

Question 1

For $G(s) = 4/(s+2)$

This is a first order system with a single pole in left half-plane.

we know that a first-order system has the form:

$$G(s) = 1/(\tau s + 1)$$

By solving $\tau = 0.5 \text{ sec}$

Rise Time (10–90%) $t_r = 2.2\tau = 1.1 \text{ sec}$

Settling Time (2%) $t_s = 4\tau = 2 \text{ sec}$

FINAL VALUE THEOREM

$$y_{ss} = \lim_{s \rightarrow 0} s \cdot G(s) \cdot 1/s$$

$$Y_{ss} = 4/2 = 2.$$

Steady-State Error

$$e_{ss} = r_{ss} - y_{ss} = 1 - 2 = -1.$$

$$e_{ss} = -1.$$

increasing gain increases final value. Since the DC gain is greater than unity, the output settles above the reference value, resulting in a negative steady-state error.

Question 2

$$G(s) = 10/(s(s+5))$$

The system contains **one pole at the origin**, which corresponds to **one integrator**.

System Type = 1

Using Final Value Theorem:

$$e_{ss} = \lim_{s \rightarrow 0} s(1/s - G(s) \cdot 1/s) = 0$$

$e_{ss} = 0$. (But this uses open circuit output $G(s)$ but steady-state error is defined for the closed-loop system)

For a Unity feed back closed system the error signal is

$$E(s) = R(s)/(1+G(s))$$

For a unit step $R(s) = 1/s$

$$E(s) = 1/(s(1+G(s)))$$

Now apply Final Value theorem $e_{ss} = \lim_{s \rightarrow 0} sE(s)$ as s tends to 0.

So, $e_{ss} = 0$. For closed loop. {as $s \rightarrow 0$, $G(s)$ tends to infinity.}

Open-Loop Steady-State Error

Given:

$$G(s) = 10 / [s(s + 5)]$$

Unit step input:

$$R(s) = 1 / s$$

For an **open-loop system**, the output is simply: $Y(s) = G(s)R(s)$

Substitute:

$$Y(s) = 10 / [s^2(s + 5)]$$

Using the Final Value Theorem:

$$y_{ss} = \lim_{s \rightarrow 0} (s \rightarrow 0) [sY(s)]$$

As $s \rightarrow 0$, the denominator goes to zero, therefore:

$$y_{ss} \rightarrow \infty$$

Steady-state error is defined as:

$$e_{ss} = r_{ss} - y_{ss}$$

So:

$$e_{ss}(\text{open-loop}) = 1 - \infty = -\infty$$

In magnitude:

$$e_{ss}(\text{open-loop}) = \infty$$

Since the steady-state error is zero, the step response will reach exactly 1.

The system perfectly tracks a step input because the presence of an integrator eliminates steady-state error.

Question 3

Design Requirements

- $t_s < 1.2$ seconds
- $e_{ss} = 0.1$

$$t_s = 1/a,$$

$$a = 0.33,$$

we know, $e_{ss} = 1/(1+K)$. So, **$K = 9$** .

So, the modified new response will be

$$G_n(s) = 9/(s+3.33)$$

Predicted Response:

Pole moved further left meaning a faster response **and**

Higher gain so higher final value.

Question 4

Given, $G(s) = 3/(s+1)$

Requirement: $C(s) = K(s+z)$

Specifications : $t_s < 2 \text{ sec}$, $M_p < 10\%$, $y_{ss} = 0.8$.

We can tell, A left half plane zero can

1. Reduce the rise time.
2. Speeds up response
3. May slightly increase overshoot.

To improve this we choose a zero as close to plant zero,

So, **$z=1$** .

Thus, $C(s) = K(s+1)$

$C(s)G(s) = 3K$.

$T(s) = C(s)G(s)/(1+C(s)G(s))$

So, $T(s) = 3K/(1+3K)$

But it only shows DC GAIN, and we know that the first order form is of order, $(A/(s+a))$

So, $T(s)=3K/(s+(1+3K))$

$s = -(1+3K)$

Applying the controller cancels the plant pole and shifts the closed-loop pole to the left, resulting in a faster first-order system with improved steady-state performance.

First order system, closed loop pole at $-(1+3K)$, Speed of response increases as K increases. Overshoot is zero. And setting time $t_s = 4/(1+3K)$.

