EE3301
ANALOG ELECTRONICS

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Filters

Time Domain Signals

Basic Characteristics

- Peak Amplitude
- Period (time)
- Frequency
- Phase
- Wavelength

Measured using:

Oscilloscope

Frequency Domain Signals

Basic Characteristics

- Peak Amplitude
- Frequency

Measured using:

Spectrum Analyzer

A Time Domain (TD) waveform is a waveform (signal) that has a variation in ***time***.

Example:

In a sine wave,

$$e(t) = A \sin(\omega t) = A \sin(2\pi f t)$$

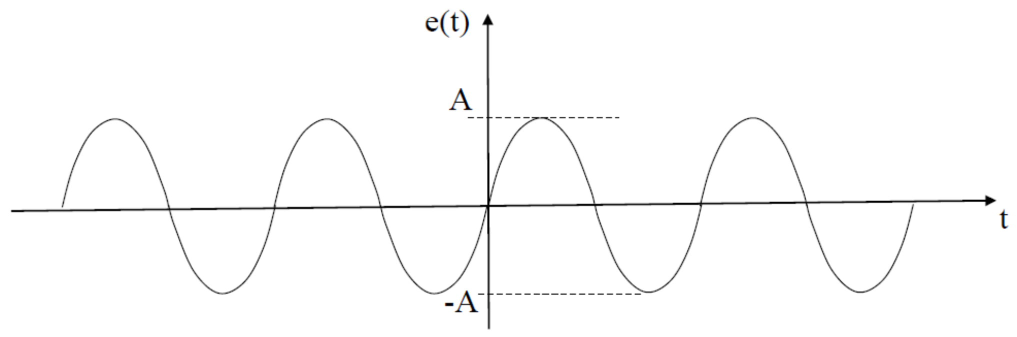
Where;

A – amplitude (V)

ω – angular (circular) frequency (rad)

f – frequency (Hz)

t –time

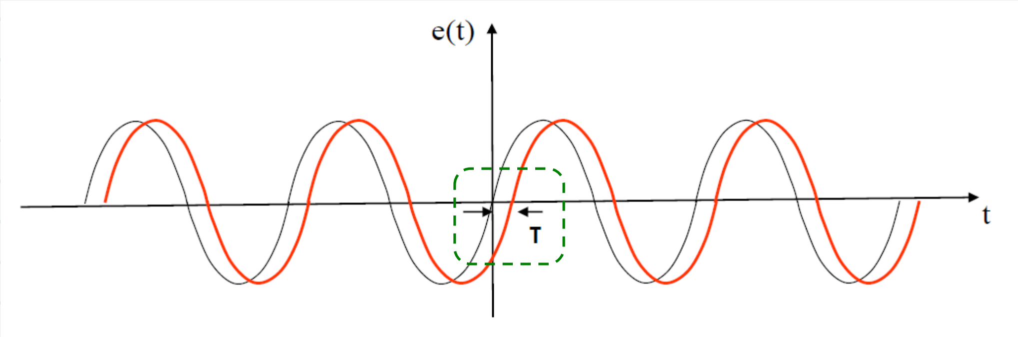


A sine wave can also have a phase angle φ,

Then;

$$e(t) = A \sin(\omega t + \varphi) = A \sin(2\pi f t + \varphi)$$

T in Figure represent the time corresponding to phase angle φ.



Time Domain → Frequency Domain

- Fourier Transform
- Laplace Transform
- Z – transform

*** *These are covered in Signals and Systems.*

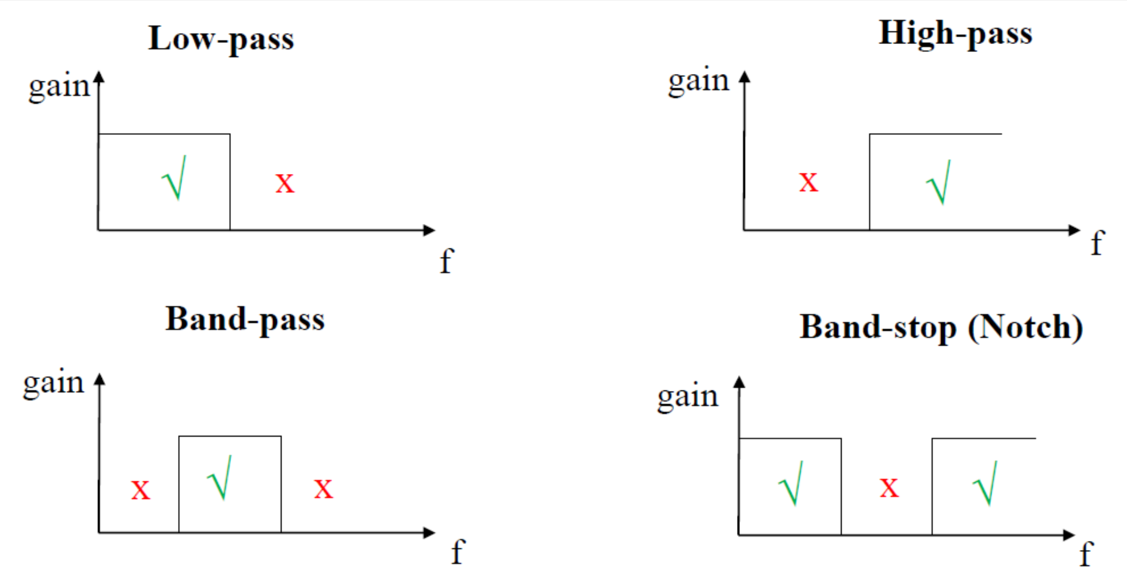
What is a filter?

A device that allows signals having frequencies in a certain range called ***passband*** to ***pass*** through ***while attenuating all other frequencies***.

What is a filter?

A device that allows signals having frequencies in a certain range called ***passband*** to ***pass*** through ***while attenuating all other frequencies***.

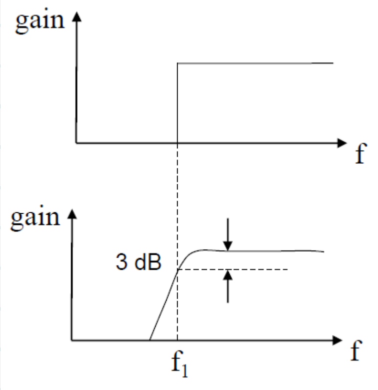
A **Ideal** filters, (sometimes) called as ***brick wall*** filters.

- 

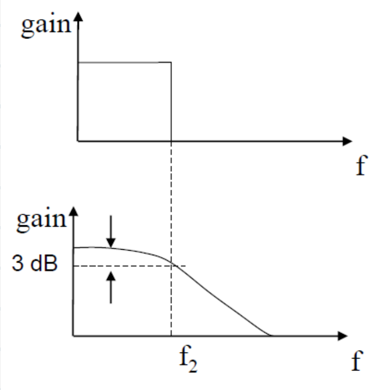
An ***ideal filter*** has *identical gain at all frequencies* **in** its pass band and *zero gain at all frequencies* **outside** of its passband.

