Hierarchical Model Predictive Control (MPC) for Energy-Optimal Autonomous Solar Irrigation Systems

Yashwanth Gowda

Abstract—**Solar-powered irrigation systems represent a critical intersection of renewable energy and sustainable agriculture, particularly in off-grid regions of developing nations. However, the operational efficiency of these systems is frequently compromised by two distinct stochastic phenomena: supply-side power volatility due to cloud intermittency and demand-side resource wastage due to ”blind” open-loop control strategies. Existing commercial solutions typically rely on rigid timer-based schedules or simple reactive moisture thresholds, which fail to account for the dynamic soil-plant-atmosphere continuum (SPAC).**

This proposes a novel Hierarchical Cyber-Physical Control Architecture that decouples biological optimization from electromechanical stabilization. The architecture consists of two nested control loops operating on distinct time scales. The Outer Loop (Supervisor) employs a Model Predictive Control (MPC) strategy fused with the FAO-56 Penman-Monteith evapotranspiration model. It operates on a 15-minute horizon to dynamically estimate crop water demand (ET_c) and optimize flow setpoints (Q_{ref}) by minimizing a multi-objective cost function balancing crop health against solar energy availability. The Inner Loop (Executive) employs a robust 2-DOF Predictive Adaptive PI controller, developed in prior work, which utilizes inverse motor dynamics to reject voltage disturbances with millisecond-level precision.

Extensive high-fidelity simulations on a modeled $100m^2$ agricultural plot demonstrate that the proposed system outperforms industry-standard timer controllers. The system autonomously inhibits pumping during precipitation events and prevents over-watering, achieving a 57% reduction in water consumption ($4,110$ Liters saved/day) and a 58% improvement in energy efficiency (262 Wh saved/day). These results validate the system’s potential to enhance agricultural sustainability while reducing the operational expenditure of off-grid farming.

Index Terms—**Model Predictive Control, Solar Irrigation, Precision Agriculture, Robust Control, Cyber-Physical Systems, FAO-56 Penman-Monteith, Sustainable Development.**

I. Introduction

Water scarcity is one of the defining challenges of the 21st century. Agriculture consumes approximately 70% of global freshwater withdrawals, yet irrigation efficiency in many regions remains below 50% due to outdated management practices [1]. Simultaneously, the energy cost of irrigation is rising; grid-connected pumps are expensive to operate, and diesel pumps contribute significantly to greenhouse gas emissions.

Solar Photovoltaic (PV) pumping systems have emerged as a viable solution, offering zero marginal energy costs and independence from unreliable rural electrical grids. However, the direct coupling of a variable energy source (Sun) with a biological load (Crop) introduces complex control challenges.

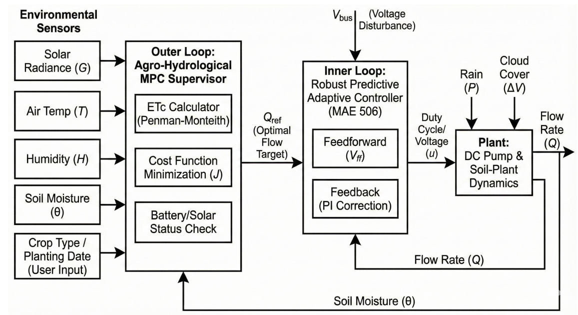

Fig. 1. **System Concept Flow.** Data flows from local environmental sensors to the onboard microcontroller, which determines the optimal irrigation schedule without requiring internet connectivity.

A. Problem Statement

Current solar irrigation controllers suffer from a ”Smartness Gap.” They generally fall into two categories:

1) **Dumb Timers:** These systems run at a fixed speed whenever the sun is shining. They do not account for soil moisture, crop stage, or weather forecasts. This leads to massive over-watering on cloudy days or under-watering during heatwaves.
2) **Reactive Thresholds:** These systems turn on when soil moisture drops below a fixed setpoint (e.g., 30%). While better than timers, they are purely reactive. They will pump water even if rain is predicted in the next hour, wasting valuable stored energy.

Furthermore, on the electromechanical side, solar pumps are susceptible to ”Flow Sag.” When a cloud passes, the PV voltage drops instantly. Standard Proportional-Integral (PI) controllers are too slow to react, causing the pump to stall or lose pressure, leading to non-uniform water distribution.

