Thursday 12/05/22

Course : Engineering Mathematics

Course code : 20SC01T

Unit - 01

Matrices and determinants

1.1 Matrix and types

1.2 Algebra of matrices (addition, subtraction, scalar multiplication and multiplication)

1.3 Evaluation of determinants of a square matrix of order 2 and 3. singular matrices

1.4 Cramer's rule for solving system of linear equations involving 2 and 3 variables.

1.5 Adjoint and inverse of the non-singular matrices of order 2 and 3

1.6 characteristics equation and eigen values of a square matrix of order 2

Unit - 02

Straight lines

2.1 slope of a straight line

2.2 Intercepts of a straight line

2.3 Intercept form of a straight line

2.4 slope - intercept form of a straight line

2.5 slope - point form of a straight line

2.6 Two - point form of a straight line

2.7 General form of a straight line

2.8 Angle between two lines and conditions for lines to be parallel and perpendicular

2.9 Equation of a straight line parallel to the given line

2.10 Equation of a straight line perpendicular to the given line.

Unit - 03

Trigonometry

3.1 Concept of angles, their measurement, Radian measure and related conversions.

3.2 Signs of trigonometric ratios in different quadrants (ASTC rule)

3.3 Trigonometric ratios of allied angles (definition and the table of trigonometric ratios of standard allied angles say $90° \pm \theta$, $180° \pm \theta$, $270° \pm \theta$ and $360° \pm \theta$)

3.4 Trigonometric ratios of compound angles (without proof)

3.5 Trigonometric ratios of multiple angles

3.6 Transformation formulae.

Unit - 04

Differential Calculus and Applications

4.1 Derivatives of continuous functions in an interval (List of formulae)

4.2 Rules of differentiation

4.3 Successive differentiation (up to second order)

4.4 Applications of differentiation

Unit - 05

Integral Calculus and Applications

5.1 List of standard integrals, and basic rules of integration

5.2 Evaluation of integrals of simple function and their combination

5.3 Methods of integration

5.4 Concept of definite integrals

5.5 Applications of definite integrals.

Friday 13/05/22

Matrices and determinants :

Introduction to matrices

The concept of matrices was original in solving the linear equations.

The word "matrixe" is the latin word for "womb", derived from matter.

The word "matrixe" was coined by James Joseph sylvester in 1850.

Matrices are used in almost all branches of science and engineering.

Defination of matrixe :

$$A = \begin{array}{ccc|l} & & & \\ \left[\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}\right] & & & \begin{matrix} \text{Frist Row} \\ \text{second Row} \end{matrix} \\ \text{Frist column} \quad \text{Second column} \quad \text{third column} & & & \end{array}$$

Defination :

A matrix is a rectangular array or arrangement of numbers in m horizontal lines [called rows] and n vertical lines [called columns] ~~and n vertical lines [called columns]~~ enclose within the brackets.

Ex:- $C = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

The numbers or objects in a matrise are called as elements of a matrix.

The brackets used to enclose elements of a matrise are big brackets, small brackets, parenthesis and square brackets.

The horizontal arrangement in matrix is called as a Row matrix

The Vertical arrangement in matrix is called as a Column matrix

The matrices are usually denoted by capital letters of English alphabets.

Order of matrix

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 6 \\ 8 & -1 \end{bmatrix}$$ no of rows = 3

no of column = 2

3×2

$$B = \begin{bmatrix} 3 & 5 & 2 \\ -8 & 6 & 2 \\ 1 & -3 & 4 \end{bmatrix}$$ no of rows = 3

no of column = 3

3×3

Types of Matrix

1. Row matrix

If a matrix has only one row and any number of column is called as Row matrix.

Ex:- $A = [4\ 5\ 6]$ is a row matrix of order 1×3.

2. Column matrix

If a matrix has only one column and any number of rows is called as <u>column matrix</u>

Ex:- $A = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ is a row matrix of order 3×1

3. Square matrix

A matrix is solid to be a square matrix if it has <u>number of rows</u> equal <u>number of columns</u>.

Ex:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 3 & 4 & 6 \end{bmatrix}$ is a square matrix of order 3×3.

4. Diagonal matrix.

If in a square matrix, all the elements except principle diagonal elements [left to right] are zeros it is called as <u>diagonal matrix</u>

Ex :- $A = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$ $B = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}$

5. Scalar matrix

A diagonal matrix in which all the diagonal elements are equal is called Scalar matrix

Ex :- $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ $B = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

6. Unit matrix or identity matrix

A scalar matrix in which all the diagonal elements are 1 is called a unit matrix

Ex :- $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Null matrix or zero matrix

A matrix in which all the elements are zero, is called a null matrix.

Ex :- $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8. Rectangular matrix

A matrix in which the number of rows is not equal to the number of columns is called as a Rectangular matrix

Ex :- $A = \begin{bmatrix} 5 & 3 & 6 \\ -2 & 5 & 2 \end{bmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ -2 & 5 \\ 4 & 6 \end{pmatrix}$

Content discussed in previous session

- Introduction
- Defination of matrix
- order of matrix
- Types of matrizes.

Contents to be discussed in this Session

- Algebra of matrices
- Scalar Multiplication of matrix
- Addition of matrices
- Subtraction of matrices
- Assignment
- Multiple choice questions.

Algebra of matrices

Scalar multiplication of a matrix

Defination: If 'A' is a matrix and 'K' is a scalar matrix then 'KA' is matrix obtained by multiplying each elements of 'A' by scalar 'K'

Ex:- $A \begin{bmatrix} 5 & 3 & 6 \\ -2 & 5 & 2 \end{bmatrix}$ then $2A = 2\begin{bmatrix} 5 & 3 & 6 \\ -2 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 6 & 12 \\ -4 & 10 & 4 \end{bmatrix}$

$B. \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ then $KB = K\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} Ka & Kb & Kc \\ Kd & Ke & Kf \end{bmatrix}$

Addition of matrices

Defination: If two matrices A and B are of the same order of matrix A + B is defined as the matrix which is obtained by the addition of the corresponding elements of A and B

Ex 1- $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ then

$$A + B = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

$A = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$

then $A+B = \begin{pmatrix} 4+2 & 8+4 \\ 12+6 & 16+10 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 18 & 26 \end{pmatrix}$

Subtraction of matrices

Defination: If A and B are two matrices of the same order then their difference A - B is defined as a matrix, each element of which is the difference of the corresponding elements of A and B.

Ex:- $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ then

$$A - B = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} - \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 - a_2 & b_1 - b_2 \\ c_1 - c_2 & d_1 - d_2 \end{bmatrix}$$

$A = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$

then $A - B = \begin{bmatrix} 4-2 & 8-4 \\ 12-6 & 16-10 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 6 \end{bmatrix}$

Contents to be discussed in this session

- product of matrices.
- Transpose of a matrix.

Multiplication of matrices.

The product AB of two matrices A and B is defined only if the number of column ~~is defined only if the number of column~~ in matrix A is equal to the number of rows in matrix B. That is if A is matrix of order $m \times n$ and B is matrix of order $n \times p$ then the product AB is a matrix of order $m \times p$

To multiply any two matrices, we should make sure that the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix. Therefore, the resulted matrix product will have a number of rows of the 1st matrix and a number of

Ex:- If $A \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $B \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ then $AB = ?$

$$AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 + 5 \times -2 & 2 \times 3 + 5 \times 4 \\ 1 \times 1 + 3 \times -2 & 1 \times 3 + 3 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 10 & 6 + 20 \\ 1 - 6 & 3 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 26 \\ -5 & 15 \end{bmatrix}$$

properties of matrix multiplication

Commutative property :

The matrix multiplication is not Commutative.

Assume that, if A and B are the two matrices, then $AB \neq BA$

Hence, the multiplication of two matrices is not Commutative

Associative property :

If A, B and C are the three matrices, the associative property of matrix multiplication states that, $(AB)C = A(BC)$

Distributive property :

If A, B and C are the three matrices, the distributive property of matrix multiplication states that, $A(B+C) = AB + AC$

$(B+C)A = BA + CA$

Multiplicative identity property :

The identity property of matrix multiplication states that,

$$A \cdot I = I \cdot A = A$$

where A is an $n \times n$ matrix and "I" is an identity matrix of order n.

Addition of matrices :

problems :-

1. $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 \\ 4 & 16 \end{bmatrix}$ then find $A + B$

$$A+B = \begin{bmatrix} 2+6 & 4+2 \\ 6+4 & 8+16 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 8 & 6 \\ 10 & 24 \end{bmatrix}$$

2. If $A = \begin{bmatrix} -4 & 6 \\ 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 6 \\ -1 & -4 \end{bmatrix}$ then find $A + B$

$$A+B = \begin{bmatrix} -4+8 & 6+6 \\ 8+(-1) & 1+(-4) \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 7 & -3 \end{bmatrix}$$

3. If $A = \begin{bmatrix} -4 & -6 \\ -2 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -1 \\ -4 & -1 \end{bmatrix}$ then find $A+B$.

$$A+B = \begin{bmatrix} -4+(-6) & -6+(-1) \\ -2+(-4) & 8+(-1) \end{bmatrix}.$$

$$= \begin{bmatrix} -10 & -7 \\ -6 & 7 \end{bmatrix}$$

Scalar Multiplication of a matrices ;

problems :-

Ex:- 1 $KA = K\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$

$$= \begin{bmatrix} Ka_1 & Kb_1 \\ Kc_1 & Kd_1 \end{bmatrix}$$

Ex:- 2 $A = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$ then find $2A$

$$2A = 2\begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 12 & 20 \end{bmatrix}$$

1. $A = \begin{bmatrix} -4 & 2 \\ 6 & 8 \end{bmatrix}$ then find $3A, -5A, 6A, \frac{1}{3}A$

$A = 3\begin{bmatrix} -4 & 2 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -12 & 6 \\ 18 & 24 \end{bmatrix}$

$A = -5\begin{bmatrix} -4 & 2 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 20 & -10 \\ -30 & -40 \end{bmatrix}$

$A = 6\begin{bmatrix} -4 & 2 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -20 & 12 \\ 36 & 48 \end{bmatrix}$

$A = \frac{1}{3}\begin{bmatrix} -4 & 2 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -4/3 & 2/3 \\ 6/3 & 8/3 \end{bmatrix}$

2. $2A + 3B$

$2A - 4B$

$2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = 2\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$

$3B = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} = 3\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 18 & 12 \end{bmatrix}$

$4B = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} = +4\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} +8 & +4 \\ +24 & +16 \end{bmatrix}$

$$2A+3B = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 18 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 4+6 & 8+3 \\ 12+18 & 16+12 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 30 & 28 \end{bmatrix}$$

$$2A-4B = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} - \begin{bmatrix} 8 & 4 \\ 24 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 4-8 & 8-4 \\ 12-24 & 16-16 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -12 & 0 \end{bmatrix}$$

3. if $A = \begin{bmatrix} -4 & -6 \\ 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ 6 & 8 \end{bmatrix}$ then

find $2A + 3B$

$2A - 4B$

$$2A = \begin{bmatrix} -4 & 6 \\ 8 & 1 \end{bmatrix} = 2\begin{bmatrix} -4 & 6 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 12 \\ 16 & 2 \end{bmatrix}$$

$$3B = \begin{bmatrix} -4 & -1 \\ 6 & 8 \end{bmatrix} = 3\begin{bmatrix} -4 & -1 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -12 & -3 \\ 18 & 24 \end{bmatrix}$$

$$4B = \begin{bmatrix} -4 & -1 \\ 6 & 8 \end{bmatrix} = 4\begin{bmatrix} -4 & -1 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 24 & 32 \end{bmatrix}$$

$$2A + 3B = \begin{bmatrix} -8 & 12 \\ 16 & 2 \end{bmatrix} + \begin{bmatrix} -12 & -3 \\ 18 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} -8 + (-12) & 12 + (-3) \\ 16 + 18 & 2 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} -20 & 9 \\ 34 & 26 \end{bmatrix}$$

$$2A - 4B = \begin{bmatrix} -8 & 12 \\ 16 & 2 \end{bmatrix} - \begin{bmatrix} -16 & -4 \\ 24 & 32 \end{bmatrix}$$

$$= \begin{bmatrix} -8 - (-16) & 12 - (-4) \\ 16 - 24 & 2 - 32 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 8 \\ -8 & -30 \end{bmatrix}$$

Transpose of a matrics

If A is any matrics than the matrics obtained by interchanging the rows and columns of A is called the transpose of a matrics.

It is denoted by A^t or A^T

Ex:- if $A = \begin{bmatrix} 1 & 3 \\ 6 & 8 \end{bmatrix}$ then A^T

$$A^T = \begin{bmatrix} 1 & 6 \\ 3 & 8 \end{bmatrix}$$

problems :-

1. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$ then find

$(A+B)^T$, $(A-B)^T$, $2A^T + 5B^T$, $2A^T - B^T$

Soln:- $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ + $B = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$

* $(A+B)^T$ $A+B = \begin{bmatrix} 1+6 & 2+1 \\ 4+3 & 6+2 \end{bmatrix}$

$$A+B = \begin{bmatrix} 7 & 3 \\ 7 & 8 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 7 & 7 \\ 3 & 8 \end{bmatrix}$$

* $(A-B)^T$

$$A - B = \begin{bmatrix} 1-6 & 2-1 \\ 4-3 & 6-2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -5 & 1 \\ 1 & 4 \end{bmatrix}$$

$$(A-B)^T = \begin{bmatrix} -5 & 1 \\ 1 & 4 \end{bmatrix}$$

* $2A^T + 5B^T$

$$2A^T = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \qquad 5B^T = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$$

$$A^T = 2\begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} \qquad B^T = 5\begin{bmatrix} 6 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 \\ 4 & 12 \end{bmatrix} \qquad = \begin{bmatrix} 30 & 15 \\ 5 & 10 \end{bmatrix}$$

$$2A^T + 5B^T = \begin{bmatrix} 2 & 8 \\ 4 & 12 \end{bmatrix} + \begin{bmatrix} 30 & 15 \\ 5 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2+30 & 8+15 \\ 4+5 & 12+10 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 23 \\ 9 & 22 \end{bmatrix}$$

* $2A^T - B^T = \begin{bmatrix} 2 & 8 \\ 4 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2-6 & 8-3 \\ 4-1 & 12-2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 5 \\ 3 & 10 \end{bmatrix}$$

Multiplication of a matrics :

problems :-

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 \\ 1 & 4 \end{bmatrix}$ then find AB

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 8 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+2 & 8+8 \\ 15+4 & 24+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 16 \\ 19 & 40 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$ then find AB

$$AB = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10+4 & -2+1 \\ 20+20 & -4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -1 \\ 40 & 1 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$ then find

$A^2, A^3, B^2, B^3, AB, BA, A^2+3A, A^2-3A, B^2+3A,$
$B^2-3A, (AB)^T, (BA)^T, (A+B)^T, (A-B)^T, A^T+B^T$
A^T-B^T

Sol^n:- $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$

* A^2

$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 4+6 & 12+24 \\ 2+4 & 6+16 \end{bmatrix}$

$= \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix}$

* A^3

$A^2 = \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 20+36 & 60+144 \\ 12+22 & 36+88 \end{bmatrix}$

$= \begin{bmatrix} 56 & 204 \\ 34 & 124 \end{bmatrix}$

* B^2

$$B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 64+6 & 8+4 \\ 48+24 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix}$$

* B^3

$$B^2 = \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 560+72 & 70+48 \\ 576+132 & 72+88 \end{bmatrix}$$

$$= \begin{bmatrix} 632 & 188 \\ 708 & 160 \end{bmatrix}$$

* AB

$$AB = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 16+36 & 2+24 \\ 8+24 & 1+16 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 26 \\ 32 & 17 \end{bmatrix}$$

* BA

$$BA = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 16+1 & 48+4 \\ 12+4 & 36+16 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 52 \\ 16 & 52 \end{bmatrix}$$

* $A^2 + 3A$

$$A^2 = \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix} \qquad 3A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$$A^2 + 3A = \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix} + \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix} \qquad A = 3\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & 36+18 \\ 6+3 & 22+12 \end{bmatrix} \qquad = \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 54 \\ 9 & 34 \end{bmatrix}$$

* $A^2 - 3A$

$$A^2 = \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix} \qquad 3A = \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$A^2 - 3A = \begin{bmatrix} 10 & 36 \\ 6 & 22 \end{bmatrix} - \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 10-6 & 36-18 \\ 6-3 & 22-12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 18 \\ 3 & 10 \end{bmatrix}$$

* $B^2 + 3A$

$$B^2 = \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix} \quad 3A = \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$B^2 + 3A = \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix} + \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 70+6 & 12+18 \\ 72+3 & 22+12 \end{bmatrix}$$

$$= \begin{bmatrix} 76 & 30 \\ 75 & 34 \end{bmatrix}$$

* $B^2 - 3A$

$$B^2 = \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix} \quad 3A = \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$B^2 - 3A = \begin{bmatrix} 70 & 12 \\ 72 & 22 \end{bmatrix} - \begin{bmatrix} 6 & 18 \\ 3 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 70-6 & 12-18 \\ 72-3 & 22-12 \end{bmatrix}$$

$$= \begin{bmatrix} 64 & -6 \\ 69 & 10 \end{bmatrix}$$

* $(AB)^T$

$$AB = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 16+36 & 2+24 \\ 8+24 & 1+16 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 26 \\ 32 & 17 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 52 & 32 \\ 26 & 17 \end{bmatrix}$$

* $(BA)^T$

$$BA = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 16+1 & 48+4 \\ 12+4 & 36+16 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 52 \\ 16 & 52 \end{bmatrix}$$

$$(BA)^T = \begin{bmatrix} 17 & 16 \\ 52 & 52 \end{bmatrix}$$

* $(A+B)^T$

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} + B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2+8 & 6+1 \\ 1+6 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 7 \\ 7 & 8 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 10 & 7 \\ 7 & 8 \end{bmatrix}$$

* $(A-B)^T$

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} - B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2-8 & 6-1 \\ 1-6 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 5 \\ -5 & 0 \end{bmatrix}$$

$$(A-B)^T = \begin{bmatrix} -6 & -5 \\ 5 & 0 \end{bmatrix}$$

* $A^T + B^T$

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} + B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} + B^T = \begin{bmatrix} 8 & 6 \\ 1 & 4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 6 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 1+6 \\ 6+1 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 7 \\ 7 & 8 \end{bmatrix}$$

* $A^T - B^T$

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} - B = \begin{bmatrix} 8 & 1 \\ 6 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} - B^T = \begin{bmatrix} 8 & 6 \\ 1 & 4 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-8 & 1-6 \\ 6-1 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -5 \\ 5 & 0 \end{bmatrix}$$

Multiplication of a matrices of order. 3X3

Ex! -

$$A = \begin{array}{c} \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & a_2 \\ a_3 & b_3 & b_3 \end{bmatrix} \end{array} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array} \quad B = \begin{array}{ccc} & & \\ \end{array}\begin{array}{c} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array}$$

$$= \begin{bmatrix} a_1x_1 + b_1x_2 + c_1x_3 \quad a_1y_1 + b_1y_2 + c_1y_3 \\ a_1z_1 + b_1z_2 + c_1z_3 \\ a_2x_1 + b_2 + x_2 + c_2x_3 \quad a_2y_1 + b_2y_2 + c_2y_3 \\ a_2z_1 + b_2z_2 + c_2z_2 \end{bmatrix}$$

$$\begin{bmatrix} a_3x_1 + b_3x_2 + c_3x_3 & a_3y_1 + b_3y_3 + c_3y_3 \\ a_3z_1 + b_3z_2 + c_3z_3. & \end{bmatrix}$$

problem :-

1. ~~Find~~ if $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 5 \\ 6 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 5 \\ 6 & 4 & 6 \\ 1 & 2 & 5 \end{bmatrix}$

then find AB.

$$AB = \begin{bmatrix} 2+24+6 & 4+16+12 & 10+24+30 \\ 1+12+5 & 2+8+10 & 5+12+25 \\ 6+6+3 & 12+4+6 & 30+6+15 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 32 & 64 \\ 18 & 20 & 42 \\ 15 & 22 & 51 \end{bmatrix}$$

2. if $A = \begin{bmatrix} 1 & 4 & 6 \\ 5 & 1 & 2 \\ 6 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 6 \\ 5 & 1 & 3 \\ 2 & 4 & 1 \end{bmatrix}$ then find AB.

$$= \begin{bmatrix} 2+20+12 & 1+4+24 & 6+12+6 \\ 10+5+4 & 5+1+8 & 30+3+2 \\ 12+5+8 & 6+1+16 & 36+3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 34 & 29 & 24 \\ 19 & 14 & 35 \\ 25 & 23 & 43 \end{bmatrix}$$

3. Verify whether $AB = BA$ for the matrices

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 & 4 \\ 0 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3+0+10 & -1+0+20 & 4+0+(-10) \\ -3+0+2 & 1+(-2)+4 & -4+2+(-2) \\ 15+0+6 & -5+(-4)+12 & 20+4+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 19 & -6 \\ -1 & 3 & -4 \\ 21 & 3 & 18 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & -1 & 4 \\ 0 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1+20 & 0+(-2)+16 & 15+(-1)+12 \\ 0+1+5 & 0+(-2)+4 & 0+(-1)+3 \\ 2+(-4)+(-10) & 0+8+(-8) & 10+4+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 14 & 26 \\ 6 & 2 & 2 \\ -12 & 0 & 8 \end{bmatrix}$$

$\therefore AB \neq BA$

Applications of matrices :-

1. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ find the product matrix AB

$$AB = \begin{bmatrix} 3+4 & 1+2 \\ 3+8 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 11 & 5 \end{bmatrix}$$

2. A manufacturer produces 100 units of product x, 200 units of product y, 800 units of product z and sells in an open market. if the unit sell price of product x is ₹ 2, product y is ₹ 4 and product z is ₹ 10, find the total revenue earned by the seller with the help of product of 2 matrices.

Sol^n:- let $A = \begin{bmatrix} 100 & 200 & 800 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix}$

$$AB = \begin{bmatrix} 100 & 200 & 800 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 200 + 800 + 8000 \end{bmatrix}$$

$$AB = \begin{bmatrix} 9000 \end{bmatrix}$$

Total revenue earned ₹ 9000

3. A manufacturer produces 800 units of product x, 900 units of product y and 100 units of product z and sells in an open market. if the unit sell price of product x is ₹ 10, product y is ₹ 12 and product z is ₹ 14 and Find the total revenue earned by the sell

with the help of product of 2 matrices.

Sol^n:- AB = product × price.

$$AB = \begin{bmatrix} 800 & 900 & 100 \end{bmatrix} \begin{bmatrix} 10 \\ 12 \\ 14 \end{bmatrix}$$

$$= \begin{bmatrix} 8000 + 10800 + 1400 \end{bmatrix}$$

$$= \begin{bmatrix} 20200 \end{bmatrix}$$

∴ Total revenue earned ₹ 20200.

