

MULTI-OBJECTIVE OPTIMIZATION APPLIED TO DIET PLANNING FOR PEOPLE WITH DIABETES

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ABSTRACT. Diabetes represents a significant global public health challenge. Effective diabetes management depends on adopting nutrient-controlled diets. This paper proposes a multi-objective mathematical optimization model for crafting appropriate diets for individuals with diabetes and ensuring adherence to nutritional requirements. Integrating integer variables, the model prioritizes minimizing carbohydrate and lipid consumption, recognizing the inherent trade-offs between these objectives. Balancing these objectives is critical considering the potential adverse health effects of excessive fat intake. In addition, the model facilitates the structure of the meal-based diet and enforces dietary diversity, promoting patient adherence while meeting vital nutritional criteria for diabetes control.

Keywords: diabetes mellitus, diet planning, multi-objective optimization, integer programming, carbohydrates consumption, lipid consumption.

1 INTRODUCTION

Diabetes mellitus, commonly known as diabetes, is a chronic disease that results from defects in insulin secretion or action, leading to hyperglycemia (Egan & Dinneen, 2019). It is one of the most significant public health challenges of the 21st century, affecting all organic systems of the human body. The global increase in the prevalence of diabetes is due to demographic factors, changes in dietary patterns and lifestyles, and increased consumption of processed foods rich in sugars and saturated fats (Sun et al., 2022). Type 2 diabetes is the most common form of diabetes worldwide, caused by reduced physical activity and unhealthy eating habits (Egan & Dinneen, 2019).

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Currently, more than 460 million adults live with diabetes and this number is expected to increase to 700 million by 2045 (Sun et al., 2022). To help prevent and manage diabetes, it is crucial to adopt a restrictive diet with controlled amounts of nutrients (Filippo et al., 2021). The nutritional composition of the food is the determining factor for reasonable glycemic control and prevention of chronic hyperglycemia. Therefore, optimizing the diet is essential to effectively manage the disease.

The existing literature reveals a variety of approaches to dietary planning in diabetes (Bas, 2014; Dhoruri et al., 2017; Ahourag et al., 2022). These works often employ mathematical optimization models that provide information on creating diets that align with nutritional restrictions and consider food preferences and cost, offering a balanced and economically viable diet plan for diabetics. However, only some studies have adapted to the dietary diversity available in Brazil (Paulino, 2017; Silva et al., 2020).

In this context, this paper proposes a multi-objective mathematical optimization model to find an adequate diet for people with diabetes, respecting the required nutritional demands. The proposed model considers integer variables and deals with two objectives, that is, minimizing carbohydrates and lipid consumption. It is essential to note that these goals can conflict: minimizing carbohydrates can lead to selecting foods richer in fat to meet energy needs. However, this behavior can increase the risks associated with excessive fat intake, such as increased cholesterol and the development of cardiovascular disease. Therefore, the challenge lies in finding an appropriate balance between these objectives, considering individual nutritional needs and possible long-term health impacts.

In addition, the model allows for the structuring of diets by meal and maintains a penalty system that guarantees dietary diversity on different days. Thus, the objective is also to facilitate patients' adherence to the proposed diets while meeting the nutritional criteria required for effective diabetes control. We perform two a priori approaches to solve the multi-objective model: the weighted sum method and the lexicographical method. As a result, we generated diet plans that met nutritional criteria and favored the variability of the diets generated.

The remainder of this paper is organized as follows. Section 2 provides background information on diabetes and multi-objective optimization. Section 3 reviews the literature on the optimization of diet for people with diabetes. The proposed model is detailed in Section 4 and the results are presented in Section 5. Section 6 concludes this paper.

2 BACKGROUND

2.1 Diabetes mellitus

Diabetes mellitus (DM) is a chronic metabolic disorder characterized by hyperglycemia due to abnormalities in insulin production and/or function (Egan & Dinneen, 2019). This condition has become an important global health issue. In 2021, the International Diabetes Federation estimated that 463 million adults worldwide have diabetes, a number that could rise to 700 million

by 2045 (Sun et al., 2022). The increase in DM prevalence is related to factors such as changes in diet, patterns of physical activity, population growth, aging, and increased consumption of processed foods high in sugar and fats (Filippo et al., 2021).

The main types of DM include Type 1 diabetes (T1D), an autoimmune disease that destroys insulin-producing beta cells; Type 2 diabetes (T2D), characterized by insulin resistance and often associated with obesity and a sedentary lifestyle; and gestational diabetes, which occurs during pregnancy due to hormonal changes (Egan & Dinneen, 2019). Each type presents specific characteristics and challenges that require individualized treatment approaches.

Diabetes complications can be macrovascular, such as coronary artery disease, cerebrovascular disease, and peripheral artery disease, or microvascular, including retinopathy, nephropathy, and diabetic neuropathy (Egan & Dinneen, 2019). Effective management of DM involves lifestyle modifications, regular monitoring of blood glucose, and often medication. Patient education is also crucial to enable active participation in the management of your health and minimize the risks of serious complications. In this sense, the American Diabetes Association (ADA) recommends an individualized approach to diet, with an emphasis on foods rich in nutrients and low in fats, calories, and carbohydrates (Filippo et al., 2021).

