Assignment_1

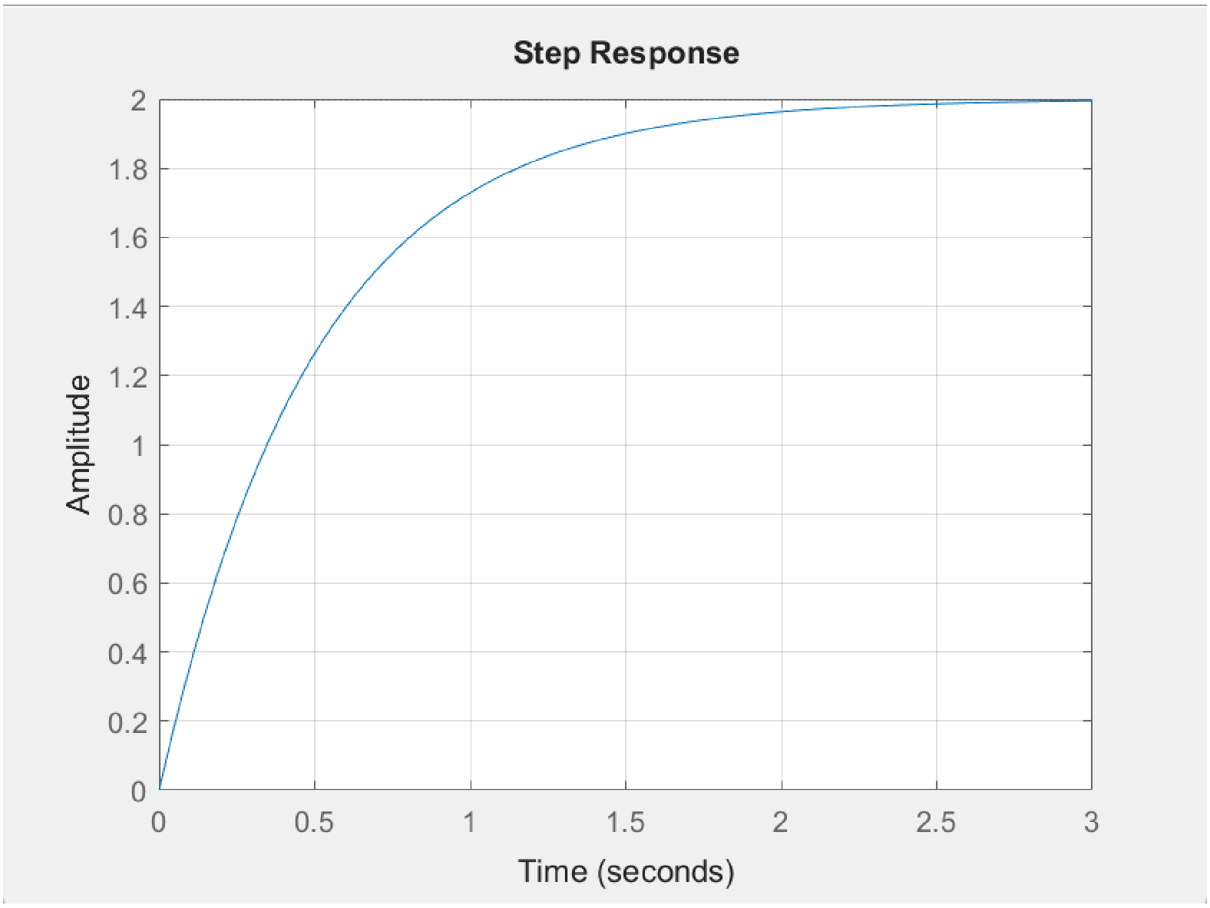
Smart throttle Control

Q1. Understanding a First-Order Plant Using Step Response

Consider the first-order plant:

$$G(s) = \frac{4}{s+2}$$

1. Plot the unit step response in MATLAB.



2. Using the plot or `stepinfo`, determine:

- Time constant τ
- Rise time t_r
- Settling time t_s
- Final value (using Final Value Theorem)
- Steady-state error e_{ss}

```
        RiseTime: 1.0985
   TransientTime: 1.9560
    SettlingTime: 1.9560
     SettlingMin: 1.8090
     SettlingMax: 1.9987
```

```
   Overshoot: 0
  Undershoot: 0
        Peak: 1.9987
    PeakTime: 3.6611
```

Final Value=2

3. Compare MATLAB's final value with:

$$y_{ss} = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s}.$$

$$Y_{ss}=\lim_{s \to 0} \frac{4s}{s^2+2s} = \frac{4s}{2s} = 2$$

The steady state value is same as the DC gain of transfer function.