4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ then can we perform AB and BA? If so, write the order of AB and BA

Sol^n:- order of $A = 2 \times 3$ $\quad (m \times n)$

order of $B = 3 \times 2$ $\quad (n \times p)$

order of $AB = 2 \times 2$

order of $BA = 3 \times 3$

5. If for a matrix, $A+I=0$ were I is identity matrix of order 3×3 and 0 is null matrix of order Corresponding to A. then find A

Sol^n:- $A+I=0$

$A=0-I$

$$A=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$$

6. If for a matrix, $A+I=0$ were I is identity matrix of order 2×2 and 0 is null matrix of order Corresponding to A then find A

Sol^n:- $A+I=0$

$A=0-I$

$$A=\begin{bmatrix}0&0\\0&0\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

Determinants

Every square matrix can be associated with a number called as determinant of a square matrix

Evaluation of 2^{nd} order determinant

let $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ be a square matrices of order 2×2, then the

determinant of $|A| = \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} = [a_1 d_1 - b_1 c_1]$

Evaluation of 3^{rd} order determinant

the value of determinant of order 3 is evaluated by the expansion of determinant along a row or column. Thus, the value of determinant of order 3 can be obtained in 6 ways by expanding a determinant Corresponding to each of 3 rows and 3 column

$$\begin{array}{cc} & \begin{array}{ccc} + & - & + \end{array} \\ A = & \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} + & - & + \end{array} \\ |A| = & \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \end{array}$$

$$= +a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 [b_2 c_3 - c_2 b_3] - b_1 [a_2 c_3 - c_2 a_3] + c_1 [a_2 b_3 - b_2 a_3]$$

problems on Evaluation of 2^{nd} and 3^{rd} order determinant

1. Evaluate $\begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}$

$= [10 - (-3)]$

$= [10 + 3]$

$= [13]$

2. Find the value of $\begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix}$

$= [28 - 30]$

$= [-2]$

3. If $A = \begin{bmatrix} 1 & 6 & 7 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ Find determinant of $|A|$

$$|A| = \begin{vmatrix} \overset{+}{1} & \overset{-}{6} & \overset{+}{7} \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix} - 6\begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix}$$

$= 1[12-0] - 6[8-0] + 7[2-0]$

$= 1[12] - 6[8] + 7[2]$

$= 12 - 48 + 14$

$= -22$

4. Find the value of $\begin{bmatrix} 3 & 1 & -9 \\ -1 & 2 & 3 \\ 2 & 1 & -6 \end{bmatrix}$

5 problems on finding unknown quantity. find x. if $\begin{vmatrix} x & 1 \\ 4 & x \end{vmatrix} = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

6. Find x if $\begin{vmatrix} x & 1 \\ 16 & x \end{vmatrix} = 0$.

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 3$

$\begin{vmatrix} x & 1 \\ 9 & x \end{vmatrix} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

$\begin{vmatrix} x & 1 \\ 25 & x \end{vmatrix} = 0$ $\quad x^2 = 25$

$x = \pm 5$

$x^2 - 25 = 0$

Singular matrix :

A square matrix is set to be singular if

$|A| = 0$. Ex:- $\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 4 - 4 = 0$ is a singular

matrix because it is singular matrix.

Non - singular matrix

A square matrix is set to be non - singular

if $|A| \neq 0$. Ex:- $\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 4 + 4 = 8$ is a non

singular matrix.

problems on singular matrices.

1. If the matrix $A = \begin{bmatrix} x & 6 \\ 6 & x \end{bmatrix}$ is singular find x

$|A| = \begin{vmatrix} x & 6 \\ 6 & x \end{vmatrix} = 0$

$= x^2 - 36 = 0$

$= x^2 = 36$

$= x = \pm 6$

2. If the matrix $A = \begin{bmatrix} x & 5 \\ 5 & x \end{bmatrix}$ is singular find x.

$$|A| = \begin{vmatrix} x & 5 \\ 5 & x \end{vmatrix} = 0$$

$$= x^2 - 25 = 0$$

$$= x^2 = 25$$

$$= x = \pm 5$$

3. Find the value of x. if the matrix

$\begin{bmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{bmatrix}$ is a singular matrix.

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{vmatrix} = 0$$

$$= 2\begin{vmatrix} x & -8 \\ 6 & 7 \end{vmatrix} - 3\begin{vmatrix} -4 & -8 \\ 5 & 7 \end{vmatrix} + 4\begin{vmatrix} -4 & x \\ 5 & 6 \end{vmatrix} = 0$$

$= 2[7x - (-48)] - 3[-28 - (-40)] + 4[-24 - 5x] = 0$

$= 2[7x + 48] - 3[-28 + 40] + 4[-24 - 5x] = 0$

$= 14x + \cancel{96} - 36 - \cancel{96} - 20x = 0$

$= -6x - 36 = 0$

$= -6x = -36$

$= x = -\frac{\cancel{36}^{6}}{\cancel{6}_{1}} = -6$

$\boxed{x = -6}$

4. Find the value of x. if the matrix $\begin{bmatrix} -2 & 3 & -4 \\ -4 & x & -8 \\ -5 & -6 & -7 \end{bmatrix}$ is singular.

5. If matrix $\begin{bmatrix} 2 & 4 & 6 \\ 2 & x & 2 \\ 6 & 8 & 14 \end{bmatrix}$ is singular then find the value of x.

6. find the value of x $\begin{bmatrix} 1 & 2 & 8 \\ 9 & 10 & 12 \\ 6 & -8 & x \end{bmatrix}$ is singular.

7. Show that the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 9 \end{bmatrix}$ is singular

8. If the determinant value of the matrix

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & x \end{pmatrix} = 8$$

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & x \end{vmatrix} = 8$$

$$2[4x - 0] - 0[\ \ ] + 0[\ \ ] = 8$$

$$2[4x] = 8$$

$$8x = 8$$

$$\boxed{x = 1}$$

9. find the value of x $\begin{pmatrix} 1 & 2 & 9 \\ 2 & x & 0 \\ 3 & 7 & -6 \end{pmatrix} = 0$

$$\begin{vmatrix} 1 & 2 & 9 \\ 2 & x & 0 \\ 3 & 7 & -6 \end{vmatrix} = 0$$

$$= 1\begin{bmatrix} x & 0 \\ 7 & -6 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 3 & -6 \end{bmatrix} + 9\begin{bmatrix} 2 & x \\ 3 & 7 \end{bmatrix} = 0$$

$= 1\left[-6x - 0\right] - 2\left[-12 - 0\right] + 9\left[14 - 3x\right] = 0$

$= \cancel{6x - 0}\ 1\left[-6x\right] - 2\left[-12\right]\ 9\left[14 - 3x\right] = 0$

$= -6x + 24 + 126 - 27x$

$= -6x - 27x + 24 + 126$ $\qquad x = \frac{\cancel{150}^{\,50}}{-33}$

$= -33x + 150 = 0$ $\qquad x = \frac{50}{11}$

$= -33x = -150$

10. Evaluate $\begin{bmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} \overset{+}{2} & \overset{-}{3} & \overset{+}{-1} \\ 3 & -2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= 2\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} + (-1)\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$$

$= 2\left[-4 - 1\right] - 3\left[6 - 1\right] - 1\left[3 - (-2)\right]$

$= 2\left[-5\right] - 3\left[5\right] - 1\left[5\right]$

$= -10 - 15\ \cancel{-5}\ -5$

$= -30$

Cramer's rule

For solving system of Linear equation involving 2 and 3 variables.

4. Find the Value of $\begin{bmatrix} 3 & 1 & -9 \\ -1 & 2 & 3 \\ 2 & 1 & -6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 1 & -9 \\ -1 & 2 & 3 \\ 2 & 1 & -6 \end{vmatrix}$$

$$= 3\begin{vmatrix} 2 & 3 \\ 1 & -6 \end{vmatrix} - 1\begin{vmatrix} -1 & 3 \\ 2 & -6 \end{vmatrix} + (-9)\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= 3(-12-3) - 1(6-6) - 9[-1-4]$$

$$= 3[-15] - 1[0] - 9[-5]$$

$$= -45 - 0 + 45$$

$$= 0$$

4. Find the Value of x if the matrix $\begin{bmatrix} -2 & 3 & -4 \\ -4 & x & -8 \\ -5 & -6 & -7 \end{bmatrix}$ is Singular.

$$|A| = \begin{vmatrix} -2 & 3 & -4 \\ -4 & x & -8 \\ -5 & -6 & -7 \end{vmatrix}$$

$$= -2\begin{vmatrix} x & -8 \\ -6 & -7 \end{vmatrix} - 3\begin{vmatrix} -4 & -8 \\ -5 & -7 \end{vmatrix} + (-4)\begin{vmatrix} -4 & x \\ -5 & -6 \end{vmatrix}$$

$$= -2[-7x - 48] - 3[28 - 40] - 4[24 - (-5x)]$$

$$= 14x + \cancel{96} - 3[-12] - \cancel{96} - 20x$$

$$= -6x + 36$$

$$x = \frac{\cancel{36}^{6}}{\cancel{6}_{1}} = 6$$

5. If matrix $\begin{bmatrix} 2 & 4 & 6 \\ 2 & x & 2 \\ 6 & 8 & 14 \end{bmatrix}$ is singular then find the value of x.

$$|A| = \begin{vmatrix} 2 & 4 & 6 \\ 2 & x & 2 \\ 6 & 8 & 14 \end{vmatrix}$$

$$= 2\begin{vmatrix} x & 2 \\ 8 & 14 \end{vmatrix} - 4\begin{vmatrix} 2 & 2 \\ 6 & 14 \end{vmatrix} + 6\begin{vmatrix} 2 & x \\ 6 & 8 \end{vmatrix}$$

$$= 2[14x - 16] - 4[28 - 12] + 6[16 - 6x]$$

$$= 28x - 32 - 4[+16] + 96 - 36x$$

$$= -8x - 32 - 64 + 96$$

$$= -8x - \cancel{96} + \cancel{96}$$

$$\boxed{x = 8}$$

6. Find the value of x $\begin{bmatrix} 1 & 2 & 8 \\ 9 & 10 & 12 \\ 6 & -8 & x \end{bmatrix}$ is singular

$$|A| = \begin{vmatrix} 1 & 2 & 8 \\ 9 & 10 & 12 \\ 6 & -8 & x \end{vmatrix} = 0$$

$$\Rightarrow 1 \begin{vmatrix} 10 & 12 \\ -8 & x \end{vmatrix} - 2 \begin{vmatrix} 9 & 12 \\ 6 & x \end{vmatrix} + 8 \begin{vmatrix} 9 & 10 \\ 6 & -8 \end{vmatrix} = 0$$

$$\Rightarrow 1\,[10x - (-96)] - 2[9x - 72] + 8[-72 - 60] = 0$$

$$\Rightarrow 10x + 96 - 18x + 144 + 8[-132] = 0$$

$$\Rightarrow -8x + 240 - 1056 = 0$$

$$\Rightarrow -8x + 816 = 0$$

$$\Rightarrow x = \frac{\cancel{816}^{102}}{\cancel{8}_{1}} = 102$$

7. Show that the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is singular.

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 1\,[45 - 48] - 2[36 - 42] + 3[32 - 35]$$

$$= 1[-3] - 2[-6] + 3[-3]$$

$$= -3 + 12 - 9$$

$$= 9 - 9 = 0$$

Cramer's rule :-

1. $a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = [a_1 b_2 - b_1 a_2] \qquad x = \frac{\Delta x}{\Delta}$$

$$\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = [c_1 b_2 - b_1 c_2] \qquad y = \frac{\Delta y}{\Delta}$$

$$\Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = [a_1 c_2 - c_1 a_2]$$

2. $a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad x = \frac{\Delta x}{\Delta}$$

$$y = \frac{\Delta y}{\Delta}$$

$$\Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \qquad Z = \frac{\Delta Z}{\Delta}$$

problems on solving saimultainious linear equation with 2 unknowns.

1. Solve the equations $2x + y = 1$

$3x + 2y = 1$

$2x + y = 1$

$a_1 x + b_1 y = c_1$

$$\Delta = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = [4 - 3] = 1$$

$3x + 2y = 1$

$a_2 x + b_2 y = c_2$

$$\Delta_x = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = [2 - 1] = 1$$

$$\Delta_y = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = [2 - 3] = -1$$

$$x = \frac{\Delta x}{\Delta}$$

$$x = \frac{1}{1} = 1$$

$$y = \frac{\Delta y}{\Delta}$$

$$= \frac{-1}{1} = -1$$

2. Solve the equations $x + y = 3$

$2x + 3y = 8$

$x + y = 3$

$a_1 x + b_1 y = c_1$

$2x + 3y = 8$

$a_2 x + b_2 y = c_2$

$\Delta = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = [3 - 2] = 1$

$\Delta_x = \begin{vmatrix} 3 & 1 \\ 8 & 3 \end{vmatrix} = (9 - 8) = 1$

$\Delta_y = \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} = [8 - 6] = 2$

$x = \dfrac{\Delta x}{\Delta}$

$= \dfrac{1}{1} = 1$

$y = \dfrac{\Delta y}{\Delta}$

$= \dfrac{2}{1} = 2$

3. Solve the equation $2x - 3y = 5$

$7x - y = 8$

$\Delta = \begin{vmatrix} 2 & -3 \\ 7 & -1 \end{vmatrix} = [-2 - (-21)] = (-2 + 21) = 19$

$\Delta_x = \begin{vmatrix} 5 & -3 \\ 8 & -1 \end{vmatrix} = (-5 - (-24)) = (-5 + 24) = 19$

$\Delta y = \begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix} = (16 - 35] = -19$

$$x = \frac{\Delta_x}{\Delta} = \frac{19}{19} = 1$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-19}{19} = -1$$

4. Solve the equation $3I_1 + 4I_2 = 10$
$2I_1 - 3I_2 = 1$

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix} = [-9 - 8] = -17$$

$$I_1 = \frac{\Delta_{I_1}}{\Delta} = \frac{\cancel{-34}^{2}}{\cancel{-17}_{1}} = 2$$

$$\Delta_{I_1} = \begin{vmatrix} 10 & 4 \\ 1 & -3 \end{vmatrix} = (-30 - 4) = -34$$

$$I_2 = \frac{\Delta_{I_2}}{\Delta} = \frac{\cancel{-17}^{1}}{\cancel{-17}_{1}} = 1$$

$$\Delta_{I_2} = \begin{vmatrix} 3 & 10 \\ 2 & 1 \end{vmatrix} = (3 - 20) = -17$$

5. Solve the equation $2I_1 - I_2 = 3$
$I_1 + 2I_2 = 4$

$$\Delta = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = (4 - (-1)) = (4 + 1) = 5$$

$$\Delta_{I_1} = \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = (6 - (-4)) = [6 + 4] = 10$$

$$I_1 = \frac{\Delta_{I_1}}{\Delta} = \frac{\cancel{10}^{2}}{\cancel{5}_{1}} = 2$$

$$\Delta_{I_2} = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = [8 - 3] = 5$$

$$I_2 = \frac{\Delta_{I_2}}{\Delta} = \frac{\cancel{5}^{1}}{\cancel{5}_{1}} = 1$$

problems on solving saimultanious with 3 Unknowns

solve the following equations using cramer's rule.

$$x + y + z = 7$$

$$2x + 3y + 2z = 17$$

$$4x + 9y + z = 37$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 9 & 1 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} 7 & 1 & 1 \\ 17 & 3 & 2 \\ 37 & 9 & 1 \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 17 & 2 \\ 4 & 37 & 1 \end{vmatrix} \qquad \Delta_z = \begin{vmatrix} 1 & 1 & 7 \\ 2 & 3 & 17 \\ 4 & 9 & 37 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 9 & 1 \end{vmatrix} = 1 \begin{vmatrix} 3 & 2 \\ 9 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix}$$

$$= 1[3-18] - 1[2-8] + 1[18-12]$$

$$= -15 + 1(-6) + 1(6)$$

$$= -15 + 6 + 6$$

$$= -3$$

$$\Delta_x = \begin{vmatrix} 7 & 1 & 1 \\ 17 & 3 & 2 \\ 37 & 9 & 1 \end{vmatrix} = 7\begin{vmatrix} 3 & 2 \\ 9 & 1 \end{vmatrix} - 1\begin{vmatrix} 17 & 2 \\ 37 & 1 \end{vmatrix} + 1\begin{vmatrix} 17 & 3 \\ 37 & 9 \end{vmatrix}$$

$$= 7[3-18] - 1[17-74] + 1[153-111]$$

$$= 7[-15] - 1[-57] + 1[42]$$

$$= -105 + 57 + 42$$

$$= -105 + 99$$

$$= -6$$

$$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 17 & 2 \\ 4 & 37 & 1 \end{vmatrix} = 1\begin{vmatrix} 17 & 2 \\ 37 & 1 \end{vmatrix} - 7\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 17 \\ 4 & 37 \end{vmatrix}$$

$$= 1[17-74] - 7[2-8] + 1[74-68]$$

$$= 1[-57] - 7[-6] + 1[6]$$

$$= -57 + 42 + 6$$

$$= -15 + 6$$

$$= -9$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 7 \\ 2 & 3 & 17 \\ 4 & 9 & 37 \end{vmatrix} = 1\begin{vmatrix} 3 & 17 \\ 9 & 37 \end{vmatrix} - 1\begin{vmatrix} 2 & 17 \\ 4 & 37 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix}$$

$= 1[111 - 153] - 1[74 - 68] + 7[18 - 12]$

$= 1[-42] - 1[6] + 7[6]$

$= \cancel{-42} - 6 + \cancel{42}$

$= -6$

$x = \frac{\Delta x}{\Delta} = \frac{\cancel{-6}^{2}}{\cancel{-3}_{1}} = 2$

$y = \frac{\Delta y}{\Delta} = \frac{\cancel{-9}^{3}}{\cancel{-3}_{1}} = 3$

$z = \frac{\Delta z}{\Delta} = \frac{\cancel{-6}^{2}}{\cancel{-3}_{1}} = 2$

2. Solve the equation

$$\begin{aligned} 2x + y - z &= 3 \\ x + y + z &= 1 \\ x - 2y - 3z &= 4 \end{aligned}$$

$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix}$ $\quad \Delta x = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix}$

$\Delta y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix}$ $\quad \Delta z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2\,[-3-(-2)] - 1\,[-3-1] - 1\,[-2-1]$$

$$= 2\,[-1] - 1\,[-4] - 1\,[-3]$$

$$= -2 + 4 + 3$$

$$= 5$$

$$\Delta_x = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix} = 3\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 4 & -2 \end{vmatrix}$$

$$= 3\,(-3-(-2)) - 1\,[-3-4] - 1\,[-2-4]$$

$$= 3\,[-1] - 1\,[-7] - 1\,[-6]$$

$$= -3 + 7 + 6$$

$$= 10$$

$$\Delta_y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix}$$

$$= 2\,[-3-4] - 3\,[-3-1] - 1\,[4-1]$$

$$= 2\,[-7] - 3\,[-4] - 1\,[3]$$

$$= -14 + 12 - 3 = -5$$

$$\Delta_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 \\ -2 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2[4-(-2)] - 1[4-1] + 3[-2-1]$$

$$= 2[6] - 1[3] + 3[-3]$$

$$= 12 - 3 - 9$$

$$= 9 - 9 = 0$$

$$x = \frac{\Delta_x}{\Delta} = \frac{\cancel{10}^{2}}{\cancel{5}_{1}} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-\cancel{5}^{1}}{\cancel{5}_{1}} = -1$$

$$Z = \frac{\Delta_z}{\Delta} = \frac{0}{5} = 0$$

In a mesh analysis formulation the following equatio[ns] are obtained $4i_1 + 2i_2 = 4$

$i_1 + i_2 = 2$

$4i_1 + 2i_2 = 4$

$a_1 x + b_1 y = c_1$

$1i_1 + 1i_2 = 2$

$a_2 x + b_2 x = c_2$

$$\Delta = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} = [4-2] = 2 \qquad i_1 = \frac{\Delta i}{\Delta} = \frac{0}{2} = 0$$

$$\Delta_i = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} = [4-4] = 0 \qquad i_2 = \frac{\Delta i_2}{\Delta} = \frac{\cancel{4}^{2}}{\cancel{2}_{1}} = 2$$

$$\Delta_{i_2} = \begin{bmatrix} 4 & 4 \\ 1 & 2 \end{bmatrix} = [8-4] = 4$$

Adjoint of a matrix :

The adjoint of a square matrix $A = [a_{ij}]$ of order n is the transpose of the matrix of order n with (A_{ij}) is the co-factor of the element $A = [a_{ij}]$ adjoint of the matrix A is denoted by adjoint of A

Ex:- 1. $A = \begin{bmatrix} \overset{+}{2} & \overset{-}{4} \\ \overset{-}{1} & \overset{+}{8} \end{bmatrix}$ $\quad$ $adj(A) = \begin{bmatrix} 8 & -1 \\ -4 & 2 \end{bmatrix}^T$

C.F of 2 = +8

C.F of 4 = -1

C.F of 1 = -4

C.F of 8 = +2

$adj(A) = \begin{bmatrix} 8 & -4 \\ -1 & 2 \end{bmatrix}$

problems on finding adjoint of a matrices.

1. If $A = \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$

$adj(A) = \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$

2 $A = \begin{bmatrix} 4 & -5 \\ 3 & -2 \end{bmatrix}$

$\operatorname{adj}(A) = \begin{bmatrix} -2 & 5 \\ -3 & 4 \end{bmatrix}$

3 for the matrix $A = \begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$ to find adjoint of A

$\operatorname{adj}(A) = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

4. Find the adjoint of the matrix $A = \begin{bmatrix} 4 & 2 & 6 \\ 1 & 8 & 5 \\ 4 & 2 & 9 \end{bmatrix}$

$$A = \begin{bmatrix} \overset{+}{4} & \overset{-}{2} & \overset{+}{6} \\ \overset{-}{1} & \overset{+}{8} & \overset{-}{5} \\ \overset{+}{4} & \overset{-}{2} & \overset{+}{9} \end{bmatrix}$$

$$\text{C.F of } 4 = + \begin{vmatrix} 8 & 5 \\ 2 & 9 \end{vmatrix} = +[72 - 10] = 62$$

$$\text{C.F of } 2 = - \begin{vmatrix} 1 & 5 \\ 4 & 9 \end{vmatrix} = -[9 - 20] = -[-11] = +11$$

$$\text{C.F of } 6 = + \begin{vmatrix} 1 & 8 \\ 4 & 2 \end{vmatrix} = +[2 - 32] = -30$$

$$\text{C.F of } 1 = -\begin{vmatrix} 2 & 6 \\ 2 & 9 \end{vmatrix} = -[18-12] = -6$$

$$\text{C.F of } 8 = +\begin{vmatrix} 4 & 6 \\ 4 & 9 \end{vmatrix} = +[36-24] = +12$$

$$\text{C.F of } 5 = -\begin{vmatrix} 4 & 2 \\ 4 & 2 \end{vmatrix} = -[8-8] = 0$$

$$\text{C.F of } 4 = +\begin{vmatrix} 2 & 6 \\ 8 & 5 \end{vmatrix} = [10-48] = -38$$

$$\text{C.F of } 2 = -\begin{vmatrix} 4 & 6 \\ 1 & 5 \end{vmatrix} = -[20-6] = -14$$

$$\text{C.F of } 9 = +\begin{vmatrix} 4 & 2 \\ 1 & 8 \end{vmatrix} = [32-2] = 30$$

$$\text{adj}(A) = \begin{bmatrix} 62 & 11 & -30 \\ -6 & 12 & 0 \\ -38 & -14 & 30 \end{bmatrix}^T$$

$$\text{adj}(A) = \begin{bmatrix} 62 & -6 & -38 \\ 11 & 12 & -14 \\ -30 & 0 & 30 \end{bmatrix}$$

Find the adjoint of the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & -3 & 1 \\ 0 & 4 & 2 \end{bmatrix}$.

