Chapter – 3

Motion in a Straight Line

NCERT Back Exercises:

Ques 3.1: In which of the following examples of motion, can the body be considered approximately a point object:

(i) A railway carriage moving without jerks between two stations.

(ii) A monkey sitting on top of a man cycling smoothly on a circular track.

(iii) A spinning cricket ball that turns sharply on hitting the ground.

(iv) A tumbling beaker that has slipped off the edge of a table.

Ans 3.1:

(i), (ii) can be treated as a point sized object.

(i) The size of a carriage is very small as compared to the distance between two stations. Therefore, the carriage *can be treated as a point sized object*.

(ii) The size of a monkey is very small as compared to the size of a circular track. Therefore, the monkey *can be considered as a point sized object on the track.*

(iii) The size of a spinning cricket ball is comparable to the distance through which it turns sharply on hitting the ground. Hence, the cricket ball *cannot be considered as a point object.*

(iv) The size of a beaker is comparable to the height of the table from which it slipped. Hence, the beaker *cannot be considered as a point object.*

Ques 3.2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.19. Choose the correct entries in the brackets below;

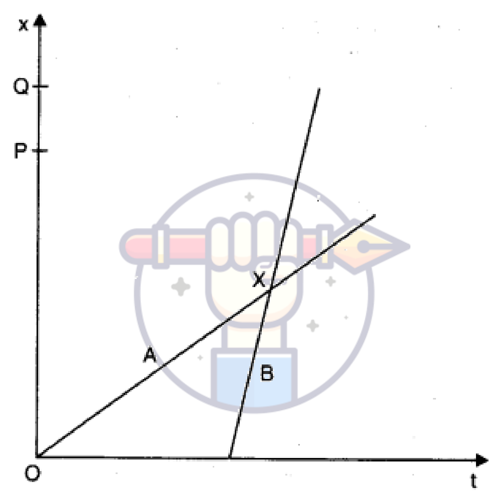
(i) (A/B) lives closer to the school than (B/A)

(ii) (A/B) starts from the school earlier than (B/A)

(iii) (A/B) walks faster than (B/A)

(iv) A and B reach home at the (same/different) time

(v) (A/B) overtakes (B/A) on the road (once/twice).



Ans 3.2:

(i) A lives closer to school than B, because B has to cover higher distances $[OP < OQ]$,

(ii) A starts earlier for school than B, because $t = 0$ for A but for B, t has some finite time.

(iii) As slope of B is greater than that of A, thus B walks faster than A.

(iv) A and B reach home at the same time.

(v) At the point of intersection (i.e., X), B overtakes A on the roads once.

Ques 3.3: A woman starts from her home at 9.00 am, walks with a speed of 5 $km\ h^{-1}$ on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 $km\ h^{-1}$. Choose suitable scales and plot the *x*-*t* graph of her motion.

Ans 3.3: Speed of the woman = 5 km/h

Distance between her office and home = $2.5\ km$

$$Time\ Taken = \frac{Distance}{Speed}$$

$$= \frac{2.5}{5} = 0.5\ h = 30\ min$$

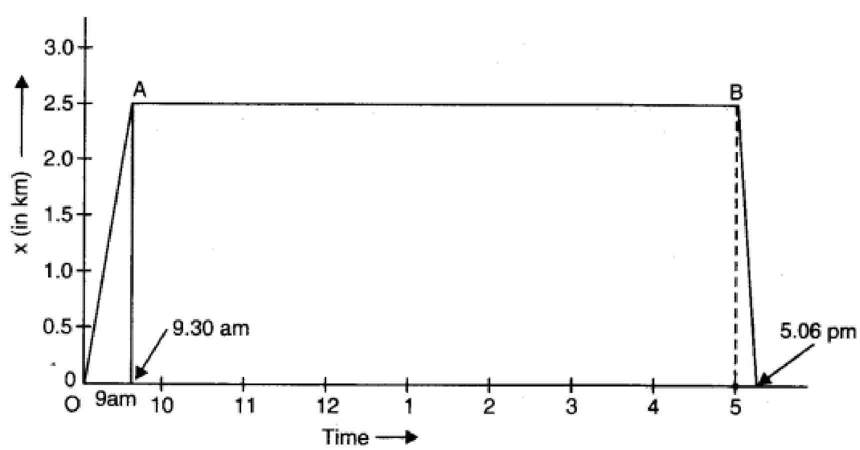
It is given that she covers the same distance in the evening by an auto.

Now, speed of the auto = $25\ km/h$

$$Time\ Taken = \frac{Distance}{Speed}$$

$$= \frac{2.5}{25} = \frac{1}{10} = 0.1\ h = 6\ mins$$

The suitable *x*-*t* graph of the motion of the woman is shown in the given figure.



