**Nature, Definition & Characteristics of Operations research**

The term Operation Research (OR) related to military operations during the Second World War. Scientists used various techniques to deal with strategic and tactical problems during the war. After the war military OR group scientists tried to apply OR techniques to civilian problems relating to business, industry and research development. During the 1950s educational institutions introduced OR in their curricula. Today service organizations such as airlines, railways, hospitals, libraries and banks employ OR to improve their efficiency.

1. **According to H.M. Wagner**

O.R. is a scientific approach to problem solving for executive management.

1. **According to Operations Research Society of America**

O.R. is an experimental and applied science devoted to observing, understanding and predicting the behaviour of purposeful man-machine systems and operations research workers are actively engaged in applying this knowledge to practical problems in business, government and society.

1. **According to E.L. Arnoff and M.J. Netzorg**

O.R. is the systematic, method-oriented study of the basic structure, characteristics, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making.

1. **According to C.W. Churchman**

Operation Research is the application of scientific methods, techniques and tools to problems involving the operation of a system so as to provide those in control of the system with optimum solution to the problem.

1. **According to C. Kittel**

O.R. is an aid for the executive in making his decision by providing him with the needed quantitative information based on the scientific method of analysis.

**It can be used for solving different types of problems, such as:**

1. Problems dealing with the waiting line, the arrival of units or persons requiring ser­vice.
2. Problems dealing with the allocation of material or activities among limited facilities.
3. Equipment replacement problems.
4. Problems dealing with production processing i.e., production control and material ship­ment.

But it may be remembered that operation research never replaces a manager as decision maker. The ultimate and full responsibility for analysing all factors and making decision will be of the manager.

In the more wide sense, operation research does not deal with the everyday problems such as output by the one worker or machine capacity; instead it is concerned with the overall aspect of business operation such as something as the relationship between inventory, sales, production and scheduling. It may also deal with the overall flow of goods and services from plants to consumers.

The team doing operation research may have statisticians, psychologists, labour specialists, mathematicians and others depending upon the requirement for the problems.

**Nature of Operation Research:**

In its recent years of organised development, O.R. has solved successfully many cases of research for military, the government and industry. The basic problem in most of the develop­ing countries in Asia and Africa is to remove poverty and hunger as quickly as possible. So there is a great scope for economist, statisticians, administrators, politicians and technicians working in a team to solve this problem by an O.R. approach.

On the other hand, with the explosion of population and consequent shortage of food, every country is facing the problem of optimum allocation of land for various crops in accordance with climatic conditions and available facilities. The problem of optimal distribution of water from a resource like a canal for irrigation purposes is faced by developing country. Hence a good amount of scientific work can be done in this direction.

In the field of Industrial Engineering, there is a claim of problems, starting from the pro­curement of material to the despatch of finished products. Management is always interested in optimizing profits.

Hence in order to provide decision on scientific basis, O.R. study team con­siders various alternative methods and their effects on existing system. The O.R. approach is equally useful for the economists, administrators, planners, irrigation or agricultural experts and statisticians etc.

Operation research approach helps in operation management. Operation management can be defined as the management of systems for providing goods or services, and is concerned with the design and operation of systems for the manufacture, transport, supply or service. The operating systems convert the inputs to the satisfaction of customers need.

Thus the operation management is concerned with the optimum utilisation of resources i.e. effective utilisation of resources with minimum loss, under utilisation or waste. In other words, it is concerned with the satisfactory customer service and optimum resource utilisation. Inputs for an operating system may be material, machine and human resource.

O.R. study is complete only when we also consider human factors to the alternatives made available. Operation Research is done by a team of scientists or experts from different related disciplines.

For example, for solving a problem related to the inventory management, O.R. team must include an engineer who knows about stores and material management, a cost ac­countant a mathematician-cum-statistician. For large and complicated problems, the team must include a mathematician, a statistician, one or two engineers, an economist, computer program­mer, psychologist etc.

1. **Finance, Budgeting and Investment:**

* Cash flow analysis, long range capital requirement, investment portfolios, divi­dend policies,
* Claim procedure, and
* Credit policies.