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & -3 & 1 \\ 0 & 4 & 2 \end{bmatrix}$$

$$\text{C.F of } 3 = +\begin{vmatrix} -3 & 1 \\ 4 & 2 \end{vmatrix} = +[-6-4] = -10$$

$$\text{C.F of } -1 = -\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = -[4-0] = -4$$

$$\text{C.F of } 2 = +\begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix} = +[8-0] = 8$$

$$\text{C.F of } 2 = -\begin{vmatrix} -1 & 2 \\ 4 & 2 \end{vmatrix} = -[-2-8] = +10$$

$$\text{C.F of } -3 = +\begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix} = +[6-0] = 6$$

$$\text{C.F of } 1 = -\begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix} = -[12-0] = -12$$

$$\text{C.F of } 0 = +\begin{vmatrix} -1 & 2 \\ -3 & 1 \end{vmatrix} = +[-1+6] = 5$$

$$C.F \text{ of } 4 = -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -[3-4] = +1$$

$$C.F \text{ of } 2 = +\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} = +[-9+2] = -7$$

$$\operatorname{adj}(A) = \begin{bmatrix} -10 & -4 & 8 \\ 10 & 6 & -12 \\ 5 & 1 & -7 \end{bmatrix}^T$$

$$\operatorname{adj}(A) = \begin{bmatrix} -10 & 10 & 5 \\ -4 & 6 & 1 \\ 8 & -12 & -7 \end{bmatrix}$$

6 Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & 10 \\ 2 & 4 & 2 \end{bmatrix}$

$$C.F \text{ of } 1 = +\begin{vmatrix} 8 & 10 \\ 4 & 2 \end{vmatrix} = [16-40] = -24$$

$$C.F \text{ of } 2 = -\begin{vmatrix} 4 & 10 \\ 2 & 2 \end{vmatrix} = -[8-20] = -[-12] = 12$$

$$C.F \text{ of } 6 = +\begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = [16-16] = 0$$

$$C.F \text{ of } 4 = -\begin{vmatrix} 2 & 6 \\ 4 & 2 \end{vmatrix} = -[4-24] = -[-20] = 20$$

$C.F \text{ of } 8 = +\begin{vmatrix} 1 & 6 \\ 2 & 2 \end{vmatrix} = [2-12] = -10$

$C.F \text{ of } 10 = -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -[4-4] = 0$

$C.F \text{ of } 2 = +\begin{vmatrix} 2 & 6 \\ 8 & 10 \end{vmatrix} = [20-48] = -12$

$C.F \text{ of } 4 = -\begin{vmatrix} 1 & 6 \\ 4 & -10 \end{vmatrix} = -[10-24] = -[-14] = 14$

$C.F \text{ of } 2 = +\begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = [8-8] = 0$

$$\text{adj}(A) = \begin{bmatrix} -24 & 12 & 0 \\ 20 & -10 & 0 \\ -12 & 14 & 0 \end{bmatrix}^T$$

$$\text{adj}(A) = \begin{bmatrix} -24 & 20 & -12 \\ 12 & -10 & 14 \\ 0 & 0 & 0 \end{bmatrix}$$

Inverse of a matrix :-

Formula of inverse of a matrix

$$= A^{-1} = \frac{adj(A)}{|A|}$$

problems on finding inverse of a matrix

1. $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

$adj(A) = \begin{bmatrix} 8 & -4 \\ -6 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$

$= 16 - 24 = -8$

$A^{-1} = \frac{adj(A)}{|A|}$

$= \frac{\begin{bmatrix} 8 & -4 \\ -6 & 2 \end{bmatrix}}{-8}$

$= \begin{bmatrix} 8/-8 & -4/-8 \\ -6/-8 & 2/-8 \end{bmatrix} = \begin{bmatrix} -1 & 1/2 \\ -3/4 & -1/4 \end{bmatrix}$

2. if $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$adj(A) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= [4-(-1)]\ 4+1 = 5$

$A^{-1} = \frac{adj(A)}{|A|}$

$= \frac{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}{5}$

$= \begin{bmatrix} 2/5 & 1/5 \\ 1/5 & 2/5 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 4 & -6 \end{bmatrix}$

$adj(A) = \begin{bmatrix} -6 & -2 \\ -4 & +1 \end{bmatrix}$

$|A| = \begin{bmatrix} 1 & 2 \\ 4 & -6 \end{bmatrix}$

$= (-6 - 8) = -14$

$A^{-1} = \dfrac{adj(A)}{|A|}$

$= \dfrac{\begin{bmatrix} -6 & -2 \\ -4 & 1 \end{bmatrix}}{-14}$

4. if $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ find the product matrix AB and hence find inverse matrix if it exists

$A = \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$AB = \begin{bmatrix} 3+4 & 1+2 \\ 3+8 & 1+4 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 11 & 5 \end{bmatrix}$

$adj(AB) = \begin{bmatrix} 5 & -3 \\ -11 & 7 \end{bmatrix}$

$|AB| = \begin{bmatrix} 7 & 3 \\ 11 & 5 \end{bmatrix} = 35 - 33 = 2$

$(AB)^{-1} = \dfrac{adj(AB)}{|AB|}$

$= \dfrac{\begin{bmatrix} 5 & -3 \\ -11 & 7 \end{bmatrix}}{2}$

5. If $A = \begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix}$ find the inverse of the matrix

$adj(A) = \begin{bmatrix} 2 & -6 \\ -1 & 5 \end{bmatrix}$

$A^{-1} = \frac{adj(A)}{|A|}$

$= \frac{\begin{bmatrix} 2 & -6 \\ -1 & 5 \end{bmatrix}}{4}$

$|A| = \begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix} = (10-6) = 4$

Find the inverse of the matrix

1. If $A = \begin{bmatrix} \overset{+}{1} & \overset{-}{-1} & \overset{+}{2} \\ \overset{-}{2} & \overset{+}{1} & \overset{-}{1} \\ \overset{+}{4} & \overset{-}{-1} & \overset{+}{-2} \end{bmatrix}$ find A^{-1}.

C.F of $1 = +\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = [-2-(-1)] = -2+1 = -1$

C.F of $-1 = -\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} = -[-4-4] = -[-8] = 8$

C.F of $2 = +\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = [2-4] = -2$

C.F of $2 = -\begin{vmatrix} -1 & 2 \\ -1 & -2 \end{vmatrix} = -[2-(-2)] = -(2+2) = -4$

C.F of $1 = +\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = [-2-8] = -10$

C.F of $1 = -\begin{vmatrix} \cancel{1} & -1 \\ 4 & -1 \end{vmatrix} = -[-1-(-4)] = -[-1+4] = -[3]$

$$C.F \text{ of } 4 = +\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = [-1-2] = -3$$

$$C.F \text{ of } -1 = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -[1-4] = -[-3] = 3$$

$$C.F \text{ of } -2 = +\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = [1-(-2)] = 1+2 = 3$$

$$adj(A) = \begin{bmatrix} -1 & 8 & -2 \\ -4 & -10 & -3 \\ -3 & 3 & 3 \end{bmatrix}^T \quad adj(A) = \begin{bmatrix} -1 & -4 & -3 \\ 8 & -10 & 3 \\ -2 & -3 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 8 & -2 \\ -4 & -10 & -3 \\ -3 & 3 & 3 \end{vmatrix} = -1\begin{vmatrix} -10 & -3 \\ 3 & 3 \end{vmatrix} - 8\begin{vmatrix} -4 & -3 \\ -3 & 3 \end{vmatrix} + (-2)\begin{vmatrix} -4 & -10 \\ -3 & 3 \end{vmatrix}$$

$$= -1(-30-(-9)) - 8(-12-9) - 2(-12-30)$$

$$= -1(-29) - 8(-21) - 2(-42)$$

$$= 29 + 168 + 84$$

$$|A| = 281$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$= \frac{\begin{bmatrix} -1 & -4 & -3 \\ 8 & -10 & 3 \\ -2 & -3 & 3 \end{bmatrix}}{281}$$

2 $A = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$

$C.F \text{ of } 3 = +\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = [2-0] = 2$

$C.F \text{ of } 1 = -\begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} = -[-4-3] = +7$

$C.F \text{ of } 2 = +\begin{vmatrix} -2 & 1 \\ 3 & 0 \end{vmatrix} = [0-3] = -3$

$C.F \text{ of } -2 = -\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = -(2-0] = -2$

$C.F \text{ of } 1 = +\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = [6-6] = 0$

$C.F \text{ of } 1 = -\begin{vmatrix} 3 & 1 \\ 3 & 0 \end{vmatrix} = -[0-3] = +3$

$C.F \text{ of } 3 = +\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = [1-2] = -1$

$C.F \text{ of } 0 = -\begin{vmatrix} 3 & 2 \\ -2 & 1 \end{vmatrix} = -[3+4] = -7$

$$C.F \text{ of } 2 = +\begin{vmatrix} 3 & 1 \\ -2 & 1 \end{vmatrix} = (3+2) = 5$$

$$\operatorname{adj}(A) = \begin{bmatrix} 2 & 7 & -3 \\ -2 & 0 & 3 \\ -1 & -7 & 5 \end{bmatrix}^T$$

$$\operatorname{adj}(A) = \begin{bmatrix} 2 & -2 & -1 \\ 7 & 0 & -7 \\ -3 & 3 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & 2 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} + 2\begin{vmatrix} -2 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= 3\,[2-0] - 1\,[-4-3] + 2\,[0-3]$$

$$= 6 + 7 - 6$$

$$= 7$$

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & -2 & -1 \\ 7 & 0 & -7 \\ -3 & 3 & 5 \end{bmatrix}}{7}$$

3. If $A = \begin{vmatrix} -1 & 0 \\ 5 & 3 \end{vmatrix}$ & $B = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$ prove that adj(AB) = adj (B) + (A)

$$AB = \begin{bmatrix} -3+0 & -5+0 \\ 15+6 & 25+12 \end{bmatrix} \qquad \text{adj}(B) = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -5 \\ 21 & 37 \end{bmatrix} \qquad \text{adj}(A) = \begin{bmatrix} 3 & -0 \\ -5 & -1 \end{bmatrix}$$

$$\text{adj}(AB) = \begin{bmatrix} 37 & 5 \\ -21 & -3 \end{bmatrix} \qquad \text{adj}(B)+\text{adj}(A) = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 12+25 & 0+5 \\ -6-15 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 37 & 5 \\ -21 & -3 \end{bmatrix}$$

adj(AB) = adj(B) adj(A)

$$\begin{bmatrix} 37 & 5 \\ -21 & -3 \end{bmatrix} = \begin{bmatrix} 37 & 5 \\ -21 & -3 \end{bmatrix}$$

hence proved.

Characteristic equation and eigen values of square matrix of order 2

* Characteristic equation of matrix : Let A be a square matrix of order n then $|A - \lambda I| = 0$ is called characteristic equation of matrix A where I is identity matrix of order n and λ is a constant.

* Characteristic roots of matrix :- Let A be a square matrix of order n then the roots of the characteristic

equation $|A - \lambda I| = 0$ of A are called characteristic roots or eigen values or latent roots of matrix A.

Find the characteristic equation and characteristic roots

1. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ Find the characteristic equation.

$|A - \lambda I| = 0$

$\left| \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$

$\left| \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$

$\begin{vmatrix} (1-\lambda) & 2 \\ 4 & (6-\lambda) \end{vmatrix} = 0$

$(1-\lambda)(6-\lambda) - 8 = 0$

$6 - 1\lambda - 6\lambda + \lambda^2 - 8 = 0$

$\lambda^2 - 7\lambda - 2 = 0$

2. Find the characteristic equation $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$

$\left| \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$

$\begin{vmatrix} (1-\lambda) & 2 \\ -3 & (4-\lambda) \end{vmatrix} = 0$

$(1-\lambda)(4-\lambda) - (-6) = 0$

$4 - 1\lambda - 4\lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 5\lambda + 10 = 0$

3\. Find the characteristic equation and roots of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$A - \lambda I = 0$$

$$\left|\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 0 & 4-\lambda \end{vmatrix} = 0$$

$$\left|\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix} - \lambda \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 0 & (4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - 0 = 0$$

$$8 - 2\lambda - 4\lambda + \lambda^2 = 0$$

$$1\lambda^2 - 6\lambda + 8 = 0$$

$$\lambda^2 - 2\lambda - 4\lambda + 8 = 0$$

$$\lambda(\lambda - 2) - 4(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 4) = 0$$

8

$-2 \quad -4$

$$\lambda - 2 = 0 \qquad \lambda - 4 = 0$$

$$\boxed{\lambda = 2} \qquad \boxed{\lambda = 4}$$

$ax^2 + bx + c = 0$

$1\lambda^2 - 6\lambda + 8 = 0$

$ax^2 + bx + c = 0$

$a = 1, b = -6, c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

$$\lambda = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$\lambda = \frac{6 \pm \sqrt{4}}{2}$$

$$\lambda = \frac{6 \pm 2}{2}$$

$$\lambda = \frac{6+2}{2} = \cancel{8}^4/\cancel{2}, = 4$$

$\boxed{\lambda = 4}$

$$\lambda = \frac{6-2}{2} = \cancel{4}^2/\cancel{2}_1 = 2$$

$\boxed{\lambda = 2}$

Find the characteristic equation and eigen values for the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

$$A\lambda I - \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2-\lambda & -1 \\ -3 & 1-\lambda \end{bmatrix}$$

$[A - \lambda I] = 0$

$$\begin{bmatrix} 2-\lambda & -1 \\ -3 & 1-\lambda \end{bmatrix} = 0$$

$(2-\lambda)(1-\lambda) - 3 = 0$

$2 - 2\lambda - \lambda + \lambda^2 - 3 = 0$

$\lambda^2 - 3\lambda - 1 = 0$

$a = 1, b = -3$ & $c = -1$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$\lambda = \frac{3 \pm \sqrt{9+4}}{2} - \frac{3 \pm \sqrt{13}}{2}$$

$$\lambda = \frac{3+\sqrt{13}}{2} \ \& \ \lambda = \frac{3-\sqrt{13}}{2}$$

1/6/2022 Wednesday

STRAIGHT LINES

Slope of a straight line :- To define that first it is essential to understand the meaning of straight line and inclination of a line

Straight line :- is a curve such that the line segment joining any two point on it lies completely on it.

$m = \tan\theta$

Inclination of a line : inclination of a line of an angle θ made by it with positive direction of x-axis measured in anti-clock-wise

The angle of inclination of a line takes the values given by zero less than equal to a less than or equal to π

Slope of a line : If the angle θ is the inclination of a line then the tangent of an angle θ i.e $\tan\theta$ is called as the slope of line usually slope of a line is denoted by m ∴ slope of the line

$m = \tan\theta$

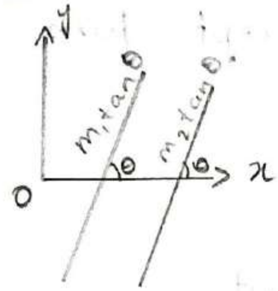
Identifications:-

* If θ is given then slope $m = \tan\theta$.
* If the given straight line passing through two point $A(x_1, y_1)$ and $B(x_2, y_2)$

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

* If θ is $0°$ then $m = \tan\theta$ $\therefore m = \tan 0, m = 0$ and
* $\therefore$ the given line is parallel to x axis and perpendicular to y axis.
* If $\theta = 90°$ then $m = \tan\theta$ $\therefore m = \tan 90°, m = ND$.
* The given line is parallel to y axis and $\perp^{er}$ to x-axis
* If 2 straight parallel then this slopes are equal

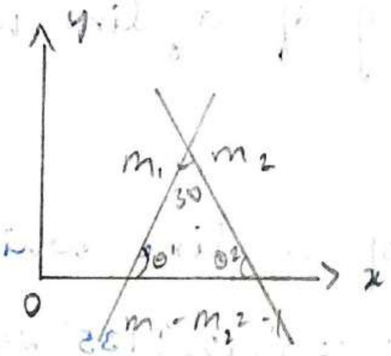
i.e. $m_1 = m_2$



$m_1 = m_2$

* If 2 straight line are perpendicular then the product of this slope is equal to $-m$

i.e $m_1 m_2 = -1$



Equation of the straight line:

- If the slope is given of an angle and Y intercept is given then $y = mx + c$

  where, m is slope

  c is y intercept

- If slope of an angle and point (x_1, y_1) is given then $(y - y_1) = m(x - x_1)$

- If two points are given $A(x_1, y_1)$ and $B(x_2, y_2)$ then $(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) (x_2 - x_1)$

- Intercept form of a straight line. the equation of straight line in intercept form is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

where, a - x intercept

b - y intercept

type - 1

problems on finding of a line when inclination or angle is given. 1.

Find the slope of a line which makes an angle $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 135^\circ, 240^\circ, 210^\circ$

$m = \tan\theta$ $\quad$ $m = \tan 30$ $\quad$ $\tan\theta = 0$

$m = \tan 0$ $\quad$ $m = \frac{1}{\sqrt{3}}$ $\quad$ $\tan 30 = \frac{1}{\sqrt{3}}$

$m = 0$

$\tan 45^\circ = 1$

$m = \tan 45^\circ$ $\quad$ $m = \tan 60$ $\quad$ $\tan 60 = \sqrt{3}$

$m = 1$ $\quad$ $m = \sqrt{3}$ $\quad$ $\tan 90 = ND$

$\tan(120)$ $\quad$ $\tan(150)$ $\quad$ $\tan(135)$

$\tan(180^\circ - 60)$ $\quad$ $\tan(180^\circ - 30)$ $\quad$ $= \tan(180 - 45)$

$= -\tan 60^\circ$ $\quad$ $= -\tan 30^\circ$ $\quad$ $= -\tan 45$

$= -\sqrt{3}$ $\quad$ $= -\frac{1}{\sqrt{3}}$ $\quad$ $= -1$

$\tan(240)$ $\quad$ $\tan(210)$

$= \tan 60$ $\quad$ $= \tan(180^\circ - 30) = \tan 30^\circ$

$= \sqrt{3}$ $\quad$ $= \frac{1}{\sqrt{3}}$

type - 2
problems on finding slope of a line joining two points

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

1. Find the slope of line joing two point $(2, 4)$ & $(6, 8)$

$(x_1, y_1) = (2, 4)$

$(x_2, y_2) = (6, 8)$

$$m = \frac{(8-4)}{(6-2)}$$

$$= \frac{4}{4} = 1$$

2. Find the slope of line passing throw the points $(2,4)$ and $(8,7)$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$(x_1, y_1) = (2, 4)$

$(x_2, y_2) = (8, 7)$

$$= \frac{7-4}{8-2} = \frac{\cancel{3}^1}{\cancel{6}_2} = \frac{1}{2}$$

3. Find the slope of the joining to the points $(1,-2)$ & $(3,4)$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(4-(-2))}{(3-1)} = \frac{4+2}{2} = \frac{\cancel{6}^3}{\cancel{2}_1} = 3$$

Find the equation of the straight line passing throw the points $(1,2)$ & $(3,5)$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{5-2}{3-1} = \frac{3}{2}$$

Type - 3

problems on finding of a line when inclination of a line is given

1. Find x & y intercept of the line

$$3x + 2y = 6$$

$$\div 6$$

$$\frac{3}{6}x + \frac{2}{6}y = \frac{6}{6}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{3} = 1$$

$a \to x$ - intercept

$b \to y$ - intercept

$\therefore$ 2 is ~~xy~~ intercept &

3 is y intercept

Find the x ~~intecp~~ intercept of the line

$$2x + 3y = 10$$

$$\div 10$$

$$\frac{2}{10}x + \frac{3}{10}y = \frac{10}{10}$$

$$\frac{x}{5} + \frac{y}{(10/3)} = 1$$

x - intercept $= 5$

y - intercept $= 10/3$

Write the standard intercept form of the straight line and hence find the equation of the straight line whose x & y intercept are 2 & 3 respectively.

standard intercept form ~~of~~ straight line $= \frac{x}{a} + \frac{y}{b} = 1$

$$= \frac{x}{2} + \frac{y}{3} = 1$$

$$\frac{x}{2} + \frac{y}{3} = 1$$

$$\frac{3x+2y}{6} = 1$$

$$3x + 2y = 6$$

$2\,|\,2, 3$
$3\,|\,1, 3$
$1, 1$
$2 \times 3 = 6$

2) $6\,(\,3x_1$; $\frac{6}{3}$

3) $6\,(\,2x_1$; $\frac{6}{2}$

4 Find the equation of straight line whose x intercept and y intercept or 3 & 4 respectively. by writing the standard form of it.

$$m = \frac{x}{a} + \frac{y}{b} = 1 \rightarrow \text{general Form}$$

$$= \frac{x}{3} + \frac{y}{4} = 1$$

$$= \frac{4x+3y}{12} = 1$$

$$= 4x + 3y = 12$$

$3\,|\,3, 4$
$4\,|\,1, 4$
$1, 1$
$3 \times 4 = 12$

5 Find the equation of the straight line ~~who~~. in intercept form with x intercept 2 units & y intercept 3 units

$$= \frac{x}{2} + \frac{y}{3} = 1$$

$$= \frac{3x+2y}{6} = 1$$

$$= 3x + 2y = 6$$

$2\,|\,2, 3$
$3\,|\,1, 3$
$1, 1$
$2 \times 3 = 6$

6. Find the equation of the straight line whose x intercept is 4 and y intercept is 1

$$\frac{x}{4} + \frac{y}{1} = 1$$

$$\frac{x + 4y}{4} = 1$$

$$x + 4y = 4$$

$4 \underline{(4, 1}$

$1, 1$

$4 \times 1 = 4$

7. Find the equation of a straight line whose x intercept is -2 and y intercept is 4

$$\frac{x}{-2} + \frac{y}{4} = 1$$

$$\frac{4x + (-2y)}{-4} = 1$$

$$4x - 2y = 4$$

$2 \underline{(2, 4}$

$2 \underline{(1, 2}$

$1 \quad 1$

$-2 \times 2 = -4$

8. Find the x intercept and y intercept of the line

$$3x + 2y = 6$$

$$-6$$

$$\frac{3}{6}x + \frac{2}{6}y = \frac{6}{6}$$

$$\frac{x}{2} + \frac{y}{3} = 1$$

x - intercept $\to$ 2

y = intercept $\to$ 3

problems on slope intercepted of straight form line $y = mx + c$

1. Find the equation of the line whose slope is 2 and y intercept 4.

$$y = mx + c$$

$$y = 2x + 4$$

$$m = 2$$

$$c = 4$$

2. Find the equation of the side line whose slope is $\frac{1}{3}$ and y intercept is 6.

$$y = mx + c$$

$$y = \frac{1}{3}x + 6$$

$$y = \frac{x + 18}{3}$$

$$3y = x + 18$$

$$x + 18 = 3y$$

$$x + 18 - 3y = 0$$

$$x - 3y + 18 = 0$$

3. Find the equation of the straight line whose intercept is 5 and makes an angle of 45.

$c = 5$ $\quad$ $y = mx + c$

$\theta = 45^\circ$ $\quad$ $y = 1x + 5$

$m = \tan\theta$ $\quad$ $y = x + 5$.

$m = \tan 45^\circ$

$m = 1$

4. The equation of a line whose inclination is 45° with positive x-axis and passing through the origin.

Note ! when the line is passing through the origin on x-axis its y intercept is zero.

$y = mx + c$ $\quad$ $y = 1x + 0$

$m = \tan\theta$ $\quad$ $y = x$.

$m = \tan 45$

$m = 1$

5. Find the equation of the straight line which has angle of inclination 45° with x-axis and y intercept of 2 units by writing its standard form.

$\theta = 45^\circ$ $\quad$ $y = mx + c$

$c = 2$ $\quad$ $y = 1x + 2$

$m = \tan\theta$ $\quad$ $y = x + 2$

$m = 1$ $\quad$ $x - y + 2 = 0$

problems on slope-~~form~~ point form of straight line 3

$(y - y_1) = m(x - x_1)$

1. Find the equation of line passing through the point (2,4) having slope 9.

$(x_1, y_1) = (2,4)$

$m = 9$

$(y - y_1) = m(x - x_1)$

$(y - 4) = 9(x - 2)$

$y - 4 = 9x - 18$

$9x - 18 = y - 4$

$9x - 18 - y + 4 = 0$

$9x - y - 14 = 0$

2 Find the equation of the straight line passing through the points (4,5) with Y-axis having slope 4.

$(y - y_1) = m(x - x_1)$

$(y - 5) = 4(x - 4)$

$y - 5 = 4x - 16$

$4x - 16 = y - 5$

$4x - 16 - y + 5 = 0$

$4x - 11 - y = 0$

$-\frac{a}{b}$

$-\frac{c}{a}$

$-\frac{c}{b}$

$ax + by + c = 0$

3

3. Find the equation of the straight line passing through the points (1, 2) with x - axis and slope $\theta = 45^\circ$.

$m = \tan 45^\circ$

$= 1$

$(y - y_1) = m(x - x_1)$

$(y - 2) = 1(x - 1)$

$y - 2 = x - 1$

~~y-2~~ $x - 1 = y - 2$

$x - 1 - y + 2 = 0$

$x - y + 1 = 0$

Find the equation of the straight line whose angle of inclination and passing through the origin. $(x_1, y_1) = (0, 0)$ slope $m = \tan 45^\circ$

$(y - y_1) = m(x - x_1)$

$(y - 0) = 1(x - 0)$

$y - 0 = 1x - 0$

$1x - 0 - y + 0 = 0$

$1x - y = 0$

If a straight line is inclined at an angle of 135° with the positive direction of x - axis than what is its slope. Further if the same line passes the point (1, 2) find its equation.

