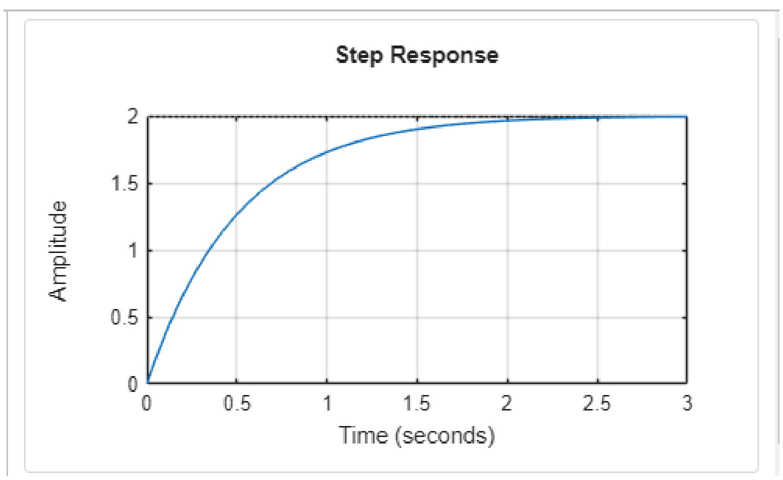
Assignment-1

Question 1

untitled *

```
s = tf('s');
G = 4/(s+2);
step(G), grid on
stepinfo(G)
```




```
ans = struct with fields:

        RiseTime: 1.0985
   TransientTime: 1.9560
    SettlingTime: 1.9560
     SettlingMin: 1.8090
     SettlingMax: 1.9987
       Overshoot: 0
      Undershoot: 0
            Peak: 1.9987
        PeakTime: 3.6611
```

① $t_r = 2.2\tau = 1.0985$ s
$t_s = 4\tau = 1.9560$ s

$\Rightarrow \tau = 0.4993$ s.

final value :
Obtained from plot $= 1.9987 \simeq 2$

Theoretical : $y_{ss} = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s \to 0} G(s)$

$y_{ss} = \frac{4}{2} = 2.$

for, $e_{ss} = \frac{1}{1 + K_p}$

$K_p = \lim_{s \to 0} G(s) H(s)$

since no block diagram is given
Assume $H(s) = 1$
$K_p = 2$ $\boxed{e_{ss} = \frac{1}{3}}$

Question 2.

(2) $G(s) = \dfrac{10}{s(s+5)}$

(i) System type = no. of poles at origin in $G(s)$

Here, one s in denominator (at origin)

Type-1 system (one integrator)

(ii) $e_{ss} = \lim_{s \to 0} s\left(\frac{1}{s} - G(s)\frac{1}{s}\right) = \lim_{s \to 0} (1 - G(s))$

$e_{ss} = (1 - \infty)$

Here, ∞ refer a very large quantity just less than 1

As $E(s) = \dfrac{R(s)}{1+G(s)}$ for $R(s) = \frac{1}{s}$ [unit step]

$e_{ss} = \lim_{s \to 0} s E(s) = \lim_{s \to 0} \dfrac{1}{1+G(s)} = 0$

$\boxed{\text{Steady state error} = 0}$

(iii) $\lim_{s \to \infty} y(s) = \lim_{s \to 0} \dfrac{sG(s)R(s)}{1+G(s)}$ [FVT]

$y(\infty) = \lim_{s \to 0} \dfrac{G(s)}{1+G(s)} = 1$

For unity feedback system, closed loop output is –

$Y(s) = \dfrac{G(s)}{1+G(s)} \cdot R(s)$

→ system is Type-1 → reaches exactly 1.

→ Poles at 0 & −5 → stable & not oscillatory

→ No steady state error

→ No need to settle below 1

Question 3:

(3) Given $t_s < 1.2\,s$ $\qquad e_{ss} = 0.1$

(i) $t_s \simeq \frac{4}{a} < 1.2$

$a > 3.33$

$\boxed{\text{Taken at } a = 4}$

(ii) $e_{ss} = 0.1 = \frac{1}{1+K}$

$0.1K + 0.1 = 1$

$\boxed{K = 9}$

(iii) $G_{new}(s) = \frac{K}{s+a} = \frac{9}{s+4}$

(iv) a) Q1 pole was at $s = -2$

New pole is at $s = -4$

More negative pole ⇒ faster Response.

b) Final value ⇒

$y(\infty) = \frac{K}{a} = \frac{9}{4} = 2.25$

in Q1, final value $= 2$.

Q3 has higher final value than Q1.

