Chapter – 12

Introduction to 3-D Geometry

NCERT Exercises:

Exercise 12.1

Ques 1: A point is on the *x*-axis. What are its *y*-coordinates and *z*-coordinates?

Ans 1: If a point is on the *x*-axis, then its *y*-coordinates and *z*-coordinates are zero.

Ques 2: A point is in the XZ-plane. What can you say about its *y*-coordinate?

Ans 2: If a point is in the XZ plane, then its *y*-coordinate is zero.

Ques 3: Name the octants in which the following points lie:
$(\mathbf{1,2,3}), (\mathbf{4,-2,3}), (\mathbf{4,-2,-5}), (\mathbf{4,2,-5}), (\mathbf{-4,2,-5}), (\mathbf{-4,2,5}), (\mathbf{-3,-1,6}), (\mathbf{2,-4,-7})$

Ans 3: The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive.

Therefore, this point lies in octant I.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, –2, 3) are positive, negative, and positive respectively.

Therefore, this point lies in octant IV.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, –2, –5) are positive, negative, and negative respectively.

Therefore, this point lies in octant VIII.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, 2, –5) are positive, positive, and negative respectively.

Therefore, this point lies in octant V.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (–4, 2, –5) are negative, positive, and negative respectively.

Therefore, this point lies in octant VI.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (–4, 2, 5) are negative, positive, and positive respectively.

Therefore, this point lies in octant II.

x-coordinate, y-coordinate, and z-coordinate of point (–3, –1, 6) are negative, negative, and positive respectively.

Therefore, this point lies in octant III.

The x-coordinate, y-coordinate, and z-coordinate of point (2, –4, –7) are positive, negative, and negative respectively.

Therefore, this point lies in octant VIII.

Ques 4: Fill in the blanks:

(i) The *x*-axis and *y*-axis taken together determine a plane known as____________.

(ii) The coordinates of points in the XY-plane are of the form_______________.

(iii) Coordinate planes divide the space into_______________octants.

Ans 4:

(i) The x-axis and y-axis taken together determine a plane known as xy - plane.

(ii) The coordinates of points in the XY-plane are of the form (x, y, 0).

(iii) Coordinate planes divide the space into eight octants.

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Exercise 12.2

Ques 1: Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) **(ii) (–3, 7, 2) and (2, 4, –1)**

(iii) (–1, 3, –4) and (1, –3, 4) **(iv) (2, –1, 3) and (–2, 1, 3)**

Ans 1: The distance between points $P(x_1, y_1, z_1)$and $P(x_2, y_2, z_2)$is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$

$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$

$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$

$$= \sqrt{25+9+9}$$

$$= \sqrt{43}$$

(iii) Distance between points (–1, 3, –4) and (1, –3, 4)

$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$

$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$

(iv) Distance between points (2, –1, 3) and (–2, 1, 3)

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Ques 2: Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Ans 2: Let points (–2, 3, 5), (1, 2, 3), and (7, 0, –1) be denoted by P, Q, and R respectively. Points P,Q, and R are collinear if they lie on a line.

$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{36+4+16}$

$= 2\sqrt{14}$

$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{81 + 9 + 36}$

$= 3\sqrt{14}$

Here, $PQ + QR = 2\sqrt{14} + \sqrt{14} = 3\sqrt{14} = PR$,

Hence points P,Q,R are collinear

Ques 3: Verify the following:

(i) (0, 7, –10), (1, 6, –6) and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (–4, 9, 6) are the vertices of a right angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Ans 3:

(i) Let points (0, 7, –10), (1, 6, –6), and (4, 9, –6) be denoted by A, B, and C respectively.

$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$

$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

$= 6$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{(3)^2 + (0)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36} = 6$

Now $AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = CA^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(ii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$

$= \sqrt{4+4+16}$

$= \sqrt{36}$

$= 6$

$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$

$= \sqrt{9+9+25}$

$= \sqrt{43}$

$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

$= 6$

$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$

$= \sqrt{9+25+25} = \sqrt{36}$

$= \sqrt{43}$

Here, $AB = CD = 6, BC = AD = \sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Ques 4: Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Ans 4: Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, –1). Accordingly, PA = PB

$PA^2 = PB^2$

$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Ques 5: Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (–4, 0, 0) is equal to 10.

Ans 5: Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10.