$\theta = 135$

$m = \tan\theta$

$m = \tan 135$

$m = \tan(180 - 45)$

$m = -\tan 45$

$m = -1$

$(x_1, y_1) = (1, 2)$

$(y - y_1) = m(x - x_1)$

$(y - 2) = -1(x - 1)$

$y - 2 = -1x + 1$

$-x + 1 = y - 2$

$-x + 1 - y + 2 = 0$

$-x - y + 3 = 0 \times -$

$x + y - 3 = 0$

* problems on ~~equation~~ of two point form of straight line $(y - y_1) = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

1. Find the equation of the straight line passing through the point (2, 4) and (5, 9)

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$(x_1, y_1) = 2, 4$

$(x_2, y_2) = 5, 9$

$$(y - 4) = \frac{(9 - 4)}{(5 - 2)}(x - 2)$$

$$(y - 4) = \frac{5}{3}(x - 2)$$

$$3(y - 4) = 5(x - 2)$$

$$3y - 12 = 5x - 10$$

$$5x - 10 = 3y - 12$$

$$5x - 3y - 10 + 12 = 0$$

$$5x - 3y + 2 = 0$$

Find the equation of the straight line passing through the points $(2, 5)$ and $(-3, 2)$

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$$

$$(y - 5) = \frac{(2 - 5)}{(-3 - 2)} (x - 2)$$

$$(y - 5) = \frac{-3}{-5} (x - 2)$$

$$-5(y - 5) = -3(x - 2)$$

$$-5y + 25 = -3x + 6$$

$$-3x + 6 = -5y + 25$$

$$-3x + 6 + 5y - 25 = 0$$

$$-3x + 5y - 19 = 0 \quad (\times -)$$

$$3x - 5y + 19 = 0$$

problems on general form of straight line $ax + by + c = 0$

Find the slope, x - intercept and y - intercept of line $2x + 3y - 11 = 0$

m.

$2x + 3y - 11 = 0$

slope $= \frac{-a}{b}$

$ax + by + c = 0$

x - intercept $= \frac{-c}{a}$

$a = 2,\ b = 3,\ c = -11$

y - intercept $= \frac{-c}{b}$

slope $= \frac{-a}{b} = \frac{-2}{3}$

x - intercept $= \frac{-c}{a} = \frac{-(-11)}{2} = \frac{11}{2}$

y - intercept $= \frac{-c}{b} = \frac{-(-11)}{3} = \frac{11}{3}$

Find the slope, x - intercept and y - intercept of the line $3x - 4y - 2 = 0$

$ax + by + c = 0$

$3x - 4y - 2 = 0$

$a = 3,\ b = -4,\ c = -2$

slope $= \frac{-a}{b} = \frac{+3}{+4} = \frac{3}{4}$

x - intercept $= \frac{-c}{a} = \frac{-(-2)}{3} = 2/3$

y - intercept $= \frac{-c}{b} = \frac{-(-2)}{-4} = \frac{\cancel{2}}{\cancel{-4}} = \frac{1}{-2}$

Angle between two lines and conditions for lines parallel and perpendicular.

$(4, 2)\ (1, 3)$

$$(y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$$

$$(y-2) = \frac{3-2}{1-4}\,x-4$$

$$y-2 = \frac{1}{-3}\,x-4$$

$$-3(y-2) = x-4$$

$$-3y+6 = x-4$$

$$x-4 = -3y+6$$

$$x-4+3y-6 = 0$$

$$x+3y-10 = 0$$

problems on finding angle between two lines

1. Find the angle between the lines $x + 2y + 9 = 0$ and $3x + y - 7 = 0$

$x + 2y + 9 = 0$

$a = 1, b = 2$

$m_1 = \frac{-a_1}{b_1} = \frac{-1}{2}$

$3x + y - 7 = 0$

$a = 3, b = 1$

$m_2 = \frac{-a_2}{b_2} = \frac{-3}{1} = -3$

$m_1 = \frac{-a_1}{b_1}$

$m_2 = \frac{-a_2}{b_2}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\frac{1}{2} - (-3)}{1 + \left(-\frac{1}{2}\right)(-3)}\right| = \left|\frac{-\frac{1}{2} + \frac{3}{1}}{1 + \frac{3}{2}}\right|$$

$$= \left|\frac{\frac{-1+6}{2}}{\frac{2+3}{2}}\right| = \left|\frac{\frac{5}{2}}{\frac{5}{2}}\right| = 1$$

$\tan\theta = \tan 45$

$\theta = 45$

Find the angle between the lines $7x - 4y = 0$ and $3x - 11y + 5 = 0$

$$
\begin{array}{r|l}
4 & 4,\ 11 \\
\hline
11 & 1,\ 11 \\
\hline
 & 1,\ 1
\end{array}
$$

$4 \times 11 = 44$

$4)\ 44\ (\ \frac{11 \times 7}{77}$, $\frac{44}{}$

$11)\ 44\ (\ \frac{4 \times 3}{12}$, 44

$7x - 4y = 0$

$a = 7,\ b = -4$

$$m_1 = \frac{-a_1}{b_1} = \frac{+7}{+4}$$

$3x - 11y + 5 = 0$

$a = 3,\ b = -11$

$$m_2 = \frac{-a_2}{b_2} = \frac{+3}{+11}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{+7}{+4} - \left(\frac{+3}{+11}\right)}{1 + \left(\frac{7}{4}\right)\left(\frac{3}{11}\right)}\right| = \left|\frac{\frac{77 - 12}{44}}{1 + \frac{21}{44}}\right|$$

$$= \left|\frac{\frac{65}{\cancel{44}}}{\frac{44 + 21}{\cancel{44}}}\right| = \left|\frac{65}{65}\right| = 1$$

$\theta = 45^{\circ}$

Find the angle between the lines $2x+4y+6=0$ & $6x+2y+1=0$

$2x+4y+6=0$

$a=2,\ b=4$

$m_1 = \frac{-a_1}{b_1} = \frac{-\cancel{2}^1}{\cancel{4}_2} = \frac{-1}{2}$

$6x+2y+1=0$

$a=6,\ b=2$

$m_2 = \frac{-a_2}{b_2} = \frac{-\cancel{6}^3}{\cancel{2}_1} = \frac{-3}{1}$

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{\frac{-1}{2}-(-3)}{1+\frac{-1}{2}\times -3}\right|$$

$$= \left|\frac{\frac{-1+6}{2}}{\frac{2+3}{2}}\right|$$

$$= \left|\frac{\frac{5}{\cancel{2}}}{\frac{5}{\cancel{2}}}\right|$$

$= 1$

$\tan\theta = \tan 45^\circ$

$\theta = 45^\circ$

Find the angle between the lines $4x+6y+8=0$ & $6x+4y+2=0$

$4x+6y+8=0$

$a=4,\ b=6$

$m_1 = \frac{-a_1}{b}$

$= \frac{-\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{-2}{3}$

$6x+4y+2=0$

$a=6,\ b=4$

$m_2 = \frac{-a_2}{b_2}$

$= \frac{-\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{-3}{2}$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{\frac{-2}{3} - \frac{-3}{2}}{1+\left(\frac{-2}{3}\right)\left(\frac{-3}{2}\right)}\right|$

$= \left|\frac{\frac{-4-(-9)}{6}}{1+\frac{6}{6}}\right|$

$= \left|\frac{\frac{-4+9}{6}}{\frac{6+6}{6}}\right| = \left|\frac{\frac{5}{6}}{\frac{\cancel{12}^{2}}{\cancel{6}_{1}}}\right| = \left|\frac{\frac{5}{6}}{\frac{2}{1}}\right|$

3 | 3 , 2
2 | 1   2
  | 1   1

$3 \times 2 = 6$

2) 6 (3×3
6 9
0

3) 6 (2×2
6
0

problems on conditions for two lines to be parall and perpendicular.

1. show that the lines $5x-2y+8=0$ and $10x-4y+3=0$ are parallel.

$5x-2y+8=0$

$a=5,\ b=-2$

$m_1 = \frac{-a_1}{b_1}$

$m_1 = \frac{-5}{-2} = \frac{5}{2}$

$10x-4y+3=0$

$a=10,\ b=-4$

$m_2 = \frac{-a_2}{b_2}$

$= \frac{-\cancel{10}^{5}}{-\cancel{4}_{2}} = \frac{5}{2}$

2. problems on conditions of two lines to be perpendicular show that the line.

$3x-2y+2=0$

$2x+3y+7=0$

$m_1 = \frac{-a}{b}$

$= \frac{-3}{-2}$

$= 3/2$

$m_2 = \frac{-a}{b}$

$= \frac{-2}{3}$

$m_1 m_2 = 3/2 \times -2/3$

$= -1$

what are the Conditions for the line $y = m_1 x + c_1$ and $y = m_2 x + c_2$ 1- to be parallel 2 - to be perpendicular also check wheather the lines $x - 2y = 4$ & $2x + y = 3$ are parallel or perpendicular

parallel = $m_1 = m_2$

perpendicular = $m_1 m_2 = -1$

$x - 2y = 4$

$2x + y = 3$

$m_1 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

$m_2 = \frac{-a}{b} = \frac{-2}{1}$.

$\therefore m_1 m_2 = \frac{1}{2} \times -2$

$= -1$

$\therefore$ The given straight lines are perpendicular.

Find the slope of the line perpendicular to $5x + 6y + 8 = 0$

$m_1 m_2 = -1$

$\left(\frac{-5}{6}\right) \times m_2 = -1$

$m_2 = \frac{-6}{-5}$

$m_2 = \frac{6}{5}$

problems on finding equation of line parallel to given line

1. Find the equation of the straight line parallel to $4x+8y+12=0$ and passing through the points $(1,2)$

$(x_1, y_1) = (1, 2)$

$4x+8y+12=0$

$m_1 = \frac{-4}{8} = \frac{-1}{2}$

$(y-y_1) = m(x-x_1)$

$(y-2) = \frac{-1}{2}(x-1)$

$2(y-2) = -1(x-1)$

$2y-4 = -1x+1$

$-x+1 = 2y-4$

$-x+1-2y+4=0$

$-x-2y+5=0$

$x+2y-5=0$

3.

2 ☆ Find the equation of the straight line parallel to $2x+4y+8=0$ and passing through the point $(-2,1)$

$x-y=5.$

$m = \frac{-a}{b} = \frac{-1}{-1} = 1$ 4

$(x_1, y_1) = (-2, 1)$

$(y-y_1) = m(x-x_1)$

$2x+4y+8=0$

$(y-1) = 1(x-1)$

$y-1 = x-1$

$m_1 = \frac{-2}{4} = -1/2$

$x-1 = y-1$

$x-1-y+1=0$

$x-y=0$

$$(y-y_1) = m(x-x_1)$$

$$(y-1) = -\frac{1}{2}(x-(-2))$$

$$2(y-1) = -1(x+2)$$

$$2y-2 = -1x-2$$

$$-x-2 = 2y-2$$

$$-x-2-2y+2 = 0$$

$$-x-2y = 0$$

$$x-2y = 0$$

Find the equation of the straight line passing through the point $(3,4)$ & parallel to the line $3x+4y=8$

$(x_1,y_1) = (3,4)$

$3x+4y = 8$

$m_1 = \frac{-3}{4}$

$(y-y_1) = m(x-x_1)$

$(y-4) = -\frac{3}{4}(x-3)$

$4(y-4) = -3(x-3)$

$4y-16 = -3x+9$

$4y-16-3x-9 = 0$

$3x+4y+25 = 0$

4. Find the equation of the straight line $8x+12y+10=0$ and passing throug the point $(-4,6)$

Find the equation of the line passing through $(1,-2)$ and parallel to line joining the points $(2,1)$ and $(-1,3)$

$(1,-2)$

$(2,1) = (x_1, y_1)$

$(-1,3) = (x_2, y_2)$

$$(y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$$

$$(y-1) = \frac{(3-1)}{-1-2}(x-2)$$

$$(y-1) = \frac{-2}{3}(x-2)$$

$$3(y-1) = -2(x-2)$$

$$3y-3 = -2x+4$$

$$-2x+4 = 3y-3$$

$$-2x+4-3y+3 = 0$$

$$-2x-3y+7 = 0 \quad (\ldots\times -)$$

$$2x+3y-7 = 0$$

$$m = -2/3$$

$(3,-1)$ & $(4,-2)$

$$(y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$$

$$(y+1) = \frac{-2+1}{4-3}(x-3)$$

$$(y+1) = \frac{-1}{1}(x-3)$$

$$y+1 = -x+3$$

$$-x+3 = y+1$$

$$-x+3-y-1 = 0$$

$$-x+4-y = 0 \quad (\ldots\times -)$$

$$x-4+y = 0$$

$$m = \frac{-a}{b} = \frac{-1}{1}$$

$$(y-y_1) = m(x-x_1)$$

$$(y-(-2)) = -2/3(x-1) \quad (y-y_1)$$

$$3y+6 = -2x+2$$

$$-2x+2 = 3y+6$$

$$-2x+2-3y-6 = 0$$

$$-2x-3y-4 = 0 \quad (\ldots\times -)$$

$$2x+3y+4 = 0$$

Find the equation of straight line parallel to line joining the points (1, 3) & (4, 6)

$(1, 3) = (x_1, y_1)$

$(4, 6) = (x_2, y_2)$

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$$

$$(y - 3) = \frac{(6 - 3)}{(4 - 1)} (x - 1)$$

$$(y - 3) = \frac{\cancel{3}^{1}}{\cancel{3}_{1}} (x - 1)$$

$1(y - 3) = 1(x - 1)$

$y - 3 = x - 1$

$x - 1 = y - 3$

$x - 1 - y + 3 = 0$

$x - y + 2 = 0$

$$y + 1 = \frac{2 + 1}{4 - 3} x - 4$$

$$y + 1 = \frac{3}{1} x - 4$$

$y + 1 = 3x - 12$

$3x - 12 - y - 1 = 0$

$3x - y - 13 = 0$

Find the equation of the straight line passing through the points (6, 2) & (8, 4)

$(6, 2) = (x_1, y_1)$

$(8, 4) = (x_2, y_2)$

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$$

$$(y - 2) = \frac{(4 - 2)}{(8 - 6)} (x - 6)$$

$(y-2) = \frac{\cancel{2}^{1}}{\cancel{2}_{1}} (x-6)$

$1(y-2) = 1(x-6)$

$y-2 = x-6$

$y-2-x+6=0$

$-x+y+4=0$

Find the equations of the line parallel to the line joining the points A $(-2, 5)$ & B $(2, -5)$

$A = (-2, 5) - (x_1, y_1)$

$B = (2, -5) - (x_2, y_2)$

$$(y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)} (x-x_1)$$

$$(y-5) = \frac{(-5-5)}{(2-(-2))} (x-(-2))$$

$$(y-5) = \frac{-10}{4} (x+2)$$

$4(y-5) = -10(x+2)$

$4y-20 = -10x-20$

$4y-20+10x+20=0$

$10x+4y=0$

4. Find the equation of the straight line $8x + 12y - 10 = 0$ and passing through the point $(-4, 6)$.

$(-4, 6) = (x_1, y_1)$ $\qquad$ $x - y = 5$ $\quad (1, 1)$

$8x + 12y - 10 = 0$

$$m = \frac{-8}{12} = \frac{-2}{3}$$

$(y - y_1) = m(x - x_1)$

$(y-1) = 1(x-1)$ $\qquad$ $y - 1 = x - 1$

$(y - 6) = \frac{-2}{3}(x - (-4))$ $\qquad$ $x - 1 - y + 1 = 0$

$x - y = 0$

$3(y - 6) = -2(x + 4)$

$3y - 18 = -2x - 8$

$3y - 18 + 2x + 8 = 0$

$2x + 3y - 10 = 0$

problems on finding equation of line perpendicular to given line

1. Find the equation of the straight line passing through the point $(6, -4)$ & perpendicular to the line $7x - 6y + 3 = 0$

$7x - 6y + 3 = 0$

$$m = \frac{-a}{b}$$

$$m = \frac{-7}{-6} = 7/6$$

perpendicular Condition :-

$(y-y_1) = -\frac{1}{m}(x-x_1)$ to find the equation of a straight line. If the slope is in perpendicular Condition then.

$$(y-y_1) = \frac{-1}{m}(x-x_1)$$

$$-1/6/7 = -7/6$$

$$(y+4) = -6/7\,(x-6)$$

$$7(y+4) = -6\,(x-6)$$

$$7y+28 = -6x+36$$

$$-6x+36 = 7y+28$$

$$-6x+36-7y-28 = 0$$

$$-6x-7y+8 = 0$$

$$(\because \times -)$$

$$6x+7y-8 = 0$$

Find the equation of the line passing through the point (1,2) & perpendicular to the line $2x+5y+9=0$

$$2x+5y+9=0$$

$$m = \frac{-a}{b} = \frac{-2}{5}$$

$$(y-y_1) = \frac{-1}{m}(x-x_1)$$

$$(y-2) = 5/2\,(x-1)$$

$$2(y-2) = 5\,(x-1)$$

$2y - 4 = 5x - 5$

$2y - 4 - 5x + 5 = 0$

$-5x + 2y + 1 = 0$

$(\times \ldots -)$

$5x - 2y - 1 = 0$

Find the equation of the line passing through the point (1, 3) & perpendicular to the line $2x + y = 1$

$(1, 3) = (x_1, y_1)$

$2x + y = 1$

$m = \frac{-a}{b} = \frac{-2}{1}$

$(y - y_1) = \frac{-1}{m}(x - x_1)$

$(y - 3) = 1/2\ (x - 1)$

$2(y - 3) = 1(x - 1)$

$2y - 6 = x - 1$

$2y - 6 - x + 1 = 0$

$-x + 2y - 5 = 0$

$(\ldots \times -)$

$x - 2y + 5 = 0$

4. Find the equation of the line passing through the point (6, -4) & perpendicular to the line joining to th points (-8, -6) & (2, -4)

$(6, -4)$

$(-8, -6) = (x_1, y_1)$

$(2, -4) = (x_2, y_2)$

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$(y + 6) = \frac{(-4 + 6)}{(2 + 8)}(x + 8)$$

$$(y + 6) = \frac{\cancel{2}^{1}}{\cancel{10}_{5}}(x + 8)$$

$5(y + 6) = 1(x + 8)$

$5y + 30 = x + 8$

$5y + 30 - x - 8 = 0$

$-x + 5y + 22 = 0$

$(\cdots \times)$

$x - 5y - 22 = 0$

$$m = \frac{-a}{b} = \frac{+1}{+5} = \frac{1}{5}$$

$$(y - y_1) = \frac{-1}{m}(x - x_1)$$

$(y + 4) = -5(x - 6)$

$y + 4 = -5x + 30$

$y + 4 + 5x - 30 = 0$

$5x + y - 26 = 0$

Application Level questions.

1. Find the equation of line passing through $(-3, -2)$ and perpendicular to x-axis (any line perpendicula x-axis is given by $x = h$) — eq (1). Since one passes through the point $-3 = h$ Therefore $h = -3$. Substituting the value of h is eq -(1)

$x = -3$. Therefore $x + 3 = 0$

2. Find the equation of the straight line perpendicular to the line $2x + 6y = 3$ & with the y-intercepts 2 units

Sol: The given line is $2x + 6y = 3$

Any line perpendicular to given line is

$$2x + 6y = 3$$
$$+6x - 2y + K = 0$$

where, K = Constant.

comparing with $ax + by + c = 0$

where, $a = 6$, $b = -2$, $K = C$

given that the required line has y-intercept 2 units $\therefore$ y-intercept $= 2$.

$y = \frac{-c}{b} = 2$

$\therefore \frac{+K}{+2} = \frac{K}{2} = 2$

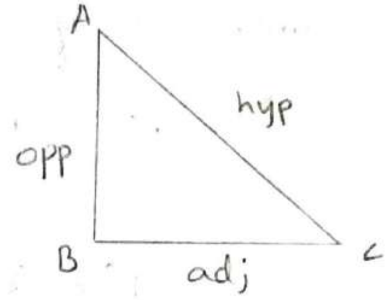
$\therefore K = 4$

$\therefore$ Substituting the value of $K = 6x - 2y + 4 = 0$

20/6/22

TRIGONOMETRY

$\sin\theta = \frac{opp}{hyp}$ $\cot\theta = \frac{adj}{opp}$



$\cos\theta = \frac{adj}{hyp}$ $\sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\text{cosec}\,\theta = \frac{hyp}{opp}$

Identities resiprocal functions

$\sin\theta = \frac{1}{\text{cosec}\,\theta}$ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin\theta \cdot \text{cosec}\,\theta = 1$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos\theta \cdot \sec\theta = 1$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\tan\theta \cdot \cot\theta = 1$

Identities :-

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$

Concepts of angles and measurements.

Conversion of radian into degree

$$1^c = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180^\circ}$$

problems :-

1. Express 75°, 300°, 105°, 180°, 240°, 360°, 125° in radian measure

$$1^\circ = \frac{\pi}{180}$$

$$= \frac{\pi}{\cancel{180}_{12}} \times \cancel{75}^{5} = \frac{5\pi}{12}$$

$$= \frac{\pi}{\cancel{180}_{3}} \times \cancel{300}^{5} = \frac{5\pi}{3}$$

$$= \frac{\pi}{\cancel{180}_{12}} \times \cancel{105}^{7} = \frac{7\pi}{12}$$

$$= \frac{\pi}{180} \times 180 = \frac{1\pi}{1} = \pi$$

$$\frac{\pi}{\cancel{180}_{3}} \times \cancel{240}^{4} = \frac{4\pi}{3}$$

$$= \frac{\pi}{\cancel{180}_{1}} \times \cancel{360}^{2} = \frac{2\pi}{1} = 2\pi$$

$$= \frac{\pi}{\cancel{180}_{36}} \times \cancel{125}^{25} = \frac{25\pi}{36}$$

$$= \frac{\pi}{\cancel{180}_{60}} \times 135 = \frac{45\pi}{60}$$

Convert radian into degree

Express $\frac{3\pi}{2}$, $\frac{6\pi}{4}$, $\frac{7\pi}{8}$, $\frac{4\pi}{6}$, $\frac{8\pi}{3}$, $\frac{9\pi}{5}$, $\frac{5\pi}{9}$

$\frac{2\pi}{8}$

$1^c = \frac{180}{\pi}$

$= \frac{\cancel{180}^{90}}{\cancel{\pi}} \times \frac{3\cancel{\pi}}{\cancel{2}}$

$= 270°$

$1^c = \frac{180}{\pi}$

$= \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{6\cancel{\pi}}{\cancel{4}}$

$= 290°$

$= \frac{\cancel{180}^{22.5}}{\cancel{\pi}} \times \frac{7\cancel{\pi}}{\cancel{8}} = 157.5°$

$= \frac{\cancel{180}^{30}}{\cancel{\pi}} \times \frac{4\cancel{\pi}}{\cancel{6}} = 120°$

$= \frac{\cancel{180}^{60}}{\cancel{\pi}} \times \frac{8\cancel{\pi}}{\cancel{3}} = 480°$

$= \frac{\cancel{180}^{36}}{\cancel{\pi}} \times \frac{9\cancel{\pi}}{\cancel{5}} = 324°$

$= \frac{\cancel{180}^{20}}{\cancel{\pi}} \times \frac{5\cancel{\pi}}{\cancel{9}} = 100°$

$= \frac{\cancel{180}^{22.5}}{\cancel{\pi}} \times \frac{2\cancel{\pi}}{\cancel{8}} = 45°$

Allied angles:

S	A
sin / cosec	
II - Quadrant	I - Quadrant
90° to 180°	0 to 90°
(90° + θ)	(90° - θ)
(180° - θ)	(360° + θ)
	All Trigonometric ratio's are positive
T	**C**
tan / cot	cos / sec
III Quadrant	IV - Quadrant
180° to 270°	270° to 360°
(180° + θ)	(270° + θ)
(270° - θ)	(360° - θ)

Y, X', O, X, Y'

Ist Quadrant.

$\sin(90^\circ-\theta) = \sin(1\times 90^\circ-\theta) = \cos\theta$

$\cos(90^\circ-\theta) = \cos(1\times 90^\circ-\theta) = \sin\theta$

$\tan(90^\circ-\theta) = \tan(1\times 90^\circ-\theta) = \cot\theta$

$\cot(90^\circ-\theta) = \cot(1\times 90^\circ-\theta) = \tan\theta$

$\sec(90^\circ-\theta) = \sec(1\times 90^\circ-\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ-\theta) = \operatorname{cosec}(1\times 90^\circ-\theta) = \sec\theta$.

$\sin(360^\circ+\theta) = \sin(4\times 90^\circ+\theta) = \sin\theta$

$\cos(360^\circ+\theta) = \cos(4\times 90^\circ+\theta) = \cos\theta$

$\tan(360^\circ+\theta) = \tan(4\times 90^\circ+\theta) = \tan\theta$

$\cot(360^\circ+\theta) = \cot(4\times 90^\circ+\theta) = \cot\theta$

$\sec(360^\circ+\theta) = \sec(4\times 90^\circ+\theta) = \sec\theta$

$\operatorname{cosec}(360^\circ+\theta) = \operatorname{cosec}(4\times 90^\circ+\theta) = \operatorname{cosec}\theta$

IInd Quadrant.