Q2. System Type, Step Error, and Final Value Theorem

Given the plant:

$$G(s) = \frac{10}{s(s+5)}$$

1. Identify the **system type** (count the number of integrators).

Type-1 system as there is only one integrator(s=0)

2. Using the Final Value Theorem, find the steady-state error to a unit step:

$$e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s} - G(s)\frac{1}{s}\right).$$

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2) b)

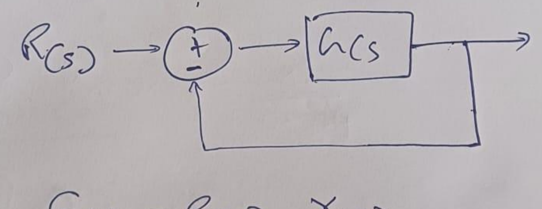
$$e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s} - \frac{G(s)}{s}\right]$$

$$\lim_{s\to 0} (1 - G(s)] = (1 - \infty)$$

So it can be taken that:

$\boxed{e_{ss} = 0}$ as ~~multiplication~~

c) we consider closed-loop system
(as it is unstable for an open-loop system)



$$E(s) = R(s) - Y(s)$$

$$E(s) = \frac{Y(s)}{G(s)}$$

$$\frac{Y(s)}{G(s)} = R(s) - Y(s) \Rightarrow Y(s)\left(\frac{1+G(s)}{G(s)}\right) = R(s)$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Steady state value

* Since $e_{ss} = 0$, the respons must reach 1 exactly

Overshoot

the open loop transfer function $G(s)$ contains:

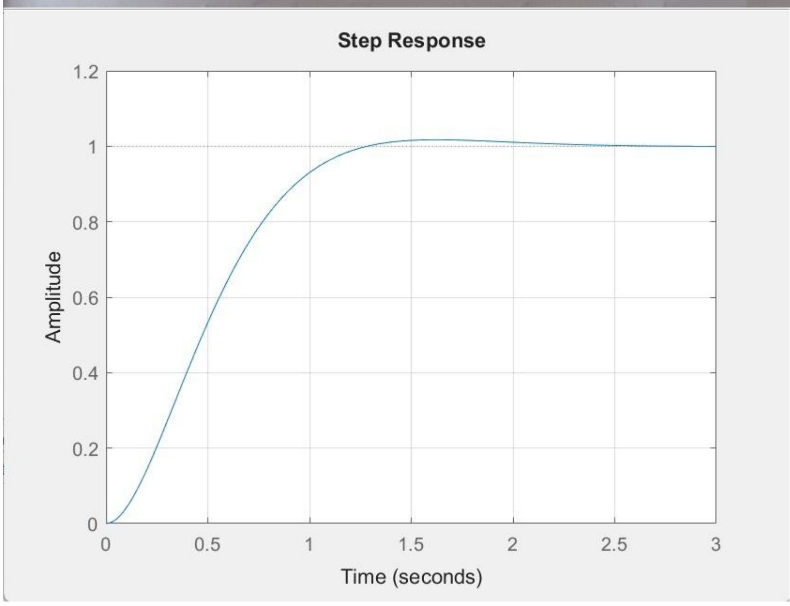
* an integrator: slows the rise, but increases overshoot.
* stable pole at -5: gives moderate damping

for large gain K, root locus tends to cross imaginary axis → overshoot likely

*overshoot above 1

Settling

as the dominant pole is close to zero, response will be slow to settle.



Q3. Required Specifications → Modify Transfer Function

Your goal is to design a first-order system that satisfies:

$$t_s < 1.2 \text{ seconds}, \qquad e_{ss} = 0.1$$

1. Using the first-order formulas:

$$t_s \approx \frac{4}{a}, \qquad e_{ss} = \frac{1}{1+K}$$

   determine:

   - the required pole location a,
   - the required static gain K.

2. Construct the modified plant:

$$G_{\text{new}}(s) = \frac{K}{s+a}.$$

3. Predict the shape of the step response before running MATLAB:

   - Should it be faster or slower than Q1?
   - Should the final value be higher or lower?