But in practical filters;

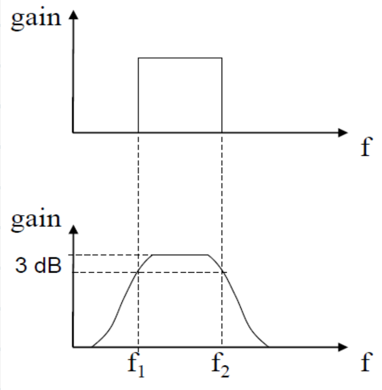
High Pass Filter



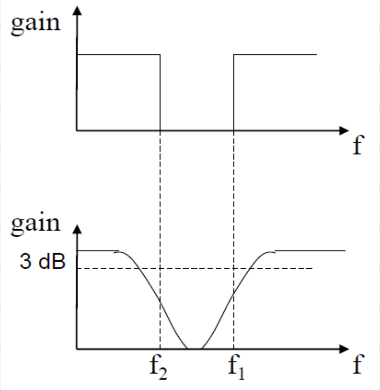
Low Pass Filter



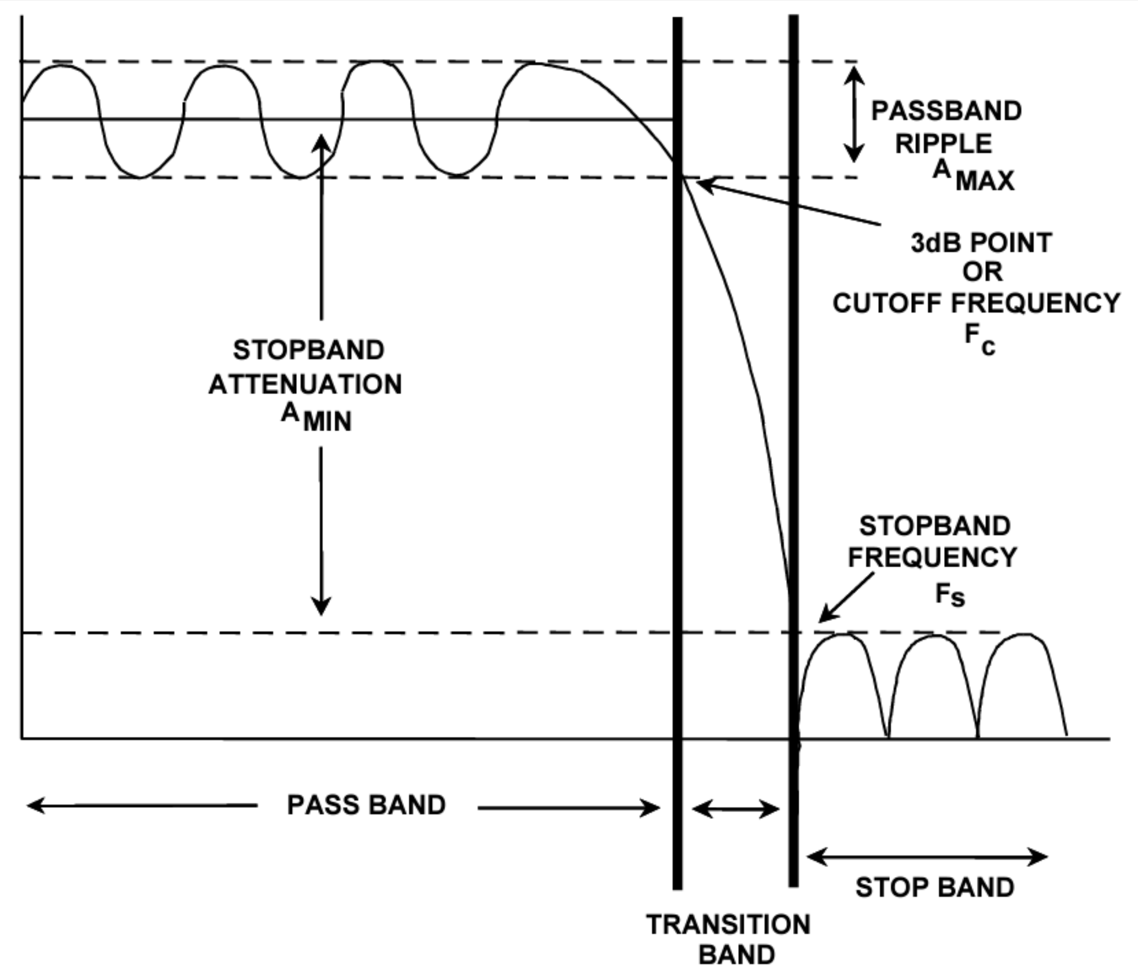
Band Pass Filter



Band Stop Filter



$\boldsymbol{f_1}$ and $\boldsymbol{f_2}$ are cut-off frequencies at which the response is down 3 dB from its maximum value in the passband.


PASSBAND
RIPPLE
A_{MAX}
3dB POINT
OR
CUTOFF FREQUENCY
F_C
STOPBAND
ATTENUATION
A_{MIN}
STOPBAND
FREQUENCY
F_S
PASS BAND
STOP BAND
TRANSITION
BAND

Filter Classification

- **Filters**
  - **Analog Filters**
    - **Element Type**
      - Active
      - Passive
    - **Frequency Band**
      - Low Pass
      - High Pass
      - Band Pass
      - Band Reject (Band Elimination)
      - All Pass (Phase Shift)
  - **Digital Filters**

Types of Filters

1. Passive Filters

– Designed with passive components.

Examples:

Capacitors, Resistors, Inductors

2. Active Filters

– Designed with active components.

Examples:

Transistors, op-amps and RC networks

– **Needs external DC supply

1. Passive Filters

- Contains only **R**, **L**, and **C** components (***not*** necessary that ***all three*** be present).
- L is often omitted (on purpose) from passive filter designs.
  Because,
  - The size and cost
  - Adds an internal resistance to the which cannot be ignored
- Q is the ***quality factor*** of the filter. Sometimes given as **α**.

  Where;

$$\alpha = \frac{1}{Q}$$

- The higher the Q;
  - Better the filter
  - Lower the losses
  - Closer to being perfect filter

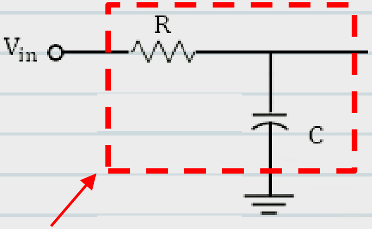
$$Q = 2\pi \text{ x } \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$$

- Filters are classified by their ***order*** (an integer number), also called the ***number of poles*** ***(a resistor + Capacitor pair)***.
  - The order here refers to the order of the ***polynomial(s)*** that are used to define the filter.

- Higher the order of a filter;
  - Response will be closer to the ideal case
  - More complex the circuitry

******Means an Ideal filter, can be approached by making the order of the filter higher and higher.***

- Frequency response outside the passband of a filter of order *n* has a slope of ***20n dB/decade*** or ***6n dB/octave***.



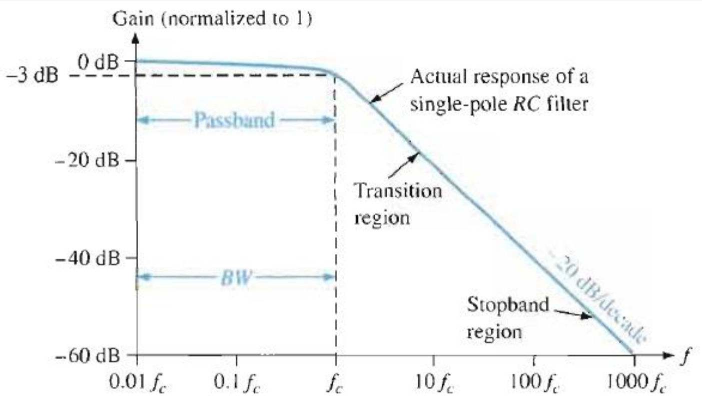
First order or single pole

Roll – off Rate

The slope of the transition region of a response curve;

$$Roll - off\ rate = \frac{gain\ (dB)}{frequency\ (decade)}$$

*****Magnitude changes 20 dB when frequency changes tenfold or one decade.***



Response curve of a first Order (Single pole) low pass filter

The # of poles determine the roll – off rate of a filter.