$\sin(90^\circ+\theta) = \sin(1\times 90^\circ+\theta) = -\cos\theta$

$\cos(90^\circ+\theta) = \cos(1\times 90^\circ+\theta) = \sin\theta$

$\tan(90^\circ+\theta) = \tan(1\times 90^\circ+\theta) = -\cot\theta$

$\cot(90^\circ+\theta) = \cot(1\times 90^\circ+\theta) = -\tan\theta$

$\sec(90^\circ+\theta) = \sec(1\times 90^\circ+\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ+\theta) = \operatorname{cosec}(1\times 90^\circ+\theta) = -\sec\theta$

$\sin(180^\circ - \theta) = \sin(2\times 90^\circ - \theta) = \sin\theta$.

$\cos(180^\circ - \theta) = \cos(2\times 90^\circ - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = \tan(2\times 90^\circ - \theta) = -\tan\theta$

$\cot(180^\circ - \theta) = \cot(2\times 90^\circ - \theta) = -\cot\theta$

$\sec(180^\circ - \theta) = \sec(2\times 90^\circ - \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}(2\times 90^\circ - \theta) = \operatorname{cosec}\theta$

III[rd] Quadrant.

$\sin(180^\circ + \theta) = \sin(2\times 90^\circ + \theta) = -\sin\theta$

$\cos(180^\circ + \theta) = \cos(2\times 90^\circ + \theta) = -\cos\theta$

$\tan(180^\circ + \theta) = \tan(2\times 90^\circ + \theta) = \tan\theta$

$\cot(180^\circ + \theta) = \cot(2\times 90^\circ + \theta) = \cot\theta$

$\sec(180^\circ + \theta) = \sec(2\times 90^\circ + \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ + \theta) = \operatorname{cosec}(2\times 90^\circ + \theta) = -\operatorname{cosec}\theta$.

$\sin(270^\circ - \theta) = \sin(3\times 90^\circ - \theta) = -\cos\theta$.

$\cos(270^\circ - \theta) = \cos(3\times 90^\circ - \theta) = -\sin\theta$

$\tan(270^\circ - \theta) = \tan(3\times 90^\circ - \theta) = \cot\theta$.

$\cot(270^\circ - \theta) = \cot(3\times 90^\circ - \theta) = \tan\theta$

$\sec(270^\circ - \theta) = \sec(3\times 90^\circ - \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ - \theta) = \operatorname{cosec}(3\times 90^\circ - \theta) = -\sec\theta$.

IVth Quadrant.

$\sin(270^\circ+\theta) = \sin(3\times 90^\circ+\theta) = -\cos\theta$

$\cos(270^\circ+\theta) = \cos(3\times 90^\circ+\theta) = +\sin\theta$

$\tan(270^\circ+\theta) = \tan(3\times 90^\circ+\theta) = -\cot\theta$.

$\cot(270^\circ+\theta) = \cot(3\times 90^\circ+\theta) = -\tan\theta$.

$\sec(270^\circ+\theta) = \sec(3\times 90^\circ+\theta) = +\operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ+\theta) = \operatorname{cosec}(3\times 90^\circ+\theta) = -\sec\theta$.

$\sin(360^\circ-\theta) = \sin(4\times 90^\circ-\theta) = -\sin\theta$

$\cos(360^\circ-\theta) = \cos(4\times 90^\circ-\theta) = \cos\theta$

$\tan(360^\circ-\theta) = \tan(4\times 90^\circ-\theta) = -\tan\theta$

$\cot(360^\circ-\theta) = \cot(4\times 90^\circ-\theta) = -\cot\theta$

$\sec(360^\circ-\theta) = \sec(4\times 90^\circ-\theta) = \sec\theta$

$\operatorname{cosec}(360^\circ-\theta) = \operatorname{cosec}(4\times 90^\circ-\theta) = -\operatorname{cosec}\theta$.

Trigonometric ratios of $(-\theta)$ interms of θ.

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$.

Find the value of $\dfrac{\cos(360^\circ-\theta)\tan(360^\circ+\theta)}{\cot(270^\circ-\theta)\sin(90^\circ+\theta)}$.

$$= \frac{\cos(360^\circ-\theta)\tan(360^\circ+\theta)}{\cot(270^\circ-\theta)\sin(90^\circ+\theta)}$$

$\cot(270^\circ-\theta)$

$= \cot(3\times 90^\circ-\theta)$

$= \tan\theta$

$$= \frac{\cos\theta \times \cancel{\tan\theta}}{\cancel{\tan\theta} \times \cos\theta}$$

$\cos(360^\circ-\theta)$

$= \cos(4\times 90^\circ-\theta)$

$= \cos\theta$

$\sin(90^\circ+\theta)$

$= \sin(1\times 90^\circ+\theta$

$= \cos\theta$

$\tan(360^\circ+\theta)$

$= \tan(4\times 90^\circ+\theta)$

$= \tan\theta$

$= 1$

Simplify $\dfrac{\sec(360^\circ-\theta)\cot(90^\circ-\theta)}{\tan(360^\circ+\theta)\operatorname{cosec}(90^\circ+\theta)}$

$$= \frac{\sec(360^\circ-\theta)\cot(90^\circ-\theta)}{\tan(360^\circ+\theta)\operatorname{cosec}(90^\circ+\theta)}$$

$$= \frac{\cancel{\sec\theta} \times \cancel{\tan\theta}}{\cancel{\tan\theta} \times \cancel{\sec\theta}}$$

$= 1/s$

$\sec(360^\circ-\theta)$

$= \sec(4\times 90^\circ-\theta)$

$= \sec\theta$

$\cot(90^\circ-\theta)$

$= \cot(1\times 90^\circ-\theta)$

$= \tan\theta$

$\operatorname{cosec}(90^\circ+\theta)$

$= \operatorname{cosec}(1\times 90^\circ+\theta)$

$= \sec\theta$.

$\tan(360$

$=$

$=$

3. Show that $\operatorname{cosec}(180^\circ-\theta)\cdot\cos(-\theta) \div \sec(180^\circ+\theta)$
$\cancel{\cot^2\theta}\cdot\cos(90+\theta) = \cot^2\theta$.

$$= \frac{\operatorname{cosec}(180^\circ-\theta)\times\cos(-\theta)}{\sec(180^\circ+\theta)\times\cos(90+\theta)}$$

$$= \frac{\operatorname{cosec}\theta\times\cos\theta}{-\sec\theta\times-\sin\theta}$$

$$= \frac{\frac{1}{\sin\theta}\times\cos\theta}{\frac{1}{\cos\theta}\times\sin\theta} = \frac{\cot\theta}{\tan\theta} = \cot^2\theta$$

4. Simplify $\dfrac{\sin(180^\circ-\theta)\times\cos(360^\circ-\theta)\times\tan(180^\circ+\theta)}{\cos(270^\circ+\theta)\times\sin(90^\circ+\theta)\times\cot(270^\circ-\theta)}$

$$= \frac{\sin(180^\circ-\theta)\times\cos(360^\circ-\theta)\times\tan(180^\circ+\theta)}{\cos(270^\circ+\theta)\times\sin(90^\circ+\theta)\times\cot(270^\circ-\theta)}$$

$$= \frac{\cancel{\sin\theta}\times\cancel{\cos\theta}\times\cancel{\tan\theta}}{\cancel{\sin\theta}\times\cancel{\cos\theta}\times\cancel{\tan\theta}} = 1$$

5. Simplify $\dfrac{\sin(360^\circ-\theta)\times\cos(180^\circ+\theta)\times\sin(90^\circ-\theta)}{\cot(360^\circ-\theta)\times\sin(90^\circ-\theta)\times\operatorname{cosec}(90^\circ+\theta)}$

$$= \frac{-\sin\theta\times-\cos\theta\times\cancel{\sin\theta}\,\cos\theta}{-\cot\theta\times\cos\theta\times\sec\theta}$$

$$= \frac{\sin\theta \times \cos\theta}{\frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos\theta}}$$

$$= \sin^2\theta \times \cos\theta.$$

Simplify $\dfrac{\cos(360° - A) \times \tan(360° + A)}{\cot(270° - A) \times \sin(90° + A)}$

$$= \frac{\cancel{\cos\theta} \times \cancel{\tan\theta}}{\cancel{\tan\theta} \times \cancel{\cos\theta}} = 1$$

Find the value of $\dfrac{\sin(\pi - \theta) \cdot \tan(\pi - \theta)}{\cos(2\pi + \theta) \cdot \cot(2\pi + \theta)}$

$$= \frac{\sin\theta \times -\tan\theta}{-\cos\theta \times \cot\theta}$$

Simplify $\dfrac{\sin(-\theta) \times \tan\left(\frac{\pi}{2} - \theta\right) \times \cos\left(\frac{\pi}{2} + \theta\right)}{\sin(\pi - \theta) \times \cot(\pi - \theta) \times \cos\left(\frac{3\pi}{2} - \theta\right)}$

$$= \frac{-\sin\theta \times -\cot\theta \times \sin\theta}{\sin\theta \times -\cot\theta \times -\sin\theta} = 1$$

Trigonometric ratios of some Standard angles

Angles	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

1. Find the values of $\sin 120°$, $\tan 240°$, $\tan 390°$, $\sin 240°$, $\cot 2~~25~~$, $\cos 150°$, $\sin 330°$, $\sin 300°$, $\cos 210°$, $\cos 330°$, $\cot 135°$, $\cot 225°$

$\sin 120°$

$= \sin(90° + 30°)$

$= \sin(1 \times 90° + 30°)$

$= \cos 30°$

$= \sqrt{3}/2$

$1 \times 90° = 90°$

$2 \times 90° = 180°$

$\sin(120°)$

$= \sin(180° - 60°)$

$= \sin(2 \times 90° - 60°)$

$= \sin 60°$

$= \sqrt{3}/2$

$\tan 240°$

$= \tan(180° + 60°)$

$= \tan(2 \times 90° + 60°)$

$= \sqrt{3}$

$\tan(390°)$

$= \tan(360° + 30°)$

$= \tan(4 \times 90° + 30°)$

$= \tan 30°$

$= 1/\sqrt{3}$

$\sin 240°$
$= \sin(180° + 60°)$
$= \sin(2 \times 90° + 60°)$
$= -\sin 60°$
$= -\frac{\sqrt{3}}{2}$

$\cot 225°$
$= \cot(180° + 45°)$
$= \cot(2 \times 90° + 45°)$
$= \cot 45°$
$= 1$

$\cos 150°$
$= \cos(180° - 30°)$
$= \cos(2 \times 90° - 30°)$
$= -\cos 30$
$= -\sqrt{3}/2$

$= \cos(225)$
$= \cos(270 - 45)$
$= \cos(3 \times 270 - 45)$
$= \cos$

$\sin 330°$
$= \sin(360° - 30°)$
$= \sin(4 \times 90° - 30°)$
$= -\sin 30$
$= -1/2$

$\sin 300°$
$= \sin(360° - 60°)$
$= \sin(4 \times 90° - 60°)$
$= -\sin 60°$
$= -\sqrt{3}/2$

$\cos 210°$
$= \cos(270° - 60°)$
$= \cos(3 \times 90° - 60°)$
$= -\sin 60°$
$= -\sqrt{3}/2$

$\cos 330°$
$= \cos(360° - 30°)$
$= \cos(4 \times 90° - 30°)$
$= \cos 30°$
$= \sqrt{3}/2$

$\cot 135°$
$= \cot(180° - 45°)$
$= \cot(2 \times 90° - 45°)$
$= -\cot 45°$
$= -1$

$\cot 225°$
$= \cot(180° + 45°)$
$= \cot(2 \times 90° + 45$
$= \cot 45°$
$= 1$

problems on finding expression involving trigonometry ratios.

1. Find the value of $\cos 570^\circ \times \sin 510^\circ - \sin 330^\circ \times \cos 390^\circ$

$\cos(570^\circ)$

$= \cos(540^\circ + 30^\circ)$

$= \cos(6 \times 90^\circ + 30^\circ)$

$= \cos(2 \times 90^\circ + 30^\circ)$

$= -\cos 30^\circ$

$= -\sqrt{3}/2$

$\sin(510)$

$= \sin(540^\circ - 30^\circ)$

$= \sin(6 \times 90^\circ - 30^\circ)$

$= \sin(2 \times 90^\circ - 30^\circ)$

$= \sin 30^\circ$

$= 1/2$

$\sin(330^\circ)$

$= \sin(360^\circ - 30^\circ)$

$= \sin(4 \times 90^\circ - 30^\circ)$

$= -\sin 30^\circ$

$= -1/2$

$\cos(390^\circ)$

$= \cos(360^\circ + 30^\circ)$

$= \cos(4 \times 90^\circ + 30^\circ)$

$= \cos 30^\circ$

$= \sqrt{3}/2$

$= \cos 570^\circ \times \sin 510^\circ - \sin 330^\circ \times \cos 390^\circ$

$= \frac{-\sqrt{3}}{2} \times \frac{1}{2} - \left(-\frac{1}{2}\right) \times \frac{\sqrt{3}}{2}$

$= -\cancel{\frac{\sqrt{3}}{4}} + \cancel{\frac{\sqrt{3}}{4}}$

$= 0$

Find the value of $\sin 120^\circ \times \cos 330^\circ - \sin 240^\circ \times \cos 390^\circ$ with out using calculator.

$\sin 120^\circ$

$= \sin(90^\circ + 30^\circ)$

$= \sin(1 \times 90^\circ + 30^\circ)$

$= \cos 30^\circ$

$= \sqrt{3}/2$

$\cos(330^\circ)$

$= \cos(270^\circ + 60^\circ)$

$= \cos(3 \times 90^\circ + 60^\circ)$

$= \sin 60^\circ$

$= \sqrt{3}/2$

$\sin(240^\circ)$

$= \sin(180^\circ + 60^\circ)$

$= \sin(2 \times 90^\circ + 60^\circ)$

$= -\sin 60^\circ$

$= -\sqrt{3}/2$

$\cos 390^\circ$

$= \cos(360^\circ + 30^\circ)$

$= \cos(4 \times 90^\circ + 30^\circ)$

$= \cos 30^\circ$

$= \sqrt{3}/2$

$= \sin(120^\circ) \times \cos(330^\circ) - \sin(240^\circ) \times \cos(390^\circ)$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \left(\frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{9}^2}{4} - \frac{\sqrt{9}^2}{4}$$

$$= \frac{9}{4} - \frac{9}{4}$$

$= 1$

3. Find the value of $\cos(135^\circ) \times \cos(45^\circ) - \sin(135^\circ) \times \sin(45^\circ)$

4. Show that $\sin(780^\circ) \times \sin(480^\circ) - \cos(120^\circ) \times \sin(330^\circ) = 1/2$

3.

$\cos(135^\circ)$
$= \cos(180^\circ - 45^\circ)$
$= \cos(2\times 90^\circ - 45^\circ)$
$= \cos(45^\circ)$
$= -\cos 45^\circ$
$= -1/\sqrt{2}$

$\sin(135^\circ)$
$= \sin(180^\circ - 45^\circ)$
$= \sin(2\times 90^\circ - 45^\circ)$
$= \sin(45^\circ)$
$= 1/\sqrt{2}$

$= \cos(135^\circ) \times \cos(45^\circ) - \sin(135^\circ) \times \sin(45^\circ)$

$= -\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

4.

$\sin(780^\circ) \times \sin(480^\circ) - \cos(120^\circ) \times \sin(330^\circ)$

$= \frac{-\sqrt{3}}{2} \times -\frac{\sqrt{3}}{2} - \frac{1}{2} \times -\frac{1}{2}$

$= \frac{\sqrt{9}}{4} + \frac{1}{4}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4+4} = \frac{4}{8} = 1/2$

show $\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$

$= \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{\cancel{\pi}}{\cancel{4}_1}$ $\quad = \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{3\cancel{\pi}}{\cancel{4}_1}$ $\quad = \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{5\cancel{\pi}}{\cancel{4}_1}$

$= 45^\circ$ $\quad = 135^\circ$ $\quad = 225^\circ$

$\therefore \sin^2 45^\circ$ $\quad \sin^2 135^\circ$ $\quad \sin^2 225^\circ$

$= \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{7\cancel{\pi}}{\cancel{4}_1}$

$= 315^\circ$

$\sin^2 315^\circ$

$\sin 135^\circ$
$= \sin(180^\circ - 45^\circ)$
$= \sin(2\times 90^\circ - 45^\circ)$
$= \sin 45^\circ$
$= \frac{1}{\sqrt{2}}$

$\sin 225$
$= \sin(180^\circ + 45^\circ)$
$= \sin(2\times 90^\circ + 45^\circ)$
$= -\sin 45$
$= -\frac{1}{\sqrt{2}}$

$\sin 315$
$= \sin(270 + 45^\circ)$
$= \sin(3\times 90^\circ + 45^\circ)$
$= -\cos 45^\circ = -\frac{1}{\sqrt{2}}$

$\cancel{= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}$

$\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$

$= \sin^2 45^\circ + \sin^2 135^\circ + \sin^2 225^\circ + \sin^2 315^\circ.$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{1+1+1+1}{2} = \frac{\cancel{4}^2}{\cancel{2}_1} = 2$

6. prove that $\frac{\sin(-\theta)}{\sin(\pi-\theta)} - \frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi-\theta)} + \frac{\cos\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{3\pi}{2}-\theta\right)}$

$$= \frac{-\sin\theta}{\sin\theta} - \frac{\cot\theta}{-\cot\theta} + \frac{-\sin\theta}{-\sin\theta} = 1$$

7. Find x if $\frac{x \sin^2 300^\circ \ \sec^2 240^\circ}{\cos(225^\circ)\ \mathrm{cosec}^2(240^\circ)} = \frac{\cot^2(315^\circ)}{\tan^2(300^\circ)}$

8. Find x if $x \cos^2 300^\circ \cdot \sin^2 330^\circ = \tan^2 225^\circ + \sec^2 22$

v.v. Vinay.

Compound angles

Trigonometric ratios of Compound angle:

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

1. Find the values of $\sin 15°$, $\cos 15°$, $\tan 15°$; $\sin 75°$, $\cos 75°$, $\tan 75°$, $\sin 105°$, $\cos 105°$, $\tan 105°$.

$\sin(15°)$

$\sin(45° - 30°)$

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$= \sin 45° \cdot \cos 30° - \cos 45° \sin 30°$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos(15^\circ)$

$\cos(45^\circ - 30^\circ)$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30 + \sin 45^\circ \sin 30$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\tan(15^\circ)$

$\tan(45^\circ - 30^\circ)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$\tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$\tan(15^\circ) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\,\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$\sin(75^\circ)$

$\sin(45^\circ + 30^\circ)$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(45^\circ + 30^\circ) = \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\sin(75^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\cos(75^\circ)$

$\cos(45^\circ + 30^\circ)$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(45^\circ + 30^\circ) = \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\cos(75^\circ) = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$\tan(75^\circ)$

$\tan(45^\circ + 30^\circ)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(75^\circ) = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$\sin(105^\circ)$

$\sin(60^\circ + 45^\circ)$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(60^\circ + 45^\circ) = \sin 60 \cdot \cos 45^\circ + \cos 60 \cdot \sin 45^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\cos(105^\circ)$

$\cos(60^\circ + 45^\circ)$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(60^\circ + 45^\circ) = \cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\tan(105^\circ)$

$\tan(60^\circ + 45^\circ)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(105^\circ) = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$\sin 2A = 2 \sin A \cdot \cos A$

L.H.S $= \sin 2A$

$= \sin(A+A)$ $\quad \sin(A+B) = \sin A \cos B + \cos A \cdot \sin$

$= \sin A \cos A + \cos A \cdot \sin A$

$= 2 \sin A \cos A =$ R.HS.

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

L.H.S $= \sin 2A$ $\quad \sin^2 A + \cos^2 A = 1$

$\frac{ab}{c}$

$\frac{a}{c}\ \frac{b}{c}$ $\quad = \dfrac{2 \sin A \cdot \cos A}{1}$

$\frac{a+b}{c}$

$\frac{a}{c}\ \frac{b}{c}$ $\quad = \dfrac{2 \sin A \cdot \cos A}{\sin^2 A + \cos^2 A}$ $\quad$ [÷ both ... by $\cos^2 A$

$$= \frac{2 \sin A \cos A}{\cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} + \frac{\cancel{\cos^2 A}}{\cancel{\cos^2 A}}$$

$$LHS = \frac{2 \tan}{\tan^2 A}$$

$$= \frac{2 \tan}{1 + \tan^2 A}$$

prove that $\cos 2A = \cos^2 A - \sin^2 A$

$L.H.S = \cos 2A$

$\cos(A+B) = \cos A \cos B - \sin A$

$= \cos(A+B)$

$= \cos A \cdot \cos A - \sin A \cdot \sin A$

$= \cos^2 A - \sin^2 A = R.H.S$

prove that $\cos 2A = 2\cos^2 A - 1$

$L.H.S = \cos 2A$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$= \cos(A+A)$

$= \cos A \cdot \cos A - \sin A \cdot \sin A$

$= \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1 = R.H.S$

prove that $\cos 2A = 1 - \sin^2 A$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$L.H.S = \cos 2A$

$= \cos^2 A - \sin^2 A$

$= (1 - \sin^2 A) - \sin^2 A$

$= 1 - 2\sin^2 A = R.H.S$

prove that $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$L.H.S = \cos 2A$

$\frac{a-b}{c}$

$= \cos^2 A - \sin^2 A$

$a/c - b/c$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$ ($\div$ both Nr & Dr by $\cos^2 A$)

$\frac{a+b}{c}$

$\frac{a}{c} + \frac{b}{c}$

$$= \frac{\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A + \sin^2 A}{\cos^2 A}}$$

$$= \frac{\dfrac{\cos^2 A}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\cos^2 A} + \dfrac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

7. prove that $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$
\begin{aligned}
L.H.S &= \tan 2A \\
&= \tan(A+A) \\
&= \frac{\tan A + \tan A}{1 - \tan A \tan A} \\
&= \frac{2\tan A}{1-\tan^2 A} = R.H.S.
\end{aligned}
$$

8. prove that $\sin 3A = 3\sin A - 4\sin^3 A$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$$
\begin{aligned}
L.H.S &= \sin 3A \\
&= \sin(2A+A) \\
&= \sin 2A \cos A + \cos 2A \cdot \sin A \\
&= 2\sin A \cos A \cos A + (1-\sin^2 A)\cdot \sin A \\
&= 2\sin A \cos^2 A + \sin A - 2\sin^3 A \\
&= 2\sin A(1-\sin^2 A) + \sin A - 2\sin^3 A \\
&= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A \\
&= 3\sin A - 4\sin^3 A = R.H.S
\end{aligned}
$$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

9. prove that $\cos 3A = 4\cos^3 A - 3\cos A$

$L.H.S = \cos 3A$

$= \cos(2A + A)$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$= \cos 2A \cdot \cos A - \sin 2A \cdot \sin A$

$= (2\cos^2 A - 1) \cdot \cos A - 2\sin A \cdot \cos A \cdot \sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A \cdot \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A) \cdot \cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 2\cos^3 A - \cos A - 2\cos A$

$= 4\cos^3 A - 3\cos A = R.H.S.$

10. prove that $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$L.H.S = \tan 3A$

$= \tan(2A + A)$

$$= \frac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$$

$$= \frac{\dfrac{2\tan A}{1-\tan^2 A} + \tan A}{1 - \dfrac{2\tan A}{1-\tan^2 A} \cdot \tan A}$$

$$= \frac{\dfrac{2\tan A + \tan A(1-\tan^2 A)}{\cancel{(1-\tan^2 A)}}}{\dfrac{(1-\tan^2 A) - 2\tan^2 A}{(1-\tan^2 A)}}$$

$$= \frac{2\tan A + \tan A - \tan^3 A}{1 - \tan^2 A - 2\tan^2 A}$$

$$= \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = R.H.S.$$

<u>V.V.V.IMP</u>

Transformation formulae to express product of ratios into sum or difference ratios.

1. $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

2. $\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

3. $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

4. $\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$

Transformation formulae to express sum or difference of ratios into products ratios.

