

# Stack and Queue

A **stack** is a finite sequence of (zero or more) elements  $(x_1, \dots, x_n)$

Two operations, **push** and **pop** are defined on a stack.

**push**( $x_{n+1}$ ) changes a stack  $(x_1, \dots, x_n)$  to  $(x_1, \dots, x_n, x_{n+1})$

**pop**() changes a nonempty stack  $(x_1, \dots, x_n, x_{n+1})$  to  $(x_1, \dots, x_n)$   
and also returns the element  $x_{n+1}$

**Example** Start with an empty stack of integers, and perform the following operations.

```
push(5)
push(9)
push(2)
pop()
pop()
push(4)
pop()
pop()
```

What values are returned by pop() operations ?

A **stack** is a finite sequence of (zero or more) elements  $(x_1, \dots, x_n)$

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**push** $(x_{n+1})$  changes a stack  $(x_1, \dots, x_n)$  to  $(x_1, \dots, x_n, x_{n+1})$

**pop** $()$  changes a nonempty stack  $(x_1, \dots, x_n, x_{n+1})$  to  $(x_1, \dots, x_n)$   
and also returns the element  $x_{n+1}$

**Example** Start with an empty stack of integers, and perform the following operations.

push(5)	$() \rightarrow (5)$	
push(9)	$(5) \rightarrow (5, 9)$	
push(2)	$(5, 9) \rightarrow (5, 9, 2)$	
pop()	$(5, 9, 2) \rightarrow (5, 9)$	and 2 is returned
pop()	$(5, 9) \rightarrow (5)$	and 9 is returned
push(4)	$(5) \rightarrow (5, 4)$	
pop()	$(5, 4) \rightarrow (5)$	and 4 is returned
pop()	$(5) \rightarrow ()$	and 5 is returned

What values are returned by pop() operations ? 2 9 4 5

A **queue** is a finite sequence of (zero or more) elements  $(x_1, \dots, x_n)$

Two operations, **enqueue** and **dequeue** are defined on a stack.

**enqueue** $(x_{n+1})$  changes a stack  $(x_1, \dots, x_n)$  to  $(x_1, \dots, x_n, x_{n+1})$

**dequeue** $()$  changes a nonempty stack  $(x_1, x_2, \dots, x_n)$  to  $(x_2, \dots, x_n)$  and also returns the element  $x_1$

**Example** Start with an empty stack of integers, and perform the following operations.

```
enqueue(5)  ( ) → (5)
enqueue(9)  (5) → (5, 9)
enqueue(2)  (5, 9) → (5, 9, 2)
dequeue()   (5, 9, 2) → (9, 2) and 5 is returned
dequeue()   (9, 2) → (2) and 9 is returned
enqueue(4)  (2) → (2, 4)
dequeue()   (2, 4) → (4) and 2 is returned
dequeue()   (4) → ( ) and 4 is returned
```

What values are returned by dequeue() operations ? 5 9 4 2

# Implementing Stack and Queue Using Array

## Implementing a stack (of integers) using array

Use an integer array named **s** of size **MAX** to store the stack elements.

$s[0], s[1], \dots$  stores the elements  $x_1, x_2, \dots$

Notice that: (i) We cannot store more than MAX elements.

(ii) When we store only a few elements space is wasted.

Use an integer named **top** to store the index of the last element.

Notice that: If the stack contains **n** elements then **top = n-1**

To initialize the stack as empty we set **top = -1**

During **push(x)** increment top by 1 and store **x** in **s[top]**

During **pop()** let  $t = s[\text{top}]$ , decrement top by 1 and return **t**.