2.2 Multi-objective Optimization

Multi-objective Optimization (MOO), or Pareto optimization, involves optimizing two or more conflicting objectives simultaneously (Deb, 2001). A typical MOO problem with k objectives can be represented as follows:

$$\begin{aligned} &\text{minimize} \quad \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})) \\ &\text{subject to} \quad \mathbf{x} \in \mathcal{X} \end{aligned}$$

where \mathbf{x} is the decision vector in the feasible set $\mathcal{X} \subseteq \mathbb{R}^n$, and $f_i(\mathbf{x})$ for $i = 1, \dots, k$ are the objective functions to be minimized.

Unlike single-objective optimization, where the goal is to find a single optimal solution, MOO aims to determine a set of solutions that provide a trade-off among the objectives, known as Pareto optimal solutions. A solution $\mathbf{x}^* \in \mathcal{X}$ is Pareto optimal if there is no other solution $\mathbf{x} \in \mathcal{X}$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all i and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one objective j . The set of all Pareto optimal solutions forms the Pareto front or Pareto boundary. A solution \mathbf{x}_1 dominates another solution \mathbf{x}_2 if $\mathbf{F}(\mathbf{x}_1) \leq \mathbf{F}(\mathbf{x}_2)$ and $\mathbf{F}(\mathbf{x}_1) \neq \mathbf{F}(\mathbf{x}_2)$, which means that \mathbf{x}_1 is not worse in all objectives and better in at least one objective (Deb, 2001; Marler & Arora, 2004).

Methods for solving MOO problems are classified into two main categories: a priori methods, where preferences are defined before optimization, and a posteriori methods, where the selection of a solution occurs after generating a set of Pareto optimal solutions (Marler & Arora, 2004). Scalarization methods, such as the weighted sum method and the lexicographical method, fall into the first category. They require the decision-maker to specify their preferences beforehand, thus guiding the search towards a preferred region of the Pareto front.

The weighted sum method assigns weights to each objective and combines them into a single objective function:

$$\text{minimize} \quad \sum_{i=1}^k \alpha_i f_i(\mathbf{x})$$

where α_i are the weights for each objective.

The lexicographical method prioritizes the objectives according to their importance. The primary objective is optimized first, and subsequent objectives are optimized under the constraint that higher-priority objectives remain at their optimal values:

$$\begin{aligned} &\text{minimize} \quad f_1(\mathbf{x}) \\ &\text{subject to} \quad \mathbf{x} \in \mathcal{X} \end{aligned}$$

Once the optimal \mathbf{x}_1^* is found, the next problem is formulated as:

$$\begin{aligned} &\text{minimize} \quad f_2(\mathbf{x}) \\ &\text{subject to} \quad f_1(\mathbf{x}) = f_1(\mathbf{x}_1^*) \\ &\quad \mathbf{x} \in \mathcal{X} \end{aligned}$$

In contrast, a posteriori methods generate a diverse set of Pareto optimal solutions, from which the decision-maker can choose after the optimization process. Evolutionary algorithms, such as NSGA-II (Non-dominated Sorting Genetic Algorithm II) and MOEA/D (Multi-objective Evolutionary Algorithm based on Decomposition), are typical examples of a posteriori methods. These algorithms approximate the entire Pareto front, providing a comprehensive view of the trade-offs between the objectives (Deb, 2001; Marler & Arora, 2004).

3 RELATED WORK

The problem of optimal diet selection can be considered one of the oldest within Operations Research. In fact, several works have been developed over the years, focusing on issues such as minimizing the financial cost of the diet, minimizing calories or fat, and maximizing a certain nutrient, among other objectives (Xavier et al., 2023).

In the context of optimizing diets for people with diabetes, many works focus on obtaining more accurate models, while others combine optimization techniques with intelligent systems, in order to address specific issues. For example, Bas (2014) introduced a mixed integer programming model (MIP) that used glycemic load values as parameters. The model aims to create a daily diet plan that meets nutritional needs while minimizing the uncertainty of the glycemic load.

Eghbali-Zarch et al. (2017) contributed to a multi-objective mixed integer linear programming model for diabetic patients that factored in uncertainties and incorporated taste, availability, and cost to provide diverse, nutritious, and economically efficient diets. Dhoruri et al. (2017) used goal programming to optimize menus for diabetic patients, considering calories, proteins, fats, and carbohydrates. The study made assumptions about patient conditions and focused on individuals with average weight and not at gestation.

In the Brazilian context, Paulino (2017) proposed a linear programming model that considered 60 raw and cooked foods and derivatives. The author conducted simulations for three individuals with different nutritional needs to minimize carbohydrate consumption. Furthermore, Silva et al. (2020) proposed an integer model and considered more than 500 foods. As a result, they found optimal but impractical diets.

Sapri et al. (2019) emphasized the completeness of nutrients for people with diabetes, using integer programming to optimize nutrient intake. Their study considered carbohydrates, proteins, fats, vitamins, sugars, saturated fats, cholesterol, and minerals. Paidipati et al. (2021) customized menus for Indian diabetic patients, which address cultural and regional preferences. Finally, Ahourag et al. (2022) proposed a multi-objective linear programming model for Moroccan patients to balance cost and glycemic load.

