Range

Note

long ≈ long long

$$\left\{\begin{array}{llll} \text{int} & x & \approx & [-2 * 10^9,\ 2 * 10^9] \\ \text{long} & y & \approx & [-8 * 10^{18},\ 8 * 10^{18}] \end{array}\right\}$$

% → modulo/ Remainder

10 % 4 = 2

13 % 5 = 3

Divident = Divisor * Quotient + Remainder

great mult of Div <= Divident

{Remainder = {Divident − $\underbrace{\overline{\text{Divisor * Quotient}}}$}}

{greatest multiple of divisor <= Divident}

150 % 11 = 150 − 143 : 11 * 13

100 % 7 = 100 − 98 : 7 * 14

$-40 \% 7 = -40 - \left\{ \begin{array}{l} \text{greatest mul} \\ \text{of } 7 < = -40 \end{array} \right\}$

$-35 < = -40$ ✗

$-42 < = -40$ ✓

$= -40 - (-42) = 2$

$-60 \% 9 = -60 - \left\{ \begin{array}{l} \text{greate mul of} \\ 9 < = -60 \end{array} \right\}$

$-63 < = -60$

$= -60 - (-63) = 3$

Eg:

In Python

$-40 \% 7 = 2$

$-60 \% 9 = 3$

$-30 \% 4 = 2$

In C/C++/Java/JS

$-40 \% 7 = -5 + 7$

$-60 \% 9 = -6 + 9$

$-30 \% 4 = -2 + 4$

$a \% b$: if $a < 0$

$\rightarrow a \% b + b$

? Todays Doubt Session

{Sunday Doubt Session, Bit's Manipulations}

$\rightarrow$ 10:30am $\rightarrow$ 1PM

Why % ? { limit our data to a given range }

$35 \% 10 = 5$

$x \% 10 = [0 - 9]$

1) Consistent Hashing

2) Hashmap / Hashtable

Dict / map /

3) Encryption

$-\infty$

$\rightarrow \% M = [0, M-1]$

$+\infty$

// Modular arithmetic + - x / → Advanced Batch

$$\rightarrow \boxed{(a+b) \% M = (a \% M + b \% M) \% M \rightarrow [0, M-1]}$$

a = 6, b = 8, M = 10

$[0, M-1]$ $[0, M-1]$ $\rightarrow [0, 2M-2]$

$(14) \% 10 = 4$

a = 3, b = 4, m = 2

$$\rightarrow \boxed{(a * b) \% M = (a \% M * b \% M) \% M}$$

// power(a, N, p) → Calculate $(a^n) \% p$

Ex: $a = 2, n = 5, p = 7 \Rightarrow (2^5) \% 7 = (32) \% 7 = 4$

$a = 3, n = 4, p = 6 \Rightarrow (3^4) \% 6 = (81) \% 6 = 3$

```
int power(a, n, p) {

    i = 1; i <= N; i++) {
        a = a * a
    }

    return a % p
}
```

a =

//

i	a value Before loop	a value after loop : a = (a * a)
1	a	$a = a^2$
2	a^2	$a = a^4$
3	a^4	$a = a^8$
4	a^8	$a = a^{16}$

→ If we iterate for N times

$a = a^{2^N}$

// p is integer ∈ [1 - 10^9]

$\mapsto (a^n)\%p$

Input $a = 10$, $N = 40$

$ans = 10^{40}$ ✗

```
int power (a, n, p) {
    long ans = 1
    i = 1; i <= N; i++) {
        ans = (ans * a) % p
        ans = ((ans % p * a % p)) % p
    }
}
return ans % p
```

$\underbrace{(p-1) \times (p-1)}_{(p-1)}$ → $(p-1)^2 \approx 10^{18}$

not needed

TC: O(N)   SC: O(1)

→ Why no overflow in python

→ 10^5 } — — — — — — —

$\approx 10^{10^5}$ }

→ 2 Numbers

Number1 : 10^5 digits

Number2 : 10^4 digits } 10^9

Python:

	1	2	3	: N digits		
	6	2	1	: N digits		
	1	2	3	: O(N)		
2	4	6	x	: O(N)		
7	3	8	x	x	: O(N)	
7	6	3	8	3	: O(N^2)	