**2. Marketing:**

* Product selection, competitive actions
* Number of salesmen, frequencies of calling on
* Advertising strategies with respect to cost and time

**3. Purchasing:**

* Buying policies, varying prices
* Determination of quantities and timing of purchases
* Bidding policies
* Replacement policies
* Exploitation of new material resources

**4. Production Management:**

* Physical distribution: Location and size of warehouses, distribution centres and retail outlets, distribution policies.
* Facilities Planning: Number and location of factories, warehouses etc. Loading and unloading facilities.
* Manufacturing: Production scheduling and sequencing stabilisation of produc­tion, employment, layoffs, and optimum product mix.
* Maintenance policies, crew size.
* Project scheduling and allocation of resources.

**5. Personnel Management:**

* Mixes of age and skills
* Recruiting policies
* Job assignments

**6. Research and Development:**

* Areas of concentration for R&D.
* Reliability and alternate decisions.
* Determination of time-cost trade off and control of development projects.

**Characteristics of Operations research**

1. **Decision making**

OR is a decision science which helps management to make better decisions.

1. **Use of Information Technology (IT)**

O.R. often requires a computer to solve the complex mathematical model or to perform a large number of computations that ae involved. Use of digital computer has become an integral part of the operations research approach to decision making.

1. **Quantitative solution**

Operations research provides the managers with a quantitative basis for decision making. OR attempts to provide a systematic and rational approach for quantitative solution to the various managerial problems.

1. **Human factors**

In deriving quantitative solution we do not consider human factors, which doubtlessly plays a great role in the problems. So study of the OR is incomplete without a study of human factors.

1. **System orientation**

O.R. study the situation or problem as a whole. This means that an activity by any part of an organization has some effect on the activity of every other part. The optimum result of one part of a system may not be the optimum for some other part. Therefore, to evaluate a decision, one must identify all possible interactions and determine their impact on the organization as a whole.

1. **Scientific approach**

O.R. uses scientific methods to solve the problems. Most of the scientific studies such as chemistry, physics, biology etc. can be carried out in the laboratories, without much interference form the outside world. Bust same is not true in the systems under study by OR teams. So, OR is an formalized process of reasoning. Under OR the problem is to be analysed ad defined clearly. Observations are made under different conditions to  study the behavior of the system. On the basis of these observations a hypothesis describing how the various factors involved are believed to interact and the best solution to the problem is formulated. To test the hypothesis experiment is designed and executed. Observations are made and measurement s are recorded. Finally results of the experiments are studied and the hypothesis is accepted or rejected. So, OR is the use of scientific method to solve the problem under study.

1. **Inter-disciplinary team approach**

O.R. is performed by a team of scientists whose individual members have been drawn from different scientific and engineering disciplines. For example, one may find a mathematician, statistician, physicist, psychologist, economist and an engineer working together on an OR problem.

1. **Uncovering new problems**

Solution of an OR problem may uncover a number of new problems. In order to derive the maximum benefit each one of them must be solved. OR is not effectively used if It is restricted to one shot problems only. In order to derive full benefits, continuity of research must be maintained

* **Overview of the Operations Research Modelling Approach**

Diagram

Description automatically generated

‰**Modelling approach for problem solving**

* Defining the problem and gathering data
* This procedure is crucial. It is difficult to extract a “right” answer from the “wrong” problem.
* Most practical problems encountered by OR teams are initially described in a vague and imprecise way.
* Who are the decision makers?
* What are the objectives?
* What are the constraints (relationships)?
* How to collect relevant data?

‰**Formulating a mathematical model**

* Construct a mathematical model that represents the essence of the problem.
* Define decision variables.
* Define the objective function.
* Define the constraints (relations among decision variables).
* Usually, there is more than one way to formulate a problem.
  + Some degree of simplifications and approximations are inevitable.
  + Different formulations may require different solution techniques.

‰**Deriving solutions from the model**

* Need an algorithm (systematic solution procedures).
* Solve to optimality or pseudo-optimal.
  + Exact algorithm vs. heuristics algorithms.
* Conduct postoptimality (what-if) analysis.
  + What would happen to the optimal solution if different assumptions are made?

‰**Testing (validating) the model**

* Make sure there are no serious flaws.
* Make changes if necessary.
* Repeat the above procedures until satisfied.

‰**Preparing to apply the model and implementation**

* Install a well-documented system for applying the model.
  + Include the model, solution procedure, and operating procedures for implementation. Even include personnel changes.
  + This system is usually computer-based. A considerable number of computer programs often need to be used and integrated.
  + Databases and management information systems may provide up-to-date input for the model.
  + Usually take several months.