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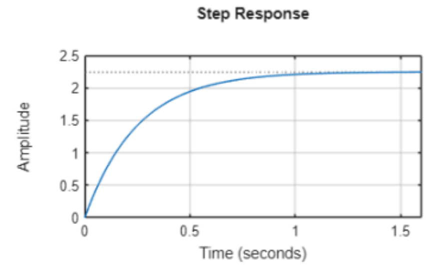
```
s = tf('s');
G = 9/(s+4);
step(G), grid on
stepinfo(G)
```




```
ans = struct with fields:
        RiseTime: 0.5493
   TransientTime: 0.9780
    SettlingTime: 0.9780
     SettlingMin: 2.0351
     SettlingMax: 2.2485
       Overshoot: 0
      Undershoot: 0
            Peak: 2.2485
        PeakTime: 1.8306
```

Question 4:

(4) $G(s) = \frac{3}{s+1}$, $C(s) = K(s+z)$

(i) ⊙ zero should be left of the pole for reducing rise time.
given pole is at $s = -1$
choose $\boxed{z = 3}$

⊙ closed loop DC gain :

$$y_{ss} = \lim_{s \to 0} s\left(\frac{C(s)G(s)}{1 + C(s)G(s)}\right) R(s)$$

$$y_{ss} = \lim_{s \to 0} \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$$C(s)G(s) = \frac{3K(s+3)}{(s+1)}$$

At $s = 0$, $y_{ss} = \frac{9K}{1+9K} = 0.8$

$$9K = 0.8 + 7.2K$$

$$1.8K = 0.8$$

$$\boxed{K = \frac{4}{9}}$$

⊙ $M_p < 10\% \Rightarrow e^{\left(\frac{-\delta\pi}{\sqrt{1-\delta^2}}\right)} < 10\%$

$$\boxed{\delta \geq 0.6}$$

Resulting system :-

$$C(s)G(s) = \frac{4}{3}\frac{(s+3)}{(s+1)}$$

iii) The step response will rise faster than the ~~prev~~ uncompensated plant due to added zero, exhibit less than 10% overshoot due to sufficient damping, settle within 2 sec and ~~read~~ reach steady state value of 0.8.

$$\cancel{(ii)\ T(s) = 1 + \frac{4(s+3)}{3(s+1)}}$$

(ii) $$T(s) = \frac{\frac{4(s+3)}{3(s+1)}}{1 + \frac{4(s+3)}{3(s+1)}} = \frac{4(s+3)}{3(s+1) + 4(s+3)}$$

$$T(s) = \frac{4(s+3)}{7s + 15}$$

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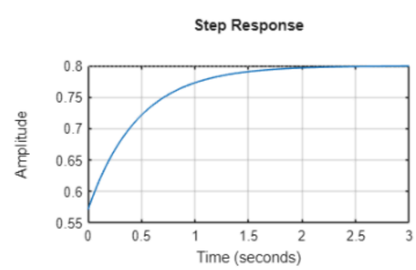
```
s = tf('s');
G = (4*s+12)/(7*s+15);
step(G), grid on
stepinfo(G)
```




```
ans = struct with fields:
        RiseTime: 0.4900
   TransientTime: 1.8256
    SettlingTime: 1.2411
     SettlingMin: 0.7207
     SettlingMax: 0.7998
       Overshoot: 0
      Undershoot: 0
            Peak: 0.7998
        PeakTime: 3.4170
```

Question 5:

(5) **Plant**

$G(s) = \frac{3}{s+1}$

Controller

$C(s) = K(s+z)$

Ramp input $\Rightarrow r(t) = t \Rightarrow R(s) = \frac{1}{s^2}$

Assume unity feedback

With controller	Without controller
(i) Open loop:	Open loop:
$T(s) = C(s)G(s) = \frac{3K(s+z)}{s+1}$	$T(s) = G(s) = \frac{1}{s+1} \Rightarrow$ Type 0
$\Rightarrow$ Type 0	
Close loop:	Close lopp:-
$T(s) = \frac{G(s)C(s)}{1+G(s)} = \frac{3K(s+z)}{s+1+3K(s+z)}$	$T(s) = \frac{G(s)}{1+G(s)} = \frac{(s+1)3}{s+4}$
$\Rightarrow$ Type 0	$\Rightarrow$ Type 0
(3) $e_{ss} = \lim_{s \to 0} \frac{1}{s} - \frac{3K(s+z)}{s(s+1+3K(s+z))}$	$e_{ss} = \lim_{s \to 0} s\left(\frac{1}{s^2} - \frac{T(s)}{s^2}\right)$
$e_{ss} = \infty$	$\lim_{s \to 0} \frac{3}{s(s+4)} = \infty$
(can't be tracked) $z \neq \frac{1}{3k}$	(can't be tracked)
(2) infinite e_{ss}	infinite e_{ss}

5.4 Adding zero at $(s+z)$ has no effect on tracking if $z \neq \frac{1}{3K}$

if $z = \frac{1}{3K}$; $e_{ss} = \left. \frac{1-3K(s+z)}{s(s+1+3K(s+z))} \right|_{s\to 0}$

$$e_{ss} = -\frac{3K}{2}$$

if $z = \frac{1}{3K}$, e_{ss} becomes finite. [with controller]