Example:

The roll – off rate of

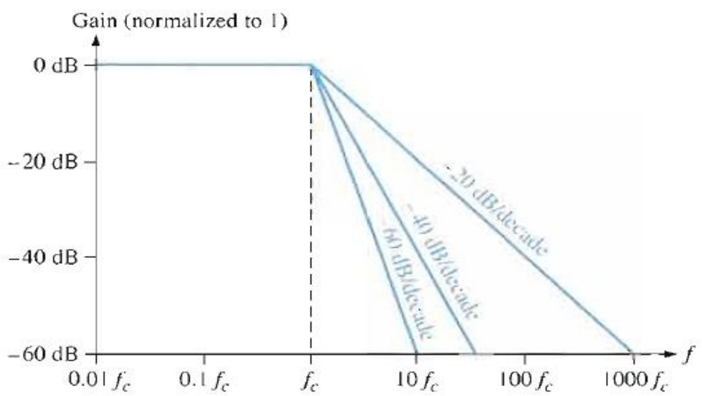
$n = 1$; a first order (or single-pole) filter = ***– 20 dB/decade***

$n = 2$; a second order (or two-pole) filter

= – 20 dB/decade x **2** = ***– 40 dB/decade***

$n = 3$; a third order (or three-pole) filter

= – 20 dB/decade x **3** = ***– 60 dB/decade***



Response curves for several order (several pole) low pass filters

Voltage Transfer Function in the frequency domain

- The main parameter in a filter.

$$H\ (j\omega) = \frac{V_O}{V_{in}}$$

Where;

$V_O - \text{Output Voltage}$

$V_{in} - \text{Input Voltage}$

- H(jω) is a complex number, so it has both the ***magnitude*** and the ***phase***.
- Filters in general, introduce a ***phase difference*** between ***input*** and ***output*** signals.

General first – order low pass filters

- In general, the voltage transfer function of a first-order low – pass filter is in the form:

$$H\,(j\omega) = \frac{K}{1 + j\frac{\omega}{\omega_C}}$$

Where;

$$|H\,(j\omega)|_{max} = |K|$$

K – A scaling factor chosen by the designer to give a specific gain.

.

Magnitude:

$$|H\,(j\omega)| = \frac{|K|}{\sqrt{1 + \left(\frac{\omega}{\omega_C}\right)^2}}$$

Phase:

$$\angle H\,(j\omega) = -\frac{|K|}{K}\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

Where;

ω_C – The cut – off frequency

RC Low pass filter

A series RC circuit act as a low pass filter.

When ***no load*** resistance R_L , (called as ***open-loop transfer function***);

V_o can be found from the voltage divider formula;

$$V_o = \left(\frac{X_C}{X_C + R}\right) V_{in}$$

Where;

V_o – Output voltage

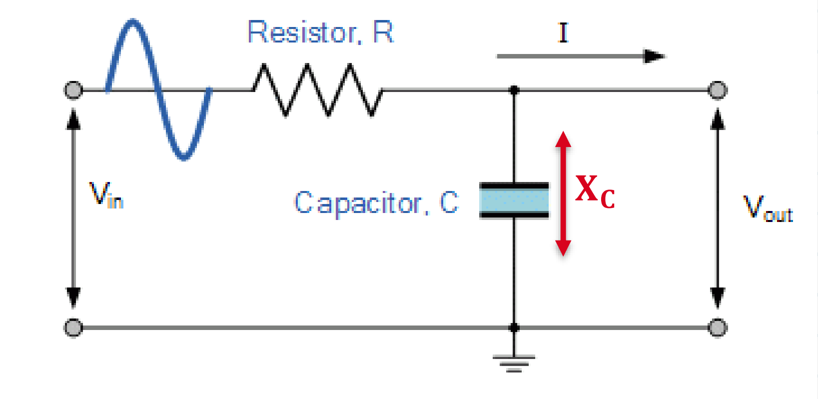
V_{in} – Input voltage

X_C – Capacitive Reactance

$$\mathbf{X_C = \frac{1}{j\omega C}}$$

☑ Impedance

☒ Resistance



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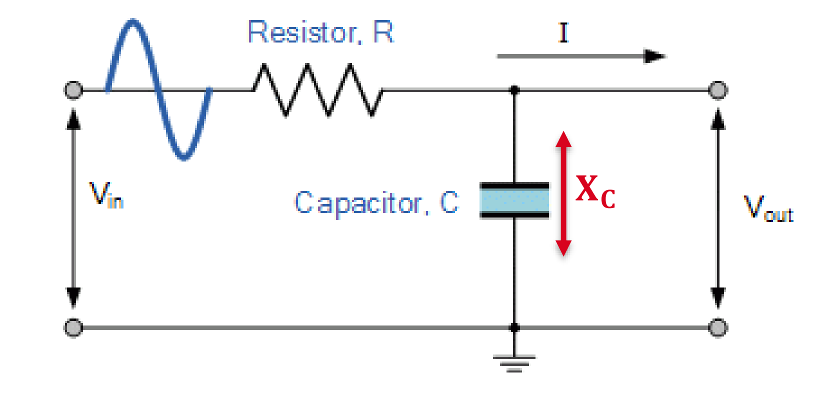
When ***no load*** resistance R_L , (called as ***open-loop transfer function***);

V_o can be found from the voltage divider formula;

$$V_o = \left(\frac{X_C}{X_C + R}\right) V_{in}$$

$$V_o = \left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right) V_{in}$$

$$H\,(j\omega) = \frac{V_o}{V_{in}} = \left(\frac{1}{1 + j\omega CR}\right)$$



If we consider the voltage transfer function of a first-order low-pass filter;

$$\mathbf{H\,(j\omega)} = \left(\frac{\mathbf{1}}{\mathbf{1 + j\omega CR}}\right) \quad \text{similar to} \quad H\,(j\omega) = \frac{K}{1 + j\dfrac{\omega}{\omega_C}}$$

Means;

$$\mathbf{K = 1} \quad \text{and} \quad \boldsymbol{\omega_C} = \frac{\mathbf{1}}{\mathbf{RC}}$$

The input impedance Z_i;

$$Z_i = R + \frac{1}{j\omega C} \quad \text{and} \quad Z_i\big|_{min} = R$$

The output impedance Z_o;

$$Z_o = R \Big\| \frac{1}{j\omega C} \quad \text{and} \quad Z_o\big|_{max} = R$$

RL Low pass filter

A series RL circuit act as a low pass filter.

V_o can be found from the voltage divider formula;

$$V_o = \left(\frac{R}{X_L + R}\right) V_{in}$$

Where;

X_L – Inductive Reactance

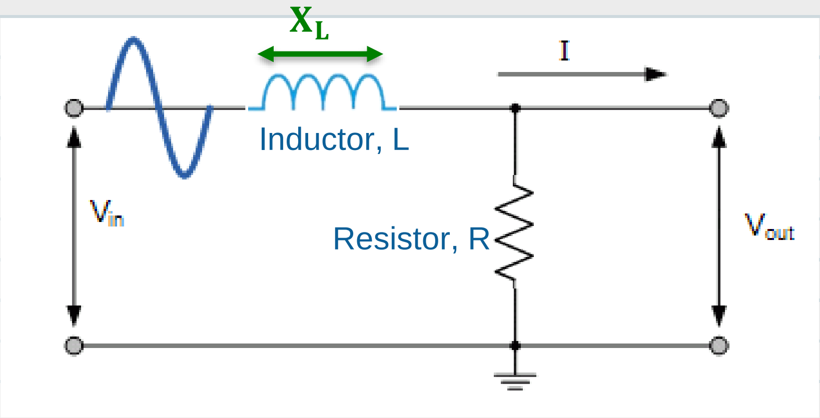
$\mathbf{X_L = j\omega L}$

Impedance

Resistance

$$V_o = \left(\frac{R}{j\omega L + R}\right) V_{in}$$

$$H\,(j\omega) = \frac{V_o}{V_{in}} = \left(\frac{1}{1 + \frac{j\omega L}{R}}\right)$$



If we consider the voltage transfer function of a first-order low-pass filter;

$$\mathbf{H\,(j\omega)} = \left(\frac{\mathbf{1}}{\mathbf{1 + j\omega\frac{L}{R}}}\right) \quad \text{similar to} \quad \mathbf{H\,(j\omega)} = \frac{\mathbf{K}}{\mathbf{1 + j\frac{\omega}{\omega_C}}}$$

Means;

$$\mathbf{K = 1} \quad \text{and} \quad \boldsymbol{\omega_C} = \frac{\mathbf{R}}{\mathbf{L}}$$

General first – order high pass filters

- In general, the voltage transfer function of a first-order high – pass filter is in the form:

$$H\,(j\omega) = \frac{K}{1 - j\,\frac{\omega_C}{\omega}}$$

Where;

ω_C − The cut − off frequency of the filter

.