Ques 3.4: A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the *x*-*t* graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

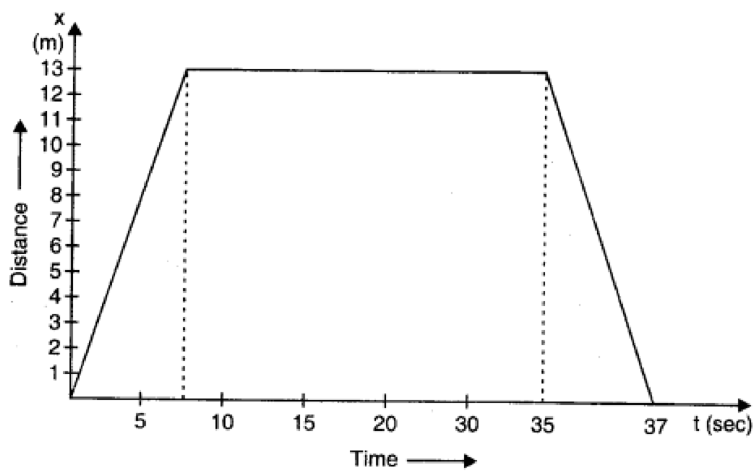
Ans 3.4: Distance covered with 1 step = $\mathbf{1\ m}$
Time taken = $1\ s$
Time taken to move first $5\ m$ forward = $5\ s$
Time taken to move 3 m backward = $3\ s$
Net distance covered = $5 - 3 = 2\ m$
Net time taken to cover $2\ m = 8\ s$

Drunkard covers $2\ m\ in\ 8\ s$.
Drunkard covered $4\ m\ in\ 16\ s$.
Drunkard covered $6\ m\ in\ 24\ s$.
Drunkard covered $8\ m\ in\ 32\ s$.

In the next 5 s, the drunkard will cover a distance of 5 m and a total distance of 13 m and falls into the pit.

Net time taken by the drunkard to cover $13\ m = 32 + 5 = 37\ s$

The *x*-*t* graph of the drunkard's motion can be shown as:



Ques 3.5: A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on ground?

Ans 3.5: Speed of the jet airplane, $\boldsymbol{v_{jet} = 500\ km/h}$

Relative speed of its products of combustion with respect to the plane,

$v_{smoke} = -\ 1500\ km/h$

Speed of its products of combustion with respect to the ground $= v'_{smoke}$

Relative speed of its products of combustion with respect to the airplane,

$v_{smoke} = v'_{smoke} - v_{jet}$

$1500 = v'_{smoke} - 500$

$v'_{smoke} = -\ 1000 km/h$

The negative sign indicates that the direction of its products of combustion is opposite to the direction of motion of the jet airplane.

Ques 3.6: A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Ans 3.6:

Initial velocity of the car, $\boldsymbol{u = 126\ km/h = 35\ m/s}$

Final velocity of the car, $v = 0$

Distance covered by the car before coming to rest, $s = 200\ m$

Retardation produced in the car $= a$

From third equation of motion, a can be calculated as:

$$v^2 - u^2 = 2as$$

$$(0)^2 - (35)^2 = 2 \times a \times 200$$

$$a = -\frac{35 \times 35}{2 \times 200} = -3.06\, m/s^2$$

From first equation of motion, $time\ (t)$ taken by the car to stop can be obtained as:

$$v = u + at$$

$$t = \frac{v-u}{a} = -\frac{35}{-3.06} = 11.44\, s$$

Ques 3.7: Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

Ans 3.7:

<u>For train A:</u>

Initial velocity, $u = 72\, km/h = 20\, m/s$

Time, $t = 50\, s$

Acceleration, $a_1 = 0$ (*Since it is moving with a uniform velocity*)

From second equation of motion, distance (s_1) covered by train A can be obtained as:

$$s_1 = ut + \frac{1}{2}a_1 t^2$$

$$s_1 = 20 \times 50 + 0 = 1000\, m$$

For train B:

Initial velocity, $u = 72\ km/h = 20\ m/s$

Acceleration, $a = 1\ m/s^2$

Time, $t = 50\ s$

From second equation of motion, distance (s_2) covered by train A can be obtained as:

$$s_2 = ut + \frac{1}{2}at^2$$

$$s_2 = 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = 2250\ m$$

Hence, the original distance between the driver of train A and the guard of train B

$= 2250 - 1000 = 1250\ m.$

Ques 3.8: On a two-lane road, car A is travelling with a speed of $36\ km\ h^{-1}$. Two cars B and C approach car A in opposite directions with a speed of $54\ km\ h^{-1}$ each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Ans 3.8:

Velocity of car A, $v_A = 36\ km/h = 10\ m/s$

Velocity of car B, $v_B = 54\ km/h = 15\ m/s$

Velocity of car C, $v_C = 54\ km/h = 15\ m/s$

Relative velocity of car B with respect to car A,

$v_{BA} = v_B - v_A = 15 - 10 = 5\ m/s$

Relative velocity of car C with respect to car A,

$v_{CA} = v_C - (-v_A) = 15 + 10 = 25\ m/s$

At a certain instance, both cars B and C are at the same distance from car A i.e.,

$s = 1\,km = 1000\,m$

Time taken (t) by car C to cover $1000\,m = \frac{1000}{25} = 40\,s$

Hence, to avoid an accident, car B must cover the same distance in a maximum of $40\,s$.

From second equation of motion, minimum acceleration (a) produced by car B can be obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$1000 = 5 \times 40 + \frac{1}{2} \times a \times (40)^2$$

$$a = \frac{1600}{1600} = 1\,m/s^2$$

Ques 3.9: Two towns A and B are connected by a regular bus service with a bus leaving in either direction every *T* minutes. A man cycling with a speed of 20 $km\,h^{-1}$ in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period *T* of the bus service and with what speed (assumed constant) do the buses ply on the road?

