MATLAB Questions: Transfer Functions, Step Response, and Basic Controller Design

Q1. Understanding a First-Order Plant Using Step Response

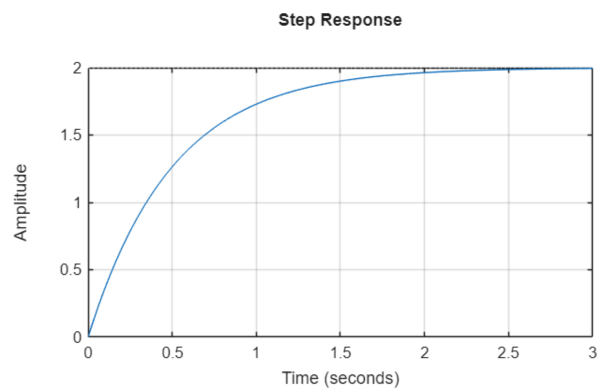
Consider the first-order plant:

$$G(s) = 4 /(s+2)$$

1. Plot the unit step response in MATLAB

Answer:

```
s = tf('s');
G = 4/(s+2);
step(G), grid on
stepinfo(G)
```




```
ans = struct with fields:
         RiseTime: 1.0985
    TransientTime: 1.9560
     SettlingTime: 1.9560
      SettlingMin: 1.8090
      SettlingMax: 1.9987
        Overshoot: 0
       Undershoot: 0
             Peak: 1.9987
         PeakTime: 3.6611
```

2.Using the plot or stepinfo, determine:

- Time constant τ
- Rise time t_r
- Settling time t_s
- Final value (using Final Value Theorem)
- Steady-state error e_{ss}

Answer :1) For a unit step response , output in frequency domain is given by,

$$Y(s) = 4/s(s+2)$$

$$Y(s) = [2/s] - [2/s+2]$$

Now the output in time domain will be given by ,

$$Y(t) = 2 - 2e^{-2t} = 2(1-e^{-2t})$$

Comparing it with the standard first-order form

$$Y(t)=K(1-e^{-t/\tau})$$

Thus, time constant (τ) = ½ seconds = 0.5 seconds

2)for first-order: Rise time $t_r \approx 2.2\tau$ {acc. to cheat sheet}

$$t_r \approx 2.2*0.5 \approx 1.1\text{seconds}$$

3)Settling time for first order $t_s \approx 4\tau \approx 4*0.5 \approx 2$seconds

4)Using Final Value Theorem,

$$Y(s) = \lim_{s \to 0} s*G(s)*1/s = \lim_{s \to 0} 4/(s+2) = 2.$$

5)steady state error for unit step input

$$e_{ss} = \lim_{t \to \infty} (1-y(t)) = 1-2 = -1 \text{ \{since, systems final value is 2\}}$$

3. Compare MATLAB's final value with:

$$\lim_{s \to 0} s*G(s)*1/s$$

Answer: Comparing with Final Value Theorem,
MATLAB's final value (around 2) matches
Y(s) = 2 computed from final value theorem
above.

Q2. System Type, Step Error, and Final Value Theorem

Given the plant: **G(s) = 10/s(s+5)**

1. Identify the system type (count the number of integrators).
Answer : Here there is exactly one pole at s=0(integrators) , so this is a type 1 system.

2. Using the Final Value Theorem, find the steady-state error to a unit step:

$$e_{ss} = \lim_{s \to 0} s(1/s - G(S)*1/s)$$

Answer: Considering the given system as open loop system , we take Y(s)=G(s)R(s) and for unit step input ,R(s) = 1/s , we get:

$$Y(s) = 10/s(s+5) * 1/s = 10/s^2(s+5)$$
$$E(s) = R(s) - Y(s) = 1/s - 10/s^2(s+5)$$

By the Final Value Theorem, $e_{ss} = \lim_{s \to 0} s[1/s - 10/s^2(s+5)]$

$$e_{ss} = \lim_{s \to 0} [1-10/s(s+5)]$$

As $s \to 0$, the second term blows up to $+\infty$, so the limit does not exist and error diverges in magnitude , rather than approaching a final value.

NOW, Considering the given system as closed loop system, so the transfer function of the plant becomes (for unity feedback)

$$T(s) = G(s)/[1+G(s)]$$

Thus, $$E(s) = R(s) /[1+G(s)]$$

For a unit step input $R(s) = 1/s$

So, $e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} 1/1+G(s) = \lim_{s \to 0} 1/1+\infty = 0.$

3. Predict whether MATLAB's step response should reach 1, overshoot it, or settle below 1.

Answer : considering unity feedback closed loop system, a type 1 system with finite poles and no zeros has zero steady-state error to a unit step, so the output should eventually settle to 1 in steady state.

