Assignment 1

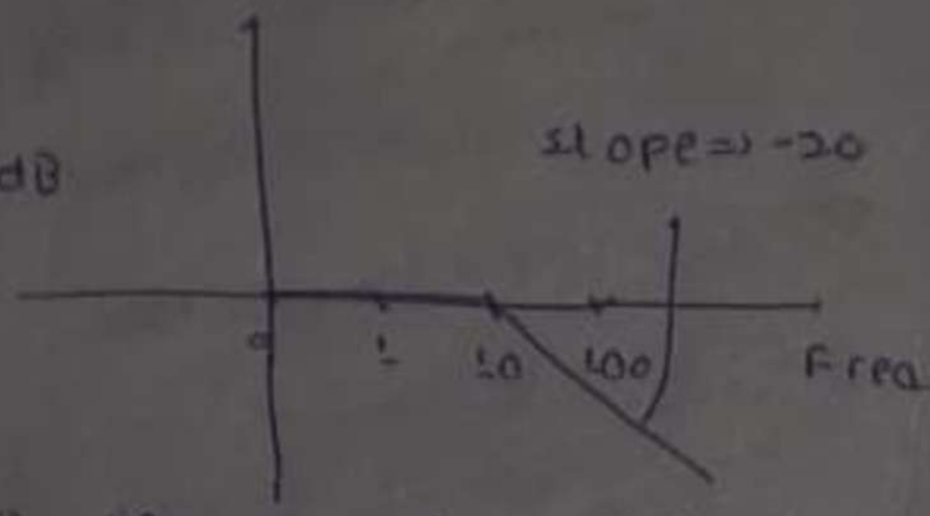
Q 1.1. $G_1(s) = \frac{10}{s+10}$; Poles $\to$ 10 ; Zeros $\to$ none

Bode plots

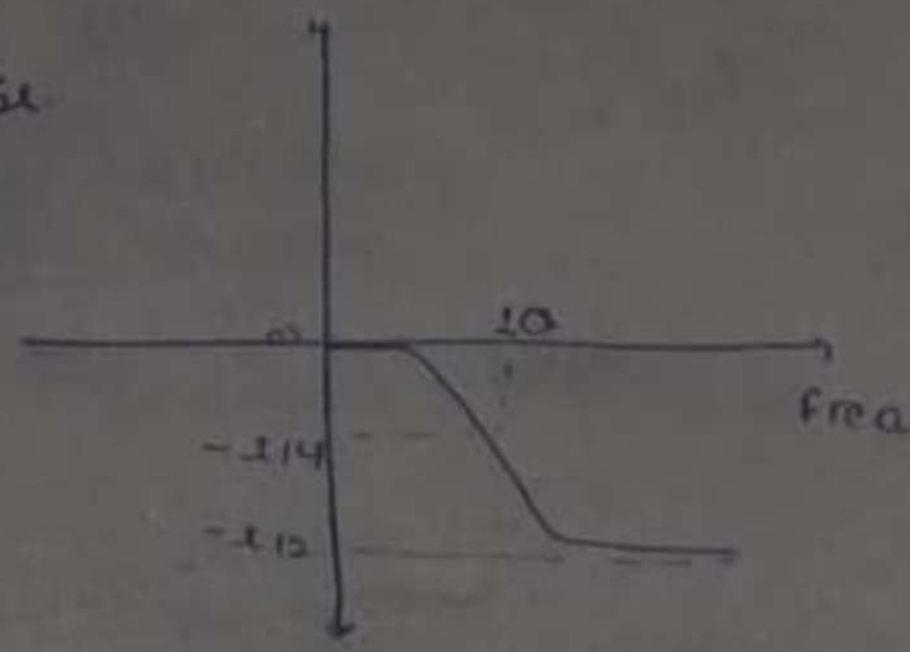
magnitude

$\uparrow M_{dB}$

slope $= -20$



Phase



magnitude $\Rightarrow \frac{10}{s+10} \Rightarrow \frac{1}{1+\frac{s}{10}}$; $s = j\omega$

$\Rightarrow \frac{1}{1+\frac{j\omega}{10}}$; corner freq $= 10$.

$0 < \omega < 10 \to$ ignored $j\omega$; $M_{dB} = 20\log|1| = 0$

$10 < \omega \to \frac{1}{\frac{j\omega}{10}} = \frac{10}{j\omega}$; $M_{dB} = 20\log\left|\frac{10}{\omega}\right|$

$= 20 - 20\log|\omega|$

Phase $\to \frac{1}{1+\frac{j\omega}{10}}$

$\Rightarrow \frac{\left(1-\frac{j\omega}{10}\right)}{\left(1+\frac{\omega^2}{100}\right)} \Rightarrow \frac{1}{1+\frac{\omega^2}{100}} - \frac{\frac{j\omega}{10}}{1+\frac{\omega^2}{100}}$

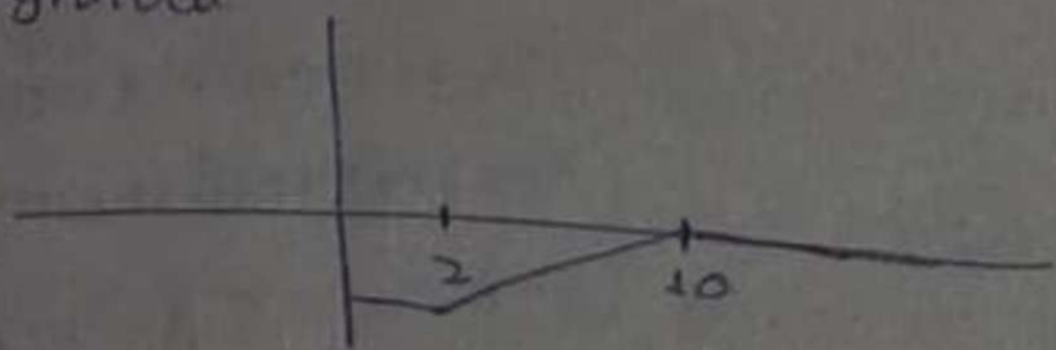
Phase $\to \tan^{-1}\left(-\frac{\omega}{10}\right)$; $\omega \to 0 \to 0$