Ans 3.9: Let V be the speed of the bus running between towns A and B.

Speed of the cyclist, $v = 20\,km/h$

Relative speed of the bus moving in the direction of the cyclist

$= V - v = \frac{(V - 20)km}{h}$

The bus went past the cyclist every 18 min i.e., $\frac{18}{60}h$

(When he moves in the direction of the bus).

Distance covered by the bus $= (V - 20)\frac{18}{60}\ km$.................................. (i)

Since one bus leaves after every T minutes, the distance travelled by the bus will be equal to:

$V \times \frac{T}{60}$ ....(ii)

Both equations (i) and (ii) are equal.

$(V - 20) \times \frac{18}{60} = \frac{VT}{60}$...$(iii)$

Relative speed of the bus moving in the opposite direction of the cyclist

$= (V + 20)\ km/h$

Time taken by the bus to go past the cyclist $= 6$ min $= \frac{6}{60}\ h$

$\therefore (V + 20)\frac{6}{60} = \frac{VT}{60}$..$(iv)$

From equations (iii) and (iv), we get

$$(V + 20) \times \frac{6}{60} = (V - 20) \times \frac{18}{60}$$

$$V + 20 = 3V - 60$$

$$2V = 80$$

$$V = 40\ km/h$$

Substituting the value of V in equation (iv), we get

$$(40 + 20) \times \frac{6}{60} = \frac{40T}{60}$$

$$T = \frac{360}{40} = 9\ min$$

Ques 3.10: A player throws a ball upwards with an initial speed of $29.4\ m\ s^{-1}$.

(i) What is the direction of acceleration during the upward motion of the ball?

(ii) What are the velocity and acceleration of the ball at the highest point of its motion?

(iii) Choose the $x = 0\ m$ and $t = 0\ s$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.

(iv) To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8\ m/s^2$ and neglect air resistance).

Ans 3.10:

(i) The direction of acceleration during the upward motion of the ball is vertically downward.

(ii) At the highest point, velocity of ball is zero but acceleration ($g = 9.8\ ms^{-2}$) in vertically downward direction.

(iii) If we consider highest point of ball motion as x = 0, t = 0 and vertically downward direction to be +ve direction of x-axis, then
   (a) During upward motion of ball before reaching the highest point position (as well as displacement) x = +ve, velocity v = -ve and acceleration a = g = +ve.
   (b) During the downward motion of ball after reaching the highest point, x, v and a = g all the three quantities are positive.

(iv) During upward motion

Initial velocity, $u = -29.4\ m/s$

$a = 9.8 m/s^2$

Final velocity, $v = 0$

Thus, using the third equation of motion, we get:

$v^2 - u^2 = 2gs$

$$s = \frac{(v^2 - u^2)}{2g}$$

$$s = \frac{(0 - 29.4^2)}{2 \times (9.8)}$$

$$s = -44.1\ m$$

Also,

$$v = u + at$$

$$t = \frac{(v - u)}{a}$$

$$= \frac{29.4}{9.8} = 3\ s$$

Thus, the total time taken by the ball to ascend and come down (air time) $= 2 \times 3 = 6\ s$ seconds.

Ques 3.11: Read each statement below carefully and state with reasons and examples, if it is true or false;

A particle in one-dimensional motion

(i) With zero speed at an instant may have non-zero acceleration at that instant

(ii) With zero speed may have non-zero velocity,

(iii) With constant speed must have zero acceleration,

(iv) With positive value of acceleration must be speeding up.

Ans 3.11:

(i) True. Consider a ball thrown up. At the highest point, speed is zero but the acceleration is non-zero.

(ii) False. If a particle has non-zero velocity, it must have speed.

(iii) True. If the particle rebounds instantly with the same speed, it implies infinite acceleration which is physically impossible.

(iv) False. True only when the chosen position direction is along the direction of motion.

Ques 3.12: A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between *t* = 0 to 12 s.

Ans 3.12:

Ball is dropped from a height, $s = 90\ m$

Initial velocity of the ball, $u = 0$

Acceleration, $a = g = 9.8\ m/s^2$

Final velocity of the ball $= v$

From second equation of motion, time (t) taken by the ball to hit the ground can be obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$90 = 0 + \frac{1}{2} \times 9.8\ t^2$$

$$t = \sqrt{18.38} = 4.29\ s$$

From first equation of motion, final velocity is given as:

$$v = u + at$$

$$= 0 + 9.8 \times 4.29 = 42.04\ m/s$$

Rebound velocity of the ball,

$$u_r = \frac{9}{10} \times v = \frac{9}{10} \times 42.04 = 37.84\ m/s$$

Time (t) taken by the ball to reach maximum height is obtained with the help of first equation of motion as:

$$v = u_r + at'$$

$$0 = 37.84 + (-9.8)\,t'$$

$$t' = \frac{-37.83}{-9.8} = 3.86\ s$$

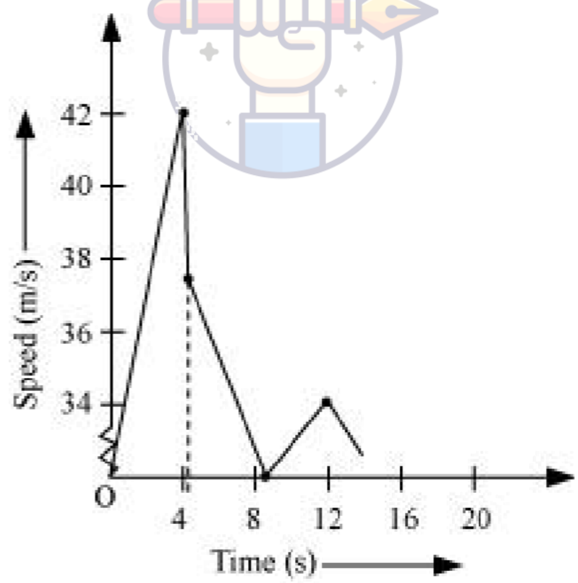
Total time taken by the ball $= t + t' = 4.29 + 3.86 = 8.15\ s$

As the time of ascent is equal to the time of descent, the ball takes $3.86\ s$ to strike back on the floor for the second time.

The velocity with which the ball rebounds from the floor $= \frac{9}{10} \times 37.84 = 34.05\ m/s$

Total time taken by the ball for second rebound $= 8.15 + 3.86 = 12.01\ s$

The speed-time graph of the ball is represented in the given figure as:



Ques 3.13: Explain clearly, with examples, the distinction between:

(i) Magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval;

(ii) Magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].

In (a) and (b) compare and find which among the two quantity is greater.

When can the given quantities be equal?
[For simplicity, consider one-dimensional motion only].

Ans 3.13:

(i) Let us consider an example of a football, it is passed to player B by player A and then instantly kicked back to player A along the same path. Now, the magnitude of displacement of the ball is 0 because it has returned to its initial position. However, the total length of the path covered by the ball $= \boldsymbol{AB + BA = 2AB}$.
Hence, it is clear that the first quantity is greater than the second.

(ii) Taking the above example, let us assume that football takes t seconds to cover the total distance.

Then, The magnitude of the average velocity of the ball over time interval

$$t = \frac{Magnitude\ of\ displacement}{time\ interval} = \frac{0}{t} = 0.$$

The average speed of the ball over the same interval = total length of the path/time interval $= \frac{2AB}{t}$

Thus, the second quantity is greater than the first.

The above quantities are equal if the ball moves only in one direction from one player to another (considering one-dimensional motion).

Ques 3.14: A man walks on a straight road from his home to a market $2.5\ km$ away with a speed of $5\ km\ h^{-1}$. Finding the market closed, he instantly turns and walks back home with a speed of $7.5\ km\ h^{-1}$. What is the

(i) Magnitude of average velocity, and

(ii) Average speed of the man over the interval of time
(a) 0 to 30 min,
(b) 0 to 50 min,
(c) 0 to 40 min?

[Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero!]

Ans 3.14:

Time taken by the man to reach the market from home, $t_1 = \frac{2.5}{5} = \frac{1}{2}h = 30\ min$

Time taken by the man to reach home from the market, $t_2 = \frac{2.5}{7.5} = \frac{1}{3}h = 20\ min$

Total time taken in the whole journey $= 30 + 20 = 50\ min$

Total displacement $= 0$

Total distance $= 2 + 2 = 4\ km$

(i) $Average\ velocity\ (0 - 30\ min) = \frac{Displacement}{Time} = \frac{2.5}{\frac{1}{2}} = 5\ km/h$

(ii) $Average\ velocity\ (0 - 50\ min) = \frac{Displacement}{Time} = \frac{2.5+2.5}{\frac{1}{2}+\frac{1}{3}} = 8\ km/h$

(iii) $Average\ velocity\ (0 - 4\ min) = \frac{Displacement}{Time} = \frac{2.5-\frac{2.5}{2}}{\frac{40}{60}h} = 1.875\ km/h$

(iv) $Average\ Speed\ (0 - 40\ min) = \frac{Distance}{Time} = \frac{2.5+\frac{2.5}{2}}{\frac{40}{60}} = 5.625\ km/h$

Ques 3.15: In Exercises 3.13 and 3.14, we have carefully distinguished between *average* speed and magnitude of *average* velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans 3.15: Instantaneous velocity is given by the first derivative of distance with respect to time i.e.,

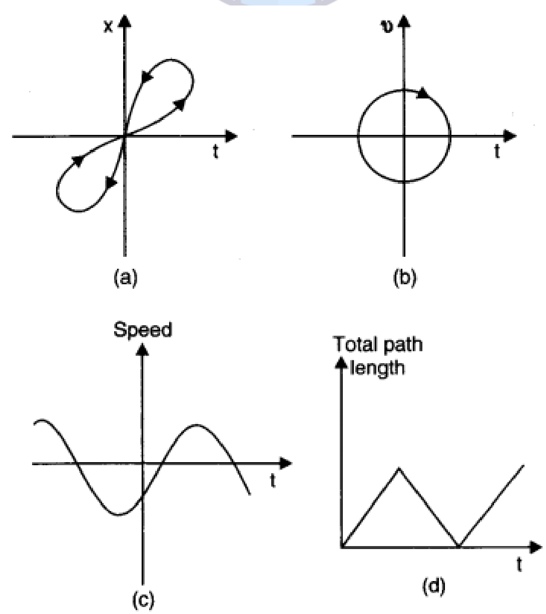
$$V_{In} = \frac{dx}{dt}$$

Here, the time interval dt is so small that it is assumed that the particle does not change its direction of motion.

