Generalized Autoregressive Score Trees and Forests

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Increase your GAS mileage

- The family of “generalized autoregressive score” (GAS) models, proposed by Creal et al. (2013) and Harvey (2013), nests many useful models for capturing time series dynamics:
  - ARMA models, see Box and Jenkins (1970)
  - ARCH/GARCH models, see Engle (1982) and Bollerslev (1986)
  - ACD models, see Engle and Russell (1998)
  - Recent surveys: Artemova et al. (2022a,b) and Harvey (2022)
- These models are good but are of course only approximations to the DGP.

★ We propose using machine learning methods to improve the forecasts from models in this class.

GAS trees and forests

- We propose a "**GAS tree**" that combines the parsimonious structure of the GAS model with the flexible, data-driven learning of decision trees, see Breiman et al. (1984, 2017)
  - The model parameters can vary across "branches" of the tree using a set of state variables
  - The resulting model can incorporate information from *outside* the GAS model, and allows for nonlinearities and interactions, while maintaining interpretability
- We also propose "**GAS forests**," analogous to the "random forests" of Breiman (2001), where many GAS trees are created using bootstrap samples of data and then averaged across trees.
  - In many applications random forests have been found to improve upon regression trees due to the reduction in variance obtained via averaging, see Hastie et al. (2009)

Applications

- We apply the proposed models in four distinct forecasting problems:
    1. Stock return volatility, baseline model is GARCH
    2. Predictive density, baseline model is t-GAS
    3. Stock-bond dependence, baseline model is t-GAS-copula
    4. Trade durations, baseline model is ACD
- In all four applications we find significantly better OOS forecasts using the tree/forest GAS model
- We can also *interpret* the source of the gains
    - The GAS tree/forest uncovers a nonlinearity that is known to improve the baseline model

Related literature

- **GAS models**: Creal et al. (2013), Harvey (2013), Creal et al. (2011), Harvey (2022)
  - www.gasmodel.com
- **ML + Econometrics**:
  - *Volatility forecasting:* Audrino and Bühlmann (2001), Christensen et al. (2022), Nguyen et al. (2022), Reisenhofer et al. (2022),Tetereva and Kleen (2022)
  - *Macro forecasting:* Goulet Coulombe (2024), Huber et al. (2020), Medeiros et al. (2021)
  - *Asset pricing:* Gu et al. (2020), Bianchi et al. (2021), Bryzgalova et al. (2023)
- **Local Estimation**: Tibshirani and Hastie (1987), Fan et al. (1998), Fan et al. (2009), Audrino and Bühlmann (2001), Oh and Patton (2024)

GAS Trees and Forests

GAS models

- The general form of the GAS Models of Creal et al. (2013) is:

$$
\begin{aligned}
y_t &\sim p(\cdot|\mathcal{F}_t; f_t, \theta, \nu) \\
\text{where} \quad f_t &= \omega + \beta f_{t-1} + \alpha s_{t-1} \\
s_t &= S_t \nabla_t
\end{aligned}
$$

where $\theta = [\omega, \beta, \alpha]'$ govern the dynamics of f_t, ν is a static parameter, ∇_t is the gradient of the log-likelihood and S_t is a scaling matrix.

- Parameter estimation is done via maximum likelihood:

$$
\hat{\theta}, \hat{\nu} = \underset{\theta,\nu}{\operatorname{argmax}} \sum_{t=1}^{T} \log p(y_t|\mathcal{F}_t; f_t, \theta, \nu)
$$

GAS Tree

- For a given tree structure with J terminal nodes, $\mathcal{P} = \{\mathcal{P}_1, ..., \mathcal{P}_J\}$, the GAS(1,1) tree is based on the evolution equation:

$$f_t = \omega(\mathbf{Z_t}) + \beta(\mathbf{Z_t}) f_{t-1} + \alpha(\mathbf{Z_t}) s_{t-1}$$

where

$$[\ \omega(\mathbf{Z_t}), \beta(\mathbf{Z_t}), \alpha(\mathbf{Z_t})\ ] = \sum_{j=1}^{J} [\ \omega_j, \beta_j, \alpha_j\ ] \times \mathbb{1}(\mathbf{Z_t} \in \mathcal{P}_j)$$

and $[\ \omega_j, \beta_j, \alpha_j\ ]$ are the GAS parameters for partition j.