- Called as a high pass filter because $|H\,(j\omega)| = 0$ for ω = 0 and $|H\,(j\omega)|$ is a constant for high frequencies.

RC High pass filter

The open-loop voltage transfer function of this filter is:

$$H\,(j\omega) = \frac{V_o}{V_{in}} = \left(\frac{R}{X_C + R}\right)$$

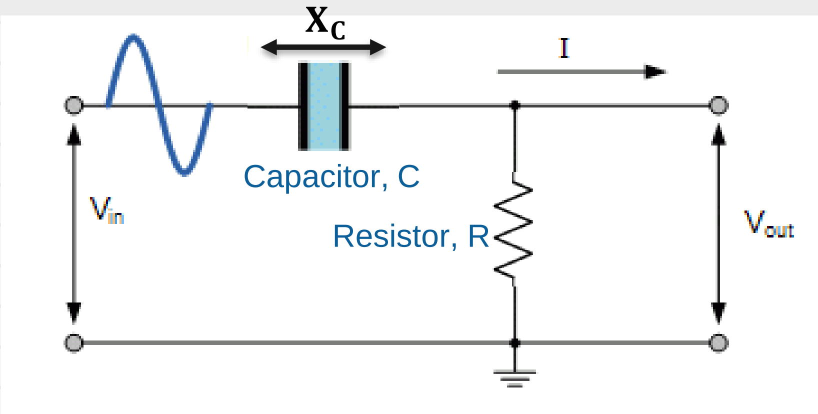
Where; $X_C = \dfrac{1}{j\omega C}$

$$H\,(j\omega) = \frac{V_o}{V_{in}} = \left(\frac{1}{1 + \frac{1}{j\omega CR}}\right) = \left(\frac{1}{1 - j\left(\frac{1}{\omega CR}\right)}\right)$$

If we consider the voltage transfer function of a first-order high – pass filter;

Where;

$\mathbf{K = 1}$ and $\boldsymbol{\omega_C = \dfrac{1}{RC}}$



Similarly;

The input impedance Z_i;

$$Z_i = R + \frac{1}{j\omega C} \qquad \text{and} \qquad Z_i\big|_{min} = R$$

The output impedance Z_o;

$$Z_o = R \left\| \frac{1}{j\omega C} \right. \qquad \text{and} \qquad Z_o\big|_{max} = R$$

RL High pass filter

The open-loop voltage transfer function of this filter is:

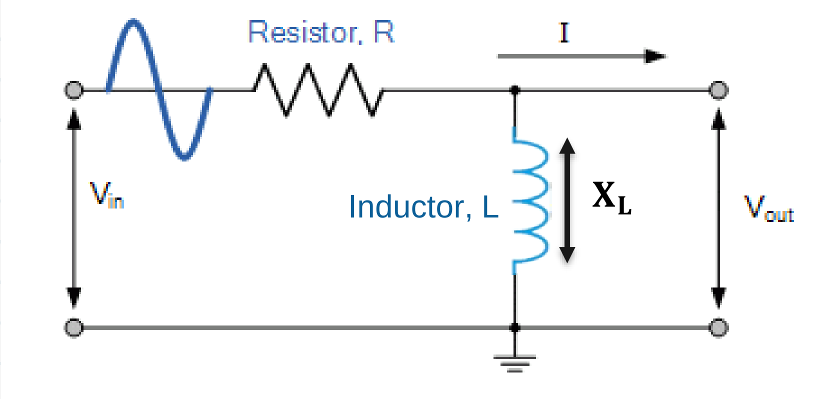
$$H\ (j\omega) = \frac{V_o}{V_{in}} = \left(\frac{X_L}{X_L + R}\right)$$

Where; $X_C = j\omega L$

$$H\ (j\omega) = \frac{V_o}{V_{in}} = \left(\frac{1}{1 + \frac{R}{j\omega L}}\right) = \left(\frac{1}{1 - j\left(\frac{R}{\omega L}\right)}\right)$$

Where;

$$\boldsymbol{\omega_C = \frac{R}{L}}$$



Similarly;

The input impedance Z_i;

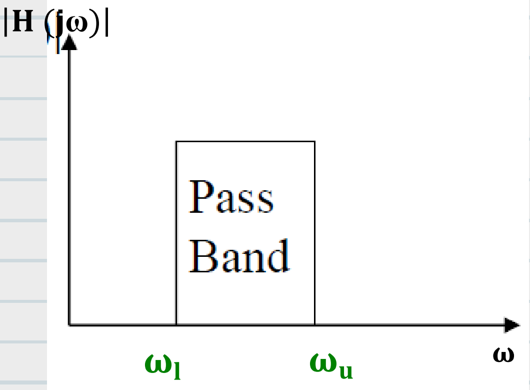
$$Z_i = R + j\omega L \quad \text{and} \quad Z_i\big|_{min} = R$$

The output impedance Z_o;

$$Z_o = R \,||\, j\omega L \quad \text{and} \quad Z_o\big|_{max} = R$$

Band Pass Filters

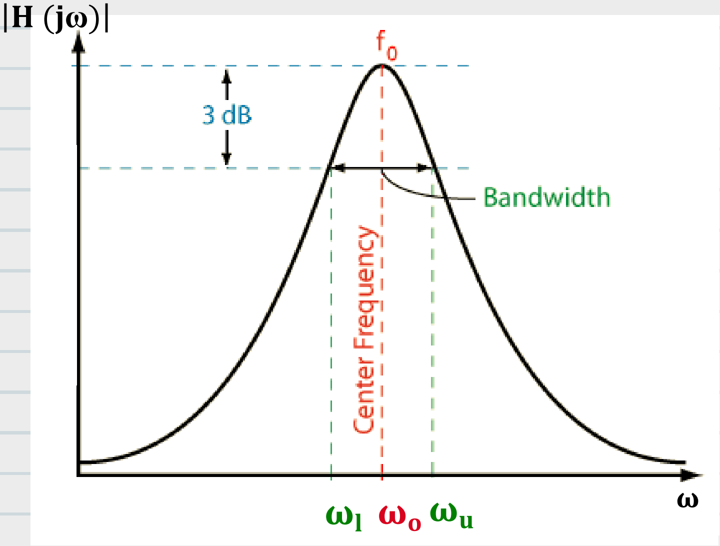
A band pass filter allows signals with a ***range of frequencies (passband)*** to pass through and attenuates signals with frequencies outside this range.



Centre Frequency, $\omega_o = \sqrt{\omega_l \omega_u}$

Bandwidth, $BW = \omega_l - \omega_u$

Quality Factor, $Q = \dfrac{\omega_o}{BW}$



Where;

ω_l – The lower cut – off frequency

ω_u – The upper cut – off frequency

ω_o – The centre frequency

$\boxed{1}$ $\boxed{2}$

Band Pass Filter = High Pass Filter + Low Pass Filter

- Band Pass filters are ***Second-order filters***.

  Second-order band pass filters include two storage elements.

  (two capacitors, two inductors, or one of each).