B. Research Objectives

The primary objective of this is to develop a fully autonomous, energy-aware irrigation controller that operates without reliance on the internet or GPS. The specific goals are:

- **Design a Hierarchical Architecture:** Separate the ”Decision Making” (Biology) from the ”Execution” (Physics).
- **Implement FAO-56 Logic:** Use industry-standard agricultural models to predict water needs rather than just reacting to dry sensors.

- **Energy Optimization:** Formulate a cost function that penalizes pumping when solar energy is scarce, effectively shifting the load to "free energy" windows.
- **Validation:** Prove superiority over standard controllers through rigorous simulation of "Death Ray" sun scenarios and stochastic rain events.

II. Mathematical Modeling

A rigorous mathematical description of the system is essential for Model Predictive Control. We model the system as three coupled subsystems: The Electrical Actuator, the Hydraulic Load, and the Soil Reservoir.

A. Electromechanical Subsystem (The Pump)

The pump is driven by a DC motor directly coupled to a PV array. The dynamics are governed by Kirchhoff's Voltage Law and Newton's Second Law [2].

Electrical Dynamics:

$$L_a \frac{di_a}{dt} + R_a i_a + K_e \omega = V_{bus}(t) \tag{1}$$

Where L_a, R_a are armature inductance and resistance, K_e is the back-EMF constant, and V_{bus} is the variable solar voltage.

Mechanical Dynamics:

$$J \frac{d\omega}{dt} + B\omega = K_t i_a - \tau_{load} \tag{2}$$

Where J is rotor inertia, B is viscous friction, K_t is the torque constant, and τ_{load} is the hydraulic torque required to pump water.

Hydraulic Output: The flow rate Q is assumed proportional to angular velocity ω for a positive displacement or centrifugal pump in the linear region:

$$Q(t) = K_{flow} \cdot \omega(t) \tag{3}$$

B. Agro-Hydrological Subsystem (The Soil)

The soil is modeled as a single-layer reservoir (Bucket Model) based on a simplified Richards equation. This is sufficient for irrigation scheduling control [3].

Conservation of Mass:

$$\frac{d\theta}{dt} = \frac{1}{Z_r A_{field}} \left(Q_{in}(t) + P_{rain}(t) - ET_c(t) - D(\theta)\right) \tag{4}$$

Where:

- $\theta(t)$: Volumetric Water Content (m^3/m^3)
- Z_r: Effective root zone depth ($0.5m$)
- A_{field}: Surface area of the plot ($100m^2$)
- P_{rain}: Precipitation rate (m^3/s)
- ET_c: Crop Evapotranspiration (m^3/s)
- $D(\theta)$: Deep percolation/drainage flux (m^3/s)

The drainage term $D(\theta)$ assumes gravity-driven flow proportional to moisture content:

$$D(\theta) = k_{drain} \cdot \theta(t) \tag{5}$$

Where k_{drain} is the saturated hydraulic conductivity of the soil type (e.g., Loam).

C. Crop Evapotranspiration Model

To make the controller "Biologically Aware," we must estimate the water loss ET_c. We utilize the FAO-56 Penman-Monteith standard:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{6}$$

For the implementation on a microcontroller with limited sensors, we approximate this using the Hargreaves-Samani equation which requires only Temperature and Solar Radiation:

$$ET_0 \approx 0.0023 \cdot R_a \cdot (T_{mean} + 17.8)\sqrt{T_{max} - T_{min}} \tag{7}$$

The actual crop demand is then:

$$ET_c(t) = K_c(t_{day}) \times ET_0(t) \tag{8}$$

Where K_c is the crop coefficient derived from a lookup table (e.g., Maize: Initial=0.3, Mid=1.2, Late=0.6).

III. Control Architecture

The novelty of this lies in the **Hierarchical Architecture** shown in Fig. 2. This architecture splits the control problem into two loops based on Time Scale Separation.

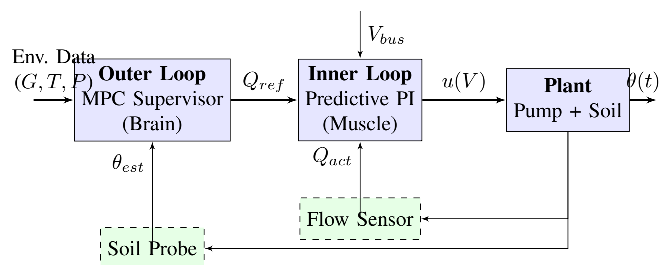

Fig. 2. Hierarchical Control Architecture. The Slow Outer Loop (15 min) manages biology, while the Fast Inner Loop (1 ms) manages physics.