As a result, both the total path length and magnitude of displacement become equal is this interval of time.

Therefore, instantaneous speed is always equal to instantaneous velocity

Ques 3.16: Look at the graphs (a) to (d) (Fig. 3.20) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.



Ans 3.16: None of the four graph represent a possible one-dimensional motion. In graphs (a) and (b) motions are definitely two dimensional.

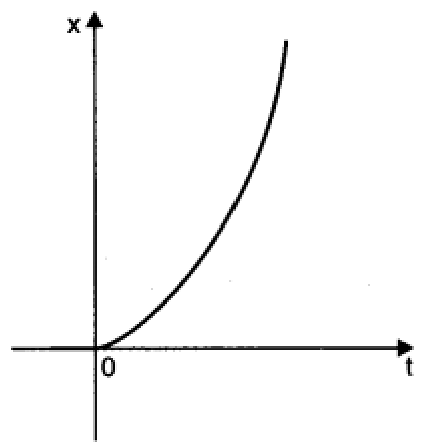
Graph (a) represents two positions at the same time which is not possible.

In graph (b) opposite motion is visible at the same time.

The graph (c) is not correct since it shows that the particle has negative speed at a certain instant. Speed is always positive.

In graph (d) path length is shown as increasing as well as decreasing. Path length never decreases.

Ques 3.17: Figure shows the $x - t$ plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.



Ans 3.17: No, from the graph we cannot say that the object moves on a parabolic path from $t > 0$ and on a straight line for $t < 0$, because the graph does not show the object's path. A suitable situation with characteristics resembling the above graph is a ball thrown from a tall building at instant $t = 0$.

Ques 3.18: A police van moving on a highway with a speed of $30\ km/h$ fires a bullet at a thief's car speeding away in the same direction with a speed of $192\ km/h$. If the muzzle speed of the bullet is $150\ m/s$, with what speed does the bullet hit the thief's car?

[Note: Obtain that speed which is relevant for damaging the thief's car].

Ans 3.18:

Speed of the police van, $v_p = 30\ km/h = 8.33\ m/s$

Muzzle speed of the bullet, $v_b = 150\ m/s$

Speed of the thief's car, $v_t = 192\ km/h = 53.33\ m/s$

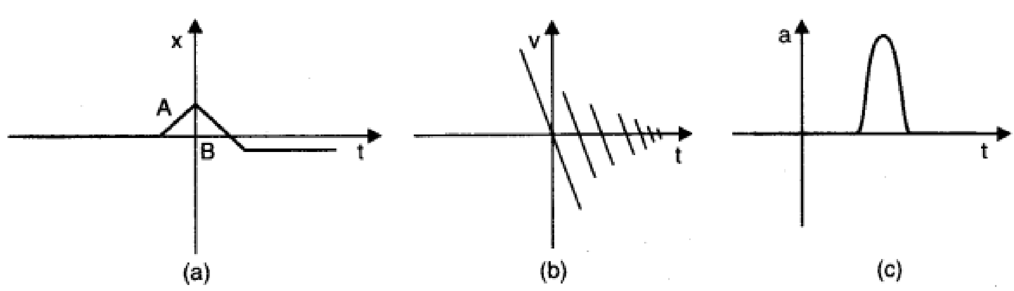
Since the bullet is fired from a moving van, its resultant speed can be obtained as:

$= 150 + 8.33 = 158.33\ m/s$

Since both the vehicles are moving in the same direction, the velocity with which the bullet hits the thief's car can be obtained as:

$v_{bt} = v_b - v_t = 158.33 - 53.33 = 105\frac{m}{s}$

Ques 3.19: Suggest a suitable physical situation for each of the following graphs:



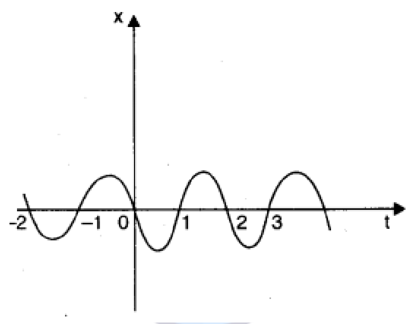
Ans 3.19:

(i) Here we can see that the velocity of the object decreases uniformly over time. An example of this situation would be a stone dropping in a pool of water.

(ii) Here we can see that initially, an object is moving with a uniform velocity, it then accelerates for a short interval of time after which its acceleration drops to zero. An example of this would be a uniformly moving hockey ball being hit by a hockey stick for a very short interval of time.

(iii) Here we see that the velocity increases initially, then it goes to 0. Again the velocity starts increasing but in the opposite direction and it attains a constant after some time. An example of this would be kicking a ball on a smooth floor. It gains velocity initially, then it stops momentarily when it hits the wall and from the wall, it rebounds and starts moving in the opposite direction with a constant speed.

Ques 3.20: Figure gives the $x - t$ plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3\ s, 1.2\ s, -1.2\ s$.