- Benefits of this approach:
  - Incorporates potential nonlinearities and interactions, Audrino and Bühlmann (2001)
  - Allows the parameters to vary across partitions, i.e. local parameters, Oh and Patton (2024)
  - Brings outside information to the model through $\mathbf{Z_t}$, Engle (2002)

GAS Forest

- Forests have been found to outperform trees (Hastie et al., 2009) in regression applications.
- Similar to bootstrap aggregation ("bagging") the GAS forest fits many trees using bootstrap samples of the original data.
  - Each sample only uses random subset of state variables, increasing variation
- Let $f_t^{(b)}(\mathbf{Z_t})$ denote the forecast from tree b at state variable $\mathbf{Z_t}$. The GAS Forest forecast is obtained simply as

$$f_t(\mathbf{Z_t}) = \frac{1}{B} \sum_{b=1}^{B} f_t^{(b)}(\mathbf{Z_t})$$

GAS Tree estimation overview

1. Estimate GAS model on full sample. Tree depth $\equiv m = 0$, partition $\boldsymbol{\mathcal{P}}^{(0)}$, parameter estimate $\hat{\theta}^{(0)}$
2. Define a new partition: $\boldsymbol{\mathcal{P}}_{j,k}^{(m+1)} = \boldsymbol{\mathcal{P}}_{-j}^{(m)} \cup \{\mathcal{P}_{j,k,L}^{(m)}, \mathcal{P}_{j,k,R}^{(m)}\}$ where $\boldsymbol{\mathcal{P}}_{-j}^{(m)} = \boldsymbol{\mathcal{P}}^{(m)} / \mathcal{P}_j$ contains all the partitions of $\boldsymbol{\mathcal{P}}^{(m)}$ except for the j^{th}, and the j^{th} partition is split into "left" and "right" subpartitions based on the k^{th} state variable and a threshold c
3. Estimate parameters for new subpartitions *taking as fixed* the parameters of the other partitions and evaluate the complete likelihood at $\left(\hat{\theta}_{-j}^{(m)}, \hat{\theta}_{j,k,L}^{(m+1)}, \hat{\theta}_{j,k,R}^{(m+1)}), \hat{\nu}^{(m)}\right)$
   - Massive saving in computation time. Also done by Athey et al. (2019).
4. Maximise step 3 across partitions j, state variables k, split points, c. Denote new set of partitions as $\boldsymbol{\mathcal{P}}^{(m+1)}$. Estimate complete model using these paritions, obtain $\hat{\theta}^{(m+1)}$.
5. Repeat until tree depth, m reaches pre-specified maximum value, M.

GAS Forest estimation overview

- "Just" repeat the previous slide for $B = 200$ trees, based on:
  - Data from a circular block bootstrap with block length 100
  - A randomly selected one-third of the state variables

Empirical Applications

Empirical applications

- We apply our new GAS tree and forest models in four out-of-sample forecasting analyses.
  1. S&P 500 index volatility
  2. S&P 500 index return predictive density
  3. Joint distribution of S&P 500 index and 10-year US govt bond returns
  4. High-frequency SPY trade durations

- We consider three benchmark models:
  - The baseline GAS specification (no tree)
  - "Distributional random forest" of Schlosser et al. (2019) (no dynamics)
  - "Small GAS Tree" similar to Audrino and Bühlmann (2001)

- We compare these models in terms of one-step-ahead predictive performance.
  - For volatility, we use the QLIKE loss function
  - For the others, we use the log-likelihood

- Data runs from 2000–2021, $T \approx 5500$
  - Split 30/30/40 for estimation/validation/testing