In summary, we observe a need for a multi-objective strategy to optimize diets for diabetic patients. Some studies focus on reducing carbohydrate consumption, while others target the glycemic load or financial cost. This study proposes an integer programming model centered on commonly found Brazilian dietary items, with the primary objective of minimizing carbohydrate intake due to its impact on blood sugar levels. The following sections elaborate on the proposed approach.

4 PROPOSED MODEL

4.1 Objective functions and constraints

This paper proposes an integer programming model that considers the different parameters involved in the diet problem. Therefore, it is possible to suggest a diet that meets the specific needs of each individual, contributing to better control of the disease. This paper focused on simultaneously minimizing two macronutrients: carbohydrates and lipids.

When eaten, carbohydrates are broken down into glucose and other simple sugars during digestion, increasing blood glucose levels. For people with diabetes who have difficulty regulating these glucose levels, excessive carbohydrate intake can cause unwanted spikes in glycemic levels (Filippo et al., 2021). However, lipids (fats), when ingested in excess, increase the risk of cardiovascular disease. These diseases are already common in people with diabetes; therefore, reducing fat intake can contribute to improving the quality of life of these individuals.

In addition, nutritional needs or limits must be represented in a specific diet. For example, we cannot eliminate carbohydrates from a dietary plan. In addition, other nutrients and minerals are essential for the human body. In this article, in addition to carbohydrates and lipids, we consider two other macronutrients: proteins and dietary fiber. In addition, we consider six minerals: sodium, calcium, phosphorus, magnesium, iron, and zinc. Finally, we incorporate the calories in the food, also called energy. Table 1 shows the notation for the proposed model.

Table 1 – Notation used in the proposed model.

\mathcal{J}	set of n foods considered to generate the diet
$j \in \mathcal{J}$	a food of the set \mathcal{J}
\mathcal{I}	set of individuals
$i \in \mathcal{I}$	an individual of set \mathcal{I}
C_j	amount of carbohydrates, in grams, in a portion of food j
L_j	amount of lipids, in grams, in a portion of food j
P_j	amount of proteins, in grams, in a portion of food j
F_j	amount of fiber, in grams, in a portion of food j
Na_j	amount of sodium, in milligrams, in a portion of food j
Ca_j	amount of calcium, in milligrams, in a portion of food j
Ph_j	amount of phosphorus, in milligrams, in a portion of food j
Mg_j	amount of magnesium, in milligrams, in a portion of food j
Fe_j	amount of iron, in milligrams, in a portion of food j
Zn_j	amount of zinc, in milligrams, in a portion of food j
E_j	amount of calories (energy), in kilocalories, in a portion of food j
C_i^{\min}	minimum amount of carbohydrates to be ingested by an individual $i \in \mathcal{I}$
L_i^{\min}	minimum amount of lipids to be ingested by an individual $i \in \mathcal{I}$
L_i^{\max}	maximum amount of lipids to be ingested by an individual $i \in \mathcal{I}$
E_i^{\min}	minimum amount of calories to be ingested by an individual $i \in \mathcal{I}$
E_i^{\max}	maximum amount of calories to be ingested by an individual $i \in \mathcal{I}$

Taking into account the notation established in Section 2.2, \mathbf{x} is the vector of decision variables that represents the number of portions of each selected food item, that is,

$$\mathbf{x} = (x_1, \dots, x_n)$$

Here, each x_j (for $j \in \mathcal{J}$) is a positive integer representing the number of portions of food j selected. Thus, \mathbf{x} is a vector in \mathbb{Z}_+^n , where each component corresponds to a different food item in the set \mathcal{J} .

Many of these nutrients (as well as calories) have a minimum amount to be consumed because they are essential for the body. In addition, calories, lipids, proteins and sodium have an upper consumption limit, as excess can cause other health problems, such as obesity and kidney disease. Finally, most of these restrictions have a fixed value; for this work, we adopted the recommendations of the Brazilian Diabetes Society (Sociedade Brasileira de Diabetes), listed in Table 2.

We define two objective functions, f_1 and f_2 , which represent the total amount of carbohydrates (Equation (1)) and lipids (Equation (2)) that an individual must consume, respectively.

$$f_1(\mathbf{x}) = \sum_{j \in \mathcal{J}} x_j C_j \quad (1)$$

$$f_2(\mathbf{x}) = \sum_{j \in \mathcal{J}} x_j L_j \quad (2)$$

Table 2 – Dietary recommendations for individuals with diabetes mellitus.

Nutrient	Recommended daily intake
Protein (g)	Between 28.4 and 38.4
Dietary Fiber (g)	Minimum of 30.0
Sodium (mg)	Maximum of 2000.0
Calcium (mg)	Minimum of 850.0
Phosphorus (mg)	Minimum of 700.0
Magnesium (mg)	Minimum of 400.0
Iron (mg)	Minimum of 18.0
Zinc (mg)	Minimum of 15.0