- The transfer function for a second-order band-pass filter can be written as;

$$H\,(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Magnitude:

$$|H\,(j\omega)| = \frac{|K|}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}$$

Phase:

$$\angle\, H\,(j\omega) = -\frac{|K|}{K}\tan^{-1}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)$$

- The maximum value of $|H(j\omega)|_{max} = |K|$ is the filter gain.
- The lower and upper cut-off frequencies can be calculated by noting that;

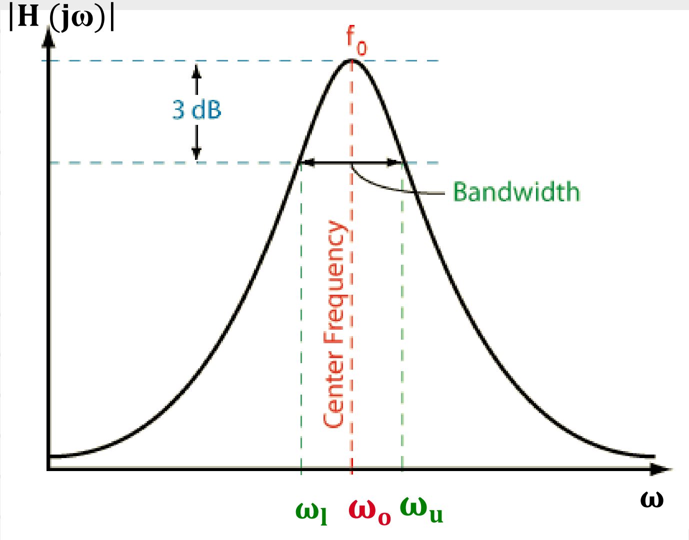
  The cutoff frequency is defined as the frequency where the amplitude of $H(j\omega)$ is $\frac{1}{\sqrt{2}}$ times the DC amplitude (approximately -3dB, half power point).

- $|H(j\omega)|_{max} = |K|$, setting $|H(j\omega_c)| = \frac{|K|}{\sqrt{2}}$ and solving for ω_C;

$$|H\,(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j\omega)|_{max} = \frac{|K|}{\sqrt{2}} = \frac{|K|}{\sqrt{1 + Q^2\left(\frac{\omega_c}{\omega_o} - \frac{\omega_o}{\omega_c}\right)^2}}$$

$$\frac{|K|}{\sqrt{2}} = \frac{|K|}{\sqrt{1 + Q^2\left(\frac{\omega_c}{\omega_o} - \frac{\omega_o}{\omega_c}\right)^2}}$$

$\omega_c = \omega_l$ or ω_u



$$|H\,(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j\omega)|_{max} = \frac{|K|}{\sqrt{2}} = \frac{|K|}{\sqrt{1 + Q^2\left(\frac{\omega_c}{\omega_o} - \frac{\omega_o}{\omega_c}\right)^2}}$$

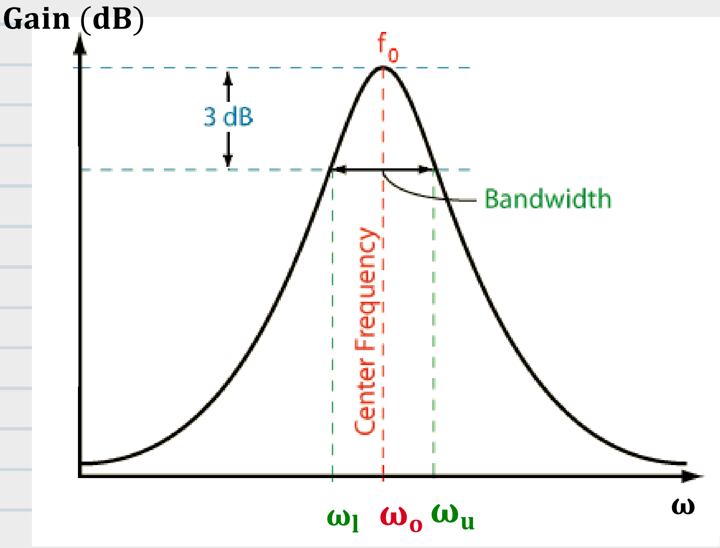
$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega_c}{\omega_o} - \frac{\omega_o}{\omega_c}\right)^2}}$$

$$1 + Q^2\left(\frac{\omega_c}{\omega_o} - \frac{\omega_o}{\omega_c}\right)^2 = 2$$

$$({\omega_c}^2 - {\omega_o}^2)^2 = \frac{(\omega_o .\ \omega_c)^2}{Q^2}$$

$$\boxed{\boldsymbol{{\omega_c}^2 - {\omega_o}^2 \pm \left(\frac{\omega_o . \omega_c}{Q}\right) = 0}}$$

This procedure will give two roots of $\boldsymbol{\omega_c}$: $\boldsymbol{\omega_l}$ and $\boldsymbol{\omega_u}$.



$$\boxed{\boldsymbol{\omega_c}^2 - \boldsymbol{\omega_o}^2 \pm \left(\frac{\boldsymbol{\omega_o . \omega_c}}{\mathbf{Q}}\right) = \mathbf{0}}$$

- This equation is really two quadratic equations.

$$\omega_c{}^2 - \omega_o{}^2 + \left(\frac{\omega_o . \omega_c}{Q}\right) = 0 \quad \text{——— } 1 \qquad\qquad \omega_c{}^2 - \omega_o{}^2 - \left(\frac{\omega_o . \omega_c}{Q}\right) = 0 \quad \text{——— } 2$$

- Solving these equation will give 4 roots (two roots per equation).
- Two of these four roots will be negative which are not physical as $\boldsymbol{\omega_c > 0}$.
- The other two roots are;

  The lower cut-off frequency, $\boldsymbol{\omega_l}$ and

  The upper cut-off frequency, $\boldsymbol{\omega_u}$.

$$\omega_l = \omega_0 \sqrt{1 + \frac{1}{4Q^2}} - \frac{\omega_o}{2Q} \qquad\qquad \omega_u = \omega_0 \sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_o}{2Q}$$

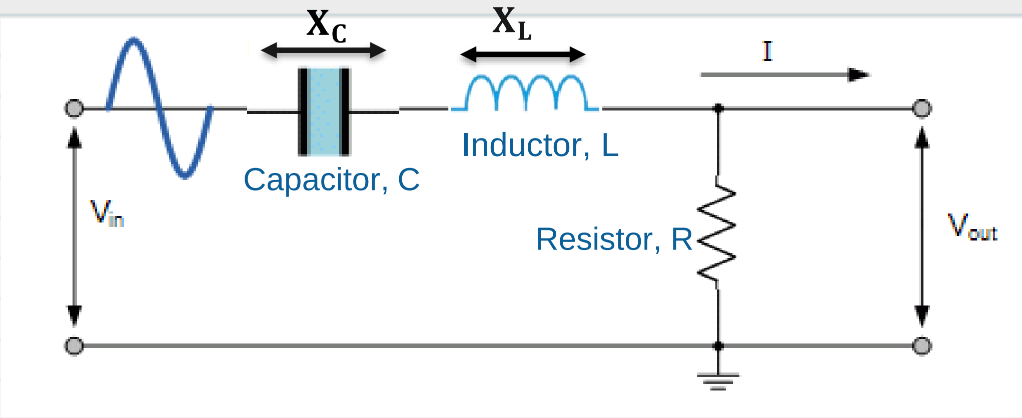
Series RLC Band Pass Filter

Using voltage divider formula:

$$H\ (j\omega) = \frac{V_o}{V_{in}} = \left(\frac{R}{R + X_L + X_C}\right) = \left(\frac{R}{R + j\omega L + \frac{1}{j\omega C}}\right)$$

$$H\ (j\omega) = \left(\frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}\right)$$

- $$H\ (j\omega) = \left(\frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{R\omega C}\right)}\right)$$



Finding filter parameters: K, ω_o, ω_u, and ω_l;

Transform the transfer function to a form similar to the general form of the transfer function for second order band pass filters;

$$H\,(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Considering RLC band pass filter;

$$H\,(j\omega) = \left(\frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{R\omega C}\right)}\right)$$

Comparing K = 1 and

$$\frac{Q\omega}{\omega_o} = \frac{\omega L}{R} \Rightarrow \frac{Q}{\omega_o} = \frac{L}{R} \quad \text{—— (1)} \qquad \frac{Q\omega_o}{\omega} = \frac{1}{R\omega C} \Rightarrow Q\omega_o = \frac{1}{RC} \quad \text{—— (2)}$$

Solving;

$$\frac{Q\omega}{\omega_o} = \frac{\omega L}{R} \Rightarrow \frac{Q}{\omega_o} = \frac{L}{R} \quad \text{——— } (1)$$

$$\frac{Q\omega_o}{\omega} = \frac{1}{R\omega C} \Rightarrow Q\omega_o = \frac{1}{RC} \quad \text{——— } (2)$$