State variables

- We consider 10 state variables (at daily frequency) for use in GAS tree and forest model.
  - **Dependent variables:** S&P500 return and 10-year government bond return
  - **Volatility measures:** S&P500 RV (daily and monthly), VIX
  - **Macro variables:** Fed Funds Rate, 10yr-3mth Treasury yield, Default spread, Policy uncertainty index (Baker et al., 2016)
  - **Time**
- For our high-frequency application we also consider:
  - **Dependent variable:** Duration
  - **Market conditions:** Return, Amihud liquidity
- This large set of state variables differentiates us from:
  - Audrino and Bühlmann (2001), which only considers dependent variables as state variables
  - Oh and Patton (2024), which can only handle two state variables at time

Volatility forecasting

- The GARCH model of Bollerslev (1986) is widely used for forecasting asset return volatility, and has been shown to be difficult to beat in a range of applications, see Hansen and Lunde (2005).

$$\begin{aligned} y_t &= \sigma_t \epsilon_t; \quad \epsilon_t \sim \textit{iid } N(0,1) \\ \sigma_t^2 &= \omega + \beta \sigma_{t-1}^2 + \alpha y_{t-1}^2, \end{aligned}$$

- Creal et al. (2013) show that this model can be interpreted as a GAS model for the scale parameter of the Normal distribution.
- The "distributional random forest" (DRF) sets $\beta = \alpha = 0$ and allows the intercept, ω to vary with the forest structure.
- The "small GAS tree" model uses a regression tree with only y_{t-1} as a state variable.

Diebold-Mariano test results

Table 1: Out-of-sample performance of GARCH models using QLIKE loss

	GARCH	DRF	Small GAS Tree	GAS Tree	GAS Forest
DRF	-1.470				
Small Tree	-2.547	-0.414			
GAS Tree	-8.651	-5.577	-8.288		
GAS Forest	-6.409	-3.429	-2.777	4.973	
Avg loss	0.393	0.375	0.367	0.303	0.343

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- GAS Tree has lowest average loss, followed by GAS Forest.

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- GAS Tree and GAS Forest significantly outperforms all benchmarks.

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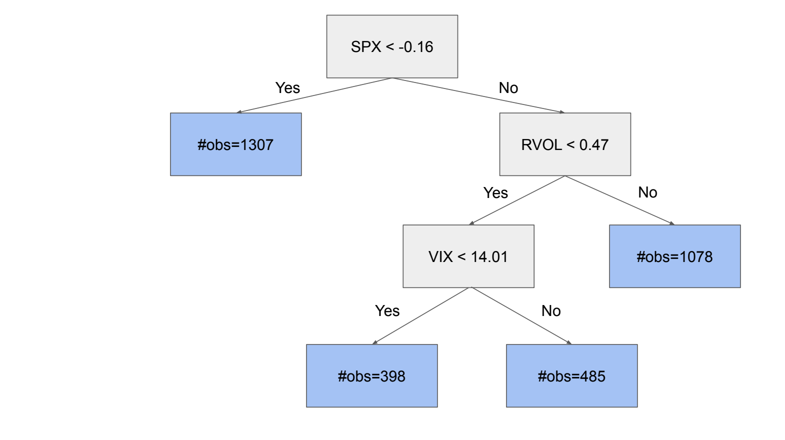
- We find that the GAS Tree outperforms the GAS Forest, in contrast with both the econometrics and the machine learning literature. $\Longrightarrow$ bias-variance tradeoff see Hastie et al. (2009)

Optimal GARCH tree

Figure 1: Estimated GARCH tree



- The optimal tree has 4 "leaves" with 3 state variables:

  SPX: neg vs pos returns

  RVOL: low vs high volatility days

  VIX: low vs high variance risk premium

Application #2: Multivariate density forecasting

- We consider S&P 500 index and 10-year Treasury Bond returns as dependent variables.
- To allow for tail dependence we use the the Student's t copula, as in Janus et al. (2014).

$$\begin{aligned} \mathbf{u}_t &\sim \mathbf{C}_{\text{Student}}(\rho_t, \nu), \quad \rho_t = \frac{\exp\{\tilde{\rho}_t\} - 1}{\exp\{\tilde{\rho}_t\} + 1} \\ \tilde{\rho}_t &= \omega + \beta \tilde{\rho}_{t-1} + \alpha s_{t-1} \end{aligned}$$

  where s_{t-1} is a complicated function of the log-likelihood (see paper for equation).

