Probability and distribution

Probability

A random experiment is an experiment known in advance

Random experiment :- A random experiment is an experiment in which all outcomes of the experiment are known in advance but the exact outcome of any specific performance of the experiment is unpridictable
ex:- Through of a dice or toss of a coin.

Sample space :- The set of all possible outcomes of an experiment is called its sample space eg:- toss of pair of coin.

$S = [HT, TH, HH, TT]$

For throw of a pair of dice

sample space ⇒ (1,1) (1,2) . . . (1,6)
(2,1) (2,2) . . . . (2,6)
:
(6,1) . . . . . . (6,6) ⇒ They are called doublets.

Event

The subset of a sample space is said to be event

NOTE

ϕ :- Impossible event. It is a subset of sample space (S).

* ϕ is a subset of S and is known as impossible event
S is also subset of S and is known as sure event.

Simple event :- If an event is a singleton subset of S it is said to be simple event.

Compound event :- A subset of sample space which contain more than one element.

Occurance of an event :- An event is said to have occured if the outcome of the experiment is an element of the event space. and if the outcome doesn't belong to it we say event has not occured.

Mutually exclusive event :- A set of event is said to be mutually exclusive if occurance of any one of them precludes/prohibits the occurance of ~~the~~ any of the remaining events. ie,

events $E_1, E_2 \ldots E_n$ are m.e if

$$E_i \cap E_j = \phi \ \forall \ i, j = 1, 2, \ldots n \quad i \neq j$$

Equally likely events :- A set of events is said to be equally likely if we have no reason to believe that one is more likely to occur as compare to other.

Exhaustive Event :- A set of event is said to be exhaustive if at least one of them must necessarily happen every time the experiment is performed.

$\therefore E_1, E_2 \ldots E_n$ are exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$

$$\bigcup_{i=1}^{n} E_i = S$$

52 cards

Spade (Black)	club (black)	Heart (Red)	Diamond
♠	♣	♡	◇

Face cards: K, Q, J

K	8	3
Q	7	2
J	6	A
10	5	
9	4	

Whole they are called Denomenation.

TTT HHH THT THH

Probability ~~Definition~~ If an experiment gives n outcomes which are equally likely mutually exclusive and exhaustive and out of these if m cases are favourable to any event A then probability of A =

Probability of A = $P(A) = \dfrac{\text{number of cases favourable to A}}{\text{Total no. of outcomes in sample space}}$

$$= \frac{n(A)}{n(S)} = \frac{m}{n}$$

* odds in favour of an event A and odds against an event A. $= \dfrac{\text{Total no. of cases favourable to A}}{\text{Total no. of cases not favourable to A}}$

$$= \frac{m}{n-m} = \frac{m/n}{1-m/n} = \frac{P(A)}{P(\bar{A})}$$

as $P(\bar{A}) = \dfrac{n-m}{n} = 1 - \dfrac{m}{n}$

$$\boxed{P(A) + P(\bar{A}) = 1}$$

*

Two important Sample Spaces

i) Throwing a pair of dice

	11	12	13	14	15	16
sum=2	21	22	23	24	25	26
sum=3	31	32	33	34	35	36
sum=4						
sum=5	41	42	43	44	45	46
	51	52	53	54	55	56
sum=6	61	62	63	64	65	66
sum=7	sum=8	sum=9	sum=10	sum=11	sum=12	

hyperbola upto D. Y.S. → 86 ...
ellipse – J.N

$P(2) = 1/36$ $P(5) = 4/36$ $P(8) = 5/36$ $P(11) = 2/36$

$P(3) = 2/36$ $P(6) = 5/36$ $P(9) = 4/36$ $P(12) = 1/36$

$P(4) = 3/36$ $P(7) = 6/36$ $P(10) = 3/36$

Q) A : Sum of 2 faces = 6 (1) $P(A \cap B)$
B : Largest of 2 face is 4 (2) $P(A \cup B)$

~~Ans~~ (1) $P(A \cap B) = 2/36$ (2) $P(A \cup B) = 10/36$

Q) Throwing a coin three times. Find the probability
i) at least one head.
ii) at most one head.
iii) alternate head and tail.
iv) both head and tail.
v) exactly one ~~tail~~ head.
vi) No ~~co~~ two consecutive heads.

TTT TTH THT HTT HHH HHT
HTH THH

i) 7/8 (ii) 4/8 = 1/2 (iii) 2/8 = 1/4 (iv) 6/8 = 3/4 (v) 3/8 (vi) 5/8

Q) Two natural number are selected from first 20 natural no. (i) Find the Probability that their sum is odd.
ii) Sum is even.

i) $P(E) = \dfrac{{}^{10}C_1 \times {}^{10}C_1}{{}^{20}C_2} = \dfrac{100 \times 2}{20 \times 19} = \dfrac{10}{19}$

$\dfrac{{}^{10}C_2 \times {}^{10}C_2}{{}^{20}C_2} = \dfrac{10! \times 10! \times 18! \times 2!}{8! \times 8! \times 2! \times 2! \times 20!}$

$\dfrac{100 \times 81}{2 \times 20 \times 19}$ $\dfrac{810}{}$

iii) Twin prime
(3,5) ~~(7,9)~~, (5,7), (11,13) (17,19)

$\frac{4}{^{20}C_2} = \frac{8}{20\times19} = \frac{4}{190} = \frac{2}{95}$

Q) What is the change that the fourth power of an integer selected randomly ends in the digit 6.

A) We only consider with last digit
∴ 2, 4, 6, 8 out of 1 to 10

$\frac{4}{10} = \frac{2}{5}$

Q) A bag ~~consist~~ contains 5 red, four white balls. Three balls are drawn randomly. Find the odd against there being all red.

⇒ |5R 4W| 3 balls drawn
Odd against all being red = ~~RR10~~. $\frac{^9C_3 - {^5C_3}}{^5C_3} = \frac{37}{5}$

* Where there is certainity there is no probability and when there is uncertainity there is probability.

Q) A leap year is selected random. Find probability that it has
i) 53 sunday, and monday.

$7\overline{)366}(52$
$\underline{35}$
16
$\underline{14}$
2

366 days = $\underbrace{52 \text{ weeks}}$ + $\underbrace{2 \text{ days}}$

52 S and
52 M

SM
MT
~~TF~~ TW
~~WT~~ T
~~Sat~~ TF
FS
S ~~S~~

$\frac{1}{7}$

ii) 53 Sundays

$= \frac{2}{7}$

iii) 53 Sundays or 53 Mondays

$= 3/7$ $\quad \frac{(SM, MT, SS)}{\text{Total}}$

Q) A pair of dice has been rolled/cast/thrown. Find the probability that at least one of the dice shows up the face one.

$11/36$

Q) A ~~B~~ has three shares in a lottery in which there are three prices and 6 blanks. B has one share in a lottery in which there is one price and two blanks. Find the ratio of A's chance of success to B's chance of success.

$= \cancel{\dfrac{{}^3C_1 \times {}^6C_0 + {}^3C_2 \times {}^6C_1 + {}^3C_3 \times {}^6C_3}{1}}$

$P(A \text{ wins}) = 1 - P(\bar{A} \text{ wins})$

$= 1 - \frac{6C_3}{{}^9C_3} = \frac{16}{21}$

$P(B \text{ wins}) = 1 - P(\text{win})$

$= 1 - \frac{{}^2C_1}{{}^3C_1} = 1/3$

$\text{Ratio} = \frac{16}{7}$

NOTE ① Let A and B be two events then

i) A' or $\bar{A}$ or A_{NOT} or A^c denotes non-occurance of A.

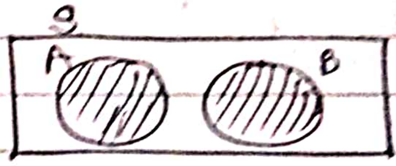
ii) $A \cup B$ or $(A \text{ or } B)$ or $(A+B)$ denotes occurance of atleast one of A or B.

iii) $(A\cap B)$ or (A and B) or (AB) denotes simultaneous occurance of A and B.

iv) $A'\cap B'$ or $\bar{A}\cap\bar{B}$, $(\overline{A\cup B})$ denotes occurance of neither A nor B.