**Example: where MAX = 7**

s[0] s[1] s[2] s[3] s[4] s[5] s[6]

12	45	23	34			
----	----	----	----	--	--	--

↑  
top=3

push(4)

s[0] s[1] s[2] s[3] s[4] s[5] s[6]

12	45	23	34	17		
----	----	----	----	----	--	--

↑  
top=4

s[0] s[1] s[2] s[3] s[4] s[5] s[6]

12	45	23	34			
----	----	----	----	--	--	--

↑  
top=3

pop()

s[0] s[1] s[2] s[3] s[4] s[5] s[6]

12	45	23				
----	----	----	--	--	--	--

↑  
top=2  
(34 is returned)

```
class stack
{
    public:
    void push(int);
    int pop(void);
    bool isempty(void);
    bool isfull(void);
    void show(void);

    stack(int);
    ~stack(void);

    private:
    int MAX, *s, top;
};
```

returns true if and only if  
the stack is empty i.e. **top is -1**

returns true if and only if  
the stack is full i.e. **top is MAX-1**

**s** should point to the beginning of the array  
**s** is just a pointer, we must allocate space  
for array elements using dynamic allocation

When an object of this class is created we should,  
(i) initialise MAX  
(ii) allocate space for array elements  $s[0], \dots, s[\text{MAX}]$   
(iii) initialise top to -1

When an object of this class is destroyed we should,  
de allocate space for array elements  $s[0], \dots, s[\text{MAX}]$

```

class stack
{
    public:
    void push(int);
    int pop(void);
    bool isempty(void);
    bool isfull(void);
    void show(void);

    stack(int);
    ~stack(void);

    private:
    int MAX, *s, top;
};

```

**The constructor takes an argument !!**  
 (constructors are automatically called  
 when an object is created )  
**How do we pass the argument ??**

↓

```

stack::stack(int size)
{
    MAX = size;
    s = new int[MAX];
    top = -1;
}

```

When an object of this class is created we should,  
 (i) initialise MAX  
 (ii) allocate space for array elements s[0],...,s[MAX]  
 (iii) initialise top to -1

When an object of this class is destroyed we should,  
 de allocate space for array elements s[0],...,s[MAX]

```

stack::~~stack(void)
{
    delete[] s;
}

```



```

class stack
{
    public:
    void push(int);
    int pop(void);
    bool isempty(void);
    bool isfull(void);
    void show(void);

    stack(int);
    ~stack(void);

    private:
    int MAX, *s, top;
};

```

**The constructor takes an argument !!**  
 (constructors are automatically called  
 when an object is created )  
**How do we pass the argument ??**



```

stack::stack(int size)
{
    MAX = size;
    s = new int[MAX];
    top = -1;
}

```

When we create an object of this class  
 we must use the following syntax.  
**stack** <object\_name> (<an integer>)

**For example**

```

int main()
{
    stack st(3);
    st.push(6); st.push(3); st.push(7);
    if(st.isfull()) cout << "stack full";
}

```

This creates an object of  
 stack class with array size 3

This returns **true**

In general if we have a class where the constructor takes n arguments,

```
class <class_name>
{
    ...
    <class_name> ( <type_1>, ... , <type_n> ) ;
    ...
};
```

then to create object of this class we must use the following syntax

```
<class_name>    <object_name> ( <arg_1>, ... , <arg_n> )
```

where <arg\_1> is of <type\_1> , <arg\_2> is of <type\_2> ... and so on.