- We complete the model with t-GAS models for the marginal distributions.

Diebold-Mariano test results

Table 2: Out-of-sample performance of t Copula GAS models using negative log-likelihood loss

	GAS	DRF	Small GAS Tree	GAS Tree	GAS Forest
DRF	2.598				
Small Tree	-1.451	-2.811			
GAS Tree	-1.451	-2.811	—		
GAS Forest	-3.680	-4.092	-0.795	-0.795	
Avg Loss	-0.079	-0.063	-0.084	-0.084	-0.087

Diebold-Mariano test results

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Avg Loss	-0.079	-0.063	-0.084	-0.084	-0.087

- Benchmark GAS model is beaten by by both the “small tree” and the GAS tree models, and significantly beaten by the GAS forest.

Diebold-Mariano test results

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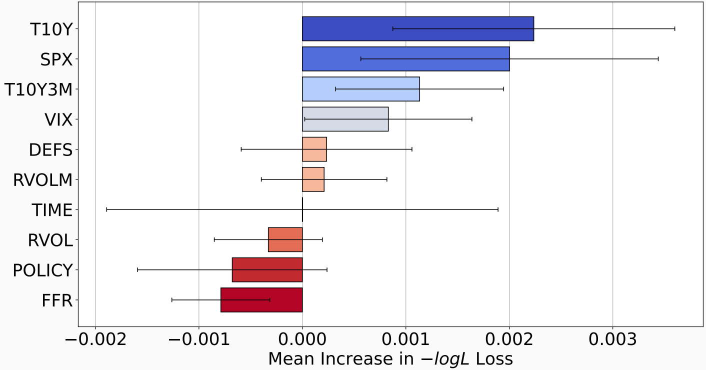
- The best performing model for t-copula is GAS forest, but does not significantly beat either of the tree models (which turn out to be identical).

GAS Forest variable importance

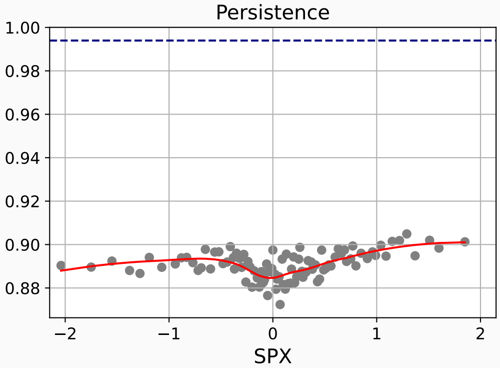
Figure 2: Leave-one-out variable importance for the Student's t copula GAS forest



- Blue means implies omitting the variable hurts performance $\implies$ the variable is important
- Horizontal lines are 95% confidence intervals from D-M test.

GAS Forest persistence parameter (β)

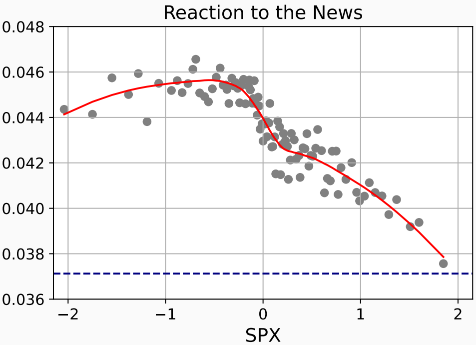
Figure 3: Parameter estimate as a function of S&P 500 returns for the Student's t copula GAS forest



- The persistence of the GAS forest model is roughly unrelated to the stock market return.

GAS Forest reaction to news parameter (α)

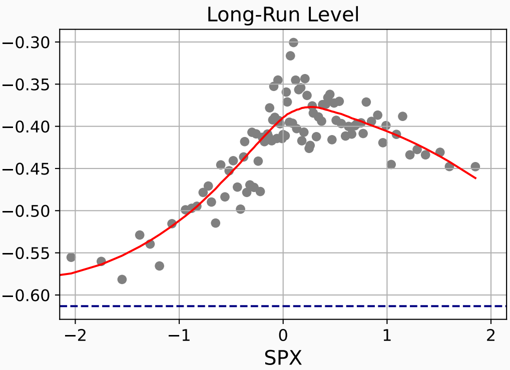
Figure 4: Parameter estimate as a function of S&P 500 returns for the Student's t copula GAS forest



- The α parameter is 0.046 when stocks are down, while it is 0.038 when stocks are up.
- This is consistent with investors paying closer attention to bad news than good news, a finding similar to that of Patton and Sheppard (2015) in a different context.