Q3. Required Specifications → Modify Transfer Function

Your goal is to design a first-order system that satisfies:

$$t_s < 1.2 \text{ seconds and } e_{ss} = 0.1$$

1. Using the first-order formulas:

$$t_s \approx 4/a \,, \quad e_{ss} = 1/(1+k)$$

determine: • the required pole location a.

• the required static gain K.

Answer : $4/a < 1.2$

$a > 4/1.2 = 10/3 \approx 3$

So any a>3.33 satisfies the spec, the minimal choice is

$a \approx 3.33$

And, $e_{ss} = 0.1 = 1/(1+k)$

$\Rightarrow 1+k = 10 \Rightarrow$ **$k=9$**

2. Construct the modified plant:

$$G_{new}(s) = K/(s+a)$$

Answer : $G_{new}(s) = K/(s+a) = 9/(s+10/3)$

3. Predict the shape of the step response before running MATLAB:

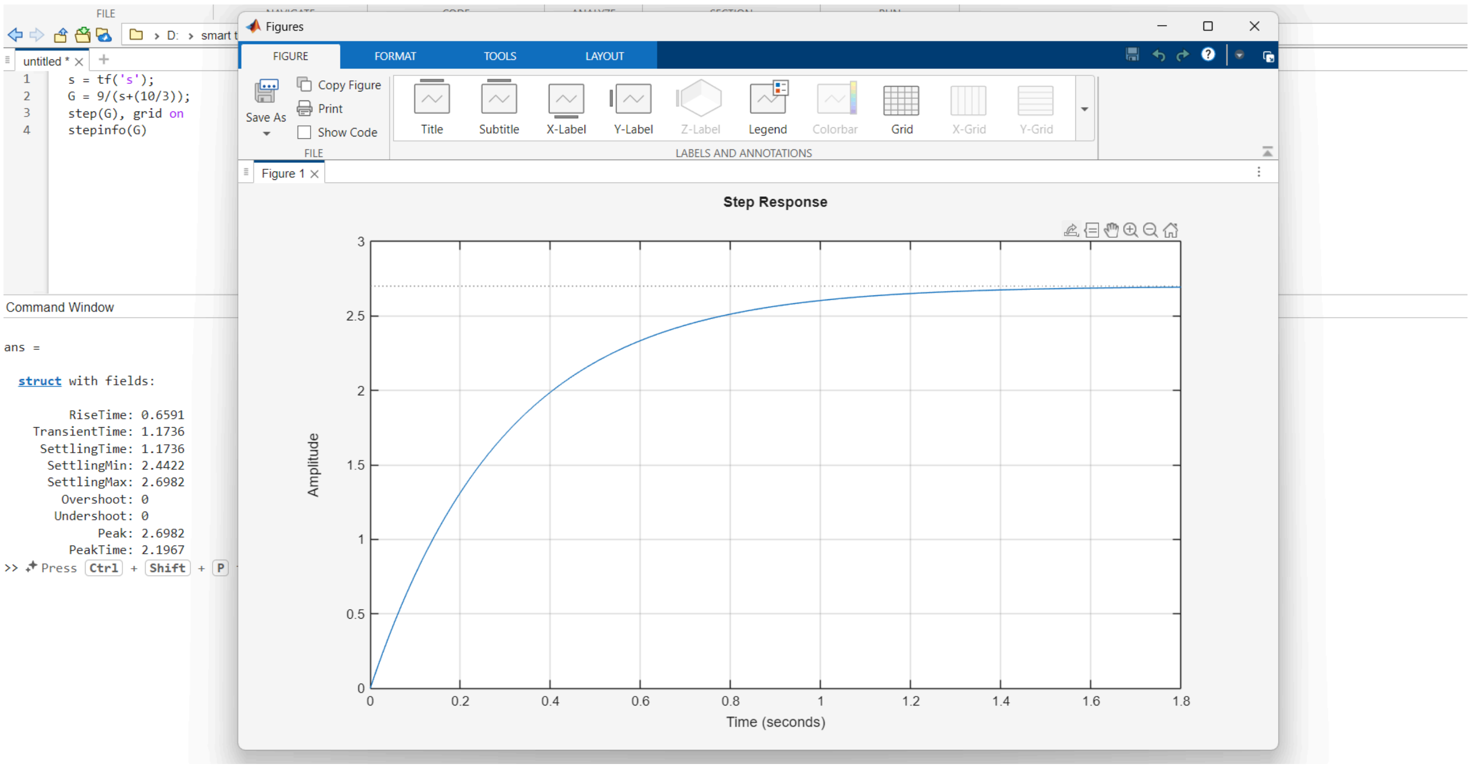
- Should it be faster or slower than Q1?
- Should the final value be higher or lower?

