Q1. First-Order Plant Step Response

Given

$$G(s) = \frac{4}{s+2}$$

(1)

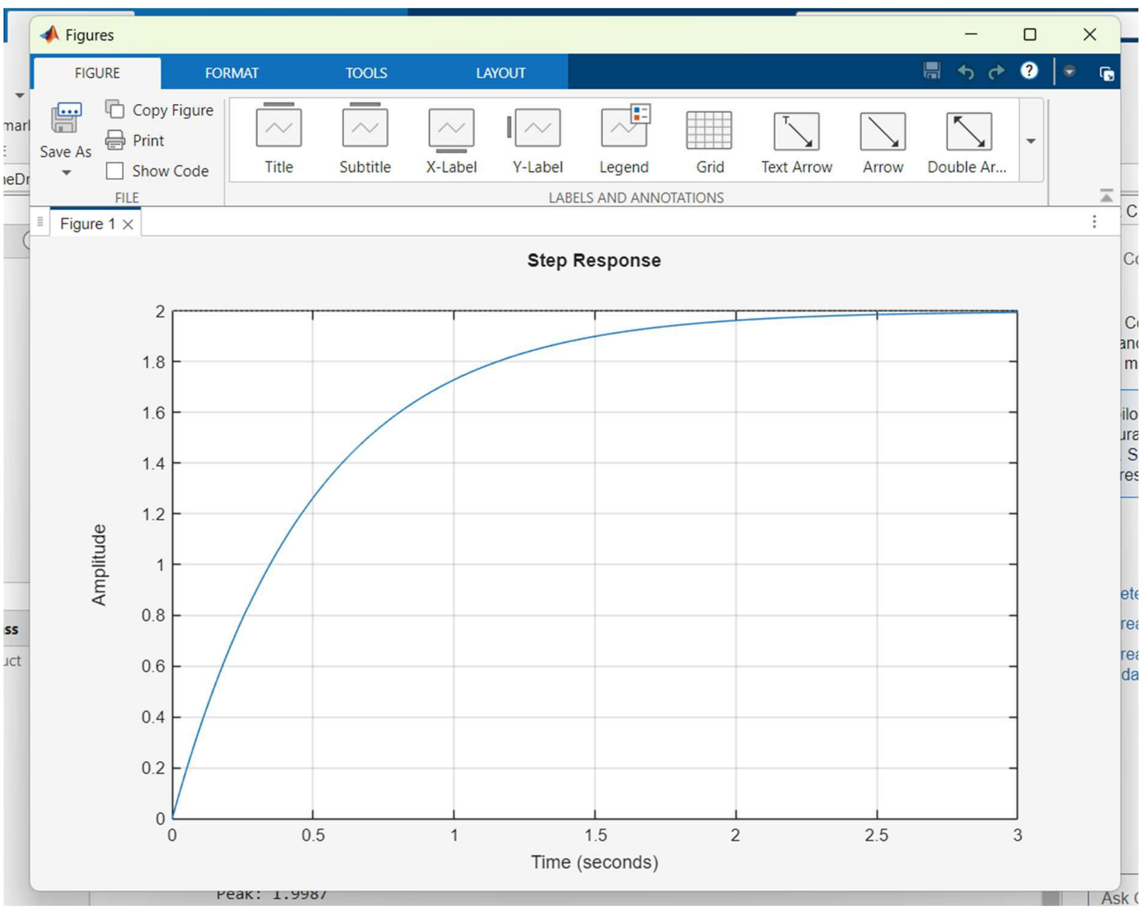
#MATLAB

```
s = tf('s');

G = 4/(s+2);

step(G), grid on

stepinfo(G)
```



(2)

From standard form:

For $G(s) = \frac{K}{\tau s+1}$. Here $K = 2 \ \ and \ \ \tau = 0.5$ s.

$$G(s) = \frac{2}{0.5s + 1}$$

- Time constant:

$$\tau = 0.5 \text{ s}$$

- Rise time:

$$t_r \approx 2.2\tau = 1.1 \text{ s}$$

- Settling time (for First Order):

$$t_s \approx 4\tau = 2 \text{ s}$$

- Final value (Using FVT):

$$y_{ss} = \lim_{s \to 0} s\, G(s) \frac{1}{s} = G(0) = 2$$

- Steady-state error (for unit step):

$$e_{ss} = 1 - y_{ss} = 1 - 2 = -1$$

(3)

MATLAB final value matches FVT.