② If $A \subseteq B$ then occurance of A implies occurance of B

3) If A and B are two mutually exclusive events associated with a sample space S, then Probability of $\boxed{A\cup B = P(A)+P(B)}$



Proof:-

$n(A) = n \quad n(B) = m \quad n(S) = N$

$n(A\cup B) = n+m$

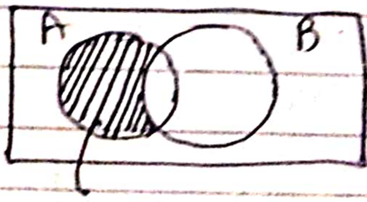
$P(A\cup B) = \frac{n+m}{N} = \frac{n}{N} + \frac{m}{N} = P(A)+P(B)$

$$\boxed{P\Big(\underset{\text{m.e}}{\bigcup_{j=1}^{n} A_j}\Big) = \sum_{j=1}^{n} P(A_i)}$$ ⇒ Generalised ~~formula~~

4) If A and B are two events then $P(A\cap\bar{B}) = P(A) - P(A\cap B)$

Proof:-

$$A = \underbrace{(A\cap\bar{B})\cup(A\cap B)}_{\text{m.e}}$$

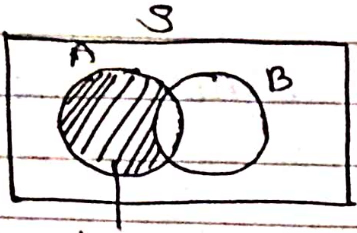


$A\cap\bar{B}$

$P(A) = P(A\cap\bar{B}) + P(A\cap B)$

$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

ADDITION RULE



$P(A\cup B) = P((A\cap\bar{B}) \cup B) = P(A\cap\bar{B}) + P(B)$

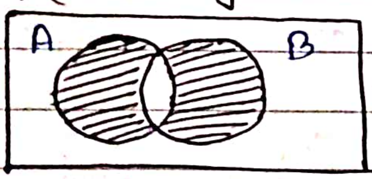
$= P(A) + P(B) - P(A\cap B)$

me

$(A\cap\bar{B})$ $\quad P(A\cap B)$

5) If A and B are two events then probability one of them occurs

P(exactly one of A and B occur)



$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$ *

$= P(A) + P(B) - 2P(A\cap B)$ *

$= P(A\cup B) - P(A\cap B)$ *

$= 1 - P(A\cap B) - (1 - P(A\cup B))$ *

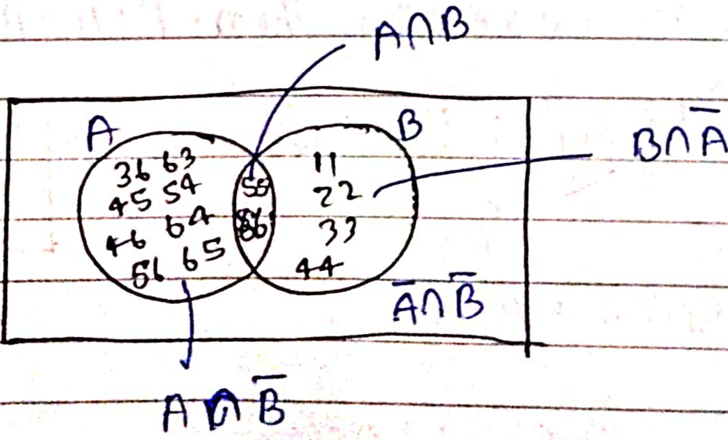
$= P(\overline{A\cap B}) - P(\overline{A\cup B})$

$= P(\bar{A}\cup\bar{B}) - P(\bar{A}\cap\bar{B})$

Experiment : Throwing a pair of dice

A : Getting sum of two faces 9 or more.

B : Getting a doublet



$\frac{4}{36}$

$P(A \cup B)$ or $P(A+B)$

$P(A \cup B)$
or
$P(A+B)$
or
$P(A \text{ or } B)$
or
P(at least one event happen).

P(exactly 1 event happen).
$= P(A \cup B) - P(A \cap B)$
$= P(A) + P(B) - 2P(A \cap B)$
$= P(\bar{A} \cap B) + P(A \cap \bar{B})$
$= 1 - P(\bar{A} \cap \bar{B}) - P(A \cap B)$
$= 1 - P((A \cup B)) - P(A \cap B)$

$P(A) + P(B) - P(A \cap B)$
$P(A) + P(\bar{A} \cap B)$
$P(B) + P(A \cap \bar{B})$
$P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)$
$1 - P(\bar{A} \cap \bar{B})$
$1 - P((A \cup B)^c)$

$\boxed{(A \cup B)^c = \bar{A} \cap \bar{B}}$ Demorgan's law

$P(A/B) = P(A) + P(B/\bar{A})$

Conditional Probability

$\boxed{* \; P(A \setminus B) = P(A \cap \bar{B})}$

$\underbrace{P(A/B) \text{ or } P(A|B) \text{ or } P\left(\frac{A}{B}\right)}$

denotes Probability of A given B or we can say probability of A if it is known that B has already occured.

here now the sample space is $P(B)$

$P(A/B)$ or $P(A|B)$ or $P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$ see from venn diagram

$P(B/A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = P(A)\, P(B/A) = P(B) * P(A/B)$

$P(A \cap B)$

$\frac{1/8}{5 \quad 1}$

$$P(A\cap B\cap C) = P(A\cap B)\cdot P\left(\frac{C}{A\cap B}\right)$$
$$= P(A)\cdot P(B/A)\, P\left(\frac{C}{A\cap B}\right)$$

$$\boxed{P(A\cap B\cap C) = P(A)\, P(B/A)\, P\left(\frac{C}{A\cap B}\right)}$$

Independence

Independent event

Two events A and B are said to be independent if ~~they are indepen~~ occurance and non-occurance of one event doen't effect the probability of the other event, otherwise the events are said to be dependent or contigent.

* If A and B are independent then $P(A/B) = P(A)$ or $P(B/A) = P(B)$

If we are not able to judge of dependency by statement then we can also find that by :-

if A and B are independent then

(i) $\boxed{P(A\cap B) = P(A)\cdot P(B)}$ converse is also true.

(ii) $P(A\cap B) = P(A)\cdot P(B/A) = P(B)\cdot P(A/B)$

From (i) and (ii) For converse

$$P(A)\cdot P(B/A) = P(A)\cdot P(B)$$
$$\Rightarrow P(B) = P(B/A)$$

NOTE: If A and B are independent event then

1) $P(A\cup B) = P(A) + P(B) - P(A)\cdot P(B)$

2) If A and B are independent event then $\bar{A}$ and B, A and $\bar{B}$, $\bar{A}$ and $\bar{B}$ are also independent.

Proof:-

$$P(\bar{A}\cap B) = P(B) - P(A\cap B)$$
$$= P(B) - P(A)\cdot P(B)$$
$$= P(B)\cdot P(\bar{A})$$

0-2 → 25 and Do.V.s all.

P(A)(PBDC)/PA P(A∩B) P C/A∩B P(B) P(A/B)

3) $P(A/B) + P(\bar{A}/B) = 1$ if $(P(B) \neq 0)$

Proof:-

$$\frac{P(A\cap B)}{P(B)} + \frac{P(\bar{A}\cap B)}{P(B)} = 1$$

4) Two possible event cannot be simultaneously mutually exclusive as well as independent.

$$P(A\cap B) = 0 = P(A)\cdot P(B).$$

Q An Important logic

Q) n whole numbers taken at random are multiplied together. Prove that the chance/probability that the digit at unit place of ~~p~~ their product is 1 or 3 or 7 or 9 is $\left(\frac{2}{5}\right)^n$.

② 2 or 4 or 6 or 8 is $\frac{4^n - 2^n}{5^n}$

③ 5 is $\frac{5^n - 4^n}{10^n}$

④ 0 is $\frac{10^n - 8^n - 5^n + 4^n}{10^n}$.