Substituting Q from 2 to 1;

$$\frac{1}{\omega_o} \text{x} \frac{1}{\omega_o RC} = \frac{L}{R} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

Substituting ω_o to 2;

$$Q\omega_o = \frac{1}{RC} \Rightarrow Q = \frac{\sqrt{LC}}{RC} = \sqrt{\frac{L}{R^2C}}$$

Substitute values of Q and ω_o the lower cut-off frequency, $\boldsymbol{\omega_l}$ and the upper cut-off frequency, $\boldsymbol{\omega_u}$;

$$\omega_l = \omega_0\sqrt{1 + \frac{1}{4Q^2}} - \frac{\omega_o}{2Q}$$

$$\omega_u = \omega_0\sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_o}{2Q}$$

Input impedance of the RLC band pass filter, Z_i;

$$Z_i = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_i\big|_{min} = R \quad \text{occurs at} \quad \omega = \omega_0$$

The output impedance of the RLC band pass filter, Z_o;

$$Z_o = R \,||\left(j\omega L + \frac{1}{j\omega C}\right) \quad \text{and} \quad Z_o\big|_{max} = R$$

Example:

In a series RLC, circuit R = 30 Ω, L = 15 mH, and C = 51 μF. Calculate the

i. **Centre frequency, ω_o**
ii. **Quality Factor, Q**
iii. **Upper cut-off frequency, ω_u**
iv. **Lower cut-off frequency, ω_l.**

Consider the transformed transfer function of the RLC circuit, to the general form of the transfer function for second order band pass filters, when K=1.

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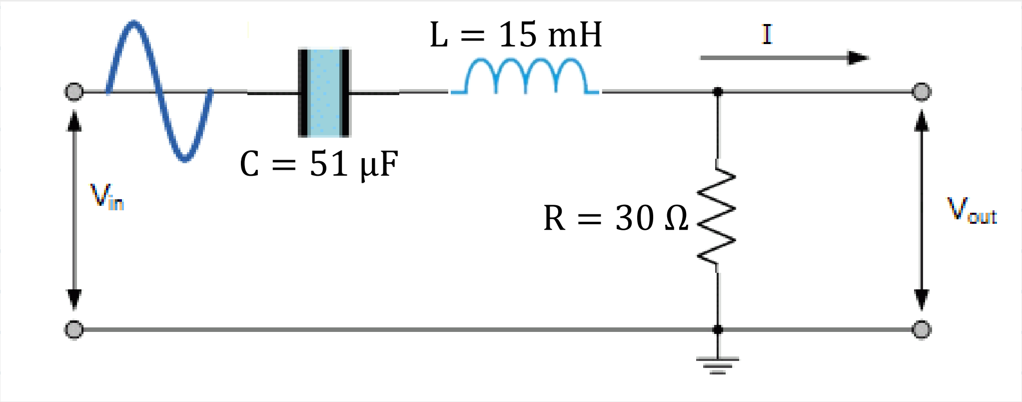


Answer:

The general form of the transfer function for second order band pass filters;

$$H\,(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \qquad \text{When } K = 1; \quad H\,(j\omega) = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

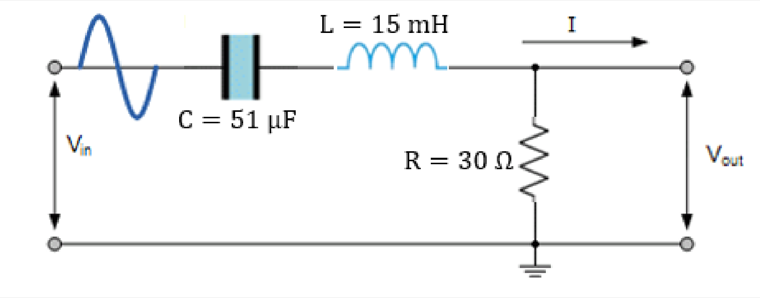
Considering RLC band pass filter;

$$H\,(j\omega) = \left(\frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{R\omega C}\right)}\right)$$

i. Calculate center frequency, ω_o

- $$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{15 \text{ x } 10^{-3} \text{ x } 51 \text{ x } 10^{-6}}} \text{ Hz}$$

$$= 8.75 \text{ x } 10^4 \text{ Hz}$$



Answer:

ii. Calculate the quality factor, Q

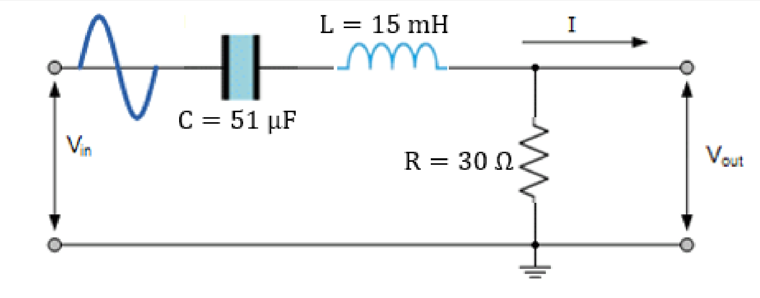
$$Q = \sqrt{\frac{L}{R^2 C}} = \sqrt{\frac{15 \text{ x } 10^{-3}}{(30)^2 \text{ x } 51 \text{ x } 10^{-6}}} = 0.572$$

iii. Calculate the upper cut-off frequency, ω_u

$$\omega_u = \omega_0 \sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_o}{2Q}$$

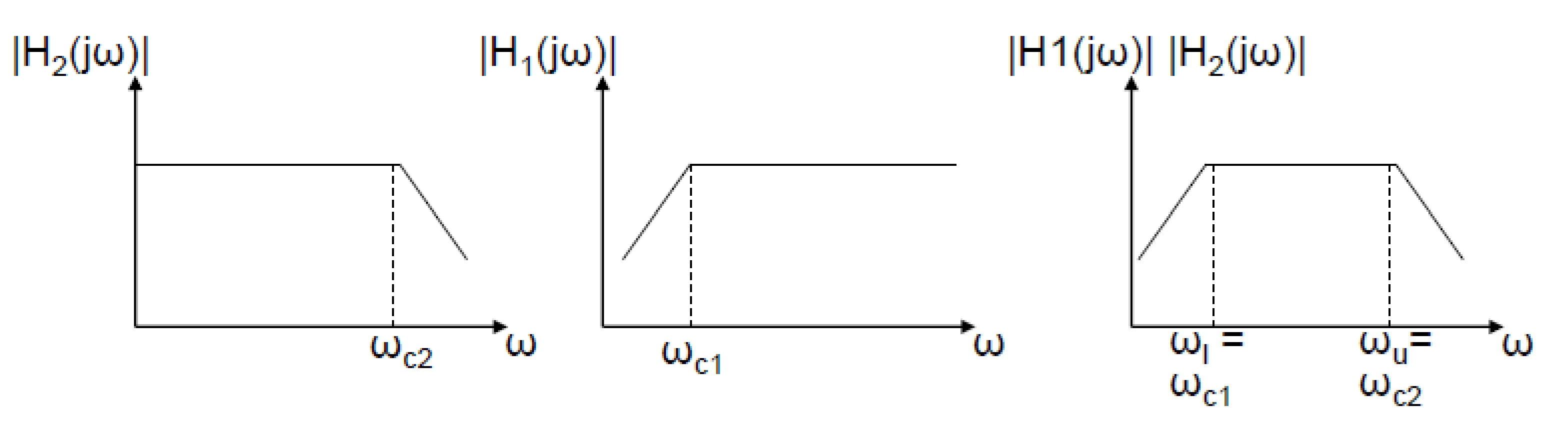
iv. Calculate the upper cut-off frequency, ω_u

$$\omega_l = \omega_0 \sqrt{1 + \frac{1}{4Q^2}} - \frac{\omega_o}{2Q}$$

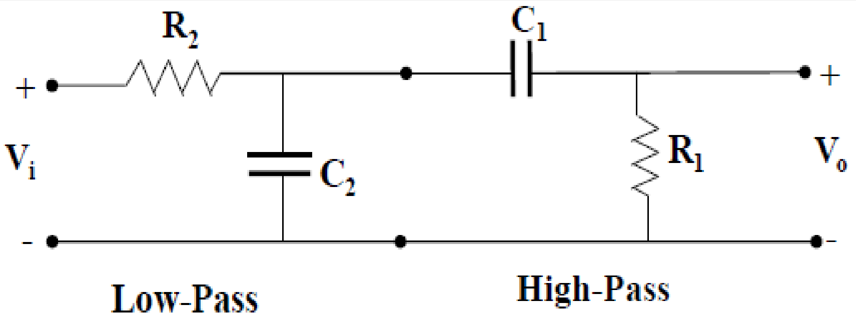


Wide Band Pass Filters

- Band-pass filters can be constructed by putting a high-pass and a low-pass filter back to back.
- The high-pass filter sets the lower cut-off frequency and the low-pass filter sets the upper cut-off frequency.



