Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.2 Practical Case

3 Feedback Current Amplifier: Example

4 Feedback Transconductance Amplifier: Series-Series

5 Feedback Problem

5.1. A two-pole amplifier for which $G_o = 10^3$ and having poles at 1 MHz and 10 MHz is to be connected as a differentiator. On the basis of the rate-of-closure rule, what is the smallest differentiator time constant for which operation is stable? What are the corresponding gain and phase margins?
Solution: The differentiator circuit is as shown in Fig. 5.1.1.Here $\tau = RC$

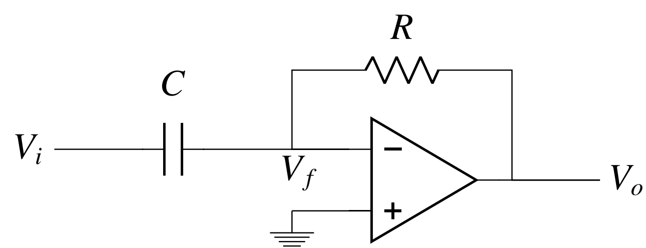

Fig. 5.1.1

For the above circuit using superspostion theorem we can write,

$$V_f = \left(\frac{R}{R + \frac{1}{SC}}\right) V_i + \left(\frac{\frac{1}{SC}}{R + \frac{1}{SC}}\right) V_o \tag{5.1.1}$$

5.2. Draw the equivalent control system.
Solution:

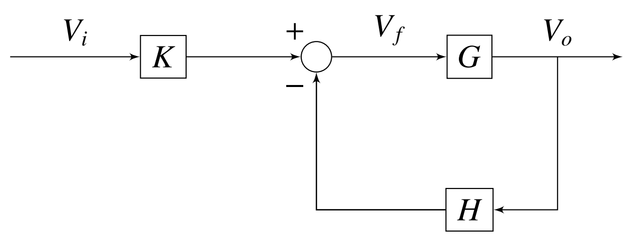

Fig. 5.2

5.3. Find feedback factor H and open loop gain G.
Solution: From the above table,

$$H(s) = \frac{1}{1 + sRC} \tag{5.3.1}$$

$$\implies \frac{1}{H(s)} = 1 + sRC \approx sRC \tag{5.3.2}$$

$$G(s) = \frac{10^3}{\left(1 + \frac{s}{2\pi \cdot 10^6}\right)\left(1 + \frac{s}{2\pi \cdot 10^7}\right)} \tag{5.3.3}$$

5.4. Rate of Closure rule
Solution: The rule states that at intersection of

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Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{1}{1+sRC}$
Loop gain	GH	$\frac{G}{1+sRC}$
Gain Factor	K	$\frac{-sRC}{1+sRC}$
Closed loop gain	$K\frac{G}{1+GH}$	$\frac{G(-sRC)}{1+G+sRC}$

TABLE 5.2

$20\log\left|\frac{1}{H(\jmath\omega)}\right|$ and $20\log\left|G(\jmath\omega)\right|$ the difference of the slopes (called the rate of closure) should not exceed 20dB/decade.
The Bode plot for $\frac{1}{|H(\jmath\omega)|}$ has a slope of +20dB/decade.Therefore , the closed-loop amplifier will be stable if $20\log\left(\frac{1}{H}\right)$ line intersects the $20\log|G|$ curve at a point on the 0dB/decade segment.
The 0dB/decade segment of the $20\log|G|$ curve is $G_o = 10^3$.We should test the stability at the pole frequency $\omega_{P1} = 2\pi\cdot 10^6$ which seperate the 0dB/decade segment and the -20dB/decade.Thus we can write that as,

$$20\log|G_o| - 20\log\left(\frac{1}{H(\jmath\omega_{P1})}\right) \le 20dB \tag{5.4.1}$$

$$\implies \left|G_o H(\jmath\omega_{P1})\right| \le 1 \tag{5.4.2}$$

Substituting,

$$G_o\frac{1}{\omega_{P1}\tau} \le 1 \tag{5.4.3}$$

$$\therefore \tau \ge \frac{G_o}{\omega_{P1}} = \frac{10^3}{2\pi\cdot 10^6} = 159.15\mu s \tag{5.4.4}$$

$$\implies \tau_{min} = 159.15\mu s \tag{5.4.5}$$

5.5. Find Phase Margin and Gain Margin for $\tau = \tau_{min}$.
Solution: From the Equation (5.4.5)

$$\text{At}, \tau = \tau_{min} \tag{5.5.1}$$

$$\left|G_o H(\jmath\omega_{P1})\right| = 1 \tag{5.5.2}$$

$$\implies P.M = \angle G(\jmath\omega_{P1})H(\jmath\omega_{P1}) + 180^\circ \tag{5.5.3}$$

So,

$$\angle G\left(\jmath\omega_{P1}\right) = -tan^{-1}\left(\frac{\omega_{P1}}{2\pi\cdot 10^6}\right) - tan^{-1}\left(\frac{\omega_{P1}}{2\pi\cdot 10^7}\right) \tag{5.5.4}$$

$$= -50.71^\circ \tag{5.5.5}$$

$$\angle H\left(\jmath\omega_{P1}\right) = -90^\circ \tag{5.5.6}$$

$$\implies P.M = 180^\circ - 50.71^\circ - 90^\circ = 39.29^\circ \tag{5.5.7}$$

$$\therefore P.M = 39.29^\circ \tag{5.5.8}$$

For Gain Margin we need to find ω_{180} such that

$$\angle G(\jmath\omega_{180})H(\jmath\omega_{180}) = -180^\circ \tag{5.5.9}$$

$$\implies -tan^{-1}\left(\frac{\omega_{180}}{2\pi\cdot 10^6}\right) - tan^{-1}\left(\frac{\omega_{180}}{2\pi\cdot 10^7}\right) - 90^\circ = -180^\circ \tag{5.5.10}$$

Solving we get,

$$\omega_{180} = 19.98Mrad/s \tag{5.5.11}$$

The Gain Margin is as follows,

$$GM = -20\log\left|G\left(\jmath\omega_{180}\right)H\left(\jmath\omega_{180}\right)\right| \tag{5.5.12}$$

$$= -20\log\left|G\left(\jmath\omega_{180}\right)\right| + 20\log\left|\frac{1}{H\left(\jmath\omega_{180}\right)}\right| \tag{5.5.13}$$

$$= -49.12 + 70.05 \tag{5.5.14}$$

$$\therefore G.M = 20.93dB \tag{5.5.15}$$