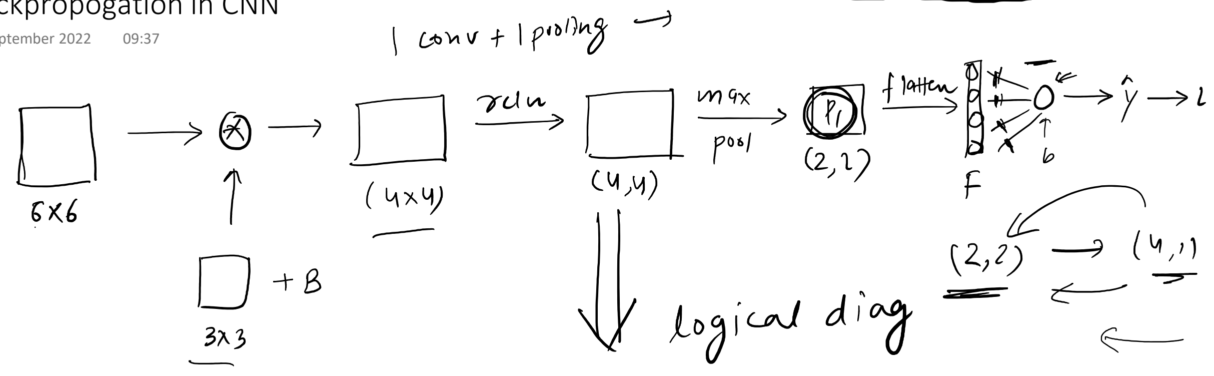
Lec-48 Backpropogation - II

Backpropogation in CNN

15 September 2022 09:37



Forward Prop

$Z_1 = conv(X, W_1) + b_1$

$A_1 = relu(Z_1)$

$P_1 = maxpool(A_1)$

$\boxed{F = flatten(P_1)}$

flatten

$Z_2 = W_2 F + b_2$

$A_2 = \sigma(Z_2)$

$L = \frac{1}{m}\sum_{i=1}^{m}\left[-y_i \log(A_2) - (1-y_i)\log(1-A_2)\right]$

6 derivatives

$$\frac{\partial L}{\partial W_1} = \underbrace{\frac{\partial L}{\partial A_2} \times \frac{\partial A_2}{\partial Z_2}}_{(A_2 - y)} \frac{\partial Z_2}{\partial F} \frac{\partial F}{\partial P_1} \frac{\partial P_1}{\partial A_1} \frac{\partial A_1}{\partial Z_1} \frac{\partial Z_1}{\partial W_1}$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial A_2} \frac{\partial A_2}{\partial Z_2} \frac{\partial Z_2}{\partial F} \frac{\partial F}{\partial P_1} \frac{\partial P_1}{\partial A_1} \frac{\partial A_1}{\partial Z_1} \frac{\partial Z_1}{\partial b_1}$$

1 2 3 4 5 6

$conv\left(X, \frac{\partial L}{\partial Z_1}\right)$

$\left[\frac{\partial Z_2}{\partial F}\right] = W_2 \rightarrow$

Shape ? → (F)

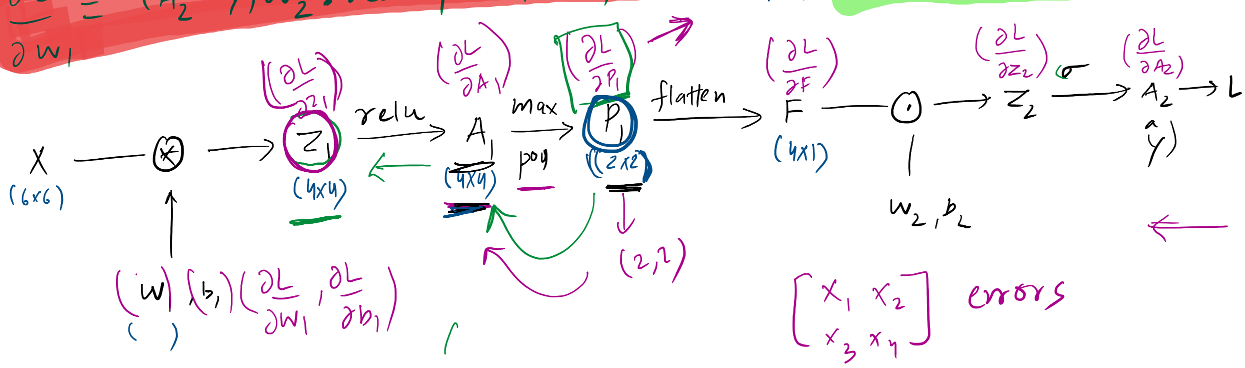
$\frac{\partial F}{\partial P_1}$ no trainable parameters

reshape(P_1.shape) → P_1 কে flatten করে F পেয়েছিলাম, back propa. এর সময় P_1 এর respect derivative করলে, F কে P_1 এ নিয়ে আসবো। (2,2) → (4,1)

$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

এখন, অর্ধেক বের করেছি

$$\frac{\partial L}{\partial W_1} = (A_2 - y) W_2 . reshape(P_1.shape)$$



$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ errors

$\frac{\partial L}{\partial A_1} = (4,4)$

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In pooling like, Flatten → no trainable parameters

max pooling → A_1

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix} \begin{bmatrix} 13 & 14 \\ 15 & 16 \end{bmatrix}$$

$\hat{y}$ → ∂L → **min value (1,2,3)**

L ← **(similarly)**

$$\frac{\partial L}{\partial P_1} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \; 2\times 2$$

(4×4)

$$\frac{\partial L}{\partial A_1} \qquad \frac{\partial L}{\partial A_1} \rightarrow \begin{bmatrix} 0 & x_1 & x_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & x_3 & x_4 & 0 \end{bmatrix}$$