As an Example;



These ***RC filters are widely used*** (when appropriate) instead of an RLC filter,

Because,

- Inductors are usually bulky
- Can take too much space on a circuit board

In order to have ***good voltage coupling,*** the input impedance of the high $-$ pass filter, should be much larger than the output impedance of the low-pass filter.

$$Z_i\big|_{min} = R_1 \gg Z_o\big|_{max} = R_2$$

or

$$R_1 \gg R_2$$

Then, can use un-terminated transfer functions:

$$H(j\omega) = H_1(j\omega) \times H_2(j\omega)$$

$$= \frac{1}{1 + j\frac{\omega}{\omega_{C2}}} \times \frac{1}{1 - j\frac{\omega_{C1}}{\omega}}$$

Where;

$$\omega_{C1} = \frac{1}{R_1 C_1} \quad \text{and} \quad \omega_{C2} = \frac{1}{R_2 C_2}$$

Considering:

$$H(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_{C2}}\right)\left(1 - j\frac{\omega_{C1}}{\omega}\right)}$$

Can find the filter parameters by transforming the transfer function to a form similar to the general form to give:

$$K = \frac{1}{1 + \frac{\omega_{C1}}{\omega_{C2}}} \qquad Q = \frac{\sqrt{\frac{\omega_{C1}}{\omega_{C2}}}}{1 + \frac{\omega_{C1}}{\omega_{C2}}} \qquad \omega_o = \sqrt{\omega_{C1} . \omega_{C2}}$$

These filters work only when $\boldsymbol{\omega_{C2} \gg \omega_{C1}}$, so they are called "***wide-band***" filters.

For these wide-band filters since $\omega_{C2} \gg \omega_{C1}$, can find from above expressions:

$$K = 1 \qquad Q = \sqrt{\frac{\omega_{C1}}{\omega_{C2}}} \qquad \omega_o = \sqrt{\omega_{C1}.\omega_{C2}}$$

$$H\,(j\omega) = \frac{1}{\left(1 + j\left(\frac{\omega}{\omega_{C2}} - \frac{\omega_{C1}}{\omega}\right)\right)}$$

Then, substitute for Q and $\boldsymbol{\omega_o}$ in the expressions for cut-off frequencies to find $\boldsymbol{\omega_u}$ and $\boldsymbol{\omega_l}$.

What are the Wide Band and Narrow Band Filters?

Typically, a wide-band filter is defined as a filter with;

$$\omega_{C2} \gg \omega_{C1} \text{ (or } \omega_{C2} \geq 10\ \omega_{C1})$$

In this case; $Q \leq 0.35$

A narrow-band filter is usually defined as a filter with;

$$B \ll \omega_o \text{ (or } B \leq 0.1\ \omega_o)$$

In this case; $Q \geq 10$

Resonance

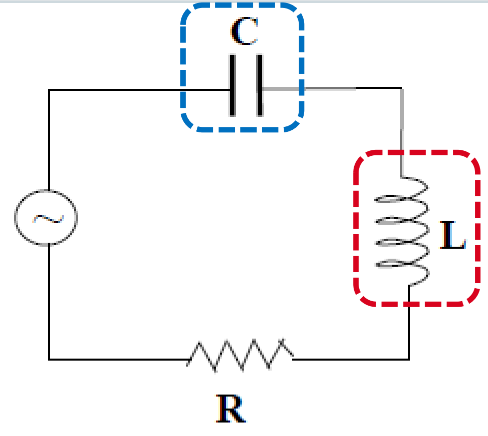
- A **capacitor** presents a **large impedance** to **low frequency ac current**, But, has a **small impedance** at **high frequency**.
- An **inductor** presents a **large impedance** to **high frequency ac current**, But, has a **small impedance** at **low frequency**.

Resonance

- A capacitor presents a large impedance to low frequency ac current,
  But, has a small impedance at high frequency.
- An inductor presents a large impedance to high frequency ac current,
  But, has a small impedance at low frequency.

Consider the series circuit with a capacitor and an inductor connected across an AC generator;

- The **impedance** is now large **both** at **high frequencies** (because of the inductor) and at **low frequencies** (because of the capacitor).

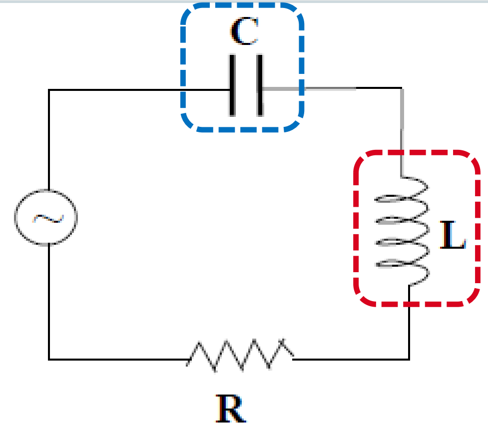


Resonance

- A capacitor presents a large impedance to low frequency ac current,
  But, has a small impedance at high frequency.
- An inductor presents a large impedance to high frequency ac current,
  But, has a small impedance at low frequency.

Consider the series circuit with a capacitor and an inductor connected across an AC generator;

- The **impedance** is now large **both** at **high frequencies** (because of the inductor) and at **low frequencies** (because of the capacitor).
- A **minimum** value of **impedance** occurs at **some intermediate frequency** called the **resonance frequency**.



The complex impedance for the circuit;

$$Z = R + j\left(\underbrace{\omega L}_{X_L} - \underbrace{\frac{1}{\omega C}}_{X_C}\right)$$

Magnitude;

$$|Z| = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

It has minimize the impedance (imaginary part) at resonant frequency;

$$Z = R + \boxed{j\left(\omega L - \frac{1}{\omega C}\right)} = \mathbf{0\ (minimum\ value)}$$

$$\omega_o L - \frac{1}{\omega_o C} = 0 \Rightarrow \omega_o L = \frac{1}{\omega_o C}$$

$$\boxed{X_L = X_C}$$

Inductive Reactance = Capacitive Reactance

The complex impedance for the circuit;

$$Z = R + j\left(\underbrace{\omega L}_{X_L} - \underbrace{\frac{1}{\omega C}}_{X_C}\right)$$

Magnitude;

$$|Z| = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

It has minimize the impedance (imaginary part) at resonant frequency;

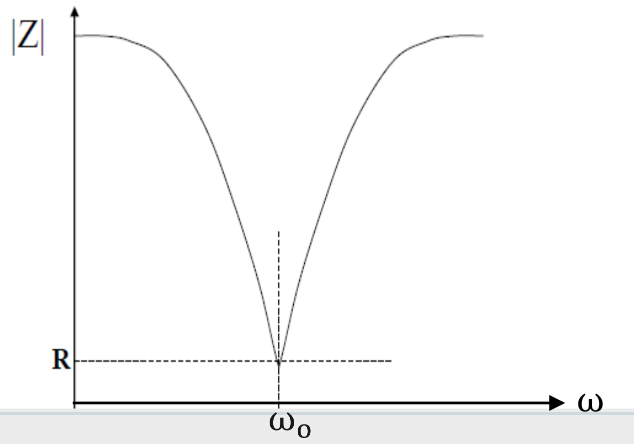
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonant frequency ω_o given by;

$$\omega_o L = \frac{1}{\omega_o C} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

