Control Systems

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***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

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4.1. What are Constant M and N circles and how can we determine closed loop frequency response using M and N circles?
Solution: M circles are called constant magnitude Loci and N circles are called as constant phase angle Loci.These are helpful in determining the closed-loop frequency response of unity negative feedback systems.

Constant-Magnitude Loci(Mcircle): Let $G(\jmath\omega)$ be complex quantity it can be written as

$$G(\jmath\omega) = X + \jmath Y \tag{4.1.1}$$

where X,Y are real quantities. Let M be magnitude of closed loop transfer function.

$$M = \left|\frac{X + \jmath Y}{1 + X + \jmath Y}\right| \tag{4.1.2}$$

$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2} \tag{4.1.3}$$

Hence,

$$X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0 \tag{4.1.4}$$

If $M=1$, then from Equation (4.1.4), we obtain $X=\frac{-1}{2}$ This is the equation of a straight line parallel to the Y axis and passing through the point $\left(\frac{-1}{2},0\right)$.
If $M\neq 1$ Equation (4.1.4) can be written as

$$X^2+\frac{2M^2}{M^2-1}X+\frac{M^2}{M^2-1}+Y^2=0 \tag{4.1.5}$$

Simplifying,

$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2} \tag{4.1.6}$$

Equation (4.1.6) is the equation of a circle with center $\left(-\frac{M^2}{M^2-1},0\right)$ and radius $\left|\frac{M}{M^2-1}\right|$
Thus the intersection of Nquist plot with M circle at a frequency(ω) results as the magnitude of closed loop transfer function as M at frequency (ω)
Constant-Phase-Angle Loci (N Circles): Finding Phase angle α from (4.1.3) we get,

$$\alpha=\tan^{-1}\left(\frac{Y}{X}\right)-\tan^{-1}\left(\frac{Y}{1+X}\right) \tag{4.1.7}$$

$$\text{Let } tan\alpha = N \tag{4.1.8}$$

$$N=tan\left(\tan^{-1}\left(\frac{Y}{X}\right)-\tan^{-1}\left(\frac{Y}{1+X}\right)\right) \tag{4.1.9}$$

Simplifying,

$$N=\frac{Y}{X^2+X+Y^2} \tag{4.1.10}$$

Further Simplifying..

$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2} \tag{4.1.11}$$

Equation (4.1.11) is the equation of a circle with center at $\left(\frac{-1}{2},\frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4}+\frac{1}{(2N)^2}}$
Thus the intersection of Nquist plot with N circle at a frequency(ω) results as the phase of closed loop transfer function as $tan^{-1}(N)$ at frequency (ω)

4.2. For unity Feedback system given below,obtain closed loop frequency response using constant M and N circles.

$$G(s)=\frac{50(s+3)}{s(s+2)(s+4)} \tag{4.2.1}$$

Solution: The following code plots Fig. 4.2

```
codes/ee18btech11017_code1.py
```

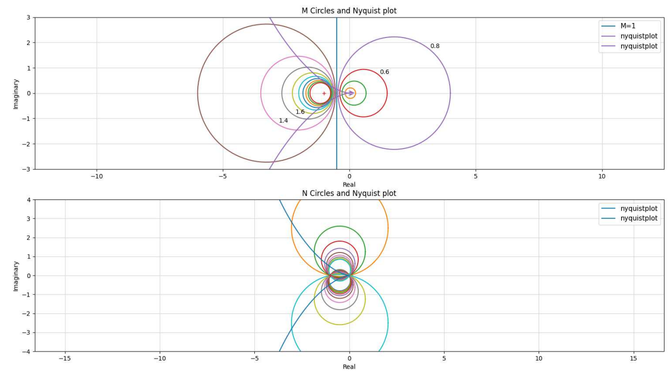

Fig. 4.2

4.3. Find the intersection of M and N circles with Nyquist plot at different frequencies.
Solution: The following code finds intersection of M and N circles with Nyquist plot at different frequencies

```
codes/ee18btech11017_code2.py
```

The points M and frequencies are listed in Table 4.3

M in dB	**M**	ω
5.15	1.81	5.08
7.64	2.41	6.34
6.48	2.11	7.58
-0.81	0.91	9.86
-10.17	0.31	14.33
-40	0.01	57.91

TABLE 4.3

The points M and frequencies are listed in Table4.3
The constant N locus for given value of α is not the entire circle but only an arc.This is beacuse tangent of angle remains same if $+180°$ or $-180°$ is added to the angle.

4.4. Plot Magnitude and Phase plot from the values obtained above.
Solution: The following code plots Fig. 4.4

α	**N**	ω
-38.65	-0.80	5.42
-66.50	-2.30	6.40
-78.60	-5.00	6.72
-101.50	4.90	7.32
-158.19	0.4	11.68
-174.28	0.1	46.01

TABLE 4.3

```
codes/ee18btech11017_code3.py
```

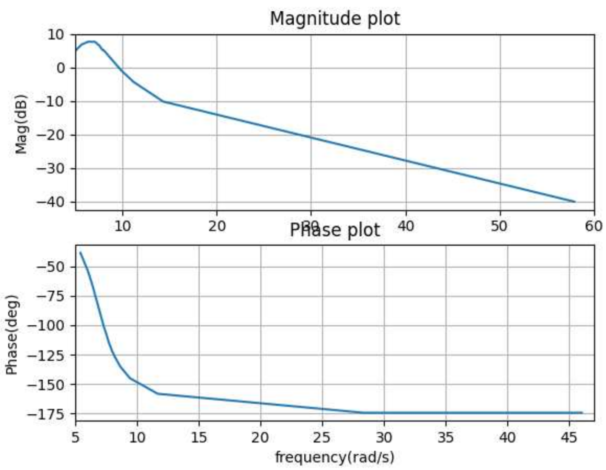

Fig. 4.4

4.5. Compare the above plot with bode plot of closed loop transfer function.
Solution: The following code plots Fig. 4.5

```
codes/ee18btech11017_code4.py
```

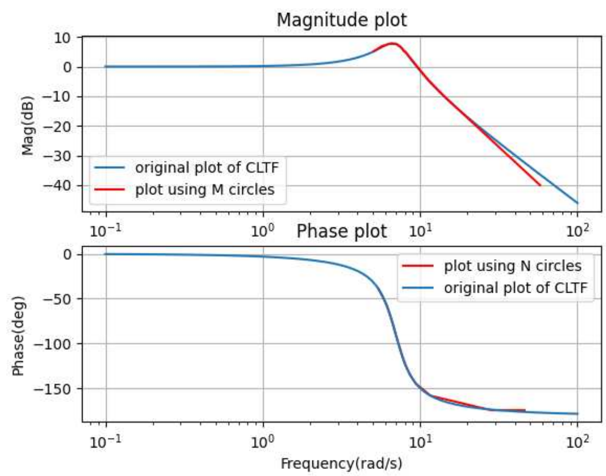

Fig. 4.5