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$P(A\cap B) = P(A/B)P(B)$

Odd Odd . . . n times odd,

1) $\left(\frac{4}{10}\right)\left(\frac{4}{10}\right) \ldots n$

here we exclude 5, 2, 4, 6, 8, 0.

every whole number selected ~~must~~ have probability $\left(\frac{4}{10}\right) = \left(\frac{2}{5}\right)$ ~~p~~ for n whole number

$$\left(\frac{2}{5}\right)^n.$$

2) $\underset{M-2}{\left(\frac{8}{10}\right)^n - \left(\frac{4}{10}\right)^n} = \frac{4^n}{5^n} - \frac{2^n}{5^n} = \frac{4^n - 2^n}{5^n}$

$\underbrace{{}^nC_1\left(\frac{4}{10}\right)^1}_{\text{even}}\underbrace{\left(\frac{4}{10}\right)^{n-1}}_{\text{odd}} + {}^nC_2\left(\frac{4}{10}\right)^2\left(\frac{4}{10}\right)^{n-2} + \ldots + {}^nC_n\left(\frac{4}{10}\right)^n$ → last case only when all are even

here a 5 is exclude. $= \left(\frac{4}{10}\right)^n(2^n - 1)$

HONOR 8X

3) First we exclud 2, 4, 6, 8, 0 as if it will come, then last digit ~~will be 0~~ ends with 0 or any even no.

$\therefore \left(\frac{5}{10}\right)\left(\frac{5}{10}\right) \ldots \text{ n } = \left(\frac{5}{10}\right)^n$

Now, we remove that case where it will ends with 1, 3, 5, 7, 9

$\therefore \left(\frac{5}{10}\right)^n - \left(\frac{4}{10}\right)^n$

$\frac{5^n - 4^n}{10^n}$

M-2

${}^nC_1\left(\frac{1}{10}\right)^1\left(\frac{4}{10}\right)^{n-1} + {}^nC_2\left(\frac{1}{10}\right)^2\left(\frac{4}{10}\right)^{n-2} + \ldots + {}^nC_n\left(\frac{1}{10}\right)^n - {}^nC_0\left(\frac{4}{10}\right)^n$

$\underbrace{\qquad}_{\left(\frac{4}{10}+\frac{1}{10}\right)^n - \left(\frac{4}{10}\right)^n}$

4) $1 - \left(\frac{2}{5}\right)^n - \frac{4^n - 2^n}{5^n} - \frac{5^n - 4^n}{10^n}$

$\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

$P(0) = 1 - P(5) - P(1,2,3,4,6,7,8,9)$

$= 1 - \left(\frac{5^n - 4^n}{10^n}\right) - \frac{8^n}{10^n}$

$\frac{10^n}{10^n} - \frac{4^n}{10^n} - \frac{8^n - 4^n}{10^n} - \frac{5^n - 4^n}{10^n}$

$\frac{10^n - 4^n - 8^n + 4^n - 5^n + 4^n}{10^n}$

Q) A problem is given to two children to solve it independently if probability of A solving it is half and B solving it is $\frac{2}{3}$ find the probability that the problem is solved.

$1 - P(\text{roblem is not solved})$

$\Rightarrow 1 - P(\bar{A} \cap \bar{B})$

$\Rightarrow 1 - P(\bar{A}) \cdot P(\bar{B})$

$\Rightarrow 1 - \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{2}{3}\right) = \frac{5}{6}$

M-2 or

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

M-3 or

$P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)$

$\frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{5}{6}$

//

Q) Two cards are drawn from a packs of 52 cards. Find the probability that either both are red or both are King.

$$\frac{{}^{26}C_2}{{}^{52}C_2} \quad \cancel{P(A \cup B)} = P(A) \cdot P(B)$$
$${}^{26}C_2 \cdot {}^{26}C_2$$

$$P(R \cup K) = P(R) + P(K) - P(R \cap K)$$
$$\frac{{}^{26}C_2 + {}^{4}C_2 - {}^{2}C_2}{{}^{52}C_2} \text{ Red King}$$

Q) A basket contains 20 Apples 10 oranges out of which five apples and three oranges are bad if a person takes out two at random. What is the Probability that either both are apples or both are good.

$$P(A \cup G) = P(A) + P(G) - P(A \cap G)$$
$$= \frac{{}^{20}C_2 + {}^{22}C_2 - {}^{15}C_2}{{}^{30}C_2} \quad 20-5$$
$$30-8$$

Q) Two integers are selected at random from integers 1 to 11 if the sum is even. Find Prob. that the numbers are odd

$$\cancel{\frac{{}^{5}C_1 \times {}^{4}C_1 + {}^{6}C_1 \times {}^{5}C_1}{{}^{11}C_2}}$$

A = both no. are odd
B = sum is even

$$P\left(\frac{A}{B}\right) = \frac{{}^{6}C_2}{{}^{6}C_2 + {}^{5}C_2}$$

Q) Probability that a teacher takes a surprise test is $\frac{1}{3}$ if a student remains absent for two days then find the probability that he misses
a) exactly one test.

$\Rightarrow P(\bar{T}T) + P(T\bar{T})$

$$\frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$$

b) at least one test.

$\Rightarrow 1 - P(\text{no test missed})$

$\Rightarrow 1 - \frac{2}{3} \times \frac{2}{3} \Rightarrow 1 - \frac{4}{9} \Rightarrow \frac{5}{9}$

M-2

$$P(T\bar{T}) + P(\bar{T}T) + P(TT)$$

$$\frac{2}{9} + \frac{2}{9} + \frac{1}{9} = \frac{5}{9}$$

c) at most one test.

$P(T\bar{T}) + P(\bar{T}T) + P(\bar{T}\bar{T})$

$$\frac{2}{9} + \frac{2}{9} + \frac{2}{3} \times \frac{2}{3} = \frac{8}{9}$$

P(at most one test missed) = P(0 test missed) + P(exc. one test missed).

Q) An urn contains one red two green and three black balls. Three people A, B, C in order draw one ball from the urn and put it back after noting its colour. They continue doing it indefinitely unless or who gets a red ball first wins the game. Find their respective probabilities of winning.

$\bar{R}\bar{R}\bar{R}R$ or $\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R \rightarrow \infty$

~~P(A wins) = 1 - P(B̄) - P(C̄)~~

~~= 1~~

a) $P(A \text{ wins}) = P(R \text{ or } \bar{R}\bar{R}\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R \rightarrow \infty)$

$= P(R) + P(\bar{R}\bar{R}\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R) + \ldots \infty$

$= \frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6} + \ldots$

$= \frac{1}{6}\left(1 + (5/6)^3 + (5/6)^6 + \ldots \infty\right)$

$= \frac{1/6}{1 - \frac{125}{216}} = \frac{36}{91}$

$P(B \text{ wins}) = P(\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}R \text{ or } \ldots)$

$= P(\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}R) + \ldots \infty$

$= \left(\frac{5}{6}\right) \times \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \ldots \infty$

$\frac{5}{6}\frac{1}{6}\left[1 + (5/6)^3 + \ldots \infty\right]$

$= \frac{30}{91}$

c) $1 - P(A \text{ wins}) - P(B \text{ wins})$

$= 1 - \frac{36}{91} - \frac{30}{91}$

$\Rightarrow \frac{25}{91}$

Q) A pair of dice is rolled untill a sum of 5 or 7 obtained. Find the probability that the total of Five comes before a total of 7. $\frac{4}{10}$

$P(S_5 \text{ occurs before } S_7)$

$= P(S_5 \text{ or } (\bar{S_5}\bar{S_7})S_5 \text{ or } (\bar{S_5}\bar{S_7})(\bar{S_5}\bar{S_7})S_5) + \ldots$

$$= \frac{4}{36} + \frac{13}{18} \times \frac{4}{36} + \left(\frac{13}{18}\right)^2 \times \frac{4}{36} + \dots \infty$$

$$= \frac{4}{36} \cdot \frac{1}{1 - 13/18}$$

$$= 2/5$$

Q) Two person A and B one by one in order draw one ball each from a purse containing five white and one red ball and retain it. The person who get the red ball wins the game. Are A and B equiprobable to win? yes.

$$P(A\text{ wins}) = P(A \text{ or } \bar{A}\bar{B}A \text{ or } \bar{A}\bar{B}\bar{A}\bar{B}A \dots)$$

$$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) \dots \infty$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} \dots$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \dots\right]$$

$$= \frac{1}{6} \cdot \frac{1}{1 - \frac{25}{36}} = \frac{1}{6} \times \frac{36}{11} = 6/11$$

$$P(B\text{ wins}) = P(\bar{A}B \text{ or } \bar{A}\bar{B}\bar{A}B \text{ or } \bar{A}\bar{B}\bar{A}\bar{B}\bar{A}B \dots \infty)$$

$$P(A\text{ wins}) = P(R \text{ or } WWR \text{ or } WWWWR)$$

$$= P(R) + P(WWR) + P(WWWWR)$$

$$= \frac{1}{6} + \frac{5}{6} \cdot \frac{4}{5} \cdot \frac{1}{4} + \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}$$

$$= \frac{1}{2}$$

$$P(B\text{ wins}) = P(R \text{ or } WR \text{ or } WWWR \text{ or } \cancel{WWWWW})$$

$$\frac{1}{6} + \frac{5}{6} \times \frac{1}{5} + \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} \quad \frac{3}{6} = \frac{1}{2}$$

∴ answer is yes

Q) A box contains 5 tubes, two of them are defective and three of them are non defective. Tubes are tested one by one till the two defective tubes are discovered. What is the Prob. that the testing procedure comes to an end of

a) Second testing

$\cancel{\frac{2}{5}} \times \cancel{\frac{4}{5}}$ $P(DD) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

b) Third testing

$P(DGD) + P(GDD) + P(GGG)$

$\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$

$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$

Q) All face cards from a pack of 52 plane cards are removed. From the remaining 40 cards, four are drawn. Find the Probability that they are of diffirent suit and diffirent denomination.