‰**Implement**

* The step yields the real benefits.
* Need support from both top management and operating management.
* It’s better to keep management well informed and encourage management’s active guidance throughout the course of the study.
* Require good communication skills.
* Continue to obtain feedback on how well the system is working and whether the assumptions of the model continue to be satisfied.
  + Need to revise or re-build models when significant deviations occur.
* Linear Programming Problem

A large number of military programming and planning problem could be formulated as maximizing/minimizing a linear form of profit/cost function whose variables were restricted to values satisfying a system of linear constraints.

A linear form is meant a mathematical expression of the type a1x1+a2x2+……..anxn where a1,a2……….an are constants and x1,x2,………xn are variables.

The term program refers to the process of determining a particular programme or plan of action.

So Linear Programming(LP) is one of the most important optimization (maximization/minimization) techniques developed in the field of Operation Research.

* Definition

The general LPP calls for optimizing (maximizing/minimizing) a linear function of variables called the OBJECTIVE FUNCTION subject to a set of linear equations and/or inequalities called the CONSTRAINTS or RESTRICTIONS.

* Formulation of LP Problem

1. Production Allocation Problem

A firm manufactures two types of products A and B and sells them at a profit of Rs 2 on type A & Rs 3 on type B. Each product is processed on two machines G & H. Type A requires one minute of processing time on G and two minutes on H; type B requires one minute on G & one minute on H. The machine G is available for not more than 6 hour 40 minutes while machine H is available for 10 hours during any working day.

Formulate the problem as a linear programming problem.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Product A | Product B |  |
| Machine G | 1 | 1 | 400 |
| Machine H | 2 | 1 | 600 |
| Profit | Rs 2 | Rs 3 |  |

X1🡪A, x2-🡪B

Max z = 2x1+3x2

Subject to constraint

X1+x2 <=400 -🡪 Machine G

2x1 + x2 <= 600 🡪 Machine h

X1, x2 >=0

1. A company produces two types of hats. Each hat of first type requires twice as much labour time as the second type. If all hats are of second type only the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming that the profit per hat are Rs 8 for type A and Rs 5 for type B. Formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Consider x1 and x2 represents type a and type B hats respectively.

Max z= 8x1 + 5x2

Subject to constraints,

2x1+x2<=500

X1<=150

X2<=250

X1,x2>=0

1. A firm manufactures 3 products A, B and C. The profits are Rs 3, Rs 2 and Rs 4respectively. The firm has 2 machines and below is the required processing time in minutes for each machine on each product

Machine G and H have 2000 & 2500 machine minutes, respectively. The firm must manufacturers 100 A’s, 200 B’s and 50 C’s but not more than 150 A’s.

Setup LP problem to maximize profit.

PRODUCT

|  |  |  |  |
| --- | --- | --- | --- |
| MACHINE | A | B | C |
| G | 4 | 3 | 5 |
| H | 2 | 2 | 4 |
|  |  |  |  |

Consider x1, x2, x3 represents A, B, C

Max Z= 3x1+2x2+4x3

Subject to constraints

4x1+3x2+5x3<=2000

2x1+2x2+4x3<=2500

100<=x1<=150

200<=x2

50<=x3

X1, x2 x3>=0

1. Diet Problem

Dieticians tells us that a balanced diet must contain quantities of nutrients such as calories, minerals, vitamins, etc. Suppose that we are asked to find out the food that should be recommended from large number of alternative sources of these nutrients so that the total cost of food satisfying minimum requirements of balanced diet is the lowest.

The medical experts and dieticians tell us that it is necessary for an adult to consume at least 75 gm of protein, 85 gm of fats and 300 gm of carbohydrates daily. The following table gives the food item (which are readily available in the market) , analysis and their respective cost.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| FOOD TYPE | FOOD VALUE(GMS) PER 100 GMS | | | COST PER KG(Rs) |
| PROTEIN | FATS | CARBOHYDRATES |
| 1 | 8.0 | 1.5 | 35.0 | 1.00 |
| 2 | 18.0 | 15.0 | - | 3.00 |
| 3 | 16.0 | 4.0 | 7.0 | 4.00 |
| 4 | 4.0 | 20.0 | 2.5 | 2.00 |
| 5 | 5.0 | 8.0 | 40.0 | 1.50 |
| 6 | 2.5 | - | 25.0 | 3.00 |
| MINIMUM DAILY REQUIREMENT | 75 | 85 | 300 |  |

Min z= x1+3x2+4x3+2x4+1.5x5+3x6 subject to

**GRAPHICAL METHOD**

Graphical Solution of Two Variable Problems

* Graphical Procedure

Simple Linear programming problem of two decision variables can be easily solved by graphical method. The outline of graphical procedure are as follows.

