Assignment 1

December 16, 2025

Q1. Understanding a First-Order Plant Using Step Response

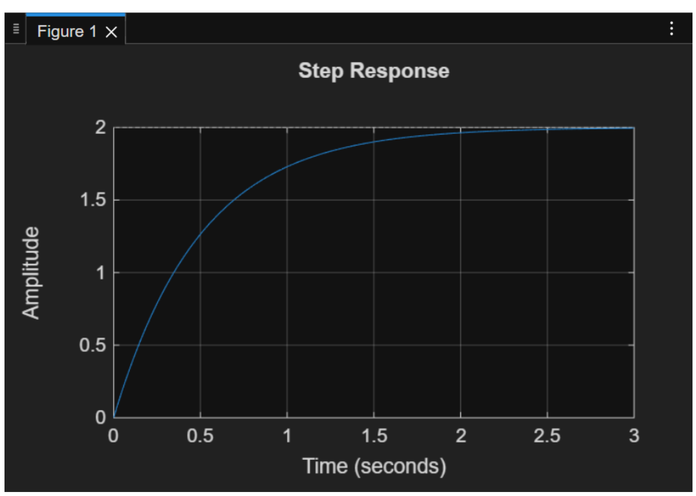
Consider the first-order plant:

$$G(s) = \frac{4}{s+2}$$

```
Command Window
>> s = tf('s');                % define Laplace variable
G = 4/(s + 2);              % transfer function

figure;
step(G)                     % unit step response
grid on

stepinfo(G)                 % step response characteristics
```



```
ans =

  struct with fields:

         RiseTime: 1.0985
    TransientTime: 1.9560
     SettlingTime: 1.9560
      SettlingMin: 1.8090
      SettlingMax: 1.9987
        Overshoot: 0
       Undershoot: 0
             Peak: 1.9987
         PeakTime: 3.6611
```

1.2 Time-Domain Analysis

The given system is a first-order system and can be compared with the standard form:

$$G(s) = \frac{k}{s+a}$$

For the given system:

$$k = 4, \qquad a = 2$$

Time Constant (τ):

$$\tau = \frac{1}{a} = \frac{1}{2} = 0.5s$$

Rise Time (t_r):

$$t_r = 2.2\tau = 2.2 \times 0.5 = 1.1s$$

Settling Time (t_s):

$$t_s = 4\tau = 4 \times 0.5 = 2s$$

Final Value (Using Final Value Theorem)
For a unit step input:

$$R(s) = \frac{1}{s}$$

The output in the Laplace domain is:

$$Y(s) = G(s)R(s) = \frac{4}{s(s+2)}$$

Applying the Final Value Theorem:

$$y(\infty) = \lim_{s \to 0} sY(s)$$

$$y(\infty) = \lim_{s \to 0} s \cdot \frac{4}{s(s+2)}$$

$$y(\infty) = \lim_{s \to 0} \frac{4}{s+2}$$

$$y(\infty) = \frac{4}{2} = 2$$

Steady-State Error
For a unit step input, the steady-state error is given by:

$$e_{ss} = 1 - y(\infty)$$

$$e_{ss} = 1 - 2 = -1$$

Steady-state error = -1

1.3)compare matlab final value

For a unit step input,

$$y_{ss} = \lim_{s \to 0} sG(s)\frac{1}{s} = \lim_{s \to 0} \frac{4}{s+2} = \frac{4}{2} = 2$$

From the MATLAB step response plot,

$$y_{ss} = 2$$

Hence, the MATLAB result matches the theoretical final value.

Q2.

The given plant is:

$$G(s) = \frac{10}{s(s+5)}$$

2.1)

Since,one factor of s in denominator

$$Number of integrators = 1$$

$$System Type = Type - 1$$

2.2)

$$e_{ss} = \lim_{s \to 0} s \left(\frac{1}{s} - G(s)\frac{1}{s} \right)$$

Substituting the value of $G(s)$:

$$e_{ss} = \lim_{s \to 0} s \left(\frac{1}{s} - \frac{10}{s^2(s+5)} \right)$$

$$e_{ss} = \lim_{s \to 0} \left(1 - \frac{10}{s(s+5)} \right)$$

As $s \to 0$:

$$\frac{10}{s(s+5)} \to \infty$$

$$e_{ss} = 0$$

2.3)

From the steady-state error calculation, $e_{ss} = 0$, hence the output must finally reach 1. Since the system is Type-1, it does not settle immediately and shows overshoot.

Therefore, the MATLAB step response will overshoot 1 and finally settle at 1.

Q3

Given: $t_s < 1.2$ **seconds,** $e_{ss} = 0.1$

3.1)

For a first-order system,

$$t_s \approx \frac{4}{a}$$

$$\frac{4}{a} < 1.2$$

$$a > \frac{4}{1.2}$$

$$a > 3.33$$

For k,

$$e_{ss} = \frac{1}{1+K}$$

$$0.1 = \frac{1}{1+K}$$

$$1 + K = 10$$

$$K = 9$$

3.2)

From the design requirements, the selected values are:

$$a = 4, \quad K = 9$$

Therefore, the new plant is given by:

$$G_{new}(s) = \frac{9}{s+4}$$

3.3)

Settling Time:

$$t_s \approx \frac{4}{a} = \frac{4}{4} = 1 \ s$$

The settling time for Q1 was $2\ s$. Hence, the new system response is faster than Q1.