Q2. System Type, Step Error & FTV

Given

$$G(s) = \frac{10}{s(s + 5)}$$

(1) System type

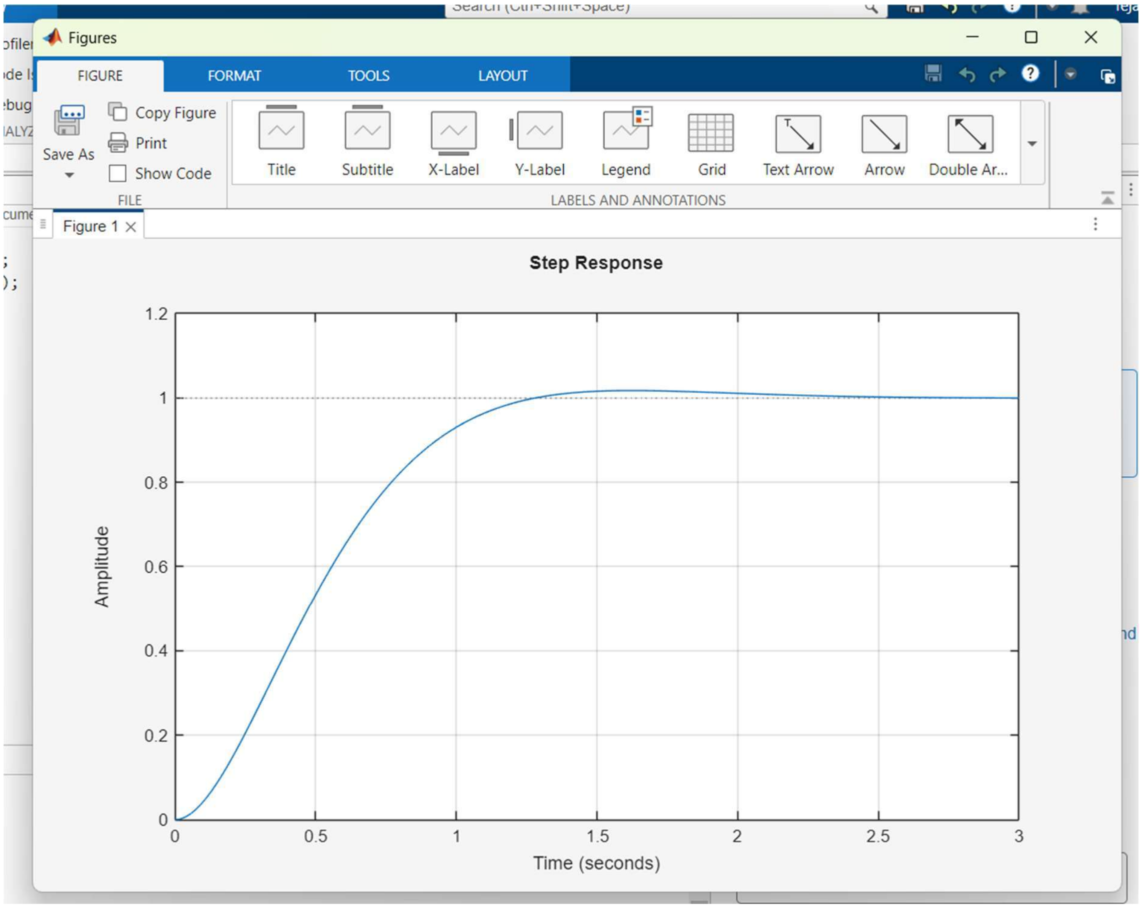
One pole at origin so it is Type-1 system

#Matlab Code

```
s = tf('s'); G = 10/(s*(s+5));

T = feedback(G,1);step(T)

grid on
```



(2) Steady-state error (for unit step)

R(s)=1/s,

$$e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s} - \frac{10}{s(s+5)}\frac{1}{s}\right) =$$

$$= \lim_{s\to 0}\left[1 - \frac{10}{s(s+5)}\right]$$

Now take the limit:

As $s \to 0$,

$$\frac{10}{s(s+5)} \to \infty$$

But this is a Type-1 system (one integrator).

So, for a unit step input:

$$\boxed{e_{ss} = 0}$$

Theoretically, $e_{ss} = 0\ for\ type\ 1\ system.$

(3) prediction

Zero steady-state error

Output reaches 1

possible overshoot due to second-order nature

Q3. Modify Transfer Function

$$t_s < 1.2\text{ s}, \quad e_{ss} = 0.1$$

(1)

$$t_s \approx \frac{4}{a} \Rightarrow a > \frac{4}{1.2} = 3.33$$

Take:

- $Pole\ Location\ a = 4$

$$e_{ss} = \frac{1}{1+K} = 0.1$$

- $\Rightarrow Static\ gain\ K = 9$

(2)

Modified plant:

$$Substituting\ in\ the\ G(s) = \frac{K}{s+a}, we\ get$$

$$G_{new}(s) = \frac{9}{s+4}$$

(3)

Prediction:

- Faster than Q1. Ex: Pole at s=-3.33 vs. -2.
- Higher final value because it has smaller settling time.