$\omega \to \infty \to -\pi/2$

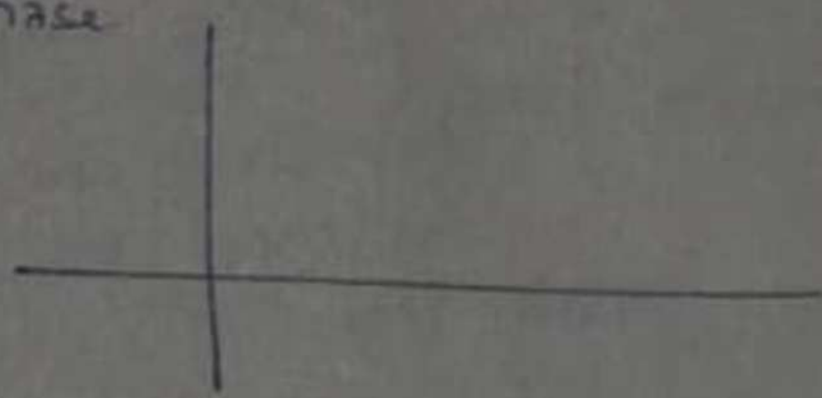
1.2 $G_2(s) = \frac{s-2}{s+10}$: Pole $= 10$

zero $\to 2$

magnitude



Phase



$\omega < 2$

$2 < \omega < 10$

$10 < \omega$

$\Rightarrow \frac{2\left(\frac{s}{2}-1\right)}{10\left(1+\frac{s}{10}\right)} \Rightarrow \frac{\frac{1}{5}\left(\frac{s}{2}-1\right)}{\left(1+\frac{s}{10}\right)}$

i) $\omega < 2 \Rightarrow \frac{\frac{1}{5}(-1)}{(1)}$

$M_{dB} \Rightarrow 20\log|1/5| = -20\log 5$

ii) $2 < \omega < 10$; $M_{dB} \Rightarrow 20\log\left|\frac{1}{5}\right| + 20\log\left|\frac{\omega}{2}\right| = -20\log 5 + 20\log\left|\frac{\omega}{2}\right|$

iii) $10 < \omega$ $M_{dB} = 20\log\left|\frac{1}{5}\right| + 20\log|5| \Rightarrow 0$

Phase :

$$\frac{i\omega - 2}{i\omega + 10} \Rightarrow \frac{(i\omega - 2)(i\omega + 10)}{(\omega^2 + 100)} \Rightarrow \frac{-20 + 10 i\omega + 2 i\omega + \omega^2}{(\omega^2 + 100)}$$

$$\Rightarrow \frac{\omega^2 - 20}{\omega^2 + 100} + \frac{(2+10) j\omega}{\omega^2 + 100}$$

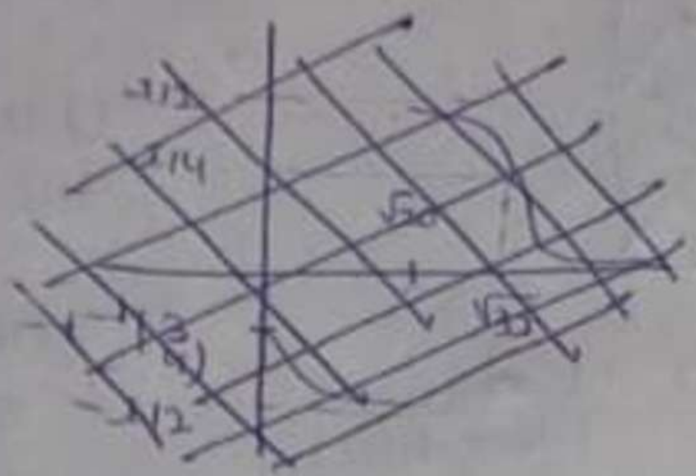
$$\phi = \tan^{-1}\left(\frac{12\omega}{\omega^2 - 20}\right)$$

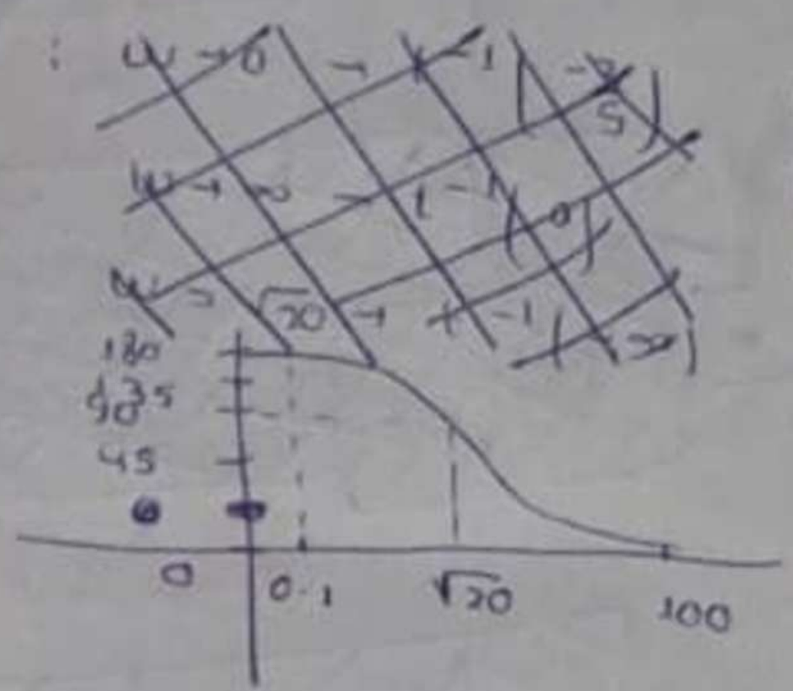
$\omega \to 0 \quad \phi \to 0°$

$\omega \to \infty \quad \phi \to 0$

$\omega \to 100 \to 0$

$\omega \to \sqrt{20} \quad \phi \to \pi/2$





1.3. $G_3(s) = \dfrac{100}{s^2 + 10s + 100}$ poles $\Rightarrow \dfrac{-10 \pm \sqrt{100 - 400}}{2} \Rightarrow -5 \pm 5\sqrt{3}j$

no zeros. imaginary

$\Rightarrow \dfrac{100}{(s - (-5 - 5\sqrt{3}j))(s - (-5 + 5\sqrt{3}j))}$

no. corner frequencies

$$m_{dB} = 20\log\left(\left\|\frac{100}{100 - \omega^2 + 10j\omega}\right\|\right)$$