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

5M

1. If $\tan A = 1/3$ and $\tan B = 1/2$ find the value of $\tan(A+B)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\frac{1}{3} + \frac{1}{2}}{1 - 1/3 \times 1/2}$$

$$= \frac{\frac{2+3}{\cancel{6}}}{\frac{6-1}{\cancel{6}}} = \frac{\cancel{5}}{\cancel{5}} = 1$$

2. If $\tan A = 1/6$ and $\tan B = 1/4$ find the value of $\tan(A+B)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\frac{1}{6} + \frac{1}{4}}{1 - \frac{1}{6} \times \frac{1}{4}}$$

6)12(2×1
12
2

$$= \frac{\frac{2+3}{12}}{1 - \frac{1}{24}} = \frac{\frac{5}{12}}{\frac{24-1}{24}}$$

4)12(3×1
12
3

$$= \frac{5}{\cancel{12}_{1}} \times \frac{\cancel{24}^{2}}{23} = \frac{10}{23}$$

v.v. IMP

* problems on expressing sum or differences of ratios to product of

3.

1. prove that $\sin 5x + \sin 4x = 2 \cdot \sin\left(\frac{9}{2}x\right) \cdot \cos\left(\frac{x}{2}\right)$

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$C = 5x, \; D = 4x$

$L.H.S = \sin 5x + \sin 4x$

$$= 2 \sin\left(\frac{5x + 4x}{2}\right) \cdot \cos\left(\frac{5x - 4x}{2}\right)$$

$$= 2 \sin\left(\frac{9x}{2}\right) \cdot \cos\left(\frac{x}{2}\right) = R.H.S$$

2. prove that $\sin 5x - \sin 3x = 2 \cdot \cos(4x) \cdot \sin(x)$

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$L.H.S = \sin 5x - \sin 3x$

$$= 2 \cos\left[\frac{5x + 3x}{2}\right] \sin\left[\frac{5x - 3x}{2}\right]$$

$$= 2 \cos\left[\frac{\cancel{8}^{4}x}{\cancel{2}}\right] \sin\left[\frac{\cancel{2}x}{\cancel{2}}\right]$$

$$= 2 \cos(4x) \cdot \sin(x) = R.H.S.$$

3. prove that $\cos 12x + \cos 2x = 2\cos(7x)\cdot\cos(5x)$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$= 2\cos\left[\frac{12x+2x}{2}\right]\cos\left[\frac{12x-2x}{2}\right]$$

$$= 2\cos\left[\frac{\cancel{14}^{7}x}{\cancel{2}_{1}}\right]\cos\left[\frac{\cancel{10}^{5}x}{\cancel{2}_{1}}\right]$$

$$= 2\cos(7x)\cdot\cos(5x) = R.H.S$$

4. prove that $\cos 10x - \cos 6x = -2\sin(8x)\sin(2x)$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin$$

$$LHS = -2\sin\left[\frac{10x+6x}{2}\right]\sin\left(\frac{10x-6x}{2}\right)$$

$$= -2\sin\left[\frac{\cancel{16}^{8}x}{\cancel{2}_{1}}\right]\sin\left(\frac{\cancel{4}^{2}x}{\cancel{2}}\right]$$

$$= -2\sin[8x]\sin(2x]$$

prove that $\cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0$

$L.H.S = \cos 80^\circ + \cos 40^\circ - \cos 20^\circ$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)$$

$$= 2\cos\left(\frac{80^\circ+40^\circ}{2}\right)\cos\left(\frac{80^\circ-40^\circ}{2}\right) - \cos 20^\circ$$

$$= 2\cos\left(\frac{120}{2}\right)\cdot\cos\left(\frac{40^\circ}{2}\right) - \cos 20^\circ$$

$$= 2\cos 60^\circ\cdot\cos 20^\circ - \cos 20^\circ$$

$$= \cancel{2}\times \frac{1}{\cancel{2}}\cos 20^\circ - \cos 20^\circ$$

6. $\sin 40^\circ + \sin 20^\circ - \cos 10^\circ = 0$

$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

$= 2 \sin\left(\frac{40^\circ + 20^\circ}{2}\right) \cdot \cos\left(\frac{40^\circ - 20^\circ}{2}\right) - \cos 10$

$= 2 \sin\left(\frac{\cancel{60}^{30}}{\cancel{2}}\right) \cdot \cos\left(\frac{\cancel{20}^{10}}{\cancel{2}}\right) - \cos 10^\circ$

$= 2 \sin 30^\circ \cdot \cos 10^\circ - \cos 10^\circ$

$= \cancel{2} \times \frac{1}{\cancel{2}} \cdot \cos 10^\circ - \cos 10^\circ$

$= 0$

Find the values of $\sin 8x + \sin 4x = 1$.

Find the values of $\sin 12x - \sin x = 1$

$\cos 6x + \cos 2x = 1$

$\cos 4x = \cos x = 1$

$(3, -1) \cdot (4, -2)$

$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$.

$y -$

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TRIGONOMETY

1. Express $\sin 6x \cdot \cos 4x$ · sum or difference formula.

$\sin 6x \cdot \cos 4x$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A = 6x,\ B = 4x$

$\sin 6x \cdot \cos 4x = \frac{1}{2}[\sin(6x+4x) + \sin(6x-4x)]$

$= \frac{1}{2}[\sin 10x + \cos 2x]$

2. $\sin 7x \cdot \cos 3x$

$\sin 7x \cdot \cos 3x$

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A = 7x,\ B = 3x$

$\sin 7x \cdot \cos 3x = \frac{1}{2}[\sin(7x+3x) + \sin(7x-4x)]$

$= \frac{1}{2}[\sin 10x + \cos 3x]$

3. $\cos 80 \cdot \sin 30^\circ$

$\cos 80 \cdot \sin 30^\circ$

$\cos A \cdot \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$A = 80^\circ,\ B = 30^\circ$

$\cos 80 \cdot \sin 30^\circ = \frac{1}{2}[\sin(80^\circ+30^\circ) - \sin(80^\circ-30^\circ)]$

$= \frac{1}{2}[\cos 110^\circ - \sin 50^\circ]$

4. $\cos 80 \cdot \cos 60^\circ$

$\cos 80^\circ \cdot \cos 60^\circ$

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 80,\ B = 60^\circ$

$\cos 80^\circ \cdot \cos 60^\circ = \frac{1}{2}[\cos(80^\circ+60^\circ) + \cos(80^\circ-60^\circ)]$

$= \frac{1}{2}[\cos 140^\circ + \cos 20^\circ]$

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5. $\sin 40^\circ \cdot \sin 10^\circ$

$\sin 40^\circ \cdot \sin 10^\circ$

$\sin A \cdot \sin B = 1/2\left[\cos(A+B) - \cos(A-B)\right]$

$A = 40^\circ,\ B = 10^\circ$

$\sin 40 \cdot \sin 10^\circ = 1/2\left[\cos(40+10) - \cos(40-10)\right]$

$= 1/2\left[\cancel{\sin} 50^\circ - \cancel{\sin} 30^\circ\right]$

imp. 1. prove that $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \sqrt{3}/8$

$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$

$= -1/2\left[\cos(20^\circ+40^\circ) - \cos(20^\circ-40^\circ)\right] \cdot \sin 80^\circ$

$= -1/2\left[\cos 60^\circ - \cos(-20^\circ)\right] \cdot \sin 80^\circ$

$= -1/2\left[1/2 - \cos 20^\circ\right] \cdot \sin 80^\circ$

$= -1/4 \sin 80^\circ + 1/2 \sin 80^\circ \cdot \cos 20^\circ$

$= -1/4 \sin 80^\circ + 1/2 \cdot 1/2\left[\sin(80^\circ+20^\circ) + \sin(80^\circ-20^\circ)\right]$

$= -1/4 \sin 80^\circ + 1/4\left[\sin 100^\circ + \sin 60^\circ\right]$

$= -1/4 \sin 80^\circ + 1/4\left[\sin(180^\circ - 80^\circ) + \sqrt{3}/2\right]$

$= \cancel{-1/4 \sin 80^\circ} + \cancel{1/4 \sin 80^\circ} + \sqrt{3}/8$

$= \sqrt{3}/8 = R.H.S$

2. prove that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = 1/8$

$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ$

~~= 1/2 [co~~ $\cos A \cdot \cos B = 1/2\left[\cos(A+B) - \cos(A-B)\right]$

$A = 20,\ B = 40^\circ$

$\cos 20^\circ \cdot \cos 40^\circ = 1/2\left[\cos(20+40) - \cos(20-40)\right] \cdot \cos$

$= 1/2\left[\cos 60^\circ - \cos(-20^\circ)\right] \cdot \cos 80^\circ$

$= 1/2\left[1/2 - \cos 20\right] \cdot \cos 80^\circ$

$= 1/4 \cos 80^\circ - 1/2 \cdot \cos 80^\circ \cdot \cos 20^\circ$

$= 1/4 \cos 80^\circ - 1/2, = 1/2 \cos(80+20) - \cos(80-20$

$= \frac{1}{4}\cos 80 - \frac{1}{4}[\cos 100 + \cos 60]$

$= \frac{1}{4}\cos 80 - \frac{1}{4}[\cos(180-80) + \frac{1}{2}]$

$= \cancel{\frac{1}{4}\cos 80} - \cancel{\frac{1}{4}\cos 80} + \frac{1}{8}$

$= \frac{1}{8} = RHS.$

3. prove that $\cos 80^\circ \cdot \cos 60^\circ \cdot \cos 40^\circ \cdot \cos 20 = \frac{1}{16}$

$\cos 80^\circ \cdot \cos 60^\circ \cdot \cos 40 \cdot \cos 20^\circ$

$\cos A \cdot \cos B = \frac{1}{2}(\cos(A+B) - \cos(A-B))$

$A = 40^\circ, \ B = 20$

$\cos 40^\circ \cdot \cos 20^\circ = \frac{1}{2}[\cos(40+20) - \cos(40-20)] \cos 80^\circ - \cos$

$= \frac{1}{2}[\cos 60 - \cos 20] \cos 80^\circ \cdot \frac{1}{2}$

$= \frac{1}{2}[\frac{1}{2} - \cos 20] \cos 80^\circ \cdot \frac{1}{2}$

$= \frac{1}{4}\cos 80^\circ - \frac{1}{2}\cos 80^\circ \cdot \cos 20^\circ \cdot \frac{1}{2}$

$= \frac{1}{2}\cos 80^\circ - \frac{1}{2} \cdot \frac{1}{2}[\cos(80+20) + \cos(80-20)] \cdot \frac{1}{2}$

$= \frac{1}{4}\cos 80^\circ - \frac{1}{4}[\cos 100 + \cos 60] \cdot \frac{1}{2}$

$= \frac{1}{4}\cos 80^\circ - \frac{1}{4}[\cos(180^\circ - 80^\circ) + \frac{1}{2}] \cdot \frac{1}{2}$

$= \cancel{\frac{1}{4}\cos 80^\circ} - \cancel{\frac{1}{4}\cos 80^\circ} + \frac{1}{8} \cdot \frac{1}{2}$

$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = RHS.$

4M 15, 75, 105

6M = 0 show that

5M Find the value of θ lying b/w 0 & 2π which statisfy the eqn $2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\cos\theta = \cos 60$

$\theta = 60^\circ$

1. $\sin 8x + \sin 4x$

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$C = 8, \quad D = 4$

$$\sin 8x + \sin 4x = 2 \sin\left(\frac{8+4}{2}\right) \cos\left(\frac{8-4}{2}\right)$$

$$= 2 \sin\left(\frac{\cancel{12}^{6}}{\cancel{2}_{1}}\right) \cos\left(\frac{\cancel{4}^{2}}{\cancel{2}_{1}}\right)$$

$$= 2 \sin 6x \cos 2x$$

2. $\sin 12x - \sin 2x$

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= 2 \cos\left(\frac{12+2}{2}\right) \sin\left(\frac{12-2}{2}\right)$$

$$= 2 \cos\left(\frac{\cancel{14}^{7}}{\cancel{2}_{1}}\right) \sin\left(\frac{\cancel{10}^{5}}{\cancel{2}_{1}}\right)$$

$$= 2 \cos 7x \cdot \sin 5x$$

3. $\cos 6x + \cos 2x$

$$\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$= 2 \cos\left(\frac{6+2}{2}\right) \cos\left(\frac{6-2}{2}\right)$$

$$= 2 \cos\left(\frac{\cancel{8}^{4}}{\cancel{2}_{1}}\right) \cos\left(\frac{\cancel{4}^{2}}{\cancel{2}_{1}}\right)$$

$$= 2 \cos 4x \cdot \cos 2x$$

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4. $\cos 4x - \cos 2x$

$\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$= 2 \sin\left(\frac{4+2x}{2}\right) \sin\left(\frac{4-2}{2}\right)$

$= 2 \sin\left(\frac{\cancel{6}^{3}}{\cancel{2}}\right) \sin\left(\frac{\cancel{2}}{\cancel{2}}\right)$

$= 2 \sin 3x \sin x$

Saturday

DIFFERENTIAL CALCULUS AND APPLICATIONS

* List of derivatives of standard functions

• Derivatives of algebraic functions

1. $\frac{d}{dx}(x^n) = nx^{n-1}$ where $n \in R$

2. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$ where $n \in R$

3. $\frac{d}{dx}(k) = 0$ where k is a constant

4. $\frac{d}{dx}(x) = 1$

5. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

6. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

7. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2x\sqrt{x}}$

• Derivatives of Exponential functions

8. $\frac{d}{dx}(a^x) = a^x \log a$

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9. $\frac{d}{dx}(e^x) = e^x$

- Derivatives of logarithmic functions

10. $\frac{d}{dx}(\log x) = \frac{1}{x}$

- Derivatives of trigonometric functions

11. $\frac{d}{dx}(\sin x) = \cos x$

12. $\frac{d}{dx}(\cos x) = -\sin x$

13. $\frac{d}{dx}(\tan x) = \sec^2 x$

14. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

15. $\frac{d}{dx}(\sec x) = \sec x \tan x$

16. $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$

- Derivatives of inverse trigonometric functions

17. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

18. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

19. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

20. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

21. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$

* Rules of differentiation

- Rule 1 : Derivative of a Constant

  If k is a constant then $\frac{d}{dx}(k) = 0$

  $$\boxed{\frac{d}{dx}(\text{constant}) = 0}$$

- Rule 2 : Derivative of product of a constant and a function.

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If K is a constant and $u = f(x)$ is a function of x then $\frac{d}{dx}(Ku) = K\frac{du}{dx}$

$$\boxed{\frac{d}{dx}(\text{constant} \times \text{function}) = \text{constant} \times \frac{d}{dx}(\text{function})}$$

- Rule 3 : Derivative of sum of two functions (sum rule)

If $u = f(x)$ and $v = g(x)$ are two functions of x then
$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\boxed{\frac{d}{dx}(\text{I function} + \text{II function}) = \frac{d}{dx}(\text{I function}) + \frac{d}{dx}(\text{II function})}$$

- Rule 4 : Derivative of difference of two functions (Difference rule)

If $u = f(x)$ and $v = g(x)$ are two functions of x then
$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

$$\boxed{\frac{d}{dx}[\text{I function} - \text{II function}] = \frac{d}{dx}(\text{I function}) - \frac{d}{dx}(\text{II function})}$$

- Rule 5 : Derivative of product of two functions (product rule)

If $u = f(x)$ and $v = g(x)$ are two functions of x then
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(\text{I function} \times \text{II function}) = \text{I function} \times \frac{d}{dx}(\text{II function}) + \text{II function} \times \frac{d}{dx}(\text{I function})$$

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- Rule 6 : Derivative of quotient of two functions (Quotient rule)

If $u = f(x)$ and $v = g(x)$ are two functions of x th

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{(v)^2}$$

$$\boxed{\frac{d}{dx}\left(\frac{Nr}{Dr}\right) = \frac{Dr \times \frac{d}{dx}(Nr) - Nr\, \frac{d}{dx}(Dr)}{(Dr)^2}}$$

Nr → Numerator
Dr → Denominator

$y = x^n$
$$\boxed{\frac{dy}{dx} = n \cdot x^{n-1}}$$

problems on finding derivatives simple functions

1. $y = x^4$
$n = 4$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 4x^{4-1} = \frac{dy}{dx} = 4x^3$

2. $y = x^2$
$n = 2$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 2x^{2-1} = \frac{dy}{dx} = 2x^1$

3. $y = x^6$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 6x^{6-1} = \frac{dy}{dx} = 6x^5$

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4. $y = x^8$
$n = 8$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 8x^{8-1} = \frac{dy}{dx} = 8x^7$

5. $y = x^{12}$
$n = 12$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 12.x^{12-1} = \frac{dy}{dx} = 12x^{11}$

6. $y = x^{16}$
$n = 16$
$\frac{dy}{dx} = nx^{n-1} = \frac{dy}{dx} = 16x^{16-1} = \frac{dy}{dx} = 16x^{15}$

7 Differentiate $12x^8$ with respective x
$y = 12x^8$
$\frac{dy}{dx} = k.nx^{n-1} = \frac{dy}{dx} = 12 \times 8 \times x^{8-1} = \frac{dy}{dx} = 96x^7$

8. Differentiate $4x^{-2}$ with respective x
$y = 4x^{-2}$
$\frac{dy}{dx} = k.nx^{n-1} = \frac{dy}{dx} = 4 \times (-2) \times x^{-2-1} = \frac{dy}{dx} = -8x^{-3}$

9. If $y = n(2^x)$, find derivative where n is a constant
$y = a^2$ $\quad y = n2^x$
$\frac{dy}{dx} = a^x . \log a = \frac{dy}{dx} = n \cdot 2^x . \log 2$

10. Find the derivative $y = \frac{2}{x}$ with respect to x

$$y = \frac{2}{x}$$

$$\frac{1}{x} = x^{-1}$$

$$y = 2 \cdot x^{-1}$$

11. If $y = 5 \log x$ then find derivatives.

$$y = 5 \cdot \log x$$

$$\frac{dy}{dx} = 5 \cdot \frac{1}{x}$$

problems on finding derivatives of sum and difference of functions

1. If $y = x^4 + x^2 + x + 1$, then find $\frac{dy}{dx}$
2. If $y = x^8 + x^5 + x^2 + x$, then find $\frac{dy}{dx}$
3. If $y = x^{10} + x^9 + x^6 + x^5 + x + 4$, then find $\frac{dy}{dx}$
4. If $y = x + 1 \times x + 2$, then find $\frac{dy}{dx}$
5. If $y = x + 1 \times x - 6$, then find $\frac{dy}{dx}$
6. If $y = x + 1 \times x + 2 \times x + 5$, then find $\frac{dy}{dx}$

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1. $y = x^4 + x^2 + x + 1$

$\frac{dy}{dx} = \frac{d}{dx}(x^4) + \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$

$= 4x^3 + 2x + 1 + 0$

$= 4x^3 + 2x + 1$

2. $y = x^8 + x^5 + x^2 + x$

$\frac{dy}{dx} = \frac{d}{dx}(x^8) + \frac{d}{dx}(x^5) + \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$

$= 8x^7 + 5x^4 + 2x + 1$

3. $y = x^{10} + x^9 + x^6 + x^5 + x + 4$

$\frac{dy}{dx} = \frac{d}{dx}(x^{10}) + \frac{d}{dx}(x^9) + \frac{d}{dx}(x^6) + \frac{d}{dx}(x^5) + \frac{d}{dx}(x) + \frac{d}{dx}$

$= 10x^9 + 9x^8 + 6x^5 + 5x^4 + 1 + 0$

$= 10x^9 + 9x^8 + 6x^5 + 5x^4 + 1$

4. $y = x + 1 \times x + 2$

$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(1) \cdot \frac{d}{dx}(x) + \frac{d}{dx}(2)$

$= 1 + 0 \cdot 1 + 0$

$= 1 \cdot 1$

$= 1.$

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5. $y = x+1 \times x-6$

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(1) \cdot \frac{d}{dx}(x) - \frac{d}{dx}(6)$$

$= 1 + 0 \cdot 1 - 0$

$= 1 \cdot 1$

$= 1$

6. $y = x+1 \times x+2 \times x+5$

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(1) \cdot \frac{d}{dx}(x) + \frac{d}{dx}(2) \cdot \frac{d}{dx}(x) + \frac{d}{dx}(5)$$

$= 1 + 0 \cdot 1 + 0 \cdot 1 + 0$

$= 1 \cdot 1 \cdot 1$

$= 1$

$y = K$ (constant)

$\frac{dy}{dx} = 0$

$y = K x$ → constant → function

$\frac{dy}{dx} = K \cdot \frac{d}{dx}(x)$

$\frac{dy}{dx} = K(1)$

$\frac{dy}{dx} = K$

$y = 2$

$\frac{dy}{dx} = 0$

$y = x^n$

$\frac{dy}{dx} = n \cdot x^{n-1}$

$y = x^1$

$\frac{dy}{dx} = 1 \cdot x^{1-1}$

$\frac{dy}{dx} = 1$

(1) $y = u + v$ $\qquad u \to x^2$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\qquad v \to x$

$y = x^2 + x$

$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$

$\frac{dy}{dx} = 2x + 1$

$y = x^1$

$\frac{d}{dx}(y) = \frac{d}{dx}(x)$

$= 1 \cdot x^{1-1}$

$= 1$

$y = x^n$

$\frac{dy}{dx} = n \cdot x^{n-1}$

$y = x^2$

$\frac{d}{dx}(y) = \frac{d}{dx}(x^2)$

$= 2 \cdot x^{2+1}$

$= 2x$

(2) $y = \sin x + \cos x$ $\qquad u \to \sin x$

$\qquad v \to \cos x$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos x)$

$= \cos x - \sin x$

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3. $y = x^2 + \sin x.$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin x)$$

$$= \frac{d}{dx}(x^n) = nx^{n-1} + \frac{d}{dx}(\sin x)$$

$$= \frac{d}{dx}(x^2) = 2x^{2-1} + \frac{d}{dx}(\sin x)$$

$$= \frac{d}{dx}(x^2) = 2x + (\cos x)$$

4. $y = \cos x + \tan x + \cos x$

$$y = \frac{d}{dx}(\cos x) + \frac{d}{dx}(\tan x) + \frac{d}{dx}(\cos x)$$

$$= -\sin x + \sec^2 x + (-\sin x)$$

$$= -\sin x + \sec^2 x - \sin x$$

5 $y = \sin x + \cos^{-1} x$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos^{-1} x)$$

$$= \cos x + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$= \cos x - \frac{1}{\sqrt{1-x^2}}$$

6 $y = e^x + x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) + \frac{d}{dx}(x^2)$$

$$= e^x + 2x$$

7 $y = \log x + \tan x$

$$\frac{dy}{dx} = \frac{d}{dx}(\log x) + \frac{d}{dx}(\tan x) = \frac{1}{x} + \sec^2 x$$

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Defferentiat with respect to x

1. If $y = x^4 + 4 \cdot x^3$ find $\frac{dy}{dx}$ at $x = 1$

$y = x^4 + 2x^2$

$\frac{dy}{dx}(y) = \frac{d}{dx}(x^4) + 2 \cdot \frac{d}{dx}(x^2)$

$\frac{dy}{dx} = 4x^3 + 2 \cdot 2x$

$= 4x^3 + 4x$

$= 4(1)^3 + 4(1)$

$= 4 + 4 = 8$

$y = x^4 + 4 \cdot x^3$

$\frac{dy}{dx}(y) = \frac{d}{dx}(x^4) + 4 \cdot \frac{d}{dx}(x^3)$

$= 4x^3 + 2 \cdot 3x^2$

$= 4x^3 + 6x^2$

$= 4(1)^3 + 6(1)^2$

$= 4x^3 + 12x^2$

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1. $y = \sin x + \cos x + \cos^{-1} x + 2$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos x) + \frac{d}{dx}(\cos^{-1}(x)) + 0$$

$$= \cos x - \sin x - \frac{1}{\sqrt{1-x^2}}$$

2 $y = x^6 + \cos x + \sin x + e^x + 1 \cdot x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^6) + \frac{d}{dx}(\cos x) + \frac{d}{dx}(\sin x) + \frac{d}{dx}(e^x) + \frac{d}{dx}(1$$

$$= \frac{d}{dx}(x^6) = n x^{n-1}$$

$y = 1 + \sin x$
$y = \sin x - \cos x$
$y = 1 + x^2$
$y = 1 + \tan x$
$y = \sin x + \cos x$
$y = 1 + x$
$y = 1 x.$

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IMP

4 or 5 M

product rule

Example — (1)

(1) $y = u \cdot v$ $\qquad$ $u \to$ I - function

$\frac{dy}{dx} = u \frac{d}{dx}(v) + v \cdot \frac{d}{dx}(u)$ $\qquad$ v — II - function

- $y = x^2 \cdot \sin x$

$\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2)$

$= x^2 \cdot \cos x + \sin x \cdot 2x$

- If $y = x^4 \cdot \cos x$

$\frac{dy}{dx} = x^4 \cdot \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(x^4)$

$= x^4 - \sin x + \cos x \cdot 4x^3$

- If $y = \tan x \cdot \sin x$

$\frac{dy}{dx} = \tan x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(\tan x)$

$= \tan x \cdot \cos x + \sin x \cdot \sec^2 x$

- If $y = \cos^{-1} x \cdot x^2$

$\frac{dy}{dx} = \cos^{-1} x \cdot \frac{d}{dx}(x^2) + x^2 \cdot \frac{d}{dx}(\cos^{-1} x)$

$= \cos^{-1} x \cdot 2x + x^2 \cdot -\frac{1}{\sqrt{1 - x^2}}$

- If $y = \operatorname{cosec} x \cdot x^4$

$\frac{dy}{dx} = \operatorname{cosec} x \cdot \frac{d}{dx}(x^4) + x^4 \cdot \frac{d}{dx}(\operatorname{cosec} x)$

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$= \operatorname{cosec} x \cdot 4x^3 + x^4 \cdot -\operatorname{cosec} x \cot x$.