Answer : a) In Q1, the pole was at -2 , but here the pole is at -3.33 , which is more in left than Q1 . Thus the time constant decreases in Q2 , so the new system responds faster and has a smaller settling time than Q1.

b) the final value $y_{ss} = 1 - e_{ss} = k/(1+k)$

if you increase k then, y_{ss} increases and gets closer to 1 and the final output goes up and error goes down.

The closed loop DC gain gain is lower for Q1 than in Q2 (k=9) , so the new system final output with unit step Input is higher.



Q4. Designing a Simple Controller to Meet Specifications

You are given the following plant:

$$G(s) = 3/(s+1)$$

You must design a simple controller:

$$C(s) = K(s+z)$$

to meet these desired characteristics:

$$t_s < 2\text{ s}, \qquad M_p < 10\%, \qquad y_{ss} = 0.8$$

1. Using formulas from the class cheat sheet:
   - Choose a zero z that reduces rise time.
   - Choose a gain K that sets the desired steady-state value.
   - Estimate the resulting damping ratio ζ from the overshoot requirement.

Answer :The steady-state value for a unit step input is found using the Final Value Theorem ($s \to 0$). now $y_{ss} = 0.8$:

$$y_{ss} = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s * \{C(s)G(s)/[1+C(s)G(s)]\} * 1/s$$
$$= C(0)G(0)/[1+C(0)G(s)]$$

$y_{ss} = 3kz/(1+3kz) = 0.8$

$\Rightarrow 3kz/(1+3kz) = 0.8$

$\Rightarrow 3kz = 0.8(1+3kz) \Rightarrow 3kz - 2.4kz = 0.8 \Rightarrow 0.6kz = 0.8 \Rightarrow$ **$kz = 3/4$**

$\Rightarrow$ **$k = 3/(4z)$**

Now, For a first-order system, the settling time is approximated by $t_s = 4/|pole|$

For the closed loop pole, $1+C(s)G(s) = 0$

$$1 + 3k(s+z)/(s+1) = 0$$
$$pole(s) = -(1+3kz)/(1+3k)$$

Substituting the value of kz and taking the constraint $t_s<2$

We get **$k<0.5$**

Choose a $K < 0.5$ (e.g., **$K=0.4$**). Then calculate z using $Kz = 4/3$

We get **$z = 1.33/0.4 = 3.33$**

Also, Using standard second-order system approximation charts $M_p = 10$ % corresponds to a damping ratio $\zeta \approx 0.6$.

2. Write the resulting closed-loop transfer function

Answer : For the closed loop transfer function , using the general form we get : T(s) = C(s)G(s)/1+C(s)G(s)