Ans 3.20:

(i) Negative, Negative, Positive (at $t = 0.3\ s$)

(ii) Positive, Positive, Negative (at $t = 1.2\ s$)

(iii) Negative, Positive, Positive (at $t = -1.2\ s$)

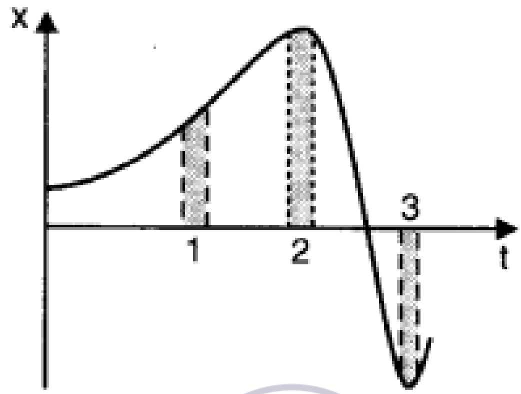
In Simple Harmonic motion $a = -kx$

(i) At $t = 0.3s, x$ (position) is $-ve$, and on increasing time x becomes more negative so velocity is $-ve$. And acceleration, a is $+ve$ (Since $a = -k(-x)$)

(ii) At $t = 1.2s, x$ is $+ve$, velocity is $+ve$ and acceleration is $-ve$

(iii) At $t = -1.2s, x$ is $-ve$ and on increasing time x becomes less $-ve$. Thus, velocity is $+ve$ and acceleration is $+ve$ (Since $a = -k(-x)$)

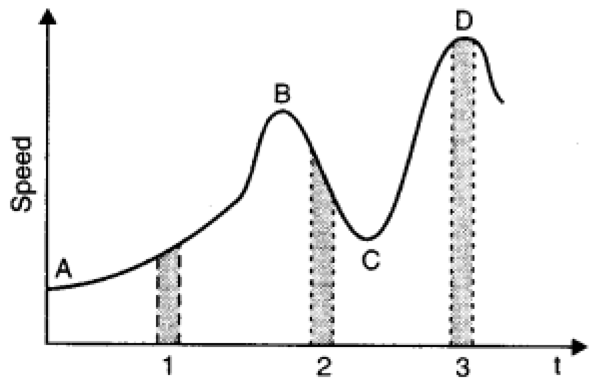
Ques 3.21: The figure given below the $x - t$ plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



Ans 3.21: The slope of the graph is minimum at 2 and maximum at 3. Thus, the least average speed is at time interval 2 and the greatest average speed is at time interval 3. Average velocity is positive in interval 1 and 2 and negative in 3.

Ques 3.22: Figure gives a speed-time graph of a particle in motion along a

constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of *v* and *a* in the three intervals. What are the accelerations at the points A, B, C and D?



Ans 3.22: The acceleration is greatest in magnitude in interval 2 as the change in speed in the same time is maximum in this interval.

The average speed is greatest in interval 3 (peak D is at maximum on speed axis).

The sign of v and a in the three intervals are:

$v > 0$ in $1, 2$ and 3; $a > 0$ in 1

$a < 0$ in 2, $a = 0$ in 3.

Acceleration is $zero$ at A, B, C and D.

Motion in a Straight Line

Additional Exercises:

Ques 3.23: A three-wheeler starts from rest, accelerates uniformly with $1\ m\ s^{-2}$ on a straight road for $10\ s$, and then moves with uniform velocity. Plot the distance covered by the vehicle during the n^th^ second ($n = 1, 2, 3 \ldots$) versus n. What do you expect this plot to be during accelerated motion: a straight line or a parabola?

Ans 3.23: Distance covered by a body in n^{th} second is given by the relation

$D_n = u + \frac{1}{2}a\,(2n - 1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

Where,

$u = Initial\ velocity$

$a = Acceleration$

$n = Time = 1, 2, 3 \ldots\ldots\ldots n$

In the given case, $u = 0$ and $a = 1\ m/s^2$

$\therefore D_n = \frac{1}{2}(2n - 1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

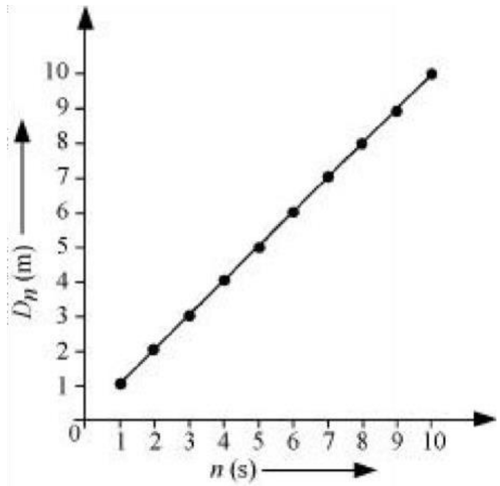
This relation shows that:

$D_n \propto n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$

Now, substituting different values of *n* in equation (iii), we get the following table:

n	1	2	3	4	5	6	7	8	9
D_n	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5

The plot between n and D_n will be a straight line as shown:



Since the given three-wheeler acquires uniform velocity after 10 s, the line will be parallel to the time-axis after $n = 10\ s$.

Ques 3.24: A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Ans 3.24:

Initial velocity of the ball, $u = 49\ m/s$

Acceleration, $a = -g = -9.8\ m/s^2$

Case I:

When the lift was stationary, the boy throws the ball.

Taking upward motion of the ball, Final velocity, v of the ball becomes zero at the highest point.