Example — (2)

(1) $y = \dfrac{u}{v}$

$$\frac{dy}{dx} = \frac{v \frac{d}{dx}(u) - u \cdot \frac{d}{dx}(v)}{(v)^2}$$

• $y = \dfrac{x^2}{\sin x}$

$$\frac{dy}{dx} = \frac{\sin x \cdot \frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$\frac{dy}{dx} = \frac{\sin x \cdot 2x - x^2 \cdot \cos x}{\sin^2 x}$$

• If $y = \dfrac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x \cdot \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x - \sin x\,(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

- If $y = \dfrac{1+x}{1-x}$ find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(1-x)\frac{d}{dx}(1+x) - (1+x)\cdot\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$= \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$$

$$= \frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^2}$$

$$= \frac{2}{(1-x)^2}$$

$y = 1+x$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(1+x)$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(1) + \dfrac{d}{dx}($

$= 0+1$

$= 1$

- If $y = \dfrac{1+x^2}{1-x^2}$ find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(1-x^2)\frac{d}{dx}(1+x^2) - (1+x^2)\cdot\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)\ 2x - (1+x^2)\ -2x}{(1-x^2)^2}$$

$$= \frac{2x - \cancel{2x^3} + 2x + \cancel{2x^2}}{(1-x^2)^2}$$

$$= \frac{4x}{(1-x^2)^2}$$

- If $y = \dfrac{1+\sin x}{1-\sin x}$ find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(1-\sin x)\frac{d}{dx}(1+\sin x) - (1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$= \frac{(1-\sin x).(\cos x) - (1+\sin x).-\cos x}{(1-\sin x)^2}$$

$$= \frac{2\cos x}{(1-\sin x)^2}$$

- If $y = \dfrac{e^x}{1+e^x}$ find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(1+e^x)\frac{d}{dx}(e^x) - e^x \cdot \frac{d}{dx}(1+e^x)}{(1+e^x)^2}$$

$$= \frac{(1+e^x)\ e^x - e^x\,(e^x)}{(1+e^x)^2}$$

$$= \frac{ex \cancel{+ ex^2} - \cancel{ex^2}}{(1+e^x)^2}$$

$$= \frac{e^x}{(1+e^x)^2}$$

- If $y = \dfrac{x\,e^x}{1+\sin x}$

$$\frac{dy}{dx} = \frac{(1+\sin x)\cdot\frac{d}{dx}(x e^x) - x\cdot e^x \cdot \frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x)\ (x\cdot e^x - e^x) - (x e^x)\ (\cos x)}{(1+\sin x)^2}$$

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$$\frac{dy}{dx} = x\frac{d}{dx}(e^x) - e^x \cdot \frac{d}{dx}(x)$$

$$= \frac{dy}{dx} = x \cdot e^x - e^x \cdot 1$$

$$= \frac{dy}{dx} = x \cdot e^x - e^x$$

$$y = 1 + \sin x$$

$$\frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}(\sin x)$$

$$= 0 + \cos x$$

$$= \cos x$$

* Chain Rule.

1. $y = (x+1)^2$

$$\frac{dy}{dx} = 2(x+1)^{2-1}$$

$$\frac{dy}{dx} = 2(x+1) \cdot \frac{d}{dx}(x+1)$$

$$\frac{dy}{dx} = 2(x+1)(1)$$

$$\frac{dy}{dx} = 2(n+1)$$

2. $y = (2x+4)^6$

$$\frac{dy}{dx} =$$

I) $y = (2x+6)^6$ find dy/dx

II) $y = (2x+6)^2$ find dy/dx

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1. $y = \sqrt{\sin 3x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin 3x}} \frac{d}{dx}(\sin 3x)$$

$$= \frac{1}{2\sqrt{\sin 3x}} \cdot \cos(3x)\, 3.$$

2. If $y = \log(\sin x)$

$$\frac{dy}{dx} = \frac{d}{dx} \log(\sin x)$$

$$= \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)$$

$$= \frac{1}{\sin x} \cdot \cos x$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

3. If $y = \operatorname{cosec}(\log x)$

$$\frac{dy}{dx} = \frac{d}{dx} \operatorname{cosec}(\log x)$$

$$= -\operatorname{cosec}(\log x) \cdot \cot(\log x) \frac{d}{dx}(\log x)$$

$$= -\operatorname{cosec}(\log x) \cdot \cot(\log x) \cdot \frac{1}{x}$$

4. If $y = \log(\sin(x^3))$

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$$\frac{dy}{dx} = \frac{d}{dx} \log(\sin(x^3))$$

$$= \frac{1}{\sin(x^3)} \frac{d}{dx}(\sin(x^3))$$

$$= \frac{1}{\sin x^3} \cos(x^3) \cdot \frac{d}{dx}(x^3)$$

$$= \frac{1}{\sin x^3} \cos(x^3) \cdot 3x^2$$

Successive Differentiation.

$\mathrm{I} - \frac{dy}{dx}$ $\qquad$ y', y''

$\qquad$ $\frac{dy}{dx}, \frac{d^2y}{dx^2}$

$\mathrm{II} = \frac{d^2y}{dx^2}$ $\qquad$ y_1, y_2

$\qquad$ $f'(x), f''(x)$

Q) $y = x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2\frac{d}{dx}(x)$$

$$\frac{d^2y}{dx^2} = 2$$

$y = x^3 + 3x^2 + 4$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + 3\frac{d}{dx}(x^2) + \frac{d}{dx}(4)$$

$$\frac{dy}{dx} = 3x^2 + 3 \cdot 2x + 0$$

$$\frac{dy}{dx} = 3x^2 + 6x$$

$$\frac{d^2y}{dx^2} = 3\frac{d}{dx}(x^2) + 6\frac{d}{dx}(x)$$

$$\frac{d^2y}{dx^2} = 3 \cdot 2x + 6(1)$$

$$\frac{d^2y}{dx^2} = 6x + 6$$

3. If $y = 4x^3 + 16x^2 + 8$ find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 4\frac{d}{dx}(x^3) + 16\frac{d}{dx}(x^2) + \frac{d}{dx}(8)$$

$$= 4 \cdot 3x^2 + 16 \cdot 2x + 0$$

$$= 12x^2 + 32x$$

$$\frac{d^2y}{dx^2} = 12\frac{d}{dx}(x^2) + 32\frac{d}{dx}(x)$$

$$= 12 \cdot 2x + 32 \cdot 1$$

$$\frac{d^2y}{dx^2} = 24x + 32$$

4. If $y = 2\sin x + 3\cos x$

$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx}(\sin x) + 3 \cdot \frac{d}{dx}(\cos x)$$

$$= 2\cos x - 3\sin x$$

$$\frac{d^2y}{dx^2} = 2 \cdot \frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$\frac{d^2y}{dx^2} = 2(-\sin x) - 3\cos x$$

$$d^2y = -2\sin x - 3\cos x$$

If $y = \sin^2 x$

$y = (\sin x)^2$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^2$$

$$= 2(\sin x)^{2-1} \cdot \frac{d}{dx}(\sin x)$$

$$\frac{dy}{dx} = 2\sin x \cdot \cos x$$

$$\frac{d^2y}{dx^2} = 2\left[\sin x \cdot \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(\sin x)\right]$$

$$\frac{d^2y}{dx^2} = 2\left[\sin x(-\sin x) + \cos x(\cos x)\right]$$

$$= 2\left[-\sin^2 x + \cos^2 x\right]$$

Equation of tangent :

① slope of tangent $= m = \left[\frac{dy}{dx}\right]_{(x_1, y_1)}$

② slope of normal $= -\frac{1}{m} = -\left[\frac{1}{dy/dx}\right]_{(x_1, y_1)}$

③ Equation of tangent $= (y - y_1) = m(x - x_1)$

④ Equation of normal $= (y - y_1) = -1/m\,(x - x_1)$

1. find the slope of the tangent to the curv $y = x^3 - x_{(x=2)}$

$y = x^3 - x$

$$\frac{dy}{dx} = 3x^2 - 1$$

$\therefore$ slope of the tangent $= \left[\frac{dy}{dx}\right]_{(x=2)}$

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$= 3x^2 - 1$

$= 3(2)^2 - 1$

$= 3 \times 4 - 1$

$= 12 - 1$

$= 11$

2 find the slope of the tangent $y = x^2 + x \, (x=1)$

$y = x^2 + x$

$\frac{dy}{dx} = 2x + 1$

$\therefore$ slope of the tangent $= \left(\frac{dy}{dx}\right)_{(x=1)}$

$= 2x + 1$

$= 2(1) + 1$

$= 2 + 1$

$= 3$

3. find the slope of the normal to the curv $y = 2x^3 \, (1,5)$

$\frac{dy}{dx} = 2\frac{d}{dx}(x^3)$

$= 6x^2$

$\left[\frac{dy}{dx}\right]_{(1,5)} = 6(1)^2$

$= 6$

$\therefore$ slope of the normal $= -1/6$

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4. find the slope of the normal to the curve
$y = 4x^2_{(2,8)}$

$$\frac{dy}{dx} = 8x$$

$$\left(\frac{dy}{dx}\right)_{(2,8)} = 8(2)$$

$$= 16$$

$\therefore$ slope of the normal $= -1/16$.

5. Find the slope of the tangent and normal to the curve $y = 3x^2 - 2x_{(1,1)}$

$$\frac{dy}{dx} = 6x - 2$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 6(1) - 2$$

$$= 6 - 2$$

$$= 4$$

$\therefore$ slope of tangent $= 4$

slope of normal $= -1/4$

Equation of tangent.

1. Find the equation of tangent to the curve
$y = x^3 - 2_{(1,-2)}$

$$\frac{dy}{dx} = 3x^2$$

$$m = \left[\frac{dy}{dx}\right]_{(1,-2)} = 3(1)^2$$

$$= 3$$

$\therefore$ ~~slope of~~ equation of tangent $= (y - y_1) = m(x - x_1)$

$$[y - (-2)] = 3(x-1)$$
$$y + 2 = 3x - 3$$
$$3x - 3 = y + 2$$
$$3x - 3 - y - 2 = 0$$
$$3x - y - 5 = 0$$

2. Find the equation of the tangent to the curve $y = x^2 + 2x_{(2,4)}$

$$\frac{dy}{dx} = 2x + 2(1)$$
$$= 2x + 2$$
$$= 2x + 2$$
$$= 2(2) + 2$$
$$= 4 + 2$$
$$= 6$$

$\therefore$ equation of the tangent $= (y - y_1) = m(x - x_1)$
$$= (y - 4) = 6(x - 2)$$
$$= y - 4 = 6x - 12$$
$$= 6x - 12 = y - 4$$
$$= 6x - 12 - y + 4 = 0$$
$$= 6x - y - 8 = 0$$

3. Find the equation of the normal to the curve $y = x^2 + 3x - 1_{(1,3)}$

$$\frac{dy}{dx} = 2x + 3$$

$$\left(\frac{dy}{dx}\right)_{(1,3)} = 2(1) + 3$$
$$= 2 + 3 = 5$$

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$\therefore$ Equation of normal $= (y-y_1) = -1/m\,(x-x_1)$

$$(y-3) = -1/5\,(x-1)$$
$$5(y-3) = -1(x-1)$$
$$5y-15 = -x+1$$
$$5y-15+x-1=0$$
$$x+5y-16=0$$

4 Find the equation of tangent and normal to the curve $y = x^2 + x_{(1,8)}$

Velocity and Accelaration

$$\text{Velocity} = \frac{ds}{dt}$$
$$\text{Acceleration} = \frac{d^2s}{dt^2}$$

1. The Law of motion of a particle is $s = 5t^2 + 6t + 3$ where s is the distance in meters and t time in seconds. find the velocity when $t = 2$ seconds

$$s = 5t^2 + 6t + 3$$
$$\frac{ds}{dt} = 5\frac{d}{dt}(t^2) + 6\cdot\frac{d}{dt}(t) + \frac{d}{dt}(3)$$
$$v = \left(\frac{ds}{dt}\right) = 10t + 6$$
$$\left(\frac{ds}{dt}\right)_{t=2} = 10(2) + 6$$
$$= 20 + 6$$
$$= 26\ m/s$$

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2. Find the velocity if $s = 8t^2 + 2t + 6$ at $t = 2$ sec

$$s = 8t^2 + 2t + 6$$

$$\frac{ds}{dt} = 8\frac{d}{dt}(t^2) + 2\frac{d}{dt}(t) + \frac{d}{dt}(6)$$

$$v = \left(\frac{ds}{dt}\right) = 16t + 2$$

$$\left(\frac{ds}{dt}\right)_{t=2} = 16(2) + 2$$

$$= 32 + 2$$

$$= 34 \text{ m/s}$$

3. Equation of motion is given by $s = 5t^3 - 3t^2 + 2t +$ find the velocity at $t = 1$ sec and acceleration $t = 2$ sec.

$$s = 5t^3 - 3t^2 + 2t + 8$$

$$\frac{ds}{dt} = 5\frac{d}{dt}(t^3) - 3\frac{d}{dt}(t^2) + 2\frac{d}{dt}(t) + \frac{d}{dx}($$

$$= 15t^2 - 6t + 2$$

$$v = \left(\frac{ds}{dt}\right)_{t=1} = 15(1)^2 - 6(1) + 2$$

$$= 15 - 6 + 2$$

$$= 11 \text{ m/s}$$

$$a = \frac{d^2s}{dt^2} = 30t - 6$$

$$\left(\frac{d^2s}{dt^2}\right)_{t=2} = 30(2) - 6$$

$$= 60 - 6$$

$$= 54 \text{ m/s}^2$$

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4. Find the velocity and acceleration equation of motion is given by $s = t^3 - t^2 + 9t +$ find initial velocity and acceleration where s in meters and t = seconds.
[for acceleration $t = 2$ sec]
Note = initial velocity $t = 0$

$$s = t^3 - t^2 + 9t + 8$$

$$\frac{ds}{dt} = \frac{d}{dt}(t^3) - \frac{d}{dt}(t^2) + 9\frac{d}{dt}(t) + \frac{d}{dt}(8)$$

$$= 3t^2 - 2t + 9 + 0$$

$$v = \left(\frac{ds}{dt}\right)_{t=0} = 3(0)^2 - 2(0) + 9 \cancel{+}$$

$$= 9$$

$$a = \frac{d^2s}{dt^2} = 6t - 2 + 9$$

$$\left(\frac{d^2s}{dt^2}\right)_{t=2} = 6(2) - 2 + \cancel{0}$$

$$= 12 - 2$$

$$= 10\ m/s^2$$

6 Mark

Maximum and Minimum.

- putting at $x = 0$
- $\frac{d^2y}{dx^2} < 0$, a maxima.
- $\frac{d^2y}{dx^2} > 0$. a minima.

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1. Find the maximum value of the function $y = x^3 - 3x + 4$

$$y = x^3 - 3x + 4$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - 3\frac{d}{dx}(x) + \frac{d}{dx}(4)$$

$$= 3x^2 - 3 + 0 \text{ —— } (1)$$

$$\frac{d^2y}{dx^2} = 6x \rightarrow (2)$$

put $dy/dx = 0$

$$3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

At $x = 1$

$$\frac{d^2y}{dx^2} = 6x$$

$$\frac{d^2y}{dx^2} = 6(1) = 6 > 0, \text{ a minima.}$$

At $x = -1$,

$$\frac{d^2y}{dx^2} = 6(1)$$

$$= 6(-1) = -6 < 0, \text{ a maxima.}$$

\# 2 Find the maximum and minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$

$$\frac{dy}{dx} = 2\frac{d}{dx}(x^3) - 21\frac{d}{dx}(x^2) + 36\frac{d}{dx}(x) - \frac{d}{dx}(20)$$

$$= 6x^2 - 42x + 36$$

$$= 12x - 42$$

$= 6x^2 - 42x + 36 = 0$

$= 6(x^2 - 7x + 6) = 0$

$= x^2 - 7x + 6 = 0$

$a = 1,\ b = -7,\ c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 6}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{2}$$

$$x = \frac{7 \pm \sqrt{25}}{2}$$

$$x = \frac{7 \pm 5}{2}$$

$$x = \frac{7+5}{2},\quad x = \frac{7-5}{2}$$

$$x = \frac{\cancel{12}^{6}}{\cancel{2}_{1}},\quad x = \frac{\cancel{2}^{1}}{\cancel{2}_{1}}$$

$x = 6,\quad x = 1$

for $x = 6$

$$\frac{d^2y}{dx^2} = 12x - 42$$

$= 12(6) - 42$

$= 72 - 42$

$= 30 > 0$ minimum

for $x = 1$

$$\frac{d^2y}{dx^2} = 12x - 42$$

$= 12(1) - 42$

$= 12 - 42$

$= -30 < 0$ maximum

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$y = 2x^3 - 21x^2 + 36x - 20$

$y\,max = 2(1)^3 - 21(1)^3 + 36(1) - 20$

$= 2 - 21 + 36 - 20$

$= 38 - 41$

$y\,max. = -3$

$y = 2x^3 - 21x^2 + 36x - 20$

$y\,min = 2(6)^3 - 21(6)^2 + 36(6) - 20$

$= 2(216) - 21(36) + 216 - 20$

$= 432 - 756 + 216 - 20$

$y\,min = -128$

3. Find the maximum and minimum value of the function. $y = x^5 - 5x^4 + 5x^3 - 1$

$y = x^5 - 5x^4 + 5x^3 - 1$

$\frac{dy}{dx} = 5x^4 - 20x^3 + 15x^2$

$\frac{d^2y}{dx^2} = 20x^3 - 60x^2 + 30x$

put, $\frac{dy}{dx} = 0$

$5x^4 - 20x^3 + 15x^2 = 0$

$5x^2(x^2 - 4x + 3) = 0$

$x^2 - 4x + 3 = 0$

$a = 1,\ b = -4,\ c = 3$

Formula

$x - \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 3}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 - 12}}{2}$

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$$x = \frac{4 \pm \sqrt{4}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4+2}{2}, \quad x = \frac{4-2}{2}$$

$$= \frac{\cancel{6}^{3}}{\cancel{2}_{1}}, \quad \frac{\cancel{2}^{1}}{\cancel{2}_{1}}$$

$$x = 3, \quad x = 1$$

For $x = 3$

$$\frac{d^2y}{dx^2} = 20x^3 - 60x^2 + 30x$$

$$= 20(3)^3 - 60(3)^2 + 30(3)$$

$$= 20(27) - 60(9) + 30(3)$$

$$= \cancel{540} - \cancel{540} + 90$$

$$= 90 > 0 \text{ minimum}$$

$$\frac{d^2y}{dx^2} = x = 1$$

$$= 20x^3 - 60x^2 + 30x$$

$$= 20(1)^3 - 60(1)^2 + 30(1)$$

$$= 20 - 60 + 30$$

$$= 50 - 60$$

$$\frac{d^2y}{dx^2} = -10 < 0 \text{ maximum}$$

$$y = x^5 - 5x^4 + 5x^3 - 1$$

$$y_{max} = (1)^5 - 5(1)^4 + 5(1)^3 - 1$$

$$= 1 - 5 + 5 - 1$$

$$y_{max} = 0$$

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$y = x^5 - 5x^4 + 5x^3 - 1$

$y\,min = (3)^5 - 5(3)^4 + 5(3)^3 - 1$

$= 243 - 5(81) + 5(27) - 1$

$= 243 - 405 + 135 - 1$

$= 378 - 406$

$= -28$

4. Find the maximum and minimum value of the function $y = x^3 - 12x^2 - 27x + 16$.

$y = x^3 - 12x^2 - 27x + 16$

$\frac{dy}{dx} = 3x^2 - 24x - 27$

$\frac{d^2y}{dx} = 6x - 24$

put $\frac{dy}{dx} = 0$

$3x^2 - 24x - 27 = 0$

$3(x^2 - 8x - 9) = 0$

$x^2 - 8x - 9 = 0$

$a = 1,\ b = -8,\ c = -9$

Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times -9}}{2(1)}$

$x = \frac{+8 \pm \sqrt{64 + 36}}{2}$

$x = \frac{+8 \pm \sqrt{100}}{2}$

$x = \frac{+8 \pm 10}{2}$

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$x = \frac{8+10}{2}$, $x = \frac{8-10}{2}$

$= \frac{\cancel{18}^{9}}{\cancel{2}_{1}}$, $\frac{-\cancel{2}^{1}}{\cancel{2}_{1}}$

$x = 9$, $x = -1$

For $x = 9$

$\frac{d^2y}{dx^2} = 6x - 24$

$= 6(9) - 24$

$= 54 - 24$

$\frac{d^2y}{dx^2} = 30 > 0$ minimum

For $x = -1$

$\frac{d^2y}{dx^2} = 6x - 24$

$= 6(-1) - 24$

$= -6 - 24$

$= -30 < 0$ maximum

$y = x^3 - 12x^2 - 27x + 16$

$y_{max} = (-1)^3 - 12(-1)^2 - 27(-1) + 16$

$= -1 - 12 + 27 + 16$

$= -13 + 43$

$y_{max} = 30$

$y = x^3 - 12x^2 - 27x + 16$

$y_{min} = 9^3 - 12(9)^2 - 27(9) + 16$

$= 729 - 12(81) - 243 + 16$

$= 745 - 1215$

$y_{min} = -467$

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Integral Calculus and Applications.