**Step1:**

Consider each inequality constraint as equation.

**Step2:**

Plot each equation on the graph as each one will geometrically represent a straight line.

**Step3:**

Shade the feasible region. Every point on the line will satisfy the equation of the line.

If the inequality constraint corresponding to that line is <= then the region below the line lying in the first quadrant (due to nonnegativity of variables) is shaded.

For the inequality constraint with >= sign, the region above the line in the first quadrant is shaded.

The points lying in common region will satisfy all the constraints simultaneously.

The common region thus obtained is called feasible region.

**Step4:**

Choose the convenient value of z (say its 0) and plot the objective function line.

**Step 5:**

Pull the objective function line until the extreme points of the feasible region. In the maximization case, this line will stop furthest from the origin & passing through at least one corner of the feasible region.

**Step 6:**

Read the coordinates of the extreme points selected in Step 5 and find the maximum or minimum (as the case may be) value of z.

Example:

* 1. Find a geometrical interpretation and solution as well for the following LP problem.

Maximize z=3x1+5x2

subject to constraint

* 1. A company manufactures two products X & Y using three machines A, B & C. Machine A has 4 hrs of capacity, machine B has 24 hrs of capacity & machine C has 35 hrs of capacity. One unit of product of X requires 1 hr on A, 3 hrs on B and 10 hrs on machine C. Product Y requires 1 hr on machine A, 8 hrs on machine B and 7 hrs on machine C. When 1 unit of X sold to the market it gives a profit of Rs 5 and one unit of Y gives a profit of Rs 7. Solve the problem using graphical method.



|  |  |  |  |
| --- | --- | --- | --- |
| Product 🡪  Machine | X | Y | Available  Capacity |
| A | 1 | 1 | **4** |
| B | 3 | 8 | **24** |
| C | 10 | 7 | **35** |
| Profit | **5** | **7** |  |

* 1. A machine component requires a drill operation followed by welding and assembling. Two versions of product are produced: ordinary service and heavy duty. A single unit of ordinary design requires 10 minutes of drilling, 5 minutes of welding and 15 mins of assembling. A single unit of heavy duty component require 5 mins of drilling, 15 mins of welding and 5 mins of assembling. The profit for ordinary design is Rs 100 per unit & for heavy duty it is Rs 150 per unit. The total capacity drilling is 1500 mins, 1000 mins for welding and 2000 mins for assembling. What is the optimum mins so as to achieve maximum profit.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Ordinary Service | Heavy Duty | Available Capacity |
| Drilling | 10 | 5 | 1500 |
| Welding | 5 | 15 | 1000 |
| Assembling | 15 | 5 | 2000 |
| Profit | 100 | 150 |  |

Consider x as ordinary service y as a heavy duty

Max z= 100x+150 y

10x+5y<=1500

5x+15y<=1000

15x+5y<=2000

X,y>=0

* 1. 10x+5y<=1500

10x+5y=1500

If Y=0

X=150

A(150,0)

If x=0

Y=300

B(0,300)

* 1. 5x+15y<=1000

5x+15y=1000

Y=0

X=200

C(200,0)

X=0

Y=66.67

D(0,66.67)

* 1. 15x+5y<=2000

Y=0

X=133.33

E(133.33,0)

F(0,400)

* 1. A manufacturer of furniture makes two products chair and tables. Processing of these products is done on 2 machines A & B.

A chair requires 2 hrs on machine A & 6 hrs on machine B. A table requires 5 hrs on machine A

There is 16 hrs of time per day available on machine A 30 hrs on machine B.

Profit gained from a chair is Rs 2 & for table is Rs 10. What should be daily production of both the products.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Chair | Table | Available Time |
| A | 2 | 5 | 16 |
| B | 6 | - | 30 |
| Profit | 2 | 10 |  |
|  |  |  |  |

* 1. A dietician wants to decide a breakfast menu for certain patients. The menu is to include 2 items of A & B. Suppose each spoon of A divides 2 unit of Vitamin C and 2 units of iron & each spoon of B provide 1 unit of vitamin C and 2 units of iron. The cost of A is Rs 4 per spoon and cost of B is Rs 3 per spoon. The breakfast menu must provide at least 8 units of Vitamin C & 10 units of iron.