$\omega \Rightarrow 0.1 \to 20\left(\log\left\|\frac{100}{100 + j}\right\|\right)$

$= 20 \times 0 \to 0$

$\omega \Rightarrow 10 \to 20\left|\log\left(\frac{100}{100 j}\right)\right|$

$\to 20\log(1) \to 0$

$\omega \Rightarrow 100 \to 20\left(\log\left\|\frac{100}{10^4}\right\|\right)$

$\Rightarrow -20\log(100)$

$\Rightarrow -200$

For maximum output $\omega = \sqrt{50}$

$\|100 - \omega^2 + 10 j\omega\| \to$ minimum

$(100 - \omega^2)^2 + 100\omega^2(1) \to$ minimum

$(-)(100 - \omega^2)2\omega(2) + 200\omega$

$-(4\omega(100 - \omega^2) - 200\omega)$

$4\omega^3 - 200\omega \; ; \; \omega = \sqrt{50}$

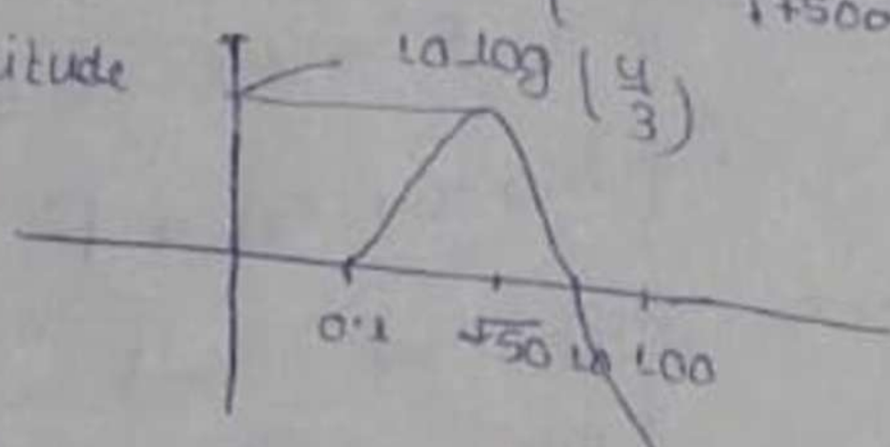
$(100 - 50)^2 + 100 \times 50$

$\Rightarrow (50)^2 + 50$

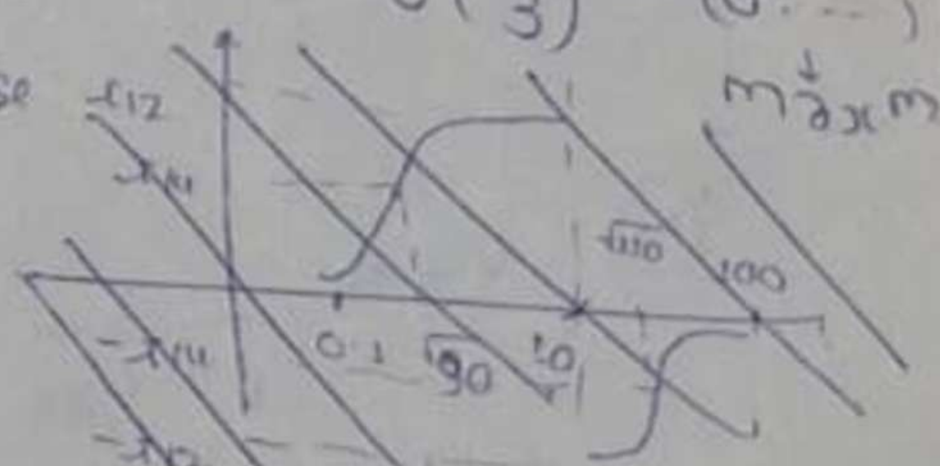
$\Rightarrow 7500$

output $= 20\log\left(\dfrac{100}{\sqrt{7500}}\right) \Rightarrow 20\log\left(\dfrac{10}{\sqrt{75}}\right) \Rightarrow 10\log\left(\dfrac{4}{3}\right) \to (0.\ldots)$

magnitude

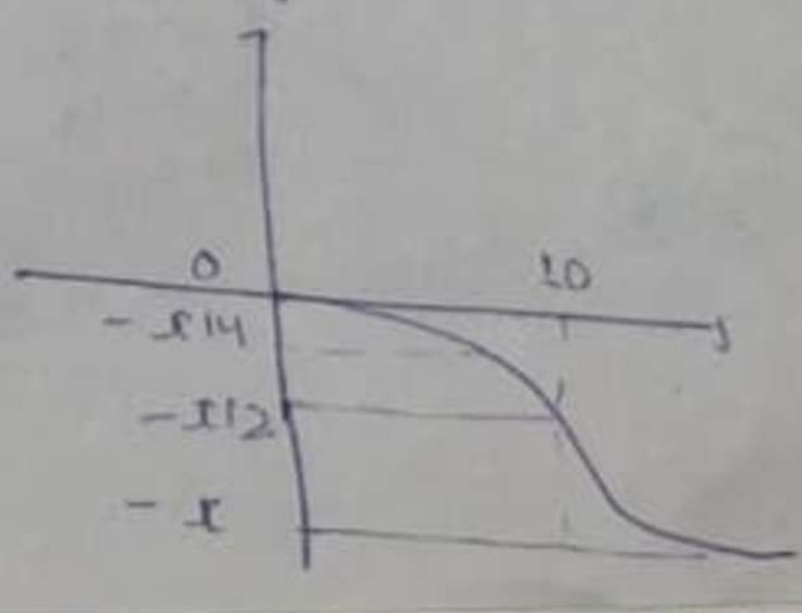


real Phase



Phase $\to \dfrac{100}{100 - \omega^2 + 10 j\omega}$

ratio $= -\dfrac{10\omega}{-\omega^2 + 100}$

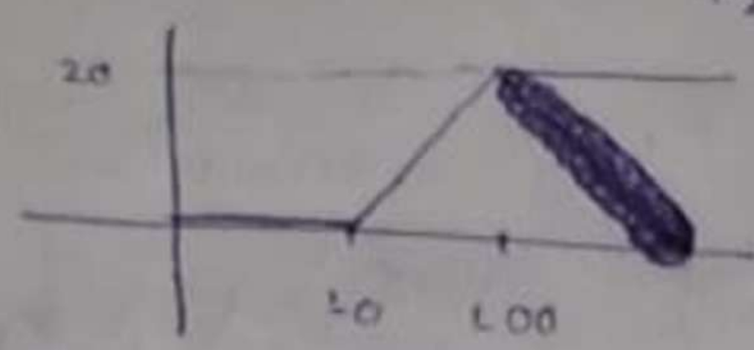


1.4. $G_H(s) \Rightarrow \dfrac{0.1s+1}{0.01s+1}$ i) zero $\Rightarrow$ 10 ; pole $\Rightarrow$ 100