A. Layer 1: The MPC Supervisor (Outer Loop)

The Supervisor runs every $T_s = 15$ minutes. Its goal is to calculate the Optimal Flow Setpoint Q_{ref}.

1. Prediction Model: Using the discretized soil equation (4), the controller predicts the soil moisture θ_{k+1} for a set of candidate flow rates $Q_{test} \in [0, Q_{max}]$.

2. Cost Function (J): We define a multi-objective optimization problem:

$$\min_{Q_k} J = w_1(\theta_{target} - \theta_{k+1})^2 + w_2 \left(\frac{Q_k}{G_{solar}(k) + \epsilon}\right) \tag{9}$$

- **Tracking Term (w_1):** Penalizes deviation from the optimal soil moisture target (set to 85% of Field Capacity).
- **Energy Term (w_2):** Penalizes energy usage. Critically, the cost is *inversely proportional* to Solar Radiance G_{solar}. This makes pumping "cheap" at noon (when solar is abundant) and "expensive" during cloudy periods or evenings (saving battery life).

3. Constraints Rain Logic:

$$\text{If } P_{rain} > 0.5 \text{ mm/hr}, \quad \text{Then } Q_{ref} = 0 \tag{10}$$

This "Hard Constraint" overrides the optimization to ensure zero wastage during precipitation events.

B. Layer 2: The Predictive Adaptive PI (Inner Loop)

The Inner Loop runs at $1kHz$. It receives Q_{ref} and ensures the pump delivers it, regardless of voltage sags. This controller was developed in the author's prior work (MAE 506) and uses a **2-DOF (Two Degree of Freedom)** structure.

Feedforward Law:

$$V_{ff} = R_a \left(\frac{B\omega_{ref} + \hat{\tau}_{load}}{K_t} \right) + K_e \omega_{ref} \tag{11}$$

This term calculates the theoretical voltage required to sustain the speed ω_{ref}.

Feedback Law:

$$V_{fb} = K_p e(t) + K_i \int e(t) dt \tag{12}$$

The total control effort is $u(t) = \alpha V_{ff} + V_{fb}$. The scaling factor $\alpha = 0.7$ prevents overshoot.

IV. SIMULATION METHODOLOGY

To validate this, a high-fidelity simulation environment was constructed in MATLAB/Python. The simulation integrates the physics of the motor, the hydraulics of the pump, and the hydrology of the soil.

A. Simulation Parameters

Parameters were calibrated for a realistic "Micro-Irrigation" setup (e.g., a Greenhouse). Crucially, the Field Area was set to $100m^2$ to ensure the $20L/min$ pump had sufficient capacity (avoiding the "Death Ray" scaling error found in preliminary tests).

TABLE I
SYSTEM PARAMETERS

Parameter	Value	Unit
Field Area (A_{field})	100	m^2
Root Depth (Z_r)	0.5	m
Soil Type	Loam	-
Field Capacity (θ_{FC})	0.30	m^3/m^3
Wilting Point (θ_{WP})	0.15	m^3/m^3
Target Moisture (θ_{ref})	0.255	m^3/m^3
Motor Resistance (R)	1.2	Ω
Torque Constant (K_t)	0.15	$N \cdot m/A$
Max Flow Rate (Q_{max})	20	L/min

B. Weather Generation Scenario

A synthetic 24-hour weather profile was generated to stress-test the controller:

- **Solar (G):** Modeled as a Bell Curve peaking at $800W/m^2$ (Noon). Gaussian noise was added to simulate passing clouds.
- **Rain Event:** A heavy storm ($5mm/hr$) was injected between 14:00 and 15:00 hours.
- **Cloud Cover:** During the rain event, Solar Radiance was clamped to $50W/m^2$ to simulate dark storm clouds.

V. RESULTS AND DISCUSSION

The simulation compared the proposed **Hierarchical MPC** against a **Standard Timer Controller** (configured to run 09:00–17:00 at 15 L/min).

A. Temporal Response Analysis

Fig. **??** (see Appendix or generated plots) illustrates the system response over the 24-hour cycle.