→ int $a = 10^5$, $b = 10^6$

int $c = a \times b$

× print (c) × {overflow} ✓

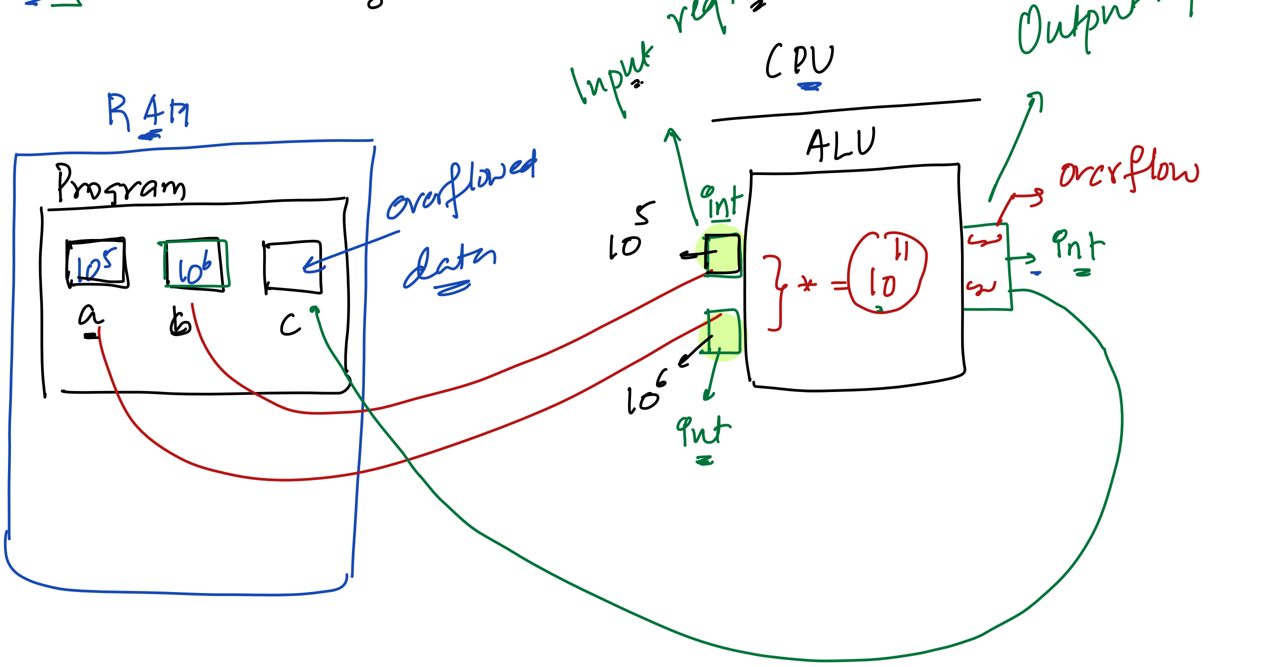
long $c = \boxed{a * b}$

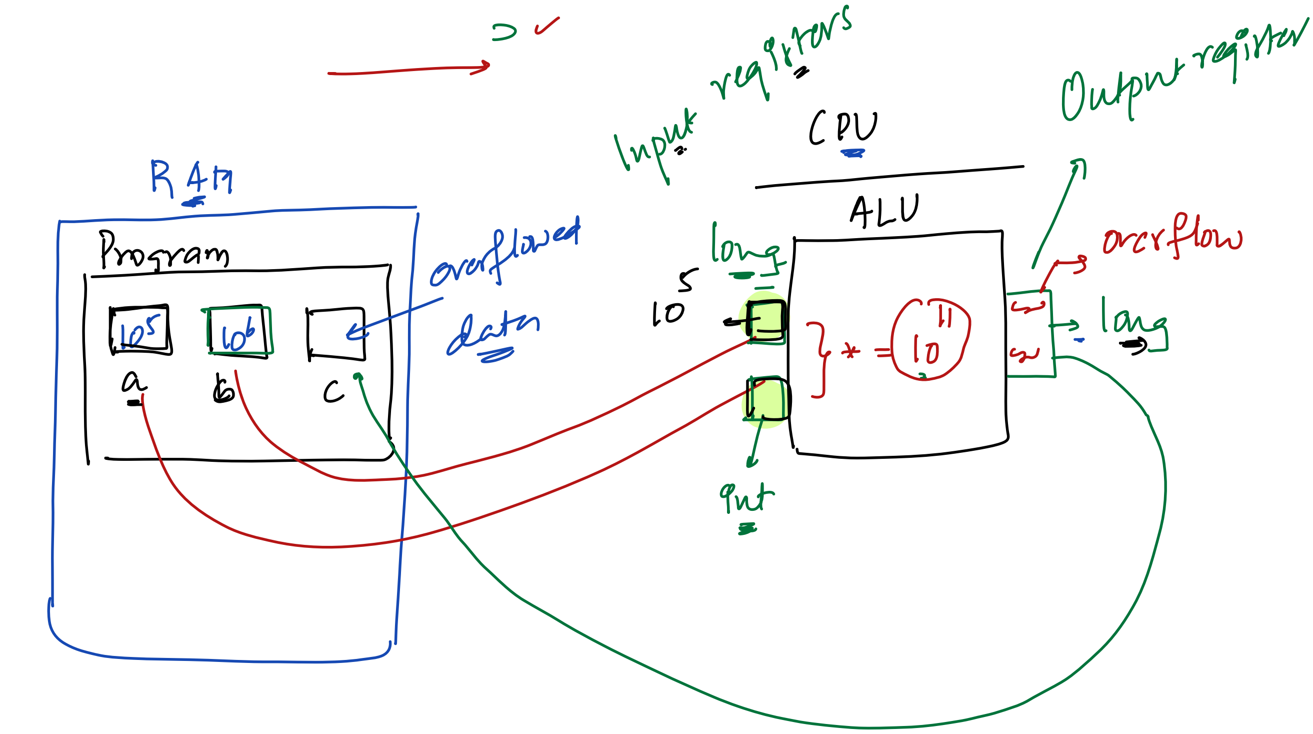
× print (c) {overflow} ×

long $c =$ long $(a * b)$ → overflowed

× print (c) = {overflow} ×

long $c = (\text{long})\, a * b$





//

1Q) Given N array elements, calculate sum of array elements

Constraints: $1 \le N <= 10^5$

$1 \le ar[i] \le 10^6$

$1 \le Sum <= 10^{11}$

Input

N = 1

[1]

15 Tcs

8 Tc → Fail

10:35 PM

```
long ~~int~~ sum = 0
i = 0; i < N; i++) {
    sum = sum + ar[i]
}
return sum
```

Doubts

$$(a^5)\,\%\,p = (a^4 * a)\,\%\,p$$

$$= (a^4\,\%\,p * a\,\%\,p)\,\%\,p$$

$$a^4\,\%\,p = (a^3 * a)\,\%\,p$$

$$= (a^3\,\%\,p * a\,\%\,p)\,\%\,p$$

$$((a^3\,\%\,p * a\,\%\,p)\,\%\,p * (a\,\%\,p))\,\%\,p$$

Divisibility rule of 3 = ?

⇒ Sum of all digits should be divisible by 3

$1 \% 3 = 1$

$10 \% 3 = 1$

$10^2 \% 3 = 1$

$10^n \% 3 = 1$

$(4\ 3\ 7\ 2) = 4*10^3 + 3*10^2 + 7*10 + 2$