List of standard Integrals

1. $\int 0\, dx = c$ [constant]

2. $\int 1\, dx = x + c$

3. $\int k\, dx = kx + c$ where k is constant

4. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$ where $n \neq -1$

5. $\int \frac{1}{x^n}\, dx = \frac{-1}{(n-1)x^{n-1}} + c$ $n \neq 1$

6. $\int 1/x\, dx = \log x + c$ $n \neq 1$

7. $\int 1/x^2\, dx = -1/x + c$

8. $\int 1/\sqrt{x}\, dx = 2\sqrt{x} + c$

9. $\int \sqrt{x}\, dx = 2/3\, x^{3/2} + c$

10. $\int a^x\, dx = \frac{a^x}{\log a} + c$

11. $\int e^x\, dx = e^x + c$

12. $\int \sin x\, dx = -\cos x + c$

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13. $\int \cos x = \sin x + c$

14. $\int \tan x \, dx = \log \sec x + c$

15. $\int \cot x \, dx = \log \cos x + c$

16. $\int \sec x \, dx = \log(\sec x + \tan x) + c$

17. $\int \operatorname{cosec} x \, dx = \log(\operatorname{cosec} x - \cot x) + c$

18. $\int \sec^2 x \, dx = \tan x + c$

19. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

20. $\int \sec x \tan x \, dx = \sec x + c$

21. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

22. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$

23. $\int \frac{1}{\sqrt{1+x^2}} \, dx = \tan^{-1} x + c$

24. $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c$

25. $\int \sin ax \cdot dx = \frac{-\cos ax}{a} + c$

Standard integrals when 'x' is replaced by (ax+b)

1. $$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

2. $$\int \frac{1}{(ax+b)} \, dx = \frac{\log(ax+b)}{a} + c$$

3. $$\int \frac{1}{\sqrt{(ax+b)}} \, dx = \frac{2\sqrt{(ax+b)}}{a} + c$$

4. $$\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + c$$

5. $$\int \cos(ax+b) \, dx = \frac{\sin(ax+b)}{a} + c$$

6. $$\int \tan(ax+b) \, dx = \frac{\log(\sec(ax+b))}{a} + c$$

7. $$\int \cot(ax+b) \, dx = \frac{\log[\sin(ax+b)]}{a} + c$$

8. $$\int \sec(ax+b) \, dx = \frac{\log[\sec(ax+b) + \tan(ax+b)]}{a} + c$$

9. $$\int \sec^2(ax+b) \, dx = \frac{\tan(ax+b)}{a} + c$$

10. $$\int \operatorname{cosec}^2(ax+b) \, dx = -\frac{\cot(ax+b)}{a} + c$$

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11. $\int \sec x \tan x \, dx = \frac{\sec(ax+b)}{a} + c$

12 $\int \operatorname{cosec} x \cot x \, dx = -\frac{\operatorname{cosec}(ax+b)}{a} + c$

problems :

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C_1$$

1. Integrate w.r.t. x

$$\int x^4 dx = \frac{x^{4+1}}{4+1} + C_1$$
$$= \frac{x^5}{5} + C_1$$

2 $$\int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + C_1$$
$$= \cancel{\frac{x^{-1}}{-1}} = \frac{x^{-1}}{-1} + C_1$$

3. $$\int x^5 dx = \frac{x^{5+1}}{5+1} + C_1$$
$$= \frac{x^6}{6} + C_1$$

4. $$\int x^{3/2} dx = \frac{x^{3/2+1}}{3/2+1} + C_1$$
$$= \frac{x^{4/2}}{4/2} + C_1$$

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5. $\int x^6\,dx = \frac{x^{6+1}}{6+1} = \frac{x^7}{7} + C_1$

1. Evaluate integral of $(x+1)(x+2)$

$\int (x+1)(x+2)\,dx$

$= (x+1)(x+2)$

$= x^2 + 2x + 1x + 2$

$= x^2 + 3x + 2$

$= \int (x^2 + 3x + 2)\,dx$

$= \int x^2\,dx + 3\int x\,dx + 2\int dx$

$= \frac{x^3}{3} + \frac{3x^2}{2} + 2x + C_1$

$\int x^n\,dx$

$= \int x^2\,dx$

$= \frac{x^{2+1}}{2+1} + C_1$

$= \frac{x^3}{3} + C_1$

2. Evaluate integral of $(x^5 + x^4)(-x^2 + 2)\,dx$

$\int (x^5 + x^4)(-x^2 + 2)\,dx$

$= (x^5 + x^4)(-x^2 + 2)$

$= -x^7 + 2x^5 - x^6 + 2x^4$

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Rules of Integration

If u and v are functions of x

$$\int k u \, dx = k \int u \, dx$$

$$\int (u+v) \, dx = \int u \, dx + \int v \, dx$$

$$\int (u-v) \, dx = \int u \, dx - \int v \, dx$$

1. Evaluate integral of $\int x^{14} \, dx$

$$\int x^{14} \, dx = \frac{x^{14+1}}{14+1} + C_1$$

$$= \frac{x^{15}}{15} + C_1$$

2. Integrate $\int x^{-8}$ w.r.t. x

$$\int x^{-8} \, dx = \frac{x^{-8+1}}{-8+1} + C$$

$$= \frac{8^{-7}}{-7} + C_1$$

3. Evaluate integral of $\int 1/x^{5/3} \, dx$

$$\int x^{-5/3} \, dx = \frac{x^{-5/3+1}}{-5/3+1} + C$$

$$= \frac{x^{-2/3}}{-2/3} + C_1$$

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4. Evaluate integral of $\int \sqrt{x^6}\, dx$

$\int \sqrt{x^6}\, dx$ $\qquad\qquad$ $\sqrt{x} = x^{1/2}$

$= \int (x^6)^{1/2}\, dx$ $\qquad\qquad$ $(x^m)^n = x^{m \times n}$

$= \int x^3\, dx$ $\qquad\qquad$ $x^{6/2} = 6 \times 1/2$

$= \dfrac{x^4}{4} + C_1$ $\qquad\qquad$ $= 6/2 = 3$

5. Integrate $\sqrt[3]{x^2}$ $\qquad\qquad$ $\sqrt[3]{x} = 1/3$

$= \int \sqrt[3]{x^2}\, dx$ $\qquad\qquad$ $\dfrac{2}{3} + \dfrac{1}{1}$

$= \int x^{2/3}\, dx$ $\qquad\qquad$ $\dfrac{2+3}{3} = \dfrac{5}{3}$

$= \dfrac{x^{2/3+1}}{2/3+1} + C_1$

$= \dfrac{x^{5/3}}{5/3} + C_1$

6. Evaluate integral of $(x^3 + 2x + 1)\, dx$

$\int (x^3 + 2x + 1)\, dx$

$= \int x^3\, dx + 2\int x\, dx + \int 1\, dx$

$= \dfrac{x^4}{4} + 2\dfrac{x^2}{2} + 1x + C$

$= \dfrac{x^4}{4} + x^2 + 1x + C$

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7. Evaluate integral of $\int (x + 1/x)^2 \, dx$

$$\int \left(x + \frac{1}{x}\right)^2 dx$$

$$= \int \left(x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x}\right) dx$$

$$= \int \left(x^2 + \frac{1}{x^2} + 2\right) dx$$

$$= \int x^2 dx + \int x^{-2} dx + 2 \int dx$$

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + C_1$$

8. Evaluate integral of $\int (x - 1/x)^2 \, dx$

$$\int \left(x - \frac{1}{x}\right)^2 dx$$

$$= \int \left(x^2 - \frac{1}{x^2} + \right.$$

9. Evaluate $\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + C$

$$\int (2x+1)^7 \, dx$$

$$= \frac{(2x+1)^{7+1}}{(7+1)\cdot 2} + C$$

$$= \frac{(2x+1)^8}{16} + C$$

10. Integrate $(8x+4)^8 \, dx$

$$\int (8x+4)^8 \, dx$$

$$= \frac{(8x+4)^{8+1}}{(8+1)\cdot 8} + C$$

$$= \frac{(8x+4)^9}{9\cdot 8} + C$$

$$= \frac{(8x+4)^9}{72} + C$$

11. Evaluate $\int \dfrac{1}{(ax+b)^n} \, dx$ $\quad \int \dfrac{1}{(8x+1)^6} \, dx$

$$= \int (ax+b)^{-n} \, dx$$

$$= \frac{(ax+b)^{-n+1}}{a(-n+1)} + C$$

$$\int \frac{1}{(2x+1)^2} \, dx$$

$$= \int (2x+1)^{-2} \, dx$$

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$= \frac{(2x+1)^{-2+1}}{2(-2+1)}$

$= \frac{(2x+1)^{-1}}{2(-1)} + C_1 \quad = \frac{(2x+1)^{-1}}{-2} + C_1$

12 Evaluate $\int \frac{1}{(ax+b)} dx$

$= \frac{\log(ax+b)}{a} + C_1$

$\int \frac{1}{(2x+16)} dx$

$= \frac{\log(2x+16)}{2} + C_1$

$= \int \frac{1}{(2x+1)} dx$

$= \frac{\log(2x+1)}{2} + C_1$

problems :-

4m 1. Evaluate intergal of $\int (2\sin x + 4\cos x + 2x)\, dx$

$\int (2\sin x + 4\cos x + 2a)\, dx$

$= 2\int \sin x\, dx + 4\int \cos x + 2\int x\, dx$

$= -2\cos x + 4\sin x + \frac{\cancel{2}x^2}{\cancel{2}} + C_1$

$= 2\cos x + 4\sin x + x^2 + C_1$

2 Evaluate the intergal of $\int x^5 + \frac{1}{x} + \operatorname{cosec}^2 x + e^{2x}$ $+ \sin x)\, dx$.

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$= \int (x^2 + \sin 2a + e^{-ax} + 4)\, dx$

$= \int (x^4 - 2e^{4x} + \sin x + 2)\, dx$

$= \int \left(\frac{1}{\sqrt{x}} + a^x + x^4 + 2x\right) dx$

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6M 1. Evaluate intergal of $\int (\tan^2 x)\, dx$

$$= \int (\sec^2 x - 1)\, dx \qquad 1 + \tan^2 x = \sec^2 x$$

$$= \int \sec^2 x\, dx - \int 1\, dx \qquad \tan^2 x = \sec^2 x - 1$$

$$= \tan x - x + C$$

2. Evaluate intergal of $\int (\cot^2 x)\, dx$

$$= \int (\operatorname{cosec}^2 x - 1)\, dx \qquad 1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$= \int \operatorname{cosec}^2 x\, dx - \int dx \qquad \cot^2 x = \operatorname{cosec}^2 x - 1$$

$$= -\cot x - x + C$$

3. Evaluat intergal of $\int \frac{1}{1+\sin x}\, dx$

$$= \int \frac{1}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)}\, dx$$

$$= \int \frac{1-\sin x}{1-\sin^2 x}\, dx$$

$$= \int \frac{1-\sin x}{\cos^2 x}\, dx$$

$$\sin^2 x + \cos^2 x = 1$$
$$\cos^2 x = 1 - \sin^2 x$$

HW

$\int \frac{1}{1-\sin}$

$$= \int \frac{1}{\cos^2 x}\, dx - \int \frac{\sin x}{\cos^2 x}\, dx$$

$\int \frac{1}{1+\cos}$

$$= \int \sec^2 x\, dx - \int \sec x \tan x\, dx$$

$$= \tan x - \sec x + C$$

$\int \frac{1}{1-\cos}$

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^2x

$^2x-1$

sec^2x

c^2x-1

dx

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1. Evaluate $\int \sin^3 x\, dx$

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$3\sin x - 4\sin^3 x = \sin 3x$$
$$-4\sin^3 x = \sin 3x - 3\sin x$$
$$-\sin^3 x = \frac{\sin 3x - 3\sin x}{4}$$
$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$\int \sin^3 x\, dx$$
$$= \int \left(\frac{3\sin x - \sin 3x}{4}\right) dx$$
$$= \int \frac{3\sin x}{4}\, dx - \int \frac{\sin 3x}{4}\, dx$$
$$= \frac{3}{4}\int \sin x\, dx - \frac{1}{4}\int \sin 3x\, dx$$
$$= \frac{-3}{4}\cos x + \frac{1}{4}\,\frac{\cos 3x}{3} + C$$

2. Evaluate intergal of $\int \sqrt{1+\sin 2x}\, dx$

$$\int \sqrt{1+\sin 2x}\, dx$$
$$= \int \sqrt{\sin^2 x + \cos^2 x + \sin 2x}\, dx$$
$$= \int \sqrt{(\sin x + \cos x)^2}\, dx$$
$$= \int (\sin x + \cos x)\, dx$$

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$$= \int \sin x \, dx + \int \cos x \, dx$$

$\sin^2 x + \cos^2 x = 1$
$\sin 2x = 2 \sin x \cos x$

$$= -\cos x + \sin x + C$$

3. Evaluate $\int \sqrt{1+\sin x} \, dx$

$$= \int \sqrt{\left(\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \right)} \, dx$$

$$= \int \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2} \, dx$$

$$= \int \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) dx$$

$\sin^2 x/2 + \cos^2 x/2 = 1$
$\sin x = 2 \sin x/2 \cdot \cos x/2$

$$= \int \sin\left(\frac{x}{2}\right) dx + \int \cos\left(\frac{x}{2}\right) dx$$

$$= \frac{-\cos(x/2)}{1/2} + \frac{\sin(x/2)}{1/2}$$

4. Evaluate intergal of $\int \sin 4x \cdot \cos 2x \, dx$

$\int \sin 4x \cdot \cos 2x \, dx$

$\sin A \cos B =$
$1/2 \{ \sin(A+B) + \sin(A-B) \}$

$$= \frac{1}{2} \int \left[\sin(4x+2x) + \sin(4x-2x) \right] dx$$

$$= \frac{1}{2} \int \left(\sin 6x + \sin 2x \right) dx$$

$$= \frac{1}{2} \left[\int \sin 6x \, dx + \int \sin 2x \, dx \right]$$

$$= \frac{-1}{2} \left[\frac{\cos 6x}{6} - \frac{\cos 2x}{2} \right] + C$$

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4. Evaluate intergal of $\int \cos 5x \cos 2x \, dx$

$\int \cos 5x \cdot \cos 2x$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}\int \left[\cos(5x+2x) + \cos(5x-2x)\right] dx$

$= \frac{1}{2}\int (\cos 7x + \cos 3x)\, dx$

$= \frac{1}{2}\left[\int \cos 7x \, dx + \int \cos 3x \, dx\right.$

$= \frac{1}{2}\left(\frac{\sin 7x}{7} + \frac{\sin 3x}{3}\right) + C_1$

- Integration by parts.

If u and v are 2 functions of x then intergal of

$\int uv \, dx = u\int v\, dx - \int\int v \cdot \frac{d}{dx}(u)\, dx$

1. Evaluate integral of $\int x \cos x \, dx$

$\int x \cos x \, dx = x\int \cos x \, dx - \int\int \cos x \frac{d}{dx}(x) \cdot dx$

$= x \sin - \int \sin x (1)\, dx$

$= x \sin x + \cos x + C_1$

2. Evaluate integral of $\int x \sin x \, dx$

$\int x \sin x \, dx = x\int \sin x \, dx - \int\int \sin x \frac{d}{dx}(x) \cdot dx$

$= x - \cos x - \int -\cos x (1)\, dx$

$= -x\cos x + \sin x + C_1$

3. Evaluate $\int x \cdot e^{x}\, dx$

$$\int x \cdot e^{x}\, dx = x\int e^{x}\, dx - \int\int e^{x} \frac{d}{dx}(x) \cdot dx$$

$$= x e^{x} - \int e^{x} \cdot (1)\, dx$$

$$= x e^{x} - e^{x} + C_1$$

4. Evaluate intergal of $\int x \log x \, dx$

$$= \int \log x \cdot x \, dx$$

$$= \log x \int x \, dx - \int\int x \cdot \frac{d}{dx}(\log x)\, dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\left(\frac{1}{x}\right) \cdot dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\int x \cdot dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2} \cdot \frac{x^2}{2} + C_1$$

$$= \frac{x^2}{2}\log x - \frac{x^2}{4} + C_1$$

5. Evaluate Intergal of $\int \log x \, dx$

$$\int \log x \, dx$$

$$= \int \log x \cdot 1 \, dx = \log x \int dx - \int\int 1 \cdot \frac{d}{dx}(\log x)\, dx$$

$$= \log x \cdot x - \int x \frac{1}{x}\, dx$$

$$= x \log x - \int dx$$

$$= x \log x - x + C_1$$

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6. Evaluate : $\int x \sin 2x \, dx$

$$= x\int \sin 2x \, dx - \int\int \sin 2x \frac{d}{dx}(x) \, dx$$

$$= \frac{x(-\cos 2x)}{2} - \int -\frac{\cos 2x}{2}(1) \, dx$$

$$= \frac{-x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \, dx$$

$$= \frac{-x\cos 2x}{2} + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$$

7. Evaluate : $\int x e^{2x} \, dx$

$$= x\int e^{2x} \, dx - \int\int e^{2x} dx \frac{d}{dx}(x)$$

$$= x\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}(1) \, dx$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x} \, dx$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\left(\frac{e^{2x}}{2}\right) + C$$

8. Integration by substitution method
6M

1. Evaluate intergal of $\int \sin x \cdot \cos x \, dx$

put $\sin x = t$

$\cos x = dt$

$\int \sin x \cdot \cos x \, dx$

$= \int t \, dt$

$= \frac{t^2}{2} + C$

$y = \sin x$

$\frac{dy}{dx} = \sin x$

$t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \cdot dx$

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$= \frac{(\sin x)^2}{2} + C_1$

2 Evaluate intergal of $\int \sin x^2 \cdot \cos\ dx$

put $\sin^3 x^2 = t$

$\cos x = dt$

$\int \sin x^2 \cdot \cos x\ dx$

$= \int t^2 \cdot dt$

$= \int \frac{t^3}{3} + C_1$

$= \frac{(\sin x)^3}{3} + C_1$

3. Evaluate : $\int \sin^6 x \cos x\ dx$

put

$\sin x = t$

$\cos x = \frac{dt}{dx}$

$\cos x\, dx = dt$

$= \int t^6\, dt$

$= \frac{t^7}{7} + C_1$

$= \frac{(\sin x)^7}{7} + C_1$

4. Evaluate : $\int \frac{6x-5}{\sqrt{3x^2-5x+2}}\, dx$

$= \int \frac{1}{\sqrt{t}}\, dt$

$= \int (t)^{-1/2}\, dt$

$= \frac{t^{-1/2+1}}{-1/2+1} + C$

$= \frac{(3x^2-5x+2)^{1/2}}{1/2} + C$

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5. Evaluate : $\int e^{x} \cos(e^{x})\, dx$

$= \int \cos(e^{x})\, e^{x}\, dx$

$= \int \cos t\, dt$

$= \sin t + c$

$= \sin(e^{x}) + c$

put

$e^{x} = t$

$e^{x} = \frac{dt}{dx}$

$e^{x}\, dx = dt$

6. Evaluate : $\int \frac{(1+\log x)^2}{x}\, dx$

$= \int t^2\, dt$

$= \frac{t^3}{3} + c$

$= \frac{(1+\log x)^3}{3} + c$

put

$1+\log x = t$

$0 + \frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x}\, dx = dt$

7. Evaluate : $\int \frac{(\tan^{-1}x)^3}{1+x^2}\, dx$

$= \int t^3\, dt$

$= \frac{t^4}{4} + c$

$= \frac{(\tan^{-1}x)^4}{4} + c$

8. Evaluate : $\int (3x^2+2x-7)^{10} (6x+2)\, dx$

$= \int t^{10}\, dt$

$= \frac{t^{11}}{11} + c$

$= \frac{(3x^2+2x-7)^4}{11} + c$

7. Evaluate : $\int \frac{\cos x}{1+\sin x} dx$

$= \int \frac{dt}{t}$

$= \log t + C$

$= \log (1 + \sin x) + C$

definite Integrals

if $f(x)$ be continuous function and a is the lower limit and b is the upper limit than integral of

$\int_a^b f(x)\, dx$

$= [F(x)]_a^b$

$= [F(b) - F(a)]$

1. Evaluate integral of $\int_1^2 x^4 dx$

$\int_1^2 x^4 dx$

$= \left[\frac{x^5}{5}\right]_1^2$

$= \frac{1}{5}\left[(2)^5 - (1)^5\right]$

$= \frac{1}{5}[32 - 1] = \frac{31}{5}$

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2 Evaluate integral of $\int_1^2 x^2 \, dx$

$$\int_1^2 x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3}\left[(2)^3 - (1)^3\right]$$

$$= \frac{1}{3}\left(8 - 1\right) = \frac{7}{3}$$

3. Evaluate integral of $\int_0^4 x^4 \, dx$

$$\int_0^4 x^4 \, dx$$

$$= \left[\frac{x^5}{5}\right]_0^4$$

$$= \frac{1}{5}\left[(4)^5 - (0)^4\right].$$

$$= \frac{1}{5}\left(34 - 0\right) = \frac{64}{5}.$$

4. Evaluate integral of $\int_0^{2\pi} \sin x \, dx$.

$$\int_0^{\pi/2} \sin x \, dx$$

$$= \left[-\cos x\right]_0^{\pi/2}$$

$$= -\left[\cos\frac{\pi}{2} - \cos 0\right]$$

$$= -\left(0 - 1\right)$$

$$= +1$$

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5. Evaluate integral $\int_0^{\pi/4} \frac{\cancel{\tan}}{\sec^2} x \cdot dx$

$$\int_0^{\pi/4} \frac{\cancel{\tan}}{\sec^2} x \cdot dx$$

$$= \left(\tan x\right)_0^{\pi/4}$$

$$= \left(\tan \frac{\pi}{4} - \tan 0\right)$$

$=$

Applications of Integration.

Area enclosed by the curve by integral method if $y = f(x)$ be a given function and a is the lower ~~upper~~ limit and b is ~~lower limit~~. upper limit then area ~~x~~ $= \int_a^b y\,dx$

1. Find the area bounded by the curve $y = 3x^2 + 2x$ x-axis and the ordinants is $x = 0$ and $x = 1$

3 me se Zarathobi Aatha.

$$A = \int_a^b y\,dx$$

$$A = \int_0^1 (3x^2 + 2x)\,dx$$

$$= \left[\cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}}\right]_0^1$$

$$= \left(x^3 + x^2\right)_0^1$$

$$= \left[\left((1)^3 + (1)^2\right) - 0\,(\,)\right]$$

$$= 1 + 1$$

$$= 2 \text{ sq units.}$$

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2 Find the area bounded by the curve $y = x^2+1$ x-axis and the ordinants at $x = 1$ and $x = 2$

$$A = \int_a^b y\, dx$$

$$= \int_1^2 (x^2+1)\, dx$$

$$= \left[\frac{x^3}{3} + x\right]_1^2$$

$$= \left[\left(\left(\frac{2}{3}\right)^3 + (2)\right) - \left(\left(\frac{1}{3}\right)^3 + (1)\right)\right]$$

$$= \left[\left(\frac{8}{3} + 2\right) - \left(\frac{1}{3} + 1\right)\right]$$

$$= \left[\left(\frac{8+6}{3}\right) - \left(\frac{1+3}{3}\right)\right]$$

$$= \frac{14}{3} - \frac{4}{3} = \frac{14-4}{3} = \frac{10}{3} \text{ sq unit}$$

3. Calculate the area converging due to radio active dk of an element given by the equation $y = x^3+1$ in b/w the ordinants $x = 0$ and $x = 1$ bounded by x - axis.

$$A = \int_0^1 (x^3+1)\, dx$$

$$= \left[\frac{x^4}{4} + x\right]_0^1$$

$$= \left[\left(\frac{1}{4} + 1\right) - 0(\ )\right]$$

$$= \frac{1}{4} + 1$$

$= \frac{1+4}{4}$

$= \frac{5}{4}$ sq. units

If $y = f(x)$ be a function given and a is the lower limit and b is the Upper limit then $V = \pi \int_a^b y^2 \, dx$

1. Find the volume of the solid generated by the revolution of the curve $y = \sqrt{x^3 + 5x}$ b/w the Ordinants $x = 2$ and $x = 4$ about x-axis.

$$V = \pi \int_a^b y^2 \, dx$$

$y = \sqrt{x^3 + 5x}$

$y^2 = \left[(x^3 + 5x)^{1/2}\right]^2$

$y^2 = x^3 + 5x$

$$= \pi \int_2^4 (x^3 + 5x) \, dx$$

$$= \pi \left[\left(\frac{x^4}{4} + 5 \cdot \frac{x^2}{2}\right)\right]_2^4$$

$$= \pi \left[\left(\frac{(4)^4}{4} + \frac{5}{2}(4)^2\right) - \left(\frac{(2)^4}{4} + \frac{5}{2}(2)^2\right)\right]$$

$$= \pi \left[\left(\frac{256}{4} + \frac{5}{2} \times 16\right) - \left(\frac{16}{4} + \frac{5}{2} \times 4\right)\right]$$

$$= \pi \left[(64 + 40) - (4 + 10)\right]$$

$$= \pi (104 - 14)$$

$$= \pi (90)$$

$= 90\pi$ cubic units.

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2 Find the volume generated by rotating the curve $y = \sqrt{x+2}$ about x-axis b/w $x = 0$ and $x = 2$

$$V = \pi \int_a^b y^2 \, dx$$

$y = \sqrt{x+2}$
$y^2 = \left((x+2)^{1/2}\right)^2$
$y^2 = x + 2\cancel{x}$

$$= \pi \int_0^2 (x+2) \, dx$$

$$= \pi \left[\frac{x^2}{2} + 2x\right]_0^2$$

$$= \pi \left[\frac{(2)^2}{2} + 2(2)\right] - \left(0\,(\,)\right)$$

$$= \pi \left(\frac{4}{2} + 4\right)$$

$$= \pi \left[\frac{4+8}{2}\right]$$

$$= \pi \left[\frac{\cancel{12}^{\,6}}{\cancel{2}_{\,1}}\right] = 6\pi \text{ cubic units}$$

3.