$\cancel{\frac{{}^{10}C_4 \times 10}{{}^{40}C_4}}$

from any of four suit

$\frac{{}^{10}C_4 \times 4!}{{}^{40}C_4}$ → 4 out of 10 of any suit

or

$1 \cdot \frac{27}{39} \times \frac{16}{38} \times \frac{7}{37}$

H	S	C	D
10	10	10	10
9	9	9	9
8	8	8	8
7	7	7	7
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
A	A	A	A

13
11
24
7
31

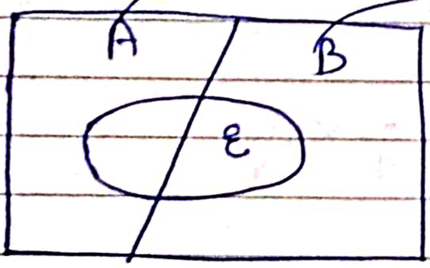
0

Q) A lot contains 20 articles. The Probability that the lot contains exactly two defective articles is 0.4 and the probability that the lot contain exactly three defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all the defective articles are formed. Find the probability that the testing procedure ends at the twelth testing.

$99/1900$

exactly 2 dif

exactly 3 def



E : Testing prodcure end at 12^{th} testing

$P(E) = P(E \cap A) + P(E \cap B)$

$= P(A)\, P(E/A) + P(B) \cdot P(E/B)$

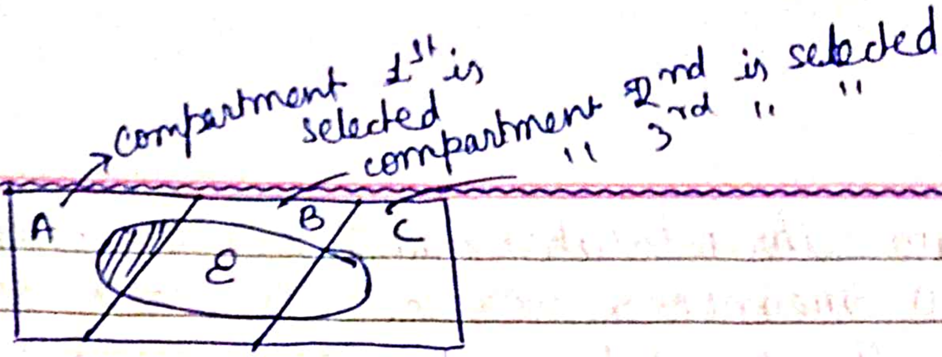
$= 0.4 \dfrac{{}^{11}C_1 \cdot 2!}{{}^{20}C_2 \cdot 2!} + 0.6 \dfrac{{}^{11}C_2 \cdot 3!}{{}^{20}C_3 \cdot 3!}$

Q) A lady has three compartment in her purse. First compartment contain 1 ₹ coin and 2 50 paise coin. Second compartment contains one ₹ coin and 5 50 paise and 3rd compartment contains 3 1₹ coin and four 50 paise coin. She randomly selected a compartment to draw a coin. What is the probability that the draw coin is rupee coin.

$\cancel{\dfrac{1}{3}\, {}^3C_1 \left(\dfrac{1}{3} + \dfrac{2}{5} + \dfrac{3}{7} \right)} = \cancel{\dfrac{1 + \frac{6}{5} + \frac{9}{7}}{35}}$

1

1



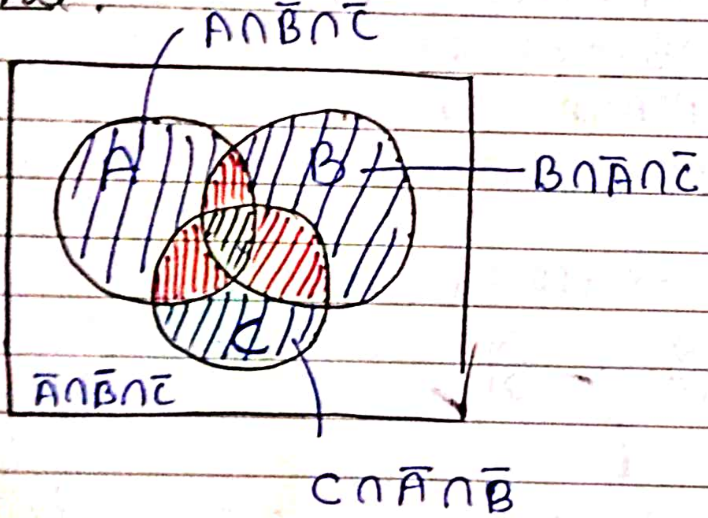
ε: she randomly select a compartment and draws 1 Rs coin.

$P(\varepsilon) = P(\varepsilon \cap A) + P(\varepsilon \cap B) + P(\varepsilon \cap C)$

$= P(A)\, P\left(\varepsilon/_A\right) + P(B)\left(\varepsilon/_B\right) + P(C) \times \left\{P(\varepsilon/_C)\right)$

$= \frac{1}{3}\left(\frac{1}{3} + \frac{2}{5} + \frac{3}{7}\right)$

Three events defined on an experimental performance.



$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

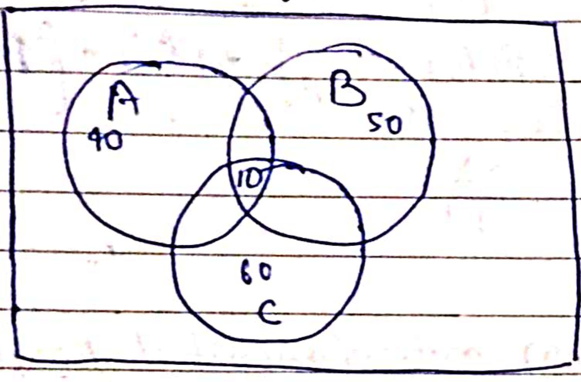
Prob. (at least 1 event happen)

P(exactly 1 event happen) $= \Sigma P(A) - 2\Sigma P(A \cap B) + 3P(A \cap B \cap C)$

P(" 2 " ") $= \Sigma P(A \cap B) - 3P(A \cap B \cap C)$

P(at least 2 " ") $= \Sigma P(A \cap B) - 2P(A \cap B \cap C)$

Q> There are three clubs A, B, C in a town with 40, 50, 60 members respec. 10 People are members of all the three clubs, 70 are members in only one club. A member is randomly selected. Find the probability he has memership of two clubs.



~~$P(A\cup B\cup C)=70$~~

$P(A\cap B\cap C)=10$

$n(A)=40 \quad n(B)=50 \quad n(C)=60$

$\cancel{P(A)+P(B)+P(C)-2\sum P(A\cap B)+3P(A\cap B\cap C)}$ $\to 0$

$150-2\sum P(A\cap B)+30=70$

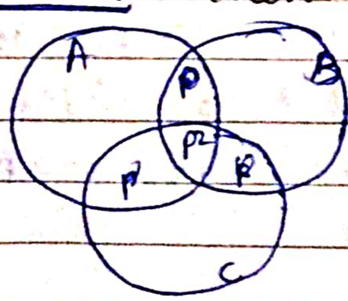
$\Rightarrow \sum P(A\cap B)=55$

$\Rightarrow n(\text{ex } 2)=55-30=25$

$n(A\cup B\cup C)=150-55+10=105$

Ans $=\frac{25}{105}=\frac{5}{21}$

Q> For three events A, B, C P(exactly one of the events A or B occurs) = P ; P(ex. one of events B or C occurs) = P , P(ex. one of events C or A occurs) = P and P(all the 3 events occur simultaneously) = P^2. If A, B, C are exhaustive. Then Find the value of P.



