Assignment 1

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Download all python codes from

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https://github.com/yashwanthguguloth24/EE3025-DSP-lab/tree/main/Assignment1/codes
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and latex-tikz codes from

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https://github.com/yashwanthguguloth24/EE3025-DSP-lab/tree/main/Assignment1
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1 Problem

1.1. Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{1.1.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.1.2}$$

1.2. Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.2.1}$$

and $H(k)$ using h(n).

1.3. Compute

$$Y(k) = X(k)H(k) \tag{1.3.1}$$

2 Solution

2.1. We know that,the Impulse Response of the LTI system is the output of the system when Unit Impulse Signal is given as input to the system. So, using Eq (1.1.2) the Impulse Response of the System can be found as,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

where $h(n)$ is an IIR Filter.

2.2. DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.2.1}$$

For N = 6, The above expression can be written in matrix form as below:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 \\ \omega_6^0 & \omega_6^1 & \omega_6^2 & \omega_6^3 & \omega_6^4 & \omega_6^5 \\ \omega_6^0 & \omega_6^2 & \omega_6^4 & \omega_6^6 & \omega_6^8 & \omega_6^{10} \\ \omega_6^0 & \omega_6^3 & \omega_6^6 & \omega_6^9 & \omega_6^{12} & \omega_6^{15} \\ \omega_6^0 & \omega_6^4 & \omega_6^8 & \omega_6^{12} & \omega_6^{16} & \omega_6^{20} \\ \omega_6^0 & \omega_6^5 & \omega_6^{10} & \omega_6^{15} & \omega_6^{20} & \omega_6^{25} \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \end{bmatrix} \tag{2.2.2}$$

Where $\omega_N = e^{-j2\pi/N}$

$$\implies \omega_6 = e^{-j2\pi/6} = \frac{1 - j\sqrt{3}}{2} \tag{2.2.3}$$

Using $x(n)$ from Eq(1.1.1), we get

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 \\ \omega_6^0 & \omega_6^1 & \omega_6^2 & \omega_6^3 & \omega_6^4 & \omega_6^5 \\ \omega_6^0 & \omega_6^2 & \omega_6^4 & \omega_6^6 & \omega_6^8 & \omega_6^{10} \\ \omega_6^0 & \omega_6^3 & \omega_6^6 & \omega_6^9 & \omega_6^{12} & \omega_6^{15} \\ \omega_6^0 & \omega_6^4 & \omega_6^8 & \omega_6^{12} & \omega_6^{16} & \omega_6^{20} \\ \omega_6^0 & \omega_6^5 & \omega_6^{10} & \omega_6^{15} & \omega_6^{20} & \omega_6^{25} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{bmatrix} \tag{2.2.4}$$

Simplifying the above equation we get,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 13+0j \\ -4-1.732j \\ 1+0j \\ -1+0j \\ 1+0j \\ -4+1.732j \end{bmatrix} \tag{2.2.5}$$

2.3. DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.3.1}$$

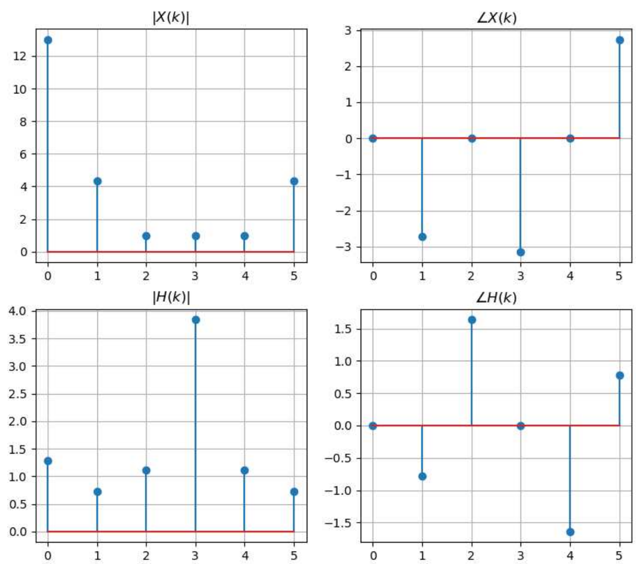
Similarly, converting the above expression in matrix form to find $H(k)$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} \omega_6^0 \omega_6^0 \omega_6^0 & \omega_6^0 & \omega_6^0 & \omega_6^0 \\ \omega_6^0 \omega_6^1 \omega_6^2 & \omega_6^3 & \omega_6^4 & \omega_6^5 \\ \omega_6^0 \omega_6^2 \omega_6^4 & \omega_6^6 & \omega_6^8 & \omega_6^{10} \\ \omega_6^0 \omega_6^3 \omega_6^6 & \omega_6^9 & \omega_6^{12} & \omega_6^{15} \\ \omega_6^0 \omega_6^4 \omega_6^8 & \omega_6^{12} & \omega_6^{16} & \omega_6^{20} \\ \omega_6^0 \omega_6^5 \omega_6^{10} & \omega_6^{15} & \omega_6^{20} & \omega_6^{25} \end{bmatrix} \begin{bmatrix} h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \\ h(6) \end{bmatrix} \tag{2.3.2}$$

Simplying we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125 + 0j \\ 0.51625 - 0.51418j \\ -0.07812 + 1.10956j \\ 3.84375 + 0j \\ -0.07182 - 1.10956j \\ 0.51625 + 0.51418j \end{bmatrix} \tag{2.3.3}$$

2.4. The magnitude and phase plots of $X(k)$ and $H(k)$



2.5. We can now compute $Y(k)$ using Eq (2.5.1)

$$Y(k) = X(k)H(k) \tag{2.5.1}$$

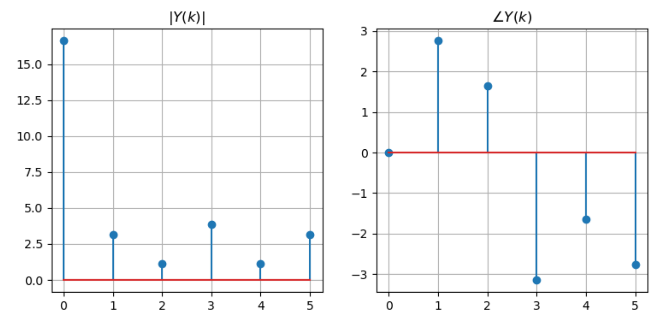
So, $Y(k)$ is obtained element wise multiplication of $X(k)$ and $H(k)$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \end{bmatrix} \tag{2.5.2}$$

Computing the above expression we get,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix} \tag{2.5.3}$$

The magnitude and phase plots of $Y(k)$ are



2.6. The following code plots all the above figures.

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https://github.com/yashwanthguguloth24/
    EE3025−DSP−lab/tree/main/
    Assignment1/codes/ee18btech11017.py
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