Question 5

From Question 4, the controller and plant are:

$$G(s) = 3 / (s + 1)$$

$$C(s) = (4/3)(s + 1)$$

The input is a unit ramp:

$$r(t) = t$$

Open loop transfer function.

$$C(s)G(s) = (4/3)(s + 1) \times 3 / (s + 1)$$

Cancel $(s + 1)$:

$$C(s)G(s) = 4$$

The open-loop transfer function is a constant (4).

There is:

- No $1/s$ term
- No pole at $s = 0$

Therefore:

$$\text{System Type} = 0$$

From control system theory:

Type-0 system \rightarrow finite step error, infinite ramp error

For a unit ramp:

$$r(t) = t$$

$$R(s) = 1 / s^2$$

Steady-state error is given by:

$$e_{ss} = \lim (s \rightarrow 0) [s \times E(s)]$$

Where:

$$E(s) = R(s) - T(s)R(s)$$

So for a ramp input:

$$e_{ss}(\text{ramp}) = \lim (s \rightarrow 0) \{ s \times [(1 / s^2) - T(s)(1 / s^2)] \}$$

Factor $1/s^2$:

$$e_{ss}(\text{ramp}) = \lim (s \rightarrow 0) [(1 - T(s)) / s]$$

From Question 4, the closed-loop transfer function is:

$$T(s) = 3K / [s + (1 + 3K)]$$

With $K = 4/3$:

$$T(s) = 4 / (s + 5)$$

Substitute $s = 0$:

$$T(0) = 4 / 5 = 0.8$$

Substitute into the error expression:

$$e_{ss}(\text{ramp}) = \lim (s \rightarrow 0) [(1 - 0.8) / s]$$

$$e_{ss}(\text{ramp}) = \lim (s \rightarrow 0) [0.2 / s]$$

As $s \rightarrow 0$, the denominator goes to zero while the numerator is constant.

Therefore:

Steady-state ramp error = Infinite

The controller includes a zero $(s + 1)$.

- Zeros affect transient response (rise time, overshoot)
- System type depends only on integrators

Since the zero does not introduce a pole at $s = 0$:

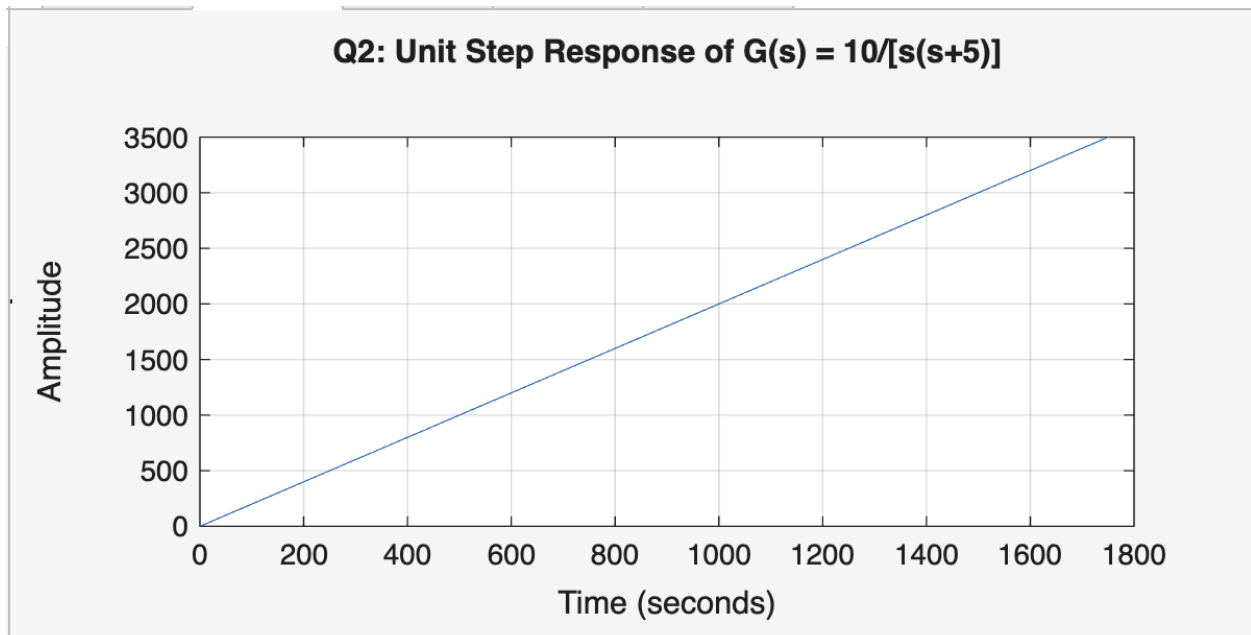
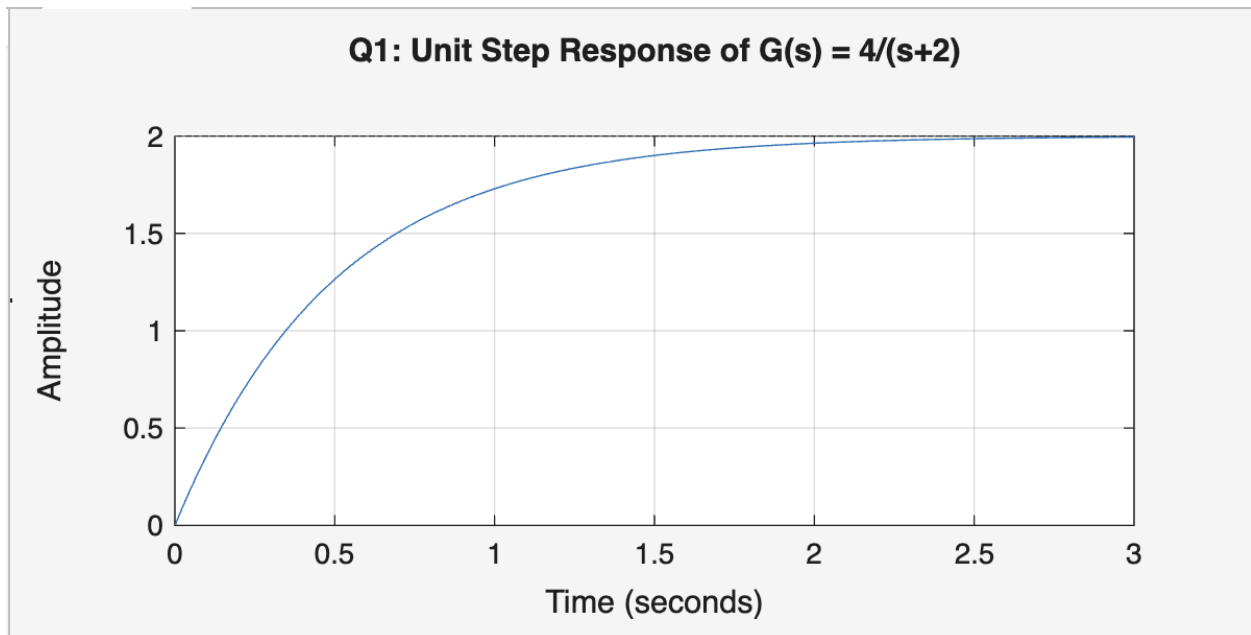
- System type does not change
- Ramp tracking does not improve

Overall answer-

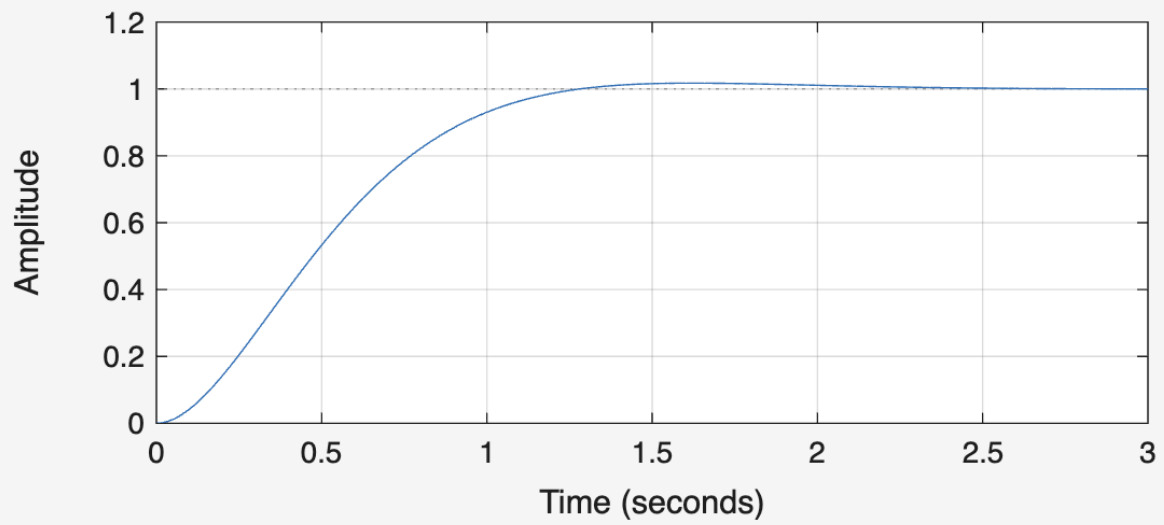
- The open-loop system has no integrators
- The system is Type-0
- Type-0 systems have infinite steady-state error for ramp inputs
- The Final Value Theorem confirms this result

