Chapter – 2

Relation and Functions

NCERT Exercises:

Exercise 2.1

Ques 1: $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ **, find the values of** x **and** y**.**

Ans 1: It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

Since the ordered pairs are equal, the corresponding elements will also be equal.
Therefore,

$\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$=> \frac{x}{3}=\frac{5}{3}-1, y-\frac{2}{3}=\frac{1}{3}$

$=> \frac{x}{3}=\frac{2}{3} \quad , y=\frac{1}{3}+\frac{2}{3}$

$=> x=2 \quad , y=1.$

$\therefore x = 2$ and $y = 1$

Ques 2: If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Ans 2: It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.
$\Rightarrow$ Number of elements in set B $= 3$

Number of elements in $(A \times B)$
$=$ (Number of elements in A) $\times$ (Number of elements in B)
$= 3 \times 3 = 9$

Thus, the number of elements in $(A \times B)$ is 9.

Ques 3: If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Ans 3: $G = \{7, 8\}$ and $H = \{5, 4, 2\}$

We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as

$P \times Q = \{(p, q): p \in P, q \in Q\}$

$\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

Ques 4: State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) $\mathbf{If\ P = \{m, n\} and\ Q = \{n, m\}, then\ P \times Q = \{(m, n), (n, m)\}.}$

(ii) $\mathbf{If\ A\ and\ B\ are\ non-empty\ sets, then\ A \times B\ is\ a\ non-empty\ set\ of\ ordered\ pairs\ (x, y) such\ that\ x \in A\ and\ y \in B.}$

(iii) $\mathbf{If\ A = \{1, 2\}, B = \{3, 4\}, then\ A \times (B \cap \Phi) = \Phi.}$

Ans 4:

(i) False

If $P = \{m, n\}$ and $Q = \{n, m\}$, then

$P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$

(ii) True

(iii) True

Ques 5: If A = {–1, 1}, find A × A × A.

Ans 5: It is known that for any non-empty set A, A × A × A is defined as

$A \times A \times A = \{(a, b, c): a, b, c \in A\}$

It is given that A = {–1, 1}

$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$

$(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

Ques 6: If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Ans 6: It is given that A × B = {(a, x), (a, y), (b, x), (b, y)}

We know that the Cartesian product of two non-empty sets P and Q is defined as

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

∴ A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Ques 7: Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) $\mathbf{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

(ii) **$A \times C$ is a subset of $B \times D$**

Ans 7: (i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$ L.H.S. $= A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$\therefore$ R.H.S. $= (A \times B) \cap (A \times C) = \Phi$

$\therefore$ L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$A \times D = \{(1,5), (1,6), (1,7), (1,8), (2,5), (2,6), (2,7), (2,8),$

$(3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8)\}$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

Ques 8: Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Ans 8: $\mathrm{A} = \{1,2\}$and $\mathrm{B} = \{3,4\}$

$\therefore\ \mathrm{A} \times \mathrm{B} = \{(1,3),(1,4),(2,3),(2,4)\}$

$\Rightarrow\ \mathrm{n}(\mathrm{A} \times \mathrm{B}) = 4$

We know that if C is a set with $\mathrm{n}(\mathrm{C}) = \mathrm{m}$, then $\mathrm{n}[\mathrm{P}(\mathrm{C})] = 2^{\mathrm{m}}$.

Therefore, the set $\mathrm{A} \times \mathrm{B}$ has $2^4 = 16$ subsets.

These are: –

$\Phi, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}, \{(1,3),(1,4)\}, \{(1,3),(2,3)\},$

$\{(1,3),(2,4)\}, \{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3),(2,4)\},$

$\{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),(2,4)\}, \{(1,3),(2,3),(2,4)\},$

$\{(1,4),(2,3),(2,4)\}, \{(1,3),(1,4),(2,3),(2,4)\}$

Ques 9: Let A and B be two sets such that *n*(A) = 3 and *n* (B) = 2. If (*x*, 1), (*y*, 2), (*z*, 1) are in A × B, find A and B, where *x*, *y* and *z* are distinct elements.

Ans 9: It is given that $n(A) = 3$ and $n(B) = 2$; and $(x,1),(y,2),(z,1)$ are in $A \times B$.

We know that

A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B.

∴ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A) = 3$ and

$n(B) = 2$,

It is clear that $A = \{x,y,z\}$ and $B = \{1,2\}$

Ques 10: The Cartesian product A × A has 9 elements among which are found (–1,0) and (0, 1). Find the set A and the remaining elements of A × A.