Screenshots for reference:

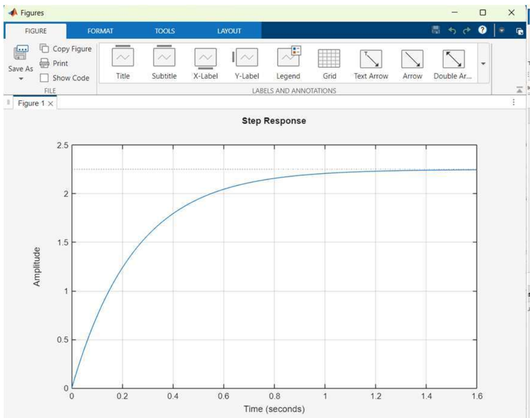


Fig: for Q3

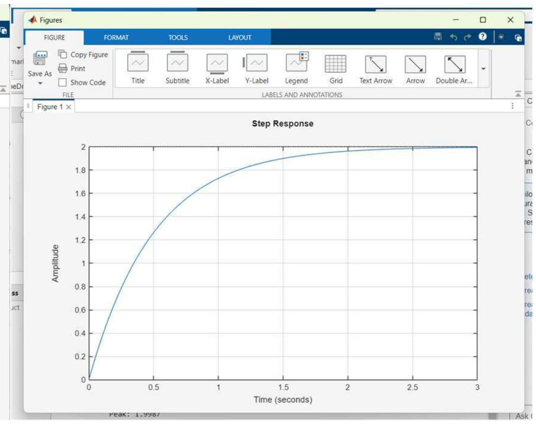


fig: for Q4

Q4.

Given plant:

$$G(s) = \frac{3}{s+1}$$

Controller:

$$C(s) = K(s+z)$$

Desired:

$$t_s < 2\text{ s},\ \ M_p < 10\%,\ \ y_{ss} = 0.8$$

(1)

- For unit step:

$$y_{ss} = \lim_{s\to 0} T(s) = \frac{3Kz}{1+3Kz}$$

Given $y_{ss} = 0.8$:

$$\frac{3Kz}{1+3Kz} = 0.8 \Rightarrow 3Kz = 4$$

Choose a simple zero:

$$z = 2 \Rightarrow K = \frac{2}{3}$$

- Standard estimate:

  From M_p=*exp*(−ζπ/sqrt(1−ζ^2)) <0.1

  Therefore ζ>0.59

  Let take:

$$\boxed{\zeta \approx 0.6}$$

(2)

Closed-loop Transfer Function:

$$T(s) = \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{K(s+z)\cdot\frac{3}{s+1}}{1+\frac{3K(s+z)}{s+1}}$$

Substituting the values: K=2/3, z=2 then we get,

$$T(s) = \frac{\frac{2}{3}(s+2).3}{s+1+3.\frac{2}{3}(s+2)}$$

$$T(s) = \frac{2(s+2)}{3s+5}$$

And Controller C(s):

$$C(s) = \frac{2}{3}(s+2)$$

(3)

Qualitative predictions:

- Adding zero increases overshoot.
- Increasing K increases y_{ss}.

- Response faster than original.

Q5. Ramp Tracking & System Type

Using system from Q4.

Closed-loop transfer function T(s):

$$T(s) = \frac{2(s+2)}{3s+5}$$

And Controller C(s):

$$C(s) = \frac{2}{3}(s+2)$$

Ramp input:

r(t)=t ⇒R(s)=1/s^2

(1) System type

System type = number of poles at origin in $T(s)$.

There is **no pole at** $s = 0$.

$$\boxed{\text{System Type = 0}}$$

(2) Ramp error

Type-0 ⇒ infinite ramp error

$$\boxed{e_{ss}^{\text{ramp}} = \infty}$$

(3) Verification (FVT)

$$e_{ss}^{ramp} = \lim_{s \to 0} s \left(\frac{1}{s^2} - \frac{T(s)}{s^2} \right)$$

Since $T(0)$is finite and there is **no integrator**,

$$\boxed{e_{ss}^{\text{ramp}} = \infty}$$

(4) Effect of adding zero at (s+z)

- Does not improve ramp tracking
- Integrator is required for ramp tracking