Finally, the bi-objective optimization model is represented as follows:

$$\text{minimize } \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x})) \quad (3a)$$

$$\text{subject to } \sum_{j \in \mathcal{J}} x_j C_j \geq C_i^{\min}, \quad i \in \mathcal{I} \quad (\text{Minimum carbohydrates}) \quad (3b)$$

$$\sum_{j \in \mathcal{J}} x_j L_j \leq L_i^{\max}, \quad i \in \mathcal{I} \quad (\text{Maximum lipids}) \quad (3c)$$

$$\sum_{j \in \mathcal{J}} x_j E_j \geq E_i^{\min}, \quad i \in \mathcal{I} \quad (\text{Minimum calories}) \quad (3d)$$

$$\sum_{j \in \mathcal{J}} x_j E_j \leq E_i^{\max}, \quad i \in \mathcal{I} \quad (\text{Maximum calories}) \quad (3e)$$

$$\sum_{j \in \mathcal{J}} x_j P_j \geq 28.4 \quad (\text{Minimum proteins}) \quad (3f)$$

$$\sum_{j \in \mathcal{J}} x_j P_j \leq 38.4 \quad (\text{Maximum proteins}) \quad (3g)$$

$$\sum_{j \in \mathcal{J}} x_j F_j \geq 30.0 \quad (\text{Minimum fiber}) \quad (3h)$$

$$\sum_{j \in \mathcal{J}} x_j Na_j \leq 2000.0 \quad (\text{Maximum sodium}) \quad (3i)$$

$$\sum_{j \in \mathcal{J}} x_j Ca_j \geq 850.0 \quad (\text{Minimum calcium}) \quad (3j)$$

$$\sum_{j \in \mathcal{J}} x_j Mg_j \geq 400.0 \quad (\text{Minimum magnesium}) \quad (3k)$$

$$\sum_{j \in \mathcal{J}} x_j Ph_j \geq 700.0 \quad (\text{Minimum phosphorus}) \quad (3l)$$

$$\sum_{j \in \mathcal{J}} x_j Fe_j \geq 18.0 \quad (\text{Minimum iron}) \quad (3m)$$

$$\sum_{j \in \mathcal{J}} x_j Zn_j \geq 15.0 \quad (\text{Minimum zinc}) \quad (3n)$$

$$x_j \in \mathbb{Z}_+, \quad \forall j \in \mathcal{J} \quad (3o)$$

We highlight that carbohydrates, lipids and calories have flexible restrictions, depending on the specific characteristics of each individual, as shown in constraints (3b), (3c), (3d), and (3e). The other values came from Table 2.

The model presented here aims to minimize the consumption of carbohydrates and lipids. However, it only generates a diet for a single day, which cannot reflect long-term eating habits. Another limitation is that the model does not restrict the frequency with which a specific food can appear on the daily menu. Consequently, it repeatedly includes the same food, which may not be nutritionally or gastronomically recommended. Furthermore, the model must differentiate between breakfast, lunch, dinner or snacks, which may not be suitable for cultural dietary practices and diurnal metabolic patterns. To counter these limitations, we propose two strategies: a penalty system and the categorization of foods, described in the next sections.

4.2 Penalty system

We have implemented a penalty system to promote variety in the diet generated on different days and to reduce excessive repetition of certain foods. The system aims to increase the diversity of meals generated on different days by reducing the frequency of certain foods. Initially, all foods are assigned a zero penalty to place them on an equal footing with the optimization model's options. As the model simulates and plans different days, the foods selected to make up the meals receive an increase of one unit in their respective weights, indicating their inclusion in the daily diet.

To promote dietary diversity and avoid frequent selection of the same items, a cumulative penalty restriction has been introduced. This restriction ensures that the sum of penalties for all foods included in a diet plan does not exceed a certain value denoted by W . The penalty restriction can be mathematically formalized as follows:

$$\sum_{j=1}^n \omega_j x_j \leq W \quad (4)$$

with ω_j been the penalty for the food j .

The penalty system for formulating diets reduces the penalty by one unit for a food that is not consumed on a given day, provided that its penalty is greater than zero. This ensures a greater likelihood of including such foods in the diet on subsequent days, promoting a continuous variation in the food spectrum consumed.

Thus, the penalty system acts as a regulator in the formulation of diets, balancing the frequency of food consumption and ensuring that nutritional recommendations are met not just on a single day but over a longer period. This mechanism is particularly effective in creating eating plans that include the principles of a healthy and varied diet, which are essential for medium- and long-term nutritional strategies.

4.3 Categorization

Another strategy used in this work is to categorize foods into different daily meals. This division is based on the work of Xavier et al. (2023) and follows the categories listed below.

- Beverages (B): encompasses items mainly liquids, water, teas, and coffees. Milk drinks and juices are in other groups.
- Carbohydrates: subdivided into C1, such as refined bread and cereals (common in snacks), and C2, including whole grains and tubers (more common in main meals, such as lunch and dinner).
- Fruit (F): covers a variety of fresh or prepared fruits without added sugar or preservatives.
- Grains (G): contains various grains rich in fiber and other essential nutrients.
- Juices (J) refer to beverages derived from fresh or processed fruits without added sugar.
- Milk (M): includes milk and its derivatives, which provide a source of calcium and proteins.
- Proteins (P): consisting of lean meats, fish, eggs and legumes, which are essential for the maintenance and repair of body tissues.
- Vegetables (V): offering a spectrum of leafy vegetables and other types of vegetables, essential to provide vitamins, minerals and fiber.

These categories are needed for an equitable distribution of nutrients in meals, promoting a diet pattern that meets the dietary recommendations and adapts to metabolic needs throughout the day. It is important to note that Xavier et al. (2023) worked to develop low-calorie diets that are not related to diabetes. However, this subdivision proved to be adequate for this work.

Table 3 – Items present in each meal.