$P(A)+P(B)-2P(A\cap B)=P$

$P(B)+P(C)-2P(B\cap C)=P$

$P(A)+P(C)-2P(A\cap C)=P$

$2[P(A)+P(B)+P(C)-\sum P(A\cap B)]=3P$

$P(A\cup B\cup B)=1$

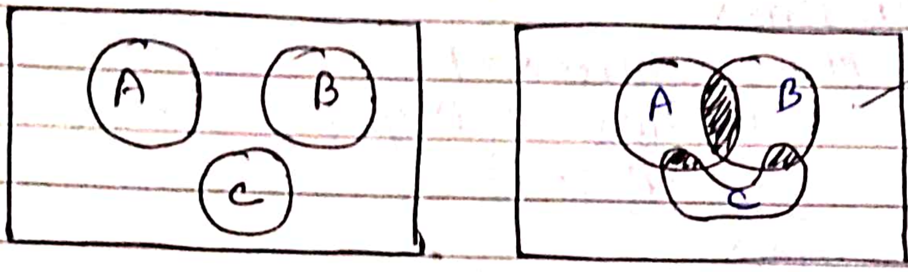
$\frac{3P}{2}+P^2=1$

$P=\frac{1}{2}$ or -2 ✗

~~$P(A\cup B\cup C)=1 \Rightarrow \sum P(A)-\sum P(A\cap B)-3P(A\cap B\cap C)=1$~~

~~$\Rightarrow \sum P(A)-3P-3P^2=1$~~

$\boxed{\text{NOTE}}$ i) If A, B and C are pair wise mutually exclusive then A, B and C must be mutually exclusive but converse is not true i.e, if A, B, C are mutually exclusive they need not be pair wise mutually exclusive.



here simultaneously all 3 can occur.

mtlb teeno ek saath nhi ho sakta

ii) For three events to be independent following conditions must be satisfied

a) $P(A\cap B) = P(A)P(B)$ $\quad P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$

b) $P(B\cap C) = P(B)P(C)$

c) $P(C\cap A) = P(C)P(A)$

d) ~~$P(A\cap B\cap C\cap D) = P(A)\cdot P(B)\cdot P(C)\cdot P(D)$~~

eg:- E: A coin has been tossed twice

A: head on first ~~class~~ toss

B: " " 2nd "

C: " " exactly one toss

$P(A) = \frac{1}{2} = P(B) = P(C)$

why all four condition

eg: $S = \{1,2,3,4,5,6,7,8\}$

$A = \{1,2,3,4\}$ $B = \{2,3,4,5\}$

$C = \{4,6,7,8\}$

$P(A\cap B\cap C) = 1/8 = P(A)\times P(B)\times P(C)$

$P(A\cap B) = 3/8$

$P(A\cap B) = 1/4 = P(A)\cdot P(B)$

$P(B\cap C) = 1/4 = P(B)\cdot P(C)$

$P(C\cap A) = 1/4 = P(C)\,P(A)$ and $P(A\cap B\cap C) = 0$

iii) For n events to be independent we need $2^n - n - 1$ condition.

n ($^nC_1$), 1 ($^nC_0$)

Binomial Probality distribution.

eg:- a baised coin is tossed such that $P(H) = 3P(T)$

$P(H) = \frac{3}{4}$ $P(T) = \frac{1}{4}$

If it is tossed 10 times

Let $P(H)$ = success = P

Failure $= 1-P = q$

$\boxed{P+q=1}$

If it is tossed 10 times.
For exactly three times Head
Total possibles are

HTTTTHTTTH $\longrightarrow \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^7$

$\vdots$ $\longrightarrow (3/4)^3 (1/4)^7$

$\vdots$

${}^{10}C_3$

Total Prob $= {}^{10}C_3 \left(3/4\right)^3 \left(1/4\right)^7$

Q) An experiment has performed for n- independent trials and the prob. of getting success is P and failure is q ($q = 1-P$) in a trial then Prob of getting success in 'r' trials out of n is given by $P(X=r) = {}^nC_r \, p^r q^{n-r}$

P(at least one success) = means $r=0$

$\therefore (1 - {}^nC_0 \, p^0 q^{n-0}) \Rightarrow (1-q^n) = {}^nC_1 \, p^1 q^{n-1} + \cdots$
$\cdots + {}^nC_n \, p^n q^0$

Q) 1) 100 identical coins each falling headwise with the probability $P(0<P<1)$ are tossed once, if the prob of 50 coins showing up the heads is equal to the prob. of 51 coins showing up the heads. Find the value of P.

$\Rightarrow {}^{100}C_{50}\, p^{50} q^{50} = {}^{100}C_{51}\, p^{51} q^{49}$

$= \left(\frac{1-P}{P}\right) \quad \frac{q}{p} = \frac{{}^{100}C_{51}}{{}^{100}C_{50}} = \frac{100-50}{51} = \frac{50}{51}$

$\Rightarrow 51 - 51P = 50P \Rightarrow P = \frac{51}{101}$

2) A pair of dice is thrown 6 times getting a doublet is considered as success. compute the probability of a) no success (b) Exactly one success. c) at least one success (d) at most one success.

a) failure $= 1 - \frac{6}{36} = \frac{5}{6}$

$(q)^6 = (5/6)^6$

(d) ${}^6C_0\, p^0 q^6 + {}^6C_1\, p^1 q^5$

$(5/6)^6 + 6(1/6)(5/6)^5$

(b) ${}^6C_1 (P)^1 (q)^5$

${}^6C_1 (1/6)(5/6)^6$

c) $1-(q)^6 = 1-(5/6)^6$

3) A coin is twice as likely to land heads as tails. In a sequence of five independent trials, find the prob. that the third head occurs on fifth toss.

$= ({}^4C_2\, p^2 q^2)\, P = {}^4C_2\, p^3 q^2$

$p = 2/3$

$q = 1/3$

$= {}^4C_2 (2/3)^3 (1/3)^2$

$p = 2x \quad q = x$

$3x = 1 \quad x = 1/3 \quad p = 2/3,\ q = 1/3$

Q) A fair coin is flipped n times. Let E be the event "a head is obtained on the first flip", and let F_K be the event "exactly k heads are obtained. For which of the following pairs (n,K) are E and F_K independent?

A) 12, 4 (B) (20, 10) (C) 40, 10 (d) (100, 51)

$P(H) = p \quad P(T) = q$

$P(E) = p\left(\sum_{r=0}^{n-1} {}^{n-1}C_r \, p^r q^{n-r-1}\right)$

$P(E) = p$

$P(F_K) = {}^nC_K \, p^K q^{n-K}$

$P(E \cap F_K) = P(E) \cdot P(F_K)$

$\Rightarrow p \, {}^{n-1}C_{K-1} \, p^{K-1} \times q^{n-K} = p \, {}^nC_K \, p^K q^{n-K}$

$1 = p \, n/K \quad (n = 2K)$

Q) In a hurdle race a man has to clear 9 hurdles. Probability that he clears a hurdle is 2/3 and the prob. that he knocks down the hurdle is 1/3. Find the Probability that he knocks down fewer than two hurdles.

$n = 9 \mid q = 2/3 \quad p = 1/3$

$P(X < 2) = P(X=1) + P(X=0)$

$= {}^9C_1 (1/3)^1 (2/3)^8 + {}^9C_0 (1/3)^0 (2/3)^9$

Q) A drunkard takes a step forward and backward. The probability that he takes a step forward is 0.4. Find the Prob. that the end of 11 steps he is one step away from the starting point.

${}^{11}C_5 (0.4)^5 (0.6)^6 + {}^{11}C_5 (0.4)^6 (0.6)^5$

Ans. ${}^{11}C_5 (0.4)^5 (0.6)^6 + {}^{11}C_5 (0.4)^6 (0.6)^5$

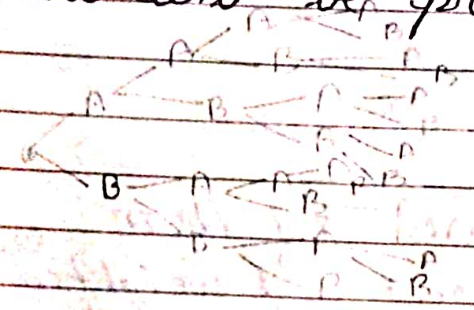
$\frac{4}{10}$ V.Good $\frac{6}{5}$

Mathematical expectation

If 'P' represents a person's whose chances of success in any venture and 'M' is the sum of money which he will receive in case of success tho sum of money denoted by PM is called the expectations.