- Adding a zero improves speed but does not help ramp tracking

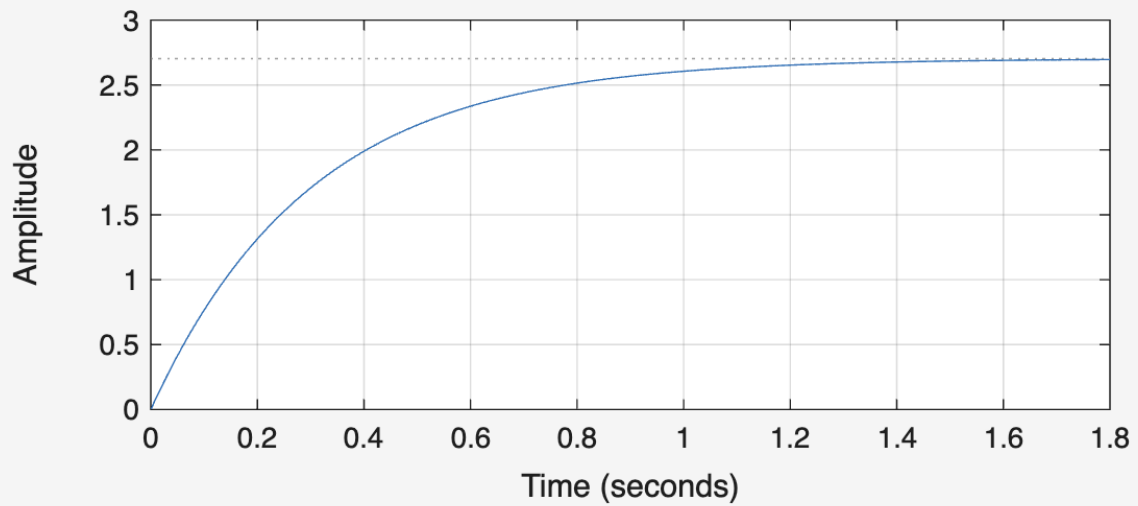
Steady-state ramp error = Infinite.



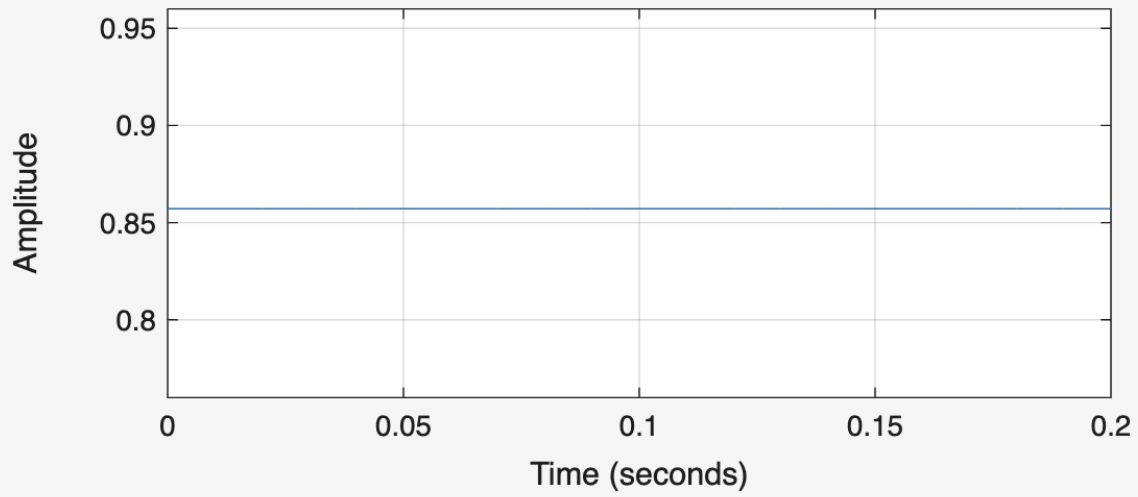
Closed-Loop Step Response (Unity Feedback)



Q3: Step Response of Modified Plant



Q4: Closed-Loop Step Response



Q5: Ramp Response of Closed-Loop System