$(4\ 3\ 7\ 2)\%3 = (4*10^3 + 3*10^2 + 7*10 + 2)\%3$

$((4*10^3)\%3 + (3*10^2)\%3 + (7*10)\%3 + 2\%3)\%3$

$(4\ 3\ 7\ 2)\%3 = (4 + 3 + 7 + 2)\%3$

TODO: divisibility of 8, 9, 5, 2, 6

Divisibility rule of 4 = ?

→ last 2 digits should be divisible by 4

$10^2 \% 4 = 0$

$10^3 \% 4 = 0$

$10^4 \% 4 = 0$

$(3\ 4\ 8\ 4)\%4 = (3*10^3 + 4*10^2 + 8*10 + 4)\%4$

$((3*10^3)\%4 + (4*10^2)\%4 + [(80)\%4 + 4\%4])\%4$

$0 \quad 0$

$= (80+4)\%M = ((80)\%4 + 4\%4)\%4$

$(84)\%M$

$(a+b)\%M = (a\%M + b\%M)\%M$

→ 10^5 → 6 digits

10^{10} →
10^{100}
10^{100000} = $(10^5 + 1)$ digits.

Given N & P calculate N % P

Constraints: $1 \le N \le 10^{100000}$ → long x → int x

10^5 ≈ Number of digits (k)

$1 \le P <= 10^9$ int

Data is given in char[] a string
Every char represents a digit

Ex:

N <= 10^{100000}

Number

len (N) <= $(10^5 + 1)$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ N: & 3 & 8 & 4 & 3 & 6 & 8 & 9 \end{array}$$

$(3843689)\%15 = \_\_$

$P = 15$

$\Rightarrow n = 1\%p$

Can we get $10\%p$?

