# EE3025 DSP LAB

EE18BTECH11017

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## Problem:

Let,

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2}$$

Compute,

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{3}$$

and $H(k)$ using h(n). Also Compute,

$$Y(k) = X(k)H(k) \tag{4}$$

## Solution:

Impulse Response of the LTI system is the output of the system when Unit Impulse Signal is given as input. So,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{5}$$

Also, expressing Eq.(3) in terms of matrices for N = 6.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \end{bmatrix} \tag{6}$$

# Assignment1 - Solution(contd..)

So,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 13 + 0j \\ -4 - 1.732j \\ 1 + 0j \\ -1 + 0j \\ 1 + 0j \\ -4 + 1.732j \end{bmatrix} \tag{7}$$

Similarly, for H(k) we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125 + 0j \\ 0.51625 - 0.51418j \\ -0.07812 + 1.10956j \\ 3.84375 + 0j \\ -0.07182 - 1.10956j \\ 0.51625 + 0.51418j \end{bmatrix} \tag{8}$$

# Assignment1 - Solution(contd..)

Computing Y(k) using Eq (4),(7),(8), we get

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix} \tag{9}$$

# Assignment1 - Plots

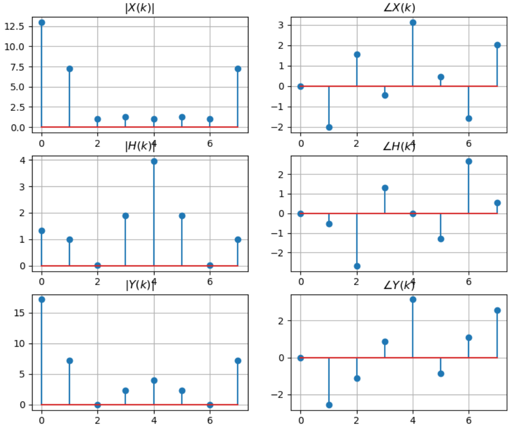



Figure 1: Plots of X(k),H(k),Y(k)

***Properties:***

1. Symmetric Property :

$$W_N^{k+N/2} = -W_N^k$$

2. Periodic Property :

$$W_N^{k+N} = W_N^k$$

3. Square Property :

$$W_N^2 = W_{N/2}$$

We use these properties to reduce Eq(3) to

$$X(k) = \underbrace{X_e(k)}_{\text{N/2 point DFT with even inputs}} + W_N^k \underbrace{X_o(k)}_{\text{N/2 point DFT with odd inputs}} \tag{10}$$

# Assignment1 - DFT Properties(contd...)

Expressing Eq(10) in terms of matrices and simplying we get,

$$F_N = \begin{bmatrix} I_{N/2} & D_{N/2} \\ I_{N/2} & -D_{N/2} \end{bmatrix} \begin{bmatrix} F_{N/2} & 0 \\ 0 & F_{N/2} \end{bmatrix} P_N \tag{11}$$

where $F_N$ is a N-Point DFT Matrix , $I_3$ is NxN identity matrix, $D_N = diag(1, W_N, W_N^2, ...., W_N^{N-1})$ and $P_N$ is an odd-even Permutation Matrix.
So,if $N = 2^M$ where $M \in \mathbb{Z}^+$ then we can recursively breakdown N/2 point DFT Matrix to N/4 point DFT Matrix ..so on till we reach 2-point DFT Matrix.This recursive approach is known as **FFT**(Fast Fourier Transform).

# Assignment1 - Time Complexity

***Time Complexity:***
In DFT - Matrix multiplication of NxN matrix with Nx1 vector. Hence it has $O(N^2)$ time complexity which is very slow for high $N$. In FFT - N-point FFT is broken down recursively into 2 N/2-point FFTs recursively. Additionally $O(N)$ operation of Vector multiplication is performed on the N/2 point FFTs.

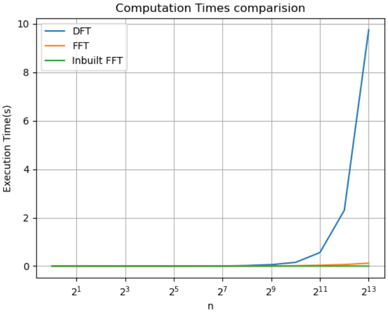
$$T(n) = 2T(n/2) + O(n) \tag{12}$$

Solving this recurrence relation gives $O(NlogN)$ time complexity.