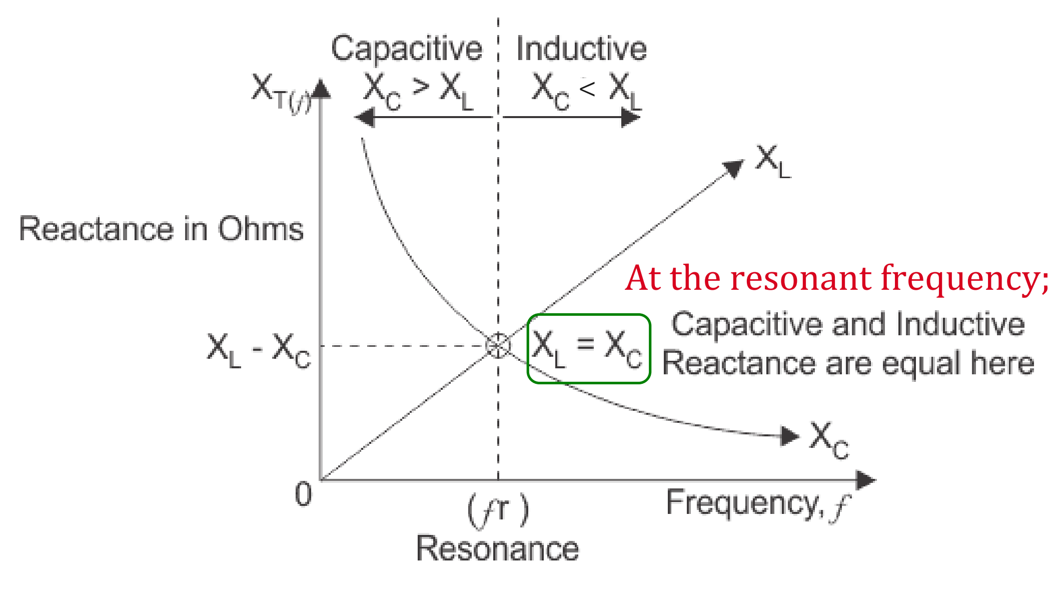
The impedance at the resonance is real, having the value of R.

Frequency response of |Z|



- If the resistance of the **inductor** is **very low**,

  The impedance increases rapidly on either side of the minimum, as the frequency moves away from the resonant frequency.



- If the resistance of the inductor is very low,

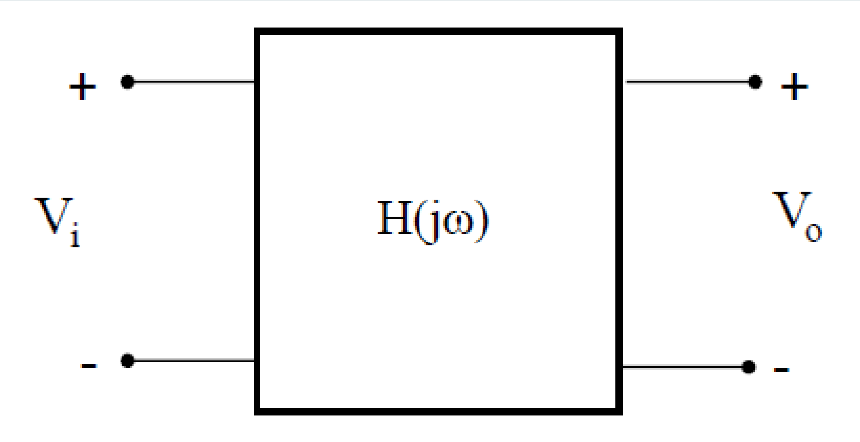
  The impedance increases rapidly on either side of the minimum,
  as the frequency moves away from the resonant frequency.

- A inductor (coil) and capacitor connected in series can thus select signals near their resonant frequency and reject signals of other frequencies for which the impedance is high.

- The circuit is then said to be **tuned** to the **resonant frequency**.

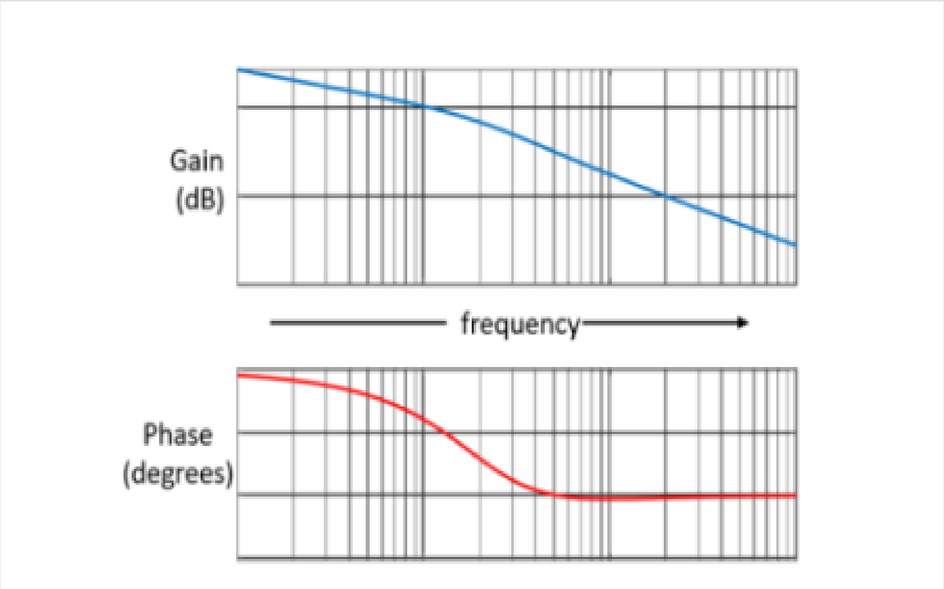
Two port network representation

- The transfer function characterize a two port network.



$$H\,(j\omega) = \frac{V_o}{V_{in}}$$

Bode Plots and Decibels



Bode plots show the frequency response, that is, the changes in magnitude and phase as a function of frequency.

> This is done on two semi-log scale plots. The top plot is typically magnitude or "gain" in dB. The bottom plot is phase, most commonly in degrees.

Bode Plots and Decibels

- The voltage transfer function of a two-port network is usually expressed in Bels:

$$\text{Number of Bels} = \log_{10}\frac{P_O}{P_i} \quad \text{or} \quad = 2\log_{10}\frac{V_O}{V_i} \qquad \left[\because P = \frac{V^2}{R} \text{ and } \therefore P \,\alpha\, V^2\right]$$

- But, usually use decibels (dB);

$$\text{Number of decibels} = 20\log_{10}\left|\frac{V_O}{V_i}\right| \quad \text{or} \quad \left|\frac{V_O}{V_i}\right|_{dB} = 20\log_{10}\left|\frac{V_O}{V_i}\right|$$

.

- Historically, the analog systems were developed first for audio equipment.
- The human ear was constructed to hear both very quiet and very loud sounds of very small and very large frequencies. Therefore the ***human ear hears logarithmically***.

- If several two-port networks are placed in a cascade (output of one is attached to the input of the next);

The overall transfer function, ***H***, is equal to the ***product of all transfer functions***.

$$|H\ (j\omega)| = |H_1\ (j\omega)|\ x\ |H_2(j\omega)|\ x\ \ldots$$

$$20\ \log_{10}|H\ (j\omega)| = 20\ \log_{10}|H_1\ (j\omega)| + 20\ \log_{10}|H_2(j\omega)| + \ \ldots$$

$$|H\ (j\omega)|_{dB} = |H_1\ (j\omega|_{dB} + |H_2\ (j\omega|_{dB} + \cdots$$

… makes it easier to find the overall response of the system.

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THANK YOU!