From first equation of motion, time of ascent (t) is given as:

$v = u + at$

$$t = \frac{v - u}{a}$$

$$= \frac{-49}{-9.8} = 5\ s$$

But, the time of ascent is equal to the time of descent.

Hence, the total time taken by the ball to return to the boy's hand $= 5 + 5 = 10\ s$.

Case II:

The lift was moving up with a uniform velocity of 5 m/s.

In this case, the relative velocity of the ball with respect to the boy remains the same i.e., 49 m/s.

Therefore, in this case also, the ball will return back to the boy's hand after 10 s.

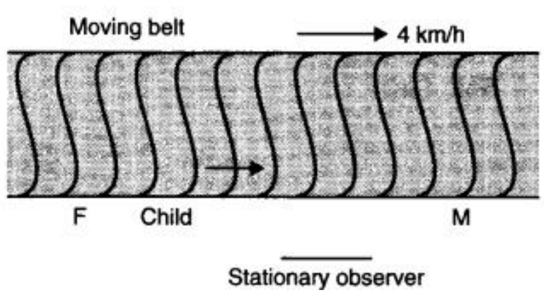
Ques 3.25: On a long horizontally moving belt, a child runs to and fro with a speed 9 $km\ h^{-1}$ (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 $km\ h^{-1}$. For an observer on a stationary platform outside, what is the

(i) Speed of the child running in the direction of motion of the belt?

(ii) Speed of the child running opposite to the direction of motion of the belt?

(iii) Time taken by the child in (i) and (ii) ?

Which of the answer alter if motion is viewed by one of the parents?



Ans 3.25:

Speed of the belt, $v_B = 4\ km/h$

Speed of the boy, $v_b = 9\ km/h$

(i) Since the boy is running in the same direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:

$$v_{bB} = v_b + v_B = 9 + 4 = 13\ km/h$$

(ii) Since the boy is running in the direction opposite to the direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:

$$v_{bB} = v_b + (-v_B) = 9 - 4 = 5\ km/h$$

(iii) Distance between the child's parents = 50 m

As both parents are standing on the moving belt, the speed of the child in either direction as observed by the parents will remain the same i.e., $9\ km/h = 2.5\ m/s$.

Hence, the time taken by the child to move towards one of his parents is $\frac{50}{2.5} = 20s$.

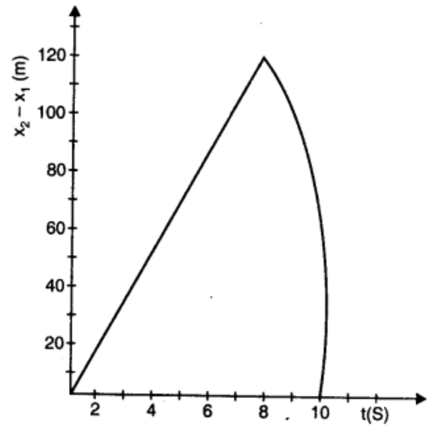
If the motion is viewed by any one of the parents, Answer obtained in (i) and (ii) get altered. This is because the child and his parents are standing on the same belt and hence, are equally affected by the motion of the belt.

Therefore, for both parents (irrespective of the direction of motion) the speed of the child remains the same i.e., $9\ km/h$.

For this reason, it can be concluded that the time taken by the child to reach any one of his parents remains unaltered.

Ques 3.26: Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m/s and 30 m/s. Verify that the graph shown in Figure correctly represents the time variation of the relative position of the second stone with respect to the first.

[Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m/s^2. Give the equations for the linear and curved parts of the plot.]



Ans 3.26:

For first stone:

Initial velocity, $u_I = 15\, m/s$

Acceleration, $a = -g = -\,10\, m/s^2$

Using the relation,

$$x_1 = x_0 + u_I t + \frac{1}{2}at^2$$

Where, the height of the cliff, $x_0 = 200\, m$

$x_1 = 200 + 15t - 5t^2$ ....(1)

When this stone hits the ground, $x_1 = 0$

$\therefore 5t^2 + 15t + 200 = 0$

$t^2 - 3t - 40 = 0$

$t^2 - 8t + 5t - 40 = 0$

$t(t - 8) + 5(t - 8) = 0$

$t = 8\, s\ or\ t = -5\, s$

Since, the stone was projected at time, $t = 0$, the negative sign before time is of no use.

$\therefore t = 8\, sec$

For second stone:

Initial velocity, $u_{II} = 30\, m/s$

Acceleration, $a = -g = -10\, m/s^2$

Using the relation,

$$x_2 = x_0 + u_{II}t + \frac{1}{2}at^2$$

$$= 200 + 30t - 5t^2 \ldots\ldots(2)$$

At the moment when this stone hits the ground; $x_2 = 0$

$5t^2 + 30\,t + 200 = 0$

$t^2 - 6t - 40 = 0$

$t^2 - 10t + 4t + 40 = 0$

$t(t - 10) + 4\,(t - 10) = 0$

$t(t - 10)\,(t + 4) = 0$

$t = 10\, s \text{ or } t = -4\, s$

Here again, the negative sign is of no use.

$\therefore t = 10\, s$

Subtracting equations (1) and (2), we get

$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$

$x_2 - x_1 = 15t \ldots\ldots(3)$

Equation (3) represents the linear path of both stones.