Ans 10: We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

$\therefore\ n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$\therefore\ n(A) \times n(A) = 9$

$\Rightarrow\ n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

We know that $A \times A = \{(a, a) : a \in A\}$.

Therefore, $-1, 0$, and 1 are elements of A.

Since $n(A) = 3$, it is clear that $A = \{-1, 0, 1\}$.

The remaining elements of set $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0)$,

$(1, -1), (1, 0)$, and $(1, 1)$.

Chapter – 2

Relation and Functions

Exercise 2.2

Ques 1: Let A = {1, 2, 3… 14}. Define a relation R from A to A by R = {(x, y): 3x – y = 0, where x, y ∈ A}. Write down its domain, co domain and range.

Ans 1: The relation R from A to A is given as $R = \{(x, y): 3x - y = 0, \text{where } x, y \in A\}$

$\text{i.e.}, R = \{(x, y): 3x = y, \text{where } x, y \in A\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{1, 2, 3, 4\}$

The whole set A is the codomain of the relation R.

$\therefore$ Codomain of $R = A = \{1, 2, 3 \ldots 14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{3, 6, 9, 12\}$

Ques 2: Define a relation R on the set N of natural numbers by R = {(x, y): y = x+ 5, x is a natural number less than 4; x, y ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

Ans 2: $R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4, x, y \in N\}$

The natural numbers less than 4 are 1, 2, and 3.

$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.
$\therefore$ Domain of $R = \{1, 2, 3\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{6, 7, 8\}$

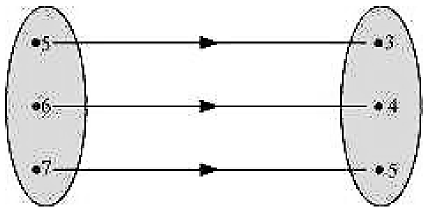
Ques 3: A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R={(*x*, *y*): the difference between *x* and *y* is odd; *x* ∈ A, *y* ∈ B}. Write R in roster form.

Ans 3: $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$

$R = \{(x, y)\text{: the difference between x and y is odd; } x \in A, y \in B\}$

$\therefore\ R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

Ques 4: The given figure shows a relationship between the sets P and Q. write this relation (i) in set-builder form (ii) in roster form. What is its domain and range?



Ans 4: According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) $R = \{(x, y): y = x - 2;\ x \in P\}$or $R = \{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$

(ii) $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Ques 5: Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(*a*, *b*): *a*, *b*∈ A, *b* is exactly divisible by *a*}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Ans 5: $A = \{1, 2, 3, 4, 6\}, R = \{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$

(i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3),$

$(3, 6), (4, 4), (6, 6)\}$

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

Ques 6: Determine the domain and range of the relation R defined by R = {(x, $x + 5$): $x \in$ {0, 1, 2, 3, 4, 5}}.

Ans 6: $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

$\therefore\ \mathrm{R} = \{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

$\therefore$ Domain of R $= \{0,1,2,3,4,5\}$

Range of R $= \{5,6,7,8,9,10\}$

Ques 7: Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Ans 7: $R = \{(x, x^3): x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore\ \mathrm{R} = \{(2,8),(3,27),(5,125),(7,343)\}$

Ques 8: Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Ans 8: It is given that A = {x, y, z} and B = {1, 2}.

$\therefore\ A \times B = \{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Since $n(A \times B) = 6$, the number of subsets of $A \times B$ is 2^6.

Therefore, the number of relations from A to B is 2^6.

Ques 9: Let R be the relation on Z defined by R = {(a, b): a, $b \in$ Z, $a - b$ is an integer}. Find the domain and range of R.

Ans 9: $R = \{(a,b): a, b \in Z, a - b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

$\therefore$ Domain of R = Z

Range of R = Z

Chapter – 2

Relation and Functions

Exercise 2.3

Ques 1: Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) **{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}**

(ii) **{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}**

(iii) **{(1, 3), (1, 5), (2, 5)}**

Ans 1:

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $= \{2,5,8,11,14,17\}$ *and range* $= \{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $= \{2,4,6,8,10,12,14\}$ *and range* $= \{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Ques 2: Find the domain and range of the following real function:

(i) $f(x) = -|x|$

(ii) $f(x) = \sqrt{9 - x^2}$

Ans 2:

(i) $f(x) = -|x|, x \in R$

We know that $|x| = \{ , if\ x \geq 0$

$-x, x < 0$

$\therefore\ (x) = -|x| = \{ -x, if\ x \geq 0$

$x, i\quad f\ x < 0$

Since $f(x)$ is defined for $x \in$ R, the domain of f is R.