Meal	Quantity of Foods
Breakfast	B, F, and C1
First Snack	F or L
Lunch	C2, G, V, P and S
Second Snack	B or S, C1
Dinner	C2, G, V and P
Supper	F or L

In addition to these categories, Xavier et al. (2023) organized the diet into six daily meals, each with a set of foods. In this work, we chose to follow the same division. Table 3 illustrates the practical application of food categories in daily meals. This table shows that, for example, breakfast consists of three portions: a drink, a fruit, and a type 1 carbohydrate. The first snack must

contain only one item: a fruit or a milk derivative. This daily organization of food and the penalty system enabled the development of a more robust mathematical optimization model, which allows the generation of a diet for different days, separated into specific meals, and also seeks to minimize the consumption of carbohydrates and lipids.

5 RESULTS AND DISCUSSION

The objective of this study was to use the weighted sum method to develop personalized diet recommendations for people with diabetes. In this sense, it is necessary to consider a set of foods and proper multi-objective optimization approaches to perform the proposed model.

To simulate the proposed model, we use the Brazilian Table of Food Composition (TACO) database (Lima et al., 2011). This database includes an extensive catalog of different foods in various stages of preparation: fresh, cooked, or canned. TACO data are based on 100-gram portions of each food item, allowing us to analyze caloric, protein, lipid content, cholesterol, carbohydrates, dietary fiber, vitamins, and minerals. In this paper, we select 294 items from TACO, representing a common set of foods in Brazil.

We have chosen the profiles of three hypothetical individuals to construct various instances of the dietary problem, as described in (Paulino, 2017). These profiles allow us to evaluate the efficacy of our proposed models in various scenarios, reflecting the unique nutritional needs of each individual. Table 4 presents the caloric needs and intake restrictions of these individuals for essential macronutrients, including carbohydrates and lipids. In this table, we observe the minimum values for calories and carbohydrates and the maximum values for lipids, setting the dietary limits for the individuals considered.

Table 4 – Minimum values for calories and carbohydrates and maximum values for lipids for the three individuals.

Individual	Calories	Carbohydrates	Lipids
$i = 1$	1800	202.5	60
$i = 2$	2100	236.5	70
$i = 3$	3000	337.5	100

We have compared two a priori multi-objective optimization methods to provide a complete decision-making mechanism. The first method is the weighted-sum method, which assigns weighted values to each objective function. This approach is more flexible and simultaneously addresses carbohydrate and lipid minimization goals; consequently, it produces a compromise solution that considers the relative importance of each of these macronutrients, determined by the assigned weights. The second method is the lexicographic approach that prioritizes the objective functions sequentially. Thus, it ensures minimizing the first objective before considering the second. This approach is functional when there is a clear hierarchy of importance between the objectives.

These methods and the proposed multi-objective model were implemented in the Python 3 programming language using the PuLP library. This library provides a programming interface with different linear programming solvers. By default, it uses COIN-OR Linear Programming (CBC), which allows solving large-scale optimization problems. The source code used in this article is available at <https://github.com/Thiagofs1211/Dieta.Python>.

5.1 Results for the Weighted Sum Method

The weighted sum method considers a coefficient, denoted by α , representing the relative weight assigned to each objective function. The objective function, denoted as $\mathbf{F}(\mathbf{x})$, is formulated as follows:

$$\text{minimize } \mathbf{F}(\mathbf{x}) = \alpha f_1(\mathbf{x}) + (1 - \alpha)f_2(\mathbf{x})$$

In the context of this article, $f_1(x)$ represents the total carbohydrate content (see Equation (1)), while $f_2(x)$ represents the total lipid content (Equation (2)). The parameter α specifies the trade-off between the competing objectives. A higher value of α increases the importance of minimizing $f_1(x)$ (carbohydrates), while a lower value prioritizes $f_2(x)$ (lipids). This flexibility allows for exploring different solution spaces based on the specific requirements and preferences of the problem. To illustrate the trade-off between carbohydrates and lipids, we present Figure 1. This graph shows the relationship between the two objectives for the individual $i = 1$ with different values of α . Each point in the graph represents a solution obtained by varying α within the range of $[0, 1]$ in increments of 0.1.

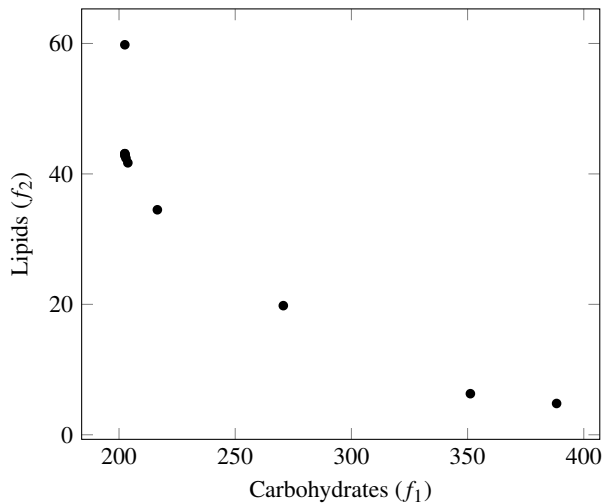


Figure 1 – Trade-off between carbohydrates and lipids using the weighted sum method. Each point represents a solution obtained by varying the coefficient α .