+20 slope ahead contribⁿ

−20 slope contribⁿ ahead

iii)

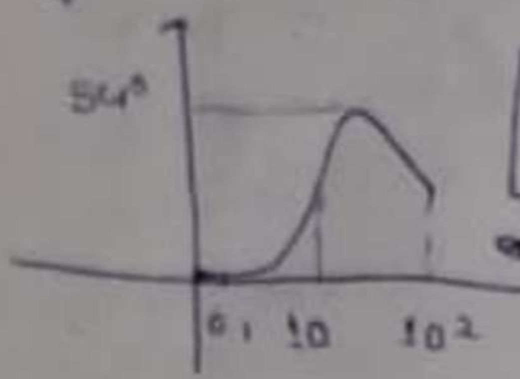


Phase → $\dfrac{0.1j\omega+1}{(0.01j\omega+1)}$

→ $\dfrac{10(j\omega+10)\;(-j\omega+100)}{(j\omega+100)(100-j\omega)}$

→ $\dfrac{10\,(j\omega\,90 + 1000 + \omega^2)}{10000+\omega^2}$

→ $\dfrac{10}{\omega^2+10000}\left(1000+\omega^2+90\,j\omega\right)$



phase $\Rightarrow \tan^{-1}\left(\dfrac{90\omega}{1000+\omega^2}\right)$

$100+\omega^2 = 90\omega$

$\omega^2 - 90\omega + 100 = 0$

$\omega = \dfrac{90 \pm \sqrt{8100-400}}{2}$

$= 45 + 5\sqrt{77}$

$\omega \to 0$
$\phi \to 0$

$\omega \to$ maxm → derivate

ii) $90(1000+\omega^2) - 90\omega \times 2\omega = 0$

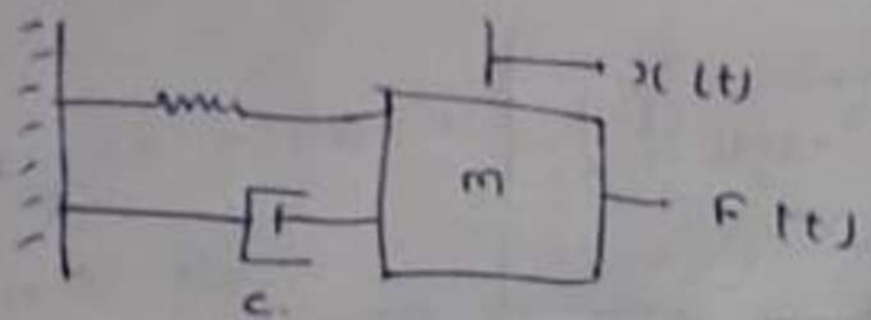
$90000 + 90\omega^2 - 180\omega^2 = 0$

$\omega = \sqrt{1000} \approx 31.62$

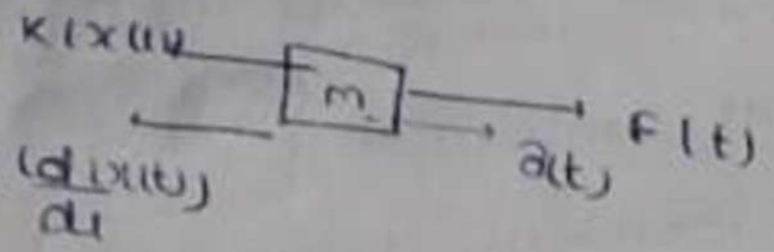
$\log(31.62) = 1.5$

iv) positive for some time −ve for some time

2.



① FBD



⇒

$m \times a(t) = F(t) - k(x(t)) - c\dfrac{d}{dt}(x(t))$

② taking laplace transform.
↳ since zero initial condⁿ is given.