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

$\therefore$ The range of f is $(-\infty, 0]$.

(ii)$f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3.

$\therefore$The range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

Ques 3: A function f is defined by $f(x) = 2x – 5$. Write down the values of

(i) $\mathbf{f(0)}$,

(ii) $\mathbf{f(7)}$,

(iii) $\mathbf{f(-3)}$

Ans 3: The given function is $f(x) = 2x - 5$.

Therefore,

(i) f(0) = 2 × 0 – 5 = 0 – 5 = – 5

(ii) f(7) = 2 × 7 – 5 = 14 – 5 = 9

(iii) f(– 3) = 2 × (– 3)– 5 = – 6 – 5 = – 11

Ques 4: The function ‘t’ which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $f(C) = \frac{9C}{5} + 32$. Find

(i) $\mathbf{t\,(0)}$

(ii) $\mathbf{t\,(28)}$

(iii) $\mathbf{t\,(-10)}$

(iv) **The value of C, when t(C) = 212**

Ans 4: The given function is $(C) = \frac{9C}{5} + 32$.

Therefore,

(i) $t(0) = \frac{9\times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9\times 28}{5} + 32 = \frac{252+160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9\times(-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) It is given that $t(C) = 212$

$212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow 9C = 180 \times 5$

$\Rightarrow C = \frac{180\times 5}{9} = 100$

Thus, the value of t, when $t(C) = 212, is\ 100$.

Ques 5: Find the range of each of the following functions.

(i) $f(x) = 2 - 3x, x \in R, x > 0$.

(ii) $f(x) = x^2 + 2, x$, *is a real number*.

(iii) $f(x) = x, x$ *is a real number*

Ans 5:

(i) $f(x) = 2 - 3x, x \in R, x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.
i.e., range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2, x$, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f = [2, \infty)$

(iii) $f(x) = x, x$ is a real number

It is clear that the range of f is the set of all real numbers.

∴ Range of $f = R$

Chapter – 2

Relation and Functions

Miscellaneous Exercise on Chapter 2

Ques 1: The relation *f* is defined by $f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

The relation *g* is defined by $g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

Show that *f* is a function and *g* is not a function.

Ans 1: The relation *f* is defined as

$f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

It is observed that for

$0 \le x < 3, f(x) = x^2$

$3 < x \le 10, f(x) = 3x$

Also, at $x = 3, f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$ i.e., at $x = 3, f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation g is defined as

$g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

It can be observed that for $x = 2, g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

Ques 2: If $f(x) = x^2$, find $\frac{f(1.1)-f(1)}{1.1-1}$

Ans 2: $f(x) = x^2$

$$\frac{f(1.1)-f(1)}{(1.1-1)} = \frac{(1.1)^2-(1)^2}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Ques 3: Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Ans 3: The given function is $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12} = \frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function *f* is defined for all real numbers except at $x = 6$ *and* $x = 2$.

Hence, the domain of *f* is R – {2, 6}.

Ques 4:Find the domain and the range of the real function *f* defined by $f(x) = \sqrt{x - 1}$

Ans 4: The given real function is $f(x) = \sqrt{x-1}$

It can be seen that $\sqrt{x - 1}$ is defined for $x \geq 1$.

Therefore, the domain of *f* is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1,)$.

As $x \geq 1 \Rightarrow (x-1) \geq 0 \Rightarrow \sqrt{x-1} \geq 0$

Therefore, the range of *f* is the set of all real numbers greater than or equal to 0 i.e., the range of $f = (0, \infty)$.

Ques 5: Find the domain and the range of the real function *f* defined by *f* (*x*) = |*x* – 1|.

Ans 5: The given real function is $f(x) = |x-1|$.

It is clear that $|x-1|$ is defined for all real numbers.

$\therefore$ Domain of $f = R$

Also, for $x \in R$, $|x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non – negative real numbers.

Ques 6: Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right); x \in R\right\}$ be a function from R into R. Determine the range of f.