# Assignment1 - Computation Times

***Computation Times:***

We can compare the computation times for DFT,FFT and Inbuit-FFT algorithms for $N = 2^M$, for $N = 1$ $(2^0)$ to 8192 $(2^{13})$.




From above plot we can observe that, computation time for DFT rises exponentially as we increase N in powers of 2 but FFT and Inbuilt-FFT are much faster.

## Problem:

We are supposed to design the equivalent FIR and IIR filter realizations for filter number 114.

## Filter Design:

**Filter Type:** We are supposed to design filters whose stopband is monotonic and passband equiripple. Hence, we use the *Chebyschev approximation* to design our bandpass IIR filter.

**Low Pass Analog Chebyschev Filter Design:**

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+\epsilon^2 c_N^2(\Omega_L/\Omega_{Lp})} \tag{13}$$

On solving using given design parameters we get $N \geq 4$, $0.3184 \leq \epsilon \leq 0.6197$.

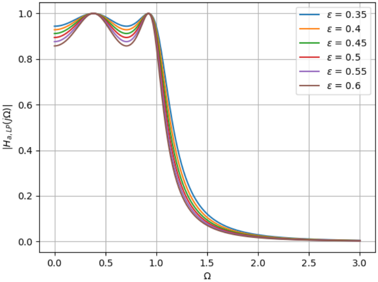



Figure 2: Varying $\epsilon$

# Assignment2 - IIR Analog Filter

**Low Pass Chebyschev Filter :**

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + 0.16c_4^2(\Omega_L)} \tag{14}$$

Equation of Low Pass stable Chebyschev filter from poles with $N = 4$,$\epsilon = 0.5$,

$$H_{a,LP}(s_L) = \frac{0.3125}{s_L^4 + 1.12s_L^3 + 1.61s_L^2 + 0.91s_L + 0.34} \tag{15}$$

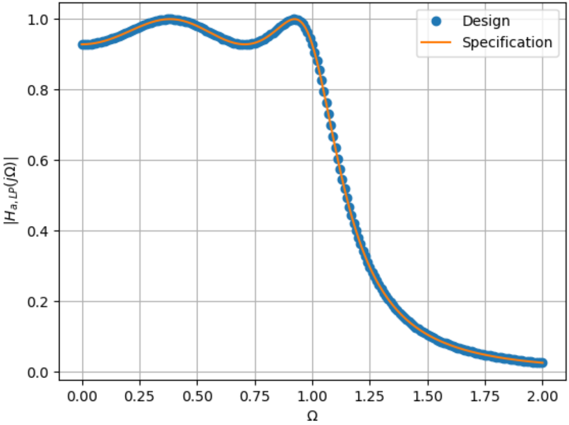



Figure 3: Chebyschev Low Pass Filter. Specification (14), Design (15)

**Band Pass Chebyschev Filter Design :**

The analog bandpass filter is obtained from (15) by substituting $s_L = \frac{s^2+\Omega_0^2}{Bs}$. Hence

$$H_{a,BP}(s) = G_{BP} H_{a,LP}(s_L)\Big|_{s_L=\frac{s^2+\Omega_0^2}{Bs}}, \tag{16}$$

where $G_{BP}$ is the gain of the bandpass filter with $H_{a,BP}(j\Omega_{p1}) = 1$. So,

$$H_{a,BP}(s) = \frac{2.78 \times 10^{-5} s^4}{s^8 + 0.11s^7 + 0.8s^6 + 0.07s^5 + 0.3s^4 + 0.01s^3 + 0.04s^2 + 0.001s + 0.002} \tag{17}$$