Our study focused on the management of carbohydrates due to their significant impact on blood glucose levels; therefore, we assign a higher value to α . Based on the values in Figure 1, we set

$\alpha = 0.7$ to reflect this preference. We applied the weighted sum method to develop personalized diet interventions for three individuals over five days. Table 5 summarizes the total nutrient intake for each individual on each day, obtained using our proposed approach. The results illustrate the effectiveness of our approach in maintaining the desired levels of nutrients, particularly carbohydrate intake.

Table 5 – Nutrient intake per day using the weighted sum method.

Individual	Nutrient	Day 1	Day 2	Day 3	Day 4	Day 5
$i = 1$	Calories	1800.0	1800.0	1807.0	1802.0	1816.0
	Carbohydrates	202.6	202.5	202.5	202.6	202.5
	Lipids	42.8	47.1	50.9	55.0	56.2
	Proteins	151.4	152.1	139.1	140.3	130.2
	Fiber	38.7	35.4	40.3	35.3	31.6
	Sodium	1997.0	1652.0	1978.0	1982.0	1993.0
	Calcium	1294.0	2660.0	1325.0	874.0	1915.0
	Phosphorus	1654.0	2721.0	1831.0	1806.0	2041.0
	Magnesium	406.0	578.0	465.0	563.0	409.0
	Iron	24.06	18.68	18.09	18.04	18.02
	Zinc	24.4	20.1	21.2	20.9	19.9
$i = 2$	Calories	2100.00	2100.00	2102.00	2100.00	2106.00
	Carbohydrates	236.50	236.50	236.50	236.50	236.60
	Lipids	54.00	57.80	59.80	63.50	69.00
	Proteins	175.00	165.00	152.20	156.00	138.00
	Fiber	35.00	34.10	38.70	39.80	40.70
	Sodium	1720.00	1778.00	1934.00	1965.00	1991.00
	Calcium	2265.00	1220.00	2140.00	1623.00	2125.00
	Phosphorus	2522.00	1985.00	1814.00	2261.00	2391.00
	Magnesium	573.00	481.00	427.00	504.00	532.00
	Iron	18.03	18.30	25.18	18.11	18.02
	Zinc	19.40	24.20	18.20	18.00	15.00
$i = 3$	Calories	3000.00	3001.00	3000.00	3001.00	3001.00
	Carbohydrates	337.50	337.50	351.70	370.40	378.20
	Lipids	96.60	97.60	99.50	100.10	100.00
	Proteins	215.10	208.90	189.20	174.50	164.50
	Fiber	39.10	32.30	52.30	51.00	37.60
	Sodium	1441.00	1792.00	1981.00	1982.00	1995.00
	Calcium	2241.00	1869.00	2120.00	1838.00	1568.00
	Phosphorus	2666.00	2485.00	2100.00	2630.00	2169.00
	Magnesium	728.00	652.00	591.00	715.00	613.00
	Iron	19.50	18.01	18.43	19.17	18.09
	Zinc	22.30	28.90	17.80	19.30	27.30

We obtained these results considering the penalty system described in Section 4.2. Here, we set the weight $W = 5$ (Equation (4)). Empirically, we observed that higher values did not significantly affect daily diets, while lower values made it difficult for the model to generate adequate diets for multiple days. Lower values resulted in fewer food options to repeat between days, reducing the diversity and viability of the proposed diets. Finally, Table 6 presents a three-day diet plan for the individual $i = 1$.

Table 6 – Foods per day using the weighted sum method.

Day 1	Day 2	Day 3
Breakfast Foods:		
Black Coffee	Fennel Tea	Mate Tea
Poncã Tangerine	Fresh Pineapple	Fresh Surinam Cherry
White Bread	White Bread	Whole Wheat Bread
First Snack Foods:		
Ricotta Cheese	Petit Suisse Cheese	Half-Cured Minas Cheese
Lunch Foods:		
Orange Pear Juice	Lime Orange Juice	Concentrated Grape Juice
Fresh Pasta Lasagna	Sautéed White Potato	Cooked Whole Grain Rice
Green Peas	Cooked Purple Beans	Cooked Pink Beans
Raw Basil	Raw Butterhead Lettuce	Raw Beetroot
Juçara Palm Hearts in Brine	Juçara Palm Hearts in Brine	Juçara Palm Hearts in Brine
Lean Grilled Beef Sirloin	Grilled Pintado Fish	Lean Grilled Beef Sirloin
Second Snack Foods:		
Black Coffee	Coconut Water	Mate Tea
White Bread	Green Corn Curau	Green Corn Curau
Dinner Foods:		
Boiled White Potato	Boiled White Potato	Cooked Whole Grain Rice
Green Peas	Soy Extract Powder	Cooked Speckled Beans
Juçara Palm Hearts in Brine	Raw Butterhead Lettuce	Raw Beetroot
Lean Grilled Beef Round	Grilled Pintado Fish	Lean Grilled Beef Sirloin
Bedtime Snack Foods:		
Ricotta Cheese	Petit Suisse Cheese	Ricotta Cheese

We have noticed that it was possible to maintain carbohydrate values consistently close to the minimum desired for $i = 1$ and $i = 2$, reflecting the priority given to this macronutrient. However, for $i = 3$, a significant increase in carbohydrates was observed in the last few days, which can be attributed to limitations in the variety of foods available in the database. Meanwhile, there was a gradual increase in the amount of lipids for all individuals over the days, demonstrating the model's adaptability to the restrictions imposed by the weights.