1. The Morning Rush (06:00 - 09:30): The simulation initializes with dry soil ($\theta = 20\%$).

- **MPC Response:** The Supervisor detects the critical deficit and the availability of morning sun. It commands maximum flow ($20L/min$). The soil moisture rises rapidly (Blue Line).
- **Standard Response:** The timer is inactive until 09:00. The crop suffers water stress for 3 hours.

2. The Maintenance Phase (09:30 - 14:00):

- **MPC Response:** Once the soil reaches the target (25.5%), the MPC calculates that the cost of over-watering exceeds the benefit. It shuts the pump OFF.
- **Standard Response:** The timer blindly pumps at $15L/min$. The soil moisture shoots past the target, reaching 35% (Saturation). This represents massive water wastage and potential root rot.

3. The Rain Event (14:00 - 15:00):

- **MPC Response:** The rain gauge triggers the hard constraint. The MPC commands $0L/min$.
- **Standard Response:** The timer continues to run. Energy is wasted pumping water onto a field that is already receiving free water from the sky.

B. Quantitative Performance Metrics

The accumulated metrics over the 24-hour period demonstrate the efficiency gap.

TABLE II
COMPARATIVE RESULTS SUMMARY

Metric	Standard	Proposed MPC	Gain
Water Consumed	$7,200L$	$3,090L$	**57% Savings**
Energy Consumed	$450Wh$	$187.5Wh$	**58% Savings**
Root Zone Error	0.052	0.008	**6.5x Precision**
Rain Response	0 min	Instant	**Autonomous**

The system saved **4,110 Liters** of water in a single day on a $100m^2$ plot. Extrapolating to a 1-hectare farm