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$

On squaring both sides, we obtain

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2 + (x-4)^2 + y^2 + z^2}$

$\Rightarrow \sqrt{x^2 - 8x + 16 + y^2 + z^2}$

$= 100 - 20\sqrt{x^2 - 8x + 16 + y^2 + z^2 + x^2 - 8x + 16 + y^2 + z^2}$

$\Rightarrow 20\sqrt{x^2 - 8x + 16 + y^2 + z^2} = 100 + 16x$

$\Rightarrow 5\sqrt{x^2 - 8x + 16 + y^2 + z^2} = 25 + 4x$

On squaring both sides again, we obtain

$25\,(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

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Exercise 12.3

Ques 1: Find the coordinates of the point which divides the line segment joining the points (–2, 3,5) and (1, –4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Ans 1:

(i) The coordinates of point R that divides the line segment joining points P ($x1$, $y1$, $z1$) and Q($x2$, $y2$, $z2$) internally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny}{m+n}, \frac{mz_2 + nz_1}{m+n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points (–2, 3, 5) and (1,–4, 6) internally in the ratio 2:3

$$x = \frac{2(1) + 3(-2)}{2+3}, y = \frac{2(-4) + 3(3)}{2+3}, x = \frac{2(6) + 3(5)}{2+3}$$

$$\text{i. e } x = -\frac{4}{5}, y = \frac{1}{5} \text{ and } z = \frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P ($x1$, $y1$, $z1$) and Q ($x2$, $y2$, $z2$) externally in the ratio m: n are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)..$$

Let R (x, y, z) be the point that divides the line segment joining points (–2, 3, 5) and (1,–4, 6) externally in the ratio 2:3

$$x = \frac{2(1) - 3(-2)}{2-3}, y = \frac{2(-4) - 3(3)}{2-3}, x = \frac{2(6) - 3(5)}{2-3}$$

$$\text{i. e. } x = -8, y = 17 \text{and } z = 3$$

Thus, the coordinates of the required point are (–8, 17, 3).

Ques 2: Given that P (3, 2, – 4), Q (5, 4, –6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Ans 2: Let point Q (5, 4, –6) divide the line segment joining points P (3, 2, –4) and R (9, 8, –10) in the ratio *k*:1.

Therefore, by section formula,

$$(5,4,-6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$$

$$=> \frac{9k+3}{k+1} = 5$$

$$=> 9k + 3 = 5k + 5$$

$$=> 4k = 2$$

$$=> k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Ques 3: Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Ans 3: Let the YZ plane divide the line segment joining points (–2, 4, 7) and (3, –5, 8) in the ratio *k*:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8) \mp 7}{k+1}\right)$$

On the YZ plane, the x − coordinate of any point is zero.

$$\frac{3k-2}{k+1} = 0$$

$$=> 3k - 2 = 0$$

$$=> k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Ques 4: Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and C $\left(0, \frac{1}{3}, 2\right)$ are collinear.

Ans 4: The given points are A $(2, -3, 4)$, B $(-1, 2, 1)$, and C $(0, \frac{1}{3}, 2)$.

Let P be a point that divides AB in the ratio *k*:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1) \mp 4}{k+1}\right)$$

Now, we find the value of *k* at which point P coincides with point C.

By taking $\frac{-k+2}{k+1} = 0$ we obtain $k = 2$

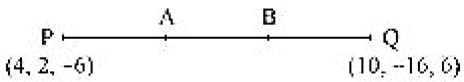
For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$

i.e., C $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio $2:1$ and is the same as point P.

Hence, points A, B, and C are collinear.

Ques 5: Find the coordinates of the points which trisect the line segment joining the points P (4, 2, –6) and Q (10, –16, 6).

Ans 5: Let A and B be the points that trisect the line segment joining points



P (4, 2, – 6) and Q (10, – 16, 6)

Point A divides PQ in the ratio 1:2.

Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2},\frac{1(-16)+22}{1+2},\frac{1(6)\mp 2(-6)}{1+2}\right)=(6,-4,-2)$$

Point B divides PQ in the ratio 2:1.

Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{1+2},\frac{2(-16)+1.2}{1+2},\frac{2(6)\mp 1(6)}{1+2}\right)=(8,-10,2)$$

Thus, (6, –4, –2) and (8, –10, 2) are the points that trisect the line segment joining points P (4, 2, –6) and Q (10, –16, 6).

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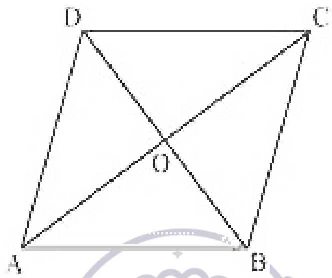
Miscellaneous Exercise on Chapter 12

Ques 1: Three vertices of a parallelogram ABCD are A (3, –1, 2), B (1, 2, –4) and C (–1, 1, 2). Find the coordinates of the fourth vertex.