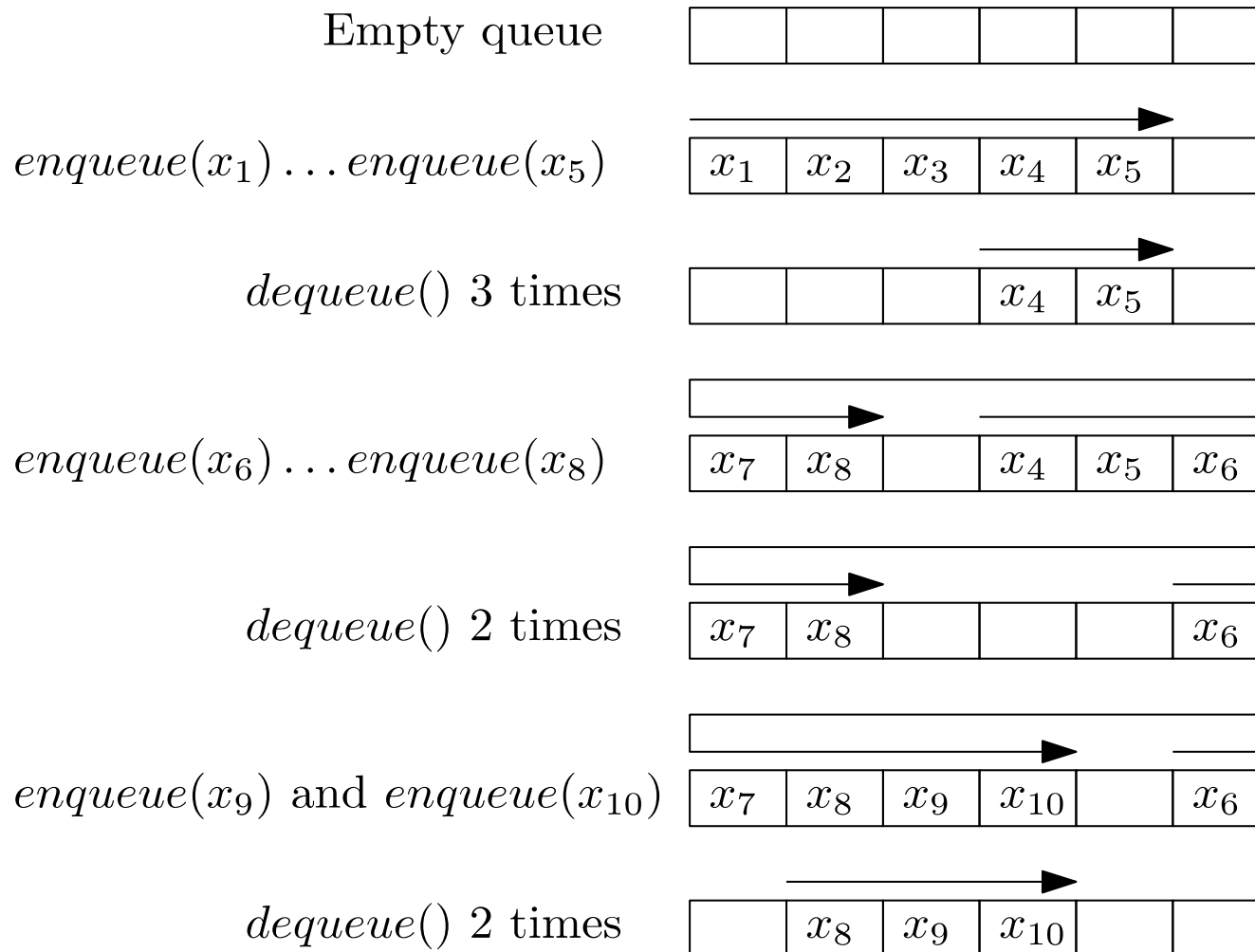
# Implementing Queue (of integers) using array

Use an integer array, say  $q[\text{MAX}]$ , to store the queue elements, where MAX is a large integer.

Notice that: (i) We cannot store more than MAX elements.

(ii) When we store only a few elements space is wasted.

Here is an example illustrating the idea where  $\text{MAX}=6$ .



When end of the array is reached, go to the beginning of the array

When end of the array is reached, go to the beginning of the array

Use an integer array, say **q[**MAX**]** , to store the queue elements, where **MAX** is a large integer. Use two integer variable **start** and **end** to store the starting and ending position of queue.