How many units of A & B should a person intake? How much will the breakfast cost?

|  |  |  |  |
| --- | --- | --- | --- |
|  | A🡪x | B🡪y | Requirement |
| Vitamin C | 2 | 1 | 8 |
| Iron | 2 | 2 | 10 |
| Cost | 4 | 3 |  |

* 1. A small business makes gear and non-gear bikes at two factories. Factory A produces 16 geared & 20 non geared bikes daily. While factory B produces 12 geared & 20 non geared bikes daily. It costs Rs 1000 per day to operate factory A and Rs 800 per day to operate factory B. An order for 96 geared and 140 non geared bikes has just arrived. How many days should each factory be operated in order to fulfil the order at minimum cost.

|  |  |  |  |
| --- | --- | --- | --- |
|  | A🡪x | B🡪y | Requirement |
| Geared | 16 | 12 | 96 |
| Nongeared | 20 | 20 | 140 |
| Cost | 1000 | 800 |  |

**Special cases in Graphical method**

* 1. Solve the following LPP using graphical method

Max Z=3x+2y

Subject to

x + y <= 1

2x + 2y >= 6

x, y >= 0

Definition:

**Infeasibility:**

Infeasibility is a case where there is no solution which satisfies all the constraints simultaneously.

* 1. Max Z = 3x+5y

Subject to

2x + y >= 7

x + y >= 6

x + 3y >= 9

x, y >= 0

2x + y >= 7

A(3.5,0)

B(0,7)

x + y >= 6

C(6,0)

D(0,6)

x + 3y >= 9

E(9,0)

F(0,3)

Definition:

**Unbounded:**

An LPP can fail to have an optimum solution if the objective is infinitely large without violating any of the constraints. An unbounded solution occurs in minimization problem if all the constraints are less than (<) type and it will occur in maximization problem if all the constraints are greater than (>) type.

* 1. Solve the LPP using graphical method.

Max Z=6x+2y

Subject to constraints

x + y <= 4

4x + 3y <= 12

-x + y >=1

x + y <= 6

x,y >= 0

Definition:

**Redundancy:**

A constraint which does not affect the feasible region is called as redundant constraints. Therefore, it can be admitted, and the problem can still be solved.

* 1. Max Z=18x +6y

Subject to constraints,

3x + y <= 120

X + 2y <= 160

x <= 35

x, y >= 0

Definition:

**Alternate optima:**

When there is more than one solution with the same optimal value then we say that we have alternative optima.

**SIMPLEX METHOD**

It has not been possible to obtain the graphical solution to the LP problem of more than two variables.

The simplex method provides an algorithm (a rule of procedure usually involving repetitive application of prescribed operation) which is based on fundamental theorem of linear programming.

The simplex method is iterative (step by step) procedure for solving LP problems. It consists of :-

* + 1. Having a trial basic feasible solution to constraint equation
    2. Testing whether it is optimal solution
    3. Improving first trial solution by a set of rules and repeating the process till optimal solution is obtained.

The computational procedure requires at most m (equal to the number of equations) nonzero variables in the solution at any step.

In case of less than m nonzero variables at any stage of computations the degeneracy arises in LP problem.

A feasible solution at any iteration is related to the feasible solution at the successive iteration in the following way:

One of the non-basic variables (which are zero now) at one iteration becomes basic(nonzero) at the following iteration and is called an entering variable. To compensate, one of the basic variables (which are nonzero now) at one iteration becomes non-basic (zero) at the following iteration and is called a departing variable.

The other non-basic variable remain zero and other basic variable remain non zero(though their values may change)

Consider LP problem in standard form:

Max z=c1x1+c2x2+-------------+cnxn+0xn+1+0xn+2+------------0xn+m

subject to constraints

a11x1+a12x2+-----------a1nxn+xn+1+ =b1

a21x1+a22x2+-----------a2nxn +xn+2+ =b2

…

….

am1x1+am2x2+-----------amnxn +xm+n =bm

x1,x2,……………..xn,xn+1,…………xn+m>=0

The starting basic feasible solution of m equations is usually taken as

x1=x2=,……………..=xn=0,

xn+1=b2,…………..…xn+m=bm

For this solution value of objective function is zero.