Ans 6: $f = \left\{\left(x, \frac{x^2}{1+x^2}\right); x \in R\right\}$

$= \left\{(0,0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots \ldots.\right\}$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1. [Denominator is greater numerator]

Thus, range of f = (0, 1)

Ques 7: Let f, g: R ⟶ R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

Ans 7: $f, g: R \to R$ is defined as $f(x) = x + 1, g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \ \ g(x) \neq 0, x \in R$

$\left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, \ \ 2x - 3 \neq 0, 2x \neq 3$

$\left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

Ques 8: Let *f* = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a function from Z to Z defined by *f*(*x*) = *ax* + *b*, for some integers *a*, *b*. Determine *a*, *b*.

Ans 8: $f = \{(1,1),(2,3),(0,-1),(-1,-3)\} and\ f(x) = ax + b$

$(1,1) \in f \Rightarrow f(1) = 1 \Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$

$(0,-1) \in f \Rightarrow f(0) = -1 \Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting $b = -1$ in $a + b = 1$,

We obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and – 1.

Ques 9: Let R be a relation from N to N defined by R = {(*a*, *b*): *a*, *b* ∈ N and *a* =*b*^{2}}. Are the following true?

(i)(*a*, *a*) ∈ R, for all *a* ∈ N

(ii)(*a*, *b*) ∈ R, implies (*b*, *a*) ∈ R

(iii)(*a*, *b*) ∈ R, (*b*, *c*) ∈ R implies (*a*, *c*) ∈ R. Justify your answer in each case.

Ans 9: $R = \{(a,b): a,b \in N\ and\ a = b^2\}$

(i)It can be seen that $2 \in N$;

however, $2 \neq 2^2 = 4$.

Therefore, the statement "$(a,a) \in R, for\ all\ a \in N$" is not true.

(ii) It can be seen that $(9,3) \in N$ because $9,3 \in N\ and\ 9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3,9) \notin N$

Therefore, the statement "$(a,b) \in R, implies\ (b,a) \in R$" is not true.

(iii) It can be seen that $(9,3) \in R, (16,4) \in R$ because $9,3,16,4 \in N$ and $9 = 3^2$ and $16 = 4^2$.

Now, $9 \neq 4^2 = 16$; therefore, $(9,4) \notin N$

Therefore, the statement "$(a,b) \in R, (b,c) \in R$ *implies* $(a,c) \in R$" is not true.

Ques 10: Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) *f* is a relation from A to B

(ii) *f* is a function from A to B. Justify your answer in each case.

Ans 10: $A = \{1,2,3,4\}$ *and* $B = \{1,5,9,11,15,16\}$

∴ A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2,9), (2, 11),(2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4,5), (4, 9), (4,11), (4, 15), (4, 16)}

It is given that $f = \{(1,5),(2,9),(3,1),(4,5),(2,11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.

It is observed that *f* is a subset of A × B. Thus, *f* is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation *f* is not a function.

Ques 11: Let *f* be the subset of Z × Z defined by *f* = {(*ab*, *a* + *b*): *a*, *b* ∈ Z}. Is *f* a function from Z to Z: justify your answer.

Ans 11: The relation *f* is defined as $f = \{(ab, a+b): a,b \in Z\}$

We know that a relation *f* from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since,

$2,6,-2,-6 \in Z, (2 \times 6, 2+6), (-2 \times -6, -2+(-6)) \in f$ $i.e., (12,8),(12,-8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and –8. Thus, relation *f* is not a function.

Ques 12: Let A = {9, 10, 11, 12, 13} and let *f*: A → N be defined by *f*(*n*) = the highest prime factor of *n*. Find the range of *f*.

Ans 12: $A = \{9, 10, 11, 12, 13\}$ f: $A \rightarrow N$ is defined as

$f(n) =$ The highest prime factor of n

Prime factor of 9 $= 3$

Prime factors of 10 $= 2, 5$

Prime factor of 11 $= 11$

Prime factors of 12 $= 2, 3$

Prime factor of 13 $= 13$

$\therefore$ $f(9) =$ The highest prime factor of 9 $= 3$

$f(10) =$ The highest prime factor of 10 $= 5$

$f(11) =$ The highest prime factor of 11 $= 11$

$f(12) =$ The highest prime factor of 12 $= 3$

$f(13) =$ The highest prime factor of 13 $= 13$

The range of f is the set of all f(n), where $n \in A$.

$\therefore$ Range of $f = \{3, 5, 11, 13\}$