- For empty queue we set **start=-1** and **end=-1**
- enqueue changes **end** to **(end+1)%MAX** (if the list is initially empty set start=end=0)
- dequeue changes **start** to **(start+1)%MAX**

Empty queue 

--	--	--	--	--	--

 start=-1 end=-1

*enqueue*( $x_1$ ) ... *enqueue*( $x_5$ ) 

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
-------	-------	-------	-------	-------	--

 start=0 end=4

*dequeue*() 3 times 

			$x_4$	$x_5$	
--	--	--	-------	-------	--

 start=3 end=4

*enqueue*( $x_6$ ) ... *enqueue*( $x_8$ ) 

$x_7$	$x_8$		$x_4$	$x_5$	$x_6$
-------	-------	--	-------	-------	-------

 start=3 end=1

*dequeue*() 2 times 

$x_7$	$x_8$				$x_6$
-------	-------	--	--	--	-------

 start=5 end=1

*enqueue*( $x_9$ ) and *enqueue*( $x_{10}$ ) 

$x_7$	$x_8$	$x_9$	$x_{10}$		$x_6$
-------	-------	-------	----------	--	-------

 start=5 end=3

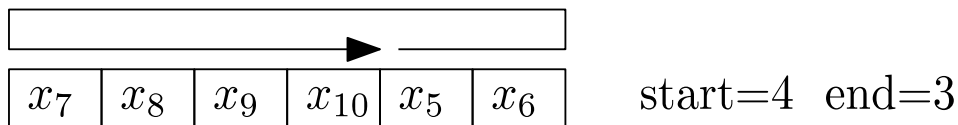
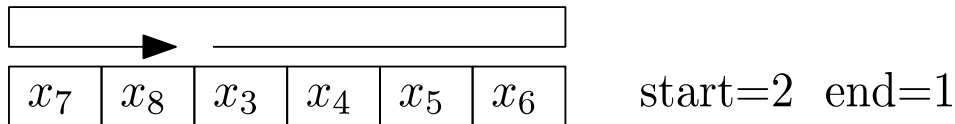
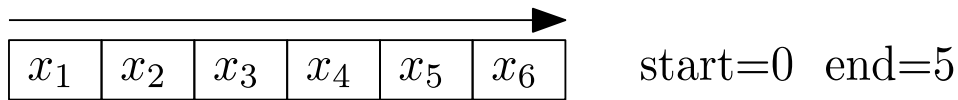
*dequeue*() 2 times 

	$x_8$	$x_9$	$x_{10}$		
--	-------	-------	----------	--	--

 start=1 end=3

The queue is **full** if and only if  $(\text{end}+1)\% \text{MAX}$  is  $\text{start}$ .

### Example




If **dequeue** makes the queue **empty**, we explicitly set  $\text{start}=-1$  and  $\text{end}=-1$ . Otherwise the following may happen.

Queue with one element 

		$x_3$			
--	--	-------	--	--	--

 start=2 end=2

dequeue()  dequeue() changes  
start to  $(\text{start}+1)\% \text{MAX}$

Empty queue 

--	--	--	--	--	--

 start=3 end=2

**Wrong**

```

class queue
{
    public:
    void enqueue(int);
    int dequeue(void);
    bool isempty(void);
    bool isfull(void);
    void show(void);

    queue(int);
    ~queue(void);

    private:
    int MAX, *q, start, end;
};

```

enqueue(x) changes **end to  $(end+1)\%MAX$**   
and stores **x at  $q[end]$**

( if the queue is initially empty explicitly set start=0 and end=0 )

dequeue(x) **returns  $q[start]$**

also changes **start to  $(start+1)\%MAX$**

( if the queue becomes empty explicitly set start=-1 and end=-1 )

returns true if and only if the queue is empty  
i.e. **start is -1 and end is -1**

returns true if and only if the queue is full  
i.e.  **$(end+1)\%MAX$  is start**

**q** should point to the beginning of the array  
q is just a pointer, we must allocate space  
for array elements using dynamic allocation

When an object of this class is created we should,  
(i) initialise MAX  
(ii) allocate space for array elements  $q[0], \dots, q[MAX]$   
(iii) initialise start and end to -1

When an object of this lass is destroyed we should,  
de allocate space for array elements  $q[0], \dots, q[MAX]$

# Inheritance

In Object Oriented Programming,  
we know that the user of a class can use the functionalities of the class  
without knowing / bothering about the implementation.