Here x1,x2,……………..xn (each equal to zero) are non-basic variables and remaining variables (xn+1,…………xn+m) are basic variables (some of them may also have value zero)

**Definitions:**

* 1. **Simplex method**

It is based on the property that the optimal solution to the LPP will be found in one of the basic feasible solution.

* 1. **Slack variable**

A variable which is used to convert less than equal to constraints into equality constraints.

It is added to the left-hand side (LHS) of the constraint.

* 1. **Surplus variable**

A variable which is used to convert greater than or equal to constraint into equality constraint. It is subtracted from LHS of the constraint.

* 1. **Artificial variable**

A variable which is added to convert equal to and greater than equal to constraint into equality constraint.

* 1. **Canonical form of LPP**

A LPP is said to be canonical form if

* + 1. The objective function is to maximize or minimize.
    2. All constraints must be less than equal to or greater than equal to
    3. All variable should be nonnegative
  1. **Standard form of LPP**

A LPP is said to be standard form if

* + 1. The objective function is to maximize or minimize.
    2. All conditions except the nonnegativity condition should be equal to type
    3. All variables should be nonnegative.
  1. **Solution Of LPP**

Any set of variables which satisfy the given constraint is called solution of LPP.

* 1. **Basic Solution**

By setting any m variables equal to zero and solving the remaining n variables, we get a basic solution.

The m variables are called as basic variables and n variables are called as non basic variables.

* 1. **Basic feasible solution**

A basic solution that is feasible (all basic variables are non negative) is called as basic feasible solution. There are 2 types of basic feasible solutions

* + 1. **Degenerate basic feasible solution**

If any of the basic variable of a basic feasible solution are zero. Then it is called as degenerate basic feasible solution.

* + 1. **Nondegenerate basic feasible solution**

In this solution all basic variables should be positive and the remaining variables should be zero.

* 1. **Optimum basic feasible solution**

A basic feasible solution is said to be optimum if it optimizes (maximize/minimize) the objective function.

* Computational Procedure of Simplex Method

1. Consider the linear programming problem

Max z= 3x1 + 2x2

Subject to constraints

x1 + x2 <= 4

x1 - x2 <= 2

x1 , x2 >=0

**Step 1:** First observe whether all the right-side constants of the constraints are non-negative. If not it can be changed into positive value on multiplying both side of the constraints by -1

In this example, all the bi’s (right side of the constraint) are already positive.

**Step 2:** Next covert the inequality constraints to equations by introducing the non negative slack or surplus variables.

The coefficient of slack or surplus variables are always taken zero in the objective function.

In this example all inequality constraints being <= , only slack variables x3 and x4 are needed.

Therefore given problem now becomes

Max z= 3x1 + 2x2 + 0x3 + 0x4

Subject to constraints

x1 + x2 + x3 + 0x4 = 4

x1 - x2 + 0x3 + x4 = 2

x1 , x2 ,x3,x4 >= 0

**Step 3**: Now present the constraint equations in matrix form:

**Step 4:** Construct the starting simplex table as follows:

It should be remembered that the values of non basic variables are always zero at each iteration.

So x1 = x2 = 0 here.

The complete starting basic feasible solution can be immediately read from following table as

x1 = 0, x2 =0,x3 =4, x4 =2

Note:

In this step the variables x3 and x4 are corresponding to the columns of basis matrix (identity matrix), so will be called basic variables.

Other variables, x1 , x2 are non basic variables which always have the value zero.

Starting simplex table:

Incoming outgoing

Step 5: Now proceed to test basic feasible solution for optimality by the rules given below. This is done by computing the net evaluation ∆j for each variable xj (column vector xj) by the formula

∆j= **Zj - Cj =CB Xj –** Cj

Optimality Test:

* 1. If all ∆j >=0 then the solution under test will be optimal. Alternative optimal solutions will exist, if any non basic ∆j is also zero.
  2. If atleast one ∆j is negative then the solution under test is not optimal then proceed to improve the solution in the next step.
  3. If corresponding to any negative ∆j all elements of the column Xj are negative or zero (<= 0) then the solution under test will be unbounded.

Applying these rules for testing the optimality of starting basic feasible solution , it is observed that ∆1 & ∆2 both are negative. Hence we have to proceed to improve this solution in step 6.