$m \times s^2 X = F - kX - csX$

③ $X(s^2m + k + cs) = F$

$G(s) \Rightarrow \dfrac{X(s)}{F(s)} \Rightarrow \dfrac{1}{s^2m + k + cs}$

④ $G(s) \Rightarrow \dfrac{1}{s^2+16+4s}$

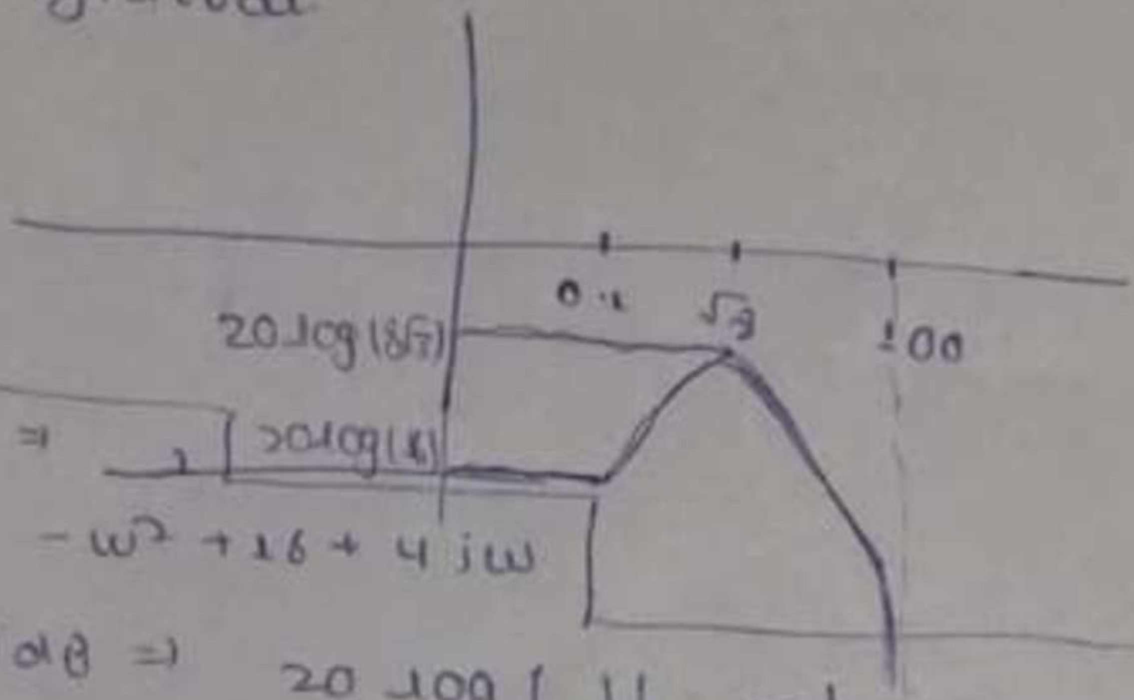
$k = 16$
$c = 4$
$m = 1$

⑤⑥ Poles $\Rightarrow \dfrac{-4 \pm \sqrt{16-64}}{2} \Rightarrow -2 \pm 2\sqrt{3}\,i$ → imaginary

no corner frequency.

Bode plot

(b) magnitude



$G(j\omega) = \dfrac{1}{-\omega^2 + 16 + 4j\omega}$

$M_{dB} = 20 \log \left(\left| \dfrac{1}{16 - \omega^2 + 4j\omega} \right| \right)$