OOP also provides a mechanism so that,  
user can create his own class which **inherits** functionalities of a pre-existing class  
without knowing / bothering about the implementation of the original class.

This is known as **inheritance**.

The pre-existing class is called **base-class**.

The newly created class is called the **derived class**.

Suppose we already have a class B with 20 public member functions.

class B is defined inside header B.h

### **B.h**

```
class B
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);
    private:
    ...
};
```

**B.h gives only the declarations of member functions**

We do not know anything about the implementation of the member functions func1() ... func20()

We can still use the class by including the header and creating object.

### **TestB.cpp**

```
#include "B.h"
int main()
{
    B t;
    t.func1(5);
    t.func2();
    ...
    t.func20();
};
```



Suppose we already have a class B with 20 public member functions.  
class B is defined inside header B.h

### B.h

```
class B
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);
    private:
    ...
};
```

**B.h gives only the declarations of member functions**

We do not know anything about the implementation of the member functions func1() ... func20()

We can still use the class by including the header and creating object.

### TestB.cpp

```
#include "B.h"
int main()
{
    B t;
    t.func1(5);
    t.func2();
    ...
    t.func20();
};
```

### Now we want to create a new class D s.t.

- D also has the member functions func1()...func20()
- Additionally, D has another member function foo() (we shall define foo() ourselves)
- We shall use class D by creating objects and calling the member functions

**<definition of class D>**

**<definition of D::foo() >**

```
int main()
{
    D x;
    x.func1(5); x.func2();
    ...
    x.foo();
};
```

A crude solution (DO NOT do this)

```
class D
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);

    void foo(void)
};
```

```
void D::func1(int i)
{
    <implementation of func1>
}
```

```
.....
.....
void D::func20(void)
{
    <implementation of func20>
}
```

```
void D::foo(void)
{ cout<<"hello"; }
```

```
class B
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);
    private:
    ...
};
```

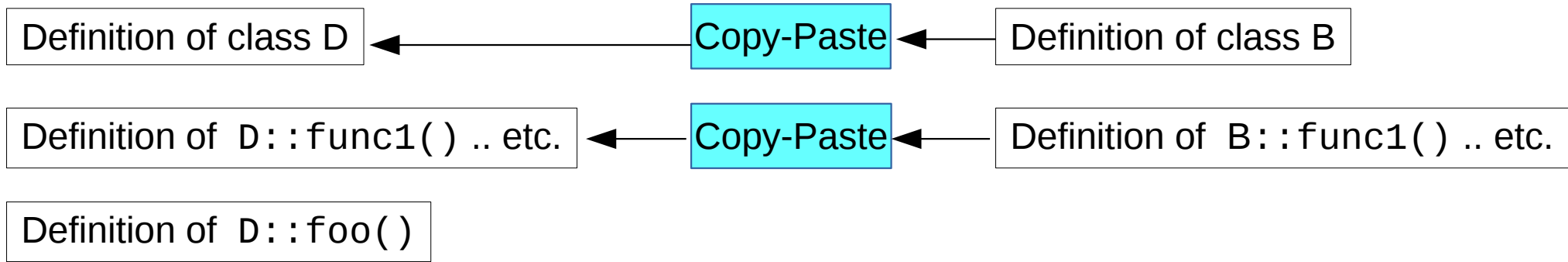
```
void B::func1(int i)
{
    <implementation of func1>
}
```

```
.....
.....
void B::func20(void)
{
    <implementation of func20>
}
```

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### Problem 1:

Source code for B::func1() etc. may not be available.  
It could be proprietary and may come as a precompiled binary file.  
Developer of class B may not allow you to know the actual implementation.

Even when you have access to the code,

### Problem 2:

Either you need to understand how those functions work,  
or you have a chunk of code in your own program and you have no idea what it is doing.