**Step 6:** In order to improve this basic feasible solution, the vector entering the basis matrix and the vector to be removed from the basis matrix are determined by the following rules. Such vectors are usually named as “incoming vector” and “outgoing vector” respectively.

**Incoming vector:**

The incoming vector Xk is always selected corresponding to the most negative value of ∆j (say ∆k)

Here, ∆k = min [∆1, ∆2]= min [ , ] =

Therefore, k=1 and hence column vector x1 must enter the basis matrix, The column x1 is marked by an upward arrow (↑)

**Outgoing vector:**

The outgoing vector Br is selected corresponding to the minimum ratio elements of XB by the corresponding positive elements of predefined incoming vector Xk. This rule is called **Minimum ratio rule.**

Here,

XB / Xk = min [XB1/X11, XB2/X21]

The X4 marked with downward arrow ( ↓) should be removed from basis matrix.

Now table 1 will be modified to table 2

Step 7: The element at intersection of minimum ratio arrow and incoming vector arrow is called key element or **pivot element.**

In order to bring pivot element equal to 1, unity must occupy in the marked \* position and 0 at all other places of xi

If the number in the marked \* position is other than unity then divide all elements of that row by the key element.

Then substract appropriate multiplies of this new row from the other (remaining) rows, so as to obtain zeros in the remaining positions of the column incoming vector(X1)

Thus process can be fortified by simple matrix transformation as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Cj🡪 |  |  |  |  |  |  |
| **Basic Variables** | **CB** | **XB** | **X1** | **X2** | **X3** | **X4** | **Min Ratio** |
| **X3** |  |  |  |  |  |  |  |
| **X4** |  |  |  |  |  |  |  |
|  |  | ∆j= **CB XB -** Cj |  |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Cj🡪 |  |  |  |  |  |  |
| **Basic Variables** | **CB** | **XB** | **X1** | **X2** | **X3** | **X4** | **Min Ratio** |
| **X3** |  |  |  |  |  |  |  |
| **X4** |  |  |  |  |  |  |  |
|  |  | ∆j= **CB XB -** Cj |  |  |  |  |  |

As all ∆j values are greater than zero. So by definition of optimality test, As all ∆j>= 0 therefor the solution under test will be optimal

At

Max Z =

1. Max z= 3x1 + 2x2

Subject to

x1 + x2 <= 4

x1 - x2 <= 2

X1, x2 >= 0

1. Min z= x1-3x2+2x3

Subject to

3x1 -x2 +3x3 <= 7

-2x1 + 4x2 <= 12

-4x1 + 3x2 + 8x3 <= 10

X1, x2, x3 >= 0

1. Max z= 3x1 + 2x2+5x3

Subject to

x1 + 2x2 +x3 <= 430

3x1 + 2x3 <= 460

x1 + 4x2 <= 420

X1, x2, x3 >= 0

1. Max z= 5x1 + 3x2

Subject to

3x1 + 5x2 <= 15

5x1 + 2x2 <= 10

1. Max z= 2x1 +3x2 + x3

Subject to

3x1 + 6x2 + x3 <= 6

4x1 + 2x2 + x3 <= 4

x1 - x2 + x3 <= 3

X1, x2, x3 >= 0

1. Min z= x1 -3x2 + 2x3

Subject to

3x1 - x2 + 3x3 <= 7

-2x1 + 4x2 <= 12

-4x1 + 3x2 + 8x3 <= 10

X1, x2, x3 >= 0

**Big M method**

Computational steps of Big – M method are as stated below:

**Step 1:**

Express the problem in standard format.

**Step 2:**

Add nonnegative artificial variables to the left side of each of the equations corresponding to constraints of type >= or =.

Whhen artificial variable are added, it causes violation of the corresponding constraints. This difficulty is removed by introducing a condition which ensures that artificiall variable will be zero in the final solution (provided solution of problem exists.)

On the other hand, if the problem does not have a solution at least one of the artificial variable will appear in final solution with positive value. This is achieved by assigning a very large price (per unit penalty) to these variables in the objective function. Such large price will be designated by -M for maximization problems and +M for minimization problems where M>0.

**Step 3:**

In the last, use the artificial variable for the starting solution and proceed with usual simplex routine until the optimal solution is obtained.