$$= \frac{3k(s+z)/(s+1)}{1 + 3k(s+z)/(s+1)}$$

$$T(s) = \frac{3k(s+z)}{s(1+3k) + (1+3kz)}$$

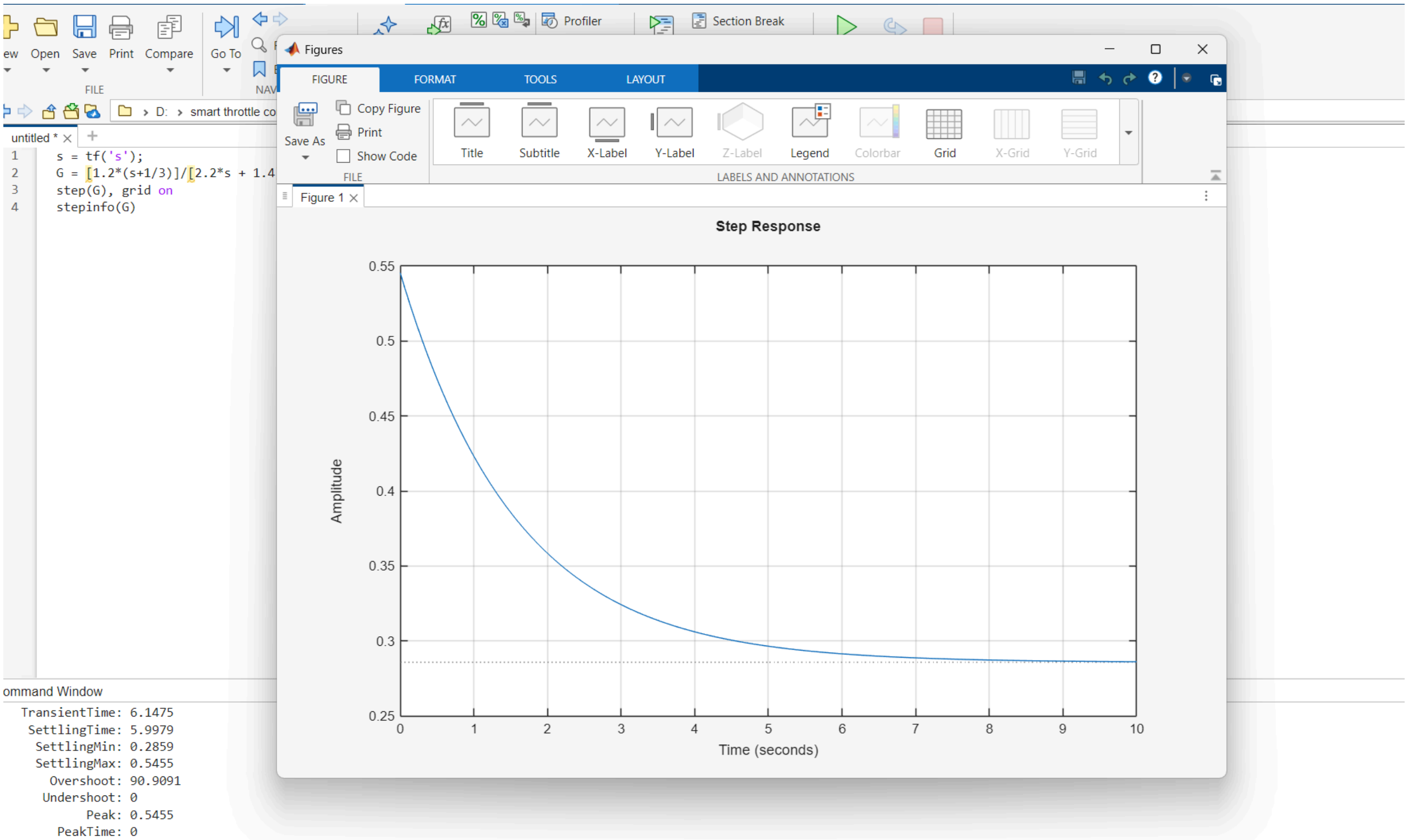
3. Before using MATLAB, predict qualitatively:
   - Will adding the zero increase or decrease overshoot?
   - Will increasing K increase or decrease y_{ss}?
   - Will the response be faster than the original plant?

<u>Answer</u>: a) Adding a zero to the left half-plane generally increases the overshoot because it adds a derivative component to the system response.

b) From the equation, $y_{ss} = 3kz /(1+3kz)$ as k becomes larger, the fraction approaches 1 and 1>0.8 so, y_{ss} increases

c) The original plant pole is at -1.If we choose z > 1 (as we did with z ≈ 3.33), the closed-loop pole moves further to the left (more negative), making the system response

Faster.



Q5. Ramp Tracking and System Type

Using the controller and closed-loop system from Q4:

$$r(t) = t \text{ (unit ramp)}$$

1. Determine the system type of the closed-loop system.

<u>Answer</u>: Since the number of integrators is 0, this is a **Type 0 System**.

2. Using system type rules, predict whether the ramp error will be:

- infinite
- finite non-zero
- or zero.

Answer: for a type 0 system, if we give ramp input the error is infinite.

3. Verify using the Final Value Theorem for ramp input:

Answer : $e_{ss}^{ramp} = \lim_{s \to 0} s(1/s^2 - T(s)1/s^2) = \lim_{s \to 0} 1/s(1\text{-}T(s))$

The closed-loop transfer function at DC (s=0) corresponds to the steady-state value for a step input. In Q4 $y_{ss} = 0.8$, therefore T(s) = 0.8.

$[e_{ss}^{ramp} = \lim_{s \to 0} 1/s(1\text{-}0.8) = \lim_{s \to 0} 0.2/s\ ]$

As s→0 the denominator vanishes, $e_{ss}^{ramp} = \infty$.

This calculation confirms the prediction from Part 2.

4. Explain whether adding the zero at (s +z) helps or hurts ramp Tracking.

Answer : It does not help.While the zero improves transient response (like reducing rise time or adjusting damping), it does not change the System Type.