Due to this linear relation between $(x_2 - x_1)$ and t, the path remains a straight line till $8\ s$.

Maximum separation between the two stones is at $t = 8\ s$.

$(x_2 - x_1)_{\max} = 15 \times 8 = 120\ m$

This is in accordance with the given graph.

After $8\ s$, only second stone is in motion whose variation with time is given by the quadratic equation:

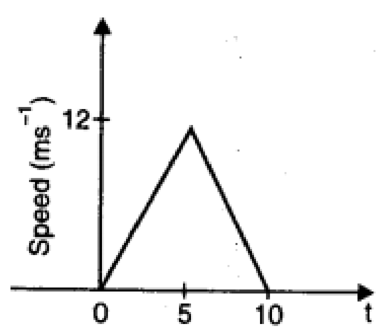
$x_2 - x_1 = 200 + 30t - 5t^2$

Hence, the equation of linear and curved path is given by

$x_2 - x_1 = 15t$ (Linear path)

$x_2 - x_1 = 200 + 30t - 5t^2$ (Curved path)

Ques 3.27: The speed-time graph of a particle moving along a fixed direction is shown in Fig. Obtain the distance traversed by the particle between (a) $t = 0\ s$ to $10\ s$, (b) $t = 2\ s$ to $6\ s$.



What is the average speed of the particle over the intervals in (a) and (b)?

Ans 3.27:

Distance travelled by the particle = Area under the given graph

$\frac{1}{2} \times (10 - 0) \times (12 - 0) = 60\ m$

$Average\ speed = \frac{Distance}{Time} = \frac{60}{10} = 6m/s$

Let s_1 be the distance covered by the particle between time $t = 2\ s\ to\ 5\ s$.

Let s_2 be the distance covered by the particle between time $t = 5\ s\ to\ 6\ s$.

Total distance (s) covered by the particle in time

$t\ =\ 2\ s\ to\ 6\ s\ \ =\ s_1\ +\ s_2$..$(i)$

For distance s_1:

Let u' be the velocity of the particle after 2 s and a' be the acceleration of the particle in $t = 0\ to\ t\ =\ 5\ s$.

Since the particle undergoes uniform acceleration in the interval t = 0 to t = 5 s, from first equation of motion, acceleration can be obtained as:

$v\ =\ u\ +\ at$

Where,

v =Final velocity of the particle

$12\ =\ 0\ +\ a'\ \times\ 5$

$a' = \frac{12}{5} = 2.4\ m/s^2$

Again, from first equation of motion, we have

$v\ =\ u\ +\ at$

$=\ 0\ +\ 2.4\ \times\ 2\ =\ 4.8\ m/s$

Distance travelled by the particle between time 2 $s\ and$ 5 s i.e., in 3 s

$s_1 = u't + \frac{1}{2}a't^2$

$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2$

$= 25.2\, m$……………………………………………………………………..(ii)

<u>For distance s_2:</u>

Let a'' be the acceleration of the particle between time $t = 5\, s\ and\ t = 10\, s$.

From first equation of motion,

$v = u + at$ (Where $v = 0$ as the particle finally comes to rest)

$a" = \frac{-12}{5} = -2.4\, s$

$0 = 12 + a'' \times 5$

Distance travelled by the particle in $1s$ (i.e., between $t = 5\, s\ and\ t = 6\, s$)

$s_2 = u"t + \frac{1}{2}at^2$

$s_2 = 12 \times a + \frac{1}{2}(-2.4) \times (1)^2$

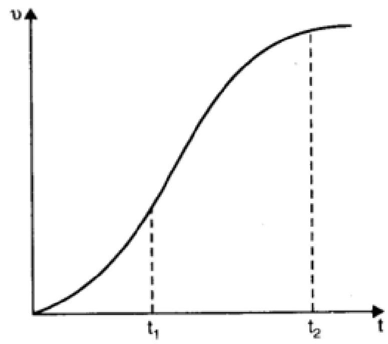
$12 - 1.2 = 10.8\, m$…………………………………………………………....(iii)

From equations (i), (ii), and (iii), we get

$s = 25.2 + 10.8 = 36\, m$

$\therefore Average\ speed = \frac{36}{4} = 9\, m/s$

Ques 3.28: The velocity-time graph of a particle in one-dimensional motion is shown in Fig.:



Which of the following formulae are correct for describing the motion of the particle over the time-interval $t2$ to $t1$?

(i) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \frac{1}{2}\, a\,(t_2 - t_1)^2$

(ii) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(iii) $v_{Average} = \frac{(x(t_2) - x(t_1))}{t_2 - t_1}$

(iv) $a_{Average} = \frac{(v(t_2) - v(t_1))}{t_2 - t_1}$

(v) $x(t_2) = x(t_1) + v_{Average}(t_2 - t_1) + \left(\frac{1}{2}\right) a_{Average}(t_2 - t_1)^2$

(vi) $x(t_2) - x(t_1) =$ **Area under the $v-t$ curve bounded by the t-axis and the dotted line shown.**

Ans 3.28: The correct formulae describing the motion of the particle are (iii), (iv) and, (vi) The given graph has a non-uniform slope.

Hence, the formulae given in (i), (ii), and (v) cannot describe the motion of the particle. Only relations given in (iii), (iv), and (vi) are correct equations of motion.