$20 \log \left(\frac{1}{16} \right) \quad \omega \to 0.1$

$20 \log (0) \quad \omega \to 100$

magnitude $= \sqrt{256 + \omega^4 - 32\omega^2 + 16\omega^2}$

$\Rightarrow \sqrt{\omega^4 - 16\omega^2 + 256}$

$\sqrt{4\omega^3 - 32\omega}$

$\sqrt{\omega^2 = 8}$ cor root

$\omega = \sqrt{8}$

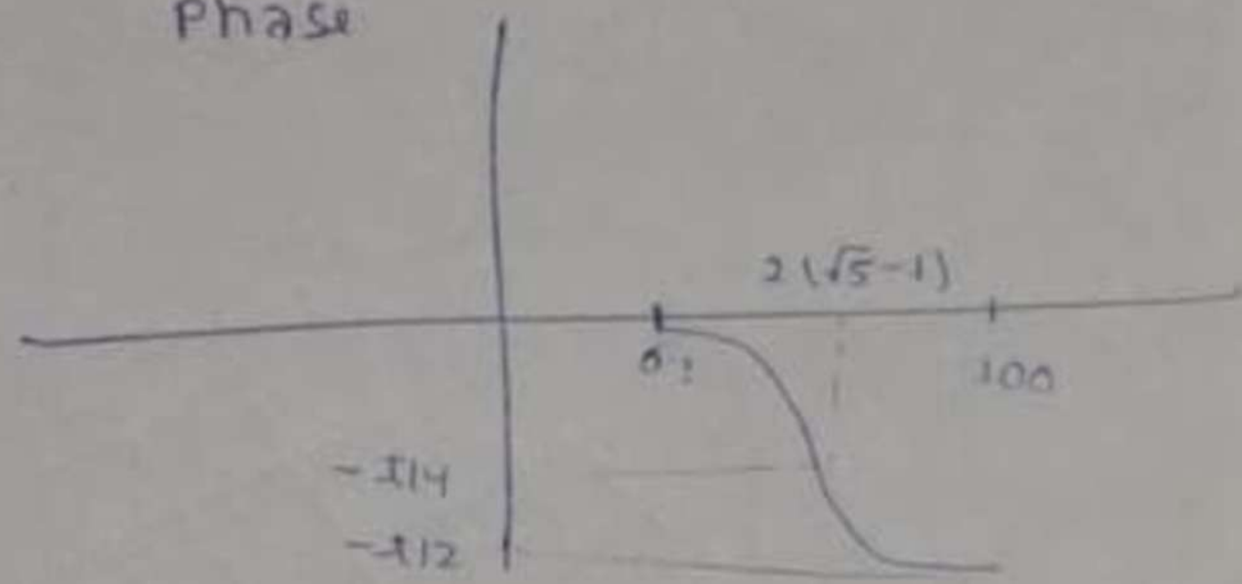
$\sqrt{256 + 64 - 128}$

$\sqrt{192}$

mag $= \sqrt{192}$

$= 8\sqrt{3}$

Phase



$\phi = \tan^{-1} \left(\dfrac{-4\omega}{16 - \omega^2} \right)$

$\omega \to 0.1 : \phi \to 0$

$\omega \to 100 : \phi \to -\pi/2$

$16 - \omega^2 = 4\omega \quad$ for $\phi = \frac{\pi}{4}$

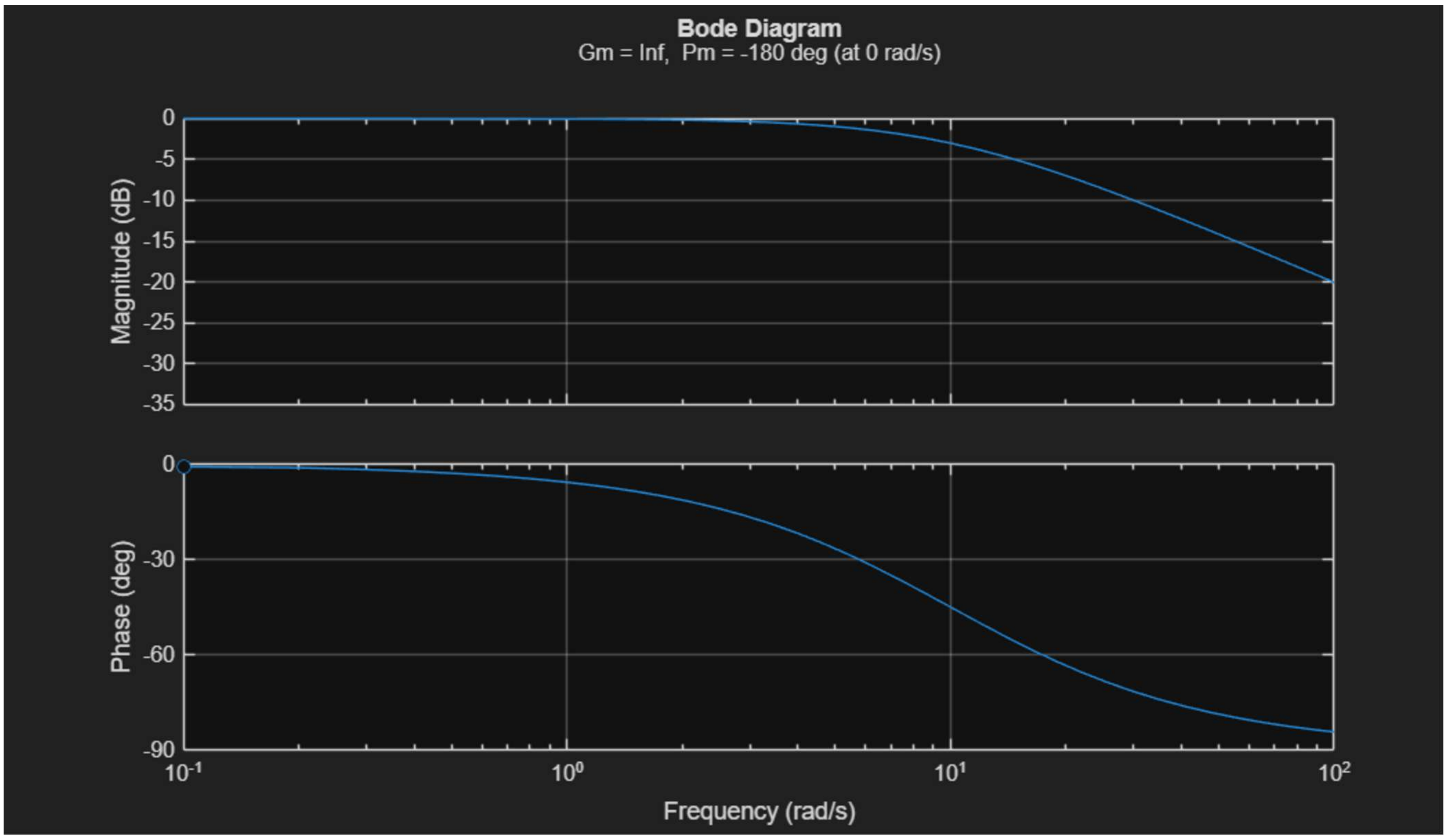
$\omega^2 + 4\omega - 16 = 0$

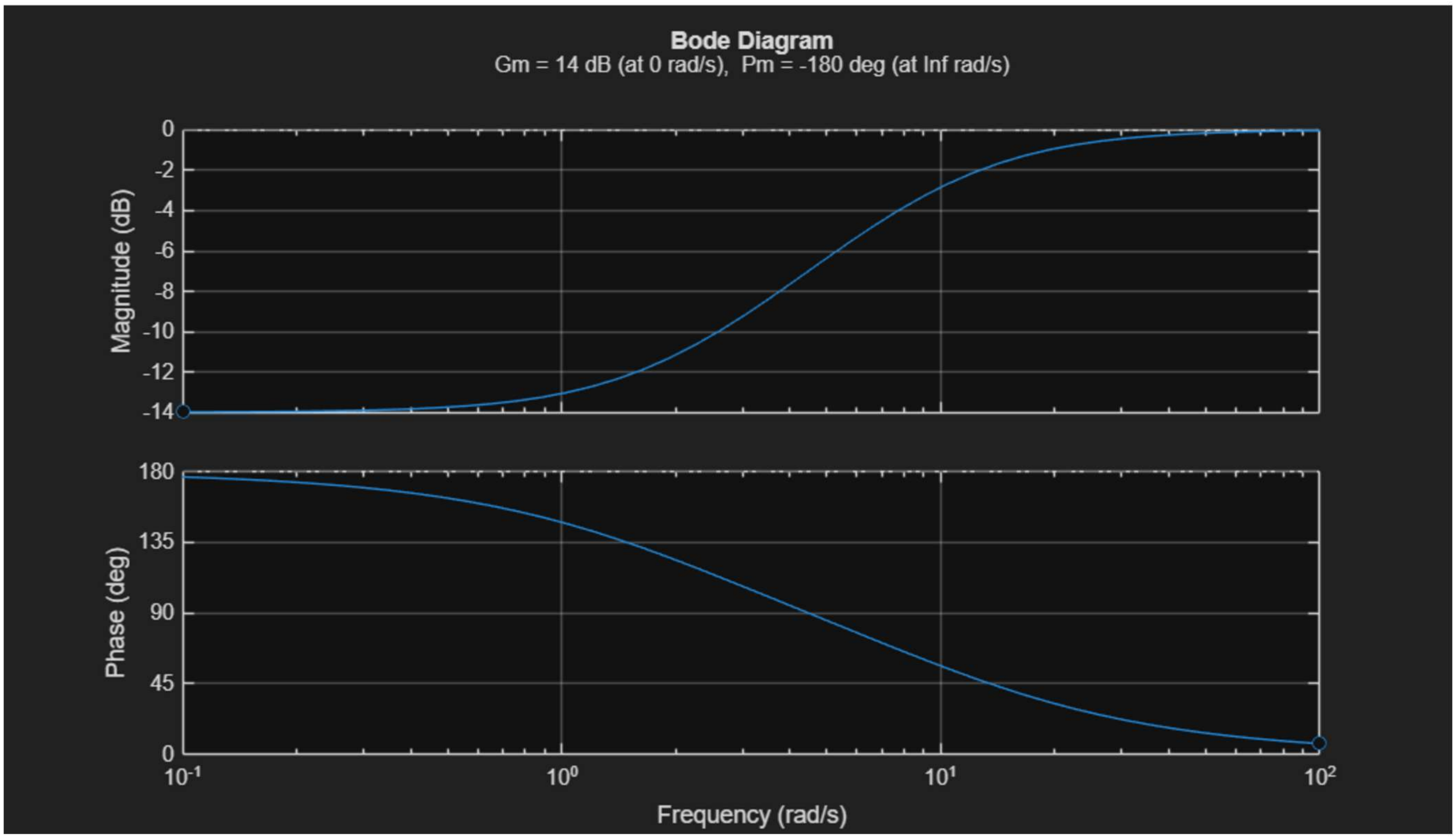
$\omega = \dfrac{-4 \pm \sqrt{16 + 64}}{2}$

