Blatt5 - Einführung in die Logik

Viradia, Yash - Informatik - 5275038 - Gruppe 01 y.viradia@tu-braunschweig.de

25.06.2023

```
HA6
```

(Vx Jy. P(x, H(y))) 1 (7 Jy Vx Jz. (Q(g(z), f(x)) V P(y, z))) Bereinige die Formel: Bennene die Variablen x und y im a und b um, also im ersten Teil, (Va Fb. P(a, f(b))) 1 (774 4x Fz. (Q(g(z), f(x)) V P(y,z))) Durch die Anwendung von De Morgan'sche Regel (Ya=b. P(a, f(b)) 1 (Yy=x VZ. (-Q(g(z), f(x)) 1-P(y, E)))

Mit der Rechenregel (2) eine PNF gewinnen:

Va∃byy Jx Vz. (P(a, f(b)) ∧ ¬d(g(z), f(x)) ∧ ¬P(y, z))

Funktion 9/1 für b und h/2 für a Fahre die

Hat.

∀a ∀y ∀z. (P(a, f(a))) 1 ¬ l(g(z), f(h(a,y))) 1 7 P(y,z))

HA5

- a) x and y sind Primzahlewillige. $T(x,y) := \exists z : z = x \cdot y$
- c) Jede gerade Zahl 24 lässt sich als Summe zweier Primzahl darstellen.

Zuerst bestimme ob die Zahl eine Primzahl ist.

d) Alle Zahlen mit geradem Quadrat sind gerade.

$$\forall x: (\forall y: x \cdot x = y \cdot y) \longrightarrow \forall z: x = z + z$$

1) ê (- YxA) = 1 - inf { 6 {x/d} (A) : de D}

be Morganische

= sup{1 - 6{x/d}(A): d ∈ D}

= Sup { = {x/d} (-A) : d ∈ D}

= 6 (3x.7A)

€(73×A) = 1 - sup{ 6{x/d}(A): d∈0}

De Morganische

= inf $\{1 - 5\{x | d\}(A) : d \in D\}$

= inf $\{ 6\{x/d\}(\neg A): d \in D \}$

EN TANELY.

= 6 (\x 7 A)

Assoziativgesetz int { \(\frac{\chi/d?}{\chi/d?} \(\beta \) : d \(\eartilder \) }

= inf { inf { 5 { x/d} } (A) , 5 { x/d} 3 (B) } : d & D}

= Inf { 6 {x/d} (A AB): deD}

2 & (Vx. (AAB))

```
\frac{6}{(\exists x \land v \exists x \land B)} = \sup \{ \sup \{ \underbrace{5 \{x / d\}^2(A)} : d \in D \} , \\
\sup \{ \underbrace{5 \{x / d\}^2(B)} : d \in D \} \} , \\
\sup \{ \sup \{ \underbrace{5 \{x / d\}^2(A)}, \underbrace{5 \{x / d\}^2(B)} \} : d \in D \} \} \\
= \sup \{ \underbrace{5 \{x / d\}^2(A \lor B)} : d \in D \} \} \\
= \frac{6}{(\exists x \lor \forall y \land A)} = \inf \{ \underbrace{5 \{x / d\}^2(\forall y \land A)} : d \in D \} \} \\
= \inf \{ \underbrace{5 \{x / d\}^2(A \lor B)} : d \in D \} \}
```

 $\hat{b}(J_{1}J_{2}A) = \sup \{ \frac{1}{2} \{ \frac{1}{2} \{ A \} : d \in D \} \}$ $= \sup \{ \frac{1}{2} \{ \frac{1}{2} \{ A \} : d \in D \} \} : d \in D \}$ $= \sup \{ \frac{1}{2} \{ \frac{1}{2} \{ A \} : d \in D \} \} : d \in D \}$ $= \sup \{ \frac{1}{2} \{ \frac{1}{2} \{ \frac{1}{2} \{ A \} : d \in D \} \} : d \in D \}$ $= \sup \{ \frac{1}{2} \{ \frac{1}{2} \{ \frac{1}{2} \{ \frac{1}{2} \{ A \} : d \in D \} \} : d \in D \}$

JOSE CHAN (MAR) + deop

Mit CamScanner gescan

```
= sup { 5 {y/2} (3xA) ; ZED} = 3y 3xA.
```

1 Beweise jetzt (4)

 $\frac{\partial}{\partial A} (A \wedge \forall x.B) = \inf \left\{ \frac{\partial}{\partial A} , \inf \left\{ \frac{\partial}{\partial A} (A) : d \in D \right\} \right\}$ Distributivg coeft

= $\inf \left\{ \inf \left\{ \frac{\partial}{\partial A} (A) , \left\{ \frac{\partial}{\partial A} (A) : d \in D \right\} \right\} \right\}$ = $\inf \left\{ \inf \left\{ \frac{\partial}{\partial A} (A) , \left\{ \frac{\partial}{\partial A} (A) : d \in D \right\} \right\} \right\}$ = $\inf \left\{ \frac{\partial}{\partial A} (A \wedge B) , d \in D \right\}$ = $\frac{\partial}{\partial A} (A \wedge B) = \frac{\partial}{\partial A} (A \wedge B) = \frac{\partial}{\partial A} (A \wedge B)$