Q) Two players of equal skills A and B are playing a game. They leave off playing (due to some force major condition) when A wants 3 points and B wants 2 points to win. If prize money is ₹16.~~000~~ How will be prize.

A = 5000 B => 11000
$P(A) = 5/16$ $P(B) = 11/16$



My ans.

$P(A) = \frac{2}{5}$ $P(B) = \frac{3}{5}$

$3x + 2x = 1$
$x = 1/5$
$P(A) = 3/5$
$P(B) = 2/5$

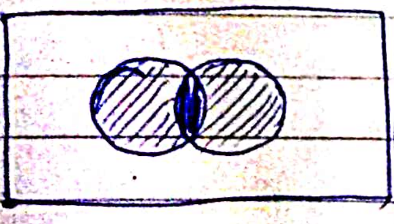
Money of A = 6400
" " B = 9600

AAA + BAAA + ABAA + AABA +

Q) Two hunters A and B shot at a bear simultaneously The bear was shot death with only one bullett in its body. Prob. of A shooting the bear is 0.8 and that of B " " " is 0.4
The hide was sold for ₹ 280 divide the money fare way.

240, 40

$P(A) = \cancel{P(A \text{ or } A\bar{B}A \text{ or } \bar{A}\bar{B}\bar{A}\bar{B}A \text{ or } \ldots)}$
$= \cancel{0.8 + 0.2 \times 0.6 \times 0.8 + 0.2 \times 0.6 \times 0.2 \times 0.6 \times} \; 1$



$\frac{0.8 \times 0.6}{0.8 \times 0.6 + 0.4 \times 0.2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B) = 0.2$
$P(\text{only } A) = 0.8 - 0.2 = 0.6$
$P(\text{only } B) = 0.4 - 0.2 = 0.2$

Prob. ki sirf A mara hai if it is given that A mara hai

Baye's Theorem

If $B_1, B_2, B_3 \ldots B_n$ are ~~ass~~ n mutually exclusive and exhaustive set of events and $P(B_1)$, $P(B_2)$, $P(B_3) \ldots P(B_N)$ are known or can be calculated also. $P(E/B_1)$, $P(E/B_2) \ldots$, $P(E/B_N)$ are known to be calculated then a/c Baye's theorem

$$P\left(B_i/E\right) = \frac{P(B_i \cap E)}{P(E)}$$

where

$$P(E) = \sum_{i=1}^{n} P(B_i \cap E)$$

Q> A lady has 10 coins in her purse 8 of them are normal coins, one doubly headed coin, 1 weighted coin ($P(H) = 1/3$) she randomly drawn a coin and tosses it 5 times and coin was found to fall headwise all the 5 times. Find the prob. that the drawn coin was a doubly headed coin.

8	1 d.H	1 W $P(1/3)$

$$\frac{\frac{1}{10}(1)^5 \text{ probability of coming head}}{\left(\frac{1}{10}\right)\left(\frac{1}{3}\right)^5 + \frac{1}{10}(1)^5 + \frac{8}{10}\left(\frac{1}{2}\right)^5}$$

$$\frac{\cancel{1/10}}{\left(\cancel{\frac{1}{10}}\right)\left(\frac{1}{243}\right) + \frac{1}{\cancel{10}} + \frac{4}{\cancel{10}}\left(\frac{1}{16}\right)}$$

3x3x3x3x

$$\frac{1}{\frac{1}{243} + 1 + \frac{1}{4}} = \frac{972}{1219}$$

Q) A bag contains 6 balls of unknown colours. 3 balls are drawn from the bag at random and found to be all black. Find the probability that no black balls are left in the bag now. Assume all no. of black balls in the bag initially equally likely.

$\cancel{3/6} \cdot \cancel{1/2}$

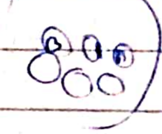
$$\frac{\frac{1}{7}\,{}^3C_3/{}^6C_3}{\frac{1}{7}(0+0+0)+\frac{1}{7}\frac{{}^3C_3}{{}^6C_3}+\frac{1}{7}\cdot\frac{{}^4C_3}{{}^6C_3}+\frac{1}{7}\frac{{}^5C_3}{{}^6C_3}+\frac{1}{7}\frac{{}^6C_3}{{}^6C_3}} = \frac{1}{35}$$

B_0
B_1
B_2
B_3
$\vdots$
B_6

$1/7$

${}^0C_3 \quad {}^1C_3 \quad {}^2C_3$

Those probabilities are zero as we know that there is 3 black ball is present.



Q) The contents of urn 1 and urn 2 are as follows

urn	I	II
W	4	3
B	5	6

One urn is selected at random and the ball is drawn, and its colour noted and replaced back in the same urn. Again a ball is drawn from same urn colour noted and replaced. The process is repeated four times and as a result one ball of white colour and 3 of black colour noted. What is the probability that the selected urn was urn I.

Q) In a test utkarsh either guesses or copies or knows the answer to a multiple choice question with four choices. The prob. that he makes a guess is 1/3 and the prob. that copies the answer is 1/6. The prob that his answer is correct given that he copied is 1/8. Find the prob. that he knows the answer to the question given that he correctly answered it.

Solve in both the cases.

a) If question is single correct.

b) one or more than 1 correct.

1) $\dfrac{\frac{1}{2}\cdot {}^4C_1\left(\frac{4}{9}\right)^1\left(\frac{5}{9}\right)^3}{\frac{1}{2}\times {}^4C_1\left(\frac{4}{9}\right)^1\left(\frac{5}{9}\right)^3+\frac{1}{2}\,{}^4C_1\left(\frac{3}{9}\right)^1\left(\frac{6}{9}\right)^3}$

→ 4 baar uthane me se koi 1 one white aane ka tarika.

2) a) $\dfrac{\frac{1}{2}\times 1}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}\times 1}$ $\;\;\dfrac{24}{29}$

→ ∵ four option

5) → doubt

$\dfrac{\frac{1}{2}\times 1}{\frac{1}{3}\left[\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+1\right]+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}} = \dfrac{1}{\frac{1}{3}\left\{\frac{105}{4}+\frac{1}{6}\times\frac{1}{4}+1\right.}$

$\dfrac{1}{\frac{20+1+12}{12}} = \dfrac{12}{33} = \dfrac{4}{11}$

Extended Baye's Theorem

Q) A bag contains five balls of unknown colour. A ball is drawn twice with replacement from the bag and found to be red on both the occasions. Now the contents of the bag was restored. Now two balls are drawn simultaneously from the bag. ~~and found to be~~ Find the probability that they will be both red. Initially assume all the no. of red balls in the bag to be equally likely.

0-1 → 12 elem → (1)

$\frac{1131}{1650}$

$\left.\begin{matrix} B_0 \\ B_1 \\ \vdots \\ B_5 \end{matrix}\right\} \frac{1}{6}$

$$\frac{P(A)}{\frac{1}{6}\times 0+\frac{1}{6}\cdot\frac{1}{25}+\frac{1}{6}\times\frac{4}{25}+\frac{1}{6}\times\frac{9}{25}+\frac{1}{6}\times\frac{16}{25}+\frac{1}{6}\times\frac{25}{25}}$$

$P(B_0/E)=0 \quad P(B_2/E)=4/25 \quad P(B_4/E)=\frac{16}{25}$

$P(B_1/E)=1/25 \quad P(B_3/E)=\frac{9}{25} \quad P(B_5/E)=\frac{25}{25}$

E = occurance of red ball two times or sample space

$$P(A)=\frac{4}{25}\cdot\frac{{}^2C_2}{{}^5C_2}+\frac{9}{25}\cdot\frac{{}^3C_2}{{}^5C_2}+\frac{16}{25}\frac{{}^4C_2}{{}^5C_2}+\frac{25}{25}\frac{{}^5C_2}{{}^5C_2}$$

Probability ki dono ball red hai and hai to unme se 2 aane ka probab.. $=\frac{377}{550}$