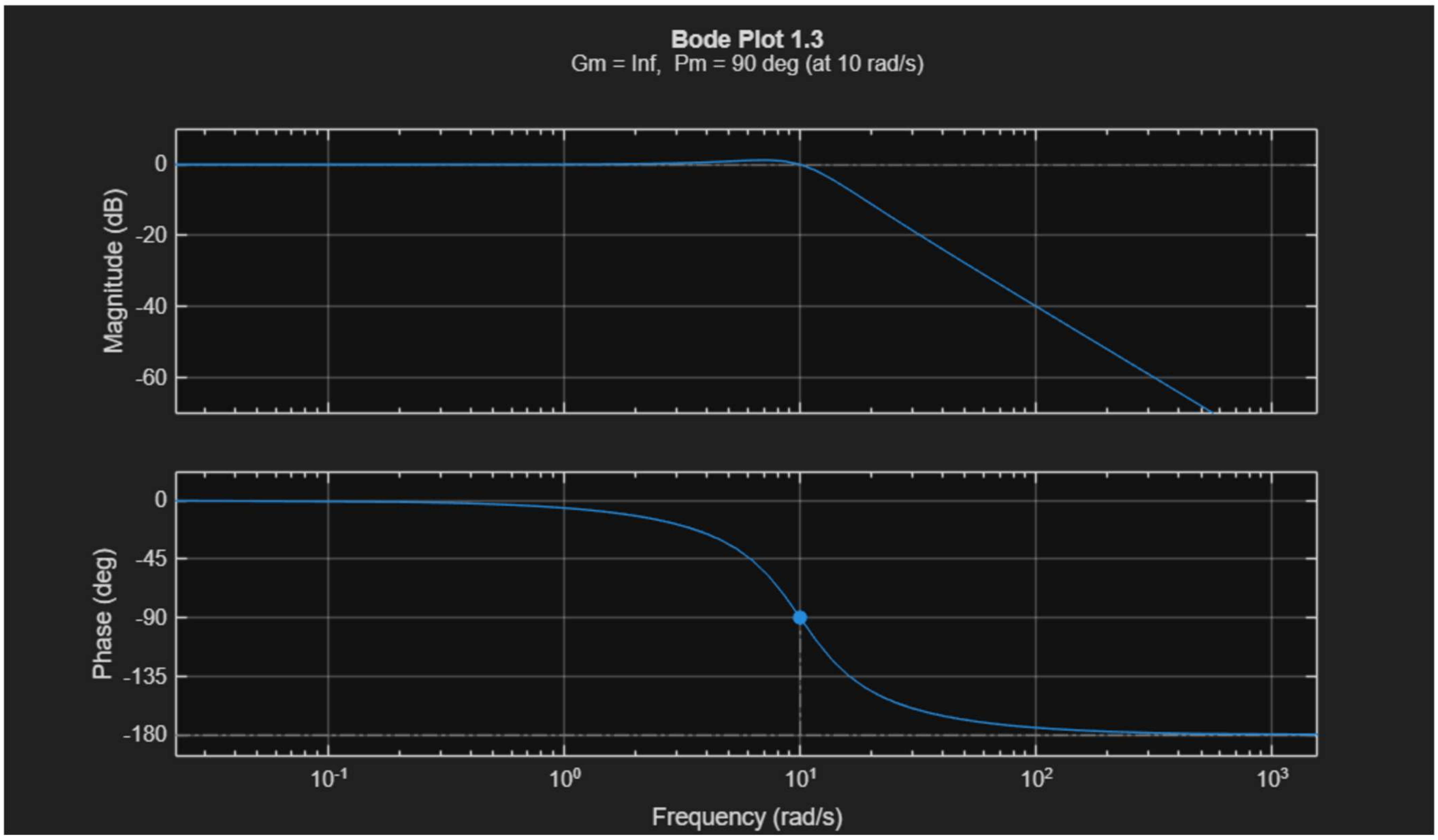
$= -2 + 2\sqrt{5}$

$= (\sqrt{5} - 1)2$

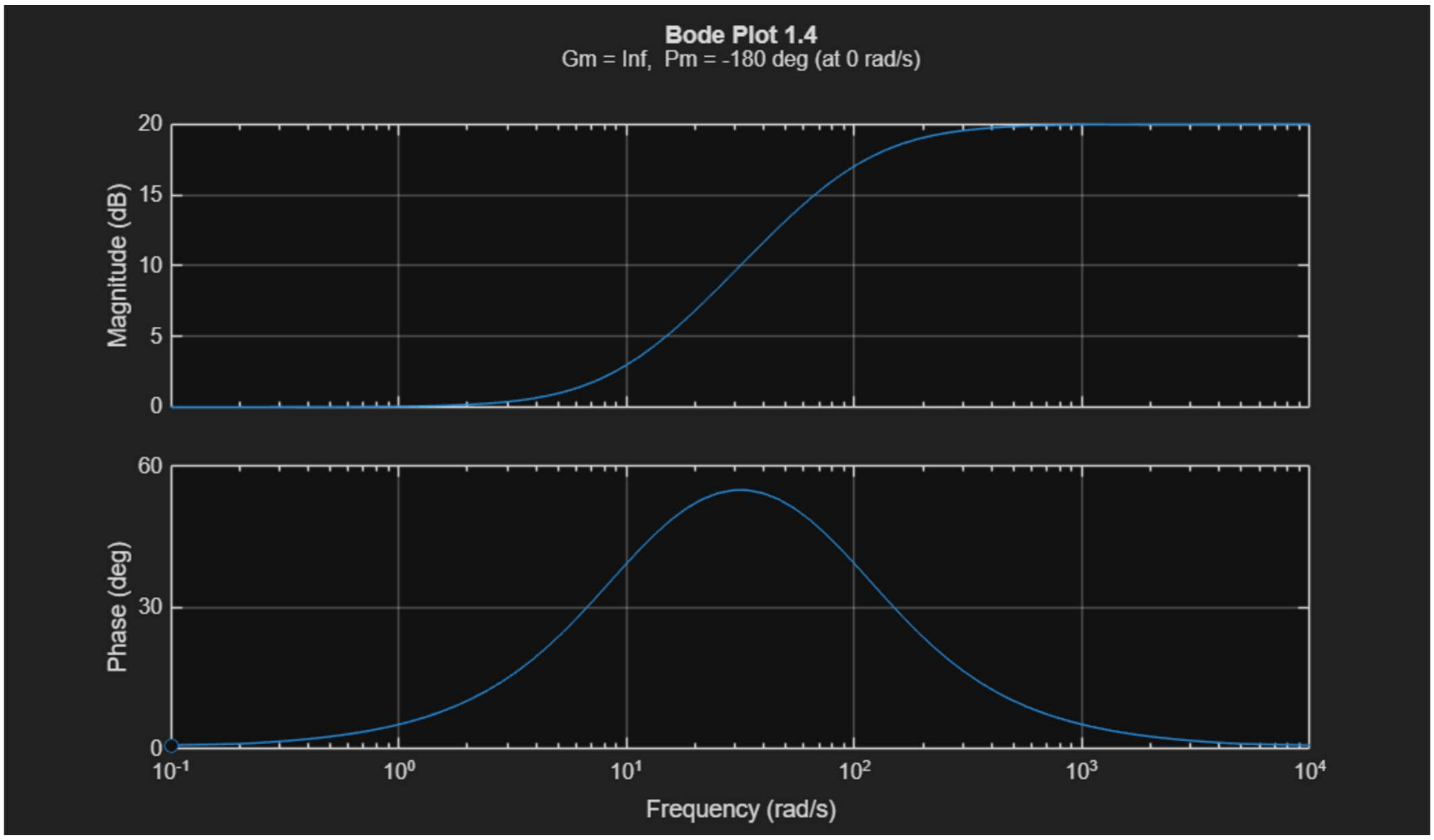
$= 2(\sqrt{5} - 1)$


Bode Diagram
Gm = Inf, Pm = -180 deg (at 0 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

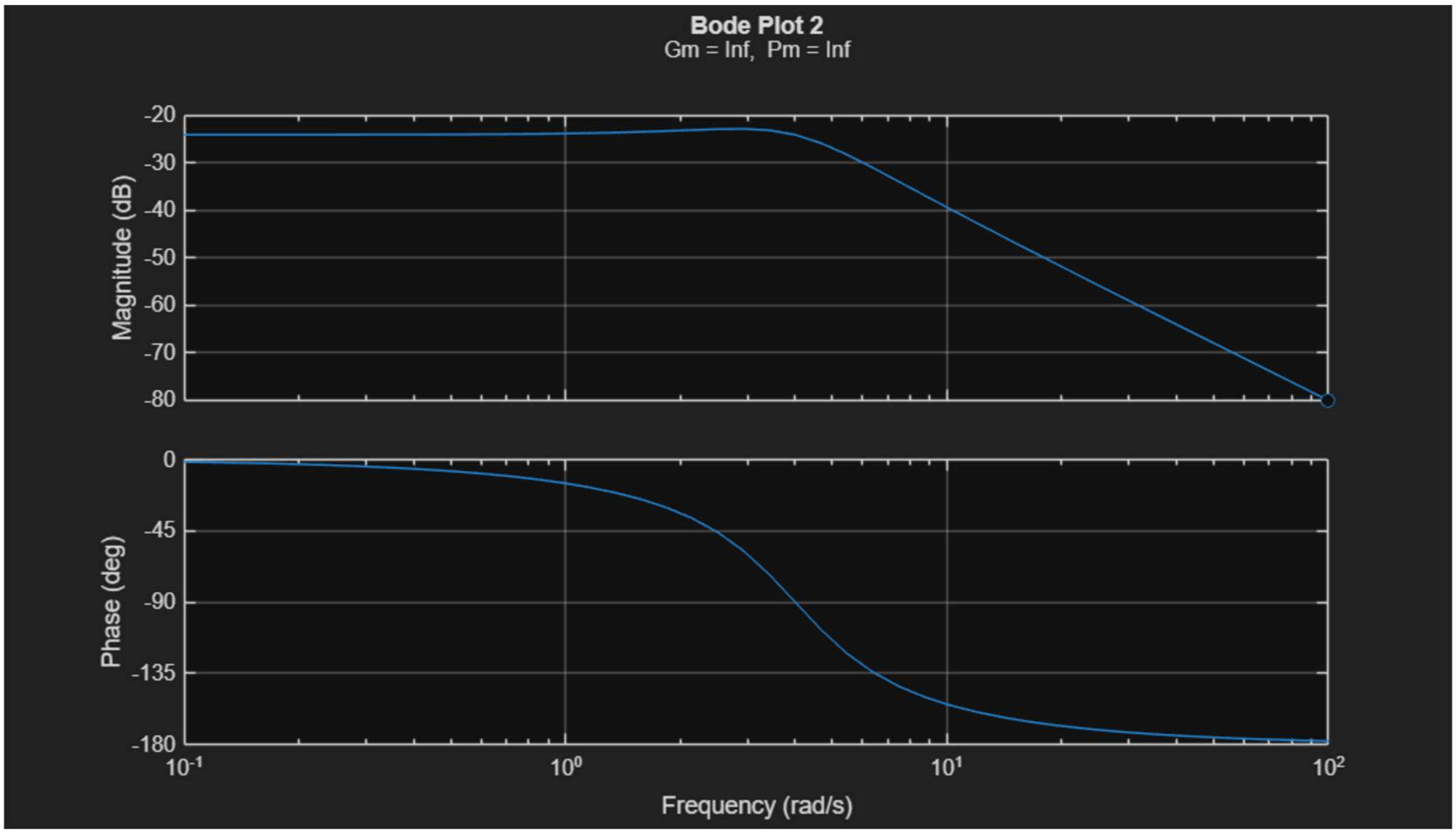

Bode Diagram
Gm = 14 dB (at 0 rad/s), Pm = -180 deg (at Inf rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



Bode Plot 1.3
Gm = Inf, Pm = 90 deg (at 10 rad/s)
Magnitude (dB)
0
-20
-40
-60
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)



Bode Plot 1.4
Gm = Inf, Pm = -180 deg (at 0 rad/s)
Magnitude (dB)
20
15
10
5
0
Phase (deg)
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)


Bode Plot 2
Gm = Inf, Pm = Inf
Magnitude (dB)
-20
-30
-40
-50
-60
-70
-80
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)