 $f(A \vee J_{X}.B) = \sup \{f(A), \sup \{f(X), B\} : d \in D\}\}$ $= \sup \{\sup \{f(A), f(X), f(X)\} : d \in D\}\}$ $= \sup \{\sup \{f(A), f(X)\} : d \in D\}\}$ $= \sup \{f(A), f(X)\} : d \in D\}$ $= \sup \{f(A), f(X)\} : d \in D\}$ $= f(f(A), f(X)\} : d \in D\}$ $= f(f(A), f(X)\} : d \in D\}$

 $\hat{G}(A \land \exists x.B) = \inf \{ \hat{G}(A), \sup \{ \hat{G}(X) \} (B) : d \in D \} \}$ $= \sup \{ \inf \{ \hat{G}(A), \widehat{G}(X) \} : d \in D \}$ $= \sup \{ \hat{G}(A), \widehat{G}(X) \} : d \in D \}$ $= \sup \{ \hat{G}(A), \widehat{G}(A) \} : d \in D \}$ $= \hat{G}(A) = \hat{G}($

 $f(A \lor \forall x.B) = \sup \{f(A), \inf \{f(x)\}\} (B) : d \in D\}\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\}\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \forall x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \exists x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \exists x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \exists x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \exists x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor \exists x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$ $f(A) \lor x.B = \sup \{f(A), \inf \{f(A)\} (B)\} : d \in D\}$

HA4

Bew:

Sei u eine von a unterschiedlichen Variablen.

$$\dot{\xi}(u\{x/t\}) = \dot{\xi}(u)$$

$$\dot{\xi}(x/t)\}(u)$$

$$\dot{\xi}(x\{x/t\}) = \ddot{\xi}(t)$$

$$\dot{\xi}(x\{x/t\})(x)$$

$$\frac{1}{6} \left(f(u) \{ x/t \} \right) = f\left(\frac{1}{6} (u) \{ x/t \} \right)$$

$$= f\left(\frac{1}{6} \{ x/6(t) \} (u) \right)$$

$$= \frac{1}{6} \{ x/6(t) \} f(u).$$

$$\frac{Z}{5} = \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \right) = \frac{1}{5} \left(\frac{1}{2} \left(\frac{1}{4} \right) \right) \\
= \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right) \\
= \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{2} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \right) \\
\Rightarrow \frac{1}{5} \left(\frac{1}{4} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right$$

Sei A eine Prädikatsauswertung P(u)

Dann.

$$\overline{b}(P(u)\{x/t\}) = 1 \quad \Leftrightarrow \quad \langle \overleftarrow{b}(u\{x/t\}) \rangle \in P^{\overleftarrow{b}}$$

$$\Leftrightarrow \quad \langle \overleftarrow{b}(\{x/\overleftarrow{b}(t)\}(u)) \rangle \in P^{\overleftarrow{b}}$$

$$\Leftrightarrow \quad \overline{b}(\{x/\overleftarrow{b}(t)\}(u)) = 1.$$

Sei A cine Konjunktion also BAC $\hat{\psi}((BAC)\{x/t\}) = \inf\{\hat{\varepsilon}(B\{x/t\}), \hat{\varepsilon}(C\{x/t\})\}$ $= \inf\{\hat{\varepsilon}(\{x/\xi(t)\}(B)), \hat{\varepsilon}(\{x/\xi(t)\}(C))\}$ $= \hat{\phi}(\{x/\xi(t)\}(BAC)\}$

See A eine Disjunktion also BVC $\hat{\phi}((BVC)\{x/t\})$ z Sup $(\hat{\sigma}(B\{x/t\}), \hat{\sigma}(C\{x/t\}))$ z Sup $(\hat{\sigma}(\{x/t\})\}(B)), \hat{\sigma}(\{x/t\})\}(C))$ z Sup $(\hat{\sigma}(\{x/t\})\}(B)), \hat{\sigma}(\{x/t\})\}(C))$ z $\hat{\phi}(\{x/t\})\}(BVC)$.

Sei A eine V-Quanter Formel also tyB. $\hat{E}(\forall y.B\{x/t\}) = \hat{E}(\forall z.B\{y|z\}\{x/t\})$ $= \inf \{ \widehat{E\{z|d\}} (B\{y|z\}\{x|t\}) : d \in D\}$ $= \inf \{ \widehat{E\{z/d\}} \{x/5\{z/d\}\} (x/5\{z/d\}(t)\} B\{y|z\} : d \in D\}$ $= \widehat{E}\{x/5(t)\} (\forall z.B\{y|z\})$ $= \widehat{E}\{x/5(t) (\forall y.B)$

Wenn A eine I-quantifizierte Formel ist, dann lässt sie sich analog wie oben beweisen. Nehme sup anstatt inf.