Q) A bag contains three biased coins B_1, B_2 and B_3 whose probability of falling head wise is $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{3}{4}$ respectively. A coin is drawn randomly and tossed fell head wise. Find the probability that the same coin when tossed again will fall headwise.

$$P(B_1/E)=\frac{\frac{1}{3}\times\frac{1}{3}}{\cancel{\frac{1}{3}}\times\frac{1}{3}+\cancel{\frac{1}{3}}\times\frac{2}{3}+\cancel{\frac{1}{3}}\times\frac{3}{4}}=\frac{\cancel{\times 3}\ 1}{1+2+9/4}=\frac{4}{4+8+9}=\frac{4}{21}$$

E → fell head wise

$$P(B_2/E)=\frac{\frac{2}{3}\times\cancel{\frac{1}{3}}}{\cancel{\frac{1}{3}}\times\frac{1}{3}+\cancel{\frac{1}{3}}\times\frac{2}{3}+\cancel{\frac{1}{3}}\times\frac{3}{4}}=\frac{2}{1+2+9/4}=\frac{8}{21}$$

$$P(B_3/E)=\frac{\frac{3}{4}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times\frac{3}{4}}=\frac{9/4^{\times 4}}{21}=\frac{9}{21}=\frac{3}{7}$$

$$P(\quad)=\frac{1}{3}\times\frac{4}{21}+\frac{2}{3}\times\frac{8}{21}+\frac{3}{4}\times\frac{3}{7}$$

$$=\frac{23}{36}$$

Q) A bag contains 6 red 4 white balls. 4 balls are drawn one by one without replacement and were found to be at least 2 white. Find probability that the next draw of a ball from this bag will give a white ball.

$\frac{17}{105}$

$\cancel{\frac{3}{56}}$

6R 4W

$\frac{2}{10}\times\frac{1}{9}+\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}$

$\cancel{\frac{1}{10}\times\frac{2}{10}+\frac{3}{10}+\frac{4}{10}}$

$P\left(\frac{B_2}{E}\right)=\frac{2}{10}\times\frac{1}{9}+\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}+\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}$

$\frac{2W}{A}$ 2W 2R $\left\{\frac{{}^4C_2\times{}^6C_2}{{}^{10}C_4}\right\}\left\{\frac{{}^2C_1}{{}^6C_1}\right\}=\frac{6\times15\times2\times24}{10\times9\times8\times7\times6}=\frac{1}{7}$

3W 1R $\left\{\frac{{}^4C_3\times{}^6C_1}{{}^{10}C_4}\right\}\left\{\frac{1}{6}\right\}$ $\frac{4\times24}{10\times9\times8\times7}=\frac{1}{\cancel{1050}}\ \frac{4}{210}$

4W 0R

Total(P) $=\frac{34}{210}=\frac{17}{105}$

Q) A purse contains four coins each coin is either a rupee or a 50 paise coin. Two coins are drawn successively without replacement and were found to be both rupee coins. If both these coins are replaced in the bag what is the probability that the next draw will give a 50 paise coin.

C_1
C_2
$C3$
C_4 $\Big\}\ \frac{1}{2}$

$\cancel{\frac{3}{17}}$

$\cancel{\frac{\frac{1}{2}\left[\frac{1}{2}\times\frac{1}{3}+\frac{1}{4}\times\frac{2}{2}\times\frac{2}{3}\right]}{\frac{1}{2}\times\frac{{}^2C_2}{4}\times\frac{1}{3}+\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}+\frac{1}{2}\times1}}$

$\frac{\frac{1}{2}\left[\frac{1}{3}+1\right]}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}+2}$

$\frac{4/3}{\frac{1+3+6}{3}}$ $\frac{4}{10}=\frac{2}{5}$

$\frac{3}{16}$

$\frac{15}{17}$ final ans.

Coincidence Testimony :-

If P_1 and P_2 are probabilities of speaking the truth of two independent witnesses A and B who give the same statement then, probability that their combine statement is true $= P\left(\frac{H_1}{H_1 \cup H_2}\right)$

Where H_1 means both speak truth and H_2 means both are lie.

$$P\left(\frac{H_1}{H_1 \cup H_2}\right) = \frac{P_1 P_2}{P_1 P_2 + (1-P_2) * (1-P_1)}$$

In this case it has been assumed that we have no knowledge about the event except the event made by A and B. however if $\underline{P}$ is the probability of the happening of the event before their statement then probability that their combined statement is true. is $= \frac{P P_1 P_2}{P P_1 P_2 + (1-P)(1-P_1)(1-P_2)}$

here it is given in the court that the murder has commited already.

here it has been assumed that the statement given by all independent witnesses can be given in two ways ~~only~~ so that if all the witnesses tell false. they agree in telling the same false hood.

same jhut bolne ka probability

* If this is not the case and 'C' is the change of their coincidence testimony then probability that their combined statement is true.

$$= \frac{P_0 P_1 P_2}{P P_1 P_2 + (1-P)(1-P_1)(1-P_2) C}$$

Q1) A speaks truth 3 out of four times and B speaks truth 5 out of 6 times. Find the probability that they contradict each other stating the same fact.

$$\cancel{\frac{\frac{3}{4}\times\frac{5}{6}}{\frac{3}{4}\times\frac{5}{6}+\frac{1}{6}\times\frac{1}{4}}} = \cancel{\frac{15}{3+15}}$$

$$\frac{3}{4}\times\frac{1}{6}+\frac{1}{4}\times\frac{5}{6}=\frac{1}{3}$$

Q) A speaks truth 3 out of 4 times B 7 out of 10 times. They both assert that a white ball has been drawn from the bag containing 6 balls of different colours. Find the probability of truth of their assertion.

same. $\frac{3}{4}\times\frac{7}{10}$

$P(\text{lies}) = \frac{1}{5}\times\frac{1}{5}$ They will tell lie if ~~they~~ both of them tell any one of the remaining 5 balls

$$\frac{\frac{1}{6}\times\frac{3}{4}\times\frac{7}{10}}{\frac{1}{6}\times\frac{3}{4}\times\frac{7}{10}+\frac{5}{6}\times\frac{1}{4}\times\frac{3}{10}\times\left(\frac{1}{25}\right)}$$ → for same lie.

Q) A 10 witness each of whom has probability of speaking the truth as 5/6 asserts that certain event took place. If the probability of this event before their statement is $\frac{1}{1+(5)^9}$ Find the

Prob of the truth of their assertion. 5/6

Continuous Sample Space

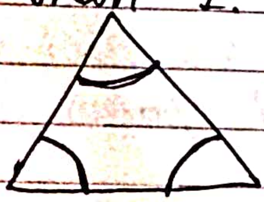
Axioms 1: If a point is randomly selected from line segment AB. Then the chance that it falls on the line segment PQ included in AB is given ~~by length of PQ~~

$$\frac{\text{length of PQ}}{\text{length of AB}}$$

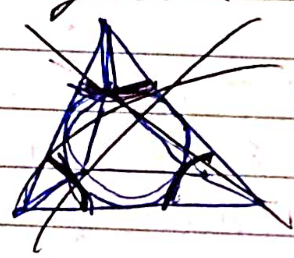
$$P = \frac{l(PQ)}{l(AB)}$$

* If a point is randomly selected from an area (S) then the chance that it falls on area σ is $\frac{\sigma}{S}$.

Q) A point is selected at random inside an equilateral traingle whose side is 3. Find the Probability that its distance from any corner is greater than 1.

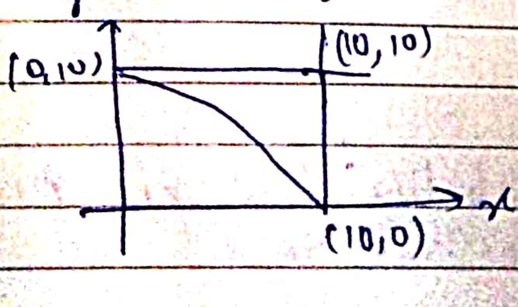


$$\frac{\frac{\sqrt{3}}{4}9 - 3\frac{1}{2}1^2\pi/3}{\frac{\sqrt{3}}{4}\times 9} = \frac{4\pi}{9\sqrt{3}} \quad 1 - \frac{2\pi}{9\sqrt{3}}$$



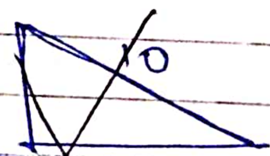
Q) The sides of a rectangle are selected at random each less than 10 cm all such lengths being equally likely. Find the probability that the diagonal of rectangle is less than 10 cm.