GAS Forest long-run level parameter ($\omega/(1-\beta)$)

Figure 5: Parameter estimate as a function of S&P 500 returns for the Student's t copula GAS forest



- This is can be interpreted as a "flight-to-quality" effect, with low stock market returns leading to more negative comovements between the stock and bond markets.

Summary

- We propose methods to improve the forecasts from GAS models using ML methods
  - **GAS trees** combine the parsimonious structure of the GAS model with the flexibility of decision trees (Breiman et al., 1984, 2017)
  - **GAS forests**, analogous to the random forests of Breiman (2001), average the forecasts from many GAS trees each produced on a bootstrap sample of the original data.
- We apply the proposed GAS tree and GAS forest models in four distinct forecasting applications
  - Tree/Forest models significantly outperform benchmark models in all cases
  - The source of the improvements is from capturing well-known empirical regularities, e.g. the leverage effect in volatility and the flight-to-quality effect in stock-bond correlations.
  - Trees and forests may be used to uncover new features not yet incorporated into models.

Additional Materials

Estimation Algorithm

- The estimation of GAS trees and GAS forests is computationally demanding.
  - finding the optimal state variables and thresholds from the set of candidate variables
  - given these estimates, estimating the parameters of the GAS model
- We use cluster computing and a "greedy" estimation algorithm related to that of Breiman et al. (1984) and Audrino and Bühlmann (2001).
- Standard tree estimation algorithms involves estimating a regression separately for each terminal node.
- We modify this aspect of the algorithm, and retain the autoregressive nature of the GAS model.

Estimation Algorithm: Step 1

- We use the following estimation algorithm to estimate the tree structure or, equivalently, to find the optimal partition $\mathcal{P}$. Estimation of the GAS tree involves Steps 1–5 below, and the GAS forest additionally uses Step 6.

Step 1: Denote the entire sample as the trivial partition $\mathcal{P}^{(0)}$. Estimate the parameters of the model using MLE since there is no tree structure in this case, and denote these as $(\hat{\theta}^0, \hat{\nu}^0)$:

$$(\hat{\theta}^0, \hat{\nu}^0) \quad = \quad \underset{\theta,\nu}{\operatorname{argmax}} \frac{1}{T} \sum_{t=1}^{T} \log p(y_t; f_t(\theta), \nu)$$

Estimation Algorithm: Step 2

Step 2: Let say $\boldsymbol{\mathcal{P}}^{(m)}$ is the partition leftover from previous step. Define a new partition:

$$\boldsymbol{\mathcal{P}}_j^{(m+1)} = \boldsymbol{\mathcal{P}}_{-j}^{(m)} \cup \{\mathcal{P}_{j,L}^{(m)}, \mathcal{P}_{j,R}^{(m)}\}$$

- $\boldsymbol{\mathcal{P}}_{-j}^{(m)} = \boldsymbol{\mathcal{P}}^{(m)} / \mathcal{P}_j$ contains all the partitions of $\boldsymbol{\mathcal{P}}^{(m)}$ except for the j^{th}
- the j^{th} partition is split into "left" and "right" subpartitions based on the k^{th} state variable and a threshold c

$$\begin{aligned}
\mathcal{P}_{j,L}^{(m)} &= \{\mathbf{Z_t} : \mathbf{Z_t} \in \mathcal{P}_j^{(m)} \quad \text{and} \quad Z_{t,k} \leq c\} \\
\mathcal{P}_{j,R}^{(m)} &= \{\mathbf{Z_t} : \mathbf{Z_t} \in \mathcal{P}_j^{(m)} \quad \text{and} \quad Z_{t,k} > c\}
\end{aligned}$$