Final Value:

$$y_{new}(\infty) = \frac{k}{a} = \frac{9}{4} = 2.25$$

The final value for Q1 was $y_1(\infty) = 2$. Hence, the final value of the new system is higher than Q1.

Q4.

$$G(s) = \frac{3}{s+1}$$

Given:

$$t_s < 2 \ s, \quad M_p < 10\%, \quad y_{ss} = 0.8$$

We have to design a controller of the form:

$$C(s) = K(s + z)$$

4.1)

- **LHP zero my reduce time but increase overshoot So, we have to place a zero in LHP.Hence, we choose:-**

$$z = 1$$

- **we Know,**

$$e_{ss} = \frac{1}{1 + K_p}$$

where

$$K_p = \lim_{s \to 0} C(s)G(s)$$

$$K_p = \lim_{s \to 0} K(s + z)\frac{3}{s + 1} = 3Kz$$

Given:

$$y_{ss} = 0.8 \Rightarrow e_{ss} = 1 - 0.8 = 0.2$$

Substituting:

$$0.2 = \frac{1}{1 + 3Kz}$$

$$1 + 3Kz = 5$$

With $z = 1$:

$$3K = 4 \Rightarrow K = \frac{4}{3}$$

$$K \approx 1.33$$

- **Given,**

$$M_p$$

should less than 10%

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

For,

$$\zeta = 0.5 \Rightarrow M_p \approx 16\% \quad (toohigh)$$
$$\zeta = 0.6 \Rightarrow M_p \approx 9\% \quad (acceptable)$$
$$\zeta = 0.7 \Rightarrow M_p \approx 5\% \quad (toolow)$$

Hence, the estimated damping ratio is:

$$\zeta \geq 0.6$$

Hence,Final Designed Controller is:

$$C(s) = 1.33(s+1)$$

4.2)

The closed-loop transfer function:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

Substituting:

$$C(s) = 1.33(s+1), \quad G(s) = \frac{3}{s+1}$$

$$C(s)G(s) = 1.33(s+1)\frac{3}{s+1} = 3.99$$

Hence,

$$T(s) = \frac{3.99}{1 + 3.99}$$

$$T(s) = \frac{3.99}{4.99} \approx 0.8$$

4.3)

- **Adding a left-half-plane zero increases the initial speed of response. Due to this faster rise, the output crosses the final value before settling, which results in overshoot. Thus, adding a zero increases the overshoot.**
-
  $$e_{ss} = \frac{1}{1+K}$$

  Hence, the steady-state error is inversely proportional to the gain K. Increasing K decreases e_{ss}.

  Since the steady-state value is:

  $$y_{ss} = 1 - e_{ss}$$

  a decrease in e_{ss} leads to an increase in the steady-state value.Hence, y_{ss} increases.
- **Due to the added zero and higher gain, the closed-loop system responds faster than the original plant.**

Q5.

5.1) Without Controller,

$$G(s) = \frac{3}{s+1}$$

The system type is equal to the number of integrators (poles at $s = 0$).

Since $G(s)$ has no pole at $s = 0$,

$$Number of integrators = 0$$

system type = Type-0

(can't track RAMP)

With Controller,

the controller is:

$$C(s) = 1.33(s+1)$$

The transfer function becomes:

$$C(s)G(s) = 1.33(s+1)\frac{3}{s+1} = 3.99$$

There is still no pole at $s = 0$.

$$Number of integrators = 0$$

$$Closed-loop system type = Type-0$$

(can't track RAMP input)

5.2)

According to system type rules:

- **Type-0 system $\rightarrow$ infinite ramp error**
- **Type-1 system $\rightarrow$ finite non-zero ramp error**
- **Type-2 system $\rightarrow$ zero ramp error**

Since both the plant and the closed-loop system are Type-0, the steady-state error for a unit ramp input is infinite.

$$Ramp steady-state error = Infinite$$

5.3)

$$e_{ss}^{ramp} = \lim_{s \to 0} s \left[\frac{1}{s^2} - T(s) \frac{1}{s^2} \right]$$

From Q4, the closed-loop transfer function is:

$$T(s) = \frac{3.99}{4.99}$$

Substituting:

$$e_{ss}^{ramp} = \lim_{s \to 0} s \left[\frac{1}{s^2} \left(1 - \frac{3.99}{4.99} \right) \right]$$

$$e_{ss}^{ramp} = \left(1 - \frac{3.99}{4.99} \right) \lim_{s \to 0} \frac{1}{s}$$

Since,

$$\lim_{s \to 0} \frac{1}{s} = \infty$$

$$e_{ss}^{ramp} = \infty$$

Hence,verified

5.4)

Ramp tracking depends on the system type, which is determined by the number of integrators (poles at $s = 0$) in the function.

Adding zero does not introduce an integrator into the system. Therefore, adding $(s + z)$ does not change the system type,which is type-0, and the steady-state error for a ramp input remains infinite.

Hence, adding the zero neither helps nor hurt ramp tracking.