Ans 1: The three vertices of a parallelogram ABCD are given as A (3, –1, 2), B (1, 2, –4), and C (–1, 1, 2).
Let the coordinates of the fourth vertex be D (*x*, *y*, *z*).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴ Mid-point of AC = Mid-point of BD

$$\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2 \mp 2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$=> (1,0,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

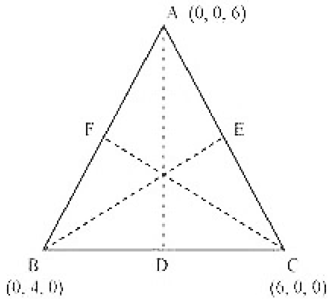
$$=> \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = -2, \text{and } z = 8$$

Thus, the coordinates of the fourth vertex are (1, –2, 8).

Ques 2: Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Ans 2: Let AD, BE, and CF be the medians of the given triangle ABCKK.



Since AD is the median, D is the mid-point of BC.

$$\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0\mp0}{2}\right)=(3,2,0)$$

$\therefore$ Coordinates of point D $=$ AD $=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}$

$=\sqrt{9+4+36}=\sqrt{49}=7$

Since BE is the median, E is the midpoint of AC

Coordinates of point E $=\frac{0+6}{2},\frac{0+0}{2},\frac{6+0}{2}=(3,0,3)$

$\text{BE}=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}=\sqrt{9+16+9}=\sqrt{34}$

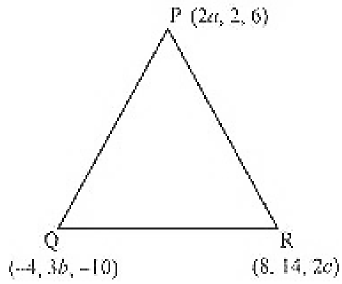
Since CF is the medium, F is the mid $-$ pt of AB

Coordinates of point E $=\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}=(0,2,3)$

Thus, the lengths of the medians of ΔABC are7, $\sqrt{34}$and 7

Ques 3: If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (–4, 3*b*, –10) and R (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Ans 3:



It is known that the coordinates of the centroid of the triangle, whose vertices are

$(x1, y1, z1), (x2, y2, z2)$and $(x3, y3, z3)$, are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR.

$$(0,0,0) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\frac{2a-4}{3} = 0, \frac{3b+16}{3} = 0, \text{and } \frac{2c-4}{3} = 0$$

$$a = -2, b = -\frac{16}{3} \text{ and } c = 2$$

Thus, the respective values of a, b, and c are $-2, -\frac{16}{3}$,2

Ques 4: Find the coordinates of a point on *y*-axis which are at a distance of 5√2 from the point P (3, –2, 5).

Ans 4: If a point is on the *y*-axis, then *x*-coordinate and the *z*-coordinate of the point are zero. Let A (0, *b*, 0) be the point on the *y*-axis at a distance of 5√2 from point P (3, –2, 5).

Accordingly, AP $= 5\sqrt{2}$

$AP^2 = 50$

$\Longrightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$

$\Longrightarrow 9 + 4 + b^2 + 4b + 25 = 50$

$\Longrightarrow b^2 + 4b - 12 = 0$

$\Longrightarrow b^2 + 6b - 2b - 12 = 0$

$\Longrightarrow (b+6)(b-2) = 0$

$\Longrightarrow b = -6 \; or \; 2$

Thus, the coordinates of the required points are (0, 2, 0) and (0, –6, 0).

Ques 5: A point R with *x*-coordinate 4 lies on the line segment joining the pointsP (2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

$[\boldsymbol{Hint\ suppose\ R\ divides\ PQ\ in\ the\ ratio\ k:1. The\ coordinates\ of\ the\ point\ R\ are\ given\ by}$ $\boldsymbol{\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}}]$

Ans 5: The coordinates of points P and Q are given as P (2, –3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio *k*:1.

Hence, by section formula, the coordinates of point R are given by

$$\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)\mp 4}{k+1} = \frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}]$$

It is given that the *x*-coordinate of point R is 4.

$$\frac{8k+2}{k+1} = 4$$

$8k + 2 = 4k + 4$

$4k = 2$

$k = \frac{1}{2}$

Therefore, the coordinates of point R are $\left(4, -\frac{3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

Ques 6: If A and B be the points (3, 4, 5) and (–1, 3, –7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Ans 6: The coordinates of points A and B are given as (3, 4, 5) and (–1, 3, –7) respectively.

Let the coordinates of point P be (*x*, *y*, *z*).

On using distance formula, we obtain

$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$

$= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$

$= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$

$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

$= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59$

Now, if $PA^2 + PB^2 = k^2$, then

$= (x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) = k^2$

$=> 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$=> 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$=> x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$