Q3

1) • required pole location a, : $t_s < 1.2s$

$\frac{4}{a} < 1.2 \Rightarrow a > 3.33$

take $\boxed{a = 3.4}$

• $e_{ss} = \frac{1}{1+K}$

$\Rightarrow 0.1 = \frac{1}{1+K} \Rightarrow K = 9$

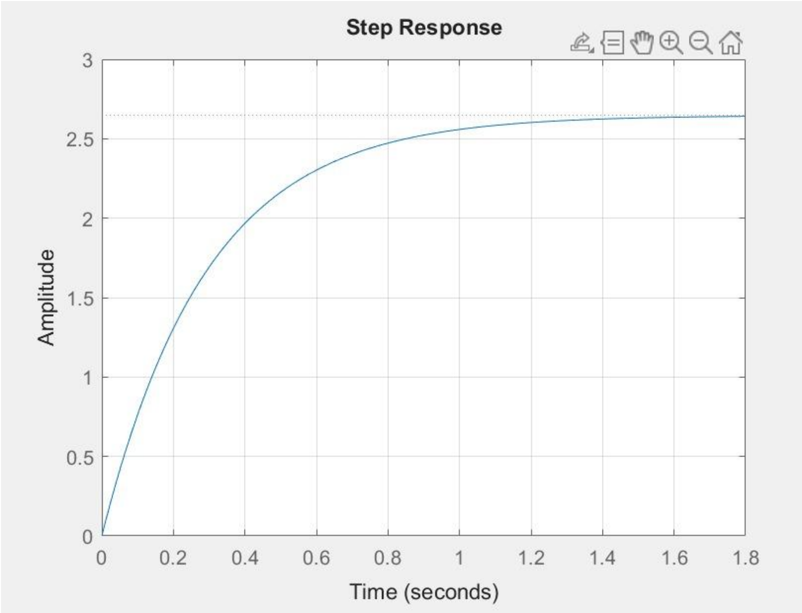
2) $G_{new}(s) = \frac{9}{s+3.4}$

3) a) since pole is further left on the s-plane → system reacts faster

→ ie new step response will be faster than Q1.

b) $\lim_{s \to 0} G_{new}(s) = \frac{9}{3.4} \simeq 2.65$

~~2.65 is higher~~ Final value is higher



Q4. Designing a Simple Controller to Meet Specifications

You are given the following plant:

$$G(s) = \frac{3}{s+1}$$

You must design a simple controller:

$$C(s) = K(s+z)$$

to meet these desired characteristics:

$$t_s < 2 \text{ s}, \qquad M_p < 10\%, \qquad y_{ss} = 0.8$$

1. Using formulas from the class cheat sheet:
   - Choose a zero z that reduces rise time.
   - Choose a gain K that sets the desired steady-state value.
   - Estimate the resulting damping ratio ζ from the overshoot requirement.

Q4

1) * a small positive zero $(0 < z < 3)$ reduces rise time.

* a zero too close to origin adds overshoot.

So we choose $\boxed{z = 1}$.

* $y_{ss} = \lim_{s \to 0} T(s)$

$$T(s) = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

$C(0) = kz, \quad G(0) = 3$

$$T(0) = \frac{3kz}{1 + 3kz} = 0.8$$

$$\Rightarrow kz = 4/3 \Rightarrow \boxed{k = 4/3}$$

Overshoot

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$M_p < 0.10$

$$0.1 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\Rightarrow \ln(0.1) = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 2.302$$

$$\Longrightarrow \boxed{\zeta = 0.59}$$ → moderate damping small overshoot.

2) $$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{3K}{1+3K}$$

3)

* a) adding the zero slightly increases overshoot but <10%, as we chose small $z = 1$, so overshoot should remain controlled.

b) $$y_{ss} = \frac{3Kz}{1+3Kz}$$

as K increase numerator increases.

⟹ increase in final value.

c) zero + gain will push effective pole leftward.

⟹ Response will be faster.