# Assignment2 - IIR Analog Filter

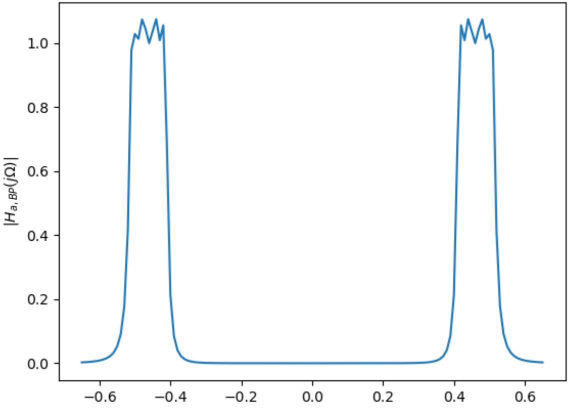



Figure 4: Chebyschev Band Pass Filter.From (17)

# Assignment2 - IIR Digital Filter

**Digital Band Pass Chebyschev Filter Design :** From the bilinear transformation, we obtain the digital bandpass filter from the corresponding analog filter as

$$H_{d,BP}(z) = GH_{a,BP}(s)\big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} \tag{18}$$

$$H_{d,BP}(z) = G\frac{N(z)}{D(z)} \tag{19}$$

$$N(z) = 1 - 4z^{-2} + 6z^{-4} - 4z^{-6} + z^{-8} \tag{20}$$

and

$$\begin{aligned} D(z) = 2.36 - 12z^{-1} + 31.88z^{-2} - 53.75z^{-3} + 62.81z^{-4} \\ - 51.47z^{-5} + 29.23z^{-6} - 10.53z^{-7} + 1.98z^{-8} \end{aligned} \tag{21}$$

# Assignment2 - IIR Digital Filter

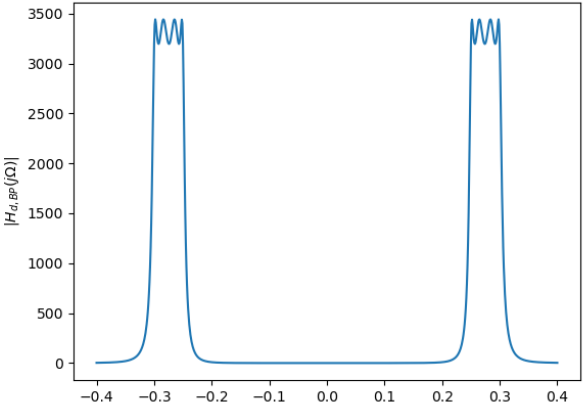



Figure 5: Digital Chebyschev Band Pass Filter.From (18)

# Assignment2 - FIR Filter

**FIR Low Pass Filter:** We design the FIR filter by first obtaining the (non-causal) lowpass equivalent using the Kaiser window($w(n)$) and then converting it to a causal bandpass filter.For the desired parameters we get, $w(n) = 1$ .

$$
\begin{aligned}
h_{lp}(n) &= \frac{\sin(\frac{n\pi}{40})}{n\pi}, -100 \leq n \leq 100 \\
&= 0, \qquad\qquad \text{otherwise}
\end{aligned}
\tag{22}
$$

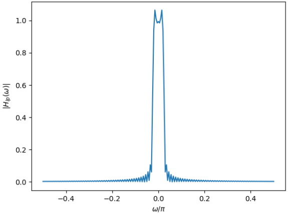



Figure 6: FIR Low Pass Filter.From (22)

**FIR Band Pass Filter:**

$$h_{bp}(n) = 2h_{lp}(n)cos(n\omega_c) \tag{23}$$

$$\begin{aligned} h_{bp}(n) &= \frac{2\sin(\frac{n\pi}{40})\cos(\frac{11n\pi}{40})}{n\pi}, -100 \leq n \leq 100 \\ &= 0, \qquad \text{else} \end{aligned} \tag{24}$$

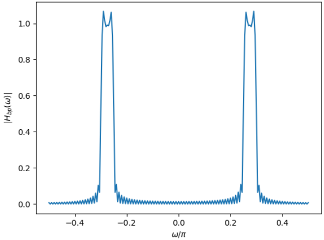



Figure 7: FIR Band Pass Filter.From (24)

# Links

**Github Links:**

[1] Assignment1 - https://github.com/yashwanthguguloth24/EE3025-DSP-lab/tree/main/Assignment1

[2] Assignment2 - https://github.com/yashwanthguguloth24/EE3025-DSP-lab/tree/main/Assignment2