$n = (n*10)\%p = 10\%p$

Can we get $100\%p$?

$n = (n*10)\%p = 100\%p$

Can we get $1000\%p$

$n = (n*10)\%p = 10^3\%p$

$(3*10^6 +$

$10^6\%p$

$8*10^5 +$

$10^5\%p$

$4*10^4 +$

$10^4\%p$

$3*10^3 +$

$10^3\%p$

$6*10^2 +$

$10^2\%p$

$8*10 +$

$10\%p$

$9*10^0\%p$

$n = 1\%p = 1$

$\Rightarrow (3*10^6 + 8*10^5 + 4*10^4 + 3*10^3 + 6*10^2 + 8*10 + 9)\%p$

$\Rightarrow (3*10^6)\%p + (8*10^5)\%p \; . \; . \; . \; \_\_ \; . \; \_$

// Generalized

$\vec{N}$: $S_0 \; S_1 \; S_2 \; \ldots \; S_{k-1}$

(len = k)

$$\left((S_0 * 10^{k-1}) + (S_1 * 10^{k-2}) + (S_2 * 10^{k-3}) + \ldots \right) \% p$$

$$\Rightarrow (S_0 * 10^{k-1}) \% p = \left((S_0)\%p * \underbrace{(10^{k-1})\%p}_{} \right) \% p$$

$(a^N)\%p$ = power(

ans = 0;

k is length of string

'0' - 48
'1' - 49
⋮
9 - 57

i = 0; i < k ; i++) { → { k }

ans = ans + $\left(\underbrace{N[i] * 10^{k-i-1}}_{} \right) \% p$

$\hookrightarrow \left(N[i] \% p * \underbrace{(10^{k-i-1})}_{} \% p \right) \% p$

$\hookrightarrow (N[i] \% p$ * power(10, k-i-1, p)) % p

$\hookrightarrow$ char to int $\qquad$ ↳) 10, 100

$$\text{ans} = \text{ans} + \boxed{\left((N[i]-48)\%p * \text{power}(10, k-i-1, p) \right) \% p}$$

}

return ans%p } TC: O(k * k)

k is length of Number

$k^2 \approx 10^{10}$ iteration, TLE

Pseudo Code } k = Number of digits.

$r = 1$, $ans = 0$

TC: $O(k)$ SC: $O(1)$

$i = k-1;\ i >= 0;\ i--)$ {

$ans = ans + (N[i] - 48) * r$

$r = (r * 10) \% p$

$ans = ans \% p$

}

return ans;

↳ N[i] is char

↳ (N[i] - 48) it can convert char to int

↳ (N[i] - '0')

↳ '0' is character

↳ (int) N[i]

Doubts

0	1	2	3	4
3	8	4	3	6

$(3 * 10^4) \% p + (8 * 10^3) \% p + (4 * 10^2) \% p + (3 * 10) \% p + (6) \% p$

power(10, 4, p)   power(10, 3, p)

0	1	2	3	4
3	8	4	3	6

$(6 * 10^0) \% p$

$(3 * 10) \% p$

rem = 1

rem = 1

(rem * 10) % p

rem = 10 % p

ans

ans = (6)(1) % p

If $x = (10^5)\%p$

we need $10^6 \% p =$ $(10^5 * 10)\%p$

$\Rightarrow (10^5\%p * 10\%p)\%p$

$\Rightarrow (x * 10\%p)\%p$

$x \Rightarrow (x*10)\%p \Rightarrow 10^6\%p$

$x = (x*10)\%p \Rightarrow 10^7\%p$

$\rightarrow (303)\%3$

$\Rightarrow (3*10^2 + 0*10 + 3)\%3$

$\Rightarrow ((3*10^2)\%3 + (0*10)\%3 + (3)\%3)\%3$

$((3)\%3 + (0\%3) + (3)\%3)\%3 = (3+0+3)\%3$

$\Rightarrow (a\%M + b\%M + c\%M)\%M \Rightarrow (a+b+c)\%M$

$$\Rightarrow (a * b) \% p = \left\{ \begin{array}{l} (a \% p * b) \% p \checkmark \\ (a * b \% p) \% p \checkmark \\ (a \% p * b \% p) \% p \checkmark \end{array} \right\}$$

$$(a \% p * b) \% p$$

$$(c * b) \% p \Rightarrow (\underbrace{c \% p}_{} * b \% p) \% p$$

$$\Rightarrow (\underbrace{a \% p \% p}_{} * b \% p) \% p$$

$$(a \% p * b \% p) \% p$$