Q5. Ramp Tracking and System Type

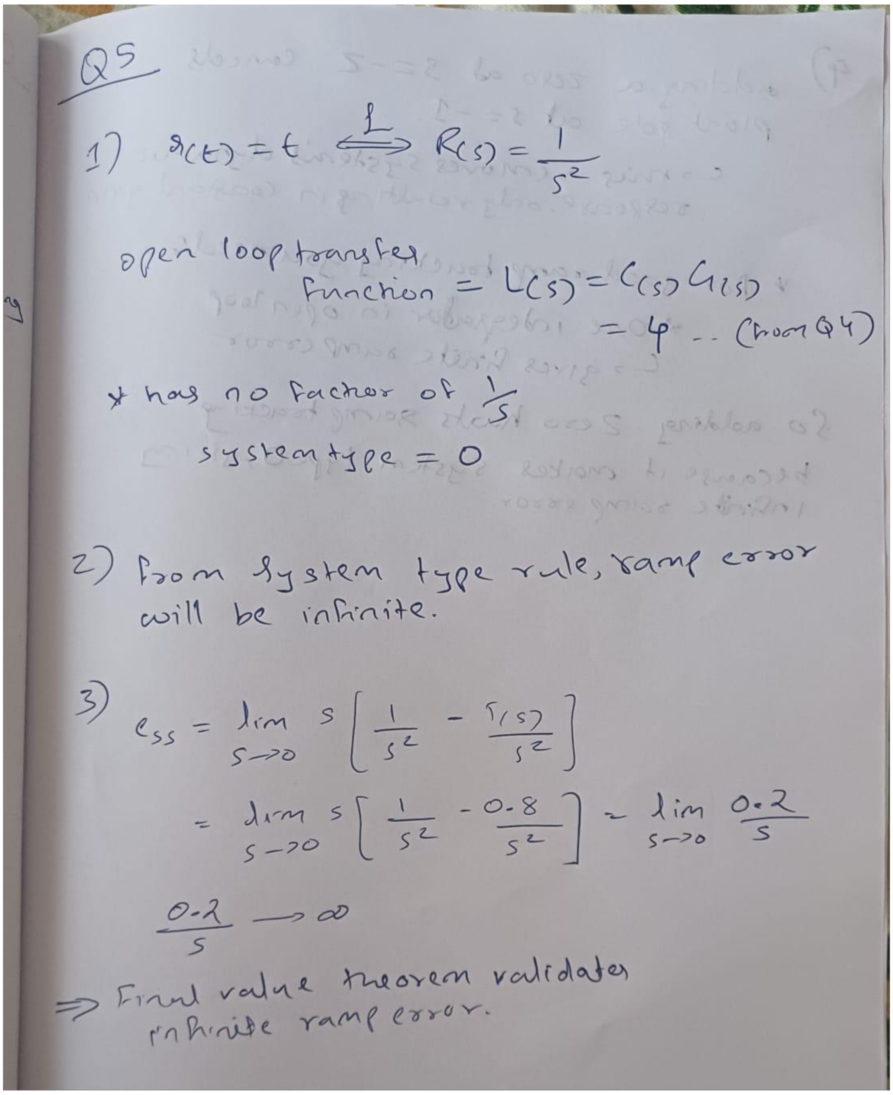
Using the controller and closed-loop system from Q4:

$$r(t) = t \quad \text{(unit ramp)}$$

1. Determine the **system type** of the closed-loop system.
2. Using system type rules, predict whether the ramp error will be:
   - infinite,
   - finite non-zero,
   - or zero.
3. Verify using the Final Value Theorem for ramp input:

$$e_{ss}^{\text{ramp}} = \lim_{s \to 0} s \left(\frac{1}{s^2} - T(s) \frac{1}{s^2} \right).$$

4. Explain whether adding the zero at $(s + z)$ helps or hurts ramp tracking.



Q5

1) $r(t) = t \xLeftrightarrow{\mathcal{L}} R(s) = \frac{1}{s^2}$

open loop transfer function $= L(s) = C(s)G(s)$

$= 4$ -- (from Q4)

* has no factor of $\frac{1}{s}$

system type $= 0$

2) From system type rule, ramp error will be infinite.

3) $e_{ss} = \lim_{s \to 0} s \left[\frac{1}{s^2} - \frac{T(s)}{s^2} \right]$

$= \lim_{s \to 0} s \left[\frac{1}{s^2} - \frac{0.8}{s^2} \right] = \lim_{s \to 0} \frac{0.2}{s}$

$\frac{0.2}{s} \to \infty$

$\Rightarrow$ Final value theorem validates infinite ramp error.

4) adding a zero at $s = -2$ cancels plant pole at $s = -1$.

↳ this removes system's dynamic response, only resulting in constant gain

* for good ramp tracking you need:
  * one integrator in open loop
    ↳ gives finite ramp error.

So adding zero hurts ramp tracking because it makes system type 0, giving infinite ramp error.