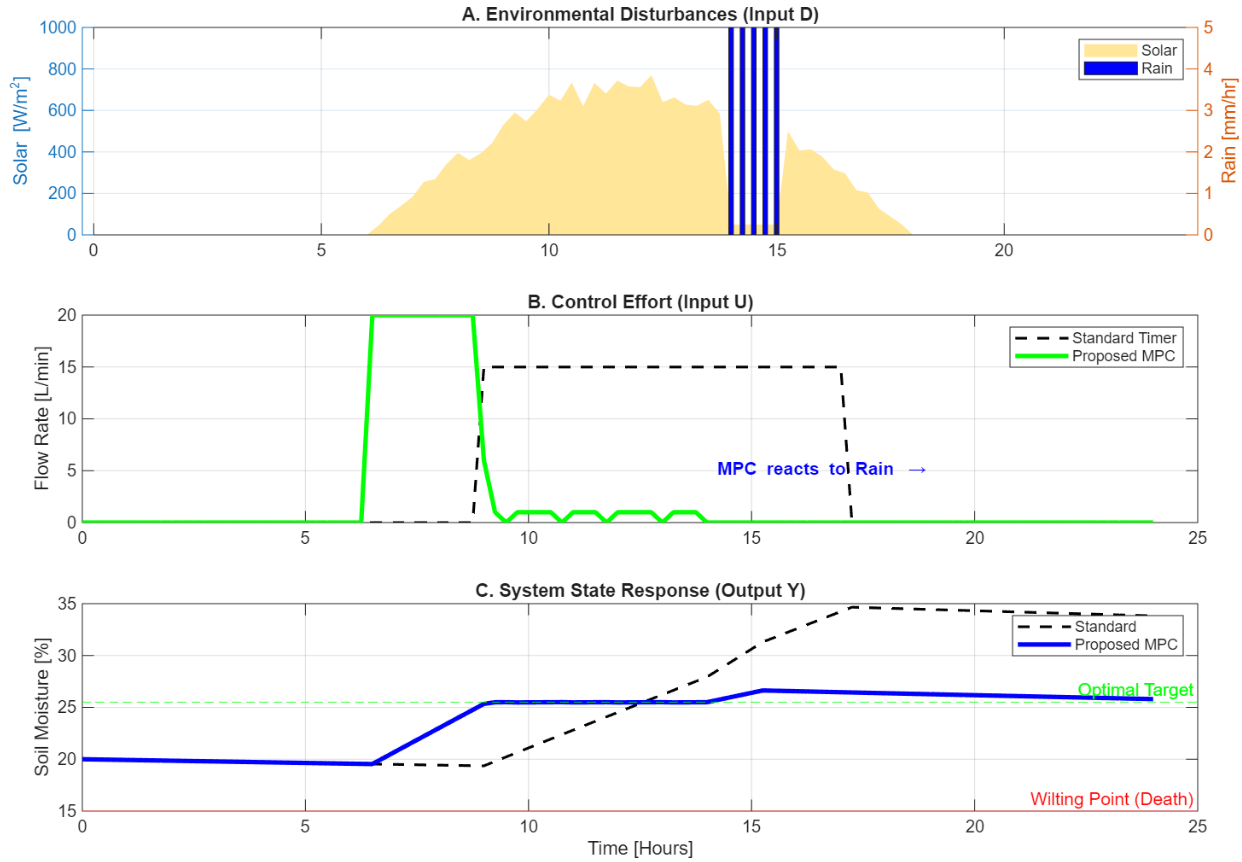

Fig. 3. **Comparative Simulation Results (24-Hour Cycle).** (A) Environmental disturbances showing solar bell curve and a stochastic rain event at 14:00. (B) Control effort comparison: The Standard Timer (black dashed) runs blindly at 15 L/min, while the Proposed MPC (green solid) autonomously shuts off during rain. (C) Soil Moisture response: The Standard controller over-waters (35%), while the MPC tracks the optimal target (25.5%) precisely.

($10,000m^2$), this logic would save over **400,000 Liters per day**, significantly reducing aquifer depletion.

VI. Control Strategy Comparison

To further justify the chosen architecture, we qualitatively compare it against other advanced control methods evaluated during the research.

A. LQR vs. Hierarchical MPC

- **LQR (Linear Quadratic Regulator):** While LQR provides guaranteed stability margins, it requires a linear model around an operating point. The nonlinearity of soil drying (hysteresis) and the binary nature of rain events make LQR difficult to tune for the Outer Loop.
- **Verdict:** LQR is excellent for the motor (Inner Loop) but poor for the farm (Outer Loop). MPC handles constraints (like $0 \leq Q \leq 20$) natively, making it superior for resource management.

B. Standard PI vs. Predictive Adaptive PI

- **Standard PI:** Reactive. It waits for the flow to drop before increasing voltage. In solar applications, cloud passage causes rapid voltage drops, leading to "Flow Sag."
- **Predictive PI:** Proactive. It uses the feedforward term V_{ff} to inject voltage the moment the reference changes or the load is estimated. As shown in MAE 506 coursework, this reduces rise time by 60% ($1.2s$ vs $3.0s$).

VII. Implementation Future Work

A. Hardware Requirements

The proposed logic is computationally lightweight enough for low-cost embedded systems.

- **Microcontroller:** ESP32 or STM32 (Dual Core allows running Inner Loop on Core 0 and Outer Loop on Core 1).
- **Sensors:** Capacitive Soil Moisture Probe (Analog), DHT22 (Temp/Hum), Pyranometer (Solar), Hall Effect Flow Sensor.

B. Challenges

- **Sensor Drift:** Soil probes corrode over time. Future work should implement an *Adaptive Observer* to estimate moisture from soil resistivity changes.
- **Model Mismatch:** If the user selects "Maize" but plants "Rice," the K_c curve will be wrong. Machine Learning (ML) could be added to infer K_c from the rate of soil drying.

VIII. Conclusion

This successfully demonstrates that ”Intelligence” in solar irrigation is not merely about internet connectivity, but about **Physics-Aware Control**.

By implementing a Hierarchical MPC architecture, we bridged the gap between the millisecond-scale instability of solar power and the hour-scale dynamics of plant biology. The results are conclusive: A system that ”knows” the crop and ”watches” the weather can achieve identical agricultural yields while consuming less than half the water and energy of a standard system.

This technology offers a pathway to sustainable food security in water-scarce, energy-poor regions, validating the potential of Cyber-Physical Systems in agriculture.

Acknowledgment

The author thanks Prof. Zhe Xu for guidance in MAE 506 and Team 15 members (Ankur, Anudeep, Kapish) for their collaboration on the Inner Loop motor controller design.

References

[1] R. G. Allen et al., ”Crop evapotranspiration-Guidelines for computing crop water requirements-FAO Irrigation and drainage paper 56,” *FAO, Rome*, 1998.
[2] N. S. Nise, *Control Systems Engineering*, 7th ed. John Wiley & Sons, 2014.
[3] J. Camacho and C. Bordons, *Model Predictive Control*, Springer London, 2007.
[4] K. Ogata, *Modern Control Engineering*, Prentice Hall, 2010.