Estimation Algorithm: Step 3 and 4

Step 3: Estimate the parameters for new subpartitions, taking the parameters of the other partitions, $\hat{\theta}_{-j}^{(m)}$, as fixed:

$$(\hat{\theta}_{j,L}^{(m+1)}, \hat{\theta}_{j,R}^{(m+1)}) \quad = \quad \underset{\theta_L, \theta_R}{\operatorname{argmax}} \frac{1}{T} \sum_{t=1}^{T} \log p(y_t; f_t(\hat{\theta}_{-j}^{(m)}, \theta_L, \theta_R), \hat{\nu}^{(m)})$$

Compute the log-likelihood value at estimated parameter values.

$$\log p(y; \mathcal{P}_j^{(m+1)}) = \frac{1}{T} \sum_{t=1}^{T} \log p(y_t; f_t(\hat{\theta}_{-j}^{(m)}, \hat{\theta}_{j,L}^{(m+1)}, \hat{\theta}_{j,R}^{(m+1)}), \hat{\nu}^{(m)})$$

Step 4: Maximize the likelihood in step 3 over the partition j, state variable k, and threshold c. Denote the optimized new partition as $\mathcal{P}^{(m+1)}$ and estimate all model parameters and denote $\hat{\theta}^{(m+1)}$.

Estimation Algorithm: Step 5 and 6

Step 5: Repeat steps 2-4 until the depth of the tree, m, reaches a prespecified maximum value, M. The depth of the tree controls the model complexity, and we consider values for M between one and six. We tune this parameter using a validation sample.

Step 6: For the GAS forest, repeat steps 2-5 for B trees. Each tree in the forest uses bootstrap data obtained from a circular block bootstrap (see, e.g., Politis et al., 1999), with block length of 100 observations, and a random selection of one-third of the total state variables. One-third is a common choice in the machine learning literature, see Hastie et al. (2009) for example.

Univariate Density Forecasting

- Our baseline model is the t-GAS model introduced by Creal et al. (2013), which captures both excess kurtosis, (the Student's t distribution), and time-varying volatility (the GAS structure).

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t; \quad \epsilon_t \sim \ i.i.d.\ t(v) \\
\sigma_t^2 &= \omega + \beta \sigma_{t-1}^2 + \alpha(1+3v^{-1}) \left(\frac{1+v^{-1}}{1-2v^{-1}} \left\{ 1 + \frac{v^{-1}}{1-2v^{-1}} \frac{y_{t-1}^2}{\sigma_{t-1}^2} \right\}^{-1} y_{t-1}^2 - \sigma_{t-1}^2 \right)
\end{aligned}
$$

- The $\{\cdot\}$ term implies a more moderate reaction to a large past return than in the GARCH model, as large returns are more common under the t distribution than the Normal distribution.

Univariate Density Forecasting: Diebold-Mariano Test Results

Table 3: Out-of-sample performance of t-GAS models using negative log-likelihood loss.

	t-GAS	DRF	Small GAS Tree	GAS Tree	GAS Forest
DRF	-5.396				
Small Tree	-3.555	1.571			
GAS Tree	-6.517	-1.240	-4.924		
GAS Forest	-5.485	1.555	-1.048	2.755	
Avg Loss	1.179	1.141	1.153	1.132	1.147

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- t-GAS model is significantly out-performed by all four competing models, with Diebold-Mariano *t*-statistics less than -3.5 in all cases.

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- The GAS tree significantly outperforms the “small GAS tree” and also the GAS forest.

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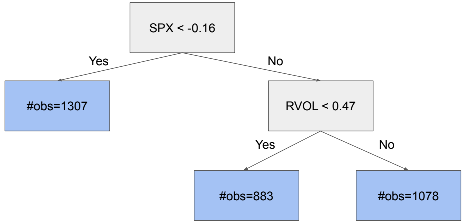
Avg Loss	1.179	1.141	1.153	1.132	1.147

- The GAS tree also outperforms the DRF forecast, but the difference is not statistically significant at the 5% level.

Figure 6: The estimated t-GAS tree model.



- The optimal tree has 3 subsamples with 2 different state variables.
- These can be interpreted as
  (1) negative returns,
  (2) positive returns and low realized volatility,
  (3) positive returns and high realized volatility.

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