5.2 Results for the Lexicographic Approach

The lexicographic method is a systematic approach that prioritizes minimizing the first objective, in this case carbohydrate consumption, before considering the second objective of reducing lipids. This sequential optimization strategy ensures strict control over the dietary components that directly influence blood glucose levels, thus promoting better management of metabolic health. In addition, the lexicographic approach simplifies the model, facilitating the interpretation of results and the application of the generated diet plans.

Analysis of carbohydrate values reveals consistent stability in the values for individuals $i = 1$ and $i = 2$, aligning closely with the established minimum levels. An observation lies in the adjustment procedure following the initial optimization of carbohydrates. When the focus changed to minimizing lipids, carbohydrate values increased by approximately five grams over the previous result. This adjustment allowed us to establish a new threshold for the second objective while maintaining an efficient optimization strategy. This flexibility facilitated a more pronounced optimization of lipids and ensured that carbohydrate intake remained within acceptable limits.

Table 7 presents the daily nutrient intake of each individual, reflecting the results of our lexicographic optimization approach over five days. These results underscore the effectiveness of our method in achieving targeted nutrient levels while accommodating individual dietary requirements and metabolic constraints. We also consider the penalty system as described in Section 5.1.

As observed, it is possible to design customized diet interventions that promote metabolic health and general well-being by sequentially minimizing carbohydrate consumption before addressing lipid reduction. We also present (Table 8) a three-day diet plan considering the nutritional need of the individual $i = 1$.

Table 7 – Nutrient intake per day using the lexicographic approach.

Individual	Nutrient	Day 1	Day 2	Day 3	Day 4	Day 5
<i>i</i> = 1	Calories	1800.00	1800.00	1801.00	1802.00	1847.00
	Carbohydrates	203.00	203.40	203.40	203.50	223.80
	Lipids	42.40	46.20	50.40	51.60	59.90
	Sodium	1999.00	1645.00	1852.00	2000.00	1930.00
	Proteins	152.00	152.60	134.20	150.70	106.60
	Fiber	39.00	31.30	48.70	40.50	40.10
	Calcium	1163.00	2650.00	1338.00	1068.00	3036.00
	Magnesium	436.00	563.00	516.00	521.00	450.00
	Phosphorus	1560.00	2716.00	1830.00	1993.00	2487.00
	Iron	24.78	19.00	18.00	18.07	18.02
	Zinc	15.80	20.20	20.70	18.40	15.10
<i>i</i> = 2	Calories	2100.00	2100.00	2101.00	2101.00	2100.00
	Carbohydrates	237.50	237.50	237.30	237.20	237.50
	Lipids	53.50	57.50	59.50	62.90	63.80
	Sodium	1721.00	1651.00	1999.00	1824.00	1999.00
	Proteins	175.30	165.70	153.50	156.00	145.80
	Fiber	34.30	47.20	44.80	33.90	40.30
	Calcium	2280.00	1226.00	2112.00	1188.00	1590.00
	Magnesium	560.00	691.00	407.00	621.00	626.00
	Phosphorus	2539.00	2103.00	1894.00	1929.00	1806.00
	Iron	18.35	18.08	18.07	18.02	18.04
	Zinc	19.50	20.30	15.50	15.50	27.80
<i>i</i> = 3	Calories	3000.00	3001.00	3000.00	3000.00	3002.00
	Carbohydrates	338.30	338.00	351.70	374.90	378.40
	Lipids	96.30	97.60	99.80	99.40	99.30
	Sodium	1605.00	1830.00	1839.00	1990.00	1997.00
	Proteins	217.40	207.90	188.00	171.00	164.30
	Fiber	38.70	35.30	44.20	39.60	39.40
	Calcium	2214.00	1904.00	2180.00	1854.00	1159.00
	Magnesium	731.00	680.00	721.00	634.00	708.00
	Phosphorus	2737.00	2506.00	2275.00	2489.00	2071.00
	Iron	19.57	18.62	18.82	18.16	18.23
	Zinc	21.10	27.90	18.00	31.50	21.40

Table 8 – Foods per day using the lexicographic approach.

Day 1	Day 2	Day 3
Breakfast Foods:		
Black Coffee	Fennel Tea	Black Tea
Passion Fruit	Pineapple	Watermelon
White Bread	White Bread	Soy Bread
First Snack Foods:		
Ricotta Cheese	Mozzarella Cheese	Ricotta Cheese
Lunch Foods:		
Orange Pear Juice	Earth Orange Juice	Orange Pear Juice
Fresh Pasta Lasagna, cooked	Boiled White Potato	Boiled Baroa Potato
Green Peas	Cooked Purple Beans	Soy Extract Powder
Raw Basil	Raw Butterhead Lettuce	Raw New Zealand Spinach
Juçara Palm Hearts in Brine	Juçara Palm Hearts in Brine	Juçara Palm Hearts in Brine
Lean Grilled Beef Round, Fat-trimmed	Grilled Pintado Fish	Grilled Sardine
Second Snack Foods:		
Black Coffee	Mate Tea	Black Tea, infusion
White Bread	Oat Bread	Green Corn Curau
Dinner Foods:		
Sautéed White Potato	Boiled White Potato	Cooked Type 2 Rice
Green Peas	Cooked Lentils	Green Peas
Juçara Palm Hearts in Brine	Raw Broccoli	Raw Beetroot
Lean Grilled Beef Sirloin, fat-trimmed	Grilled Pintado Fish	Grilled Sardine
Bedtime Snack Foods:		
Ricotta Cheese	Pineapple	Watermelon