Example

Solve by using big – M method the following linear programming problems

1. Max z= - 2x1 -x2

Subject to

3x1 + x2 = 3

4x1 + 3x2 >= 6

x1 + 2x2  <= 4

X1, x2, x3 >= 0

1. Max z= 3x1 + 2x2

Subject to

2x1 + x2 <= 1

3x1 + 4x2 >= 4

X1, x2, x3 >= 0

1. Max z= 3x1 -x2

Subject to

2x1 + x2 >=2

x1 + 3x2 <= 3

x2  <= 4

X1, x2 >= 0

1. Min z= 4x1 + x2

Subject to

3x1 + 4x2 >= 20

x1 + 5x2 >= 15

x1, x2 >= 0

1. Min z= 2x1 + 3x2

Subject to

3x1 + 5x2 >= 30

5x1 + 3x2 >= 60

x1, x2 >= 0

**Two phase simplex method**

Comparing the Big M simplex method and the Two-Phase simplex method, we observe the following:

• The basic approach to both methods is the same. Both add the artificial variables to get the initial canonical system and then derive them to zero as soon as possible.

• The sequence of tableaus and the basis changes are identical.

• The number of iterations are the same.

• The Big M simplex method solves the linear problem in one pass while the Two-Phase simplex method solves it in two stages as two linear program.

Algorithm:

***Phase 1:***

Step 1:

Form a new objective function by assigning zero to every original variable including slack and surplus variables

e.g. Max z= -a1 – a2

Step 2:

Using simplex method, try to eliminate the artificial variables from the basis.

Step 3:

The solution at the end of the phase 1 is the initial basic feasible solution for phase 2.

***Phase 2:***

Step 1:

The original objective function is used, and coefficient of artificial variable is 0. (So artificial variable is removed from the calculation process)

Step 2:

Then use simplex method as usual way to find optimal solution.

Example:

* 1. Find optimal solution using two phase method

Min z = x1+x2

2x1+ x2 >= 4

x1+ 7x2 >= 7

x1,x2 >=0

* 1. Use two phase simplex method to maximize

Z=2x1 + 3x2- 5x3

Subject to

x1 + x2 + x3 = 7

2x1 - 5x2 >= 10

x1,x2,x3 >= 0

* 1. Use two phase method to maximize

Z= 3x1 – x2

Subject to

2x1 + x2 >= 2

x1 + 3x2 <= 2

x1, x2 >= 0

* 1. Use two phase method

Min z = 5x1 - 2x2 + 5x3

Subject to

2x1 + 2x2 - x3 >= 2

3x1 - 4x2 + x3 <= 3

x2 + 3x3 <= 5

x1, x2 ,x3 >= 0

***Special cases in Simplex Method***

* 1. Degeneracy in Simplex Method

In some cases there may be ambiguity in selecting the variable that should be introduced in the basis, i.e. there is a tie between the replacement ratio of two variables.

To resolve degeneracy in simplex method, we select one of them arbitrarily.

Example:

Max z= 3x1 + 9x2

subject to

x1 + 4x2 <= 8

x1 + 2x2 <= 4

x1, x2 >= 0

* 1. No feasible Solution

If in course of Simplex method computation, one or more artificial variables remain in the basis at positive level at the end of the phase 1computation, the problem has no feasible solution or it is called as Infeasible solution

Example

Max Z=200x1 - 300x2

2x1 + 3x2 >= 1200

x1 + x2 <=400

2x1 + 3/2x2 >=900

x1, x2 >=0

* 1. Unbounded Solution

If in course of simplex computation zj-cj < 0 but minimum ratio value is <= 0 then problem has unbounded solution

Max z= 5x1 + 4x2

x1 <=7

x1 - x2 <=8

x1, x2 >=0

* 1. Unrestricted/ Unconstrained Variables

Sometimes decision variables are unrestricted in sign(positive, negative or zero). In all such cases, the decision variables can be expressed as the difference between two non negative variables. For example x1is unrestricted in sign then put x1 = x1’-x1’’

e.g.

Max z=2x1 + 3x2

-x1 + 2x2 <=4

x1 + x2 <= 6

x1 + 3x2 <=9

where x1, x2 are unrestricted variables

* 1. Multiple optimal solutions

The optimal solution may not be unique, if the non-basic variables have a zero coefficient in the index row(zj-cj)

This implies that bringing the non-basic variable into the basis will neither increase nor decrease the value of objective function

Thus the linear program problem has multiple optimal solutions or alternative solutions

Max z=2000x1 + 3000x2

6x1 + 9x2 <=100

2x1 + x2 <= 20

x1,x2 >=0