=>



$$\frac{\frac{1}{4}\times\pi\times 100}{100}$$

$$\frac{\pi}{4}$$



~~$\sqrt{a^2+b^2} < 10$~~

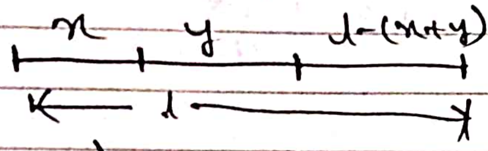
~~$a^2+b^2 < 100$~~

Q) A line is divided into 3 parts what is the chance that they form the sides of possible traingle.

~~$\frac{{}^3C_2 + {}^2C_1}{8}$~~

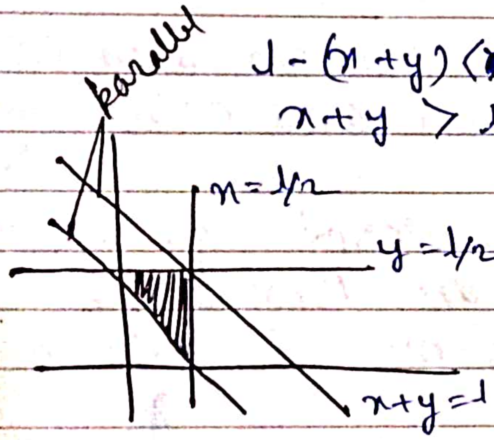
~~$1({}^3C_1) + 1({}^2C_1) : 1 - 1({}^3C_1) + 1({}^2C_1)$~~

~~$1({}^3C_1) + 1({}^2C_1) > 1/2$~~



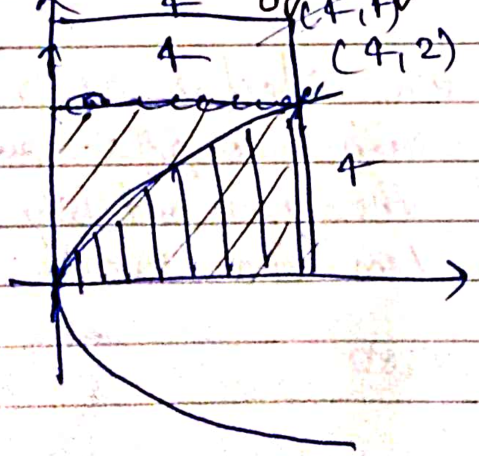
$1-(x+y) < x+y$ $\qquad$ $1-y > y$

$x+y > 1/2$ $\qquad$ $y < 1/2$ $\qquad$ $x < 1/2$



$$\frac{\frac{1}{2} \times \frac{1^2}{4}}{\frac{1}{2} \times 1^2} = 1/4$$

Q) Two numbers $x \in [0, 4]$ and $y \in [0, 4]$ are selected Find the Probability that the selected pairs number satisfy $y^2 \leq x$.



$$\frac{2/3 \times 4 \times 2}{4 \times 4} = \frac{1}{3}$$

Q) Let $a \in [-20, 0]$ Find the Probability that the graph of the function $y = 16x^2 + 8(a+5)x - 4a - 5$ is strictly above x-axis.

$64(a+5)^2 + 4(7a-5)16 < 0$

~~$64a + 320 + (28a - 20)16 < 0$~~

~~$64a + 320 +$~~

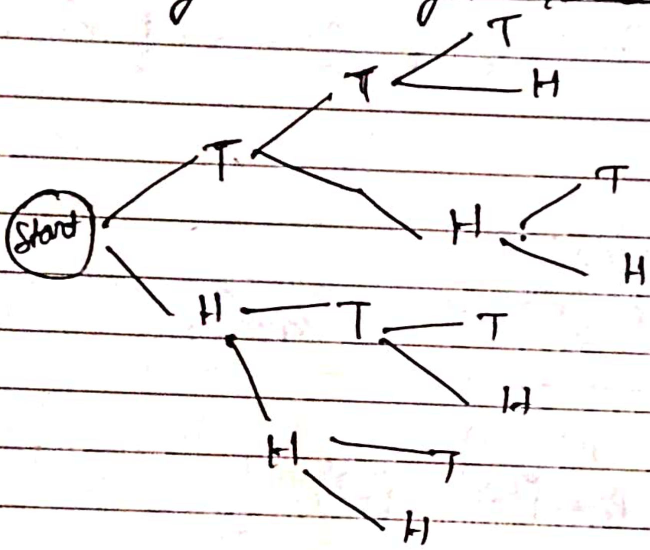
$a^2 + 17a + 30 < 0$

$a \in (-15, -2)$

$-20 \quad -15 \quad -2 \quad 0$

$P = \frac{13}{20}$

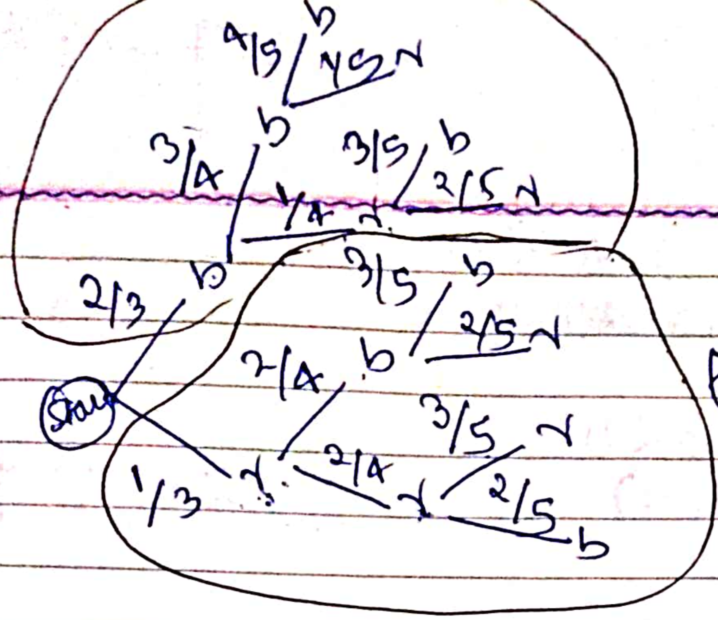
Probability through tree diagram



Q) A bag initially contains one red balls and two blue balls. A trail consist of selecting the a ball at random noting its colour and replacing it togather with an additional ball of the same colour. Given that three trails are made. Draw the tree diagram of the process and also find the probability of

i) At least blue ball is drawn.
ii) exactly one blue ball is drawn
iii) All the three ball drawn are of same colour.

Find the probability that they are all red.



$\cancel{(i)\ \frac{1}{3}\times\frac{2}{4}\times\frac{2}{5} = \frac{1}{15}}$

+

i) P(at least 1 blue) $= 1 - P(RRR)$

$= 1 - \frac{1}{3}\times\frac{1}{2}\times\frac{3}{5} = \frac{9}{10}$

ii) $P(BRR + RBR + RRB) = \frac{2}{3}\times\frac{1}{4}\times\frac{2}{5} + \frac{1}{3}\times\frac{1}{2}\times\frac{2}{5} + \frac{1}{3}\times\frac{1}{2}\times\frac{2}{5}$

$= 1/5$

iii) $P\left(\frac{RRR}{RRR \cup BBB}\right) = \frac{1/3 \cdot 1/2 \cdot 3/5}{1/3\times1/2\times3/5 + 2/3\times3/4\times4/5} = \frac{1}{5}$

NOT in tree

$\mu = np \qquad \sigma^2 = npq$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

x_i	0	1	2	...	n
$P(x)$	${}^nC_0 q^n$	${}^nC_1 q^{n-1} p$			

$$\mu = \sum_{i=0}^{n} x_i\, P(x=i) = \sum_{r=0}^{n} r\cdot {}^nC_r\, q^{n-r} p^r$$

$$(q+p)^n = \sum_{r=0}^{n} {}^nC_r\, q^{n-r} p^r$$

$$pn\,(q+p)^{n-1} = \sum_{r=0}^{n} r\cdot {}^nC_r\, q^{n-r} p^{r-1} p$$

$$np = \sum_{r=0}^{n} r\cdot {}^nC_r\, q^{n-r} p^r = \mu$$

$$\sigma^2 = \sum_{i=1}^{n} (x_i-\mu)^2\, P(x_i)$$