5.3 Discussion

The results presented in this paper underscore the practical utility and effectiveness of the proposed approach in optimizing diet plans. This methodology serves as a valuable tool for health professionals and individuals looking to manage their nutritional intake in a targeted and efficient manner. Incorporating the penalty system (Section 4.2) and food categorization (Section 4.3) introduces diversity in diet planning, facilitating the generation of multi-day meal plans.

Among the foods selected, black coffee, white bread, and a variety of fruits such as pineapple and tangerine emerged as regular components of breakfast and snacks. These choices not only satisfy the minimum nutritional requirements for carbohydrates and calories, but also have a low lipid content, which is crucial for glycemic control in diabetic patients. The consistent inclusion of fruits underscores the importance of natural sources of carbohydrates and fiber in a diabetic diet. Furthermore, opting for whole grain bread instead of refined alternatives can help manage glycemic spikes by providing a slower and more stable glucose release.

Another critical aspect is the impact of food prices on the feasibility of planned diets. Excessive food diversification can lead to high costs as the purchase of small quantities of numerous products is impractical. In fact, typically food is bought in larger quantities, such as packages of 500 g or 1.0 kg. For example, recommending different types of cheese on separate days could substantially increase the overall cost of the diet. To mitigate these expenses, the model should consider repeating certain foods throughout the week, maintaining affordability without compromising nutritional quality. Future research could improve the model by integrating cost variables directly into the optimization process, thus generating more practical and economically viable solutions.

The penalty system within the model plays a crucial role in ensuring dietary variety over multiple days. By penalizing the repetition of the same foods, the model encourages a wider selection of food choices, which is essential for a balanced diet. However, this approach has its challenges. Excessively high penalties could lead to the exclusion of nutritionally significant foods that may lack suitable alternatives. Analysis of diets over multiple days revealed that while staple foods were frequently selected, there was considerable substitution among other items. This balance is crucial to prevent dietary monotony and ensure adherence to the prescribed diet.

The constraints of the database also affected the model's ability to optimize effectively, particularly for $i = 3$, where minimizing carbohydrates and lipids proved challenging. Standardizing food portions to 100 g or 100 mL may not accurately reflect realistic eating practices or individual nutritional needs, which often require customized portions. In addition, diets tend to be based on portions based on home measurements, such as cups and spoons. These limitations highlight the need for future model improvements, such as expanding the food database and introducing a flexible portion system adaptable to individual requirements, particularly in diabetic diets where precise macronutrient control is essential.

Finally, it is important to acknowledge that the proposed methodology is confined to measurable properties within a food prescription table. To enhance the computational model, it is crucial to consider constraints based on specific objectives and individuals. Therefore, further analysis with patient-specific considerations will be necessary to refine the proposed approach.

6 CONCLUSIONS

This paper has proposed a mathematical optimization model that facilitates the development of nutritionally adequate diets that are practical and sustainable for individuals diagnosed with diabetes. Specifically, our goal is to meet the nutritional requirements imposed by the condition of diabetes and also address the practical aspects of daily nutrition. Adopting a multi-objective approach, this model integrates the minimization of carbohydrates and lipid consumption. Another contribution was the design of a penalty system, allowing diets to be customized for multiple days without significant repetition of foods, an important step to avoid dietary monotony and increase adherence to the nutritional plan.

Finally, the meal schedule was adapted by category, promoting an equitable distribution of nutrients throughout the day. The multi-objective models were then solved using two approaches: the weights method and the lexicographic method. The solutions met established nutritional criteria and favored the variability of the generated diets, an essential aspect for accepting the proposed plan. Therefore, this article contributes to the existing literature by providing empirical evidence of the benefits of a mathematical optimization approach in clinical dietetics, offering new perspectives for the effective management of diabetes through diet interventions.

A limitation of the final model is the adoption of integer portions of 100 g or 100 mL for all foods, which may not accurately reflect the recommended or preferred portions in daily eating practice. Furthermore, although the multi-objective model has advanced in incorporating structured and varied meals, the challenge of balancing carbohydrates and lipids persists, especially on consecutive days of meal planning.

In this sense, future research can explore more flexible models that allow for variations in food quantities, better adapting to individual nutritional needs and preferences. Another area of interest for future work is the inclusion of fiber maximization as an objective function within the model. Given the importance of fiber in the diet of people with diabetes to help with glycemic control and promote satiety, models that prioritize dietary fiber can offer eating plans even more aligned with nutritional recommendations and clinical guidelines.

Finally, machine learning and predictive analytics techniques could be implemented to improve diet personalization, allowing models not only to react to real-time data input but also to anticipate user needs and preferences based on pattern based consumption. These suggestions for future work have the potential to substantially improve the dietary management of diabetes and other health conditions, leading to more effective, personalized, and practical interventions for patients and healthcare professionals.

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