5),"\n")
```

Output:

E[W(2)] = 5.075252
E[W(5)] = 5.301343

The corresponding plot of "Y v/s t" is shown below:



(c) "Y v/s t" plot