**A better solution:** **Derive** a class D from the **base class** B and **inherit** its members.

class B is defined  
inside header B.h

### **B.h**

```
class B
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);
    private:
    ...
};
```

We may define our class D  
as follows

### **D.h**

```
#include "B.h"
class D : public B
{
    public:
    void foo(void);
};
```

We need to define our  
own member foo()

### **D.cpp**

```
void B::foo(void)
{ cout<<"hello"; }
```

Finally we can use  
class D as intended

```
#include "D.h"
int main()
{
    D x;
    x.func1(5);
    x.func2();
    ...
    x.foo();
};
```

We do not need to know anything about the implementation of the base class

class B is defined  
inside header B.h

**B.h**

```
class B
{
    public:
    void func1(int);
    int func2(void);
    ...
    ...
    void func20(void);
    private:
    ...
};
```

We may define our class D  
as follows

**D.h**

```
#include "B.h"
class D : public B
{
    public:
    void foo(void);
};
```

We need to define our  
own member foo()

**D.cpp**

```
void B::foo(void)
{ cout<<"hello"; }
```

Finally we can use  
class D as intended

```
#include "D.h"
int main()
{
    D x;
    x.func1(5);
    x.func2();
    ...
    x.foo();
};
```

Syntax for  
public inheritance

Public members (both data and function) of B automatically becomes public members of D

**Private members of B are NOT inherited**

We do not need to know anything about the implementation of the base class

In C++ we have 3 kinds of inheritance - public, private and protected. For all these kinds, **Private members of base class are never inherited.**

We may add new data members or member functions to the derived class

### Public Inheritance

To derive a class <der> from a base class <base> we may use the syntax

```
class <der> : public <base>
{
    ....
    ....
};
```

**All the public members of base class become public members of derived class.**

### Private Inheritance

To derive a class <der> from a base class <base> we may use the syntax

```
class <der> : private <base>
{
    ....
    ....
};
```

**All the public members of base class become private members of derived class.**

## Implementing Stack and Queue Using Linked List

## **Implementing a stack (of integers) using linked list**

Use a linked list to store the stack elements

push(x) should be same as push\_back(x)

pop() should be same as pop\_back()

**How can we make a new class which re-uses some functionalities of a pre-existing class ?**



## Implementing a stack (of integers) using linked list

Use a linked list to store the stack elements

push(x) should be same as push\_back(x)

pop() should be same as pop\_back()

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```
class stack
{
    ...
};
```

```
void stack::push(int x)
{
    ...
}
```

```
int stack::pop(void)
{
    ...
}
```

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```
class list
{
    ...
};
```

```
void list::push_back(int x)
{
    ...
}
```

```
int list::pop_back(void)
{
    ...
}
```

## Implementing a stack (of integers) using linked list

Use a linked list to store the stack elements

push(x) should be same as push\_back(x)

pop() should be same as pop\_back()

We may derive our class from previously defined class for linked list

### list.h

Contains definition of class **list**  
Does not contain implementation

### Implementation of class **list**

May be available as source code,  
for example **list.cpp**

may be available as precompiled  
binary, for example **list.o**

### stack.h (definition of class stack)

```
#include "list.h"

class stack : public list
{
    public:
    void push(int);
    int pop(void);
};
```

### stack.cpp (implementation of class stack)

```
#include "stack.h"

void stack::push(int x)
{ push_back(x); }

int stack::pop(void)
{ return pop_back(); }
```

# Compilation

**list.h** Contains definition of class list

**list.cpp** or **list.o** Contains implementation of class list  
(or some other form of binary)

**stack.h** Contains definition of class stack

**stack.cpp** Contains implementation of class stack

**main.cpp** Contains main() function which uses class stack

**Compilation:**      g++ main.cpp stack.cpp list.cpp      (if list.cpp is available)  
                 or    g++ main.cpp stack.cpp list.o      (if list.o is available)

## **Implementing a queue (of integers) using linked list**

Use a linked list to store the queue elements

enqueue(x) should be same as push\_back(x)

dequeue() should be same as pop\_front()

You may derive a class for queue using class list as base class