Control Systems

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Abstract—**The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

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4.1. What are Constant M and N circles and how can we determine closed loop frequency response using M and N circles? **Solution:** M circles are called constant magnitude Loci and N circles are called as constant phase angle Loci.These are helpful in determining the closed-loop frequency response of unity negative feedback systems.

Constant-Magnitude Loci(Mcircle): Let $G(\jmath\omega)$ be complex quantity it can be written as

$$G(\jmath\omega) = X + \jmath Y \tag{4.1.1}$$

where X,Y are real quantities. Let M be magnitude of closed loop transfer function.

$$M = \left|\frac{X + \jmath Y}{1 + X + \jmath Y}\right| \tag{4.1.2}$$

$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2} \tag{4.1.3}$$

Hence,

$$X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0 \tag{4.1.4}$$

If $M=1$, then from Equation (4.1.4), we obtain $X=\frac{-1}{2}$ This is the equation of a straight line parallel to the Y axis and passing through the point $\left(\frac{-1}{2},0\right)$.
If $M\neq 1$ Equation (4.1.4) can be written as

$$X^2+\frac{2M^2}{M^2-1}X+\frac{M^2}{M^2-1}+Y^2=0 \tag{4.1.5}$$

Simplifying,

$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2} \tag{4.1.6}$$

Equation (4.1.6) is the equation of a circle with center $\left(-\frac{M^2}{M^2-1},0\right)$ and radius $\left|\frac{M}{M^2-1}\right|$
Thus the intersection of Nquist plot with M circle at a frequency(ω) results as the magnitude of closed loop transfer function as M at frequency (ω)

Constant-Phase-Angle Loci (N Circles): Finding Phase angle α from (4.1.3) we get,

$$\alpha=\tan^{-1}\left(\frac{Y}{X}\right)-\tan^{-1}\left(\frac{Y}{1+X}\right) \tag{4.1.7}$$

$$\text{Let } tan\alpha=N \tag{4.1.8}$$

$$N=tan\left(\tan^{-1}\left(\frac{Y}{X}\right)-\tan^{-1}\left(\frac{Y}{1+X}\right)\right) \tag{4.1.9}$$

Simplifying,

$$N=\frac{Y}{X^2+X+Y^2} \tag{4.1.10}$$

Further Simplifying..

$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2} \tag{4.1.11}$$

Equation (4.1.11) is the equation of a circle with center at $\left(\frac{-1}{2},\frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4}+\frac{1}{(2N)^2}}$
Thus the intersection of Nquist plot with N circle at a frequency(ω) results as the phase of closed loop transfer function as $tan^{-1}(N)$ at frequency (ω)

4.2. For unity Feedback system given below,obtain closed loop frequency response using constant M and N circles.

$$G(s)=\frac{50(s+3)}{s(s+2)(s+4)} \tag{4.2.1}$$

Solution: The following code plots Fig. 4.2

```
codes/ee18btech11017_1.py
```

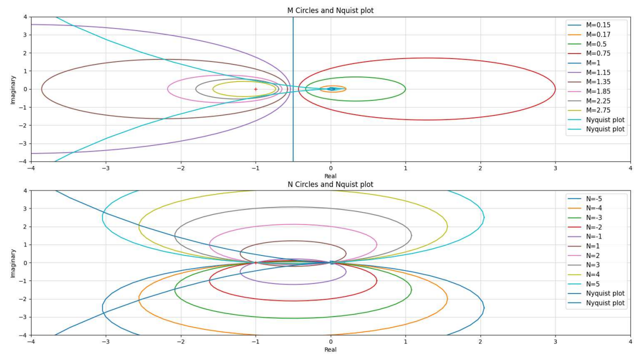

Fig. 4.2

4.3. The following code finds the intersection of M and N circles with Nyquist plot at different frequencies in Table 4.3

```
codes/ee18btech11017_2.py
```

M(dB)	M	ω
1.58	1.2	2.95
7.04	2.25	5.96
-2.49	0.75	10.48
-20.9	0.09	24.42
-26.0	0.05	32.37

TABLE 4.3

4.4. The following code plots figure(4.4), the magnitude plot from the values obtained above in the Table 4.3 along with the original bode plot of the closed loop transfer function.

```
codes/ee18btech11017_3.py
```

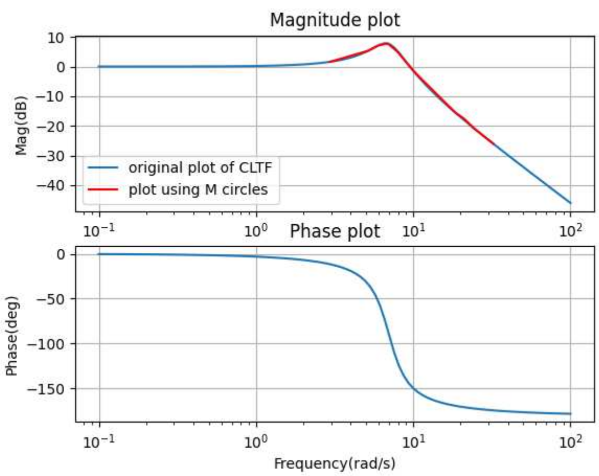
Magnitude plot
Mag(dB)
original plot of CLTF
plot using M circles
Phase plot
Phase(deg)
Frequency(rad/s)

Fig. 4.4