$2\times 2 \rightarrow (4\times 4)$

error

$\hat{y}$ র loss নির্ণয়ে min value গুলোর কোন contribution নেই। তাই max বাদে বাকী গুলো শূন্য দিয়ে দিব।

A, P

$$\frac{\partial L}{\partial W_2} = \frac{\partial L}{\partial A_2}\frac{\partial A_2}{\partial Z_2}\frac{\partial Z_2}{\partial F}\frac{\partial F}{\partial P_1}\frac{\partial P_1}{\partial A_1}\frac{\partial A_1}{\partial Z_1}\frac{\partial Z_1}{\partial W_1}$$

$$\boxed{\frac{\partial L}{\partial A_1}} = \begin{cases} \frac{\partial L}{\partial P_{1xy}}, & \text{if } A_{mn} \text{ is the max element} \\ 0, & \text{otherwise} \end{cases}$$

$(A_2 - Y)W_2.\text{reshape}(P_1.\text{shape})$ → $\frac{\partial L}{\partial A_1}$

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial A_2}\frac{\partial A_2}{\partial Z_2}\frac{\partial Z_2}{\partial F}\frac{\partial F}{\partial P_1}\frac{\partial P_1}{\partial A_1}\frac{\partial A_1}{\partial Z_1}\frac{\partial Z_1}{\partial b_1}$$

$$\frac{\partial L}{\partial P_1} \qquad \frac{\partial L}{\partial A_1}$$

$$\frac{\partial A_1}{\partial Z_1} = \begin{cases} 1 & \text{if } Z_{1xy} > 0 \\ 0 & \text{if } Z_{1xy} < 0 \end{cases}$$

relu $\max(0,x)$ $\quad \frac{dy}{dx}(x) = 0$ $\quad \frac{dy}{dx}(0) = 0$

Last step

$$\left(\frac{\partial L}{\partial Z_1}\right)$$

Convolution
↳ Backprop

→ max pooling
↘ Flatten

$$\frac{\partial L}{\partial Z_1} = \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} \end{bmatrix}$$

Backprop on Convolution

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial Z_1} \times \frac{\partial Z_1}{\partial b_1}$$

assume input image (3×3)

$X \rightarrow \circledast \rightarrow Z_1$ $(2,2)$

$(3,3)$ $\quad W_1, b_1$

$\left(\frac{\partial L}{\partial Z_1}\right) \rightarrow (2,2)$

$$Z_1 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

W_1, b_1

$(2,2)$

$[Z_{21} \quad Z_{22}]$

$$\underbrace{X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}}_{\text{img}} \circledast \underbrace{W_1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}}_{\text{filter}} \qquad b_1$$

$$\boxed{\frac{\partial L}{\partial b_1}} = \frac{\partial L}{\partial Z_1} \times \frac{\partial Z_1}{\partial b_1} = \boxed{\frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial b_1} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial b_1} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial b_1} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial b_1}}$$

$$= \left(\frac{\partial L}{\partial Z_{11}} + \frac{\partial L}{\partial Z_{12}} + \frac{\partial L}{\partial Z_{21}} + \frac{\partial L}{\partial Z_{22}} \right) = \text{sum}\left(\frac{\partial L}{\partial Z_1} \right)$$

$$\boxed{\frac{\partial L}{\partial b_1}} = \text{sum}\left(\frac{\partial L}{\partial Z_1} \right) \rightarrow \text{scalar}$$

bias

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \qquad W_1 = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} + b_1$$

$X \rightarrow \circledast \rightarrow Z_1$, with W_1, b_1 (2×2) going into $\circledast$; X is (3×3); $\rightarrow \left(\frac{\partial L}{\partial Z_1}\right)$

$$\frac{\partial L}{\partial W_1} = \begin{bmatrix} \frac{\partial L}{\partial W_{11}} & \frac{\partial L}{\partial W_{12}} \\ \frac{\partial L}{\partial W_{21}} & \frac{\partial L}{\partial W_{22}} \end{bmatrix} \qquad \frac{\partial L}{\partial Z_1} = \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial Z_1} \times \frac{\partial Z_1}{\partial W_1}$$

$$\frac{\partial L}{\partial W_{11}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{11}}$$

$$\frac{\partial L}{\partial W_{12}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{12}}$$

$$\ldots \times \frac{\partial Z_{12}}{\ldots} + \frac{\partial L}{\ldots} \times \frac{\partial Z_{21}}{\ldots} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{21}}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{21}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{21}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{21}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{21}}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{22}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{22}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{22}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{22}}$$

$$\begin{cases} \frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{11}} x_{11} + \frac{\partial L}{\partial z_{12}} x_{12} + \frac{\partial L}{\partial z_{21}} x_{21} + \frac{\partial L}{\partial z_{22}} x_{22} \\ \frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{11}} x_{12} + \frac{\partial L}{\partial z_{12}} x_{13} + \frac{\partial L}{\partial z_{21}} x_{22} + \frac{\partial L}{\partial z_{22}} x_{23} \\ \frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} x_{21} + \frac{\partial L}{\partial z_{12}} x_{22} + \frac{\partial L}{\partial z_{21}} x_{31} + \frac{\partial L}{\partial z_{22}} x_{32} \\ \frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} x_{22} + \frac{\partial L}{\partial z_{12}} x_{23} + \frac{\partial L}{\partial z_{21}} x_{32} + \frac{\partial L}{\partial z_{22}} x_{33} \end{cases}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \qquad \frac{\partial L}{\partial z_1} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} \\ \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial w_1} = conv\left(X, \frac{\partial L}{\partial z_1}\right)$$

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$$\frac{\partial L}{\partial b_1} = sum\left(\frac{\partial L}{\partial z_1}\right)$$